How to Become a Pure Mathematician:

a List of Undergraduate and Basic Graduate Textbook

怎樣研習純數學

本科與基礎研究課程參考書目

PDF Version of http://hk.geocities.com/mathphyweb/puremath.htm

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It's like if you want to be a good pianist,
you have to do a lot of scales and a lot of practice,
and a lot of that is kind of boring, it's work.
But you need to do that before you can really be very expressive and really play beautiful music.
You have to go through that phase of practice and drill.

- Terry Tao

Content:

About this article:

- What for?
  OK, there is a way to become a good theoretical physicist (http://www.phys.uu.nl/~thooft/theorist.html). Here is a guide to study pure mathematics, or even more. This list is written for those who want to learn mathematics but have no idea how to start. Yup, a list for beginners. I don't claim that this list makes you a *good* pure mathematician, since I belong to the complement of good pure mathematician. I make no attempt to define what pure mathematics is, but hopefully it will be clear as you proceed. I also highlighted several books that you would really like to keep in your own library. You probably like to read those books again and again in your life. Free material excluded. Note the highlighted list does NOT indicate those books are good for beginners. I shall try to keep this list up to date whenever I exist.
- Assumed knowledge.
  I assumed you have high school mathematics background (i.e. basic trigonometry, Euclidean geometry, etc). The aim of this page is to introduce what different branches of mathematics are; and recommended a few notes or texts.
  Scientists in other fields and engineers may skip first or second stages and begin at later stages, according to their prior knowledge.
- Time.
  It takes approximately one year for each stage (except for stage 4, I list more material in each field for more advanced studies), for a full time student. Part time students may double the time. But its better for anyone to understand most parts of stage n before proceeding to n+1, for some integer n in {1, 2, 3, 4}. If you decided to attend a class, don't expect the professor can teach, it always happen, especially in higher level courses. What's the order of courses to study within a stage doesn't really matter, usually. One doesn't need to read every listed book within a subject to master the subject. I listed more than enough so that you can scout around to find one that you feel comfortable with. Some people like to consult a few books, beware of the symbols from different books in such cases.
  Moreover, it often happens that you couldn't solve a problem within an hour. It's not surprise to spend a week or more to tackle one problem. Things may come to your head suddenly. Shouting eureka is the high point of a mathematician.
- "Axiom of choice".
  My selection will not be bounded by any publication press, author's nationality or religion. It relies on two factors: well written or cheap. These two factors are not mutually exclusive. I treat "free" as an element in "cheap". Note the price factor may by irrelevant, sometimes I get a HK $2xx book and Amazon says its US $1xx (~HK $7xx)..... with the only difference is, perhaps, I got the international edition. Moreover, some Chinese press in mainland China

published photocopied of English text with a relatively cheap price.

Bear in mind that, just because one is a good mathematician doesn't imply he's a good author or educator. Perhaps Terry Tao is an exceptional case. To study science, reading the classics (the *Elements*, *Dialogo sopra i due massimi sistemi del mondo*, the *Principia*, *Disquisitiones Arithmeticae*, *Principia Mathematica*, etc) is optional. While for literature or philosophy, I wonder if any well educated student has never study Shakespeare or Plato.

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- Comments.

  Links to Amazon for most of the listed book are included, so that you have an easy access to other users' comments. Note that the comments are sometimes quite extreme: for the same book, one rated it with 5 star (with dozens of people supporting) and at the same time another rated 1 star (with dozens of people supporting again), especially for introductory discrete mathematics, probability and statistics books. It seemed to me that lots of people study these subjects because they need to, they want to apply mathematics. Large proportion of these readers are lack of mathematical maturity. If they can't pass the exam, you know... In contrast, most pure mathematics students study because they like the subject and enjoy it. So, ask yourself, why do you study?

- Other resources.

  Although I'm not into reading books online, I should remind you that MIT's open recourse (http://ocw.mit.edu/OcwWeb/Mathematics/index.htm), the Archimedeans (http://www.archim.org.uk/notes/) and Wikibooks (http://en.wikibooks.org/wiki/Category:Mathematics) provide another great sources of materials. These are excluded in the following list. The list below aimed to recommend books or (usually) printable notes. Google books (http://books.google.com/) allow you to preview sections from a book. Schaum's Outlines (http://www.mhprofessional.com/category/?cat=4234) series are cheap, but I seldom include them, you may search the relevant if you like.

- Me.

  A product of School of Mathematics and Statistics, UNSW, Sydney, Australia. I've taken all undergraduate core pure mathematics and statistics courses there, with all pure mathematics courses in higher level, whenever they exist. Also, I was Terry Tao's teacher's student, Michael Artin's student's student, Gottfried Leibniz's student Jacob Bernoulli's student Johann Bernoulli's student Leonhard Euler's student Joseph Lagrange's student Simeon Poisson's student's student's student's student's student's student's student's student's student, Max Planck's student's student's student's student and Thomas Kuhn's student's student. Just feel

like I'm an idiot.

- Disclaimer.

  I'm not responsible for any external link.

- Comments/suggestion for a book, etc. just email me, contact can be found in the main page (http://hk.geocities.com/mathphyweb/).

Stage 1

For many reasons, high school mathematics is taught in an informal way, we'll fix things up here. The first stage contains the basic material that is required for most scientists and engineers, discrete mathematics is particularly meant for computer scientists.

Elementary Stuff:

Here is a list that helps you to refresh and enrich your high school mathematics.

- Barnard S. and Child J.M. *Higher Algebra*
- Beecher J.A., Penna J.A. and Bittinger M.L. *Algebra and Trigonometry*
- Chen W.W.L. and Duong X.T. Elementary Mathematics (FREE! http://www.maths.mq.edu.au/~wchen/lnemfolder/lnem.html)
- Gibson C.G. *Elementary Euclidean Geometry: An Undergraduate Introduction*
- Gow M.M. *A Course in Pure Mathematics* - Contains exercises taken from British Universities exams.
- Ikenaga B. Basic Algebra and Trigonometry Notes (FREE! http://marauder.millersville.edu/~bikenaga/basicalg/balgnote.html)
- Joyce D.E. A Short Course on Trigonometry (FREE! http://www.clarku.edu/~djoyce/trig/)
- Kedlaya K. A<B (FREE! http://www.artofproblemsolving.com/Resources/Papers/KedlayaInequalities.pdf)
- Kubota K.K. College Algebra (FREE! http://www.msc.uky.edu/ken/ma109/notes.htm)
- Meyer W.A. *Geometry and Its Applications*
- Mildorf T.J. Olympiad Inequalities (FREE! http://www.artofproblemsolving.com/Resources/Papers/MildorfInequalities.pdf)
- Santos D. Arithmetic Lecture Notes (FREE! http://www.openmathtext.org/lecture_notes/a-course-in-arithmetic.pdf)
- Santos D. Elementary Algebra Lecture Notes (FREE! http://www.openmathtext.org/lecture_notes/elementary_algebra_book.pdf)
- Santos D. Precalculus I and II Lecture Notes (FREE! http://www.openmathtext.org/lecture_notes/david_santos_precalculus.pdf)
- Sharipov R. *Foundations of Geometry for University Students and High-school Students* (FREE! http://uk.arxiv.org/abs/math/0702029/)
- Veeh J.A. Lecture Notes on Algebra and Trigonometry (FREE! http://javeeh.net/lecnotes/algtrg.pdf)
- Ward T.B. Basic Mathematics Covering Algebra and Trigonometry (FREE! http://www.mth.uea.ac.uk/~h720/lecturenotes/OB81lectures.pdf)

- Zakon E. *Basic Concepts of Mathematics* (FREE! http://www.trillia.com/zakon1.html)

Introductory Discrete Mathematics:

You will study logic and set theory in an introductory discrete mathematics. The concepts you learnt here will play a key role in your later studies and improve your thinking skill. You will also study proofs, functions (injective, surjective, bijective inverse), introductory graph theory and number theory, Euclid's algorithm, discrete probability (counting, nCr, nPr) etc in this course.

Only for Proof, Logic and Set Theory:

- Bilaniuk S. *A Problem Course in Mathematical Logic* (FREE! http://euclid.trentu.ca/math/sb/pcml/)
- Connell E.H. Background and Fundamentals of Mathematics (FREE! http://www.math.miami.edu/~ec/book/ch01.pdf) - A short chapter from the author's algebra book.
- Daoud A. and Franklin J. *Proof in Mathematics: An Introduction*
- Düntsch I. Sets, Relations, Functions (FREE! http://www.cosc.brocku.ca/~duentsch/papers/methprimer1.html)
- Enderton H.B. *Elements of Set Theory*
- Halmos P.R. *Naive Set Theory* - Don't misinterpret the word naive, though he propose the honest title as *An Outline of the Elements of Naive Set Theory*. This is not those books for dummies nor complete idiot's guide. Set theory plays a significant role in mathematics, if you are serious about it. Halmos is one of the better expositors of mathematics.
- Halmos P.R. and Givant S. *Logic as Algebra*
- Herrmann R.A. *Logic for Everyone* (FREE! http://arxiv.org/abs/math.GM/0601709)
- Ikenaga B. Notes on Math Proof (FREE! http://marauder.millersville.edu/~bikenaga/mathproof/mathproofnotes.html)
- Skolem T.A. *Abstract Set Theory* (FREE! http://projecteuclid.org/euclid.ndml/1175197470)
- Suppes P. *Introduction to Logic*
- Velleman D.J. *How to Prove It: A Structured Approach* - Let me warn you: proof is a crucial, I repeat, crucial, part in higher mathematics. Velleman did a good job for beginners here.

In general:

- Anderson I. *A First Course in Discrete Mathematics*
- Chen H.T. Discrete Mathematics (see the Lecture Notes section) (FREE! http://mcu.edu.tw/~htchen/dm/index.html)
  陳旭東：離散數學
- Chen W.W.L. Discrete Mathematics (FREE! http://www.maths.mq.edu.au/~wchen/lndmfolder/lndm.html)

- Epp S.S. *Discrete Mathematics with Applications* - I enjoy reading Epp's text when I first learn this topic. At this stage, some author makes good effort to spark student's interest. Don't forget to check out the errata (http://condor.depaul.edu/~sepp/DMwA3e.htm).
- Finan M.B. Lectures Notes in Discrete Mathematics (FREE! http://syssci.atu.edu/math/faculty/finan/main2.pdf)
- Graham R.L., Knuth D.E. and Patashnik O. *Concrete Mathematics: A Foundation for Computer Science*
- Kemeny J.G., Snell J.L. and Thompson G.L. *Introduction to Finite Mathematics* (FREE! http://www.math.dartmouth.edu/~doyle/docs/finite/cover/cover.html)
- Lerma M.A. Notes on Discrete Mathematics (FREE! http://www.math.northwestern.edu/~mlerma/papers/discrete_mathematics-2005.pdf)
- Lovasz L., Pelikan J. and Vesztergombi K.L. *Discrete Mathematics*
- Lovasz L. and Vesztergombi K. Discrete Mathematics (FREE! http://www.cs.tau.ac.il/~odedr/teaching/discrete_math_fall_2005/dmbook.pdf)
- MacGillivray G UVic Discrete Mathematics Study Guide (FREE! http://www.math.uvic.ca/faculty/gmacgill/guide/)
- Rosen K.H. *Discrete Mathematics and Its Applications* - Another nice introductory text on this topic. See the companion website (http://www.mhhe.com/rosen).
- Ross S.M. *Topics in Finite and Discrete Mathematics*
- Santos D. Discrete Maths Lecture Notes (FREE! http://www.openmathtext.org/lecture_notes/discrete_math_lecture_notes.pdf
  Santos D. Discrete Maths II (FREE! http://www.openmathtext.org/lecture_notes/discrete_math_lecture_notesII.pdf)

Introductory Algebra:

Complex number, polynomials, matrix, system of equation, Gaussian elimination, vector space, linear transformation, etc. T.M. Aposotol's *Calculus Vol. 1* in the calculus sections also introduce topics that is relevant to this area.

A few Americans (exchanged to Sydney) in my third year abstract algebra class told us they couldn't follow because they haven't learnt much complex number. In order to have fun with linear algebra and complex variables in stage 2, it's better to learn complex number now.

- Anton H. and Rorres C. *Elementary Linear Algebra* - The application version introduced how linear algebra can be applied to Markov Chains, Graph Theory, Games of Strategy, Computer Graphics, Fractals, Chaos, Cryptography, Genetics, etc.
- Chen W.W.L. Linear Algebra (FREE! http://www.maths.mq.edu.au/~wchen/lnlafolder/lnla.html) - Read Chapter n's, for all positive integer n<8.
- Chen W.W.L. Miscellaneous Topics in First Year Mathematics (FREE!

http://www.maths.mq.edu.au/~wchen/lnmtfymfolder/lnmtfym.html)

- Dawkins P. Algebra (FREE! http://tutorial.math.lamar.edu/classes/alg/alg.aspx)
- Gardner R.B. Linear Algebra (FREE! http://www.etsu.edu/math/gardner/2010/notes.htm) - Leave Chapter 6 and 7 to the next stage.
- Joyce D.E. A Short Course on Complex Numbers (FREE! http://www.clarku.edu/~djoyce/complex/)
- Lang S. *Undergraduate Algebra* - He's a member of the Bourbaki and wrote many books. Not friendly for beginners, but you should get use to it.
- Matthews K. Elementary Linear Algebra (FREE! http://www.numbertheory.org/book/)
- Santos D. Linear Algebra Notes (FREE! http://www.openmathtext.org/lecture_notes/new_linearalgebra.pdf)
- Strang G. *Linear Algebra and Its Applications*
- Wedderburn J.H.M. *Lectures on Matrices* (FREE! http://www.ams.org/online_bks/coll17/)

Introductory Calculus:

As you may have seen, calculus is crucial. It is not only important in advanced mathematics but also can be applied to many other fields. If you learn first year calculus in a college, your suggested text is probably J. Stewart's *Calculus*. This is a standard modern text, it is colorful and quite detailed. If you prefer it, that's fine. This book is thick, you would like to leave a few section, say vector calculus, double, triple, line and surface integrals, to the next stage. I would suggest a few non-standard texts and notes.

The first thing we need to fix up is the definition of limit, learn the epsilon and delta proof. Other key concepts are the mean value theorem, fundamental theorem of calculus, logarithm and exponential functions, inverse trigonometric functions, hyperbolic functions, basic differential equations, limit of sequences, indeterminate form, infinite series, basic idea of several variables.

- Apostol T.M. *Calculus Vol. 1*
- Burn R.P. *Numbers and Functions: Steps into Analysis*
- Chen W.W.L. First Year Calculus (FREE! http://www.maths.mq.edu.au/~wchen/lnfycfolder/lnfyc.html)
- Courant R. and John F. *Introduction to Calculus and Analysis I* - One of the better calculus text in print.
- Crowell B. *Calculus* (FREE! http://www.lightandmatter.com/calc/)
- Dawkins, P Calculus I and Calculus II (FREE! http://tutorial.math.lamar.edu/classes/calcI/calcI.aspx and http://tutorial.math.lamar.edu/classes/calcII/calcII.aspx)
- Garrett P. Calculus (FREE! http://www.math.umn.edu/~garrett/calculus/)
- Ghorpade S.R. and Limaye B.V. *A Course in Calculus and Real Analysis* - See their web (http://www.math.iitb.ac.in/~srg/acicara/) for errata.

- Gill G.S. *Calculus Bible* (FREE! http://www.math.byu.edu/Math/CalculusBible/)
- Hardy G.H. *A Course of Pure Mathematics* - Contains Cambridge Mathematical Tripos' exam paper, have fun!
- Hwang A.D. Calculus for Mathematicians, Computer Scientists, and Physicists: An Introduction to Abstract Mathematics (FREE! http://mathcs.holycross.edu/~ahwang/print/calc.pdf)
- Ikenaga B. Notes on Calculus (FREE! http://marauder.millersville.edu/~bikenaga/calculus/calculusnotes.html)
- Santos D. Differential Calculus (FREE! http://www.openmathtext.org/lecture_notes/jose_diff_calc.pdf)
- Shapiro B.E. Calculus and Analysis 2 (FREE! http://www.bruce-shapiro.com/math150B/spring2002/)
- Spivak M. *Calculus* - To paraphrase J.R. Giles, Spivak presents a first course in real analysis or a course in calculus that has been carefully developed with attention given to the real analysis foundations with an eye on rigour.
- Strang G. *Calculus* (FREE! http://ocw.mit.edu/ans7870/resources/Strang/strangtext.htm)
- Thomas G.B. and Finney R.L. *Calculus and Analytic Geometry*
- Tranter C.J. *Advanced Level Pure Mathematics*
- Tranter C.J. *Techniques of Mathematical Analysis*
- Veeh J.A. Lectures Notes on Calculus (FREE! http://javeeh.net/lecnotes/calc1.pdf)
- 張筑生：《數學分析新講》第一、二冊，北京大學出版社

Stage 2

Mathematics program in the University are constructed in the way that courses are offered in favor of other schools (Physics, Chemistry, Biology, Engineering, Economic, etc). For example, Physics students have to take linear algebra, several variable calculus, mathematical analysis and differential equation. As a consequence, textbooks are written and catalog like the following way. Several authors focus on the application and often do calculation without justifying, beware of it.

Linear Algebra:

Vectors, vector spaces, linear transformations, multilinear map, inner product spaces, norms, orthogonality, Gram-Schmidt algorithm, QR-factorisation, least square, Householder algorithm, normal matrices, Jordan canonical forms, Cayley-Hamilton theorem, minimal and characteristic polynomials, direct sum decompositions, generalised eigenspaces, functions of matrices, exponentials of matrices, etc will be studied in this course. Such material can be applied to linear programming, computer graphics, fractals, and many areas in natural sciences and social sciences.

- Beezer R. *A First Course in Linear Algebra* (FREE! http://linear.ups.edu/)
- Bowen R.M. and Wang C-C. *Introduction to Vectors and Tensors* (FREE! http://www1.mengr.tamu.edu/rbowen/)
- Chen W.W.L. Linear Algebra (FREE! http://www.maths.mq.edu.au/~wchen/lnlafolder/lnla.html) - Read Chapter n's, for integer n>7.
- Dawkins P. Linear Algebra (FREE! http://tutorial.math.lamar.edu/classes/linalg/linalg.aspx)
- Fraleigh J.B. and Beauregard R.A. *Linear Algebra* - Not excellent for a first course.
- Halmos P.R. *Finite-Dimensional Vector Spaces*
- Halmos P.R. *Linear Algebra Problem Book*
- Hefferon J. *Linear Algebra* (FREE! http://joshua.smcvt.edu/linalg.html/)
- Herod J.V. Linear Algebra, Infinite Dimensions, and Maple (FREE! http://www.math.gatech.edu/~herod/Hspace/Hspace.html)
- Ikenaga B. Notes on Linear Algebra (FREE! http://marauder.millersville.edu/~bikenaga/linearalgebra/linearalgebranotes.html)
- Kuttler K. An Introduction To Linear Algebra (FREE! http://www.math.byu.edu/~klkuttle/Linearalgebra.pdf)
- Lang S. *Linear Algebra* - As Lang had said this book isn't aimed for introductory linear algebra, it goes beyond standard first course of linear algebra. Make sure you have a solid foundation before you read this.
- Meyer C.D. *Matrix Analysis and Applied Linear Algebra* (FREE! http://www.matrixanalysis.com/)

- Sharipov R. Course of Linear Algebra and Multidimensional Geometry (FREE! http://uk.arxiv.org/abs/math.HO/0405323/)
- Shilov G.E. *Linear Algebra* - Professor Shilov at the Moscow State University states that this text "considers spaces over an arbitrary field, with the real and complex spaces being considered as closely related special cases of the general theory."

Introductory Higher Algebra:

I've taken this course entitled "Finite Mathematics", a course for computer scientists, software engineers and pure mathematicians (optional but useful). Assuming you have learnt the very basic of number theory in the discrete mathematics section, you are ready to get a taste of higher algebra here. Things like prime numbers, tests for primality, Fundamental Theorem of Arithmetic, Fermat's little theorem, Gauss' lemma, Euler's theorem, Chinese remainder theorem and their applications (coding, RSA) etc are concerned in this section. This is a bridge that connects first stage basic algebra, discrete mathematics and the third stage abstract algebra.

- Chapman R. A Guide to Arithmetic (FREE! http://www.secamlocal.ex.ac.uk/people/staff/rjchapma/notes/arith.pdf)
- Childs L.N. *A Concrete Introduction to Higher Algebra* - This book isn't terribly good.
- Smith R. Elementary Algebra Course Notes (FREE! http://www.math.uga.edu/~roy/)

Calculus (Introductory Real Analysis, Several Variables Calculus, Vector Calculus, etc):

Learn the very basic concept of real analysis, like open set, close set, boundary point, closure, limit point, bounded set, connected set, compact set, Bolzano-Weierstrass theorem. To get yourself ready for the stage 3 analysis, make sure you understand those stuff. Next, the Lagrange multiplier, inverse and implicit function theorems can also be studied. For the several variables section, gradients, double, triple and surface integrals, cylindrical coordinates, Green's theorem and divergence theorem are the key things. Also learn basic Fourier series.

- Apostol T.M. *Calculus Vol. 2*
- Bartle R.G. *The Elements of Real Analysis*
- Cain C. and Herod J. Multivariable Calculus (FREE! http://www.math.gatech.edu/~cain/notes/calculus.html)
- Chen W.W.L. Fundamentals of Analysis (FREE! http://www.maths.mq.edu.au/~wchen/lnfafolder/lnfa.html)
- Chen W.W.L. Multivariable and Vector Analysis (FREE! http://www.maths.mq.edu.au/~wchen/lnmvafolder/lnmva.html)
- Courant R. and John F. *Introduction to Calculus and Analysis II/1*, *II/2* - If you want to get *one* several variables calculus text.
- Dawkins P. Calculus III (FREE! http://tutorial.math.lamar.edu/classes/calcIII/calcIII.aspx)

- Demidovich B.P. *Problems in Mathematical Analysis*
- Gaughan E.D. *Introduction to Analysis* - Expensive. Have a look in the library.
- Ikenaga B. Calculus Notes (FREE! http://marauder.millersville.edu/~bikenaga/calc/calcnote.html)
- Kaplan W. *Advanced Calculus*
- Krantz S.G. *Real Analysis and Foundations* - The author is a fan of Rudin and he was trying to bridge between less rigorousX books and hard-edged classical books like baby Rudin.
- Keisler H.J. *Elementary Calculus: An Approach Using Infinitesimals* (FREE! http://www.math.wisc.edu.nyud.net:8090/~keisler/calc.html) - Check it out, can you believe it's free?
- Lewin J. *An Interactive Introduction to Mathematical Analysis* - Come with a CD-ROM. Very friendly, aimed particularly for non-mathematicians who need to study this hardest ever course (the author means analysis) in their life.
- Protter M.H. *Basic Elements of Real Analysis*
- Rosenlicht M. *Introduction to Analysis*
- Santos D. Vector Calculus Notes (FREE! http://www.openmathtext.org/lecture_notes/vector_calculus_book3.pdf)
- Shilov G.E. *Elementary Real and Complex Analysis* - Republication of Mathematical Analysis Vol. I, MIT Press, Cambridge, Mass.
- Shapiro B.E. Multivariate Calculus in 25 Easy Lectures (FREE! http://www.bruce-shapiro.com/math250/math250.pdf)
- Sloughter D. The Calculus of Functions of Several Variables (FREE! http://math.furman.edu/~dcs/mvbook/)
- Tao T. Analysis 1 (FREE! http://www.math.ucla.edu/~tao/resource/general/131ah.1.03w/) - Not only Terry is a good mathematician, but also a good educator.
- Thomson B.S., Bruckner J.B. and Bruckner A.M. *Elementary Real Analysis* (FREE! http://classicalrealanalysis.com/download.aspx)
- Veeh J.A. Lectures Notes on Multivariate Calculus (FREE! http://javeeh.net/lecnotes/calc3.pdf)
- Wachsmuth B.G. *IRA: Interactive Real Analysis* (FREE! http://pirate.shu.edu/~wachsmut/ira/index.html)
- Wilde I.F. Real Analysis: An Introduction (FREE! http://www.mth.kcl.ac.uk/~iwilde/notes/jha/index.html)
- Zakon E. *Mathematical Analysis I* (FREE! http://www.trillia.com/zakon-analysisI.html)
- 張筑生：《數學分析新講》第三冊，北京大學出版社

Complex Variables (Introductory Complex Analysis):

Another beautiful branch of mathematics. While you're in high school, you may wonder what's the

point of introducing imaginary number i, does it really exists? Is it really useful? In an introductory complex analysis course, you will see the beauty of this construction. Gauss, Riemann, Weierstrass and Cauchy are key figures in this area. Key concepts to learn are: analytic functions, Cauchy-Riemann equations, contour integral, Cauchy-Goursat theorem, residues and poles.

- Ablowitz M.J. and Fokas A.S. *Complex Variables: Introduction and Applications*
- Bass R.F. Complex Analysis (FREE! http://www.math.uconn.edu/~bass/ca.pdf)
- Beck M., Marchesi G. and Pixton D. A First Course in Complex Analysis (FREE! http://math.sfsu.edu/beck/complex.html)
- Brown J.W. and Churchill R.V. *Complex Variables and Applications* - Study up to chapter 7.
- Cain G. Complex Analysis (FREE! http://www.math.gatech.edu/~cain/winter99/complex.html)
- Carne T.K. Complex Analysis (FREE! http://www.dpmms.cam.ac.uk/site2002/Teaching/IB/ComplexAnalysis/2005-2006/Notes.pdf)
- Chen W.W.L. Introduction to Complex Analysis (FREE! http://www.maths.mq.edu.au/~wchen/lnicafolder/lnica.html)
- Kuttler K. Complex Analysis (FREE! http://www.math.byu.edu/~klkuttle/math532notes01.pdf)
- Marsden J.E. and Hoffman M.J. *Basic Complex Analysis* - See Marsden's page (http://www.cds.caltech.edu/~marsden/books/Basic_Complex_Analysis.html) for errata.
- Ponnusamy S. and Silverman H. *Complex Variables with Applications*
- Saff E.B. and Snider A.D. *Fundamentals of Complex Analysis*
- Spiegel M.R. *Schaum's Outline of Complex Variables*
- Tao T. Complex Analysis for Applications (FREE! http://www.math.ucla.edu/~tao/resource/general/132.1.00w/)
- Wilkins D.R. Functions of a Complex Variable (FREE! http://www.maths.tcd.ie/~dwilkins/Courses/214/)
- Wunsch D.A. *Complex Variables with Applications*

Differential Equation:

Isaac Newton originally studies behaviour of dynamical systems using differential equations. Mathematical models are often described with differential equations and they are widely used in many fields, including physics, chemistry, biology, economics etc. Mathematical methods used in solving differential equation also play an important in advanced studies. Key topics to learn are: Wronskians, series solutions, reduction of order, variation of parameters, Frobenius normal form, Bessel's equation, Legendre's equation, two points boundary problem, Fredholm alternative, Green's function, complete orthogonal system, Sturm-Liouville problems (inhomogeneous, singular), basic partial differential equation, elliptic eigen-problems, heat equation and wave equation.

- Boyce W.E. and DiPrima R.C. *Elementary Differential Equations and Boundary Value Problems*
- Dawkins P. Differential Equations (FREE! http://tutorial.math.lamar.edu/classes/de/de.aspx)
- Forsyth A.R. *A Treatise on Differential Equations*
- Ikenaga B. Differential Equations Notes (FREE! http://marauder.millersville.edu/~bikenaga/diffeq/deqnote.html)
- Shapiro B.E. Lecture Notes on Differential Equations (FREE! http://www.bruce-shapiro.com/math351/351notes/)
- Simmons G.F. and Krantz S.G. *Differential Equations: Theory, Technique, and Practice* - Both Simmons and Krantz are writers for mathematicians.
- Veeh J.A. Lectures Notes on Ordinary Differential Equations (FREE! http://javeeh.net/lecnotes/odes.pdf)

Probability and Statistics:

Well, being a well educated mathematician, you should have basic knowledge of statistics. Things to learn: basic probability theory (independent events, conditional probability, Bayes' Theorem), random variables (r.v.), expectation, convergence of r.v., maximum likelihood estimator, basic hypothesis testing, p-values are the basic. If you want to go a bit further, check out linear regression, linear model, residual, categorical predictors, logistic regression, ANOVA (analysis of variance) etc.

Probability:

- Bass R.F. Undergraduate Probability (FREE! http://www.math.uconn.edu/~bass/elemprob.pdf)
- Bertsekas D.P. and Tsitsiklis J.N. *Introduction to Probability* - Leave chapter 5 and 6 to next stage. Solution: http://www.athenasc.com/probsolved.pdf
- Boik R. J. Probability (FREE! http://www.math.montana.edu/~rjboik/classes/502/notes.420.02.pdf)
- Gnedenko B.V. *The Theory of Probability* - First six chapters.
- Grinstead C.M. and Snell J.L.'s *Introduction to Probability* (FREE! http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/book.html) - Read first ten chapters.
- Lebanon G. Tutorial Notes (FREE! http://www.stat.purdue.edu/~lebanon/notes/) - Read P1 - P14.
- Lipschutz S. and Lipson M. *Schaum's Outline of Probability* - Read chapter 1-6.
- Mosteller F. *Fifty Challenging Problems in Probability with Solutions* - Later stages topics included.
- Ross S. *A First Course in Probability* - First eight chapters.
- Santos D. Probability Lecture Notes (FREE! http://www.openmathtext.org/lecture_notes/probability_book2.pdf)
- Stirzaker D. *Elementary Probability* - Leave the last chapter.

Statistics:

- Berry D.A. *Statistics: A Bayesian Perspective*
- Boik R. J. Elementary Statistics (FREE! http://www.math.montana.edu/~rjboik/classes/216/lecture.notes/)
- Boik R. J. Statistics (FREE! http://www.math.montana.edu/~rjboik/classes/502/notes.424.03.pdf)
- Bulmer M.G. *Principles of Statistics*
- DeCoster J. Introductory Statistics Notes (FREE! http://www.stat-help.com/intro.pdf)
- Dekking F.M., Kraaikamp C., Lopuhaä H.P. and Meester L.E. *A Modern Introduction to Probability and Statistics: Understanding Why and How*
- El-Taha M. MBA604 Introduction Pobabiltity and Statistics Lecture Notes (FREE! http://www.math.leidenuniv.nl/~redig/lecturenotesstatistics.pdf)
- Gerstman B.B. StatPrimer (FREE! http://www.sjsu.edu/faculty/gerstman/StatPrimer/)
- Hodges J.L. and Lehmann E.L. *Basic Concepts of Probability and Statistics* - Non-calculus based text by two renowned statisticians.
- Lane D., Lu J., Peres C. and Zitek E. Online Statistics: An Interactive Multimedia Course of Study (FREE! http://www.onlinestatbook.com/)
- Larsen R.J. and Marx M.L. *An Introduction to Mathematical Statistics and Its Applications*
- Lavine M.L. *Introduction to Statistical Thought* (FREE! http://www.stat.duke.edu/~michael/book.html)
- Ross S.M. *Introduction to Probability and Statistics for Engineers and Scientists*
- Simon J.L. Resampling: The New Statistics (FREE! http://www.resample.com/content/text/index.shtml)
- Spiegel M.R. *Schaum's Outline of Statistics* - As a reference. Chapters up to 12 are core, interested reader may read up to chapter 16 at this stage.
- Stockburger D.W. Introductory Statistics: Concepts, Models, and Applications (FREE! http://www.psychstat.missouristate.edu/sbk00.htm)

Second course in Statistics:

- Janke S.J. and Tinsley F. *Introduction to Linear Models and Statistical Inference* - Examples illustrated with Minitab.
- Myers R.H. *Classical and Modern Regression with Applications*
- Miller J. and Haden P. *Statistical Analysis with the General Linear Model* (FREE! http://www.geocities.com/milleratotago/)
- Taylor J. Introduction to Applied Statistics (FREE! http://www-stat.stanford.edu/~jtaylo/courses/stats191/index.html)
- Taylor J. Introduction to Regression Models and Analysis of Variance (FREE!

http://www-stat.stanford.edu/~jtaylo//courses/stats203/index.html)

Stage 3

At this stage, you begin to learn modern Pure Mathematics. Yup, this is *the beginning*. Focus on the proof. Don't expect you can solve a problem by plugging numbers into formulae. It often happens in advanced text that the author skips some steps in a proof or calculation, while elementary text gives detail explanations. But as G.F. Simmons has said, '[t]he serious student will train himself to look for gaps in proofs, and should regard them as tacit invitations to do a little thinking on his own.' Moreover, '[i]t is a basic principle in the study of mathematics, and one too seldom emphasized, that a proof is not really understood until the stage is reached at which one can grasp it as a whole and see it as a single idea. In achieving this end, much more is necessary than merely following the individual steps in the reasoning. This is only the beginning. A proof should be chewed, swallowed, and digested, and this process of assimilation should not be abandoned until it yields a full comprehension of the overall pattern of though.'

Also get maturity and learn how to write mathematics, by reading good books (?). I don't think Gauss or Galois can get full mark in mathematics assignments these days, for 1) something obvious to them may not be obvious to everyone else, and 2) they don't bother to explain to others. The point is, every steps in a proof should be logically related and everyone can go easily from the previous step to the next. This is not the case for non-pure mathematics. If you insist, I can show you a copy of full mark statistics assignment consists work of mere calculation and computation. (Of course not done by me!)

I had tried to list more books so that you can compare and choose one or two that fit you. Despite that most of following courses are 'pure', they can be applied to other graduated level science subjects. In the analysis class, Ian Doust even showed us an article in the *Econometrica* (Hildenbrad W. and Metrens J.F. Upper Hemi-continuity of the Equilibrium-set Correspondence for Pure Exchange Economies, Vol. 40, No. 1) talks about liminf, limsup, measure, weak topology and stuffs like that. If one insists that the pure stuff is useless, one is just ignorant, I don't bother to argue with those people anymore.

Introductory Analysis:

At this stage you should have learnt limits, continuity and convergence, over the reals. These are central concepts of calculus in both one and several variables. These can be generalised. You will study the idea of several spaces (metric space, topological space, Banach, Hilbert), compactness, connectedness, linear operators, elementary Lebesgue theory and measure theory, etc.

Contents of analysis courses varies almost everywhere (a.e.). As you may see, the term analysis appear in the first and second year calculus to forth year analysis. Kolmogorov and Fomin's text fits this stage

well, they cover most of the above topics. Apostol, Marsden and Hoffman, and Simmons make good references. Several books entitled analysis are indeed advanced calculus (stage two calculus) or not aimed for mathematicians. Stay away from Mathematical analysis for business and finance or the like.

Only for metric spaces and topological spaces (also consult the list of Topology below):

- Burkill J.C. and Burkill H. *A Second Course in Mathematical Analysis*
- Giles J.R. *Introduction to the Analysis of Metric Spaces* - One of the Australian Mathematical Society Lecture Series, gives nice discussion on limit processes, continuity, compactness and metric topology.
- Searcóid M.Ó. *Metric Spaces*
- Sutherland W.A. *Introduction to Metric and Topological Spaces*

Also deals with Lebesgue theory, Banach spaces and Hilbert spaces, etc:

- Apostol T.M. *Mathematical Analysis* - If you enjoy his *Calculus Vol. 1* in stage 1 and *Calculus Vol. 2* in stage 2, then you should go for this. Solution by Shin-Yi Lee can be found here.
- Bass R.F. Real Analysis (Measure Theory) (FREE! http://www.math.uconn.edu/~bass/meas.pdf)
- Beals R. *Analysis: An Introduction* - It covers a lot of stuff and hence too concise for an introductory text, maybe. Treat this as a reference.
- Bruckner A.M., Bruckner J.B.and Thomson B.S. *Real Analysis* (FREE! http://classicalrealanalysis.com/download.aspx)
- Chen W.W.L. Introduction to Lebesgue Integration (FREE! http://www.maths.mq.edu.au/~wchen/lnilifolder/lnili.html)
- Chen W.W.L. Linear Functional Analysis (FREE! http://www.maths.mq.edu.au/~wchen/lnlfafolder/lnlfa.html)
- Cohen G.L. *A Course in Modern Analysis and its Applications*
- Davidson K.R. and Donsig A.P. *Real Analysis with Real Applications*
- Jost J. *Postmodern Analysis*
- Kolmogorov A.N. and Fomin S.V. *Elements of the Theory of Functions and Functional Analysis* - The material is based on lectures given by the authors in the Moscow State University.

  Kolmogorov A.N. and Fomin S.V. *Introductory Real Analysis* - This version is translated and edited by R.A. Silverman. It seems that this is a more popular edition. See the typo by Edgar (http://www.math.ohio-state.edu/~edgar/752_06/errata.pdf).
- Marsden J.E. and Hoffman M.J. *Elementary Classical Analysis* - This book sits somewhere between the stage 2 calculus and stage 3 analysis in this list. One feature is that proofs are often left behind examples.

- Rudin W. *Principles of Mathematical Analysis* - Baby Rudin. A modern classic. Get a copy and play around with it. See also George M. Bergman's supplement exercises (http://math.berkeley.edu/~gbergman/ug.hndts/m104_Rudin_exs.ps), notes (http://math.berkeley.edu/~gbergman/ug.hndts/m104_Rudin_notes.ps) and q&a (http://math.berkeley.edu/~gbergman/ug.hndts/06x2+03F_104_q+a.txt); and Evelyn Silvia's Companion Notes (http://www.math.ucdavis.edu/~emsilvia/math127/math127.html).
- Santos D. Real Analysis/Advanced Calculus (FREE! http://www.openmathtext.org/lecture_notes/david_santos_calculus1.pdf)
- Saxe K. *Beginning Functional Analysis* - Large proportion of this book can be studied at this stage.
- Simmons F.G. *Introduction to Topology and Modern Analysis* - Nice mathematical writing.
- Sternberg S. Advanced Calculus (FREE! http:/www.math.harvard.edu/~shlomo/docs/Advanced_Calculus.pdf)
- Tao T. Analysis 2 (FREE! http://www.math.ucla.edu/~tao/resource/general/131bh.1.03s/)
- Wilkins D.R. General Topology and Real Analysis (FREE! http://www.maths.tcd.ie/~dwilkins/Courses/221/)

Abstract Algebra:

One of the beautiful branches of mathematics. You will see how we study symmetry. Topics to be covered are basic group, ring and field theory.

Only discuss Groups:

- Armstrong M.A. *Groups and Symmetry* - 'Numbers measure size, groups measure symmetry,' said the author. This book deals with several topics, which introductory abstract algebra texts usually skip, like Platonic solid and wallpaper patterns.
- Baker A. Groups, Symmetry and Fractals (FREE! http://www.maths.gla.ac.uk/~ajb/2q/index.phtml)
- Milne J.S. Group Theory (FREE! http://www.jmilne.org/math/CourseNotes/math594g.html)

Rings and Fields:

- Chambert-Loir A. *A Field Guide to Algebra*
- Ellis G. *Rings and Fields*
- Mitchell J. Rings and Fields (FREE! http://www-history.mcs.st-and.ac.uk/~jamesm/teaching/MT4517/)
- Skorobogatov A.N. Rings and Fields Lecture Notes (FREE! http://www.ma.ic.ac.uk/~anskor/notesM2P4.pdf)

In general:

- Anderson M. and Feil T. *A First Course in Abstract Algebra: Rings, Groups, and Fields*
- Arapura D. Abstract Algebra Done Correctly (FREE! http://www.math.purdue.edu/~dvb/algebra/algebra.pdf)
- Artin M. *Algebra* - Not aimed at this level, but this is the one you like to keep in your own library. As M. Artin states in "A Note for the Teacher" that several sections would make a coherent course, you may simply follow his list at this stage. My lecturer recommended this partly because M. Artin was his supervisor, I recommend this but I have no direct relationship with M. Artin.
- Beachy J.A. and Blair W.D. *Abstract Algebra* Online Study Guide (FREE! http://www.math.niu.edu/~beachy/abstract_algebra/study_guide/contents.html)
- Birkhoff G. and Mac Lane S. *A Survey of Modern Algebra*
- Blomqvist C. Algebraic Systems (FREE! http://hemsidor.torget.se/users/m/mauritz/math/alg/index.htm)
- Chan D. Higher Algebra Lecture Notes (FREE! http://web.maths.unsw.edu.au/~danielch/algebra07a/qinnotes.pdf) - Diagram omitted, it's best to attend Daniel's lecture.
- Clark A. *Elements of Abstract Algebra* - Kind of handbook.
- Connell E.H. *Elements of Abstract and Linear Algebra* (FREE! http://www.math.miami.edu/~ec/book/)
- Dummit D.S. and Foote R.M. *Abstract Algebra* - If you could afford this. Standard recommended text in the U.S. Read sections recommended by the authors in the preface, leave others to the next stage. See Foote's web (http://www.emba.uvm.edu/~foote/) for errata.
- Fraleigh J.B. *A First Course in Abstract Algebra* - Fraleigh made it, he gives a good introduction abstract algebra for newbie. Ideal for a first coures. See also Afra's incomplete notes (http://www.cs.dartmouth.edu/~afra/goodies/abstract.pdf).
- Garrett P. Intro to Abstract Algebra (FREE! http://www.math.umn.edu/~garrett/m/intro_algebra/notes.pdf)
- Gilbert W.J. *Modern Algebra with Applications*
- Gillian J.A. *Contemporary Abstract Algebra* - As the author have said, this book includes 'lines from popular songs, poems, quotations, biographies, historical notes, hundreds of figures, dozens of photographs, and numerous tables and charts." If you prefer those things. See the author's website (http://www.d.umn.edu/~jgallian/) for resources.
- Goodman F.M. *Algebra: Abstract and Concrete* (FREE! http://www.math.uiowa.edu/~goodman/algebrabook.dir/algebrabook.html)
- Herstein I.H. *Abstract Algebra*
- Ikenaga B. Abstract Algebra (FREE! http://marauder.millersville.edu/~bikenaga/abstractalgebra/abstractalgebranotes.html)
- Jacobson N. *Basic Algebra I* - New edition of his *Lectures in Abstract Algebra*.

- Knapp A.W. *Basic Algebra* - See the web (http://www.math.sunysb.edu/~aknapp/books/b-alg.html) for correction.
- Rotman J.J. *A First Course in Abstract Algebra* - Check out Rotman's page (http://www.math.uiuc.edu/~rotman/) for errata.
- Shahriari S. *Lectures on Algebra I* (FREE! http://pages.pomona.edu/~sshahriari/shahalg1-16.pdf)
- 李華介：大學基礎代數 (FREE! http://math.ntnu.edu.tw/~li/algebra-html/) - The best free material on introductory abstract algebra I have seen so far, provided you can read Chinese.

Introductory Number Theory:

Gauss once said "mathematics is the queen of sciences and number theory the queen of mathematics." G..H. Hardy once said in *A Mathematician's Apology* that, "If the theory of numbers could be employed for any practical and obviously honourable purpose, if it could be turned directly to the furtherance of human happiness or the relief of human suffering, as physiology and even chemistry can, then surely neither Gauss nor any other mathematician would have been so foolish as to decry or regret such applications. But science works for evil as well as for good (and particularly, of course, in time of war); and both Gauss and less mathematicians may be justified in rejoicing that there is one science at any rate, and that their own, whose very remoteness from ordinary human activities should keep it gentle and clean." He wrote it on November 1940. It turned out coding theory, where algebra and number theory are applied, played a key role in the World War II.

- Adler A. and Cloury J.E. *Theory of Numbers: A Text and Source Book of Problems*
- Baker A. Algebra and Number Theory (FREE! http://www.maths.gla.ac.uk/~ajb/dvi-ps/3q-notes.pdf)
- Burton D.M. *Elementary Number Theory* - One of the better books for an introduction.
- Chen W.W.L. Elementary Number Theory (FREE! http://www.maths.mq.edu.au/~wchen/lnentfolder/lnent.html)
- Clark W.E. *Elementary Number Theory* (FREE! http://www.math.usf.edu/~eclark/elem_num_th_book.pdf)
- Coppel W.A. *Number Theory: An Introduction to Mathematics: Part A, B* - You may leave Part B to the next stage.
- Dantzig T. and Mazur J. *Number: The Language of Science* - Non-mathematical treatment indeed.
- Dirichlet P.G.L. Supplements by Dedekind R. *Lectures on Number Theory*
- Erdos P. and Suranyi J. *Topics in the Theory of Numbers*
- Everest G. and Ward T. *An Introduction to Number Theory*
- Fine B. and Rosenberger G. *Number Theory: An Introduction via the Distribution of Primes*
- Ikenaga B. Number Theory (FREE! http://marauder.millersville.edu/~bikenaga/numbertheory/numbertheorynotes.html)

- Joens G.A. and Jones J.M. *Elementary Number Theory*
- Moser L. *An Introduction to the Theory of Numbers* (FREE! http://www.trillia.com/moser-number.html)
- Nathanson M.B. *Elementary Methods in Number Theory*
- Niven I., Zuckerman H.S. and Montgomery H.L. *An Introduction to the Theory of Numbers*
- Parent D.P. *Exercises in Number Theory*
- Redmond D *Number Theory*
- Rosen K.T. *Elementary Number Theory* - See the website (http://www.aw-bc.com/rosen/) for supplementary material.
- Santos D. Number Theory Notes (FREE! http://www.openmathtext.org/lecture_notes/number_theory_book.pdf)
- Sato N. Number Theory (FREE! http://www.artofproblemsolving.com/Resources/Papers/SatoNT.pdf) - IMO level.
- Shoup V. *A Computational Introduction to Number Theory and Algebra* (FREE! http://www.shoup.net/ntb/)
- Silverman J.H. *A Friendly Introduction to Number Theory* - See the web (http://www.math.brown.edu/~jhs/frint.html) for extra material.
- Stein W. An Explicit Approach to Elementary Number Theory (FREE! http://modular.fas.harvard.edu/edu/Fall2001/124/lectures/)
- Stein W. *Elementary Number Theory* (FREE! http://modular.math.washington.edu/ent/)
- 李華介：基礎數論 (FREE! http://math.ntnu.edu.tw/~li/ent-html/)

Introductory Topology:

Topology is an extension of geometry and built on set theory, it's about continuity. There is a joke that topologists are those who cannot distinguish between a doughnut and a coffee or tea cup. Wikipedia has a [.gif] to demonstrate this (http://en.wikipedia.org/wiki/Image:Mug_and_Torus_morph.gif). You may have studied open sets, open neighbourhood, interior, closure, boundary, basis, continuity, compactness and connectedness etc. in the analysis course, move on to homemorphism, homotopy and fundamental group.

As in abstract algebra and analysis, several selected books or notes cover more than enough. You may leave a few sections to study at the next stage.

- Crossley M.D. *Essential Topology* - Undergraduate introductory level, detailed explanation.
- Davis S.W. *Topology* - It's like lecture notes, many proofs are left as exercises and many exercises are proofs.
- Dugundji J. *Topology* - Another topology text with quality.
- Engelking R. *General Topology*
- Gemignani M.C. *Elementary Topology*
- Henle M. *A Combinatorial Introduction to Topology* - Plenty of illustrations. Indeed many

concepts in this subject can and should be visualised.

- Ikenaga B. Notes on Topology (FREE! http://marauder.millersville.edu/~bikenaga/topology/topnote.html)
- McCluskey A. and McMaster B. Topology Course Lecture Notes (FREE! http://at.yorku.ca/i/a/a/b/23.htm)
- Mendelson B. *Introduction to Topology* - Very nice introduction to the metric spaces, topological spaces, connectedness and compactness. Recommended for first reading.
- Moller J.M. General Topology (FREE! http://www.math.ku.dk/~moller/e03/3gt/notes/gtnotes.pdf)
- Morris S.A. *Topology Without Tears* (FREE! http://uob-community.ballarat.edu.au/~smorris/topology.htm)
- Munkres J.R. *Topology* - Another nice introduction. Go further than Mendelson's one. Also cover topics like the Tychonoff theorem, metrization theorems, complete metric spaces and function spaces, the fundamental group and covering spaces.
- Singer I.M. and Thorpe J.A. *Lecture Notes on Elementary Topology and Geometry*
- Steen L.A. and Seebach J.A. *Counterexamples in Topology*
- Strickland N. Topology Notes (FREE! http://neil-strickland.staff.shef.ac.uk/courses/topology/)
- Thurston W.P. *The Geometry and Topology of Three-Manifolds* (FREE! http://msri.org/publications/books/gt3m/)
- Ward T.B. Topology Lecture Notes (FREE! http://www.mth.uea.ac.uk/~h720/teaching/topology/materials/topology.pdf)
- Wilkins D.R. Topology (FREE! http://www.maths.tcd.ie/~dwilkins/Courses/212/)

Differential Geometry:

You may wonder, geometry takes up a large portion in high school mathematics, why isn't there any geometry course in the first two stages? In fact, geometry is kind of imbedded in stage two calculus (several variables) and linear algebra courses, they are usually assumed and will be used for this course. Here differential calculus is used to study geometry. Key things to study: multilinear algebra, curvature and torsion, Serret-Frenet equation, fundamental theorem of curves, Poincaré Index theorem (plane and surface), exterior calculus, Gauss' *theorema egregium*, geodesics, Gauss-Bonnet theorem.

Moving frame, due to Élie Cartan, is an approach to geometry of surface. According to my lecturer John Steele, it is "computationally the easiest, notationally the neatest, aesthetically the best, makes the definitions more natural and the proofs of the two major theorems easier. The moving frame method also points the way towards several important ideas in modern differential geometry and theoretical physics. The downside (if there is one) is the reliance on exterior calculus of differential forms." O'Neill, for example, uses this approach and he manages to prove Gauss' *theorema egregium* in half page, see p.281.

In general:

- Animov Y. *Differential Geometry and Topology of Curves*
- Csikós B. Differential Geometry (FREE! http://www.cs.elte.hu/geometry/csikos/dif/dif.html)
- do Carmo M.P. *Differential Geometry of Curves and Surfaces* - Quite popular for introductory level. Beware that ^ means cross product, and <a, b> means a dot b or inner product in this text. Check out the errata list by Bjorn Poonen (http://math.berkeley.edu/~poonen).
- Hicks N.J. *Notes on Differential Geometry* (FREE! http://www.wisdom.weizmann.ac.il/~yakov/scanlib/hicks.pdf)
- Kreyszig E. *Differential Geometry* - Neither do Carmo nor O'Neill introduce the matrix notation when they first discuss the Frenet formulae, Kreyszig does that, which is nice.
- Millman R.S. and Parker G.D. *Elements of Differential Geometry*
- O'Neill B. *Elementary Differential Geometry*
- Pressley A. *Elementary Differential Geometry* - Solution at the back.
- Sharipov R. *Course of Differential Geometry* (FREE! http://arxiv.org/abs/math/0412421)
- Struik D.J. *Lectures on Classical Differential Geometry*
- Zaitsev D. Differential Geometry: Lecture Notes (FREE! http://www.maths.tcd.ie/~zaitsev/ln.pdf)

Tensor Analysis and Manifolds:

- Abraham R., Marsden J.E. and Ratiu T. Manifolds, Tensors, Analysis and Applications (FREE! http://www.cds.caltech.edu/oldweb/courses/2002-2003/cds202/textbook/)
- Bishop R.L. and Goldberg S.I. *Tensor Analysis on Manifolds*
- Lebedev L.P. and Cloud M.J. *Tensor Analysis*
- Munkres J.R. *Analysis on Manifolds*
- Sharipov R. *Quick Introduction to Tensor Analysis* (FREE! http://uk.arxiv.org/abs/math/0403252/)
- Spivak M. *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus* - Not as excellent as his books on single variable calculus and the five volumes differential geometry, still better than many authors.

Differential Forms:

- Arapura D. Introduction to Differential Forms (FREE! http://www.math.purdue.edu/~dvb/preprints/diffforms.pdf)
- Bachman D. *A Geometric Approach to Differential Forms*
- Cartan H. *Differential Forms* - Member of Bourbaki, get a taste of the so called French style.
- Darling R.W.R. *Differential Forms and Connections*
- do Carmo M.P. *Differential Forms and Applications*
- Flanders H. *Differential Forms with Applications to the Physical Sciences*

- Morita S. *Geometry of Differential Forms*

Mathematical Modelling (optional):

"All models are wrong, some are useful." George Box. Using mathematics to model real world phenomena is useful, but not the main concern of pure mathematician.

- Fowler A.C. An Introduction to Mathematical Modelling (FREE! http://www.maths.ox.ac.uk/~fowler/courses/o11intro/o11notes.pdf)
- Fowler A.C. Techniques in Applied Mathematics (FREE! http://www.maths.ox.ac.uk/~fowler/courses/tech/technotes.pdf)
- Gershenfeld N. *The Nature of Mathematical Modeling*
- Giordano F.R., Weir M.D. and Fox W.P. *A First Course in Mathematical Modeling*
- Meerschaert M.M. *Mathematical Modeling*
- Veeh J.A. Lectures Notes on Mathematical Modeling (FREE! http://javeeh.net/lecnotes/model.pdf)

Statistical Inference (optional):

Build upon stage 2 statistics, you will learn here the Cramer-Rao bound, uniform minimum variance unbiased estimators, Neyman-Pearson theory, Bayesian inference, basic bootstrap and robustness, introductory non-parametric (including the sign test, Wilcoxon signed rank test, McNemar's chi-square test, Wald-Wolfowitz runs test, Mann-Whitney U test, Kolmogorov-Smirnov two-samples test, Kruskal-Wallis analysis of ranks, Spearman's R and Kendal's Tau), etc. Note a few texts say they are aimed at graduated level, partly because people from other fields only learn this material in graduated schools, as a result the texts have to set up basic probability and statistics (you have done it, stage 2 stuff) for them.

- Ash R.B. Lectures on Statistics (FREE! http://www.math.uiuc.edu/~r-ash/Stat.html)
- Bain L.J. and Engelhardt M. *Introduction to Probability and Mathematical Statistics*
- Casella G. and Berger R.L. *Statistical Inference* - A standard text, sort of. Used widely in grad school. Solutions Manual available on web (http://www.math.ntnu.no/~ushakov/emner/ST2201/v07/files/sol.pdf).
- DeGroot M.H. and Schervish M.J. *Probability and Statistics* - Detail explanation. Common misconceptions are mentioned, for example: "Probability Zero Does Not Mean Impossible" (p.17) and Statistical Swindles (p.45). Check out the typo page (http://www.stat.cmu.edu/~mark/degroot/typos/typos.html).
- Garthwaite P.H., Jolliffe I.T. and Jones B. *Statistical Inference* - Made a few good comments, but perhaps too concise as a textbook. Can be served as a reference. Unlike the rest recommended texts for this course, stage 2 probability and statistics is the assumed knowledge for this text.

- Hogg R.V., McKean J.W. and Craig A.T. *Introduction to Mathematical Statistics* - Classical text modernised. If you are serious about Statistics. Quite professional, not so user friendly for people from other fields. 11 pages of errata (http://www.stat.wmich.edu/mckean/HMC/)... it's a pain for customers to correct such long list of typos.
- Lebanon G. Tutorial Notes (FREE! http://www.stat.purdue.edu/~lebanon/notes/) - Read S1 - S12.
- Lowry R. Concepts and Applications of Inferential Statistics (FREE! http://faculty.vassar.edu/lowry/webtext.html)
- Mood A.M., Graybill F.A. and Boes D.C. *Introduction to the Theory of Statistics* - As my lecturer Spiridon Penev have said, this text is a bit outdated but it contains useful exercise problems.
- Mukhopadhyay N. *Probability and Statistical Inference*
- Rice J.A. *Mathematical Statistics and Data Analysis* - The main feature of this book is that it contain lots of figures. Check the at: http://stat-www.berkeley.edu/~rice/Book3ed/index.html
- Roussas G.G. *A Course in Mathematical Statistics*
- Roussas G.G. *An Introduction to Probability and Statistical Inference*
- Sprott D.A. *Statistical Inference in Science*
- Wald A. *On the Principles of Statistical Inference* (FREE! http://projecteuclid.org/euclid.ndml/1175196978)
- Weber R. Statistics (FREE! http://www.statslab.cam.ac.uk/~rrw1/stats/notes.pdf)
- Young G.A. and Smith R.L. *Essentials of Statistical Inference* - A bit advanced for a first course, perhaps.

Probability and Stochastic (Random) Processes (optional):

Conditional expectation, Poisson process, Markov chains, renewal theory, queueing theory, reliability theory, Brownian motion and stationary processes. This topic is particularly useful for electrical and computer engineers, actuarial study, finance or things like that. See also left material from stage 2 probability. If you want to learn this with measure, see stage 4 probability list.

- Baclawski K. and Rota G.-C. *An Introduction to Probability and Random Processes* (FREE! http://www.ellerman.org/Davids-Stuff/Maths/Rota-Baclawski-Prob-Theory-79.pdf)
- Feller W. *An Introduction to Probability Theory and Its Applications Vol. 1* - "A colorful and rich introduction to probability theory and its applications" according to Karlin and Taylor, but it "is limited in that it deals only with discrete probabilities."
- Gray R.M. and Davisson L.D. *Introduction to Statistical Signal Processing* (FREE! http://ee.stanford.edu/~gray/sp.html)
- Grimmett G.R. and Stirzaker D.R. *Probability and Random Processes*
- Hsu H. *Schaum's Outline of Probability, Random Variables, and Random Processes*
- Karlin S. and Taylor H.M. *A First Course in Stochastic Processes*

- Krishnan V. *Probability and Random Processes*
- Lebanon G. Tutorial Notes (FREE! http://www.stat.purdue.edu/~lebanon/notes/) - Read R1 - R5.
- Mörters P. Lecture Notes on Probability Theory (FREE! http://people.bath.ac.uk/maspm/prob.ps)
- Papoulis A. *Probability, Random Variables and Stochastic Processes* - See their web (http://www.mhhe.com/engcs/electrical/papoulis/) for resources.
- Ross S.M. *Introduction to Probability Models* - Note this is an introduction to probability and stochastic processes, not an introduction to basic probability theory. This text could be quite hard to follow without knowing very basic probability theory. Also note that, no graph has been used when the author introduce Markov chains for the first time.
- Walrand J. Lecture Notes on Probability Theory and Random Processes (FREE! http://robotics.eecs.berkeley.edu/~wlr/126notes.pdf)
- Yates R.D. and Goodman D.J. *Probability and Stochastic Processes: An Friendly Introduction for Electrical and Computer Engineers*

Statistical Computing (optional):

These days, you can't do statistics without computer, in practice. R is a freeware (http://www.r-project.org/), S-PLUS uses the same language, so I recommend books of this kind.

- Dalgaard P. *Introductory Statistics with R* - The author is a biostatistician, so he includes the topic survival analysis in this introductory text.
- Everitt B.S. and Hothorn T. *A Handbook of Statistical Analyses Using R* - Many functions, data sets, analyses and examples from the book are available at http://cran.r-project.org/web/packages/HSAUR/index.html , in particular the section "an Introduction to R" is completely included free on web.
- Krause A. and Olson M. *The Basics of S and S-Plus*
- Kuhnert P. and Venables B. *An Introduction to R: Software for Statistical Modelling and Computing* (FREE! http://www.et.bs.ehu.es/~etptupaf/pub/R/Rlecturenotes.pdf)
- Ripley B.D. Introductory Guide to S-Plus (FREE! http://www.ats.ucla.edu/stat/SPLUS/library/sguide.pdf)
- Spector P. An Introduction to R (FREE! http://www.stat.berkeley.edu/users/rice/Stat230A/SpectorIntroR.pdf)
- Spector P. R Tutorial (FREE! http://www.stat.berkeley.edu/users/rice/Stat230A/SpectorRtutorial.pdf)
- Venables B. and Smith D.M. An Introduction to R (FREE! http://cran.r-project.org/doc/manuals/R-intro.pdf)
- Venables B. and Smith D.M. R 丁國徽譯：《R 導論》，《R 文檔》(FREE! http://www.biosino.org/pages/newhtm/r/tchtml/ and http://www.biosino.org/R/R-doc/)

- Venables W.N. and Ripley B.D. *Modern Applied Statistics with S* - More advanced topics are covered. See their web (http://www.stats.ox.ac.uk/pub/MASS4/) for extra material. See also B. Yakir's notes (http://www.ces.clemson.edu/~calvinw/MthSc885/yakirsplus.pdf).

Stage 4

Equivalent to the (fourth) honours year in Australia, or the basic graduated year in other places. I shall outline the key topics and give short lists of references, you may scout around to see what makes you interest. I'm far from being an expert, let me know if you have a suggestion! Note that at this stage, the levels or difficulties of books within a certain topic may vary a lot. A few subfields may be highly related, e.g. further reading in Number Theory and algebraic geometry in Geometry. Moreover, further reading should belong to stage 5.

Foundations and Discrete Mathematics:

Foundation:

- Courant R. and Robbins H. *What Is Mathematics? An Elementary Approach to Ideas and Methods*
- Mac Lane S. *Mathematics: Form and Function*
- Russell B. *The Principles of Mathematics* (FREE! http://fair-use.org/bertrand-russell/the-principles-of-mathematics/)
- Simpson S.G. Foundations of Mathematics (FREE! http://www.math.psu.edu/simpson/courses/math558/fom.pdf) - Interested reader may find his paper Logic and Mathematics (http://www.math.psu.edu/simpson/foundations.html) worth reading.
- Whitehead A.N. and Russell B. *Principia Mathematica* (FREE! http://name.umdl.umich.edu/AAT3201.0001.001)
  Whitehead A.N. and Russell B. *Principia Mathematica to *56* - First part of the landmark work.
- Wittgenstein L. *Remarks on the Foundations of Mathematics*

Logic, Set Theory, etc.:

- Ash C.J., Crossely J.N., Brickhill C.J., Stillwell J.C. and Williams N.H. *What is Mathematical Logic?*
- Barwise J. (ed.) *Handbook of Mathematical Logic* - Not for beginners, mind you.
- Bornat R. *Proof and Disproof in Formal Logic: An Introduction for Programmers* - Oxford Texts in Logic 2.
- Chiswell I. and Hodges W. *Mathematical Logic* - Oxford Texts in Logic 3.
- Church A. *Introduction to Mathematical Logic*
- Dixon P.G. Set Theory (FREE! http://www.peter-dixon.staff.shef.ac.uk/teaching/STDN.PDF)

- Hausdorff F. *Set Theory*
- Hedman S. *A First Course in Logic: An Introduction to Model Theory, Proof Theory, Computability, and Complexity* - Oxford Texts in Logic 1.
- Hrbacek K. and Jech T. *Introduction to Set Theory*
- Jech T. *Set Theory*
- Komjáth P. and Totik V. *Problems and Theorems in Classical Set Theory*
- Mac Lane S. and Moerdijk I. *Sheaves in Geometry and Logic: A First Introduction to Topos Theory*
- Manin Y.I. *A Course in Mathematical Logic*
- Rautenberg W. *A Concise Introduction to Mathematical Logic*
- Simpson S.G. Mathematical Logic (FREE! http://www.math.psu.edu/simpson/courses/math557/logic.pdf)
- Srivastava S.M. *A Course on Mathematical Logic*
- Suppes P. *Axiomatic Set Theory*
- Takeuti G. and Zaring W.M. *Introduction to Axiomatic Set Theory*
- Tourlakis G. *Lectures in Logic and Set Theory. Volume I: Mathematical Logic*
  Tourlakis G. *Lectures in Logic and Set Theory. Volume 2: Set Theory*
- Walicki M. Introduction to Logic (FREE! http://www.ii.uib.no/~michal/und/i227/book/book.pdf)

Graph Theory:

- Agnarsson G. and Greenlaw R. *Graph Theory: Modeling, Applications, and Algorithms*
- Bang-Jensen J. and Gutin G. *Digraphs: Theory, Algorithms and Applications* (FREE! http://www.cs.rhul.ac.uk/books/dbook/)
- Bollobas B. *Modern Graph Theory*
- Bondy J.A. and Murty U.S.R. *Graph Theory* (FREE! http://www.ecp6.jussieu.fr/pageperso/bondy/books/gtwa/gtwa.html) - Discuss in their blog (http://blogs.springer.com/bondyandmurty/).
- Diestel R. *Graph Theory* (FREE! http://www.math.uni-hamburg.de/home/diestel/books/graph.theory/)
- Harary F. *Graph Theory*
- Hell P. and Nesetril J. *Graphs and Homomorphisms* - See their web (http://www.cs.sfu.ca/~pavol/hombook.html) for errata.
- Scheinerman E.R. and Ullman D.H. *Fractional Graph Theory: A Rational Approach to the Theory of Graphs*
- Tutte W.T. *Graph Theory As I Have Known It*
- West D.B. *Introduction to Graph Theory*

Combinatorics:

- Bender E.A. and Williamson S.G. *Foundations of Combinatorics with Applications* (FREE! http://www.math.ucsd.edu/~ebender/CombText/)
- Berman G. *Introduction to Combinatorics*
- Bona M. *Combinatorics of Permutations*
- Kisacanin B. *Mathematical Problems and Proofs: Combinatorics, Number Theory, and Geometry*
- Lovasz L. *Combinatorial Problems and Exercises*
- Niven I.M. *Mathematics of Choice: Or, How to Count Without Counting*
- Ryser H. *Combinatorial Mathematics*
- Stanton D. and White *Constructive Combinatorics*
- Tucker A. *Applied Combinatorics*
- van Lint J.H. and Wilson R.M. *A Course in Combinatorics*
- Wilf H. *Generatingfunctionology* (FREE! http://www.math.upenn.edu/~wilf/DownldGF.html)

Cryptography, Coding and Information Theory (not so pure):
First stage introductory discrete mathematics and basic linear algebra are usual assumed knowledge.

- Ash R.B. *Information Theory* - An introductory text.
- Bellare M. and Rogaway P. Introduction to Modern Cryptography (FREE! http://www-cse.ucsd.edu/users/mihir/cse207/classnotes.html)
- Berlekamp E.R. *Algebraic Coding Theory*
- Coutinho S.C. *The Mathematics of Ciphers: Number Theory and RSA Cryptography*
- Goldreich O. The Foundations of Cryptography (FREE! http://www.wisdom.weizmann.ac.il/~oded/foc-drafts.html)
- Goldwasser S. and Bellare M. Lecture Notes on Cryptography (FREE! http://www-cse.ucsd.edu/~mihir/papers/gb.html)
- Hamming R.W. *Coding and Information Theory* - If you are serious about this field (I mean you are not merely interested in cracking ciphers like those the Zodiac Killer used), you must learn Hamming error correcting codes, this is the origin.
- Hill R. *A First Course in Coding Theory*
- Humphreys J.F. and Prest M.Y. *Numbers, Groups and Codes* - Friendlier than Roman.
- Johnson O. *Information Theory and the Central Limit Theorem*
- Koblitz N. *A Course in Number Theory and Cryptography*
- Lysyanskaya A. Cryptography and Cryptanalysis (FREE! http://courses.csail.mit.edu/6.875/fall01/index.html)
- MacKay D. *Information Theory, Inference, and Learning Algorithms* (FREE! http://www.inference.phy.cam.ac.uk/mackay/itila/) - Take a look, and give me a reason why

you shouldn't download it if you are studying this subject.

- Menezes A.J., van Oorschot P.C. and Vanstone S.A. *Handbook of Applied Cryptography* (FREE! http://www.cacr.math.uwaterloo.ca/hac/)
- Mollin R.A. *An Introduction to Cryptography*
- Pless V. *Introduction to the Theory of Error-Correcting Codes*
- Pretzel O. *Error-Correcting Codes and Finite Fields*
- Roman S. *Coding and Information Theory* - More advanced text, basic abstract algebra required.
- Salomaa A. *Public-Key Cryptography*
- Schneier B. *Applied Cryptography: Protocols, Algorithms, and Source Code in C* - See author's site (http://www.schneier.com/book-applied.html).
- Smith L.S. *Cryptography: The Science of Secret Writing* - Elementary, suitable for non-mathematicians.
- van Lint J.H. *Introduction to Coding Theory* - Algebra and probability required.
- Vaudenay S. *A Classical Introduction to Cryptography: Applications for Communications Security* - See the web (http://www.vaudenay.ch/crypto/) for extra material.

Analysis:

Materials listed in stage 3, which have not been covered there, may be studied here.

Functional Analysis:

- Belton A.C.R. Functional Analysis (FREE! http://www.ucc.ie/en/euclid/StaffProfiles/BeltonAlex/Teaching/)
- Conway J.B. *A Course in Functional Analysis*
- Eidelman Y., Milman V. and Tsolomitis A. *Functional Analysis: An Introduction*
- Garrett P. Functional Analysis (FREE! http://www.math.umn.edu/~garrett/m/fun/)
- Kreyszig E. *Introductory Functional Analysis with Applications*
- Riesz F. and Sz.-Nagy B. *Functional Analysis* - Riesz is a key figure in this field.
- Rudin W. *Functional Analysis*
- Schaefer H.H. with Wolff M.P. *Topological Vector Spaces*
- Serfaty S. Functional Analysis Notes (FREE! http://www.math.nyu.edu/phd_students/vilensky/Functional_Analysis.pdf)
- Shilov G.E. *Elementary Functional Analysis* - Republication of Mathematical Analysis Vol. II, MIT Press, Cambridge, Mass.
- Teschl G. Functional Analysis (FREE! http://www.mat.univie.ac.at/~gerald/ftp/book-fa/index.html)
- Treves F. *Topological Vector Spaces, Distributions and Kernels*
- Ward T.B. Functional Analysis Lecture Notes (FREE! http://www.mth.uea.ac.uk/~h720/teaching/functionalanalysis/materials/FAnotes.pdf)

- Wilde I.F. C*-algebras (FREE! http://www.mth.kcl.ac.uk/~iwilde/notes/calg/index.html)
- Wilde I.F. Functional Analysis (FREE! http://www.mth.kcl.ac.uk/~iwilde/notes/fa1/index.html)
  Wilde I.F. Functional Analysis (topological vector space version) (FREE! http://www.mth.kcl.ac.uk/~iwilde/notes/fa2/index.html)
- Yosida K. *Functional Analysis*

Measure Theory:

See also the probability section below.

- Bogachev V.I. *Measure Theory*
- Cohn D.L. *Measure Theory*
- Fremlin D.H. *Measure Theory* (FREE! http://www.essex.ac.uk/maths/staff/fremlin/mt.htm)
- Halmos P.R. *Measure Theory* - Another nice text by Halmos.
- Hjorth G. Measure Theory (FREE! http://www.math.ucla.edu/~greg/measure.html)
- Liskevich V. Measure Theory (FREE! http://www-maths.swan.ac.uk/staff/vl/c98.pdf) - Elementary notes.
- Oxtoby J.C. *Measure and Category: A Survey of the Analogies between Topological and Measure Spaces*
- Sattinger D.H. Measure Theory & Integration (FREE! http://www.math.usu.edu/~dhs/MeasTheory.pdf)
- Schilling R.L. *Measures, Integrals and Martingales* - A friendly introduction to these topics. Additional material, hints and solutions at http://www.motapa.de/measures_integrals_and_martingales/index.html

Hilbert Spaces:

- Halmos P.R. *A Hilbert Space Problem Book*
- Halmos P.R. *Introduction to Hilbert Space And the Theory of Spectral Multiplicity*
- Young N. *An Introduction to Hilbert Space*

Real and Complex Analysis:

- Ahlfors L. *Complex Analysis* - Classic in complex analysis.
- Ash R.B. and Novinger W.P. *Complex Variables* (FREE! http://www.math.uiuc.edu/~r-ash/CV.html)
- Baggett L. *Analysis of Functions of a Single Variable* (FREE! http://spot.colorado.edu/~baggett/.analysis/analysis.html)
- Bartle R.G. *A Modern Theory of Integration*
- Bear H.S. *A Primer of Lebesgue Integration*

- Conway J.B. *Functions of One Complex Variable I, II*
- Duistermaat J.J. and Kolk J.A.C. *Multidimensional Real Analysis I, II*
- Folland G.B. *Real Analysis: Modern Techniques and Their Applications*
- Freitag E. and Busam R. *Complex Analysis*
- Hahn L.-S. and Epstein B. *Classical Complex Analysis*
- Krantz S.G. *Function Theory of Several Complex Variables*
- Krantz S.G. *Partial Differential Equations and Complex Analysis*
- Kurtz D.S. and Swartz C.W. *Theories Of Integration: The Integrals Of Riemann, Lebesgue, Henstock-Kurzweil, and Mcshane*
- Lang S. *Real Analysis*
- Markushevich A.I. *Theory of Functions of a Complex Variable*
- Needham T. *Visual Complex Analysis* - Plenty of diagrams, as the title suggested, make this book beautiful. See the web for extra information.
- Pedersen G.K. *Analysis Now*
- Royden H.L. *Real Analysis* - Fairly good text.
- Rudin W. *Real and Complex Analysis* - Big Rudin!
- Scheidemann V. *Introduction to Complex Analysis in Several Variables*
- Stein E.M. and Shakarchi R. *Complex Analysis*
- Stein E.M. and Shakarchi R. *Real Analysis: Measure Theory, Integration, and Hilbert Spaces*
- Sternberg S. *Theory of Functions of Real Variable* (FREE! http://www.math.harvard.edu/~shlomo/docs/Real_Variables.pdf)
- Whitney H. *Geometric Integration Theory*

Fourier and Harmonic Analysis:

- Axler S., Bourdon P. and Ramey W. *Harmonic Function Theory* (FREE! http://www.axler.net/HFT.html)
- Brown J.W. and Churchill R.V. *Fourier Series and Boundary Value Problems*
- Dahlberg B.E.J. and Kenig C.E. Harmonic Analysis and Partial Differential Equations (FREE! http://www.math.chalmers.se/Math/Research/GeometryAnalysis/Lecturenotes/HAPE.ps)
- Dales H.G., Aiena P. Eschmeier J. Laursen K. and Willis G.A. *Introduction to Banach Algebras, Operators, and Harmonic Analysis*
- Hardy G.H. and Rogosinski W.W. *Fourier Series*
- Hewitt E. and Ross K.A. *Abstract Harmonic Analysis: Volume 1: Structure of Topological Groups. Integration Theory. Group Representations*
  Hewitt E. and Ross K.A. *Abstract Harmonic Analysis: Volume 2: Structure and Analysis for Compact Groups. Analysis on Locally Compact Abelian Groups*
- Howell K.B. *Principles of Fourier Analysis*
- Katznelson Y. *An Introduction to Harmonic Analysis*

- Stein E.M. and Shakarchi R. *Fourier Analysis: An Introduction*
- Stein E.M. *Harmonic Analysis*
- Tao T. Fourier Analysis 1, 2 (FREE! http://www.math.ucla.edu/~tao/247a.1.06f and http://www.math.ucla.edu/~tao/247b.1.07w)
- Varadhan S.R.S. Harmonic Analysis (FREE! http://www.math.nyu.edu/faculty/varadhan/harmonic.html)
- Wolff T. Lectures on Harmonic Analysis (FREE! http://www.math.ubc.ca/~ilaba/wolff/)

Algebra:

Materials mentioned in stage 3, which have not been covered there, can be studied at this stage, for example, Artin, Dummit and Foote.

My lecturer Daniel Chan's web page (http://web.maths.unsw.edu.au/~danielch/) contains lots of algebra stuffs.

In general:

- Artin E. Notes by A.A. Blank *Modern Higher Algebra (Galois Theory)*
- Ash R.B. Abstract Algebra: The Basic Graduate Year (FREE! http://www.math.uiuc.edu/~r-ash/Algebra.html)
- Bourbaki N. *Elements of Mathematics: Algebra I, II* - Nicolas Bourbaki is a group of French mathematician aimed to write rigour mathematic which based on set theory. Not recommended for a beginner, but if you like, search around for topics.
- Burris S.N. and Sankappanavar H.P. *A Course in Universal Algebra The Millennium Edition* (FREE! http://www.math.uwaterloo.ca/~snburris/htdocs/ualg.html)
- Cohn P.M. *Algebra, Vol 1, 2, 3*
- Garrett P. Abstract Algebra (FREE! http://www.math.umn.edu/~garrett/m/algebra/) - Over 400 pages.
- Grillet P.A. *Abstract Algebra*
- Herstein I.N. *Topics in Algebra*
- Hungerford T.W. *Algebra* - James Wilson has a solutions manual: http://www.uoregon.edu/~jwilson7/math/Hungerford.pdf.
- Isaccs I.M. *Algebra: A Graduate Course* - Non-standard treatment.
- Jacobson N. *Lectures in Abstract Algebra I, II, III* - Classic.
- Lang S. *Algebra* - My first Springer GTM. Only a few of us dare to ignore it. Ignoring it may look cool, but I'm not going to do so. To quote an Amazon customer, '[t]his will teach you how to run if you know how to walk.' Some readers think that if one wanna study algebraic number theory, one should be comfortable with Lang's *Algebra* first. See George M. Bergman's Companion (http://math.berkeley.edu/~gbergman/.C.to.L/).
- Smith R. Algebra Course Notes (FREE! http://www.math.uga.edu/~roy/)
- Spindler K. *Abstract Algebra with Applications Vol I, II*

- Surowski D. *Workbook in Higher Algebra* (FREE! http://www.math.ksu.edu/~dbski/book.pdf)
- Rotman J.J. *Advanced Modern Algebra* - See author's web (http://www.math.uiuc.edu/~rotman/) for errata.
- van der Waerden B.L. *Algebra: Volume I*, *II* - For historical interest, perhaps.
- Wickless W.J. *A First Graduate Course in Abstract Algebra*
- Wilkins D.R. Abstract Algebra (FREE! http://www.maths.tcd.ie/~dwilkins/Courses/311/)

Advanced Linear Algebra:

- Golan J.S. *The Linear Algebra - A Beginning Graduate Student Ought to Know*
- Greub W.H. *Linear Algebra*
- Matthews K.R. and Fama C. Linear Algebra Notes (FREE! http://www.numbertheory.org/courses/MP274/)
- Roman S. *Advanced Linear Algebra*

Groups and Lie Algebras:

- Alperin J.L. and Bell R.B. *Groups and Representations*
- Baker A. *Matrix Groups: An Introduction to Lie Group Theory*
- Chevalley C. *Theory of Lie Groups*
- Erdmann K. and Wildon M.J. *Introduction to Lie Algebras*
- Hall B.C. An Elementary Introduction to Groups and Representations (FREE! http://xxx.lanl.gov/abs/math-ph/0005032)
- Hall B.C. *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction* - See his web for information.
- Humphreys J.F. *A Course in Group Theory*
- Humphreys J.F. *Linear Algebraic Groups*
- Jacobson N. *Lie Algebras*
- Kurzweil H. and Stellmacher B. *The Theory of Finite Groups: An Introduction*
- Milne J.S. Algebraic Groups and Arithmetic Groups (FREE! http://www.jmilne.org/math/CourseNotes/aag.html)
- Lang S. *$SL_2(R)$*
- Rossmann W. *Lie Groups: An Introduction through Linear Groups*
- Rotman J.J. *An Introduction to the Theory of Groups*
- Schmidt O.U. *Abstract Group Theory*
- Sepanski M.R. *Compact Lie Groups*
- Serre J.-P. *Linear Representations of Finite Groups*
- Serre J.-P. *Trees* - More advanced work.
- Skorobogatov A.N. Lie Algebras Lecture Notes (FREE!

http://www.ma.ic.ac.uk/~anskor/LIE.PDF)

- Sternberg S. *Lie Algebras* (FREE! http://www.math.harvard.edu/~shlomo/docs/lie_algebras.pdf)
- Tauvel P. and Yu R.W.T. *Lie Algebras and Algebraic Groups*
- Varadarajan V.S. *Lie Groups, Lie Algebras, and Their Representation*
- Weyl H. *The Classical Groups: Their Invariants and Representations*

Rings, Fields and Galois Theory:

Note several stage 3 algebra texts also introduce Galois Theory. For example, J.B. Fraleigh's one.

- Artin E. *Galois Theory* (FREE! http://projecteuclid.org/euclid.ndml/1175197041)
- Baker A. Galois Theory (FREE! http://www.maths.gla.ac.uk/~ajb/dvi-ps/Galois.pdf)
- Borceux F. and Janelidze G. *Galois Theories*
- Edwards H.M. *Galois Theory*
- Escofier J.-P. *Galois Theory* - Assumed knowledge is standard algebra, i.e. stage 3 algebra above. Despite the nice treatment of the subject itself, the author also gives historical remarks for the interesting life of Galois.
- Ghorpade S.R. Lectures on Field Theory and Ramification Theory (FREE! http://www.math.iitb.ac.in/~srg/Lecnotes/isant.pdf)
- Ghorpade S.R. Notes on Galois Theory (FREE! http://www.math.iitb.ac.in/~srg/Lecnotes/galois.pdf)
- Hadlock C.R. *Field Theory and Its Classical Problems*
- Herstein I.N. *Noncommutative Rings*
- Howie J.M. *Fields and Galois Theory*
- Lam T.Y. *A First Course in Noncommutative Rings*
- Lam T.Y. *Exercises in Classical Ring Theory*
- Milne J.S. Fields and Galois Theory (FREE! http://www.jmilne.org/math/CourseNotes/math594f.html)
- Morandi P. *Field and Galois Theory*
- Roman S. *Field Theory*
- Stewart I. *Galois Theory* - Another well written book on this subject.
- Swallow J. *Exploratory Galois Theory*
- Weintraub S.H. *Galois Theory* - See author's website (http://www.lehigh.edu/~shw2/) for errata.
- 李華介：簡介 Galois 理論 (FREE! http://math.ntnu.edu.tw/~li/galois-html/)

Modules and Representation Theory:

- Fulton W. and Harris J. *Representation Theory: A First Course*

- Ghorpade S.R. and Verma J. K. Primary Decompositions of Modules (FREE! http://www.math.iitb.ac.in/~srg/Lecnotes/primary_decomp.pdf)
- Humphreys J.E. *Introduction to Lie Algebras and Representation Theory*
- Lam T.Y. *Lectures on Modules and Rings*
- Li W.C.W Lectures on Modular Forms and Galois Representations (FREE! http://math.cts.nthu.edu.tw/Mathematics/english/lecnotes/2006/Lectures on Modular Forms and Galois Representations.pdf)
- Woit P. Lie Groups and Representations (FREE! http://www.math.columbia.edu/~woit/repthy.html)

Commutative Algebra:

- Ash R.B. A Course In Commutative Algebra (FREE! http://www.math.uiuc.edu/~r-ash/ComAlg.html)
- Atiyah M.F. and MacDonald I.G. *Introduction to Commutative Algebra* (http://modular.math.washington.edu/scans/252/Atiyah-MacDonald/index.html a scanned version for FREE!) - Solutions by Jinhyun: http://www.math.purdue.edu/~jinhyun/sol2/comm.html
- Cox D.A., Little J.B. and O'Shea D.B. *Ideals, Varieties, and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra* - See their web (http://www.cs.amherst.edu/~dac/iva.html) for typo, etc.
- Eisenbud D. *Commutative Algebra: with a View Toward Algebraic Geometry*
- Fesenko I. Introduction to Commutative Algebra (FREE! http://www.maths.nott.ac.uk/personal/ibf/als3/leno.pdf)
- Ghorpade S.R. A Quick Review of Commutative Algebra (FREE! http://www.math.iitb.ac.in/~srg/Lecnotes/commalg.pdf)
- Ghorpade S.R. Lectures on Commutative Algebra (FREE! http://www.math.iitb.ac.in/~srg/Lecnotes/AfsPuneLecNotes.pdf)
- Reid M. *Undergraduate Commutative Algebra*
- Zariski O. and Samuel P. *Commutative Algebra*

Homological Algebra and Category:

- Adámek J., Herrlic H. and Strecker G.E. *Abstract and Concrete Categories: The Joy of Cats* (FREE! http://katmat.math.uni-bremen.de/acc/)
- Awodey S. *Category Theory*
- Barr M. and Wells C. *Toposes, Triples and Theories* (FREE! http://www.case.edu/artsci/math/wells/pub/ttt.html)
- Cartan H. and Eilenberg S. *Homological Algebra*

- Fesenko I. Introduction to Homological Algebra (FREE! http://www.maths.nott.ac.uk/personal/ibf/ha/ha.pdf)
- Fokkinga M.M. A Gentle Introduction to Category Theory: The Calculational Approach (FREE! http://wwwhome.cs.utwente.nl/~fokkinga/mmf92b.html)
- Freyd P. *Abelian Categories* (FREE! http://www.tac.mta.ca/tac/reprints/articles/3/tr3abs.html) - This site (http://www.tac.mta.ca/tac/reprints/index.html) consists dozens of Reprints in Theory and Applications of Categories for further reading.
- Hilton P.J. and Stammbach U. *A Course in Homological Algebra*
- Kashiwara M. and Schapira P. *Categories and Sheaves*
- Lawvere F.W. and Schanuel S.H. *Conceptual Mathematics: A First Introduction to Categories*
- Mac Lane S. *Categories for the Working Mathematician*
- Mac Lane S. *Homology*
- Rotman J.J. *Introduction to Homological Algebra*
- Weibel C.A. *An Introduction to Homological Algebra* – Correction: http://www.math.rutgers.edu/~weibel/Hbook-corrections.html Solution by Jinhyun: http://www.math.purdue.edu/~jinhyun/sol2/weibel.html

Number Theory:

There are several branch of Number Theory and we focus in algebraic and analytic here. As the title indicated, algebraic number theory employs algebra result to number theory, analytic one applies analysis theory. It is recommended you have a solid background of algebra before you begin to study algebraic number theory. Riemann hypothesis is one of the Millennium Prize Problems. Fermat's last theorem and the Poincaré conjecture have been solved, this is probably the biggest challenge to date. http://www.numbertheory.org/ntw/lecture_notes.html consists links to online number lecture notes. Algebraic Number Theory Archives (http://www.math.uiuc.edu/Algebraic-Number-Theory/) provide papers for further study.

In general:

- Baker A. *A Concise Introduction to the Theory of Numbers*
- Cohen H. *Number Theory: Volume I: Tools and Diophantine Equations*
  Cohen H. *Number Theory: Volume II: Analytic and Modern Tools* - See his web (http://www.math.u-bordeaux1.fr/~cohen/) for errata list.
- Crandall R. and Pomerance C. *Prime Numbers: A Computational Perspective*
- Guy R.K. *Unsolved Problems in Number Theory*
- Hardy G.H. and Wright E.M. *An Introductory to the Theory of Numbers* - A must have for this field.
- Ireland K. and Rosen M. *A Classical Introduction to Modern Number Theory*
- Manin Y.I. and Panchishkin A.A. *Introduction to Modern Number Theory: Fundamental*

Problems, Ideas and Theories

- Rademacher H. *Lectures on Elementary Number Theory*
- Serre J.-P. *A Course in Arithmetic* - English translation of *Cours d'arithmétique*.
- Weil A. *Basic Number Theory*

Algebraic Number Theory:

- Achter J. and Chai C.L. Algebraic Number Theory (FREE! http://www.math.upenn.edu/~chai/coursenotes.html)
- Alaca S. and Williams K.S. *Introductory Algebraic Number Theory*
- Ash R.B. A Course In Algebraic Number Theory (FREE! http://www.math.uiuc.edu/~r-ash/ANT.html)
- Chai C.-L. Algebraic Number Theory and Introduction to Automorphic L-functions (FREE! http://www.math.upenn.edu/~chai/coursenotes.html)
- Chapman R. Algebraic Number Theory (FREE! http://www.secamlocal.ex.ac.uk/people/staff/rjchapma/notes/ant2.pdf)
- Chapman R. Notes on Algebraic Numbers (FREE! http://www.secamlocal.ex.ac.uk/people/staff/rjchapma/notes/algn.pdf)
- Cohen H. *A Course in Computational Algebraic Number Theory* - See his web (http://www.math.u-bordeaux1.fr/~cohen/) for errata list.
- Cohn H. *Advanced Number Theory*
- Fesenko I. Introduction to Algebraic Number Theory (FREE! http://www.maths.nott.ac.uk/personal/ibf/aln/aln.pdf)
- Ghorpade S.R. Lectures on Topics in Algebraic Number Theory (FREE! http://www.math.iitb.ac.in/~srg/Lecnotes/KielNotes.pdf)
- Gross D. Algebraic Number Theory Notes (FREE! http://www.math.utah.edu/~sather/notes.html)
- Lang S. *Algebraic Number Theory*
- Milne J.S. Algebraic Number Theory (FREE! http://www.jmilne.org/math/CourseNotes/math676.html)
- Mollin R.A. *Algebraic Number Theory*
- Murty R.M. and Esmonde J. *Problems in Algebraic Number Theory*
- Neukirch J. *Algebraic Number Theory*
- Pollard H. and Diamond H.G. *The Theory of Algebraic Numbers*
- Reid L.W. *The Elements of the Theory of Algebraic Numbers*
- Stein W. A Brief Introduction To Classical and Adelic Algebraic Number Theory (FREE! http://modular.math.washington.edu/papers/ant/)
- Stewart I. and Tall D. *Algebraic Number Theory and Fermat's Last Theorem*
- Swinnerton-Dyer H. P. F. *A Brief Guide to Algebraic Number Theory*

- Weyl H. *Algebraic Theory of Numbers*

Class Field Theory:

- Artin E. and Tate J. *Class Field Theory*
- Fesenko I.B. and Vostokov S.V. *Local Fields and Their Extensions* (FREE! http://www.maths.nott.ac.uk/personal/ibf/book/book.html)
- Gras G. *Class Field Theory: From Theory to Practice*
- Milne J.S. Class Field Theory (FREE! http://www.jmilne.org/math/CourseNotes/math776.html)
- Neukirch J. *Class Field Theory*
- Serre J.-P. *Algebraic Groups and Class Fields*
- Serre J.-P. *Local Fields*

Analytic Number Theory:

- Apostol T.M. *Introduction to Analytic Number Theory* - Another nice book by Apostol.
- Bateman P.T. and Diamond H.G. *Analytic Number Theory: An Introductory Course*
- Chandrasekharan K. *Introduction to Analytic Number Theory*
- Chen W.W.L. Distribution of Prime Numbers (FREE! http://www.maths.mq.edu.au/~wchen/lndpnfolder/lndpn.html)
- Davenport H. and Montgomery H.K. *Multiplicative Number Theory*
- Elkies N.D. An Introduction to Analytic Number Theory (FREE! http://www.math.harvard.edu/~elkies/M259.98/index.html)
- Everest G. Analytic Number Theory (FREE! http://www.mth.uea.ac.uk/~h090/antmain.pdf)
- Hildebrand A.J. Introduction to Analytic Number Theory (FREE! http://www.math.uiuc.edu/~hildebr/ant/)
- Kedlaya K.S. Analytic Number Theory (FREE! http://www-math.mit.edu/~kedlaya/18.785/calendar.html)
- Knopfmacher J. *Abstract Analytic Number Theory* - He developed this from analytic number theory and applied it to other areas.
- Martin G. and Leung D. Analytic Number Theory (FREE! http://www.math.sfu.ca/~desmondl/ANT Notes with Greg.pdf)
- Murty M.R. *Problems in Analytic Number Theory*
- Newman D.J. *Analytic Number Theory*
- Stopple J. *A Primer of Analytic Number Theory: From Pythagoras to Riemann*
- Tijdeman R. Combinatorial and Analytic Number Theory (FREE! http://www.math.leidenuniv.nl/~tijdeman/cant.pdf)

Riemann Zeta Function and the Hypothesis:

- Borwein P., Choi S., Rooney B. and Weirathmueller A. *The Riemann Hypothesis: A Resource for the Afficionado and Virtuoso Alike*
- Edwards H.M. *Riemann's Zeta Function*
- Koblitz N. *p-adic Numbers, p-adic Analysis, and Zeta-Functions*
- Lapidus M.L. and van Frankenhuijsen M. *Fractal Geometry, Complex Dimensions and Zeta Functions: Geometry and Spectra of Fractal Strings* - More advanced work.
- Mazur B. and Stein W. What is Riemann's Hypothesis? (FREE! http://modular.math.washington.edu/edu/2007/simuw07/notes/rh.pdf) - See Stein's page (http://modular.math.washington.edu/edu/2007/simuw07/index.html) for extra material, like Riemann's paper.
- Patterson S.J. *An Introduction to the Theory of the Riemann Zeta-Function*
- Rockmore D. *Stalking the Riemann Hypothesis: The Quest to Find the Hidden Law of Prime Numbers*
- Titchmarsh E.C. *The Theory of the Riemann Zeta-Function*

Further readings (including Modular Forms, Elliptic Curves, etc.):
See Franz Lemmermeyer's page (http://www.rzuser.uni-heidelberg.de/~hb3/LN/ellc-intro.html) for further study.

- Apostol T.M. *Modular Functions and Dirichlet Series in Number Theory*
- Armitage J.V. and Eberlein W.F. *Elliptic Functions*
- Baker A. *Transcendental Number Theory*
- Borwein P. *Computational Excursions in Analysis and Number Theory*
- Borwein J.M. and Borwein P.B. *PI and the AGM: A Study in Analytic Number Theory and Computational Complexity*
- Cassels J.W.S. and Frohlich A. *Algebraic Number Theory: Proceedings of an Instructional Conference Organized by the London Mathematical Society* (http://modular.math.washington.edu/scans/252/cassels-frohlich/ a scanned version for FREE!)
- Diamon F. and Shurman J. *A First Course in Modular Forms*
- Fesenko I. Lectures on Complete Discrete Valuation Fields (FREE! http://www.maths.nott.ac.uk/personal/ibf/lf/lf.pdf)
- Greaves G. *Sieves in Number Theory*
- Husemöller D. *Elliptic Curves*
- Knapp A.W. *Elliptic Curves* (http://modular.math.washington.edu/scans/252/Knapp/index.html a scanned version for FREE!)

- Knopp M.I. *Modular Functions in Analytic Number Theory*
- Koblitz N.I. *Introduction to Elliptic Curves and Modular Forms*
- Lang S. *Elliptic Functions*
- Lang S. *Introduction to Modular Forms* (http://modular.math.washington.edu/scans/252/lang-intro_modform/ a scanned version for FREE!)
- Lang S. (ed.) *Number Theory III: Diophantine Geometry* (http://modular.math.washington.edu/scans/252/lang-nt3/ a scanned version for FREE!)
- Miyake T. *Modular Forms* (http://modular.math.washington.edu/scans/252/Miyake/index.html a scanned version for FREE!)
- Sándor J., Mitrinovic D.S. and Crstici B. *Handbook of Number Theory I, II*
- Shimura G. *Abelian Varieties with Complex Multiplication and Modular Functions*
- Shimura G. *Elementary Dirichlet Series and Modular Forms* - Has a supplementary nature to his *Introduction to Arithmetic Theory of Automorphic Functions*, to some extend, as he says.
- Shimura G. *Introduction to Arithmetic Theory of Automorphic Functions* (http://modular.math.washington.edu/scans/252/Shimura-Intro/index.html a scanned version for FREE!)
- Silverman J.H. *Advanced Topics in the Arithmetic of Elliptic Curves*
- Silverman J.H. *The Arithmetic of Elliptic Curves*
- Silverman J.H. and Tate J. *Rational Points on Elliptic Curves*

Geometry:

Just like number theory, algebraic geometry applies algebra result to study geometry, blah, blah, blah. Algebraic geometry is related to algebra and number theory.

See Geometry & Topology Monographs (http://www.msp.warwick.ac.uk/gtm/) for further study.

In general:

- Berger M., Pansu P., Berry J.-P. and Saint-Raymond X. *Problems in Geometry*
- Bredon G.E. *Topology and Geometry*
- Eves H. *Survey of Geometry*
- Coxeter H.S.M. *Introduction to Geometry*
- Hartshorne R. *Geometry: Euclid and Beyond* - He wrote an excellent text on algebraic geometry. Begin with this, see if you are with him.
- Hilbert D. *The Foundations of Geometry* (FREE! http://www.gutenberg.org/etext/17384)
- Hilbert D. and Cohn-Vossen S. *Geometry and the Imagination*
- Hvidsten M. *Geometry with Geometry Explorer* - See the web (http://homepages.gac.edu/~hvidsten/geom-text/) for extra material.
- Nikulin V.V. and Shafarevich I.R. *Geometries and Groups*

Algebraic Geometry:

- Bump D. *Algebraic Geometry*
- Dolgachev I.V. Introduction to Algebraic Geometry (FREE! http://www.math.lsa.umich.edu/~idolga/AG1.pdf)
- Dolgachev I.V. Topics in Classical Algebraic Geometry (FREE! http://www.math.lsa.umich.edu/~idolga/topics1.pdf)
- Drozd Y. *Introduction to Algebraic Geometry* (FREE! http://bearlair.drozd.org/~yuriy/ageng.pdf)
- Ewald G. *Combinatorial Convexity and Algebraic Geometry*
- Fulton W. *Algebraic Curves: An Introduction to Algebraic Geometry*
- Griffiths P. and Harris J. *Principles of Algebraic Geometry*
- Harris J. *Algebraic Geometry: A First Course*
- Hartshorne R. *Algebraic Geometry* (http://modular.math.washington.edu/scans/252/hartshorne/ a scanned version for FREE!) - Solutions by William Stein (http://modular.fas.harvard.edu/AG.html), Richard Borcherds (http://math.berkeley.edu/~reb/courses/alggeom/), Jinhyun Park (http://www.math.purdue.edu/~jinhyun/sol2/hart.html) and Chris (http://lomont.org/Math/Solutions.pdf).
- Hassett B. *Introduction to Algebraic Geometry*
- Lang S. *Introduction to Algebraic Geometry*
- Lefschetz S. *Algebraic Geometry*
- Milne J.S. Algebraic Geometry (FREE! http://www.jmilne.org/math/CourseNotes/math631.html)
- Reid M. *Undergraduate Algebraic Geometry*
- Skorobogatov A.N. Algebraic Geometry Lecture Notes (FREE! http://www.ma.ic.ac.uk/~anskor/AG.PDF)
- Smith K.E., Kahanpää L., Kekäläinen P. and Traves W. *An Invitation to Algebraic Geometry*
- Ueno K. *An Introduction to Algebraic Geometry*
- Walker R.J. *Algebraic Curves*

Further Algebraic Geometry:

- Basu S., Pollack R. and Roy M.-F. *Algorithms in Real Algebraic Geometry*
- Bochnak J., Coste M. and Roy M.-F. *Real Algebraic Geometry*
- Cox D.A., Little J.B. and O'Shea D.B. *Using Algebraic Geometry* - Don't forget to check out the typological error at http://www.cs.amherst.edu/~dac/uag.html.
- Debarre O. *Higher-Dimensional Algebraic Geometry*
- Eisenbud D. *The Geometry of Syzygies: A Second Course in Algebraic Geometry and*

Commutative Algebra

- Hirzebruch F. *Topological Methods in Algebraic Geometry*
- Lang S. *Abelian Varieties* (http://modular.math.washington.edu/scans/252/lang-abvar/ a scanned version for FREE!)
- Lang S. *Introduction to Algebraic and Abelian Functions* (http://modular.math.washington.edu/scans/252/lang-algab/ a scanned version for FREE!)
- Le Potier J. *Lectures on Vector Bundles*
- Liu Q. *Algebraic Geometry and Arithmetic Curves*
- Mumford D. *Abelian Varieties* (http://modular.math.washington.edu/scans/252/mumford-abvar/ a scanned version for FREE!)
- Reid M. Chapters on Algebraic Surfaces (FREE! http://arxiv.org/abs/alg-geom/9602006)
- Semple J.G. and Kneebone G.T. *Algebraic Projective Geometry*
- Ueno K. *Algebraic Geometry 1: From Algebraic Varieties to Schemes*
  Ueno K. *Algebraic Geometry 2: Sheaves and Cohomology*
  Ueno K. *Algebraic Geometry 3: Further Study of Schemes*
- Voisin C. *Hodge Theory and Complex Algebraic Geometry Vol 1*, *2*
- Yang K. *Complex Algebraic Geometry*

Differential Geometry:

Also consult mathematical physics section below.

- Chern S.S. Chen W.H. and Lam K.S. *Lectures on Differential Geometry* - Basic topological concspts assumed. A chapter on Finsler geometry is added in the English translation.
  陳省身，陳維桓：《微分幾何講義》，北京大學出版社
- Helgason S. *Differential Geometry, Lie Groups, and Symmetric Spaces*
- Hwang A.D. Complex Manifolds and Hermitian Differential Geometry (FREE! http://cs.holycross.edu/~ahwang/print/HDG.pdf)
- Ivey T.A. and Landsberg J.M. *Cartan for Beginners: Differential Geometry Via Moving Frames and Exterior Differential Systems*
- Kennington A.U. *Differential Geometry Reconstructed: A Unified Systematic Framework* (FREE! http://www.topology.org/tex/conc/dg.html) - Comprehensive.
- Kobayashi S. and Nomizu K. *Foundations of Differential Geometry Vol 1*, *2*
- Kobayashi S. *Transformation Groups in Differential Geometry*
- Lang S. *Fundamentals of Differential Geometry*
- Lang S. *Introduction to Differentiable Manifolds*
- Lee J.M. *Introduction to Topological Manifolds*
- Madore K. *An Introduction to Noncommutative Differential Geometry and its Physical Applications* - More advanced work.

- Michor P.W. Topics in Differential Geometry (FREE! http://www.mat.univie.ac.at/~michor/dgbook.pdf)
- Montiel S. and Ros A. *Curves and Surfaces*
- Schoen R. and Yau S.T. *Lectures on Differential Geometry*
  丘成桐，孫理察：《微分幾何講義》，高等教育出版社
- Spivak M. *A Comprehensive Introduction to Differential Geometry 1*, *2*, *3*, *4*, *5* - An epic. Read the first two volumes at least. Content can be found at: http://www.mathpop.com/bookhtms/dg.htm
- Warner F.W. *Foundations of Differentiable Manifolds and Lie Groups* (FREE! http://www.wisdom.weizmann.ac.il/~yakov/scanlib/Warner.djvu)

Riemannian Geometry:

- Boothby W.M. *An Introduction to Differentiable Manifolds and Riemannian Geometry* (FREE! http://www.wisdom.weizmann.ac.il/~yakov/scanlib/Boothby.djvu)
- Chavel I. *Riemannian Geometry: A Modern Introduction* - Check the correction and addition (http://math.gc.cuny.edu/faculty/chavel/CORR-R.pdf).
- Chern, S.S. and Shen Z. *Riemann-Finsler Geometry*
- do Carmo M.P. *Riemannian Geometry*
- Gallot S., Hulin D. and Lafontaine J. *Riemannian Geometry*
- Gudmundsson S. An Introduction to Riemannian Geometry (FREE! http://www.matematik.lu.se/matematiklu/personal/sigma/Lecture-Notes.html)
- Jost J. *Riemannian Geometry and Geometric Analysis*
- Lang S. *Differential and Riemannian Manifolds*
- Moller J.M. Riemannian Geometry (FREE! http://www.math.ku.dk/~moller/f05/genotes.pdf)
- O'Neill B. *Semi-Riemannian Geometry With Applications to Relativity*
- Petersen P. *Riemannian Geometry*
- Wilkins D.R. A Course in Riemannian Geometry (FREE! http://www.maths.tcd.ie/~dwilkins/Courses/425/RiemGeom.pdf)
- 伍鴻熙、沈純理、虞言林：《黎曼幾何初步》，北京大學出版社

Fractals:

- Barnsley M.F. *Fractals Everywhere*
- Barnsley M.F. *SuperFractals* - See the web (http://www.superfractals.com/).
- Edgar G.A. *Measure, Topology, and Fractal Geometry* - See author's page (http://www.math.ohio-state.edu/~edgar/books/mtfg.html) for errata.
- Falconer K. *Fractal Geometry: Mathematical Foundations and Applications*
- Mandelbrot B.B. *The Fractal Geometry of Nature*

- Mörters P. Lecture Notes on Fractal Geometry (FREE! http://people.bath.ac.uk/maspm/fract.ps)
- Peitgen H.-O., Jürgens H. and Saupe D. *Chaos and Fractals: New Frontiers of Sciences* (FREE! http://www.springerlink.com/content/978-0-387-21823-6/)

Further readings:

- Audin M. *Geometry*
- Gruber P.M. *Convex and Discrete Geometry*
- Huybrechts D. *Complex Geometry: An Introduction*
- Jennings G.A. *Modern Geometry with Applications*
- Matousek J. *Lectures on Discrete Geometry*

Topology:

See Geometry & Topology Monographs (http://www.msp.warwick.ac.uk/gtm/) for further study.

In general:

- Hocking J.G. and Young G.S. *Topology*
- Kelley J.L. *General Topology* - "What every young analyst should know" was the originally intended title. Morse-Kelley set theory was introduced in this book.
- Steenrod N. *The Topology of Fibre Bundles*
- Stillwell J. *Classical Topology and Combinatorial Group Theory*

Geometric Topology:

- Firby P.A. and Gardiner C.F. *Surface Topology* - One quote by Ian Stewart is sufficient. "This book is in the top ten for books conveying the dreadful secret that mathematics is fascinating."
- Gay D. *Explorations in Topology: Map Coloring, Surfaces and Knots*
- Ranicki A. History of Knot Theory (FREE! http://www.maths.ed.ac.uk/~aar/knots/index.htm) - Contains dozens of historical material.
- Roberts J.D. Knots Knotes (FREE! http://math.ucsd.edu/~justin/Papers/knotes.pdf)

Algebraic Topology:

- Bott R. and Tu L.W. *Differential Forms in Algebraic Topology*
- Dold A. *Lectures on Algebraic Topology*
- Eilenberg S. and Steenrod N.E. *Foundations of Algebraic Topology*
- Fulton W. *Algebraic Topology: A First Course*
- Hatcher A. *Algebraic Topology* (FREE!

http://www.math.cornell.edu/~hatcher/AT/ATpage.html) - Very beautiful book.

- Lefschetz S. *Algebraic Topology* (FREE! http://www.ams.org/online_bks/coll27/) - Lefschetz developed new result in this monograph.
- Maunder C. R. F. *Algebraic Topology*
- Massey W.S. *A Basic Course in Algebraic Topology*
- Massey W.S. *Algebraic Topology: An Introduction*
- Mislin G. Algebraic Topology (FREE! http://shamir.vmp.ethz.ch/svnbuild/math/at/0506-ws/at.pdf)
- Moller J.M. Algebraic Topology Notes (FREE! http://www.math.ku.dk/~moller/f04/algtop/AlgTopnotes.html)
- Munkres J.R. *Elements of Algebraic Topology*
- Switzer R.M. *Algebraic Topology - Homology and Homotopy*
- Vick J.W. *Homology Theory: An Introduction to Algebraic Topology*
- Wallace A.H. *Algebraic Topology: Homology and Cohomology*
- Wallace A.H. *An Introduction to Algebraic Topology*
- Whitehead G.H. *Elements of Homotopy Theory*
- Wilkins D.R. Algebraic Topology (FREE! http://www.maths.tcd.ie/~dwilkins/Courses/421/)

K-theory:

- Hatcher A. *Vector Bundles and K-theory* (FREE! http://www.math.cornell.edu/~hatcher/VBKT/VBpage.html)
- Karoubi M. *K-theory: An Introduction*
- Rosenberg J. *Algebraic K-Theory and Its Applications*
- Weibel C. *The K-book: An Introduction to Algebraic K-theory* (FREE! http://math.rutgers.edu/~weibel/Kbook.html)

Differential Topology:

- Brin M.G. Introduction to Differential Topology (FREE! ftp://ftp.math.binghamton.edu/pub/matt/diff.pdf or ftp://ftp.math.binghamton.edu/pub/matt/diff.ps.gz)
- Guillemin V. and Pollack A. *Differential Topology*
- Hirsch M.W. *Differential Topology*
- Kosinski A.A. *Differential Manifolds* - An appendix by John W. Morgan on the work of Grigory Perelman is included in the 2007 Dover edition.
- Lang S. *Introduction to Differentiable Manifolds*
- Milnor J. *Morse Theory* - More advanced work, influential. Notes taken by M. Spivak and R. Wells.

- Milnor J.W. *Topology from the Differentiable Viewpoint* - As an introduction.
- Wallace A.H. *Differential Topology: First Steps*
- 張筑生：《微分拓樸》

Further Calculus (optional):

Ordinary Differential Equations:

- Aronld V.I. *Ordinary Differential Equations*
- Ince E.L. *Ordinary Differential Equations*
- Miller R.K. and Michel A.N. *Ordinary Differential Equations*
- Tenenbaum M. and Pollard H. *Ordinary Differential Equations*

Partial Differential Equation:

Differential equation with several variables, i.e. equations with partial derivatives. Particularly useful in physics, e.g. heat, electrostatics, electrodynamics, etc.

- Arnold V.I. *Lectures on Partial Differential Equations*
- Bass R.F. PDE From a Probability Point of View (FREE! http://www.math.uconn.edu/~bass/pdeprob.pdf)
- Driver B. *Analysis Tools with Applications* and PDE Notes (FREE! http://math.ucsd.edu/~driver/231-02-03/lecture_notes.htm)
- Evans G., Blackledge J. and Yardley P. *Analytic Methods for Partial Differential Equations*
- Evans L.C. Entropy and Partial Differential Equations (FREE! http://math.berkeley.edu/~evans/entropy.and.PDE.pdf)
- Evans L.C. *Partial Differential Equations* - As Evans emphasise, PDE is not a branch of functional analysis; many interesting equations are non-linear; understanding generalised solutions is fundamental; good theory is as useful as exact formulas; and this is a textbook, not a reference book. Check out his page (http://math.berkeley.edu/~evans/) for errata.
- Jost J. *Partial Differential Equations*
- King A.C., Billingham J. Otto S.R. *Differential Equations: Linear, Nonlinear, Ordinary, Partial*
- Kuttler K. Notes for Partial Differential Equations (FREE! http://www.math.byu.edu/~klkuttle/547notesB.pdf)
- Markowich P.A. *Applied Partial Differential Equations: A Visual Approach*
- Pinchover Y. and Rubinstein J. *An Introduction to Partial Differential Equations*
- Pivato M. *Linear Partial Differential Equations and Fourier Theory* (FREE! http://xaravve.trentu.ca/305/305.pdf)
- Sattinger D.H. Partial Differential Equations of Applied Mathematics (FREE! http://www.math.usu.edu/~dhs/pde.pdf)

- Sauvigny F. *Partial Differential Equation Vol 1, 2*
- Showalter R.E. A PDE Primer (FREE! http://www.math.oregonstate.edu/~show/docs/pde.html)
- Sneddon I.N. *Elements of Partial Differential Equations*
- Strauss W.A. *Partial Differential Equations: An Introduction* - The author's principle: "Motivate with physics but then do mathematics."
- Symes W.W. Partial Differential Equations of Mathematical Physics (FREE! http://www.caam.rice.edu/~caam436/caam436notes.pdf)
- 郇中丹、黃海洋：《偏微分方程》，高等教育出版社

Calculus of Variations:

- Dacorogna B. *Introduction To The Calculus Of Variations*
- Gelfand I.M. and Fomin S.V. *Calculus of Variations*
- Jost J. and Li-Jost X. *Calculus of Variations*
- van Brunt B. *The Calculus of Variations*
- Weinstock R. *Calculus of Variations*

Mathematical Physics (optional):

Advanced studies of modern physics rely heavily on mathematics. In fact, throughout history, these two subjects are highly related.

Introductory level:

- Crowell B. Light and Matters (FREE! http://www.lightandmatter.com/area1.html) - Including *Newtonian Physics*, *Conservation Laws*, *Vibrations and Waves*, *Electricity and Magnetism*, *Optics* and *the Modern Revolution in Physics*, etc.
- Dolgachev I.V. Introduction to Physics 1, 2, 3, 4 and literature (FREE! http://www.math.lsa.umich.edu/~idolga/lecturenotes.html)
- Schnick J.W. *Calculus-Based Physics* (FREE! http://www.anselm.edu/internet/physics/cbphysics/index.html)

Relatively advanced level:

- Eisberg R.M. *Fundamentals of Modern Physics*
- Eisberg R. and Resnick R. *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles* - A very nice quantum physics text.
- Feynman R.P. *Feynman Lectures On Physics* - Modern classics.
- Fowler M. Modern Physics (FREE! http://galileo.phys.virginia.edu/classes/252/)
- Goldstein H. *Classical Mechanics*

- Firk W.K. *Introduction to Groups, Invariants & Particles* (FREE! http://www.physicsforfree.com/intro.html)
- Fitzpatrick R. Courses Taught at UT Austin (FREE! http://farside.ph.utexas.edu/teaching.html) - Quantum mechanics, classical electromagnetism, introduction to plasma physics, thermodynamics and statistical mechanics, analytical classical dynamics, etc.
- Hawking S. Public Lectures (FREE! http://www.hawking.org.uk/lectures/public.html)
- Heinbockel, J.H *Introduction to Tensor Calculus and Continuum Mechanics* (FREE! http://www.math.odu.edu/~jhh/counter2.html)
- Heisenberg W. *Physical Principles of the Quantum Theory*
- Joos G. and Freeman I.M. *Theoretical Physics*
- Morin D. *Introductory Classical Mechanics, with Problems and Solutions* (FREE! http://www.courses.fas.harvard.edu/~phys16/Textbook/)
- Rzchowski M. and Kern K. Ideas of Modern Physics (FREE! http://uw.physics.wisc.edu/~rzchowski/phy107/lectures.htm)
- Schiller C. *Motion Mountain* (FREE! http://motionmountain.dse.nl/index.html)
- Tatum J.B. Physics Topics (FREE! http://astrowww.phys.uvic.ca/~tatum/index.html) - Stellar atmospheres, celestial mechanics, classical mechanics, geometric optics, electricity and magnetism, heat and thermodynamics, planetary photometry notes.

Mathematical methods in physics:

- Arnold V.I. *Mathematical Methods of Classical Mechanics*
- Boas M.L. *Mathematical Methods in the Physical Sciences*
- Butkov E. *Mathematical Physics*
- Byron F.W. and Fuller R.W. *Mathematics of Classical and Quantum Physics* - This text covers a wide range of mathematics topics that are often used in physics, e.g. calculus of variations, Hilbert space, Green's function and group theory.
- Courant R. and Hilbert D. *Methods of Mathematical Physics Vol 1, 2*
- Fecko M. *Differential Geometry and Lie Groups for Physicists*
- Hamermesh M. *Group Theory and Its Application to Physical Problems*
- Hassani S. *Mathematical Physics: A Modern Introduction to Its Foundations*
- Isham C.J. *Modern Differential Geometry for Physicists*
- Jeffreys H. and Jeffreys B. *Methods of Mathematical Physics*
- Koks D. *Explorations in Mathematical Physics: The Concepts Behind an Elegant Language*
- Kreyszig E. *Advanced Engineering Mathematics* - The bible of many engineers.
- Nakahara M. *Geometry, Topology and Physics*
- Nash C. and Sen S. *Topology and Geometry for Physicists*
- Nearing J. Mathematical Tools for Physics (FREE! http://www.physics.miami.edu/~nearing/mathmethods/) - ODE, Fourier Series, Multivariable

Calculus, Vector Calculus, PDE, Numerical Analysis, Tensors, Complex Variable, Fourier Analysis, Calculus of Variations, etc.

- Szekeres P. *A Course in Modern Mathematical Physics: Groups, Hilbert Space and Differential Geometry*
- Teschl G. Ordinary Differential Equations and Dynamical Systems (FREE! http://www.mat.univie.ac.at/~gerald/ftp/book-ode/index.html)
- Wyld H.W. *Mathematical Methods for Physics*

Relativity:

- Carroll S.M. Lecture Notes on General Relativity (FREE! http://pancake.uchicago.edu/~carroll/notes/ or http://arxiv.org/abs/gr-qc/9712019)
- Einstein A. *Relativity* - For general reader.
- Einstein A. *The Meaning of Relativity* - Stanford Little Lectures.
- Frauendiener J, Giulini D.J.W. and Perlick V. *Analytical and Numerical Approaches to Mathematical Relativity* - More advanced work.
- Gardner R.B. Differential Geometry (and Relativity) (FREE! http://www.etsu.edu/math/gardner/5310/notes.htm)
- Lugo G. Differential Geometry in Physics (FREE! http://people.uncw.edu/lugo/COURSES/DiffGeom/dg1.htm) - Aimed to built up mathematical background to study general theory of relativity.
- Pauli W. *Theory of Relativity* - Published at the young age of relativity by a 21 years old young man, still stand as a good exposition today.
- Sachs R.K. and Wu H.-H. *General Relativity for Mathematicians*
- Sharipov R. *Classical Electrodynamics and Theory of Relativity* (FREE! http://uk.arxiv.org/abs/physics/0311011)
- 't Hooft G. Introduction to General Relativity (FREE! http://www.phys.uu.nl/~thooft/lectures/genrel.pdf)
- Waner S. Introduction to Differential Geometry and General Relativity (FREE! http://people.hofstra.edu/Stefan_Waner/diff_geom/tc.html)
- Weyl H. *Space, Time, Matter* - Weyl describes a failed attempt of gauge invariance in this book.

Quantum Mechanics:

- Griffiths D.J. *Introduction to Quantum Mechanics*
- Liboff R. *Introductory Quantum Mechanics* - A very nice quantum mechanics text.
- Sakurai J.J. *Advanced Quantum Mechanics*
- Sakurai J.J *Modern Quantum Mechanics*

- Schulten K. Notes on Quantum Mechanics (FREE! http://www.ks.uiuc.edu/Services/Class/PHYS481/lecture.html)
- Shankar R. *Principles of Quantum Mechanics*
- Teschl G. Mathematical Methods in Quantum Mechanics (FREE! http://www.mat.univie.ac.at/~gerald/ftp/book-schroe/index.html)
- von Neumann J. *Mathematical Foundations of Quantum Mechanics*

Quantum Field Theory:

- de Wit B. Lecture Notes Quantum Field Theory 2007, 2006, 2005 (FREE! http://www.phys.uu.nl/~bdewit/qft07.pdf, http://www.phys.uu.nl/~bdewit/qft06.pdf, http://www.phys.uu.nl/~bdewit/qft05.pdf)
- Forshaw J. Quantum Field Theory (FREE! http://www.hep.man.ac.uk/u/forshaw/NorthWest/QFT.html)
- IAS Quantum Field Theory (FREE! http://www.math.ias.edu/QFT/)
- Siegel W. Introduction to String Field Theory (FREE! http://arxiv.org/abs/hep-th/0107094)
- Siegel W. Fields (FREE! http://xxx.lanl.gov/abs/hep-th/9912205) - Over 800 pages of comprehensive text.
- Weinberg S. *The Quantum Theory of Fields, Vol. 1: Foundations*
  Weinberg S. *The Quantum Theory of Fields, Vol. 2: Modern Applications*
  Weinberg S. *The Quantum Theory of Fields, Vol. 3: Supersymmetry*

String Theory:

- Polchinski J. What is String Theory? (FREE! http://xxx.lanl.gov/abs/hep-th/9411028)
- Siopsis G. String Theory (FREE! http://aesop.phys.utk.edu/strings/)
- 't Hooft G. Lectures String Theory (FREE! http://www.phys.uu.nl/~thooft/lectures/string.html)
- Uranga A.M. Graduate Course in String Theory
- Zwiebach B. *A First Course in String Theory*

Chaos:

- Cvitanović P., Artuso R., Mainieri R., Tanner G., Vattay G., Whelan N. and Wirzba A. *Chaos: Classical and Quantum* (FREE! http://chaosbook.org/)
- Cross M. Introduction to Chaos (FREE! http://www.cmp.caltech.edu/~mcc/Chaos_Course/Outline.html)
- Smith L. *Chaos: A Very Short Introduction*

Probability (optional):

These days, studying probability without measure is like studying physics without calculus. If you have done some baby measure theory in stage 3, you are probably ready for the followings.

Check out Probability Theory As Extended Logic (http://bayes.wustl.edu/) for collected probability papers.

Probaility built upon Measure Theory:

- Ash R.B. and Doleans-Dade C.A. *Probability & Measure Theory*
- Athreya K.B. and Lahiri S.N. *Measure Theory and Probability Theory*
- Bass R.F. Probability Theory (FREE! http://www.math.uconn.edu/~bass/prob.pdf)
- Billingsley P. *Probability and Measure*
- Bobrowski A. *Functional for Analysis for Probability and Stochastic Processes: An Introduction*
- Capinski M. and Kopp P.E. *Measure, Integral and Probability*
- Dembo A. Probability Theory (FREE! http://www-stat.stanford.edu/~adembo/stat-310a/lnotes.pdf)
- Dudley R.M. *Real Analysis and Probability*
- Feller W. *An Introduction to Probability Theory and Its Applications Vol. 2*
- Gray R.M. *Probability, Random Processes, and Ergodic Properties* (FREE! http://www-ee.stanford.edu/~gray/arp.html)
- Gut A. *Probability: A Graduate Course*
- Jaynes E.T. *Probability Theory: The Logic of Science* (FREE! http://omega.math.albany.edu:8008/JaynesBook.html)
- Jorgensen P.E.T. *Analysis and Probability: Wavelets, Signals, Fractals*
- Kallenberg O. *Foundations of Modern Probability* - Nice historical and bibliographical notes are included at the back.
- Knill O. *Probability and Stochastic Processes with Applications* (FREE! http://abel.math.harvard.edu/~knill/teaching/math144_1994/probability.pdf)
- Loève M. *Probability Theory I, II* - Fairly comprehensive.
- Kolmogorov A.N. *Foundations of the Theory of Probability* (FREE! http://www.mathematik.com/Kolmogorov/index.html) - For historical interest.
- Shiryaev A.N. *Probability* - The material is based on a three-semester course of lectures, under the title *Probability, Statistics, Stochastic Processes, I, II*, given by the author in the Moscow State University. He draws nice comparison between set-theoretic interpretation and interpretation in probability theory, see p.136-137.
- Strasser H. Introduction to Probability Theory and Stochastic Processes (FREE! http://statmath.wu-wien.ac.at/~strasser/STATS 25-01-2006.pdf)
- Tsirelson B. Advanced Probability Theory (FREE!

http://www.math.tau.ac.il/~tsirel/Courses/AdvProb03/syllabus.html)

- Wilde I.F. Measures, Integration and Probability (FREE! http://www.mth.kcl.ac.uk/~iwilde/notes/mip/index.html)
- Williams D. *Probability with Martingales*

Stochastic Processes:

- Brzezniak Z. and Zastawniak T. *Basic Stochastic Processes*
- Doob J.L. *Stochastic Processes* - Considered to be classics in the field.
- Dembo A. Stochastic Processes (FREE! http://www-stat.stanford.edu/~adembo/math-136/nnotes.pdf)
- Jacod J. and Shiryaev A.N. *Limit Theorems for Stochastic Processes*
- Karlin S. and Taylor H.M. *A Second Course in Stochastic Processes* - Amazon user Yan Zhang comments that this is a must-have if you wanna go to Wall Street.
- Mörters P. and Peres Y. Brownian Motion (FREE! http://www.stat.berkeley.edu/users/peres/bmbook.pdf)
- Ross S.M. *Stochastic Processes* - See also Russell Lyons' course notes (http://mypage.iu.edu/~rdlyons/pdf/StochProc.pdf).
- Shalizi S. Stochastic Processes (FREE! http://www.stat.cmu.edu/~cshalizi/754/)
- Todorovic P. *An Introduction to Stochastic Processes and Their Application*
- van Zanten H. An Introduction to Stochastic Processes in Continuous Time (FREE! http://www.cs.vu.nl/~rmeester/onderwijs/stochastic_processes/sp_new.pdf)
- Varadhan S.R.S. Stochastic Processes (FREE! http://www.math.nyu.edu/faculty/varadhan/processes.html)

Stochastic Analysis:

- Bain A. Stochastic Calculus (FREE! http://www.chiark.greenend.org.uk/~alanb/)
- Chung K.L. and Williams R.J. *An Introduction to Stochastic Integration*
- Evans L.C. An Introduction to Stochastic Differential Equations (FREE! http://math.berkeley.edu/~evans/SDE.course.pdf) - Evans wrote a popular book on PDE.
- Karatzas I. and Shreve S.E. *Brownian Motion and Stochastic Calculus*
- Klebaner F.C. *Introduction to Stochastic Calculus with Applications*
- Kurtz T.G. Lectures on Stochastic Analysis (FREE! http://www.math.wisc.edu/~kurtz/735/main735.pdf)
- Malliavin P. *Stochastic Analysis*
- Mörters P. Lecture Notes on Stochastic Analysis (FREE! http://people.bath.ac.uk/maspm/stoa.ps)
- Seppalainen T. Basics of Stochastic Analysis (FREE!

http://www.math.wisc.edu/~seppalai/sa-book/etusivu.html)

- Varadhan S.R.S. Stochastic Analysis (FREE! http://www.math.nyu.edu/faculty/varadhan/fall06.html)
- Wilde I.F. Stochastic Analysis (FREE! http://www.mth.kcl.ac.uk/~iwilde/notes/sa/)

Statistics (optional):

In general:

- Balakrishnan N. and Nevzorov V.B. *A Primer on Statistical Distributions*
- Dallal G.E. The Little Handbook of Statistical Practice (FREE! http://www.tufts.edu/~gdallal/LHSP.HTM)
- Davidian M. Preparation for Statistical Research (FREE! http://www4.stat.ncsu.edu/~davidian/st810a/index.html)
- Gentle J.E., Härdle W. and Mori Y. (ed.) *Handbook of Computational Statistics* (FREE! http://www.quantlet.com/mdstat/scripts/csa/html/index.html or http://www.quantlet.com/mdstat/scripts/csa/pdf/csa.pdf)
- Good P.I. *Permutation, Parametric, and Bootstrap Tests of Hypotheses*
- Kardaun O.J.W.F. *Classical Methods of Statistics*
- Lehmann E.L. *Elements of Large-Sample Theory*
- Lehmann E.L. and Romano J.P. *Testing Statistical Hypotheses* - Classical work on hypotheses testing.
- Kish L. *Statistical Design for Research*
- Ramsay J.O. and Silverman B.W. *Applied Functional Data Analysis*
- Ramsay J.O. and Silverman B.W. *Functional Data Analysis*
- Rao C.R. *Linear Statistical Inference and Its Application*
- Wasserman L. *All of Statistics: A Concise Course in Statistical Inference* - The author aimed to write a short course in this area. It's kind of encyclopedic, rather than a textbook that explains everything in detail. But if it's a handbook, it's kind of incomplete. And if you want a handbook, why don't use just get a handbook? Wasserman states in page 25 that if the reader is unfamiliar with "inf" then just treat it as the minimum. I can't accept this for if "inf" can be treated as minimum there is no point to introduce "sup" and "inf". Check the errata list (http://www.stat.cmu.edu/~larry/all-of-statistics/index.html).
- Zwillinger D. and Kokoska S. *CRC Standard Probability and Statistics Tables and Formulae*

Statistical Models and Regression:

- DeCoster J. Applied Linear Regression Set 1, Set 2 (FREE! http://www.stat-help.com/cohen 2006-09-26.pdf, http://www.stat-help.com/linreg.pdf)
- Draper N. and Smith H. *Applied Regression Analysis*

- Fox J. *Applied Regression Analysis, Linear Models, and Related Methods*
- Goldstein H. *Multilevel Statistical Models* (FREE! http://www.ats.ucla.edu/stat/examples/msm_goldstein/)
- Härdle W. *Applied Nonparametric Regression* (FREE! http://www.quantlet.com/mdstat/scripts/anr/html/anrhtml.html or http://www.quantlet.com/mdstat/scripts/anr/pdf/anrpdf.pdf)
- Härdle W., Müller M., Sperlich S. and Werwatz A. *Nonparametric and Semiparametric Models* (FREE! http://www.quantlet.com/mdstat/scripts/spm/html/spmhtml.html or http://www.quantlet.com/mdstat/scripts/spm/spm.pdf)
- Harrell F.E.Jr. *Regression Modeling Strategies*
- Jiang J. *Linear and Generalized Linear Mixed Models and Their Applications*
- Kutner M.K., Nachtsheim C.J., Neter J. and Li W. *Applied Linear Statistical Models* - New edition of the old title *Applied Linear Regression Models*. Well explanation, with reasonably amount of illustration. Solution, data set, errata etc. for 5th ed. (http://apps.csom.umn.edu/Nachtsheim/5th/); for 4th ed. (http://apps.csom.umn.edu/Nachtsheim/).
- Lindsey J.K. *Applying Generalized Linear Models*
- Mendenhall W. and Sincich T.L. *A Second Course in Statistics: Regression Analysis*
- Rao C.R. and Toutenburg H. *Linear Models: Least Squares and Alternatives*
- Rao C.R., Toutenburg H., Shalabh and Heumann C. *Linear Models and Generalizations: Least Squares and Alternatives*
- Schroeder L., Sjoquist D.L. and Stephan P.E. *Understanding Regression Analysis: An Introductory Guide*
- Seber G.A. and Lee A.J. *Linear Regression Analysis*
- Sengupta S. and S.R. Jammalamadaka *Linear Models: An Integrated Approach* - Data set and solution on web.
- Song P.X.-K. *Correlated Data Analysis: Modeling, Analytics, and Applications*
- Weisberg S. *Applied Linear Regression*

Multivariate Analysis:

- Bilodeau M. and Brenner D. *Theory of Multivariate Statistics*
- Härdle W. and Simar L. *Applied Multivariate Statistical Analysis* (FREE! http://www.quantlet.com/mdstat/scripts/mva/htmlbook/mvahtml.html or http://www.quantlet.com/mdstat/scripts/mva/pdf/mvapdf.pdf)
- Härdle W. and Hlavka Z. *Multivariate Statistics: Exercises and Solutions* (FREE! http://www.karlin.mff.cuni.cz/~hlavka/sms/html/index.html or http://www.karlin.mff.cuni.cz/~hlavka/sms/pdf/smspdf.pdf)
- Jackson J.E. *A User's Guide to Principal Components*

- Johnson R.A. and Wichern D.W. *Applied Multivariate Statistical Analysis*
- Kachigan S.K. *Multivariate Statistical Analysis: A Conceptual Introduction*
- Lattin J., Carroll D. and Green P. *Analyzing Multivariate Data*
- Mardia K.V., Kent J.T. and Bibby *Multivariate Analysis*
- Miles D.B. and McCarthy B.C. BIOS870 Multivariate Statistics (FREE! http://oak.cats.ohiou.edu/~milesd/multivar.htm)
- Stockburger D.W. Multivariate Statistics: Concepts, Models, and Applications (FREE! http://www.psychstat.missouristate.edu/MultiBook/mlt00.htm)
- Tabachnick B.G. and Fidell L.S. *Using Multivariate Statistics* - Check out their page for the 4th edition here (http://www.ablongman.com/tabachnick/stats/index.html) and the 5th here (http://wps.ablongman.com/ab_tabachnick_multistats_5/).
- Tharp I. Selected Multivariate Statistical Techniques (FREE! http://homepages.gold.ac.uk/aphome/multivariate.html)
- Wendorf C.A. Manual for Univariate and Multivariate Statistics (FREE! http://www.uwsp.edu/psych/cw/statmanual/index.html)

Bayesian:

- Bernardo J.M. and Smith A.F.M. *Bayesian Theory*
- Bolstad W.M. *Introduction to Bayesian Statistics*
- Carlin B.P. and Louis T.A. *Bayes and Empirical Bayes Methods for Data Analysis* - Data set can be found at: http://www.biostat.umn.edu/~brad/data.html
- Congdon P. *Bayesian Statistical Modelling*
- Gelman A., Carlin J.B., Stern H.S. and Rubin D.B. *Bayesian Data Analysis* - Check out Gelman's website (http://www.stat.columbia.edu/~gelman/book/) for extra information, including data sets, solution and errata etc.
- Ghosh J., Delampady M. and Samanta T. *An Introduction to Bayesian Analysis: Theory and Methods*
- Koch K.-R. *Introduction to Bayesian Statistics*
- Rowe D.B. *Multivariate Bayesian Statistics: Models for Source Separation and Signal Unmixing*

Categorical Data Analysis:

- Agresti A. *An Introduction to Categorical Data Analysis* - Correction to 1st (http://www.stat.ufl.edu/~aa/intro-cda/corrections/corrections.html) / 2nd (http://www.stat.ufl.edu/~aa/intro-cda/corrections.pdf) edition.
- Agresti A. *Categorical Data Analysis* - See author's website for this book (http://www.stat.ufl.edu/~aa/cda/cda.html), that provides datasets used for examples, solutions,

etc.

- Friendly M. *Categorical Data Analysis with Graphics* (FREE! http://www.math.yorku.ca/SCS/Courses/grcat/)
- Lloyd C.J. *Statistical Analysis of Categorical Data*
- Powers D.A. and Xie Y. *Statistical Methods for Categorical Data Analysis*
- Williams R. Categorical Data Analysis (FREE! http://www.nd.edu/~rwilliam/xsoc73994/index.html)

Data Mining:

- Hastie T. Tibshirani R. and Friedman J.H. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction* - See their web (http://www-stat.stanford.edu/~tibs/ElemStatLearn/) for data, errata, etc.
- Myatt G. *Making Sense of Data: A Practical Guide to Exploratory Data Analysis and Data Mining*

Time Series:

- Box G., Jenkins G.M. and Reinsel G. *Time Series Analysis: Forecasting & Control*
- Brockwell P.J. and Davis R.A. *Introduction to Time Series and Forecasting*
- Brockwell P.J. and Davis R.A. *Time Series: Theory and Methods*
- Chair of Statistics, University of Würzburg *A Firset Course on Time Series Analysis: Examples with SAS* (FREE! http://statistik.mathematik.uni-wuerzburg.de/timeseries/index.php?id=book)
- Chatfield C. *The Analysis of Time Series: An Introduction*

Simulation and the Monte Carlo Method:

- Hammersley J.M. and Handscomb D.C. *Monte Carlo Methods* - (FREE! http://www.cs.fsu.edu/~mascagni/Hammersley-Handscomb.pdf)
- Haugh M. Monte Carlo Simulation (FREE! http://www.columbia.edu/~mh2078/MCS04.html)
- Rubinstein R.Y. and Kroese D.P. *Simulation and the Monte Carlo Method*
- Ross S.M. *Simulation*
- Rummukainen K. Monte Carlo Simulation Methods (FREE! http://cc.oulu.fi/~tf/tiedostot/pub/montecarlo/)
- Sokal A.D. Monte Carlo Methods in Statistical Mechanics: Foundations and New Algorithms (FREE! http://www.math.nyu.edu/faculty/goodman/teaching/Monte_Carlo/Sokal.ps)

Biostatistics (optional):

When mathematics is applied to biology or medical science, a subject with significant importance. For

more free online material, see also John Hopkins Bloomberg School of Public Health's biostatistics section (http://ocw.jhsph.edu/Topics.cfm?topic_id=33).
In general:

- Brant R. Biostatistics I, II (FREE! http://www.stat.ubc.ca/~rollin/teach/643.f02/index.html, http://www.stat.ubc.ca/~rollin/teach/643w04/index.html)
- Broemeling L.D. *Bayesian Biostatistics and Diagnostic Medicine*
- Feinstein A.R. *Principles of Medical Statistics*
- Forthofer R.N., Lee E.S. and Hernandez M. *Biostatistics: A Guide to Design, Analysis and Discovery* - See their web (http://www.biostat-edu.com/).
- Glantz S.A. *Primer of Biostatistics*
- Härdle W., Mori Y. and Vieu P. *Statistical Methods for Biostatistics and Related Fields* - More advanced, collection of research papers.
- Larget B. Statistical Methods for Bioscience I, II, 2007 (FREE! http://www.stat.wisc.edu/courses/st571-larget/stat571.html, http://www.stat.wisc.edu/courses/st572-larget/, http://www.stat.wisc.edu/courses/st572-larget/Spring2007)
- Moye L.A. *Statistical Reasoning in Medicine: The Intuitive P-Value Primer*
- Norman G.R. and Streiner D.L. *Biostatistics: The Bare Essentials*
- Paciorek C. Bayesian Methods in Biostatistics (FREE! http://isites.harvard.edu/icb/icb.do?keyword=k16579&pageid=icb.page99868)
- Petrie A. and Sabin C. *Medical Statistics at a Galance* - See the online companion (http://www.medstatsaag.com/).
- van Belle G. and Fisher L.D. *Biostatistics: A Methodology for the Health Sciences* - See their site (http://www.biostat-text.info/).
- Vittinghoff E., Glidden D.V., Shiboski S.C. and McCulloch C.E. *Regression Methods in Biostatistics: Linear, Logistic, Survival, and Repeated Measures Models*
- Zar J.H. *Biostatistical Analysis*

Statistical Methods in Epidemiology:

- Bhopal R. *Concepts of Epidemiology: An Integrated Introduction to the Ideas, Theories, Principles and Methods of Epidemiology*
- Gerstman B.B. *Epidemiology Kept Simple: An Introduction to Traditional and Modern Epidemiology* - Review questions, exercises, PowerPoints, etc. at: http://www.sjsu.edu/faculty/gerstman/eks/index.htm
- Jewell N.P. *Statistics for Epidemiology* - Amazon reader Kalantar-zadeh comments it as a historical masterpiece in the field.
- Newman S.C. *Biostatistical Methods in Epidemiology*

- Rothman K.J. *Epidemiology: An Introduction*
- Twisk J.W.R *Applied Longitudinal Data Analysis for Epidemiology: A Practical Guide*
- Wassertheil-Smoller S. *Biostatistics and Epidemiology: A Primer for Health and Biomedical Professionals*
- Woodward M. *Epidemiology: Study Design and Data Analysis* - Mark insists that this book is written for both researcher and statistician, proofs are omitted. CRC's Instruction and Download at: http://www.crcpress.com/e_products/downloads/download.asp?cat_no=C415X

Longitudinal Data Analysis:

- Davidian M. ST 732, Applied Longitudinal Data Analysis (FREE! http://www.stat.ncsu.edu/people/davidian/st732/)
- Diggle P., Heagerty P., Liang K.-Y. and Zeger S. *Analysis of Longitudinal Data* - Considered to be classics in the field. Data and errata page at: http://faculty.washington.edu/heagerty/Books/AnalysisLongitudinal/
- Fitzmaurice G.M., Coull B. and Ware J. Applied Longitudinal Analysis (FREE! http://biosun1.harvard.edu/~fitzmaur/ala/lectures.html) - Lecture slides used at the Harvard School of Public Health for BIO226.
- Fitzmaurice G.M., Laird N.M. and Ware J.H. *Applied Longitudinal Analysis*
- Hedeker D. BSTT513 Longitudinal Data Analysis (FREE! http://www.uic.edu/classes/bstt/bstt513/index.html)
- Hedeker D. and Gibbons R.D. *Longitudinal Data Analysis*

Survival Analysis:

- Hosmer D.W. and Lemeshow S. *Applied Survival Analysis: Regression Modeling of Time to Event Data*
- Hougaard P. *Analysis of Multivariate Survival Data*
- Ibrahim J.G., Chen M.-H. and Sinha D. *Bayesian Survival Analysis*
- Klein J.P. and Moeschberger M.L. *Survival Analysis* – See Klein's page (http://www.biostat.mcw.edu/homepgs/klein/book.html) for data sets and errors.
- Kleinbaum D.G. and Klein M. *Survival Analysis: A Self-Learning Text* - Data files at: http://www.sph.emory.edu/~dkleinb/surv2.htm#data
- Laws N. Survival Analysis MT (FREE! http://www.stats.ox.ac.uk/~laws/survival/)
- Tsiatis A. and Zhang D. Analysis of Survival Data (FREE! http://www4.stat.ncsu.edu/~dzhang2/st745/)

Stage 5

By completing stage 4, you should be ready for graduated schools, a.e. My list ends here. You've completed an undergraduate mathematics program (who cares about certificate?), and possibly more than that. Believe it or not, I've seen mathematics graduates haven't taken any analysis, abstract algebra or topology courses. This is (partly) because the entry requirement of mathematics degree is usually not so high (but don't expect it's easy) and folks fall into such program accidentally and they don't have to pick those abstract courses. Now you probably know what's going on and you should have an idea what you can read. Several books in the list above give suggestions for further reading, you may refer to them according to your interest. Moreover, choices decrease as stages increase.

For more advanced FREE! materials, check out:

- American Mathematical Society Books Online: http://www.ams.org/online_bks/online_subject.html
- MIT's graduate courses list: http://ocw.mit.edu/OcwWeb/Mathematics/index.htm#grad
- The Electronic Library of Mathematics http://www.emis.de/ELibM.html
- FreeScience http://freescience.info/Mathematics.php
- Scanned text (mostly Chinese, e.g. Baby Rudin and Big Rudin, Riesz and Sz.-Nagy, Yosida, Harthsorne, Halmos, etc) http://202.38.126.65/mathdoc/
- Stein W.'s Modular Abelian Varieties page http://modular.math.washington.edu/scans/252/ (I've listed a few of them in the above list)

List of books:

- American Mathematical Society: Graduate Studies in Mathematics http://www.ams.org/cgi-bin/bookstore/bookpromo/gsmseries
- Springer: Graduate Text in Mathematics GTM http://www.springer.com/west/home/math?SGWID=4-10042-69-173621337-0
- Springer: Grundlehren der mathematischen Wissenschaften http://www.springer.com/west/home/math?SGWID=4-10042-69-173621309-0
- Springer: Universitext http://www.springer.com/west/home/math?SGWID=4-10042-69-173621316-0
- Cambridge University Press: Cambridge Studies in Advanced Mathematics http://www.cambridge.org/uk/series/sSeries.asp?code=CSAM
- Cambridge University Press: London Mathematical Society Student Texts http://www.cambridge.org/uk/series/sSeries.asp?code=LMST
- Cambridge University Press: London Mathematical Society Lecture Notes Series http://www.cambridge.org/uk/series/sSeries.asp?code=LMSN

- Dover books in mathematics
  http://store.doverpublications.com/by-subject-science-and-mathematics-mathematics.html

Want challenging questions? See:

- Mathematical Tripos Examination Papers http://www.maths.cam.ac.uk/teaching/pastpapers/
- Harvard Mathematics Qualifying Exam
  http://www.math.harvard.edu/graduate/index.html#past

After several extra stages, if you feel like you are ready to read journals, try:

- Annals of Mathematics http://annals.princeton.edu/
- arXiv.org http://arxiv.org/archive/math
- Project Euclid http://projecteuclid.org/

After all, if it turned out that you're not going to further your study in pure mathematics, what can you do with your mathematical knowledge? There are a few options (ideas by James Franklin, given in the 'Professional Issues and Ethics in Mathematics' course):

- Modelling of environment (related areas are global warming, weather prediction, etc)
- Optimization for resources planning
- Statistical research on effectiveness of drugs and medical procedures, better diagnosis
- Teaching, to inspire (or corrupt?) the next generation