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Dale Jacquette

Meinongian Logic

The Semantics of Existence and Nonexistence

de Gruyter

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for Tina
critic and muse

Do I by painting what I see
tell myself what I see?

—Wittgenstein

Preface

I would like to thank Roderick M. Chisholm for direction and encouragement. I have benefited from discussions and correspondence about particular topics with John N. Findlay, Reinhardt Grossmann, Jacek Paśniczek, William J. Rapaport, Robin D. Rollinger, Stephen G. Simpson, Barry Smith, and Richard (Routley) Sylvan. Rudolf Haller, Peter Simons, and Liliana Albertazzi and Roberto Poli offered opportunities to rethink central themes in object theory logic and semantics by extending invitations to participate in conferences and contribute to recent anthologies on Meinong's philosophy. The problem of ontological commitment in an extensional versus Meinongian semantic framework was discussed in an informal lecture, "Logik, Philosophie der Psychologie, und Meinongs Gegenstandstheorie", presented to the Philosophy of Mind seminar at the Universität Würzburg, January 1990, by invitation of Wilhelm Baumgartner. The ontological economy of Meinongian semantics was examined in a talk on "Virtual Relations" at the Marvin Farber Conference on the Ontology and Epistemology of Relations, State University of New York (SUNY) at Buffalo, NY, September 1994, and in a lecture on "Meinongian Objects" given at the Filozofska Fakulteta, Ljubljana, Slovenia, December 1994, organized by Matjaž Potrč. The many nonexistent errors in the present version of the book are largely due to the criticisms and advice of these and other colleagues and commentators, and I can claim responsibility only for whatever existent mistakes may yet remain. The work was made possible in part by generous research and publication grants from the Alexander von Humboldt-Stiftung, for whose continuing support I am greatly appreciative. I would like to thank Robert Dixon for his original typesetting and graphic text design. The editors of *Philosophical Studies, History and Philosophy of Logic, Grazer Philosophische Studien, Pacific Philosophical Quarterly, Journal of the History of Ideas, The British Journal of Aesthetics, The Philosophical Forum*, *Man and World*, *Brentano Studien*, *Logique et Analyse*, *Conceptus: Zeitschrift für Philosophie*, *Wittgenstein Studien*, *Axiomathes*, *Acta Analytica*, *The School of Franz Brentano*, and *Proceedings of the Eleventh International Wittgenstein Symposium*, kindly gave permission to reprint portions of previously published or forthcoming essays.

Dale Jacquette

Contents

Introduction

Alexius Meinong and his circle of students and collaborators at the Philosophisches Institut der Universität Graz formulated the basic principles for a general theory of objects.[1] They developed branches and applications of the theory, outlined programs for further research, and answered objections from within and outside their group, revising concepts and sharpening distinctions as they proceeded. The object theory that emerged as the result of their efforts combines important advances over traditional systems of logic, psychology, and semantics.

The fate of object theory in the analytic philosophical community has been unfortunate in many ways. With few exceptions, the theory has not been sympathetically interpreted. It has often met with unfounded resistance and misunderstanding under the banner of what Meinong called "The pre-

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[1] I refer to Meinong's *Gegenstandstheorie* as a theory of objects, but alternative English equivalents have been proposed which should also be considered. Reinhardt Grossmann argues that the theory must be called a theory of *entities* because it includes not merely objects (*Objekte*), but objectives or states of affairs (*Objektive*). Grossmann, *Meinong* [1974], pp. 111–12: "If we keep in mind that Meinong will eventually divide all entities (other than so-called dignitatives and desideratives) into objects on the one hand and objectives on the other, we cannot speak of a theory of *objects* as the all-embracing enterprise, but must speak— as I have done and shall continue to do — of a theory of *entities*." This argument is inconclusive, since objectives are also objects of a kind, which Meinong describes as objects of higher order (*höherer Ordnung*), *superiora* founded on *inferiora* or lower order objects. An objective in any case can be as much an object of thought as any other nonobjective object, as when someone thinks about the fact that Graz is in Austria, and thereby makes that state of affairs an object of thought. In this sense, the theory of objects, of lower and higher order, is already all-embracing in the way Grossmann thinks Meinong's *Gegenstandstheorie* is meant to be. Nicholas Griffin identifies a further difficulty in Grossmann's terminological recommendation. In "The Independence of *Sosein* from *Sein*" [1979], p. 23, n. 2, Griffin writes: "Grossmann standardly uses the term 'entity' for Meinong's '*Gegenstand*', which is usually translated as 'object'. Since the *Oxford English Dictionary* defines 'entity' as 'thing that has real existence', this switch is unsatisfactory. Accordingly I have switched back either to 'object' or to the even more neutral term 'item'." Griffin's choice of translation agrees with Richard Routley's in *Exploring Meinong's Jungle and Beyond* [1981], where Routley refers to a theory of items distinct in some respects from but directly inspired by Meinong's theory of objects. Routley's 'theory of items' is perhaps better used to designate his own special version of object theory, which he also denotes 'noneism'. Neither Grossmann's nor Routley's terminology carries the intentional force of '*Gegenstand*', which as Meinong explains is etymologically related to '*gegenstehen*', to stand against or confront, as objects of thought are supposed to confront and present themselves to the mind.

judice in favor of the actual".[2] The idea of nonexistent objects has wrongly been thought to be incoherent or confused, and there are still those who mistakenly believe that the theory inflates ontology with metaphysically objectionable quasi-existent entities.[3] These criticisms are dealt with elsewhere by object theory adherents, and are not considered here. In what follows, the intelligibility of an object theory such as Meinong envisioned is assumed, and ultimately vindicated by the construction of a logically consistent version. The inadequacies of extensionalist theories of ontological commitment and definite description, hallmarks of the Russell-Quine axis in recent analytic philosophy, justify an alternative intentional Meinongian object theory logic. Analytic philosophy survives the rejection of extensionalist treatments of definite description and ontological commitment, since analytic methods are not inherently limited to any particular set of extensional or intentional assumptions.

A comprehensive historical treatment of Meinong's philosophy is not attempted in these chapters, though some historical issues are addressed. Some of Meinong's most important philosophical writings have now been translated or are expected to appear in the near future, and there are several recent commentaries on Meinong's work, including Richard Routley's *Exploring Meinong's Jungle and Beyond*, Terence Parsons' *Nonexistent Objects*, and Karel Lambert's *Meinong and the Principle of Independence*. These studies have contributed to renewed interest in and unprejudiced reappraisal of object theory. Analyses of the subtle turnings in Meinong's thought over several decades may be found in J. N. Findlay's *Meinong's Theory of Objects and Values*, Reinhardt Grossmann's *Meinong*, Robin Rollinger's *Meinong and Husserl on Abstraction and Universals*, and Janet Farrell Smith's essay "The Russell-Meinong Debate". These works trace the complex development of Meinong's early nominalism or moderate Aristotelian realism in the *Hume-Studien* to his mature realistic

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2 Alexius Meinong, "The Theory of Objects" ("Über Gegenstandstheorie") [1904], pp. 78—81.

3 In his early work, Meinong expressed the belief that nonexistent objects have what he then called *Quasisein*. "The Theory of Objects", pp. 84—5. Meinong here refers to the first edition of his *Über Annahmen* [1902], p. 95. See J.N. Findlay, *Meinong's Theory of Objects and Values* [1963], pp. 47—8. Routley, *Exploring Meinong's Jungle and Beyond* [1981], pp. 442, 854. Routley reports that Meinong renounced the theory of *Quasisein* in favor of the *Aussersein* thesis by 1899 (presumably with the publication in that year of his essay "Über Gegenstände höherer Ordnung und deren Verhältnis zur inneren Wahrnehmung"). As a statement of the frequent misinterpretations of Meinong's object theory that persist today, see P.M.S. Hacker, *Insight and Illusion: Themes in the Philosophy of Wittgenstein*, revised edition [1986], p. 8: "The Theory of Descriptions... enabled Russell to thin out the luxuriant Meinongian jungle of entities (such as the square circle) which, it had appeared, must *in some sense* subsist in order to be talked about..."

interpretation of relations and factual objectives or states of affairs as subsistent entities, the theory of objects of higher order, and the doctrine of the *Aussersein* of the pure object. I have relied on these among other sources, I cannot hope to improve on them in some respects, and my topic in any case is somewhat different. I am concerned exclusively with the logic, semantics, and metaphysics or ontology and extraontology of Meinong's theory. Accordingly, I shall not discuss Meinong's epistemology, theory of perception, or value theory, which I nevertheless regard as essential to an understanding of his philosophy as a whole. The logic, semantics, and metaphysics of object theory are in a sense the most fundamental aspects of Meinong's thought, and therefore require the most careful preliminary investigation.

The formal system I develop is a variation of Meinong's vintage *Gegenstandstheorie*, refined and made precise by the techniques of mathematical logic. The proposal offers an integrated three-valued formalization of Meinongian object theory with existence-conditional abstraction, and modal and non-Russellian definite description subtheories. The logic is motivated by considerations about the need for an object theory semantics in the correct analysis of ontological commitment and definite description. Applications of the logic are provided in phenomenological psychology, Meinongian mathematics and metamathematics, criticism of ontological proofs for the existence of God in rationalist theodicy, the interpretation of fiction and scientific law, and formal resolutions of Wittgenstein's private language argument and the paradox of analysis. In some areas it has been necessary to depart from Meinong's official formulation of the theory. But I have tried to make these differences explicit, justifying them by argument and evaluating alternative interpretations. This I believe is in keeping with the spirit of the first exponents of object theory, who did not advance their views as a fixed body of doctrine, but maintained an openminded scientific attitude, and continually sought to achieve a more accurate approximation of the truth.

I. Elements of Object Theory

1. Data and Theory

If there is anything of philosophical significance to be taken at face value in ordinary thought and language it is the reference and attribution of properties to existent and nonexistent objects. We regularly speak of the creatures of fiction and myth, nonexistent idealizations, and objects falsely believed to exist in science and mathematics. We are understood when we suppose that nonexistent objects are distinguishable one from another, that they satisfy identity conditions whereby particular reference is achieved and confusion with other existents and nonexistents avoided, and that the objects so identified and uniquely designated have the properties predicated of them.

The unimpeded projectile is different than the ideal gas, even though neither exist, because their properties are different — the unimpeded projectile unlike the ideal gas is unimpeded and a projectile and not a gas. The man in the street and the expert on classical mythology find it natural and entirely unproblematic to distinguish Cerberus from Pegasus, since Cerberus but not Pegasus is a nonexistent multiheaded dog, while Pegasus but not Cerberus is a nonexistent winged horse. The history of science is replete with reference and attribution of properties to distinct nonexistent objects wrongly thought to exist at least for a time. Phlogiston is not mistaken for the ether, caloric, or vortices, nor Leverrier's nonexistent planet Vulcan for Frege's nonexistent reduction of mathematics to logic.

A semantic theory adequate and requiring minimal departure from or reinterpretive violence to this pretheoretical data must be intensional rather than extensional, and permit the reference and predication of constitutive properties to existent and nonexistent intentional objects. This is sufficient motivation to begin serious examination of Meinong's object theory. A revisionary elaboration of Meinongian logic is later defended against certain kinds of objections, and an argument offered to show that a Meinongian intentional semantics is required in order to advance an intuitively correct account of ontological commitment and definite description.

2. Meinongian Semantics

Meinong's object theory provides the basis for intentional semantics, and establishes the psychological foundations for a phenomenological theory of thought, emotion, and propositional attitude. Meinong accepted a version of phenomenological psychology developed by Kazimierz Twardowski, according to which every psychological experience consists of an act, its content, and the object toward which the act is directed by virtue of its content.[1]

If I think about the Taj Mahal, there are three elements of the experience discernible by phenomenological introspection. These are a mental act by which the Taj Mahal is apprehended or thought about; the lived-through content of the act which directs it toward its object, which may but need not be the psychological equivalent of a description or mental image of the Taj Mahal; and the object toward which the thought is directed by virtue of its content, which in this case is the Taj Mahal. The existence or nonexistence of the objects of thought cannot always be determined phenomenologically, and may be presented to the mind in much the same way whether one believes that the Taj Mahal is a mausoleum, that triangles are three-sided plane geometrical figures, that the golden mountain is golden, or the round square round and square. Something more is required, beyond the introspective description of psychological states in which these objects are presented, to decide whether or not they exist.

For this reason, a phenomenological theory of thought and language must be independent of ontological assumptions about the existence of the objects of thought. Meinong's theory accordingly subsumes not only existent objects, but also nonexistents and nonsubsistents. Included are incomplete or fictional objects that contingently fail to exist, and impossible objects that cannot exist as a matter of metaphysical necessity. Meinong's theory is meant to accommodate this generous variety of objects, regardless of their ontological status, as the intentional objects of possible psychological experiences. Meinong proposed a science of objects, which he regarded as a neglected branch of philosophy, wrongfully denigrated to the field of epistemology in his day.

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1 Kazimierz Twardowski, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen* [1894]. The intentionality thesis has its roots in Franz Brentano's *Psychologie vom empirischen Standpunkt* [1874]. See also Thomas Reid, *The Works of Thomas Reid* [1895], p. 292: "In perception, in remembrance, and in conception, or imagination, I distinguish three things — the mind that operates, the operation of the mind, and the object of that operation... In all these, the act of the mind about the object is one thing, the object is another thing. There must be an object, real or imaginary, distinct from the operation of the mind about it." See Dale Jacquette, "The Origins of *Gegenstandstheorie*: Immanent and Transcendent Intentional Objects in Brentano, Twardowski, and Meinong" [1990].

He hoped to restore the theory of objects to its rightful place alongside metaphysics and mathematics.[2]

3. Principles of Meinong's Theory

Meinong's object theory and Meinongian object theories generally are constructed from the following core of basic principles.

(1) Any thought or corresponding expression can be assumed.
(Principle of unrestricted free assumption or *Annahmen* or *unbeschränkten Annahmefreiheit* thesis)

(2) Every assumption is directed toward an intended object.
(Intentionality thesis)

(3) Every intentional object has a nature, character, *Sosein*, 'how-it-is', 'so-being', or 'being thus-and-so', regardless of its ontological status.
(Independence or independence of *Sosein* from *Sein* thesis)

(4) Being or nonbeing is not part of the *Sosein* of any intentional object, nor of the object considered in itself.
(Indifference thesis or doctrine of the *Aussersein* of the homeless pure object)

(5) There are two modes of being or *Sein* for intentional objects:
(a) Spatio-temporal existence
(b) Platonic subsistence
(*Existenz/Bestand* thesis)

(6) There are some intentional objects which do not have *Sein*, but neither exist nor subsist (objects of which it is true to say that there are no such objects).

Some object theories Meinongian in all other respects do not recognize or make use of the distinction between existence and subsistence.[3] For present purposes it is appropriate to relax principle (5) in order to permit theories of this kind also to qualify as Meinongian. Thesis (6) is similarly expendable when derived from (1)—(4). Meinong's exposition of the theory includes additional postulates, but the present amended list of assumptions

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[2] Meinong, "The Theory of Objects", pp. 97—117.
[3] Terence Parsons, *Nonexistent Objects* [1980], p. 45, n. 9.

is sufficient for a minimal Meinongian object theory. Alternative versions can be formulated by supplementing the core with further assumptions consistent with it.

4. Meinongian Ontology and Extraontology

The Meinongian framework opens up unexpected possibilities for the interpretation of traditional ontologies.

A nominalist or purely materialist ontology can be expressed in Meinongian terms as the view that so-called universals, relations, real numbers, and other abstract entities, do not exist, but are nonexistent Meinongian objects. The materialist working outside of Meinongian theories has usually been driven to implausible attempts to reduce abstract entities to material existents, or else to deny that such putative entities have any theoretical significance.[4] But in Meinongian semantics it is possible to attribute constitutive properties to abstract and immaterial objects without admitting their existence.

At the opposite extreme of the ontological spectrum, a radically Platonist ontology can be transposed in the Meinongian mode as the theory that abstract entities alone exist or subsist, and that material things in sense experience are mere nonexistent Meinongian objects that imitate or participate in their corresponding real abstracta. These may nevertheless have interesting physical and historical as well as logical and metaphysical properties, which in Meinongian semantics can truly be predicated of objects despite their nonexistence.

It should be emphasized that Meinong did not make either of these radical applications of object theory. To do so requires special assumptions about the domains of existence and subsistence which Meinong would probably have been unwilling to support. In his mature theory, Meinong adopts the moderate realist position that material objects exist and abstract objects subsist, and augments the theory by including incomplete and impossible objects in an extraontology of nonexistent and nonsubsistent objects. The moderately realist ontology and extravagant extraontology are further subdivided into objecta and objectives or states of affairs, and the normative objects Meinong called dignitatives and desideratives. Objectives finally are divided into *Seinsobjektive, Nichtseinsobjektive*, and *Soseinsobjektive*, and these latter

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4 Meinong, "The Theory of Objects", pp. 90—92. Routley, *Exploring Meinong's Jungle and Beyond*, pp. 29—30, 750—89, 791—805.

into *Wassein-* and *Wieseinsobjektive*, to distinguish the states of affairs of *what* an object is from *how* it is.

5. Program for a Revisionary Object Theory

All this is anathema to mainstream analytic extensionalist traditions in reference theory and the philosophy of language. These systems downplay the importance of nonexistents in ordinary and scientific thought and discourse, and rely on ingenious but ultimately implausible technical devices to restrict reference to existent entities.

Meinongian object theory must be defended against two broad kinds of objections. If Meinongian intentional semantics are inconsistent or otherwise unintelligible, or if they are strictly unnecessary in an adequate account of language, then on grounds of explanatory competence, simplicity, or economy, the theory cannot compete as an acceptable alternative to received accounts of extensional logic and semantics. It is important critically to reexamine Meinong's object theory with two corresponding purposes in mind: (1) To sort out by argument and discard assumptions and distinctions that are unsound or superfluous; (2) To establish a satisfactory rationale for adopting some version of Meinongian object theory that can sufficiently motivate a formally more rigorous reformulation.

The discussion to follow isolates and justifies the elements of a revisionary Meinongian object theory. The most characteristic assumptions of Meinong's theory are retained, while others more peripheral to his central concerns are shown to be inessential and eliminated. What remains is a Meinongian object theory stripped of unneeded premises but firmly upheld by semantic considerations in the philosophy of logic, language, and science.

II. Formal Semantic Paradox in Meinong's Object Theory

1. Clark-Rapaport Paradox

William J. Rapaport argues that Meinong's concept of being or *Sein* is subject to an object theory paradox. He defines a set of *Sein*-correlates as the objects corresponding to Meinongian objects that have *Sein.* Then he formulates a paradox in terms of self-*Sein*-correlates and non-self-*Sein*-correlates. The paradox is similar to and was inspired by Romane Clark's antinomy in naive predication theory, and may therefore be called the Clark-Rapaport paradox.

> ...the M-object $<F,G,\ldots>$ *has Sein* (*or exists*) iff $\exists\alpha[\alpha$ is an actual object & α ex F & α ex G &....].
>
> ...if the M-object o has Sein, then we call $\{\alpha$: α is actual & $\forall F[F \text{ c } o \rightarrow \alpha \text{ ex } F]\}$ *the set of Sein-correlates of o*, and we write
>
> α SC o
>
> when α *is a Sein-correlate of o.*[1]

Here the relation symbolized by 'c' is constituency, and the relation symbolized by 'ex' is exemplification, adapted in part from Hector-Neri Castañeda's notation for 'internal' and 'external' predication in the theory of particulars, guises, and consubstantiations.[2] Rapaport's distinction between constituency and exemplification as dual modes of predication for existent and nonexistent objects is central to his formalization and criticism of Meinong's object theory.

The paradox is given in terms of *Sein*-correlates for putative Meinongian objects. Rapaport offers a definition by truth conditions for the state of

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[1] William J. Rapaport, "Meinongian Theories and a Russellian Paradox" [1978], p. 165.

[2] Hector-Neri Castañeda, "Thinking and the Structure of the World" [1974], pp. 3—40; "Identity and Sameness" [1975], pp. 121—51; "Individuation and Non-Identity: A New Look" [1975], pp. 131—40; "Philosophical Method and the Theory of Predication" [1978], pp. 189—210.

affairs in which a Meinongian object *o* is its own *Sein*-correlate. (An object that is its own *Sein*-correlate by the above definition must also be actual, though the requirement is not explicitly emphasized.)

$$o \text{ SC } o \text{ iff } \forall F\,[F \text{ c } o \rightarrow o \text{ ex } F\,]$$

Next, by lambda abstraction, Rapaport defines the properties of being a self-*Sein*-correlate and of being a non-self-*Sein*-correlate.[3] These are, respectively:

(R1) $\lambda x \forall F\,[F \text{ c } x \rightarrow x \text{ ex } F\,] =_{df} \overline{\text{SSC}}$

(R2) $\lambda x \exists F\,[F \text{ c } x \;\&\; -(x \text{ ex } F\,)] =_{df} \overline{\text{SSC}}$

The paradox is brought about by assuming that $\langle\overline{\text{SSC}}\rangle$ ex SSC, and showing that this implies $\langle\overline{\text{SSC}}\rangle$ ex SSC, when '$\langle\overline{\text{SSC}}\rangle$' is substituted for '*x*' and '$\overline{\text{SSC}}$' for '*F*' in (R1). If $\langle\overline{\text{SSC}}\rangle$ exemplifies SSC, then by definition it exemplifies all of its constituting properties, and $\overline{\text{SSC}}$ is its one and only constituting property. From this and (R2) it follows that –($\langle\overline{\text{SSC}}\rangle$ ex SSC), since again by definition under substitution of '$\overline{\text{SSC}}$' for '*F*' and '$\langle\overline{\text{SSC}}\rangle$' for '*x*', –($\langle\overline{\text{SSC}}\rangle$ ex SSC). This flatly contradicts the previous conclusion, reducing the assumption to absurdity.

The opposite assumption that –($\langle\overline{\text{SSC}}\rangle$ ex SSC) implies –($\langle\overline{\text{SSC}}\rangle$ ex SSC) by (R2), where '$\langle\overline{\text{SSC}}\rangle$' substitutes for '*x*' and 'SSC' for '*F*', satisfying the definition of $\overline{\text{SSC}}$. This in turn implies by (R2) that there is a constituting property of $\langle\overline{\text{SSC}}\rangle$ that $\langle\overline{\text{SSC}}\rangle$ does not exemplify. But since the one and only constituting property of $\langle\overline{\text{SSC}}\rangle$ is $\overline{\text{SSC}}$, it follows again by (R2) that –($\langle\overline{\text{SSC}}\rangle$ ex $\overline{\text{SSC}}$). From this and (R1) it is inferred that $\langle\overline{\text{SSC}}\rangle$ ex $\overline{\text{SSC}}$. If $\langle\overline{\text{SSC}}\rangle$ does not exemplify $\overline{\text{SSC}}$, then by bivalence it must exemplify SSC, which by definition requires that $\langle\overline{\text{SSC}}\rangle$ exemplify all of its constituting properties, the only one of which is $\overline{\text{SSC}}$, reducing the second assumption to absurdity. The conclusion Rapaport draws is that $\langle\overline{\text{SSC}}\rangle$ both does and does not exemplify SSC.

$$\langle\overline{\text{SSC}}\rangle \text{ ex SSC} \equiv -(\langle\overline{\text{SSC}}\rangle \text{ ex SSC})$$

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[3] Rapaport, "Meinongian Theories and a Russellian Paradox", p. 172.

Rapaport examines several ways of avoiding the paradox, but concludes that none is entirely satisfactory. He maintains that: "Rather than give up such theories in a wholesale way, [the paradox] behooves us to search more deeply for the source of the trouble."[4]

2. Mally's Heresy and Nuclear-Extranuclear Properties

The Clark-Rapaport paradox can be defeated. Rapaport describes a number of ways around it, each with undesirable consequences. But there is another formal semantic object theory paradox that is not amenable to any of the solutions Rapaport considers.[5] This suggests that the revised paradox is philosophically more problematic than the Clark-Rapaport. Since the paradox is resolvable by application of what has come to be known as the nuclear-extranuclear property distinction, it also suggests that the proper direction for development of Meinongian object theory requires the nuclear-extranuclear property distinction.

It may be doubted whether the concepts of *Sein* and *Sein*-correlates could provide the basis for a powerful semantic paradox in Meinong's theory. The *Sosein* concept seems more fundamental, since Meinong believes that all objects have *Sosein* even if they do not have *Sein.* Clark's paradox was originally offered as a proof to the effect that not every object of thought has being in naive predication theory.[6] But this does not produce an interesting result when applied directly to Meinong's object theory, since Meinong admits as a basic tenet that not every intentional object has being, and that some objects neither exist nor subsist, but fall entirely outside of either mode of being.

Clark nevertheless believes that his paradox has immediate implications for Meinongian theories. He writes: "...with respect to a rising, contemporary interest in Meinongian and other intensionalist theories [the paradox] shows that to think of a putative object is not a guarantee that there is in some sense (perhaps other than actual existence) such an object..."[7] Yet even this should not disturb the Meinongian, since according to Meinong:

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4 Ibid., p. 177.

5 In this criticism I will consider only solutions to the paradox proposed by Rapaport in his [1978]. He offers another approach to the paradoxes in an unpublished paper, "Meinongian Analyses of Some (Psycho-) Logical Paradoxes", which I discuss in Chapter III.

6 Romane Clark, "Not Every Object of Thought has Being: A Paradox in Naive Predication Theory" [1978], p. 181.

7 Ibid.

"There are objects of which it is true to say that there are no such objects."[8] By this Meinong apparently means that some objects of thought do not have being in any sense at all. In that case, Clark's paradox does not contradict or uncover any difficulties in Meinong's theory. The paradox is beside the point, for on Meinong's theory it is not supposed to be true that every object of predication has being. Clark's effort to show that not every object of thought has being reappears in Rapaport's paradox in his emphasis on the concept of *Sein* and the *Sein*-correlates of Meinongian objects that have *Sein*.

Attempts by logicians to formalize versions of Meinong's object theory have fallen into two categories, each based on a different distinction suggested by Meinong's student Ernst Mally.

The first is the nuclear-extranuclear property distinction, which Meinong accepted as part of object theory, and the second is the distinction between two different modes of predication for existent and nonexistent objects. Meinong did not accept this second distinction, perhaps because it contradicts his central thesis that every object of thought has whatever constitutive properties are predicated of it independently of its ontological status.

The second approach may therefore be called Mally's heresy. The idea of dual modes of predication is that Hume's nonexistent golden mountain has the property of being golden in a different way and in a different sense than the golden funeral mask of Tutankhamen. The dual modes of predication approach is elaborated by Rapaport, but most notably and completely by Edward N. Zalta.[9] The nuclear-extranuclear property distinction by contrast implies that nonexistent objects have the nuclear constitutive properties in their *Soseine* just as fully and in the very same sense as existent objects. The theory distinguishes between the two categories of constitutive and nonconstitutive properties which both existent and nonexistent objects may have. This approach is explicitly approved by Meinong, and is defended in formalizations of object theory by Terence Parsons and Richard Routley.

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8 Meinong, "The Theory of Objects", p. 83. Meinong states: "...es gibt Gegenstände, von denen gilt, daß es dergleichen Gegenstände nicht gibt."

9 Ernst Mally, *Gegenstandstheoretische Grundlagen der Logik und Logistik* [1912], p. 76. Findlay expresses sympathy with Mally's proposal in *Meinong's Theory of Objects and Values*, pp. 110—12, 182—84, and 340—42. The attitude pervades Grossmann's *Meinong*, and contributes to his main criticism of Meinong's theory. I am largely in agreement with Routley's countercriticism of Mally's heresy in *Exploring Meinong's Jungle and Beyond*, pp. 457—70, and with Griffin's objections to Grossmann's analysis in "The Independence of *Sosein* from *Sein*". Meinong's object theory is eviscerated and the independence thesis contradicted if Mally's plural modes of predication are foisted on it.

In *Über Möglichkeit und Wahrscheinlichkeit*, Meinong introduces the distinction between *konstitutorische* and *ausserkonstitutorische* properties, adapting a new terminology for the distinction suggested by Mally's discussion of *formale* and *ausserformale* properties. (J. N. Findlay originated the English equivalents 'nuclear' and 'extra-nuclear'.) Meinong explains:

> One sees from this that the property 'simple' evidently does not obey the rules which are decisive for the *constitutiva* and *consecutiva* of an object. E. Mally for this reason has distinguished properties of this special character as 'extra-formal' from the ordinary 'formal' properties; however, in view of the traditional denotation of the word 'formal', these designations hardly have the appropriate force. Therefore I propose for the whole of the constitutive and consecutive properties the appelation 'nuclear' ['*konstitutorische*'], and for the remainder the appelation 'extranuclear property' ['*ausserkonstitutorische Bestimmungen*'].[10]

The properties collected under the nuclear category are ordinary properties like being red, round, ten centimeters in diameter, and their complements. They belong to the uniquely identifying *Sosein* or character, nature, being-thus-and-so, or so-being of an object. Extranuclear properties by contrast include special properties that supervene on the totality of an object's nuclear properties, and include the properties of being existent, determinate, incomplete, impossible, and their complements. Extranuclear properties are strictly excluded from the *Sosein* of any existent or nonexistent Meinongian object, and from the *Aussersein* of the pure object considered in itself as constituted by its nuclear properties.

Zalta acknowledges that the dual modes of predication thesis is Mallyan rather than strictly or historically Meinongian when he writes:

> I discovered, indirectly, that Mally, who had originated the nuclear/extranuclear distinction among relations (a seminal distinction adopted by both Meinong and Parsons), had had another idea which could be developed into an alternative axiomatic theory. This discovery was a result of reading both a brief description of Mally's theory in J. N. Findlay's book, *Meinong's Theory of Objects and Values*... and what appeared to be an attempt to reconstruct Mally's theory by W. Rapaport in his paper "Meinongian Theories and a Russellian Paradox."[11]

Rapaport on the other hand gives no indication that his project is Mallyan

[10] Meinong, *Über Möglichkeit und Wahrscheinlichkeit: Beiträge zur Gegenstandstheorie und Erkenntnistheorie* [1915], pp. 176—77. (My translation.)

[11] Edward N. Zalta, *Abstract Objects: An Introduction to Axiomatic Metaphysics* [1983], pp. *xi—xii.*

rather than Meinongian, but seems to regard his work as part and parcel and perfectly in keeping with Meinong's original object theory. Kit Fine similarly presents the two approaches as competing alternative methods of modifying naive abstraction principles to produce consistent comparable Meinongian object theories, in light of difficulties surrounding Russell's problem of the existent round square. Fine maintains:

> The domain of objects and properties cannot be extended in the ways jointly prescribed by the naive principles of object and property abstraction. The naive theory must be modified. There are basically two ways in which this can be done, both suggested by Meinong's pupil, Mally. The first depends upon introducing two copulas: one is an ordinary copula and may be called 'exemplification'; the other is a special copula, which we may follow Zalta... in calling 'encoding'... The second method depends upon introducing two kinds of property: the ordinary or 'nuclear' properties, and the special or 'extranuclear' ones... Very roughly, we may say that Castañeda..., Rapaport... and Zalta... favour the dual copula approach, whilst Parsons and Routley... favour the dual property approach.[12]

This is not the place to split hairs about whether or not the Castañeda-Rapaport-Zalta dual copula or dual modes of predication approach to object theory is 'Meinongian' in the true, historically accurate sense of the word. There is a very broad sense in which the term 'Meinongian' may be applied to any theory of nonexistent objects, in which Rapaport's and Zalta's formalizations can also be described as Meinongian.

A more interesting argument can be made to show that Meinongian object theory based on Meinong's choice of the nuclear-extranuclear property distinction is more fundamental than the Mallyan dual copula or dual modes of predication approach, in the sense that the dual copula or dual modes of predication distinction can be reduced to the nuclear-extranuclear property distinction, but not conversely, and that there are problems and object theory paradoxes avoided by the nuclear-extranuclear property distinction that cannot be as satisfactorily solved by the dual copula or dual modes of predication distinction. These considerations justify Meinong's decision not to accept dual modes of predication, and set constraints for continued efforts to reconstruct a formal Meinongian object theory logic.

The reduction of the dual copula or dual modes of predication distinction to the nuclear-extranuclear property distinction is easy to accomplish, since the two predication modes arise entirely in connection with whether or not

[12] Kit Fine, "Critical Review of Parsons' *Non-Existent Objects*" [1984], p. 97.

an object has the extranuclear property of existence. In Rapaport's terminology, the (nuclear) properties of existent and nonexistent objects are *constituents* or bear only the predication mode of *constituency* to objects, while some (extranuclear) properties of existent and nonexistent objects are *exemplified* alike by some existent and nonexistent objects, but are constituents of or bear the relation of constituency only to nonexistent objects.[13] Zalta similarly but somewhat differently distinguishes between the properties *encoded* by a nonexistent object and those *exemplified* by an existent or nonexistent object, elegantly exploiting the argument places left and right of monadic or *n*-adic predicate symbols.[14] The respective formalizations state:

Rapaport—

$M_0(x,F)$ (or, F c x) $=_{df}$ 'F is a *constituent* of x'
$M_1(x,F)$ (or, x ex F) $=_{df}$ 'x *exemplifies* F'

Zalta—

$x_1F^1 =_{df}$ 'x_1 *encodes* F^1' ($x_1 \ldots x_nF^n =_{df}$ '$x_1, \ldots, x_n$ *encode* F^n')
$F^n x_1 \ldots x_n =_{df}$ '$x_1, \ldots, x_n$ *exemplify* F^n'

These distinctions, despite their formal appearance, are philosophically obscure. Rapaport says that Meinongian objects are constituted by properties, *whereas* actual objects exemplify them. Then he adds: "...there are two distinct types of objects: Meinongian and actual."[15] Later in the essay he declares: "In our terms, [Parsons] is holding that all actual objects are M-objects; our theory holds the reverse: all M-objects are actual."[16] And again: "Since M-objects are among the furniture of the world, they are actual objects... Indeed, not only are they constituted by properties, they also *exemplify* properties, e.g., being an M-object, being thought of by person *S* at time *t*, being constituted by redness, etc."[17]

The claim that Meinongian objects are actual does not necessarily dissolve the distinction between Meinongian and actual objects, since even if all Meinongian objects are actual, not all actual objects need be Meinongian. But if all Meinongian objects are actual, and if actual objects exemplify their categorically undifferentiated properties, then Meinongian objects like the golden

13 Rapaport, "Meinongian Theories and a Russellian Paradox", p. 162.
14 Zalta, *Abstract Objects*, p. 12.
15 Rapaport, "Meinongian Theories and a Russellian Paradox", p. 162.
16 Ibid., p. 167.
17 Ibid., p. 171.

mountain and round square presumably exemplify being golden and a mountain, round and square. Yet this is precisely what Rapaport elsewhere wants to deny, when he argues: "Meinongian objects may or may not exemplify properties, but *whatever* the Meinongian object, *my gold ring*, may exemplify, it *doesn't* exemplify the property of being gold... My actual gold ring, on the other hand, *does* exemplify this property."[18]

The Clark-Rapaport paradox requires that some Meinongian objects both exemplify and have at least some properties as constituents, in order intelligibly to formulate the postulate that a Meinongian object *o* is its own *Sein*-correlate, o SC o iff $\forall F[F \text{ c } o \rightarrow o \text{ ex } F]$ (where *o* exemplifies and has property *F* as c-constituent). But it is hard to reconcile Rapaport's claims that constituency is the mode of predication of Meinongian objects, exemplification the mode of predication for actual objects, that there are two types of objects, Meinongian and actual, and finally that all Meinongian objects are actual. Nor is it clear what could be meant by the implication that the round square or golden mountain as Meinongian objects are actual and part of the 'furniture of the world'. Equally, if Meinongian objects are actual, and Meinongian objects have properties as constituents, then at least some actual objects not only exemplify properties but have properties as constituents, even though constituency is the mode of predication appropriate to nonexistent Meinongian objects.

Which objects, then, are both constituted by and exemplify which properties? There is no unequivocal answer to this important question in Rapaport's theory. The properties he mentions as being exemplified by a Meinongian object are being an M-object, being thought of by person *S* at time *t*, and being constituted by redness. These are all arguably extranuclear. Significantly, Rapaport does not include the (nuclear) property of being red as exemplified by an object constituted by redness, but only the (extranuclear) property of being constituted by redness, and he excludes exemplification of the property of being gold from the example of the Meinongian object my gold ring, even when there is an existent *Sein*-correlated gold ring that exemplifies the property. It seems correct to suppose that for Rapaport nonexistent Meinongian objects exemplify only the extranuclear properties truly univocally predicated of them, though this is at odds with his statement that all Meinongian objects are actual, and that actual objects exemplify their nuclear-extranuclear undifferentiated properties.

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[18] Ibid., p. 162. See ibid.: "Hence, M_0 is the mode of predication appropriate to Meinongian objects, and M_1 is the appropriate mode for actual objects. Put otherwise, Meinongian objects are constituted by properties, whereas actual objects exemplify them."

Zalta's logic is somewhat different and more formally developed, but equally inexplicit about when nonexistent or abstract objects encode and exemplify properties. Zalta forbids existent A-objects from encoding properties, but nonexistent or 'abstract' objects are permitted both to encode and exemplify certain kinds of properties. Zalta's logic deliberately offers no general principle for determining when an abstract object encodes or exemplifies a property. He extends this liberty to his theory when he writes:

> ...a question arises as to what properties A-objects exemplify. Strictly speaking, the theory doesn't say (other than the property of being non-ordinary). For the most part, we can rely on our intuitions of ordinary properties, such as being non-round, being non-red, etc. A-objects also exemplify intentional properties and relations, such as being thought about (by so and so), being searched for, etc. These intuitions serve well for most purposes, but there may be occasions where we might want to disregard some of them, in return for theoretical benefits. Since the theory is neutral about what properties A-objects exemplify, we are free, from the standpoint of the theory, to decide this according to theoretical need.[19]

This makes the question whether an object encodes or exemplifies a property depend in part on *ad hoc* decision on a case-by-case basis. But how can such a fundamental semantic distinction be determined by decision? Surely abstract objects encode or exemplify properties independently of the existence of decision-makers.

The differences between Rapaport's and Zalta's versions of the dual copula or dual modes of predication approach to object theory are sorted out in more detail below. Here for comparison are the proposed reductions of Zalta's and Rapaport's dual modes of predication distinction to the nuclear-extranuclear property distinction.

The extranuclear existence property is symbolized '$E!$'. Here and throughout, extranuclear properties are distinguished from nuclear properties by the exclamation or 'shriek' sign; F is nuclear, $G!$ extranuclear. Where reference to either a nuclear or extranuclear property indifferently is intended, the predicate encloses the exclamation mark in parentheses, '$F(!)$'. Let $A(F^n(!), x_1 \ldots x_n) =_{df}$ 'property $F^n(!)$ is *attributed to the nature of* $x_1 \ldots x_n$', understood either as the abstract mind-independent association of an object with a set of either nuclear or extranuclear properties, or the (true or false) occurrent psychological predication of nuclear or extranuclear properties to the object via the *Annahmen* or free assumption thesis. The extranuclear property

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19 Zalta, *Intensional Logic and the Metaphysics of Intentionality* [1988], pp. 30—1.

$A!$, adapted from Zalta's notation, is the property of being actual. Theory Z is the application of Zalta's intensional logic, and the connective $\stackrel{R}{\Rightarrow}$ is relevant entailment as Zalta introduces it.[20]

Rapaport-Constituency—

$$(\forall x)[M_0(x,F) \equiv ((\overline{E}!x \,\&\, A(F(!), x)) \vee (E!x \,\&\, Fx))]$$

$$(\forall x)[(F \text{ c } x) \equiv ((\overline{E}!x \,\&\, A(F(!), x)) \vee (E!x \,\&\, Fx))]$$

Rapaport-Exemplification—

$$(\forall x)[M_1(x, F) \equiv (F!x \vee (A!x \,\&\, Fx))]$$

$$(\forall x)[(x \text{ ex } F) \equiv (F!x \vee (A!x \,\&\, Fx))]$$

Zalta-Encoding—

$$(\forall x_1)[x_1F^1 \equiv (\overline{E}!x_1 \,\&\, A(F^1(!),x_1))]$$

$$(\forall x_1)\ldots(\forall x_n)[x_1\ldots x_nF^n \equiv (\overline{E}!x_1 \,\&\ldots\&\, \overline{E}!x_n \,\&\, (A(F^n(!),x_1\ldots x_n))]$$

Zalta-Exemplification—

$$(\forall x_1)\ldots(\forall x_n)[F^nx_1\ldots x_n \equiv (F^n!x_1\ldots x_n \vee (Z \stackrel{R}{\Rightarrow} F^nx_1\ldots x_n))]$$

The equivalences effect a straightforward reduction of Rapaport's and Zalta's dual copula or dual modes of predication distinctions to a univocal mode of predication based on the distinction between nuclear and extranuclear properties, featuring the extranuclear property of existence, the attribution of properties to an object by free assumption, ordinary univocal predication, quantification, and logical connectives.

The idea of the Rapaport reductions is that an object has a property as c-constituent just in case either the object is nonexistent and the (nuclear or extranuclear) property is attributed to the object, or the object exists, the property is nuclear, and the property is truly univocally predicated of the object. This allows both existent and nonexistent objects to have constituents, though nonexistent objects can have either nuclear or extranuclear properties as constituents, while existent objects are constituted only by nuclear properties. An object exemplifies a property on the other hand if and only if the property is extranuclear and truly univocally predicated of the existent or nonexistent object, or the object is actual and the (nuclear or extranuclear) property is truly univocally predicated of it. The formulation

[20] Ibid., pp. 124—25.

reflects the unresolved relation between existent and actual objects in Rapaport's exposition.

The Zalta reductions are more complicated for reasons discussed below. The basis for eliminating encoding is that according to Zalta it is the mode of predication unique to nonexistent objects, where objects have (nuclear or extranuclear) properties attributed to them. Zalta-exemplification is similarly reduced by the fact that an object exemplifies a property if and only if either the property is extranuclear and is truly univocally predicated of the existent or nonexistent object, or, by virtue of the freedom Zalta extends to the distinction, the theory relevantly entails that the (nuclear) property is truly univocally predicated of the object. This disallows the encoding of properties by existent objects, permits existent and nonexistent objects alike to exemplify certain properties, but prevents nonexistent objects from exemplifying nuclear properties unless the theory in application specifically requires it.

Zalta officially restricts the encoding of properties to monadic or 1-ary qualities, but in the above Zalta-encoding has been characterized both as monadic and fully generalized for *n*-ary relations.[21] Zalta claims that his theory can be expanded to include encoded as well as exemplified relations, but he is reluctant to do so unless or until the encoding of relations is justified by sufficient data.[22] There is however as much data to justify the encoding of relations as properties. If it is important to the intensional logic of abstract objects to express nonrelational qualities of nonexistents, like Sherlock Holmes' property of being a detective, then it should be equally important to be able to express relations, like the fact that Holmes is admired by Watson, or has the 2-place relation of being admired by Watson. Holmes must also encode the relation of having solved the Hound of the Baskervilles case, being the enemy of Moriarity, smoking a pipe, using cocaine, playing the violin, and so on. The distinction between encoded qualities and relations is so tenuous that it is extraordinary to find Zalta drawing a line between them. If his logic is not or for any reason cannot be enriched to permit encoding of relations for abstract objects, then it may be too impoverished to be of interest in the study of nonexistent objects, and cannot hope to command the philosophical regard of its competitors. Accordingly, Zalta's theory of encoding is reduced in both styles, his original monadic version, and the projected expansion to include encoding of *n*-ary relations.

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21 Zalta, *Abstract Objects*, pp. 12—4. Zalta, *Intensional Logic and the Metaphysics of Intentionality*, pp. 15—7.

22 Zalta, *Intensional Logic and the Metaphysics of Intentionality*, pp. 36—7.

Zalta's encoding is distinct from Rapaport's constituency relation in that for Zalta actual objects do not encode but only exemplify properties, whereas for Rapaport actual objects both exemplify and have properties as c-constituents. The differences are most apparent in Zalta's AXIOM 2. ("NO-CODER"): $E!x \rightarrow \sim(\exists F)xF$, and Rapaport's definition of a self-*Sein*-correlate, on Rapaport's assumption that all Meinongian objects are actual.[23]

There is an abstract object in Zalta's theory that encodes the property of being existent, golden, and a mountain, but the existent golden mountain only encodes and does not exemplify the properties of being existent, golden, or a mountain. Zalta explains:

> We also find an abstract object which encodes just existence, goldenness, and mountainhood. ($(\exists x)(A!x \mathbin{\&} (F)(xF \equiv F = E! \vee F = G \vee F = M))$). Although the theory presupposes that this object fails to exemplify existence, this is compatible with the contingent fact that no existent object exemplifies all the properties this abstract object encodes (which is how we will read the ENGLISH nonexistence claim).[24]

If we share Zalta's insight that there may be some logically important difference in the way existent versus abstract objects can have properties attributed to them, then we might begin with a more intuitive distinction. Informally, the difference between the two sensed modes of predication for existent and abstract objects can be marked by a neutral index for a distinction in the ordinary language copula:

(1) Existent or abstract object *a* is^1 F (*a really is* or *really has* property F).

(2) Abstract object *b* is^2 F (*b* 'is' or 'has' *in some sense* but *is not really* and *does not really* have property F).

When he defines or applies the property of being abstract, instead of writing '$xA!$', to indicate that an object encodes rather than exemplifies the property of being abstract, Zalta typically writes '$A!x$' (though an abstract object

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[23] Zalta, *Abstract Objects*, p. 33; Zalta, *Intensional Logic and the Metaphysics of Intentionality*, p. 21: Principle 1 $(\forall x)(O!x \rightarrow \Box\blacksquare\sim(\exists F)xF)$. (This is the counterpart of NO-CODER in the more recent version of Zalta's logic.) Zalta's modal temporal operator '$\blacksquare$' ('*Always* ϕ ("$\blacksquare$ ϕ")') is explained in *Intensional Logic and the Metaphysics of Intentionality*, pp. 20—1. Rapaport, "Meinongian Theories and a Russellian Paradox", p. 165.

[24] Zalta, *Abstract Objects*, p. 13. Zalta has responded to some of the criticisms I raised in "Mally's Heresy and the Logic of Meinong's Object Theory" [1989] in his [1992] "On Mally's Alleged Heresy: A Reply". The present account is intended to meet his counterarguments by proposing a reduction that better satisfies the formal requirements of his theory.

also encodes the property of being abstract, via Axiom or Principle 2). The definition itself in the most recent version of his theory tells us that an abstract object really has^1 or exemplifies the property of being abstract.[25]

$$\textit{being abstract } (\text{`}A!\text{'}) =_{df} [\lambda x {\sim} \Diamond \blacklozenge E!x]$$

The same flexibility of the exemplification-encoding distinction is seen in some of Zalta's more elaborate principles.[26]

Principle 2

For every formula f in which x doesn't occur free, the following is an axiom: $\Box\blacksquare(\exists x)(A!x \mathbin{\&} (\forall F)(xF \equiv \phi))$

Principle 3

$$\mathrm{x = y} =_{df} (\mathrm{O!x} \mathbin{\&} \mathrm{O!y} \mathbin{\&} \Box\blacksquare(\forall \mathrm{F})(\mathrm{Fx} \equiv \mathrm{Fy})) \vee (A!x \mathbin{\&} A!y \mathbin{\&} \Box\blacksquare(\forall F)(xF \equiv yF))$$

Here there is a mixture of encoding and exemplification of properties for abstract objects in the second disjunct. Abstract objects x and y merely have^2 or encode property F (for *all* properties F $(\forall F)(xF\ldots)$), indicated by 'xF' and 'yF', but they really have^1 or exemplify the property of being abstract, as indicated by '$A!x$' and '$A!y$', when abstract object $x = y$.

The closest thing to a principle determining what abstract objects both encode and exemplify properties is Zalta's informal stipulation that an abstract object fails to exemplify (specifically) existence if no existing object exemplifies all the properties the abstract object encodes. This is a problematic basis for the distinction in pure logic and semantics, since it depends on empirical facts about what properties actual objects do or do not happen to exemplify. But why should the possession of properties by existent objects determine whether or not a nonexistent object merely encodes or encodes and exemplifies a given (nuclear) property?

The limitation of Zalta's informal principle is that while it rules out exemplification by an abstract object when no existent object exemplifies the properties an abstract object encodes, it does not work in the opposite direction to determine when an abstract object exemplifies the properties it encodes. Making the informal criterion into a biconditional is evidently inadequate, because by Zalta's own axiom, $(\forall x)(A!x \equiv {\sim}E!x)$.[27] It would be mis-

25 Zalta, *Intensional Logic and the Metaphysics of Intentionality*, p. 21.

26 Ibid.

27 Zalta, *Abstract Objects*, p. 12: "...*x is abstract* ('$A!x$') iff x fails to exemplify existence."

taken to expand on Zalta's negative criterion by suggesting that an abstract object positively exemplifies the properties it encodes if the properties are actually exemplified by an existent object. No existent object exemplifies the property of being abstract, but every abstract object exemplifies both the property of being abstract and the property of being nonexistent. Abstract objects therefore exemplify some properties that no existent object exemplifies. By Zalta's Principle (1):

> *For every expressible condition on properties, there is an abstract object which encodes just the properties meeting the condition*:
>
> $(\exists x)(A!x \;\&\; (F^1)(xF^1 \equiv \phi))$, where ϕ has no free x's.[28]

This means that there is an abstract object that encodes just the property of being existent. Every existent object exemplifies the property of being existent.

But from this it cannot be concluded that an abstract object that encodes the property of being existent also exemplifies that property, because no abstract object exemplifies existence. Even possible exemplification by an existent object of the properties encoded by abstract objects is too strong to determine when the abstract object exemplifies as well as encodes its properties. This follows from the above argument *a fortiori*, and also because it is *possible* for an existent object to exemplify the properties of being existent, golden, and a mountain, though Zalta insists that the existent golden mountain merely encodes and does not exemplify these properties on the grounds that there is *in fact* no existent object exemplifying just these properties.

Whether or not there are other properties besides being abstract and nonexistent that nonexistent objects exemplify as well as encode is undetermined in Zalta's theory. In particular, whether abstract objects can exemplify as well as encode what on the nuclear-extranuclear property distinction are nonrelational unary nuclear properties like roundness and goldenness is a matter about which his logic is silent.[29]

Zalta's flexibility about the nonexclusiveness of property encoding and exemplification by abstract objects is puzzling in light of his own avowed historical precedents. He claims to have derived the basic principles of his

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[28] Ibid.

[29] The ambivalence in Zalta's theory is evident in ibid., p. 11: "...the properties roundness and squareness *can* determine an abstract object which satisfies neither roundness nor squareness. The properties of existence, goldenness, and mountainhood *can* determine an abstract object which does not satisfy any of these properties." (Emphases added.)

theory from Findlay's description of Mally's distinction between an object being *determined* (encoded) by a set of properties, and *satisfying* (exemplifying) the properties. Findlay states:

> On the theory of Mally, the object 'something that is blue' is merely the determinate of the determination 'being blue'; it does not *satisfy* this determination. The only objects which satisfy this determination of being blue are concrete blue existents. 'Something that is blue' is not really blue; the only property it really possesses is that of being determined by the determination 'being blue'. This property or determination, which is one of higher order, it satisfies.[30]

What is noteworthy in this characterization of Mally's later theory is that it rules out the possibility of nonexistent objects exemplifying the nuclear properties they encode, like the property of being blue. Findlay goes so far as to limit the exemplification of properties by nonexistent objects to the single case of the (extranuclear) property of being determined by the properties that determine the object, or, in Zalta's terminology, being encoded by the properties the object encodes.

Zalta further attributes inspiration for the basic distinction between encoding and exemplification to Rapaport's theory. But Rapaport's theory in at least some places seems to divide the constituency-exemplification of properties on the basis of existence and nonexistence, allowing existent objects to have constituents, but not permitting nonexistents to exemplify nuclear properties.

> ... it seems to me, non-existing golden mountains cannot be made of gold in the same way that existing golden rings are... But the only relevant difference between the entities is that one exists and the other doesn't, which does not help solve the problem of how non-existents can *have* properties. That can be done by taking the other alternative: There are *two* modes of predication.[31]

It is not enough for Zalta to reform his use of what might now be called extranuclear predicates '*E*!' and '*A*!'. Even if he were to agree that abstract objects can only encode properties, and no abstract object exemplifies or really has a property, and write '$xA!$' or '$\sim xE!$' instead of '$A!x$' or '$\sim E!x$',

[30] Findlay, *Meinong's Theory of Objects and Values*, p. 183. Findlay is usually regarded as an authoritative source on Mally's ideas, since Mally was Findlay's dissertation director at the Universität Graz.

[31] Rapaport, "Meinongian Theories and a Russellian Paradox", p. 161.

the philosophical problem remains. Surely Zalta *needs* to say that an abstract object *really* is^1 abstract, and that a nonexistent object *really* is^1 nonexistent. If it is not true that the golden mountain is^1 nonexistent, then why should logic treat it any differently than ordinary existent mountains? To make a special case for properties like 'abstract' and 'nonexistent' is tacitly to rely on a distinction between constitutive and nonconstitutive or nuclear and extranuclear properties, which categories of properties Zalta indeed refers to respectively as 'nontheoretical' and 'theoretical'.

As Meinongian logics like Parsons', Routley's, and the system developed below serve to show, if the nuclear-extranuclear property distinction is adopted, then there is no need for a neo-Mallyan distinction between dual copulas or dual modes of predication. What remains difficult to understand on either Zalta's or Rapaport's dual copula or dual modes of predication theories is how impossible objects can really be^1 impossible, and so exemplify the property of being impossible, when they do not really have^1 or exemplify such metaphysically incompatible properties as being globally and simultaneously round and square, but at most encode, are constituted by, or have^2, the properties of being round and square.

If object theory is equipped with the nuclear-extranuclear property distinction it is possible to define and recover the dual copula or dual modes of predication distinction. But the opposite is not true. If an object theory does not already contain the nuclear-extranuclear property distinction, there is no way to define or recover it by means of the dual copula or dual modes of predication distinction (whether or not there is any reason to do so). It may be possible to define 'theoretical' properties like existence, determinateness, or impossibility, but they will not be definable *as* extranuclear properties if the dual copula or dual modes of predication object theory does not already contain the distinction. As a result, the nuclear-extranuclear property distinction is conceptually more fundamental than the dual copula or dual modes of predication distinction.

This would still leave choice of one kind of distinction over the other at a draw in which either alternative might be made the foundation of a distinct but comparable object theory, were it not for the fact that there are problems and paradoxes more satisfactorily resolved by the nuclear-extranuclear property distinction than by the dual copula or dual modes of predication distinction.

The Clark-Rapaport paradox depends essentially on rejection of the nuclear-extranuclear property distinction, and therefore arises not in Meinong's own theory, but only in revised object theories like Rapaport's, in which the dual modes of predication distinction replaces the nuclear-extranuclear property distinction. Rapaport reconstructs the Clark paradox by defining the state of affairs in which a Meinongian object is its own *Sein*-correlate.

$$o \text{ SC } o \text{ iff } \forall F[F \text{ c } o \rightarrow o \text{ ex } F]$$

As an example of something which is supposed to satisfy these truth conditions, Rapaport offers <being a Meinongian object>. He explicitly requires that <being a Meinongian object> exemplify its *only* constituting property. This is the property of being a Meinongian object, which enables the object to qualify as its own *Sein*-correlate. The paradox is easily avoided by insisting that being a Meinongian object is an extranuclear nonconstitutive property, and not a nuclear constitutive property, and that being a self-*Sein*-correlate or non-self-*Sein*-correlate is an extranuclear nonconstitutive rather than constitutive nuclear property.[32]

3. Sosein and the Sosein Paradox

A philosophically and semantically deeper object theory paradox than the Clark-Rapaport that depends on the *Sosein* concept alone can be defined. It provides a more difficult problem in the sense that it avoids the solutions Rapaport considers to his version of the Clark paradox involving self-*Sein*-correlates and non-self-*Sein*-correlates.

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[32] Ibid., p. 162. Rapaport also writes, p. 158: "The related question whether all Meinongian objects have Aussersein or only those which lack Sein may be answered in favor of the former alternative." Rapaport seems to believe that when an object exists or is actual, then there are really two objects, the Meinongian object and its *Sein*-correlate. In a more recent essay, "How to Make the World Fit Our Language: An Essay in Meinongian Semantics" [1981], Rapaport introduces the concept of a *Sein*-correlate as follows: "In addition to the *Meinongian* object of a psychological act, I suggest that there is also, in some cases, an 'actual' (usually physical) object which is distinct from the Meinongian object and of which..." This raises difficulties about the relation between the two objects, and about which object a person is thinking about when thinking about an existent object. A more accurate interpretation of the *Aussersein* thesis in Meinong would hold that the pure Meinongian object considered in itself or qua object is beyond being and nonbeing, *jenseits von Sein und Nichtsein*. This is not to postulate an additional object, but to regard the very same object from a certain perspective, or with reference to one of its metaphysical aspects, divorced from its ontological status. If on the other hand Rapaport means by 'Sein-correlate' this ontologically neutral perspective or aspect of an object, then it would appear much simpler and more appropriate to use a predicate such as '*E!*' to indicate that an object is actual or exists (or subsists). Actual existence in a Meinongian theory is an extranuclear *property*, rather than a *relation* like Rapaport's SC or *Sein*-correlation. See Routley, *Exploring Meinong's Jungle and Beyond*, pp. 883--85.

We begin by designating an object's *Sosein* as a nuclear property or properties in the set of properties produced as value of a *Sosein* function applied to the object, recalling that sets and properties are themselves objects that have *Soseine.*

The independence of *Sosein* from *Sein* is formalized by a *Sosein* principle, which states that the *Sosein* function applied to an object produces a set of properties if and only if all and only those properties are true (truly predicated) of the object. The *Sosein* function produces a set of nuclear properties, and the application of the *Sosein* function to an object is always identical to a set of nuclear properties. The *Sosein* of an object is not this set, but the property or properties in the set. For convenience, the extranuclear relational properties identity and nonidentity are written as '=' in familiar notation, rather than the strictly correct '=!'.

(*Sosein*) $(\forall x)[S(x) = \{\ldots P, Q\ldots\} \equiv (\ldots Px \;\&\; Qx\ldots)]$

A standard principle of λ-abstraction introduction and elimination is also required.

(λ-Conversion) $(\forall y)(\lambda x[\ldots x\ldots]y \equiv (\ldots y\ldots))$

Two jointly paradoxical properties are designated: the property of being an object identical to its own *Sosein*, and the property of being an object nonidentical to its own *Sosein*. The remaining steps of the proof are justified by the ordinary inference rules of propositional and predicate logic.[33]

The *Sosein* Paradox

$$(S1)\; o_i = \lambda x[S(x) = \{x\}]$$
$$(S2)\; o_j = \lambda x[S(x) \neq \{x\}]$$

1. $o_i = \lambda x[S(x) = \{x\}]$ (S1)
2. $o_j = \lambda x[S(x) \neq \{x\}]$ (S2)
3. $\lambda x[S(x) = \{x\}]o_j$ Assumption
4. $\lambda x[S(x) = \{x\}]o_j \equiv S(o_j) = \{o_j\}$ λ-Conversion
5. $S(o_j) = \{o_j\}$ (3,4)
6. $S(\lambda x[S(x) \neq \{x\}]) = \{\lambda x[S(x) \neq \{x\}]\}$ (2,5)

[33] The inference principles are close enough to those in standard logic to be readily intelligible. They are explained in more detail in Part Two, Chapter I, 'Syntax, Formation and Inference Principles'.

7.	$S(\lambda x[S(x) \neq \{x\}]) = \{\lambda x[S(x) \neq \{x\}]\} \supset$ $\lambda x[S(x) \neq \{x\}](\lambda x[S(x) \neq \{x\}])$	*Sosein*
8.	$\lambda x[S(x) \neq \{x\}](\lambda x[S(x) \neq \{x\}])$	(6,7)
9.	$S(\lambda x[S(x) \neq \{x\}]) \neq \{\lambda x[S(x) \neq \{x\}]\}$	(8,λ-Conversion)
10.	$S(o_j) \neq \{o_j\}$	(2,9)
11.	$\lambda x[S(x) \neq \{x\}]o_j$	(10)
12.	$\lambda x[S(x) = \{x\}]o_j \supset \lambda x[S(x) \neq \{x\}]o_j$	(12)
13.	$\lambda x[S(x) \neq \{x\}]o_j$	Assumption
14.	$\lambda x[S(x) \neq \{x\}]o_j \supset S(o_j) \neq \{o_j\}$	l-Conversion
15.	$S(o_j) \neq \{o_j\}$	(13,14)
16.	$S(\lambda x[S(x) \neq \{x\}]) \neq \{\lambda x[S(x) \neq \{x\}]\}$	(2,15)
17.	$S(\lambda x[S(x) \neq \{x\}]) \neq \{\lambda x[S(x) \neq \{x\}]\} \supset$ $\lambda x[S(x) \neq \{x\}](\lambda x[S(x) \neq \{x\}])$	*Sosein*
18.	$\lambda x[S(x) \neq \{x\}](\lambda x[S(x) \neq \{x\}])$	(16,17)
19.	$\lambda x[S(x) \neq \{x\}](\lambda x[S(x) \neq \{x\}]) \supset$ $S(\lambda x[S(x) \neq \{x\}]) = \{\lambda x[S(x) \neq \{x\}]\}$	*Sosein*
20.	$S(\lambda x[S(x) \neq \{x\}]) = \{\lambda x[S(x) \neq \{x\}]\}$	(18,19)
21.	$S(o_j) = \{o_j\}$	(2,20)
22.	$\lambda x[S(x) = \{x\}]o_j$	(21)
23.	$\lambda x[S(x) \neq \{x\}]o_j \ldots \lambda x[S(x) = \{x\}]o_j$	(22)
24.	$\lambda x[S(x) = \{x\}]o_j \int \lambda x[S(x) \neq \{x\}]o_j$	(12,23)
25.	$(\exists y)(\lambda x[S(x) = \{x\}]y \equiv \lambda x[S(x) \neq \{x\}]y)$	(24)

The paradox depends essentially on the *Annahmen* or unrestricted freedom of assumption thesis in the core of Meinongian object theory. The paradox involves an untyped construction of self-*Sosein*-application and self-non-*Sosein*-application.[34] Freedom of assumption permits thought to entertain the proposition $F(F)$, and even, in typed notation, $F^i(F^i)$, so that a Meinongian

[34] The *Sosein* paradox is naturally formulated in combinatory notation without abstraction devices, since it involves the application of a property to another property. Let S be a *Sosein* function combinator that takes a Meinongian object o_i as argument into its *Sosein*.

(i) $Z = (S[o_i](= (S\ o_i)\ o_i))$
(ii) $Z' = (S[o_i](\neq (S\ o_i)\ o_i))$

The paradoxical conclusion which obtains can then straightforwardly be expressed as $(\equiv (\sim(Z\ Z'))\ (Z\ Z'))$; that is to say, "$Z'$ is a Z (or, Z is true of Z', Z' has property Z) if and only if it is not the case that Z' is a Z."

1. Assume $(Z\ Z')$
2. From (1) and the definition of Z and Z' in (i) and (ii): $((S[o_i]\ (= (S\ o_i)\ o_i))\ (S[o_i]\ (\neq (S\ o_i)\ o_i)))$

logic in the true sense of the word, faithful to the unrestricted freedom of assumption (*unbeschränkten Annahmefreiheit*),[35] must be able to represent the formal structure of untyped predications. Any theory that lacks the ability sacrifices a crucial assumption in the core of Meinongian object theory, but any theory without the nuclear-extranuclear property distinction that permits such predications is subject to the *Sosein* paradox.

Property o_j on Meinong's theory is not an incomplete or impossible object, but is in some sense an abstract object or universal, a subsistent entity in a Platonic realist ontology. It might even be held that most if not all Meinongian objects are distinct from their *Soseine*, so that o_j is a property intelligibly shared by many objects. This indicates that o_j is *prima facie* a determinately subsistent object to which excluded middle must apply. If o_j has *Sosein* at all, it must either have a self-identical or self-non-identical *Sosein*. Yet either alternative leads to outright contradiction. Property object o_j therefore appears not to have *Sosein* at all, in violation of the independence thesis.

Meinong replied to an early objection of Russell's by claiming that excluded middle and the law of contradiction should not be expected to apply

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3. By hypothesis, the *Sosein* of Z' is identical to Z':
   $(= (S\ (S[o_i]\ (\neq (S\ o_i))))\ (S[o_i]\ (\neq (S\ o_i)\ o_i))))$
4. By definition, Z' is the *Sosein* of the property of being anything which is not identical to its own *Sosein*:
   $(\neq (S\ (S[o_i]\ (\neq (S\ o_i))))\ (S[o_i]\ (\neq ((S\ o_i)\ o_i))))$
5. Hence by (3) and (4), *reductio ad absurdum*:
   $(\sim (Z\ Z'))$
6. From (1) through (5) by conditional proof:
   $(\supset (\sim (Z\ Z'))\ (Z\ Z'))$
7. Assume $(\sim (Z\ Z'))$
8. From (7) and the definition of Z and Z' in (i) and (ii):
   $(\sim ((S[o_i]\ (= (S\ o_i)\ o_i))\ (S[o_i]\ (\neq (S\ o_i)\ o_i)))$
9. By hypothesis, the *Sosein* of Z' is not identical with Z':
   $(\neq (S\ (S[o_i]\ (\neq (S\ o_i))))\ (S[o_i]\ (\neq (S\ o_i)\ o_i))))$
10. By definition, Z' is not identical with the *Sosein* of the property of anything that is not identical with its own *Sosein*:
   $(= (S\ (S[o_i]\ (\neq (S\ o_i))))\ (S[o_i]\ (\neq (S\ o_i)\ o_i))))$
11. Hence by (9) and (10), *reductio ad absurdum*:
   $(\sim (\sim (Z\ Z')))$
12. From (11) by double negation and (7) through (11) by conditional proof:
   $(\supset (Z\ Z')\ (\sim (Z\ Z')))$
13. Therefore, from (6) and (12) by a rule of biconditional introduction:
   $(\equiv (\sim (Z\ Z'))\ (Z\ Z'))$

The basic combinatorial operation is the application of a function to an argument. See Frederick B. Fitch, *Elements of Combinatory Logic* [1974]. J. R. Hindley, B. Lercher, and J. P. Seldin, *Introduction to Combinatory Logic* [1972].

[35] Meinong, *Über Annahmen*, pp. 346ff. Meinong, *Über Möglichkeit und Wahrscheinlichkeit*, p. 283.

to impossible objects.[36] If it could be shown that o_j is impossible, that it has a *Sosein* of metaphysically incompatible properties, then the counterexample to the independence thesis might be avoided. In that case, it need not be said of o_j that if it has a *Sosein* it must either have a self-identical or self-non-identical *Sosein.* Yet o_j arguably does not have a *Sosein* of metaphysically incompatible properties, since it is not clear whether or in what sense o_j can be said to have *Sosein* at all. By construction, o_j has *Sosein* if and only if it does not have *Sosein.* This indicates that for Meinong it would not be possible to avoid the paradox by pleading that excluded middle or the law of contradiction does not apply to o_j (at least not for the same reason that these classical logical principles are supposed not to apply to nonexistent impossible objects).

4. *Dual Modes of Predication*

The Clark-Rapaport paradox has weaker object theory consequences than the *Sosein* paradox, since the Clark-Rapaport paradox can be avoided by methods that are inadequate for the *Sosein* paradox. Rapaport writes concerning his application of the Clark paradox to Meinong's theory:

> ... it might be held that not every 'well-formed propositional form' yields a property... While the definition of SSC involves quantification over *all* properties, it might be reconstructible in terms of *bounded* quantification over all properties of an antecedently given and well-defined kind. Should this not be possible, then this way out of the paradox is perhaps the most promising.[37]

Rapaport is right to insist that the Clark paradox involves quantification over the set of all properties. But this occurs in the abstract that defines the property of being a self-*Sein*-correlate, $\lambda x \forall F[F \mathrm{\ c\ } x \rightarrow x \mathrm{\ ex\ } F]$ and not in the lambda abstract which, as his solution requires, defines the property $\overline{\mathrm{SSC}}$ of being a *non*-self-*Sein*-correlate, $\lambda x \exists F[F \mathrm{\ c\ } x \ \& - (x \mathrm{\ ex\ } F)]$. Perhaps the solution could be made to depend on the universal quantification over properties in the definition of SSC. But the *Sosein* paradox is untyped, and does not require higher order quantification, so it cannot be avoided by the method, even if properly amended, which Rapaport regards as the most promising solution to the Clark paradox. The proposed solution is also defeated by Rapaport's own admission, in keeping with the argument above about the free assumption of

[36] Meinong, *Über die Stellung der Gegenstandstheorie im System der Wissenschaften* [1907], pp. 14—20.
[37] Rapaport, "Meinongian Theories and a Russellian Paradox", p. 174.

untyped self-non-predications, that "... it is not immediately clear how [this analysis] would account for the apparent fact that we can think of <$\overline{\text{SSC}}$>."[38]

Rapaport suggests that the "... second major way to block the paradox is to deny that M-objects are actual, for then they would not exemplify any such properties."[39] But although this blocks the Clark-Rapaport paradox, it is ineffective against the *Sosein* paradox. It defeats the Clark-Rapaport paradox only because the *Sein*-correlates in terms of which the paradox is formulated are defined for M-objects that have *Sein*. But it is Meinong's contention that all intentional objects have *Sosein* regardless of their ontological status. This is enough to support the *Sosein* paradox in the absence of the nuclear-extranuclear property distinction, even if it is agreed that Rapaport's M-objects are not actual. Whether an object is actual or not, it must have *Sosein* according to the independence thesis. This, together with the free assumption of untyped self-applications and self-non-applications, is sufficient to generate the *Sosein* paradox.

In a third attempt to avoid the Clark-Rapaport paradox, Rapaport considers the possibility of abandoning the principle of free assumption or *Annahmen* thesis. The principle invoked is: $\forall F \forall F'[F \neq F' \rightarrow \exists o[F \mathrm{\ c\ } o \;\&\; F' \not\mathrm{c}\ o]]$. This does not require that <SSC> is an M-object, which in turn blocks the paradox. But by violating the *Annahmen* thesis, the approach contradicts an important principle in the core of Meinongian object theory assumptions, and so cannot be regarded as a satisfactory way to preserve Meinongian object theory from inconsistency.[40]

5. Extranuclear Solution

The nuclear-extranuclear property distinction provides a solution to both the Clark-Rapaport and *Sosein* paradoxes. By Meinong's intuitive criteria, the paradoxical Clark-Rapaport properties of being a self-*Sein*-correlate and non-self-*Sein*-correlate are extranuclear rather than nuclear. There is therefore no Meinongian object <SSC> and no Meinongian object <$\overline{\text{SSC}}$>, since SSC and $\overline{\text{SSC}}$ if extranuclear are not assumptible and do not constitute an object by free assumption. It follows that the Clark-Rapaport paradox cannot be intelligibly formulated if the nuclear-extranuclear property distinction is enforced.

This also defeats the *Sosein* paradox, which as already shown, cannot be

38 Ibid.
39 Ibid.
40 Ibid., pp. 174—75.

solved by any of the solutions proposed by Rapaport to the Clark-Rapaport paradox. The properties o_i and o_j, the '*Soseine*' of being a self-*Sosein* or non-self-*Sosein*, invoked in the construction of the *Sosein* paradox, are evidently extranuclear rather than nuclear. This means that the paradox is improperly formulated if object theory contains the nuclear-extranuclear property distinction. Meinong need not be troubled by the Clark-Rapaport or *Sosein* paradoxes, provided only that object theory is formulated on the nuclear-extranuclear property distinction rather than the dual copula or dual modes of predication distinction.

It might be thought that the *Sosein* paradox can be solved within the Mallyan Rapaport-Zalta dual modes of predication theory by denying the *Sosein*-predication assumptions at steps (7), (17), and (19). In effect, this is to insist that $S(o_i) =^2 o_i$ rather than $S(o_i) =^1 o_i$, under the informal is^1-is^2 characterization of the dual copula. But this amounts to claiming that o_i only encodes or has as c-constituent the property of being its own *Sosein.* Yet intuitively the membership of constitutive properties in an object's *Sosein* is itself not merely a matter of encoding or c-constituency. The object really is^1 determined by its determining properties, even as Findlay allows in his explanation of Mally's later predication distinction, making it the one and only exception in which a nonexistent object can also satisfy (exemplify) its determining (encoded) properties. The solution cannot work at all in Zalta's official theory, since identity is relational, and his formalization of encoding is limited to nonrelational monadic qualities. The *Sosein* function applied to an object moreover is said to be identical to a set of properties. If sets exemplify and do not merely encode properties like the property of being identical to a function applied to an object, then the c-constituency or encoding rather than exemplification of self-*Sosein* and self-non-*Sosein* identities is intuitively implausible, and cannot be used to avoid the paradox.

Adherence to the dual modes of predication approach to Meinongian semantics finally leads Rapaport to claim that Meinong's logic is classically bivalent despite the predication of nuclear properties – to nonexistent incomplete objects. Rapaport holds that there are no truth value gaps for objectives or states of affairs on Meinong's theory. "According to Meinong," he writes, "there are two kinds of objectives: Sein-objectives (e.g., *x has Sein*) and Sosein-objectives (e.g., *x is F*). In general, there are no truth-value gaps among objectives."[41] Rapaport observes that Parsons had insisted on truth

41 Ibid., p. 166. Meinong's student Rudolf Ameseder asserted that objectives not only *have* being if they obtain, but literally *are* being. See Meinong, *Über Annahmen* [1910], p. 61: "On the other hand, the indirect path taken by R. Ameseder serves in an excellent way to give a

value gaps in his early reconstructions of Meinong's object theory. But he insists that this is at odds with Meinong's pronouncements. He reports: "...Parsons' theory allows truth-value gaps...Here, he differs sharply from both our theory and Meinong's; following Meinong, our theory holds that every objective either has or lacks Sein, *tertium non datur*..."[42] In his first treatments of Meinong's theory, Parsons had said: "Notice that some sentences can lack truth value; this will happen whenever *a* names an object that is indeterminate with respect to the property that *P* stands for."[43] But in *Nonexistent Objects*, a more recent formalization of object theory, Parsons for convenience rejects nonstandard truth value semantics, and installs a classically bivalent propositional logic.[44]

It is possible that Rapaport and others have been misled by the following passages in Meinong's essay "Über Gegenstandstheorie". There Meinong states:

> If the opposition of being and non-being is primarily a matter of the Objective and not of the Object, then it is, after all, clearly understandable that neither being nor non-being can belong essentially to the Object in itself. This is not to say, of course, that an Object can neither be nor not be...The Object is by nature indifferent to being (*ausserseiend*), although at least one of its two Objectives of being, the Object's being or non-being, subsists.[45]

These passages do indeed commit Meinong to a kind of bivalence, but only with respect to the specifically *extranuclear* predications of an object's being or nonbeing, the object's supervenient possession of either the extranuclear property of being, existence or subsistence, or the exclusively complementary extranuclear property of nonbeing, nonexistence or nonsubsistence. It is not surprising that Rapaport, in his rejection of the nuclear-extranuclear property distinction and espousal of an alternative Mallyan dual modes of predication analysis, should fail to distinguish the bivalence of Meinong's extranuclear subtheory from the nonstandard many-valued semantics of the nuclear predication component of the remainder of Meinong's object

precise description of the facts about object and objective, by which usage one could say: Every object has Being (or Nonbeing). But there are objects that not only *have* Being (in this broadest sense), but also *are* Being, and these objects are the objectives, while that which *has* Being, without *being* Being, is thereby characterized as an object." (My translation.) This has a striking, almost anticipatory resemblance to Wittgenstein's later pronouncements in the *Tractatus Logico-Philosophicus* [1922], 1—1.2.

42 Rapaport, "Meinongian Theories and a Russellian Paradox", p. 168.

43 Parsons, "A Prolegomenon to Meinongian Semantics" [1974], p. 571.

44 Parsons, *Nonexistent Objects*, p. 116.

45 Meinong, "The Theory of Objects", p. 86.

theory. That the two parts of Meinong's theory need not and should not be conflated is sufficiently indicated by the consideration that the nuclear predication and corresponding *Soseinsobjektiv* for the indeterminate or incomplete nonexistent object Pegasus in 'Pegasus is a mudder' (in racetrack *argot*) is intuitively neither true nor false but undetermined. The indeterminacy of nuclear properties attributable to an incomplete object carries over into an indeterminacy of truth value for nuclear predications about whether or not the object has the nuclear properties for which it is indeterminate.[46]

[46] See Findlay, *Meinong's Theory of Objects and Values*, p. 162: "Meinong distinguishes therefore between objects which are subject to the law of excluded middle in its narrowest form, i.e., which are determined in respect of every object, and those which are not. The former are called *completely determined* or *complete objects*, the latter *incompletely determined* or *incomplete objects*."

III. Meinong's Theory of Defective Objects

1. Mally's Paradox

In his difficult work *Über emotionale Präsentation*, Meinong introduces the concept of defective objects. These are meant to provide part of the solution to Mally's paradox about the impossibility of self-referential thought. They also suggest an alternative general method for avoiding the *Sosein* and Clark-Rapaport paradoxes.

But Meinong's discussion of defective objects is ambiguous in ways which give rise to a dilemma. It is not clear whether defective objects are supposed to be a special kind of intentional object on Meinong's theory, or whether they are not intentional objects at all. If defective objects are a special kind of intentional object, then it is possible to put forward a strengthened version of Mally's paradox which cannot be resolved by the theory of defective objects. The strengthened paradox also represents a putative counterexample to the intentionality thesis. But if defective 'objects' are not really objects at all, then experiences that have or are directed toward defective objects constitute immediate counterexamples to the intentionality thesis. In either case, the intentionality thesis cannot be consistently upheld. This means that defective objects do not provide a solution to Mally's paradox within Meinong's object theory. As an answer to the *Sosein* and Clark-Rapaport paradoxes, defective object theory is unacceptable for even more fundamental reasons.

Meinong argues that the notion of self-presentation should not be applied in philosophy unless or until Mally's paradox is resolved. The paradox calls attention to a seeming difficulty in the concept of self-presentation which may have far-reaching implications for the object theory.

The paradox can be formulated in terms of self-referential thought. It is similar in construction to other diagonalized semantic paradoxes, like the Liar or Epimenides, and the Russell paradox in set theory. Meinong writes: "The problem is to determine whether a thought (D') about a thought (D) (*Denken*) which is not about itself (*sich selbst nicht trifft*) is about itself."[1] If D

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1 Meinong, *On Emotional Presentation* (*Über emotionale Präsentation*) [1916], p. 13. Meinong refers to Mally's essay "Über die Unabhängigkeit der Gegenstände vom Denken" [1914], pp. 37ff.

is a thought which is not about itself, and if D' is a thought about D, then Meinong maintains that D' is a thought about itself if and only if it is not about itself. But as Meinong formulates the problem, there is no paradox. Let D be a thought which is not about itself, such as the thought that Graz is a city. D' may then be a thought about D. Suppose that yesterday I had the thought that Graz is a city on the Mur, and today in a reflective moment I recall that thought or think to myself that yesterday I had the thought that Graz is a city on the Mur. Where is the paradox? Is D' about itself if and only if it is not about itself? Evidently not. D' is not about itself in any sense at all; it is only about thought D, that Graz is a city on the river Mur. The construction is not complex enough to circle back in self-reference to D'.

Rapaport also observes that Mally's paradox as explained by Meinong is not really paradoxical. In an unpublished criticism, he offers a reconstruction of Meinong's exposition of the paradox, and gives a somewhat different explanation of its failure to constitute a genuine paradox.

(i) Let D be a thought which is not about itself.
(ii) That is, D is about O, and $O \neq D$.
(iii) Let D' be a thought which is about D.
(iv) Assume D' is not about D.
(v) Therefore, D' "is subsumable under the concept 'thoughts not about themselves'".
(vi) Therefore, D' *is* about D', contradicting (iv).
(vii) Assume D' is about D.
(viii) Therefore, D' is not about D', "since it is subsumable under the concept 'thought which is about itself'". This contradicts (vi).

Rapaport adds: "But this paradox makes no sense, for (iii) and (iv) together imply that $D' \neq D$; so how does (vi) follow? It would, if $D' = D$; but by hypothesis $D' \neq D$."[2]

The reconstruction may do justice to Meinong's exposition of the paradox, but Rapaport's criticism of the paradox is inaccurate. The paradox *does* make *sense*, even where $D' \neq D$. It is just that the paradox reconstructed in this way is explicitly unsound and therefore trivial or formally and philosophically uninteresting. Another version of the paradox can be given, based in

[2] Rapaport, "Meinongian Analyses of Some (Psycho-)Logical Paradoxes" [undated], pp. 8–9. See the published version of this essay which contains a different reconstruction of Mally's paradox, in Rapaport, "Meinong, Defective Objects, and (Psycho-)Logical Paradox" [1982], pp. 17–39.

part on Mally's original presentation, and not merely on Meinong's unsatisfactory description.[3]

Marie-Luise Schubert Kalsi translates Mally's and Meinong's verb '*treffen*' as 'about'. But the English word is ambiguous, and it is difficult to see how a genuine paradox could arise on such an interpretation. '*Treffen*' might be better translated as 'directed upon' or 'directed toward' (Meinong also uses the verb '*gerichten*'). It has connotations of reference, naming, picking out a thing, or hitting the mark, as in targetry.

With this substitution, an informal proof of Mally's paradox can be offered which avoids Rapaport's objection to Meinong's rendition.

1.	Thought *D'* is directed toward and only toward thought *D*.	
2.	*D* is any thought which is not directed toward itself.	
3.	*D'* is directed toward and only toward any thought which is not directed toward itself.	(1,2)
4.	*D'* is directed toward itself.	
5.	*D'* is not directed toward *D'*.	(3,4)
6.	*D'* is not directed toward itself.	(5)
7.	If *D'* is directed toward itself, then *D'* is not directed toward itself.	(4,6)
8.	*D'* is not directed toward itself.	
9.	*D'* is directed toward *D'*.	(3,8)
10.	*D'* is directed toward itself.	(9)
11.	If *D'* is not directed toward itself, then *D'* is directed toward itself.	(8,10)
12.	*D'* is directed toward itself if and only if *D'* is not directed toward itself.	(7,11)

Conclusion (5) follows from steps (3) and (4) because, according to (4), *D'* is directed toward itself, or, that is, toward *D'*. But (3) states that *D'* is directed toward and only toward any thought which is *not* directed toward itself. If, therefore, *D'* is directed toward itself, then it is directed toward a thought which is not directed toward itself. But then *D'* is not directed toward *D'*, which is to say that *D'* is not directed toward itself. Conclusion (9) follows from steps (3) and (8) because, according to (8), *D'* is not directed

[3] See also Chapter V.

toward itself, or, that is, not toward *D*'. But again, (3) says that *D*' is directed toward and only toward any thought that is not directed toward itself. Since by (8) *D*' is a thought not directed toward itself, it must be a thought toward which *D*' is directed. But then *D*' *is* directed toward *D*', which is to say that *D*' is directed toward itself.

This reformulation has the advantage of producing the paradox and avoiding Rapaport's objection to Meinong's admittedly faulty exposition. The argument does not fall under Rapaport's criticism because it does not assume that *D*' is not about *D*, but instead that *D*' is not about or not directed toward itself. In this way, no trivializing overt contradiction occurs in the assumptions.

Mally understood the paradox to imply that the concept of a thought which refers to itself and the concept of a thought which does not refer to itself are meaningless. Meinong argues that by analogy the same would apply to the concepts of self-presenting judgments and self-presenting assumptions. This alone would threaten Meinong's project of accounting for human values as given through emotional self-presentation. But he reports that the paradox has more devastating consequences for the object theory.

> ...even such commonplace, familiar statements as that each judgment has an object, or that each judgment is either affirmative or negative in 'quality', and so on, are not compatible with the thesis in question [*viz.*, that the concept of self-referential thought is meaningless]. For nobody would wish to hold that what is asserted of judgments in general is not asserted of the asserting judgment in question. The impression that in such cases one is confronted with something 'meaningless', in whatever sense this word may be understood, is not in accord with direct experience (*Empirie*).[4]

From this it is clear that Meinong cannot simply dismiss Mally's paradox as an anomaly of certain kinds of psychological experience. It entails conceptual difficulties that would cripple many fundamental object theory principles.

2. Russellian Hierarchy of Ordered Objects

To prepare the way for his analysis of Mally's paradox, Meinong considers the Russell and Burali Forti paradoxes in set theory. He suggests that a solution to the Russell paradox may clarify the problems encountered in Mally's.

The Russell paradox is similar in design. A set *R* is defined as the set of

[4] Meinong, *On Emotional Presentation*, pp. 10—11.

all sets which contain themselves as members. Another set R' is defined as the set of all sets which do not contain themselves as members. It is easy to show on the basis of these definitions that R' is itself a member of R if and only if it is not a member of R.

Russell's Paradox—

$$R = \{x \mid x \in x\}$$
$$R' = \{x \mid x \notin x\}$$
$$R' \in R \equiv R' \notin R$$

Meinong attempts to block the paradox by arguing on what he takes to be intuitive grounds that the definition of R is incoherent. He believes that it is impossible, though not inconceivable, for a set to contain itself as a member.[5]

> There is one striking presupposition on which the preceding considerations are founded. Is it possible for a set to contain itself as an element? As far as I can see, this is no more possible than for a whole to contain itself as a part or for a difference to be its own object of reference or its own foundation... An object of higher order can never be its own subordinate.[6]

The final remark is reminiscent of Russell's own solution to the paradox, establishing an ordered hierarchy of set theoretical types and ramifications within a type. Both Meinong and Mally offer this as a remedy to some of the theoretical implications of formal semantic paradoxes in object theory.

It may be significant that Meinong embellishes his intuitive rationale for the impossibility of sets that contain themselves as members by referring to wholes or concrete things and collectives, which admittedly could not contain themselves as proper parts. But abstract sets, as usually understood outside of type theory, are another matter. The ordinary conception of sets seems to be entirely compatible with the idea that a set may contain itself among its members. The set of all sets is itself a set, and must therefore contain itself — otherwise it is not really the set of *all* sets. The set of all nonphysical entities is itself presumably a nonphysical entity, and so must also contain itself.

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5 Ibid., p. 12: "The set that contains itself as an element is after all conceivable, and we must seek to determine in what follows what could possibly be meant by such a set."

6 Ibid., p. 11. Meinong refers to his essay, "Über Gegenstände höherer Ordnung und deren Verhältnis zur inneren Wahrnehmung" [1899], pp. 189f. See Marie-Luise Schubert Kalsi's translation, *Alexius Meinong on Objects of Higher Order and Husserl's Phenomenology* [1978].

Nevertheless, Meinong believes that denying the coherence or possibility of *R* is an *intuitively* justified approach to the solution of the paradox. He holds that although this is sufficient to defeat the paradox, more philosophical mileage may be gained from a careful examination of the confusions on which it rests. "We have to admit", he writes, "that in this way we can cut the knot, which, however, for the sake of theoretical interest, should be disentangled."[7]

The solution he recommends, like Russell's type theory, is to postulate an ascending order of collectives and derivative collectives. He defines a derivative collective as one that contains a collective together with all its members.[8] On the assumption that it is impossible for a set to contain itself, Meinong offers an amended version of the Russell paradox in terms of derivative sets and natural derivative sets. Natural derivative sets are those whose members, the original nonderivative set and its members, bear the relation of similarity one to another.[9]

The Russell-like construction which results is anomalous on this interpretation, but free of genuine contradiction or logical antinomy. Meinong maintains: "The alternative is now concerned with the question as to whether a set of sets which do not have any natural derivative sets can itself constitute a natural derivative set."[10] The reformulation of the paradox, in keeping with what Meinong regards as the intuitive limitations of logical possibility for set membership, yields the result that if a set of sets lacking in natural derivative sets can itself constitute a natural derivative set of the kind, then the set is similar in this respect to the sets which are its constituents.

The conclusion is that the specially constructed set is similar to its constituents if and only if it is dissimilar. Meinong rightly holds that this is neither contradictory nor genuinely paradoxical.

> ...we note that here too the similarity permits in the one case the formation of a new derivation, i.e., set, whereas dissimilarity in the other case does not permit this. No doubt we have here some complicated and perhaps somewhat subtle state of affairs, but is there anything self-contradic-

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7 Meinong, *On Emotional Presentation*, pp. 11—12.

8 Ibid., p. 12: "...no collective can contain itself as a part. But there are circumstances in which it is natural to form a new collective out of the collective itself and its parts, which we will provisionally call a 'derivative collective'."

9 Ibid.: "The alternative to Russell's paradox concerns derivative sets, that is, natural derivative sets. The principle of naturalness is then a relationship of similarity between the original (non-derivative) set and its elements."

10 Ibid.

> tory in them? All that it means, in the end, is that two objects may be similar in one respect and dissimilar in others.[11]

He compares the conclusion of the amended alternative version of Russell's paradox with the ordinary conception of color properties and their application. The property of being black and the property of being white are dissimilar in obvious respects, but are similar in their common inability to be predicated of red or blue.

> If similar things can form a collective, but dissimilar things cannot, then two objects considered in one way can form a collective, while considered in another way they cannot. The paradox, therefore, is easily resolved if a rigorous interpretation of the situation is given.[12]

Meinong believes that the same sort of analysis can be offered to undermine the construction and theoretical implications of Mally's paradox. As a solution, he proposes treatment analogous to that given Russell's. He argues that, correctly interpreted, the 'paradox' results in a peculiar or anomalous proposition, but not in anything genuinely contradictory or paradoxical. He attempts to show that Mally's construction merely establishes the claim that a thought may be about itself in one respect and not about itself in another.

> A thought which is at once about itself and not about itself is indeed peculiar. But these two adequacy-relationships (*Adäquatheitsverhältnisse*), i.e., the thought being at once about itself and not about itself, can coexist as long as they relate to different foundations (*sich auf verschiedene Grundlagen beziehen*). There is no more incompatibility here than in the analogous coexistence of exact likeness and unlikeness (as obtained in the case of colors) (*Gleichheit und Ungleichheit*).[13]

Meinong introduces the concept of defective objects by distinguishing between immediate and remote intentional objects. The difficulty in Mally's paradox, according to Meinong, by analogy with the Liar or Epimenides, is that it attempts to describe a situation in which the apprehending experience is supposed to refer to itself as an immediate rather than merely as a remote intentional object.[14] There is no paradox if such constructions are understood to refer back to themselves as remote objects, and it is in this sense that,

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[11] Ibid., p. 13.
[12] Ibid.
[13] Ibid., p. 14.

contrary to Mally's conclusion, thought can be about itself. In constructions where thoughts putatively refer to themselves as immediate objects of intention, Meinong claims that the psychological apprehending experiences in question have defective objects (*defektere Gegenstände*).

The distinction between immediate and remote intentional objects is grounded in turn on a difference in their respective characteristic modes of apprehension. Meinong holds that the mode of apprehending immediate intentional objects is primarily by way of reference to their being or *Sein*, whereas the mode of apprehending a remote object is always by way of reference to its being thus-and-so or *Sosein*. He admits that any immediate intentional object can also be apprehended by way of reference to its *Sosein* as well as its *Sein*. But he insists that immediate reference to the being or *Sein* of an object is possible only in the case of immediate intentional objects.[15]

These distinctions, together with their implications for Meinong's object theory, are presented in the following diagram.

For any putative self-referential
construction in thought or language

If the object of self-reference is immediate	If the object of self-reference is remote
... then it is a defective object, a genuine paradox results, and *no* self-reference is achieved.	... then, contra Mally, an anomalous but innocuous self-reference *is* achieved.

[14] Ibid., p. 15: [In the expressions 'What I think or what I apprehend is false' and 'What I think or what I apprehend is correct'] "... one is confronted with a peculiar *defectiveness in the object of thought*, which always becomes evident when an apprehending experience tries to refer to itself as immediate object. This point of view is clearly different from Mally's position, which we rejected above, in that he is opposed to any apprehension that is about itself, no matter what the circumstances may be." (Emphasis added.)

[15] Ibid., p. 17: "One can better understand these matters by considering that the mode of apprehending immediate objects is primarily an immediate reference to the object's being, whereas the mode of apprehending more remote objects is primarily a reference to an object's being thus-and-so. In principle, any immediate object can be apprehended by reference to the object's being thus-and-so, but there is seldom occasion to resort to this approach. On the other hand, immediate reference to the object's being is possible only in the case of immediate (*nächste*) objects, whereas any reference to an object as being thus-and-so has no limitations in this respect. Thus it is apparent at once that an immediate reference to an object's being, and a reference to an object as being thus-and-so, involve grasping the object by two entirely different modes of apprehension."

Meinong explains:

> ...E. Mally has not proved that thinking cannot under any circumstances apply to itself. The difficulties which he has pointed out are traceable to the special nature of defective objects and to the fact that the domain of what can rationally be believed is to some extent restricted by factors involved in every judgment. The emerging peculiarity of defective objects is not, however, only of object-theoretic interest, but is also valuable as marking off a typical case in which self-reference is denied to an intellectual experience, since this would unavoidably amount to the apprehension of a defective object.[16]

Yet the analysis does not provide a satisfactory solution to Mally's paradox if the distinction between immediate and remote intentional objects cannot be sustained.[17]

3. Dilemmas of Intentionality and a Strengthened Paradox

The most unacceptable thing about the theory of defective objects as a solution to Mally's paradox is that Meinong does not explain how the distinction between remote and immediate objects of putative self-reference is supposed to avoid the paradox and make self-reference possible for remote objects. Meinong may be able to distinguish these two kinds of objects of purported self-reference by appealing to their distinctive modes of apprehension. But once they are distinguished, it remains entirely mysterious why putatively self-referential thoughts directed toward themselves as immediate objects are paradoxical and preclude genuine self-reference, while putatively self-referential thoughts directed toward themselves as

[16] Ibid., pp. 21—2. On p. 18, Meinong writes: "E. Mally's expression 'meaningless' (*sinnleer*) may appropriately apply to such defective objects, so that it might be presumed, as previously noted, that E. Mally's and his predecessor's attention to these defective objects was well placed."

[17] Meinong mentions the so-called identity restriction as distinguishing immediate from remote intentional objects; ibid., p. 17. He maintains that: "It is thus not surprising that the identity-restriction is essential for one mode of apprehension, but not so for the other." Presumably, the identity concerned is that between the act of apprehension and its object. But the identity-restriction is not essential to the mode of apprehending *all* immediate objects of intention, but only to those which purport to be the immediate objects of self-referential thoughts or acts of apprehension. This renders the identity-restriction circular or uninformative and therefore useless as a criterion for distinguishing between immediate and remote objects of intention. It is also phenomenologically or introspectively inscrutable.

remote objects are nonparadoxical and can be involved in presentations that are in some sense genuinely self-referential. It may also appear unsatisfactory to conclude that nonexistent incomplete and impossible intentional objects in nonparadoxical constructions must always be remote rather than immediate intentional objects just because they lack being and so cannot be apprehended by mode of reference to their *Sein*. This has the intuitively incorrect result that fictional characters are inherently incapable of self-referential thought.

There is further no phenomenological difference to be discerned introspectively between the two kinds of objects. According to Meinong, the Fountain of Youth is a remote intentional object because it has no being or *Sein* by reference to which it may be apprehended. The Eiffel Tower, on the other hand, is an immediate object of intention for some psychological presentations. Since it exists, it can be apprehended by mode of reference to its being. But if I mistakenly believe that both the Fountain of Youth and the Eiffel Tower exist, and if I alternatively think of each and thereby make them the objects of distinct psychological states, then there appears to be no phenomenological feature of my apprehending experiences which would enable me to distinguish between them as existent or nonexistent. I may believe that I am apprehending the objects in precisely the same way, but in fact, according to Meinong's principle, this would be impossible. Yet the only way to determine these ontological discrepancies is to undertake empirical investigation of the external world. This alone will enable me to learn that the Fountain of Youth does not exist, and that therefore, despite the introspective inscrutability of my respective internal states with respect to the existence of their objects, I could not have been apprehending the Fountain of Youth and the Eiffel Tower by the very same mode of apprehension. But if external evidence must be resorted to in object theory semantics, then at least some of the philosophical appeal of a purely phenomenological methodology over traditional extensional approaches will be sacrificed.

Even if these deficiencies were corrected, Meinong could not consistently appeal to the concept of defective objects to avoid Mally's paradox without contradicting the intentionality thesis. Meinong's discussion does not make clear whether defective objects are supposed to be a special kind of intentional object, or whether the term 'defective object' is meant instead to indicate the lack of any intentional object in the description or phenomenological reduction of a particular psychological experience. In the latter case, it would be more appropriate to speak of defective *acts* — those which in fact are not directed toward any intentional object — rather than defective 'objects' as such.[18] If defective objects are a special kind of intentional object, more or less on a par with incomplete and impossible objects, then a strengthened version of Mally's paradox can be advanced, the logically inconsistent conclusion of which cannot be dismissed as harmless in object theory, since it presents a counterexample to the intentionality thesis.[19] But if defective objects are not really intentional objects at all, then any psychological experience with a defective object will automatically represent a counterexample to the intentionality thesis.

Meinong holds that the mode of apprehending remote intentional objects is by reference to their *Sosein* or so-being. This suggests that a remote object may be defined as the object satisfying some particular set of nuclear properties. In the strengthened version of Mally's paradox, on the assumption that defective objects are a special kind of intentional object, 'o_D'' can be defined as the remote intentional object of a thought D' if and only if that thought is about or directed toward itself (in the supposedly harmless nondefective sense of 'being about or directed toward itself' implied in Meinong's analysis).[20] From this it follows that a thought could not have o_D' as its remote object unless the thought were about or directed toward itself.

The definition makes it possible to produce the following strengthened version of Mally's paradox. The steps indicated below are added to the argument outlined above.

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[18] This terminology was suggested to me by Roderick M. Chisholm. A similar recommendation is made by Rapaport in "Meinongian Analyses of Some (Psycho-) Logical Paradoxes", p. 20.

[19] Rapaport, "Meinongian Theories and a Russellian Paradox", p. 154: "(M1) *Thesis of Intentionality*: Every psychological experience is 'directed' toward something called its object (*Gegenstand*) . . ."

[20] This is intimated by much of Meinong's commentary. It is difficult to understand why, if immediate defective objects were not a special kind of intentional object, he would otherwise distinguish between immediate and remote objects of self-reference. Findlay also seems to imply that defective objects according to Meinong are a special kind of object. In the "Forward" to the Schubert Kalsi translation of Meinong's *On Emotional Presentation*, pp. *xviii*—*xix*, Findlay writes: "Meinong discusses various legitimate and illegitimate senses and cases of self-reference and arrives among other things at the view that the objects projected by certain viciously circular references, e.g., certain forms of the Liar paradox, are certainly worse than self-contradictory: they are so ill-formed that they do not even achieve the status of the absurd. Meinong does not, however, hold that it is senseless to talk of them: the class of all classes not members of themselves is not to be compared with an ill-formed structure like *beautiful in truly*. Meinong speaks of defective objects in such a case; and, while we may demur at his allowing such objects to parade the world unbracketed, they certainly enter into the description of the intentions that conceive them and reject them and the logical principles that rule them out."

13. Object $o_{D'}$ is the remote intentional object of thought D' (or, thought D' has remote intentional object $o_{D'}$) if and only if D' is directed toward itself. (Df)
14. If D' is directed toward itself, then it has remote intentional object $o_{D'}$. (13)
15. If D' is not directed toward itself, then it does not have remote intentional object $o_{D'}$. (13)
16. Thought D' has remote intentional object $o_{D'}$ if and only if thought D' does not have remote intentional object $o_{D'}$. (12,14,15)

Propositions (14) and (15) are consequences of the definition of '$o_{D'}$' in step (13). The conclusion of the strengthened Mally paradox in (16) follows from steps (12), (14), and (15). If D' has remote intentional object $o_{D'}$, then by (15), D' is directed toward itself. But by (12), if D' is directed toward itself, then it is not directed toward itself. Again by (15), if D' is not directed toward itself, then D' does not have remote intentional object $o_{D'}$. If, on the other hand, D' does not have remote intentional object $o_{D'}$, then by (14), D' is not directed toward itself. But by (12), if D' is not directed toward itself, then it is directed toward itself. By (14), if D' is directed toward itself, then D' has remote intentional object $o_{D'}$. Therefore, D' has remote intentional object $o_{D'}$ if and only if D' does not have remote intentional object $o_{D'}$.

The conclusion in (16) cannot be dismissed as innocuous for the object theory in the way that Meinong tries to dismiss the reinterpretations of the Russell and original Mally paradoxes. The revised conclusion states that a particular thought both has and does not have an arbitrarily designated remote intentional object. This is logically inconsistent and contradicts the intentionality thesis. It cannot be said that the thought has the object in one respect, but not in another. Nor will it do to say that thought D' has some object other than remote intentional object $o_{D'}$ as its intentional object. Meinong admits that a thought which is about itself in the harmless, nondefective sense has a remote intentional object. But '$o_{D'}$' is simply defined as the remote intentional object of a particular harmlessly self-referential thought. If thought D' is harmlessly self-referential, as Meinong maintains, then by definition it has remote intentional object $o_{D'}$ as its object of intention. Yet D' has object $o_{D'}$ if and only if it does not have object $o_{D'}$. The result is genuinely paradoxical for Meinong's object theory.

It appears more appropriate in any case to interpret Meinong's defective object theory as entailing the lack of any intentional object in situations where a psychological experience is correctly described as having a defective object. This would be a kind of Aristotelian position, since, in his first philosophy, Aristotle often says that a defective thing incapable of performing its natural or man-made function is not really a thing of the kind.[21] It might be held on this view that Meinong may regard defective objects as other than genuine intentional objects, in much the same way that Aristotle claims that a dead hand is not really a hand, except in name only, and that a stone flute in a work of sculpture is not really a flute, unable to perform its musical function. 'Defective objects' could then be understood to refer to the complete absence of any intentional object in the phenomenological reduction of a particular psychological experience. The experience might then be more accurately described as having a defective intentional act, rather than a defective intentional object, since the mental act in question by hypothesis does not connect up with and is not actually directed toward any intentional object.

The interpretation is supported in part by Meinong's own discussion when he writes:

> Let us once more return to the thinking which is about itself and to its analogues, the objects which do not satisfy the requirements stipulated for the identity-restriction[22] in regard to immediate objects. It is a striking feature of these cases that they are experiences which do not have objects in the way other experiences do, which in a sense lack objects altogether.[23]

But if defective 'objects' are not really intentional objects at all, then psychological experiences with defective objects do not have and are not directed toward any intentional objects.

In either case, regardless of whether defective objects are a special kind

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[21] Aristotle, *De Anima*, 412^{b}13–22; *De Partibus Animalium*, 640^{b}35–641^{a}5; *Meteorologica*, 389^{b}32–390^{a}1.

[22] Note supra 19.

[23] Meinong, *On Emotional Presentation*, p. 18. In discussing the possibility of defective objects which lack even *Aussersein*, Meinong again indicates that at least some kinds of defective objects are not really intentional objects at all. He maintains, p. 20: "If [the defective object itself is apprehended and not some nondefective object in apprehending 'I think that I think'] . . . , then one is confronted with defective objects which lack even *Aussersein*, though this expression is indeed peculiar. In this case *one is not really confronted with an object*, and experiences of apprehension in this instance *lack a proper object*." (Emphases added.) See also, Schubert Kalsi, "On Meinong's Pseudo-Objects" [1980], p. 120: "Then Meinong says [an expression which does not stand for any idea or for one which we do not understand] denotes a defective object, which is no object at all . . ." Routley, *Exploring Meinong's Jungle and Beyond*, pp. 501–2.

of intentional object or are not genuine objects of intention, the dilemma shows that the intentionality thesis is incompatible with defective object theory. The theory of defective objects cannot be made logically consistent with both the possibility of self-referential thought and the fully generalized intentionality thesis.

This is also seen in Rapaport's solution to Mally's paradox. Rapaport holds that defective acts are not actual psychological experiences of a given kind, but nonexistent M-object counterparts. This is supposed to be justified by a further distinction between having a particular psychological experience, and thinking falsely that one is having the experience, or merely going through the motions of having that kind of psychological experience.[24] Yet it also entails either that defective acts lack an object, so that the intentionality thesis is again contradicted, or else that the free assumption or *Annahmen* thesis is false. As Rapaport describes the solution, a person cannot freely assume the paradoxical constructions, but only mistakenly believe that they are assumed, merely going through the motions of assuming them. Rapaport here appears to soften the *Annahmen* thesis in order to make his solution work, as he also proposes in one of his recommended solutions to the Clark-Rapaport paradox.

Mally's paradox and the strengthened Mally paradox, like the Clark-Rapaport paradox, is solved by application of the nuclear-extranuclear property distinction. If the semantic property of being directed or self-directed toward an object is extranuclear rather than nuclear (as is intuitively plausible and follows from both formal and informal characterizations of the nuclear-extranuclear property distinction), then the paradox cannot arise. In that case, there is no existent or nonexistent thought with the *Sosein* of being intentionally directed toward itself or toward a particular remote intentional object, or defined in terms of any extranuclear properties as part of its identifying *Sosein*, as required in the formulation of both the Mally and strengthened Mally paradoxes. If thought D' is not constituted by the property of being directed toward thought D as part of its *Sosein*, and if D is not constituted by self-non-directedness, then the object theory does not authorize the inferences in steps (3)—(12) of the reconstruction of Mally's paradox, and blocks the definition of remote intentional object o_D' in (13) of the strengthened Mally paradox. This makes it unnecessary to adopt Meinong's theory of defective objects, and avoids the dilemma which otherwise threatens the intentionality thesis.

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[24] Rapaport, "Meinongian Analyses of Some (Psycho-)Logical Paradoxes", pp. 22—3.

4. *The Soseinlos Mountain*

A problem occurs in object theory which first appears to contradict the independence thesis, and which similarly cannot be resolved by appeal to the theory of defective objects. It also casts doubt on the generality of the intentionality thesis. But like the Clark-Rapaport, *Sosein*, Mally, and strengthened Mally paradoxes, it is refuted by proper application of the nuclear-extranuclear property distinction.

Russell's objection about the existent round square sought to determine whether the existent round square considered as an intentional object has the property of being existent as part of its *Sosein*.[25] Of course, the existent round square is an impossible object, which as a matter of metaphysical necessity neither exists nor subsists. Parsons points out that Meinong inexplicably gave his reply to Russell's question in terms of the existent golden mountain rather than in terms of the existent round square.[26] But the philosophical problem for the independence thesis in object theory is clearly much the same. Meinong tried to answer Russell's objection by saying that the golden mountain has the property of being existent as part of its *Sosein* even though it does not exist. This reply is made more intelligible and intuitively plausible by Rapaport's discussion of plural modes of predication, and by Parsons' use of the nuclear-extranuclear property distinction.[27]

Rapaport writes:

> Meinong's reply to Russell can be maintained and given some substance: The existent round square is existent (i.e., $E \text{ c } <E,R,S>$) but does not exist (i.e., $\neg\exists\alpha[\alpha \text{ SC } <E,R,S>]$).[28]

The expression '$E \text{ c } <E,R,S>$' indicates that the property E of being existent is a c-constituent of the Meinongian object $<E,R,S>$, the existent round square. The expression '$\neg\exists\alpha[\alpha \text{ SC } <E,R,S>]$' indicates in the usual quantificational notation that the existent round square does not exist. This

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25 Russell, "On Denoting", p. 45. Russell refers here to the problem of the existent King of France. See Russell, "Meinong's Theory of Complexes and Assumptions" [1904], pp. 21—76; "Review of A. Meinong, *Untersuchungen zur Gegenstandstheorie und Psychologie*" [1905], pp. 77—88; "Review of A. Meinong, *Über die Stellung der Gegenstandstheorie im System der Wissenschaften*" [1907], pp. 89—93.

26 Parsons, "A Prolegomenon to Meinongian Semantics", p. 562, n. 1.

27 Rapaport, "Meinongian Theories and a Russellian Paradox", pp. 159—65. Parsons, "A Prolegomenon to Meinongian Semantics", pp. 561—80. See Parsons, "Nuclear and Extranuclear Properties, Meinong, and Leibniz" [1978], pp. 137—51.

28 Rapaport, "Meinongian Theories and a Russellian Paradox", p. 165.

means that there is or exists nothing which has the set of properties defining the existent round square as its SC or *Sein*-correlate. It provides the distinction needed to make sense of Meinong's reply, because the *Sein*-correlate of an intentional object can only be the object or set of objects corresponding to a Meinongian object that has being or *Sein*.[29]

Parsons proposes less formally:

> Suppose we use 'is existent' for the nuclear sense of existence, and 'exists' for the extranuclear sense. Then our conclusion is: Although the existent gold mountain is existent, the existent gold mountain does not exist.[30]

Parsons and Rapaport agree that Meinong's reply to Russell can be understood by introducing a distinction between existence interpreted as a constituent or constitutive nuclear property, and as a nonconstitutive quantifier or term of ontological status.

But it is possible to raise an objection similar to Russell's for which Rapaport's solution unlike Parsons' is unsatisfactory. This is a counterexample in which the *soseinlos* mountain is offered as an intentional object which by construction apparently cannot have *Sosein*, and which therefore seems to contradict the independence of *Sosein* from *Sein* thesis. It might be asked, by parity of formulation with Russell's objection, whether or not the *soseinlos* mountain is *soseinlos*, or, that is, whether or not the *soseinlos* mountain has the property of being *soseinlos* as part of its *Sosein*.

The *soseinlos* mountain is evidently at least *prima facie* a Meinongian object, by the usual psychological and linguistic semantic criteria that serve as an informal comprehension principle for the object theory domain. But if we try to say that the *soseinlos* mountain is constituted by or has among its properties the property of being *soseinlos* (as included in its *Sosein* — part of which is to be *soseinlos* or entirely lacking in *Sosein*), then we are immediately placed in absurdity. For 'the *Sosein* of encoding or being constituted by properties that include the property of being *soseinlos*', like 'the property of having no properties', is a blatant contradiction in terms. This entails a direct conflict with the independence of *Sosein* from *Sein* thesis.

It might be thought that the problem of the *soseinlos* mountain could be avoided if it were held that the *soseinlos* mountain is a defective object. The category of defective objects seems to apply in some sense to the concep-

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[29] Ibid.

[30] Parsons, "A Prolegomenon to Meinongian Semantics", p. 574. Also, Parsons, *Nonexistent Objects*, pp. 42—4.

tual peculiarities of the *soseinlos* mountain. If it is defective, then it need not fall under the independence thesis. But the *soseinlos* mountain cannot be a defective object. In order to qualify as defective, the *soseinlos* mountain would have to be an immediate object of intention. According to Meinong, this means it would need to be capable of being apprehended by mode of reference to its being or *Sein*. But the *soseinlos* mountain is entirely lacking in being or *Sein*, and is therefore precluded from this mode of apprehension. Nor is it possible to say that the *soseinlos* mountain may be apprehended by mode of reference to its being thus-and-so or *Sosein*. It has already been established that the *soseinlos* mountain if constituted by the property of being *soseinlos* does not have *Sosein*, or has *Sosein* if and only if it does not have *Sosein*.

The only way to reconcile the *soseinlos* mountain with the independence thesis is to admit that the *soseinlos* mountain is not really an intentional object at all. But then Meinong's intentionality thesis again is contradicted. It is thereby acknowledged that thoughts ostensibly about the *soseinlos* mountain do not have or are not directed toward any genuine object of intention. The alternative would be to deny unrestricted free assumption, and insist that the mind cannot assume or entertain in thought ideas about the *soseinlos* mountain, but, as Rapaport suggests, only appear to entertain or go through the motions of entertaining such thoughts. These solutions are all unsatisfactory because they violate the central core of Meinongian object theory assumptions.

The problem is overcome by invoking the nuclear-extranuclear property distinction. To be *soseinlos* is to have an extranuclear rather than nuclear property. Like the *Sosein*, Clark-Rapaport, Mally, and strengthened Mally paradoxes, the problem of the *soseinlos* mountain is defeated by judicious application of the nuclear-extranuclear property distinction. The solution reinforces the importance of the nuclear-extranuclear property distinction. It is only by recognizing the property of being *soseinlos* as a nonconstitutive extranuclear property that independence of *Sosein* from *Sein* and intentionality can be fully preserved. Thought may be free to assume that there is a *soseinlos* mountain, but the independence thesis guarantees only that an object has whatever nuclear constitutive properties it is assumed to have. It cannot be inferred that the freely assumed *soseinlos* mountain is actually *soseinlos*.

5. Nuclear Converse Intentionality

There is a further question raised by the problem of the *soseinlos* mountain. It is assumed that to be *soseinlos* means, among other things, to have no converse intentional or psychological properties, such as the properties of being thought about, feared, doubted, and the like. If the *soseinlos* mountain is the intentional object of any psychological experience, then presumably it must at least have the property of being the intentional object of that experience.

Yet the *soseinlos* mountain by definition is supposed to be truly *soseinlos*, or entirely lacking in *Sosein* properties. This implies that the *soseinlos* mountain cannot even be the intentional object of any psychological experience, for then it would have *Sosein.* It is controversial whether or not objects can contain converse intentional or psychological properties in their individuating *Soseine.* Later it will be shown that in order to provide an adequate object theory logic of intention it is necessary to include private mental objects in the semantic domain or universe of discourse of the object theory, and for this converse intentional or psychological properties must enter into an object's *Sosein.*

Without reference to existent or nonexistent private mental objects, the object theory is unable to represent the deep formal structure of intentional philosophical theories rooted in private psychological experiences of privileged epistemic access. This opens the door to degenerate constructions like the *soseinlos* mountain, which seem in the absence of the nuclear-extranuclear property distinction to contradict the independence and intentionality theses. But it also makes possible the intelligible designation of private mental objects, which restores intentionality in a more important sense to the theory.

Meinong's theory of defective objects evidently yields an unsatisfactory solution to the problem of the *Sosein* paradox. The propositions of the paradox are not acts, which means that Meinong's theory does not directly apply to them. Even if the *Sosein* paradox were revised so that the theory of defective objects would pertain to them (perhaps by referring to judgments instead of propositions as a kind of mental act), the theory would still fail on its own merits to provide an adequate solution. The theory of defective objects is accordingly eliminated from the revised object theory. It is necessary to turn elsewhere for a satisfactory remedy to the object theory paradoxes. The answer may be found, as previously indicated, in enforcement of the nuclear-extranuclear property distinction. To show that the endeavor is worth the effort of untangling these problems, it must now be demonstrated that a semantics of nonexistent Meinongian objects is required in order to provide an intuitively correct analysis of ontological commitment.

IV. The Object Theory Intentionality of Ontological Commitment

1. The Poverty of Extensionalism

Arthur N. Prior in his posthumous *Objects of Thought* describes the limitations of the extensional outlook in philosophy as something like the limitations of Newtonian mechanics.[1] It is not that Newtonian mechanics or extensional theories of ontological commitment are false in and of themselves, but rather that they are unable to account for everything that needs to be explained in a complete theory of the kind. In physics, the recalcitrant phenomena are events at the quantum or microphysical and astronomical levels. In semantic philosophy, the recalcitrant phenomena are theories with putative ontological commitments to nonexistent objects. The extensional theory of ontological commitment needs to be absorbed by or embedded in a more complete intentional theory, of which it will then be but a fragment or proper part, in much the same way that Newtonian mechanics needs to be absorbed by or embedded in a unified field theory.

There is a limited range of unproblematic cases to which extensional theories of ontological commitment apply. In determining the ontological commitments of scientific and philosophical theories committed to existent entities, extensional theories of ontological commitment work perfectly well. Problems occur when extensional theories of ontological commitment are brought to bear on the ontological commitments of theories that contain non-(existent object)-designating singular terms or extensionless predicates.

The limitations of extensional theories of ontological commitment have been independently established by Noam Chomsky and Israel Scheffler, Richard Cartwright, and, most recently, Michael Jubien. Although these critics of the extensional theory of ontological commitment have emphasized the need to abandon the theory in favor of an intentional (or, nonequivalently, 'intensional') account, they have stopped short of endorsing anything

1 A. N. Prior, *Objects of Thought* [1971], pp. 48—9. Prior rejects Meinongian semantics.

like a Meinongian object theory. But the sort of non-Meinongian intentional or intensional theories of ontological commitment which they have proposed are also inadequate. This is shown by eliminating alternatives, criticizing both extensional and non-object-theoretical intentional and intensional theories.

Assume that theory *T* is a scientific or philosophical theory which falsely claims that things of kind *P* exist, and that theory *T** is a scientific or philosophical theory which falsely claims that things of kind *Q* exist. Further, *T* truly claims that things of kind *Q* do not exist, and *T** truly claims that things of kind *P* do not exist. For example, let *T* be a theory which says that unicorns exist, but that winged horses do not; and let *T** be a theory which says that winged horses exist, but that unicorns do not. It appears that if '*P*' and '*Q*' are extensionless predicates, then no extensional theory of ontological commitment can adequately account for the fact that theory *T* is ontologically committed to things of kind *P* rather than *Q*, and that theory *T** is committed to things of kind *Q* rather than *P*. The extensional theory has three alternatives, none of which is acceptable.

1. The theory might hold that *T* and *T** are ontologically committed to the extensions of predicates '*P*' and '*Q*', respectively. But by hypothesis, the extensions of '*P*' and '*Q*' are empty or null. This means that *T* and *T** would simply fail to have any ontological commitment at all. But the preanalytic assumption is that *T* is committed to things of kind *P* rather than *Q*, and that *T** is committed to things of kind *Q* rather than *P*. On this alternative, the ontological commitments of *T* and *T** are indistinguishable.

2. The theory might hold that *T* and *T** are ontologically committed to the universal set containing all existent entities. This could be made somewhat plausible by the consideration that in standard first order logic a false proposition logically implies any and every proposition. In particular, for any arbitrary property *P*, false theories *T* and *T** alike imply $(\exists x)Px$. But this also contradicts the preanalytic assumption that *T* is not ontologically committed to whatever *T** is committed to, and vice versa.

3. The theory might hold that *T* and *T** are ontologically committed to the existence of an arbitrary existent entity other than the universal set. This is similar to a semantic strategy for interpreting nondesignating terms devel-

[2] Gottlob Frege, "Über Sinn und Bedeutung" [1892], pp. 70—1. See Karel Lambert, *Meinong and the Principle of Independence: Its Place in Meinong's Theory of Objects and its Significance in Contemporary Philosophical Logic* [1983], pp. 95—6.

oped by Frege.[2] Theoretically, the ontological commitments of *T* and *T** could be distinguished in this way, if each were assigned a different arbitrary existent entity. But this also seems wrong. *T* and *T** are not ontologically committed to something that exists, but to things or kinds of things that do *not* exist. That is why the theories are false.

2. *Parsons' Criticisms*

The difficulties of extensional theories of ontological commitment can be made more precise by comparing them to a related formal criticism. Parsons has attempted to refute three distinct extensional theories of ontological commitment. In these theories, ontological commitment is construed as a relation between a sentence and an entity, or between a sentence and a reference class (where the members of the class are within the range of bound variables contained in the sentence). Three different relations are considered.

Parsons' objections are directed primarily at Quine. He offers the second theory as a version of the account which Quine accepts, and tries to show that it is inadequate by the application of two criteria. The refutation of the first and third theories proves that it would not be worthwhile to reformulate the extensional position as either of these, though they are the most likely amendments of Quine's view.

Parsons defines two general kinds of ontological commitment statements. The first has the form 'ϕ oc x'. This is a relation of ontological commitment that holds between sentences and entities. The second relation has the form 'ϕ OC α'. This is an ontological commitment relation that holds between sentences and unique classes. Parsons defines these relations as extensional

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[3] Parsons, "Extensional Theories of Ontological Commitment" [1967], pp. 446—47: "I exclude theories which hold ontological commitment to be a relation between a sentence and a property or some kind of intension. I also want to discuss only extensional relations; that is, both open places in the relation 'x is ontologically committed to y' are to be referentially transparent. These restrictions place the theories in question within the realm of what Quine calls the 'theory of reference' and, thus, makes my discussion relevant to the kind of theory he would favor." Parsons' analysis deals specifically with the ontological commitments of sentences, but he claims that his conclusions have immediate implications for theories construed as sets of sentences. See p. 466, n. 1: "Some versions speak of the relation as holding between *theories*, construed as sets of sentences, and objects (or sets). My arguments carry over to such versions in a straightforward manner." The ontological commitments of theories are discussed in greater detail in Parsons, "Various Extensional Notions of Ontological Commitment" [1970], pp. 65 ff.

in the sense that by stipulation 'ϕ' and 'x' in statements of both forms are referentially transparent.[3]

Where 'α' ranges over classes or sets, 'x' over entities, and 'ϕ' over sentences, the two relations can be formally interdefined as follows.

(D1) $\phi \text{ oc } x =_{df} (\exists\alpha)(x \in \alpha \,\&\, \phi \text{ OC } \alpha)$
(D2) $\phi \text{ OC } \alpha =_{df} \alpha = \{x\colon \phi \text{ oc } x\}$

A commitment operator on sentences '$\mathscr{C}(\phi)$' may be regarded as an abbreviation for '$\{x\colon \phi \text{ oc } x\}$'. This is equivalent to the unique α such that $\phi \text{OC} \alpha$. $\mathscr{C}(\phi)$ is thus the set of all entities to which 'ϕ' is ontologically committed.

Parsons offers two criteria for the adequacy or acceptability of any theory of ontological commitment.

(CR1) There are *atomic predicates* 'P' and 'Q', such that $\mathscr{C}((\exists x)Px) \neq \mathscr{C}((\exists x)Qx)$.
(CR2) If ψ is a logical consequence of ϕ, then $\mathscr{C}(\psi) \subseteq \mathscr{C}(\phi)$.

If an extensional theory of ontological commitment fails to satisfy criterion (CR1), then, according to Parsons, the theory will be philosophically uninteresting. The point is that on any adequate theory of ontological commitment, at least some commitment sets of sentences containing interpreted predicates must be distinct. Parsons maintains that any theory of ontological commitment which fails to meet criterion (CR2) is simply false. He holds that both (CR1) and (CR2) are close enough to our intuitive understanding of the concept of ontological commitment to require no argument.

The analysis proceeds by considering the class of sentences of the form '$(\exists x)\phi$', where 'ϕ' contains no quantifiers. These include sentences like 'There are men', 'There are even integers'. Parsons argues that an adequate theory of ontological commitment must say 'something simple' about the relation between the ontological commitment of sentences such as 'There are men' and the class of men in the extension of the predicate. In other words, a simple relation must obtain between the commitment set $\mathscr{C}((\exists x)\phi)$ and the reference class $\{x\colon \phi\}$. He claims that there are just three candidates for this simple relation, the three extensional theories of ontological commitment he seeks to refute.

Theory (1) $\mathscr{C}((\exists x)\phi) = \{x\colon \phi\}$
Theory (2) $\mathscr{C}((\exists x)\phi) \subseteq \{x\colon \phi\}$
Theory (3) $\mathscr{C}((\exists x)\phi \supseteq \{x\colon \phi\}$

It is easy for Parsons to show that theories (T1)—(T3) are inconsistent with criteria (CR1) and (CR2). He rightly holds that a complete refutation need only deal with (T2) and (T3), since if either of these is false, so is (T1).

In order to refute (T3) (and (T1)), Parsons offers the following argument.

1. $\{x\colon Ax\} \cup \{x\colon Bx\} = \{x\colon Ax \vee Bx\} \supseteq ((\exists x)Ax \vee Bx)$ (T3)
2. $\mathcal{C}((\exists x)Ax \vee Bx) \subseteq \mathcal{C}((\exists x)Ax)$ (CR2)

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3. $\{x\colon Ax\} \cup \{x\colon Bx\} \subseteq \mathcal{C}((\exists x)Ax)$ (1,2)

If '~Ax' is permitted as a substitution instance for 'Bx', then we have:

3'. $\{x\colon Ax\} \cup \{x\colon \sim Ax\} \subseteq \mathcal{C}((\exists x)Ax)$

But this amounts to a proof that $\mathcal{C}((\exists x)Ax)$ is the universal set, no matter how atomic predicate '*A*' is interpreted, in violation of (CR1).

To refute theory (T2), Parsons offers a different inference.

1. $\mathcal{C}((\exists x)Ax \,\&\, Bx) \subseteq \{x\colon Ax \,\&\, Bx\} = \{x\colon Ax\} \cap \{x\colon Bx\}$ (T2)
2. $((\exists x)Ax \,\&\, Bx) \supset ((\exists x)Ax)$
3. $\mathcal{C}((\exists x)Ax) \subseteq \mathcal{C}((\exists x)Ax \,\&\, Bx)$ (CR2)

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4. $\mathcal{C}((\exists x)Ax) \subseteq \{x\colon Ax\} \cap \{x\colon Bx\}$ (1,3)

If '~*Ax*' is again permitted to stand as a substitution instance for '*Bx*', then we have:

4'. $\mathcal{C}((\exists x)Ax) \subseteq \{x\colon Ax\} \cap \{x\colon \sim Ax\}$

But $\{x\colon Ax\} \cap \{x\colon \sim Ax\} = \varnothing$, the null set. Therefore, $\mathcal{C}((\exists x)Ax) = \varnothing$, no matter how atomic predicate '*A*' is interpreted, again in violation of (CR1).

Parsons' criticism may appear to be unfairly directed against a degenerate case. His objection to extensional theories of ontological commitment holds only if '~*Ax*' is substituted for '*Bx*' in the arguments he presents. In effect, this means that it applies only in the strange situation where it is necessary to determine the ontological commitments of theories that contain quantified logically self-contradictory sentences of the form '$((\exists x)Ax \,\&\, \sim Ax)$'. The criticism is easily avoided by an *ad hoc* restriction of the extensional principles to non-self-contradictory theories. This blocks Parsons' objection by refusing to permit the substitution of '~*Ax*' for '*Bx*' in the inference sequence.

3. Extensional Alternatives

Another approach that avoids Parsons' argument against Theory (2) is suggested by Mario Bunge. Parsons gives the commitment set of the existentially quantified conjunction $((\exists x)Ax \; \& \; {\sim}Ax)$ as an *intersection* of reference classes, $\mathcal{C}((\exists x)Ax \; \& \; {\sim}Ax) \subseteq \{x\colon Ax\} \cap \{x\colon {\sim}Ax\}$. In this example, the intersection is null. No matter how atomic predicate 'A' is interpreted, the commitment set of any existentially quantified sentence in the language is empty. This means that the commitment sets of all sentences of that form are identical. Bunge proposes an analysis of commitment reference class membership that is entirely insensitive to propositional connectives. His recommendation is that in every case the commitment reference class of a sentence be identified as the union, and never the intersection, of reference classes of the extensions of atomic predicates in the sentence. In a revision of Parsons' argument, this produces $\mathcal{C}((\exists x)Ax \; \& \; {\sim}Ax) \subseteq \{x\colon Ax\} \cup \{x\colon {\sim}Ax\}$, instead of $\mathcal{C}((\exists x)Ax \; \& \; {\sim}Ax) \subseteq \{x\colon Ax\} \cap \{x\colon {\sim}Ax\}$. The resulting union set is not null, but equivalent to the universal set, and there is no difficulty in accepting the conclusion that the commitment set of any existentially quantified sentence is a subset of or equivalent to the universal set, especially if the sentence is self-contradictory.

Bunge does not propose a new theory of ontological commitment, but tries instead to explain the relationships between logic and ontology. He offers a general reference class principle and applies it to tautologies in an effort to show that they may be ontologically neutral. But the union set analysis of reference class membership which he proposes might be adapted in the construction of an extensional theory of ontological commitment that avoids Parsons' objection to Theory (2). Bunge writes:

> Clearly, the denial of a statement does not change its reference class *even though it alters its truth value.* And if a second statement combines with the first *either disjunctively or conjunctively*, it contributes its own referents. That is, the reference function $\mathcal{R}$ is insensitive to the propositional connectives: $\mathcal{R}(\neg p) = \mathcal{R}(p)$, $\mathcal{R}(p \vee q) = \mathcal{R}(p \; \& \; q) = \mathcal{R}(p) \cup \mathcal{R}(q)$, for any propositions p and q. Similarly for predicates. Not surprisingly, $\mathcal{R}$ is also insensitive to the precise kind of quantifier.[4]

Bunge applies the reference function in two definitions.

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4 Mario Bunge, "The Relations of Logic and Semantics to Ontology" [1974], p. 201. (Emphasis added.)

DEFINITION 6. The reference class of a predicate is the set of its arguments. More precisely, let $\mathbb{P}$ be a family of n-ary predicates with domain $A_1 \times A_2 \times \ldots \times A_n$. The function

$$\mathcal{R}_{\mathrm{p}}: \mathbb{P} \rightarrow \rho(\cup_{1 \leq i \leq n} A_i)$$

from predicates to the set of subsets of the union of the cartesian factors of the domains of the former is called the *predicate reference function* iff it is defined for every P in $\mathbb{P}$, and its values are

$$\mathcal{R}_{\mathrm{p}}(P) = \cup_{1 \leq i \leq n} A_i.$$ [5]

In DEFINITION 7 (ii), he adds:

The reference class of an *arbitrary propositional compound* equals the *union* of the reference classes of its components. More precisely, if $s_1, s_2, \ldots, s_m$ are statements in S and if ω is an m-ary propositional operation, then

$$\mathcal{R}_s(\omega(s_1, s_2, \ldots, s_m)) = \cup_{1 \leq j \leq m} \mathcal{R}(s_j).$$ [6]

He further maintains that the union set formulation should be applied regardless of the kind of quantifiers occurring in the commitment sentence.

The reference class of a quantified formula equals the reference class of the predicate occurring in the formula. More explicitly, if P is an n-ary predicate in $\mathbb{P}$, and the Q_j (for $1 \leq i \leq n$) are arbitrary quantifiers,

$$\mathcal{R}_s((Q_1 x_1)(Q_2 x_2)\ldots(Q_n x_n) P x_1 x_2 \ldots x_n) = \mathcal{R}_{\mathrm{p}}(P).$$ [7]

The union set interpretation of reference class membership can be introduced in order to express the belief that in any commitment sentence ontological commitment is made to each and every reference class of entities in the extension of each and every predicate occurring within the scope of its quantifiers. Commitment in this sense can only be expressed as a union of reference classes, not an intersection.

But the adaptation of Bunge's proposal invites additional difficulties. It is possible to criticize the union set analysis of ontological commitment by formalizing the previous objection about theories and sentences with non-(existent object)-designating singular terms and extensionless predicates. For this it is necessary to consider another criterion like Parsons' (CR1).

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5 Ibid.

6 Ibid., p. 202. (Emphasis added.)

7 Ibid., pp. 202—3.

(CR3) $(\forall P)(\forall Q)[(P \neq Q) \supset \mathcal{C}((\exists x)Px) \neq \mathcal{C}((\exists x)Px \,\&\, Qx)]$

This says that if properties P and Q are distinct, then the commitment set of quantified sentences containing the predicate 'P' is not the same as the commitment set of quantified sentences containing predicates 'P' and 'Q'. The criterion is intuitively justified by the consideration that if property P is not the same as property Q, then the ontological commitment of a theory which entails just $(\exists x)Px$ may be a proper part of, but never strictly identical to, the ontological commitment of a theory which entails $(\exists x)(Px \,\&\, Qx)$. The latter must always include something more.[8]

The following argument applies to all Bungean extensional theories of ontological commitment based on the union set analysis of reference class membership. Substitution of identicals in extensional contexts is assumed. The principle (B) adapts Bunge's union set analysis of reference class membership to ontological commitment operators. It cannot be directly attributed to Bunge, but may be referred to as 'Bungean', since it is suggested by his formal ontology. An assumption is also made to the effect that for all sets, if a set is a subset of or equivalent to the null set, then it is equivalent to the null set.

1.	$P \neq Q$	
2.	$\{x: Px\} = \varnothing \,\&\, \{x: Qx\} = \varnothing$	
3.	$(\forall S)[(S \subseteq \varnothing) \supset (S = \varnothing)]$	
4.	$\mathcal{C}((\exists x)Px \,\&\, Qx) \subseteq \{x: Px\} \cup \{x: Qx\}$	(B)
	———————	
5.	$\{x: Px\} \cup \{x: Qx\} = \varnothing \cup \varnothing = \varnothing$	(2)
6.	$\mathcal{C}((\exists x)Px \,\&\, Qx) \subseteq \varnothing$	(4,5)
7.	$\mathcal{C}((\exists x)Px) \subseteq \varnothing$	(2,B)
8.	$\mathcal{C}((\exists x)Px \,\&\, Qx) = \varnothing$	(3,6)
9.	$\mathcal{C}((\exists x)Px) = \varnothing$	(3,7)
10.	$\mathcal{C}((\exists x)Px) = \mathcal{C}((\exists x)Px \,\&\, Qx)$	(8,9)
11.	$(P \neq Q) \supset \mathcal{C}((\exists x)Px) \neq \mathcal{C}((\exists x)Px \,\&\, Qx)$	(CR3)

[8] It might be objected that (CR3) makes an unwarranted distinction between the ontological commitments of sentences involving predicates that represent distinct properties, particularly where the properties are *a priori* knowably and necessarily coextensive. But even in such cases, the criterion may be defended as upholding the intuitively justified position that the ontological commitments of the theories are different. The comparison of alternative competing theories often depends on fine distinctions between their respective ontological commitments.

12. $\mathcal{C}((\exists x)Px) \neq \mathcal{C}((\exists x)Px \,\&\, Qx)$ (1,11)
13. $\mathcal{C}((\exists x)Px) = \mathcal{C}((\exists x)Px \,\&\, Qx) \,\&\, \mathcal{C}((\exists x)Px) \neq \mathcal{C}((\exists x)Px \,\&\, Qx)$ (10,12)

The assumption that property P is not the same as property Q and the assumption that 'P' and 'Q' are extensionless leads to a contradiction with criterion (CR3).

The contradiction might be said to show that P *is* the same property as Q if both 'P' and 'Q' are extensionless. But this has the counterintuitive consequences noted in the informal discussion of the objection. The property of being a man is intuitively different than the property of being a woman, even if there are no men and no women. It would be wrong to conclude that a theory is without ontological commitment just because it entails sentences that contain extensionless predicates. But this is the result when a Bungean union set analysis of ontological commitment is applied to propositionally compound quantified sentences with extensionless predicates.

Another problem in the Bungean extensional theory results from the insensitivity of the union set analysis to propositional connectives including negation. Let T and T^* be theories construed as sentences or conjunctions of sentences. Then we have:

1. $T \vdash (\exists x)Px$
2. $T^* \vdash \neg(\exists x)Px$

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3. $\mathcal{R}_s((\exists x)Px) = \mathcal{R}_p(P)$ (B)
4. $\mathcal{R}_s(\neg(\exists x)Px) = \mathcal{R}_s((\exists x)Px) = \mathcal{R}_p(P)$ (B)
5. $\mathcal{R}_s((\exists x)Px) = \mathcal{R}_s(\neg(\exists x)Px)$ (3,4)

This clearly implies in Parsons' notation that $\mathcal{C}(T) = \mathcal{C}(T^*)$. If a theory entails that there are no entities of kind P, then paradoxically on the Bungean extensional union set analysis of ontological commitment, the theory is ontologically committed to the existence of things of kind P. A theory that entails that there are no unicorns, for example, is ontologically committed to the existence of unicorns. This is drastically counterintuitive, and dissolves all the important distinctions between the ontological commitments of contrary and mutually contradictory theories. Although an extensional Bungean theory of ontological commitment avoids Parsons' criticism of Theory (2), it is unacceptable for more fundamental reasons.

4. Non-Object-Theoretical Intensional Methods

This dispenses with extensional theories. What about intentional and intensional alternatives? Chomsky, Scheffler, Cartwright, Jubien, and Parsons agree, for different but related reasons, that extensional theories of ontological commitment are unsatisfactory. They hold that only an intentional or intensional theory can avoid the difficulties encountered in extensional accounts. Parsons writes:

> The only possible conclusion, then, is that, even for syntactically atomic ϕ, there is no systematic relation between $\mathcal{C}((\exists x)\phi)$ and $\{x: \phi\}$. But this is tantamount to saying that $\mathcal{C}((\exists x)\phi)$ doesn't depend on any aspects of ϕ that fall within the domain of the theory of reference. The only recourse for a meaningful notion of ontological commitment is to move into the domain of the theory of meaning.[9]

The others reach similar conclusions, emphasizing the need to develop theories of ontological commitment along lines that are not purely extensional or referential, but in some sense intentional or intensional.

The trouble is finding the right kind of intentionality or intensionality required for an acceptable account of ontological commitment. Although Parsons has become an advocate of object theory intentionality, he does not explicitly draw connections between this and his earlier rejection of extensional theories of ontological commitment.[10] Chomsky and Scheffler, Cartwright, and Jubien maintain that a satisfactory theory of ontological commitment must be intentional or intensional in the sense that it make use of concepts from semantics and the theory of meaning, in agreement with Parsons' conclusion. But it can be shown that an intuitively correct account of ontological commitment must be intentional in the object theory sense, and not merely in the generic semantic or theory of meaning sense.

Jubien holds that Chomsky and Scheffler have mistakenly interpreted the failure of extensional theories of ontological commitment as due to referential opacity. He acknowledges the intensionality of ontological commitment, but attempts to find a comparatively harmless sense of its intensionality which would not be so blatantly at odds with the main lines of Quine's philosophy. He writes:

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9 Parsons, "Extensional Theories of Ontological Commitment", p. 450.

10 In *Nonexistent Objects*, Parsons does not even mention his previously published criticisms of extensional theories of ontological commitment.

> Having ruled out intentional objects[11] and there being no *other* objects possibly denoted by '*a*' [where '*a*' fails to designate an existent object], we can only conclude that there is nothing *but* the form of the name for the statement to depend on... We can only infer, then, that '*T* assumes *a*' expresses some relation between *T* and the *expression* '*a*' (R (*T*, '*a*')) or between *T* and a sentence in which '*a*' occurs essentially (such as '$\exists x(x = a)$'). It is easy to see, however, that a relation between *T* and such a sentence can readily be reinterpreted as a relation between *T* and the expression '*a*'.[12]

This sense of intensionality also fails to provide an adequate account of ontological commitment.

A satisfactory theory of ontological commitment must give an intuitively correct answer to the question, 'What thing or kind of thing is any given theory ontologically committed to?' Quine has repeatedly maintained that correct principles of ontological commitment should entail not what exists, but only what a particular theory *says* exists.[13] Extensional theories of ontological commitment fail because they do not yield intuitively correct answers to this question. In the case of false theories containing sentences with non-(existent object)-designating singular terms or extensionless predicates they entail that the theories have no ontological commitments at all, or give other more inventive but equally implausible answers. If a theory says that there are unicorns, then intuitively it is committed to the existence of unicorns — whether or not unicorns happen to exist — and not to the same thing or kind of thing as a theory which says that there are winged horses. That the two theories are not the same in ontological commitment is shown by the fact that they would be true by virtue of their respective ontological commitments in different logically possible worlds.

By this standard and in this respect, Jubien's intensional analysis of ontological commitment is no more satisfactory than the extensional theories he

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11 The only thing that 'rules out' intentional objects in Jubien's discussion is the fact that Quine has expressed suspicion and disapproval of them. Jubien has no arguments to offer against the object theory intentionality of ontological commitment. His purpose is to reconcile as well as possible the undeniable intensionality of ontological commitment with Quine's philosophy.

12 Michael Jubien, "The Intensionality of Ontological Commitemnt" [1972], pp. 384—85. See also, Noam Chomsky and Israel Scheffler, "What is Said to Be" [1958–1959]; Richard Cartwright, "Ontology and the Theory of Meaning" [1954], pp. 316 —25; Routley, *Exploring Meinong's Jungle and Beyond*, pp. 411—18; "On What There is Not" [1982], pp. 151—77.

13 W.V.O. Quine, "On What There Is" [1963], pp. 15—16. Quine, *Word and Object* [1960], pp. 119—20, 241—45. See Douglas Browning, "Quine and the Ontological Enterprise" [1973], p. 500.

criticizes. It seems just as wrong to answer the question what thing or kind of thing a theory about unicorns is ontologically committed to by saying that an ontological commitment relation relates the theory to linguistic entities or bits of language like the word 'unicorn', or expressions in which the non-(existent object)-designating term or extensionless predicate 'unicorn' essentially occurs. Yet Jubien's intensional theory of ontological commitment offers no other way to answer the question.

The criticism also applies to formally more sophisticated attempts to invoke semantic ascent to explain ontological commitment, such as B. M. Taylor's:

($\exists'$) Entities y which are members of $\{x\colon Ax\}$ are assumed by L iff '$\vdash_{SML}(\forall x)(Exp(x) \rightarrow (Pr(x) \rightarrow Tr(x))).\rightarrow.(\exists y)(P_D(y)\ \&\ y \in \{x\colon Ax\})$'

Here $Exp(x) =_{df}$ 'x is an expression', $Pr(x) =_{df}$ 'x is provable', $Tr(x) =_{df}$ 'x is true'; $P_D(y) =_{df}$ 'y is a value in the domain D', and $SML =_{df}$ 'the syntactical metalanguage of L'.[14] Trading on the use-mention distinction, and placing the entire expression in quotations is more complex than but affords no philosophical advantage or advance over Jubien's Quinean quotation-context semantic ascent approach.

A counterfactual analysis might be proposed, according to which theories T and T^* are ontologically committed to the non-Meinongian objects that *would* exist *if* the theories were true. Because of substitutivity failure in some modal contexts, a counterfactual interpretation of ontological commitment based on an underlying modal theory could also be described as intensional. The counterfactual proposal is unobjectionable when applied to theories like T or T^*, which are ontologically committed to contingently nonexistent objects. But it is inadequate in distinguishing between the ontological commitments of theories committed to impossible objects like the round square and the rectangular triangle, Hobbes' squaring the circle, or Frege's reduction of mathematics to logic (unless it is assumed that counterfactual conditionals with necessarily false antecedents are not trivially true).

There is yet another intensional interpretation of ontological commitment which should finally be considered. It might be thought that ontological commitment could be understood in terms of properties or concepts construed as intensional entities. The ontological commitment of a theory that

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[14] B. M. Taylor, *Universals and Predication* (unpublished thesis, University of Melbourne), p. 55; cited in Paul Gochet, *Ascent to Truth: A Critical Examination of Quine's Philosophy* [1986], pp. 74—5.

there are unicorns might then be explained as a relation obtaining between the theory or sentences of the theory and the property or concept of unicornicity (or, if unicorns are not a natural kind, the more basic properties or concepts of being equine and one-horned). But this is unacceptable for the same reason that justified rejection of the extensional and other non-object-theoretical intentional and intensional analyses of ontological commitment. Property and concept interpretations of ontological commitment equally fail satisfactorily to answer the question to what thing or kind of thing the proponents of these theories are ontologically committed.

It cannot reasonably be held that the theories are committed to the property or concept of unicornicity or of being equine and one-horned, because the theory might be radically nominalistic, or, say, behavioristic, or otherwise anti-conceptualistic. The idea of a property or concept might be entirely alien and antithetical to its ontology, and the corresponding predicate terms might have no place in its theoretical vocabulary. To adopt the property or concept account of ontological commitment would be automatically to saddle each and every scientific and philosophical theory with a realist or conceptualist ontology, even if the theory is specifically designed to avoid ontological commitments to properties or concepts.

5. Ontological Commitment and the Object Theory Rationale

In order to make intuitively correct sense of the ontological commitments of false theories that contain sentences with non-(existent object)-designating singular terms or extensionless predicates, some other more appropriate sense of 'intentionality' must be found. The only remaining alternative which presents itself is the sense of intentionality in which nonexistent Meinongian objects are introduced. If a Meinongian approach is adopted, then the commitment relation can be said to hold between sentences or theories and unique sets of existent or nonexistent Meinongian objects. This may be part of what Cartwright and others wish to affirm when they conclude that theories of ontological commitment must be made intentional or intensional by incorporating concepts from or moving into the domain of the theory of meaning.[15] But there are many theories of meaning, and most have nothing to do with Meinongian objects.

It is customary after Frege to distinguish at least two elements in the the-

[15] Cartwright, "Ontology and the Theory of Meaning", pp. 319—22. Parsons, "Extensional Theories of Ontological Commitment", p. 450.

ory of meaning. Thought or propositional content and reference or denotation are usually regarded as its parts (though problems of reference are sometimes entirely excluded from the theory of meaning). But reference to existent non-Meinongian objects alone has already been shown to be inadequate in determining the ontological commitments of some false theories. Thought or propositional content on the other hand does not help to answer the fundamental question for theories of ontological commitment, because not all theories are ontologically committed to the existence of thoughts or propositional contents, but to things like numbers, electrons, and unicorns. If thought is regarded as requiring reference to intentional objects as components, then it might be said that a satisfactory theory of ontological commitment could be intentional in the theory of meaning sense. But this is acceptable only if it implies that the theory is also intentional in the object theory of meaning sense.

If ontological commitment is intentional in the object theory sense, it is easy to see how the fundamental question for theories of ontological commitment can be answered. Here ontological commitment is not conveyed merely by quantification over objects, since the object theory domain contains both existent and nonexistent objects within the range of bound variables, but by extranuclear existence predications. The approach has an intuitive appeal and sense of rightness about it that is hard to discount, especially in light of the difficulties encountered by extensional and alternative intentional and intensional hypotheses. A theory about unicorns on this analysis turns out to be ontologically committed to the existence of unicorns, even though no unicorns exist. This provides a more powerful motivation for object theory logic than typical concerns about the meaning of fiction. It offers at once an intuitively satisfying way of explaining the ontological commitments of scientific theories to such strange nonexistent objects as phlogiston, the planet Vulcan, the ideal gas, the unimpeded projectile, the frictionless surface, the average brickmason, and infinitesimals in the metaphysics of the calculus.

VI. Meinong's Doctrine of the Modal Moment

1. The Annahmen Thesis

Meinong's object theory is built on three pillars. First, that thought is free to assume that there are such objects as a golden mountain or round square; second, that when these assumptions are entertained in thought they are directed toward the intentional objects the golden mountain and round square; and third, that the golden mountain is in fact golden and a mountain, and the round square round and square, even though neither the golden mountain nor the round square exist, subsist, or have any mode of being.[1]

Without all three principles, the round square and golden mountain cannot enter into Meinongian semantics. But in a sense the free assumption or *Annahmen* thesis is the most important, since the others depend on it as a source of phenomenological raw material. If the intentionality thesis were restricted, then thoughts ostensibly about the golden mountain and round square might not actually be directed toward these or any other intentional objects. If the independence thesis were limited, then although psychological presentations ostensibly about the round square and golden mountain might actually be about or directed toward the round square and golden mountain, the round square would not be round and square, nor the golden mountain golden and a mountain. Yet to restrict free assumption potentially eliminates nonexistent Meinongian objects from object theory semantics, even if the intentionality and independence theses have unrestricted application. If thought is not free to assume the golden mountain and round square, then incomplete and impossible nonexistent objects have no entry into the impoverished Meinongian domain. This gives freedom of assumption a certain priority in Meinong's object theory. Its unrestricted generality must be preserved against unnecessary incursions.

[1] Meinong, "The Theory of Objects". Chisholm, "Homeless Objects" and "Beyond Being and Nonbeing" [1982], pp. 37—67. Lambert, *Meinong and the Principle of Independence*, pp. 13—93.

2. Russell's Problem of the Existent Round Square

Whether or not Meinong's *Annahmen* thesis can be maintained without limitation, it was early recognized by the *gegenstandstheoretischen Philosophen* that the independence thesis cannot stand without significant qualification. Unrestricted free assumption makes it possible to entertain in thought ideas about the existent round square, the logically possible round square, and the necessarily complete round square, just as it does about the (plain, unadorned) round square. By parity of form it appears that if the independence thesis implies that the round square is round and square, then it ought also to imply that the existent round square is existent, round, and square. But since anything that is both round and square is nonexistent, the conclusion lands Meinong's theory in contradiction.

To solve the problem, the independence thesis is not so much restricted as reinterpreted to apply only to nuclear properties, and not to extranuclear properties like existence. The independence thesis must then be understood to say that the *Sosein* of an object has whatever nuclear properties (and not whatever extranuclear properties) are attributed to it in assumption, independently of its ontological status. Routley describes the role of the distinction between nuclear and extranuclear properties in the problem of the existent round square by insisting that extranuclear properties are not assumptible.

> The difference indicated, between properties which can be part of the nature of an object and those which cannot be (but which are, for instance, founded on the nature of the object), is consolidated in Mally's and Meinong's distinction between nuclear and extra-nuclear properties. Extra-nuclear properties, such as existence, determinateness and simplicity, are not, to put it bluntly, assumptible: the Characterisation Postulate [independence thesis] does not apply without important restriction where extra-nuclear properties figure.[2]

The existent round square is round and square, or has the nuclear properties of being round and square in its *Sosein.* But the existent round square is not existent. Nothing that it is round and square (everywhere at once) can exist. The round square is a metaphysically impossible necessarily nonexistent Meinongian object. The property of existence or existing is an extranuclear nonconstitutive property that is not assumptible and not part of the uniquely identifying or characterizing *Sosein* of any Meinongian object. Existence there-

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2 Routley, *Exploring Meinong's Jungle and Beyond*, p. 496.

fore does not fall under the province of the properly interpreted independence thesis in Meinong's object theory.

Russell in his essay "On Denoting" and review of Meinong's anthology *Untersuchungen zur Gegenstandstheorie und Psychologie*, advances the problem of the existent round square.[3] From previous argument it might appear that Meinong's reply to Russell's objection would be immediate. The *Sosein* of an object contains only the object's nuclear properties, and none of its nonassumptible extranuclear properties. Since existence is an extranuclear property, the existent round square does not have the property of being existent as part of its *Sosein.* The existent round square is therefore not existent. To assume that there is an existent round square is not to be directed in thought toward an existent object.

Yet this is only part of the answer Meinong gives to Russell's criticism. The core of his solution to the problem is to say that the existent round square is existent, even though it does not exist.[4] Russell claimed he was unable to make sense of this reply, and others have since interpreted Meinong's retort as a desperate reduction to absurdity of the object theory as a whole.[5] Meinong's response involves a technical distinction that in some ways drives an already complicated semantic philosophy from the baroque to the rococo.

Meinong maintains that for every extranuclear property there corresponds a 'watered-down' (*depotenzierte*) nuclear counterpart, deprived of 'full-strength factuality' because it lacks the 'modal moment' (*das Modalmoment*).[6] When Meinong answers Russell's objection by stating that the existent round square is existent even though it does not exist, he means that the *Sosein* of the existent round square includes a watered-down nuclear counterpart of the extranuclear property of existence, but that the existent round square does not exist because its surrogate nuclear existence property lacks the modal moment. This eliminates contradiction by introducing an equivocation in two senses of 'existence'. The implications of Meinong's doctrine of the modal

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3 Russell, "On Denoting", p. 45; "Review of A. Meinong, *Untersuchungen zur Gegenstandstheorie und Psychologie*".

4 Meinong, *Über die Stellung der Gegenstandstheorie im System der Wissenschaften*, pp. 16—7; *Über Möglichkeit und Wahrscheinlichkeit*, pp. 278—82.

5 Rudolf Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic* [1956], p. 65. Ryle, "Intentionality-Theory and the Nature of Thinking" [1970], p. 7.

6 Meinong, *Über Möglichkeit und Wahrscheinlichkeit*, p. 266. Meinong's modal moment is also supposed to contribute full-strength factuality to the truth or subsistence of objectives, propositions, or states of affairs, as well as to extranuclear properties or determinations. For simplicity, I have confined discussion to the modal moment of properties. The proposal to eliminate the modal moment from revisionary Meinongian object theory applies equally with appropriate qualifications to the modal moment of subsistent objectives.

moment, and the question of whether watering-down, full-strength factuality, and the modal moment are essential to Meinong's object theory must be critically examined.

3. Watering-Down

Findlay describes Meinong's doctrine of the modal moment in arithmetical terms.

> Meinong holds that there must be a factor, which he calls the *modal moment*, in which the difference between full-strength factuality and watered-down factuality consists. Full-strength factuality minus the modal moment yields watered-down factuality. Watered-down factuality plus the modal moment yields full-strength factuality.[7]

This suggests that the basic principles of Meinong's distinction can be represented algebraically to describe a problem about the absolute inability of the modal moment to be watered-down to a weakened nuclear counterpart.

Meinong permits a watered-down nuclear existence property to enter into the *Sosein* of the existent round square, since the surrogate existence property lacks the modal moment of full-strength factuality. An indefinitely ascending hierarchy of orders of watering-down is engendered if the modal moment itself is subject to watering-down. Russell's problem of the existent round square might then be reformulated as the problem of the existent-cum-modal-moment round square. If this assumption is not to posit an actually existent impossible object, then the property of existence-cum-modal-moment, and therefore the modal moment itself, must admit of watering-down in a successive ordering of strengths or modalities of factuality. The existent round square in that case may lack the modal moment order $i+1$, while the existent-cum-modal-moment round square has the watered-down modal moment i. From this an infinite regress follows. Terms designating the modal moment of any particular order can be indexed with an appropriate superscript or similar device to indicate their precise place in the hierarchy of ordered degrees of factuality. The problem of the existent-cum-modal-moment^{i} round square, for any factuality order i, is outdistanced by the problem of the existent-cum-modal-moment^{i+1} round square. For every kindred

[7] Findlay, *Meinong's Theory of Objects and Values*, pp. 103—4.

problem in the series, the existent-cum-modal-moment^{i} round square has the watered-down modal moment^{i} in its *Sosein*, but lacks the higher-order relatively extranuclear modal moment^{i+1}. The proposal therefore affords no final characterization of factuality or real existence. The concept is always just out of reach, limited by the possibility of further watering-down. This is the difficulty Findlay refers to as the second and third waves of Russellian objections to Meinong's theory.[8]

Meinong avoids the regress of orders of watering-down for possession of the modal moment by stipulating that the modal moment is exempt from watering-down. Findlay explains:

> Suppose I assume that the objective 2+2=5 has factuality *plus* the modal moment, then it is clear that I am assuming something more than that 2+2=5 has watered-down factuality. Shall we hold that the modal moment is itself capable of being watered down, that it too has a ghostly counterpart which requires a second modal moment to lend it full reality? It is clear that this path leads to the infinite regress; we should have an infinite series of strengthless modal moments, each appealing to another moment which was equally feeble.[9]
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> From this situation Meinong saves himself by holding that we *cannot*, by means of a judgment or assumption, attribute the modal moment to an objective which does not possess it.[10]

Meinong's solution to the regress problem involves an otherwise unwarranted restriction of the *Annahmen* or freedom of assumption thesis. Any proposition can be entertained in thought or held before the mind for consideration *except* the attribution of the modal moment to an object that does not have it.[11] This violates unlimited free assumption, but there may be no theoretically more acceptable alternative. The exception and its restriction on free assumption are upheld to prevent the watering-down regress. Meinong blocks the flood of infinite orders of watered-down modal moments by limiting assumption to properties other than possession of the modal moment. The modal moment is distinguished as unique in this regard, an absolutely extranuclear constant in a special category of its own. Despite its historical importance in Meinong's object theory, free assumption cannot be totally unrestricted, but must be appropriately qualified.

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8 Ibid., pp. 106—10.
9 Findlay, *Meinong's Theory of Objects and Values*, pp. 106—7.
10 Ibid.
11 Meinong, *Über Möglichkeit und Wahrscheinlichkeit*, p. 283.

4. Eliminating the Modal Moment

Meinong's doctrine of the modal moment is burdened by vague concepts and terminology. Parsons writes:

> Existing is not the only extranuclear property which has a watered-down version; being possible also has one. This tempts one to wonder if *all* extranuclear properties have nuclear watered-down versions. That will depend, of course, on what 'watered-down' means. In Meinong's theory it is not clear (at least to me). He speaks of the watered-down version of a property as got by removing the 'modal moment' from 'full-strength factuality'. I am not sure what this means.[12]

Admittedly, Meinong's doctrine is obscure. It must either be made precise, or eliminated in favor of some more satisfactory alternative. Meinong's object theory can be streamlined by dropping the modal moment and its accompanying machinery of full-strength factuality and watering-down. The following analysis eliminates the modal moment from Meinong's theory, and solves Russell's problem about the existent round square in a simpler and more economical way, with no restriction of the *Annahmen* or freedom of assumption thesis.

The fact that Meinong must draw the line against watering-down somewhere, holding the modal moment as uniquely outside the watering-down of extranuclear properties, suggests that the line might as well be drawn at the distinction between nuclear and extranuclear properties. The implication then is that no extranuclear property can be watered-down, and no surrogate or watered-down nuclear counterpart of any extranuclear property can enter into the *Sosein* of any object as a constitutive or identifying property. Instead, the distinction between nuclear and extranuclear properties is rigidly enforced. The answer to Russell's problem on this interpretation is that the existent round square is not existent, and that no object, even if existent, has the extranuclear property or any supposedly watered-down counterpart of any extranuclear property in its constitutive uniquely characterizing *Sosein*.[13]

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[12] Parsons, *Nonexistent Objects*, p. 44.

[13] Routley, *Exploring Meinong's Jungle and Beyond*, p. 496: "...logically important though the modal moment is, the property [nuclear/extranuclear] distinction alone, properly applied, is enough to meet all objections to theories of objects based on illegitimate appeals to the Characterisation Postulate [Independence of *Sosein* from *Sein*]. The Meinong whose theory includes an unrestricted Characterisation Postulate is accordingly, like Meinong the super-platonist, a mythological Meinong."

The modal moment is eliminated by answering Russell's objection in terms of the prior distinction between nuclear and extranuclear properties. To do so has several advantages. The proposal banishes the obscure concepts and terminology of the modal moment, watering-down, and full-strength factuality from the object theory and its vocabulary. The revised account relies instead on the more deeply entrenched and intuitively justified distinction between nuclear and extranuclear properties. The resulting theory is simpler, without the unnecessary complexities and Ptolemaic epicycles of watering-down. Eliminating the modal moment is also more economical. It is not clear whether there is only one modal moment, or whether each extranuclear property or watering-down of an extranuclear property requires its own distinct modal moment. The elimination of the modal moment reduces the high-level theoretical domain of the object theory by at least one element if there is only one modal moment, and by many more if each watering-down of an extranuclear property is supposed to have its own distinct modal moment. Nuclear and extranuclear properties are already in the object theory domain, so the elimination of the modal moment is not compromised by the compensating introduction of any other elements to do its work. If the modal moment must be held constant anyway, if it represents a point beyond which free assumption and watering-down is not permitted, then it is preferable to regard nuclear properties themselves as discrete points beyond which watering-down cannot occur.

It might be objected that eliminating the modal moment involves a trade-off of one kind of simplicity for another kind of complexity. Perhaps the advantage of Meinong's doctrine of the modal moment is that it provides a single focus for the restriction of free assumption and a concentrated barrier to regressive watering-down. The alternative proposal for eliminating the modal moment distributes these frontiers of restriction to the distinctions between nuclear and extranuclear properties. The elimination proposal may thus appear to sacrifice simplicity and unity of explanation.

The criticism has force only if it is assumed that Meinong's doctrine requires no more than a single modal moment to give full-strength factuality to each and every distinct extranuclear property. But this assumption is doubtful for a number of reasons. Meinong does not say whether there is just one modal moment or a pluralilty of distinct modal moments. It may turn out upon analysis that many modal moments are needed to solve Russell's problem and avoid regressive watering-down. If there are many modal moments, then there is no single focus advantage in retaining the modal moment as an essential part of object theory. But even if there is just one modal moment lending full-strength factuality to so many different and diverse kinds of extranuclear properties, the purported advantage of positing the modal moment as a single focus for the restriction of free assumption and regressive watering-down is outweighed by its disadvantages. It is preferable to halt the advance of internal inconsistency on many secure fronts than at a single insecure front. The simplicity expected of the elimination proposal is assured by the fact that it implements a single principle that no extranuclear property or watered-down nuclear counterpart of an extranuclear property can enter into the uniquely identifying constitutive nature or characterizing so-being of a Meinongian object. The applications of the principle are as numerous as the extranuclear properties, but the principle itself is simple and one.

5. Intentional Identity and Assumptive Generality in Meinong's Object Theory

The remaining advantage in eliminating the modal moment from Meinong's object theory is its compatibility with a completely unrestricted *Annahmen* or freedom of assumption thesis. It was observed at the outset that it is desirable to uphold unlimited free assumption without exception or qualification. Meinong seems to restrict free assumption by refusing to allow the intelligible assumption of the existent-cum-modal-moment round square, and forbidding the watering-down of the modal moment.

Findlay interprets Meinong's doctrine of the modal moment as limiting free assumption when he writes:

> ...the freedom of our assumptions is limited in one important respect: we cannot by any mental feat lift out of *Aussersein* a fact that two straight lines should enclose a space, *in which the modal moment is present.* The most fantastic and insane assumptions can present genuine objects, but the attempt to assume the presence of the modal moment where it is not present is necessarily abortive, and apprehends no object whatever.[14]

Reinhardt Grossmann similarly argues that Meinong requires the modal moment in order to preserve the *Annahmen* thesis. Since Grossmann finds the concept unsatisfactory, he concludes that Meinong's object theory is entangled in insuperable difficulties generated by Russell's problem of the existent round square.

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[14] Findlay, *Meinong's Theory of Objects and Values*, p. 107.

> Meinong ... introduced a distinction ... between existence and existing and claimed that the existing round square does indeed have the determination of existing, but does not exist. Why did Meinong simply not claim, in answer to Russell, that existence is so different from ordinary properties that Russell's argument breaks down on this account alone? Meinong subscribed also to the principle of unlimited freedom of assumption, so he had to admit that one can indeed conceive of an existing round square just as well as of a round square. He held, furthermore, that whatever one thus conceives of has all the features which it is conceived to have.[15]

> Meinong ... can solve the problem [of the existent round square] more easily, on the level of existence and factuality, if he is willing to restrict the principle of unlimited freedom of assumption.[16]

Grossmann's argument that Meinong needs the modal moment to salvage unlimited free assumption is obviously mistaken, since as Findlay testifies, the doctrine itself limits freedom of assumption by prohibiting the assumption and regressive watering-down of the existent-cum-modal-moment round square. The restriction of free assumption entailed by Meinong's doctrine of the modal moment may also threaten its proposed elimination. If thought is not free to assume the existent-cum-modal-moment round square, then if anything the limitation is compounded on the elimination proposal, according to which thought appears unable freely to assume the existent round square, the necessary round square, or the possible round square (all of which are permitted by the doctrine of the modal moment).

An interpretation of intentional identity conditions can be given under which the proposal to eliminate the modal moment does not in any way restrict, limit, or qualify the absolute generality of free assumption in Meinong's object theory. There is an important distinction between assuming an object by thinking of or making reference to its constitutive nuclear properties, and assuming the object and attributing additional nonconstitutive nuclear or extranuclear properties to it. These assumptions and superadded predications are sometimes grammatically indistinguishable, and analysis is required to sort them out before philosophical implications of the distinction can be appreciated. When this is done, fine-grained intentional identity conditions in the two kinds of constructions make it possible to avoid conflict between unrestricted freedom of assumption and the elimination of the modal moment.

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15 Grossmann, *Meinong*, p. 221.
16 Ibid., p. 222.

Thinking about the round square is undoubtedly different than thinking about the existent round square. But this does not mean that the existent round square is a different intentional object than the round square. Twardowski's distinction between the act, content, and object of psychological presentations supports an alternative analysis.[17] The content of an assumption about the round square is certainly different than the content of an assumption about the existent round square. The lived-through psychological experience of each of these assumptions is phenomenologically distinct. But the intentional object of the assumptions may be identical.

If the round square is intentionally identical with or the very same object of thought as the existent round square, then a question naturally arises about the role of the term 'existent' in the assumption of the existent round square. What purpose does it serve? In the assumption of the existent round square, the (plain, unadorned) round square is the intentional object to which the extranuclear property of existence is (falsely) predicated. The attributional nature of the assumption is grammatically obscured by the common form 'existent round square', which, like 'red round square', superficially appears to qualify the object in much the same way that makes the red round square a different intentional object than the (plain, unadorned) round square. A person thinking about the existent round square is thinking about or directed in thought toward the very same intentional object as a person thinking about or assuming the (plain, unadorned) round square. But a person thinking about the existent round square incorrectly attributes the extranuclear property of existence to the (necessarily nonexistent plain, unadorned) round square. This is why the contents of thought of the two kinds of assumptions are qualitatively different. An assumption about the round square is just an assumption about the round square, since being round and square are constitutive nuclear properties. But an assumption about the existent round square is an assumption about the round square *as* or *considered as* existent.

The disambiguation of assumption and superadded predication contexts is required even for certain nuclear predications. Without the analysis disagreements about the properties of intentional objects are unintelligible. If someone believes that the Parthenon is made of granite, and another believes that it is made of marble, then the two appear to have a substantive dis-

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[17] Twardowski, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen.* Meinong adopts Twardowski's phenomenological reduction of psychological experience to the act-content-object structure in his essay "Über Gegenstände höherer Ordnung und deren Verhältnis zur inneren Wahrnehmung", pp. 181—271. See also Meinong, "Über Inhalt und Gegenstand (Fragment aus dem Nachlass)" [1977], pp. 67—76.

agreement about the stone from which the temple was cut. But the difference of beliefs constitutes a genuine disagreement only if the beliefs are intentionally directed toward an identical object of thought, about which different properties are truly or falsely predicated. If the beliefs are about different Parthenons, disagreement is impossible. Since being made of granite or marble are nuclear properties, there is some risk of interpreting the intentional objects of these beliefs as distinct rather than identical. If the granite Parthenon is a different object of thought than the marble Parthenon, in the way that the red round square is a different intentional object than the blue round square, then the first putative disputant accepts the logically necessary proposition that the granite Parthenon is granite, and the second accepts the equally logically necessary proposition that the marble Parthenon is marble, leaving no room for contradiction. The hypothesis of distinct intentional objects in these beliefs also has the unacceptable consequence of implying that the propositions accepted by the supposed antagonists are logically necessary rather than contingent. But surely their opinions about the materials of which the Parthenon is built are contingent. The beliefs are incompatible, and at least one is false.

The remedy for these misinterpretations is to recognize that beliefs about the granite Parthenon and marble Parthenon are to be analyzed as intentionally directed toward one and the same Parthenon (the existent columned structure atop the Acropolis in Athens), to which the properties of being made of granite or marble are superadded as nonconstitutive nuclear predications. The solution is similar to that suggested for the false superadded extranuclear predication of existence to the round square in the analysis of thoughts ostensibly about the existent round square or existent-cum-modal-moment round square as thoughts about the (plain, unadorned) round square *as* existent or *as* existent-cum-modal-moment.

The elimination of the modal moment disarms Russell's objection without appeal to the concept of the modal moment, watering-down, or full-strength factuality. It accounts for the difference in thought between assumptions about the round square and existent round square as a difference in content rather than object. The difference in content is explained as the superadded predication or lack of superadded predication of extranuclear existence to an identical intentional object of thought. The analysis complements the elimination proposal in its reliance on a rigidly enforced distinction between nuclear and extranuclear properties, and in its refusal to countenance the watering-down of any extranuclear properties to assumptible constitutive surrogate nuclear counterparts. The important advantage resulting from this approach is that there is no violation, restriction, or qualification of the *Annahmen* or unlimited freedom of assumption thesis. There are no exceptions, absolutely no assumptive limit or boundary to thought. The existent round square can be assumed; it is just that on the present interpretation, to assume the existent round square is falsely to assume that the round square has the superadded extranuclear property of existence. The assumption is directed toward the same intentional object as the true assumption that the round square is round, square, or nonexistent. The analysis does not contradict free assumption because it in no way prohibits or precludes any intentional objects from being presented, posited, or entertained by the mind. The thoughts in question on this account are about or directed toward the same intentional object, despite the superaddition of denotatively superfluous nonconstitutive extranuclear predicates to their natural language designators. The independence thesis, that an object has whatever properties it is conceived to have, must be understood in an appropriately qualified way. The principle implies that an intentional object has whatever constitutive nuclear properties it is conceived to have, and not whatever superadded nonconstitutive properties whether nuclear or extranuclear.

I. Syntax, Formation and Inference Principles

1. The Logic

The formalization provides an exact interpretation of object theory logic and semantics. The system developed is ontologically neutral in areas where traditional logic is intolerably partisan. It is more encompassing and flexible than alternative standard and nonstandard logics of comparable power. The logic is paradox-free and untyped. It preserves intact almost all the notation and inferences of standard systems, replacing the usual propositional semantics with three truth values and a Meinongian domain of existent and nonexistent objects.

In Part One the basic requirements for an object theory logic were identified. It was shown that nonexistent objects must enter into semantic analysis in order to provide an intuitively correct interpretation of ontological commitment. The logic must formalize the nuclear-extranuclear property distinction, the *Annahmen* or unrestricted free assumption thesis, intentionality, independence, and indifference theses, without defective objects, the modal moment, full-strength factuality, or watering-down. The semantics of the system are three-valued to accommodate in the most straightforward way the nonstandard truth values of propositions attributing nuclear properties to incomplete nonexistent objects. This permits the logic to do justice to the fact that such propositions as 'The golden mountain is taller than Mt. Analogue', and 'The golden mountain is not taller than Mt. Analogue', are intuitively neither true nor false, but undetermined in truth value.

Quantificational semantics permit bound variables to range over an ontology and extraontology of existent and nonexistent Meinongian objects. The 'existential' quantifier with bound variables ranging over existent and nonexistent objects in the Meinongian domain cannot serve as a criterion of ontological commitment. Instead the test must be to examine canonical formulations of a theory for occurrences of extranuclear existence predications to determine which objects the theory claims to exist. This is implied by previous criticism of extensional theories of ontological commitment, in which the extensionalist slogan 'To be is to be the value of a bound variable' is rejected.[1] Additional machinery, such as the distinction between sentence negation and predicate complementation (sometimes known as external and internal negation), and the definitions of three kinds of identity, are introduced with appropriate justification in the formal exposition of the logic. The preceding discussion has prepared the way for development of the logic by clarifying its purpose, defining some of its inherent limitations, and setting a number of problems for it to solve. Paradoxes were described which the object theory logic must avoid to preserve internal consistency. The paradoxes are defeated by an existence restriction on abstraction, nonstandard three-valued truth value semantics, and correct application of the nuclear-extranuclear property distinction. In the logic it must be shown exactly how these solutions work.

The underlying philosophy of logic supports a system that avoids antinomy and preserves the intuitive expectations of naive mathematical languages prior to the discovery of formal diagonalization techniques.[2] Logic cannot depend on a preferred ontology, nor on any merely contingent facts about the world, or particular science or metaphysics, since it must mediate between and impartially represent the formal structure of incompatible scientific and metaphysical theories. In the *Philosophical Investigations*, Wittgenstein writes: "[Logic] is *prior* to all experience, must run through all experience; no empirical cloudiness or uncertainty can be allowed to affect it — It must rather be of the purest crystal."[3] Meinongian object theory uniquely satisfies this ideal. No alternative approach offers the required distance from empirical contingencies about actual existents while addressing such a wide range of pretheoretical semantic data. The logic aspires to nothing less than a framework for the complete consistent formal characterization of thought and language.

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[1] Quine, "On What There Is", p. 13; *Word and Object*, pp. 192—93.

[2] The philosophy of logic and mathematics developed here is most closely connected to Wittgenstein's. The paradoxes of logic, semantics, and set theory, and the metatheoretical limitations of mathematical languages demonstrated by diagonalization techniques, are problems to be overcome, not inevitable obstacles to completeness and consistency of systematic thought that must be lived with and from which we must draw pessimistic conclusions about the limits of reason and knowledge.

[3] Wittgenstein, *Philosophical Investigations* [1958], § 97, p. 44[e]. See also, Wittgenstein, *Remarks on the Foundations of Mathematics* [1956], V, § 36, p. 186[e].

2. Syntax

Object theory O contains the following items of primitive syntax.

1. propositional negation: '$\sim$'
2. predicate complementation: '$\overline{}$'
3. conditional: '$\supset$'
4. universal quantification: '$\forall$'
5. object variable: 'x'
6. object constant: 'o'
7. predicates: 'P^n' (nuclear); '$P^n!$' (extranuclear) ('$P^n(!)$' denotes an n-ary nuclear or extranuclear property indifferently; in practice the superscript is implicit)
8. functor: 'f^n'
9. denumerable variable, constant, predicate, and functor indices:
   '1', '2', '3', ...
10. punctuation, including:
    parentheses: '(', ')'
    square brackets: '[', ']'
    curly brackets: '{', '}'
    comma: ','
    ellipsis: '...'
    blank space: ' '

In addition to the primitive syntax, defined expressions, including connectives, predicates, functors, and operators, are introduced as abbreviations or particular applications of more complex expressions in the primitive syntax. In principle, the primitive syntax can supply all needed variables, constants, predicates, and functors, by subscripting natural number indices to the 'x', 'o', '$P(!)$', and 'f' expressions, as in: 'x_1', 'x_2', 'x_3',...; 'o_1', 'o_2', 'o_3',...; '$P(!)_1$', '$P(!)_2$', '$P(!)_3$'...; and 'f_1', 'f_2', 'f_3',... But in practice it is more convenient to use variables like 'x', 'y', 'z', predicates like 'P ', '$Q!$', '$R(!)$', or those with mnemonic significance, and function symbols from traditional logical and mathematical notations, such as '$\in$', '$\leq$', '$\neq$'.

Object constants are given special interpretations for convenience and readability. In particular, an object constant o_i $(1 \leq i \leq n)$ may be chosen to represent an arbitrary proposition, though in practice the familiar propositional variables 'p', 'q', and 'r' are used. Three object constants are chosen to represent the truth values 'T', 'F', and 'U'. They are understood as the values true, false, undetermined. All such conventions are translatable into the official primitive syntax.

3. Formation Principles

The terms of object theory *O* are recursively inductively defined as follows.

1. The $x_1, \ldots, x_n, \ldots$ are terms.
2. The $o_1, \ldots, o_n, \ldots$ are terms.
3. If f^n is *n*-ary, then $f^n(o_1, \ldots, o_n)$ is a term.
4. Nothing else is a term (closure).

The well-formed formulas or wffs of object theory *O* are recursively inductively defined as follows.

1. If $P^n(!)$ is *n*-ary, then $P^n(!)o_1 \ldots o_n$ is a wff.
2. If $P^n(!)$ is *n*-ary, then $\bar{P}^n(!)o_1 \ldots o_n$ is a wff.
3. If p is a wff, then $\sim p$ is a wff.
4. If p and q are wffs, then $p \supset q$ is a wff.
5. If p is a wff, then $(\forall x)p$ is a wff.
6. Nothing else is a wff (closure).

The following connectives and operators are introduced as abbreviations for the primitive connectives and operators to which they are logically equivalent.

E1	$p \,\&\, q$	for	$\sim((\sim p \supset \sim q) \supset \sim q)$
E2	$p \vee q$	for	$\sim(\sim p \,\&\, \sim q)$
E3	$p \equiv q$	for	$(p \supset q) \,\&\, (q \supset p)$
E4	$(\exists x)p$	for	$\sim(\forall x)\sim p$

In later developments, the formation principles are expanded to incorporate modality, definite description, and lambda abstraction.

4. Inference Principles

A proof in the logic is a consecutively numbered sequence of well-formed formulas. The inferences are justified by analogues of familiar axioms and rules of propositional and predicate logic.

The intent of inference structures is to permit the deduction from true assumptions of true conclusions only, and to prevent the deduction of false or undetermined conclusions. For present purposes, it will suffice to advance a number of logical axioms generally recognized as deductively valid, from which derived inference principles can be obtained. The turnstile '$\vdash$'

is used to indicate the valid deduction of a conclusion q from logical and nonlogical axioms, definitions, and theorems $p_1, \ldots, p_n$ in $p_1, \ldots, p_n \vdash q$. This means that it is deductively valid to infer q from $p_1, \ldots, p_n$. In a proof this is written:

$$\begin{array}{rll} 1. & p_1 & \\ & \vdots & \\ n. & p_n & \\ \hline n+1. & q & (1-n) \end{array}$$

The turnstile ($\vdash$) is a metatheoretical device, a (non-truth-functional) function from propositions to propositions. When the turnstile appears with a proposition to its right but none to its left, it indicates that the proposition is unconditionally true, a theorem or tautology of the logic.

The primitive axioms are presented as inference schemata, uniform substitution instances of which may enter into any proof. Inference is by uniform substitution of propositions for propositional variables, and a rule of detachment, *modus ponendo ponens*.

A1 $p \supset (q \supset p)$
A2 $(p \supset q) \supset ((q \supset r) \supset (p \supset r))$
A3 $(\sim p \supset p) \supset p$
A4 $(\sim p \supset \sim q) \supset (q \supset p)$

A5 $(\forall x_1) \ldots (\forall x_n) P^n(!)x_1 \ldots x_n \vdash P^n(!)x_i / o_i$ (similarly for $\ldots \overline{\mathrm{P}}^n(!) \ldots$)
(selectively substituting constants 'o_i' for variables 'x_i')

A6 $P^n(!)x_i / o_i \vdash (\forall x_1) \ldots (\forall x_n) P^n(!)x_1 \ldots x_n$ (similarly for $\ldots \overline{\mathrm{P}}^n(!) \ldots$)
(where 'x_i' does not occur free in '$P^n(!)x_i / o_i$', '$\overline{\mathrm{P}}^n(!)x_i / o_i$')

A7 $(\forall x_1) \ldots (\forall x_n)(p \supset q) \vdash (p \supset (\forall x_1) \ldots (\forall x_n) q)$
(where 'x_i' does not occur free in 'p')

A8 $(\forall x_1) \ldots (\forall x_n)(P^n(!)x_1 \ldots x_n \equiv \overline{\overline{P}}^n(!)x_1 \ldots x_n)$

The logic is type-unordered, but distinguishes between propositional negation and predicate complementation. This requires the special double complementation principle in (A8). The axiom states that an arbitrary predication of given length involving the complement of the complement of a property is equivalent to a predication of the same length of the property itself. If it is not the case that an object has a nuclear property, it does not follow that the object has the complement of the property. If an object has the complement of a nuclear property, it does not follow that the object does not have the property. The importance of these restrictions in maintaining syntactical consistency is appreciated when the logic of incomplete and impossible objects is presented.

II. Semantics

1. Intended Interpretation

The intended interpretation of the logic can be described by explaining the meaning of each term and well-formed formula. The semantics divides into two parts. If p, q are wffs of O, then truth matrix interpretations are provided for negation in $\sim p$, and implication in $p \supset q$. If p is of the form $P^n(!)o_1 \ldots o_n$ or $(\forall x)p$, it is interpreted in a semantic model, expressed as an ordered pair of reference class designators, or as an equivalent function on the Cartesian product of objects in designated reference classes.

The most important primitive semantic relation is designation, not of existents only, but of any object or set of objects in the semantic domain of existent and nonexistent Meinongian objects. The dictum attributed to Quine that there can be 'No entity without identity'[1] is expanded to provide the more inclusive principle that there can be no entity *or* nonentity without identity. Nonexistent objects have *Sosein* identity conditions specifiable as identities of unordered sets of nuclear properties. Existent and nonexistent objects alike are distinguishable elements of the Meinongian semantic domain, and can therefore stand as the values of bound variables (which in Meinongian quantificational logic carry no real existential or ontological import). This removes philosophical obstacles to the claim that nonexistent objects are designatable.

The domain or universe of discourse includes a wide variety of objects, existent as well as incomplete and impossible nonexistent objects. Meinong identified the major divisions among kinds of intentional objects, but he did not recognize all interesting subdivisions. These can be mapped as intersections, unions, and exclusions of partitioned reference classes of nonexistent objects.

There are three truth values in the logic's formal semantics. They are required because intentional discourse sometimes attributes properties to in-

[1] The thesis is ascribed to Quine by Charles D. Parsons in "Frege's Theory of Number" [1965], p. 182. See Quine, *Word and Object*, pp. 200—5; "Ontological Relativity" [1969], pp. 32—4, 45—6.

complete objects in predications that are intuitively neither true nor false. A third truth value must therefore be introduced for expressions in the logic undetermined in truth value. The golden mountain is neither taller than nor not taller than René Daumal's Mt. Analogue. Macbeth neither spoke nor was unable to speak Italian. Truth values are distinguished as special objects of the semantic domain, and an identity predicate is used to express identity relationships between truth values and truth value functions applied to propositions.[2]

2. Formal Semantics

Let p, q be any well-formed formulas. Three objects in the semantic domain are designated as truth values, symbolized in the usual way as 'T' for 'true', 'F' for 'false', and 'U' for 'undetermined'. A function V is defined for wffs of the logic that gives the truth value of any wff taken as argument. If $V(p)=T$ then $V(\sim p)=F$; if $V(p)=T$ and $V(q)=F$, then $V(p\supset q)=F$; $V(p)=U$ if and only if $V(\sim p)=U$. The complete truth functional relations for negation and the conditional are represented in truth tables.

[2] Routley, *Exploring Meinong's Jungle and Beyond*, p. 170: "It is objects, in the first place, that are incomplete with respect to certain features, though this may be reflected back into incompleteness of statements." Terence Parsons for simplicity sake formalizes Meinongian object theory within standard bivalent truth value semantics. See his *Nonexistent Objects*, p. 116: "Once we allow truth-value gaps, then all kinds of complications creep in. For example, we need to say what happens to complex sentences whose parts lack truth value, and there is no concensus as to how to do this. My enterprise is complicated enough without burdening the reader with additional complications such as these." But the complications may have to be endured in order to provide an adequate formal object theory, and the lack of consensus should not deter logicians from seeking a workable answer. Although Parsons wants to restrict his theory to a bivalent semantics, he argues in several places that incomplete objects are by definition lacking in some nuclear properties and their complements. Thus, on p. 182 Parsons observes in an example that seems to be inspired by John Woods' *The Logic of Fiction* [1974], that Sherlock Holmes neither has nor fails to have a mole on his back, since the Arthur Conan Doyle stories are silent on this point. But then the sentence 'Sherlock Holmes has a mole on his back', the complement predication 'Sherlock Holmes does not have a mole on his back', and their negations 'It is not the case that Sherlock Holmes has a mole on his back' and 'It is not the case that Sherlock Holmes does not have a mole on his back', are intuitively neither true nor false but undetermined in truth value.

p	$\sim q$
T	F
F	T
U	U

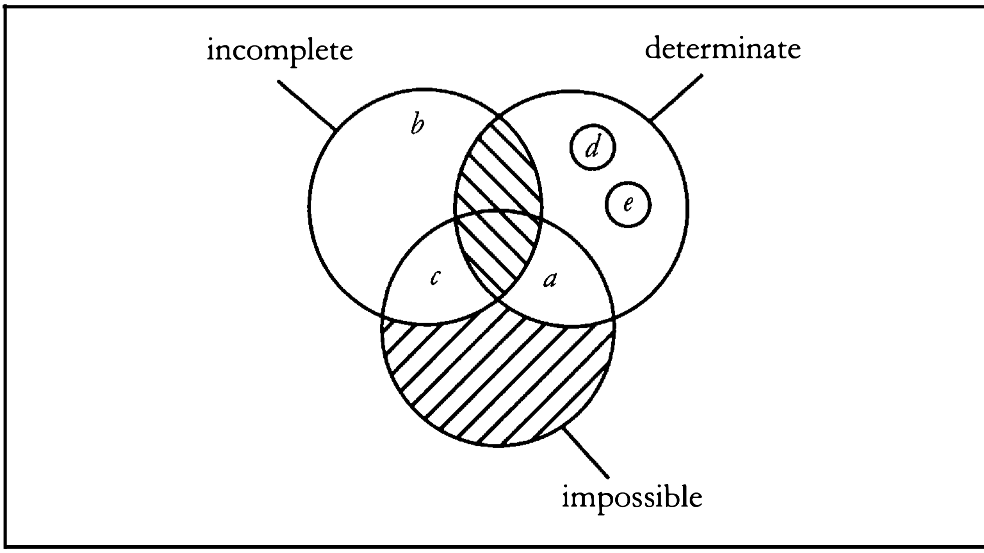
p	q	$p \supset q$
T	T	T
T	F	F
T	U	U
F	T	T
F	F	T
F	U	T
U	T	T
U	F	U
U	U	T

The undetermined truth value *U* is preserved through negation and through conjunction with anything that has truth value *T*. It is natural to suppose that if a wff is undetermined in truth value, then its negation is also undetermined. If it is undetermined that the golden mountain is taller than Mt. Analogue, then it is also undetermined that the golden mountain is not taller than Mt. Analogue. If one conjunct of a conjunction is true and the others undetermined, then the conjunction as a whole cannot unqualifiedly be said to be true or false. The undetermined truth value is not preserved through conjunction with any proposition that has truth value *F*, since a conjunction is rightly regarded as false if even one of its conjuncts is false. The consequences of these evaluations for the conditional are recorded in the truth tables. The propositional semantics are identical to the trivalent system proposed by Jan Łukasiewicz.[3]

Predicate expressions of the form $P^n(!)o_1 \ldots o_n$ and complementary form $\bar{P}^n(!)o_1 \ldots o_n$ are interpreted by an extension of the truth value function.[4] It is necessary to describe a domain of existent and nonexistent objects. Among nonexistent objects are impossible, incomplete, and impossible incomplete objects, overdetermined objects, and a maximally impossible object. These are represented diagrammatically:

[3] Jan Łukasiewicz, "Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls" [1930], pp. 51—77 (translation in Storrs McCall, editor, Polish Logic: 1920—1939 [1967], pp. 40—65). See Nicholas Rescher, Many-Valued Logic [1969], p. 23. The truth tables are similar to those presented by Stephen Cole Kleene, Introduction to Metamathematics [1952], pp. 334–35. The difference is that for Łukasiewicz, when V(p) = U, V($p \supset p$) = T; whereas for Kleene, when V(p) = U, V($p \supset p$) = U.

[4] The distinction between propositional negation and predicate complementation enables the logic to make intuitively correct sense of the metaphysically incompatible nuclear properties of impossible objects without compromising the system's soundness or consistency. Parsons



In this hybrid Venn-Euler diagram, intentional objects are found within every sphere, and no objects occur outside the interlocking spheres. Clearly, no objects belong to the intersection of determinate and incomplete objects, nor to the domain of neither incomplete nor determinate impossible objects, and in the diagram this is indicated by hatching. Object *a* is the maximally impossible object which has every nuclear constitutive property and its complement. Object *b* is the (incomplete fictional) golden mountain, and *c* the (impossible) round square. Subsets *d* and *e* might distinguish Meinong's ontological categories of existent and subsistent determinate objects, though the distinction need not be observed.

To interpret '$P^n(!)o_1 \ldots o_n$' and '$\overline{P}^n(!)o_1 \ldots o_n$', a semantic model is provided that can be directly transcribed from the logical syntax. The model consists of three ordered components: a domain D, an interpretation I on the domain, and a truth valuation of propositions under the interpretation V, $<D, I, V>$. D is the Meinongian domain of existent and nonexistent intentional objects, $D = \{o_1, o_2, o_3, \ldots\}$. Every term designates an existent or nonexistent object, by the *Annahmen* or unrestricted free assumption thesis. Do-

in *Nonexistent Objects* pp. 104—5 offers the distinction only to introduce '$\overline{P}$' as the negation of P. But on p. 105, he requires that $\Box(x)(\overline{P}x \equiv {\sim}Px)$. Despite this superficial similarity of notation, Parsons does not include in his logic anything corresponding to the distinction between propositional negation and predicate complementation. Routley acknowledges the need to draw the distinction in *Exploring Meinong's Jungle and Beyond*, pp. 89—92, 192—97. He posits a distinction between 'internal' and 'external' negation ($x{\sim}f/{\sim}xf$), which is later elaborated into a distinction between sentence negation and predicate (extended to property and attribute) negation.

main D thus contains an object corresponding to every grammatically well-defined term, constant, predicate, functor, definite descriptor, and lambda abstract. Some of the objects are sets of objects, where every condition on any objects determines a set.

A nonstandard version of Zermelo-Fraenkel set theory drives the semantics. Meinongian set membership, inclusion and exclusion, are nuclear relational properties in atomic predications undetermined in truth value. The philosophical justification for this is that incomplete objects like Pegasus intuitively are undetermined with respect to nuclear properties not included in their so-beings. The proposition that Pegasus is blue-eyed is neither true nor false, so that the corresponding membership statement that Pegasus $\in$ {blue-eyed things} is also undetermined.

The classical set identity axiom states, for sets φ, ψ, membership '$\in$' undefined:

$$(\forall\varphi)(\forall\psi)[\varphi = \psi \equiv (\forall x)(x \in \varphi \equiv x \in \psi)]$$

The principle cannot be introduced without modification in trivalent semantics, since it can happen that it is true or false that an object is a member of set φ but undetermined that the object is a member of set ψ, or the reverse. The truth matrix in that case dictates that it is also undetermined that $\varphi = \psi$, though intuitively identity unlike set membership is a bivalent extranuclear relation.

There are several ways of refining the standard extension axiom for set identity to preserve bivalence of identity for three-valued membership. The most straightforward method is to require that membership of an object in sets φ and ψ always has the same truth value when and only when set identity holds. The amended set identity axiom states:

$$(\forall\varphi)(\forall\psi)(\varphi = \psi \equiv (\forall x)[(x \in \varphi \equiv x \in \psi) \,\&\, V(x \in \varphi) = V(x \in \psi)])$$

This assures that identity predications for sets are always true or false and never undetermined, precluding identity when it is true or false that an object is a member of φ but undetermined that it is a member of ψ, or the reverse.

The principle may appear to be circular by its explicit reference to the valuations of membership predications in the second conjunct, since the truth values of predicate expressions are about to be interpreted in terms of set theoretical operations on the domain of objects. Vicious circularity is avoided by the consideration that interpretations of predicate expressions in the logic presuppose but do not explicitly involve set identities. There is no more circularity in the nonstandard set identity axiom than in standard applications of set theory in the semantics of predicate logic, where predicate and quantificational expressions are interpreted in set theoretical models, and the axioms of set theory are themselves expressed as predicate or quantificational expressions. The loophole here as in standard semantics lies in a hierarchy of object and metalanguages. The semantics are given in a metalanguage to interpret predicate expressions in a subordinate object language; the metalanguage is expressed by means of its own higher-order object language, interpreted in an even higher-order metametalanguage.

The appearance of circularity in the nonstandard set identity axiom is entirely avoided by introducing a new propositional operator '$\Leftrightarrow$' according to the following matrix, collapsing three truth values to two:

Strong Equivalence

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
T	U	F
F	T	F
F	F	T
F	U	F
U	T	F
U	F	F
U	U	T

This makes it possible to redefine the Meinongian set identity axiom without hint of predicate valuation circularity.

$$(\forall\varphi)(\forall\psi)[\varphi = \psi \equiv (\forall x)(x \in \varphi \Leftrightarrow x \in \psi)]$$

The complete axioms of the Meinongian set theory underlying the predicate semantics can now be given; again for any sets φ, ψ, membership '$\in$' and the null set '$\emptyset$' undefined.

Concept of Set

$$(\forall x)[Set(x) \equiv (\exists y)(y \in x \vee x = \emptyset)]$$

Nuclear (Unrestricted) Comprehension

$$(\exists\varphi)(\forall P)(\forall x)(x \in \varphi \equiv Px)$$

Meinongian Set Identity

$$(\forall\varphi)(\forall\psi)(\varphi = \psi \equiv (\forall x)[(x \in \varphi \equiv x \in \psi) \,\&\, V(x \in \varphi) = V(x \in \psi)])$$
$$(\forall\varphi)(\forall\psi)[\varphi = \psi \equiv (\forall x)(x \in \varphi \Leftrightarrow x \in \psi)]$$

Subsets

$$(\forall x)(\forall P(!))(\exists\varphi)[x \in \varphi \equiv P(!)x \,\&\, (\exists y)(y \in x)]$$

Pairing

$$(\forall\varphi)(\forall\psi)(\exists x)(\exists y)[(x \in \varphi \,\&\, y \in \psi) \supset (\exists\gamma)(\varphi \in \gamma \,\&\, \psi \in \gamma)]$$

Union

$$(\forall\varphi)(\exists\psi)(\exists x)(\forall y)[(x \in \varphi \,\&\, y \in x) \supset y \in \psi]$$

Power Set

$$(\forall\varphi)(\exists\psi)(\forall x)(x \in \psi \equiv x \subseteq \varphi)$$

Infinity

$$(\exists\varphi)[\varnothing \in \varphi \,\&\, (\forall x)(x \in \varphi \supset x \cup \{x\} \in \varphi)]$$

Choice

$$(\forall\varphi)(\exists f)(D(f) = \{x \mid x \supseteq \varphi \,\&\, x \neq \varnothing\} \,\&\, (\forall\psi)[(\psi \subseteq \varphi \,\&\, \psi \neq \varnothing) \supset f(\psi) \in \psi])$$

(*Replacement* and *Restriction* axioms are excluded as optional to set theoretical semantics.)

The unrestricted set determination principle associates with every nuclear or extranuclear property of objects a set in the Meinongian domain. The principle is philosophically justified as the set theory counterpart of the *Annahmen* or unrestricted free assumption thesis in Meinongian object theory. If assumption is free, then it can freely postulate sets as objects of thought corresponding to any condition on objects. The naive determination of sets is standardly subject to set theory paradoxes involving self-non-membership diagonalizations. But these are neutralized in Meinongian set theory and logic by restrictions on set abstraction, as described in Chapter III, Section 9 on 'Meinongian Mathematics and Metamathematics'.

The naive comprehension principle or unrestricted determination of sets and the fact that Meinongian domain D is itself an object, implies that D as the set of all objects is also (harmlessly) one of the objects and sets of objects in D. The set of all sets in D, the power set of D, $\mathcal{P}(D)$, is itself an object, and so must also belong to D. This entails by comprehension and the power set axiom that D contains its own power set; the power set of D is therefore a member and subset of D, $\mathcal{P}(D) \in D$ & $\mathcal{P}(D) \subseteq D$.

The predicate semantics for the object language of the logic can now be set theoretically defined. The function I is an interpretation that assigns objects as elements of the domain to items of syntax, terms, functors, predicates, descriptors, and abstracts. To each constant ‘o_i’ of O, I assigns an object in D. I(‘o_i’) $= o_i$ indicates that I assigns object o_i to object constant ‘o_i’. To each n-ary functor or function symbol ‘$f^n{}_i$’ of O, I assigns an n-ary function from D to D, I(‘$f^n{}_i$’) $= f^n{}_i$. To each n-ary predicate ‘$P^n(!)_i$’ of O, I assigns an n-ary property $P^n(!)_i$, I(‘$P^n(!)_i$’) $= P^n(!)_i$. More precisely, I assigns to each n-ary predicate ‘$P^n(!)_i$’ of O a set of ordered n-tuples of the objects in D, such that I(‘$P^n(!)_i$’) $= \{<o_{i_1},\ldots,o_{i_n}>, <o_{j_1},\ldots,o_{j_n}>,\ldots\}$. Similarly for ‘$\bar{P}^n(!)$’.

This provides an intensional counterpart of the identity of a property with the existent and nonexistent objects in the *intensions* of corresponding predicates, parallel to the identity of a property with the existent objects in the *extensions* of corresponding predicates in standard extensional semantics. The intension of a predicate is non-Fregean, though indirectly related to Frege's property ‘senses’ by virtue of the identity conditions for objects, determined by their *Soseine* or associated unordered sets of nuclear constitutive properties.

Function V is a truth valuation on the well-formed formulas of O, which gives a unique truth value for every proposition as interpreted under I for D. The role of truth valuation function V for the propositional fragment of the logic has already been described. The valuations for predicate or quantificational theory can be explained by means of I(‘$P^n(!)_i$’), I(‘$\bar{P}^n(!)_i$’), and I(‘o_i’), as previously defined.

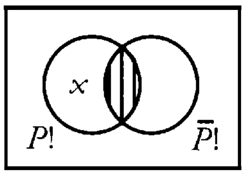
1. $V(P^n(!)o_1\ldots o_n) = T \equiv <I(‘o_1’),\ldots,I(‘o_n’)> \in I(‘P^n(!)’)$.
   $V(P^n(!)o_1\ldots o_n) = F \equiv <I(‘o_1’),\ldots,I(‘o_n’)> \notin I(‘P^n(!)’)$.
   $V(P^no_1\ldots o_n) = U \sim(<I(‘o_1’),\ldots,I(‘o_n’)> \in I(‘P^n’)$ &
   $<I(‘o_1’),\ldots,I(‘o_n’)> \notin I(‘P^n’))$ (for nuclear properties only).

2. $V(\bar{P}^n(!)o_1\ldots o_n) = T \equiv <I(‘o_1’),\ldots,I(‘o_n’)> \in I(‘\bar{P}^n(!)’)$.
   $V(\bar{P}^n(!)o_1\ldots o_n) = F \equiv <I(‘o_1’),\ldots,I(‘o_n’)> \notin I(‘\bar{P}^n(!)’)$.
   $V(\bar{P}^no_1\ldots o_n) = U \equiv \sim(<I(‘o_1’),\ldots,I(‘o_n’)> \in I(‘\bar{P}^n’)$ &
   $<I(‘o_1’),\ldots,I(‘o_n’)> \notin I(‘\bar{P}^n’))$ (for nuclear properties only).

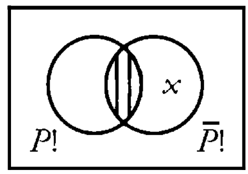
3. $V(P^no_1\ldots o_n) = U \equiv (V(\sim P^no_1\ldots o_n) = U$ & $V(\bar{P}^no_1\ldots o_n) = U$ &
   $V(\sim\bar{P}^no_1\ldots o_n) = U)$ (for nuclear properties only).

The valuations for nuclear predications are diagrammed below. The extranuclear predications are classical, so their corresponding diagrams are standard with closed intersections that permit no membership outside the interlocking circles.

EXTRANUCLEAR PREDICATIONS

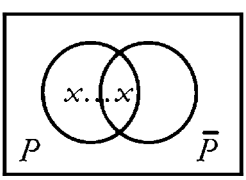


$V(P^n!o_1 \ldots o_n) = T$
$V(\sim P^n!o_1 \ldots o_n) = F$
$V(\bar{P}^n!o_1 \ldots o_n) = F$
$V(\sim \bar{P}^n!o_1 \ldots o_n) = T$

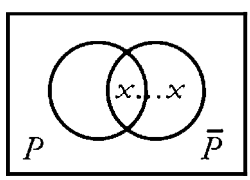


$V(\bar{P}^n!o_1 \ldots o_n) = T$
$V(\sim P^n!o_1 \ldots o_n) = T$
$V(P^n!o_1 \ldots o_n) = F$
$V(\sim \bar{P}^n!o_1 \ldots o_n) = F$

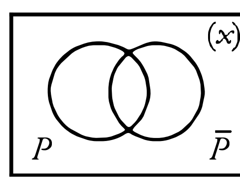
NUCLEAR PREDICATIONS



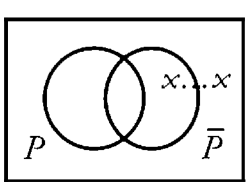
$V(P^n o_1 \ldots o_n) = T$
$V(\sim P^n o_1 \ldots o_n) = F$



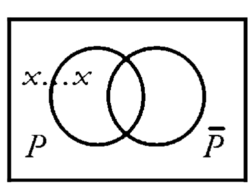
$V(\bar{P}^n o_1 \ldots o_n) = T$
$V(\sim \bar{P}^n o_1 \ldots o_n) = F$



$V(P^n o_1 \ldots o_n) = U$
$V(\bar{P}^n o_1 \ldots o_n) = U$
$V(\sim P^n o_1 \ldots o_n) = U$
$V(\sim \bar{P}^n o_1 \ldots o_n) = U$



$V(P^n o_1 \ldots o_n) = F$
$V(\sim P^n o_1 \ldots o_n) = T$



$V(\bar{P}^n o_1 \ldots o_n) = F$
$V(\sim \bar{P}^n o_1 \ldots o_n) = T$

The diagrams for nuclear predications distinguish the intensions of property P and its complement $\bar{P}$. Since some impossible objects have both property P and its complement in their *Soseine*, the spheres that represent the intensions of properties P and $\bar{P}$, or, the set of all existent and nonexistent objects with properties P or $\bar{P}$, are interlocking or overlapping. Unlike the previous diagram for the entire domain of O, in these diagrams objects can occur outside both spheres, to allow for incomplete objects lacking property P and its complement. The token 'x' indicates the location, inclusion or exclusion, of the ordered n-tuple of objects in $<o_1,\ldots,o_n>$ in or from or with respect to the intensions of predicates 'P' and '$\bar{P}$'. The use of '$x\ldots x$', as in Venn diagrams, indicates that $<o_1,\ldots,o_n>$ may be in either of the intensions where either 'x' in '$x\ldots x$' occurs. This enables a single diagram to represent several possibilities for the inclusion or exclusion of objects in or from or with respect to the intensions of predicates with the same truth valuational consequences. In the third diagram, '(x)' indicates that $<o_1,\ldots,o_n>$ is neither included in nor excluded from either the intension of nuclear predicate 'P' or the intension of its complement '$\bar{P}$'. This means that it is undetermined with respect to inclusion in or exclusion from the intensions of either predicate or complement, or, in other words, from the sets of existent and nonexistent objects with the corresponding properties.

4. $V((\forall x)p) = T$ just in case for every truth valuation in $<D,I,V>$, $V(p) = T$. $V((\forall x)p = F$ just in case there is at least one truth valuation in $<D,I,V>$ such that $V(p) = F$. $V((\forall x)p) = U$ just in case there is at least one truth valuation in $<D,I,V>$ such that $V(p) = U$, and there is no truth valuation in $<D,I,V>$ such that $V(p) = F$.

There is no formal contradiction in $\bar{P}^n(!)o_1\ldots o_n$ & $\sim P^n(!)o_1\ldots o_n$, $\sim\bar{P}^n o_1\ldots o_n$ & $\sim P^n o_1\ldots o_n$, or $P^n o_1\ldots o_n$ & $\sim\bar{P}^n o_1\ldots o_n$, just as there is no formal contradiction in $P^n o_1\ldots o_n$ & $\bar{P}^n o_1\ldots o_n$. By the semantics of sentence negation and predicate complementation, it is invalid to infer either conjunct from the other.

3. *Validity*

Validity of deductive inference is defined by cases. The valid inference patterns are:

$$
\begin{array}{lll}
T & \Rightarrow & T \\
F & \Rightarrow & F \\
F & \Rightarrow & T \\
F & \Rightarrow & U \\
U & \Rightarrow & T \\
U & \Rightarrow & U
\end{array}
$$

The remaining possible constructible combinations of truth values yield formally invalid inference patterns.

$$
\begin{array}{lll}
T & \not\Rightarrow & F \\
T & \not\Rightarrow & U \\
U & \not\Rightarrow & F
\end{array}
$$

A proposition p is also said to be valid, a theorem or tautology of the logic, $\vdash p$, if and only if, for every semantic model $<D,I,V>$, $V(p) = T$.

Comparison with the truth matrix for the conditional shows an exact correspondence between valid inferences and true conditionals, and invalid inferences and false or undetermined conditionals. This makes it possible to prove a nonstandard version of the Deduction Theorem, offered in Chapter III, Section 10, on 'Consistency, Completeness, Compactness'.

It is important to see that the logic is internally determinate, and that although some propositions are logically true or logically false, no propositions of the theory are logically undetermined. The only possible source of undetermined truth value assignments is in extralogical scientific or philosophical applications of the logic. These cannot occur except when nuclear properties are predicated of nonexistent incomplete objects lacking both the nuclear properties and the complements of the nuclear properties attributed to them. This means that the logic considered in itself and independently of its applications, and the extranuclear component of the logic, is classically bivalent, and never includes propositions undetermined in truth value. The axioms and theorems of the logic are logically true, and are validity-preserving even in extralogical applications, in the sense that instantiating any of the axioms with undetermined nuclear predications never leads from true assumptions to false or undetermined conclusions, but produces true or undetermined conclusions from undetermined premises. An axiom or theorem of the logic is always logically true, the extranuclear fragment is classical, and derivations from the logic are internally logically true and validity-preserving within the logic and in all formal extralogical applications.

4. Ambiguity and Translation from Ordinary Language

Formal semantic principles define the meaning of well-formed expressions in the logic, but there are difficulties in translating statements from natural languages into the symbolic notation. If it is said that all horses are quadrupeds, this may mean that all existent horses are actual quadrupeds, all existent and fictional horses are actual or fictional quadrupeds, all impossible and fictional horses are impossible quadrupeds, and so on. The truth value of the quantified formula is clearly at stake in these alternative interpretations, for while all existing horses might be quadrupeds, some fictional ones need not be. The ambiguities can be multiplied for all the various kinds and combinations of kinds of objects and properties, so that the total number of senses that may be intended by the subject and predicate terms in natural language is indefinitely large. This makes it important to settle philosophical ambiguities before translating natural language statements into the formal symbolism of Meinongian logic.

The range of object theory interpretations within ordinary language is suggested by a few examples. Unqualified predicates are understood as ambiguous with respect to any of the several kinds of objects indicated.

All horses are quadrupeds.
All horses are actual quadrupeds.
All horses are fictional quadrupeds.
All horses are impossible quadrupeds.
All horses are maximally impossible quadrupeds.
All horses are overdetermined quadrupeds.
All existent horses are quadrupeds.
All existent horses are actual quadrupeds.
All existent horses are fictional quadrupeds.
All existent horses are impossible quadrupeds.
.
.
.
etc.

Further combinations, such as 'All fictional horses are quadrupeds', 'All impossible horses are fictional quadrupeds', and the like, must also be considered. The truth value of each is determined in the semantic model by the inclusion or exclusion of corresponding intensional reference classes uniquely designated by and associated with appropriately disambiguated object and predicate terms.

'All horses are quadrupeds' would have value F, ignoring mutations and mutilations of naturally four-legged horses, because some incomplete and impossible horses are not quadrupeds. 'All existent horses are actual quadrupeds', on the other hand, again discounting mutations and mutilations, would have value T, since the reference class or intension of all existent horses is contained in the reference class or intension of all actual quadrupeds. 'All fictional horses are quadrupeds' is subject to further ambiguity. If it is understood to express a quantification over all fictional horses as incomplete objects, none of whose properties other than being equine are specified, then the statement has value U. But if it is understood to express a quantification over all fictional horses, including quadrupedal and nonquadrupedal fictional horses, then the generalization has value F by an easily imagined counterexample. 'All maximally impossible horses are quadrupeds' (non-quadrupeds, impossible quadrupeds, or impossible nonquadrupeds) unqualifiedly has truth-value T, since the maximally impossible horse has all nuclear properties and their complements, including the properties and complements of properties mentioned. 'All horses are maximally impossible impossible quadrupeds' has value F by an obvious counterexample.

The semantic principles are adequate for all of these intuitive interpretations. The careful disambiguation of natural language expressions into canonical and derived notations is not a task for the logic itself.

III. Developments of the Logic

1. *Nuclear and Extranuclear Properties*

The distinction between nuclear and extranuclear properties is represented in the notation as a distinction between 'P' and '$P!$' predicates. Reference to nuclear or extranuclear properties indifferently is indicated by placing the exclamation in parentheses, '$P(!)$'. Quantification over nuclear or extranuclear properties is indicated by expressions of the form:

$$\ldots(\forall x)\ldots(\forall P^n(!))\ldots(\ldots P^n(!)x_1,\ldots,x_n\ldots)\ldots$$

The predications are higher-order in appearance only. The logic is type-unordered, and propositions of the kind are instances of bounded quantification, convenient abbreviations for the more explicitly type-unordered $\ldots(\forall x)\ldots(\forall y(!))\ldots(\ldots y(!)x\ldots)\ldots$ The leveling of types is appropriate in Meinongian logic because properties like anything that can be thought of are also intentional objects.

Nuclear P properties and extranuclear $P!$ properties are distinguished by the following critera:

(C1) $\sim(\forall x_1)\ldots(\forall x_n)(\forall P^n)(\sim P^n x_1,\ldots,x_n \equiv \bar{P}^n x_1,\ldots,x_n)$

(C2) $(\forall x_1)\ldots(\forall x_n)(\forall P^n!)(\sim P^n!x_1,\ldots,x_n \equiv \bar{P}^n!x_1,\ldots,x_n)$

These can also be called the negation/counterpart-complementation criteria for the nuclear-extranuclear property distinction. It would clearly suffice to rely on (C1) or (C2) alone, categorizing extranuclear properties as properties that do not satisfy (C1), or nuclear properties as properties that do not satisfy (C2).

The criteria mark an important distinction between nuclear and extranuclear properties. Nuclear property and property complement pairs can fail to hold of incomplete or indeterminate nonexistent objects. Hume's golden mountain (to belabor a familiar example) is neither taller than nor not taller than Daumal's Mt. Analogue. Criterion (C1) reflects the fact that the property of being taller than Mt. Analogue is nuclear rather than extranuclear by denying the logical equivalence of sentence negation and nuclear predicate complementation counterpart expressions.[1] To say that the round square is nonsquare (because it is round) is *not* to say that it is not the case that the round square is square (since by the independence thesis it is both round *and* square).

Extranuclear properties on the contrary do not permit this latitude even when predicated of nonexistent incomplete indeterminate or impossible objects. An object, whether determinate or indeterminate, is either existent or not existent, possible or impossible, self-identical or not self-identical, with no middle ground or room for extranuclear incompleteness or indeterminacy. This means, as (C2) requires, that extranuclear predicate complementation and sentence negation counterparts are logically equivalent and extensionally intersubstitutable. For this reason, the extranuclear subtheory of Meinongian logic, unlike the object theory of nuclear predications, is classically bivalent and subject to excluded middle. Nuclear constitutive properties alone belong to the distinguishing or uniquely individuating *Sosein* of an object, to the absolute exclusion of any extranuclear properties.

Criteria (C1) and (C2) by themselves do not determine which properties are nuclear and which extranuclear. The criteria state distinguishing features of the two kinds of properties, and in application require additional information about whether or not the complement of a particular property predicated of any object is logically equivalent to its sentence negation counterpart. An independent informal criterion for the distinction can be provided to determine intuitively which properties satisfy (C1) rather than (C2), and which (C2) rather than (C1). A property is extranuclear on this conception if and only if it is definable in terms of logical operators and uninterpreted predicate symbols alone. It is nuclear if and only if it is not extranuclear, or if it is not definable in this way, but requires for its definition the interpreta-

[1] In *Nonexistent Objects*, pp. 168—69, Parsons relegates comparatives to the category of extranuclear properties. He acknowledges that this goes somewhat against the grain of his own informal characterization of the nuclear-extranuclear distinction, but he accepts the hypothesis to preserve bivalence. The intuitive argument for the classification is that relations between existent and nonexistent objects are problematic, as when we consider whether Parsons is taller than Hercule Poirot (in Agatha Christie's novels). Parsons attributes the problem to the fact that he nowhere appears in the stories. But if Parsons is, say, six-feet tall, and Poirot is consistently described as five-feet-ten-inches, then presumably there would be no difficulty in establishing the truth of the comparative. This suggests that Parsons' theory lacks a general intuitive foundation for distinguishing trans-ontic-categorical comparatives as nuclear or extranuclear. Parsons' proposal, with its appearance of *ad hoc* adjustment in any case does not solve the problem of nonbivalence for noncomparatives in predications of monadic nuclear properties or their complements to incomplete objects for which they are indeterminate. A contextualist solution to these problems (inspired by Routley) is presented in Part Three, Chapter V on 'Aesthetics and Meinongian Logic of Fiction'.

tion of at least some predicate symbols. Examples of the distinction are given in the following section.

2. Definitions

(D1) $E!o_i =_{df}$ 'Object o_i *exists*'
$(\forall x)(E!x \equiv (\forall P)[(Px \vee \overline{P}x) \; \& \; \sim(Px \; \& \; \overline{P}x)])$

This is a definition of the extranuclear existence property $E!$, and is intended to subsume both of Meinong's existence and subsistence categories. To say that an object exists means only that the object is complete or determinate and possible.[2] When Meinong provocatively maintains that "There are objects of which it is true that there are no such objects,"[3] his assertion can be formalized on the basis of this definition as $(\exists x)\overline{E}!x$.

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[2] Routley, *Exploring Meinong's Jungle and Beyond*, pp. 420—21, 720—26, challenges the adequacy of determinate consistency as a definition of existence. He argues that some existents like waves, forests, clouds, and mountain ranges are indeterminate with respect to at least some properties, since they cannot unambiguously be counted or identified. The objection is answered by distinguishing indeterminacy of concept from the closure and determinacy of existent objects. See Meinong, *Über Möglichkeit und Wahrscheinlichkeit*, p. 171: "If we say that an object A is determined in respect of an object B when we are entitled to say either that it is B or that it is not B, then 'something blue' is undetermined in respect of extension, and the principle embodied in the law of *excluded middle* (which, as we saw, *holds in the case of all that is actual or subsistent*), a principle by virtue of which every object must be determined in respect of every object, has no longer a justifiable application to the object 'something blue in the abstract'." (Emphasis added.) See ibid., p. 169. Findlay, *Meinong's Theory of Objects and Values*, p. 57: "All the objects in the actual world are fully determined, and we can pass beyond the determinations which we know to others which we do not know"; p. 206: "The objects that exist or subsist are all...completely determined in every respect...". It would be possible to distinguish formally as Meinong does informally between existence and subsistence. For present purposes, there is not much more to be done with the distinction when it is drawn because existence and subsistence are mutually exclusive and logically unrelated except as alternative modes of being. We could therefore define the general concept of being in this way: $(\forall x)(B!x \equiv ((E!x \vee S!x)) \; \& \; \sim(E!x \; \& \; S!x)))$. But unless the distinction between existence and subsistence is required for nuanced metaphysical reasoning in the ontology of physical or spatio-temporal versus abstract objects, there is no pressing need in object theory logic to observe the distinction. I have accordingly followed the expedient of other intensional logicians of using '$E!$' for predications of being generally, ignoring the existence-subsistence distinction. Where the distinction makes a difference, I would use '$B!$' for being generally, '$E!$' exclusively for spatio-temporal being or existence in the narrow sense, and '$S!$' exclusively for subsistence. The formal theory developed here makes no application of the distinction, consistently treating the two modes of being as one.

[3] Meinong, "The Theory of Objects", p. 83. See above, Part One, Chapter II, note 8.

(D2) $\mathit{Det}!o_i =_{df}$ 'Object o_i is *determinate*'
$(\forall x)(\mathit{Det}!x \equiv (\forall P)(Px \vee \overline{P}x))$

A determinate object has every nuclear property or its complement, and in this sense is not incomplete with respect to any nuclear property or property complement pair.

(D3) $\mathit{Inc}!o_i =_{df}$ 'Object o_i is *incomplete*'
$(\forall x)(\mathit{Inc}!x \equiv (\exists P){\sim}(Px \vee \overline{P}x))$

An incomplete object fails to have some nuclear property or its complement, and is therefore not determinate with respect to every nuclear property and property complement pair.

(D4) $\mathit{Imp}!o_i =_{df}$ 'Object o_i is *impossible*'
$(\forall x)(\mathit{Imp}!x \equiv (\exists P)(Px \;\&\; \overline{P}x))$

An impossible object has both at least one nuclear property and its complement. The occurrence of impossible objects in the object theory domain explains the need to distinguish between sentence negation and predicate complementation, for without it impossible objects would imply outright logical inconsistency. The round square may be round and not round. But this does not mean both that it is round and it is not the case that it is round. To say that a Meinongian object is impossible is not to say that it both has and does not have a particular nuclear property; it is only to say that the object has some metaphysically incompatible combination of nuclear properties.

(D5) $\mathit{Pinc}!o_i =_{df}$ 'Object o_i is *possible and incomplete*'
$(\forall x)(\mathit{Pinc}!x \equiv [{\sim}(\exists P)(Px \;\&\; \overline{P}x) \;\&\; (\exists P){\sim}(Px \vee \overline{P}x)])$

(D6) $\mathit{Iinc}!o_i =_{df}$ 'Object o_i is *impossible and incomplete*'
$(\forall x)(\mathit{Iinc}!x \equiv [(\exists P)(Px \;\&\; \overline{P}x) \;\&\; (\exists P){\sim}(Px \vee \overline{P}x)])$

(D7) $\mathit{Maximp}!o_i =_{df}$ 'Object o_i is *maximally impossible*'
$(\forall x)(\mathit{Maximp}!x \equiv (\forall P)(Px \;\&\; \overline{P}x))$

The maximally impossible object has every nuclear property and its complement. Meinong does not mention an object of this kind, but it seems entitled to a place in his taxonomy of nonexistent objects. In the section on 'Theorems' it is demonstrated (T42), (T43), that there is just one, unique maximally impossible object.

(D8) $Odet!o_i =_{df}$ 'Object o_i is *overdetermined*'
$(\forall x)(Odet!x \equiv [(\forall P)(Px \vee \bar{P}x) \; \& \; (\exists P)(Px \; \& \; \bar{P}x)])$

An overdetermined object is determinate and impossible. Not all impossible objects are determinate, since some like the round square are also incomplete. An overdetermined object is a determinate object to the *Sosein* of which at least one nuclear property or property complement is added.[4]

(D9) $o_i =_i o_j =_{df}$ 'Object o_i is *intentionally identical* to object o_j'
$(\forall x)(\forall y)[(x =_i y) \equiv (\forall P)(Px \equiv Py)]$

For any objects x and y, x is intentionally identical to y if and only if, for all nuclear properties P, x has P if and only if y has P. This principle is sometimes known as Leibniz's Law, or the identity of indiscernibles and indiscernibility of identicals. If $o_i =_i o_j$, then o_i and o_j have all nuclear properties in common, including converse intentional properties. Identity is an extranuclear relation which either holds or does not hold exclusively and without possibility of indeterminacy. Since the identity sign is already a distinguished predicate, it is convenient to drop the exclamation mark indicating that identity is an extranuclear relation that would otherwise be part of its official formulation, and in what follows it is understood that identity is always extranuclear.

It is standardly said that if psychological or converse intentional properties are included in the reference class of properties within the range of property quantifiers in Leibniz's Law, then entities preanalytically extensionally identical turn out to be intentionally nonidentical. This provides the basis for a distinction between intentional, referential, and extensional identity. (If there is a philosophical objection to distinguishing three kinds of *identity*, the distinction may alternatively be described as a distinction between (unrestricted Leibnizian or intentional) identity, and referential and extensional *codesignation*.)

The intentional identity conditions are very strong. Many objects that satisfy the requirements for extensional or referential identity fail to be intentionally identical. Cicero $=_i$ Cicero, but Cicero $\neq_i$ Marcus Tully. Intentional identities obey an unrestricted version of Leibniz's Law, and may therefore be intersubstituted in any linguistic context, intentional or nonintentional, without change or loss of truth value.

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[4] The term 'overdetermined' was suggested to me by Chisholm.

(D10) $Cint!(P_i) =_{df}$ 'P_i is a *converse intentional property*'
$(\forall P)Cint!(P) \equiv (\exists x)(\forall y)[(x \neq_i y) \equiv \sim(Px \,\&\, Py)]$

Converse intentional properties include being believed by Socrates, feared by Plato, worshipped by Aristotle. The definition trades on the fact that intentionally nonidentical but otherwise indiscernible existent or nonexistent objects are distinguished only by their lack of shared converse intentional properties.

If the principle were false, then intentionally distinct objects like the Evening Star and Morning Star would nevertheless share every converse intentional property, so that counterfactually every intelligence would have the same intentional attitudes toward them, or at least some intentionally identical objects would fail to have all converse intentional properties in common, contrary to the definition of intentional identity. The definition can be used to define referentially identical objects.

(D11) $o_i =_{rf} o_j =_{df}$ 'Object o_i is *referentially identical* to object o_j'
$(\forall x)(\forall y)[(x =_{rf} y) \equiv (\forall P)(\overline{Cint}!(P) \supset (Px \equiv Py))]$

The definition says that objects are referentially identical if and only if they share all non-converse-intentional nuclear properties. Cicero and Marcus Tully are referentially identical because they share all non-converse-intentional nuclear properties. But by definition (D10) they are intentionally nonidentical if one has a converse intentional property the other lacks, as when Cicero is believed while Tully is not believed by someone to have denounced Cataline.

(D12) $o_i =_e o_j =_{df}$ 'Object o_i is *extensionally identical* to object o_j'
$(\forall x)(\forall y)[(x =_e y) \equiv [(E!x \,\&\, (x =_{rf} y)) \vee (\overline{E}!x \,\&\, \overline{E}!y)]]$

The definition says that objects are extensionally identical if and only if either the objects exist and are referentially identical, or are nonexistent. This entails that Cicero and Marcus Tully are extensionally identical, even as the objects of thoughts in which incompatible properties are attributed to them, since Cicero exists and is referentially identical to Tully.

An arbitrary nonexistent impossible or incomplete object is extensionally identical with any other. This accords with the standardly accepted view that non-(existent object)-designating terms and extensionless predicates are extensionally identical. It happens that Cicero and Tully are what might be called significantly extensionally identical. But it would be mistaken to conclude that the objects of all incompatible thoughts in some sense about the same object are significantly extensionally identical though intentionally nonidentical. Superman and Clark Kent are intentionally nonidentical, since one is believed to be an alien from the doomed planet Krypton, while the other is not. But although they are extensionally identical, they are not significantly extensionally identical, since, as nonexistent fictional objects, Superman and Clark Kent are also extensionally identical to Sherlock Holmes, the golden mountain, and the round square. The definition of referential identity makes it possible to say that incompatible thoughts about Cicero and Marcus Tully are about the same object in the sense that they are about or directed toward a referentially identical object, whether or not the object exists. But for objects to be significantly extensionally identical, they must either also be intentionally identical, or existent rather than nonexistent. In the section on 'Theorems' it is shown (T34) that whatever objects are intentionally identical are also extensionally identical, but not conversely (T35).

Although extensional identity is the kind most often at issue in mathematics and science, referential and even intentional identity is equally important in determining as precisely as possible the object of a scientist's or philosopher's thought, or the exact intent of a legal document. We may want to draw as close as we can to the very same object of thought as the one we are trying to understand, and for this extensional, and sometimes even referential, identity conditions are insufficient. Existent objects alone nontrivially satisfy extensional identity principles, and these are often the things we are concerned with. The previously undifferentiated '=' associated especially with interpretation I and truth-valuation function V of model $<D,I,V>$ may now be interpreted either as extensional or referential identity, depending on whether a realist assumption about the existence of abstract properties is accepted. The disjunctive definition of referential identity accommodates both and leaves open the philosophical question of ontic realism or nominalism. When utmost ontological neutrality is required, '$=_{rf}$' is explicitly adopted. The undistinguished identity operator '=' is therefore used throughout as a convenient, more readable abbreviation for extranuclear referential identity in '$=_{rf}$'. The triad of identity relations '$=_i$', '$=_e$', and '$=_{rf}$' is explicit only when the difference between them must be emphasized, when the distinction is put to work.

(D13) $S(o_i) = \{P_1, \ldots, P_n, \ldots\} =_{df}$ 'The *Sosein* of object o_i is referentially identical to the unordered set of nuclear properties $\{P_1, \ldots, P_n, \ldots\}$' $(\forall x)(\forall P_1) \ldots (\forall P_n) \ldots (S(x) = \{P_1, \ldots, P_n, \ldots\} \equiv (P_1 x \,\&\ldots\&\, P_n x \,\&\ldots))$

This says that the *Sosein* or distinguishing or uniquely identifying set of an object's nuclear constitutive properties is referentially identical to an unordered collection of nuclear properties if and only if the object has all and only the nuclear properties in the set (if and only if they are all truly predicated of or attributed to the object).

In the section on 'Theorems' it is proved (T38) that if the *Soseine* of objects are referentially identical, then the objects are intentionally identical, but not conversely (T39). Well-formed identity expressions involving the *Sosein* function do not have undetermined truth value when the function takes incomplete or indeterminate nonexistent objects as arguments, because identity is always extranuclear and therefore bivalent. The *Sosein* of an incomplete object does not satisfy the closure condition, $(\forall x)(\forall P)(S(x) = \{\ldots P \ldots\} \vee S(x) = \{\ldots \overline{P} \ldots\})$. But by virtue of the classical bivalence of the extranuclear identity subtheory, it *does* satisfy the consistency or excluded middle condition, $(\forall x)(\forall P)(S(x) = \{\ldots P \ldots\} \vee S(x) \neq \{\ldots P \ldots\})$.

3. *Nonlogical Axioms*

(N1) Every term designates an object.
$(\forall x)[(\ldots x \ldots) \supset O!x]$

This axiom introduces the primitive extranuclear property $O!$ of being an object or belonging to the Meinongian domain of existent and nonexistent objects. The principle derives from Russell's syntactical interpretation of Meinong's *Annahmen* or free assumption thesis.[5] The principle implements unrestricted assumption; any term in any well-formed context can enter into an assumption, so that every term designates an object.

(N2) $(\forall x)(x =_i x)$

(N3) $(\forall x)(\forall y)([(x =_i y) \;\&\; (\ldots x \ldots)] \supset (\ldots y \ldots))$

These are identity postulates. They state (N2) that every object is intentionally self-identical, and (N3) that if objects are intentionally identical, then they may be validly intersubstituted in any propositional context. Additional identity axioms for extensional and referential identity relations (N8)—(N11) are provided in the following section on 'Theorems'.

(N4) $(\forall x)(\exists P!)P!x$

(N5) $(\forall x)(\exists P)Px$

(N6) $(\exists x)Inc!x$

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[5] Russell, "On Denoting", p. 45.

These are domain postulates. They state (N4) that every object has at least one extranuclear property; (N5) that every object (in accord with the independence of *Sosein* from *Sein* thesis) has at least one nuclear property; and (N6) that the domain includes an incomplete object. No postulate about the inclusion of an existent object $(\exists x)E!x$ is needed or made. Axiom (N6) is expendable if an incomplete object is introduced via axiom (N1) from extralogical discourse. The *Sosein* function applied to it would then reveal it to be incomplete, and existential generalization would produce the equivalent of (N6). The same is true of another useful axiom.

(N7) $(\exists x)Maximp!x$

4. *Theorems*

The axioms, definitions, and nonlogical axioms make it possible to derive a number of interesting theorems. Some of these have to do with the further formal characterization of logical interrelationships between various kinds of objects. The axioms support a natural deduction format for theorem derivation, which in many ways is more informative and accords more closely with ordinary reasoning about Meinongian objects than axiomatic demonstrations. The theorems are meant to be representative of formal proofs available in the logic. They illustrate basic proof techniques in several natural deduction styles, and suggest additional applications of the system. Blank entries in the right-hand justification column indicate that the corresponding proposition is an assumption.

(T1) $(\forall x)O!x$

1.	$\sim(\forall x)O!x$	
2.	$(\exists x)\sim O!x$	(1)
3.	$\sim O!o_i$	
4.	$(\forall x)[(\ldots x\ldots) \supset O!x]$	(N1)
5.	$(\ldots o_i\ldots) \supset O!o_i$	(4)
6.	$O!o_i$	(3,5)
7.	$O!o_i \ \& \sim O!o_i$	(3,6)
8.	$\sim\sim(\forall x)O!x$	(1,7)
9.	$(\forall x)O!x$	(8)
10.	$(\forall x)O!x$	(2,3,9)
11.	$(\forall x)O!x \ \& \sim(\forall x)O!x$	(1,10)

12. $\sim\sim(\forall x)O!x$ (1,11)
13. $(\forall x)O!x$ (12)

(T2) $(\forall x)(\forall P!)(\sim\bar{P}!x \equiv P!x)$

1. $(\forall x)(\forall P!)(\sim P!x \equiv \bar{P}!x)$ (C2)
2. $(\forall P!)(\sim P!o_i \equiv \bar{P}!o_i)$ (1)
3. $\sim P!_j o_i \equiv \bar{P}!_j o_i$ (2)
4. $P!_j o_i$
5. $\sim\bar{P}!_j o_i$ (3,4)
6. $P!_j o_i \supset \sim\bar{P}!_j o_i$ (5)
7. $\sim\bar{P}!_j o_i$
8. $\sim\sim P!_j o_i$ (3,7)
9. $P!_j o_i$ (8)
10. $\sim\bar{P}!_j o_i \supset P!_j o_i$ (9)
11. $\sim\bar{P}!_j o_i \equiv P!_j o_i$ (6,10)
12. $(\forall P!)(\sim\bar{P}!o_i \equiv P!o_i)$ (11)
13. $(\forall x)(\forall P!)(\sim\bar{P}!x \equiv P!x)$ (12)

(T3) $\sim(\forall x)(\forall\bar{P})(\sim\bar{P}x \equiv Px)$

1. $\sim(\forall x)(\forall P)(\sim Px \equiv \bar{P}x)$ (C1)
2. $(\forall x)(\forall P)(\sim Px \equiv \bar{P}x)$
3. $\sim\bar{\bar{P}}_j o_i \equiv \bar{P}_j o_i$ (2)
4. $\sim P_j o_i \equiv \bar{P}_j o_i$ (3)
5. $(\forall x)(\forall P)(\sim Px \equiv \bar{P}x)$ (4)
6. $(\forall x)(\forall P)(\sim Px \equiv \bar{P}x)$ & $\sim(\forall x)(\forall P)(\sim Px \equiv \bar{P}x)$ (1,5)
7. $\sim(\forall x)(\forall P)(\sim Px \equiv \bar{P}x)$ (2,6)

(T4) $(\forall x)(\forall P!)(P!x \equiv \bar{\bar{P}}!x)$

1. $P!_j o_i$
2. $\sim\sim P!_j o_i$ (1)
3. $(\forall x)(\forall P!)(\sim P!x \equiv \bar{P}!x)$ (C2)
4. $\sim P!_j o_i \equiv \bar{P}!_j o_i$ (3)
5. $(\sim P!_j o_i \supset \bar{P}!_j o_i)$ & $(\bar{P}!_j o_i \supset \sim P!_j o_i)$ (4)
6. $\bar{P}!_j o_i \supset \sim P!_j o_i$ (5)
7. $\sim\sim P!_j o_i \supset \sim\bar{P}!_j o_i$ (6)
8. $\sim\bar{P}!_j o_i$ (2,7)
9. $\sim\bar{P}!_j o_i \equiv \bar{\bar{P}}!_j o_i$ (3)

10.	$(\sim P!_j o_i \supset \bar{\bar{P}}!_j o_i)$ & $(\bar{\bar{P}}!_j o_i \supset \sim P!_j o_i)$	(9)
11.	$\sim \bar{P}!_j o_i \supset \bar{\bar{P}}!_j o_i$	(10)
12.	$\bar{\bar{P}}!_j o_i$	(8,11)
13.	$P!_j o_i \supset \bar{\bar{P}}!_j o_i$	(12)
14.	$\bar{\bar{P}}!_j o_i$	
15.	$\bar{\bar{P}}!_j o_i \supset \sim \bar{P}!_j o_i$	(10)
16.	$\sim \bar{P}!_j o_i$	(14,15)
17.	$\sim P!_j o_i \supset \bar{P}!_j o_i$	(5)
18.	$\sim \bar{P}!_j o_i \supset \sim\sim P!_j o_i$	(17)
19.	$P!_j o_i$	(16,18)
20.	$\bar{\bar{P}}!_j o_i \supset P!_j o_i$	(19)
21.	$P!_j o_i \equiv \bar{\bar{P}}!_j o_i$	(13,20)
22.	$(\forall x)(\forall P!)(P!x \equiv \bar{\bar{P}}!x)$	(21)

(T5) $(\forall x)(\forall P!)(P!x \vee \bar{P}!x)$

1.	$(\forall x)(\forall P!)(\sim P!x \equiv \bar{P}!x)$	(C2)
2.	$\sim P!_j o_i \equiv \bar{P}!_j o_i$	(1)
3.	$(\sim P!_j o_i \supset \bar{P}!_j o_i)$ & $(\bar{P}!_j o_i \supset \sim P!_j o_i)$	(2)
4.	$\sim P!_j o_i \supset \bar{P}!_j o_i$	(3)
5.	$P!_j o_i \vee \bar{P}!_j o_i$	(4)
6.	$(\forall x)(\forall P!)(P!x \vee \bar{P}!x)$	(5)

(T6) $(\forall x)(\forall P!)(P!x \vee \sim P!x)$

1.	$(\forall x)(\forall P!)(\sim P!x \equiv \bar{P}!x)$	(C2)
2.	$\sim P!_j o_i \equiv \bar{P}!_j o_i$	(1)
3.	$(\forall x)(\forall P!)(P!x \vee \bar{P}!x)$	(T5)
4.	$P!_j o_i \vee \bar{P}!_j o_i$	(3)
5.	$P!_j o_i$	
6.	$P!_j o_i \vee \sim P!_j o_i$	(5)
7.	$\bar{P}!_j o_i$	
8.	$\sim P!_j o_i$	(2,7)
9.	$P!_j o_i \vee \sim P!_j o_i$	(8)
10.	$P!_j o_i \vee \sim P!_j o_i$	(4,6,9)
11.	$(\forall x)(\forall P!)(P!x \vee \sim P!x)$	(10)

(T7) $(\forall x)(Maximp!x \supset Imp!x)$

1. $Maximp!o_i$

2. $(\forall x)(Maximp!x \equiv (\forall P)(Px \;\&\; \overline{P}x))$ (D7)
3. $Maximp!o_i \equiv (\forall P)(Po_i \;\&\; \overline{P}o_i)$ (2)
4. $(\forall P)(Po_i \;\&\; \overline{P}o_i)$ (1,3)
5. $(\exists P)(Po_i \;\&\; \overline{P}o_i)$ (4)
6. $(\forall x)(Imp!x \equiv (\exists P)(Px \;\&\; \overline{P}x))$ (D4)
7. $Imp!o_i \equiv (\exists P)(Po_i \;\&\; \overline{P}o_i)$ (6)
8. $Imp!o_i$ (5,7)
9. $Maximp!o_i \supset Imp!o_i$ (8)
10. $(\forall x)(Maximp!x \supset Imp!x)$ (9)

If anything is maximally impossible, it is also impossible. From this and the domain postulate for the maximally impossible object (N7), it follows that:

(T8) $(\exists x)(\exists P)(Px \;\&\; \overline{P}x)$
(T9) $(\exists x)(Imp!x)$

These theorems state that the object theory domain contains an impossible object with metaphysically incompatible complementary nuclear properties.

(T10) $(\forall x)(Maximp!x \supset Odet!x)$

1. $Maximp!o_i$
2. $(\forall x)(Maximp!x \equiv (\forall P)(Px \;\&\; \overline{P}x))$ (D7)
3. $Maximp!o_i \equiv (\forall P)(Po_i \;\&\; \overline{P}o_i)$ (2)
4. $(\forall P)(Po_i \;\&\; \overline{P}o_i)$ (1,3)
5. $P_jo_i \;\&\; \overline{P}_jo_i$ (4)
6. P_jo_i (5)
7. $P_jo_i \vee \overline{P}_jo_i$ (6)
8. $(\forall P)(Po_i \vee \overline{P}o_i)$ (7)
9. $(\exists P)(Po_i \;\&\; \overline{P}o_i)$ (4)
10. $(\forall P)(Po_i \vee \overline{P}o_i) \;\&\; (\exists P)(Po_i \;\&\; \overline{P}o_i)$ (8,9)
11. $(\forall x)(Odet!x \equiv [(\forall P)(Px \vee \overline{P}x) \;\&\; (\exists P)(Px \;\&\; \overline{P}x)])$ (D8)
12. $Odet!o_i \equiv [(\forall P)(Po_i \vee \overline{P}o_i) \;\&\; (\exists P)(Po_i \;\&\; \overline{P}o_i)]$ (11)
13. $Odet!o_i$ (10,12)
14. $Maximp!o_i \supset Odet!o_i$ (13)
15. $(\forall x)(Maximp!x \supset Odet!x)$ (14)

If anything is maximally impossible, it also overdetermined. This proves that the object theory domain contains an overdetermined object.

(T11) $(\exists x)Odet!x$

The same type of proof establishes:

(T12) $(\forall x)(Odet!x \supset Imp!x)$

An overdetermined object is also impossible.

(T13) $(\forall x)(Odet!x \supset Det!x)$

An overdetermined object is determinate. It follows that:

(T14) $(\forall x)(Maximp!x \supset (Odet!x \; \& \; Det!x \; \& \; Imp!x))$

A number of interesting results can now be established about the interrelationships between existent and nonexistent objects. Here is a proof of the theorem that whatever exists is an object.

(T15) $(\forall x)(E!x \supset O!x)$

1.	$E!o_i$	
2.	$(\forall x)[(\ldots x \ldots) \supset O!x]$	(N1)
3.	$(\ldots o_i \ldots) \supset O!o_i$	(2)
4.	$O!o_i$	(1,3)
5.	$E!o_i \ldots O!o_i$	(4)
6.	$(\forall x)(E!x \supset O!x)$	(5)

The derivation is straightforward, since '$E!o_i$' is well-formed and contains 'o_i' as a term. The domain comprehension postulate in (N1) is so liberal that it admits an object into the domain for any well-formed ostensibly designating expression. It permits enlargement of the domain in practical application as new thoughts are entertained and new ostensibly designating expressions formulated, though in a more fundamental sense the object theory domain is independent of thought and language. The converse of (T15) is not a theorem, but its negation is.

(T16) $\sim(\forall x)(O!x \supset E!x)$

This means that there are some nonexistent objects in the Meinongian domain. It directly implies:

(T17) $(\exists x)(O!x \; \& \; \sim E!x)$
(T18) $(\exists x)(O!x \; \& \; \overline{E}!x)$

According to these theorems the Meinongian domain includes at least one nonexistent object. The conclusion can be derived from the postulate (N6) that $(\exists x)Inc!x$, or (N7) that $(\exists x)Maximp!x$.

1. $(\exists x)Inc!x$ (N6)
2. $Inc!o_i$
3. $(\forall x)[(\ldots x \ldots) \supset O!x]$ (N1)
4. $(\ldots o_i \ldots) \supset O!o_i$ (3)
5. $O!o_i$ (2,4)
6. $(\forall x)(Inc!x \equiv (\exists P){\sim}(Px \vee \overline{P}x))$ (D3)
7. $Inc!o_i \equiv (\exists P){\sim}(Po_i \vee \overline{P}o_i)$ (6)
8. $(\exists P){\sim}(Po_i \vee \overline{P}o_i)$ (2,7)
9. $(\forall x)(E!x \equiv (\forall P)[(Px \vee \overline{P}x) \;\&\; {\sim}(Px \;\&\; \overline{P}x)])$ (D1)
10. $E!o_i \equiv (\forall P)[(Po_i \vee \overline{P}o_i) \;\&\; {\sim}(Po_i \;\&\; \overline{P}o_i)]$ (9)
11. ${\sim}(P_jo_i \vee \overline{P}o_i)$
12. ${\sim}(P_jo_i \vee \overline{P}o_i) \vee (P_jo_i \;\&\; \overline{P}o_i)$ (11)
13. ${\sim}[(P_jo_i \vee \overline{P}o_i) \;\&\; {\sim}(P_jo_i \;\&\; \overline{P}o_i)]$ (12)
14. $(\exists P){\sim}[(Po_i \vee \overline{P}o_i) \;\&\; {\sim}(Po_i \;\&\; \overline{P}o_i)]$ (13)
15. ${\sim}(\forall P)[(Po_i \vee \overline{P}o_i) \;\&\; {\sim}(Po_i \;\&\; \overline{P}o_i)]$ (14)
16. ${\sim}E!o_i$ (10,15)
17. $(\forall x)({\sim}E!x \equiv \overline{E}!x)$ (C2)
18. ${\sim}E!o_i \equiv \overline{E}!o_i$ (17)
19. $\overline{E}!o_i$ (16,18)
20. $O!o_i \;\&\; \overline{E}!o_i$ (5,19)
21. $(\exists x)(O!x \;\&\; \overline{E}!x)$ (20)
22. $(\exists x)(O!x \;\&\; \overline{E}!x)$ (1,2,21)

The proof is much the same for $(\exists x)Maximp!x$, using (N7) and definition (D7) instead.

(T19) $(\forall x)(E!x \supset Det!x)$

1. $E!o_i$
2. $(\forall x)(E!x \equiv (\forall P)[(Px \vee \overline{P}x) \;\&\; {\sim}(Px \;\&\; \overline{P}x)])$ (D1)
3. $E!o_i \equiv (\forall P)[(Po_i \vee \overline{P}o_i) \;\&\; {\sim}(Po_i \;\&\; \overline{P}o_i)]$ (2)
4. $(\forall P)[(Po_i \vee \overline{P}o_i) \;\&\; {\sim}(Po_i \;\&\; \overline{P}o_i)]$ (1,3)
5. $(\forall x)(Det!x \equiv (\forall P)(Px \vee \overline{P}x))$ (D2)
6. $Det!o_i \equiv (\forall P)(Po_i \vee \overline{P}o_i)$ (5)
7. $(P_jo_i \vee \overline{P}o_i) \;\&\; {\sim}(P_jo_i \;\&\; \overline{P}o_i)$ (4)
8. $P_jo_i \vee \overline{P}o_i$ (7)
9. $(\forall P)(Po_i \vee \overline{P}o_i)$ (8)
10. $Det!o_i$ (6,9)
11. $E!o_i \supset Det!o_i$ (10)
12. $(\forall x)(E!x \supset Det!x)$ (11)

Every existent object is determinate. The converse does not hold. It is a theorem that:

(T20) $\sim(\forall x)(Det!x \supset E!x)$

1.	$(\exists x)Maximp!x$	(N7)
2.	$Maximp!o_i$	
3.	$(\forall x)(Maximp!x \equiv (\forall P)(Px \ \&\ \bar{P}x))$	(D7)
4.	$Maximp!o_i \equiv (\forall P)(Po_i \ \&\ \bar{P}o_i)$	(3)
5.	$(\forall P)(Po_i \ \&\ \bar{P}o_i)$	(2,4)
6.	$(\forall x)(E!x \equiv (\forall P)[(Px \vee \bar{P}x) \ \&\ \sim(Px \ \&\ \bar{P}x)])$	(D1)
7.	$E!o_i \equiv (\forall P)[(Po_i \vee \bar{P}o_i) \ \&\ \sim(Po_i \ \&\ \bar{P}o_i)]$	(6)
8.	$P_jo_i \ \&\ \bar{P}_jo_i$	(5)
9.	$\sim(P_jo_i \vee \bar{P}o_i) \vee (P_jo_i \ \&\ \bar{P}o_i)$	(8)
10.	$\sim[(P_jo_i \vee \bar{P}o_i) \ \&\ \sim(P_jo_i \ \&\ \bar{P}o_i)]$	(9)
11.	$(\exists P)\sim[(Po_i \vee \bar{P}o_i) \ \&\ \sim(Po_i \ \&\ \bar{P}o_i)]$	(10)
12.	$\sim(\forall P)[(Po_i \vee \bar{P}o_i) \ \&\ \sim(Po_i \ \&\ \bar{P}o_i)]$	(11)
13.	$\sim E!o_i$	(7,12)
14.	$(\forall x)(Maximp!x \supset (Odet!x \ \&\ Det!x \ \&\ Imp!x))$	(T14)
15.	$Maximp!o_i \ldots (Odet!o_i \ \&\ Det!o_i \ \&\ Imp!o_i)$	(14)
16.	$Odet!o_i \ \&\ Det!o_i \ \&\ Imp!o_i$	(2,15)
17.	$Det!o_i$	(16)
18.	$Det!o_i \ \&\ \sim E!o_i$	(13,17)
19.	$\sim(Det!o_i \supset E!o_i)$	(18)
20.	$(\exists x)\sim(Det!x \supset E!x)$	(19)
21.	$(\exists x)\sim(Det!x \supset E!x)$	(1,2,20)
22.	$\sim(\forall x)(Det!x \supset E!x)$	(21)

Not every determinate object exists. The maximally impossible object is determinate but necessarily nonexistent.

(T21) $(\forall x)(\bar{E}!x \equiv (Inc!x \vee Imp!x))$

1.	$\sim E!o_i$	
2.	$(\forall x)(E!x \equiv (\forall P)[(Px \vee \bar{P}x) \ \&\ \sim(Px \ \&\ \bar{P}x)])$	(D1)
3.	$E!o_i \equiv (\forall P)[(Po_i \vee \bar{P}o_i) \ \&\ \sim(Po_i \ \&\ \bar{P}o_i)]$	(2)
4.	$\sim(\forall P)[(Po_i \vee \bar{P}o_i) \ \&\ \sim(Po_i \ \&\ \bar{P}o_i)]$	(1,3)
5.	$(\exists P)\sim[(Po_i \vee \bar{P}o_i) \ \&\ \sim(Po_i \ \&\ \bar{P}o_i)]$	(4)
6.	$\sim[(P_jo_i \vee \bar{P}o_i) \ \&\ \sim(P_jo_i \ \&\ \bar{P}o_i)]$	
7.	$\sim(P_jo_i \vee \bar{P}o_i) \vee (P_jo_i \ \&\ \bar{P}o_i)$	(6)

8.	$(\forall x)(Inc!x \equiv (\exists P){\sim}(Px \vee \bar{P}x))$	(D3)
9.	$Inc!o_i \equiv (\exists P){\sim}(Po_i \vee \bar{P}o_i)$	(8)
10.	${\sim}(P_j o_i \vee \bar{P}o_i)$	
11.	$(\exists P){\sim}(Po_i \vee \bar{P}o_i)$	(10)
12.	$Inc!o_i$	(9,11)
13.	$Inc!o_i \vee Imp!o_i$	(12)
14.	${\sim}E!o_i \supset (Inc!o_i \vee Imp!o_i)$	(13)
15.	$(\forall x)(Imp!x \equiv (\exists P)(Px \,\&\, \bar{P}x))$	(D4)
16.	$Imp!o_i \equiv (\exists P)(Po_i \,\&\, \bar{P}o_i)$	(15)
17.	$P_j o_i \,\&\, \bar{P}_j o_i$	
18.	$(\exists P)(Po_i \,\&\, \bar{P}o_i)$	(17)
19.	$Imp!o_i$	(16,18)
20.	$Inc!o_i \vee Imp!o_i$	(19)
21.	$Inc!o_i \vee Imp!o_i$	(7,13,20)
22.	${\sim}E!o_i \supset (Inc!o_i \vee Imp!o_i)$	(21)
23.	$Inc!o_i \vee Imp!o_i$	
24.	$Inc!o_i$	
25.	$(\forall P){\sim}(Po_i \vee \bar{P}o_i)$	(9,24)
26.	${\sim}(P_j o_i \vee \bar{P}o_i)$	
27.	${\sim}(P_j o_i \vee \bar{P}o_i) \vee (P_j o_i \,\&\, \bar{P}o_i)$	(26)
28.	${\sim}[(P_j o_i \vee \bar{P}o_i) \,\&\, {\sim}(P_{jj} o_i \,\&\, \bar{P}o_i)]$	(27)
29.	$(\exists P){\sim}[(Po_i \vee \bar{P}o_i) \,\&\, {\sim}(Po_i \,\&\, \bar{P}o_i)]$	(28)
30.	${\sim}(\forall P)[(Po_i \vee \bar{P}o_i) \,\&\, {\sim}(Po_i \,\&\, \bar{P}o_i)]$	(29)
31.	${\sim}E!o_i$	(3,30)
32.	$Imp!o_i$	
33.	$(\exists P)(Po_i \,\&\, \bar{P}o_i)$	(16,32)
34.	$P_j o_i \,\&\, \bar{P}_j o_i$	
35.	${\sim}(P_j o_i \vee \bar{P}o_i) \vee (P_j o_i \,\&\, \bar{P}o_i)$	(34)
36.	${\sim}[(P_j o_i \vee \bar{P}o_i) \,\&\, {\sim}(P_j o_i \,\&\, \bar{P}o_i)]$	(35)
37.	$(\exists P){\sim}[(Po_i \vee \bar{P}o_i) \,\&\, {\sim}(Po_i \,\&\, \bar{P}o_i)]$	(36)
38.	${\sim}(\forall P)[(Po_i \vee \bar{P}o_i) \,\&\, {\sim}(Po_i \,\&\, \bar{P}o_i)]$	(37)
39.	${\sim}E!o_i$	(3,38)
40.	${\sim}E!o_i$	(23,31,39)
41.	$(Inc!o_i \vee Imp!o_i) \supset {\sim}E!o_i$	(40)
42.	${\sim}E!o_i \equiv (Inc!o_i \vee Imp!o_i)$	(22,41)
43.	$(\forall x)({\sim}E!x \equiv (Inc!x \vee Imp!x))$	(42)
44.	$(\forall x)({\sim}E!x \equiv (Inc!x \vee Imp!x))$	(5,6,43)
45.	$(\forall x)(\bar{E}!x \equiv Inc!x \vee Imp!x))$	(44,C2)

An object is nonexistent if and only if it is either impossible or incomplete. The theorem has an alternative formulation based on previously established interrelations between impossible objects.

(T22) $(\forall x)(\overline{E}!x \equiv (Inc!x \vee Imp!x \vee Maximp!x \vee Odet!x))$
(T23) $(\forall x)((Det!x \,\&\, {\sim}Imp!x) \equiv E!x)$

An object exists if and only if it is determinate and possible, or if its *Sosein* is consistent and complete.

(T24) $(\forall x)(Imp!x \equiv (\exists P)(S(x) = \{P, \ldots, \overline{P}, \ldots\}))$

1.	$Imp!o_i$	
2.	$(\forall x)(Imp!x \equiv (\exists P)(Px \,\&\, \overline{P}x))$	(D4)
3.	$Imp!o_i \equiv (\exists P)(Po_i \,\&\, \overline{P}o_i)$	(2)
4.	$(\exists P)(Po_i \,\&\, \overline{P}o_i)$	(1,3)
5.	$(\forall x)(\forall P_1)\ldots(\forall P_n)\ldots(S(x) = \{P_1, \ldots, P_n, \ldots\} \equiv (P_1x \,\&\ldots\&\, P_nx\ldots))$	(D13)
6.	$(\forall P_1)\ldots(\forall P_n)\ldots(S(o_i) = \{P_1, \ldots, P_n, \ldots\} \equiv (P_1o_i \,\&\ldots\&\, P_no_i\ldots))$	(5)
7.	$S(o_i) = \{P_i, \ldots, P_k, \ldots\} \equiv (P_io_i \,\&\ldots\&\, P_ko_i\ldots)$	(6)
8.	$P_jo_i \,\&\, \overline{P}_jo_i$	
9.	$S(o_i) = \{P_j, \ldots, \overline{P}_j, \ldots\}$	(7,8)
10.	$(\exists P)(S(o_i) = \{P, \ldots, \overline{P}, \ldots\})$	(9)
11.	$Imp!o_i \supset (\exists P)(S(o_i) = \{P, \ldots, \overline{P}, \ldots\})$	(10)
12.	$(\exists P)(S(o_i) = \{P, \ldots, \overline{P}, \ldots\})$	
13.	$S(o_i) = \{P_k, \ldots, \overline{P}_k, \ldots\}$	
14.	$S(o_i) = \{P_k, \ldots, \overline{P}_k, \ldots\} \equiv (P_ko_i \,\&\ldots\&\, \overline{P}_ko_i\ldots)$	(6)
15.	$P_ko_i \,\&\, \overline{P}_ko_i\ldots$	(13,14)
16.	$(\exists P)(Po_i \,\&\, \overline{P}o_i\ldots)$	(15)
17.	$(\exists P)(Po_i \,\&\, \overline{P}o_i\ldots)$	(12,13,16)
18.	$Imp!o_i$	(3,17)
19.	$(\exists P)(S(o_i) = \{P, \ldots, \overline{P}, \ldots\}) \supset Imp!o_i$	(18)
20.	$Imp!o_i \equiv (\exists P)(S(o_i) = \{P, \ldots, \overline{P}, \ldots\})$	(11,19)
21.	$(\forall x)(Imp!x \equiv (\exists P)(S(x) = \{P, \ldots, \overline{P}, \ldots\}))$	(20)

An object is impossible if and only if its *Sosein* contains at least one nuclear property and property complement pair. Similarly:

(T25) $(\forall x)[(Imp!x \vee Maximp!x \vee Odet!x) \equiv (\exists P)(S(x) = \{P, \ldots, \overline{P}, \ldots\})]$
(T26) $(\forall x)(Pinc!x \equiv (Inc!x \,\&\, {\sim}Imp!x))$
(T27) $(\forall x)(Iinc!x \equiv (Inc!x \,\&\, Imp!x))$
(T28) $(\forall x)[(Det!x \vee Inc!x) \,\&\, {\sim}(Det!x \,\&\, Inc!x)]$

Theorem (T28) states that all Meinongian objects are either determinate or incomplete, but not both.

(T29) $\sim(\exists x)(E!x \;\&\; (Imp!x \vee Inc!x))$
(T30) $(\forall x)(E!x \supset \sim(\exists P)(S(x) = \{P, \ldots, \bar{P}, \ldots\}))$

No existent object is either impossible or incomplete, and no existent object has a *Sosein* containing a nuclear property and property complement pair.

Intentional identity is symmetrical:

(T31) $(\forall x)(\forall y)[(x =_i y) \supset (y =_i x)]$

1.	$o_i =_i o_j$	
2.	$(\forall x)(x =_i x)$	(N2)
3.	$o_i =_i o_i$	(2)
4.	$(\forall x)(\forall y)([(x =_i y) \;\&\; (\ldots x \ldots)] \supset (\ldots y \ldots))$	(N3)
5.	$[(o_i =_i o_j) \;\&\; (\ldots o_i \ldots)] \supset (\ldots o_j \ldots)$	(4)
6.	$o_j =_i o_i$	(1,3,5)
7.	$(o_i =_i o_j) \supset (o_j =_i o_i)$	(6)
8.	$(\forall x)(\forall y)[(x =_i y) \supset (y =_i x)]$	(7)

Intentional identity is transitive:

(T32) $(\forall x)(\forall y)(\forall z)([(x =_i y) \;\&\; (y =_i z)] \supset (x =_i z))$

1.	$(o_i =_i o_j) \;\&\; (o_j =_i o_k)$	
2.	$o_i =_i o_j$	(1)
3.	$o_j =_i o_k$	(1)
4.	$(\forall x)(\forall y)([(x =_i y) \;\&\; (\ldots x \ldots)] \supset (\ldots y \ldots))$	(N3)
5.	$[(o_j =_i o_i) \;\&\; (\ldots o_j \ldots)] \supset (\ldots o_i \ldots)$	(4)
6.	$(\forall x)(\forall y)[(x =_i y) \supset (y =_i x)]$	(T31)
7.	$(o_i =_i o_j) \ldots (o_j =_i o_i)$	(6)
8.	$o_j =_i o_i$	(2,7)
9.	$o_i =_i o_k$	(3,5,8)
10.	$[(o_i =_i o_j) \;\&\; (o_j =_i o_k)] \supset (o_i =_i o_k)$	(9)
11.	$(\forall x)(\forall y)(\forall z)([(x =_i y) \;\&\; (y =_i z)] \supset (x =_i z))$	(10)

The logic cannot prove the symmetry or transitivity of extensional or referential identity without additional nonlogical axioms. There is no unrestricted substitution inference principle for extensional and referential identity, as there is for intentional identity. Symmetry and transitivity of extensional and referential identity are therefore postulated as additional nonlogical axioms.

(N8) $(\forall x)(\forall y)[(x =_e y) \supset (y =_e x)]$
(N9) $(\forall x)(\forall y)(\forall z)([(x =_e y) \ \& \ (y =_e z)] \supset (x =_e z))$
(N10) $(\forall x)(\forall y)[(x =_{rf} y) \supset (y =_{rf} x)]$
(N11) $(\forall x)(\forall y)(\forall z)([(x =_{rf} y) \ \& \ (y =_{rf} z)] \supset (x =_{rf} z))$

It is possible to prove the reflexivity of extensional identity. For this it is useful to establish some formal relations between intentional and extensional identity.

(T33) $(\forall x)(\forall y)[(x =_i y) \supset (x =_{rf} y)]$

1. $o_i =_i o_j$
2. $(\forall x)(\forall y)[(x =_i y) \equiv (\forall P)(Px \equiv Py)]$ (D9)
3. $(o_i =_i o_j) \equiv (\forall P)(Po_i \equiv Po_j)$ (2)
4. $(\forall P)(Po_i \equiv Po_j)$ (1,3)
5. $P_i o_i \equiv P_i o_j$ (4)
6. $(\forall x)(\forall y)[(x =_{rf} y) \equiv (\forall P)(\overline{Cint}!(P) \supset (Px \equiv Py))]$ (D11)
7. $(o_i =_{rf} o_j) \equiv (\forall P)(\overline{Cint}!(P) \supset (Po_i \equiv Po_j))$ (6)
8. $\overline{Cint}!(P_i) \supset (P_i o_i \equiv P_i o_j)$ (5)
9. $(\forall P)(\overline{Cint}!(P) \supset (Po_i \equiv Po_j))$ (8)
10. $o_i =_i o_j$ (7,9)
11. $(o_i =_i o_j) \supset (o_i =_{rf} o_j)$ (10)
12. $(\forall x)(\forall y)[(x =_i y) \supset (x =_{rf} y)]$ (11)

(T34) $(\forall x)(\forall y)[(x =_i y) \supset (x =_e y)]$

1. $o_i =_i o_j$
2. $(\forall x)(\forall y)[(x =_e y) \equiv [(E!x \ \& \ (x =_{rf} y)) \vee (\overline{E}!x \ \& \ \overline{E}!y)]]$ (D12)
3. $(o_i =_e o_j) \equiv [(E!o_i \ \& \ (o_i =_{rf} o_j)) \vee (\overline{E}!o_i \ \& \ \overline{E}!o_j)]$ (2)
4. $(\forall x)(\forall P!)(P!x \vee \overline{P}!x)$ (T5)
5. $E!o_i \vee \overline{E}!o_i$ (4)
6. $E!o_i$
7. $(\forall x)(\forall y)[(x =_i y) \supset (x =_{rf} y)]$ (T33)
8. $(o_i =_i o_j) \supset (o_i =_{rf} o_j)$ (7)
9. $o_i =_{rf} o_j$ (1,8)
10. $E!o_i \ \& \ (o_i =_{rf} o_j)$ (6,9)
11. $(E!o_i \ \& \ (o_i =_{rf} o_j)) \vee (\overline{E}!o_i \ \& \ \overline{E}!o_j)$ (10)
12. $o_i =_e o_j$ (3,11)
13. $\overline{E}!o_i$

14.	$(\forall x)(E!x \equiv (\forall P)[(Px \vee \bar{P}x) \ \& \ {\sim}(Px \ \& \ \bar{P}x)])$	(D1)
15.	$E!o_i \equiv (\forall P)[(Po_i \vee \bar{P}o_i) \ \& \ {\sim}(Po_i \ \& \ \bar{P}o_i)]$	(14)
16.	$(\forall x)(\forall P!)({\sim}P!x \equiv \bar{P}!x)$	(C2)
17.	${\sim}E!o_i \equiv \bar{E}!o_i$	(16)
18.	${\sim}E!o_i$	(13,17)
19.	${\sim}(\forall P)[(Po_i \vee \bar{P}o_i) \ \& \ {\sim}(Po_i \ \& \ \bar{P}o_i)]$	(15,18)
20.	$(\exists P){\sim}[(Po_i \vee \bar{P}o_i) \ \& \ {\sim}(Po_i \ \& \ \bar{P}o_i)]$	(19)
21.	${\sim}[(P_j o_i \vee \bar{P}_j o_i) \ \& \ {\sim}(P_j o_i \ \& \ \bar{P}_j o_i)]$	
22.	${\sim}(P_j o_i \vee \bar{P}_j o_i) \vee {\sim}{\sim}(P_j o_i \ \& \ \bar{P}_j o_i)$	(21)
23.	${\sim}(P_j o_i \vee \bar{P}_j o_i)$	
24.	${\sim}P_j o_i \ \& \ {\sim}\bar{P}_j o_i$	(23)
25.	${\sim}P_j o_i$	(24)
26.	$(\forall x)(\forall y)[(x =_i y) \equiv (\forall P)(Px \equiv Py)]$	(D9)
27.	$(o_i =_i o_j) \equiv (\forall P)(Po_i \equiv Po_j)$	(26)
28.	$(\forall P)(Po_i \equiv Po_j)$	(1,27)
29.	$P_j o_i \equiv P_j o_j$	(28)
30.	${\sim}P_j o_j$	(25,29)
31.	$\bar{P}_j o_i \equiv \bar{P}_j o_j$	(28)
32.	${\sim}\bar{P}_j o_i$	(24)
33.	${\sim}\bar{P}_j o_j$	(31,32)
34.	${\sim}P_j o_j \ \& \ {\sim}\bar{P}_j o_j$	(30,33)
35.	${\sim}(P_j o_j \vee \bar{P}_j o_j)$	(34)
36.	${\sim}(P_j o_j \vee \bar{P}_j o_j) \vee (P_j o_j \ \& \ \bar{P}_j o_j)$	(35)
37.	${\sim}{\sim}(P_j o_i \ \& \ \bar{P}_j o_i)$	
38.	$P_j o_i \ \& \ \bar{P}_j o_i$	(37)
39.	$P_j o_i$	(3)
40.	$P_j o_j$	(29,39)
41.	$\bar{P}_j o_i$	(38)
42.	$\bar{P}_j o_j$	(31,41)
43.	$P_j o_j \ \& \ \bar{P}_j o_j$	(40,42)
44.	${\sim}(P_j o_j \vee \bar{P}_j o_j) \vee (P_j o_j \ \& \ \bar{P}_j o_j)$	(43)
45.	${\sim}(P_j o_j \vee \bar{P}_j o_j) \vee (P_j o_j \ \& \ \bar{P}_j o_j)$	(22,36,44)
46.	${\sim}[(P_j o_j \vee \bar{P}_j o_j) \ \& \ {\sim}(P_j o_j \ \& \ \bar{P}_j o_j)]$	(45)
47.	$(\exists P){\sim}[(Po_j \vee \bar{P}o_j) \ \& \ {\sim}(Po_j \ \& \ \bar{P}o_j)]$	(46)
48.	$(\exists P){\sim}[(Po_j \vee \bar{P}o_j) \ \& \ {\sim}(Po_j \ \& \ \bar{P}o_j)]$	(20,21,47)
49.	${\sim}(\forall P)[(Po_j \vee \bar{P}o_j) \ \& \ {\sim}(Po_j \ \& \ \bar{P}o_j)]$	(48)
50.	$E!o_j \equiv (\forall P)[(Po_j \vee \bar{P}o_j) \ \& \ {\sim}(Po_j \ \& \ \bar{P}o_j)]$	(14)
51.	${\sim}E!o_j$	(49,50)
52.	${\sim}E!o_j \equiv \bar{E}!o_j$	(16)
53.	$\bar{E}!o_j$	(51,52)

54. $\overline{E}!o_i \;\&\; \overline{E}!o_j$ (13,53)
55. $(E!o_i \;\&\; (o_i =_{rf} o_j)) \vee (\overline{E}!o_i \;\&\; \overline{E}!o_j)$ (54)
56. $o_i =_e o_j$ (3,55)
57. $(o_i =_i o_j) \supset (o_i =_e o_j)$ (56)
58. $(o_i =_i o_j) \supset (o_i =_e o_j)$ (5,8,57)
59. $(\forall x)(\forall y)[(x =_i y) \supset (x =_e y)]$ (58)

(T35) $\sim(\forall x)(\forall y)[(x =_e y) \supset (x =_i y)]$

1. $(\exists x)Maximp!x$ (N7)
2. $(\exists x)Inc!x$ (N6)
3. $Maximp!o_i$
4. $Inc!o_j$
5. $(\forall x)(Maximp!x \equiv (\forall P)(Px \;\&\; \overline{P}x))$ (D7)
6. $Maximp!o_i \equiv (\forall P)(Po_i \;\&\; \overline{P}o_i)$ (5)
7. $(\forall P)(Po_i \;\&\; \overline{P}o_i)$ (3,6)
8. $(\forall x)(Inc!x \equiv (\exists P)\sim(Px \vee \overline{P}x))$ (D3)
9. $Inc!o_j \equiv (\exists P)\sim(Po_j \vee \overline{P}o_j)$ (8)
10. $(\exists P)\sim(Po_j \text{ v } \overline{P}o_j)$ (4,9)
11. $\sim(P_jo_j \vee \overline{P}o_j)$
12. $P_jo_i \;\&\; \overline{P}_jo_i$ (7)
13. $\sim P_jo_j \;\&\; \sim\overline{P}_jo_j$ (11)
14. P_jo_i (12)
15. $\sim P_jo_j$ (13)
16. $P_jo_i \;\&\; \sim P_jo_j$ (14,15)
17. $\sim(P_jo_i \equiv P_jo_j)$ (16)
18. $(\exists P)\sim(Po_i \equiv Po_j)$ (17)
19. $(\exists P)\sim(Po_i \equiv Po_j)$ (10,11,18)
20. $\sim(\forall P)(Po_i \equiv Po_j)$ (19)
21. $(\forall x)(\forall y)[(x =_i y) \equiv (\forall P)(Px \equiv Py)]$ (D9)
22. $(o_i =_i o_j) \equiv (\forall P)(Po_i \equiv Po_j)$ (21)
23. $o_i \neq_i o_j$ (20,22)
24. $(\forall x)(\sim E!x \equiv (Inc!x \vee Imp!x \vee Maximp!x \vee Odet!x))$ (T22,C2)
25. $\sim E!o_i \equiv (Inc!o_i \vee Imp!o_i \vee Maximp!o_i \vee Odet!o_i)$ (24)
26. $Inc!o_i \vee Imp!o_i \vee Maximp!o_i \vee Odet!o_i$ (3)
27. $\sim E!o_i$ (25,26)
28. $\overline{E}!o_i$ (27,C2)
29. $\sim E!o_j \equiv (Inc!o_j \vee Imp!o_j \vee Maximp!o_j \vee Odet!o_j)$ (24)
30. $Inc!o_j \vee Imp!o_j \vee Maximp!o_j \vee Odet!o_j$ (4)
31. $\sim E!o_j$ (29,30)

32. $\overline{E}!o_j$ (31,C2)
33. $\overline{E}!o_i \,\&\, \overline{E}!o_j$ (28,32)
34. $(\forall x)(\forall y)[(x =_e y) \equiv [(E!x \,\&\, (x =_{rf} y)) \vee (\overline{E}!x \,\&\, \overline{E}!y)]]$ (D12)
35. $(o_i =_e o_j) \equiv [(E!o_i \,\&\, (o_i =_{rf} o_j)) \vee (\overline{E}!o_i \,\&\, \overline{E}!o_j)]$ (34)
36. $(E!o_i \,\&\, (o_i =_{rf} o_j)) \vee (\overline{E}!o_i \,\&\, \overline{E}!o_j)$ (33)
37. $o_i =_e o_j$ (35,36)
38. $(o_i =_e o_j) \,\&\, (o_i \neq_i o_j)$ (23,37)
39. $\sim[(o_i =_e o_j) \supset (o_i =_i o_j)]$ (38)
40. $(\exists x)(\exists y)\sim[(x =_e y) \supset (x =_i y)]$ (39)
41. $\sim(\forall x)(\forall y)[(x =_e y) \supset (x =_i y)]$ (40)

Not all extensionally identical objects are intentionally identical. The reflexivity of extensional identity can now be proved.

(T36) $(\forall x)(x =_e x)$

1. $(\forall x)(x =_i x)$ (N2)
2. $(\forall x)(\forall y)[(x =_i y) \supset (x =_e y)]$ (T34)
3. $o_i =_i o_i$ (1)
4. $(o_i =_i o_i) \supset (o_i =_e o_i)$ (2)
5. $o_i =_e o_i$ (3,4)
6. $(\forall x)(x =_e x)$ (5)

It is instructive to consider why the same sort of argument involving (T34) could not be used to prove the symmetry and transitivity of extensional identity.

Nonexistent objects though extensionally identical and extensionally indistinguishable are individuated by intentional identity conditions. This is demonstrated by adding to the proof of (T35), after step (33), the following alternative inferences:

(T37) $(\exists x)(\exists y)(\overline{E}!x \,\&\, \overline{E}!y \,\&\, (x \neq_i y))$

34'. $\overline{E}!o_i \,\&\, \overline{E}!o_j \,\&\, (o_i \neq_i o_j)$ (23,33)
35'. $(\exists x)(\exists y)(\overline{E}!x \,\&\, \overline{E}!y \,\&\, (x \neq_i y))$ (34')

A similar result is obtained by indirect proof, on the assumption that $(\forall x)(\forall y)(x =_i y)$, where the golden mountain is chosen as an o_i, and the maximally impossible object as an o_j, and the *Sosein* function applied to each. The intentional nonidentity of some objects is established in a more matter of fact way by constructing a counterexample to the general assumption of intentional identity in which some converse intentional property attaches to a particular object but not to another, as in Cicero $\neq_i$ Marcus Tully, or Superman $\neq_i$ Clark Kent.

(T38) $(\forall x)(\forall y)(S(x) = S(y) \supset (x =_i y))$

1.	$S(o_i) = S(o_j)$	
2.	$o_i \neq_i o_j$	
3.	$(\forall x)(\forall y)[(x =_i y) \equiv (\forall P)(Px \equiv Py)]$	(D9)
4.	$(o_i =_i o_j) \equiv (\forall P)(Po_i \equiv Po_j)$	(3)
5.	$\sim(\forall P)(Po_i \equiv Po_j)$	(2,4)
6.	$(\exists P)(Po_i \; \& \; \sim Po_j)$	(5)
7.	$P_k o_i \; \& \; \sim P_k o_j$	
8.	$(\forall x)(\forall P_1) \ldots (\forall P_n) \ldots (S(x) = \{P_1, \ldots, P_n, \ldots\} \equiv (P_1 x \; \& \ldots \& \; P_n x \ldots))$	(D13)
9.	$(\forall P_1) \ldots (\forall P_n) \ldots (S(o_i) = \{P_1, \ldots, P_n, \ldots\} \equiv (P_1 o_i \; \& \ldots \& \; P_n o_i \ldots))$	(8)
10.	$S(o_i) = \{P_k, \ldots\} \equiv P_k o_i$	(9)
11.	$P_k o_i$	(7)
12.	$S(o_i) = \{P_k, \ldots\}$	(10,11)
13.	$(\forall P_1) \ldots (\forall P_n) \ldots (S(o_j) = \{P_1, \ldots, P_n, \ldots\} \equiv (P_1 o_j \; \& \ldots \& \; P_n o_j \ldots))$	(8)
14.	$S(o_j) = \{P_k, \ldots\} \equiv P_k o_j$	(13)
15.	$\sim P_k o_j$	(7)
16.	$S(o_j) \neq \{P_k, \ldots\}$	(14,15)
17.	$(\forall x)(\forall y)(\forall z)([(x =_{rf} y) \; \& \; (y =_{rf} z)] \supset (x =_{rf} z))$	(N11)
18.	$[S(o_j) = S(o_i) \; \& \; S(o_i) = \{P_k, \ldots\}] \supset S(o_j) = \{P_k, \ldots\}$	(17)
19.	$\sim[S(o_j) = S(o_i) \; \& \; S(o_i) = \{P_k, \ldots\}]$	(16,18)
20.	$S(o_j) \neq S(o_i) \vee S(o_i) \neq \{P_k, \ldots\}$	(19)
21.	$S(o_j) \neq S(o_i)$	
22.	$(\forall x)(\forall y)[(x =_{rf} y) \supset (y =_{rf} x)]$	(N10)
23.	$S(o_i) = S(o_j) \supset S(o_j) = S(o_i)$	(22)
24.	$S(o_j) = S(o_i)$	(1,23)
25.	$S(o_j) = S(o_i) \; \& \; S(o_j) \neq S(o_i)$	(21,24)
26.	$S(o_i) \neq S(o_j)$	(25)
27.	$S(o_i) \neq \{P_k, \ldots\}$	
28.	$S(o_i) = \{P_k, \ldots\} \; \& \; S(o_i) \neq \{P_k, \ldots\}$	(14,27)
29.	$S(o_i) \neq S(o_j)$	(28)
30.	$S(o_i) \neq S(o_j)$	(20,26,29)
31.	$S(o_i) \neq S(o_j)$	(6,7,30)

32.	$S(o_i) = S(o_j)$ & $S(o_i) \neq S(o_j)$	(1,31)
33.	$o_i =_i o_j$	(2,32)
34.	$S(o_i) = S(o_j) \supset (o_i =_i o_j)$	(33)
35.	$(\forall x)(\forall y)(S(x) = S(y) \supset (x =_i y))$	(34)

If the *Soseine* of objects are referentially identical, then the objects are intentionally identical (where converse intentional properties are nuclear). But not conversely.

(T39) $\sim(\forall x)(\forall y)((x =_i y) \supset S(x) = S(y))$

1.	$(\forall x)(\forall y)((x =_i y) \supset S(x) = S(y))$	
2.	$(o_i =_i o_j) \supset S(o_i) = S(o_j)$	(1)
3.	$(\forall x)(\forall y)[(x =_i y) \equiv (\forall P)(Px \equiv Py)]$	(D9)
4.	$(o_i =_i o_j) \equiv (\forall P)(Po_i \equiv Po_j)$	(3)
5.	$(\forall P)(Po_i \equiv Po_j) \supset S(o_i) = S(o_j)$	(2,4)
6.	$(P_k o_i \equiv P_k o_j) \supset S(o_i) = S(o_j)$	(5)

The assumption in (1) is made for purposes of indirect proof. The conclusion in (6) is refuted by counterexample. Let P_k be the nuclear property of redness or being red, and let o_i be intentionally identical to the golden mountain, and o_j the round square. Then $P_k o_i \equiv P_k o_j$ is true, since neither the round square nor the golden mountain is red. But $S(o_i) = S(o_j)$ is false, since the *Sosein* of the round square is not the same as nor referentially identical with, but rather quite different from, the *Sosein* of the golden mountain.

7.	$\sim[(P_k o_i \equiv P_k o_j) \supset S(o_i) = S(o_j)]$	
8.	$[(P_k o_i \equiv P_k o_j) \supset S(o_i) = S(o_j)]$ & $\sim[(P_k o_i \equiv P_k o_j) \supset S(o_i) = S(o_j)]$	(6,7)
9.	$\sim(\forall x)(\forall y)((x =_i y) \supset S(x) = S(y))$	(1,8)

Theorems can now be proved to show that the logic is not subject to certain kinds of triviality. The peculiar ontological nature of impossible objects makes it imperative to establish that not all nonexistent objects are referentially identical, and that objects with metaphysically incompatible or incomplete combinations of nuclear properties are nonetheless distinguishable. This avoids insignificance in the logic because it guarantees individuation and distinct designation of existent and nonexistent objects. The following demonstration implies that not every impossible object is a maximally impossible object, justifying the claim that at least some impossible nonexistent objects are referentially distinct. It establishes the negative converse of (T7), that every maximally impossible object is an impossible object.

(T39) $\sim(\forall x)(Imp!x \supset Maximp!x)$

(T40) $(\exists x)(Imp!x \mathbin{\&} \sim Maximp!x)$

1.	$(\forall x)(Imp!x \supset Maximp!x)$	
2.	$Imp!o_i \supset Maximp!o_i$	(1)
3.	$(\forall x)(Maximp!x \equiv (\forall P)(Px \mathbin{\&} \bar{P}x))$	(D7)
4.	$Maximp!o_i \equiv (\forall P)(Po_i \mathbin{\&} \bar{P}o_i)$	(3)
5.	$(\forall x)(Imp!x \equiv (\exists P)(Px \mathbin{\&} \bar{P}x))$	(D4)
6.	$Imp!o_i \equiv (\exists P)(Po_i \mathbin{\&} \bar{P}o_i)$	(5)
7.	$(\exists P)(Po_i \mathbin{\&} \bar{P}o_i) \supset Imp!o_i$	(6)
8.	$(\exists P)(Po_i \mathbin{\&} \bar{P}o_i) \supset Maximp!o_i$	(2,7)
9.	$(\exists P)(Po_i \mathbin{\&} \bar{P}o_i) \supset (\forall P)(Po_i \mathbin{\&} \bar{P}o_i)$	(4,8)
10.	$\sim[(\exists P)(Po_i \mathbin{\&} \bar{P}o_i) \supset (\forall P)(Po_i \mathbin{\&} \bar{P}o_i)]$	(A5,A6)
11.	$[(\exists P)(Po_i \mathbin{\&} \bar{P}o_i) \supset (\forall P)(Po_i \mathbin{\&} \bar{P}o_i)] \mathbin{\&}$ $\sim[(\exists P)(Po_i \mathbin{\&} \bar{P}o_i) \supset (\forall P)(Po_i \mathbin{\&} \bar{P}o_i)]$	(9,10)
12.	$\sim(\forall x)(Imp!x \supset Maximp!x)$	(1,11)
13.	$(\exists x)(Imp!x \mathbin{\&} \sim Maximp!x)$	(12)

The importance of (T40) and (T41) can be appreciated when the consequences of their negations are considered. If the theorems could not be proved, or if their negations were theorems, it would mean that all impossible objects in Meinongian logic would collapse into one. Despite ostensible differences as discriminable objects of thought, nonexistent impossible objects would be referentially as well as extensionally indistinguishable.

(T42) $(\forall x)(\forall y)[(Maximp!x \mathbin{\&} Maximp!y) \supset (x =_i y)]$

1.	$Maximp!o_i \mathbin{\&} Maximp!o_j$	
2.	$Maximp!o_i$	(1)
3.	$(\forall x)(Maximp!x \equiv (\forall P)(Px \mathbin{\&} \bar{P}x))$	(D7)
4.	$Maximp!o_i \equiv (\forall P)(Po_i \mathbin{\&} \bar{P}o_i)$	(3)
5.	$(\forall P)(Po_i \mathbin{\&} \bar{P}o_i)$	(2,4)
6.	$Maximp!o_j$	(1)
7.	$Maximp!o_j \equiv (\forall P)(Po_j \mathbin{\&} \bar{P}o_j)$	(3)
8.	$(\forall P)(Po_j \mathbin{\&} \bar{P}o_j)$	(6,7)
9.	$(\forall x)(\forall y)[(x =_i y) \equiv (\forall P)(Px \equiv Py)]$	(D9)
10.	$(o_i =_i o_j) \equiv (\forall P)(Po_i \equiv Po_j)$	(9)
11.	$(\exists P)(Po_i \mathbin{\&} \sim Po_j)$	
12.	$P_k o_i \mathbin{\&} \sim P_k o_j$	
13.	$\sim P_k o_j$	(12)
14.	$P_k o_j \mathbin{\&} \bar{P}_k o_j$	(8)

15. $P_k o_j$ (14)
16. $P_k o_j \ \& \ {\sim}P_k o_j$ (13,15)
17. ${\sim}(Maximp!o_i \ \& \ Maximp!o_j)$ (1,16)
18. ${\sim}(Maximp!o_i \ \& \ Maximp!o_j)$ (11,12,17)
19. $(Maximp!o_i \ \& \ Maximp!o_j) \ \& \ {\sim}(Maximp!o_i \ \& \ Maximp!o_j)$ (1,18)
20. ${\sim}(\exists P)(Po_i \ \& \ {\sim}Po_j)$ (11,19)
21. $(\forall P)(Po_i \supset Po_j)$ (20)
22. $(\exists P)(Po_j \ \& \ {\sim}Po_i)$
23. $P_k o_j \ \& \ {\sim}P_k o_i$
24. ${\sim}P_k o_i$ (23)
25. $P_k o_i \ \& \ \overline{P}_k o_i$ (5)
26. $P_k o_i$ (25)
27. $P_k o_i \ \& \ {\sim}P_k o_i$ (24,26)
28. ${\sim}(Maximp!o_i \ \& \ Maximp!o_j)$ (1,27)
29. ${\sim}(Maximp!o_i \ \& \ Maximp!o_j)$ (22,23,28)
30. $(Maximp!o_i \ \& \ Maximp!o_j) \ \& \ {\sim}(Maximp!o_i \ \& \ Maximp!o_j)$ (1,29)
31. ${\sim}(\exists P)(Po_j \ \& \ {\sim}Po_i)$ (22,30)
32. $(\forall P)(Po_j \supset Po_i)$ (31)
33. $P_k o_j \supset P_k o_i$ (32)
34. $P_k o_i \supset P_k o_j$ (21)
35. $P_k o_i \equiv P_k o_j$ (33,34)
36. $(\forall P)(Po_i \equiv Po_j)$ (35)
37. $o_i =_i o_j$ (10,36)
38. $(Maximp!o_i \ \& \ Maximp!o_j) \supset (o_i =_i o_j)$ (37)
39. $(\forall x)(\forall y)[(Maximp!x \ \& \ Maximp!y) \supset (x =_i y)]$ (38)

Theorem (T42) also entails:

(T43) $(\forall x)(\forall y)[(Maximp!x \ \& \ Maximp!y) \supset (x =_e y)]$

(T44) $(\forall x)(\forall y)[(Maximp!x \ \& \ Maximp!y) \supset (x =_{rf} y)]$

The green maximally impossible object is intentionally, and therefore extensionally and referentially, identical to the blue maximally impossible object, even though the terms by which they are designated are not lexically or orthographically identical. A person might desire the green maximally impossible object and despise the blue maximally impossible object. But since the property of being maximally impossible is extranuclear rather than nuclear, free assumption does not guarantee that thoughts ostensibly about the green maximally impossible object or blue maximally impossible object are actually about a green or blue maximally impossible object, or green or blue object with the extranuclear property of being maximally impossible in its *Sosein.* At most these would be thoughts about the green object and the blue object to which the extranuclear nonconstitutive property of being maximally impossible is superadded. The desire for the green maximally impossible object and despite of the blue maximally impossible object by some particular person is not sufficient to distinguish the green maximally impossible object from the blue maximally impossible object, even where converse intentional properties like being desired or despised are included as constitutive uniquely identifying or *Sosein* nuclear properties. The green maximally impossible object and the blue maximally impossible object alike have the converse intentional properties of being desired and despised by everyone, together with the complements of these properties.

The following theorems can also be proved.

(T45) $(\forall x)(\sim E!x \equiv \overline{\overline{E}}!x)$
(T46) $(\forall x)(Det!x \equiv \sim Inc!x)$
(T47) $(\forall x)(\sim Det!x \equiv Inc!x)$
(T48) $\sim(\forall x)(Imp!x \supset Odet!x)$
(T49) $\sim(\forall x)(Det!x \supset Odet!x)$
(T50) $\sim(\forall x)(Odet!x \supset Maximp!x)$
(T51) $(\forall x)(\forall y)[(x =_{rf} y) \supset (x =_{e} y)]$
(T52) $\sim(\forall x)(\forall y)[(x =_{e} y) \supset (x =_{rf} y)]$
(T53) $\sim(\forall x)(\forall y)[(x =_{rf} y) \supset (x =_{i} y)]$
(T54) $(\forall x)(\forall y)[S(x) = S(y) \supset (x =_{e} y)]$
(T55) $(\forall x)(\forall y)[S(x) = S(y) \supset (x =_{rf} y)]$
(T56) $(\forall x)[(Imp!x \;\&\; \sim Inc!x) \equiv Odet!x]$
(T57) $(\forall x)[(Det!x \;\&\; Imp!x) \equiv (Maximp!x \vee Odet!x)]$
(T58) $\sim(\forall x)[(Odet!x \;\&\; Det!x \;\&\; Imp!x) \supset Maximp!x]$

5. Definite Description

Russell's theory of descriptions, or what may in retrospect be called the standard or classical theory of definite description, was intended in part as a refutation of Meinong's object theory.[6] For this reason, classical definite description theory cannot be incorporated into the object theory logic without revision.

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[6] Ibid., pp. 41—56. Leonard Linsky, *Referring* [1967], especially p. 87; *Names and Descriptions* [1977].

The sentence 'The golden mountain is golden' is analyzed on Russell's theory as an existentially quantified conjunction of three components: (i) an existence condition; (ii) a uniqueness condition; (iii) a predication. On this analysis, the sentence turns out to be false, since the existence condition is unsatisfied. This renders the entire existentially quantified conjunction false. In Meinongian object theory, on the contrary, the sentence is true. The golden mountain *is* golden, for it has the nuclear constitutive property of being golden as part of its *Sosein*, and by the independence thesis has that property just as surely and in the same sense as an existent object made of gold. From an object theory perspective, Russell may be said to have formulated a specialized extensional theory of definite description or extensional fragment of the complete theory of definite description, with limited application to descriptors for existent entities. The object theory logic provides an unambiguous way of expressing the limitations of Russell's theory, and of supplementing it with descriptors for nonexistent Meinongian objects.

The distinction between intentional, referential, and extensional identity makes it possible to adapt Russell's uniqueness and predication conditions in a Meinongian intentional definite description theory that excludes Russell's existence condition. Intentional or referential identity is required in the object theory analysis of definite description, since all incomplete and impossible objects are extensionally identical. If extensional rather than intentional or referential identity were built into the analysis, then the sentence 'The golden mountain is a maximally impossible quadruped' would be true, though intuitively it is false. Rejecting Russell's existence condition is necessary in order to make the analysis of definite description fully general with respect to the entire Meinongian semantic domain or ontology and extraontology of existent and nonexistent objects. Russell's theory is subsumed as a proper part of the complete object theory analysis.

Let '℩' (inverted Greek letter 'iota') be a definite descriptor. Russell's theory states (for $1 \leq i \leq n$):

$$\text{(DD1)}\quad (\forall P(!)_1) \ldots (\forall P(!)_n)[P(!)_1(\text{℩}_r x(P(!)_i x \;\&\ldots\&\; P(!)_n x)) \equiv (\exists x)[(E!x \;\&\; P(!)_i x) \;\&\ldots\&\; P(!)_n x) \;\&\; (\forall y)[(P(!)_i y \;\&\ldots\&\; P(!)_n y) \equiv (x =_e y)] \;\&\; P(!)_1 x]]$$

A more generalized version of definite description is required for intentional logic.

$$\text{(DD2)}\quad (\forall P_1) \ldots (\forall P_n)(\forall P!_1) \ldots (\forall P!_n)[P(!)_1(\text{℩}_m x(P_i x \;\&\ldots\&\; P_n x \;\&\; P!_i x \;\&\ldots\&\; P!_n x)) \equiv (\exists x)[(P_i x) \;\&\ldots\&\; P_n x) \;\&\; (\forall y)[(P(!)_i y \;\&\ldots\&\; P(!)_n y) \equiv (x =_{rf} y)] \;\&\; P(!)_1 x]]$$

(DD3) $(\forall P_1)\ldots(\forall P_n)(\forall P!_1)\ldots(\forall P!_n)[P(!)_1(\iota_m x(P_i x \mathbin{\&}\ldots\mathbin{\&} P_n x \mathbin{\&} P!_i x \mathbin{\&}\ldots\mathbin{\&} P!_n x)) \equiv (\exists x)[(P_i x) \mathbin{\&}\ldots\mathbin{\&} P_n x) \mathbin{\&} (\forall y)[(P(!)_i y \mathbin{\&}\ldots\mathbin{\&} P(!)_n y) \equiv (x =_i y)] \mathbin{\&} P(!)_1 x]]$

These alternative versions of the theory provide a choice of identity relations for the Meinongian definite descriptor. The intentional identity requirement in (DD3) limits the object of a particular description to objects that are intentionally identical, having all nuclear constitutive properties in common, including converse intentional or psychological properties. The referential identity requirement in (DD2) limits the object of a particular description to existent or nonexistent objects that are referentially identical in the sense that they share all non-converse-intentional constitutive nuclear properties. For present purposes, the identity relation of (DD3) is too strong, and (DD2) is offered as the correct theory of ι_m.

The definitions in (DD2) and (DD3) permit free assumption to consider thoughts about objects to which extranuclear properties are superadded, as in the *soseinlos* mountain and Russell's existent round square. But the semantics of definite description determine the denotation of an object only by the nuclear properties contained in the description, not by designatively superfluous extranuclear properties truly or falsely attributed to it. This is seen in the first domain-membership conjunct of the modified 'Russellian' three-part analysis in (DD2) and (DD3), where the extranuclear properties are eliminated in fixing descriptor reference. The properties predicated of definitely described objects can be nuclear or extranuclear, as indicated by the exclamation enclosed in parentheses in the description under analysis, and in the final predication component of the analysis. This provides a fully general semantic principle for the denotation of any definite description.

In a sense, (DD2) embodies a more demanding set of descriptor conditions than Russell's (DD1). When Meinongian description theory is formulated as (DD2) or (DD3) in terms of referential or intentional identity, it places higher demands on what can count as *the* object of a certain kind, since intentional or referential identity implies extensional identity, but not conversely. Russell's theory on the other hand imposes the more stringent requirement that definitely described objects to which properties are truly predicated actually exist. Meinongian description theory requires only that descriptors designate objects, whether existent or nonexistent, provided that the uniqueness and predication conditions are fulfilled. The domain membership principle of the logic is satisfied by any term or grammatically well-formed definite description.

It follows for $(1 \leq i \leq n)$ that:

(T59) $(\forall P(!)_1) \ldots (\forall P(!)_n)(P(!)_1(\iota_r x(P(!)_i x \; \& \ldots \& P(!)_n x)) \supset$
$(\forall P(!)_1) \ldots (\forall P(!)_n)(P(!)_1(\iota_m x(P(!)_i x \; \& \ldots \& \; P(!)_n x)))$

But not conversely:

(T60) $\sim[(\forall P(!)_1) \ldots (\forall P(!)_n)(P(!)_1(\iota_m x(P(!)_i x \; \& \ldots \& \; P(!)_n x)) \supset$
$(\forall P(!)_1) \ldots (\forall P(!)_n)(P(!)_1(\iota_r x(P(!)_i x \; \& \ldots \& \; P(!)_n x)))]$

(T61) $(\exists P(!)_1) \ldots (\exists P(!)_n)(P(!)_1(\iota_m x(P(!)_i x \; \& \ldots \& \; P(!)_n x)) \; \&$
$\sim(\forall P(!)_1) \ldots (\forall P(!)_n)(P(!)_1(\iota_r x(P(!)_i x \; \& \ldots \& \; P(!)_n x)))$

To see informally that (T60) and (T61) are theorems, let '$P_i(\iota_m x(P_j x))$' represent the sentence 'The$_m$ golden mountain is golden'. This is obviously true in Meinongian logic, though 'The$_r$ golden mountain is golden' is false.

The intentional theory of definite description may be applied to Russell's problem of the existent round square, and to a strengthened version of the problem, previously characterized as the problem of the *soseinlos* mountain. The *Sosein* function is defined in (D13):

$$(\forall x)(\forall P_1) \ldots (\forall P_n) \ldots (S(x) = \{P_1, \ldots, P_n, \ldots\} \equiv (P_1 x \; \& \ldots \& \; P_n x \ldots))$$

By instantiation this yields:

$$S(\iota_m x(\textit{golden-mountain}(x))) = \{\textit{goldenness, mountainhood}\} \equiv (\textit{Golden}(\textit{golden-mountain})) \; \& \; \textit{Mountainous}(\textit{golden-mountain}))$$

On the assumption of the analytically true antecedent that $S(\iota_m x(\textit{golden-mountain}(x))) = \{\textit{goldenness, mountainhood}\}$, it is implied that the golden mountain is golden and a mountain.

Russell's objection about the existent round square can be made formally precise as a problem about the *Sosein* function applied to a definite description. Russell wanted to know whether the existent round square is existent. In a way, he is asking if there should not be a distinction between kinds of properties like being existent on the one hand, and roundness or squareness on the other. This need is met in the distinction between nuclear and extranuclear properties. The most straightforward answer to Russell's objection is that *no* object, not even an existent object, has the property of being existent as part of its *Sosein*. The *Sosein* of an object contains only the object's nuclear constitutive properties, and none of its extranuclear nonconstitutive properties. Since existence is an extranuclear property, the existent round square does not have the property of being existent in its *Sosein*, and is decidedly nonexistent by virtue of its metaphysically incompatible complementary nuclear properties of being both round and square. The expression 'The$_m$ existent round square is existent' has the form, '$E!x(\iota_m(E!x \,\&\, Rx \,\&\, Sx))$'. As such it is clearly false, even on ι_m principles. This does not mean that the term, 'The$_m$ existent round square' does not designate an object or cannot be part of an assumption. But by strict enforcement of the nuclear-extranuclear property distinction, it does not designate an object distinct from or other than 'The$_m$ round square'. This is clearly reflected in (DD2), where any extranuclear properties in the description drop out on analysis in determining whether an object with the exclusively nuclear properties contained in the description belongs to the Meinongian domain. It remains true that the$_m$ existent round square does not exist, so that the extranuclear predication external to the description $\bar{E}!x(\iota_m(E!x \,\&\, Rx \,\&\, Sx))$ holds by satisfaction of the third analysis component, $(\forall P_1)\ldots(\forall P_n)(\forall P!_1)\ldots(\forall P!_n)$ $[\ldots P(!)_1x\ldots]$. The proposition in Russell's problem, $E!x(\iota_m(E!x \,\&\, Rx \,\&\, Sx))$, is therefore false.

The exclusion of extranuclear properties from the *Soseine* of objects does not threaten free assumption. Nor does it endanger the intentionality thesis. There is no reason to believe that the description fails to designate an object. It is just that the object designated is referentially identical to the (plain, unadorned round square) object designated by the term without superaddition of constitutively and designatively superfluous extranuclear existence predicates. Extranuclear properties are not assumptible, so to assume that there is an existent round square is semantically no different than to assume that there is a (plain, unadorned) round square.

Objections similar to Russell's can be raised for any extranuclear property. In their formidable strengthened versions, these criticisms involve the object theory in deep paradox if the range of the *Sosein* function is not limited to nuclear properties. The problem of the *soseinlos* mountain is a difficulty of this kind that is similarly resolved. It is an extranuclear property of an object that it has a particular *Sosein*. To say that the *soseinlos* mountain is *soseinlos* is not to say that it has the *Sosein* of being *soseinlos*, but rather that its '*Sosein*' contains no properties. If the *soseinlos* mountain is *soseinlos*, then it may have the extranuclear property of being *soseinlos*, being an object, and so on. But this does not introduce any nuclear properties into its supposedly empty *Sosein*. The *soseinlos* mountain is not *soseinlos* in any case, but has the *Sosein* of being a mountain. To assume that there is a *soseinlos* mountain is not to be semantically directed toward a *soseinlos* object, but toward an incomplete nonexistent mountain to which the superadded nonconstitutive extranuclear property of being *soseinlos* is falsely attributed.

The problem of the *soseinlos* mountain is really no different than Russell's problem of the existent round square. In each, the misguided criticism involves an attempt to attribute an extranuclear property as one of the object's assumptible properties. The range restriction of the *Sosein* function to nuclear properties entails that 'the *soseinlos* mountain' does not designate an object distinct from that designated by 'the mountain', 'the existent mountain', 'the nonexistent mountain', 'the impossible mountain', or 'the maximally impossible mountain'.

The 'problem' of the *soseinlos* mountain poses no real difficulty for Meinong's theory. But it suggests another class of difficulties involving predications of extranuclear properties to definite descriptions that do not contain or explicitly make reference to any nuclear properties. Consider the proposition, 'The$_m$ nonexistent object is nonexistent', $\overline{E}!x(\iota_m(\overline{E}!x))$. There is a temptation to regard the predication as true, but of course it is false, since it fails the uniqueness requirement; the Meinongian semantic domain contains indefinitely if not infinitely many distinct nonexistent objects. It may be useful to compare the construction with another proposition that might also mistakenly be judged true, 'The$_r$ orangutan is an orangutan.' This is false even on Russell's definite description theory, by virtue of the existence (at time of writing) of multiple existent orangutans. The same is true of other attempts to designate an object by definite description without reference to nuclear properties.

The only conceivable exception might be the maximally impossible object, which has already been proved (T44) to be referentially unique. We should therefore consider the proposition, 'The maximally impossible object is maximally impossible', expressed by analogy with the previous example as $Maximp!(\iota_m(Maximp!x))$. This proposition is true, but it poses no real difficulty for the Meinongian semantics of definite description in (DD2), because far from having no nuclear properties, the maximally impossible object according to definition (D7) has every nuclear property and its complement. By equivalence, uniqueness via referential identity, and the fact that the external extranuclear predication *Maximp*! is satisfied, it follows by (DD2) that the maximally impossible object is indeed maximally impossible.

Russell's theory of descriptions has been so influential in the widespread analytic disapprobation of Meinong's object theory that it may be worthwhile to conclude by considering an argument against Russell in support of Meinongian description theory. Consider the proposition, 'The winged horse is mythological'. Intuitively, the proposition is true. Let '*W*' represent the property of being a winged horse, and '*M*!' the property of being mythological. Then on Russell's analysis in the recommended notation the proposition reads:

(1) $M!(\iota_r x(Wx)) \equiv (\exists x)[(E!x \ \& \ Wx) \ \& \ (\forall y)(Wy \equiv (x =_e y)) \ \& \ M!x]$

The interpretation is unsound since it converts a true into a false proposition. The biconditional fails and the equivalence is rendered false because the existence conjunct does not hold.

Defenders of Russell's theory will not hesitate to point out that there is something special about the predicate 'mythological' on which the counterexample turns. For the winged horse to be mythological is for it to be nonexistent (and described in a myth or to have the words 'the winged horse' or their equivalents inscribed in the writings of storytellers). If for convenience we ignore the second component concerning linguistic ascent or inscriptional occurrence, then to say that the winged horse is mythological is just to say that the winged horse does not exist. Parting the waters of surface grammar, the first step toward a correct analysis of the proposition might then be:

(2) $M!(\iota_r x(Wx)) \equiv \bar{E}!(\iota_r x(Wx))$

The equivalence is true, since both constituent propositions are true (assuming that the winged horse does not exist, and that nonexistence exhausts the property of being mythological). But when Russell-style analysis is applied to the definite description in the right half of the biconditional, the equivalence is counterintuitively made false, and with it the original proposition that the winged horse is mythological. Now we have:

(3) $\bar{E}!(\iota_r x(Wx)) \equiv (\exists x)[(E!x \ \& \ Wx) \ \& \ (\forall y)(Wy \equiv (x =_e y)) \ \& \ \bar{E}!x]$
(4) $M!(\iota_r x(Wx)) \equiv (\exists x)[(E!x \ \& \ Wx) \ \& \ (\forall y)(Wy \equiv (x =_e y)) \ \& \ \bar{E}!x]$

Russell's analysis suffers from the defect of reducing an intuitively true proposition about the mythology of the winged horse to the false proposition that a mythological winged horse exists. It further converts the contingent truth that the winged horse is mythological (an empirical question to be settled by explorers, scientists, historians, and literary scholars), to the logical inconsistency or necessary falsehood that a winged horse both exists and does not exist. Armed with Russell's theory of descriptions an investigator need only logically analyze propositions about the nonexistent creatures of myth and fiction ostensibly designated by definite descriptions in order to determine *a priori* that all such objects are *logically* impossible.

What is worse, if suitable precautions against standard inference rules are not taken, the analysis permits (by detachment from the truth that the winged horse is mythological) deduction of the logical inconsistency that there is something which exists and does not exist. This introduces semantic chaos of a much greater magnitude than anything envisioned in Meinong's position that there are nonexistent impossible objects whose *Soseine* contain both a nuclear property and its complement. Meinong's theory does not generate formal contradiction, provided that the independence thesis is restricted to nuclear predications, and the distinction between sentence negation and predicate complementation is observed. Russell's analysis on the imagined interpretation by contrast involves the contradictory extranuclear proposition that if the winged horse is mythological, then a winged horse exists and it is not the case that a winged horse exists.

Russell's description theory runs up against the dilemma that it must either interpret intuitively true propositions like 'The winged horse is mythological' as false, or else misconstrue certain contingently true or false propositions as logically necessary false. The problem lies in the extensionalist demand that definite description entails existence, reflected in the first conjunct of Russell's analysis. The difficulty is avoided in ontologically neutral Meinongian description theory, in which no existence requirement is made. Meinongian description theory is preferable to the extensionalist Russellian account, wherewith the historically impressive argumentative force of Russell's analysis against Meinong's object theory evaporates.

6. *Lambda Abstraction*

The theory of lambda abstraction provides a method for generating terms, typically producing propositionally complex property terms from predicate expressions. There are several abstraction operations, including set and relation abstraction, but all are reducible to property abstraction, which makes it convenient to consider property abstraction as abstraction *per se.*

The notation is enlarged to include a defined lambda operator 'λ', that binds object variables in much the same way as a quantifier or the definite description operators 'ι_m' and 'ι_r'. If p is an otherwise well-formed formula that contains n free object variables $x_1, \ldots, x_n$, then $\lambda x_1 \ldots x_n[p]$ is its lambda transform.[7] Since lambda abstracts are terms, like definite descriptions, that designate complex 'propositional' properties, they may be nuclear or extranuclear, and must accordingly be distinguished as $\lambda x[Px]$ or $\lambda x[P!x]$ abstracts. Compound abstracts containing even one extranuclear property are themselves extranuclear, $\lambda x[\ldots Px \ldots P!x \ldots]$; they are nuclear if and only if the properties to which the lambda operation is applied are exclusively nuclear.

[7] Alonzo Church, *The Calculi of Lambda Conversion* [1941]. Parsons, *Nonexistent Objects*, pp. 103–11. Zalta, *Abstract Objects*, pp. 18–9.

The following restricted equivalence principles for lambda abstracts are given in truth functional terms as definitions of the lambda operator. The predication contexts in truth functional combinations indicated by the ellipses may be nuclear or extranuclear, making the abstract nuclear or extranuclear by the above requirement.

(L1) $E!(\lambda x_1 \ldots x_n[\ldots x_1 \ldots x_n \ldots]) \supset$
$(\forall y_1) \ldots (\forall y_n)(\lambda x_1 \ldots x_n[\ldots x_1 \ldots x_n \ldots]y_1, \ldots, y_n \equiv (\ldots y_1 \ldots y_n \ldots))$

The equivalence of lambda abstracts to nuclear, extranuclear, or mixed predications is restricted to existent abstracts. The unrestricted standard counterpart is:

$$(\forall y_1) \ldots (\forall y_n)(\lambda x_1 \ldots x_n[\ldots x_1 \ldots x_n \ldots]y_1, \ldots, y_n \equiv (\ldots y_1 \ldots y_n \ldots))$$

The restriction is necessary to avoid syntactic and formal semantic paradox. The solution which this makes possible is described below in Section 9, 'Meinongian Mathematics and Metamathematics'. The paradoxes, interpreted so as to require abstraction elimination at some stage of their derivation, are blocked by the restriction of the elimination principle to existent abstracts. When the inconsistency is deduced in the attempt to prove a paradox, it reflects back on the false assumption that the abstract exists, required for valid detachment of the corresponding predication. This establishes by indirect proof that the abstract which would otherwise lead to paradox does not exist, precluding it from applications of abstraction elimination. It also provides an indirect proof criterion for distinguishing existent from nonexistent abstracts, where nonexistent abstracts are just those that lead to contradiction. This method is available in Meinongian logic only because it permits the intelligible designation of nonexistent objects, including nonexistent properties.

There is a reduction of lambda abstracts to definite descriptions in extensional logic which does not hold without modification in Meinongian object theory. The standard transformation states, in the simplest unary case:

$$(\forall x)(\lambda y[\ldots y \ldots]x \equiv \imath_r z(\forall y)(zy \equiv (\ldots y \ldots))x)$$

For nonexistent abstracts, the principle is amended to require Meinongian definite description.

$$(\forall x)(\lambda y[\ldots y \ldots]x \equiv \imath_m z(\forall y)(zy \equiv (\ldots y \ldots))x)$$

In keeping with the existence restriction on abstraction equivalence, it is accordingly stipulated that:

(L2) $E!x(\iota_m z(\forall y)(zy \equiv (\ldots y \ldots))) \supset$
$(\forall x)(\iota_m z(\forall y)(zy \equiv (\ldots y \ldots))x \equiv (\ldots x \ldots))$

The complete reduction states in the general case:

(L3) $E!(\iota_m z(\forall y_1)\ldots(\forall y_n)[zy_1 \ldots y_n \equiv (\ldots y_1 \ldots y_n \ldots)]) \supset$
$(\forall x_1)\ldots(\forall x_n)\ (\lambda y_1 \ldots y_n [\ldots y_1 \ldots y_n \ldots]\ x_1, \ldots, x_n \equiv$
$\iota_m z\ (\forall y_1)\ldots(\forall y_n)\ [zy_1 \ldots y_n \equiv (\ldots y_1 \ldots y_n \ldots)]\ x_1 \ldots x_n\ (n \geq 1)$

This translates the existence restriction on abstraction equivalence to the reduction of lambda abstracts into Meinongian definite descriptions.

7. Alethic Modality

Meinong's object theory provides the basis for an informal modal logic. The semantics of Meinongian objects describe properties of actual, merely possible, and metaphysically impossible intentional objects. This is a *de re* theory of Meinongian modalities, involving the modal status of objects of thought (though not every Meinongian '*res*' *cogitabilis* exists, subsists, or has any mode of being in any logically possible world). Meinong developed an elaborate informal theory of possibility and probability that is different in many ways from contemporary model set theoretical semantics. To formally express *de dicto* and *de re* modalities in a possible worlds context it is necessary to extend Meinong's object theory to construct a Meinongian counterpart of standard non-object-theoretical modal logic.

There have been several attempts to develop modal Meinongian logics. But these do not always take sufficient account of the ontic peculiarities of Meinongian systems, and thereby fail to demonstrate semantic connections between the Meinongian domain and model set theoretical operations on domains, worlds, and models. Without this philosophical groundwork modal Meinongian logic remains an empty formalism that cannot contribute to a more thorough understanding of Meinong's object theory.

The semantic structures of standard systems of alethic modal logic standardly involve a Henkin-type recursive procedure for assembling maximally consistent sets of propositions, each of which constitutes or at least completely describes what for heuristic purposes is sometimes referred to as a logically possible world.[8] A model for standard modal logic is an ordered

[8] For convenience maximally consistent sets of propositions are referred to as 'worlds'. The problem of whether nonactual logically possible worlds exist has motivated attempts to elim-

quadruplet $<\Sigma,\Gamma,R,V>$, consisting of the set Σ of all maximally consistent sets of propositions, $\Sigma = \{W_1, W_2, W_3, \ldots\}$; an element or member $\Gamma \in \Sigma$, sometimes distinguished as or as representing the actual world; a (usually minimally reflexive) relation R on Σ; and a valuation function V, which for $V(p, W_i)$ assigns truth value T or F to each proposition p of each maximally consistent set of propositions or world $W_i \in \Sigma$ $(i \geq 1)$. For truth functionally complex propositions, $V(\sim p, W_i) = T$ if and only if $V(p, W_i) = F$; $V((p \supset q), W_i) = T$ if and only if $V(p, W_i) = F$ or $V(q, W_i) = T$. Logical necessity is defined on the Leibnizian conception as truth in every logically possible world, $V(\Box p, W_i) = T$ if and only if $V(p, W_j) = T$ for every W_j such that $R(W_i, W_j)$. Logical possibility is reducibly defined in terms of necessity, $\Diamond p \equiv \sim\Box\sim p$ (or conversely by duality; $\Box p \equiv \sim\Diamond\sim p$, where $V(\Diamond p, W_i) = T$ if and only if $V(p, W_j) = T$ in at least some W_j such that $R(W_i, W_j)$). Relation R is often interpreted as world-accessibility. When a world W_j is accessible from world W_i, then any proposition true in W_j is logically possible in accessible world W_i. Distinct systems of modal logic are semantically determined by distinct models with distinct accessibility relations, such as combinations of reflexivity with symmetry, transitivity, and other more exotic variants.

Quantificational or predicate modal logic is interpreted by means of an expanded semantic model, an ordered quintuplet $<\Sigma,\Gamma,R,D,V>$, in which Σ, Γ, and R are as before, and where D is a function which for $D(W_i)$ assigns a domain of existent objects to each world $W_i \in \Sigma$, and valuation function V assigns a set of n-tuples of members of $D(W_i)$ to n-ary predicate 'P^n' if $n > 0$, and otherwise if $n = 0$ assigns T or F to P^n in $V(P^n, W_i)$.

Truth functional valuations are defined as before for propositional connectives. If $p = P^n x_1 \ldots x_n$, then $V(p, W_i) = T$, relative to an assignment of $a_1, \ldots, a_n$ to the x_i if and only if the n-tuple $a_1, \ldots, a_n \in V(P^n, W_i)$; if $p = (\forall x)q(x, y_1, \ldots, y_n)$, then $V(p, W_i) = T$ relative to an assignment of $b_1, \ldots, b_n$ to the y_i if and only if $V(q(x, y_1, \ldots y_n), W_i) = T$ for every assignment of a member $d \in D(W_i)$ to x.[9] Axiom schemata are devised to assure convergence of semantic models and deductive inference methods for logically valid propositions in each modal system.[10]

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inate reference to worlds in standard modal semantics. See Hugues Leblanc, "On Dispensing with Things and Worlds" [1973], pp. 241—59. In Meinongian semantics, logically possible worlds need not exist or subsist in order meaningfully to enter into interpretations of modal logic.

9 The model set theoretical semantics are derived from Saul A. Kripke, "A Completeness Theorem in Modal Logic" [1959], pp. 1—14; "Semantical Analysis of Modal Logic I, Normal Propositional Calculi" [1963], pp. 67—96; "Semantical Considerations on Modal Logic" [1963], pp. 83—94.

10 Axioms for standard modal systems are found in Robert Feys, "Les logiques nouvelles des

Modal Meinongian logics can now be similarly defined. Non-quantificational versions have precisely the same kind of semantic model, but are different because of differences in the propositions included in and excluded from their worlds. In Meinongian semantics, the proposition that the round square is round is true, and so belongs to maximally consistent sets of true propositions in modal Meinongian semantic models, but not to the models of standard non-Meinongian modal logics.[11] The independence thesis in Meinongian, unlike standard propositional semantics, permits the true predication of properties to nonexistent objects. A further distinction that complicates modal Meinongian logics occurs because of the three-valued interpretation of Meinongian propositional logic for some nuclear predications to indeterminate nonexistent objects.[12] The proposition that an incomplete Meinongian object has (or does not have) a (nuclear) property for which the object is indeterminate (such as 'Macbeth spoke Italian', 'Macbeth did not speak Italian') is most naturally classified as neither true nor false but undetermined in truth value. Standard propositional semantics in model set theoretical interpretations of standard modal logics on the contrary are classically bivalent. A maximally consistent set of propositions in a modal Meinongian model might include the proposition that not every proposition is true or false, which no standard model would contain. There are also true propositions of the models of standard modal logic that are not true and therefore not part of the models of modal Meinongian logic, such as the proposition that every object exists, or that every proposition is true or false. Quantificational modal Meinongian logic also parallels to some extent the formalization of standard quantificational modal logic. But here important differences emerge. The domain function D^m of a modal Meinongian quantificational model assigns the same domain consisting of both existent and nonexistent Meinongian objects to each Meinongian world $W^m_i \in \Sigma^m$. This guarantees uniform population or homogenous distribution of objects across every logically possible world in every model for each distinct system of modal Meinongian logic. The identity of Meinongian domains and nonselective occurrence of Meinongian objects in every world of every model has impor-

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modalitiés" [1937], pp. 517—53; [1938], pp. 217—52. Kurt Gödel, "Eine Interpretation des intuitionistischen Aussagenkalküls" [1933], pp. 34—40. Boleslaw Sobocinski, "Note on a Modal System of Feys-von Wright" [1953], pp. 171—78. C. I. Lewis and C. H. Langford, *Symbolic Logic* [1932]. See G. E. Hughes and M. J. Cresswell, *An Introduction to Modal Logic* [1980], pp. 31, 46, 49, 58. E. J. Lemmon (with Dana Scott), *An Introduction to Modal Logic* [1977], pp. 20—78.

11 Meinong, "The Theory of Objects", p. 82.

12 Ibid., pp. 83—6. Lambert, *Meinong and the Principle of Independence.*

tant formal and philosophical consequences. The result of these distinctions is that logical necessity, possibility, and impossibility, do not coincide in standard and Meinongian modal logics. Standard modal logics cannot embed and are not embeddable in modal Meinongian logics.

There is a plethora of systems of modal logic, just as there is a continuum of inductive methods, and of standard and nonstandard deductive logics.[13] It would not be appropriate to undertake the formalization of each and every system of modal Meinongian logic, since there are indefinitely many. For most philosophical, scientific, and mathematical purposes, only a few modal logics are needed. It will therefore suffice to outline Meinongian counterparts of the four most common and useful systems of modal logic, and to provide Meinongian semantic models for their interpretation. Modal Meinongian logic in its formal treatment of extranuclear necessity, possibility, and impossibility, is part of the classically bivalent extranuclear subtheory of the otherwise nonstandardly three-valued Meinongian logic. The modal propositions of modal Meinongian logic are exclusively either true or false, even though they are about or involve modal operations on at least some nuclear predications that are neither true nor false but undetermined in truth value. The nonmodal logical truths of object theory are also truths of every modal Meinongian logic.

The four basic systems of alethic modal logic are T (Feys-Gödel), S_4 (Lewis), S_5 (Lewis), and B ('Brouwersche'). Listed here are characteristic definitions and inference principles for the four corresponding systems of modal Meinongian logic.

Axioms and Necessitation Rule

If p, q are wffs of O:

T^m (Meinongian variant of Feys-Gödel T)

(M1) $\Box p \equiv {\sim}\Diamond{\sim}p$

(M2) $\Diamond p \equiv {\sim}\Box{\sim}p$

(M3) $(p \dashv q) \equiv \Box(p \supset q)$

(M4) $\Box p \supset p$

(M5) $\Box(p \supset q) \supset (\Box p \supset \Box q)$

(NR) $\vdash p \supset \vdash \Box p$

[13] D. Paul Snyder, *Modal Logic and its Applications* [1971], pp. 166—89.

S_4^m(Meinongian variant of Lewis S_4)
(M1)—(M5)—(NR)
(M6) $\Box p \supset \Box\Box p$

S_5^m(Meinongian variant of Lewis S_5)
(M1)—(M6)—(NR)
(M7) $\Diamond p \supset \Box\Diamond p$

B^m (Meinongian variant of Brouwersche system B)
(M1)—(M5)—(NR)
(M8) $p \supset \Box\Diamond p$

The inference structures of these systems are the same as those of their corresponding standard non-Meinongian modal logics. Differences between the two kinds of systems are hidden away in the formal semantics. The models of modal Meinongian logic constitute a DeMorgan lattice. For any Meinongian worlds $W^m{}_i$, $W^m{}_j$, $W^m{}_k \in \Sigma^m$, the following conditions are obviously satisfied.[14]

$$W^m{}_i \cap W^m{}_i = W^m{}_i$$
$$W^m{}_i \cup W^m{}_i = W^m{}_i$$
$$W^m{}_i \cap W^m{}_j = W^m{}_j \cap W^m{}_i$$
$$W^m{}_i \cup W^m{}_j = W^m{}_j \cup W^m{}_i$$
$$W^m{}_i \cap (W^m{}_j \cap W^m{}_k) = (W^m{}_i \cap W^m{}_j) \cap W^m{}_k$$
$$W^m{}_i \cup (W^m{}_j \cup W^m{}_k) = (W^m{}_i \cup W^m{}_j) \cup W^m{}_k$$
$$W^m{}_i \cap (W^m{}_i \cup W^m{}_j) = W^m{}_i \cup (W^m{}_i \cap W^m{}_j) = W^m{}_i$$

This makes it possible to define Boolean set theoretical relations on the lattice of all Meinongian worlds or maximally consistent sets of Meinongian propositions.[15]

Truth valuation $V^m(p, W^m{}_i) = T\ (F \text{ or } U)$ if and only if proposition p has Meinongian truth valuation $V^m(p) = T\ (F \text{ or } U)$ in world $W^m{}_i$. The modal Meinongian semantic models for T^m, $S_4{}^m$, $S_5{}^m$, and B^m can be formally defined. Combinations of accessibility relations holding between worlds and propositions true in worlds within a model are indicated by '+'.

[14] Garret Birkoff, *Lattice Theory* [1967], pp. 244—45.
[15] Rescher and Robert Brandom, *The Logic of Inconsistency: A Study in Non-Standard Possible-World Semantics and Ontology* [1979], pp. 92—8, 158—59.

T^m	$<\Sigma^m, \Gamma^m, \mathit{Reflexivity}, V^m>$
$S_4{}^m$	$<\Sigma^m, \Gamma^m, \mathit{Reflexivity}+\mathit{Transitivity}, V^m>$
$S_5{}^m$	$<\Sigma^m, \Gamma^m, \mathit{Reflexivity}+\mathit{Transitivity}+\mathit{Symmetry}, V^m>$
B^m	$<\Sigma^m, \Gamma^m, \mathit{Reflexivity}+\mathit{Symmetry}, V^m>$

Nonstandard truth valuations for primitive propositional connectives negation and the conditional are defined. $V^m(\sim p, W^m{}_i) = T$ if and only if $V^m(p, W^m{}_i) = F$; $V^m(\sim p, W^m{}_i) = U$ if and only if $V^m(p, W^m{}_i) = U$; $V^m((p \supset q), W^m{}_i) = T$ if and only if $V^m(p, W^m{}_i) = F$, or $V^m(p, W^m{}_i) = T$ and $V^m(q, W^m{}_i) = T$ or $V^m(p, W^m{}_i) = U$ and $V^m(q, W^m{}_i) = U$; $V^m((p \supset q), W^m{}_i) = F$ if and only if $V^m(p, W^m{}_i) = T$ and $V^m(q, W^m{}_i) = F$; $V^m((p \supset q), W^m{}_i) = U$ if and only if $V^m(p, W^m{}_i) = T$ $V^m(q, W^m{}_i) = U$, or $V^m(p, W^m{}_i) = U$ and $V^m(q, W^m{}_i) = F$.

Modal truth conditions under V^m can be described in a completely general way for any system of modal Meinongian logic. Let Σ^m represent the set of all maximally consistent sets of propositions in modal Meinongian theory m, and let T^m, F^m, U^m represent the truth values of propositions in m. Modal Meinongian theory m can be defined by its corresponding model. These conventions save rewriting the truth value conditions for each system of modal Meinongian logic when only the accessibility relations of a particular model differ.

The following simplified principles for alethic modal Meinongian operators may be advanced. Truth conditions of the semantic model serve the same purpose as Kripke's 'models', intercalating a truth value function into a so-called normal model. Quantification in $(\forall W^m)(\ldots W^m \ldots)$ ranges over the Meinongian worlds of a particular Meinongian model. Relation R is any specialized (complex of) accessibility relation(s). Modal expressions in which a necessity operator applies to a proposition are classically bivalent, logically equivalent to corresponding extranuclear necessity predications.

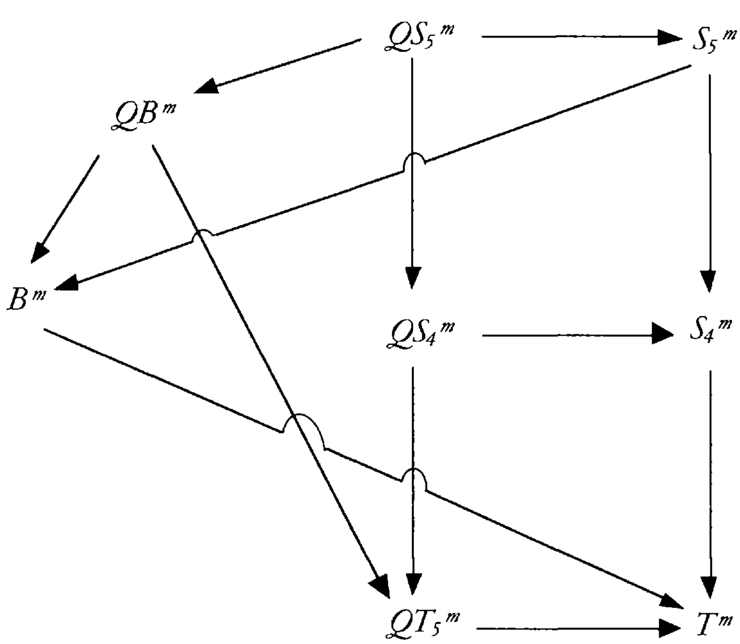
$$V^m(\Box p) = T^m \equiv (\forall W^m)[(W^m \in \Sigma^m \ \& \ R(\Gamma^m, W^m)) \equiv V^m(p, W^m) = T]$$
$$V^m(\Box p) = F^m \equiv (\exists W^m)[(W^m \in \Sigma^m \ \& \ R(\Gamma^m, W^m)) \ \& \ V^m(p, W^m) \neq T]$$

Quantificational versions of modal Meinongian logic are obtained by adding the characteristic axioms of nonquantificational modal systems to quantificational Meinongian object theory. It is important to recall that 'existential' quantification in Meinongian logic has no real existential or ontic import, but merely indicates membership in the Meinongian domain of existent and nonexistent objects.[16]

[16] That the $\exists$ 'existential' quantifier has no real existential or ontological import in Meinongian semantics is also affirmed by Parsons, *Nonexistent Objects*, pp. 69—70, and Routley, *Exploring Meinong's Jungle and Beyond*, p. 174.

A quantificational model for quantificational modal Meinongian logic is an ordered quintuplet $<\Sigma^m, \Gamma^m, R, D^m, V^m>$, in which Σ^m, Γ^m, and R are as before in semantic models for nonquantificational modal Meinongian logic. Meinongian domain function D^m in $D^m(W^m_i)$ uniformly assigns the same domain of existent and nonexistent Meinongian objects to each and every world $W^m_i \in \Sigma^m$. Meinongian valuation function V^m combines the previously described effects of V in $<D,I,V>$ for nonmodal quantificational Meinongian logic, and of V^m in $<\Sigma^m, \Gamma^m, R, V^m>$ for nonquantificational modal Meinongian logic.

Inference relations between quantificational and nonquantificational systems of modal Meinongian logic are represented diagrammatically. The arrow indicates a transitive theoremhood containment relation, where $L \rightarrow L'$ means that system L contains all the theorems of system L' (and may contain more). Here at a glance are the formal interconnections among alternative modal Meinongian logics.



The formal systems T^m, $S_4{}^m$, $S_5{}^m$, and B^m are distinct nonquantificational modal Meinongian logics. But the most straightforward unqualified quantificational versions of these systems are distinct only because of their inferentially distinct nonquantificational fragments. The nonvacuously quantified theorems of quantificational modal Meinongian logics QT^m, $QS_4{}^m$, $QS_5{}^m$, and

QB^m are identical. Even such standardly distinguishing propositions as $(\forall x)(\Diamond P(!)x \supset \Box\Diamond P(!)x)$, true in ordinary S_5, but not in ordinary T or S_4, are true in every quantified modal Meinongian system. The semantic collapse of quantificational modal Meinongian logics is determined by quantification over an identical domain of existent and nonexistent Meinongian objects in the models of unqualified quantificational systems. This dissolves the semi-permeable accessibility membranes of accessibility relations that otherwise hold between worlds in standard modal semantics.

Standard quantificational modal logics are inferentially distinguished on the basis among other things of whether or not they contain as theorems the Barcan or converse Barcan formulas.[17] The converse Barcan formula $\Box(\forall x)P(!)x \supset (\forall x)\Box P(!)x$ is a theorem of standard modal systems QT, QS_4, QS_5, and QB. But the Barcan formula $(\forall x)\Box P(!)x \supset \Box(\forall x)P(!)x$ is a theorem only of QS_5 and QB, and not of QT or QS_4.[18]

These inferential asymmetries do not provide a satisfactory method of distinguishing any of the quantificational systems of modal Meinongian logic. There is a difficulty in the construction of quantificational modal Meinongian logic which must now be resolved. The reason why the converse Barcan but not the Barcan formula is a theorem of most standard quantificational modal logics is more easily seen in the modal-quantificational duals of these propositions. The converse Barcan formula is logically equivalent to its dual, $(\exists x)\Diamond P(!)x \supset \Diamond(\exists x)P(!)x$. This standardly states that if there actually exists an entity that in some logically possible world has property $P(!)$, then it is possible or there is another accessible logically possible world in which there exists an entity that has property $P(!)$. But the Barcan formula $\Diamond(\exists x)P(!)x \supset (\exists x)\Diamond P(!)x$ under standard interpretation states that if in some logically possible world there exists an entity that has property $P(!)$, then there actually exists an entity that possibly or in some other accessible logically possible world has property $P(!)$. The truth of this proposition depends on the accessibility relations in the semantic models of particular systems of standard modal logic, since it need not follow that an entity that possibly exists or exists in some other world also exists in the actual world where possibly it has property $P(!)$. Accessibility gradients and uneven distribution of existent entities across logically possible worlds in standard modal semantics determine whether or not standard modal logics contain the Barcan or converse Barcan

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[17] Ruth Barcan Marcus, "A Functional Calculus of First Order Based on Strict Implication" [1946], pp. 1—16.

[18] Hughes and Cresswell, *An Introduction to Modal Logic*, p. 142. Lemmon, "Quantified S_4 and the Barcan Formula" (Abstract) [1960], pp. 391—92.

formulas as theorems. If there were a uniform distribution of existent entities in every world of every modal semantic model, if any entity existing in any logically possible world existed in every logically possible world, then the Barcan and converse Barcan formulas would be theorems of every standard system of quantified modal logic.[19]

In modal Meinongian logic there is an entirely uniform distribution of existent and nonexistent Meinongian objects in every Meinongian domain of every world of every modal Meinongian model, so that every modal Meinongian semantic model has precisely the same Meinongian domain. The combined biconditional $(\forall x)\Box P(!)x \equiv \Box(\forall x)P(!)x$ $(\Diamond(\exists x)P(!)x \equiv (\exists x)\Diamond P(!)x)$ under modal Meinongian interpretation says no more than that the domain of a logically possible world contains an existent or nonexistent Meinongian object with property $P(!)$ if and only if the domain of the actual world contains an existent or nonexistent Meinongian object which in some accessible logically possible world has property $P(!)$. The truth of the proposition is trivially guaranteed by the construction of modal Meinongian semantic models. The domains of the actual world and all other logically possible worlds are identical, and '$(\exists x)\Diamond P(!)x$' carries no real existential or ontological import in Meinongian quantificational semantics.[20]

As things stand, it is not possible validly to deduce the Barcan formula in quantificational modal Meinongian systems QT^m and $QS_4{}^m$. The Barcan formula is true in QT^m and $QS_4{}^m$ as determined by their modal quantificational semantic models, but the inference schemata of the logics are not powerful enough to derive the Barcan formula as a theorem. (The axioms of QT^m and $QS_4{}^m$ are the same as those of QT and QS_4, from which the Barcan formula standardly cannot be derived.)

The situation must be corrected to regain convergence of semantic and inference structures for QT^m and $QS_4{}^m$. The Barcan formula can be added as a nonlogical axiom to QT^m and $QS_4{}^m$ to produce QT^{m+} and $QS_4{}^{m+}$, without strengthening these systems to $QS_5{}^m$.[21] The validly deducible theorems of

19 Snyder, *Modal Logic and its Applications*, pp. 143—51. The modal semantic theories of some versions of logical atomism also posit a uniform distribution of existents across every logically possible world (usually in different terminology). See Wittgenstein, *Tractatus Logico-Philosophicus*, 2.014—2.0231. Wittgenstein's theory does not provide transworld uniform populations of complex existents.

20 The predominance of modal Meinongian versions of S_5 is suggested by Parsons, *Nonexistent Objects*, pp. 100—3. Zalta, *Intensional Logic and the Metaphysics of Intentionality*, pp. 61—7. Routley favors a quantificational version of Lewis' S_2 in *Exploring Meinong's Jungle and Beyond*, pp. 207—21.

21 Adding the Barcan formula to standard quantificational versions of T and S_4 without strengthening them to quantificational S_5 is proposed by Hughes and Cresswell, *An Introduction to Modal Logic*, p. 144.

QT^{m+} and $QS_4{}^{m+}$ will then have perfect congruity with their semantic models. QT^{m+} and $QS_4{}^{m+}$ accordingly must replace QT^m and $QS_4{}^m$ as the legitimate modal Meinongian counterparts of QT and QS_4. This undermines the inferential isomorphism between quantificational standard and modal Meinongian logics, but in a sense provides the most direct solution to the problem. Another method is to modify the semantic models for QT^m and $QS_4{}^m$ so that only the converse Barcan and not the Barcan formula remains true. This can be done by placing restrictions on the domain function D^m, limiting it to D^{m-}, which in $D^{m-}(W^m{}_i)$ assigns to $W^m{}_i$ a domain of existent objects only, rather than a full Meinongian domain of existent and nonexistent objects. This proposal also restores convergence of inferential and semantic structures to QT^{m-} and $QS_4{}^{m-}$, preserving intact the inferential isomorphism between quantificational standard and Meinongian modal logics. (None of these solutions to the semantic and inferential incongruities of QT^m and $QS_4{}^m$ over the truth and derivability of the unmodified Barcan formula require adjustment to the theoremhood containment relations among quantificational and non-quantificational modal Meinongian logics.)

The existential intent of the Barcan formula cannot be expressed in unqualified quantificational modal Meinongian logic except by an extranuclear existence predication:

$$\Diamond(\exists x)(E!x \;\&\; P(!)x) \supset (\exists x)(E!x \;\&\; \Diamond P(!)x)$$

This complex expression cannot be derived in modal Meinongian logic without supplementary addition of further specific nonlogical axioms about the nature of the extranuclear existence property $E!$. It remains to be seen whether and in which systems of quantificational modal Meinongian logic this version of the Barcan formula or its converse are theorems.

The unmodified converse Barcan formula can be proved in QT^m (and hence in $QS_4{}^m$, $QS_5{}^m$, and QB^m). The implementation of necessitation rule (NR) in a natural deduction environment is that whenever there is a proof of p (not subordinate to any other proof), then there is a proof of $\Box p$.[22]

(T63) $(\exists x)\Diamond P(!)x \supset \Diamond(\exists x)P(!)x$

1. $\Box(\forall x){\sim}P(!)x$
2. $\Box(\forall x){\sim}P(!)x \supset (\forall x){\sim}P(!)x$ (M4)
3. $(\forall x){\sim}P(!)x$ (1,2)

[22] Ibid., Appendix I, 'Natural Deduction and Modal Systems', p. 333.

4.	$\sim P(!)o_i$	(3)
5.	$\Box\sim P(!)o_i$	(1—4,NR)
6.	$(\forall x)\Box\sim P(!)x$	(5)
7.	$\Box(\forall x)\sim P(!)x \supset (\forall x)\Box\sim P(!)x$	(6)
8.	$\sim(\forall x)\Box\sim P(!)x \supset \sim\Box(\forall x)\sim P(!)x$	(7)
9.	$(\exists x)\Diamond P(!)x \supset \Diamond(\exists x)P(!)x$	(8,M1,M2)

The unmodified Barcan formula is derivable in unqualified quantificational modal Meinongian logic QS_5^m. The QS_5^m proof is unavailable in QT^m and QS_4^m, as indicated by appeal to characteristic QS_5^m axiom (M7) in steps (13) and (33). This completes discussion of the problem of quantifying into modal contexts in modal Meinongian logic.

(T64) $\Diamond(\exists x)P(!)x \supset (\exists x)\Diamond P(!)x$

1.	$(\forall x)\Box\sim P(!)x$	
2.	$\Box\sim P(!)o_i$	(1)
3.	$(\forall x)\Box\sim P(!)x \supset \Box\sim P(!)o_i$	(2)
4.	$\sim\Box\sim P(!)o_i \supset \sim(\forall x)\Box\sim P(!)x$	(3)
5.	$\Box(\sim\Box\sim P(!)o_i \supset \sim(\forall x)\Box\sim P(!)x)$	(1—4,NR)
6.	$\Box(\sim\Box\sim P(!)o_i \supset \sim(\forall x)\Box\sim P(!)x) \supset$ $(\Box\sim\Box\sim P(!)o_i \supset \Box\sim(\forall x)\Box\sim P(!)x)$	(M5)
7.	$\Box\sim\Box\sim P(!)o_i \supset \Box\sim(\forall x)\Box\sim P(!)x$	(5,6)
8.	$\sim\Box\sim(\forall x)\Box\sim P(!)x \supset \sim\Box\sim\Box\sim P(!)o_i$	(7)
9.	$\Diamond(\forall x)\Box\sim P(!)x \supset \sim\Box\sim\Box\sim P(!)o_i$	(8,M2)
10.	$\Box\sim P(!)o_i \supset \sim P(!)o_i$	(M4)
11.	$\sim\sim P(!)o_i \supset \sim\Box\sim P(!)o_i$	(10)
12.	$P(!)o_i \supset \Diamond P(!)o_i$	(11,M2)
13.	$\Diamond P(!)o_i \supset \Box\Diamond P(!)o_i$	(M7)
14.	$P(!)o_i \supset \Box\Diamond P(!)o_i$	(12,13)
15.	$\Box\Diamond P(!)o_i \supset \sim\Diamond\Box\sim P(!)o_i$	(M1,M2)
16.	$P(!)o_i \supset \sim\Diamond\Box\sim P(!)o_i$	(14,15)
17.	$\sim\sim\Diamond\Box\sim P(!)o_i \supset \sim P(!)o_i$	(16)
18.	$\Box\Diamond P(!)o_i \supset \Diamond P(!)o_i$	(M4)
19.	$\sim\Diamond P(!)o_i \supset \sim\Box\Diamond P(!)o_i$	(18)
20.	$\Box\sim P(!)o_i \supset \Diamond\Box\sim P(!)o_i$	(19,M1,M2)
21.	$\Diamond\Box\sim P(!)o_i$	(2,20)
22.	$\sim\sim\Diamond\Box\sim P(!)o_i$	(21)
23.	$\sim P(!)o_i$	(16,22)
24.	$(\forall x)\sim P(!)x$	(23)
25.	$\Diamond(\forall x)\Box\sim P(!)x \supset (\forall x)\sim P(!)x$	(24)

26.	$\Box(\Diamond(\forall x)\Box{\sim}P(!)x \supset (\forall x){\sim}P(!)x)$	(6—25,NR)
27.	$\Box(\Diamond(\forall x)\Box{\sim}P(!)x \supset (\forall x){\sim}P(!)x) \supset$ $(\Box\Diamond(\forall x)\Box{\sim}P(!)x \supset \Box(\forall x){\sim}P(!)x)$	(M5)
28.	$\Box\Diamond(\forall x)\Box{\sim}P(!)x \supset \Box(\forall x){\sim}P(!)x$	(26,27)
29.	$\Box{\sim}(\forall x)\Box{\sim}P(!)x \supset {\sim}(\forall x)\Box{\sim}P(!)x$	(M4)
30.	${\sim}{\sim}(\forall x)\Box{\sim}P(!)x \supset {\sim}\Box{\sim}(\forall x)\Box{\sim}P(!)x$	(29)
31.	$(\forall x)\Box{\sim}P(!)x \supset \Diamond(\forall x)\Box{\sim}P(!)x$	(30,M2)
32.	$\Diamond(\forall x)\Box{\sim}P(!)x$	(1,31)
33.	$\Diamond(\forall x)\Box{\sim}P(!)x \supset \Box\Diamond(\forall x)\Box{\sim}P(!)x$	(M7)
34.	$\Box\Diamond(\forall x)\Box{\sim}P(!)x$	(32,33)
35.	$\Box(\forall x){\sim}P(!)x$	(28,34)
36.	$(\forall x)\Box{\sim}P(!)x \supset \Box(\forall x){\sim}P(!)x$	(35)
37.	${\sim}\Box(\forall x){\sim}P(!)x \supset {\sim}(\forall x)\Box{\sim}P(!)x$	(36)
38.	$\Diamond(\exists x)P(!)x \supset (\exists x)\Diamond P(!)x$	(37,M2)

The possible existence of incomplete Meinongian objects presents a special problem for modal object theory semantics. Consider the proposition that the golden mountain is possible or possibly exists. The semantics for modal Meinongian logic interprets possible existence as existence in some world or worlds accessible to the actual world. In the actual world the golden mountain is incomplete, lacking many nuclear properties and their complements in its uniquely identifying *Sosein*. But in a world containing the golden mountain as actual and not a mere Meinongian object, the golden mountain exists and is complete, with a full selection of nuclear properties including exclusively every nuclear property or its complement, featuring especially the nuclear properties of being golden and a mountain.

This suggests that some worlds may contain complete existent objects that are incomplete in other logically possible worlds. There seems nothing paradoxical or metaphysically unacceptable about this. It is natural to suppose that if a square table had also been round, then instead of existing it would be an impossible round square table. If the round square table had not been square, it might exist. Again there appears to be no limit (beyond essential property or natural kind restrictions) to any combination of nuclear properties among possible, actually existent, or nonexistent Meinongian objects in different worlds. But it might be objected that this latitudinarian approach to transworld identity for incomplete and impossible Meinongian objects in the domains of alternative accessible Meinongian worlds implies that an impossible object like the round square is possible after all, in the sense that there are worlds in which the round square is not round or not square. If this were true, it might preclude the intelligible categorization of any Meinongian objects as impossible.

Routley has challenged the intuitive picture of transworld identity among existent, incomplete, and impossible Meinongian objects, by arguing that an object incomplete in a given world is essentially incomplete or incomplete in every logically possible world. He writes:

> Consider, for instance, the round squash: as a pure deductively (unclosed) object this is round and a squash and has no other properties. Thus it is incomplete, e.g. it is neither blue nor not blue. Hence it does not exist. Nor can *it* exist: to exist it would have to be completed, but any such completion is a different object.[23]

The modal Meinongian counterpart theory developed by Routley in accord with this criticism is like the standard counterpart modal logics described by Leibniz and David Lewis.[24] But Routley's version of counterpart modal Meinongian logic is different in that he seems to permit transworld identity of existent and nonexistent objects, provided that no existent object in a given world is nonexistent in another accessible logically possible world, or conversely. This posits an ordinary counterpart theory for Meinongian objects restricted to contingently existent or nonexistent objects.[25]

Routley's proposal contradicts well-entrenched beliefs about the possible existence of contingently nonexistent objects. When someone says that Pym in Edgar Allan Poe's *The Narrative of Arthur Gordon Pym* is possible, that he is a person who might have lived and had the adventures attributed to him in Poe's story, it is undoubtedly meant that the very same object described by Poe and not merely another relevantly like him is possible, even though Pym in the actual world is incomplete and indeterminate with respect to many nuclear properties and their complements. If it were true that actually incomplete objects are incomplete in every logically possible world, then modal object theory would be exceedingly uninteresting. It would then be possible only for actually existent or nonexistent objects to have different complete or incomplete sets of nuclear properties than they happen to have in the actual world (and even this might be prohibited by strict adherence to Routley's criterion). But if objects are identified and distinguished by the unique unordered sets of constitutive nuclear properties in their *Soseine*, then it remains at least a technical problem to explain how Meinongian objects could be incomplete or impossible in some logically possible worlds, but complete and existent in others.

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[23] Routley, *Exploring Meinong's Jungle and Beyond*, p. 247.

[24] G.W. Leibniz, *Discours de métaphysique* [1685]; *Correspondence with Arnauld* [1846]. David Lewis, *Counterfactuals* [1973].

[25] Routley, *Exploring Meinong's Jungle and Beyond*, pp. 247—53.

The difficulty is removed by indexing an object's nuclear properties to particular worlds.[26] An analogous problem arises for the indiscernibility of identicals over time. The objection is sometimes made that a man cannot be identical to his youthful self if the man is bald and the youth is not. But this is a superficial criticism of the identity principle overcome by requiring that properties are incompletely and incorrectly specified unless indexed to time. The man does not have the property of being bald *simpliciter*, but the property of being bald at time t. The youth does not have the complement of the property of being bald *simpliciter*, but has the complement of the property at time t' ($\neq t$). The indiscernibility of identicals is not contradicted by the example on this reformulation because both the old man and the youth have the properties of being bald at t and not bald at t'.

The same idea enables modal Meinongian logic to include objects in the domains of its semantic models that are complete in some worlds, but incomplete or even impossible in others. According to the world-indexing proposal, Arthur Gordon Pym does not simply have the nuclear property of being a shipwrecked cannibal, he has the nuclear property of being a shipwrecked cannibal in Meinongian world W^m_{Poe} (and other worlds of the modal Meinongian semantic model). Pym does not simply lack the nuclear property of speaking Italian, he lacks both this property and its complement in the actual world, and in some but not all alternative logically possible worlds. The world-indexing solution to the transworld identity problem for nonexistent Meinongian objects does not entail that the round square is not impossible, but only that the Meinongian object which in or relative to some logically possible worlds is an impossible round square is a possible round object in or relative to other logically possible worlds in which it is not square, and in other worlds a possible square object that is not round.[27]

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26 World-indexing is proposed as a solution to problems of transworld identity for standard modal logics by Alvin Plantinga, *The Nature of Necessity* [1974], pp. 92—7.

27 Meinong has a different approach to the possible existence of actually nonexistent objects that avoids the need for transworld identity of incomplete objects. Meinong argues that incomplete but possible objects have implexive being or are implected [*implektiert*] in existent or possible complete objects. The possibility of the incomplete golden mountain is explained on this proposal by the claim that all nuclear properties of the golden mountain are shared by another possible complete existent object, subsumed in its larger complete set of properties. The incomplete object is not literally a part of the possible complete object in which it is implected, but its possibility is accounted for by the claim that the possible complete object absorbs the incomplete object's smaller complement of nuclear properties as a subset. Meinong, *Über Möglichkeit und Wahrscheinlichkeit*, pp. 211—24. Findlay, *Meinong's Theory of Objects and Values*, pp. 168—70, 181—82, 209—15. Although Meinong's thesis is in some sense an alternative to transworld identity and counterpart modal semantics, it resembles

In this way, the very same object, the man described in Poe's tales, can correctly be said to be possible, or such that he might have existed in the actual world. He is an incomplete object in or relative to some worlds, but in others he exists and is fully determinate. Pym has both the incomplete set of properties completely characterized by Poe in the actual world, and the complete set of properties partially characterized by Poe in some of the fictional logically possible worlds in which Pym exists. By similar token, Edgar Allan Poe, though complete and existent in the actual world, is in some worlds a fictional, incomplete, and nonexistent but logically possible Meinongian object — in some worlds he is the literary invention of Pym![28]

8. The Sosein Paradox

The *Sosein* paradox was introduced in Part One, Chapter II. The properties of being an object referentially identical to its own *Sosein* and of being an object referentially nonidentical to its own *Sosein* are defined. They are presented as abstracts.

(S1) $o_i = \lambda x[S(x) = \{x\}]$
(S2) $o_j = \lambda x[S(x) \neq \{x\}]$

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counterpart theory in that the (incomplete) golden mountain is not literally identical to any possible complete object nor to any complete object in any logically possible world. The same arguments raised against counterpart semantics therefore also apply to Meinong's theory of implexive being.

28 It might be objected that the world-indexing solution to transworld identity of actually nonexistent objects invites a certain kind of confusion. Consider three worlds, W, W', W''. World W contains the round square table, T_1. By stipulation in W' it might lack the property of being square while gaining other compatible nuclear properties, so that in W' it exists as an actual complete round table, or at least as an incomplete but possible round table. In W'', the table might lack the property being round while gaining other compatible nuclear properties, so that in W'' it exists as an actual complete square table, or at least as an incomplete but possible square table. There presumably is also a nonexistent incomplete object T_2 that has just the nuclear properties of being round, square, and a table, in all three worlds. The round square table T_1 is arguably referentially identical to the round square table T_2 in W where they share all nuclear non-converse-intentional properties, but nonidentical to T_2 in W' and W'', where they do not. This would violate intuitive conceptions of identity, especially if 'T_1' and 'T_2' are supposed to be rigid designators. If we take the world-indexing approach seriously, then there is an easy solution to the apparent problem. The world-indexed properties of the two objects keep them distinct, where $T_1 = RST_W—RT_{W'}—ST_{W''}$ and $T_2 = RST_W—RST_{W'}—RST_{W''}$. These complex rigidly de-signative terms preserve transworld distinctions between T_1 and T_2, while accounting for their exact coincidence of properties in some logically possible worlds.

It appears to follow that:

1. $\lambda x[S(x) = \{x\}]o_j \equiv \lambda x[S(x) \neq \{x\}]o_j$
2. $(\exists y)(\lambda x[S(x) = \{x\}]y \equiv \lambda x[S(x) \neq \{x\}]y)$
3. $S(o_j) = \{o_j\} \equiv S(o_j) \neq \{o_j\}$
4. $(\exists x)(S(x) = \{x\} \equiv S(x) \neq \{x\})$

The proof is blocked by the requirement in (D13) that no extranuclear property, including especially the property of being referentially identical or nonidentical to a particular set of properties, can enter into an object's *Sosein*. The extranuclear property o_j or $\lambda x[S(x) \neq \{x\}]$ is not in the range of *Sosein* function S. This necessarily falsifies any application of the *Sosein* function in $S(\lambda x[S(x) \neq \{x\}]) = \{\lambda x[S(x) \neq \{x\}]\}$ (required for inference of the paradox), rendering any attempted proof of the *Sosein* paradox unsound. The informal refutation is reinforced by the existence restriction on abstraction equivalence. When inconsistency is deduced in a derivation of the paradox, it merely reflects back on the falsehood of the existence assumption for the abstract, $E!(\lambda x[S(x) \neq \{x\}])$. This undermines the paradox by falsifying the existence condition required for valid detachment of any formal antithetical self- and self-non-*Sosein* predications.

9. Meinongian Mathematics and Metamathematics

Meinong's object theory permits the true predication of constitutive nuclear properties of nonexistent objects, and implies that intentional objects have the constitutive properties attributed to them regardless or independently of whether or not they exist. The pure object considered in itself as constituted by its nuclear properties is homelessly placed beyond being and nonbeing.

Extensional logic and semantics by contrast limit the true predication of constitutive properties to existent objects. The restriction is enshrined in Russell's theory of definite descriptions, and has the metaphysically interesting consequence that mathematical objects must exist in order to admit true predications of constitutive properties.[29] That $2 = |\sqrt{4}|$ (the absolute value of $\sqrt{4}$) is not true in extensional semantics unless 2 and $|\sqrt{4}|$ (whatever their constitutive analysis) actually exist. If numbers and mathematical construc-

[29] Russell, "On Denoting". Russell's earlier account of descriptions, prior to his rejection of Meinong's object theory, appears in *Principles of Mathematics* [1938], pp. 62—5.

tions generally can be reduced to other entities, then those entities must exist as the ultimate referents to which true predications of constitutive mathematical properties attach, to the nonempty extensions of whose predicates mathematical objects necessarily belong. Mathematics in an extensional semantic framework standardly requires a realist ontology in which at least some abstract nonphysical nonmaterial and non-spatio-temporal entities exist.[30]

Quine has developed an extensional ontology in which mathematical objects are reduced to classes under operations, and in which classes therefore exist as irreducibly abstract mathematical entities.[31] This is undoubtedly among the most economical extensional ontologies capable of supporting true constitutive predications in mathematics. Quine is known for his theoretical and aesthetic preference for desert landscape ontology, and for his commitment to classical mathematics as an adjunct to modern physics.[32] But in Meinongian semantics it is possible to advance an even more austere ontology in which no abstract mathematical objects exist, but only concrete particulars or material spatio-temporal entities.

Meinong did not make this application of the theory, but instead endorsed a moderate realist account of the subsistence of at least some mathematical abstracta.[33] Yet it is easy to see that Meinong's realism is not strictly entailed by object theory, however compatible the addition. The central task in philosophy of mathematics is to explain the necessary truth of mathematical propositions. In extensional semantics this can only be done by acknowledging the existence of abstract mathematical objects, perhaps minimally classes. But in Meinongian object theory true attributions of constitutive properties can be made to nonexistent mathematical objects, leaving a nominalist-like ontology of concrete particulars immersed in a larger Meinongian extraontology of nonexistent mathematical objects alongside the notorious golden mountain and round square. There is no need even for classes actually to exist or subsist. Meinong did not plant a jungle — quite the opposite.[34]

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[30] Frege, *Die Grundlagen der Arithmetik: Eine logisch mathematische Untersuchung über den Begriff der Zahl* [1884]. Alfred North Whitehead and Bertrand Russell, *Principia Mathematica* [1927], p. 74. Carnap, *Meaning and Necessity*.

[31] Quine, *Methods of Logic* [1972], pp. 240—49; "Foundations of Mathematics" [1976], pp. 22—32; "Ontological Reduction and the World of Numbers" [1976], pp. 212—20; "New Foundations for Mathematical Logic" [1963]; *Word and Object*, pp. 262—70.

[32] Quine, "On What There Is", pp. 1—19; *Philosophy of Logic* [1970], pp. 85—6, 96—102.

[33] Meinong, "Zur Gegenstandstheorie" [1921]; "Über Gegenstände höherer Ordnung und deren Verhältnis zur inneren Wahrnehmung", pp. 181—271.

[34] Routley, *Exploring Meinong's Jungle and Beyond*, pp. 791—808.

Despite the allure of a nominalist Meinongian ontology, it is preferable to adopt a moderate realist theory in which most but not all abstract properties exist, even if mathematical objects are excluded. The distinction might be upheld by an argument to the effect that existent particulars must be material spatio-temporal entities, and that mathematical objects unlike universals are (abstract nonmaterial non-spatio-temporal) particulars.

If at least some properties exist, then a uniquely Meinongian solution can be given to forestall formal semantic and set theoretical paradoxes and Gödel-Church incompleteness and undecidability results. In Meinongian semantics, well-defined terms need not designate existent objects, and predicate terms need not represent existent properties. Existence-restricted principles for lambda abstraction make abstraction equivalence conditional on the existence of the abstract, or of the property designated by the abstraction. If the consequent equivalence does not hold, then the abstract or abstracted property does not exist. The intuitive rationale is that no object exists if inconsistency or metaphysical incompatibility obtains among its properties. When inconsistency is uncovered in the attempt to derive a paradox by diagonalization, the contradiction reflects back on the false assumption that the abstracted property exists. This precludes the abstract from legitimate applications of the abstraction elimination principle required to produce antinomy.

It remains to reduce formal semantic and set theoretical paradoxes and metatheoretical diagonalizations to abstraction formulations in which abstraction elimination must be invoked to derive inconsistency. The Gödel-Church metatheorems are first examined in untyped abstraction counterpart formulations, and then in their original type-sensitive arithmetizations. The reason for this unorthodox approach to the classical metatheorems will become apparent as the argument proceeds. All such constructions depend on denial of self-application, or denial of a special metatheoretical predication of a self-application.

(1) Semantic Paradox

$$Z = \lambda x[\sim(xx)]$$
$$(ZZ \vee \sim(ZZ)) \supset (ZZ \,\&\, \sim(ZZ))$$

(2) Set Theoretical Paradox

$$R = \{y \mid \lambda x[\in (xx)]\,y\}$$
$$R' = \{y \mid \lambda x[\notin (xx)]\,y\}$$
$$R' \in R \equiv R' \notin R$$

(3) Gödel-Church Metatheory

D = derivable or demonstrable

$G = \lambda x[\sim D(xx)]$

$GG \supset \sim D(GG)$(incompleteness)

$\sim(GG) \supset D(GG)$(inconsistency, where $D(p) \supset p$)

The existence restriction on abstraction elimination prevents these paradoxes from occurring by transforming what would otherwise be proofs of inconsistency into indirect proofs of the nonexistence of diagonal properties. Here is the proof structure of paradox neutralization in Meinongian logic. The self-application formula ZZ is assumed to be a proposition where type theory does not hold, and $ZZ \vee \sim(ZZ)$ is a well-formed substitution instance of $p \vee \sim p$. Diagonalized paradoxes reduced to this formulation are effectively neutralized by the existence condition. The Liar or classical semantic paradox in this notation requires the entailment of $\sim(ZZ)$ from $\lambda x[\sim(xx)]Z$. This is blocked by the existence condition.

1.	$Z = \lambda x[\sim(xx)]$	
2.	$E!(\lambda x[\sim(xx)]) \supset (\forall y)(\lambda x[\sim(xx)]y \equiv \sim(yy))$	(L1)
3.	$ZZ \vee \sim(ZZ)$	(T6)
4.	$E!(\lambda x[\sim(xx)])$	
5.	$(\forall y)(\lambda x[\sim(xx)]y \equiv \sim(yy))$	(2,4)
6.	$\lambda x[\sim(xx)]Z \equiv \sim(ZZ)$	(5)
7.	ZZ	
8.	$\lambda x[\sim(xx)]Z$	(1,7)
9.	$\sim(ZZ)$	(6,8)
10.	$ZZ \,\&\, \sim(ZZ)$	(7,9)
11.	$\sim(ZZ)$	
12.	$\sim(\lambda x[\sim(xx)]Z)$	(1,11)
13.	$\sim(\lambda x[\sim(xx)]Z) \equiv ZZ$	(6)
14.	ZZ	(12,13)
15.	$ZZ \,\&\, \sim(ZZ)$	(11,14)
16.	$ZZ \,\&\, \sim(ZZ)$	(3,10,15)
17.	$\overline{E}!(\lambda x[\sim(xx)])$	(4,16)

The conclusion is not that object theory logic is inconsistent, but that abstract Z (= $\lambda x[\sim(xx)]$) does not exist. If the abstract does not exist, then by the restricted abstraction elimination principle, $\lambda x[\sim(xx)]Z$ does not imply and cannot be used to derive $\sim(ZZ)$. An analogous solution undermines set theoretical paradoxes in negated double variable or self-non-application formulations.

The limiting metatheorems of Gödel and Church can now be addressed. To begin, consider untyped counterparts of the arithmetized metatheorems. Where $G = \lambda x[\sim D(xx)]$, it is argued that GG is true if and only if $\sim D(GG)$, if and only if GG is not derivable or demonstrable as a theorem. If there are no restrictions on abstraction elimination, the proof proceeds as follows.

1. GG
2. $\lambda x[\sim D(xx)]G$
3. $\sim D(GG)$
4. $GG \supset \sim D(GG)$

If GG is true, it is not decidable or demonstrable. This means that the formal system containing GG is incomplete, incapable of deducing all its truths.

5. $\sim(GG)$
6. $\sim(\lambda x[\sim D(xx)]G)$
7. $D(GG)$
8. $\sim(GG) \supset D(GG)$

If GG is not true, it is forthcoming as a theorem. In that case the logic is inconsistent, since it produces a falsehood as a theorem. As in Gödel-Church metatheory, the untyped abstraction metatheorems imply that the logic is either inconsistent or incomplete and undecidable.[35]

The limitations are avoided by the existence restriction on abstraction elimination, which invalidates the inferences in steps (2)—(3) and (6)—(7). The assumption that $\sim(GG)$ leads to inconsistency on the thesis that $D(p) \supset p$ by (8) for $D(GG) \supset GG$. The conclusion is that $\sim(GG)$ is false and GG true. This constitutes a proof of GG, supporting the truth of $D(GG)$. But by (4), $GG \supset \sim D(GG)$. The contradiction $D(GG)$ & $\sim D(GG)$ obtains, justifying rejection of the abstraction existence condition, $E!(\lambda x[\sim D(xx)])$. The self-denial of theoremhood or decidability is not an existent property. If the property does not exist, then existence-conditional abstraction equivalence cannot be used validly to derive $\sim D(GG)$ from $\lambda x[\sim D(xx)]G$, nor $D(GG)$ from $\sim(\lambda x[\sim D(xx)]G)$. The assumption that $D(p) \supset p$ is much stronger than the assumption of ω-consistency used in Gödel's original argument, or of ordinary syntactical consistency assumed in Rosser's later proof, and the differences these make in generating their respective metatheoretical dilemmas are discussed below.

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[35] Gödel, "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I" [1931], pp. 173—198. Church, "An Unsolvable Problem of Elementary Number Theory" [1935], pp. 332—33.

Gödel-Church arithmetized undecidable sentences hold if and only if their unarithmetized counterparts do (in the untyped logic containing both). The conclusions entailed by paradox neutralization for the three kinds of untyped unarithmetized diagonalization in the semantic Liar or Epimenides and set theoretical paradox and Gödel-Church metatheory are: $\overline{E}!(\lambda x[\sim(xx)])$ (the property of self-non-application does not exist); $\overline{E}!(\lambda x[\not\in(xx)])$ (set theoretical self-non-membership does not exist); $\overline{E}!(\sim D(xx)])$ (self-non-derivability does not exist). The classical formal semantic and set theoretical paradoxes and limitations of untyped abstraction versions of Gödel-Church-style metatheory are avoided in Meinongian logic with existence-conditional abstraction.

The Gödel-Church metatheorems in their original presentations are arithmetized to avoid type restrictions on paradoxical syntax combinations. The arithmetization codes each expression in the logic so that logical formulas can be translated into Gödel number equivalents, and recovered again from their numerical codings in exact syntax-item-for-syntax-item reconstruction. Each element of syntax is assigned a natural number, which is then made the exponent of a corresponding prime number base taken in the same order of increasing magnitude as the syntax (standardly left-to-right) in the expression to be coded. The Gödel number of the expression is the product of these primes raised to the powers of the corresponding syntax item code numbers. The Fundamental Theorem of Arithmetic guarantees that every number can be decomposed into a unique factorization of prime number bases raised to certain powers, and when these are put in ascending order, the expression mapped into Gödel-numbered space can be read directly from the exponents of each prime, and translated back into logical syntax by the glossary of initial natural number assignments.[36]

The open sentence $\overline{Provable}(sub(\ ))$ is introduced. It says that the Gödel-coded proposition substituted for the code number in innermost parentheses is unprovable. A glossary of syntax item numbers is assigned, one of which is temporarily attached to the blank space (alternatively, a free variable) within innermost parentheses. The unprovability predicate '$\overline{Provable}$' can be assigned Gödel number 1, $g(\ulcorner \overline{Provable} \urcorner) = 1$; the open parenthesis '(', 2,

[36] Gödel presupposes but does not explicitly mention the Fundamental Theorem of Arithmetic, which guarantees that every number can be decomposed into a unique factorization of prime number bases raised to certain powers. When this is done to Gödel number n and the factors put in ascending order, the expression mapped into Gödel-numbered space can be read directly from the exponents of each prime, and translated back into logical syntax by the glossary of natural number assignments. See, among other sources, Elliott Mendelson, *Introduction to Mathematical Logic* [1964], pp. 131, 135—42.

$g(\ulcorner (\urcorner) = 2$; the substitution function '*sub*', 3, $g(\ulcorner sub \urcorner) = 3$; the blank space or free variable, 4, $g(\ulcorner \; \urcorner) = 4$; the close parenthesis ')', 5, $g(\ulcorner) \urcorner) = 5$. These become the exponents for prime number bases taken in sequence in the same order as the syntax items in the expression to be coded, the product of which Gödel-codes the open sentence.

$$\begin{array}{ccccccccccccc} \overline{Provable} & & (& & sub & & (& & & &) & &) \\ | & & | & & | & & | & & | & & | & & | \\ 2^1 & \times & 3^2 & \times & 5^3 & \times & 7^2 & \times & 11^4 & \times & 13^5 & \times & 17^5 = n \end{array}$$

It is important to see that although Gödel code number 4 is not yet assigned to any syntax item, or is temporarily assigned the blank space or free variable, the Gödel number of the entire expression is calculable. It is a large number, which can simply be designated *n*. When *n* is computed, it is plugged into the blank space or replaces the free variable in the innermost parentheses of the sentence. This completes the Gödel diagonalization. The proposition then says that the proposition substituted for the Gödel number in innermost parentheses is unprovable. But by construction, the Gödel code number *n* in innermost parentheses codes the entire proposition. The proposition in effect says of itself that it is unprovable. The result is the Gödel sentence:

$$\overline{Provable}(sub(n)) \; \& \; g(\ulcorner \overline{Provable}(sub(n)) \urcorner) = n$$

The Gödel sentence is metatheoretically limiting, but involves no violation of type theory restrictions, because it does not require self-application or self-non-application as in the untyped abstraction version. The unprovability predicate or other syntax item of order *i* does not attach to another predicate or syntax item of the same order, but instead the unprovability predicate applies to an object or constant term order *i*—1, to a numeral that Gödel-codes the proposition in which it is included. The Gödel arithmetization thereby circumvents type theory restrictions.

In classical bivalent logic, every well-formed formula is either true or false. This gives rise to the same dilemma described for the untyped unarithmetized abstraction version of the Gödel-Church metatheorems. It is shown that the logic is inconsistent if the Gödel sentence is provable, and ω-inconsistent if the negation of the Gödel sentence is provable. The logic is therefore either inconsistent, ω-inconsistent, or incomplete. To preserve consistency, first order logic with identity, addition, and multiplication, or with binary predicates powerful enough to represent the axioms of Dedekind-Peano arithmetic, is judged deductively incomplete and formally undecidable, recursively enumerable by closure on definitions, axioms, and inference rules, but not recursive.

The idea of ω-consistency is that if the conjunction $\overline{Provable}(\overline{P}(0))$ & $\overline{Provable}(\overline{P}(1))$ & $\overline{Provable}(\overline{P}(2))$ &..., holds, then it follows that $\overline{Provable}((\exists x)Px$. Gödel's ω-consistency assumption is required to demonstrate that if the negation of the Gödel sentence is provable, then the logic is inconsistent. Without the assumption, there is no guarantee that the Gödel number asserted to exist by the negation of the Gödel sentence is any one of 0, 1, 2, 3, ...

The metatheoretical dilemma of the original Gödel proof is informally explained, abbreviating the entire Gödel sentence as 'G'. If $Provable(G)$, then since G asserts its own unprovability, it follows immediately that $Provable(\overline{G})$. But the logic is inconsistent if both G and $\overline{G}$ are provable. Suppose then that $Provable(\overline{G})$, and assume the logic is ω-consistent. If the logic is ω-consistent, then it is consistent (though not conversely). By consistency, if $Provable(\overline{G})$, then $\overline{Provable}(G)$. If $\overline{Provable}(\overline{G})$, then there is no Gödel number of any proof of G; that is, $Provable(g(\ulcorner G \urcorner)) \neq 0$ & $Provable(g(\ulcorner G \urcorner)) \neq 1$ & $Provable\ (g(\ulcorner G \urcorner)) \neq 2$ &... By ω-consistency, it follows that $\overline{Provable}(\exists x)(g(\ulcorner G \urcorner) = x)$. This contradicts the assumption, because, where G asserts its own unprovability, $Provable(\overline{G})$ by previous Gödel-coding entails $Provable(Provable(G)$ & $g(\ulcorner G \urcorner) = n)$, which by standard conjunction elimination and existential generalization implies $Provable(\exists x)(g(\ulcorner G \urcorner) = x)$.

Rosser's Theorem reaches a similar conclusion, but without assuming ω-consistency.[37] Instead of offering an arithmetized sentence that says in effect, 'I am unprovable', Rosser constructs an arithmetized sentence that asserts of itself, 'If I am provable, then there is a shorter proof (determined by corresponding Gödel-number cardinalities) of my negation'. Rosser builds his proof on Gödel's foundation, and proposes substitute expressions to be inserted into Gödel's original demonstration in certain places. Adapting the notation above to Rosser's metatheorem, the following Rosser sentence is introduced:

$$(Provable(sub(n)) \supset ((\exists x)(g(\ulcorner Provable\ (\overline{sub(n)})\urcorner) = x \ \& \ sub(x) \ \& \ x \leq n)) \ \& \ g(\ulcorner Provable(sub(n))\urcorner) = n)$$

The Rosser metatheoretical dilemma has this form. As in the Gödel metatheorem, it is shown that the logic is inconsistent if either the Rosser sentence or its negation is provable. The ω-consistency assumption is not needed, because the assertion in the consequent that proof of the sentence negation is shorter than proof of the sentence itself sets an upper bound to

[37] J. Barkley Rosser, "Extensions of Some Theorems of Gödel and Church" [1936].

assure that the Gödel number of the sentence negation is less than or equal to the Gödel number of the sentence. Abbreviating the Rosser sentence as 'R', if *Provable*(R), then it follows directly that *Provable*($\overline{R}$). The assumption permits detachment of the consequent, which contains the conjunct *sub*(x) that Gödel-codes the equivalent of *Provable*($\overline{R}$), asserting that the negation of the Rosser sentence is provable. The sentence *says* that if it is provable, then its negation is (as or more economically) provable. If *Provable*($\overline{R}$), on the other hand, it follows in this case that :

$$(\exists x)(g(\ulcorner \mathit{Provable}\ (\overline{\mathit{sub}(n)})\urcorner) = x \ \&\ \mathit{sub}(x) \ \&\ x \leq n)) \ \&\ g(\ulcorner \mathit{Provable}(\mathit{sub}(n))\urcorner) = n$$

This merely expresses the fact that if $\overline{R}$ is provable, then the Gödel number of its proof has a certain cardinality, stipulated to be less than or equal to the cardinality of the Gödel number of R itself. By propositional logic, using the paradox of material implication, the material conditional is then derived:

$$(\mathit{Provable}(\mathit{sub}(n)) \supset (\exists x)(g(\ulcorner \mathit{Provable}\ (\overline{\mathit{sub}(n)})\urcorner) = x \ \&\ \mathit{sub}(x) \ \&\ x \leq n)) \ \&\ g(\ulcorner \mathit{Provable}(\mathit{sub}(n))\urcorner) = n.$$

To prove the conditional is just to prove R, from which it follows that *Provable*(R). This contradicts the assumption, where by standard consistency *Provable*($\overline{R}$) implies $\overline{\mathit{Provable}}(R)$.

Gödel's and Church's limiting metatheorems are directed specifically at Whitehead and Russell's *Principia Mathematica* and related systems ('*und verwandter Systeme*'). But there are many *un*related systems of logic to which the incompleteness and undecidability results do not apply. Gödel's limiting metatheorem applies to all ω-consistent systems with finitistic proof methods, and therefore has implications limiting Hilbert's program in constructive mathematics. If, however, a theory is consistent, but not ω-consistent, then the proof does not go through. Finitist and intuitionist logics for this reason are unaffected, and arithmetics with addition but not multiplication have been shown to be complete and effectively decidable.[38] The importance of Rosser's metatheorem in part is to prove that such a finitist limitation is not inherent in Gödel-type incompleteness proofs. The object theory is not finitist, though it posits three truth values like most intuitionist logics, and can include both addition and multiplication with strong infinitary induction. Yet

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[38] Mojżesz Presburger, "Über die Vollständigkeit eines gewissen Systems der Arithmetik ganzer Zahlen in welchem die Addition als einzige Operation hervortritt" [1929], translated as "On the Completeness of a Certain System of Arithmetic of Whole Numbers in which Addition Occurs as the Only Operation" by Jacquette, *History and Philosophy of Logic*, 12, 1991, 225—33.

it is so nonstandard and unrelated to classical logics in other ways that it avoids even arithmetized Gödel-Rosser-Church limitations.

To see this, consider that in the untyped abstraction version of the metatheorems, $\lambda x[\sim D(xx)]G$ is undetermined in truth. This follows from the nonexistence of the abstract $\lambda x[\sim D(xx)]$ by existence restricted abstraction equivalence and the argument for diagonalization. The abstract is therefore impossible because it leads to contradiction, but like the round square and many other impossible objects, it is also incomplete. Self-non-derivability formulated by the abstract neither has nor fails to have the property of belonging to or being a member of set φ, where:

$$\varphi = \{\text{properties in atomic predications } p \text{ in a decidable logic} \mid p \equiv (p \equiv p)\}$$

The diagonalization proves that: $\lambda x[\sim D(xx)] \in \varphi \equiv \lambda x[\sim D(xx)] \notin \varphi$. This immediately implies that $V(\lambda x[\sim D(xx)]G) = U$, since in that case both $V(\lambda x[\sim D(xx)] \in \varphi) = U$ and $V(\lambda x[\sim D(xx)] \notin \varphi) = U$.

The untyped unarithmetized abstraction version of the Gödel-Church metatheorem cannot appear in a typed or type-restricted logic, but there is nothing to prevent arithmetization in the untyped Meinongian object theory, where the expressions are logically equivalent.

$$(\overline{Provable}(sub(n)) \;\&\; g(\ulcorner \overline{Provable}(sub(n)) \urcorner) = n) \equiv \lambda x[\sim D(xx)]G$$

Intuitively, these alternative formulations of classical metatheory diagonalization say the same thing in typed and untyped notations; indeed, the arithmetization is immediately available by free assumption in Meinongian logic.

Their equivalence follows directly from the propositional tautology: $((p \supset r) \;\&\; (q \supset r) \;\&\; (\sim p \supset \sim r) \;\&\; (\sim q \supset \sim r)) \supset (p \equiv q)$. If the untyped and unarithmetized abstraction version is true, then logic is incomplete (of course, it is *not* true, but undetermined); if the arithmetized version is true, then logic is also incomplete; if the untyped unarithmetized version is not true, then logic is not incomplete (though trivially so only because logic is then supposedly inconsistent); if the arithmetized typed version is not true, then logic is also not incomplete. Therefore, the untyped abstraction version of the metatheoretical diagonalization is true if and only if the classical arithmetized typed version is true.

But the untyped unarithmetized abstraction version is undetermined in three-valued Meinongian logic. The nonstandard truth matrix implies that if the above equivalence between untyped unarithmetized abstraction and arithmetized typed Gödel-Church sentences holds in Meinongian logic, and if the untyped unarithmetized abstraction version is undetermined in truth value, then the arithmetized type-sensitive version is also undetermined. It follows that either $\overline{\mathit{Provable}}(\mathit{sub}(n))$ or $g(\ulcorner \overline{\mathit{Provable}}(\mathit{sub}(n)) \urcorner) = n$ is undetermined. But $g(\ulcorner \overline{\mathit{Provable}}(\mathit{sub}(n)) \urcorner) = n$ cannot be undetermined, because it is an extranuclear referential identity predication, which must be either true or false exclusively, and is guaranteed true by stipulation. Therefore, the first proposition, $\overline{\mathit{Provable}}(\mathit{sub}(n))$, is undetermined in Meinongian logic whenever the second, $g(\ulcorner \overline{\mathit{Provable}}(\mathit{sub}(n)) \urcorner) = n$, is true.

If $\overline{\mathit{Provable}}(\mathit{sub}(n))$ is undetermined, then the untyped arithmetized Gödel-Church metatheorem is disabled, since the diagonalization implies a limitation only if the logic contains a true unprovable proposition. There is no incompleteness or undeciability about a logic that includes an unprovable sentence that is undetermined in truth. The neutralization effectively avoids strengthened reformulations of the metatheorems, because any such strengthenings will be logically equivalent to a predication involving an untyped abstraction, which by the existence restriction on abstraction equivalence is certain to be nonexistent if diagonal. This guarantees, first, that the untyped unarithmetized abstraction constructions for strengthened metatheory are undetermined in truth, and second, that their logically equivalent arithmetized counterparts are also undetermined, again blocking any conceivable strengthened arithmetized limiting metatheorem. Similar reasoning undermines the Rosser metatheorem.

The consequences of forestalling paradox and limiting metatheorems in Meinongian logic are likely to seem liberating or disorienting, depending on one's philosophical and mathematical temperament. The fact that some mathematical propositions are undetermined in truth value in Meinongian logic might be regarded as conceding to Gödel, Rosser, and Church the undecidability of the theory. But in trivalent semantics, decidability implies the existence of finite mechanical algorithms or recursive decision procedures for tripartitioning the semantic field, determining of any proposition whether it is true, false, or undetermined. The decidable tripartioning of the semantic domain is theoretically available to the logic.

To be complete and decid*able* does not mean to be complet*ed* and decid*ed*. Unproven propositions like Goldbach's conjecture are not thought to threaten the decid*ability* of standard first order logic with arithmetic, and can similarly be regarded as posing no deep or special challenge to the decidability of Meinongian logic. If necessary, unproven and undisproven mathematical propositions can decidably be assigned the undetermined value, on modified intuitionist principles. More sophisticated results like Gödel's and Cohen's proofs of the consistency and independence of the continuum hypothesis in classical set theory[39] also go by the board in Meinongian logic, because of paradox and limiting metatheorem neutralization, since its underlying set theory is equally nonstandard and nonclassical. Classical extensional theories are not strictly embeddable but can be modeled within the Meinongian intensional logic, and within these circumscribed modelings the classical paradoxes and limiting metatheorems are reconstructible. As such they are clearly not true subtheories of the system, and do not extend beyond these constraints to jeopardize the integrity of the logic as a whole.

The advantage of the above approach to the paradoxes and limiting metatheories is that only the undesirable potentially paradoxical properties and their complements are rejected as nonexistent. There is no need to give up Gödel-style arithmetization techniques that have proven so useful in mathematical logic. The usual type theory and Tarskian semantic hierarchy responses to the paradoxes throw out the baby with the bathwater, by rejecting harmless objects like the set of all sets that contain themselves as members as well as the paradoxical set of all sets that do not contain themselves as members, properties like self-application along with self-non-application, and predications like harmless self-truth-ascriptions along with Liar sentences. The innocuous items in these categories are preserved by the paradox and limiting metatheorem neutralization in the logic. Self-non-application and $\overline{\text{self-non-application}}$ do not exist, even though self-application and $\overline{\text{self-application}}$ remain, where by criterion (C1) and definition (D11) self-non-application ≠ self-application and $\overline{\text{self-application}}$ ≠ self-non-application, even though $\overline{\text{self-non-application}}$ = non-self-non-application, and $\overline{\text{self-application}}$ = non-self-application.

The existence-conditional restrictions on abstraction provide a uniquely Meinongian solution to the standard paradoxes. The neutralization technique cannot be introduced in extensional logic because in those systems well-defined terms must designate existent entities. The concept of a nonexistent property, if not extensionally unintelligible, is at least unavailable to the method. A nonexistent property has neither existent nor nonexistent objects in its intension, which intuitively precludes it from involvement in abstraction introduction or elimination. This provides a satisfactory rationale for restricting abstraction from the standpoint of object theory predicate semantics. The technically comparable extensional device of restricting abstraction introduction and elimination to abstracts with the conditional property of being *non-*

[39] Gödel, *The Consistency of the Axiom of Choice and of the Generalized Continuum Hypothesis with the Axioms of Set Theory* [1940]. Paul J. Cohen, *Set Theory and the Continuum Hypothesis* [1966].

paradoxical (or the like) would be uninformative and *ad hoc*. For the extensionalist a nonexistent property is no more a property than a nonexistent detective is a detective. The counterpart extensional proposal would therefore require an unwarranted divergence of properties and abstracts. The Meinongian approach preserves the propertyhood of all abstracts, but distinguishes between existent and nonexistent properties, limiting valid abstraction equivalence to existent properties. Routley and Lambert have also recognized the need to restrict abstraction in Meinongian object theory. Lambert argues that "... Meinong must reject [unrestricted or classical abstraction] if the... principle of independence... is sound."[40] He speaks of consigning abstraction to 'logical oblivion' or doing away with it entirely, but does not anticipate the compromise possibility proposed here of restricting the principle to existent abstracts.

Cantor's theory of transfinite cardinalities and the generalized power set axiom are not undermined by Meinongian paradox neutralization. If that were true the method would be too powerful. The proof of Cantor's theorem proceeds by diagonalization. But that does not mean that it establishes a genuine paradox. The demonstration is an argument by indirect proof against the assumption that the power set of a denumerably infinite set has the same cardinality as the set itself.[41] The existence-conditional abstraction principle does not require rejecting the assumption that the corresponding abstract exists. Instead, the one-one cardinality assumption is rejected when inconsistency is deduced. Classical mathematics with Cantor's ascending orders of transfinite cardinalities can be modeled in Meinongian object theory logic and mathematics, either as existent or nonexistent objects. The paradox neutralization procedure is therefore selective. It eliminates only genuine paradoxes, and does not limit diagonalized indirect proof.

Meinongian metamathematics with these qualifications avoids formal semantic and set theoretical paradox and scope-restricting diagonalizations without type theory. Meinongian logic and mathematics with classical transfinite subtheory intact is so nonclassically defined that the theory does not support the classical limiting metaproofs of inconsistency, incompleteness, or undecidability for standard first-order logic.

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[40] Lambert, *Meinong and the Principle of Independence*, p. 159. The existence-conditional restrictions on lambda abstraction are predicate counterparts of analogous restrictions placed on von Neumann-Gödel-Bernays set theory in order to avoid the Russell paradox. See also George Bealer, *Quality and Concept* [1982], pp. 97—100.

[41] Georg Cantor, "Mitteilungen zur Lehre vom Transfiniten I, II" [1887], pp. 81—125, 252—70; [1888], pp. 240—65.

10. Consistency, Completeness, Compactness

This is a consistency, Henkin-type strong and weak completeness, and compactness proof, based on a version first extended to three-valued logic by Leblanc, Goldberg, and Weaver.[42]

Internal Determinacy

It is useful to establish an internal determinacy theorem for Meinongian logic. Although some propositions of *O* are logically true, and others logically false, no propositions of *O* are logically undetermined. The truth valuations under which an undetermined proposition can be validly inferred prevent deduction of an undetermined proposition from true propositions. The semantics of *O* do not require that any truth functionally complex proposition be undetermined in truth value. If the undetermined truth valuation of a proposition can only come from outside the logic in a particular scientific or philosophical application, then within the logic inference always carries from true propositions to true propositions, and never to undetermined propositions. This preserves an exact semantic isomorphism with the inference conditions of classical logic and the classical deduction theorem.

Internal Determinacy Theorem for *O*

No proposition of *O* is logically undetermined.

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[42] Leblanc, Harold Goldberg, and George Weaver, "A Strong Completeness Theorem for Three-Valued Logic: Part I"; Leblanc, "A Strong Completeness Theorem for Three-Valued Logic: Part II", in Leblanc, *Existence, Truth, and Provability* [1982], pp. 240—57. Leon Henkin, "The Completeness of the First-Order Functional Calculus" [1949], pp. 159—66. The internal determinacy metatheorem for the object theory logic might simplify its metatheoretical characterization by limiting metaproofs to the more classical bivalent semantic pure theory of the logic. Since, however, the main point of developing a Meinongian logic is for the sake of problem-solving applications, it seems more appropriate to proceed by adapting the Leblanc-Goldberg-Weaver strategy for three-valued propositional and predicate systems to the Meinongian system.

Proof.

By mathematical induction on the length of wffs in *O*. Let p be a wff of *O*.

1. If $p = (q \supset r)$, then by truth tables, $V(p) = U$ if and only if either $V(q) = T$ and $V(r) = U$ or $V(q) = U$ and $V(r) = F$. But then p cannot be logically undetermined unless q or r is logically undetermined.
2. If $p = \sim q$, then by truth tables, $V(p) = U$ if and only if $V(q) = U$. But then p cannot be logically undetermined unless q is logically undetermined.
3. If p is not truth functionally complex, then $V(p) = T$, $V(p) = F$, or $V(p) = U$. But then p cannot be logically undetermined. ⊠

It follows that no proposition of *O* is logically or internally undetermined. The truth valuation of a proposition of *O* as undetermined must come from outside the logic. Undetermined nuclear predications of certain properties to incomplete nonexistent objects can only be introduced as substitution instances for propositional compoments of axioms and theorems in extralogical applications.

Propositional Object Theory Logic

Definitions

1. Let *O*' be the propositional fragment of *O*. A set ξ of wffs of *O*' is *syntactically consistent* if there is no wff p of *O*' such that both p and $\sim p$ are provable from ξ. Set x is *syntactically inconsistent* if there is a wff p of *O*' such that both p and $\sim p$ are deducible from ξ. As explained in the section on 'Inference Structures', to be deducible in *O*' means to be derivable in a sequence of wffs by the logical axioms, definitions, nonlogical axioms, and inference principles of *O*'. Set ξ is *maximally consistent* if (i) ξ is syntactically consistent; (ii) $\xi \vdash p$ for any wff p of *O*' such that $\xi \cup \{p\}$ is syntactically consistent.
2. Set ξ of wffs of *O*' is *semantically consistent* if there is a nonstandard truth value assignment for which all members of ξ have truth value *T*. Set x *entails* a wff p of *O*' ($\xi \vDash p$) if, under any truth value assignment in *O*', p has truth value *T* when every member of ξ has truth value *T*. Proposition p is *valid* ($\vDash p$ or $\varnothing \vDash p$) if, under any truth value assignment in *O*', p unconditionally has truth value *T*.

Lemma I

1. If $\xi \vdash p$, then $\xi' \vdash p$ for every superset ξ' of ξ. By definition. In particular, if $\vdash p$, then $\xi \vdash p$ for every set ξ of wffs of O'.
2. If $\xi \vdash p$, then there is a finite subset ξ' of ξ such that $\xi' \vdash p$. By definition.
3. If $p \in \xi$, then $\xi \vdash p$.
4. If $\xi \vdash p$ and $\xi \vdash (p \supset q)$, then $\xi \vdash p$.
5. If $\xi \cup \{p\} \vdash p$, then $\xi \vdash (p \supset (p \supset q))$.

Proof.

Assume that the wff sequence made up of $r_1, r_2, r_3, \ldots, r_n$ constitutes a proof of q from $\xi \cup \{p\}$. By mathematical induction on i it is shown that $\xi \vdash (p \supset (p \supset r_i))$ for each i from 1-n, and hence in particular that $\xi \vdash (p \supset (p \supset q))$. There are three cases to consider.

Case 1. r_i is an axiom or member of ξ ($r_i \in \xi$).

Then $\xi \vdash r_i$ by (1) or (3). But $\xi \vdash (r_i \supset (p \supset r_i))$ by (1). Hence, $\xi \vdash (p \supset r_i)$ by (4). But $\xi \vdash ((p \supset r_i) \supset (p \supset (p \supset r_i)))$ by (1). Hence, $\xi \vdash (p \supset (p \supset r_i))$ by (4). In particular, then, $\xi \vdash (p \supset (p \supset q))$.

Case 2. $r_i = p$.

Then $\xi \vdash (p \supset (p \supset r_i))$ by (1). In particular, then, $\xi \vdash (p \supset (p \supset q))$.

Case 3. r_i is derived from r_j and $r_j \supset r_i$.

Then $\xi \vdash (p \supset (p \supset r_i))$ and $\xi \vdash (p \supset (p \supset (r_j \supset r_i)))$, by the hypothesis of the induction. Hence $\xi \vdash (p \supset (p \supset r_i))$ by (3) and (4). In particular, then, $\xi \vdash (p \supset (p \supset q))$. ⊠

6. If ξ is syntactically inconsistent, then $\xi \vdash p$, for every wff p of O'.

Proof.

Assume that $\xi \vdash q$ and $\xi \vdash \sim q$ for some wff q of O'. Then by truth tables and (4), $\xi \vdash p$ for any wff p of O'. ⊠

7. ξ is syntactically inconsistent if and only if $\xi \vdash (p \ \& \sim p)$.

Proof.

By truth tables and internal determinacy, $\xi \vdash (p \supset p)$ for any wff p. Hence, if $\xi \vdash (p \; \& \; {\sim}p)$, then by truth tables and the definition, ξ is syntactically inconsistent. The converse follows by (6). ⊠

8. If $\xi \cup \{p\}$ is syntactically inconsistent, then $\xi \vdash {\sim}p$.

Proof.

Assume that $\xi \cup \{p\}$ is syntactically inconsistent. Then $\xi \cup \{p\} \vdash {\sim}p$ by (6). Thus, $\xi \vdash (p \supset {\sim}p)$ by (5). Thus, $\xi \vdash {\sim}p$ by truth tables and (4). ⊠

9. If $\xi \cup \{{\sim}p\}$ is syntactically inconsistent, then $\xi \vdash p$.

Proof.

From (8), (1), and (4). ⊠

Lemma II

If any set ξ of wffs of O' is syntactically consistent, then ξ is also semantically consistent.

The proof involves the assumption for purposes of indirect proof that ξ is syntactically consistent, and the construction of a ξ extension into a two-valued superset ξ^*. The members of ξ^*, and hence of ξ, are shown to have truth value T under a special assignment that constitutes a Henkin model for ξ in O'. The construction of ξ^* requires several steps. Let ξ_0 be ξ. Assume that the wffs of O' are alphabetically well-ordered in some unambiguous enumeration, and consider p_i to be, for each $i \geq 1$, the i-th wff of O'. Then ξ_i is $\xi_{i-1} \cup \{p_i\}$ if $\xi_{i-1} \cup \{p_i\}$ is syntactically consistent, and otherwise ξ_i is ξ_{i-1}. Finally, $\xi^* = \sum_{i=0}^{\infty}(\xi_i)$.

It is shown that:

1. ξ^* is syntactically consistent.
2. ξ^* is maximally consistent.

Proof.

1. Assume for purposes of indirect proof that ξ^* is syntactically inconsistent. Then by (7) and (2) of Lemma I at least one finite subset ξ' of ξ^* is syntactically inconsistent. But ξ' must either be a subset of $\xi_0, \xi_1, \xi_2, \xi_3, \ldots$, each of which is syntactically consistent by condition of membership in ξ^*. ⊠

2. Assume that $\xi^* \nvdash p_i$, where p_i is the i-th wff of O' in the same unambiguous alphabetical ordering as before. Then by (3) of Lemma I, $p_i \notin \xi^*$. Thus, $p_i \notin x_i$, and $\xi_{i-1} \cup \{p_i\}$ is syntactically inconsistent. Hence, by (7) and (1) of Lemma I, $\xi^* \cup \{p_i\}$ is also syntactically inconsistent. ⊠

Consistency Theorem for O'

Let there be a truth value assignment in O', $V_{O'}$, that assigns to each propositional variable 'p_i' of O' the value T if $\xi^* \vdash p_i$ (so that by the syntactical consistency of ξ^*, $\xi^* \nvdash \sim p_i$); the value F if $\xi^* \vdash \sim p_i$ (so that by the syntactical consistency of ξ^*, $\xi^* \nvdash p_i$); and otherwise assigns p_i the undetermined truth value U.

For any wff p_i of O' it is shown that:

1. If $\xi^* \vdash p_i$ (where $\xi^* \nvdash \sim p_i$), $V(p_i) = T$.
2. If $\xi^* \vdash \sim p_i$ (where $\xi^* \nvdash p_i$), $V(p_i) = F$.
3. If $\xi^* \nvdash p_i$ and $\xi^* \nvdash \sim p_i$), $V(p_i) = U$.

Proof.

The proof is by mathematical induction on the length of p_i. The length $L(p_i)$ of a propositional variable 'p_i' is 1. The length $L(\sim p_i)$ of a negation '$\sim p_i$' is $L(p_i)+1$. The length $L(p_i \supset p_j)$ of a conditional '$p_i \supset p_j$' is $L(p_i)+L(p_j)+1$. It is assumed as confirmable by truth table analysis and the recursive inductive definition of the wffs of O' that any wff of O' can be truth functionally reduced to equivalent form in the primitive notation, involving only negation and the conditional.

Basis: $L = 1$.

Here p_i is a propositional variable. The proof follows from the definition of $V_{O'}$.

Inductive step: $L > 1$.
There are two case to consider.

Case 1. $p_i = \sim q$

1. Assume that $\xi^* \vdash \sim q$. Then it is false that $\xi^* \vdash q$. Hence, by the hypothesis of the induction, $V(q) = T$ and $V(\sim q) = F$.

2. Assume that $\xi^* \vdash \sim\sim q$. Then by truth tables and (4) of Lemma I, $\xi^* \vdash q$. Hence, by the hypothesis of the induction, $V(q) = T$, and $V(\sim q) = F$.

3. Assume that neither $\xi^* \vdash \sim q$ nor $\xi^* \vdash \sim\sim q$. If q were provable from ξ^*, then by truth tables and (4) of Lemma I, $\sim\sim q$ would also be provable from ξ^*. Hence, neither $\xi^* \vdash q$ nor $\xi^* \vdash \sim q$. Thus, by the hypothesis of the induction, $V(q) = U$ and $V(\sim q) = U$.

Case 2. $p_i = (q \supset r)$

1. Assume that $\xi^* \vdash (q \supset r)$. If $\xi^* \vdash q$, then $V(q) = T$ by the hypothesis of the induction. If $\xi^* \vdash r$, then $V(r) = T$, again by the hypothesis of the induction. Thus, if $V(q) = T$ and $V(r) = T$, then $V(q \supset r) = T$.

2. Assume that $\xi^* \vdash \sim(q \supset r)$. Then by truth tables $\xi^* \vdash q$ and $\xi^* \vdash \sim r$. Thus, by the hypothesis of the induction, $V(q) = T$ and $V(r) = F$. Therefore $V(q \supset r) = F$.

3. Assume that neither $\xi^* \vdash (q \supset r)$ nor $\xi^* \vdash \sim(q \supset r)$. Then neither $V(r) = T$ nor $V(q) = F$. For by the hypothesis of the induction, q, r, $\sim q$, or $\sim r$ would then be deducible from ξ^*, and thus, by the above, so would $q \supset r$ or $\sim(q \supset r)$, contrary to the assumption. Suppose that $V(q) = T$. Then $V(r) \neq T$ and $V(r) \neq F$. For then by the hypothesis of the induction, $q \supset r$ or $\sim(q \supset r)$ would be deducible from ξ^*, contrary to the assumption. $V(q) \neq T$ and $V(q) \neq F$, so $V(q) = U$. But if $V(q) = U$, $V(r) \neq T$, for then by truth tables and the hypothesis of the induction, $q \supset r$ would be deducible from ξ^*, contrary to the assumption. Then $V(r) \neq U$, for then by truth tables and the hypothesis of the induction, $q \supset r$ would be deducible from ξ^*, again contrary to the assumption. Therefore, $V(q) = T$ and $V(r) = U$ or $V(q) = U$ and $V(r) = F$; and in either case, $V(q \supset r) = U$ and $V(\sim(q \supset r)) = U$.

Since every member of ξ is a member of ξ^*, every member of ξ is provable from ξ^* by (3) of Lemma I. This means that every member of ξ has truth value T under truth value assignment $V_{O'}$. Meinongian propositional object theory logic O' therefore has a Henkin model. It follows that:

If ξ is syntactically consistent, then ξ is semantically consistent. ☒

Strong Completeness Theorem for O'

If $\xi \vdash p$, then $\xi \vDash p$.

Proof.

Assume that $\xi \vDash p$. Then $\xi \cup \{\sim p\}$ is semantically inconsistent. Hence, by Lemma II, $\xi \cup \{\sim p\}$ is syntactically inconsistent. From this and (9) of Lemma I, it follows that $\xi \vdash p$. ⊠

Weak Completeness Theorem for O'

If $\vDash p$, then $\vdash p$.

Proof.

From strong completeness, where $\xi = \emptyset$. ⊠

Compactness Theorem for O'

If every finite subset of ξ is semantically consistent, then ξ is semantically consistent.

Proof.

If ξ is syntactically inconsistent, then by (7) of Lemma I, there is a proposition p of O' such that $\xi \vdash (p \;\&\; \sim p)$. But then ξ is semantically inconsistent, since by the hypothesis of the induction in the proof of Lemma II, $V(p) = T$ and $V(\sim p) = F$. Thus, if ξ is syntactically inconsistent, then ξ is semantically inconsistent. This implies by contraposition that if ξ is semantically consistent, then ξ is syntactically consistent (the converse of Lemma II). It follows from this and (2) and (7) of Lemma I that if every finite subset of ξ is semantically consistent, then ξ is syntactically consistent. This, with Lemma II, entails the theorem. ⊠

Predicate and Modal Object Theory Logic

Definitions

1. A set ξ of wffs of O is *syntactically decidable* with respect to truth value if there is a truth value assignment in O under which all members of ξ are assigned truth value T.

2. A set ξ of wffs of O is *semantically consistent* if either ξ or some set isomorphic to ξ is syntactically decidable with respect to truth value. Set ξ *entails* a wff p if $\xi \cup \{\sim p\}$ is semantically inconsistent. A wff p is *valid* if $\varnothing$ entails p.

Let ξ be a syntactically consistent and infinitely extendible set of wffs of O. Set ξ is extended into another set ξ^{**}, then ξ^{**} is extended into ξ^{***}, and ξ^{***} extended into a set ξ^{*}, like that of the propositional metatheory, to which predicate and modal wffs are added. It is shown that there is a truth value assignment according to which every wff in set ξ receives truth value T. Set ξ^{**} is defined as follows. Let ξ^0 be ξ. Assume that the quantified wffs of O are ordered alphabetically so that $(\forall \xi_i)p_i$ is the i-th wff of O for each $i \geq 1$. Take ξ^i to be $\xi^{i-1} \cup \{p_i(o_i/\xi_i) \supset (\forall \xi_i)p_i\}$, where '$o_i$' designates any object of the domain not designated in ξ^{i-1} such that $(\forall \xi_i)p_i$. Let ξ^{**} be the union set $\xi^0 \cup \xi^1 \cup \xi^2 \cup \xi^3 \cup \ldots \cup \ldots$ Set ξ^{***} is defined as the extension of ξ^{**}. Let $\xi_m{}^0 = \xi^{**}$. Assume that the modal wffs of O are alphabetically ordered in a unique way so that $\Box p_i$ is the i-th wff of O for each $i \geq 1$. Take $\xi_m{}^i$ to be $\xi_m{}^{i-1} \cup \{\Box p_i\}$ if $\xi_m{}^{i-1} \cup \{\Box p_i\}$ is syntactically consistent, and otherwise take $\xi_m{}^i$ to be $\xi_m{}^{i-1}$. Set ξ^* is defined as the extension of ξ^{***}. Let $\xi_0 = \xi^{***}$. Proposition p_i is the i-th wff of O in the expanded ordering of wffs of ξ^{***}. Let $\xi_i = \xi_{i-1} \cup \{p_i\}$ if $\xi_{i-1} \cup \{p_i\}$ is syntactically consistent, and otherwise take ξ_i to be ξ_{i-1}. Let ξ^* be the union set $\xi_0 \cup \xi_1 \cup \xi_2 \cup \xi_3 \cup \ldots \cup \ldots$

It is shown that:

1. ξ^{***} is syntactically consistent.
2. ξ^{**} is syntactically consistent.
3. ξ^{*} is syntactically consistent.
4. ξ^{*} is maximally consistent.

Proof.

1. Assume for purposes of indirect proof that ξ^{***} is syntactically inconsistent. Then by (7) and (2) of Lemma I, at least one finite subset $\xi^{**'}$ of ξ^{***} is syntactically inconsistent. But $\xi^{**'}$ must either be $\xi_m{}^0$, $\xi_m{}^1$, $\xi_m{}^2$, $\xi_m{}^3, \ldots$, each of which is syntactically consistent by membership conditions for the construction of ξ^{***}.

2. Assume that ξ^i is syntactically inconsistent. Then by (8) of Lemma I, $\sim(p_i(o_i/x_i) \supset (\forall x_i)p_i)$ is provable from ξ^{i-1}. Let $x_{i'}$ be the i-th variable ranging over individuals of O not designated in $(\forall x_i)p_i$. Then $\xi^{i-1} \vdash (\forall x_{i'})\sim(p_i(x_{i'}/x_i) \supset (\forall x_i)p_i)$. From this it follows that:

$\xi^{i-1} \vdash (\forall x_{i'})\sim(p_i(x_{i'}/x_i) \supset (\forall x_i)p_i) \supset \sim(\exists x_{i'})(p_i(x_{i'}/x_i) \supset (\forall x_i)p_i)$
$\xi^{i-1} \vdash \sim(\exists x_{i'})(p_i(x_{i'}/x_i) \supset (\forall x_i)p_i)$
$\xi^{i-1} \vdash (\exists x_{i'})(p_i(x_{i'}/x_i) \supset (\forall x_i)p_i) \supset \sim(\exists x_{i'})(p_i(x_{i'}/x_i) \supset (\forall x_i)p_i)$
$\xi^{i-1} \vdash (\exists x_{i'})(p_i(x_{i'}/x_i) \supset (\forall x_i)p_i)$

Thus, ξ^{i-1} is syntactically inconsistent. But ξ^i is syntactically consistent if ξ^{i-1} is. By hypothesis, ξ^0 is syntactically consistent. Thus, each of ξ^0, ξ^1, ξ^2, $\xi^3, \ldots$ is syntactically consistent. Hence, by (1) and (2) above, ξ^{**} is syntactically consistent.

3. Assume that ξ^* is syntactically inconsistent. Then by (7) and (2) of Lemma I, at least one finite subset $\xi^{***'}$ of ξ^* is syntactically inconsistent. But $\xi^{***'}$ must either be a subset of ξ_0, ξ_1, ξ_2, $\xi_3, \ldots$, each of which is syntactically consistent by membership conditions for construction of ξ^*.

4. Assume it is not the case that $\xi^* \vdash p_i$, where p_i is the i-th wff of O in the unique ordering of ξ^*. Then by (3) above, $p_i \notin \xi^*$. Hence, $p_i \notin \xi^{**'}$. Thus, $\xi^{**} \cup \{p_i\}$ is syntactically inconsistent, and by (7) and (1) of Lemma I, $\xi^* \cup \{p_i\}$ is also syntactically inconsistent. Thus, ξ^* is maximally consistent. ⊠

Let V_O be a truth value assignment in O that assigns to each propositional variable 'p_i' of O the truth value T if $\xi^* \vdash p_i$, F if $\xi^* \vdash \sim p_i$, and U if neither $\xi^* \vdash p_i$ nor $\xi^* \vdash \sim p_i$. It is proved for any wff p_i of O:

1. If $\xi^* \vdash p_i$, then $V(p_i) = T$.
2. If $\xi^* \vdash \sim p_i$, then $V(p_i) = F$.
3. If neither $\xi^* \vdash p_i$ nor $\xi^* \vdash \sim p_i$, then $V(p_i) = U$.

Proof.

The proof is by mathematical induction on the length of wffs. The length L of a wff, already defined for propositional formulas, is now defined for predicate and modal formulas. The length $L((\forall x)p_i)$ of a universally quantified formula '$(\forall x)p_i$' is $L(p_i(o_i/x))+1$, where '$(p_i(o_i/x)$' indicates as before the replacement of object term 'o_i' for all occurrences of 'x' in p_i. The length $L(\Box p_i)$ of a modal formula '$\Box p_i$' is $L(p_i)+1$. The subtheories of lambda abstraction and definite description do not affect induction on length of wffs in O, since they only reductively introduce new kinds of object- and property-designating terms.

Basis: $L = 1$

Here p_i is a propositional variable. The proof follows from the definition of V_O.

Inductive step: $L > 1$

There are four cases to consider.

Case 1. $p_i = \sim q$

Proof as in propositional metatheory.

Case 2. $p_i = (q \supset r)$

Proof as in propositional metatheory.

Case 3. $p_i = (\forall x)q$

1. Assume that $\xi^* \vdash (\forall x)q$. Then $\xi^* \vdash q(o_i/x)$ for every object o_i of O. Thus, by the hypothesis of the induction, $V(q(o_i/x)) = T$ for every o_i, and hence, $V((\forall x)q) = T$.

2. Assume that $\xi^* \vdash \sim(\forall x)q$. Let o_i be the i-th object of O, such that $q(o_i/x) \supset (\forall x)q$. Hence, $\xi^* \vdash \sim(\forall x)q \supset \sim q(o_i/x)$. Thus, $\xi^* \vdash \sim q(o_i/x)$, and, by the hypothesis of the induction, $V(q(o_i/x)) = F$, so that $V((\forall x)q) = F$.

3. Assume that neither $\xi^* \vdash (\forall x)q$ nor $\xi^* \vdash \sim(\forall x)q$. If $V(q(o_i/x)) = F$ for any object o_i of O, then by the hypothesis of the induction, $\sim q(o_i/x)$

would be provable from ξ^* for some o_i, and hence, $\sim(\forall x)q$ would also be provable from ξ^* for some o_i, contrary to the assumption. But if $V(q(o_i/x)) = T$ for every object o_i of O, then by the hypothesis of the induction, $q(o_i/x)$ would be provable from ξ^* for every o_i. But $q(o_i/x) \supset (\forall x)q$ belongs to ξ^*, and by (3) above is provable from ξ^* for at least some object o_i of O. Thus, if $V(q(o_i/x)) = T$ for every object o_i of O, then by (4), $(\forall x)q$ would be provable from ξ^*, also contrary to the assumption. Hence, $V((\forall x)q) = U$.

Case 4. $p_i = \Box q$

1. Assume that $\xi^* \vdash \Box q$. Then $\xi^* \vdash q\text{-}W^m{}_i$, for every $W^m{}_i \in \Sigma^m$ of the modal semantic model $<\Sigma^m, \Gamma^m, R, D^m, V^m>$ for O. Hence, by the hypothesis of the induction, $V(q\text{-}W^m{}_i) = T$ for every $W^m{}_i \in \Sigma^m$, and hence, $V(\Box q) = T$.

2. Assume that $\xi^* \vdash \sim\Box q$. Let $W^m{}_j$ be the j-th world of Σ^m in O, such that $q\text{-}W^m{}_j$ is a wff of ξ^*. Then by (3) of the propositional metatheory, $\xi^* \vdash q\text{-}W^m{}_j \supset \Box q$. Hence, by (1) and (4), $\xi^* \vdash \sim\Box q \supset \sim q\text{-}W^m{}_j$. By (4), $\xi^* \vdash \sim q\text{-}W^m{}_j$, which by the hypothesis of the induction entails that $V(q, W^m{}_j) = F$, and hence that $V(\Box q) = F$.

3. By virtue of the fact that modal Meinongian logic is part of the classical bivalent extranuclear subtheory of object theory, there is no need to consider the assumption that neither $\xi^* \vdash \Box q$ nor $\xi^* \vdash \sim\Box q$.

Since every member of ξ is a member of ξ^*, every member of ξ^* is deducible from ξ^* by (3) of Lemma I. This means that every member of ξ has truth value T under truth value assignment V_O. Meinongian predicate and modal object theory O therefore has a Henkin model.

It follows that:

If ξ is syntactically consistent and infinitely extendible, then ξ is syntactically decidable with respect to truth value, and hence semantically consistent. ⊠

Strong Completeness Theorem for O

If $\xi \vDash p$, then $\xi \vdash p$.

Proof.

Assume that $\xi \vDash p$. Then $\xi \cup \{\sim p\}$ is semantically inconsistent. Hence, by Lemma II, $\xi \cup \{\sim p\}$ is syntactically inconsistent. From this and (9) of Lemma I it follows that $\xi \vdash p$. ⊠

Weak Completeness Proof for *O*

If $\vDash p$, then $\vdash p$.

Proof.

From strong completeness, where $\xi = \varnothing$. ⊠

Compactness Theorem for *O*

If every finite subset of ξ is semantically consistent, then ξ is semantically consistent.

Proof.

By the converse of Lemma II, (2), and (7). ⊠

The compactness theorem in Meinongian object theory is unobtainable in most predicate systems, including nonstandard three-valued logics, subject to the decidability limitations of Gödel-Rosser-Church metatheory. This is because compactness requires the preliminary result that if a set of wffs is semantically consistent, then it is also syntactically consistent. The requirement is not met in classical logics, in which there are always sets ξ of wffs which, though true semantically, contain undecidable Gödel-Rosser-Church sentences p such that $\xi \nvdash p$. Meinongian logic avoids these difficulties by imposing existence restrictions on abstraction equivalence, thereby removing standard metatheoretical obstacles to compactness. The logic unlike traditional systems is therefore recursively enumerable and recursive, naively complete and mechanically decidable.

Deduction Theorem for O

If p is a wff of O, then for every wff q of O, under internal determinacy, $\vdash (p \supset q)$ if and only if q is a theorem of O/p (where O/p is the theory obtained from O by adding p as a nonlogical axiom).

Proof.

1. If $\vdash (p \supset q)$, then p and $p \supset q$ are theorems of O/p. Then q is a theorem of O/p.

2. If q is an axiom of O/p, then if $q = p$, then $(p \supset q) = (p \supset p)$, which by internal determinacy is a theorem of O, and therefore of O/p.

3. If q is an axiom of O/p, but $q \neq p$, then q is an axiom of O, and not just of O/p. Then q is a theorem of O. It follows that $\vdash (p \supset q)$.

4. If $(r_1, \ldots, r_n) \vdash q$, then $(p \supset r_1, \ldots, p \supset r_n) \vdash (p \supset q)$. By the hypothesis of the induction, $\vdash (p \supset r_1), \ldots, \vdash (p \supset r_n)$. It follows that $\vdash (p \supset q)$. ⊠

The conditional version of the deduction theorem holds for internally determinate Meinongian object theory logic, and for its bivalent extranuclear fragment, even in scientific and philosophical applications involving undetermined nuclear predications to incomplete objects.[43]

Consistency and Free Assumption

The *Annahmen* or unrestricted freedom of assumption thesis in Meinongian logic underwrites a powerful comprehension principle for the object theory semantic domain. It is not so powerful, however, as to permit the ingress of outright logical inconsistency or logically inconsistent Meinongian objects.

From a phenomenological standpoint, the assumption that there is an object with both nuclear properties P and not-P is most naturally formalized as $(\exists x)(Px \mathbin{\&} \overline{P}x)$, and as such standardly comprehends an impossible but by no means logically inconsistent Meinongian object o_i such that $Po_i \mathbin{\&} \overline{P}o_i$ or

[43] This formulation of the deduction theorem is adapted from Joseph R. Shoenfield, *Mathematical Logic* [1967], p. 33.

$\lambda x[Px \ \& \ \bar{P}x]o_i$. Here the object is freely assumed to have the above-named metaphysically impossible but logically consistent nuclear property or conjunction of nuclear properties in its *Sosein*.

Logical inconsistency is avoided where free assumption and the comprehension principle might otherwise be thought to introduce it in exploiting the syntactical resources of the logic, by the restriction of the *Annahmen* thesis to the determination of objects exclusively by their nuclear properties. The assumption that there is an object such that it has nuclear property P and it is *not the case that* it has nuclear property P, $(\exists x)(Px \ \& \ {\sim}Px)$, does not comprehend a logically inconsistent Meinongian object. Although 'P' and '$\bar{P}$' are (nuclear) property terms, '${\sim}P$', according to the formation rules, is not a term at all, but an ill-formed symbol combination that represents no nuclear or extranuclear property. The logical inconsistency abstract '$\lambda x[Px \ \& \ {\sim}Px]$', on the other hand, is a term, but for a decidedly *extranuclear* property to which the logic's comprehension principle via free assumption does not apply. The abstract is correctly written '$\lambda x[Px \ \& \ {\sim}Px]!$', as in the assumption $(\exists y)(\lambda x[Px \ \& \ {\sim}Px]!y)$, since it evidently satisfies criterion (C2) for extranuclear properties:

$$(\forall y)(\overline{\lambda x[Px \ \& \ {\sim}Px]}!y \equiv {\sim}(\lambda x[Px \ \& \ {\sim}Px]!y))$$

The property of being logically inconsistent, like the property of being possible, impossible, or logically consistent, is intuitively extranuclear rather than nuclear. That an object has the complement of the property of being logically inconsistent (or, is logically consistent) is logically equivalent to the object's not being logically inconsistent or to it not being the case that the object is logically inconsistent.

I. Twardowski On Content and Object

1. Phenomenological Psychology

Twardowski developed a version of Brentano's phenomenological psychology according to which every thought or psychological presentation is reduced to a mental act, its content, and object. The content of a thought presents the object; in some cases the content may be a mental image of the object, or the noematic psychological equivalent of its description. Content is lived-through, psychological, or experiential. But the object of thought is seldom and then only accidentally psychological, as when a thought happens to be about another thought.

Meinong, Mally, and others, developed Twardowski's distinction between act, content, and object, and made it a fundamental part of object theory. The distinction is sometimes regarded as a necessary presupposition of phenomenology and other intentional philosophical theories. Grossmann writes: "Twardowski... distinguished between the content of an act of presentation— what I shall sometimes call an 'idea'— and the object of this act. Without this distinction, I am convinced, there would be neither phenomenology nor a theory of entities."[1] It is therefore worthwhile to examine Twardowski's distinction not only for its own sake, but as background to a number of important philosophical traditions.

In *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*, Twardowski advances four arguments for the distinction between content and object. The first, second, and fourth arguments are in some ways less interesting than the third. The third argument attempts to show that the contents of presentations are distinct from their objects because presentations with distinct contents sometimes have the same object. Twardowski concludes that if the content and object of presentations were identical, then presentations with

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[1] Grossmann, *Meinong*, p. 48. Grossmann does not explain why phenomenology requires Twardowski's distinction between content and object. But it may be conjectured that without the distinction phenomenology would amount to no more than a highly subjective idealistic version of phenomenalism.

distinct contents could not be directed toward the same object.[2] The appeal to a lawlike principle of this kind suggests that the argument aspires to full generality, and that it may be intended to justify the conclusion that the content of every psychological presentation is always distinct from its object. Twardowski claims: "...a brief consideration shows that the differences between content and object of a presentation which can be ascertained when the object exists also are present when the object does not exist."[3] This may be interpreted as asserting that the content and existent or nonexistent object of any arbitrary presentation are necessarily distinct. Twardowski's distinction has also been described by Grossmann as applying with full generality to the content and object of every presentation: "Twardowski... argues, in defense of his distinction, that the content and object of a presentation could not possibly be the same."[4]

But Twardowski offers only inductive evidence to support the conclusion of the third argument. He considers a thought about the birthplace of Mozart, and another about the city located at the site of the Roman Juvavum.[5] These thoughts presumably have the same object (Salzburg), but different contents (though Grossmann has also disputed the assumption that their objects are precisely identical).[6] The contents of presentations in both thoughts are clearly distinct from their shared object, which defeasibly justifies Twardowski's thesis. But the example does not show that the distinction always holds. It may turn out that most thoughts with distinct contents and identical objects are such that their contents are not identical with their objects. But the conclusion is not fully generalizable.

2. *Diagonal Content-Object Coincidence*

To construct a counterexample to Twardowski's argument, it is necessary and sufficient to describe a logically possible set of circumstances in which:

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2 Twardowski, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*, pp. 29—31.

3 Ibid., p. 27. Twardowski writes: "That the content and object of a presentation are different from each other will hardly be denied when the object exists." This implies that the distinction between content and object has full generality at least with respect to presentations about or directed toward existent objects. But even this restricted thesis is contradicted by the counterexample outlined in the section on 'Diagonal Content-Object Coincidence'. In an enactment of the counterexample in which the assumptions are true of actual presentations, the existent object of an actual thought is identical to its existent content.

4 Grossmann, *Meinong*, p. 48.

5 Twardowski, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*, p. 29.

6 Grossmann, *Meinong*, pp. 51—2.

(i) the contents of two thoughts or presentations are distinct; (ii) their objects are identical; (iii) the content of at least one thought or presentation is identical to its object.

The content (*noema*) of presentation P is designated by a term for an existent or nonexistent object enclosed in stars, $C(P) = *T*$.[7] The notation makes it possible to formalize three useful principles about the contents and objects of presentations.

(P1) $(C(P_i) = *T_i* \;\&\; C(P_j) = *T_j*) \supset (C(P_i) = C(P_j) \equiv (`T_i' = `T_j'))$

This states that the contents of presentations are identical if and only if they are designated by identical terms in a star quotation context. Distinct terms can designate the same object, but identity of contents holds only when the very same term is used correctly to complete the content context $*\ldots* = *\ldots*$.

(P2) $C(P_i) \neq C(P_j) \supset P_i \neq P_j$

If the contents of presentations are distinct, then the presentations themselves must also be distinct. The principle is intuitively justified by the consideration that one and the same presentation cannot have more than one psychological or experiential content.

(P3) $C(P_i) = *T* \supset O(P_i) = T$

If the content of a presentation is designated by a singular term in the star quotation context, then the object of the presentation is identical to the existent or nonexistent referent of the term. The converse of the principle does not obtain, because the object of the presentation can be designated by a coreferential term other than that used in the star quotation context specification of its content.

The counterexample can now be given. Two assumptions are made.

1.	$C(P_1) = *O(P_2)*$	
2.	$C(P_2) = *C(P_1)*$	
3.	$C(P_1) \neq C(P_2)$	(1,2,P1)
4.	$P_1 \neq P_2$	(3,P2)
5.	$C(P_2) = *C(P_1)* \ldots O(P_2) = C(P_1)$	(P3)

[7] The star quotation convention for designating thought contents or *noemata* is adapted from Castañeda, *Thinking and Doing: The Philosophical Foundations of Institutions* [1975], pp. 19—20.

6.	$O(P_2) = C(P_1)$	(2,5)
7.	$O(P_2) = {*}O(P_2){*}$	(1,6)
8.	$C(P_1) = {*}O(P_2){*} \ldots O(P_1) = O(P_2)$	(P3)
9.	$O(P_1) = O(P_2)$	(1,8)
10.	$O(P_1) = {*}O(P_2){*}$	(7,9)
11.	$C(P_1) = O(P_1)$	(1,10)
12.	$C(P_1) = O(P_1) \vee C(P_2) = O(P_2)$	(11)
13.	$\sim[C(P_1) \neq O(P_1)\ \&\ C(P_2) \neq O(P_2)]$	(12)

The assumptions in (1) and (2) are logically possible and jointly consistent. Thoughts can be about other thoughts, and in particular they can be about the objects and contents of other thoughts. Assumption (1) states that the content of presentation 1 is referentially identical to *the object of presentation 2* (the *noema* or psychological thought content equivalent of the description 'the object of presentation 2'). Assumption (2) states that the content of presentation 2 is referentially identical to *the content of presentation 1* (the *noema* or psychological thought content equivalent of the description 'the content of presentation 1'). Conclusions (3)—(4), (9), and (11), satisfy counterexample requirements (i), (ii), and (iii), respectively. Presentation 1 indirectly refers to its own content (and is therefore directed toward its content, has its own content as its object), by referring to the object of presentation 2, which by stipulation is identical to the content of presentation 1. The counterexample construction is indirectly self-referential like some diagonalized versions of the Liar or Epimenides paradox.

A: Sentence *B* is true.
B: Sentence *A* is false.

The counterexample refutes Twardowski's argument. But in many ways it is too abstract. Here is a scenario in which the assumptions describe two logically possible thoughts or presentations in an imaginable concrete real life situation.

Suppose that subjects *A* and *B* are instructed deliberately to entertain certain thoughts on cue. When the clock strikes 12:00, *A* is to think *the object of *B*'s thought*, and *B* is to think *the content of *A*'s thought*. In this enactment, the contents of *A*'s and *B*'s distinct presentations are distinct, but their objects are exactly the same. The object of *B*'s thought is the content of *A*'s thought. But the object of *A*'s thought is the object of *B*'s thought, which once again is the content of *A*'s thought. Whether *A* knows it or not, *A*'s thought is about its own content. This is rather like someone thinking *whoever is now in the Vienna Opernhaus*, not realizing that he himself is the only person in the Opernhaus, and so, without knowing it, is thinking about himself. The content and object of *A*'s thought in that case are precisely identical.[8]

A's thought ≠ *B*'s thought.
The content of *A*'s thought ≠ the content of *B*'s thought.
The object of *A*'s thought = the object of *B*'s thought
(the object of *B*'s thought = the content of *A*'s thought, *the object of *B*'s thought*).
The object of *A*'s thought = the content of *A*'s thought.

This provides a counterexample to Twardowski's thesis. It defeats the principle that if the objects but not the contents of distinct presentations are identical, then their contents are distinct from their objects. The counterexample proves that the content and object of psychological presentations are not always distinct. In some logically possible circumstances they are strictly identical.

3. Reinterpreting Twardowski's Reduction

Twardowski correctly maintains that the contents of some presentations are distinct from their objects. But this existential conclusion is established with less difficulty by his first and second arguments.

The first argument trades on the claim that the content of a presentation can exist even when its object does not exist.[9] In a thought or presentation about the golden mountain, the content through which the golden mountain is presented exists, even though the object itself, the golden mountain, does not. But the argument is not fully generalizable, since the contents and objects of some presentations coexist.

The second argument is based on an observation attributed to Benno Kerry. Kerry argues that the properties of an intentional object do not always attach to the content of the thought or presentation through which the object is presented.[10] The golden mountain may be golden, but the content of a thought about the golden mountain is not. By appeal to a Leibnizian

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[8] See Chisholm, "Beyond Being and Nonbeing" [1972], pp. 245—55. Chisholm offers a similar indirectly self-referential construction, involving my wish that your wish come true, when your wish is merely that my wish come true.

[9] Twardowski, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*, pp. 27—8.

[10] Ibid., pp. 28—9. Twardowski refers to Kerry, "Über Anschauung und ihre psychische Verarbeitung", *Vierteljahrsschrift für wissenschaftliche Philosophie*, X, p. 428.

identity principle, it follows that the contents of at least some presentations are distinct from their objects. Again, the conclusion is not fully generalizable, because the properties of the contents of some presentations are also properties of their objects. The content of a presentation about a psychological phenomenon has the property of being a psychological phenomenon, and so does its object. Whether or not the content and object share all properties may be difficult to determine for many presentations. But if a Leibnizian identity principle is assumed, then the counterexample shows that the contents and objects of at least some presentations have all their properties in common.

In a fourth argument, which Twardowski also attributes to Kerry, the content and object of psychological presentations are distinct because a presentation with unitary content can be directed toward a plurality of objects. But Twardowski rejects the argument, along with the assumption that a plurality of objects can fall under a general unitary presentation.[11] Even if the argument were satisfactory, it also plainly lacks the generality necessary to establish the conclusion that the content and object of psychological presentations are never identical.

The philosophically interesting claim would be that the content and object of a thought or presentation are always distinct. If the text is correctly interpreted as implying that Twardowski's arguments for the distinction between content and object are supposed to apply with full generality to the content and object of every presentation, then the conclusion is contradicted by the counterexample. If Twardowski did not intend the argument to have full generality, the counterexample nevertheless establishes a limit to any induction on the contents and objects of psychological presentations that might support an empirical thesis about the distinction.[12]

Yet there is a sense in which Twardowski can be said to have marked an important distinction between the content and object of every thought or presentation. It is probably not the kind of distinction Twardowski meant to offer, nor is it quite the same distinction as that usually attributed to him. But the counterexample and inductive limitations of Twardowski's arguments require reevaluation of the distinction between act, content, and object.

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[11] Twardowski, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*, p. 31.

[12] Meinong in "Über Gegenstände höherer Ordnung und deren Verhältnis zur inneren Wahrnehmung" accepts Twardowski's distinction between content and object, but rejects his third and fourth arguments. See Findlay, *Meinong's Theory of Objects and Values*, p. 11. Grossmann, *Meinong*, pp. 48—54. Phenomenological coincidence of content and object is recognized by Edmund Husserl, *Logical Investigations* (*Logische Untersuchungen*) [1913], pp. 287—91, and especially pp. 290—91.

We may proceed by analogy. There are exclusive and nonexclusive kinds of distinctions. The distinction between being an even number and being an odd number is exclusive, because nothing can be both even and odd. But there are also nonexclusive distinctions among the properties of numbers, such as the distinction between being even and having an irrational square root. The properties are unquestionably distinct. But since not every even number fails to have an irrational square root, it follows that the property of being even does not logically exclude the property of having an irrational square root.

These elementary considerations can now be applied to Twardowski's arguments for the distinction between content and object. The counterexample shows that Twardowski's distinction is not exclusive, but at most nonexclusive. Twardowski's arguments prove that content and object are sometimes distinct. But they do not exclude the possibility that one and the same component of thought is both its own content and object. The conflation of content and object is also implied in intentional analyses of sensation.[13]

Twardowski would probably be dissatisfied with this reinterpretation of his distinction. But the counterexample suggests that it cannot be made any stronger. The distinction remains useful nonetheless, just as it is useful to distinguish between the properties of being an even number and having an irrational square root. The nonexclusive distinction between content and object does not preclude the overlap or intersection of content and object. But neither does it forfeit any of its significance for phenomenology or the object theory. Thought may still admit of an act-content-object structure, as in Twardowski's classical phenomenological reduction. It is just that in some extraordinary cases the content and object of thought turn out to be precisely the same.

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[13] Jacquette, "Sensation and Intentionality" [1985], pp. 429—40.

II. Private Language and Private Mental Objects

1. *Wittgenstein's Private Language Argument*

Wittgenstein's private language argument is scattered over a number of related philosophical remarks. There is disagreement about whether Wittgenstein intended to support a thesis concerning the impossibility of private language, and about the significance of the private language argument in his later philosophy. It has been suggested that the private language argument has important negative consequences for phenomenology, phenomenalism, sense data theory, foundationalist epistemology, solipsism or skepticism about other minds, the existence of so-called private mental objects, and intentionality.

The private language argument contradicts Meinongian object theory, because the logic makes it possible to designate private mental objects. If the private language argument is sound, then, according to some interpretations, there cannot be any private mental objects, nor a language in which private mental objects are intelligibly designated. The classification of converse intentional or psychological properties as nuclear rather than extranuclear enables the logic to designate (existent or nonexistent) objects that are necessarily thought about by only one person. Even if private mental objects do not exist, the fact that they can be intelligibly designated at least as nonexistent objects in a Meinongian language, and that inferences can be drawn from propositions in which they are designated, contradicts the conclusions of the private language argument as it is usually understood. Either the logic must be amended in such a way that it is prevented from designating existent or nonexistent private mental objects, or else the private language argument must be refuted. The attempt is made here to interpret the private language argument, explain its consequences, and finally argue that although the argument is valid, it is unsound.

Wittgenstein denied that he was offering philosophical theses,[1] and while

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[1] Wittgenstein, *Philosophical Investigations* § 109, p. 47^{e}: "And we may not advance any kind of theory. There must not be anything hypothetical in our considerations. We must do away

some commentators have taken him at his word for this,[2] others have found it difficult to believe that he does not in fact, despite verbal protests to the contrary, argue in favor of the thesis that private language is impossible.[3] What is not open to doubt is that Wittgenstein's remarks in the *Blue and Brown Books* and *Philosophical Investigations* have led others to adopt the view that there cannot be a private language. The argument suggested by these remarks therefore needs to be evaluated on its own merits, without concern for the historical problem of Wittgenstein's actual intentions.[4]

In the *Philosophical Investigations*, Wittgenstein writes:

> But could we also imagine a language in which a person could write down or give vocal expression to his inner experiences — his feelings, moods, and

with all *explanation*, and description alone must take its place." Also § 128, p. 50[e]: "If one tried to advance *theses* in philosophy, it would never be possible to debate them, because everyone would agree to them."

2 Judith Jarvis Thompson, "Private Languages" [1964], p. 20: "On the contrary, the disservice was done by those who credited the [anti-private-language] thesis to him [Wittgenstein]. If nothing else, they failed utterly to take seriously his claim that he held no opinions and put forward no theses in philosophy." Timothy Binkley, *Wittgenstein's Language* [1973], p. 172: "The discussion of private language is ... contrapuntal, and not the development of a theory which claims that private language is impossible." And, p. 173: "If [Wittgenstein] does not want to put forward theses, he avoids negative as well as positive claims: his task is ultimately neither to affirm nor to deny philosophic claims or theories. If p is a philosophic thesis, so is $-p$, and Wittgenstein will have nothing to do with the assertion of either one." F.A. Siegler, "Comments" (on Newton Garver's "Wittgenstein on Criteria"), in *Knowledge and Experience: Proceedings of the 1962 Oberlin Colloquium in Philosophy*, edited by C. D. Rollins [1963], p. 77: "Has Wittgenstein a logical theory? He disavows having any sort of theory at all... the fact that Wittgenstein disavows any logical theory should lead one carefully to question assertions that he does have one." Colin McGinn, *Wittgenstein on Meaning* [1984], p. 2: "We must avoid the temptation to regard the text as a sort of cipher through which we must penetrate to reveal the linearly ordered argument beneath. It is not that Wittgenstein really has an argument of orthodox form which for some inscrutable reason he chose to present in a disguised fashion." Jaakko Hintikka, "Wittgenstein on Private Language: Some Sources of Misunderstanding" [1969], pp. 423—25. Warren B. Smerud, *Can There Be a Private Language? An Examination of Some Principal Arguments* [1970], pp. 14—5.

3 Norman Malcoln, "Wittgenstein's *Philosophical Investigations*" [1962], pp. 74—100. James D. Carney, "Private Language: The Logic of Wittgenstein's Argument" [1960], pp. 389—96. C. W. K. Mundle, "'Private Language' and Wittgenstein's Kind of Behaviourism" [1966], p. 35. Castañeda, "The Private Language Argument", in *Knowledge and Experience*, pp. 88—132. John Turk Saunders and Donald F. Henze, *The Private Language Argument: A Philosophical Dialogue* [1967], p. 5. Anthony Kenny, *Wittgenstein* [1973], pp. 178—202.

4 Smerud, *Can There Be a Private Language?*, p. 15, adopts a similar provision. On some interpretations, Wittgenstein's remarks on private language are not even supposed to suggest the impossibility of private language, but show only that a private language, if there were such a thing, could not provide the basis for a phenomenalistic reduction of public language. See Moltke S. Gram, "Privacy and Language" [1971], pp. 298—327. Andrew Oldenquist, "Wittgenstein on Phenomenalism, Skepticism, and Criteria" [1971], pp. 394—422.

> the rest — for his private use? — Well, can't we do so in our ordinary language? — But that is not what I mean. The individual words of this language are to refer to what can only be known to the person speaking; to his immediate private sensations. So another person cannot understand the language.[5]

A private language is a language that can only be understood by the person who uses it to refer to his own private or internal psychological experiences. It must be such that it is impossible for anyone but the user to understand. That private languages are necessarily unintelligible to others at once rules out extraneous interpretations of the argument. The idea that a private language in Wittgenstein's sense of the word might be a secret code referring to sensations that could in principle be taught to others is eliminated, since it would not be logically impossible for another person to decipher. Also excluded are interpretations of the argument based on mere difficulties in learning to speak a language in circumstances of total social isolation. These problems are not central to Wittgenstein's concerns.[6]

Another preliminary matter of interpretation has to do with the purity or mixture of putative private languages. It appears that for Wittgenstein, a private language may be embedded in or a proper part of a larger public language.

> Let us imagine the following case. I want to keep a diary about the recurrence of a certain sensation. To this end I associate it with the sign "S" and write this sign in a calendar for every day on which I have the sensation.[7]

If a publicly understood calendar, with its square blocks or separate pages conventionally representing days of the year, is used as a medium or framework for recording purported private sensations, then the private language in question must be a mixed or impure private language, in which signs supposedly designating private incommunicable sensations are presented within the conventions of a public grammar. Castañeda writes:

> It is not clear that Wittgenstein's is the issue between a public and an *ab-*

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5 Wittgenstein, *Philosophical Investigations* § 243, pp. 88ᶜ—9ᶜ.

6 A. J. Ayer may have inadvertently misled the private language controversy in his essay, "Can There Be a Private Language?" [1971], pp. 50—61. Ayer complicates Wittgenstein's example by asking whether a Robinson Crusoe, left alone on a desert island, could invent a language in total social isolation in which private or internal sensations are described.

7 Wittgenstein, *Philosophical Investigations* § 258, p. 92ᶜ.

> *solutely* private language. Very naturally, one would expect to find many cases of private language all linked up by a series of family resemblances, ranging off from a language *all* of whose individual words refer only to private objects.[8]

Wittgenstein insists that language does not exist in a vacuum, but in a context of social institutions that can be described as a form of life. He frequently emphasizes the background and stage setting without which the definition and use of a sign in a language are meaningless. Thus he asks:

> But what does it mean to say that he has 'named his pain'?— How has he done this naming of pain?! And whatever he did, what was its purpose?— When one says "He gave a name to his sensation" one forgets that a great deal of stage-setting in the language is presupposed if the mere act of naming is to make sense. And when we speak of someone's having given a name to pain, what is presupposed is the existence of the grammar of the word "pain"; it shews the post where the new word is stationed.[9]

Since Wittgenstein's discussion of the diary of private sensations follows immediately after these observations, it may be concluded that the objections to the possibility of a private language suggested by his remarks apply to the concept of a mixed or impure private language.

The reason for disputing the possibility of mixed private languages in Wittgenstein's argument depends on what might be called the criterion requirement for the meaningful application of terms in a language. Wittgenstein maintains that the person who attempts to keep a diary of private sensations does not have a criterion of correctness for the use of sign 'S'.

> I will remark first of all that a definition of the sign cannot be formulated. — But still I can give myself a kind of ostensive definition. — How? Can I point to the sensation? Not in the ordinary sense. But I speak, or write the sign down, and at the same time I concentrate my attention on the sensation — and so, as it were, point to it inwardly. — But what is this ceremony for? for that is all it seems to be! A definition surely serves to establish the meaning of a sign. — Well, that is done precisely by the concentrating of my attention; for in this way I impress on myself the connexion between the sign and the sensation. — But "I impress it on myself" can only mean: this process brings it about that I remember the connexion *right* in the future. But in the present case I have no criterion of correctness.

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8 Castañeda, "The Private Language Argument", p. 136.

9 Wittgenstein, *Philosophical Investigations* § 257, p. 92[c].

> One would like to say: Whatever is going to seem right to me is right. And that only means that here we can't talk about 'right'.[10]

Language is a rule-governed activity. But according to Wittgenstein, there can be no rules for the use of a term in a language if there are no criteria of correct or incorrect use of the term. To explain this, it is necessary to understand what Wittgenstein means by a criterion of correctness, and what difference the criterion or lack of criterion is supposed to make in the attempt to define or understand terms.

Wittgenstein's concept of criterion is sometimes mistakenly regarded as having only epistemological significance. This may be the result of his frequent references to memory and the limitations of memory in attempts to reidentify a recurring sensation. Wittgenstein implies that an individual's memories, however internally coherent, are inadequate as a criterion for the correct or incorrect application of ostensible private language signs. He claims that memory is ordinarily relied upon with good justification only because of the logical possibility that memory can be verified or corroborated by checking it against information in nonprivate, nonmnemic sources. But this is impossible in the attempted use of private language terms, because the sensation in question by hypothesis has no external or publicly distinguishing behavioral manifestations by which its occurrence could be known even in principle by anyone but the individual who privately experiences it.

It may then be inferred that since there is no way for the person to determine outside of his memories whether or not another sensation is the same or of the same kind as that originally supposed to have been designated by sign 'S', the use of the sign is not actually governed by a linguistic rule, and therefore cannot be part of a genuine language.

> Let us imagine a table (something like a dictionary) that exists only in our imagination. A dictionary can be used to justify the translation of a word X by a word Y. But are we also to call it a justification if such a table is to be looked up only in the imagination?— "Well, yes; then it is subjective justification."
>
> — But justification consists in appealing to something independent. — "But surely I can appeal from one memory to another. For example, I don't know if I have remembered the time of departure of a train right and to check it I call to mind how a page of the time-table looked. Isn't it the

[10] Ibid. § 258, p. 92e.

same here?" — No; for this process has go to produce a memory which is actually *correct*. If the mental image of the time-table could not itself be *tested* for correctness, how could it confirm the correctness of the first memory? (As if someone were to buy several copies of the morning paper to assure himself that what it said was true.)

Looking up a table in the imagination is no more looking up a table than the image of the result of an imagined experiment is the result of an experiment.[11]

This appears to support the epistemological or verificationist interpretation of the criterion of correctness objection to private language. Yet it is not simply the difficulty of relying on memories to verify that a sensation is of the same kind as that supposedly designated by the first use of 'S' that prevents there from being a criterion of correctness or linguistic rule governing the private language term in the diary, but rather the lack of any criterion or linguistic rule is the result of deeper semantic trouble, of which the epistemological, memory, and verification problems are mere symptoms. The reason why memory fails to provide verification of the occurrence of sensations like that supposedly designated by the diarist's use of sign 'S' is that the sign fails to designate even in its very first attempted use.

The epistemological interpretation of the criterion objection says in effect that the application of sign 'S' by the diarist stands in need of justification, which an individual's memory, the only possible source of justification under the circumstances, is unable to provide. This, in some unspecified way, is supposed to preclude the possibility of a linguistic rule governing the diarist's use of the sign in his putative private language. But this is not an accurate interpretation of Wittgenstein's argument. Rush Rhees writes:

Wittgenstein did not say that the ascription of meaning to a sign is something that needs justification. That would generally be as meaningless as it would if you said that language needs justification. What Wittgenstein did hold was that if a sign has meaning it can be used wrongly.[12]

The difficulty in the diarist's attempted use of sign 'S' is not just that memory alone is an inadequate epistemological basis for justifying subsequent applications of the sign, but rather that the sign has no meaning in *any* of its attempted applications, including the first, because there is no sense in which the sign can be used incorrectly.

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[11] Ibid. §265, pp. 93^{e}—4^{e}.

[12] Rush Rhees, "Can There Be a Private Language?" [1954], p. 68.

Another way of expressing the criterion objection is to say that there is no way satisfactorily to distinguish between instances in which an individual has correctly followed a linguistic rule involving the putative private language sign and instances in which the individual merely believes that he is correctly following a linguistic rule for the application of the sign.[13]

> Are the rules of the private language *impressions* of rules?— The balance on which impressions are weighed is not the *impression* of a balance. "Well, I believe that this is sensation S again."— Perhaps you *believe* that you believe it![14]

It is not enough for an individual such as the diarist to believe or be under the impression that he is following a rule when using sign 'S' in attempting to record the occurrences of an incommunicable sensation. It must be possible to distinguish between situations in which the individual merely believes or is under the mistaken impression that he is following a linguistic rule for the putative private language sign from situations in which he is actually following a rule. But this, Wittgenstein suggests, because of the nature of the case, is precisely what it is impossible to do. Without an independent or external check on the application of the private language sign, it is not possible even in principle for the diarist to use the sign correctly or incorrectly.[15]

If the argument is sound, it entails that the very first application of sign 'S' is equally ungoverned by any linguistic rule, despite its apparent stipulative character. This is different from the epistemological interpretation of the criterion objection, since the latter provides no basis for the claim that the original or very first use of a private language term also lacks meaning. The diarist does not actually make use of or act in accord with a linguistic rule in any of his attempts, including the first, but at best has the mistaken impression that he is following a rule.

> Then did the man who made the entry in the calendar make a note of *nothing whatever*? — Don't consider it a matter of course that a person is making a note of something when he makes a mark — say in a calendar. For a note has a function, and this "S" so far has none.[16]

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[13] Carney, "Private Language: The Logic of Wittgenstein's Argument", p. 563. Smerud, *Can There Be a Private Language?*, pp. 28f. Ross Harrison, *On What There Must Be* [1974], pp. 158—59.

[14] Wittgenstein, *Philosophical Investigations* §§ 259—260, p. 92^{e}.

[15] Castañeda, "The Private Language Argument", pp. 144—45.

[16] Wittgenstein, *Philosophical Investigations* § 260, pp. 92^{e}—3^{e}.

It is clear that for Wittgenstein, in the absence of whatever he means by criteria of correctness, the terms of a putative private language are without sense. This is worse than the epistemological inability to verify that subsequent occurrences of sensations are or are not of the same kind as a single instance originally supposed to be stipulatively designated by a purported private language sign.

If the epistemological rather than the deeper semantic interpretation of the criterion objection to private language were accepted, then it would still be possible to formulate a restricted version of a private language in which individual sensations are designated by signs, but in which no terms are offered as predicate or sensation kind terms, and no attempt is made to reidentify sensations as falling under the same sensation kind term or private language predicate. Each sensation or sensation instance could then receive its own private logically proper name, and the epistemological restrictions of verification by memory comparison in the attempt to classify sensations by kinds would be taken as limiting reidentification beyond the capabilities of language. Nevertheless, the individually designated private sensations could still be said to have been named in a limited private language consisting only of individually designating terms. The semantic interpretation of the private language argument suggested by Wittgenstein's remarks avoids even this attenuated sort of private language by reflecting back on the meaninglessness of every attempted application of the sign. The absence of a criterion of correctness for the terms invalidates even the first attempted or original ostensibly stipulative designation of any private sensation.[17]

[17] See Kripke, *Wittgenstein on Rules and Private Language* [1982]. Kripke's interpretation is mistaken in several ways. Kripke does not accept, and notes that Wittgenstein would not endorse, outright skepticism about the meaningfulness of language. But unless Kripke's community of language users solution to Humean skepticism about language is satisfactory, his exposition leads inevitably to this conclusion. That the proposal is unworkable is sufficiently indicated by the fact that absolutely any reaction of the language community can be regarded by the Humean skeptic as expressing approval or disapproval of the use of a term (say, the continuation of the series 2, 4, 6,...), just as any number placed after 6 in the series can be regarded by the Humean skeptic as correctly continuing the series. Thus, if Kripke's analysis is correct, it provides the basis for an immediate *reductio ad absurdum* of the private language argument. Further criticisms of Kripke's interpretation are given by McGinn, *Wittgenstein on Meaning*.

2. Phenomenology, Intentionality, and Psychological Privacy

The most interesting byproduct of the private language argument is the claim that if private language is impossible, then there cannot be private mental objects. The inference seems to be that if there cannot be a language in which so-called private mental objects are designated, then necessarily there are no objects of the sort to be designated. O. R. Jones writes:

> The implication for the Cartesian viewpoint should be clear. Sensations, feelings, and so forth are not private in the way supposed in the Cartesian view. If they were, then only a private language involving private rules would be possible to talk about them. But such a language is impossible. Conversely, since we are able to talk about our sensations and feelings, they are not Cartesian private objects.[18]

This would also be a serious consequence for object theory, since the domain of objects in the logic is determined by the terms and well-formed expressions it contains. If there are no terms designating existent or nonexistent private mental objects in the syntax of the logic, then there can be no corresponding existent or nonexistent private mental objects in the object theory domain. Yet there are terms in the object theory which do ostensibly designate private mental objects, and which appear to obey the linguistic rules of the formal system. If the private language argument is sound, drastic revision is required.

The basic presuppositional foundations of phenomenology, sense data theory, phenomenalism, and intentional and subjective theories of many kinds, may also be undermined in the same way by the private language argument. The threat to phenomenology and the intentional outlook posed by the private language argument is acknowledged by Harry P. Reeder.

> Phenomenology often comes into criticism from linguistic analysts of the Wittgensteinian tradition who claim that the language used in phenomenological description must be an unacceptable form of language, due to the fact that the reflexive shift of the phenomenological reduction bars one from criteria of consistency of language use.[19]

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[18] Smerud, *Can There Be a Private Language?*, p. 107f.

[19] Harry P. Reeder, "Language and the Phenomenological Reduction: A Reply to a Wittgensteinian Objection" [1979], p. 35.

And Alan Paskow observes:

> Despite the enormous body of commentary by analytic philosophers on Wittgenstein's theory of psychological privacy, I know of no phenomenologist who has attempted to deal with and respond to this theory, which is a threat to the very foundations of any philosophy that accords an honorific status to the data of subjectivity... What features of Wittgenstein's view of privacy constitute this challenge? Essentially those which point to the two following conclusions (which, if true, logically entail the falsity of critical suppositions of several varieties of phenomenological methodology): (a) that it is not possible to intuit *apodictically* the essences or patterns of sensory presentations; and further, (b) that it is not even possible to formulate with respect to such presentations true (or false) descriptions of fact. Indeed, Wittgenstein implies that subjective claims do not denote one's own private experiences and that the communicative function of such statements is different from what most ('pre-analytic') philosophers thought it to be.[20]

Any philosophical theory based on data held to be of privileged epistemic access, or that attempts to reconstruct knowledge from phenomenological descriptions of the private contents of consciousness, is plainly contradicted if the private language argument is sound. The kinds of philosophical theories consistent with the argument by contrast then are extensional, behaviorist, physicalist, and materialist. Phenomenology and intentional philosophy are excluded.

Philosophical behaviorism is supported if it can be shown that there are no private mental objects or private psychological experiences. It must then be possible at least in principle to find external or publicly observable distinguishing criteria for every so-called private mental event.[21] No thought, feeling, or occurrence in the mind could be concealed from the behavioral scientist equipped with sufficiently sophisticated monitoring equipment. It would always be possible for an outsider to know a person's innermost

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20 Alan Paskow, "A Phenomenological View of the Beetle in the Box" [1974], p. 277.

21 Mundle, "'Private Language' and Wittgenstein's Kind of Behaviourism", p. 35: "Though Wittgenstein professed to eschew philosophical theories, he seems to have accepted a theory which could not be confirmed simply by observing how people talk when not doing philosophy, namely a form of Behaviourism... If Wittgenstein had formulated this view explicitly, it would presumably have run something like this: that words which are ostensibly used to name or refer to private experiences can have meaning only by referring to overt behaviour (as in 'he is in pain') or by deputising for other, and 'natural' forms of behaviour (as in 'I am in pain'). Wittgenstein's main reason for putting forward this account seems to have been his acceptance of a questionable theory of meaning, namely that statements can be meaningful only if they are *publicly* verifiable."

thoughts and sensations, no matter how carefully disguised by self-control or effort of will. The office of mind in the phenomenal world, a place of freedom and resort, would be as open to public empirical inspection as any external entity.

It is also maintained that the private language argument refutes dualism, solipsism, and phenomenalist epistemology. Warren B. Smerud writes:

> Turning now to the claim that the anti-private-language thesis is philosophically important, the sort of significance which it is alleged to have is indicated by Norman Malcolm, who informs us that the possibility of a private language is presupposed in the formulation of a number of long-standing philosophical problems as well as in the sorts of attempts which have commonly been made to resolve them; for example, all traditional problems concerning inferring the existence of other minds, phenomenalism, and sense-data theory, presuppose the possibility of the sort of langauge which the anti-private-language thesis denies.[22]

It is not worthwhile to exhibit all of the inferences needed to obtain these further results. In general, it must first be shown that a particular problem presupposes the possibility of private language. The 'problem' is then made unproblematic by the consideration that if the private language argument is sound, then private language is impossible.

These consequences may be accepted by adherents of the private language argument, even if Wittgenstein would not have approved of them. But it should be observed that several philosophers have interpreted Wittgenstein's remarks throughout the later writings as strongly hinting that he would indeed have supported at least some of the secondary results of the private language argument. It may appear that as usual, Wittgenstein has deliberately left the most important things unsaid. He insists that readers of his unconventional philosophical investigations think for themselves and draw their own conclusions from the ideas he presents.[23]

It has already been observed that if the private language argument is sound, then the domain of objects cannot contain any private mental objects. But there are well-formed expressions and well-defined terms in the logic which ostensibly designate objects that could only be the objects of thought of particular minds. The object necessarily thought about only by per-

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[22] Smerud, *Can There Be a Private Language?*, p. 16. See also, Malcolm, "Wittgenstein's *Philosophical Investigations*", p. 75.

[23] Wittgenstein, *Philosophical Investigations* "Preface", p. x^e: "I should not like to spare other people the trouble of thinking. But, if possible, to stimulate someone to thoughts of his own."

son A may be designated in the logic by the definite description, '$\iota_m x(\forall y)\Box(Thinks(y,x) \equiv y = A)$'. If the argument is sound, then not only is there no such existent private mental object, but, since terms ostensibly designating the object are not governed by linguistic rules, the terms themselves cannot belong to a genuine language. This implies that the object theory is not a genuine language. To amend the theory in conformity with the private language argument would require either rejecting the syntactical criterion for domain membership based on the *Annahmen* thesis, or else revising the formation principles of the logic to exclude all possible terms ostensibly designating private mental objects.

But any constant term 'o_i' potentially designates a private mental object if it is permitted to abbreviate a term such as the definite description mentioned above. Generalizations of the form '$(\forall x)(\ldots x \ldots)$' apply to all o_i, and there is no suitable technical way of eliminating ostensibly designating private mental object terms by syntactical stipulation, unless converse intentional or psychological properties are shifted from the category of nuclear constitutive properties to the extranuclear category, as in Parsons' and Routley's formulations.[24]

These measures are unnecessary if the private language argument is invalid or unsound. Private mental objects may then be admitted into the domain of the logic, and the object theory can be used to formalize phenomenological and intentional theories so completely as to include their very foundations in private mental experience. Whether or not these theories are true, it must at least be possible to represent their distinctive logical structures.[25]

3. A Diary of Private Sensations and the Beetle in the Box

If I am not in pain, I find that I do not know or rightly remember what pain is. If pain and sensation language generally were public rather than private, how could I not know what another person means when he says that he is in pain? I may be in great sympathy with the person, and I may have access to all the external behavior associated with pain, such as verbal reports, wincing, muscle tension, blood pressure. But it would be wrong to conclude that I know what the person is experiencing, except in the most general terms.

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24 Parsons, *Nonexistent Objects*, pp. 22—6. Routley, *Exploring Meinong's Jungle and Beyond*, p. 266.

25 Nicholas F. Gier, *Wittgenstein and Phenomenology: A Comparative Study of the Later Wittgenstein, Husserl, Heidegger, and Merleau-Ponty* [1981].

The situation is not improved if I am told that the person has a sharp pain, excruciating pain, stabbing pain, flowing, burning, or traveling pain. I may know that the person is in pain, but I do not know what the pain is like, no matter what descriptive efforts are made in the public sensation language to help me understand. This is something that can only be known to the individual in the privacy of thought, and that can be expressed, if at all, only in a private language. The public sensation language used to communicate limited information about psychological states does not embody knowledge of what L. C. Holborow has appropriately called the *infima species* of sensation.[26]

A man feels a twinge of pain and opens his diary to the page for that day. He writes down, 'burning twinge'. Then he scratches it out and rewrites, 'stabbing twinge'. He frowns, erases the words, and tries, 'wrenching twinge'. This too he rejects. He decides that no word in the language he has learned from others since his childhood adequately expresses the feeling he has experienced, and decides, arbitrarily, to write down the sign 'S' in the diary calendar space to indicate that on this day he experienced the sensation which was something but not quite like a burning, stabbing, wrenching twinge. He resolves for diagnostic purposes to write down the sign whenever he feels the pain.

The next day he experiences another pain, and writes 'S' in the journal. Alongside it he makes the remark, 'I think it was the same as S, but cannot be sure. I will call it "S?".' He corrects the entry by writing 'S/S?' instead. Over a period of time he finally sees a doctor. He has not yet found a more apt way to designate the pain he experiences than by calling it 'S', 'S?', or indicating his uncertainty about whether or not the pain sensation is just like the first experience he chose to record. He tells the doctor that on a particular occasion he had a long S-seizure, and that he has no better way to describe it than to say it is something but not quite like a burning, stabbing, wrenching twinge. He says that for lack of exact description he calls it his 'S' pain.

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26 L.C. Holborow, "Wittgenstein's Kind of Behaviourism" [1967], p. 128: "We must now return to Mundle's diarist and his stomach pains. It is now clear that his claim that this pain is just like the others that have troubled him all morning is a 'result of observation' in Wittgenstein's sense. But the claim rests on the contention that this is a specific type of pain which can be discriminated, but not publicly described. Others can be told that it is a sharp type of stomach pain, but this does not give its *infima species*." Wittgenstein, "Wittgenstein's Notes for Lectures on 'Private Experience' and 'Sense Data'" [1968], p. 233: "It is as though, although you can't tell me exactly what happens inside you, you can nevertheless tell me something general about it. By saying e.g. that you are having an impression which can't be described. As it were: There is something further about it, only you *can't say* it; you can only make the general statement. It is this idea which plays hell with us."

The doctor asks questions about the pain, and runs through his own vocabulary of sensation terms, attempting to explain to the man what each kind of pain is supposed to be like. But the man remains unwilling to accept any of these terms as more accurately naming the pain he feels. He does not give up 'S', and insists that the doctor also refer to it as 'S' when discussing the case. The doctor does not know exactly what is meant by the sign, but he complies with the request in order to humor his patient, and designates the pain, whatever it is, by 'S', even though he secretly believes that the man is stubbornly refusing to admit that the pain is in fact like one of the kinds listed in his professional lexicon.

Several years later the doctor himself has a peculiar twinge of pain. In trying to identify it, he discovers that none of the terms he is accustomed to use adequately describes it, for the sensation is of a sort he had never experienced or heard of before. He feels the pain as something but not quite like a burning, stabbing, wrenching twinge, but admits that the description is not entirely correct. At last he recalls his former patient who had a pain he could not describe in ordinary terms, but was something not quite like a burning, stabbing, wrenching twinge. He remembers that the man had called it 'S', and he decides to call his own recent pain by the same name.

The shared use of 'S' to designate their respective sensations does not mean that the doctor's pain and the patient's pain are of the same kind, nor that the doctor has finally learned the meaning of the patient's putative private language term. There is no way to tell, for as Wittgenstein and others have argued, there are no external criteria for private sensations. The doctor begins to use the same sign, and may even believe that in doing so he is identifying or designating the very same kind of pain as that experienced by his patient. But this does not entail that he has learned the meaning of the sign. It is a well-known linguistic phenomenon that the very same sign type can be used with widely divergent denotations and connotations. The fact that the doctor takes over the patient's sign to designate what he believes to be the same kind of pain does not establish that what was once private has now become public. Nor is it implied that the sign became public when the doctor first patronized the patient by referring to his pain by sign 'S'.

It appears that 'S' may constitute a term in a mixed or impure private language, the precise meaning of which must remain obscure to everyone but the individual for whom it privately designates a particular kind of sensation. From this it follows that any sensation word, even in the so-called public sensation language, may be subject to the same indeterminacy of reference. In what is generally regarded as the public sensation language, we share signs that are believed to designate at least relevantly similar kinds of sensations.

We teach portmanteau sensation words such as 'pain', 'excruciating pain', 'stabbing pain', and the like, color words, words for auditory sensations, and others, by teaching the use of signs that we have also learned under appropriate publicly observable circumstances that are thought to be causally connected to the occurrence of the sensations. But it may be that even these uses of language involve nothing more than a shared sign and an unjustified belief in the codesignation of public sensation language.

The ordinary sensation language learning situation is perhaps no different in important ways from the story of the doctor and patient. At first, the doctor uses the same sign as the patient more or less to satisfy the patient's whim, though he is not really sure what the patient means to refer to by 'S'. Later, he adopts the same sign to classify his own sensations on the belief that they are the same. By analogy, the child says 'blue' when sufficiently coaxed to respond verbally in this way in the presence of what others have similarly been trained to recognize as blue. But who can speak for the color sensation which the child has when a blue object enters his visual field? Even if the experience is experimentally determined to be something we would otherwise like to call a shade of blue, who can speak for the child's experience in its *infima species*? This is concealed from all but the individual, who learns a public language to express inadequately what is in fact hidden away from public inspection.[27]

A diary of private sensations like that described in the discussion above is thought by some to be outlawed by Wittgenstein's private language argument. But the objection must be critically examined. For this, an analysis is required of the concepts of following a rule, the recognition and reidentification of recurring sensation kinds, and the Wittgensteinian criterion of correctness. The criterion objection to private language states that putative private language terms do not belong to a genuine language because there are no criteria for the correct or incorrect application of private language terms. The validity of the argument is not in doubt, but its soundness is another matter. The assumption that if a putatively designating term belongs to a genuine language, then there must be criteria to determine correct and incorrect applications of the term, is sometimes regarded as an

[27] This is the problem of partial spectrum inversion. See Keith Campbell, *Body and Mind* [1980], p. 74: "When I see that the traffic light has changed, more has happened than just the acquisition of a new set of dispositions to acts in which I discriminate one state of the traffic light from another. If I have a curious sort of color blindness, in which I see as many different shades of color as you do, but different ones, then when we both see the traffic light (or anything else) we will each acquire the very same discriminative dispositions. Yet there are great differences in our mental lives..."

expression of the view that language or language use is a rule-governed activity. Wittgenstein's obsession with the practice of following a rule in the *Blue and Brown Books*, *Philosophical Investigations*, *Philosophical Remarks*, *Remarks on the Foundations of Mathematics*, *Zettel*, and *Nachlass*, indicates that the concept of a criterion, the concept of following a rule, and language games construed as forms of life, are intimately connected in his later thought. It is commonly held that Wittgenstein established the dependence of rule-governed forms of life on criteria of correctness, and that this logically excludes private languages.

In order to challenge the soundness of the private language argument, it is best to begin with an understanding of what Wittgenstein means by the concept of a criterion. There is general agreement that a criterion is a kind of decision procedure, which in principle makes it possible to determine beyond reasonable doubt whether or not a particular term is correctly applied to an object. In a critical exposition of Wittgenstein's concept of criterion, Carl Wellman writes:

> To describe something is to specify what it is like and what is unlike. In this sense descriptive terms are always used to classify or divide things into kinds. This seems to imply that to use descriptive language a person must be able to distinguish between the different kinds of things. Unless a person were able to recognize members of a given class, he could hardly use the class name very effectively. To be able to use or understand descriptions one must be able to tell which objects fit a given description and which descriptive expressions fit a given object. But how does one know whether or not a specified description fits a given object? This is the question which Wittgenstein's conception of a criterion is intended to answer.[28]

In what he takes to be the spirit of Wittgenstein's later philosophy, Norman Malcolm asserts:

> ... one may be inclined to think that there cannot be a criterion (something that settles a question with certainty) of someone's having a sore foot or having dreamt, but merely various 'outer' phenomena that are empirically correlated with sore feet and dreams. This view, however, is self-contradictory: without criteria for the occurrence of these things the correlations could not be established. Without criteria the sentences 'His foot is sore', 'He had a dream', would have no use, either correct or incorrect.[29]

[28] Carl Wellman, "Wittgenstein's Conception of a Criterion" [1962], p. 435.
[29] Malcolm, *Dreaming* [1959], p. 60. (Emphasis added.)

But it is uncertain whether or not Wittgenstein meant to give 'criterion' a univocal technical sense. Paul Ziff observes:

> [Wittgenstein] meant by 'criterion' something like test, or standard or way of telling. That is, he meant what any speaker of his dialect would have meant if he were using the word in familiar ways. I am inclined to suppose that most likely his use of the word 'criterion' would fit his use of the word 'game', that is, one might be able to discern a family of cases.[30]

Yet the private language argument requires a technical or quasi-technical definition of the concept. It may be regarded as suggested if not actually explicit in Wittgenstein's remarks about criteria of correctness.

For purposes of evaluation, the following definition of 'criterion' is proposed. The concept of nondefective determination of the truth value of a predication must first be defined. This is necessary in order to avoid trivializing counterexamples involving the paradoxes of material and strict implication.[31]

> For any x and any predicate proposition p, x *nondefectively determines the truth-value of* $p =_{df} x$ is a nonempty set of propositions describing the steps of a finite procedure, together with the results of each step of the procedure in a particular application, that entails p or $\sim p$, exclusively, and that does not imply any false or undetermined proposition.

Let '$P(!)$' represent either a nuclear or extranuclear property, indifferently. Then a special concept of criterion for this interpretation of the private language argument can be defined.

> For any x and any predicate term '$P(!)$', x is a *W-criterion of or for* $P(!) =_{df}$ for any object y, x in principle nondefectively determines that $P(!)y$ or

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[30] Paul Ziff, "Comments" (on Garver's "Wittgenstein on Criteria"), in *Knowledge and Experience*, p. 84. See also Rogers Albritton, "On Wittgenstein's Use of the Term 'Criterion'" [1959], pp. 845—57. Albritton emphasizes the apparent plurality of meanings of 'criterion' in Wittgenstein's writings, and collates passages from many places throughout the later works to support a multiplicity of interpretations of the concept, and in this sense agrees with Ziff. But compare Kenny, *Freewill and Responsibility: Four Lectures* [1978], p. 11: "To use Wittgenstein's technical term, the physical expression of a mental process is a *criterion* for that process: that is to say, it is part of the concept of a mental process of a particular kind (a sensation such as pain, for instance, or an emotion such as grief) that it should have a characteristic manifestation."

[31] The term 'nondefective determination' is borrowed from and suggested by Chisholm's definition of 'non-defectively evident' in his solution to Gettier-type counterexamples to the traditional analysis of knowledge. Chisholm, *Theory of Knowledge* [1977], p. 109.

$\sim P(!)y$, exclusively (determining nondefectively whether the term is truly predicated of the object, or not truly predicated of the object).

This definition can be provisionally used in assessing the soundness of the private language argument on the criterion of correctness interpretation.

Here is a reconstruction of the private language argument that accords with Wittgenstein's remarks, and with the most authoritative commentaries of critics and defenders of the argument. It is presented by means of the definition of a *W*-criterion.

> 1. If a putatively designating term belongs to a genuine language, then there must be *W*-criteria that determine in principle whether or not the term is correctly applied to a particular object under particular circumstances.
>
> 2. But there are no *W*-criteria that determine even in principle whether or not a putatively designating private language term is ever correctly applied to any purported private mental object.
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> ---
>
> 3. No putatively designating private language term belongs to any genuine language. There cannot be a private language.

This version of the argument is valid, but not sound.[32] There are several ways to show that on this definition of 'criterion' or '*W*-criterion' the premises need not, and perhaps cannot, be accepted as true.

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[32] Here is a simplified formal proof of the validity of the private language argument.

'*L*' = 'the property of belonging to a genuine language'
'*W*' = 'the property of being a *W*-criterion for a particular term'
'*P*' = 'the property of being a (putatively) designating private language term'

1.	$(\forall x)(Lx \supset (\exists y)\, Wyx)$	
2.	$\forall x)(Px \supset \sim(\exists y)\, Wyx)$	
3.	$(\exists x)(Px \;\&\; Lx)$	
4.	$Po_i \;\&\; Lo_i$	
5.	$Lo_i \supset (\exists y)\, Wyo_i$	(1)
6.	Lo_i	(4)
7.	$(\exists y)\, Wyo_i$	(5,6)
8.	$Po_i \supset \sim(\exists y)\, Wyo_i$	(2)
9.	Po_i	(4)
10.	$\sim(\exists y)\, Wyo_i$	(8,9)
11.	$(\exists y)\, Wyo_i \;\&\; \sim(\exists y)\, Wyo_i$	(7,10)
12.	$\sim(\forall x)(Ix \supset (\exists y)\, Wyx)$	(1,11)
13.	$\sim(\forall x)(Lx \supset (\exists y)\, Wyx)$	(3,4,12)

Defenders of the private language argument usually devote most of their energies to establishing something like premise (2). Reference is made to Wittgenstein's remarks about the inadequacy of memory in the reidentification of purported kinds of private sensations. Wittgenstein seems to believe that the internal comparison of sensation memories is the only thinkable criterion candidate, but that as a criterion it is as useless as consulting multiple copies of the same morning newspaper in an effort to determine the truth of reports published in any particular copy. This may be true. We can simply grant premise (2), and challenge the soundness of the argument by disputing the truth of premise (1), an assumption most accounts take for granted.

A consequence of modern quantum physics and the indeterminacy principle is that, in the proposed terminology, there are no *W*-criteria for the reidentification of subatomic particles. The reidentification of subatomic particles requires the determination of their precise location in space and time. This in turn presupposes the ability to fix both the position and velocity of a particular subatomic particle at a given time. But such an exact determination cannot be made, because position can only be experimentally verified by interacting with the particle in a way that disturbs its velocity, and velocity can only be verified by interacting with the particle in a way that disturbs its position. Position and velocity can therefore never be jointly determined even in principle for a particular subatomic particle. The best that can be achieved is a statistical approximation of its location, insufficient for exact reidentification.

The problem of reidentifying subatomic particles is much like the problem of reidentifying recurring sensations. Suppose that an arbitrary individual subatomic particle could be stipulatively named. It persists at least for a time in a swarm of subatomic particles, undergoing random alterations of velocity and position. According to the indeterminacy principle, there is no equivalent of a *W*-criterion for determining later whether or not any chosen particle is the same as the original, because it cannot be reidentified. But it would be strange to conclude that subatomic particles are anything but public objects. They are obviously not private mental objects, but are

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14. $(\forall x)(Lx \supset (\exists y)Wyx) \ \& \sim(\forall x)(Lx \supset (\exists y)Wyx)$ (1,13)
15. $\sim(\exists x)(Px \ \& \ Lx)$ (3,14)

This formalization has the advantage of presenting the structure of the private language argument as an indirect proof, as Castañeda and others have recommended. See O.R. Jones, editor, *The Private Language Argument* [1971], Part V, 'The Private Language Argument as a *Reductio ad Absurdum*', pp. 132—82.

rightly thought to constitute the very stuff of the physical, objective public world.[33]

There are other terms designating other kinds of public nonprivate and nonmental objects for which there are no *W*-criteria to determine even in principle whether or not the terms are correctly applied to objects.[34] Wittgenstein discusses family resemblance predicates as those for which there is nothing essential in common between objects falling under a given predicate.[35] But there are some indisputably public objects that fall in the shadowy areas between family resemblance predicates, where there is clearly no *W*-criterion definitely establishing them as belonging under one family resemblance predicate rather than another.

As an illustration of the kind of public language terms in genuine languages for which there are no *W*-criteria, consider Wittgenstein's discussion of the public language family resemblance term 'game'. Wittgenstein allows great latitude in the kinds of objects that may reasonably fall under this predicate. But there are controversial cases of public objects for which there is no *W*-criterion that determines even in principle whether or not the predicate 'is a game' is correctly or incorrectly applied to the objects. Law offenders may describe their wrongdoing as a kind of game, in which the purpose is to violate civil or criminal statutes, or as many statutes as possible, without being detected, captured, or punished by the authorities. A number of different kinds of 'games' of the sort are possible. If the criminals are caught and convicted, they might regard the matter as of no further moral consequence than the loss of a game of chess or cards. Those who take a more serious moral attitude toward the law on the contrary may staunchly deny that crime is ever a game of any kind or in any sense at all, despite its family resemblance to certain kinds of games. (Is Russian roulette a *game*?)

If someone learning English as a foreign language encounters the term 'game', and wants to know whether or not the term is correctly applied to crimes, there will evidently be no *W*-criterion to settle the question. If he asks criminals, they will tell him one thing; civil authorities may say just the opposite. There is no test or finite objective decision procedure which he can then apply to determine which of the two groups of language users is

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33 The indeterminacy thesis in quantum physics is sometimes said to have idealistic implications, but this contradicts Wittgenstein's private language argument construed as a refutation of idealism and methodological solipsism. See Hacker, *Insight and Illusion*, pp. 245—75.

34 Robert J. Richman, "Concepts Without Criteria" [1965], pp. 65—85.

35 Wittgenstein, *The Blue and Brown Books*, pp. 17—20, 87, 124; *Philosophical Investigations* §§ 23, 65—73, 77—8, 87; *Remarks on the Foundations of Mathematics*, I, §§ 64—7, IV, § 8, V, §§ 26, 36; *Philosophical Grammar* [1967], §§ 326, 472, 474—76.

using the word 'game' (or 'crime') correctly, and which incorrectly. But the terms 'crime' and 'game' are undoubtedly public language terms in a genuine language.

Similar cases arise in areas of dispute about moral and aesthetic value in public objects. There seems to be no *W*-criterion that determines even in principle whether or not the public language term 'person' is correctly applied to fetuses, and there is no *W*-criterion that determines even in principle whether or not the public language term 'beautiful' applies to particular artworks.[36] Some language users will say one thing, and others another. It would be extravagant in the least to insist that these persons are therefore speaking nonequivalent idiolects of a language rather than expressing substantive disagreements of opinion in the same language. Is there no single language in which this sort of disagreement can take place? If not, then either all language is private, or there is no genuine language. If there is such a language, then it is sufficient to refute premise (1) and the private language argument based on it. If public languages can be understood despite the lack of *W*-criteria for certain predicates, then purported private languages should not be expected to satisfy a more demanding requirement. Adherence to premise (1) of the reconstructed version of Wittgenstein's private language argument would have the unreasonable consequence that subatomic particles and many other more familiar kinds of things are not public and cannot be designated in any genuine language.

But if these consequences are unacceptable, then the private language argument is unsound, and provides no compelling reason for restricting private mental objects from the object theory domain. The possibility of a mixed or impure private language in the object theory then provides the

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36 Wellman, "Wittgenstein's Conception of a Criterion", p. 438: "Since there is no sharp line between essential and nonessential characteristics, it is a mistake to look for some essence common to all instances of a term. Instead, a term is usually applied on the basis of many overlapping characteristics which form a family likeness. As a rule there is no such thing as *the* criterion for the use of a descriptive expression. This implies that in justifying the use of an expression by giving its criteria one will normally have to give more than one criterion... Upon occasion these various criteria may even conflict with one another. Which criteria are relevant to the use of a term on any particular occasion will depend primarily upon the circumstances under which it is used." See also Leon Pompa, "Family Resemblance" [1967], pp. 66—8; Richman, "'Something Common'" [1962], p. 828. Griffin, "Wittgenstein, Universals and Family Resemblances" [1974], pp. 644—46; Hubert Schwyzer, "Essence Without Universals" [1974], pp. 69—78; R. I. Aaron, "Wittgenstein's Theory of Universals" [1965], p. 251. An example involving art objects is suggested by Maurice Mandelbaum, "Family Resemblances and Generalizations Concerning the Arts" [1965], pp. 220—22. Wittgenstein in *Philosophical Investigations* § 66, p. 31, mentions *Kampfspiele*, which Anscombe translates as 'Olympic games'.

necessary (but not sufficient) presuppositional basis for phenomenology, phenomenalism, sense data theory, and other kinds of intentional philosophical theories.

Wittgenstein writes:

> Let us now imagine a use for the entry of the sign "S" in my diary. I discover that whenever I have a particular sensation a manometer shews that my blood-pressure rises. So I shall be able to say that my blood-pressure is rising without using any apparatus. This is a useful result. And now it seems quite indifferent whether I have recognized the sensation *right* or not. Let us suppose I regularly identify it wrong, it does not matter in the least. And that alone shews that the hypothesis that I make a mistake is mere show. (We as it were turned a knob which looked as if it could be used to turn on some part of the machine; but it was a mere ornament, not connected with the mechanism at all.)[37]

The passage is difficult to interpret. But it suggests that a monitoring device such as a manometer might supersede an individual's subjective report about his own internal mental states.

The transition is subtle. At first Wittgenstein seems to imply that the diary of private sensations might replace the external monitoring equipment because of its correlation with the person's subjective reports. The diary can be used as an indication of rising blood pressure, so the manometer becomes obsolete. In the sentences immediately following this, however, Wittgenstein maintains that recognizing the sensation incorrectly does not matter. All that appears to be crucial in this sudden change of emphasis is the correlation of sensation and manometer reading. The monitor somehow gains authority over the individual's interpretation of his own sensations, since, as Wittgenstein says, it eventually becomes *indifferent* whether or not the sensation is *recognized* 'correctly'. This is supposed to show that the concept of mistaken recognition is without meaning or application in an individual's assessment of his own psychological experiences. Wittgenstein's thought takes an unexpected turn here, since presumably it is the prior regular coincidence of sensation and manometer reading, based on a reliable recognition of sensations, that first justifies elimination of the monitoring device in favor of sensation reports recorded in the diary.

A more sophisticated monitor is conceivable, which a behavioral scientist might use to read slight changes in the central nervous system of a subject. It could be like Wittgenstein's manometer, but atuned to factors more

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37 Wittgenstein, *Philosophical Investigations* § 270, pp. 94ᶜ—5ᶜ.

informative than blood pressure alone in evaluating a subject's so-called internal states. A machine of the sort might be connected to a person, so that a behaviorist could study the dials and readouts, interpreting them on the basis of his training in some experimental method, by translation manual, or in accord with external indicators, like a needle that rotates to positions on a panel marked with the names of different kinds of sensations.

It is always at least logically possible that the machine and the person's introspective reports disagree, even on the assumption that the machine never malfunctions. The behaviorist's central nervous system monitoring equipment may indicate that a subject is experiencing sensations which the subject would positively deny. But there seems to be no better reason for saying that the machine is right about the *infima species* of sensation experienced, than that the person is right. Of course the monitoring equipment is sure to be right about something. It will have measured a parameter of the central nervous system of the subject to be interpreted by the operator of the machine. But if the machine indicates that the subject is in pain, and a needle or readout specifies a kind of pain called 'burning', though the person insists that it is not quite like a burning sensation but something different, then, leaving mechanical malfunction and subject dishonesty aside, it is always possible that the machine is wrong, and has not accurately evaluated the precise quality or kind of sensation which the person is experiencing.

The machine is capable of measuring only parameters of the subject's experiences that are already part of the public language, since private experiences by definition have no distinguishing external behavioral manifestations. Whatever the physical mechanism used, it must always be interpreted by someone in a public sensation language. This means that it will necessarily fail to distinguish any private sensation from some other kind described in the public sensation language. The machine cannot settle the question of whether or not a person is having a private sensation of a particular sort, because its inherent limitations of interpretation and design logically exclude it from the mechanical determination of any but publicly defined sensations. The correlation of subjective sensation recognition and monitoring equipment results in the case of the manometer or more sophisticated imaginary devices does not make subjective sensation reports obsolete, epistemically inferior, or less trustworthy than the mechanical testimony of a behavioral monitoring device. The external monitoring equipment cannot provide evidence against the existence of private sensations or private mental objects. It is not to be relied on over and above introspective reports.

In the *Philosophical Investigations*, Wittgenstein outlines the problem of the beetle in the box.

> Suppose everyone had a box with something in it: we call it a "beetle". No one can look into anyone else's box, and everyone says he knows what a beetle is only by looking at *his* beetle. — Here it would be quite possible for everyone to have something quite different in his box. One might even imagine such a thing constantly changing. — But suppose the word "beetle" had a use in these people's languages? — If so it would not be used as the name of a thing. The thing in the box has no place in the language-game at all; not even as a *something*: for the box might even be empty.— No, one can 'divide through' by the thing in the box; it cancels out, whatever it is. That is to say: if we construe the grammar of the expression of sensation on the model of 'object and designation' the object drops out of consideration as irrelevant.[38]

Here again Wittgenstein makes an unexpected and apparently unjustified leap. As he describes the case, there is nothing objectionable about everyone having an inscrutable, different kind of beetle, even one that constantly changes. The problem arises only on the further supposition that the word 'beetle' may have a use in public language. The conclusion drawn is that if this were true, then the word could not be the name of a thing. This is left unexplained until the following sentence, in which it is said that the word could not have a place in the language game because an individual's box might be empty.[39]

But is unclear even in this situation why the word 'beetle' would not still function at least to designate the contents of an individual's box, whether empty or not, existent or nonexistent. The Internal Revenue Service intelligibly uses the word 'income' to refer to a person's yearly earnings regardless of amount, including as a limiting case the possibility in which a person has no earnings at all. (The analogy with the beetle in the box can be reinforced by supposing that a person's income is entirely inscrutable by outsiders, that it may change constantly, and no one else, including the IRS, is permitted to know *exactly* what another person's real income is.) The analogy with the bee-

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[38] Ibid. §293, p. 100^{c}.

[39] C. A. Van Peursen, *Ludwig Wittgenstein: An Introduction to his Philosophy* [1970], p. 91: "... Wittgenstein compares this whole problem of mental images, that are supposed to give meaning to words, with the following game. A number of people have a box with a beetle inside. Each person can look in his own box, but in no one else's. They tell each other what their beetles look like, what color they are, and so on. This language game can continue smoothly, *even if all the boxes are empty*. The thing in the box, which must not be public, accessible to others, is not essential for the game." (Emphasis added.) Note that Van Peursen has distorted Wittgenstein's description of the example. But see Wittgenstein, *Philosophical Investigations*, II, *xi*, p. 207^{c}: "Always get rid of the idea of the private object in this way: assume that it constantly changes, but that you do not notice the change because your memory constantly deceives you."

tle in the box remains intact, because although there is nothing which in principle prevents a person's income from being publicly known — a disanalogy with the case of private mental objects — this is also true of physical objects like beetles kept in boxes. In this respect, both income and the beetle-box have crucial disanalogies with private mental experience. But these features are not relevant to Wittgenstein's claim that the terms in question could not be used as names of things, and that they could have no legitimate function in any kind of language game. Nonexistence of the thing is not decisive, since words like 'income', 'Pegasus', 'phlogiston', and others, function perfectly well in public language and public language games, despite the nonexistence of designated objects.

The beetle in the box presents no theoretical difficulties for object theory logic. The beetle is an appropriate metaphor for the privacy of sensations and the inclusion of private mental objects in the domain of the logic. The beetle in the box is the private and incommunicable psychological experience of a person that makes possible the learning of public language, and the development of phenomenological and related intentional theories.

The epistemological problems connected with the private language argument raise further questions about the limitations of what the mind can know by introspection. Is it possible for the mind to be mistaken about the identification or reidentification of a private sensation or private mental object? The anti-private-language thesis seems to entail that the very concept of mistaken identification of private sensations or private mental objects is meaningless, and that because of this correct identification is also meaningless and impossible, placing ostensibly designating private language terms beyond the pale of linguistic rules of application. This is expressed by premise (2) of the reconstruction of the private language argument. In previous discussion, the premise was simply granted in order to concentrate on premise (1). Now premise (2) must also be examined.

It may be objected against the premise that private experience and the recognition of private sensation kinds is a prerequisite for public language and for public or external criteria of mental phenomena, and that there is a sense in which the mind may be mistaken in hypothetical applications of private language terms to private mental objects or internal psychological experiences. Wellman writes:

> One cannot claim that the credibility of the identification of recurrent kinds of sensations depends entirely upon the possibility of public corroboration, for of what value is checking one identification against another unless each has some independent credibility? Actually, corroboration is a test of cor-

> rectness only because the identifications which support one another each have some antecedent claim to correctness.[40]

If this is true, then it must also be possible for a person to be mistaken in the application of a private mental term to a private mental object. There are conceivable circumstances in which the mind may wrongly describe a sensation in the vocabulary of its private language.

An individual uses private language term 'S' to denote an incommunicable pleasure, and another private language term 'E' to designate an incommunicable pain. It is possible for the person to mistake one for the other, and later correct himself. This is not to say that the individual could have a sensation and not know it, but that he could have a sensation of a particular kind and mistakenly identify it at least temporarily as a sensation of a different kind. Deception can also occur if a subject is preconditioned in appropriate ways. Suppose that stroking the surface of the skin with a feather-edge produces an incommunicable pleasure designated 'S' in a person's private sensation language. A paper-cut on the other hand produces an incommunicable pain, which the person designates 'E' in his private sensation language. If the person is preconditioned to expect the pain of the paper-cut as part of an experiment or initiation ceremony, but in fact receives a light feather stroke, then he may utter 'E' at the moment the skin is touched, and wince as if in pain. He may at once realize the error and correct his prematurely mistaken identification of the sensation. He may notice the faintly pleasant glow that normally follows an S-type sensation, instead of the throbbing ache of the E-type, and admit that it was not E after all, but S.

A subject might also prepare a questionnaire, in which he asks for information to be recorded in his diary of private sensations as a kind of routine introspective procedure. One of the questions asks, 'What sensation did I experience at 12:00 noon?' The person waits until what he takes to be noon, unaware that his watch is broken or that daylight savings time has been instituted that day, and so writes 'S' when in fact at noon by the correct time he does not experience an incommunicable pleasure, but an incommunicable pain. Later he may realize that his judgment of time had been inaccurate,

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[40] Wellman, "Wittgenstein's Conception of a Criterion", pp. 446—47. Wellman writes, p. 447: "It is clear... that Wittgenstein is faced with an awkward dilemma. Either there is some justification prior to corroboration for trusting one's memory or there is not. If there is, even a private identification has some claim to validity; if there is not, even a public identification has no claim to validity. Therefore, either Wittgenstein's objection to private sensations serving as criteria is mistaken, or his own theory that publicly observable characteristics serve as criteria is inadequate."

check the diary for a record of what his sensation report would have been at that time, and correct his answer on the questionnaire, erasing 'S' and writing down 'E' instead. This provides yet another sense in which the significant correction and possibility of error in private language ascription and private mental object identification can occur. The possibility of error in the application of private language terms establishes the significance of private sensation language. This in no way contradicts the presuppositions of phenomenology, phenomenalism, or foundationalist epistemology, since it remains incorrigibly true and directly evident that it *seemed* to the individual that he was in E pain, and later that it *seemed* he was not. And incorrigible directly evident *seeming* is all that philosophical theories of the sort usually require.

To define private mental objects in the object theory domain, a two-place nuclear psychological or intentional predicate is introduced to represent the property of thinking about an object.

$$(\forall x)(Private!x \equiv (\exists y)[(Thinks(y,x)) \; \& \; \Box[(\forall z)(Thinks(z,x)) \equiv (y =_e z)]])$$

This says that something is a private mental object if and only if there is someone who thinks about the object, and it is logically necessary that no other person thinks about it. The predicate '*Thinks*' may be regarded as having the broad Cartesian sense of any psychological attitude or experience, and need not be restricted to reasoning, calculating, or entertaining propositions. To allow for the private mental objects of the thoughts of fictional or otherwise nonexistent persons, the definition may be slightly revised.

$$(\forall x)(Private!x \equiv (\exists y)[(Thinks(y,x)) \; \& \; \Box[(\forall z)(Thinks(z,x)) \equiv (y =_{rf} z)]])$$

In the object theory, private mental objects can be uniquely distinguished as intentionally nonidentical by virtue of their unique converse intentional or psychological properties of being thought about or experienced by and necessarily only by a particular person.

$$(\forall w)(\forall x)(\forall y)(\forall z)([(Private!w \; \& \; Private!x) \; \& \; (Thinks(y,w)) \; \& \; (Thinks(z,w)) \; \& \; (y \neq_e z)] \supset (w \neq_i x))$$

$$(\forall w)(\forall x)(\forall y)(\forall z)([(Private!w \; \& \; Private!x) \; \& \; (Thinks(y,w)) \; \& \; (Thinks(z,x)) \; \& \; (y \neq_{rf} z)] \supset (w \neq_i x))$$

There are other circumstances under which private mental objects are intentionally nonidentical, in situations where the thought or experience of the very same person is involved.

$$(\forall x)(\forall y)([(Private!x \ \& \ Private!y) \ \& \ (\exists P)(Px \ \& \ \sim Py)] \supset (x \neq_i y))$$

$$(\forall x)(\forall y)([(Private!x \ \& \ Private!y) \ \& \ (\forall P)(Px \equiv Py)] \supset (x =_i y))$$

These statements indicate the conditions that determine intentional identity and nonidentity for private mental objects. Since the nuclear properties include psychological or intentional properties, they help to differentiate private mental objects that are qualitatively different, even for the same subject.

A private language can be defined by making reference to the domain of a subtheory that contains private mental objects. The construction can be described in terms of a containment predicate '$C!$', which represents a relation on the domain $D!$ of any private language subtheory, and the objects that $D!$ contains.

$$(\forall x)[Private\text{-}L!x \equiv (\exists y)(Private!y \ \& \ C!(D!(x),y))]$$

If a private mental object is contained in the domain of object theory O, if, that is, it is true that $(\exists x)(Private!x \ \& \ C!(D!(O),x))$, then it follows that $Private\text{-}L!O$, or that object theory O is a mixed or impure private language with at least nonexistent private mental objects in its domain.

The philosophical objection to the *existence* of private mental objects in the private language argument is that there cannot be private mental objects because such objects could not be designated in a language or fall under any linguistic rule of application. But in the object theory, there are many linguistic rules regarding the application of ostensibly designating private language terms. Whether or not private mental objects exist, they *can* intelligibly be talked about in Meinongian object theory. The linguistic argument against the *possibility* of private mental objects, that there are no linguistic rules governing their application, is ineffectual. In order to show that private mental objects do not exist, the defender of the private language argument must establish that the objects could not exist because they have some metaphysically incompatible combination of constitutive nuclear properties. Otherwise, and this would make the anti-private-language position philosophically less interesting, it may be necessary for the defender of the private language argument to prove that there is some contingent reason why private mental objects do not exist. But this again goes beyond anything that has so far been represented as a version or interpretation of the private language argument.

The linguistic argument alone will not do, because even if private mental objects are nonexistent, they can be designated in a rule-governed way in the object theory. Since there are terms ostensibly designating private mental objects, the domain of the logic contains at least nonexistent private mental objects. It remains for the anti-private-language theorist to demonstrate that there are not also existent private mental objects. The upholder of the private language argument may point out that Wittgenstein did not mean to suggest that private mental object terms could not be part of any genuine language or fall under any linguistic rules, but the less sweeping claim that private mental object terms do not fall under any linguistic rules for the application of such terms to actual objects in experience.

According to object theory semantics, a predicate is true of an object if and only if the state of affairs obtains in which the reference class or intension of the predicate includes the object. The epistemological problem of how we can tell whether or not the state of affairs in question obtains need not entail that there are no linguistic rules governing the application of private mental object terms in the logic. A private language may be called an *E*! private language if its domain contains an existent private mental object. The domain contains an object of this kind just in case there exists a person who thinks about something which, as a matter of logical necessity, is not thought about by any other person.

$$(\forall x)(E!\text{-}Private\text{-}Lx \equiv (\exists y)[(Private!y \; \& \; E!y \; \& \; C!(D!(x),y)])$$

Wittgenstein's criterion objection to private language embodies a rather jaundiced view of the reliability of the private language user. Wittgenstein maintains that there can be no criterion of correctness in the case of private sensation terms because there is no independent check on whatever a person says is true about his internal states. An individual could conceivably insist that any two sensations, no matter how phenomenologically distinct, were of the same kind, and that two sensations phenomenologically indistinguishable were actually distinct. But this is not a difficulty for the hypothesis that private sensation language subconsciously orders experiences within the mind in a rule-governed way, nor for the view that the mind is equipped with a private mental language or acquires its own private mental language prior to socialization.[41]

A less intransigent individual conscientiously recording sensations in a diary of private sensations, sensitive to fine-grained phenomenological distinctions in experience, and honestly attempting to apply private language terms as family resemblance predicates for similar kinds of sensation in an accurate description of his mental life, will not be likely to confuse private pain and private pleasure, or other distinct sensations, as incommunicable

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[41] Jerry A. Fodor, *The Language of Thought* [1975], pp. 55—205.

experiences of precisely the same kind. Nor is it probable that he will classify similar incommunicable experiences as radically different, though mistakes may occur. The objection that phenomenological misdescriptions can arise presupposes the possibility of error, which is just what the private language argument is supposed to preclude. By Wittgenstein's reasoning, the poss-ibility of error also implies the possibility of correct judgment and accurate reidentification of sensations. Epistemic limits of memory aside (which are equally problematic for the users of any public language), there is no reason to think that at least some kinds of private sensations could not be identified wrongly in a private language vocabulary. This also makes meaningful the possibility of correct sensation description in a private mental language.[42]

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[42] Wittgenstein's later diagnosis of philosophical malady identifies the grammar of a language game in its natural habitat of convention and practice, and exposes the conceptual confusions that result when such a grammar is inappropriately projected out of its depth onto another language game. Ironically, this is just what Wittgenstein seems to do in taking the criteria of correctness grammar involved in the naming and reidentification of some material objects and applying it to phenomenology. This produces havoc when private experience fails to satisfy alien grammars transferred from material-object-description language games. The affliction is marked by the private language argument itself and the philosophical confusion it fosters, the symptom of which is the wide variety of interpretations of the argument and the ingenious but usually strained efforts to reconcile it with ordinary ways of speaking about psychological experience. I assume for the sake of argument the standard account by which Wittgenstein's private language argument is intended to show that there cannot be a private language in the sense that there can be no private ascription of sensation predicates to internal psychological occurrences. I believe, however, that a more accurate reading of the private language argument passages shows Wittgenstein to be concerned exclusively with the problem of whether or not recurring private sensations can be named as particulars. See Jacquette "Wittgenstein's Private Language Argument and Reductivism in the Cognitive Sciences" [1994].

III. God an Impossible Meinongian Object

1. Anselm's Ontological Proof

Attempts to prove God's existence by appeal to existence as a necessary property of a perfect being are known collectively, despite wide-ranging differences, as variations on St. Anselm's ontological proof for the existence of God.

Leibniz gives an over-simplified but largely accurate reconstruction of this style of argument in philosophical theology in his *Monadology*:

> 44. For it must needs be that if there is a reality in essences or in possibilities or indeed in the eternal truths, this reality is based upon something existent and actual, and consequently, in the existence of the necessary Being in whom essence includes existence or in whom possibility is sufficient to produce actuality.[1]

Descartes had written earlier, in Meditation V of his *Meditations on First Philosophy*, having first established to his own satisfaction the epistemic credentials of clear and distinct conception:

> ...I clearly see that existence can no more be separated from the essence of God than can its having three angles equal to two right angles be separated from the essence of a [rectilinear] triangle...
>
> While from the fact that I cannot conceive God without existence, it follows that existence is inseparable from Him, and hence that He really exists...
>
> For it is not within my power to think of God without existence (that is of a supremely perfect Being devoid of a supreme perfection)...
>
> And this necessity suffices to make me conclude (after having recognised that existence is a perfection) that this first and sovereign Being really exists...[2]

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1 Leibniz, *Monadology* [1714], p. 260.

2 René Descartes, *Meditations on First Philosophy* [1641], pp. 181—82.

In these expositions, the ontological proof in simplest form amounts to something like this:

1. God's nature or essence includes all perfections.
2. Existence is a perfection.

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3. God's essence or nature includes existence; God exists.

This is crude when compared with Anselm's original intentional formulation. In the *Proslogium*, Anselm writes:

> Hence, if that, than which nothing greater can be conceived, can be conceived not to exist, it is not that, than which nothing greater can be conceived. But this is an irreconciliable contradiction. There is, then, so truly a being than which nothing greater can be conceived to exist, that it cannot even be conceived not to exist; and this being thou art, O Lord, our God.[3]

In this treatment, and yet another more refined version in Anselm's *Appendix in Behalf of the Fool by Gaunilon*, there is an emphasis on the concept of an entity whose essence includes or implies existence. The argument is unique only in that Anselm makes special reference to the impossibility of conceiving a being than which none greater can be conceived, on the assumption that an existent entity is in some sense 'greater' than an otherwise identical nonexistent. The underlying principle remains unchanged. The ontological proof in all these guises is defeated by Meinong's distinction between nuclear and extranuclear properties. To show this, it will suffice to concentrate on a contemporary apology for Anselm's argument.

2. Plantinga and the Free Will Defense

In his modal-theological treatise *The Nature of Necessity*, Alvin Plantinga upholds the possibility of God's existence against the problem of evil, and maintains a version of Anselm's ontological argument from the standpoint of a *de re* interpretation of standard modal logic.[4]

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[3] St. Anselm, *Proslogium, Monologium, An Appendix in Behalf of the Fool by Gaunilon, and Cur Deus Homo* [1954], pp. 8—9.

[4] Plantinga, *The Nature of Necessity*, pp. 164—221. Anselm, *Proslogium,* Chapter II.

Plantinga offers what he calls a free will defense of the possibility that God exists against traditional consistency objections. The problem of evil is that God is logically or metaphysically impossible given the moral and natural evil in the world. God by definition is omniscient, omnipotent, and perfectly benevolent. Either God does not know about evil in the world, in which case he is not omniscient; or he knows about it but cannot prevent it, in which case he is not omnipotent; or he knows about and could prevent it, but chooses not to, in which case he is not perfectly benevolent.[5]

Plantinga tries to show that propositions (1) and (2) below are logically consistent:

(1) God is omniscient, omnipotent, and wholly good.
(2) There is evil.

His strategy is to prove that (1) and (2) are consistent by finding a proposition (31) such that (1) and (31) are jointly consistent and jointly entail (2).[6]

(31) Every essence suffers from transworld depravity.[7]

(30) An essence E *suffers from transworld depravity* if and only if for every world W such that E entails the properties *is significantly free in* W and *always does what is right in* W, there is a state of affairs T and an action A such that

(1) T is the largest state of affairs God strongly actualizes in W,
(2) A is morally significant for E's instantiation in W, and
(3) if God had strongly actualized T, E's instantiation would have gone wrong with respect to A.[8]

The idea is to attribute moral evil and eventually natural evil to the free actions of agents who suffer from transworld depravity, without which 'defect' moral good would be impossible in the actual world.[9]

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5 Voltaire in the *Dictionnaire philosophique* [1769], p. 117, attributes this trilemma version of the problem of evil to Lactantius, *The Wrath of God*, Chapter 13 (in the person of Epicurus).
6 Plantinga, *The Nature of Necessity*, pp. 165, 189.
7 Ibid., p. 189.
8 Ibid., p. 188.
9 Ibid., pp. 164—67, 184—93. Plantinga offers another version of the free will defense in *God and Other Minds* [1967]; and *God, Freedom, and Evil* [1974]. See Plantinga, "Self-Profile" [1985], pp. 3—93.

(32) God actualizes a world containing moral good.[10]

Proposition (1) is supposed to be consistent with (31); but the conjunction of (1), (31), and (32) (together with auxiliary assumptions) entails (2).[11]

Not *every* essence suffers from transworld depravity, as Plantinga's (31) requires. If that were true, God's essence would not be compatible with his perfect benevolence, so that proposition (1) would not be logically consistent with (31). The necessary exception for God is accordingly assumed.

Even so, Plantinga's argument embodies a glaring circularity. The possible truth of (31) as explained and amplified in (30) presupposes the possibility of God's existence, and therefore cannot be used in an interesting proof of that conclusion. If it is impossible that God exists, then it is impossible that God actualizes *T* or any other state of affairs (see (30), clause 3). But by the definition of transworld depravity, if it is impossible that God actualizes state of affairs *T*, then it is impossible that any essence suffers from transworld depravity.

The argument in no way neutralizes the objection that the existence of evil in the actual world is logically inconsistent with the existence of a divinely omniscient, omnipotent, and perfectly benevolent God. Transworld depravity and the independently dubious free will thesis (which Plantinga in this treatment does not analyze in depth) are beside the point. Plantinga does not prove the possibility of God's existence, but merely conceals the presupposition in a series of prolix definitions. The *necessary* moral depravity of free moral agents might support Plantinga's consistency conclusion (though perhaps at the cost of free will and moral responsibility for wrongdoing). But this strengthened assumption is implausible and unavailable to Plantinga.

3. Meinongian Countercriticisms

Plantinga gives the following restatement of Anselm's ontological proof for the existence of God. He regards the argument as *reductio ad absurdum* in form, and posits the first assumption for purposes of indirect proof.

(1) God does not exist in the actual world.
(2) For any worlds *W* and *W'* and object *x*, if *x*

[10] Plantinga, *The Nature of Necessity*, p. 189.
[11] Ibid.

exists in *W* and *x* does not exist in *W'*, then the greatness of *x* in *W* exceeds the greatness of *x* in *W'*.

(3) It is possible that God exists.

(4) There is a possible world *W* such that God exists in *W*. (3)

(5) God exists in *W* and God does not exist in the actual world. (1,4)

(6) If God exists in *W* and God does not exist in the actual world, then the greatness of God in *W* exceeds the greatness of God in the actual world. (2)

(7) The greatness of God in *W* exceeds the greatness of God in the actual world. (5,6)

(8) There is a possible being *x* and a world *W* such that the greatness of *x* in *W* exceeds the greatness of God in actuality. (7)

(9) It is possible that there be a being greater than God. (8)

(10) It is possible that there be a being greater than which it is not possible that there be a greater (substituting for 'God' the analytic description 'the being than which it is not possible that there be a greater'). (9)

(11) It is not possible that there be a being greater than which it is not possible that there be a greater.

(12) It is false that God does not exist in the actual world; God exists.[12] (1,10,11)

The argument can be dismissed as unsound unless there is a more satisfactory solution to the problem of evil than Plantinga's, which has already been exposed as circular. Without this defense, the problem of evil undermines Plantinga's proposition (3). It is not possible that God exists, or at least Plantinga has not shown it to be possible.[13]

Plantinga offers the argument only to reject it as an unsatisfactory pre-

[12] Ibid., p. 202. Plantinga's numbering of propositions is revised; minor stylistic changes introduced.

[13] See Chisholm, *Brentano and Intrinsic Value* [1986], pp. 91—102.

liminary version of a later inference he acknowledges as sound in the concluding section 'A Victorious Modal Version'.[14] Even then Plantinga does not unreservedly regard the argument as proving God's existence, but more cautiously maintains that: "...these reformulated versions of St. Anselm's argument... cannot, perhaps, be said to *prove* or *establish* their conclusion. But since it is rational to accept their central premise, they do show that it is rational to *accept* that conclusion."[15] Despite such qualifications, this version of Plantinga's argument illustrates the basic problem of ontological proofs in an extensional semantic framework. The modal quibbles between this version and the formulation Plantinga finally accepts are important to his project, but irrelevant to the more fundamental semantic issue.

Another more interesting criticism of Plantinga's reconstruction of Anselm's argument is suggested by the Meinongian nuclear-extranuclear property distinction. In object theory, greatness is a nuclear property. It does not supervene on existence as Plantinga's assumption in proposition (2) (and Anselm's original argument) requires, but just the opposite, since existence as an extranuclear property supervenes on the totality of an object's nuclear properties.

The semantic framework of Anselm's ontological argument is usually taken for granted, but is of utmost importance. The selection of extensional or Meinongian semantics determines whether Anselm's argument can provide satisfactory grounds for the existence of God. If the semantic question is raised in connection with Anselm's proof, it is obvious that an extensional system is unsatisfactory. Unlike ontically neutral Meinongian semantics, an extensional semantics prejudges the existence of objects to which properties are truly predicated. To construct an Anselm-type ontological argument with the innocent-seeming predication that God is or is by definition omnipotent (omniscient, etc., or that which has all perfections, that than which none greater can be conceived) under an extensional semantics is automatically implicitly to assume that which is to be proved, that God exists. Otherwise, the predications contained in the argument are not true, and cannot be interpreted as true by the semantics, not even as a matter of apparently harmless stipulative definition about the meaning of the word 'God'. If God does not exist, then by an extensional semantics God will not be among the existent objects belonging to the extension of the predicate 'omnipotent' or the others. In an extensional semantics, Anselm's ontological argument cannot even get off the ground except by a viciously circular *petitio principii*. The only alter-

[14] Plantinga, *The Nature of Necessity*, pp. 203—17.
[15] Ibid., p. 221.

native, the only ontically neutral semantic basis from which to formulate and evaluate the ontological argument so that its meaning does not prejudge its truth, is to adopt a Meinongian intentional semantics in which the question of the existence of objects to which properties are truly predicated is left open, to be determined only in the end by the success or failure of the argument as a whole.

Here is a dilemma. Either God's greatness *consists in* his essence or *Sosein* containing the nuclear properties of omniscience, omnipotence, and perfect benevolence, or it consists *also* in his having the extranuclear property of existence. If God's greatness consists in his having the extranuclear property of existence, then Anselm-type ontological proofs for the existence of God are viciously circular when God is defined as the greatest being, or the being than which none greater is logically possible or conceivable, as critics of the argument have long maintained.[16] If, on the other hand, God's greatness consists only in having the nuclear properties of omniscience, omnipotence, and perfect benevolence, then an existent being with these qualities cannot possibly be greater than a nonexistent being with these same qualities. The greatness of an existent God on this assumption would be precisely identical to the greatness of a nonexistent God, making Plantinga's assumption (2) false and the argument as a whole unsound.

This explains Kant's 100 gold thalers objection to the ontological argument in the *Critique of Pure Reason*, and the claim, not that 'existence' is not a predicate (in Meinongian semantics '*E*!' is the extranuclear existence predicate), but instead that existence is not a constitutive nuclear property that in any way qualifies the nature or essence (in Meinongian terminology, the *Sosein*) of an object.[17] The dilemma demonstrates that Plantinga's attempt

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[16] Circularity objections to ontological arguments for the existence of God have had currency at least since the publication of Pierre Gassendi's attack on Descartes' version in Meditation V of the *Meditations on First Philosophy*. See Gassendi, *Disquisitio metaphysica* [1644], pp. 140—42, 147—48. Additional defenses of Anselm's proof are given by Malcolm, "Anselm's Ontological Arguments" [1960], pp. 41—62; Charles Hartshorne, *Anselm's Discovery* [1965], and related ancillary essays. For criticisms see Robert Brecher, *Anselm's Argument: The Logic of Divine Existence* [1985]. Also, D. P. Henry, *The Logic of St. Anselm* [1967]; Gregory Schufreider, *An Introduction to Anselm's Argument* [1978].

[17] Immanuel Kant, *Critique of Pure Reason* [1787], A599/B627—A600/B628: "A hundred real thalers do not contain the least coin more than a hundred possible thalers. For as the latter signify the concept, and the former the object, should the former contain more than the latter, my concept would not, in that case, express the whole object, and would not therefore be an adequate concept of it. My financial position is, however, affected very differently by a hundred real thalers than it is by the mere concept of them (that is, of their possibility). For the object, as it actually exists, is not analytically contained in my concept, but is added to my concept (which is a determination of my state) synthetically; and yet the conceived

to restore Anselm's ontological argument for the existence of God is doomed to circularity or unsoundness.

God as an impossible, necessarily nonexistent Meinongian object can nevertheless be defined as an object than which none greater is conceivable or logically possible. An existent God is metaphysically impossible as the unresolved problem of evil indicates, unless a more acceptable solution than Plantinga's is forthcoming. To be at once omniscient, omnipotent, perfectly benevolent, and the author of an actual world in which there is moral and natural evil is tantamount to being a round square. Meinongian logic implies that even if God exists, an existent God could not possibly be greater or more perfect than an impossible necessarily nonexistent Meinongian object God.[18]

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hundred thalers are not themselves in the least increased through thus acquiring existence outside my concept. By whatever and by however many predicates we may think a thing— even if we completely determine it— we do not make the least addition to the thing when we further declare that this thing *is*. Otherwise, it would not be exactly the same thing that exists, but something more than we had thought in the concept, and we could not, therefore, say that the exact object of my concept exists." Kant does not claim that existence cannot be a property of things, but only that it is not a 'determining' property. In Meinongian terminology, Kant's assertion is rewritten to read: 'By whatever and by however many (nuclear) properties we may assume an object to have— even if we completely determine it— we do not make the least addition to the nature or *Sosein* of the object (to the *Aussersein* of the pure object) when we further assume that the object *is*, *exists*, or has *Sein*.' Routley, *Exploring Meinong's Jungle and Beyond*, pp. 181—87. Plantinga, *The Nature of Necessity*, p. 196.

[18] Meinong, *Über Möglichkeit und Wahrscheinlichkeit*, pp. 278—82; *Über die Stellung der Gegenstandstheorie im System der Wissenschaften*, p. 18. Findlay, *Meinong's Theory of Objects and Values*, p. 105: "Meinong proposes to say that an object is existent (*existierend*) when it merely has the watered-down variety of existence, and that it exists (*existiert*) when there is no such watering-down. The God of Anselm is an existent and so is the wealth which lies before us in dreams, but neither exists in the full sense, because genuine existence does not belong to the sphere of so-being. The existence of the God of Anselm and the existence of the wealth in dreams are both existences in which the modal moment is lacking."

IV. Meinongian Models of Scientific Law

1. *Idealization in Science*

The primary motivation for development of Meinongian logic is the need to provide an intuitively correct analysis of ontological commitment. The importance of ontological commitment in the evaluation of competing scientific theories is widely appreciated in contemporary philosophy of science, particularly in problems of reduction and choice among alternative explanatory schemes of comparable explanatory adequacy but different degrees of simplicity and economy. The seemingly inescapable idealization in high-level theoretical scientific principles, and the possibility of error and unwitting reference to nonexistent objects in mistaken hypotheses, require an intentional object theory approach to the semantics of scientific language and theory.

Meinongian object theory supports a formalization of the logical and semantic structure of natural or scientific law. Scientific laws are usually thought to be universal in scope, logically contingent but causally or nomically necessary, and of uniform application at all times and places throughout the universe. A naive attempt to represent the logical form of scientific laws might involve the universal generalization: $(\forall x)(Fx \supset Gx)$. Difficulties of two kinds indicate the inadequacies of this proposal: (i) The paradoxes of confirmation, justification of induction, counterfactuals, and the explanatory role of natural laws in scientific explanations, demonstrate inherent limitations of the material conditional in representing the logical form of lawlike generalizations; (ii) Standard extensional interpretations of universal quantification and the conditional preclude the naive formulation from satisfactory expression of uninstantiated scientific laws involving nonexistent idealizations (Boyle's ideal gas, Galileo's frictionless surface, Newton's moving bodies unimpeded by impressed forces).

D. M. Armstrong in his book *What is a Law of Nature?* offers the following preliminary criticism of the naive generalization account under the heading 'Humean Uniformities with Non-Existent Subjects':

> Contemporary logic renders a Humean uniformity by expressions of the form '$(x)(Fx \supset Gx)$', or some more complex expression of a material im-

> plication preceded by a universal quantifier. It is notorious that such a formula expresses a true proposition if there are no *F*s. For the statement is a statement about everything, saying of each thing that *either* that thing is not an *F*, *or* if it is an *F*, then it is a *G*. Hence, given that everything is not an *F*, the statement is true. Given further that '*F*' and '*G*' are suitable predicates, then '$(x)(Fx \supset Gx)$' is a statement of a Humean uniformity.[1]
>
> It apparently follows that, on the Naive Regularity view of laws of nature, it is a law of nature that centaurs are peculiarly adept at philosophy, simply because, omnitemporally, there are no centaurs. It is also a law of nature that centaurs are quite unable to take in the simplest philosophical argument. There is no contradiction in the notion that both these 'uniformities' should be laws. But it is a most unwelcome conclusion.[2]

Armstrong discusses three attempts to solve the problem. The most interesting involves the existential conditionalization of naive universal generalization, which Armstrong characterizes as an implication of actualism.

> This modification questions whether a formula like '$(x)(Fx \supset Gx)$' really captures the notion of a Humean uniformity. Perhaps the formula is too liberal, and it should be restricted by requiring the actual existence of *F*s at some time:
>
> $(\exists x)(Fx)\ \&\ (x)(Fx \supset Gx)$
>
> In defence of this formula it may be said that uniformities without positive instances are really pseudo-uniformities.[3]

The existence-conjunction amendment of the naive formulation defeats the centaur objection, but is inadequate in the formalization of classical scientific laws involving idealizations.

> A case in point is Newton's First Law of Motion. The law-statement tells us what happens to a body which is not acted upon by a force. Yet it may be that the antecedent of the law is never instantiated. It may be that every body that there is, is acted upon by some force.[4]

The idealization in Newton's first law does not postulate the existence of unimpeded bodies, but attempts to explain the movement of actually impeded bodies by presenting a counterfactual principle about what would

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1 D. M. Armstrong, *What is a Law of Nature?* [1983], p. 19.
2 Ibid.
3 Ibid., p. 20.
4 Ibid., p. 21.

happen if impeded bodies were unimpeded. The distinction topically divides the first and second books of Newton's *Principia*, separating the more abstract geometrized hypothetical motion of ideal bodies, and the attempt to apply the laws of ideal bodies to real world situations 'in resisting mediums'. With the hindsight of modern physics and its revolutionary methodology, it might be said that the proposal accurately represents the logical structure of Newton's law, now known to be false, and here rendered explicitly so by failure of the existence condition in the first conjunct. But this would be an uninteresting application of the analysis, since Newton never supposed that unimpeded bodies exist, and Newtonian mechanics is not thought false by modern science because a thorough search of the cosmos has revealed that there are no projectiles unacted on by impressed forces. Armstrong is undoubtedly correct to hold that the existence-conditional revision of the naive logical form of scientific law is inadequate, at least as a key to the structure of classical laws of nature, but only under standard extensionalist interpretations of the universal and existential quantifiers.

Idealization is characteristic of many kinds of classical laws, and is almost a prerequisite of the need to avoid accidental nonlawlike or merely phenomenological generalizations in science. Archimedes' imaginary lever is supposed to have an absolutely rigid fulcrum. Galileo's ideal pendulum requires a massless string, and a bob unimpaired by air resistance, with no frictional forces resulting from different periods of motion for different segments of the string. Fourier's law of heat conduction refers to the spatial coordinates x, y, z of a point in an infinitely long material conductor.

$$\lambda\left[\frac{\delta^2\theta}{\delta x^2} + \frac{\delta^2\theta}{\delta y^2} + \frac{\delta^2\theta}{\delta z^2}\right] = \rho c \frac{\delta\theta}{\delta t}$$

Fourier's Law of Heat Conduction

Galileo is self-conscious about the importance to scientific explanation of idealization and simplification by appeal to nonexistent ideal objects and circumstances. In his *Dialogue Concerning the Two Chief World Systems*, he writes:

> . . . just as the computer who wants his calculations to deal with sugar, silk, and wool must discount the boxes, bales, and other packings, so the mathematical scientist, when he wants to recognize in the concrete the effects

which he has proved in the abstract, must deduct the material hindrances, and if he is able to do so, I assure you that things are in no less agreement than arithmetical computations.[5]

Newton makes frequent reference to ideal objects not exemplified in the actual world, but invoked for purposes of scientific explanation. This is especially conspicuous in his definition of the 'absolute magnitudes' of 'absolute space and time', which he carefully distinguishes from 'sensible measures' admitting of degrees of approximate accuracy, such as those afforded in practice by instruments like the calipers and waterclock.[6] Newton's uninstantiated laws of idealized motion in Book I of the *Philosophiae Naturalis Principia Mathematica* state:

I. Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

II. The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

III. To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.[7]

The nonexistence of the unimpeded projectile, advanced as an idealization for explanatory purposes in Newton's mechanics, is no stumbling block to the Meinongian object theory interpretation of scientific law. In object theory logic, the original naive universalization $(\forall x)(Fx \supset Gx)$ is not made trivially true for any property G if there are no existent objects with

5 Galileo Galilei, *Dialogue Concerning the Two Chief World Systems* [1632], pp. 207—8. In a similar realist, neo-Platonic or neo-Pythagorean vein, Albert Einstein maintains in "Geometry and Experience", *Sidelights on Relativity* [1923], p. 23: "...as far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." See John Losee, *A Historical Introduction to the Philosophy of Science* [1972], p. 54: "Galileo insisted on the importance to physics of abstraction and idealization, thereby extending the reach of inductive techniques. In his own work, he made use of idealizations such as 'free fall in a vacuum' and the 'ideal pendulum'. These idealizations are not exemplified directly in phenomena. They are formulated by extrapolating from serially ordered phenomena. The concept free fall in a vacuum, for example, is an extrapolation from the observed behavior of bodies dropped in a series of fluids of decreasing density."

6 Isaac Newton, *Philosophiae Naturalis Principia Mathematica* [1686], Volume I, Definition VIII, Scholium, pp. 4—12. See also those sections of Volume II on fluid dynamics.

7 Ibid., p. 13.

property F. The Meinongian universal quantifier ranges over all existent and nonexistent objects in a semantic domain or ontology and extraontology, so that the universal generalization is true only if all existent or nonexistent objects with property F also have property G. There is no need even to consider and no advantage to be found in Armstrong's revised actualist conjunction $(\exists x)Fx \mathbin{\&} (\forall x)(Fx \supset Gx)$, trivially equivalent to $(\forall x)(Fx \supset Gx)$ under Meinongian interpretation. Armstrong's counterexample about the philosophical acumen of centaurs does not threaten Meinongian semantics, because the proposition is not true unless the intension of existent and nonexistent objects with property F is completely contained in the intension of all existent and nonexistent objects with property G. The centaurs of ancient myth are probably indeterminate with respect to the nuclear property of philosophical ability and its complement, and if some centaurs are endowed by free assumption as prodigies of philosophy, others are denied it by the same assumptive freedom and Meinongian independence of so-being from being.

An intentional Meinongian object theory logic and semantics avoids objection (ii), and clears the way for additions and corrections to universal generalization that circumvent objection (i) in Meinongian models of scientific law. The naive universal generalization $(\forall x)(Fx \supset Gx)$ is immune to Armstrong's objections under nonstandard object theory interpretation in Meinongian logic.

2. Probability, Confirmation, Induction

There is another set of limitations associated with the use of universal generalization that makes it an unsuitable formulation of scientific law even under Meinongian object theory interpretation. These do not concern the limitations of extensionalist quantificational semantics, but traditional skeptical paradoxes about induction and confirmation that arise because of the truth functional definition of the material conditional. Contemporary philosophy of science is a proving ground for many such objections, of which just these three will be considered:

(1) A.J. Ayer's formulation of Hume's skeptical problem of (the justification of) induction.
(2) Carl Hempel's paradox of confirmation or 'raven' problem.
(3) Nelson Goodman's 'new riddle of induction', concerning the predicate 'grue'.

Hume's criticism of rationalist belief in the logical or conceptual necessity of causation implies skepticism about the justification of inductive methods in science. In *A Treatise of Human Nature*, Hume concludes:

> Any thing may produce any thing. Creation, annihilation, motion, reason, volition; all these may arise from one another, or from any other object we can imagine... Where objects are not contrary, nothing hinders them from having that constant conjunction, on which the relation of cause and effect totally depends.[8]

The problem of whether induction can be justified follows from the empiricist rejection of the logical or conceptual necessity of causal connections. If causation is not logically or conceptually necessary, then causal inferences can only be justified by appeal to empirical experience. This makes inevitable the question of the justification of inductive methods themselves. Induction is the distinguishing characteristic of scientific explanation, used to justify hypotheses about contingent causal connections in every field of natural inquiry. But what justifies induction? Why should anyone suppose that there is a uniformity in nature, or that the future will be anything like the past? According to Ayer, there is a dilemma about the justification of induction. In *Language, Truth and Logic*, he offers a concise formulation of the problem:

> The problem of induction is, roughly speaking, the problem of finding a way to prove that certain empirical generalizations which are derived from past experience will hold good also in the future. There are only two ways of approaching this problem on the assumption that it is a genuine problem, and it is easy to see that neither of them can lead to its solution. One may attempt to deduce the propositions which one is required to prove either from a purely formal principle or from an empirical principle. In the former case one commits the error of supposing that from a tautology it is possible to deduce a proposition about a matter of fact; in the latter case one simply assumes what one is setting out to prove.[9]

The dilemma is that attempts to justify induction are either deductive or inductive. But deductive justifications of induction require the invalid deduction of contingent inductive generalizations from necessarily true logical tautologies, and inductive justifications of induction simply assume that induction is epistemically sound and can be used to justify itself. The justification of induction is therefore either invalid or viciously circular.

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[8] David Hume, *A Treatise of Human Nature* [1740], Book I, Section XV, p. 173. Hume, *An Enquiry Concerning Human Understanding* [1748], Section IV, pp. 18—9.

[9] A. J. Ayer, *Language, Truth and Logic* [1946], p. 49.

There are several proposals for avoiding the dilemma. Ayer suggests that since the 'problem' has no solution, it is not really a problem but a pseudo-problem. He argues:

> Thus it appears that there is no possible way of solving the problem of induction, as it is ordinarily conceived. And this means that it is a fictitious problem, since all genuine problems are at least theoretically capable of being solved: and the credit of natural science is not impaired by the fact that some philosophers continue to be puzzled by it. Actually, we shall see that the only test to which a form of scientific procedure which satisfies the necessary condition of self-consistency is subject, is the test of its success in practice. We are entitled to have faith in our procedure just so long as it does the work which it is designed to do — that is, enables us to predict future experience, and so to control our environment.[10]

Ayer's remarks combine three different reactions to the problem of induction. The problem is dismissed as fictional because it seems incapable of solution. But something like a pragmatic solution is also suggested in the claim that induction and faith in induction is rational, provided that it continues to be successful in the prediction of events and the engineering or manipulation of the environment. Ayer admits that this 'justification' of induction or elimination of philosophical problems about the justification of induction does not imply the future success of inductive methods. But he adds:

> ... it is a mistake to demand a guarantee where it is logically impossible to obtain one. This does not mean that it is irrational to expect future experience to conform to the past. For when we come to define "rationality" we shall find that for us "being rational" entails being guided in a particular fashion by past experience.[11]

The facile disposal of the problem of induction as a fictional pseudo-problem, and the convenient definition of 'rationality' in terms of uncritical acceptance of inductive practice do not allay deep philosophical misgivings about the justification of induction. Insofar as considerations about past successes of induction have any weight in our understanding of what constitutes rationality, the appeal is just another disguised viciously circular attempt to justify induction inductively, involving inductive generalization over past successful applications of induction. The same criticism can be leveled against efforts to apply C. S. Peirce's concept of abduction or infer-

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[10] Ibid., p. 50.
[11] Ibid.

ence to the best explanatory principle to solve the problem of induction by riding between the deduction-induction horns of Hume's fork or Ayer's dilemma.[12] It is only by tacit appeal to inductive generalizations about the success of previously accepted explanations that such criteria as simplicity are reasonably judged to determine the 'best' explanatory principle in abductive justifications of induction by acceptance of regulative metatheories about the uniformity of nature. In *An Enquiry Concerning Human Understanding*, Hume had written:

> ... all arguments concerning existence are founded on the relation of cause and effect; that our knowledge of that relation is derived entirely from experience; and that all our experimental conclusions proceed upon the supposition that the future will be conformable to the past. To endeavor, therefore, the proof of this last supposition by probable arguments, or arguments regarding existence, must be evidently going in a circle, and taking that for granted, which is the very point in question.[13]

The path of least resistance may therefore be to deny that there is any such thing as an inductive mode of inference, or that so-called inductive inference has anything other than purely deductive logical structure, and therefore does not stand in need of special justification. Inductive logics may be regarded instead as deductive systems with probability semantics that assign probable truth values to atomic propositions. The inferences authorized within a particular development of inductive logic will then be entirely deductive, deductively entailing propositions with certain truth probabilities on the basis of the truth probabilities of the atomic propositions from which they are derived. The advantages of a deductive inference structure for inductive logic can be appreciated by comparing two different approaches to induction in Arthur W. Burks' *Chance, Cause, Reason*, and Wilfred Sellars' essay "Are There Non-Deductive Logics?".

Burks characterizes inductive arguments as unique and distinct from deductive arguments in discussing Ignaz Semmelweis' discovery of infection and the importance of disinfectants.

> Semmelweis' conclusion was that the students' hands had been spreading the infection and that his policy of disinfection caused the decrease in mortality. Clearly this conclusion does not follow deductively; it is logically possible that the premise should have been true and the conclusion false.

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[12] See C. S. Peirce, *Collected Papers of Charles Sanders Peirce* [1931—1935], 5.189, 6.522—28. K. T. Fann, *Peirce's Theory of Abduction* [1970].

[13] Hume, *An Enquiry Concerning Human Understanding*, p. 23.

> Moreover, there was a *chance* that the cause of the mortality decrease was something else, e.g., the natural abatement of the epidemic... It is not *likely* that the conclusion should be false; i.e., given the premise it is *probably* true. The inductive argument can thus be stated
>
> [42] *D, therefore probably C,*
>
> where the premise *D* contains the data... and the conclusion is
>
> (C) The students had been spreading the infection and the practice of disinfection caused the decrease in mortality.
>
> Thus as "therefore" is the characteristic mark of a deductive argument, so "therefore probably" is the characteristic mark of an inductive argument, though of course synonyms may be used and these terms may be present implicitly rather than explicitly.[14]

There is an ambiguity in the interpretation of Burks' conclusion *C*, on which his distinction between inductive and deductive logics depends. Burks seems to understand the argument in [42] as:

D, therefore-probably C

But the probability qualification might instead be said to attach to the data *D* and conclusion *C* instead of to the inference or 'therefore'. In other words, the argument can alternatively be interpreted:

probably-D, therefore probably-C

On this reading it is not the inference that is probably valid or sound, but the conclusion *C* that is probably true, given *D* or the probable truth of *D*.

Sellars favors a similar analysis when he argues that:

> ...it is reasonable to accept '2 plus 2 = 4' *and* reasonable to accept 'the moon is round,' and hence to inscribe
>
> | | | |
> |---|---|---|
> | 2 plus 2 = 4
The moon is round | but *not* | 2 plus 2 = 4
So the moon is round |
>
> ...If this line of thought is correct, then, even though the sequence
>
> *g* & *Q*
(Probably) *T*

[14] Arthur W. Burks, *Chance, Cause, Reason* [1977], p. 24.

is correct and proper, there is no such thing as a probability *argument* of which the conclusion is

So (probably) *T*.

...This suggests the possibility that in *no* case is there a probability *argument* of the form

g & *Q*
So (probably) *p*

i.e., that the concept of such probability arguments is an illusion; the division of argument into 'deductive' and 'probability' arguments a mistake. Notice that by a probability argument I mean an argument of which the conclusion is

So (probably) *p*

which *asserts p*, though in a qualified way. I do not mean to say that there are no probability arguments, if by this is meant an argument which has as its conclusion

So it is probable that *p*.

The latter conclusion does not assert '*p*'; it asserts a higher order proposition about '*p*'— perhaps the higher order proposition that it is reasonable to assert that *p*.[15]

From this it is clear that Sellars agrees with the contention that probability in the conclusions of so-called inductive or probability arguments must attach to the conclusion and not to the inference or validity of inference, and on this basis disputes the distinction between deductive and nondeductive inductive or probability arguments. He proposes the definition: "'*q*' is true = it is *ideally* ε-reasonable to accept '*q*'..." (where 'ε' abbreviates 'epistemically').[16] Then he compares deductive and so-called inductive inferences:

...we paralleled the deductive argument

(If *p*, then *q*) & *p*
So (necessarily) *q*

[15] Wilfrid Sellars, "Are There Non-Deductive Logics?" [1972], pp. 299—300.
[16] Ibid., p. 301.

with

> '(If p, then q) & p' implies 'q'
> '(If p, then q) & p' is true.
> So it is ε-reasonable (for me, now) to accept 'q'.

Let us therefore construct the higher order counterpart of what we have been construing as a probability argument with the premiss '(3/4 C_A is B) & Q.' It would be something like

> 'a_i is B' stands in R_I to '(3/4 C_A is B) & Q'
> '(3/4 C_A is B) & Q' is true
> So it is ε-reasonable (for me, now) to accept 'a_i is B.'[17]

Then he concludes:

> ... If this interpretation is correct, there is no such thing as an argument
>
> (3/4 C_A is B)
> So (Probably) a_i is B
>
> i.e., no such thing as a non-deductive probability argument. The argument by virtue of which it is reasonable to accept '(Probably) a_i is B' has as its conclusion not '(Probably) a_i is B,' but rather
>
> It is ε-reasonable (for me, now) to accept 'a_i is B'
>
> and the argument of which *this* is the conclusion is a *deductively* valid argument.[18]

The alternative interpretation of inductive inference as purely deductive inference of probable conclusions from probable data advocated by Sellars fits more readily the Hempel-Oppenheim nomological-deductive covering law model of scientific explanation.[19]

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[17] Ibid., p. 302.
[18] Ibid., p. 303. See Sellars, "Induction as Vindication" [1964], pp. 197—231.
[19] Carl Hempel and Paul Oppenheim, "Studies in the Logic of Explanation" [1948], pp. 135—75. Hempel, *Aspects of Scientific Explanation* [1965], pp. 245—95.

Hempel-Oppenheim Covering Law Model

$C_1, C_2, \ldots, C_k$	Statements of antecedent conditions
$L_1, L_2, \ldots, L_r$	General scientific laws or laws of nature
———————	
E	Description of empirical phenomena to be explained

The difference in the two approaches to the logical form of inductive inference endorsed by Burks and Sellars can be represented as follows:

Burks' Nondeductive Inductive Inference Model	Sellars' Deductive Probability Inference Model
$C_1, C_2, \ldots, C_k$	(probably) $C_1, C_2, \ldots, C_k$
$L_1, L_2, \ldots, L_r$	(probably) $L_1, L_2, \ldots, L_r$
═══════ [probably]	———————
E	(probably) E

If Sellars' deductive probability inference model of induction is accepted instead of Burks' more standard nondeductive inductive inference model, then the Hume-Ayer problem about the justification of induction does not arise, for in that case there is no nondeductive form of inductive inference requiring special justification. Inductive inference is deductive in form and probabilistic only in content or in the inductive semantic probability values of its data assumptions, scientific laws, and conclusions. The solution is not rationalist, and does not pretend to enable deductive inferences of causal connections by reason alone. The elimination of inductive inference as a distinct nondeductive inference type nevertheless permits valid deductive definitional justification of appropriate probability functions for the derivation of probable conclusions in particular empirical problems.

Probability functions are defeasibly *justified* by definition, and therefore deductively justified, *even if disconfirmed* by recalcitrant experience, if they are adjusted in a self-correcting way so as to reflect the successes and failures of the provisional ancestral probability functions from which they are derived.

The same solution cannot be used without eliminating inductive inference as a special mode of argument, because although an inductively inferred conclusion can be justified though disconfirmed, its justification is not sufficient to uphold inductive inference as a special argument form. The self-correcting adjustment of a probability function to reflect its ancestral success and failure rate in extrapolating hypotheses beyond local observation and experiment also avoids the circularity trap of using induction to justify induction. The conclusion is not: 'Probability function *f* was correct in the past, therefore it will be correct in the future'; but rather: 'Probability function *f* was correct in the past, therefore we are defeasibly justified in using it to project hypotheses about the future, regardless of whether such hypotheses are ultimately confirmed or disconfirmed.'

Carl Hempel's paradox of confirmation or raven paradox invites the rather different solution of amending the naive universal conditional formulation in $(\forall x)(Fx \supset Gx)$ by recognizing the functional component of scientific laws. The paradox is offered in terms of an accidental rather than lawlike generalization, but the same problem evidently applies to any attempt to use the material conditional in expressing general scientific correlations of natural properties. The generalization 'All ravens are black' is rendered $(\forall x)(Raven(x) \supset Black(x))$. By contraposition, this is logically equivalent to $(\forall x)(\sim Black(x) \supset \sim Raven(x))$, which by a familiar truth-functional transformation is also equivalent to $(\forall x)(Black(x) \vee \sim Raven(x))$. This solicits the objection that the original hypothesis 'All ravens are black' is equally confirmed by examination of black or nonblack nonravens as by positive observations of black ravens. Hempel clinches the problem by grafting Jean Nicod's confirmation criterion to an equivalence condition that requires the empirical confirmation of every logically equivalent formulation of a generalization to count as empirical confirmation of the generalization itself.[20]

There are many philosophical responses to Hempel's raven paradox. Hempel in *Aspects of Scientific Explanation* and earlier essays bites the bullet by admitting that the examination of black or nonblack nonravens does indeed partially confirm the hypothesis that all ravens are black, and dismisses the 'paradox' as a psychological illusion that obtains only when the tacit presupposition of relevant information about the world already confirmed by empirical experience convinces us that we need only examine positive instances of the color of particular kinds of birds to establish the generaliza-

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[20] Hempel, "A Purely Syntactical Definition of Confirmation" [1943], pp. 122—43; "Studies in the Logic of Confirmation" [1945], pp. 1—26, 97—121; *Aspects of Scientific Explanation*, pp. 3—51.

tion. In other scientific applications, and prior to the widespread accumulation of empirical information, Nicod's confirmation criterion and the equivalence condition hold true.[21] Israel Scheffler, in *The Anatomy of Inquiry*, removes the ground even for an appearance of paradox by breaking the assumed connection between the fact that a state of affairs *accords* with a generalization in any of its logically equivalent transformations, and the epistemological claim that the state of affairs therefore *confirms* the generalization.[22] The initial appeal of this solution is somewhat diminished by Scheffler's own assertion: "That an object satisfying both antecedent and consequent of a universal conditional ... *confirms* it seems the most elementary truth about confirmation ... [and that] logically equivalent statements have exactly the same weight, as elements of scientific argument, and, in particular, are identically related to instances, seems equally plain."[23]

A more persuasive solution can be given compatible with but not exclusive to Meinongian semantics. This requires expansion of the conditional generalization form of scientific law to include the functional correlation of properties. The solution to Hempel's raven paradox of confirmation agrees with the pronouncement of many philosophers of science that scientific laws are always functional laws. The trivial limiting case of the constant or identity function is also eliminated (as it must be even for nonfunctional formulations of scientific law) by a property nonidentity stipulation.

$$(\forall x)(\forall F)(\forall G)[(Fx \supset Gx) \mathbin{\&} Gx = f(Fx) \mathbin{\&} F \neq G]$$

This emendation accomplishes several things at once. It improves on Scheffler's solution and avoids the Hempel paradox by requiring that confirmatory instances offered in support of a scientific law confirm not only the universal conditional but the additional functional correlation and nonidentity conjuncts. Discovery and examination of black or nonblack nonravens may confirm the generalization that all nonblack things are nonravens, and therefore by equivalent accordance the generalization that all ravens are black. But experience of black or nonblack nonravens does not confirm the functional subcomponent of the law that being black is a (nonconstant) function of being a raven. Indeed, there is in all likelihood no scientific functional correlation between ravenhood and plumage color. Accidental generalizations do not support the essential explanatory purposes of scientific

[21] Hempel, "Studies in the Logic of Confirmation", p. 20. Jean Nicod, *Foundations of Geometry and Induction* [1930], p. 219.

[22] Israel Scheffler, *The Anatomy of Inquiry* [1963], p. 237, 292. See also pp. 240—41, 258—91.

[23] Ibid., p. 259.

law, which is satisfied only when specific functions relating natural properties are identified and confirmed. The functional model of scientific law reinforces the important distinction between accidental generalizations and genuine scientific lawlike generalizations that many theorists of scientific method have emphasized.[24]

The proposed revision in the logical form of scientific law further solves Nelson Goodman's new riddle of induction about the projectibility of contrived predicates like 'grue' and 'bleen'. Goodman describes the problem this way:

> Suppose that all emeralds examined before a certain time *t* are green. At time *t*, then, our observations support the hypothesis that all emeralds are green; and this is in accord with our definition of confirmation... Now let me introduce another predicate less familiar than "green". It is the predicate "grue" and it applies to all things examined before *t* just in case they are green but to other things just in case they are blue. Then at time *t* we have, for each evidence statement asserting that a given emerald is green, a parallel evidence statement asserting that that emerald is grue. And the statements that emerald *a* is grue, that emerald *b* is grue, and so on, will each confirm the general hypothesis that all emeralds are grue.[25]

The difficulty is that the same empirical evidence equally confirms incompatible predications. On the proposed reformulation of scientific law, Goodman's new riddle of induction cannot arise. Suppose, as Goodman's problem requires, that the examination of emeralds prior to time *t* equally confirms the universal generalization that all emeralds are green and the universal generalization that all emeralds are grue. In that event, the evidence by equivalence also paradoxically confirms the universal generalization that whatever is green is grue. But although being green is arguably a function of being grue and conversely, the induction does not confirm the nonidentity of or distinction between being a green emerald and being a grue emerald necessitated by the provision that $F \neq G$. It follows from the temporal parameter of Goodman's definition of 'grue' (which is the trick of his new riddle) that no experience or experiment could possibly serve to distinguish an emerald's being green from an emerald's being grue. But confirmation of the entire law including the nonidentity provision is strictly required in the revised model of scientific law. The unavoidable breakdown in the chain of confirmation indicates that the original assumption that pre-*t* experience of emeralds

24 For example, Stephen Toulmin, *The Philosophy of Science* [1953], pp. 34—110.
25 Nelson Goodman, "The New Riddle of Induction" [1953], pp, 73—4.

equally confirms the generalizations that all emeralds are green and all emeralds are grue is false. In the amended functional interpretation of the logical form of scientific laws Goodman's paradox is defeated.

3. Causal Explanation and Lawlike Necessity

With the solution of Ayer's dilemma, Hempel's raven paradox, and Goodman's new riddle of induction, it remains only to complete the basic logical form of functional scientific law with causal or nomic necessity and probability operators, and to show how particular scientific laws fit the Meinongian model.

Causal or nomic necessity is usually understood to be a fundamental prerequisite of any adequate concept of scientific law. It is defined by restricting accessibility relations in a suitably flexible system of modal logic, such as Meinongian system $S5^m$, limiting semantic transworld access to worlds in which the same natural laws obtain. Supplementary modal operators are added to the notation, like alethic modal symbols, but enclosing the letter '*c*' to stand for causal necessity, as in '$\boxed{c}p$'. Probability operators are also needed to indicate the mere probable truth of a scientific law, confirmed only to some degree of probability by the probable truth of propositional evidence supported by empirical observations and experiments. These are especially needed to implement Sellars' deductive model of probable inference, in which probability attaches to scientific assumptions and conclusions in purely deductive inferences, and not to any nondeductive inference scheme. There are many different methods for assigning probability to propositions, among which the most common are the standard, inverse, and random.[26] The probability operator attributes numerical value i ($0 \leq i \leq 1$) to proposition p according to probability system s, in $P_i^s(p)$.

The difference in intuitive scope of these additional items of syntax make it possible to express a distinction between non-quantum-style or classical-realist, and quantum-style positive-idealist scientific laws. To avoid counterexamples involving conditional tautologies, which are also causally necessary, and whose consequents are (truth) functions of their antecedents, it is important to characterize scientific laws as causally necessary but logically contingent. For any law L, it is implicit that $\boxed{c}L$ & $\sim\Box L$.

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26 Burks, *Chance, Cause, Reason*, pp. 99—164.

Classical-Realist Laws:

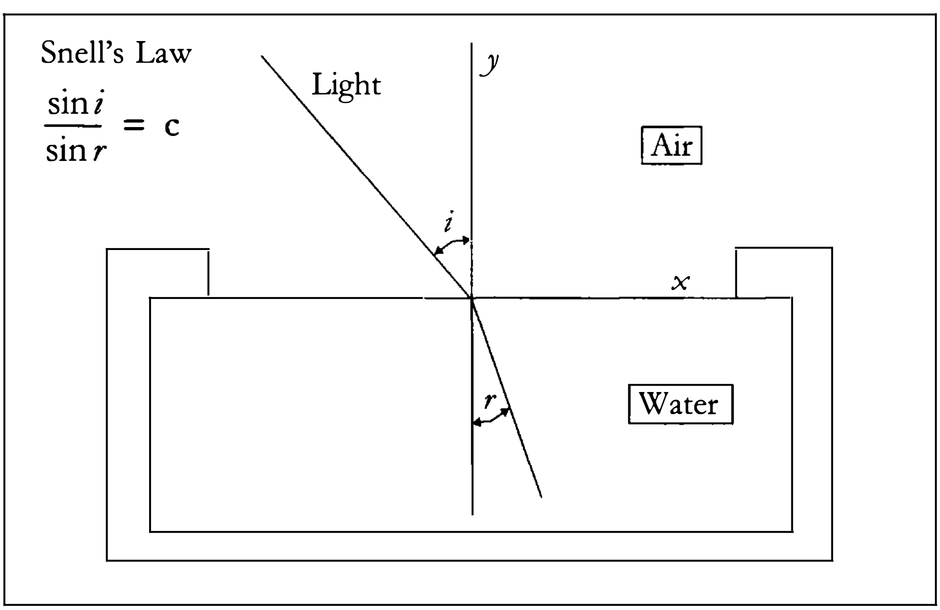
$$P_i^s \boxed{c} (\forall x)(\forall F)(\forall G)[(Fx \supset Gx) \ \&\ Gx = f(Fx) \ \&\ F \neq G]$$

Positive-Idealist Laws:

$$\boxed{c} P_i^s (\forall x)(\forall F)(\forall G)[(Fx \supset Gx) \ \&\ Gx = f(Fx) \ \&\ F \neq G]$$

The distinction between these two conceptions of scientific laws is that prior to the advent of quantum physics, classical-realist lawlike functional generalizations are understood to be causally or nomically necessary, but only probably true, while scientific laws modeled on the post-quantum conception are causally or nomically necessarily such that they are only probably true, either because of the insurmountable epistemic limitations of observers and their unavoidable cognitive interactions with and disturbances of observed systems, or because nature is fundamentally indeterminate and probabilistic.

A single example will serve to show how any classical-realist or positive-idealist lawlike generalization can be formulated to fit the revised Meinongian model of the logical form of scientific law. Snell's Law in geometrical optics holds that the ratio of the sine function of the angle of incidence of a light ray (in modern terms, the propogation or transmission of the wave-front of a photon stream) to the sine function of its angle of refraction through transparent noncrystalline media is always constant (the entry medium's refractive index).



The heart of Snell's Law can be written in terms of lambda abstraction for a universally quantified conditional in which the antecedent predicates the complex property of being a light ray or wave-front Ly that passes P through transparent T noncrystalline C medium x. The consequent of the conditional states that the ratio of the sine of incidence to the sine of refraction of y through x is constant.

$$P_i{}^{s}\boxed{c}(\forall x)(\lambda y[Ly \,\&\, (\exists z)(Tz \,\&\, Cz \,\&\, Pyz)]x \supset \lambda z\left[\frac{\sin i(z)}{\sin r(z)} = \mathrm{c}\right]x)$$

If the complex antecedent is abbreviated F, and the consequent G, then Snell's Law in its entirety, with implicit nonconstant functional correlation conjuncts $G = f(F)$ & $F \neq G$, and attached probability and causal or nomic necessity operators, will exactly conform to the proposed Meinongian model of scientific laws.[27]

Newton's First Law of Motion, in which reference to nonexistent bodies unimpeded by impressed forces is made, also fits the intentional Meinongian conditional model. Let 'B' represent the property of being a body, 'F' the property of being unaffected by impressed forces, 'R' the property of continuing in a state of rest, and 'U' the property of continuing in a state of uniform motion in a right line. The First Law can then appropriately be formulated:

$$P_i{}^{s}\boxed{c}(\forall x)(\lambda y[By \,\&\, Fy)]x \supset \lambda z\,[Rz \vee Uz]x)$$

[27] It need not be supposed that Snell intended to include functional correlation as part of the law, nor that the proposed formulation of scientific law translates a precisely identified function. If the functional reinterpretation of a scientific generalization is incompatible with its correct explication, then it must be relegated to the status of accidental nonlawlike or phenomenological generalization. This need not diminish its importance as a convenient statement of scientific data, as in the case of Johannes Kepler's three so-called 'laws' of planetary motion, which partly describe particular astronomical phenomena in the Earth's solar system, and which Newton later reduced to his own more generalized functional laws of motion.

V. Aesthetics and Meinongian Logic of Fiction

1. Story and Context

The logic of fiction raises special problems about the network of relations among existent and nonexistent objects. Reference to fictional entities in the most uncomplicated cases is no different in principle than reference to nonexistents in scientific discourse. Vulcan the nonexistent Roman god postulated by religious myth-makers, and Vulcan the nonexistent planet hypothesized by astronomy, can be given the same ontologically neutral interpretation in Meinongian semantics. When art is unconstrained by scientific explanatory obligations, it enjoys greater imaginative freedom in the introduction of complex semantic relations linking the characters of fiction and real life, actual and fictionally embellished events in historical novels, stories within stories, and fictional characters in series of stories by the same or different existent or nonexistent authors.

As a story is created, the author determines by free assumption the nuclear properties of its characters. These are what John Woods in *The Logic of Fiction* calls the characters' 'sayso' properties.[1] The author stipulates what is supposed to be true of the people, places, and events of the narrative, which by Meinongian independence of so-being from being makes it true that these fictions have the nuclear properties bestowed on them by their creator. Incomplete and impossible objects are brought into the story for any desired aesthetic effect by being named or associated with a particular set of nuclear properties. There are practical constraints on how far an author can freely violate conventional expectations about the properties of fictional characters, and a certain inevitability in art may prune away aesthetically unacceptable outcomes of events and developments of plot once a story is begun and the *dramatis personae* established. But these considerations at most determine the aesthetic success or popularity of an artwork, not its semantics.

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1 Woods, *The Logic of Fiction*, pp. 35, 38, 60.

Audience expectations in any case are progressively expanded and sometimes deliberately frustrated or outraged in creative fiction, especially in experimental literature and the avant-garde. Narrators do not usually die or disappear halfway through a novel while the story somehow continues, as in John Hawkes' *The Lime Twig*, nor does the hero typically change species like Gregor Samsa in Franz Kafka's *Metamorphosis*; or expect rescue after days of listening to someone tunnel into his prison only to have the warden and his assistants burst through the wall in a fit of laughter in the black comic play of reprieve and despair of Vladimir Nabokov's *Invitation to a Beheading*. But the author is in artistic control of these nuclear properties and even more grotesque and fantastic possibilities. The question whether or not Samsa becomes an insect has no authoritative answer outside the context of Kafka's story. The claim that Odysseus blinded Polyphemus can only be justified by appeal to Homer's text. The author is the primary and often the only source of what nuclear properties characters have or do not have.

2. *Interpenetration of Reality and Myth*

A difficulty occurs in the logic of fiction because of nuclear relational properties. The same reasoning that by application of Meinong's independence thesis implies that Sherlock Holmes was a detective makes it equally plausible to suppose that Holmes lived in London. But if it is true of Holmes that he lived in London, is it also true of London that it was lived in or has the property of having been lived in by Holmes? As Routley observes: "... a stake-out on Baker St. would have obtained no trace of Holmes ..."[2] Routley considers but rejects a range of alternative solutions to the problem, including Parsons' 'plugging-up' function, designed to transform relational nuclear into nonrelational nuclear predicates, and a concatenation solution, which attributes to Holmes the unanalyzable property of having⁀lived⁀in⁀London, from which the relational properties of London are not deducible.[3]

The solution Routley finally adopts advances an integrated contextualist analysis of nuclear predications.[4] The contextualist approach requires statements about nonexistents to be implicitly or explicitly prefaced by reference

2 Routley, *Exploring Meinong's Jungle and Beyond*, p. 563.
3 Ibid., pp. 583—85, 886. See Parsons, *Nonexistent Objects*, pp. 26—7, 59—60, 64—9, 75—7; and "A Prolegomenon to Meinongian Semantics", pp. 575—77.
4 Routley, *Exploring Meinong's Jungle and Beyond*, pp. 569—70, 595—98.

to a source of information about the properties attributed to the object. The correct formulation of the claim that Holmes lived in London is then, '[According to the detective stories of Arthur Conan Doyle] it is true that Holmes lived in London'. This avoids the problem of relational nuclear properties, for when the same context is ascribed to the counterpart relational predication involving London, a true rather than false sentence results, '[According to the detective stories of Arthur Conan Doyle] it is true of London that it was lived in by Holmes'.

Routley's contextualism is independently justified by its explanation of the nonrelational nuclear properties of fictional objects. Consider the myth of Iphigenia in the House of Atreus cycle of Greek tragedies (ignoring for the moment the fact that Iphigenia may have been a real person, the daughter of King Agamemnon and Queen Clytemnestra of Mycenae). Iphigenia is said in different versions of the fable both to have been sacrificed and to have been rescued from sacrifice. Stesichorus, Pindar, Aeschylus, and Sophocles agree that she was offered on the altar at Aulis to propitiate the gods after Agamemnon committed an unholy offense, in order to bring the winds that carried his armies across the sea to Troy. But Stasinus, Euripides, and the Latin poets Hyginus and Ovid (and Goethe in *Iphigenie auf Tauris*), send the goddess Diana on a mercy mission to save the princess at the last moment, appearing in a cloud of altar smoke and escaping to Tauris on the Black Sea. The question of whether Iphigenia was sacrificed or not receives a different answer depending on which author is consulted. According to Stesichorus it is a nonrelational nuclear property of Iphigenia to have been sacrificed, but according to Stasinus it is not. If story-context is disregarded, then Iphigenia is an impossible Meinongian object with the metaphysically incompatible combination of the nuclear properties of having been sacrificed and its complement of not having been sacrificed. Yet, preanalytically, Iphigenia is not an impossible object, since at most it is only contingently true that a girl with her variously described fate did not exist. If context is introduced, the problem disappears.

The interpenetration of reality and fiction is further exemplified in legends, historical plays and novels, and fictional depictions of actual persons, places, and events. We may think of Socrates in Aristophanes' *The Clouds*, Napoleon and the Battle of Borodinó in Tolstoy's *War and Peace*, DeSoto and LaSalle and the conquest of New Spain in Edward Dahlberg's *The Sorrows of Priapus*. Parsons provides a useful terminology to distinguish between what he calls the native and immigrant characters relative to a story context.[5]

[5] Parsons, *Nonexistent Objects*, pp. 51—3.

Native characters are those belonging only to the story, not imported from the real world or another artwork. Democritus of Abdera by this account is introduced as a real life immigrant to Dante's *Inferno*. This does not add to Democritus' so-being the nuclear property of occupying an upper circle of hell, but at most by the contextualist analysis implies that '[According to Dante's story] Democritus occupies an upper circle of hell.' Yet it is Democritus himself and not another fictional character with the same name to whom the property is attributed in Dante's poem. It must then be said, though Parsons' treatment of the nuclear-extranuclear property distinction does not permit the judgment, that Democritus has the converse intentional nuclear property of being supposed by Dante (and Dante's readers) to occupy an upper circle of hell. The semantic device of fictional world-indexing in modal Meinongian logic implements the story-contextual interpretation of a fictional object's nuclear properties, so that by Kripke-style stipulation Democritus [according to Dante] languishes in the inferno, not in the actual world, but in an accessible fictional Meinongian world. In creating a work of fiction, the author freely invents an alternative incomplete so-being for immigrant existent and native or immigrant nonexistent objects relative to a particular story-context or fictional world. The same world-indexing requirements for transworld identity of existent and nonexistent native and immigrant Meinongian objects apply across fictional worlds and the real world as between any Meinongian worlds of the modal Meinongian semantic model.[6]

Legends unlike myths have more direct basis in fact, beginning with actual persons or events and adding falsehoods to their description until they begin to take on the dimensions of total fictions. The distinction between myth and legend is continuous and admits of degree. There is an analogy between legend and myth in art and illusion and hallucination in perception. Myth and hallucination are pure fabrications without existent referents, while legend and illusion are mere distortions of something that exists. Meinongian semantics can include legends as special cases of fiction that centrally feature

[6] The story-context indexing solution applies only to the nuclear properties of fictional objects. See Parsons, ibid., p. 54: "...we don't confuse 'Holmes doesn't exist' with 'According to the story, Holmes doesn't exist.'" On p. 198, Parsons considers degenerate fictions that seem to involve nothing but extranuclear predications: "*Story*: 'Jay exists. The end.' *Story*: 'An object doesn't exist. The end.'" Parsons expresses doubt about whether these examples are genuine stories at all. From an aesthetic viewpoint this may be true, but it is hard to see what the passages lack in syntactic or semantic content that would disqualify them as (terse, uninteresting) stories.

immigrant real world objects. As between hallucination and illusion, it is often hard to discern the fine line between myth and legend.[7]

3. Philosophical Creatures of Fiction

There is a proliferation of fictional objects and characters in potentially unlimited nestings of stories within stories. The semantic principles required to explain the relations and interrelations of these freely iterative creations depend on generalizations of solutions to two basic problems.[8]

Consider the 'play within a play' in Shakespeare's *Hamlet*. The brooding prince decides to unmask his uncle's treachery against his father by substituting the script of his own play 'The Mousetrap' for the entertainment planned by the itinerant dramatists scheduled to perform before the usurper King and Queen. By having an actor imitate his father's assassination and his uncle's incestuous liaison with his mother, Hamlet hopes to shock Claudius into confession or visible sign of guilt. As he says in Act II, Scene II, 612:

> The play's the thing
> Wherein I'll catch the conscience of the King.

The King and Queen of 'The Mousetrap' are thinly disguised proxies for King Claudius and Queen Gertrude (who in turn are probably derived from historical royalty, and at least from the precursor plays Shakespeare studied for inspiration). There is a transcontextual identity or similarity relation between the King and Queen of Hamlet's 'The Mousetrap' and the King and Queen of Shakespeare's *Tragedy of Hamlet*. Shakespeare of course is the author both of *Hamlet* and 'The Mousetrap'. But as far as the drama is concerned, Hamlet rather than Shakespeare, who nowhere appears in the play, is the author of 'The Mousetrap'.

This nesting is uncomplicated, though in principle fictional author attributions can be indefinitely ramified. There could be a story about an author who writes a story about an author who writes a story about an author who writes a story, and so on, and a complex web of interconnections might be

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7 Ibid., p. 207. Routley, *Exploring Meinong's Jungle and Beyond*, pp. 537—606. Zalta, *Abstract Objects*, pp. 91—9.

8 An account of allegory, metaphor, and related textural dimensions of art and literature requires a connotative and associative semantics that is at least made possible by if not yet included in intentional systems of Meinongian logic.

established between the persons, objects, and events appearing in subsequent stories all contained in the original story like the layers of a Russian doll. The author in nested story 13 might fall in love with the daughter of the author in nested story 347; the heroine of nested story 1016 might intrude on the action of nested story 6 to save the day; or it might be claimed that the author of story 828 is really the author of the entire structure of stories, twisting back on itself in various combinatorial involutions and convolutions. Again, the application of implicit story-context indexing devices can sort out and keep track of these semantic intricacies. We do this less formally when we specify that [According to Shakespeare's *Hamlet*] Claudius pours poison in the King's ear, and that [According to Hamlet's 'The Mousetrap' in Shakespeare's *Hamlet*] it is not the case that Claudius pours poison in the King's ear, but another (strictly unidentified) regicide who pours poison in another (strictly unidentified) King's ear.

Story-context disambiguates the nuclear properties of fictional objects found in stories within stories. But there is a problem about the ontological status of objects within a story that cannot adequately be explained by indexing existence and nonexistence predications to story contexts. In Shakespeare's *Macbeth*, Macbeth has two daggers: a nonexistent hallucinatory dagger that floats before him and leads him to the murder room, and another dagger in his belt, which within or from the point of view of the story is existent, which he can grasp and wield. As fictional objects, both daggers are actually nonexistent. But within the story only one is supposed to be nonexistent, while the other exists.

It is useless to distinguish between the story-context ontological status of these two daggers by maintaining that [according to Shakespeare's *Macbeth*] Macbeth's beltworn dagger exists, but the floating dagger does not exist. These statements are true, but in Meinongian semantics they cannot satisfactorily explain the ontological difference between Macbeth's daggers. Extranuclear existence and nonexistence predications are not subject to Woods' sayso or the independence thesis by which Shakespeare may freely assume distinct objects in creating a story. The problem is not that the daggers are indistinguishable in the play, since they have different nuclear properties, one sheathed in Macbeth's belt, and the other floating and ungraspable. But this does not account for their contextual existence or nonexistence. There are possible stories identical to Shakespeare's in every respect except that in them Macbeth's floating dagger is supposed to be just as 'real' as the tangible blade he wears at his side. We need a way to distinguish Shakespeare's tale from near counterparts that mean to describe *existent* rather than hallucinatory floating ungraspable daggers.

There is an easy method of capturing these ontic distinctions within narrative contexts if Meinong's doctrine of the modal moment, full-strength factuality, and the watering-down of extranuclear properties to nuclear surrogates is accepted. Then it is possible to say that both of Macbeth's daggers are nonexistent as fictional creatures of Shakespeare's imagination, but that within the story, the beltworn dagger has the watered-down nuclear version of the extranuclear property of existence, and the floating dagger has the diluted nuclear version of the extranuclear property of nonexistence. To paraphrase Meinong's reply to Russell about the existent round square, the beltworn dagger according to Shakespeare's play unlike the floating dagger is an existent dagger, even though it does not exist. This is the solution Parsons adopts to similar problems in the logic of fiction, making use of Meinong's modal moment theory and a refined functional application of the concept of watering-down. But the problem of Macbeth's daggers can be avoided in another way, without resorting to the metaphysically dubious semantic subterfuge of watering-down and Meinong's largely discredited doctrine of the modal moment.[9]

Macbeth's floating dagger according to Shakespeare's story has the ontologically significant converse intentional nuclear property of being hallucinatory, and specifically of being hallucinated by Macbeth, while the graspable dagger in his belt does not have this property. Extranuclear properties are not assumptible but supervene on an object's totality of nuclear properties according to revisionary Meinongian semantics. Neither dagger exists or has the univocal extranuclear property of existence. But within the story it can be inferred from the information given or implied by the author about the nuclear properties and especially converse intentional nuclear properties of the two daggers that the floating hallucinatory dagger is nonexistent. The beltworn dagger is also nonexistent, but unlike the hallucinatory dagger its nonexistence cannot be inferred from the nuclear properties ascribed to it in the story, which in every way are indistinguishable from those that might be found in an incomplete description of the nuclear properties of an existent dagger. This is sufficient to establish the intended ontological distinction between Macbeth's daggers within Shakespeare's story.[10]

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9 Parsons, *Nonexistent Objects*, pp. 184—86, 192, 200—6. Parsons, "A Meinongian Analysis of Fictional Objects", pp. 83—5.

10 The attribution of converse intentional nuclear properties to fictional objects may also correct an apparent disadvantage of Meinongian semantics of fiction identified by Barry Smith. In "Ingarden vs. Meinong on the Logic of Fiction" [1980], Smith argues that Roman Ingarden's intentional approach to the semantics of fiction is superior to Meinong's in that, p. 96: "...Meinong allows no place for the crucial characteristic of fictional objects that they

Extrapolation beyond the literal text or source book of information about fictional characters and objects provides an alternative though compatible solution. Fictional nonexistents are incomplete Meinongian objects whose so-beings lack many nuclear property and property-complement pairs. An author sketches evocative details, and leaves it to the reader's imagination to further partially complete the picture. Shakespeare does not explicitly say that the floating dagger is hallucinated by Macbeth, but describes Macbeth's experience of it in such a way that it is more reasonable to conclude that it is hallucinatory than to suspend judgment or decide that his perception is veridical. If Shakespeare or Macbeth had unequivocally attributed the extranuclear property of existence or nonexistence to the floating dagger, that still would not augment its so-being of exclusively assumptible nuclear properties. At most it might then be said that the dagger has the derivative converse intentional property of being believed by Shakespeare or Macbeth to be existent or nonexistent, which the dagger acquires when Shakespeare or Shakespeare through Macbeth superadds an existence or nonexistence predication when referring to or describing it.

In drawing these conclusions we partially complete the fictional dagger characterized by Shakespeare by extrapolating beyond his sayso description of it, and adding other nuclear properties to its so-being in imagination. Unless expressly cautioned or forbidden by the author, we ordinarily expect additions to an incomplete object's nuclear properties to accord with extrapolations inductively justified by empirical experience of similar existent objects. Thus, Parsons says: "We bring a great deal of understanding of the world with us to the text, and we utilize this understanding to expand on what is explicitly stated."[11]

Woods raises the related problem of whether Sherlock Holmes has an alimentary canal, given that Conan Doyle never explicitly attributes or denies him one. Woods concludes: "...an author [in 'normal literate practice'] speaks up to his maximum, in the sense that he will declare all departures from the normal for his creatures or at least will weaken normalcy assumptions appropriately. Otherwise his creations are assumed to be normal cases of their kinds; and by these lights, Holmes will be allowed to have had an

are *created* at determinate points in time (i.e., by the sentence-forming acts of the author of the appropriate work)." The origination of fictional objects can be accommodated in revisionary Meinongian object theory logic and semantics if the so-being of an object's constitutive nuclear properties is permitted to include such converse intentional properties as being invented on Thursday by Chaucer, imagined in 1889 by Mark Twain, developed or originated in the Crimea by Fyodor Dostoyevsky.

[11] Parsons, *Nonexistent Objects*, p. 178.

alimentary canal."[12] But presumably if we were contemplating a science fiction sequel to Holmes' adventures, we might imagine him all along to have been an android lacking an alimentary canal, and consistently anneal this futuristic epilogue to the original stories without changing a word, since the question of Holmes' internal anatomy is left open by Conan Doyle in characterizing the incomplete fictional characters of his detective stories. In the case of Macbeth's two daggers it is reasonable to assume that a floating ungraspable dagger is hallucinatory and therefore nonexistent, even though Shakespeare does not explicitly say so. But in drawing this conclusion we extrapolate, go beyond, and add to the assumptible nuclear properties by which the author introduces a nonexistent object to a story-context.

Artworks of many kinds can be regarded as instruction kits for more complete subjective aesthetic experiences. Poetry, plays, short stories, and novels are rather like musical scores, architect's plans, or stage directions. They are aesthetic objects in and of themselves, but are fully realized only in another medium, on the piano keyboard or symphony performance, in constructions of glass, wood, and stone, or in a reader's imagination. The artist usually provides just enough evocative information for us to enter imaginatively into the artwork, and by an act of intentionality bring its characters and events to life. In this we are aided by criticism and transmedia comparisons of immigrant objects of fiction, as when we study Salvador Dali's, Gustave Doré's, or Picasso's paintings and drawings of Don Quixote to help us visualize Cervantes' knight of the woeful countenance. These elaborations and partial completions of creatures of fiction may remain personal and private, or inscribed for others in commentary and complementary artworks. It is possible at most to add to or partially complete an artist's originally incomplete fictional characters and events. But to do so at all or in any degree makes the experience itself an artwork, and belies the purely passive nature of aesthetic appreciation.[13]

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[12] Woods, *The Logic of Fiction*, p. 64.

[13] See Joseph Margolis, *The Language of Art and Art Criticism: Analytic Questions in Aesthetics* [1965], p. 153: "We are, in viewing fiction as fiction, interested in finding out and understanding what the story is... What needs to be emphasized here is that fiction provides us with a story, an account of a world of action that exists, as we say, in the imagination, that is, a world we imagine to exist but which we know does not and never did exist." Jean-Paul Sartre, *The Psychology of Imagination* (*L'Imaginaire*) [1948], pp. 22–5, 34.

VI. The Paradox of Analysis

1. The Langford-Moore Paradox

The object theory logic can be applied to a philosophical problem with important methodological consequences. The paradox of analysis, discussed by C. H. Langford and G. E. Moore, is not a logical antinomy implying outright contradiction, but a kind of metaphilosophical dilemma about conceptual inquiry.[1] In philosophical analysis, the *analysandum* is either the same or not the same in meaning as the *analysans*. If the *analysandum* and *analysans* are the same in meaning, then the analysis is uninformative. But if the *analysandum* and *analysans* are not the same in meaning, then the analysis is faulty.

2. Object Theory Identity

The distinction between intentional and referential identity, and the definition of the *Sosein* function, provides the basis for an intentional solution of the dilemma.[2] As a paradigm of analysis, consider the statement, 'A triangle is a three-angled plane (geometrical) figure.' In object theory, the *analysans*

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1 C. H. Langford, "Moore's Notion of Analysis", in *The Philosophy of G. E. Moore*, edited by Paul A. Schilpp [1968], I, p. 323: "It is indeed possible to deny that analysis can be a significant or logical procedure. This is possible, in particular, on the ground of the so-called paradox of analysis, which may be formulated as follows. Let us call what is to be analyzed the analysandum, and let us call that which does the analyzing the analysans. The analysis then states an appropriate relation of equivalence between the analysandum and analysans. And the paradox of analysis is to the effect that, if the verbal expression representing the analysandum has the same meaning as the verbal expression representing the analysans, the analysis states a bare identity and is trivial; but if the two verbal expressions do not have the same meaning, the analysis is incorrect." G. E. Moore, "A Reply to My Critics", Schilpp, II, pp. 660—67.

2 Compare Chisholm and Richard Potter, "The Paradox of Analysis: A Solution" [1982], pp. 100—6. See Parsons, "Frege's Hierarchies of Indirect Senses and the Paradox of Analysis" [1981], pp. 37—57. Max Black, "The Paradox of Analysis" [1944], pp. 263—67. Morton G. White, "A Note on the 'Paradox of Analysis'" [1945], pp. 71—2. Black, "The 'Paradox of Analysis' Again: A Reply" [1945], pp. 272—73. White, "Analysis and Identity: A Rejoinder" [1945], pp. 357—61. Black, "How Can Analysis Be Informative?" [1946], pp. 628—31. Langford, "Review" [1944], pp. 104—5. Church, "Review" [1946], pp. 132—33.

may be regarded as the result of applying the *Sosein* function to the concept designated in the *analysandum*. The dilemma is avoided if it is assumed that the concepts designated in the *analysandum* and *analysans* are intentionally nonidentical, but that the *analysans* is logically related to the *analysandum* as derivable from the *analysandum* or concept designated in the *analysandum* when the *Sosein* function is applied to it, and thereby referentially identical to the set of nuclear constitutive properties generated as the value of the function.[3]

3. *Solutions*

We have the general solution:

(1) $S(\textit{analysandum}) =_{rf} \{\ldots$properties specified by the $\textit{analysans}\ldots\}$
(2) $\textit{analysandum} =_{rf} \textit{analysans}$
(3) $\textit{analysandum} \neq_{i} \textit{analysans}$

The paradox arises only by equivocation when it is assumed that there is just one kind of identity relation. The distinction between intentional and referential identity makes it possible to say that in one sense *analysans* and *analysandum* are the same, while in another they are different. The referential identity of *analysandum* and *analysans* permits sound or correct analyses, while their intentional nonidentity determined by lack of shared converse intentional properties guarantees the possibility of significant informative analysis, since it is a converse intentional property for an *analysans* to be informative or uninformative.

The solution can now be applied to the paradigm test-case previously but more informally described:

3 The paradox arises because of an equivocation between at least two senses of identity or sameness. Bealer offers a similar diagnosis in *Quality and Concept*, pp. 75—7. Bealer's solution is unsatisfactory because it relies on an orthographic underlining convention to distinguish undefined from defined concepts. This solves the paradox only for analyses in which the *analysans* consists of undefined primitive concepts. It is insufficiently general for such analyses as the Aristotelian Chisholm-Potter example 'Man = rational animal', if, as seems reasonable, neither 'rational' nor 'animal' are undefined concepts. Bealer's approach also has built into it what he calls the Type 1/Type 2 distinction between kinds of intensional entity, which ultimately rests on an unsupported semantic absolutism committed to the existence of uniquely correct reductions of complex to primitive concepts in defiance of Goodman-Quine ontological relativity. See Jacquette, "Intentionality and Intentional Connections" [1987].

(1) S(triangle) $=_{rf}$ {three-angled, planar, (geometrically) figural}
(2) triangle $=_{rf}$ three-angled plane (geometrical) figure
(3) triangle $\neq_i$ three-angled plane (geometrical) figure

The analysis is informative because of the intentional nonidentity of *analysandum* and *analysans*. We may stand in different cognitive converse intentional relations or attitudes to the *analysans* than to the *analysandum*. We may be familiar with the concepts of the *analysans*, but not with that of the *analysandum*, and we may appeal to the analysis or try to arrive at a satisfactory analysis of an unfamiliar concept in order to fill in gaps and asymmetries in our understanding, as we look up unfamiliar words in a dictionary. The *analysandum* and *analysans* in that case will be distinct concepts, and therefore intentionally nonidentical, because one has at least some converse intentional properties the other lacks. The analysis is sound, because the *Sosein* function applied to the *analysandum* is referentially identical to the set of its nuclear constitutive properties presented as the *analysans* or value or product of the *Sosein* function, and because *analysans* and *analysandum* are the referentially identical objects of referentially codesignative terms.

The challenge of the paradox is to clear the way for the possibility of correct and informative analyses, and not to deliver a foolproof general procedure for inventing or discovering analyses. The solution accordingly specifies necessary but not sufficient conditions for a philosophically correct analysis.

Informativeness is in many ways relative to interest, ignorance, and cultural context. Consider Moore's example, 'brother = male sibling' or $b = ms$. If in a certain context we have all the same intentional attitudes toward b and ms, we will not offer $b =_{rf} ms$ as an analysis, since in that case and relative to that context not only is $b =_{rf} ms$, but $b =_i ms$, so that contextually the conditions for informative analysis are not satisfied.

It might be objected that the account trivializes analysis by limiting identity to situations in which tokens of inscriptionally identical term types flank the intentional identity sign. The theory allows possible though perhaps nonexistent intelligences to take up different intentional attitudes toward the objects designated by any distinct terms (or for that matter toward the objects designated by the distinct tokens of inscriptionally identical term types, perhaps *fearing* token 'T' on the left side of '$=_i$', and *not fearing* token 'T' on the right side). Objects designated by distinct tokens or types *can* indeed acquire distinct converse intentional properties, from unknowledgeable nonexistent persons if from nowhere else. But this is a desirable feature of the theory, since it holds out the extracontextual possibility that in principle if not in practice every nonprimitive concept can be soundly and significantly analyzed. In practice, trivializations of the imagined sort are contextually excluded, where the question is not whether objects designated by distinct terms *could* acquire different converse intentional properties, which is always possible, but whether in context, given the information and attitudes of analysis participants and intended audience, they actually *do* have distinct converse intentional properties.

A subject might but need not have precisely the same intentional attitudes toward inscriptionally distinct term types 'brother' and 'male sibling'. But this would be a distinction of intentional attitudes toward the *names* or *expressions* of the concepts, and not a difference of intentional attitudes toward conferring a difference of converse intentional properties on the *concepts* themselves. It seems reasonable to conclude that concepts expressed by means of inscriptionally distinct term types flanking the intentional identity sign in a nontrivial conceptual analysis can designate objects that are contextually conceptually identical, possessing the very same set of properties including converse intentional properties relative to the interests and beliefs of those for whom the analysis is intended. The possibility of nontrivial types vouchsafes the intelligibility of referential identity and contextually nontrivial intentional nonidentity of *analysandum* and *analysans*.

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