Sources in the History of Mathematics and Physical Sciences 8

PAPPUS OF ALEXANDRIA

BOOK 7 OF THE *COLLECTION*

PART 1. INTRODUCTION, TEXT, AND TRANSLATION

Edited
With Translation and Commentary by

ALEXANDER JONES

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Sources
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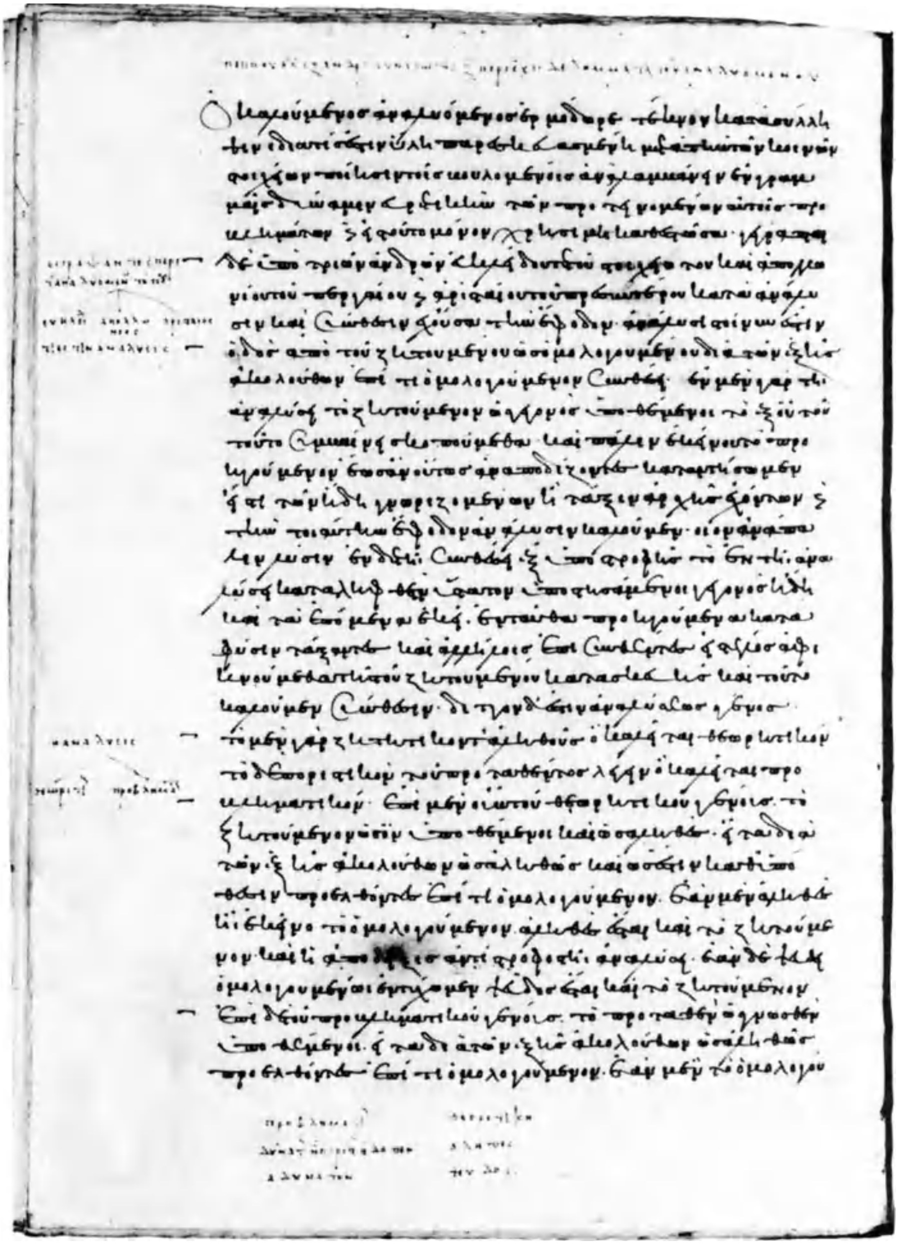
8



Vat. gr. 218, f. 118[v]
Photo: Biblioteca Vaticana

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Edited
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Alexander Jones

In Two Parts
With 308 Figures

Springer Science+Business Media, LLC

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AMS Classification: 01A20

Library of Congress Cataloging in Publication Data
Pappus, of Alexandria.
Book 7 of the Collection.
(Sources in the history of mathematics and physical sciences ; 8)
English and Greek.
Revision of thesis (Ph. D.)—Brown University, 1985.
Bibliography: p.
Includes indexes.
Contents: pt. 1. Introduction, text, and translation—
pt. 2. Commentary, index, and figures.
1. Mathematics, Greek. I. Jones, Alexander.
II. Title. III. Title: Book seven of the Collection.
IV. Series.
QA22.P3713 1986 516.2 85-27788

Originally published by Springer-Verlag New York Inc. in 1986
Softcover reprint of the hardcover 1st edition 1986

9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-9355-2 ISBN 978-1-4612-4908-5 (eBook)
DOI 10.1007/978-1-4612-4908-5

TO MY PARENTS

Preface

The seventh book of Pappus's *Collection*, his commentary on the Domain (or Treasury) of Analysis, figures prominently in the history of both ancient and modern mathematics: as our chief source of information concerning several lost works of the Greek geometers Euclid and Apollonius, and as a book that inspired later mathematicians, among them Viete, Newton, and Chasles, to original discoveries in their pursuit of the lost science of antiquity. This presentation of it is concerned solely with recovering what can be learned from Pappus about Greek mathematics. The main part of it comprises a new edition of Book 7; a literal translation; and a commentary on textual, historical, and mathematical aspects of the book. It proved to be convenient to divide the commentary into two parts, the notes to the text and translation, and essays about the lost works that Pappus discusses.

The first function of an edition of this kind is, not to expose new discoveries, but to present a reliable text and organize the accumulated knowledge about it for the reader's convenience. Nevertheless there are novelties here. The text is based on a fresh transcription of Vat. gr. 218, the archetype of all extant manuscripts, and in it I have adopted numerous readings, on manuscript authority or by emendation, that differ from those of the old edition of Hultsch. Moreover, many difficult parts of the work have received little or no commentary hitherto. In particular I believe that more sense can be recovered from several problematic passages in the important first part of the book than has been recognized. The account of the evolution and vicissitudes of the text, from its composition to the Renaissance, is largely new. In treating the lost works of Apollonius and Euclid, where so much has been done between the times of Maurolico and Zeuthen, my main work was to select what seemed to be valid scholarship; the remainder, if mentioned at all, had to be ruthlessly relegated to footnotes, without regard for intrinsic merit.

This edition is a revision of my doctoral dissertation in the Department of History of Mathematics at Brown University, which was submitted in April 1985. It was stored on and printed by Brown University's computer facilities, using experimental laser-printer typesetting software. Some minor typographical infelicities, for example the lack of an iota subscript, are I hope outweighed by the reduced cost of production. I am entirely responsible for typographical and other errors.

I have to thank the Biblioteca Apostolica Vaticana for access to its facilities and collections, and providing, through my teacher Gerald Toomer, a microfilm of the archetype. I have also profited from research in the Biblioteca Ambrosiana, Milan; the Newberry Library, Chicago; the libraries of the University of British Columbia and Simon Fraser University; and above all the libraries of Brown University. During the writing of the dissertation I held a doctoral fellowship from the Social Sciences and Humanities Research Council of Canada. The History of Mathematics Department provided a truly congenial home for four years; I mention with special gratitude the often manifested hospitality of the late Professor A. J. Sachs and Mrs J. Sachs, and many kindnesses of Professor O. Neugebauer. A summer stipend from the History of Mathematics Department enabled me to spend two months during the Summer of 1984 in Italy palpating the past. For various suggestions, information, and corrections I am indebted to Professors J. L. Berggren, A. L. Boegehold, David Pingree (who also proof-read the Greek text expertly), D. T. Whiteside, and Mr N. G. Wilson. Dr Jan Hogendijk, surpassing his function as reader of the dissertation, rescued me from numerous mathematical and logical morasses. Many of my notes on Pappus's mathematics are the better for his suggestions, and the essays (especially those on the *Porisms* and the loci) were enormously improved, in form and content, under his guidance. He also generously allowed me to read the results of his researches into the traces of lost works of Apollonius in Arabic sources; since these are, at the time of writing, not published, I have limited myself to mentioning the existence of relevant fragments at appropriate points in the essay on Apollonius. My debt to Gerald Toomer extends throughout the book, every page of which (in its earlier version) he read with the greatest care. He suggested the edition in the first place, and I can only hope that a little of his learning is reflected in it.

Providence,
September 1985.

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PART 1

General Introduction
(Pappus and the *Collection*)

§1. **Biographical Data.** In the later Hellenistic period, after several hundred years of progress, the main stream of Greek mathematics, synthetic geometry, experienced a deep and permanent decline. The subject did not stop being studied and taught, but original discoveries became less and less frequent and important. The causes and even the date of this decadence are obscured by the fewness of our sources for the period between Apollonius, about 200 B.C, and the fourth century A.D. But although the conditions under which ancient books were transmitted to us naturally favored (if we except a few 'classics' by the great Hellenistic geometers) later texts over earlier ones, we learn from reports at second hand that authors such as Geminus, Menelaus, and Heron in the first century A.D. were already excerpting, reediting, and commenting on older works.

Pappus of Alexandria is the first author in this degenerate tradition of whom we have substantial writings on higher geometry, and – for the modern historian – he is also the most important. The period around the fourth century A.D. has often been described as a 'Silver Age' of mathematics, an illusion for which the bulk of Pappus's extant work, and the abundance of information uniquely preserved in it, are largely responsible. In fact the few occasions on which Pappus claims something as his original discovery give little evidence of a fertile mind. Nevertheless his reputation, as shown by the later allusions of Proclus, Marinus, and Eutocius, was high – deservedly so, according to the debased standard of his time.

The only document concerning Pappus that can be called biographical is the short article on him in the *Souda*, a tenth-century Byzantine encyclopedia:[1]

> Pappus of Alexandria, philosopher, lived in the reign of the emperor Theodosius the elder, when Theon the philosopher who wrote on Ptolemy's *Table* [*i.e.* the *Handy Tables*] also flourished. His books are *Chorography of the Inhabited World*, Commentary on the four books of Ptolemy's *Great Syntaxis*, *The Rivers in Libya*, and *Interpretations of Dreams*.

Souda, ed. Adler, vol. 4 p. 26. The article on Theon of Alexandria (vol. 2 p. 702) repeats the claim that he and Pappus were contemporaries.

Aside from the listed writings, which we will return to later on, the article makes two assertions. The first, that Pappus was a 'philosopher', could mean that Pappus held some kind of official post as a teacher of philosophy, presumably at Alexandria, or perhaps no more than that he was interested in scientific matters. None of Pappus's known works was truly philosophical, although his extant commentary on Book 10 of Euclid's *Elements* is admittedly devoted as much to metaphysical as to purely mathematical considerations. Many later writers on mathematical subjects are known otherwise, even primarily, as philosophers: Theon of Alexandria, his daughter Hypatia, Ammonius, Heliodorus, John Philoponus, and Eutocius, all associated with the philosophical school at Alexandria, and Proclus and Marinus at Athens. One of the few contemporaries that Pappus names in his works as an acquaintance is a certain Hierius the philosopher, who may be the same as a Hierius known from other sources (see below, note 9).

The *Souda*'s other claim, that Pappus was a contemporary of Theon of Alexandria in the reign of Theodosius (379-395) is false. The correct date of Pappus's career, about the first decades of the fourth century, is delimited, at one end by a marginal note in a chronological table in the ninth-century manuscript Leiden B.P.G. 78 which places Pappus in the reign of Diocletian (284-308 according to the table), at the other end by a computed conjunction of Sun and Moon for October 18, A.D. 320 in his commentary on Book 5 of Ptolemy's *Almagest*.[2] This computation is worked out for an observer at Alexandria: the only explicit confirmation that Pappus's career was passed in his home town. It seems probable that the *Souda*, or its source (conjectured to be the sixth-century biographer Hesychius of Miletus),[3] was misled by the insertion of parts of Pappus's

[2] B.P.G. 78, f. 55r (noted by Van der Hagen [1735] p. 320 and Usener [1873]), probably copied from the manuscript's exemplar. Similar notes in the margins date other early astronomers, certainly on the basis of observations quoted in the *Almagest*; hence the scholiast may have derived his date of Pappus from a computation in one of the lost books of his commentary on Ptolemy. On the conjunction in 320 (a partial solar eclipse) and Pappus's date, see Rome, *CA* vol. 1 pp. x-xi. Rome ([1939] p. 212 and *CA* vol. 2 p. 907) found another computation, of the Sun's position on January 5, 323, in Theon's commentary on Book 3 of the *Almagest*. This, being too early for Theon's career, may have been lifted from the corresponding (lost) part of Pappus's commentary. Astronomical writers usually illustrated their rules with examples near their own time.

[3] Adler, in *Souda* vol. 4 p. 26.

commentary among the books of Theon's commentary on the *Almagest* (at least they are so combined in the medieval manuscript tradition), which might suggest to a casual examination that the two authors collaborated.

§2. **Works.** Pappus's works, extant and lost, show his varied interests in the exact sciences and other subjects. The list below includes all the writings now known, and some dubious or false attributions.[4]

§2.1. **The Collection.** The most important of the surviving works is the Συναγωγή or *Collection*, preserved in a tenth-century manuscript, Vaticanus gr. 218, and its many descendants.[5] Hultsch's edition of 1876-1878 (the first complete one) is the standard text.[6] The Vaticanus is defective at the beginning and end: we have lost (in Greek) Book 1, the first part of Book 2, and the end of Book 8. The remnants are:

a. Book 2. The text is divided at the beginning into numbered paragraphs or propositions, of which we have 16 (2.1) through 23 (2.13), with the rest (to 2.27) unnumbered in the transmitted version. From these figures it appears that about half of Book 2 survives.

The individual letters in a line of Greek verse can be interpreted as numerals from 1 to 800. Pappus wishes to multiply these numerals all together, and express the product in words, using as base the myriad (10,000). The discussion follows a lost work of Apollonius, which the reader was expected to have at hand. Pappus provides arithmetical demonstrations of propositions that Apollonius proved by geometry. We are not told the title of Apollonius's book: it was probably a *jeu d'esprit* like Archimedes's *Sand Reckoner*, perhaps itself partly in verse. Heiberg suggested that this work of Apollonius's was the same as the strangely named *Okytokion* in which, according to Eutocius, Apollonius derived limits for π more refined than those of Archimedes, but this guess is

4 Good earlier surveys include K. Ziegler, "Pappos von Alexandria", in *RE* vol. 18 (1949) cc. 1084-1106; (for mathematical contents) Heath, *HGM* vol. 2, pp. 355-439; and I. Bulmer-Thomas, "Pappus of Alexandria", *DSB* vol. 10 (1974) pp. 293-304. Ziegler's perceptive discussion of the bibliographical aspects of the *Collection* deserves more note than it has received.

5 In the heading of Book 3 the title is given as συναγωγαί.

6 Hultsch, *PAC*. I refer to passages in the *Collection* always by Hultsch's chapter numbers; for example, 7.26 will mean Book 7, chapter 26.

unsupported.[7]

b. Book 3. The heading of the book is: "Contains geometrical problems, both plane and solid." It is addressed to a woman named Pandrosion, who was a teacher of mathematics (we know nothing else about her).[8] The introduction presents an unusual variant of the normal practice of addressing a published work to some primary recipient, sometimes with an extended explanation of the circumstances of the work's 'publication' and reasons for the dedication (as in the prefaces of Archimedes and Apollonius), or with only the perfunctory insertion of a vocative in the first sentence (as is the case with Pappus's other dedications in Books 5, 7, and 8). In any event the dedication was usually a compliment; here, however, we have a rebuke. Pappus writes that he has observed several of Pandrosion's pupils, and found their mathematical education deficient. Three examples of these weaknesses follow, giving Pappus the opportunity to expand on certain topics.

A first pupil repeatedly approached Pappus with an alleged construction of the two mean proportionals between given magnitudes by compass and straight-edge methods, asking whether it is correct (magnitudes Γ and Δ are two mean proportionals between magnitudes A and B if A:Γ = Γ:Δ = Δ:B; hence if A is taken as unit, the problem of finding Γ (and Δ) is equivalent to that of finding the cube root of B).[9]

[7] Heiberg in Apollonius, *Opera* vol. 2 p. 124.

[8] The recondite name might suggest Athenian origin. Pandrosos was a legendary Athenian heroine, daughter of Cecrops; the site of the sacred olive tree of the city was called the 'Pandroseion'. Neither the name Pandrosos nor the diminutive Pandrosion seems to have been common. The only attested name that comes close, to my knowledge, is an African 'Pandroseios', *Supplementum Epigraphicum Graecum* vol. 9 no. 532, from Teuchiris-Arsinoe (courtesy of G. J. Toomer). Pappus's Pandrosion has suffered strange indignities from Pappus's editors: in Commandino's Latin translation her name vanishes, leaving the absurdity of the polite epithet κρατίστη being treated as a name, "Cratiste"; while for no good reason Hultsch alters the text to make the name masculine.

[9] Pappus mentions in passing a philosopher Hierius, friend of the geometry student and an acquaintance of Pappus, who also asked for Pappus's opinion of the construction. This Hierius may be identifiable with a student of Iamblichus and teacher of Maximus referred to by Ammonius (commentary on the *Prior Analytics*, *CAG* vol. 4.6 p. 31). He is in turn, I think, the same as a philosopher Hierius mentioned by

Pappus gives a detailed refutation of the alleged solution (which is an iterative method), showing that it gives exact results only if one assumes to be given the very mean proportionals that are sought. Then he sets out a classification of geometrical problems into 'planar', 'solid', and 'curvilinear' types, depending on the resources necessary to solve them.[10] The problem of the two mean proportionals, he says, is 'solid', and so cannot be solved using only compass and straight-edge. Pappus then describes constructions of the two means by Eratosthenes, Nicomedes, Heron, and lastly one "discovered by us", all using extra resources (either a markable ruler or a special mechanically drawn curve). None of the quoted solutions uses conic sections (we know several ancient solutions that did); Pappus perhaps considered them too advanced for the intended readers of this book.

A second pupil tried to construct the three basic means between two magnitudes (the arithmetic, geometric, and harmonic means) by a minimal construction, in which as few lines and arcs are drawn as possible. Pappus finds fault with this because the harmonic mean produced is not the mean of the same quantities as the others. He therefore embarks on a lecture on the means, giving his own minimalist solution of the original problem, and definitions and constructions of seven more means.

A third problem is to construct within a triangle and on part of its base another triangle whose other two sides together are longer than the other two sides of the containing triangle. Pappus complains that the proposition is incompetently stated and proved in an "inexperienced" way. The same proposition, paraphrased only slightly and with the same figure, appears again in Proclus's commentary to Book 1 of Euclid's *Elements*. Since Proclus shows no sign of knowing Pappus's criticisms of it, it is possible that Pappus was criticizing on a written work that Proclus later consulted, or that Pandrosion's pupil took the construction from some book.[11] In the remainder of this section Pappus quotes a series of similar problems from the "so-called paradoxes of Erycinus", of whom and which we otherwise know nothing. The last (which may be Pappus's own) constructs a triangle smaller than a given triangle, but with sides greater than or equal to given multiples of the given triangle's sides.

Libanius (Oration 14 to Julian, chapters 7, 32, 34). According to Libanius, Julian would have wanted Hierius and his brother Diogenes at his court had they still been alive (this was in 362), just as in fact he has Maximus and Priscus. The remark would have more point if there were a well known connection between the two pairs of sages.

10 For the meaning of this terminology, see the notes to 7.22.

11 Proclus, *Elements I* ed. Friedlein, pp. 326-28.

The final section of Book 3 gives a series of constructions of the regular solids in a given sphere. These problems, which follow a different procedure from that of Book 13 of Euclid's *Elements*, are solved by analysis and synthesis. There is no connection whatever between the first parts of the book, which make up the letter to Pandrosion, and this last part.

Book 3 is followed by an appendix, "The tenth theorem in the third (book) of Pappus's Collection in another way, comprising the proof and the instrumental construction of both the doubling of the cube and the two mean proportionals." Under this title we have a long treatment of a variant of Pappus's solution of the mean proportional problem in the first part of the book.

c. **Book 4.** "Consists of exquisite theorems, planar, solid, and curvilinear." We have to take the title from the subscription at the end of the book, for in the Vaticanus the book begins, without a title, immediately following the appendix to Book 3. The book has no preface or dedication, and no overall governing plan.

The first section can only be characterized as miscellaneous theorems, probably jotted down in the course of reading other treatises. First we are given a interesting generalization of the 'Pythagorean' theorem for a general triangle and parallelograms erected on its sides. Then comes a pair of theorems in which certain line segments, produced in geometrical constructions with circles, are classified according to the system of irrational magnitudes that is set out in Book 10 of the *Elements*. There follows a series of propositions that are, at least in part, concerned with tangent circles. The theme of tangencies is developed in a series of propositions concerning packing circles in the 'arbelos', the space between two externally tangent semicircles that are also tangent internally to a third semicircle, all on a common diameter.

The remainder of Book 4 is devoted broadly to special curves: the Archimedean Spiral, the Cochloid of Nicomedes, the Quadratrix of Dinostratus and Nicomedes, and a spiral on the surface of the sphere. After that, Pappus expounds on the trisection of the angle, with an introduction that again discusses the division of problems into planar, solid, and curvilinear. He mentions in passing two alleged instances of abuse of the powerful 'solid' methods (that is, using conic sections) by Archimedes in the work *On Spirals* and by Apollonius in Book 5 of the *Conics*. According to Pappus, the problems in question can be solved by ruler and compass alone, but he does not elaborate on this topic. Instead he goes on to give a solution, which he implies is an old one, of the trisection problem, using the intersection of a circle and a hyperbola. After giving another comparable solution by "some people", Pappus turns to the general problem of dividing an angle in a given ratio. This, he says, is a curvilinear problem, and was solved by the "moderns". He gives solutions "written by ourselves", using the Quadratrix and the Archimedean Spiral, and demonstrates a few applications of the method to problems involving ratios of angles and circumferences. The book ends with an analysis of the neusis construction assumed by Archimedes in the book *On Spirals*, "in order that you will not be nonplussed when you work through the book".

d. Book 5. "Contains comparisons of plane figures having equal perimeter, with respect to each other and to the circle, and comparisons of solid figures having equal perimeter, with respect to each other and to the sphere." Of all the books of the *Collection*, this is the most polished. Pappus begins with an elaborate introduction, addressed to one Megethion, on how the hexagonal cells of honeycombs show that bees, like men, have some divinely furnished knowledge of geometry. For the hexagon is, among the regular figures that can pack the plane, the one that has greatest area in proportion to its perimeter, and so the bee uses the least material to store the most honey. This preface introduces the problem of proving that the circle has the greatest area of figures of equal perimeter, and that of regular isoperimetric polygons, that which has the most angles is the greatest. The sequence of theorems that purport to prove this proposition is adapted only slightly from a lost work on isoperimetric figures by the Hellenistic mathematician Zenodorus, which we know also from versions reported by Theon and Eutocius. He adds some unimportant generalizations to circular sectors.

A heading "On solids" introduces the second part of the book. Again we have a short preface: the philosophers maintain that (the Neoplatonist) God chose the sphere to shape the universe as the fairest of shapes, and assert that among the properties that make the sphere best is that of solid figures of equal surface it is the greatest. This, Pappus says, they have not been able to prove, but only to affirm. The treatment that follows will be limited to the sphere and regular solids. Among these are the five Platonic solids, but also the thirteen Archimedean semiregular solids, which Pappus enumerates, but, not surprisingly, chooses not to include in the comparison with the sphere. On the other hand, he does include a section on the solids of rotation of regular polygons, drawing on Archimedes's *On the Sphere and Cylinder*, and only after this digression addresses the isoperimetric regular solids and derives their relative volumes.

e. Book 6. "Contains resolutions of difficulties in the little (Domain) of Astronomy". Pappus's introduction (without dedication) complains that "many of those who teach the 'Domain of Astronomy' [ἀστρονομούμενος τόπος, the division of mathematics that furnishes the equipment for astronomy], because they attend carelessly to the propositions, add some things on the grounds that they are necessary, and omit others as unnecessary." Three instances are named, in Theodosius's *Spherics*, Euclid's *Phaenomena*, and Theodosius's *On Days and Nights*. Pappus proposes to explain and correct these errors. The book contains other things too, however, though without much plan. These include a synopsis of Autolycus *On the Sphere in Motion*, and a survey of Aristarchus *On the Sizes and Distances of Sun and Moon* with some comparison with the work of Ptolemy and Hipparchus and a lemma allegedly necessary to follow Aristarchus's argument. There is also an expansion of a theorem in Euclid's *Optics* concerning the projection of a circle through a point.

f. Book 7. "Contains lemmas of the 'Domain of Analysis' [ἀναλυόμενος τόπος]". Book 7 begins with a preface, addressed to a pupil Hermodorus, explaining geometrical analysis and synthesis and listing the books that make up the 'Domain of Analysis', the branch of mathematics that provides equipment for analysis of theorems and problems. There follow, first a series of synopses of most of these books, then sets of lemmas required for the reading of them. Following Book 7 is an appendix, "lemma of the 'Domain of Analysis' ", the relationship of which to the preceding material is not explained.

g. Book 8. "Contains miscellaneous exquisite mechanical problems." This book too is dedicated to Hermodorus. The preface describes the scope and divisions of mechanics, how Archimedes was the first to write on the subject, and its association with geometry. Pappus promises to take up several matters, on the drawing of weights on inclined surfaces, on the finding of the two mean proportionals, and on the proportioning of gears, as well as other topics "useful for architect and mechanician". He begins, however, with a series of propositions on centers of gravity. Then follows a section on the inclined plane, and the power necessary to draw a weight up it. Within the same topic, he continues, belongs the problem of moving a given weight by a given power, and as illustration of this he presents Heron's geared instrument called the 'baroulkos'.

Pappus turns now from what he calls the "things particularly pertinent to the topic of mechanics" to the "instrumental" topic, which encompasses so-called 'mechanical' methods of solving geometrical problems. The advantage of these over conic sections, according to Pappus, is that conics are in practice difficult to draw. He produces as an example the same solution of finding the two mean proportionals as he gave in Book 3 as his own. Another type of instrumental problem arises when the resources of geometry are restricted, "such as constructions by a single (compass) interval and the (problem) proposed by architects of a cylinder broken off at both bases", where the object is to find the cylinder's diameter. Pappus shows how to solve this second problem, using a construction of an ellipse through five given points. As further examples of the same kind of problem, he performs certain tasks concerning an elevated solid sphere, where the practical motivation is not obvious. In another example Pappus appears to recommend instrumental methods for drawing figures for analyses of problems, since otherwise one must anticipate the solution to prepare a suitable figure; but the illustration, constructing seven adjacent hexagons inscribed in a given circle, does not make the point clear. The promised discussion of gears follows, and then one of how to make a screw; these are finally combined in a simple machine taken from Heron's *Mechanics.* Excerpts from the same book of Heron, treating the five 'powers' or elemental machines, and various apparatus for lifting and moving weights, make up the remainder of the book, to the point where the Vaticanus abruptly ends.

§2.2. **Introduction to Mechanics.** In his commentary to the second book of Archimedes *On the Sphere and Cylinder* Eutocius quotes Pappus's method of finding the two mean proportionals as from his Μηχανικαὶ Εἰσαγωγαί, and this has long been understood as a reference to Book 8 of the *Collection.* Confirmation and modification of this opinion has come with the recent discovery of Book 8 in a ninth-century Arabic translation.[12] The Arabic version agrees in all major respects with the Greek, except for two things. First, the Arabic version is entitled "Introduction of Pappus to the science of mechanics", with no suggestion of its being part of a larger work. Secondly, the Arabic text preserves, not only the end of the book which is lost in the Greek, but also a long and interesting passage that comes after the construction of the seven hexagons. In it Pappus presents a series of constructions by fixed compass and straight-edge, leading to the construction by these limited means of a triangle, given its three sides. The last part of the book, which continues the adaptations from Heron, was apparently lost in Greek only after the extant archetype, the Vaticanus, was copied, so its presence in the translation does not illuminate the relationship between the two texts. The other passage, however, although we can see from the reference in the Greek text to this kind of construction (quoted above) that it is an intended and integral part of the book, is so neatly absent from the Greek version that we can scarcely suppose a 'mechanical' cause for its dropping out (damage to a manuscript, the careless eye of a copyist), nor is there any motive to excise it deliberately. The remaining possibility is that both recensions go back independently to Pappus, and that the version in the *Collection* is an earlier one in which the author had not yet had the opportunity to insert the fixed compass propositions.

§2.3. **Commentary on the *Almagest.*** Pappus's title for this work was (with trivial variations) σχόλια εἰς τὰ Κλαυδίου Πτολεμαίου μαθηματικὰ ('Notes on Claudius Ptolemy's *Mathematics*'). We have the commentaries to the fifth and sixth books, preserved in several manuscripts.[13] Remarks in these books show that Pappus had already written commentaries to the first and fourth books,

[12] Jackson [1972], [1980]. An edition by Jackson is forthcoming.

[13] Edited in Rome, *CA* vol. 1.

while Eutocius cites the third book in the "Prolegomena" to the *Almagest*.[14] Eutocius's version of the isoperimetric theorems too was likely adapted from Pappus's first book. It is reasonable (if not necessary) to assume that Pappus commented on the whole of Ptolemy's treatise, but no evidence for the seventh through thirteenth books is known. The *Souda*'s mention (in the article on Pappus) of a "commentary [ὑπόμνημα] on the *four* books of Ptolemy's Great Syntaxis" (*i.e.* the *Almagest*) probably reflects a confusion with Ptolemy's *Tetrabiblos*.

Except for some information on lost writings of Hipparchus, the surviving parts are of small historical value. Pappus gives little more than a verbose explanation of numerous points in Ptolemy's text that might make trouble for an inexperienced reader, with supplementary proofs of cases that Ptolemy considered too obvious to set out.

§2.4. **Commentary on Book 10 of Euclid's *Elements*.** This opuscule in two parts survives in the Arabic translation of the scholar al-Dimishqī (about A.D. 1000).[15] The attribution to Pappus (transliterated in accordance with normal Arabic practice for Greek names as "b.b.s" in unvocalized script— through misplaced dotting this easily became "b.y.s" or "b.t.s") was once in doubt, but there is evidence to support it.[16] The commentary is listed in the article on Pappus in Ibn al-Nadīm's *Fihrist* (a tenth-century encyclopedia of authors known to the Arabs) as a 'commentary on the tenth book of Euclid in two books".[17] Still more authoritative is a Greek scholion to Euclid's *Data*, which declares that "both rational [ῥητόν] and irrational [ἄλογον] can be given, as Pappus says in the beginning of the (commentary) on the tenth (book) of Euclid". The reference is to Book 1 chapter 7 (about a quarter of the way into Book 1 in the Arabic text), where Pappus discusses the commensurability of pairs of

14 Rome, *CA* vol. 1 pp. xvii-xviii.

15 Edited in Thomson – Junge [1930]. An incomplete Latin translation was made from the Arabic in the twelfth century; see Junge [1936].

16 Woepcke, who discovered the text, assigned it tentatively to the second-century astrologer Vettius Valens (Woepcke [1876] p. 17), but it later turned out that his reading of the author's name (as "b.l.s") was mistaken. Suter, [1922] p. 78, was led by the philosophical content of the commentary to suspect that it was written by Proclus; but Heiberg showed convincingly that Proclus's commentary on the *Elements* never extended beyond Book 1 (Heiberg, *LSE* pp. 165-68).

17 Ibn al-Nadīm, *Fihrist* (Flügel) p. 269, (Dodge) p. 642.

given rational or irrational magnitudes.[18] Several scholia to Book 10 of the *Elements* are derived from Pappus's commentary, but without attribution.[19]

The work is not a proposition-by-proposition commentary on Book 10 of Euclid, nor does it seem to have been part of a complete exegesis of the *Elements.* In the first part, Pappus gives a short history of the study of irrational magnitudes, an argument of why one should study it, a short synopsis of Book 10 of Euclid, discussion of the possibility of irrationals and incommensurables, a long review of the relevant passages in Plato, and again a more detailed summary of Book 10. The second part is devoted entirely to the various classes of ordered irrationals in Euclid and how they can be produced from one another by geometrical procedures. The book seems to have been composed for readers versed in philosophy, especially Neoplatonism, but with little mathematical background. For us the book is of only modest historical value, mostly for its allusion to a work by Apollonius on 'unordered' irrationals (about which, however, Pappus tells us nothing substantial).

§2.5. Chorography of the Inhabited World. The *Souda* mentions among Pappus's works a χωρογραφία οἰκουμενική, and extensive fragments of this work can be extracted from a seventh-century Armenian geography (*Ašxarhac'oyc'*) of uncertain authorship.[20] From these extracts it appears that Pappus followed the arrangement of regions of Ptolemy's *Geography*, providing amusing and instructive descriptions of the lands and the wonderful things to be found in them (hippocentaurs, Amazons, man-eating and wine-loving beasts).

§2.6. The Rivers in Libya.
§2.7. Interpretation of Dreams. These two works are known only from the article in the *Souda.*

§2.8. Commentaries or notes on Euclid's *Elements*. Several times in his commentary on Book 1 of the *Elements* Proclus cites remarks of Pappus, without specifying the work in question.[21] Eutocius credits

18 Euclid, *Opera* vol. 6 p. 262; Thomson – Junge, p. 70. In his *DSB* article (p. 302 note 32) Bulmer-Thomas mistakenly writes that nothing in the opening section of the commentary corresponds to the scholion.

19 Heiberg, *LSE* pp. 170-171.

20 Hewsen [1971].

21 Proclus *Elements I* ed. Friedlein, pp. 189, 197, 249, 429. The phrase

Pappus with a commentary (ὑπόμνημα) on the *Elements*, in which he demonstrated the construction of a polygon inscribed in a given circle and similar to a given polygon inscribed in another circle.[22] These references suggest a collection of notes on specific passages in the *Elements*, not a freely composed review like the extant treatise on Book 10.

§2.9. **Commentary on Ptolemy's *Planispherium*.** We know of this only from the *Fihrist* of ibn al-Nadīm which reports that Thābit ibn Qurra translated it into Arabic.[23] The *Planispherium*, which itself is extant only in Arabic translation, is an early treatise on stereographic projection.

§2.10. **Commentary on Diodorus's *Analemma*.** The 'analemma' was a method of solving problems in spherical geometry (arising in astronomical applications such as sundial theory) by means of geometrical constructions in the plane.[24] The treatise of Diodorus (first century B.C.) on the subject is lost, although an Arabic translation of it existed in the middle ages. Only a few second-hand 'fragments' of it have been identified so far. According to *Collection* 4.40, Pappus exposed Nicomedes's trisection of the angle in his commentary on Diodorus's work (ἐν τῶι εἰς τὸ ἀνάλημμα Διοδώρου). Neugebauer points out that the trisection problem would be useful for constructions related to the length of a seasonal hour. We have no further information on his commentary. In the Milan palimpsest (Ambros. L 99 sup.) that contains what we have of the Greek text of Ptolemy's *Analemma* as well as the "Bobbio mathematical fragment", there are several pages that contain parts of a work on the analemma employing a system of coordinate angles that Ptolemy repudiates; these may belong to Diodorus's lost treatise or Pappus's commentary, but the writing has so far been deciphered only in short fragments.

"οἱ περὶ Ἥρωνα καὶ Πάππον" used on p. 429 is merely a periphrasis for "Heron and Pappus". References to Pappus in the Arabic commentator on Euclid al-Nairīzī coincide with those in Proclus (see the index *s.v.* Pappus in Curtze's edition, supplement to Euclid, *Opera*).

22 Archimedes *Opera* vol. 3 p. 28.

23 *Fihrist* (Flügel) p. 269, (Dodge) p. 642.

24 See Neugebauer, *HAMA* vol. 2 pp. 839-856.

§2.11. Ἡμεροδρόμιον Πάππου τῶν διεπόντων καὶ πολευόντων (a kind of astrological almanac relating each hour of each day of the week to the planets and to certain actions and consequences). This short piece is found in an eleventh-century compilation of astrological texts, Florence Laur. 28,34, f. 137r. Pappian authorship can not be proved or disproved. The attributions in astrological anthologies are notoriously untrustworthy, but this manuscript earns some credibility from the antiquity of other things in it, for example a horoscope for 497 plausibly ascribed to Eutocius.[25]

A curious fragment preserved in a thirteenth-century astrological manuscript, Vind. phil. gr. 115, f. 120r, appears to confirm that Pappus wrote something on astrology.[26] Embedded in excerpts from Hephaestion of Thebes is, irrelevantly, the observation "that a certain pious Pappus says that an unfortunate person (?) obtained an oracle in the Serapeum of Alexandria who was bemoaning his poverty. The oracle given him by the god Serapis was as follows: blame not fate, not gods, not spirits; but blame the hour when your father begot you."[27] Whatever the provenance of this anecdote, it must refer to a time before the destruction of the Serapeum at the end of the fourth century, and our Pappus stands a good chance of being the one in question.

§2.12. Lastly, a Greek alchemical oath and formula appear in manuscripts under the name of "Pappus the philosopher".[28] Tannery reasonably argued that an attribution to Pappus is not as likely to be fraudulent as one to an ancient or legendary authority in this kind of text.[29] But it does not follow that the whole is genuine. The recipe is certainly late; it refers to Stephanus of Alexandria. By itself, the oath says

25 Neugebauer – Van Hoesen [1959] pp. 152-157, 188-89.

26 I thank Prof. David Pingree for showing me this interesting unpublished text.

27 ὅτι φησὶ Πάππος τις θεοφιλὴς, †ἀτυχον† χρισμὸν εἴληφεν ἐν τῶι Σεραπίωι Ἀλεξανδρείας ὃς ἀπωδύρετο πενίας. ὁ δὲ χρισμὸς οὕτως εἶχεν ὁ παρὰ τοῦ θεοῦ δοθεὶς Σεράπιδος·
Μὴ μέμφου μοῖραν, μὴ θεούς, μὴ δαίμονας·
ὥραν δὲ μέμφου ἣν πατὴρ ἔσπειρέ σε.

28 Berthelot – Ruelle [1888] vol. 3 pp. 27-28.

29 Tannery [1896].

nothing about alchemy: "In oath therefore I swear the great oath to you, whoever you are, God I say, the one, the (one) in form and not in number, that made [the heaven and the earth] both the 'tetractys' of the elements and the things (that originate) from them, and that furthermore fitted our reasoning and intuitive souls to body, [borne on cherubic chariots and hymned by angelic hosts]."[30] The bracketed words are surely interpolations, inserted for obvious reasons, perhaps by the Byzantine adaptor who prefixed the oath to the alchemical material. The rest is pure Neoplatonism.[31]

§2.13. **Dubious works and false attributions.** Three works on music theory allegedly by Pappus can probably be rejected from the canon. An "introduction to harmony" is attributed in some manuscripts to Pappus, in others to Cleonides; the latter assignment is now generally accepted, though the evidence falls short of being conclusive.[32] The claim that Pappus wrote the latter part of Porphyry's commentary on Ptolemy's *Harmonics* has been repeated numerous times, on no more basis, apparently, than the misreading of ΠΑΠΠΟΥ for ΤΑΥΤΟΥ in a section title in Isaac Argyrus's recension of the work.[33] An opuscule called "Book of the elements of music" appears in the Arabic manuscript Manisa Genel 1705/9, ff. 126b – 133b as the work of "Būl.s", whom Sezgin has identified tentatively as Pappus.[34] The name should surely be read as "Paulus", and

30 ὅρκωι οὖν ὄμνυμί σοι τὸν μέγαν ὅρκον, ὅστις ἄν συ ἦι, θεόν φημι τὸν ἕνα, τὸν εἴδει καὶ οὐ τῶι ἀριθμῶι, τὸν ποιήσαντα [τὸν οὐρανὸν καὶ τὴν γῆν] τῶν τε στοιχείων τὴν τετρακτὺν καὶ τὰ ἐξ αὐτῶν, ἔτι δὲ καὶ τὰς ἡμετέρας ψυχὰς λογικάς τε καὶ νοερὰς, ἁρμόσαντα σώματι [τὸν ἐπὶ ἁρμάτων χερουβικῶν ἐποχούμενον, καὶ ὑπὸ ταγμάτων ἀγγελικῶν ἀνυμνούμενον].

31 Tannery concluded from the apparently syncretistic content of the oath that Pappus was some sort of gnostic, a theory repeated by Bulmer-Thomas in his *DSB* article, p. 301. But the Biblical language is crudely integrated with the rest, and the possibility of Byzantine meddling is too great to justify such a remarkable hypothesis.

32 *Musici scriptores graeci* ed. K. Jan (Leipzig: 1895), pp. 169-74.

33 Düring [1932] pp. xxvi and xxxvii-xxxix.

34 Sezgin, *GAS* vol. 5 p. 176. As before periods represent vowels that the

in any case since the work quotes Ammonius right at the beginning, it cannot be from the fourth century. Similarly the references to "Būl.s" in works by al-Bīrūnī are not to Pappus but to the Sanskrit *Pauliśasiddhānta*, and its putative author Paulos.[35] Chapter 5, section 7 of al-Khāzinī's *Balance of Wisdom* describes an instrument for measuring the density of liquids by "Fūf.s the Greek", who has again been supposed to be Pappus, because of a far-fetched resemblance of name.[36] The device is unquestionably of Greek origin, for Synesius gives a description of it that is perfectly compatible with the Arabic account in his letter 154 to Hypatia. Significantly, both texts say that the areometer is useful for medical applications. My guess is that al-Khāzinī's source was not "Fūf.s" but "Rūf.s", that is Rufus, many of whose medical writings were translated into Arabic (a misreading of the letter 'rā' as 'fā' is possible in some scripts).

It has been inferred from a remark of Marinus that Pappus wrote a commentary on Euclid's *Data*. We will consider whether this commentary was distinct from the relevant section of Book 7 of the *Collection* below (see page 21), together with the other testimonia for early knowledge of books of the *Collection*. Boll induced a recension by Pappus of the *Handy Tables* of Ptolemy on the basis only of a mistaken dating of the "Helios" diagram in the *Handy Tables* manuscript Vat. gr. 1291.[37]

§3. Integrity and Composition of the *Collection*. The *Collection* has often been regarded as a kind of encyclopedia of Greek mathematics, a compendium in which Pappus attempted to encompass all the most valuable accomplishments of the past.[38] However, it exhibits anomalies that are difficult to explain if that description is correct, but that become intelligible if we suppose the *Collection* to have been originally, not a single work, but in fact a collection of separate shorter works, brought together with only the most superficial effort to integrate them. The title Συναγωγή would have been exactly suited to such a volume of 'collected

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Arabic script leaves ambiguous.

35 Sezgin, p. 176. See Pingree [1969] for the correct identification of the citations.

36 Khanikoff [1860] pp. 40-52.

37 Vat. gr. 1291, f. 9r. See Neugebauer, *HAMA* vol. 2 p. 978, especially note 3.

38 Notable exceptions are Ziegler (see note 4 above) and Jackson [1972].

works'.[39]

The individual books are dissimilar in genre. For example, Books 5 and 8 and the first part of Book 3 appear as self-standing 'publishable' pieces.[40] Of these, Book 8 is an introductory textbook, while the preface to Book 3 shows it to be an occasional, polemical composition. These books avoid requiring that the reader have access to other texts to be able to follow the mathematical reasoning. Books 2, 6, and 7, on the other hand, were intended to accompany the reading of older texts, and without them become in parts unintelligible and in general useless. The latter part of Book 4 seems also to be related to the reading of Archimedes's *On Spirals*, although the topics are mostly introductory or digressive.[41] The first part of the book has no apparent plan.

Of the six books whose beginnings are extant, only four have dedications, to three different people. Since in antiquity the dedicatee was in fact the principal recipient of the work, it would make no sense to dedicate one part of a single composition to one person, another part to another, even if the various sections were completed over a long time, unless, say, the first dedicatee died (as was the case with Apollonius's *Conics*) – and in such cases an explanation would be in order.[42]

39 Among several ancient parallels is Cicero's letter to Atticus XVI, 5: "mearum epistularum nulla est συναγωγή" ("there is no συναγωγή of my letters"); from what follows it is clear that Cicero meant, not a file of duplicates (which his amanuensis had), but a comprehensive transcript.

40 'Publication' should be understood as a translation of ἔκδοσις, and may signify no more than an authorized copy that the writer or redactor permits to be reproduced. There probably would not have been a demand for large numbers of copies of advanced mathematical texts at any time in antiquity.

41 Some of Pappus's discussion may derive from an otherwise unattested Archimedean work earlier than the *On Spirals*; see Knorr [1978,2]. Some of the material that Knorr ascribes to this hypothetical work of Archimedes probably comes from later authors.

42 An exceptional instance of a change of dedication in a work of assured integrity where such an explanation is missing is the longer commentary of Theon on Ptolemy's *Handy Tables*, but that work's textual transmission is extremely problematic. See Mogenet – Tihon [1981] pp. 526-29, who believe that the tradition descends from an unauthorized copy, arguing from the state of the text. Some

The sequence of subjects in the *Collection* is disorganized and illogical if it is meant to be a survey of all mathematics. Book 5 is largely devoted to the geometry of regular solids; yet Book 3 ends with a section on inscribing the solids in a sphere, which is unrelated to the rest of that book. Tangency problems are discussed in Book 4, but no reference is made there to Apollonius's *Tangencies*, which Pappus takes up in Book 7. The typology of problems into planar, solid, and curvilinear is brought up redundantly in Books 3, 4, and 7. Moreover, the topics treated are sometimes highly specialized and of minor significance compared to subjects that are omitted (one is, of course, free to hypothesize lost books after Book 8 that contained some of these). To include Book 2's puerile number games in a work that also contains the subtle theorems on spirals in Book 4 would imply a strange sense of proportion; while the absence of discussion of the geometry of conic sections (while nevertheless expecting the reader of Books 4 and 8 to know a fair amount about them) is, to say the least, puzzling.

Considering that the books often overlap in subject matter, it is also odd that they never refer to one another (Pappus's announcement near the beginning of Book 3 of what he intends to do ἐν τῶι τρίτωι τούτωι τῆς συναγωγῆς βιβλίωι, "in this third book of the Collection", is not significant: he or his redactor would automatically have changed such a phrase as "in this letter" to one more appropriate for inclusion in a volume of collected works). One instance is notable: in 8.46 Pappus invokes a lemma (that the rectangle contained by the circumference of a circle and its radius is twice the circle's area), and refers to his own proof in the commentary to Book 1 of the *Almagest*; yet the lemma has been given already in Book 5 of the *Collection* (5.6). Much of the repetitiveness of the *Collection* could be attributed to Pappus's style and carelessness. In some instances, though, the doublets are on too large a scale to have escaped the most inattentive author. Pappus presents (at length) Nicomedes's method of finding the two mean proportionals in 3.24 and again in 4.40-44. The central parts of the two passages are, except for a few trivialities and the exchanging of two letters on the figure, word for word identical. Again, Pappus's own solution of the problem is given in 3.27 and, identically, in 8.26; we also have a variant of it (unexplained) in the appendix to Book 3. Pappus's classifications of problems in 3.20 and 4.57 are not merely similar, but often word-for-word identical, and there are similar exact verbal parallels between 3.21 and .27, and 8.25. Pappus must, in these cases, have had the one version in front of him while writing the other (or, less likely, have taken them both from a third, vanished version).

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interesting chronological problems were already signalled by Rome [1939], pp. 213-14. These suggest that our text somehow combines elements of two editions of the long commentary.

The first example given above has further convolutions. In 8.46, Pappus proves that circles' circumferences are proportional to their diameters. This proof depends on a lemma, that twice the area of a circle equals the product of its circumference and its radius; Pappus says that this was proved by Archimedes (in the *Measurement of the Circle*), and by himself, as a single theorem, in his commentary to the *Almagest*, Book 1. The lemma has, however, appeared in the *Collection* already, as 5.6, where Pappus again writes that Archimedes had proved it; it is there because 5.5 requires it. 8.46 itself is identical to 5.21. A subsequent proposition, 5.23, reappears in the commentary to Book 6 of the *Almagest* (Rome, pp. 254-58); this theorem uses the lemma 5.6 too, and in the commentary to Book 6 Pappus again says that it is to be found in Archimedes and in his commentary to Book 1. What is important to note in this tangle of cross-references and duplications is that each part of it is manifestly required by the context in which it appears, so that the repetitions cannot plausibly be ascribed to a later interpolator.[43] But if Pappus himself knowingly included these passages in more than one book, he can hardly have intended these books primarily as components of a unified work.

§4. Interpolations. Since Pappus's autographs do not survive, the question of how the text transmitted in the Vaticanus differs from them, though it cannot be answered definitely, is important to raise. We know from secondary sources (Theon, Eutocius) that ancient editors interfered with the texts of such treatises as Euclid's *Elements*, Apollonius's *Conics*, and some works of Archimedes, most conspicuously by adding new material. To decide whether the same was true of the *Collection*, we have to depend on the more precarious evidence of the text itself, supported by what we know of the work's reception in the early Byzantine period.

There is no infallible test to distinguish interpolated from authentic text (even disregarding the more insidious possibility of text revised by a later hand). A common-sense principle to assist editorial judgement is that a passage should be bracketed as interpolation only if its presence in the text is distinctly more plausible as an intrusion than as the author's work.

43 Thus Ziegler (see note 4 above) rightly rejects Rome's complicated explanation of this complex of repetitions (Rome, *CA* vol. 1 pp. 254-55 note 1). Rome regards parenthetic references, occurring identically in both 5.23 and its parallel in the commentary to the *Almagest* Book 6, to the *Elements* and to Theodosius's *Spherics*, as spurious (on the basis of an unfounded assumption about Pappus's 'normal practice', as if this would be the same in all his writings for all kinds of anticipated reader), so that consequently their presence in both places would prove that one is interpolated.

Although many passages in Pappus's *Collection*, and particularly in Book 7, are difficult to make sense of, few of these become more explicable if a later meddler is hypothetically introduced. Hultsch was very liberal with brackets in his edition of the *Collection*, and still more passages, though left unmarked in the text, are noted as suspect in his apparatus. The scope of his commentary allowed too little room for him to justify his editorial decisions, and it would be futile to discuss them all here. Some of them, however, are illustrations of how interpolations should not be identified.

For example, 7.64 shows how it is possible to construct geometrically a figure that is used in Apollonius's *Cutting off of a Ratio*. Between the enunciation and the solution of the problem, however, is a passage that makes little sense as it stands in the manuscript: it seems to stipulate certain requirements on the given magnitudes that inspection shows are neither required for the ensuing solution nor consistent with the problem. Hultsch (following Halley here) brackets the problematic sentences, making one emendation to the supposed interpolation (οἷόν τε for οἴονται) that, while probably correct, does not by itself make the meaning clear. The mere fact that certain sentences do not make sense as they are transmitted does not make them more probably spurious than genuine: in either case, whoever wrote them must have meant something, and it is only after we have recovered the meaning that we can decide on authenticity. In the present instance, the mathematical sense has been obscured by two simple corruptions in the notational letters; once these have been restored, the passage turns out to give the conditions for an alternative, arithmetical solution of the problem.

There is even less justification for Hultsch's deletion of the whole of chapters 7.41-42 (except for most of the last sentence of 7.42). Chapters 7.33-42, which bring the introductory part of Book 7 to a close, make up a great blast of Pappian invective, first against Apollonius's presumption in criticizing Euclid, then against the decadence of later mathematicians up to Pappus's own time. Pappus finishes by saying that he at least tries to do better things, and gives as proof of this claim the enunciation of a theorem about the volumes of solids of revolution (see the notes to 7.42). Hultsch seems to have judged the style of the final paragraphs to be too late for Pappus;[44] but the only real peculiarities in the transmitted text are not Byzantinisms, but probably corruptions (ἐγώ for ἔχω, πρὸς τοῖς for πρὸς ὀρθὰς τοῖς). And in any case it is difficult to see why a later hand should have wanted to foist this theorem on Pappus.[45]

[44] Hultsch, *PAC* p. 683.

[45] A curious involution of Hultsch's interpolations is Knorr's ([1982,2]) suggestion that the first part of the invective (chapters 7.32 and

In fact there is scant evidence to suggest that anyone introduced any significant interpolations in the *Collection* after it was assembled.[46] Nor is this conclusion inherently improbable. Ironically, a late, secondary commentator would have been a less attractive victim for interpolation than the much-studied Euclids and Archimedeses whose works were vulgarized by well-meaning pedagogues. Moreover, the accident of a unique manuscript's being in a place unfrequented by scholars could have protected Pappus's text from tampering during the comparatively short time separating him from the extant manuscript tradition.

§5. **The Marginalia.** The Vaticanus's margins contain annotations that have been called 'scholia', an expression that suggests prejudicially that they are all later than Pappus. These marginalia are limited to Books 5, 6, and 7.[47] Those to Book 7 are few and do indeed resemble the sort of notes that a reader might make, marking interesting points such as where Pappus says that Euclid wrote on conics. They are reproduced in Appendix 1. The more extensive notes to the other books seem to be for the greater part by Pappus, and to provide afterthoughts and expansions.[48] In Book 5 these include a lemma associated with Theodosius's *Spherics*, and additional information on the composition of the Archimedean semiregular solids, which it is not probable that a later reader would have possessed or bothered to add. In the astronomical Book 6, in addition to a number of supplements to the mathematical arguments, the marginalia include many references to propositions in Euclid's *Elements* that are invoked in the text.

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following) is really by a hypothetical Hellenistic geometer, Aristaeus the younger (on whom see the notes to 7.1), while the closing theorem might be by Dionysodorus. It may well be that certain of Pappus's phrases would sound better from another mouth; but one has to explain how such fragments could end up in the middle of the *Collection*, impersonating Pappus's opinions.

46 For the possibility that the last sentence of 7.6 is spurious, see the notes to that chapter.

47 Printed in Hultsch, *PAC* vol. 3 pp. 1166-88. Following Hultsch, I do not include in the marginalia a few insignificant contributions by late hands, nor the original proposition numbers, nor the additions of the second hand that are merely corrections to the text. For evidence that Book 3 originally had marginal notes, see the commentary to 7.6.

48 There are exceptions: the remark "pretty drawing" on f. 111r is not likely to be Pappus's self-compliment.

Such references are rare in Pappus's writings on higher geometry, but significantly they do occur in his commentary on Ptolemy's *Almagest*: it appears that students studying astronomy were not expected to be always able to provide these for themselves.

§6. Early references to the *Collection*. If the *Collection* is no more than an assembly of already written works, then at least those parts that bear dedications must have been issued publicly, or have been meant for publication. In fact we have what must be the 'published' version of Book 8 as the "Introduction to Mechanics" preserved in Arabic. A medieval notice of the *Collection* made, apparently, before the loss of the beginning suggests that the lost Book 1 was the commentary on Book 10 of the *Elements*, which again we have in an Arabic translation, with no sign that it is an excerpt of a larger work.[49] Furthermore, the few references that appear to pertain to the *Collection* in subsequent works as late as the sixth century seem to be based on the separate editions, if on Pappus at all.

Two passages in Marinus's introduction to Euclid's *Data* may refer to Book 7. The first and longer says:[50]

> Now that the (concept of) 'given' has been defined more broadly and with a view to immediate application, the next point would be to reveal how the application of it is useful. This is in fact one of the things that have their goal in something else; for the knowledge of it is absolutely necessary for what is called the 'Domain of Analysis' [ἀναλυόμενος τόπος]. What power the 'Domain of Analysis' has in the mathematical sciences and those that are closely related to it, optics and music theory, has been precisely stated elsewhere, and that analysis is the way to discover proof, and how it helps us in finding the proof of similar things, and that it is a greater thing to acquire the power of analysis than to have proofs of many particular things.

The other passage is:[51]

49 See below, page 46.

50 Euclid, *Opera* vol. 6 pp. 252-54.

51 Euclid, vol. 6 p. 256.

> (Euclid) has not followed the synthetic manner of exposition there (in the *Data*) but the analytic, as Pappus showed competently in the commentaries [ὑπομνήμασιν] to the book.

Of these references, it can only be said that they do not closely follow the text we have of Book 7, Pappus's commentary on the 'Domain of Analysis', which includes a discussion of the *Data*. This could mean that Marinus had other sources, including an otherwise unattested commentary on the *Data* different from the one in Book 7; or that he had a version of Book 7 that differed from ours in significant ways; or, in the first passage, that he had himself written an introduction to the 'Domain of Analysis'. He may also be distorting from memory.

With Eutocius we can be more sure, because his citations of other authors are usually accurate. We have already seen that he quoted Pappus's solution of the two mean proportionals problem as from the Μηχανικαὶ εἰσαγωγαί, which is certainly the separate edition of Book 8, not that of the *Collection* which omits the authentic title.[52] The quotation from Pappus is part of a series of solutions of the problem, by 'Plato', Heron, Philon, Apollonius, Diocles, Pappus, Sporus, Menaechmus, Archytas, Eratosthenes, and Nicomedes. Since this canon of authorities includes the four that Pappus drew on for the similar series in Book 3, it is tempting to see whether Eutocius shows any sign in the other solutions that he knows Pappus's Book 3. There is none. Eutocius's solution of Heron comes from the *Belopoiika*, while Pappus's Heronic version is adapted from the *Mechanics*. Eutocius quotes *in extenso* an alleged letter of Eratosthenes from which Pappus probably derived his information. The central theorem in Eutocius's section on Nicomedes is close to that in Pappus Book 3, almost identical to that in Book 4; but since Eutocius's material includes sections, apparently quoted from Nicomedes, that Pappus omits, the similarity of the two texts must be the consequence of accurate copying of a common source. In this same passage Pappus alludes to a solution by Apollonius using conic sections. This solution can be reconstructed from various sources (see the notes to 7.276); it turns out to be mathematically related to, but significantly distinct from, the solution attributed to Apollonius by Eutocius (which uses 'mechanical' methods, not conics).

Eutocius seems also to have known a work by Pappus that covered some of the same topics as our Book 7. In his commentary on Apollonius's *Conics*, Eutocius has a discussion of Apollonius's description of Book 3 of the *Conics* as pertinent to the syntheses of loci, and especially the "locus on

[52] Commentary to *Sphere and Cylinder Book 2*, in Archimedes, *Opera* vol. 3 pp. 70-74.

three and four lines". Eutocius refers to Pappus by name:[53]

> He (Apollonius) then criticizes Euclid, not, as Pappus and some others suppose, because he (Euclid) had not found two mean proportionals; for Euclid found the one mean proportional, but not, as he (Apollonius) says, infelicitously, and he did not undertake to inquire at all about the two mean proportionals in the *Elements*, while Apollonius himself does not seem to make any inquiry about the two mean proportionals in the third book. Rather, as it appears, he is referring to another book on loci written by Euclid, which has not reached us.

This is a very curious statement. One can understand how Apollonius's expression, "τὸν ἐπὶ τρεῖς καὶ τέσσαρας γραμμὰς τόπον," could be misconstrued as "the topic (τόπος) of three and four lines (in ratio)", and evidently some lost commentators to Apollonius made that mistake. In the version of Book 7 that we have, however, Pappus does not; quite the contrary: he follows an ill tempered paragraph comparing Apollonius's character unfavorably with Euclid's (7.35) with a detailed digression correctly explaining the three and four line locus (7.36), which wanders into an invective against the state of geometry in his time. It seems impossible, then, that Eutocius can have seen this passage, not only because he attributes a wrong explanation to Pappus, but because he has no detailed knowledge of the correct one. Yet immediately before his allusion to Pappus and the others he makes a remark that is probably adapted from another place in Book 7:[54]

> Plane loci, then, are like that. But the loci that are called solid have acquired the name from the fact that the curves by which the problems in them are drawn get their origin from the section of solids, such as sections of the cone and many others. *There are also other loci called on the surface* [τόποι πρὸς ἐπιφάνειαν λεγόμενοι], *which get their name from the property concerning them* [ἀπὸ τῆς περὶ αὐτοὺς ἰδιότητος].

Why does Eutocius give such a vague explanation of the name of the surface-loci? Very likely because he knows nothing about them except that they exist, a fact he would have learned from 7.22, where Pappus discusses

[53] Apollonius, *Opera* vol. 2 p. 186. See Essay C, section §7, on the locus.

[54] Apollonius, vol. 2 p. 185.

the loci in general. But the obfuscating phrase itself may betray Eutocius's source, for in the same chapter 7.22 Pappus uses nearly the same nebulous expression (ἀπὸ τῆς ἰδιότητος τῶν ὑποθέσεων) to justify the naming of Eratosthenes's "loci with respect to means."

Eutocius, then, probably had a version of Pappus's Book 7, but that version cannot have included the excursus on the multi-line locus. We are not absolutely compelled to believe that the version Eutocius saw contained the erroneous reference to the problem of the mean proportionals that Eutocius refutes; that just might have been stated by the "some others" and merely conflated with Pappus's vituperation by Eutocius (but this seems improbable). If Pappus did give this misinformation, then Eutocius must have had an earlier version of some of the material in Book 7, antedating Pappus's discovery of the true meaning of the multi-line locus.[55]

Book 7 seems to have been intended originally to accompany the works of Euclid and Apollonius for which it provides summaries and lemmas. It is not surprising, then, that Eutocius's reminiscences of Book 7, quoted above, immediately follow a theorem taken from "Apollonius in the 'Domain of Analysis' ", which is in fact a fragment of the lost *Plane Loci*.[56] Eutocius is the last Greek known to have seen this work, or the complete *Conics*, which is also among the books discussed in Book 7.

§7. **Foul Papers.** Did Pappus himself assemble the *Collection*? We have seen that the parts are put together so haphazardly and ineptly that it is difficult to believe that Pappus, for all his imperfections, could have done himself such an injustice. But if someone else was responsible for it, at what remove was this editor from Pappus, and what sort of 'copy' was

55 Knorr has objected that Eutocius's explanation of Apollonius's nomenclature of the conic sections is wrong, and so, since Pappus gives a substantially correct version, Eutocius cannot have known Book 7 (Knorr [1982,2] pp. 284-85). He does not explain what work of Pappus he thinks Eutocius meant in the other passage. But Eutocius had other authorities on the *Conics*, and we cannot presume on the wisdom of his judgement of which account to follow. The etymology of 'parabola', 'ellipse', and 'hyperbola' that Pappus gives, based on the 'application of areas' in Apollonius's standard representation of the curves as loci, is closer to the truth (it is not quite correct), but the explanations that Eutocius gives are much simpler to understand. See the commentary to 7.30.

56 See Essay A, section §8.

he working from? If all the parts could stand by themselves, the collecting of them might have occurred, in principle, at any time up to the ninth century, because the first reference unambiguously to the *Collection* could be that late.[57]

But there are outstanding reasons to believe that the editor was working, not from 'published' texts, even when, as for Book 8 and probably Books 1 and 7, they seem to have had some circulation, but from drafts and notes, in other words from Pappus's 'foul papers'. Thus the composite character of Books 3 and 4 suggests that the editor's source did not clearly mark by titles or other indications when one opuscule ended and another began. The appendices to Books 3 and 7 seem to have been stray notes that the editor inserted where they seemed appropriate. Book 4 lacks a title and preface, and the material making up the first part of the book is a random and obscure assembly, probably derived from a notebook in which Pappus recorded theorems of interest. The second part of the book, perhaps intended as an introduction to Archimedes's *On Spirals*, cannot be even nearly a finished work, to judge by its abrupt changes of topic, false starts, and general incoherence. Even Book 5 (the most straightforward of the books) has a frayed patch. In 5.14 Pappus invokes a lemma, that if $a:b = c:d$ and $e:f = g:h$, then $(a+e):(b+f) = (c+g):(d+h)$, and promises that this will be proved presently. In the place where we should expect it (5.17) there is merely a tag, "The other one of the things that were put off." Nor is that surprising, because the lemma is false. But something that looks like a futile effort to prove it appears among the marginalia, and this may be a trace of Pappus's revision.[58] The theorem (5.16) that comes before the tag, itself a digression from the main mathematical argument and intended only to prove an incidental point, is very sloppily executed, as if it were only a first attempt to work out the proof. We have seen already that the missing section on fixed compass constructions in Book 8, rather than having dropped out, appears not yet to have been inserted. This may be another instance of Pappus's habit of stitching his works together from other writings, by himself and earlier authors.

Book 7 in particular shows traits of a draft. For example, the section in the introductory part that discusses Apollonius's *Conics* makes a false start in 7.29, then begins anew in 7.30. In 7.31 Pappus carelessly writes that the pre-Apollonian conics were generated by a plane intersecting the cone parallel to a generator, when he clearly means perpendicular. Discussing Apollonius's *Tangencies* (7.11), he lists the ten possible

57 See below, page 36.

58 Hultsch, *PAC* vol. 3 p. 1168.

combinations of three things to which a circle can be required tangent, but the order in which he lists them does not agree with his indication, a few lines later, of which were treated in each book of Apollonius. Section titles, identifying the books that Pappus is commenting on, are only sometimes provided, not always in the right place. Lemmas associated with identified theorems are often not in the correct order. Some proofs are garbled by systematic confusion among the points and letters. The book ends with a fragmentary and error-tainted section on Euclid's *Loci on Surfaces*, which was not promised in the preface. Most of these anomalies must originate with the author, and it is not probable that he would have allowed them to stand in a 'published' work.

The *Collection*, therefore, appears as a volume of collected works, put together by an editor whom we could describe as a 'literary executor', and who was more concerned with faithfully preserving Pappus's various papers than with creating an intelligible or useful work. Probably compiled shortly after Pappus's death, some time in the middle of the fourth century, it would not have circulated widely, much of it being not merely useless, but unintelligible except to a reader thoroughly versed in advanced mathematical texts — and such readers were not common between the fourth century and the late Renaissance.

§8. The proarchetypes. At least two copyings separate the text in the Vaticanus from Pappus's autographs: the original transcriptions by the 'editor' of the *Collection*, about the middle of the fourth century, and the making of the Vaticanus itself perhaps five centuries and a half later. There is, as far as I know, no certain evidence that the transmission had more stages in between; caution forbids a definite judgement, because the conservative character of the earlier Greek book hands makes it very difficult, if not impossible, to separate the strata of antecedent exemplars out of the errors of a single manuscript.

It has been said of the Vaticanus's text of Pappus that "all the errors are misreadings of uncials or uncial abbreviations."[59] This is not literally true, since of course there are certain to be errors that have nothing to do with palaeography. One type of mistake in this class is the very common transposition of label letters, for example ΑΒΓ for ΑΓΒ. Misreading may sometimes account for this error, but most often it must have happened in the copyist's mind. However, it is true that there do not seem to be any traces of an exemplar in minuscule.

59 Treweek [1957] p. 208.

The label letters in the geometrical arguments provide a rough indication of the ease with which copyists confused various pairs of capitals, since they can usually be restored with certainty on purely mathematical grounds. Most of the more common confusions of letters (Α/Δ, Ε/Θ, Α/Λ, Β/Ε, Η/Ν) are typical mistakes in copying a text in capitals. The many Γ/Ε errors suggest that these letters were very narrow in an ancestral manuscript, while a hand slanting to the right would explain the numerous Β/Δ confusions.

Moreover, at least one ancestor of the Vaticanus used extensive abbreviation, especially of mathematical terminology. The text of Pappus in the Vaticanus is almost entirely free of compendia (however, the marginalia bristle with them, as is common in the scholia of early minuscule manuscripts). The few exceptions are likely to be deliberate retentions from the exemplar, usually because the copyist was uncertain of the correct resolution. Examples of this kind of compendium, including symbols for very common words and truncations of others to their initial letters, are common already in papyri from antiquity, though not in literary texts.[60] It is very probable that abbreviation was actually normal in certain kinds of technical, especially mathematical, texts by the early Byzantine period. Manuscripts of geometrical texts written before the ninth century are extremely rare. Besides a handful of papyrus and other archeologically recovered scraps of slight value, we have only eight palimpsest bifolia in the manuscript Ambrosianus L 99 sup (now S.P. II 65), known as the "Bobbio mathematical fragments" (after the medieval monastery where the manuscript was long preserved before it came to Milan).[61] The Greek texts, which lie under an eighth-century copy of Isidore's *Etymologiae*, have been dated variously to the seventh or (more plausibly) sixth century, and contain material that falls into two classes. On one group of pages are fragments of texts on sundial theory, including what remains of the Greek text of Ptolemy's *Analemma*. The copyist of these leaves used abbreviation only rarely. The other pages are from some late antique writings on centers of gravity and on catoptrics, and have extensive abbreviation. Unfortunately the original attempt to clean the parchment in the eighth century and the subsequent application of staining chemicals in the nineteenth have rendered most of these latter pages illegible; three of the more legible pages have been printed in facsimile.[62]

60 See for examples the index of Turner [1971] *s.v.* 'Abbreviation'.

61 On the manuscript, see Heiberg [1895].

62 Mai [1819] pp. 36f, reprinted in Wattenbach [1876] pl. 6 and [1883] pl. 8 (p. 124 of the manuscript). Belger [1881] plates (pp. 113-114).

Several abbreviations attested in the Bobbio fragments (not all of which remained standard in the later period) can be deduced from errors in the text of Pappus. The fragments also show the curtailing to a few initial letters of common words (the verbs ἐπεζεύχθω 'join'; ἤχθω, 'draw'; nouns such as πλευρά, 'side', and σημεῖον, 'point') that would explain the frequent errors in inflectional endings in Pappus's text.

In genre the Bobbio fragments were part of just such a work as those Pappus's *Collection* comprises, so that one might expect the same motives for using abbreviation to have pertained to him or his early copyists. By using compendia the writer not only saved his own effort, but actually made the mathematical argument easier to read too. In fact, while we have no more extant manuscripts of the same age that are quite like the Bobbio fragments, comparison of the transmitted texts of Archimedes and of Eutocius's commentaries reveals that some of the compendia preserved in the tradition of those authors are at least as old as the sixth century.[63] Some of the compendia are known from papyri to be much older still. While the abbreviations underlying the text of the Vaticanus may have originated in an intermediate, say sixth century, copy, it is by no means impossible that they were used in the original of the *Collection*, or indeed in Pappus's autograph.

These abbreviations survive in Book 7:

 ὥστε 7.151

 δοθείς etc. 7.316

 οὖν 7.232

 κοινός etc. 7.194, .198, .210

 καί Occasionally throughout.

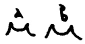
Some scraps of other pages are legible, but do not add to the repertory of compendia.

63 Heiberg in Archimedes, *Opera* 3 pp. xcii-xciii. Lost ancestral manuscripts of other writers also had much abbreviation: see for example Heiberg in Ptolemy, *Opera* vol. 2 pp. xxxiv, lix, lxxxvi-xciii. We have less satisfactory control of the dates of these manuscripts.

More appear in the other books:

 ἥλιος 6.68

 μοῖρα etc. 6.69

 μυριάς etc. Book 2 often.[64]

 μονάς etc. Book 2 often, 8.47

$\bar{\text{o}}$ symbol for zero. 6.119ff

O^c κύκλος 6.125

$\stackrel{c}{=}$ παράλληλος 6.125

$\overline{\pi\lambda}$ πλευρά 4.51

∟⁄ ἥμισυ 3.5, 5.47f, 6.122

It is evident from the errors in the text that these are mere vestiges of a much more extensive practice. A conspicuous trait of this kind of compendium is the omission of inflectional endings, or their reduction to an accent-like mark that is easily missed. The same abbreviation stands for δοθείσης and for δοθέντων, for example, and the same for κοινός and κοινόν. The ends of words are in any circumstances more liable to be mistaken than the other parts, but no mere act of copying is likely to result in the chaotic confusion of cases and numbers that we find on almost any page of the mathematical text, and most often in certain very often repeated words like κοινός, λοιπός, μείζων, ἑκάτερος, and the more common verbs, participles, and articles. These are the very sort of words that are likely to have been abbreviated. Again, certain words seem to have dropped out of the text surprisingly often: ἄρα, ἐστιν, ὡς, even παράλληλος are examples. The losses are easier to explain if these words were represented by compendia that took up about the space of a single letter.

The following are a few illustrations of errors in Book 7 in the Vaticanus (A) that probably resulted from abbreviation in an exemplar.

[64] The superscripted letter indicates the number of myriads.

7.110: ΓΖΑ Commandino ΓΖ ἀπὸ A. In the Bobbio fragments ἀπό is often written as A'. A stray mark above the A may have looked like an apostrophe to the scribe.

7.123: Γ Commandino γὰρ A. The same kind of mistake: γάρ could be written as Γ or Γ (Bobbio).

7.304: τὸ ὑπὸ Commandino τοῦ A (also 7.143, .144). ὑπό could be written Υ' (Bobbio). The difference between ΤΟΥ' and ΤΟΥ is very slight.

7.49: ἔστω Hultsch ὥστε A. **7.274:** ὥστε Halley ἔστω A. **7.292:** ἔστω τε Commandino ὥστε A. The signs for ὥστε (ꞷ) and ἔστω (ꞷ) attested in the Bobbio fragments are practically indistinguishable (see also 8.7, 8.55 for this error). ἔστω τε may have looked to the scribe like ΩΤΕ, and in haste misread as (or emended to) ὥστε. Note that in chapter 151 the scribe has preserved the compendium for ἔστω, probably unsure of the interpretation.

7.274: γωνίας Halley ΓΕ A. The scribe missed a small ω above the Γ in the standard compendium (Bobbio). The source of the spurious epsilon is not evident.

7.231: κοινοῦ Hultsch καὶ A. **7.280:** κοινὸν Halley κύβου A. **7.280:** κύβος Halley καὶ A. **7.159:** κύκλου Hultsch καὶ A. A trivial confusion between embellishments of K.

7.265: τετράκις Halley δεκάκις A. The copyist confused Δ as a numeral with Δ as an initial letter.

7.196: ἀνάπαλιν Commandino ἀνάλογον A (also 7.217, .220, .221). Probably these ratio manipulation tags were written ANA and ANA .

7.124: ἡμίσεια Commandino ἄρα A (twice). **7.143:** ἄρα Commandino ἐστιν A. The signs for ἡμίσεια (∠) and ἄρα (∠ : Bobbio) are not very different, and an oddity of the hand may have made the distinction still less clear. The most prominent feature of both the ἄρα and ἐστιν (⁄ : Bobbio) compendia is a long diagonal stroke.

7.318: τὸ Hultsch παράλληλος A. The scribe must have seen a spurious extra horizontal line in the T.

§9. **Description of the Vaticanus.** About forty manuscripts in European and American libraries contain some portion of Pappus's *Collection*, but the primary artifact in its transmission is the manuscript Vaticanus graecus 218, which dates probably from the early tenth century.[65] As Hultsch conjectured, and A. P. Treweek has proved, the

[65] *CVG* I 283. The Vatican cataloguers observe that a note "sec. XII" at the top of the title page probably led Westermann (*Paradoxographoi* p. xviii), Hultsch (*PAC* I p. vii), and Heiberg (*MGM* p. 77) to adopt a

Vaticanus is the sole independent witness to the text (except for the recently discovered Arabic version of Book 8).[66] At present the Vaticanus comprises 202 folia of parchment, each approximately 256 by 175 millimeters, disposed in twenty-five quires, uniformly of four sheets, preceded by a single sheet. Two folia at the beginning (title page and index) and three at the end, all paper, are modern (16th – 18th century) additions, including transcriptions of poorly legible passages in Book 7, of no independent textual value. The folia are numbered starting with the first parchment sheet.

The contents of the manuscript are distributed as follows:

1 (ff. 1r – 2v) **Anthemius of Tralles** *περὶ παραδόξων μηχανημάτων*, 'On paradoxical devices' (incomplete), a sixth-century A.D. quasi-geometrical discourse on trick mirrors. The text is extant without interruption from the top of 1r, where the opuscule begins, to the bottom of 2v, which ends in mid-sentence; the remainder presumably was once in this manuscript. Since all the other, much later, manuscripts of Anthemius in Greek share this abrupt end, their common ancestry in the Vaticanus is obvious.[67] Moreover, since there is no lacuna between the second and third pages, the surviving sheet must originally have stood in the middle of a quire, and so not at the beginning of the manuscript.

The text is written, 34 lines to a page (not counting the title at the top of 1r), in a rather ugly tenth-century minuscule; the Archimedes palimpsest of Heiberg may be the work of the same copyist (a

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twelfth-century dating. Some of the data on the manuscript given below come from the Vatican catalogue description.

[66] Hultsch *PAC* I p. vii; Treweek [1957].

[67] Anthemius's first editor, Dupuy, recognized the authority of the Vaticanus in his second edition, although he reserved needless doubt whether it was the archetype (Dupuy [1786] p. 399 note). There exists no complete classification of the manuscripts of Anthemius; Treweek has incidentally identified the part of them (apparently the majority) that accompany Pappus (Treweek [1957] pp. 210-11). To his list may be added MS (formerly) Honeyman 7 (private collection; see *CMRM* supp. p. 20, but the manuscript has since been sold), Marc. gr. XI, 30 (*BDM* vol. 3 pp. 155-56), Vind. phil. gr. 229 (*KGH* vol. 1 p. 340), and Copenhagen Thott MS 215 (Schartau – Smith [1974] pp. 335-36). Anthemius is edited in Heiberg, *MGM*.

mathematically-inclined patron?).[68] Abbreviations, particularly of prepositions, conjunctions, and word terminations, abound. Iota and upsilon often bear diaeresis. Accents are generally present. Proposition numbers are written in the left margins, and figures, crudely drawn, occupy spaces indented on the right. The two folia have suffered wear and moisture, and in places (for example the beginning) the script is faint or illegible.[69] This and other comparable damage elsewhere in the manuscript is older than the oldest copies made from it.

2 (ff. 3r – 202v) **Pappus of Alexandria** Συναγωγή from part way through Book 2 (beginning in mid-sentence) to near the end of Book 8 (ending at the bottom of a page with the end of a proposition). Again the text lost at the two ends of this section was once present; water damage in the early pages has left unmistakable traces of the page that originally faced 3r.[70] The extant portion is as follows:

3r:	Book 2, lacking beginning. No subscription.
8r:	Book 3.
32r:	Appendix to Book 3. No subscription.
35r:	Book 4, beginning in mid-page without title.
56r:	Book 5.
87v:	Book 6.
118v:	Book 7.
183v:	Appendix to Book 7.
184v:	Book 8, lacking end.

Two hands appear on the Pappus leaves. The main body of the text is written in a calligraphic but distinctive minuscule that is very like (if not the same as) that of a νοτάριος Baanes who copied two extant

68 Wilson, *SB* p. 139. A photograph of 1r of the Vaticanus is given by Browning [1971] p. 85; one of the Archimedes MS (formerly Metochion of the monastery τοῦ παναγίου τάφου, Istanbul, no. 355, but now inaccessibly in private hands, except for a stray leaf at Cambridge, University Library Add. 1879.23) is in Heiberg [1907] facing p. 235.

69 The title now visible on 1r is a not too skilful restoration imitating the original script, traces of which are visible.

70 Treweek [1957] p. 206.

manuscripts for Arethas, one dated 913/14.[71] The scribe, whom Hultsch named A^1,[72] wrote 33 lines to a page, except where a book ended or a large figure was needed. With extremely rare exceptions, there are no abbreviations in the text. Accents are seldom missing: when they are, the text is often difficult or corrupt. Most breathings are present, and iota adscript is applied, though neither uniformly nor always correctly. The titles, subscriptions, and scholia appear to be by the same scribe, working with a finer pen, and are sometimes in capitals or heavily abbreviated.[73] Punctuation is limited to the sign :— marking the end of sections, the 'diplē' ('>') in the left margin of lines in quotation (ff. 125r-125v, chapter 7.32, quoting Apollonius), and the 'coronis' (here reduced to a horizontal stroke) in the left margins to indicate the ends of logical divisions.

The other hand, Hultsch's A^2, is that of the Anthemius pages: thus the two sections were together very early, probably from the first. A^2 has written in the margin various bits of text that A^1 missed. These restorations were made on the authority of a manuscript (probably the same as A^1 worked from; no variants are recorded), and not by conjecture. For example in 7.116 (f. 141v), A^2 has restored two passages of necessary mathematical argument; yet there are more than twenty errors of lettering in the text of this proposition, not to speak of the figure, which would have to be repaired before one could follow, let alone complete, the argument.

The figures belonging to the text of Pappus are mostly well executed, and occupy indentations on the right side of the page, following the illustrated text. The lettering of the figures appears to be the work of both scribes, so far as it is possible to distinguish between their capitals.

§10. Disturbances in the Vaticanus. Traces survive on some pages of three series of quire numbers, and a series of folio numbers different from the present ones. These inform us about the original state and certain subsequent disorderings of the manuscript.[74] In the first place,

[71] The dated MS is Paris B.N. grec 451 (Christian apologetics), the other London B.M. Harl. 5694 (Lucian). The likeness of hand to the Vaticanus is noted by R. Barbour, *GLH* p. 27, and E. Follieri [1977] p. 148 note 43.

[72] A^1, A^2 have a different meaning in the apparatus of this edition: see the list of abbreviations used in the apparatus below.

[73] Hultsch attributed these marginalia to a third hand, A^3.

[74] The chief deductions in what follows are Treweek's ([1957] pp. 206-208), but he does not mention the folio numbers.

there were the original quire numbers, in Greek numerals, at the top right corner of the front page of each quire. Almost all were casualties of binder's clipping, but there is on f. 179r (quire 23) a kappa that was followed by a lost units digit, and on f. 11r (quire 2) traces of the bottom of either Δ or Η. Together these imply either two or six lost quires at the beginning. We exclude the Anthemius sheet from the count, since its original place in the manuscript was probably at the end.

Somewhat better attested are a series of roman numerals on the last page of each quire, at the bottom right.[75] Still decipherable are, on f. 26v (quire 3) "V", on 55v (middle sheet of quire 7) "XXVIII", on 90v (11) "XIII", on 154v (19) "XXI", on 162v (20) "XXII", on 178v (22) "XXVI", on 186v (23) "XXVII", on 194v (24) "XXIIII", and on 202v (25) "XXV". From these three things are apparent. First, there were, at the time these numbers were written, two more quires at the beginning. This supports the hypothesis that the Greek quire number on f. 11r was Δ. Secondly, the last two extant pairs of quires were exchanged. Thirdly, the middle leaf of quire 7 (ff. 54-55) became detached, and was placed immediately after the four disordered quires. The natural position for stray sheets, such as this one and probably also the Anthemius, would have been at the end of the manuscript, and so it is likely that the end of Book 8 had already been lost.

The third series, of modern numerals, was written at the bottom left of the first page of each quire. One can still read the numbers "6" and "7" for quires 5 and 6, "10" through "12" for 9 through 11, "13" through "21" for 13 through 21 (but the number for quire 18 is not visible), "24" and "25" for quires 22 and 23, and "22" and "23" for quires 24 and 25. On f. 54r is "26". The change at the beginning may be accounted for by the loss of the two first quires, compensated by the moving of the Anthemius sheet to the front. Quire 12 apparently slipped out of place here and thus missed being numbered. Otherwise the disorder remains unchanged.

Later, presumably after the manuscript was rebound, the folia were numbered on the recto at the bottom right. These numbers are legible on pages of all quires but no. 23, and they confirm the sequence 1-11, 13, 12, 14-21, 24-25, 22-23. Folia 54-55 were still out of place and presumably still at the end.[76] The present f. 3 was number 4 in this sequence, and hence was probably preceded both by the two folia of Anthemius and a

75 The MS Florence Laur. 28,18 (commentaries on the *Almagest*) exhibits the same roman numeration of quires. The two manuscripts were together at two periods: in the thirteenth century in the papal collection, and in the fifteenth in the private library of the Medici. See below, pages 52-54.

76 No folio numbers of this series can be seen on this sheet.

numbered title page.

To recapitulate, the Vaticanus was copied out, somewhere in the Byzantine world, at a date that we do not know exactly, but probably within a few decades either way of 913. It was primarily the work of a professional scribe (A^1), then checked by a second person, either a second scribe or the mathematically inclined patron who commissioned the manuscript (A^2), who in either case had access to the exemplar. This second person also contributed, most likely on unused pages at the end, a copy of the short treatise of Anthemius. There is no evidence that either text was defective at that time. Later the manuscript suffered a series of disturbances. At one stage, when it had presumably already reached the Latin world, the manuscript had lost its final quire or quires, except for a single sheet bearing the first four pages of Anthemius. The manuscript was rebound with some quires out of order, and the Anthemius fragment and another displaced sheet put at the end. Subsequently, but not before it had suffered water damage, the Vaticanus lost its first two quires (or, less likely, the fifth and sixth, with the first four having vanished earlier). In another rebinding that followed this mishap, the existing disorder was allowed to stand, aggravated now by another transposition of quires in the middle. Perhaps because it began with a prominent title, the Anthemius fragment was placed at the head of the manuscript. This was the disfigured state of the Vaticanus in the last years of the fifteenth century: about that time or shortly after the turn of the century, someone went through the manuscript, determining the correct sequence of quires, and writing, in Greek, directions at the appropriate points. On the present f. 202v, he wrote: *οὐκ οἴδαμεν βέβαιον εἰ τοῦτό ἐστι τὸ τέλος ἤ τε λείπει ἢ οὔ. ζαχαρίας διωρθώσατο καὶ τοῦτο.* "We do not know for certain if this is the end, or whether it is deficient or not. Zacharias sorted this one out too." This Zacharias can be identified from his hand as Zacharias Callierges, an expatriate Cretan printer and copyist active from the 1490's to the 1520's.[77] All the copies known to have been made from the Vaticanus must date after Zacharias's work, since they all exhibit the correct order.[78] The next rebinding of the Vaticanus most likely corrected the sequence.

77 See *RGK* Part 1 A, pp. 80-81. On Zacharias see Geanakoplos [1962] pp. 201-222; a little more information can be assembled from the literature listed in *RGK*.

78 Treweek, p. 209.

§11. Byzantine notices. We now turn from the evidence in the Vaticanus to the sparse other evidence of Pappus's medieval tradition. Not surprisingly, considering the nature of the book, the *Collection* does not seem to have been much studied in Byzantium. There are, for example, no copies, aside from the Vaticanus, of Byzantine origin. The few allusions to Pappus that we have from other sources do not evince any serious effort to understand his mathematics; and if the Vaticanus had been lost before 1450, probably no modern scholar would have deduced the *Collection*'s existence.

The earliest medieval notice is older than the Vaticanus. A ninth-century manuscript Vat. gr. 1594 of Ptolemy's *Almagest* is a copy of a lost uncial manuscript that preserved several layers of explanatory and supplementary matter from late antiquity, most prominently the "Prolegomena" to the *Almagest* that were compiled by Eutocius in the sixth century.[79] Later than Eutocius, but earlier than the copying of Vat. gr. 1594, must be a series of marginal scholia written in Vat. gr. 1594 by the original copyist, who, being surely a professional calligrapher, is much more likely to have derived them from his exemplar than from his own knowledge. These notes merit a proper study; but it is evident even from a cursory survey that their author had at his disposal both Theon's commentaries to the *Almagest* and, of more interest here, Pappus's *Collection*. Referring to the section of the "Prolegomena" that treats isoperimetric figures, the annotator remarks (f. 1v): ἐκ τῶν ζηνοδώρου σχολίων ὡς ἱστορεῖ ὁ θέων ἐν τῶι εἰς τὴν σύνταξιν ὑπομνήματι. ἐποίησε δὲ ὁ πάππος βιβλίον ὅλον περὶ τοῦ προκειμένου προβλήματος, "From the notes of Zenodorus, as Theon records in his *Hypomnemata* to the *Almagest*. Pappus composed a whole book on the present problem." More specifically, when, on f. 5r, Eutocius, with a gesture of impotence, abandons the proof that the sphere has greatest volume of solids of equal surface area, the scholiast writes: ἰστέον ὅτι ὁ μέγας πάππος ταῦτα ἀπέδειξεν ἐμμελῆ ἐν τῆι ε´ βιβλωι τῶν ἀνθηρῶν προβλημάτων. "Note that the great Pappus proved these things completely in the fifth book of the *Exquisite Problems*." There is no question here of an independent transmission of Pappus's opuscule on the solid figures, nor of a vague reference in an intermediate author: the scholiast clearly had access to the *Collection* more or less as it now stands, in which the fifth book is devoted

[79] On the MS and its parentage, see Heiberg in Ptolemy, *Opera* II pp. xxvi-xxxvii. The 'Prolegomena' are anonymous in most manuscripts (including Vat. gr. 1594), in others attributed to Theon or Diophantus; Mogenet [1956] cogently demonstrated Eutocius's authorship.

to the isoperimetric theorems for plane and solid figures. As for the title 'Exquisite Problems' (Ἀνθηρὰ Προβλήματα), we may suppose one of two things: either the original title at the beginning of the *Collection* was something like Συναγωγὴ ἀνθηρῶν προβλημάτων, 'Collection of exquisite problems', or, more likely, the scholiast, turning to the beginning of Book 5, saw the subscription to Book 4, which read πάππου συναγωγῆς ὅπερ ἐστὶν ἀνθηρῶν θεωρημάτων ἐπιπέδων καὶ στερεῶν καὶ γραμμικῶν, "Of Pappus's *Collection*, which comprises exquisite theorems, planar, solid, and curvilinear".[80]

Several centuries separate this Byzantine bibliophile from the next known reader of Pappus. Remarkably, this was a person not especially known for interest in the sciences, the prolific twelfth-century scholar John Tzetzes, and, more remarkably, he alludes several times, not only to the rare author Pappus, but also, and sometimes in the same passages, to the likewise rare Anthemius; so that there is a fair chance that he read them in the Vaticanus itself. It never required more than the most oblique provocation to incite the garrulous Tzetzes to digress. Commenting on Aristophanes's *Clouds*, for example, he finds in the phrase (line 1024) ὦ καλλίπυργον σοφίαν ("O fair-towering wisdom!") the excuse for an encomium on the power of mechanics, ending with a short list of authorities on the subject, including Philon of Byzantium, Archimedes, Heron, a certain Sostratus (the architect of the Pharos lighthouse?), and Pappus.[81] Again, in his *Allegories of the Iliad*[82] Book 5, Tzetzes interprets the fire Athena gives Diomedes's helmet and spear as a mirror to reflect the Sun, continuing (lines 10-19) with another list of mechanical writers who allegedly discuss such things. Among these are Philon, Archimedes, Heron, Dionysius (a writer on siege machines, quoted by Philon), Athenaeus (the writer on siege machines), Apollodorus (another writer on weaponry), Ctesibius, and Philetaerius (author of a lost work on harbor engineering); a few mysterious authorities including Isoes (?), Patrocles, and again Sostratus; and Pappus and Anthemius. Not all these names are really relevant to the topic, because Tzetzes tended to stretch the truth when the opportunity arose to boast of his reading.

80 The subscription in the Vaticanus has two trivial scribal errors.

81 Tzetzes, *Aristophanes Scholia* pp. 621-22.

82 Tzetzes, *Iliad Allegory* p. 105.

But the most extensive citations come from Tzetzes's *Chiliades*, a long poem written as a commentary to his letters.[83] From *Chiliades* II 106-159 ("On Archimedes and some of his contraptions") come these passages (lines 121-130, 152-159), in an account of the siege of Syracuse otherwise largely dependent on (since lost) parts of Diodorus and Dio Cassius:

ὡς Μάρκελλος δ᾽ ἀπέστησε βολὴν ἐκείνας τόξου,
ἑξάγωνόν τι κάτοπτρον ἐτέκτηνεν ὁ γέρων.
ἀπὸ δὲ διαστήματος συμμέτρου τοῦ κατόπτρου
μικρὰ τοιαῦτα κάτοπτρα θεὶς τετραπλᾶ γωνίαις
κινούμενα λεπίσι τε καὶ τισι γιγγλυμίοις
μέσον ἐκεῖνο τέθεικεν ἀκτίνων τῶν ἡλίου,
μεσημβρινῆς καὶ θερινῆς καὶ χειμεριωτάτης.
ἀνακλωμένων δὲ λοιπὸν εἰς τοῦτο τῶν ἀκτίνων
ἔξαψις ἤρθη φοβερὰ πυρώδης ταῖς ὁλκάσι,
καὶ ταύτας ἀπετέφρωσεν ἐκ μήκους τοξοβόλου.

ὁ Δίων καὶ Διόδωρος γράφει τὴν ἱστορίαν
καὶ σὺν αὐτοῖς δὲ μέμνηνται πολλοὶ τοῦ Ἀρχιμήδους,
Ἀνθέμιος μὲν πρώτιστον ὁ παραδοξογράφος,
Ἥρων καὶ Φίλων, Πάππος τε καὶ πᾶς μηχανογράφος,
ἐξ ὧνπερ ἀνεγνώκειμεν κατοπτρικὰς ἐξάψεις
καὶ πᾶσαν ἄλλην μάθησιν τῶν μηχανικωτάτων,
βαρυολκόν, πνευματικήν, τὰς ὑδροσκοπικάς τε,
κἀκ τούτου δὲ τοῦ γέροντος τῶν βίβλων Ἀρχιμήδους.

"As Marcellus kept (his ships) an arrow's shot away, the old man fashioned a hexagonal mirror. Putting small fourfold mirrors at a commensurate distance from the mirror, these moved by plates and certain hinges, he placed this in the middle of the Sun's rays, equinoctial, summer, and winter. As the rays were reflected then on it, a fearful fiery ignition started up on the ships, and reduced them to ashes from an arrow-shot's distance....

"Dion and Diodorus record the story, and many along with them tell about Archimedes, Anthemius the paradoxographer first of all, Heron and Philon, Pappus and every writer on mechanics, in whom I have read about reflective ignitions and every other lesson of mechanics, the

[83] Quotations from Tzetzes, *Chiliades*, ed. Leone.

baroulkos, pneumatics, water-clocks, and also (by reading) the books of this old man Archimedes."

The entire first part of this passage, attributing to Archimedes a 'burning mirror' composed of hinged hexagonal faces, is adapted from Anthemius. Tzetzes even inserts an irrelevant allusion to another device that Anthemius describes, an arrangement of mirrors that reflects the suns rays to a certain point at all seasons.[84]

Chiliades XII 964-990 ("On the words and works that Archimedes performed while alive, and the writings still extant") begins as follows (lines 965-971):

τινὲς βιβλίον λέγουσιν ἓν γράψαι Ἀρχιμήδην,
ἐγὼ δὲ τούτου ἀναγνοὺς διάφορα βιβλία,
< . . . > τὰ κεντροβαρικά, κατόπτρων τὰς ἐξάψεις,
καὶ τὰ ἐπιστασίδια καὶ ἕτερα βιβλία,
ἐξ ὧν Ἥρων, Ἀνθέμιος καὶ πᾶς μηχανογράφος
τὰ ὑδρικά τε ἔγραψαν καὶ τὰ πνευματικά δε,
βαρυολκά τε σύμπαντα καὶ θαλασσοδομέτρας...

"Some say that Archimedes wrote one book; but as I have read various books of his... the studies of center of gravity, mirror burning, the *Epistasidia*, and other books, on the basis of which Heron, Anthemius, and every writer on mechanics wrote hydraulics, pneumatics, everything about the *baroulkos*, and aquatic hodometry..."

The passage is less important for the implausible list of (otherwise mostly unattested) works of Archimedes that Tzetzes claims to have read (including not one geometrical work!) than for the premise that Archimedes only wrote one book, which appears, attributed to Carpus of Antioch, only in Pappus's *Collection* 8.3 (Carpus was discussing only books on mechanics).

From *Chiliades* XI 586-641 ("On geometry and optics") comes this (lines 586-610, 616-618):

[84] Dupuy ([1786] pp. 429-435) discusses this passage's relationship with Anthemius, perhaps taking Tzetzes's version a bit too seriously.

γεωμετρία χρήσιμος πολλαῖς μηχανουργίαις,
πρός τε ἑλκύσεις τῶν βαρῶν, ἀναγωγάς, ἀφέσεις
πετροπομποὺς καὶ μηχανὰς ἄλλας πορθητηρίους,
καὶ πρὸς ἐκπυρακτώσεις δὲ τὰς ἀπὸ τῶν κατόπτρων,
καὶ σωστικάς δὲ πόλεων ἄλλας μηχανουργίας,
λυσιτελὴς γεφύραις τε καὶ λιμενοποιίαις,
καὶ μηχαναῖς, αἳ θαυμασμὸν ποιοῦσιν ἐν τῶι βίωι,
ὡς τὰ χαλκᾶ καὶ ξύλινα καὶ σιδερᾶ καὶ τἄλλα
πίνειν, κινεῖσθαι, φθέγγεσθαι, καὶ ἕτερα τοιαῦτα,
καὶ τὸ μετρεῖν δὲ μηχαναῖς σταδίους τῆς θαλάσσης,
καὶ γῆν τοῖς ὁδομέτραις δὲ καὶ ἕτερα μυρία
γεωμετρίας πέφυκεν ἔργα, πανσόφου τέχνης.
πέντε δυνάμεις δὲ αὐτῆς αἷς γίνονται τὰ πάντα.
ὁ σφὴν καὶ τὰ πολύσπαστα, μοχλὸς καὶ ὁ κοχλίας
καὶ σὺν αὐτοῖς ὁ ἄξων δὲ μετὰ περιτροχίου.
Βαρυολκοὺς χελώνας με τί δέον διαγράφειν;
χελώνας ὀρυκτρίδας τε καὶ τὰς ὀπλοχελώνας
καὶ τὰς ἀμπέλους ἐλαφράς, χελώνας καλουμένας,
καὶ πᾶσαν ἄλλην μηχανὴν ἐκ τῶν πορθητηρίων,
τὰς ἀναγούσας βάρη τε, ῥύστακας μονοκώλους,
δικώλους καὶ τρικώλους τε καί γε τοὺς τετρακώλους,
ἀφετικάς τε μηχανὰς οἷον τὰς πετροβόλους,
καὶ καταπέλτας τῶν βελῶν πάντας καὶ γαστραφέτας,
καὶ τοὺς πορθοῦντας δὲ κριοὺς τῶν πόλεων τὰ τείχη,
κλίμακας καὶ καρχήσια καὶ πύργους ὑποτρόχους,
καὶ πᾶσαν ἄλλην μηχανὴν τί δέον παρεγγράφειν;

καὶ ὀπτικὴ δὲ συντελεῖ σὺν τῆι γεωμετρία
πολλαῖς μὲν ἄλλαις μηχαναῖς καὶ τέχνηι τῆι ζωγράφων
καὶ ἀγαλμάτων τέχναις δὲ καὶ ἀνδριαντουργίαις.

"Geometry is useful for many mechanical works, for lifting of weights, putting ships to sea, rock throwing, and other siege machines, and for setting things on fire by means of mirrors, and other contrivances for defending cities, useful for bridges and harbor-making, and machines that make a wonder in life, such as bronze and wooden and iron things and the rest, drinking, moving, crying out and the like, and measuring by machines the stades of the sea, and the earth by hodometers, and a myriad other works are born of geometry, the all-wise art.

> *"It has five powers by which all are accomplished: the wedge and pulleys, the lever and the screw, and with them the axle and wheel. What need for me to list the "baroulkos", "tortoises"* [weight-bearing frames with rollers], mining tortoises, armed tortoises, manoeuverable mantelets, called tortoises too, and every other machine of siege, *and the things that draw weights up, draggers of one member, of two members, of three members, and even of four members,* and shooting devices like stone-throwers and all catapults for missiles, and stomach-bows, and rams that breach city walls, ladders and universal joints and wheeled towers, and every other machine – what need to add these to the catalogue?...
>
> "And optics together with geometry contributes to, among many other mechanical matters, the *art of life-painting* and portraits, and the statuary arts...."

The passages in italics here are reminiscences of Heron's *Mechanics*, which by Tzetzes's time was almost certainly no longer extant in Greek, except as quoted in Pappus's Book 8 (8.52-61). The reference to painting as profiting from geometry seems to be inspired by Pappus's introduction to the book (8.1).

That John Tzetzes had direct access to both Anthemius and Pappus, then, is certain, and some passages of both authors evidently impressed him deeply.[85]

The only other late Greek allusions to the *Collection* that I know of are two marginal scholia in a tenth- or eleventh-century manuscript of metrological writers, Istanbul Old Serai gr. 1.[86] One, on f. 8r, remarks:[87] ἡ γὰρ ἐκ τοῦ κέντρου διπλασίων τῆς ἐκ τοῦ

[85] The apparent absence of references to the non-mechanical parts of the *Collection* is in accordance with Tzetzes's complete lack of interest in pure geometry (compare his assessment of Archimedes quoted above). Even in the Renaissance Humanist readers were drawn to Book 8, leaving the rest for the mathematicians. It is not likely that the independent version of Book 8 was still available in Greek in the twelfth century.

[86] Photographs in Bruins, *CC* vol. 1. A dating to the eleventh century has been accepted in most discussions of the manuscript since Schöne (Heron *Opera* III p. vii); Irigoin [1971] prefers the tenth century. Irigoin is mistaken in asserting that the Istanbul MS contains any of Euclid's *Elements*.

[87] Edited by Heiberg in Heron *Opera* V p. 223.

κέντρου ἐπὶ τὴν βάσιν τοῦ τριγώνου, ὡς ὁ Ὑψικλῆς ἐν τῶι πρώτωι τῶν εἰς Εὐκλείδην ἀναφερομένων ἐπορίσατο καὶ Πάππος ἀπέδειξεν. "For the radius (of a circle) is twice the (line) from the center to the base of the (equilateral) triangle (inscribed in the circle), as Hypsicles derived in the first of the (books) referring back to Euclid (*Elements* XIV, Euclid *Opera* V p. 6), and Pappus proved." *Collection* 5.76 proves this proposition.

The other scholion (f. 98r) contains the following:[88] *ἀποδέδειχε δὲ καὶ ὁ Πάππος ὡς ἡ ἐκ τοῦ κέντρου τῆς σφαίρας τῆς περιλαμβανούσης τὸ εἰκοσάεδρον ἐφ' ἓν ἐπίπεδον τοῦ εἰκοσαέδρου κάθετος μείζων ἐστὶ ἢ δυνάμει δωδεκαπλάσιον τῆς τοῦ εἰκοσαέδρου πλευρᾶς δυνάμει πενταπλάσιον*..., which is an untranslatable garbling of the enunciation of Pappus 5.81, that the perpendicular dropped from a sphere's center onto a face of an inscribed icosahedron, in square and times twelve, is greater than the side of the icosahedron, in square and times five.

Schöne dated the margin hands in the Istanbul manuscript to the early fifteenth century.[89] Unless this dating is very wrong, the scholiast's source could not have been the Vaticanus, which was in Italy before the end of the thirteenth century. The information could have come from another manuscript of the *Collection*, but the low intellectual level of the marginalia in general, and the monstrous misquotation of Pappus's second proposition, would agree better with the scholiast's using an intermediary text (an elementary treatise or scholia in another manuscript) by some earlier reader of Pappus.[90]

§12. Witelo. Our knowledge that the Vaticanus was in western Europe before 1300 derives first of all from the recent discovery that several propositions in Witelo's *Perspectiva*, written about the 1270's, are close adaptations of theorems in Book 6 of the *Collection*.[91] This dependence on Pappus had already been hinted at in the 1572 edition of Witelo by Risner, who had inserted references to parallel passages in other

[88] Transcription by Bruins, *CC* III p. 305, corrected by me from his photograph, *CC* I p. 191.

[89] Heron *Opera* III p. xi.

[90] Aside from these imprecise references to Pappus, the scholia exhibit knowledge only of the contents of the manuscript itself, supplemented by the fifteen books of the *Elements*.

[91] Unguru [1974], especially pp. 310-319.

authors, although without asserting that Witelo had used these as sources.[92] Witelo's borrowings are not strict translations of Pappus, but the variations go little beyond rephrasing, without significant changes to the mathematical argument.

All the borrowed theorems come from one section of Book 6 of the *Collection* (chapters 80-103). In the margin at the beginning of this passage (f. 107r) the scribe A^1 has written ΕΙΣ Τ(Α) ΟΠΤΙΚΑ ΕΥΚΛΕΙΔΟΥ, and in what follows Pappus indeed expands a pair of theorems from Euclid's *Optics* (44 and 45) that determine the conditions under which two diameters of a circle appear equal from a point of observation outside the circle's plane, and then, departing from Euclid, considers two problems concerning the center of the apparent ellipse seen when the circle is viewed obliquely. The actual pertinence of these theorems to Euclid is irrelevant.[93] What is important is that the marginal note would easily and naturally have attracted the eye of anyone looking for material on optics, and hence no extensive translation needs to be supposed as intermediary between Greekless Witelo and the Greek text of Pappus. Not all this material is subsumed into the *Perspectiva*, nor all in one place. The second theorem in Pappus (6.81) becomes I, 22 in Witelo, while the first and third (6.80 and 82-84) appear later as I, 38 and 39; these were identified by Risner and Unguru. But there are more: Pappus 6.85 and 86 as Witelo I, 49 and 50; Pappus 6.87 and 88 as Witelo I, 47 and 48; Pappus 6.89 as Witelo I, 51; Pappus 6.99 becomes Witelo I, 125. But Witelo does not seem to have employed the theorems of Pappus (6.90-98, 100-103) to which the foregoing are all lemmas; and when in book IV he comes to treat the projected circle, he adapts the demonstrations

[92] But see Risner's preface to Witelo, f. *3r, where he writes: "Sed ex Apollonio, Theodosio, Menelao, Theone, Pappo, Proclo & aliis firmamenta permultarum demonstrationum singulari iudicio repetiuit..." Risner's knowledge of Pappus, several years before Commandino's translation was printed, probably came through his patron and colleague Ramus, who owned a manuscript of the Greek text; see Lindberg's introduction to the 1972 reprint of Risner, p. xxviii, and below, page 59.

[93] Neugebauer, *HAMA* p. 768, writes: "It has often been said that these sections are a commentary to Euclid's 'Optics' because of a reference to Euclid in a scholion, the contents, however, do not justify such an attribution." This seems unnecessarily skeptical; the 'scholion' (quoted above), which refers explicitly to the *Optics*, is probably an original heading, and is not unsuitable for an excursus only tangential to Euclid's work.

from Euclid's *Optics*.

Witelo also knew the work of Anthemius, and cites him in VI, 65 by name:[94]

> iam autem dixit Anthemius nescio qua ductus experientia, quod solum uiginti quatuor reflexi radii concurrentes in uno puncto materiae inflammabilis, ignem in illa accendant: & coniunxit septem specula plana hexagona colligatione stabili fixa, scilicet sex extrema circa unum, quod statuit in medio illorum, & uniebantur illa specula in quibuslibet angulis hexagoni: ideo quia figurae hexagonae replent locum superficialem: ualent enim tres anguli hexagoni quatuor rectos. et dixit Anthemius quod ad quamcunque distantiam sic ignis potuit accendi...

> "Now Anthemius was led by some experimentation to say that only when twenty-four rays are reflected and meet at one point of inflammable substance, will they ignite fire in it; and he joined seven hexagonal plane mirrors held by a stable binding, that is six on the outside around one, which he placed in the middle of them, and those mirrors were joined at each angle of the hexagon, because hexagonal shapes fill the planar area, since three angles of a hexagon equal four right angles. And Anthemius said that fire can be set in this way at any distance..."

94 Risner's edition, p. 223. Discussed by Dupuy [1786] pp. 436-48, and Huxley [1959] pp. 39-43. Huxley's treatment is vitiated by his conviction that the continuation of the Anthemius fragment is to be found in the Bobbio fragments, a theory originally suggested by Heiberg [1883, 2], which examination of the substantially complete Arabic translation of Anthemius renders untenable (Toomer *Diocles* p. 20). The putative reminiscences of the alleged continuation of Anthemius that Huxley sees in Witelo IX, 44 are adapted in fact from Ibn al-Haytham's *Optics*. The spelling "Anthemius" guarantees that Witelo had a Greek source, since although Anthemius was mentioned in Ibn al-Haytham's treatise on burning mirrors, coming to Latin through Arabic the name became "Anthimus" or worse; see Heiberg — Wiedemann [1910] p. 219.

Again the conjunction of Anthemius and Pappus makes one suspect a connection with the Vaticanus.[95] Now, it is not likely that Witelo could read Greek; still less that Latin versions of the whole of Pappus and Anthemius were in circulation. But it is very easy to believe that the great translator of Greek philosophic and scientific texts William of Moerbeke, who was Witelo's friend and the dedicatee of the *Perspectiva*, and who furthermore had translated from Greek a number of works of Archimedes, Eutocius, and pseudo-Ptolemy that Witelo used, might also have extracted for his friend any passages in the Vaticanus that obviously pertained to optics, if he had access to the manuscript.[96]

§13. The papal inventories. Now we turn to another document relating to Pappus in the West, the 1311 inventory of the papal library. This is the later of two inventories (the other dates from 1295) that reveal an impressive collection of Greek manuscripts, mostly philosophical and scientific, that belonged to the Popes. William of Moerbeke made translations of many of the works that were in these manuscripts, and in several cases the translations bear colophons that date them during his years at the papal court at Viterbo. These include Proclus's commentary on Plato's *Parmenides* and *Timaeus*, Simplicius on Aristotle's *De caelo*, Themistius and John Philoponus on the *De anima*, and perhaps other philosophical works; and as well the mathematical works that Witelo used, translated in 1269 from two manuscripts in the papal collection.[97] The 1311 inventory also includes the following entry:[98]

95 This inference was drawn by Clagett, *AIMA* III p. 406 note 56.

96 Unguru [1974] pp. 322-23. An excellent brief summary of William's life and work is L. Minio-Paluello's article in *DSB* 9 (New York: 1974), pp. 434-440. The biography is based mostly on William's own subscriptions to his translations. In 1260, he was at Nicaea and Thebes. By 1267 he was at the papal court at Viterbo, and at least as early as 1272 he was papal chaplain and penitentiary. In 1278 he relinquished this office to become archbishop of Corinth. Recently published documents show that he had returned to Italy by 1284, since in January of that year he participated in the lifting of a papal interdict at Perugia (Bagliani [1972]). He died before the end of 1286.

97 See Jones, *William of Moerbeke* (forthcoming) for a more detailed account of this papal Greek library. Of the scattered literature on the subject, Heiberg [1891] is the most illuminating, although by now out of date.

98 The text of the 1311 inventory is in Ehrle, *Historia* pp. 95-99 (Pappus

> item unum librum, qui dicitur Commentum Papie super difficilibus Euclidis et super residuo geometrie, et librum de ingeniis, scriptum de lictera greca in cartis pecudinis, et est in dicto libro unus quaternus maioris forme scriptus de lictera greca, et habet ex una parte unam tabulam.

This "Commentary of Pappus on difficult things of Euclid and on the rest of geometry" can hardly be anything but the *Collection*, unless we are to imagine that there were copies not only of Pappus and Anthemius somewhere in western Europe, but also an otherwise unknown major treatise by him.[99] Furthermore, the title "liber de ingeniis" is surely a translation of Περὶ παραδόξων μηχανημάτων, so that Anthemius too was probably in this papal manuscript.[100] The coincidence of Pappus and Anthemius in Witelo's work and (probably) also in this catalogue entry, and the scarcity of knowledge of Pappus in the East after Tzetzes, are strong circumstantial evidence that the papal manuscript was the Vaticanus itself.[101]

The inventory's title for the *Collection* implies that, when complete, it contained a prominent enough discussion of something in Euclid to merit special mention. This could most easily be accounted for if Book 1 was Pappus's commentary on Book 10 of the *Elements*.[102] A crude check of

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is item 604 on p. 96), that of the 1295 inventory in Pelzer, *Addenda* pp. 23-24.

99 The conjecture that this was the *Collection* was first made by Heiberg [1891] p. 314; Ehrle (p. 96) had already correctly identified Pappus's name.

100 This is my identification. Previous conjectures have included Philon's *Pneumatics* (Heiberg p. 314), Heron's *Pneumatics* (Birkenmajer, *VU* p. 22), and Book 8 of the *Collection* itself (Grant [1971] pp. 667-68, Clagett, *AIMA* III p. 406 note 56). The first two are not supported by evidence that these works were known in the West, while the third is implausible.

101 This has been suggested by Grant (p. 668), Clagett (p. 406 note 56), and Derenzini [1976] p. 101.

102 Rose, *IRM* p. 37, has remarked that the commentary on Book 10 seems to fit the description in item 604; but by itself this work would have been too short to fill a manuscript, nor would it explain the continuation "super residuo geometrie". Considering the blunders that the cataloguers make in copying the titles, one would not be

this theory is possible. Comparing Greek mathematical texts with Arabic translations, one finds that the numbers of words in each are, very roughly, equal. From this ratio one can compute that, written in the hand and format of the Vaticanus, the Euclid commentary would take up about ten folia. We also know, from the proposition numbers, that about half of Book 2, which would be five folia or so, is lost.[103] The sum is sufficiently close to the two lost quires (sixteen folia) that were deduced from the quire numbers. By a similar argument, using the Arabic version of Book 8, we find that between two and three more folia were needed at the end. If there was no Book 9, but Anthemius's work followed immediately, it would have fallen on the middle sheet of a quire, as we know it did. An extant Arabic recension of Anthemius shows that less than a page more of Greek text followed the surviving fragment. Perhaps, then, the losses in the *Collection* amount only to the first part of Book 2 (since we have adequate Arabic translations of the commentary on Euclid and the 'Introduction to Mechanics'), and this misfortune is not very serious. We have no way to know what the "larger quire" that was with the manuscript might have been.

Unlike most of the entries in the 1311 inventory, this one does not add to the title the abbreviation "And", which, according to one explanation, identified manuscripts that had formerly belonged to the Sicilian Angevin court and had passed, perhaps after the battle of Benevento in 1266, to the Popes.[104] Hence we can only speculate on how the Vaticanus reached Italy. One possibility is that William himself acquired it while he was in the East.

The 1311 inventory is the last to contain individual descriptions of the Greek manuscripts. They are listed as a block in inventories dating from 1327 and 1339 of papal possessions deposited at Assisi.[105] What became of the Greek manuscripts after that is not clear. According to one report, about 1368 Pope Urban V had various treasures, including books, brought to Rome from Assisi, and distributed most of them among the various churches of the city.[106] Such a dispensation would easily explain

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surprised if "difficilibus" were a mistake for "decimum librum".

103 See above, page 3.

104 See Pelzer, *Addenda* pp. 92-94; Jones, *William of Moerbeke*. Bagliani [1983] presents a contrary view.

105 Pelzer, pp. 34-35.

106 Ehrle [1913] pp. 344-46, from Albanès – Chevalier [1897] p. 398: "Item, dum esset apud Urbem et audiuisset quod a tempore domini

the calamitous number of these manuscripts that vanished at that time, and the way that the few that did survive reappeared independently in the fifteenth century, in the possession of the great humanist collectors of Greek manuscripts: Cardinal Bessarion somehow obtained the present Marc. gr. 313 of the *Almagest*, and Marc. gr. 258 containing minor works of Alexander of Aphrodisias, which likely were in the papal library (the first almost certainly so);[107] Valla acquired one of the Archimedes manuscripts;[108] Poliziano, Laur. 28,18, the first half of Theon's *Almagest* commentaries, identifiable with certainty from an inscription in the manuscript that matches the inventory entries.[109] A manuscript of Dionysius the Areopagite, Vat. gr. 370, that is thought to be one of the items appears in Vatican catalogues definitely first about 1510, but possibly as early as about 1450.[110] On the other hand, at least eight manuscripts in the inventories are demonstrably lost, while many more, too imprecisely described to compare with modern collections, probably no longer exist.

§14. **False leads.** We must now consider two cases of alleged knowledge of Pappus's *Collection* in the fifteenth century. The first is Heiberg's suggestion that Giovanni Aurispa owned the Vaticanus as early as the 1420's. This theory has been repeated as established fact many times,[111] but it has only a slender foundation. In 1422 and the next year

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Bonifacii pape octavi, certi thesauri papales fuissent in ciuitate Assisii reseruati et adhuc reseruarentur, in quindecim uel uiginti saumatis, fecit coram se aportari, et reperiit quod ibi erant multe sanctorum reliquie, multi libri et alia ecclesiastica ornamenta. Tunc illa refutauit penes se retinere, sed ecclesiis Urbis omnia predicta distribuit, donauit et realiter traddidit, excepto capite beati Blasii, martiris, et quibusdam aliis reliquiis..."

107 Labowsky [1979] p. 8.

108 But, significantly, it can be traced back to Rome in about 1450, during the pontificate of Nicholas V; see Clagett, *AIMA* III part 3, p. 333.

109 Rome [1938], Pelzer [1938].

110 Devreesse, *FG* pp. 178, 24.

111 For example, R. Sabbadini, *Carteggio*, p. 13; Rose, *IRM* p. 28 and [1977] p. 131; Garin [1969] p. 495; Francheschini [1976] p. 48. Garin asserts that Aurispa traded the Vaticanus to Filelfo in 1431.

the humanist Ambroglio Traversari made several inquiries after a rumored manuscript of Archimedes alleged to have been brought to Italy from the East by Rinuccio da Castiglione.[112] Traversari wrote to, among others, Aurispa, who had been in Greece collecting manuscripts and returned at the same time as Rinuccio. In August 1423 Aurispa replied:[113]

> That Rinuccio has found Archimedes, is possible indeed, but in my view not plausible. I have never spoken to anybody who said he had seen this author. But you of course have had experience of how very adroit a hunter of these matters I once was. I have one big old book of the 'mathematician' [mathematicus] Athenaeus of Athens with illustrations of machines. This book is old, and the illustrations are not very good, but they can be understood easily. I have also another 'mathematical' book [mathematicus], incomplete, also old, whose author I do not know; in fact it lacks the beginning. I cannot say whether maybe Rinuccio attributes the name of Archimedes to that age. It may be true that he has found [? *text uncertain*] it, and neither I nor the people I have spoken to have seen it.

Heiberg guessed first that Aurispa's defective mathematical manuscript was the Archimedes that Valla later owned.[114] Returning to the question later, he decided that Aurispa could not have mistaken that manuscript if he had it, and so suggested that Aurispa's manuscript was the Vaticanus, because that was defective at the beginning.[115] The argument is very weak. We do not know, for example, how old "uetustus" means for Aurispa. Moreover, the authorship of the *Collection* would be obvious to anyone who had inspected the manuscript even superficially. But it is not even certain that Aurispa is referring to a book separate from the old

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This fatidic mistake is the harder to trace because Garin provides no reference; it originates in a misunderstanding of Sabaddini, p. 13 note 7, where the subject is in fact Diogenes Laertius, not Pappus.

112 See Heiberg [1883,1], from which the quotation translated below is taken.

113 Letter XXIV, 53, in Traversari, *Epistolae* ed. Mehus.

114 Heiberg [1883,1] p. 427.

115 In Archimedes, *Opera* III, p. lxxxii.

Athenaeus that he says he has; in a later letter Aurispa asks Traversari asks for the return of "Athenaeum ὄργανον πολεμικόν et nescio quid aliud in mathematicis", presumably only one manuscript.[116] As Heiberg observed, nothing resembling the Vaticanus appears in the catalogue of Aurispa's books made after his death in 1459, but of course he could have sold it before then.

Less has to be said of Clagett's theory that the painter Piero della Francesca was acquainted with Pappus.[117] Piero's work *De quinque corporibus regularibus*, which dates from the 1480's, ends, after three books treating the regular solids, with a fourth part, *De corporibus irregularibus*, and in this section he describes the construction of five Archimedean solids.[118] The only ancient source for these solids is Pappus Book 5, chapters 34-37, together with a marginal note describing the construction of some of them.[119] But Piero produces only some of the thirteen Archimedean solids, strictly those that can be generated by truncation, and includes with them numerous other, quite irregular, solids. Since he gives no suggestion of depending on an ancient authority, we have to grant that an independent rediscovery is very probable. Nor does Pappus seem to have influenced subsequent investigations of semiregular solids until Kepler's work on the subject.

A third supposed use of Pappus may as well be disposed of here, because although it belongs to a later time, it reflects on the library of Valla. A very obscure doctor from Piacenza, Giuseppe Ceredi, in a rare book called *Tre discorsi sopra il modo d' alzar acque da' luoghi bassi...* (Parma, 1567), made the following claim (p. 6):

> Avenga che quasi a sorte mi fur venduti da chi lor non conosceva, certi scritti di Herone, di Pappo, & di Dionisidoro [*sic*] tolti dalla libraria, che fu gia del dotissimo Giorgio Valla nostro Piacentino.... Ne' quali scritti non mai stampati, o tradotti, che si sappia; confesso di havere ritrovato molte cose di quelle, ch'io sono per dire piu di sotto, & che dopo molte

116 Letter XXIV, 49. Heiberg suggests that the Athenaeus was the present Vat. gr. 1164 (described by Wescher, *Poliorcétique*, pp. xxiv-xxvi).

117 Clagett, *AIMA* III part 3, pp. 405-406.

118 Mancini [1909].

119 Hultsch, *PAC* III pp. 1169-72.

> positioni d'Euclide, d'Archimede, d'Appollonio Pergeo, & di molti altri piu nuovi, che gia conosciute da chi ha voluto, è necessario, che s'habbiano alla mano in queste operationi; m'hanno fatto non poco lume nel camino, ch'io penso haver finito dello stabilimento di questa macchina.

> More or less by chance I was sold, by someone with whom I otherwise am not acquainted, certain writings of Heron, Pappus, and Dionysodorus, removed from the library that formerly belonged to our countryman of Piacenza, the learned Giorgio Valla.... In these writings, which so far as is known have never been printed, I admit I have rediscovered many things of theirs, which I will say more about later, and which, following many statements of Euclid, Archimedes, Apollonius of Perge, and numerous others of more recent times, which anyone who wanted to already knew, it is necessary for them to have to hand in this business. They have illuminated not a little the road that I believe I have finished in the establishment of this machine.

Surprisingly, Heiberg believed this tale, and Ceredi thus slips into the roll of collectors of mathematical texts.[120] One need only remark that no writing by the Hellenistic mathematician Dionysodorus is known to have survived antiquity, while this trio of authorities appears in a famous passage of Valla's encyclopedia *De expetendis et fugiendis rebus* (Venice, 1501), which Valla adapted from Eutocius without noting his source.[121] Any doubt that Ceredi's scholarship is fictitious vanishes when, on p. 34, he ludicrously foists on Pappus a kind of Archimedean screw, and, violating chronology, has Dionysodorus add to the description.[122] Of course one can

120 Heiberg [1896] pp. 107-108. Rose, *IRM* p. 47, accepts the story.

121 Liber. XIII cap. ii, "de duobus cubis ad unum redactis". Valla also published extracts in this work from Archimedes and Apollonius, though of course not concerning machines to raise water.

122 "Pappo, & Dionisodoro; quello nel trattato de gli istromenti mecanici, & questi in certi pezzami d'un' opera di simile materia, di cui non si legge il titolo, essendovi restato solamente il nome dell' autore [!], con facilissima brevità mostrano la vera, & piu utile via di fabricare la Chiocciola. Piglia (dice Pappo) un sostegno, che non si pieghi, tornito a sesta; lungho & alto quanto basterà a tirare duoi canali di spire

not use Ceredi as evidence that Valla owned a manuscript of Pappus, or that his library was at all dispersed before passing into the hands of Alberto Pio di Carpi.

§15. The Vaticanus in Florence and Rome.[123] In the last decades of the fifteenth century Pappus finally comes out of hiding, in Florence. An incidental clue is given in a partly preserved late fifteenth-century marginal note on f. 13v of the Theon manuscript, Laur. 28,18, that had formerly been in the papal library with Pappus, but by this time had come into Poliziano's possession:

> (...) ὅμοια τρίγωνα καὶ ἐπὶ τῶν αὐτῶν πάλιν ἔτι τρίγωνα ἀνόμοια ἑαυτοῖς καὶ τοῖς ὁμοίοις δείκνυσιν ὁ πάππος ἐν τῶι ε´ τῶν συναγωγῶν ἐν ὧι παρ[αλα]μβάνει γεωμετρικῶν θεωρηματων.
>
> "... similar triangles, and on the same (bases) furthermore triangles not similar to each other or to the similar ones, Pappus proves in the fifth (book) of the *Collections* in which he takes up geometrical theorems."

The annotator notes the parallel between Theon's (or Zenodorus's) exposition of the isoperimetric theorems, and Pappus's in Book 5, particularly chapter 13.

The identity of this marginal hand is not clear. Poliziano himself, however, certainly read parts of Pappus, as we know from his paraphrase of the generalities on mechanics at the beginning of Book 8. These are to be found in a short work of 1490/91, the *Panepistemon*.[124] In the translation below, original phrases in Pappus's chapters 8.1 and .2 that Poliziano adapts are given in brackets.

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equidistanti, capaci di tanta quantità d' acqua quanta potrà essere mossa dal motore, che hai ordinato, all' altezza, che ti fa bisogno. Vi aggiunge Dionisodoro, che l' elevatione si farà secondo la ragione del pendio de' vermi a rispetto di lei. Dio buono con quanta brevità, & chiarezza, hanno questi duoi valenti Greci compreso tutto il magistero di si utile istromento?"

123 On this section see also Jones, *William of Moerbeke.*

124 Noted by Rose, *IRM* p. 35. Basle edition of Poliziano's works (1553), pp. 466 and 467-68.

Geodesia uero, quae etiam a Pappo geomoria uocatur, et ipsa in sensilibus uertitur... [cf. Pappus 8.3]

Mechanica sequitur, cuius (ut Heron, Pappusque declarant) altera pars rationalis est, quae numerorum, mensurarum, siderum, naturaeque rationibus perficitur: altera chirurgice, cui uel maxime artes aeraria, aedificatoria, materiaria, picturaque adminiculantur. Huius autem partes manganaria, per quam pondera immania minima ui tolluntur in altum: mechanopoetice, quae facile aquas antliis extrahit: organopoetice, quae bellis accomoda instrumenta fabricatur, arietes, testudines, turres ambulatorias, helepolis, sambucas, exostras, tollenones, et quaecunque graeco uocabulo poliorcetica uocantur, tormentorumque uaria genera, quae libris Athenaei, Bitonis, Heronis, Pappi, Philonis, Apollodorique continentur, ut Latinos omiserim....

Geodesy, which is also called by Pappus 'geomoria' [γεωμορία] itself is directed at sensible things...

Mechanics comes next. As Heron and Pappus say, one part of it is rational [λογικόν], which is accomplished by considerations of numbers, measures, stars, and nature [ἔκ τε γεωμετρίας καὶ ἀριθμητικῆς καὶ ἀστρονομίας καὶ τῶν φυσικῶν λόγων]. The other part is craftsmanship [χειρουργικόν], which bronze-working [χαλκευτική], construction, [οἰκοδομική], wood-working [τεκτονική], and painting [ζωγραφική] serve. Its parts are 'manganaria' by which great weights are raised up; machine-making, which easily draws water by pumps; (war-)machine-making, which makes instruments fit for wars: ...whatever are called by the Greek word 'poliorcetica', and various kinds of weapons, which are contained in the books of Athenaeus, Biton, Heron, Pappus, Philon, and Apollodorus, to pass over the Latins....

The exact location of the manuscript that Poliziano used can be identified. One of the most important collections of manuscripts at that time in Florence was the private library of the Medici family, the Biblioteca Medicea privata, of which there exists an inventory from 1495, prepared in conjunction with the transfer of the collection of Lorenzo il Magnifico to the monastery of San Marco.[125] In the second part of this inventory the very first entry is: "Arthemius [*sic*] Grecus de paradoxis machinationibus."[126] The manuscript also is listed, less ambiguously, as "᾿Ανθέμιος καὶ Πάππος γεωμέτραι π(εργαμηνόν)" ("Anthemius and Pappus, geometers, in parchment") in Janus Lascaris's inventory of 1472.[127]

A record of a loan of Anthemius in October 1486 in a register of loans from the Privata specifies the manuscript as having formerly belonged to the humanist Filelfo.[128] The manuscript cannot have been a Renaissance copy, for Francesco Filelfo died in 1481, while the earliest Renaissance copies of Pappus cannot precede Zacharias Callierges's discovery of the correct order of quires in the Vaticanus (Zacharias's earliest known work dates from the late 1490's). Also, the 1472 listing states that the manuscript was parchment, unlike all but one of the extant *recentiores* of Pappus, or for that matter most Renaissance manuscripts. Hence Filelfo evidently found the Vaticanus, and it passed with the rest of his collection into Lorenzo's library.

The circumstances under which Filelfo obtained the Vaticanus can only be guessed; it could have come into his hands as early as the late 1420's or 1430's. Pappus and Anthemius are not mentioned in his correspondence, or, apparently, in his published writings.[129] From several letters of 1440 and 1450 we learn that Filelfo had lent Vittorino da Feltre and Jacobus Cremonensis, the translator of Archimedes, a manuscript that he calls merely "mathematici" or "mathematicorum libri", and which could be the Vaticanus.[130]

The later history of the Vaticanus can now be reconstructed in part. In 1508 Giovanni Cardinal de' Medici regained much of the Biblioteca Privata, which had been confiscated a dozen years earlier by the city, and

125 Printed in Piccolomini [1875].

126 Piccolomini, p. 97.

127 Müller [1884] p. 376.

128 Piccolomini, p. 127.

129 See Calderini [1913], who was not aware, however, of the evidence for Filefo's library in the borrowers' registers.

130 In the 1502 Venice edition of Filelfo's letters, ff. 26v, 27r, 29r, 48v. See Rose, *IRM* pp. 28 and 59 note 24. But Filelfo also owned a manuscript of Apollonius's *Conics*; and the term "mathematicus" could mean also a writer on mechanics.

the same year he had it brought with him to Rome.[131] A catalogue (Vatican Barb. lat. 3185, f. 308v) of the Cardinal's library about this time, made by Fabio Vigili, lists the Vaticanus. Zacharias Callierges, who moved to Rome from Venice at some point between 1511 and 1515, likely unravelled the manuscript after it had come to Rome, as he is not known to have worked in Florence. In addition to determining the manuscript's proper order, he attempted to restore some of the washed out writing on ff. 54 and 55. It is conceivable that he made one of the two lost direct copies of the Vaticanus.[132] In 1513 Cardinal de Medici was elected Pope as Leo X, but he kept his private library distinct from the papal collection. Shortly after Pope Leo's death in 1521, his heir, Giulio Cardinal de' Medici, instructed that the Medici library should be taken back to Florence (mostly to become part of the Biblioteca Laurenziana), but this move took place only at the end of May, 1527, after he had become Pope Clement VII.[133]

However, the manuscript went, not to Florence, but to the Vatican Library. It must have entered the Vatican before 1533, since an inventory of that year lists "Anthemii Mechanica". It had not been in the Vatican inventory of about 1511, nor was it in either of two inventories of 1518 (one of these is incomplete). The manuscript must have been transferred, then, between 1518 and 1533. During these years the most important event to effect the Vatican library was the sack of Rome on May 6, 1527. If the library did not suffer quite the enormous losses that were sometimes claimed afterward, certainly the damage was serious enough that Pope Clement authorized a vigorous effort to recover dispersed books, both in Rome and abroad.[134] The papal decree further authorized the agents in

131 Bandini, *CC* I pp. xii-xiii.

132 Either the destroyed Strasbourg MS (**R**) or the ancestor of the family **CVkD** (see below, page 57). The first copy of the Vaticanus must have been made before 1527, when Andreas Coner died, leaving a library including a manuscript "Mechanica Pappi Alexandrini greca scripta in papiro." See Mercati [1952] p. 143. The title "mechanica" does not prove that he had only Book 8; for example Vat. gr. 1725, with Books 3 to 7 (incomplete), bears this title (it cannot be Coner's MS however).

133 Bandini, p. xiii note 4. It should be observed, though, that many manuscripts belonging to the Medicea Privata passed to Leo X's nephew, Niccolò Cardinal Ridolfi, and these are now mostly in the Bibliothèque Nationale. There were clearly many opportunities for a manuscript to become detached from Pope Leo's library.

134 Devreesse, *FG* pp. 152-184 (1511), 185-263 (1518), 309 (1533),

Rome to select desirable books from the libraries of deceased collectors; but this was not applicable to the Vaticanus, which belonged to the Pope himself, and would by that time have gone to Florence if it was still among the Medici manuscripts.

From 1533 on, the Vaticanus remained almost continuously in the Vatican Library. The only recorded exception was the loan of it briefly to the copyist Valeriano da Forlì in 1547; it was returned the following year.

§16. The recentiores. Thanks to Treweek's paper on the European manuscript tradition of Pappus, we can trace the sometimes quite complicated relations between the Vaticanus's descendants.[135] The following list of manuscripts includes all those that Treweek identified and classified, with a few additions. The sigla are Treweek's, which incorporate those of Hultsch (**A**, **B**, **V**, and **S**). Column three indicates the books of the *Collection* represented partly or completely ('A' for Anthemius), column four the date of the manuscript (except for **A**, always sixteenth-century or later), column five the exemplars. This information is derived, except for the additional manuscripts at the end of the list, from Treweek's article, to which the reader is directed for details. I have added notes on the identified copyists and early owners of some of the manuscripts. References to Treweek will be given as [T *page*].

A	Vat. gr. 218	A2345678	10th c.	
U	Urb. gr. 72	7	ca. 1588	**A**

U was commissioned by Duke Francesco Maria II of Urbino on behalf of the editor of Commandino's translation, Guido Ubaldi (or Guidobaldo). Commandino had left some gaps in his translation, and the manuscript from which he worked for Book 7 (**k**) apparently was not available to Ubaldi.[136] In August, 1587, Ubaldi was awaiting a manuscript from Rome before submitting Book 7 to the press,[137] but since the printed text still shows some gaps, it seems that the new copy arrived too late. In fact

264-66 (Sack of Rome). Devreesse computes from the 1533 and 1518 inventories that the number of Greek manuscripts declined by about thirty. This figure does not attempt to account for new manuscripts that entered during the interval; and we do not know how successful the effort to recover the scattered books was.

[135] Treweek [1957]

[136] Rose, *IRM* p. 211.

[137] Rose, p. 211 and p. 221 note 151.

a letter of **U**'s copyist, Pietro Devaris, to the Duke of Urbino announcing the completion of the manuscript cannot be earlier than 1588, since it alludes also to a publication by Devaris's uncle Matteo of that year.[138]

L	Neap. III c 16	345678	<1588	**A**
Q	Par. gr. 2369	3	late 16th c.	**L A**

Omont identified the hand as that of the mathematician G. Auria [T 202]. Auria is known to have consulted manuscripts at the Vatican.[139]

F	Laur. 28,9	34567	<1588	**L**

F originally had the continuation to the 'end' of Book 8. The final pages, separated from the rest, were the exemplar for parts of **M** and **Z**, before being lost [T 215].

[R]	Strasbourg (lost)	A2345678	<1554	**A**

The manuscript entered the possession of the Strasbourg mathematician Dasypodius, apparently before 1582, when he was contemplating making a translation of it.[140] Most of Dasypodius's mathematical manuscripts are alleged to have passed through the hands of Andreas Darmarius.[141] **R** was destroyed in the bombardment of Strasbourg in 1870 [T 205].

H	Ambr. D 336 inf.	A8	16th c.	**R**
P	Oxon. Savile 9	2345678	16th c.	**R**

The later of Savile's two copies of Pappus, obtained from Dasypodius in Strasbourg [T 203].

138 *CUG* p. xxx.

139 Mogenet, *Autolycus* pp. 43-49.

140 C. Wescher, in *CRAI* N.S. 7 (1871) p. 182. Letter of Savile to Pinelli, June 12, 1582, in Ambr. D 243 inf.

141 Wescher, p. 182.

E Escorial T i 11, y i 7 A2345678 1547/48 **A**

Copied for Don Diego Hurtado de Mendoza by Valeriano da Forlì [T 200]. Manuscripts dependent on **E** have to date before 1554, when Don Diego returned to Spain [T 231]

B Par. gr. 2440 A2345678 <1554 **R E**

The earliest of the known copies of **R** [T 217-18]. It was in the collections of Cardinal Cervini (briefly Pope Marcellus II) and Cardinal Sirleto, where it still was in 1574.[142] **B** may have been Andreas Coner's manuscript (see page 55 above), since another mathematical manuscript, Ottob. lat. 1850 (William of Moerbeke's autograph of the translations of Archimedes) is known to have passed from Coner's collection to Cardinal Cervini. How it strayed later to France is not known.

Y Vind. sup. gr. 40 A2345678 >1574 **B**

Copied in Paris [T 218].

J Angelica gr. 111 34568 <1572 **A B**

Copied partly by Manuel Provataris (active from the 1540's to the early 1570's at the Vatican Library).[143]

G Edinb. Adv. 18.1.3 34568 <1572 **J B**

Apparently made at the same time as **J** [T 219-20]; **G** is partly the work of Camillus Zanettus or Venetus, apparently early in his career.[144] These manuscripts, which omit Book 7, must have been commissioned to complete manuscripts that had only Book 7. **G** was the manuscript on which Commandino based his translations of Books 3 to 6 and Book 8 [T 228-29]. It later belonged to Bullialdus, and still later to Simson [T 204].

[x] (lost) 2345678A <1554 **A**

142 Inventories in Devreesse [1968], especially p. 261, where the specification of 197 folia permits the identification.

143 *RGK* I A, p. 139.

144 *RGK* pp. 119-121.

D	Par. gr. 583	8	<1569	**x**

Copied by Angelus Vergetius [T 202] ('Αγγελος Βεργίκιος), who was active in Venice and Rome in the 1530's, and in Paris from 1539 to his death in 1569.[145]

d	Par. gr. 2871	8A	<1569	**D**

Copied by Angelus Vergetius [T 202].

V	Leiden Voss. F. 18	2345678	16th c.	**x**
C	Par. gr. 2368	2345678	1562	**x**

Copied by Nicolaus Nancellius for Ramus [T 225]. It is likely that Witelo's editor Risner obtained his information about Pappus from Ramus. Ramus himself used Pappus as a historical source in two works published in 1569.[146]

S	Leiden Scal. 3	2345678	>1562	**C**
k	Chicago Newberry 115	7	<1554	**x**

The earliest copy of **x** [T 226]. This was Commandino's manuscript for Book 7 [T 229].

K	Neap. III c 14	A2345678	16th c.	**E k**

Treweek reports that the leaf edges have the inscription "FRANCIS MAYRON SUMMA" [199]. Could this be Francisco Maurolico? Until 1570 there seems to be no sign of knowledge of Pappus in his work, although certain projects, such as the reconstruction of Books 5 and 6 of Apollonius, would have given the opportunity to mention him. The 1570 edition of Maurolico's "Index Lucubrationum", a kind of survey and program for publication of ancient mathematics, names "Pappi... mechanica et

145 *RGK* pp. 25-26.

146 Ramus [1569,1], pp. 25-26 on isoperimetry, from Book 5; [1569,2] pp. 28-30, 33 (on Apollonius, from Book 7), 35 (duplication of cube), 58 (mechanical 'powers' from Book 8), 106-107 (reports that Commandino has corresponded with Ramus about Pappus).

machinae", which had not appeared in the earlier versions of this opuscule.[147]

N Neap. III c 15 345678 16th c. **B J k**

X Vind. supl gr. 12 34567 16th c. **N**

May have belonged to Giovanni Carlo Grimani [T 205].

W Wolfenbüttel Gud. gr. 7 34567 16th c. **X**

Belonged to Matteo Macigni (active in the 1540's and 1550's), later to Nicolaus Trevisianus of Padua and Marquard Gude [T 201].

M Ambr. C 266 inf. A2345678 16th c. **W R F**

J. V. Pinelli's manuscript [T 198]. Originally there must have been two incomplete manuscripts, a copy by Pinelli's regular copyist Camillus Venetus of **W** with Books 3 to 6 and the beginning of 7, and a copy of the lost end of **F**. The rest was apparently filled in later from **R** [T 222-24].

Z B.M. Burney 105 2345678 <1588 **M F**

Another composite manuscript. Its two original parts were a copy of that part of **M** that was a copy of **W**, and another copy of the lost end of **F**. It was completed from **M** after that manuscript had been completed [T 222-23]. In 1588 it was used and emended by Barozzi during his preparation of a revision of Commandino's translation; at this time it was owned by Contarini.[148]

O Oxon. Savile 3 345678 16th c. **M**

Savile had this copy made by Camillus Venetus [T 203] in 1581-82, during his travels on the continent in search of mathematical manuscripts.[149]

T Vat. gr. 1725 34567 16th c. **M**

147 Clagett [1974] p. 190.

148 Rose [1977].

149 Letters of Savile to Pinelli, 27 December 1581 and 24 May 1582, in Ambr. D 243 inf.

Copied from the part of **M** that was copied from **W**. Belonged to Alvise Lollino [T 198].

I	Laur. Plut. 28,17	2	16th c.	**M**

A copy by Camillus Venetus of the added part of **M** that he had copied from **R** [T 222].

-	Oxon. Christ Church 86	2345678	1688-1710	**P O**
-	Neap. III c 16 bis	345678	19th c.	**N**
-	Oxon. Savile 60A	23	1772	**C**
Wa	Oxon. Savile 60B	23	1680's	**P O**

Used by Wallis for his *editio princeps* of Book 2.

-	B.M. Egerton 850	7	>1748	**C**
-	Par. gr. 2370	8A	1646	**d**
-	Par. gr. 2535	8		**D**
-	Par. gr. sup. 15	8	>1574	**B**
-	Florence Bibl. naz. II iii 37	Figures	16th c.	
-	Ambr. P 144 sup.	Figures	16th c.	

Treweek [T 228] briefly discusses these diagram books. The Milan manuscript may have been intended for **M**.

The remaining manuscripts are not listed in Treweek's article.

-	Vind. phil. gr. 229	A2	late 16th c.

Copied by Claudius Acantherus.[150] Readings reported by Dupuy in his edition of Anthemius show that the text ultimately descends from **R**.[151]

150 *KGH* vol. 1 p. 340.
151 Dupuy [1786].

- Copenhagen Thott 215	A	ca. 1560

The manuscript is partly by Manuel Provataris, and allegedly is a direct copy of **A**.[152]

- Marc. gr. XI,30	A	late 16th c.

A collection of booklets, once belonging to Pinelli, later to Contarini. Partly copied by Camillus Venetus (including Anthemius).[153] Hence probably descends from **R**.

- Saragossa 1143	A	1585
- Honeyman 7	A	1585

Both manuscripts (if they are not one and the same!) are subscribed as having been copied by Andreas Darmarius, Venice, 24 August 1585.[154] Likely descendants of **R**.

§17. **Printed editions.** Even from the little that we now know about the dispersal of the *Collection* during the sixteenth century, we can see that by the 1580's copies of the Greek text were reaching not only humanist bibliophiles, but also some mathematicians: Savile, Dasypodius, Auria, possibly Maurolico. But during that time not much was done with the book. At Urbino Bernardino Baldi and Guido Ubaldi studied the mechanics in Book 8,[155] but the great exploitation of the other books became possible only with the posthumous printing of Commandino's Latin translation of the *Collection*. Commandino's achievement as a translator was admirable, making accessible to European mathematicians the works of Euclid, Archimedes, Apollonius, and other lesser texts, in versions that were intelligible and free from most of the mistakes that had made

[152] Schartau-Smith [1974] pp. 335-36.

[153] *BDM* vol. 3 pp. 155-56.

[154] *CMRM* supp. p. 20; Graux-Martin [1892] pp. 223-24. The folio numbers listed in the two catalogues differ; but the identity of the dates and contents, together with known disturbances in the Saragossa collection, arouses suspicion.

[155] See Rose, *IRM* pp. 222-79.

nonsense of the mathematics. The history of the publication of his Pappus is unusual.[156] Commandino had left the translation at his death in 1575 in a nearly finished state, but for many years its publication was delayed because of a family dispute. Duke Francesco Maria II of Urbino eventually obtained the manuscript and sent it to Francesco Barozzi in Venice to assess. Barozzi was dissatisfied with the translation, and asked permission to revise it extensively. This request was refused. The manuscript was then passed on to Guido Ubaldi, who saw the text, more or less as Commandino left it, through the press. There were four editions of the translation: Pesaro, 1588; Venice, 1589; Pesaro, 1602; and, reset (generally to the text's detriment) by Carolus Manolessius, Bologna, 1660.

The text of Pappus was already quite corrupt in the Vaticanus, and in Commandino's manuscripts **G** and **k**, apographs at second or third remove from the archetype, it had only become worse. He had no copy of Book 2 (it would be printed only in 1688 by Wallis). For the books he had, Commandino's service was not of uniform quality. In the mathematical parts, he was able to restore the correct sense very successfully. Immense numbers of errors of lettering, omissions, erroneous repetitions, and other similar corruptions are corrected, either in the textual notes that follow the propositions, or tacitly in the translation. Where the mathematics gives way to prose, however, Commandino was much more diffident, adopting the text before him and interpreting it as best he could. The difference is very apparent in Book 7. Scarcely a page of text in Hultsch's or this edition, from chapter 43 to the end, does not preserve several emendations, correct or substantially so, by Commandino. In the first forty-two chapters, however, he made only a handful of unremarkable changes. Some of the defects of the translation can be ascribed to its being printed without Commandino's final revisions.

After Commandino several projects were begun to publish editions or translations of the *Collection*, of which the only one that calls for mention here is Barozzi's revision of Commandino, which exists in manuscript but has not been studied.[157] More important are several publications of small parts of the Greek text, especially from Book 7. None of these could be called a critical edition, but some introduced emendations of the received text, or offered original interpretations of them. Halley deserves special mention for his edition of the whole introductory part of the book and a

[156] See Rose, pp. 209-213.

[157] Par. lat. 7222. See Treweek [1957] pp. 230-31; Rose, *IRM* pp. 211-13, and [1977]; M. Boyer in *CTC* vol. 2 pp. 205-213 and vol. 3 pp. 426-431.

large fraction of the lemmas. The list of publications below draws on Hultsch's survey.[158]

Snel [1608]: Book 7, 9-10.

Wallis [1699]: Book 2.

Halley [1706] pp. i-xxvii: Book 7, 1-67. [1710]: 233-311. Used **O** and **P**.

Simson [1749]: 21-26. Used Halley [1706], **B**, and **C**.

Torelli [1769]: Book 4, 45-52.

Horsley [1770]: Book 7, 27-28, 126.

Camerer [1795] pp. 158-84: 11-12. [1796] pp. 185-92: 21-26. Used **B**, **C**, and **R**. The Greek text of the 1795 publication was reprinted in Haumann [1817].

Eisenmann [1824]: Book 5, chapters 33-105 (the only part printed of a planned edition).

Breton de Champ [1855] pp. 209-304: Book 7, chapters 13-20. Used **B** and **C**.

In 1871 C. J. Gerhardt produced an edition and German translation of Books 7 and 8, as the second volume of a projected complete edition. There is no apparatus or introduction, but he apparently used at least **B**, **C**, and **M**, which are mentioned in his notes on pages 216 and 300. Gerhardt's text improves that of the manuscripts to the extent of incorporating Commandino's improvements and other obvious corrections, but his more elaborate conjectures are few and unimpressive. Gerhardt elsewhere proposed a bizarre interpretation of the *Collection*, admitting only Books 3, 4, 7, and 8 as authentic; perhaps it is just as well that his work did not preempt a better edition.[159]

158 Hultsch, *PAC* vol. 1 pp. xv-xxii. Later excerpts, such as Heiberg's in his editions of Apollonius and Euclid, are based on Hultsch's text.

159 Gerhardt [1875], cited by K. Ziegler in *RE* vol. 18 (1949) col. 1095.

This was Friedrich Hultsch's Pappus, one of the first modern critical editions of a Greek mathematical work.[160] Hultsch gave scholars a generally reliable text, a new Latin translation with critical and historical notes, and an annotated index that remains invaluable as a lexicographical aid for the study of Greek geometry. After more than a century this work remains the standard reference for the *Collection*. Nevertheless it is unsatisfactory in some important respects. The foundation of Hultsch's text is not the primary source of trouble. It is true that Hultsch learned of the existence and guessed the importance of the Vaticanus only after having gone far in establishing his text from other manuscripts, and that in adjusting his text to stand on a new basis he introduced many errors in the reporting the archetype's readings. These mistakes, while annoying, did not lead to any significant misrepresentation of Pappus's text: for the greater part they merely caused simple and obvious emendations in the mathematical reasoning or the grammar to be credited to the Vaticanus. Much more regrettable was Hultsch's readiness to attribute almost any oddity in the received text to the intervention of interpolators; this has already been discussed above (section §4). In many cases it is difficult to see why Hultsch judged passages as inauthentic; often scribal and authorial carelessness and the derivation of our text from draft copies are the likely explanations of what Hultsch saw as intrusions. The Arabic version of Book 8 confirms that many of Hultsch's bracketings are incorrect. This translation, based on a text that probably descends from Pappus's autographs by a line independent of the Greek tradition of the *Collection*, shares with it all the many larger passages that Hultsch excised. The effect of the bracketings is not trivial; often it distorts the sense of Pappus's statements. These are the serious general faults of Hultsch's edition; however, it is no criticism of his work to add that Pappus's text remains susceptible of improvement in numerous places.

After Hultsch no edition or translation based on a new examination of the text has been printed (A. P. Treweek's edition of Books 2 to 5 [thesis, University of London, 1950] remains unpublished). The only complete translation into a modern language is the French version by Ver Eecke.[161] Like his many other translations, this is useful and competent, but it is also too faithful to Hultsch's text. Ver Eecke's commentary is sparse, though generally accurate, and the lack of page and chapter references to the Greek text makes comparison with the original inconvenient.

160 Hultsch, *PAC*.

161 Ver Eecke [1933].

Introduction to Book 7

§18. **The Domain of Analysis.** Book 7 of the *Collection* is a companion to several geometrical treatises, which by Pappus's time were alotted to a special branch of mathematics, the ἀναλυόμενος τόπος, or 'Domain of Analysis'.[162] These books were supposed to equip the geometer with a "special resource" enabling him to solve geometrical problems. More precisely, they were to help him in a particular kind of mathematical argument called 'analysis'. The nature of Greek geometrical analysis has been the subject of an enormous philosophical and metamathematical literature, to which I am reluctant to add.[163] The following remarks are meant only as a description of analysis as it actually occurs in Pappus and other ancient texts, and to show the application of the "Domain of Analysis" to it.

In ancient geometry 'analysis' had none of its modern connotations, but referred to a kind of reversal of the normal 'synthetic' method of proof or construction. Synthesis began with assumed abstract objects and statements about them, and, by a series of steps conventionally admitted to be valid, eventually arrived at a desired conclusion: the validity of an assertion in a 'theorem', the construction of a specified object in a 'problem'. A synthetic proof of any but the simplest propositions might be difficult to discover directly, so that as a preliminary approach it would be advantageous to work backwards from the goal, on the supposition that the order of the steps could be reversed to produce a valid synthesis of the proposition.

162 See the notes to 7.1.

163 One recent paper, Mahoney [1968], is notable, in spite of several misconceptions, for its refreshing emphasis on analysis as a mathematicians' tool rather than philosophical method, and for its bibliographical references. A more promising line of investigation than the meticulous hermeneusis of the same few passages in Greek authors (Pappus, Marinus, the scholiast to *Elements* XIII) might be the reception and development of Greek analysis by Arabic mathematicians, of which there survive copious theoretical discussions and examples in practice that have yet to be studied.

Pappus draws (in 7.2) an important distinction between the analysis of theorems (propositions in which the validity of an assertion is to be determined) and the analysis of problems (propositions requiring the construction of a described object from various data). Actual examples of 'theorematical' analysis in ancient texts are not numerous: they include a well known series of analyses of the first five propositions in Book 13 of the *Elements* inserted into the transmitted text at some time after Euclid,[164] and some instances in Pappus, for example 7.225, .226, .231, and .321 in Book 7. As these show, analysis as applied to theorems was a comparatively naive technique using the same kinds of logical steps as synthetic proof, but beginning with the assumption of that which is to be proved, and advancing until a conclusion is reached that is known to be true (or false) independently of the assumption. Consequently the technique guarantees neither the correctness of the proposition nor the possibility of obtaining a valid proof by inverting the steps of the argument. For example, in 7.321 the proposition is indeed correct, but the analysis that apparently verifies it is not reversible, a circumstance that explains Pappus's difficulties in attempting a synthesis of the proposition in 7.319. However, if the analysis arrives at a conclusion independently known to be false, or inconsistent with the assumption, then it is a valid disproof by *reductio ad absurdum*, and requires no inversion; such proofs are, of course, well attested.

In contrast to their counterpart for theorems, analyses of problems are very common in Greek treatises. There seem to have been two reasons for this fact: first, there existed an expandable repertory of operations that were reversible as steps in geometrical construction (so that the analysis of a problem had a degree of cogency lacking in theorematic analysis); and secondly, an analysis could yield information about the conditions of possibility and number of solutions of a problem, the determination of which, or 'diorism', was an essential part of a complete solution of a problem. Essential to the analysis of problems was the concept of being 'given', which was applied both to those objects that are assumed at the beginning of a problem, and to any other objects that are determined by the assumptions. The word 'given' had a wide range of mathematical connotations in antiquity,[165] but the most common meanings were 'assumed', 'determined', and 'determined and constructible'. The distinction between the second and third arises only in problems, such as the trisection

164 Euclid, *Opera* vol. 4 pp. 364-76.

165 They are discussed, rather confusingly, by Marinus (fifth century A.D.) in his introduction to Euclid's *Data* (Euclid, *Opera* vol. 6 pp. 234-57).

of the angle, where the postulates admitted by the geometer may not enable the actual construction of the object, although it is considered to exist.

Euclid's *Data* codifies the basic definitions and fundamental theorems required for analysis of problems. Line segments, areas, circles, circular segments, and angles are 'given in magnitude' when their equals can be constructed (πορίσασθαι). Similarly a ratio whose equal can be constructed is 'given'. Rectilineal figures, when figures similar to them can be constructed, are 'given in shape' (or 'in species'). 'Given in position', applied to points, lines, and other drawn objects, is defined as "always occupying the same place" (a not entirely satisfactory description). The propositions that follow each assert that, if various objects are assumed given, then a certain consequent object is given (that is, determined). For example, proposition 25: "If two (straight) lines given in position intersect, the point at which they intersect is given in position." Or proposition 90: "If from a given point a straight line is drawn tangent to a circle given in position (and magnitude), the (line) drawn is given in position and magnitude." This example shows that being 'given' does not always entail being unique (but there must be only a finite number of solutions). The proofs use the established arguments of synthetic geometry (as in the *Elements*), together with the foregoing propositions within the *Data*. Necessarily there are some steps that are not well defined, as in proposition 25, where the argument is that if the intersection is not given, it can be 'shifted', and therefore one of the two straight lines will 'shift', in contradiction with the assumptions. But essentially the *Data* establishes a large number of theorems about the constructibility of objects, which are extremely valuable in the analysis of a problem.

An illustration of the complete solution of a problem, with analysis and synthesis, is the first proposition (1.1.1) of Apollonius's *Cutting off of a Ratio*, translated in Appendix 3. The analysis begins by assuming the existence of the sought object, and by various constructions and arguments of the kind proved in the *Data* arrives at the conclusion that the sought object is given. Furthermore, Apollonius derives from the analysis a 'diorism' for the problem, namely a condition that one of the given objects (in this instance a ratio) must satisfy for the problem to have its unique solution.

Not surprisingly, the *Data* turns out to be the very first treatise in Pappus's list of works in the 'Domain of Analysis'. The remainder apparently were to provide help at a more advanced level. One of them, Euclid's *Porisms*, seems to have been in character rather like the *Data*, but with much more complex hypotheses.[166] With the exception of

166 See Essay B.

Apollonius's *Conics*, the remainder of the books in Pappus's list were collections of either problems or locus theorems. The five problem books were all by Apollonius: the *Cutting off of a Ratio*, the *Cutting off of an Area*, the *Determinate Section*, the *Neuses*, and the *Tangencies*. Only the first of these works survives intact (in Arabic), but their general character appears to have been uniform.[167] Apollonius chose for each a single problem or group of related problems, and gave an analysis, synthesis, and (where necessary) diorism of every conceivable case as determined by the various possible mutual relationships of the objects assumed given. This thoroughness inevitably made the books very long and monotonous, while the problems chosen for solution were sometimes not very interesting in themselves. On the other hand, they are the kind of problems to which more complicated problems might often be reducible by analysis. It appears, therefore, that Apollonius himself must have had a programmatic purpose in writing these works, and that the idea of a 'Domain of Analysis' may have originated with him. The books of loci also have a manifest utility in analysing problems. Each locus theorem proved that some object (usually a point) that satisfied certain conditions with respect to given objects lies on a given object (usually a straight or curved line, or a surface). Hence if the same point simultaneously exhibits two independent locus properties, it will be at the intersection of two given lines, and so will itself be given. The locus books in the 'Domain of Analysis' were Apollonius's *Plane Loci* (loci that are straight lines and circles), Aristaeus's *Solid Loci* (conic sections), Euclid's *Loci on Surfaces* (probably surfaces of spheres, cylinders, and cones), and Eratosthenes's 'loci with respect to means' (in his book *On Means*), of which we know nothing.[168] Apollonius's *Conics* seems oddly out of place in the 'Domain of Analysis'. While it is true that parts of its eight books are devoted to problems related to conics, much of the work is devoted to proofs of properties of the conic sections that would be of little immediate use in applications to general problems. Pappus hints in 7.29 that the *Conics* was in the 'Domain' primarily as a preparation for Aristaeus's earlier collection of loci, since that work did not prove all the basic theorems concerning conic sections that it depended on.

Pappus is our only substantial source of knowledge of the 'Domain of Analysis'. It was known later, for about A.D. 500 Marinus mentions it, and in the next century Eutocius quotes a theorem in Apollonius's *Plane*

167 See Essay A.

168 For Apollonius's book, see Essay A section §8; for the others, Essay C.

Loci as coming from it.[169] Of the works that it comprised, all those by Apollonius (except Book 8 of the *Conics*), as well as the *Data*, apparently were translated into Arabic around the ninth century.[170] However, Euclid's *Porisms* and *Loci on Surfaces*, and the treatises of Aristaeus and Eratosthenes probably were not known to Arabic mathematicians, and there is no evidence that the other works had a common mode of transmission. Perhaps the manuscript or manuscripts of the six minor works of Apollonius that gave the Arabic translators their Greek text were the last in the world, for after Eutocius no Byzantine ever alludes to them.

§19. The purpose and plan of Book 7. Book 7 is not a commentary to the works of the 'Domain of Analysis', at least in the conventional sense. It comprises three parts: a general introduction to the 'Domain', a series of introductions or 'epitomes' (περιοχαί) of nine of the treatises (omitting Aristaeus's *Solid Loci*, Euclid's *Loci on Surfaces*, and Eratosthenes's *On Means*), and a corpus of lemmas to these treatises (omitting the *Data*, but including a fragmentary section for the *Loci on Surfaces*). Where possible Pappus follows a constant formula for the epitomes: he states the problem or problems solved in the work in as general a form as he can, and then recites various statistics about the numbers of problems, cases, propositions, diorisms, and lemmas belonging to it. For the *Porisms* and *Conics*, which were to long and varied in content for such a summarization, Pappus abbreviated the account, in the one case by classifying the propositions according to a rather arbitrary scheme, in the other by quoting Apollonius's own introduction. Occasional digressions sometimes contain interesting matter; the most remarkable is in 7.33-42, in which we are given Pappian portraits of Euclid and Apollonius, the enunciation of an important locus theorem (the 'locus on three and four lines') and its unsolved generalization, a tirade against the incompetence of Pappus's contemporaries, and an unproved proposition concerning the volumes of solids of revolution. These epitomes must have been meant to be read before the treatises, and as a guide to their contents.

The lemmas, on the other hand, were to accompany the actual working through of each treatise. Pappus claims (7.3) to have identified every passage that required a lemma, that is, every passage in the geometrical reasoning that assumed steps that a reader would not be able to justify immediately from what had preceded and his elementary knowledge. Unfortunately, when Pappus included a lemma in Book 7, he

[169] See pages 21, 24.

[170] See the references in Essay A, and the notes to 7.4.

did not invariably indicate the place in the treatise to which it referred. Moreover, he often included additional theorems and problems that were not true lemmas, but rather supplements and alternative proofs. Consequently it is often difficult for us to correlate the lemmas with the treatises, even in the case of the extant parts of Apollonius's *Conics*.

§20. Mathematics in Book 7. The lemmas for the most part make dreary reading. As one might expect, the steps that Apollonius chose not to fill out in his minor works and the *Conics* are not the most advanced and interesting innovations, but usually certain frequently encountered theorems of an easily recognizable kind that the author preferred to leave to his reader to confirm. The lemmas (7.132-156) to the *Neuses* are typical: except for 7.142 and .146 they are all variations of the same moderately easy proposition, adapted to different relative dispositions of the given objects. This class of lemmas, though tedious to work through, are historically valuable as clues to confirming the identification of the actual solutions used in the lost works, either reconstructed by conjecture or recovered from second-hand sources. Moreover, the pattern of variations in a series of similar lemmas is an indication of the plan of the original work that assumed them.

A few of Pappus's lemmas surpass this level of interest. In particular, those to Euclid's *Porisms* and *Loci on Surfaces* are our best evidence for the content of those works. Many of the lemmas to the *Porisms* are either demonstrably or probably syntheses of theorems that Euclid proved by analysis; they are themselves much more sophisticated than Pappus's usual fare.[171] In the fragmentary section on the *Loci on Surfaces* Pappus presents a proof of the focus-directrix property for the general conic (the earliest preserved), which seems to have had a three-dimensional analogue in Euclid's work.[172]

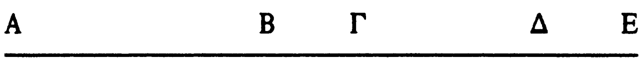
Among the more humble lemmas to Apollonius's treatises, a large and homogeneous group are related to what is conventionally called 'geometrical algebra'. These propositions (which include many of the lemmas to the *Cutting off of a Ratio*, the *Determinate Section*, and the *Conics*) prove various identities concerning sums of products of (or in Greek terms, rectangles contained by) line segments along a single straight line. For example, 7.117 to the *Determinate Section* demonstrates that, if points **A**, **B**, **C**, **D**, **E** are distributed in that order along a line, and segment **AB** equals segment **CD**, then

171 See Essay B.

172 See Essay C, section §6.

$$BE \cdot EC - AE \cdot ED = BD \cdot DC$$

(the purist will read **a·b** as "the rectangle contained by **a** and **b**"). Pappus generally proves this kind of lemma by reference to the theorems at the beginning of Book 2 of the *Elements*, where Euclid establishes several fundamental theorems in 'geometrical algebra'. The propositions most often used are summarized in the following table.



Let $AB = B\Delta$.

II,4: $A\Gamma^2 = AB^2 + B\Gamma^2 + 2\,AB \cdot B\Gamma$

II,5: $A\Gamma \cdot \Gamma\Delta + B\Gamma^2 = B\Delta^2$ (Used very often)

II,6: $AE \cdot E\Delta + B\Delta^2 = BE^2$ (Used very often)

II,7: $A\Gamma^2 + \Gamma B^2 = 2\,A\Gamma \cdot \Gamma B + AB^2$

II,8: $4\,A\Gamma \cdot \Gamma B + AB^2 = (A\Gamma + \Gamma B)^2$

II,9: $A\Gamma^2 + \Gamma\Delta^2 = 2\,(AB^2 + B\Gamma^2)$

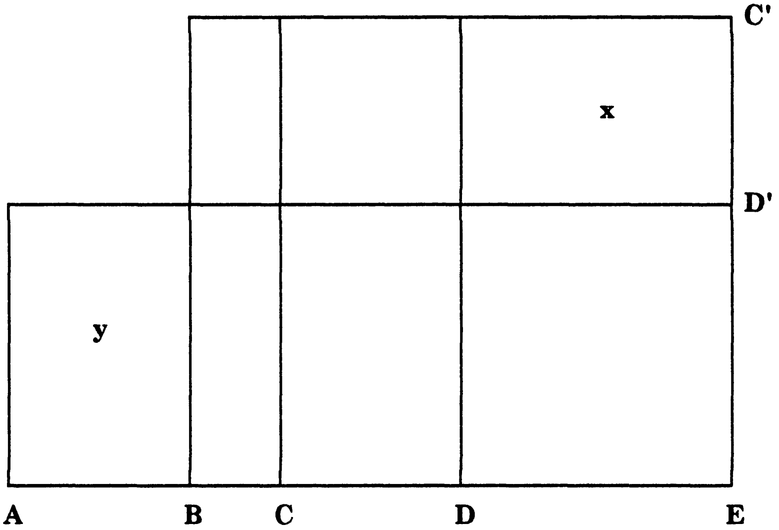
II,10: $AE^2 + E\Delta^2 = 2\,(AB^2 + BE^2)$

II,11: Problem, given $A\Delta$, to find Γ such that $A\Delta \cdot \Delta\Gamma = A\Gamma^2$

Probably when Apollonius assumed identities in 'geometrical algebra', he did not expect the reader to work out Pappian proofs based on the *Elements*, but rather to justify them directly by the same kind of proofs as Euclid uses for his fundamental theorems.[173] The technique is to erect on the line segments rectangles with heights equal to some of the segments, so that the products in the inequality are readily visible. For example, in place of 7.117 one could construct the figure below, from which the

[173] This is Zeuthen's hypothesis (Zeuthen [1886] pp. 36-38). It is independent of the much discussed question of whether such theorems should be 'interpreted' as geometry or algebra, or the separate, purely historical question of their origins.

proposition is immediately apparent because the areas labelled **x** and **y** are equal.



Manipulations of ratios also are extremely important in the lemmas of Book 7. The fundamental theorems are in Book 5 of the *Elements*, and are tabulated below.

Ratio manipulation and equalities.

Let **a:b** = **c:d**, **b:e** = **d:f**, **b:g** = **h:c**.

V,12: taking all to all **(a+c):(b+d)** = **a:b**

V,16: *alternando* **a:c** = **b:d**

V,18: *componendo* **(a+b):b** = **(c+d):d**

V,17: *separando* **(a − b):b = (c − d):d**

V,19: *convertendo* **a:(a − b) = c:(c − d)**

V,22: *ex aequali* **a:e = c:f**

V,23: *ex aequali* in disturbed ratio **a:g = h:d**

Moreover, Pappus makes considerable use of compound ratios (that is, products of ratios), which are not treated in the *Elements.* There are a large number of instances where Pappus proves the same lemma twice, once using compound ratios, once without them.

For almost all the lemmas, the first few books of the *Elements* are a sufficient basis. The exceptions, which may be result from Pappus's carelessness in adapting proofs from earlier, more complete sources, are indicated in the commentary. Although the longest series of lemmas pertains to Apollonius's *Conics*, conic sections appear in only a few propositions: 7.274-279, related to Book 5 of the *Conics*, and 7.312-318, the lemmas to Euclid's *Loci on Surfaces.* These use no advanced results of the study of conic sections, and their dependence on Apollonius's treatise and earlier sources can be deferred to the commentary.

The reader should be aware of one convention that differs from modern practice in mathematical writing. Generally, the figures that accompany the text are illustrative, and it would be extremely bad form to argue 'from the figure'. However, the order of points on a line, and the definition of points that are the intersections of lines described in the text is often left to the reader's consulting of the drawing. For example, Pappus might write, "Join ΑΓ", and subsequently discuss points Η, Θ that have not been defined in the text, but that the reader sees from the drawing are the intersections of ΑΓ with, say, a circle that has been defined earlier. Also, Pappus will often write that "line ΑΓ is given" when the points Α, Γ on it are not given, or when only one of them is given, at the time that he first mentions the line. This practice, a consequence of the fact that only points are named in most rectilinear figures, should not cause confusion so long as the reader is alerted to it.

Editorial Principles

The Greek text. The text is based on a transcription made from photographs of the Vaticanus, subsequently collated with Hultsch's text, Commandino's translation, and the partial editions. Some passages, where moisture had long ago made the manuscript difficult to read, were collated again with the Vaticanus itself; personal inspection revealed that much of the text in these places could be read from the impression of the scribe's pen. In a small number of passages the original text is practically illegible; these have been enclosed in half-brackets ("⌈", "⌉") in this edition.

The apparatus was constituted as follows. Where the adopted text diverges from the Vaticanus, the Vaticanus's reading is reported. Errors of accent and breathing, omissions of iota adscript (but not superfluous iotas), and division or joining of geometrical letters were excluded from the reporting; they almost certainly do not reflect phenomena in Pappus's own copy. Emendations adopted in the text from earlier authorities are credited in the apparatus, which also includes many, but not all, innovations of the earlier printed texts which were rejected or modified. A just treatment of the *recentiores* would have required not only collation of all the manuscripts, but also a determination of the hands and identification of those correctors who used Commandino's translation. But to omit their contribution entirely would have led to the attribution of numerous readings to Commandino, Gerhardt, and Hultsch that they merely inherited. As a compromise, the following rules were adopted.[174] Where Commandino adopts a text of his manuscript **k** that is an improvement on that of **A**, it is reported as "Co (k)". The readings of **k** come from my collation of the manuscript.[175] Where Hultsch credits an improvement to one of the manuscripts **B**, **V**, or **S**, it is reported as, for example, "Hu (S)" (or "Ge (S)" if Gerhardt adopted it). I have not collated these manuscripts, and so Hultsch's readings have to be taken on trust. When he specifies a correcting hand, in particular the hand that he believed, mistakenly, to be Scaliger's in **S**,[176] these have

174 I have not had the opportunity to collate the Savile manuscripts, so that probably some minor corrections already in them are attributed to Halley in my apparatus.

175 The numerous corrections in **k** are probably Commandino's own (if not, then they are derived from his translation). These are not reported.

176 Treweek [1957] p. 201 note 16.

been reported only when they do not coincide, exactly or substantially, with Commandino. When they do, the credit is given to him. Sometimes an incorrect reading in **A** is corrected tacitly in Hultsch's text. If the correct reading is in **k**, I report "Co (k)"; if it is in Commandino but not **k**, I report "Co". If it is in neither, I report "Hu (recc?)" or, if appropriate, "Ge (recc?)". Where an adopted innovation receives no credit, it is my own.

The text retains the orthography of the Vaticanus, and normalizations of previous editors are not reported. For typographical reasons, iota adscript instead of subscript is used.

The text figures. The figures for the geometrical propositions in Book 7, which generally occupy indented spaces at the end of the relevant theorem in the Vaticanus, are collected at the end of this edition. The reproductions are not exact facsimiles, but attempt to reconstruct Pappus's originals to the extent that that goal is possible. This object dictates, in the first place, the correction of gross errors in the relative positions of lines and the labelling of points, such as are to be expected in careless copying; in the second place, the preservation where possible of such conventions in the drawings as appear to be authentic. The most apparent, and paradoxical, convention is a pronounced preference for symmetry and regularization in figures, introducing equalities where quantities are not required to be equal in the proposition, parallel lines that are not required, right angles for arbitrary angles, and so forth. Modern practice discourages the introduction of this kind of atypicality in geometrical figures. In a translation or commentary by itself of an ancient text, it is desirable to make the figures completely general, and even in an edition it would be defensible. Since this edition is conservative in this respect, the reader of the mathematical parts must take care not to assume relations from the figure that are not explicitly stated in the proposition (a few conspicuous instances are signalled in the notes).

The apparatus for the figures is unconventional, no convention having yet been established for the reporting of variants. Describing the differences between my figures and those in **A** has not been a problem. Reproduction of all the manuscript figures is obviously impractical; but in most cases where my drawings differ significantly from the originals, the alterations can be described clearly enough. A Latin apparatus would be as inconvenient to read as laborious to write lucidly, so I have adopted a few standard and easily intelligible abbreviations ("om", "corr", "transp") from standard critical usage and for the rest used English. Reluctantly, I have decided not to compare my figures with those of Commandino and Hultsch. To report their variants would have encumbered the apparatus beyond the limits of clarity. It would also have been deceptive, since the previous editions (this is true of most other mathematical texts too) have looked upon the figures, not as part of the text, but as adjuncts to be remade at will. It goes without saying that Commandino and Hultsch identified and corrected many of the errors in the transmitted figures. I acknowledge my great debt to them here once for all.

Reference numbers. Chapter numbers are given in the Greek text and translation, as '(39)', and in the running heads on the text pages. The margins of the text give, in large type, the folio number in the Vaticanus, and, in small type, the page number in Hultsch's edition. The proposition numbers of Commandino and Hultsch are indicated in the translation, and are used to number the text figures. Their use as a method of reference should however be discouraged in favor of the chapter numbers, which have the great advantage of extending over the entire text, not merely the mathematical parts.

The translation. The translation attempts to be literal, though not lexical. It is desirable that translations of technical words should be consistent, but no useful purpose would be served by, for example, rendering each of the several Greek words meaning 'therefore' by its own special English particle. I have inserted in parentheses, which are reserved for that purpose, phrases understood in the Greek but not implicit in English. In certain frequent and conventional cases the glosses are not bracketed in order not needlessly to annoy the reader: thus ἡ ὑπὸ ΑΒΓ is translated as "angle ABG", and τὸ ὑπὸ τῶν ΑΒΓ as "the rectangle contained by AB, BG". Major restorations of the text are marked in the translation by angular brackets ("<", ">"). Passages that are probably authentic, but contain essential mistakes that require comment are placed between asterisks.

Mathematical symbols are excluded from the translation, not as inconsistent with literality or faithfulness to Greek mathematical thought (a sufficiently flexible notation, carefully used, can avoid such faults), but because a translation accompanying an edition and notes that sometimes discuss small textual points ought to represent even verbal details that are inessential to the mathematics. As an assistance to the reading of the mathematical arguments, I have provided a compressed mathematical summary in the commentary. This is meant to be read along with the translation, and omits some things that the text states; on the other hand it gives explicit statements of some steps that are only implied by Pappus. The notation, which is not a formal translation of the Greek mathematical methods, is mostly self-explanatory, but the following interpretations of symbols may be helpful:

$=$	equality of lengths, areas, ratios, angles
$\sim$	similarity of figures (triangles, etc.)
$\cong$	congruence of figures

$A - B$	the excess of A over B
$A \cdot B$	the rectangle contained by A and B
$A:B$	the ratio of A to B
$(A:B)\cdot(\Gamma:\Delta)$	the ratio compounded of $A:B$ and $\Gamma:\Delta$
tr. $AB\Gamma$	triangle $AB\Gamma$
$AB \parallel \Gamma\Delta$	line AB is parallel to line $\Gamma\Delta$
$AB \perp \Gamma\Delta$	line AB is perpendicular to line $\Gamma\Delta$
$\angle AB\Gamma$	angle $AB\Gamma$
$\perp AB\Gamma$	right angle $AB\Gamma$
$\angle AB\Gamma \perp$	angle $AB\Gamma$ is right
$\therefore$	precedes an assertion that is derived from the immediately preceding one.
?	precedes an assertion to be proved
!	follows an assertion that is false

A few references to propositions of the *Elements* tacitly invoked by Pappus are inserted in the translation, as "(IV 3)" for Book 4, proposition 3. These references are kept to a minimum, and not always repeated when analogous situations recur. References in parentheses to other chapters in Book 7, such as "(7.191)", to subsidiary lemmas in the commentary, such as "(222.1)", and to Apollonius's *Conics*, as "(Conics II 1)", are also editorial supplements.

Abbreviations Used in the Apparatus

Manuscripts

A	Vaticanus graecus 218 (10th c.)
A^1, A^2	These refer only to the putative order of writing of the manuscript within a single apparatus note, not to hands.
A alia manu	The second, corrector's hand (Hultsch's **A^2**).
B	Parisinus graecus 2440 (16th c.). Collated by Hultsch.
C	Parisinus graecus 2368 (16th c.). Collated by Hultsch.
k	Chicago Newberry 11 (16th c.)
S	Leiden Scaligeranus 3 fol. (16th c.). Collated by Hultsch. **S_2** is a Renaissance corrector's hand.
V	Leiden Vossianus 18 fol. (16th c.). Collated by Hultsch. **V_2** is a Renaissance corrector's hand.
recc	Readings of **B**, **C**, **S**, or **V**.

Editors, Translators, Commentators

Breton	Breton [1855]. Contains ch. 13-20.
$Camer_1$	Camerer [1795]. Contains ch. 11-12, 158-184.
$Camer_2$	Camerer [1796]. Contains ch. 21-26, 185-192.
Co	Commandino [1588]. Latin translation.
Friedlein	Friedlein [1871].
Ge	Gerhardt [1871]. Edition and German translation.
Greg	Gregory [1703]. Contains ch. 1-4.

Ha	Halley [1706]. Contains ch. 1-67, 233-311.
Ha_2	Halley [1710]. Contains ch. 233-311.
Haumann	Haumann [1817]. Contains ch. 11-12, 158-184.
$Heiberg_1$	Heiberg, *LSE*.
$Heiberg_2$	Apollonius, *Opera* vol. 2. Text of passages relative to Apollonius, derived from **Hu.**
$Heiberg_3$	Euclid, *Opera* vol. 8. Text of passages relative to Euclid, derived from **Hu.**
Horsley	Horsley [1770]. Contains ch. 27-28, 126.
Hu	Hultsch, *PAC.* Critical edition and Latin translation.
Hu app	Conjectures in the apparatus of **Hu.**
Hu_1	Hultsch [1873].
$Simson_1$	Simson [1776].
$Simson_2$	Simson [1749]. Contains ch. 21-26.
Snel	Snel [1608]. Contains ch. 9-10.
Tannery	Tannery [1882].
Vincent	Vincent [1859].

TEXT AND TRANSLATION

Pappus of Alexandria
The Collection
Book 7
Which contains lemmas of the *Domain of Analysis*

(1) That which is called the *Domain of Analysis*, my son Hermodorus, is, taken as a whole, a special resource that was prepared, after the composition of the *Common Elements*, for those who want to acquire a power in geometry that is capable of solving problems set to them; and it is useful for this alone. It was written by three men: Euclid the Elementarist, Apollonius of Perge, and Aristaeus the elder, and its approach is by analysis and synthesis.

Now, analysis is the path from what one is seeking, as if it were established, by way of its consequences, to something that is established by synthesis. That is to say, in analysis we assume what is sought as if it has been achieved, and look for the thing from which it follows, and again what comes before that, until by regressing in this way we come upon some one of the things that are already known, or that occupy the rank of a first principle. We call this kind of method 'analysis', as if to say *anapalin lysis* (reduction backward). In synthesis, by reversal, we assume what was obtained last in the analysis to have been achieved already, and, setting now in natural order, as precedents, what before were following, and fitting them to each other, we attain the end of the construction of what was sought. This is what we call 'synthesis'.

(2) There are two kinds of analysis: one of them seeks after truth, and is called 'theorematic'; while the other tries to find what was demanded, and is called 'problematic'. In the case of the theorematic kind, we assume what is sought as a fact and true, then, advancing through its consequences, as if they are true facts according to the hypothesis, to something established, if this thing that has been established is a truth, then that which was sought will also be true, and its proof the reverse of

ΠΑΠΠΟΤ ΑΛΕΞΑΝΔΡΕΩΣ ΣΤΝΑΓΩΓΗΣ Ζ΄.
ΠΕΡΙΕΧΕΙ ΔΕ ΛΗΜΜΑΤΑ ΤΟΤ ΑΝΑΛΤΟΜΕΝΟΤ. 634

(1) ὁ καλούμενος Ἀναλυόμενος, Ἑρμόδωρε τέκνον, κατὰ σύλληψιν ἰδία τίς ἐστιν ὕλη παρασκευασμένη μετὰ τὴν τῶν κοινῶν στοιχείων ποίησιν τοῖς βουλομένοις ἀναλαμβάνειν ἐν γραμμαῖς δύναμιν εὑρετικὴν τῶν προτεινομένων αὐτοῖς προβλημάτων, καὶ εἰς τοῦτο μόνον χρησίμη καθεστῶσα. γέγραπται δὲ ὑπὸ τριῶν ἀνδρῶν, Εὐκλείδου τε τοῦ στοιχειωτοῦ καὶ Ἀπολλωνίου τοῦ Περγαίου καὶ Ἀρισταίου τοῦ πρεσβυτέρου, κατὰ ἀνάλυσιν καὶ σύνθεσιν ἔχουσα τὴν ἔφοδον. ἀνάλυσις τοίνυν ἐστὶν ὁδὸς ἀπὸ τοῦ ζητουμένου, ὡς ὁμολογουμένου, διὰ τῶν ἑξῆς ἀκολούθων, ἐπί τι ὁμολογούμενον συνθέσει. ἐν μὲν γὰρ τῆι ἀναλύσει, τὸ ζητούμενον ὡς γεγονὸς ὑποθέμενοι τὸ ἐξ οὗ [τοῦ] τοῦτο συμβαίνει σκοπούμεθα, καὶ πάλιν ἐκείνου τὸ προηγούμενον, ἕως ἂν οὕτως ἀναποδίζοντες καταντήσωμεν εἴς τι τῶν ἤδη γνωριζομένων ἢ τάξιν ἀρχῆς ἐχόντων. καὶ τὴν τοιαύτην ἔφοδον ἀνάλυσιν καλοῦμεν οἷον ἀνάπαλιν λύσιν. ἐν δὲ τῆι συνθέσει ἐξ ὑποστροφῆς τὸ ἐν τῆι ἀναλύσει καταληφθὲν ὕστατον ὑποστησάμενοι γεγονὸς ἤδη, καὶ τὰ ἑπόμενα ἐκεῖ, ἐνταῦθα προηγούμενα κατὰ φύσιν τάξαντες καὶ ἀλλήλοις ἐπισυνθέντες, εἰς τέλος ἀφικνούμεθα τῆς τοῦ ζητουμένου κατασκευῆς· καὶ τοῦτο καλοῦμεν σύνθεσιν.

(2) διττὸν δ' ἐστὶν ἀναλύσεως γένος· τὸ μὲν γὰρ ζητητικὸν τἀληθοῦς, ὃ καλεῖται θεωρητικόν, τὸ δὲ ποριστικὸν τοῦ προταθέντος [λέγειν] ὃ καλεῖται προβληματικόν. ἐπὶ μὲν οὖν τοῦ θεωρητικοῦ γένους τὸ 636
ζητούμενον ὡς ὂν ὑποθέμενοι καὶ ὡς ἀληθές, εἶτα διὰ τῶν ἑξῆς ἀκολούθων ὡς ἀληθῶν καὶ ὡς ἔστιν καθ' ὑπόθεσιν προελθόντες ἐπί τι ὁμολογούμενον, ἐὰν μὲν ἀληθὲς ἦι ἐκεῖνο τὸ ὁμολογούμενον, ἀληθὲς ἔσται καὶ τὸ ζητούμενον καὶ ἡ

‖ 13 ante συνθέσει add ἐν Greg | γὰρ om Greg ‖ 14 τοῦ (in fine versus A) del Greg ‖ 18 τῆι om Ge ‖ 20 ἑπόμενα τὰ transp Hu ‖ 21 ἐνταῦθα secl. Hu ‖ 24 γὰρ om Ha ‖ 26 προτεθέντος Greg | λέγειν secl. Hu ‖ 29 ἀληθῶν Ha ἀληθῶς A | ἔστιν] ὄντων Hu app the analysis; but if we should meet with something established to be false, then the thing that was sought too will be false. In the case of the problematic kind, we assume the proposition as something we know, then, proceeding through its consequences, as if true, to something established, if the established thing is possible and obtainable, which is what mathematicians call 'given', the required thing will also be possible, and again the proof will be the reverse of the analysis; but should we meet with something established to be impossible, then the problem too will be impossible. Diorism is the preliminary distinction of when, how, and in how many ways the problem will be possible. So much, then, concerning analysis and synthesis.

(3) The order of the books of the *Domain of Analysis* alluded to above is this: Euclid, *Data,* one book; Apollonius, *Cutting off of a Ratio,* two; *Cutting off of an Area,* two; <*Determinate Section*>, two; *Tangencies,* two; Euclid, *Porisms,* three; Apollonius, *Neuses,* two; by the same, *Plane Loci,* two; *Conics,* eight; Aristaeus, *Solid Loci,* five; Euclid, *Loci on Surfaces,* two; Eratosthenes, *On Means* [two]. These make up 32 books. I have set out epitomes of them, as far as the *Conics* of Apollonius, for you to study, with the number of the dispositions and diorisms and cases in each book, as well as the lemmas that are wanted in them, and there is nothing wanting for the working through of the books, I believe, that have I left out.

(4) (The Data.)

The first book, which is the *Data,* contains ninety theorems in all. The first twenty-three diagrams are all about magnitudes. The twenty-fourth is on proportional lines that are not given in position. The fourteen next to these are on lines given in position. The next <ten> are on triangles given in shape without position. The next seven are on arbitrary rectilineal areas given in shape without position. The next six are on parallelograms and ἀπόδειξις ἀντίστροφος τῆι ἀναλύσει· ἐὰν δὲ ψεύδει ὁμολογουμένωι ἐντύχωμεν, ψεῦδος ἔσται καὶ τὸ ζητούμενον. ἐπὶ δὲ τοῦ προβληματικοῦ γένους τὸ προταθὲν ὡς γνωσθὲν ὑποθέμενοι, εἶτα διὰ τῶν ἑξῆς ἀκολούθων ὡς ἀληθῶν προελθόντες ἐπί τι ὁμολογούμενον, ἐὰν μὲν τὸ ὁμολογού|μενον δυνατὸν ἦι καὶ ποριστόν, ὃ καλοῦσιν οἱ ἀπὸ |119 τῶν μαθημάτων δοθέν, δυνατὸν ἔσται καὶ τὸ προταθέν, καὶ πάλιν ἡ ἀπόδειξις ἀντίστροφος τῆι ἀναλύσει· ἐὰν δὲ ἀδυνάτωι ὁμολογουμένωι ἐντύχωμεν ἀδύνατον ἔσται καὶ τὸ πρόβλημα. διορισμὸς δέ ἐστιν προδιαστολὴ τοῦ πότε καὶ πῶς καὶ ποσαχῶς δυνατὸν ἔσται [καὶ] τὸ πρόβλημα. τοσαῦτα μὲν οὖν περὶ ἀναλύσεως καὶ συνθέσεως.

(3) τῶν δὲ προειρημένων τοῦ Ἀναλυομένου βιβλίων ἡ τάξις ἐστὶν τοιαύτη. Εὐκλείδου Δεδομένων βιβλίον α̅, Ἀπολλωνίου Λόγου Ἀποτομῆς β̅, Χωρίου Ἀποτομῆς β̅, <Διωρισμένης Τομῆς> δύο, Ἐπαφῶν δύο, Εὐκλείδου Πορισμάτων τρία, Ἀπολλωνίου Νεύσεων δύο, τοῦ αὐτοῦ Τόπων Ἐπιπέδων δύο, Κωνικῶν η̅, Ἀρισταίου Τόπων Στερεῶν πέντε, Εὐκλείδου Τόπων πρὸς Ἐπιφανείαι δύο, Ἐρατοσθένους Περὶ Μεσοτήτων [δύο]. γίνεται βιβλία λ̅β̅, ὧν τὰς περιοχὰς μέχρι τῶν Ἀπολλωνίου Κωνικῶν ἐξεθέμην σοι πρὸς ἐπίσκεψιν, καὶ τὸ πλῆθος τῶν τόπων καὶ τῶν διορισμῶν καὶ τῶν πτώσεων καθʼ ἕκαστον βιβλίον, ἀλλὰ καὶ τὰ λήμματα τὰ ζητούμενα, καὶ οὐδεμίαν ἐν τῆι πραγματείαι τῶν βιβλίων καταλέλοιπα ζήτησιν, ὡς ἐνόμιζον.

(4) περιέχει δὲ τὸ πρῶτον βιβλίον, ὅπερ ἐστὶν τῶν 638 Δεδομένων, ἅπαντα θεωρήματα ἐνενήκοντα. ὧν πρῶτα μὲν καθόλου ἐπὶ μεγεθῶν διαγράμματα κ̅γ̅. τὸ δὲ δʹ καὶ [τὸ] κʹ ἐν εὐθείαις ἐστὶν ἀνάλογον ἄνευ θέσεως. τὰ δὲ ἑξῆς τούτοις ι̅δ̅ ἐν εὐθείαις ἐστὶν θέσει δεδομέναις. τὰ δὲ τούτοις ἑξῆς <ι̅> ἐπὶ τριγώνων ἐστὶν τῶι εἴδει δεδομένων ἄνευ θέσεως. τὰ δὲ ἑξῆς τούτοις ξ̅ ἐπὶ τυχόντων ἐστὶν εὐθυγράμμων χωρίων εἴδει δεδομένων ἄνευ θέσεως. τὰ δὲ ἑξῆς τούτοις ϛ̅ ἐν

‖ 3 προτεθὲν Greg ‖ 4 ἀληθῶν Ha ἀληθῶς A ‖ 7 προτεθὲν Greg ‖ 10 διορισμὸς — πρόβλημα secl Hu ‖ 11 καὶ om Greg ‖ 15 (ἀποτομῆς) β̅ om Greg ‖ 16 Διωρισμένης Τομῆς add Ha ‖ 19 ante πρὸς add τῶν Hu | ἐπιφανείαι Hu ἐπιφάνειαν A ‖ 20 λ̅β̅ A λ̅γ̅ Ha λ̅α̅ Greg ‖ 24 καταλέλοιπα Greg κατὰ δὲ λοιπὰ A ‖ 27 πρῶτα Ha πρῶτον A ‖ 28 διαγράμματα secl Hu | τὸ (κʹ) del Hu ‖ 31 ι̅ add Greg | τριγώνων Ha τριγώνου A applications of area given in shape. Of the ensuing five, the first is on figures erected upon lines, the other four on triangular areas, that the differences of the squares of the sides are in given ratio to those triangular areas. The next seven, up to the seventy-third, are on two parallelograms, that by the stipulations concerning their angles they are in given ratios to one another. Some of these have similar postscripts on two triangles. Among the next six diagrams, up to the seventy-ninth, two are on triangles, four on more (than two) lines in proportion. The next three are on two lines [that are in ratio that is, and] enclose a given area. The final eight up to the ninetieth are proved on circles, some given only in magnitude, others also in position, that when lines are drawn through a given point, the results are given.

(5) (The Cutting off of a Ratio.)

The proposition of the two books of the *Cutting off of a Ratio* is a single one, albeit subdivided; and therefore I can write one proposition, as follows: through a given point to draw a straight line cutting off from two lines given in position (abscissas extending) to points given upon them, that have a ratio equal to a given one. In fact the figures are varied and numerous, when the subdivision is made, because of the dispositions with respect to each other of the given lines and the various cases of the way that the given point falls, and because of the analyses and syntheses of them and their diorisms. **(6)** Thus the first book of the *Cutting off of a Ratio* contains seven dispositions, twenty-four cases, and five diorisms, three of which are maxima, two minima. There is a maximum in the third case of the fifth disposition, a minimum in the second of the sixth disposition, in the same (number) of the seventh disposition; those in the fourth of the sixth and seventh dispositions are maxima. The second book of the *Cutting off of a Ratio* contains fourteen dispositions, sixty-three cases, and for diorisms those of the first, because it reduces entirely to the first.

παραλληλογράμμοις ἐστὶ καὶ παραβολαῖς εἴδει δεδομένων χωρίων. τῶν δὲ ἐχομένων ε̅ τὸ μὲν <ἐπὶ> ἀναγραφομένων ἐστιν, τὰ δὲ δ̅ ἐπὶ τριγώνων χωρίων, ὅτι αἱ διαφοραὶ τῶν δυνάμεων τῶν πλευρῶν πρὸς ταῦτα τὰ τρίγωνα χωρία λόγον ἔχουσιν δεδομένον. τὰ δὲ ἑξῆς ϛ̅ ἕως τοῦ ο΄ καὶ γ΄ ἐν δυσὶ παραλληλογράμμοις, ὅτι διὰ τὰς ἐν ταῖς γωνίαις ὑποθέσεις ἐν δεδομένοις ἐστὶν λόγοις πρὸς ἄλληλα· ἔνια δὲ τούτων ἐπιλόγους ἔχει ὁμοίους ἐν δυσὶ τριγώνοις. ἐν δὲ τοῖς ἐφεξῆς ϛ̅ διαγράμμασιν ἕως τοῦ ο΄ καὶ θ΄ δύο μὲν ἐστι ἐπὶ τριγώνων, |δ̅ δὲ ἐπὶ πλειόνων εὐθειῶν ἀνάλογον οὐσῶν. τὰ δὲ ἑξῆς γ̅ ἐπὶ δύο εὐθειῶν [ἀνάλογον οὐσῶν τάδ' ἐστιν] δοθὲν [τε] |119v περιεχουσῶν χωρίον. τὰ δὲ ἐπὶ πᾶσιν η̅ ἕως ἐν κύκλοις δείκνυται, τοῖς μὲν μεγέθει μόνον δεδομένοις, τοῖς δὲ καὶ 640 θέσει, <ὅτι> ἀγομένων εὐθειῶν [ἐστιν] διὰ δεδομένου σημείου τὰ γενόμενα δεδομένα <ἐστίν>.

(5) τῆς δ' Ἀποτομῆς τοῦ Λόγου βιβλίων ὄντων β̅ πρότασίς ἐστιν μία ὑποδιῃρημένη, διὸ καὶ μίαν πρότασιν οὕτως γράφω· διὰ τοῦ δοθέντος σημείου εὐθεῖαν γραμμὴν ἀγαγεῖν τέμνουσαν ἀπὸ τῶν τῆι θέσει δοθεισῶν δύο εὐθειῶν πρὸς τοῖς ἐπ' αὐτῶν δοθεῖσι σημείοις λόγον ἐχούσας τὸν αὐτὸν τῶι δοθέντι. τὰς δὲ γραφὰς διαφόρους γενέσθαι καὶ πλῆθος λαβεῖν συμβέβηκεν, ὑποδιαιρέσεως γενομένης, ἕνεκα τῆς τε πρὸς ἀλλήλας θέσεως τῶν δεδομένων εὐθειῶν καὶ τῶν διαφόρων πτώσεων τοῦ δεδομένου σημείου καὶ διὰ τὰς ἀναλύσεις καὶ συνθέσεις αὐτῶν [τε] καὶ τῶν διορισμῶν.

(6) ἔχει γὰρ τὸ μὲν πρῶτον βιβλίον τῶν Λόγου Ἀποτομῆς τόπους ζ̅, πτώσεις κδ̅, διορισμοὺς δὲ ε̅, ὧν τρεῖς μέν εἰσιν μέγιστοι, δύο δὲ ἐλάχιστοι, καὶ ἔστι μέγιστος μὲν κατὰ τὴν τρίτην πτῶσιν τοῦ ε΄ τόπου, ἐλάχιστος δὲ κατὰ τὴν δευτέραν τοῦ ϛ΄ τόπου, <καὶ> κατὰ τὴν αὐτὴν τοῦ ζ΄ τόπου, μέγιστοι δὲ οἱ κατὰ τὰς τετάρτας τοῦ ϛ΄ καὶ τοῦ ζ΄ τόπου. τὸ δὲ δεύτερον βιβλίον Λόγου Ἀποτομῆς ἔχει τόπους ιδ̅, πτώσεις δὲ ξγ̅, διορισμοὺς δὲ τοὺς ἐκ τοῦ πρώτου. ἀπάγεται γὰρ ὅλον εἰς τὸ πρῶτον. λήμματα δὲ ἔχει τὰ λόγου ἀποτομῆς κ̅, αὐτὰ δὲ

‖ 1 ἐστὶ Hu ἔτι A ‖ 2 ἐπὶ ἀναγραφομένων] πρῶτον γραφόμενόν A ‖ 7 ἀλλήλους Greg ‖ 9 ἀφεξῆς Greg ‖ 11 ἀνάλογον — ἐστιν secl Hu | τε A τι Ha ‖ 12 χωρίον Greg χωρίων A | post ἕως add τοῦ Hu ‖ 14 ὅτι add Ha | ἀγομένων — δεδομένα secl Hu | διαγομένων Ha | ἐστιν om Greg ‖ 20 ἐπ' Ha ἀπ' A ‖ 23 δεδομένων Ha διδομένων A ‖ 24 δεδομένου Ha διδομένου A ‖ 30 καὶ add Ha | τὴν αὐτὴν Ha τῆς αὐτῆς A ‖ 32 ιδ̅ Ha κδ̅ A

The *Cutting off of a Ratio* has twenty lemmas, and the two books of the *Cutting off of a Ratio* comprise 181 theorems. But according to Pericles, more than that many.

(7) (The Cutting off of an Area.)

There are two books of the *Cutting off of an Area*, and again one problem in them, though subdivided. Hence they also have one proposition, in all other respects similar to the one above, and differing in this respect alone, that one must make the two (abscissas) that are cut off, in the former case, have a given ratio, but in the latter case, enclose a given area. This is how it will be expressed: to draw through a given point a straight line cutting off from two lines given in position (abscissas extending) to points given on them that enclose an area equal to a given one. This (proposition), for the same reasons, has obtained a large number of figures.

(8) The first book of the *Cutting off of an Area* has seven dispositions, twenty-four cases, and seven diorisms, four of which are maxima, three minima. There is a maximum in the second case of the first disposition, as is that in the first case of the second disposition and in the second of the fourth and in the third of the sixth disposition. That in the third case of the third disposition is a minimum, as is that in the fourth of the fourth disposition, and in the first in the sixth disposition. The second book of the *Cutting off of an Area* contains thirteen dispositions, sixty cases, and for diorisms those of the first, because it reduces to it. The first book contains forty-eight theorems, the second seventy-six.

(9) (The Determinate Section.)

Next after these the two books of the *Determinate Section* have been passed down, for which, as for those above, it is possible to state a single proposition, although one admitting choices, and it is this: to divide a given unbounded line by one point so that of the abscissas extending (from the point) to points given on (the line), either the square of one or the rectangle enclosed by two abscissas have a given ratio either to the <square of> one, <or to the (rectangle enclosed) by one> abscissa and another (line) given besides, or to the rectangle enclosed by two abscissas extending to τὰ δύο βιβλία τῶν Λόγου Ἀποτομῆς θεωρημάτων ἐστὶν ρπᾱ· κατὰ δὲ Περικλέα πλειόνων ἢ τοσούτων.

(7) τῆς δ' Ἀποτομῆς τοῦ Χωρίου βιβλία μέν ἐστιν δύο,
πρόβλημα δὲ κἀν τούτοις ἓν ὑποδιαιρούμενον, διὸ καὶ τούτων
μία πρότασίς ἐστιν, τὰ μὲν ἄλλα ὁμοίως ἔχουσα τῆι προτέραι,
μόνωι δὲ τούτωι διαφέρουσα τῶι δεῖν τὰς ἀποτεμνομένας δύο
εὐθείας ἐν ἐκείνηι μὲν λόγον ἐχούσας δοθέντα ποιεῖν, ἐν δὲ 642
ταύτηι χωρίον περιεχούσας δοθέν. ῥηθήσεται γὰρ οὕτως· διὰ
τοῦ δοθέντος σημείου εὐθεῖαν γραμμὴν ἀγαγεῖν τέμνουσαν
ἀπὸ τῶν δοθεισῶν θέσει δύο εὐθειῶν πρὸς τοῖς ἐπ' αὐτῶν
δοθεῖσι σημείοις χωρίον περιεχούσας ἴσον τῶι δοθέντι. καὶ
αὕτη δὲ διὰ τὰς αὐτὰς |αἰτίας τὸ πλῆθος ἔσχηκε τῶν |120
γραφομένων.

(8) ἔχει δὲ τὸ μὲν α´ βιβλίον Χωρίου Ἀποτομῆς τόπους ζ̄, πτώσεις κδ̄, διορισμοὺς ζ̄, ὧν δ̄ μὲν μέγιστοι, τρεῖς δὲ ἐλάχιστοι. καὶ ἔστι μέγιστος μὲν κατὰ τὴν δευτέραν πτῶσιν τοῦ πρώτου τόπου καὶ ὁ κατὰ τὴν πρώτην πτῶσιν τοῦ β´ τόπου καὶ ὁ κατὰ τὴν β´ τοῦ δ´ καὶ ὁ κατὰ τὴν τρίτην τοῦ ς´ τόπου, ἐλάχιστος δὲ ὁ κατὰ τὴν τρίτην πτῶσιν τοῦ τρίτου τόπου καὶ ὁ κατὰ τὴν δ´ τοῦ δ´ τόπου καὶ ὁ κατὰ τὴν πρώτην τοῦ ἕκτου τόπου. τὸ δὲ δεύτερον βιβλίον τῶν Χωρίου Ἀποτομῆς ἔχει τόπους ιγ̄, πτώσεις δὲ ξ̄, διορισμοὺς δὲ τοὺς ἐκ τοῦ πρώτου. ἀπάγεται γὰρ εἰς αὐτό. θεωρήματα δὲ ἔχει τὸ μὲν πρῶτον βιβλίον μη̄, τὸ δὲ δεύτερον ος̄.

(9) ἑξῆς <δὲ> τούτοις ἀναδέδοται τῆς Διωρισμένης Τομῆς βιβλία β̄ ὧν ὁμοίως τοῖς πρότερον μίαν πρότασιν πάρεστιν λέγειν, διεζευγμένην δὲ ταύτην· τὴν δοθεῖσαν ἄπειρον εὐθεῖαν ἑνὶ σημείωι τεμεῖν ὥστε τῶν ἀπολαμβανομένων εὐθειῶν πρὸς τοῖς ἐπ' αὐτῆς δοθεῖσι σημείοις, ἤτοι τὸ ἀπὸ μιᾶς τετράγωνον ἢ τὸ ὑπὸ δύο ἀπολαμβανομένων περιεχόμενον ὀρθογώνιον, δοθέντα λόγον ἔχειν, ἤτοι πρὸς τὸ ἀπὸ μιᾶς <τετράγωνον, ἢ πρὸς τὸ ὑπὸ μιᾶς> ἀπολαμβανομένης καὶ τῆς ἔξω δοθείσης, ἢ πρὸς τὸ ὑπὸ δύο ἀπολαμβανομένων

‖ 4 διὸ] δὶς A ‖ 8 οὕτω Ha ‖ 10 ἐπ' Ha ἀπ' A ‖ 17 β´ (τόπου) Ha δ´ A ‖ 22 ξ̄ Ha ζ̄ A ‖ 23 αὐτό Ge αὐτόν A ‖ 25 δὲ add Snel | ἀναδέδοται Hu ἀναδέδονται A ‖ 31 ἀπὸ] ὑπὸ Snel ‖ 32 τετράγωνον — μιᾶς add Simson$_1$

whichever two that you use of the given points.

And since this (proposition) admits choices twice, and has intricate diorisms, necessarily its demonstration is long. Apollonius proves it backwards on pure straight lines, trying the more beaten path – as in the second book of the *First Elements* of Euclid, who proved these things in a still more introductory way using erections of figures on lines – and (then) ingeniously by means of semicircles.

(10) The first book contains six problems, sixteen assignments, and five diorisms: four maxima, one minimum. Maxima are in the second assignment of the second problem, and in the third of the fourth problem, and in the third of the fifth, and in the third <of the sixth; and a minimum in the third> assignment of the third problem. The second (book) of the *Determinate Section* contains three problems, nine assignments, three diorisms, of which that in the third of the first and that in the third of the second are minima, and that in the third of the third problem is a maximum. The first book contains 27 lemmas, the second 24. The two books of the *Determinate Section* are in 83 theorems.

(11) (The Tangencies.)

Following these are the two books of *Tangencies*. There appear to be several propositions in them, but even for these we set down one, which is as follows: given in position any three points, straight lines, or circles, to draw a circle through each of the given points, if there be given any, and tangent to each of the given (straight or circular) lines. Because of the number of like and unlike givens in the hypotheses, necessarily there are ten propositions differing in part, since out of the three unlike kinds, ten περιεχόμενον ὀρθογώνιον ἐφ' ὁποτέραι χρῆι τῶν δοθέντων 644 σημείων. καὶ ταύτης ἅτε δὶς διεζευγμένης καὶ περισκελεῖς διορισμοὺς ἐχούσης, διὰ πλειόνων ἡ δεῖξις γέγονεν ἐξ ἀνάγκης. δείκνυσι δὲ ταύτην Ἀπολλώνιος ἀνάπαλιν ἐπὶ ψιλῶν τῶν εὐθειῶν τριβακώτερον πειρώμενος, καθάπερ καὶ ἐπὶ τοῦ δευτέρου βιβλίου τῶν Πρώτων Στοιχείων Εὐκλείδου, καὶ ταῦτα πάλιν εἰσαγωγικώτερον ἐπ' ἀναγραφῶν δείξαντος, καὶ εὐφυῶς διὰ τῶν ἡμικυκλίων.

(10) ἔχει δὲ τὸ μὲν πρῶτον βιβλίον προβλήματα ς̄, ἐπιτάγματα ις̄, διορισμοὺς δὲ ε̄, ὧν μεγίστους μὲν δ̄, ἐλάχιστον δὲ ἕνα. καὶ εἰσὶν μέγιστοι μὲν ὅ τε κατὰ τὸ δεύτερον ἐπίταγμα τοῦ δευτέρου προβλήματος καὶ ὁ κατὰ τὸ γʹ τοῦ δʹ προβλήματος καὶ ὁ κατὰ τὸ τρίτον τοῦ εʹ καὶ ὁ κατὰ τὸ τρίτον <τοῦ ἕκτου, ἐλάχιστος δὲ ὁ κατὰ τὸ τρίτον> ἐπίταγμα τοῦ τρίτου προβλήματος. τὸ δὲ δεύτερον Διωρισμένης |Τομῆς ἔχει προβλήματα τρία, ἐπιτάγματα θ̄, |120v διορισμοὺς γ̄, ὧν εἰσιν ἐλάχιστοι μὲν ὅ τε κατὰ τὸ τρίτον τοῦ πρώτου καὶ ὁ κατὰ τὸ τρίτον τοῦ δευτέρου, μέγιστος δὲ ὁ κατὰ τὸ τρίτον τοῦ τρίτου προβλήματος. λήμματα δὲ ἔχει τὸ μὲν πρῶτον βιβλίον κζ̄, τὸ δὲ δεύτερον κδ̄. θεωρημάτων δέ ἐστιν τὰ δύο βιβλία Διωρισμένης Τομῆς πγ̄.

(11) ἑξῆς δὲ τούτοις τῶν Ἐπαφῶν ἐστιν βιβλία δύο· προτάσεις δὲ ἐν αὐτοῖς δοκοῦσιν εἶναι πλείονες, ἀλλὰ καὶ τούτων μίαν τίθεμεν οὕτως ἔχουσαν· ἐξ σημείων καὶ εὐθειῶν καὶ κύκλων τριῶν ὁποιωνοῦν θέσει δοθέντων, κύκλον ἀγαγεῖν δι' ἑκάστου τῶν δοθέντων σημείων, εἰ δοθείη, ἐφαπτόμενον ἑκάστης τῶν δοθεισῶν γραμμῶν. ταύτης διὰ πλήθη τῶν ἐν ταῖς ὑποθέσεσι δεδομένων, ὁμοίων ἢ ἀνομοίων, κατὰ μέρος

‖ 1 ὁποτέραι χρῆι τῶν Snel ὁπότερα χρηστῶν A ὁποτέρ' ἂν χρῆι τῶν Hu ‖ 2 περισκελεῖς Snel περισκελις A ‖ 4 δείκνυσι — ἡμικυκλίων secl Hu | ἀνάπαλιν] μὲν πάλιν A, om Ha ‖ 7 ταῦτα Snel ταύτην A, del Heiberg$_2$ | ἐπαναγράφων A ἐπαναφέρων Snel | δείξαντος] δείξας Snel δείξας τε Ha ‖ 9 δὲ om Snel ‖ 10 δὲ (ε̄) om Ha ‖ 11 μέγιστοι Ha μέγιστον A ‖ 12 ὁ κατὰ τὸ γʹ — τρίτου προβλήματος] καὶ ὁ κατὰ τὸ τρίτον ἐπίταγμα τοῦ τρίτου προβλήματος Snel ‖ 14 τοῦ ἕκτου — τρίτον add Ha ‖ 24 ἐξ] ἑξῆς A ἐκ coni Camer$_1$ ‖ 25 κύκλον Ha κύκλων A ‖ 26 ante ἐφαπτόμενον add ἢ Heiberg$_2$ | ἐφαπτόμενον Ha ἐφαπτόμενος A different unordered groups of three result: for the givens are either (1) three points, or (2) three lines, or (3) two points and a line, or (4) two lines and a point, or (5) two points and a circle, or (6) two circles and a point, or (7) two lines and a circle, or (8) two circles and a line, or (9) a point and a line and a circle, or (10) three circles. The first two of these were proved in Book 4 of the *First Elements*, and these (Apollonius) omitted to write. Thus the "given three points not on a line" is the same as the "to circumscribe a circle about a given triangle" (IV, 5), while the "given three lines, not parallel but all three meeting one another" is the same as the "to inscribe a circle inside a given triangle" (IV, 4). For the "having two parallel lines and one meeting them", as a part of the subdivision of the next (problem), is written in these (books) before all, and the next six in the first book; the remaining two, the "given two lines and a circle" or "given three circles" only in the second book, because of the numerous placements with respect to each other of the circles and lines, and because these also require many diorisms.

(12) There is a group of problems similar to the *Tangencies* mentioned above, that has been omitted by the people who have passed them down, and one might have given them too, before the two books mentioned, – for it would be easily comprehended and more introductory, but also a whole and something to fill out the class of tangencies – again encompassing everything in one proposition, which, though shorter than the one stated above in its hypothesis, is more abundant in (the number of) assignments (that it has); and it is thus: given any two points, lines, or circles, to draw a circle given in magnitude passing through the given point or given points, if

διαφόρους προτάσεις ἀναγκαῖον γίνεσθαι δέκα. ἐκ τῶν τριῶν 646
γὰρ ἀνομοίων γενῶν τριάδες διάφοροι ἄτακτοι γίνονται ῑ.
ἤτοι γὰρ τὰ δεδομένα (1) τρία σημεῖα, ἢ (2) τρεῖς εὐθεῖαι,
ἢ (3) δύο σημεῖα καὶ εὐθεῖα, ἢ (4) δύο εὐθεῖαι καὶ σημεῖον,
ἢ (5) δύο σημεῖα καὶ κύκλος, ἢ (6) δύο κύκλοι καὶ σημεῖον, ἢ
(7) δύο εὐθεῖαι καὶ κύκλος, ἢ (8) δύο κύκλοι καὶ εὐθεῖα, ἢ
(9) σημεῖον καὶ εὐθεῖα καὶ κύκλος, ἢ (10) τρεῖς κύκλοι.
τούτων δύο μὲν τὰ πρῶτα δέδεικται ἐν τῶι δʹ βιβλίωι τῶν
Πρώτων Στοιχείων ἃ παρῆκεν γράφειν. τὸ μὲν γὰρ τριῶν
δοθέντων σημείων μὴ ἐπ' εὐθεῖαν ὄντων τὸ αὐτό ἐστιν τῶι
περὶ τὸ δοθὲν τρίγωνον κύκλον περιγράψαι, τὸ δὲ γ̄ δοθεισῶν
εὐθειῶν μὴ παραλλήλων οὐσῶν ἀλλὰ τῶν τριῶν συμπιπτουσῶν
τὸ αὐτό ἐστιν τῶι εἰς τὸ δοθὲν τρίγωνον κύκλον ἐγγράψαι.
τὸ γὰρ δύο παραλλήλων οὐσῶν καὶ μιᾶς ἐμπιπτούσης ὡς μέρος
ὂν τῆς τοῦ ἑξῆς ὑποδιαιρέσεως προγράφεται ἐν τούτοις
πάντων. καὶ τὰ ἑξῆς ϛ ἐν τῶι πρώτωι βιβλίωι, τὰ δὲ
λειπόμενα δύο, τὸ δύο δοθεισῶν εὐθειῶν καὶ κύκλου ἢ τριῶν
δοθέντων κύκλων, μόνον ἐν τῶι δευτέρωι βιβλίωι διὰ τὰς
πρὸς ἀλλήλους θέσεις τῶν κύκλων τε καὶ εὐθειῶν πλείονας
οὔσας καὶ πλειόνων διορισμῶν δεομένας.

(12) ταῖς προειρημέναις Ἐπαφαῖς ὁμογενὲς πλῆθός ἐστιν
|προβλημάτων παραλειπόμενον ὑπὸ τῶν ἀναδιδόντων, καὶ |121
προσανέδωκεν ἄν τις πρότερόν [τε] τῶν εἰρημένων δύο
βιβλίων — εὐσύνοπτόν τε γὰρ καὶ εἰσαγωγικὸν μᾶλλον ἦν,
ἐντελὲς δὲ καὶ συμπληρωτικὸν τοῦ γένους τῶν ἐπαφῶν —
πάλιν μιᾶι περιλαβὼν ἅπαντα προτάσει ἥτις τῆς 648
προειρημένης λείπουσα μὲν ὑποθέσει, περιττεύουσα δὲ
ἐπιτάγματι, οὕτως ἔχει· ἐκ σημείων καὶ εὐθειῶν καὶ κύκλων
ὁποιωνοῦν δύο δοθέντων κύκλον γράψαι τῶι μεγέθει δοθέντα

‖ 1 δέκα Ha δὲ καὶ A ‖ 2 τρίαδες Ha τρίαδε A ‖ 3 τὰ secl Hu | δεδομένα Ha διδόμενα A | εὐθεῖαι Ha εὐθείας A ‖ 4 ἢ (δύο εὐθεῖαι) Co καὶ A ‖ 5 ἢ δύο εὐθεῖαι καὶ κύκλος post σημεῖον καὶ εὐθεῖα καὶ κύκλος transp Ha (Co) ‖ 7 εὐθεῖα Ha εὐθειαι A ‖ 9 ἃ παρῆκεν γράφειν] ὁπερημεν γραφῶν A ὃ παρῆκεν γράφων Friedlein ὃ παρεῖμεν γράφειν Hu ὅπερ ἦν μὲν γράφων Ha ὃ παρίη γράφων Ge app διὸ παρίει μὴ γράφων Heiberg₂ ‖ 10 εὐθεῖαν Ha εὐθείας A ‖ 14 γὰρ] δὲ Heiberg₂ ‖ 15 ὂν τῆς Ha ὄντος A | τοῦ om Ha | ἑξῆς] ϛʹ A βʹ Ha ‖ 16 πάντα Ha | τὰ (ἑξῆς)] τῶν Camer₁ ‖ 21 ὁμογενὲς Ha ὁμογενὴς A ‖ 22 ὑπὸ Hu ἀπὸ A | καὶ om Ha ‖ 23 προσανέδωκάν τισι πρότερον A προσανέδωκα ἐν τοῖς πρότερον Hu προσανέδωκαν δέ τινες προτέρωι Ha καὶ προσανέδωκεν ἄν τις τῶι προτέρωι Friedlein | τε om Ha ‖ 24 τε om Ha | post μᾶλλον add ἂν Friedlein ‖ 25 ἐντελές τε Ha ‖ 26 περιλάβωμεν Hu ‖ 28 ἐκ] ἑξῆς Haumann ‖ 29 γράψει Ge any be given, and tangent to each of the given (straight or circular) lines. Already this contains six problems, since from three classes one obtains six different unordered pairs. For with either (1) two points being given, or (2) two lines given, or (3) two circles given, or (4) a point and a line, or (5) a point and a circle, or (6) a line and a circle, one has to draw, as was said, a circle given in magnitude. To give an analysis and synthesis and diorism case by case.

The first (book) of the *Tangencies* contains seven problems, the second four problems. The two books have twenty-one lemmas, and comprise sixty theorems.

(13) (The Porisms)

After the *Tangencies* are Euclid's *Porisms* in three books, which are for many people a very clever collection for the analysis of more weighty problems; and although nature furnishes a boundless multitude of kinds of them, (the moderns) have added none to what Euclid originally wrote, except that some tasteless predecessors of ours have inserted second constructions to a few of them, whereas each of them, as I have shown, has a fixed number of proofs, and Euclid put the proof of each that is most suggestive. They have a delicate and natural aspect, cogent and quite universal, and pleasant for people who know how to see, and how to find. All of them are in form neither theorems nor problems, but of a type occupying a sort of mean between them, so that their propositions can assume the form of theorems or problems, and it is for this reason that among the many geometers some have assumed them to be of the class of theorems, others, of problems, looking only at the form of the proposition.

διὰ τοῦ δοθέντος σημείου ἢ τῶν δοθέντων παραγινόμενον, εἰ δοθείη, ἐφαπτόμενον δὲ ἑκάστης τῶν δεδομένων γραμμῶν. αὕτη περιέχει προβλημάτων ἤδη τὸ πλῆθος ἕξ. ἐκ τριῶν γὰρ διαφόρων τινῶν δυάδες ἄτακτοι διάφοροι γίνονται τὸ πλῆθος ϛ̅· ἤτοι γὰρ (1) δύο δοθέντων σημείων, ἢ (2) δύο δοθεισῶν εὐθειῶν, ἢ (3) δύο δοθέντων κύκλων, ἢ (4) σημείου καὶ εὐθείας, ἢ (5) σημείου καὶ κύκλου, ἢ (6) εὐθείας καὶ κύκλου, τὸν δεδομένον τῶι μεγέθει κύκλον ἀγαγεῖν δεῖ ὡς εἴρηται. ταῦτα δὲ ἀναλῦσαι καὶ συνθεῖναι καὶ διορίσασθαι κατὰ πτῶσιν. ἔχει δὲ τὸ πρῶτον τῶν Ἐπαφῶν προβλήματα ζ̅, τὸ δὲ δεύτερον προβλήματα δ̅. λήμματα δὲ ἔχει τὰ δύο βιβλία κα̅, αὐτὰ δὲ θεωρημάτων ἐστὶν ξ̅.

(13) μετὰ δὲ τὰς Ἐπαφὰς ἐν τρισὶ βιβλίοις Πορίσματά ἐστιν Εὐκλείδου, πολλοῖς ἄθροισμα φιλοτεχνότατον εἰς τὴν ἀνάλυσιν τῶν ἐμβριθεστέρων προβλημάτων, καί, τῶν γενῶν ἀπερίληπτον τῆς φύσεως παρεχομένης πλῆθος, οὐδὲν προστεθείκασι τοῖς ὑπὸ Εὐκλείδου γραφεῖσι πρώτου χωρὶς εἰ 650
μή τινες τῶν πρὸ ἡμῶν ἀπειρόκαλοι δευτέρας γραφὰς ὀλίγοις αὐτῶν παρατεθείκασιν, ἑκάστου μὲν πλῆθος ὡρισμένον ἔχοντος ἀποδείξεων ὡς ἐδείξαμεν, τοῦ δ' Εὐκλείδου μίαν ἑκάστου θέντος τὴν μάλιστα ὑπεμφαίνουσαν. ταῦτα δὲ λεπτὴν καὶ φυσικὴν ἔχει θεωρίαν καὶ ἀναγκαίαν καὶ καθολικωτέραν καὶ τοῖς δυναμένοις ὁρᾶν καὶ πορίζειν ἐπιτερπῆ. ἅπαντα δὲ αὐτῶν τὰ εἴδη οὔτε θεωρημάτων ἐστὶν οὔτε προβλημάτων, ἀλλὰ μέσον πως τούτων ἐχούσης ἰδέας, ὥστε τὰς προτάσεις αὐτῶν δύνασθαι σχηματίζεσθαι ἢ ὡς θεωρημάτων ἢ ὡς προβλημάτων, παρ' ὃ καὶ συμβέβηκε τῶν πολλῶν γεωμετρῶν |τοὺς μὲν |121v
ὑπολαμβάνειν αὐτὰ εἶναι τῶι γένει θεωρήματα τοὺς δὲ

‖ 1 δοθέντων Ha δοθεν A | εἰ Ha ἢ A ‖ 3 ἕξ Ha (Co) ἥξει A ‖ 4 διαφόρων Ha διαφορῶν A | δυάδες Ha δυάδος A | διάφοροι Hu διάφοραι Ha διαφοραὶ A ‖ 5 σημείων — δοθέντων om A¹ add mg A² alia manu ‖ 6 σημείου καὶ εὐθείας ἢ σημείου Ha σημεία καὶ εὐθεῖα ἢ σημεία A ‖ 8 τὸν Ha τὸ A | διαγαγεῖν Ha | δεῖ Ha δύο A ‖ 9 ταῦτα δὲ] καὶ ταῦτα Ha | διορίζεσθαι Ha ‖ 12 θεωρήματα Ha ‖ 14 πολλοῖς secl Hu ‖ 15 καὶ secl Hu | γενῶν] γενομένων Breton ‖ 20 τοῦ Ha τὴν A ‖ 21 ἑκάστου] ἑκάστοτε Hu | ὑπεμφαίνουσαν Ha ἀπεμφαίνουσαν A πως ἐμφαίνουσαν Heiberg₁ ‖ 25 μέσον Hu μέσην A | ὥστε — προβλημάτων secl Hu ‖ 26 ἢ ὡς Ha τέως A

(14) That the ancients best knew the distinction between these three things, is clear from their definitions. For they said that a theorem is what is offered for proof of what is offered, a problem what is proposed for construction of what is offered, a porism what is offered for the finding of what is offered. This definition of porism has been altered by the moderns because they could not find everything, but applying these elements and proving only this, that what is sought exists, without finding it, they were refuted by the definition and by what they were teaching. Hence on the basis of an accidental trait they wrote as follows: a porism is what is short by a hypothesis of (being) a theorem of a locus. The form of this class of porisms is the loci, and these abound in the *Domain of Analysis*. This kind, separated from the porisms, has been accumulated and named and handed down because of its being more diffusible than the other forms. There are, in fact, ten <books> of loci, some of planar, some of solid, some of curvilinear in (the loci) with respect to means.

(15) Another accidental trait of the porisms is that they have terse propositions because of their complexity, and many things are customarily left to be understood, with the result that many of the geometers comprehend them in part, but are ignorant of the more essential of the things signified. To encompass many things by one proposition is scarcely possible in the case of (the porisms); Euclid himself, after all, did not set down many things from each form, but one or a few out of the abundance for illustration. But at the beginning of the first book he placed some of similar form, all belonging to that more abundant kind of the loci, to the number of ten.

προβλήματα, ἀποβλέποντας τῶι σχήματι μόνον τῆς προτάσεως.

(14) τὴν δὲ διαφορὰν τῶν τριῶν τούτων ὅτι βέλτιον ἤιδεσαν οἱ ἀρχαῖοι δῆλον ἐκ τῶν ὅρων. ἔφασαν γὰρ θεώρημα μὲν εἶναι τὸ προτεινόμενον εἰς ἀπόδειξιν αὐτοῦ τοῦ προτεινομένου, πρόβλημα δὲ τὸ προβαλλόμενον εἰς κατασκευὴν αὐτοῦ τοῦ προτεινομένου, πόρισμα δὲ τὸ προτεινόμενον εἰς πορισμὸν αὐτοῦ τοῦ προτεινομένου. μετεγράφη δὲ οὗτος ὁ τοῦ πορίσματος ὅρος ὑπὸ τῶν νεωτέρων μὴ δυναμένων ἅπαντα πορίζειν, ἀλλὰ συγχρωμένων τοῖς στοιχείοις τούτοις καὶ δεικνύντων αὐτὸ μόνον τοῦθ' ὅτι ἔστι τὸ ζητούμενον, μὴ ποριζόντων δὲ τοῦτο, καὶ ἐλεγχομένων ὑπὸ τοῦ ὅρου καὶ τῶν διδασκομένων, ἔγραψαν δὲ 652
ἀπὸ συμβεβηκότος οὕτως· πόρισμά ἐστιν τὸ λεῖπον ὑποθέσει τοπικοῦ θεωρήματος· τούτου δὲ τοῦ γένους τῶν πορισμάτων εἶδός ἐστιν οἱ τόποι, καὶ πλεονάζουσιν ἐν τῶι Ἀναλυομένωι, κεχωρισμένον δὲ τῶν πορισμάτων ἤθροισται καὶ ἐπιγράφεται καὶ παραδίδοται διὰ τὸ πολύχυτον εἶναι μᾶλλον τῶν ἄλλων εἰδῶν. τῶν γοῦν τόπων ἐστὶν δέκα <βιβλία> ἃ μὲν ἐπιπέδων, ἃ δὲ στερεῶν, ἃ δὲ γραμμικῶν [καὶ] ἐπὶ τῶν πρὸς μεσότητας.

(15) συμβέβηκε δὲ καὶ τοῦτο τοῖς πορίσμασιν, τὰς προτάσεις ἔχειν ἐπιτετμημένας διὰ τὴν σκολιότητα, πολλῶν συνήθως συνυπακουομένων, ὥστε πολλοὺς τῶν γεωμετρῶν ἐπὶ μέρους ἐκδέχεσθαι, τὰ δὲ ἀναγκαιότερα ἀγνοεῖν τῶν σημαινομένων. περιλαβεῖν δὲ πολλὰ μιᾶι προτάσει ἥκιστα δυνατὸν ἐν τούτοις διὰ τὸ καὶ αὐτὸν Εὐκλείδην οὐ πολλὰ ἐξ ἑκάστου εἴδους τεθεικέναι, ἀλλὰ δείγματος ἕνεκα τῆς πολυπληθείας ἓν ἢ ὀλίγα, πρὸς ἀρχὴν δὲ τοῦ πρώτου βιβλίου τέθεικεν ὁμοειδῆ, πάντ' ἐκείνου τοῦ δαψιλεστέρου εἴδους

‖ 1 τῶι σχήματι] εἰς τὸ σχῆμα vel σχηματικὸν Hu ‖ 2 τὴν δὲ διαφορὰν Ge τὴν δὲ διαφορᾶς A τὰς δὲ διαφορὰς Ha ‖ 3 ἠιδεισαν Ha ‖ 8 μετεγράφη — πρὸς μεσότητας secl Hu ‖ 12 ἐλεγχόμενοι Ha | δὲ del Ha ‖ 16 κεχωρισμένων Ha ‖ 18 δέκα del Ha ‖ 19 καὶ ἐπὶ] καὶ ἔτι Ha ‖ 23 post ἐπὶ add μὲν Hu ‖ 24 ἐκδέχεσθαι Ha ἐκδέχεται A ‖ 25 περιλαβεῖν — τὸ πλῆθος secl Hu | ἥκιστα Ha ηδιστα A ‖ 27 δείγματα Ge | post ἕνεκα add ἐκ Hu ‖ 28 πολυπληθείας Hu (BS) πολυπληθιας A | ἓν ἢ Littré apud Breton ἐν ῆι A ἔνια Breton | ὀλίγα προσαρκεῖν δεδομένα Vincent | ἀρχῆι Heiberg$_1$ | δὲ] δεδομένον A δὲ ὅμως Heiberg$_1$ δεδομένων Ge, secl Hu ‖ 29 πάντ' Hu παν A τινα Heiberg$_1$

(16) And hence, finding it to be possible to encompass these in one proposition, we have written: "If in a *'hyptios'* or *'paryptios'* three points on one (line), or both (the points) on a parallel (line) are given, while (each of) the rest except one touches a line given in position, then that one too will touch a line given in position." This is enunciated only for four lines, of which no more than two are through the same point. It is not recognized that it is true for every number put forward, if one states it thus: "If any number of lines should intersect each other, with no more than two through the same point, and all (points) on one (line) are given, and each of those on another touch a line given in position..." or more generally thus: "If any number of lines should intersect each other, not more than two through the same point, and all points on one line be given, the rest being in quantity a triangular number, the side of this having each point touching a line given in position, and no three being at the angles of a triangular area, each remaining point will touch a line given in position." **(17)** It is not likely that the Elementarist was unaware of this, but he put down only the beginning. In all the porisms he evidently sowed only the starts and seeds of many great multitudes. Their classes should be defined, not by the various hypotheses, but by the various things that result in them and are sought in them. All the hypotheses differ from each other, being very individual, but each of the results and things sought turns up exactly the same in many different hypotheses. *(unintelligible text)* τῶν τόπων, ὡς ῑ τὸ πλῆθος.

(16) διὸ καὶ περιλαβεῖν ταύτας μιᾶι προτάσει ἐνδεχόμενον εὑρόντες οὕτως ἐγράψαμεν· ἐὰν ὑπτίου ἢ παρυπτίου τρία τὰ ἐπὶ μιᾶς σημεῖα ἢ παραλλήλου ἕτερα τὰ <δύο> δεδομένα ἦι, τὰ δὲ λοιπὰ πλὴν ἑνὸς ἅπτηται θέσει δεδομένης εὐθείας, καὶ τοῦθ' ἅψεται θέσει δεδομένης 654 εὐθείας. τοῦτ' ἐπὶ τεσσάρων μὲν εὐθειῶν εἴρηται μόνων ὧν οὐ πλείονες ἢ δύο διὰ τοῦ αὐτοῦ σημείου εἰσίν, ἀγνοεῖται δὲ |ἐπὶ παντὸς τοῦ προτεινομένου πλήθους ἀληθὲς ὑπάρχον |122 οὕτω λεγόμενον· ἐὰν ὁποσαιοῦν εὐθεῖαι τέμνωσιν ἀλλήλας, <μὴ> πλείονες ἢ δύο διὰ τοῦ αὐτοῦ σημείου, πάντα δὲ ἐπὶ μιᾶς αὐτῶν δεδομένα ἦι, καὶ τῶν ἐπὶ ἑτέρας ἕκαστον ἅπτηται θέσει δεδομένης εὐθείας. ἢ καθολικώτερον οὕτως· ἐὰν ὁποσαιοῦν εὐθεῖαι τέμνωσιν ἀλλήλας, μὴ πλείονες ἢ δύο διὰ τοῦ αὐτοῦ σημείου, πάντα δὲ τὰ ἐπὶ μιᾶς αὐτῶν σημεῖα δεδομένα ἦι, τῶν δὲ λοιπῶν τὸ πλῆθος ἐχόντων τρίγωνον ἀριθμόν, ἡ πλευρὰ τούτου ἕκαστον ἔχηι σημεῖον ἁπτόμενον εὐθείας θέσει δεδομένης, τῶν τριῶν μὴ πρὸς γωνίαις ὑπαρχόντων τριγώνου χωρίου, ἕκαστον λοιπὸν σημεῖον ἅψεται θέσει δεδομένης εὐθείας.

(17) τὸν δὲ στοιχειωτὴν οὐκ εἰκὸς ἀγνοῆσαι τοῦτο, τὴν δ' ἀρχὴν μόνην τάξαι· καὶ ἐπὶ πάντων δὲ τῶν πορισμάτων φαίνεται ἀρχὰς καὶ σπέρματα μόνα πληθῶν πολλῶν καὶ μεγάλων καταβεβλημένος ὧν τὰ γένη οὐ κατὰ τὰς τῶν ὑποθέσεων διαφορὰς διαστέλλειν δεῖ ἀλλὰ κατὰ τὰς τῶν συμβεβηκότων καὶ ζητουμένων. αἱ μὲν ὑποθέσεις ἅπασαι διαφέρουσιν ἀλλήλων εἰδικώταται οὖσαι, τῶν δὲ συμβαινόντων καὶ ζητουμένων ἕκαστον ἓν καὶ τὸ αὐτὸ ὂν πολλαῖς ὑποθέσεσι διαφόροις συμβέβηκε. [τῶι ταῦτα γένη]

‖ 2 post ταύτας add ἐν Ha ‖ 4 τρία — σημεῖα post ἕτερα transp Ha | σημεῖα Ha σημεῖον A | ἢ — τὰ secl Hu | ἕτερα A ἑτέραι Ha | τὰ (δύο) om Breton ‖ 5 δύο add Simson$_1$ ‖ 7 ἐπὶ Ha ἔστιν A | μόνον Breton ‖ 10 οὕτως Hu ‖ 11 μὴ add Ha ‖ 15 σημεῖα Ha σημείων A ‖ 17 ἔχηι Hu ἔχει A ‖ 18 τῶν Hu ὧν A | τρία Breton | γωνίαις ὑπαρχόντων Hu γωνίαν ὑπάρχον A ‖ 23 πληθῶν] πλήθει Heiberg$_1$ | πληθῶν — μεγάλων secl Hu ‖ 24 καταβεβλημένος Hu -μένας A -κέναι Ha | ὧν τὰ γένη Hu ων ενη A ὧν ἕκαστον Ha ‖ 26 αἱ — συμβέβηκε secl Hu | post μὲν add γὰρ Heiberg$_1$ ‖ 27 διαφέρουσιν Ha διαφοροῦσιν A ‖ 28 ἕκαστον Ha ἑκάστην A ‖ 29 post συμβέβηκε add διαιρεῖσθαι Hu | τῶι ταῦτα γένη om Ha

(18) Thus the following kinds of things sought in the propositions are to be accomplished in the first book:

(1) In the beginning of the book is this diagram: If lines from two given points inflect on a line given in position, and one (line) cuts off (an abscissa) from a line given in position up to a point given on it, the other (line) too will cut off from another (line given in position) an (abscissa) having a given ratio (to the first).

Among those that follow:

(2) That this point touches a line given in position.
(3) That the ratio of this (line) to this (line) is given.
(4) That the ratio of this (line) to an abscissa (is given).
(5) That this (line) is given in position.
(6) That this (line) makes a neusis on a (point) given in position.
(7) That the ratio of this (line) to some one from this (point) to a given (point is given).
(8) That the ratio of this (line) to one drawn down from this (point is given).
(9) That the ratio of this area to the (rectangle contained) by a given line and this (line is given).
(10) That one part of this area is given, the other has a (given) ratio to an abscissa.
(11) That this area or this plus some area is given, and that (area) has a (given) ratio to an abscissa.
(12) That this (line) plus that to which this (line) has a given ratio, has a (given) ratio to some (line) from this (point) to a given (point).
(13) That the (rectangle contained) by a given and this (line) plus the (rectangle contained) by a given and this (line) equals the (rectangle contained) by a given and the (line) from this (point) to a given (point).

(18) ποιητέον οὖν ἐν μὲν τῶι πρώτωι βιβλίωι ταῦτα τὰ γένη τῶν ἐν ταῖς προτάσεσι ζητουμένων.

ἐν ἀρχῆι μὲν τοῦ βιβλίου διάγραμμα τοῦτο.

(1) ἐὰν ἀπὸ δύο δεδομένων σημείων πρὸς θέσει δεδομένην 656
εὐθεῖαι κλασθῶσιν, ἀποτέμνηι δὲ μία ἀπὸ θέσει δεδομένης εὐθείας πρὸς τῶι ἐπ' αὐτῆς δεδομένωι σημείωι, ἀποτεμεῖ καὶ ἡ ἑτέρα ἀπὸ ἑτέρας λόγον ἔχουσαν δοθέντα.

ἐν δὲ τοῖς ἑξῆς·

(2) ὅτι τόδε τὸ σημεῖον ἅπτεται θέσει δεδομένης εὐθείας.
(3) ὅτι λόγος τῆσδε πρὸς τήνδε δοθείς.
(4) ὅτι λόγος τῆσδε πρὸς ἀποτομήν.
(5) ὅτι ἥδε θέσει δεδομένη ἐστίν.
(6) ὅτι ἥδε ἐπὶ δοθὲν νεύει.
(7) ὅτι λόγος τῆσδε πρός τινα ἀπὸ τοῦδε ἕως δοθέντος.
(8) ὅτι λόγος τῆσδε πρός τινα ἀπὸ τοῦδε κατηγμένην.
(9) ὅτι λόγος τοῦδε τοῦ χωρίου πρὸς τὸ ὑπὸ δοθείσης καὶ τῆσδε.
(10) ὅτι τοῦδε τοῦ χωρίου ὃ μέν τι δοθέν ἐστιν, ὃ δὲ λόγον ἔχει πρὸς ἀποτομήν.
(11) ὅτι τόδε τὸ χωρίον ἢ τόδε μετά τινος χωρίου δοθέν ἐστιν, ἐκεῖνο δὲ λόγον ἔχει πρὸς ἀποτομήν.
(12) ὅτι ἥδε μεθ' ἧς πρὸς ἣν ἥδε λόγον ἔχει |δοθέντα, λόγον |122v
ἔχει πρός τινα ἀπὸ τοῦδε ἕως δοθέντος.
(13) ὅτι τὸ ὑπὸ δοθέντος καὶ τῆσδε καὶ τὸ ὑπὸ δοθέντος καὶ

‖ 2 γένη] γενόμενα coni Breton ‖ 3 ἐν — τοῦτο secl Hu ἐν ἀρχῆι μὲν τούτου ζήτει τὸ διάγραμμα Vincent | τοῦ βιβλίου Heiberg$_1$ τὸ ζ´ A τοῦ ζ´ Ha ‖ 5 εὐθεῖαι Hu εὐθεῖαν A | ἀποτέμνηι δὲ μία Ha (Co) ἀποτέμνη δὲ μίαν A ‖ 6 δεδομένωι σημείωι Ha δεδομένων σημείων A ‖ 7 ἔχουσαν Ha ἔχουσα A ‖ 13 θέσει δεδομένη] ἐν παραθέσει Breton ‖ 15 ὅτι — δοθέντος bis A om Co | ἕως Ha ὡς A ‖ 16 κατηγμένην Ha κατηγμένης A ‖ 21 δοθέν Heiberg$_1$ δοθέντος A ‖ 23 ἥδε μεθ' ἧς] μεθ' ἧς Hu ἡ μεθ' ἧς Heiberg$_1$ ‖ 24 ἕως Ha ὡς A ‖ 25 post ὅτι τῶι ὑπὸ add τοῦ Ha | δοθέντος... δοθέντος] δοθείσης... δοθείσης Heiberg$_1$ | καὶ τὸ ὑπὸ δοθέντος καὶ τῆσδε del Co

(14) That the ratio of this (line) and this (line) to some (line) from this (point) to a given (point is given).
(15) That this (line) cuts off from (lines) given in position (abscissas) containing a given.

(19) In the second book, the hypotheses are different, but of the things sought, the majority are the same as those in the first book, but there are these in addition:

(1) That this area either has a (given) ratio to an abscissa or plus a given has a (given) ratio to an abscissa.
(2) That the ratio of the (rectangle contained) by these (lines) to an abscissa (is given).
(3) That the ratio of the (rectangle contained) by these (lines) taken together plus these (lines) taken together to an abscissa (is given).
(4) That the (rectangle contained) by this (line) and the sum of this (line) and that to which this (line) has a given ratio, plus the (rectangle contained) by this (line) and that to which this line has a given ratio have a (given) ratio to an abscissa.
(5) That the ratio of a sum to a (line) from this (point) to a given (point is given).
(6) That the (rectangle contained) by these is given.

(20) In the third book most of the hypotheses are on semicircles, and a few on the circle and sectors. Of the things sought, many are analogous to those above, but there are these in addition:

τῆσδε ἴσον ἐστὶν τῶι ὑπὸ δοθέντος καὶ <τῆς> ἀπὸ τοῦδε ἕως δοθέντος.

(14) *ὅτι λόγος τῆσδε καὶ τῆσδε πρός τινα ἀπὸ τοῦδε ἕως δοθέντος.* 658

(15) *ὅτι ἥδε ἀποτέμνει ἀπὸ θέσει δεδομένων δοθὲν περιεχούσας.*

(19) *ἐν δὲ τῶι δευτέρωι βιβλίωι ὑποθέσεις μὲν ἕτεραι, τῶν δὲ ζητουμένων τὰ μὲν πλείονα τὰ αὐτὰ τοῖς ἐν τῶι πρώτωι βιβλίωι, περισσὰ δὲ ταῦτα·*

(1) *ὅτι τόδε τὸ χωρίον ἤτοι λόγον ἔχει πρὸς ἀποτομὴν <ἢ> μετὰ δοθέντος λόγον ἔχει πρὸς ἀποτομήν.*

(2) *ὅτι λόγος τοῦ ὑπὸ τῶνδε πρὸς ἀποτομήν.*

(3) *ὅτι λόγος τοῦ ὑπὸ συναμφοτέρου τῶνδε καὶ συναμφοτέρου τῶνδε πρὸς ἀποτομήν.*

(4) *ὅτι τὸ ὑπὸ τῆσδε καὶ συναμφοτέρου τῆσδέ τε καὶ τῆς πρὸς ἣν ἥδε λόγον ἔχει δοθέντα καὶ τὸ ὑπὸ τῆσδε καὶ τῆς πρὸς ἣν ἥδε λόγον ἔχει δοθέντα λόγον ἔχει πρὸς ἀποτομήν.*

(5) *ὅτι λόγος συναμφοτέρου πρός τινα <ἀπὸ> τοῦδε ἕως δοθέντος.*

(6) *ὅτι δοθὲν τὸ ὑπὸ τῶνδε.*

(20) *ἐν δὲ τῶι τρίτωι βιβλίωι αἱ μὲν πλείονες ὑποθέσεις ἐπὶ ἡμικυκλίων εἰσίν, ὀλίγαι δὲ ἐπὶ κύκλου καὶ τμημάτων. τῶν δὲ ζητουμένων τὰ μὲν πολλὰ παραπλησίως τοῖς ἔμπροσθεν, περισσὰ δὲ ταῦτα·*

‖ 1 *δοθέντος* Ha *δοθέντι* A *δοθείσης* Heiberg$_1$ | *τῆς* add Ha ‖ 3 *ἕως* Ha *ὡς* A ‖ 10 *ἤτοι*] *ἢ τόδε μετὰ δοθέντος* Hu | *ἢ* add Ha ‖ 11 *μετὰ* — *ἀποτομήν* om A^1 add mg A^2 alia manu, om Hu ‖ 12 *λόγος* Ha *λόγον* A ‖ 13 (*ὑπὸ*) *συναμφοτέρου*] *συναμφοτέρων* Hu | (*καὶ*) *συναμφοτέρου* Ha *συναμφοτέρων* A ‖ 19 *λόγος* Ha *λόγου* A | *ἀπὸ* add Ha ‖ 24 *παραπλήσια* Heiberg$_1$

(1) That the ratio of the (rectangle contained) by these to the (rectangle contained) by these (is given).
(2) That the ratio of the square of this to an abscissa (is given).
(3) That the (rectangle contained) by these equals the (rectangle contained) by a given and a (line) from this (point) to a given (point).
(4) That the square of this (line) to the (rectangle contained) by a given and what is cut off by a perpendicular as far as a given (point is given).
(5) That the sum of <this> (line) and (that) to which this line has a given ratio has a (given) ratio to an abscissa.
(6) That there is some given point from which (lines) joined to this will contain a triangle given in shape.
(7) That there is a given point from which (lines) joined to this receive equal arcs.
(8) That this (line) either is parallel to a (line given) in position, or contains a given angle with some line that makes a neusis on a given (point).

The three books of the *Porisms* contain thirty-eight lemmas. They comprise 171 theorems.

(21) Two (books) of Plane Loci:

Of the loci in general, some are fixed, as Apollonius also states before his own elements: the locus of a point being a point, a line the locus of a line, a surface of a surface, a solid of a solid; others are path loci: as a line of a point, a surface of a line, a solid of a surface; others are domain loci: as a surface of a point, a solid of a line.

(22) Among the (loci) in the *Domain of Analysis,* those of things given in position are fixed, while the so-called 'plane' and 'solid' and 'curvilinear' (loci) are path loci of points, and the loci on surfaces are domain loci of points, but path loci of lines. However, the curvilinears are demonstrated

(1) ὅτι λόγος τοῦ ὑπὸ τῶνδε πρὸς τὸ ὑπὸ τῶνδε. 660
(2) ὅτι λόγος τοῦ ἀπὸ τῆσδε πρὸς [τὸ] ἀποτομήν.
(3) ὅτι τὸ ὑπὸ τῶνδε τῶι ὑπὸ δοθείσης καὶ <τῆς> ἀπὸ τοῦδε ἕως δοθέντος.
(4) ὅτι τὸ ἀπὸ τῆσδε τῶι ὑπὸ δοθείσης καὶ ἀπολαμβανομένης ὑπὸ καθέτου ἕως δοθέντος.
(5) ὅτι συναμφότερος <ἥδε> καὶ πρὸς ἣν ἥδε λόγον ἔχει δοθέντα λόγον ἔχει πρὸς ἀποτομήν.
(6) ὅτι ἔστιν τι δοθὲν σημεῖον ἀφ' οὗ αἱ ἐπιζευγνύμεναι ἐπὶ τόδε δοθὲν περιέξουσι τῶι εἴδει τρίγωνον.
(7) ὅτι ἔστιν <τι> δοθὲν σημεῖον ἀφ' οὗ αἱ ἐπιζευγνύμεναι ἐπὶ τόδε ἴσας ἀπολαμβάνουσι περιφερείας.
(8) ὅτι ἥδε ἤτοι παρὰ θέσει ἐστὶν ἢ μετά τινος εὐθείας ἐπὶ δοθὲν νευούσης δοθεῖσαν περιέχει γωνίαν.

ἔχει δὲ τὰ τρία βιβλία τῶν Πορισμάτων λήμματα λη̅· αὐτὰ δὲ θεωρημάτων ἐστὶν ροα̅.

(21) ΤΟΠΩΝ ΕΠΙΠΕΔΩΝ ΔΥΟ

τῶν τόπων καθόλου οἱ μὲν εἰσιν ἐφεκτικοί, ὡς καὶ Ἀπολλώνιος πρὸ τῶν ἰδίων στοιχείων λέγει· σημείου μὲν τόπον σημεῖον, γραμμῆς δὲ τόπον γραμμή, ἐπιφανείας δὲ ἐπιφάνεια, |στερεοῦ δὲ στερεόν, οἱ δὲ διεξοδικοί, ὡς σημείου 662
μὲν γραμμή, γραμμῆς ἐπιφάνεια, ἐπιφανείας δὲ στερεόν, οἱ δὲ |123
ἀναστροφικοί, ὡς σημείου μὲν ἐπιφάνεια, γραμμῆς δὲ στερεόν.

(22) τῶν δὲ ἐν τῶι Ἀναλυομένωι οἱ μὲν τῶν θέσει δεδομένων ἐφεκτικοί εἰσιν, οἱ δὲ ἐπίπεδοι λεγόμενοι καὶ οἱ στερεοὶ <καὶ οἱ> γραμμικοὶ διεξοδικοί εἰσιν σημείων, οἱ δὲ πρὸς ἐπιφανείαις ἀναστροφικοὶ μέν εἰσιν σημείων,

‖ 2 λόγος Ha λόγον A | τὸ secl Ha ‖ 3 τῆς add Ha ‖ 5 δοθείσης Hu δοθέντος A ‖ 7 συναμφότερος ἥδε καὶ] τὸ ὑπὸ συναμφοτέρου καὶ τῆσδε Breton | ἥδε (καὶ) add Hu ‖ 10 τόδε Ha τὸ A τούσδε Hu τόνδε Simson$_1$ ‖ 11 τι add Ha ‖ 12 τόνδε Hu ‖ 13 ἥδε ἤτοι παρὰ θέσει coni Hu (index s.v. παράθεσις) ηδεντοι παραθέσει A ἥδε ἤτοι ἐν παραθέσει Ha ἐν τῆι παραθέσει Ge | ἐστὶν Hu ἔσται A | post ἐπὶ add τὸ Ha ‖ 18 ὡς Hu οὓς A ‖ 19 ἰδίων om Ha ‖ 20 γραμμή A γραμμὴν Ha ‖ 21 ἐπιφάνεια A ἐπιφάνειαν Ha ‖ 22 γραμμή Heiberg$_2$ γραμμὴν A | post γραμμῆς add δ' Hu | ἐπιφάνεια Heiberg$_2$ ἐπιφάνειαν A ‖ 23 ἐπιφάνεια Heiberg$_2$ ἐπιφάνειαν A ‖ 24 totum cap. 22 secl Hu | τῶν (θέσει) Ha τῶι A ‖ 26 ante διεξοδικοί add καὶ Ha ‖ 27 ἐπιφανείας Ha on the basis of the (loci) on surfaces. The loci about which we are teaching, and generally all that are straight lines or circles, are called 'plane'; all those that are sections of cones, parabolas or ellipses or hyperbolas are called 'solid'; and all those loci are called 'curvilinear' that are neither straight lines nor circles nor any of the aforesaid conic sections. The loci that Eratosthenes named 'with respect to means' are in classification among those named above, but they have been named on the basis of the characteristic of their hypotheses.

(23) The ancients compiled their elements attending to the order of these plane loci; but the people who came after them disregarded this, and added others – as if they were not boundless in number if one wanted to add some that do not belong to that order! Hence I shall put the additional ones later, and those that belong to the order first, encompassing them by one proposition, namely: (1) If two straight lines are drawn either from one given point or from two, and either in a straight line or parallel or containing a given angle, and either holding a ratio to one another or containing a given area, and the end of one touches a plane locus given in position, the end of the other will touch a plane locus given in position, sometimes of the same kind, sometimes of the other, and sometimes similarly situated with respect to the straight line, sometimes oppositely; this follows in accordance with the various assumptions.

(24) And the additional ones. First, three by Charmandrus that are harmonious:

(2) If one end of a straight line given in magnitude be given, the other will touch a concave (circular) arc given in position.

(3) If straight lines from two given points should inflect and contain a given angle, their common point will touch a concave (circular) arc given in position.

διεξοδικοὶ δὲ γραμμῶν. οἱ μέντοι γραμμικοὶ ἀπὸ τῶν πρὸς ἐπιφανείαις δείκνυνται. λέγονται δὲ ἐπίπεδοι μὲν τόποι οὗτοί τε περὶ ὧν ἐπάγομεν καὶ καθόλου ὅσοι εἰσὶν εὐθεῖαι [τε καὶ] γραμμαὶ ἢ κύκλοι, στερεοὶ δὲ ὅσοι εἰσὶν κώνων τομαί, παραβολαὶ ἢ ἐλλείψεις ἢ ὑπερβολαί, γραμμικοὶ δὲ τόποι λέγονται ὅσοι γραμμαί εἰσιν οὔτε εὐθεῖαι οὔτε κύκλοι οὔτε τις τῶν εἰρημένων κωνικῶν τομῶν. οἱ δὲ ὑπὸ Ἐρατοσθένους ἐπιγραφέντες τόποι πρὸς μεσότητας ἐκ τῶν προειρημένων εἰσὶν τῶι γένει, ἀπὸ δὲ τῆς ἰδιότητος τῶν ὑποθέσεων ἐκλήθησαν.

(23) οἱ μὲν οὖν ἀρχαῖοι <εἰς τὴν> τῶν ἐπιπέδων τούτων τόπων τάξιν ἀποβλέποντες ἐστοιχείωσαν, ἧς ἀμελήσαντες οἱ μετ᾽ αὐτοὺς προσέθηκαν ἑτέρους, ὡς οὐκ ἀπείρων τὸ πλῆθος ὄντων εἰ θέλοι τις προσγράφειν οὐ τῆς τάξεως ἐκείνης ἐχόμενα. θήσω οὖν τὰ μὲν προσκείμενα ὕστερα, τὰ δ᾽ ἐκ τῆς τάξεως πρότερα μιᾶι περιλαβὼν προτάσει ταύτηι· (1) ἐὰν δύο εὐθεῖαι ἀχθῶσιν ἤτοι ἀπὸ ἑνὸς δεδομένου σημείου ἢ ἀπὸ δύο, καὶ ἤτοι ἐπ᾽ εὐθείας ἢ παράλληλοι, ἢ δεδομένην περιέχουσαι γωνίαν, καὶ ἤτοι λόγον ἔχουσαι πρὸς ἀλλήλας ἢ χωρίον 664 περιέχουσαι δεδομένον, ἅπτηται δὲ τὸ τῆς μιᾶς πέρας ἐπιπέδου τόπου θέσει δεδομένου, ἅψεται καὶ τὸ τῆς ἑτέρας πέρας ἐπιπέδου τόπου θέσει δεδομένου, ὅτε μὲν τοῦ ὁμογενοῦς, ὅτε δὲ τοῦ ἑτέρου, καὶ ὅτε μὲν ὁμοίως κειμένου πρὸς τὴν εὐθεῖαν, ὅτε δὲ ἐναντίως. ταῦτα δὲ γίνεται παρὰ τὰς διαφορὰς τῶν ὑποκειμένων.

(24) τὰ δὲ προσκείμενα. ἐν ἀρχῆι μὲν ὑπὸ Χαρμάνδρου γ̄ συμφωνεῖ ταῦτα·

(2) ἐὰν εὐθείας τῶι μεγέθει δεδομένης τὸ ἓν πέρας ἦι δεδομένον, τὸ ἕτερον ἅψεται θέσει δεδομένης περιφερείας κοίλης.

(3) |ἐὰν ἀπὸ δύο δεδομένων σημείων κλασθῶσιν εὐθεῖαι |123v δεδομένην περιέχουσαι γωνίαν, τὸ κοινὸν αὐτῶν σημεῖον ἅψεται θέσει δεδομένης περιφερείας κοίλης.

‖ 2 ἐπιφάνειαν Ha ‖ 3 καὶ (καθόλου) om Ha ‖ 4 τε καὶ om Ha | ὅσαι Ha ‖ 5 ἐλλήψεις A corr Ha ‖ 7 τις] τινὲς Hu ‖ 10 lacunam post ὑποθέσεων coni. Ha | ἐκλήθησαν] ἐκείνοις A ‖ 11 εἰς τὴν add Hu | τούτων secl Hu τόπων τούτων transp Ha ‖ 14 οὐ] τὰ Hu ‖ 15 προσκείμενα Ge προκείμενα A | δ᾽ ἐκ] δὲ Ha ‖ 16 πρότερα Ha προτέραι A ‖ 17 ἀχθῶσιν om Ha ‖ 19 γωνίαν Ha γωνίαι A ‖ 22 τόπου Ha τόπους A ‖ 26 μὲν om Ha ‖ 27 συμφρονεῖ Ha ‖ 32 γωνίαν Ha γωνίαι A | σημεῖον Ha σημείων A

(4) If the base of a triangular area given in magnitude should be given in position and magnitude, its vertex will touch a straight line given in position.

(25) Others are like this:

(5) If one end of a straight line given in magnitude and drawn parallel to some straight line given in position, should touch a straight line given in position, the other (end) too will touch a straight line given in position.

(6) If from a point to two straight lines given in position, whether parallel or intersecting, (straight lines) are drawn at given angles, either having a given ratio to one another, or with one of them plus that to which the other has a given ratio being given, the point will touch a straight line given in position.

(7) And if there be any number whatever of straight lines given in position, and straight lines be drawn to them from some point at given angles, and the (rectangle contained) by a given and a (line) drawn upon (one of them) plus the (rectangle contained) by a given and another (line) drawn upon (one of them) equals the (rectangle contained) by a given and another (line) drawn upon (one of them), and the rest similarly, the point will touch a straight line given in position.

(8) If from some point straight lines be drawn onto parallels given in position at given angles, and either cutting off straight lines as far as points given on them that have a (given) ratio (to each other) or containing a given area, or so that the given shapes (constructed) upon the (lines) drawn upon (them) or the excess of the shapes equals a given area, the point will touch a straight line given in position.

(4) ἐὰν τριγώνου χωρίου μεγέθει δεδομένου ἡ βάσις θέσει καὶ μεγέθει δεδομένη ἧι, ἡ κορυφὴ αὐτοῦ ἅψεται θέσει δεδομένης εὐθείας.

(25) ἕτερα δὲ τοιαῦτα.

(5) ἐὰν εὐθείας τῶι μεγέθει δεδομένης καὶ παρά τινα θέσει δεδομένην εὐθεῖαν ἠγμένης, τὸ ἓν πέρας ἅπτηται θέσει δεδομένης εὐθείας, ἅψεται καὶ τὸ ἕτερον εὐθείας θέσει δεδομένης.

(6) ἐὰν ἀπό τινος σημείου ἐπὶ θέσει δεδομένας δύο εὐθείας παραλλήλους ἢ συμπιπτούσας καταχθῶσιν ἐν δεδομέναις γωνίαις ἤτοι λόγον ἔχουσαι πρὸς ἀλλήλας δεδομένον ἢ ὧν ἡ μία μεθ' ἧς πρὸς ἣν ἡ ἑτέρα λόγον ἔχει δοθέντα δεδομένη ἐστίν, ἅψεται τὸ σημεῖον θέσει δεδομένης εὐθείας.

(7) καὶ ἐὰν ὦσιν ὁποσαιοῦν εὐθεῖαι θέσει δεδομέναι καὶ ἐπ' αὐτὰς ἀπό τινος σημείου καταχθῶσιν εὐθεῖαι ἐν 666 δεδομέναις γωνίαις, ἧι δὲ τὸ ὑπὸ δοθείσης καὶ κατηγμένης μετὰ τοῦ ὑπὸ δοθείσης καὶ ἑτέρας κατηγμένης ἴσον τῶι ὑπὸ δοθείσης καὶ ἄλλης κατηγμένης καὶ τῶν λοιπῶν ὁμοίως, τὸ σημεῖον ἅψεται θέσει δεδομένης εὐθείας.

(8) ἐὰν ἀπό τινος σημείου ἐπὶ θέσει δεδομένας παραλλήλους καταχθῶσιν εὐθεῖαι ἐν δεδομέναις γωνίαις ἀποτέμνουσαι πρὸς τοῖς ἐπ' αὐτῶν δοθεῖσι σημείοις εὐθείας ἤτοι λόγον ἐχούσας ἢ χωρίον περιέχουσαι δεδομένον ἢ ὥστε τὰ ἐπ' αὐτῶν τῶν κατηγμένων δεδομένα εἴδη ἢ τὴν ὑπεροχὴν τῶν εἰδῶν ἴσην εἶναι δεδομένωι χωρίωι, τὸ σημεῖον ἅψεται θέσει δεδομένης εὐθείας.

‖ 7 θέσει om Ha ‖ 10 παραλλήλας Ha | δεδομέναις γωνίαις Ha δεδομένη γωνία A ‖ 11 ἔχουσαι Ha ἔχουσιν A ‖ 19 ἄλλης] ἑτέρας Ha ‖ 23 post γωνίαις transp ἤτοι (post εὐθείαις) Simson₂ ‖ 24 σημείοις Ha σημείων A ‖ 25 εὐθείας Ha εὐθείαις A | post ἐχούσας add δοθέντα Ha | ἢ – χωρίωι secl Ha | περιέχουσαι Simson₂ περιεχούσας A ‖ 26 ἐπ' Ha ἀπ' A ‖ 27 ἴσην Ha ἴσον A

(26) The second book contains these:

(1) If straight lines from two given points inflect and their squares differ by a given area, the point will touch a straight line given in area.

(2) But if they be in a given ratio, (the point will touch) either a straight line or an arc.

(3) If a straight line be given in position, and a point be given on it, and from this some bounded (line) be drawn, and from the end a (straight line) be drawn at right angles to the (line) <given> in position, and the square of the (first line) drawn equals the (rectangle contained) by a given and what (the perpendicular) cuts off either as far as the given point or as far as another given point on the (line) given in position, the end of this (line) will touch an arc given in position.

(4) If straight lines from two given points inflect and the square of the one is greater than the square of the other by a given amount than in ratio, the point will touch an arc given in position.

(5) If straight lines from any number of points whatever inflect at one point, and the shapes (constructed) on all of them equal a given area, the point will touch an arc given in position.

(6) If straight lines from two given points inflect, and a straight line is drawn from the point parallel (to a line given) in position and cuts off (an abscissa) from a straight line given in position (extending) as far as a given point, and the shapes (constructed) on the inflecting (lines) equal the (rectangle contained) by a given and the abscissa, the point at the inflection will touch an arc given in position.

(7) If in a circle given in position some point is given, and through it is drawn some straight line, and some point is taken on it outside (the line) and the square of the (segment) as far as the point given inside equals the (rectangle contained) by the whole and the segment outside, either (the square) by itself or this and the (rectangle contained) by the two segments inside, the point outside will touch a straight line given in position.

(26) τὸ δὲ δεύτερον βιβλίον περιέχει τάδε·

(1) ἐὰν ἀπὸ δύο δεδομένων σημείων εὐθεῖαι κλασθῶσιν καὶ ἦι τὰ ἀπ' αὐτῶν δοθέντι χωρίωι διαφέροντα, τὸ σημεῖον ἅψεται θέσει δεδομένης εὐθείας.

(2) ἐὰν δὲ ὦσιν ἐν λόγωι δοθέντι ἤτοι εὐθείας ἢ περιφερείας.

(3) ἐὰν ἦι θέσει δεδομένη εὐθεῖα καὶ ἐπ' αὐτῆς δοθὲν σημεῖον καὶ ἀπὸ τούτου διαχθεῖσά τις πεπερασμένη, ἀπὸ δὲ τοῦ πέρατος ἀχθῆι κάθετος ἐπὶ τὴν θέσει <δεδομένην> καὶ ἦι τὸ ἀπὸ τῆς διαχθείσης ἴσον τῶι ὑπὸ δοθείσης καὶ ἧς ἀπολαμβάνει ἤτοι πρὸς τῶι δοθέντι σημείωι ἢ πρὸς ἑτέρωι δοθέντι σημείωι [ἢ πρὸς ἑτέρωι δοθέντι] ἐπὶ τῆς θέσει δεδομένης, τὸ πέρας τῆσδε ἅψεται θέσει δεδομένης περιφερείας.

(4) ἐὰν ἀπὸ δύο δοθέντων σημείων εὐθεῖαι κλασθῶσιν καὶ ἦι τὸ ἀπὸ τῆς μιᾶς τοῦ ἀπὸ τῆς ἑτέρας δοθέντι μεῖζον ἢ ἐν 668
λόγωι, τὸ σημεῖον ἅψεται θέσει δεδομένης περιφερείας.

(5) ἐὰν ἀπὸ ὁσωνοῦν δεδο|μένων σημείων κλασθῶσιν εὐθεῖαι |124
πρὸς ἑνὶ σημείωι καὶ ἦι τὰ ἀπὸ πασῶν εἴδη ἴσα δοθέντι χωρίωι, τὸ σημεῖον ἅψεται θέσει δεδομένης περιφερείας.

(6) ἐὰν ἀπὸ δύο δοθέντων σημείων κλασθῶσιν εὐθεῖαι, ἀπὸ δὲ τοῦ σημείου παρὰ θέσει ἀχθεῖσα εὐθεῖα ἀπολαμβάνηι ἀπὸ θέσει δεδομένης εὐθείας πρὸς δοθέντι σημείωι, καὶ ἦι τὰ ἀπὸ τῶν κεκλασμένων εἴδη ἴσα τῶι ὑπὸ δοθείσης καὶ τῆς ἀπολαμβανομένης, τὸ πρὸς τῆι κλάσει σημεῖον ἅψεται θέσει δεδομένης περιφερείας.

(7) ἐὰν ἐν κύκλωι θέσει δεδομένωι δοθέν τι σημεῖον ἦι καὶ δι' αὐτοῦ ἀχθῆι τις εὐθεῖα καὶ ἐπ' αὐτῆς ληφθῆι τι σημεῖον ἐκτὸς καὶ ἦι τὸ ἀπὸ τῆς ἄχρι τοῦ δοθέντος ἐντὸς σημείου ἴσον τῶι ὑπὸ τῆς ὅλης καὶ τῆς ἐκτὸς ἀπολαμβανομένης ἤτοι μόνον ἢ τοῦτό τε καὶ τὸ ὑπὸ τῶν ἐντὸς δύο τμημάτων, τὸ ἐκτὸς σημεῖον ἅψεται θέσει δεδομένης εὐθείας.

‖ 9 κάθετος] καὶ A πρὸς ὀρθὰς Ha | θέσει δεδομένην Ha θέσιν A ‖ 11 ἤτοι Ha ἢ καὶ A ‖ 12 ἢ — δοθέντι del Ha ‖ 13 τὸ πέρας — δεδομένης om A^1 add mg A^2 alia manu | τῆσδε] τῆς διαχθείσης Hu app ‖ 16 δοθέντι Ha δοθὲν A | μεῖζον Ha μείζων A | ἢ Ha ἦι A ‖ 19 ἴσα Ha ἴσον A ‖ 22 post παρὰ add τὴν Ha | ἀπολαμβάνηι Hu ἀπολαμβανομένη A ‖ 24 ἴσα Ha ἴσον A ‖ 25 σημεῖον Ha σημείωι A ‖ 27 ἐν — δεδομένωι] ἐντὸς κύκλου θέσει δεδομένου Ha | δοθέν τι σημεῖον Ha δοθέντι σημείωι A ‖ 30 ὑπὸ Ha ἀπὸ A ‖ 31 ἤτοι Hu ἢ τῶι A ἢ τὸ Simson$_2$ | μόνον ἢ τοῦτό τε καὶ τὸ Simson$_2$ μόνωι ἢ τούτωι τε καὶ τῶι A

(8) And if this point touches a straight line given in position, and the circle is not assumed, the points on either side of the given (point) will touch the same arc given in position.

The two books of the *Plane Loci* contain 147 theorems or diagrams, and eight lemmas.

(27) Two (Books) of Neuses:

A line is said to make a neusis on a point if it passes through it when produced. Generally it is the same thing when (the line) is said to make a neusis on a given point, or when some (point) is (said) to be given on it, or when it is (said) to be through a given point. They named these *Neuses* on the basis of one of these expressions, the problem being generally this: given two lines in position, to place between them a straight line given in magnitude, making a neusis on a given point. Among those that have varying assumptions in the details according to this definition, some were plane, some solid, some curvilinear. Choosing from among the plane ones those that are more useful for many things, they demonstrated the following problems: given in position a semicircle and a straight line at right angles to the base, or two semicircles having their bases in a straight line, to place a straight line given in magnitude between the two curves, making a neusis on the angle of the semicircle; and given a rhombus with one side extended, to fit into the outside angle a straight line given in magnitude making a neusis on the opposite angle; and given in position a circle, to fit (in the circle) a straight line given in magnitude making a neusis on a given (point).

(28) Of these, the one for one semicircle and a straight line is proved in the first volume, with four cases, and the one for the circle, with two cases, and the one for the rhombus, with two cases. In the second volume the one for the two semicircles is proved; its hypothesis has ten cases, and in these there are numerous subdivisions with diorisms as a consequence of

(8) καὶ ἐὰν τοῦτο μὲν τὸ σημεῖον ἅπτηται θέσει δεδομένης εὐθείας, ὁ δὲ κύκλος μὴ ὑπόκειται, τὰ ἐφ' ἑκάτερα τοῦ δεδομένου σημεῖα ἅψεται θέσει δεδομένης περιφερείας τῆς αὐτῆς.

ἔχει δὲ τὰ Τόπων Ἐπιπέδων δύο βιβλία θεωρήματα ἤτοι διαγράμματα ρμζ, λήμματα δὲ η̄. 670

(27) ΝΕΥΣΕΩΝ ΔΥΟ

νεύειν λέγεται γραμμὴ ἐπὶ σημεῖον ἐὰν ἐπεκβαλλομένη ἐπ' αὐτὸ παραγίνηται. καθόλου δὲ τὸ αὐτό ἐστιν ἐάν τε ἐπὶ δοθὲν νεύειν σημεῖον λέγηται, ἐάν τέ ἐστίν τι ἐπ' αὐτῆς δοθέν, ἐάν τε διὰ δοθέντος ἐστὶν σημείου. ἐπέγραψαν δὲ ταῦτα νεύσεις ἀπὸ ἑνὸς τῶν εἰρημένων, προβλήματος δὲ ὄντος καθολικοῦ τούτου· δύο δοθεισῶν γραμμῶν θέσει, θεῖναι μεταξὺ τούτων εὐθεῖαν τῶι μεγέθει δεδομένην, νεύουσαν ἐπὶ δοθὲν σημεῖον. ἐπὶ ταύτης τῶν ἐπὶ μέρους διάφορα τὰ ὑποκείμενα ἐχόντων ἃ μὲν ἦν ἐπίπεδα, ἃ δὲ στερεά, ἃ δὲ γραμμικά. τῶν <δ'> ἐπιπέδων ἀποκληρώσαντες τὰ πρὸς πολλὰ χρησιμώτερα, ἔδειξαν τὰ προβλήματα ταῦτα· θέσει δεδομένων ἡμικυκλίου τε καὶ εὐθείας πρὸς ὀρθὰς τῆι βάσει, ἢ δύο ἡμικυκλίων ἐπ' εὐθείας ἐχόντων τὰς βάσεις, θεῖναι δοθεῖσαν τῶι μεγέθει εὐθεῖαν μεταξὺ τῶν δύο γραμμῶν, |νεύουσαν ἐπὶ γωνίαν ἡμικυκλίου. καὶ ῥόμβου δοθέντος καὶ ἐπεκβεβλημένης μιᾶς πλευρᾶς ἁρμόσαι ὑπὸ τὴν ἐκτὸς γωνίαν δεδομένην τῶι μεγέθει εὐθεῖαν νεύουσαν ἐπὶ τὴν ἀντικρὺς γωνίαν. καὶ θέσει δοθέντος κύκλου ἐναρμόσαι εὐθεῖαν μεγέθει δεδομένην νεύουσαν ἐπὶ δοθέν. |124v

(28) τούτων δὲ ἐν μὲν τῶι πρώτωι τεύχει δέδεικται τὸ ἐπὶ τοῦ ἑνὸς ἡμικυκλίου καὶ εὐθείας ἔχον πτώσεις δ̄, καὶ τὸ ἐπὶ τοῦ κύκλου ἔχον πτώσεις δύο, καὶ τὸ ἐπὶ τοῦ ῥόμβου πτώσεις ἔχον β̄. ἐν δὲ τῶι δευτέρωι τεύχει <τὸ> ἐπὶ τῶν δύο ἡμικυκλίων, τῆς ὑποθέσεως πτώσεις ἐχούσης ῑ, ἐν δὲ ταύταις

‖ 1 τὸ om Ha ‖ 2 ἑκατέραι Ha ‖ 3 σημεῖα Ha σημείου A ‖ 5 τόπων Ha τόπον A ‖ 8 γραμμὴ Hu γραμμὴν A ‖ 9 παραγίνηται Ha παραγίνεται A | καθόλου — εἰρημένων secl Hu ‖ 11 σημείου Ha σημεῖον A ‖ 15 ταύτης] τούτου Horsley ‖ 16 ἦν secl Hu ‖ 17 δ' add Ha ‖ 18 χρησιμώτερα Ha χρησιμωτέραν A | τὰ Hu τε A om Ha ‖ 23 ἐπεκβεβλημένης Hu ἐπεμβλημένης A ἐπεκβλημένης Ha | post ἐπεκβεβλημένης add μόνης Ha ‖ 28 εὐθείας Ha εὐθεῖαν A | δ̄] πέντε Horsley ‖ 30 τὸ add Ha the given magnitude of the straight line.

(29) These are the plane things in the *Domain of Analysis*, which are the earlier ones to be proved, excepting the means of Eratosthenes; these come last. The order calls for the examination of the solid ones next after the plane (problems). One calls problems solid, not that pertain to solid figures, but that cannot be demonstrated by means of the plane (figures), but are demonstrated through the three conic curves, so that it is necessary to write first about these. Five volumes of conic elements by Aristaeus the elder were passed down earlier, written rather concisely for their recipients as if they were already competent.

The two books of *Neuses* contain 125 theorems or diagrams, and 38 lemmas.

(30) Eight (Books) of Conics:

Apollonius, filled out Euclid's four books of *Conics* and added on another four, handing down eight volumes of *Conics*. Aristaeus, who wrote the five volumes of *Solid Loci,* which have been transmitted until the present immediately following the *Conics,* and Apollonius's (other) predecessors, named the first of the three conic curves 'section of an acute-angled cone', the second 'of a right-angled', the third 'of an obtuse-angled'. But since the three curves occur in each of these three cones, when cut variously, Apollonius was apparently at a loss to know why on earth his predecessors selectively named the one 'section of an acute-angled cone' when it can also be (a section) of a right-angled and obtuse-angled one, the second (cone), and the third 'of an obtuse-angled' when it can be of an

ὑποδιαιρέσεις πλείονες διοριστικαὶ ἕνεκα τοῦ δεδομένου 672
μεγέθους τῆς εὐθείας.

(29) τὰ μὲν οὖν ἐν τῶι Ἀναλυομένωι Τόπωι ἐπίπεδα ταῦτ' ἔστιν ἃ καὶ πρότερα δείκνυται, χωρὶς τῶν Ἐρατοσθένους μεσοτήτων (ὕστατα γὰρ ἐκεῖνα), τοῖς δὲ ἐπιπέδοις ἐφεξῆς τὴν τῶν στερεῶν ἡ τάξις ἀπαιτεῖ θεωρίαν. στερεὰ δὲ καλοῦσι προβλήματα οὐχ ὅσα ἐν στερεοῖς σχήμασιν προτείνεται ἀλλ' ὅσα διὰ τῶν ἐπιπέδων μὴ δυνάμενα δειχθῆναι, διὰ τῶν τριῶν κωνικῶν γραμμῶν δείκνυται, ὥστε ἀναγκαῖον πρότερον περὶ τούτων γράφειν. ἦν μὲν οὖν ἀναδεδόμενα κωνικῶν στοιχείων πρότερον Ἀρισταίου τοῦ πρεσβυτέρου ε̄ τεύχη, ὡς ἂν ἤδη δυνατοῖς οὖσι τοῖς ταῦτα παραλαμβάνουσιν ἐπιτομώτερον γεγραμμένα.

ἔχει δὲ τὰ τῶν νεύσεων βιβλία δύο θεωρήματα μὲν ἤτοι διαγράμματα ρκε̄, λήμματα δὲ λη̄.

(30) ΚΩΝΙΚΩΝ Η̄

τὰ Εὐκλείδου βιβλία δ̄ Κωνικῶν Ἀπολλώνιος ἀναπληρώσας καὶ προσθεὶς ἕτερα δ̄ παρέδωκεν η̄ Κωνικῶν τεύχη. Ἀρισταῖος δέ, ὃς γέγραφε τὰ μέχρι τοῦ νῦν ἀναδιδόμενα στερεῶν τόπων τεύχη ε̄ συνεχῆ τοῖς Κωνικοῖς, ἐκάλει καὶ οἱ πρὸ Ἀπολλωνίου τῶν τριῶν κωνικῶν γραμμῶν τὴν μὲν ὀξυγωνίου, τὴν δὲ ὀρθογωνίου, τὴν δὲ ἀμβλυγωνίου κώνου τομήν. ἐπεὶ δ' ἐν ἑκάστωι τῶν τριῶν τούτων κώνων διαφόρως τεμνομένων αἱ γ̄ γίνονται γραμμαί, διαπορήσας ὡς φαίνεται Ἀπολλώνιος τί δήποτε ἀποκληρώσαντες οἱ πρὸ αὐτοῦ ἣν μὲν
ἐκάλουν ὀξυγωνίου κώνου τομὴν δυναμένην καὶ ὀρθογωνίου 674

‖ 3 τὰ μὲν – ἐπιτομώτερον γεγραμμένα secl Hu | ἐπίπεδα Ha ἐπιπέδωι A | τοῦτ' Ha ‖ 4 δεύκνυνται Ha ‖ 6 τάξις Ha ταξεις A ‖ 10 ἀναδεδόμενα Hu ἀναδιδομένων A ‖ 11 ὡς – παραλαμβάνουσιν] ὡς ἂν τοῖς ἤδη δυνατοῖς οὖσι ταῦτα παραλαμβάνειν Ha ὡς ἂν ἤδη δυνατοῖς οὖσι τὰ τοιαῦτα παραλαμβάνειν Hu app ‖ 14 δύο βιβλία transp Hu app | μὲν om Ha ‖ 17 ἀναπληρώσας Ge ἀναπλώσας A ‖ 19 ἀρισταῖος Ha ἀρισταιας A | γέγραφε Hu γραφει A ἔγραψε Ge | τὰ Ha καὶ A ‖ 20 καὶ – ἀπολλωνίου secl Hu ‖ 23 ἐπειδὴ ἐν Ha | κώνων Ha κωνικῶν A ‖ 24 τεμνουμένων Ha ‖ 25 ἀποκληρώσαντο Ha acute-angled and a right-angled (cone), so, replacing the names, he called the (section) of an acute-angled (cone) 'ellipse', that of a right-angled 'parabola', and that of an obtuse-angled 'hyperbola', each from a certain property of its own. For a certain area applied to a certain line, in the section of an acute-angled cone, falls short by a square, in that of an obtuse-angled (cone) exceeds by a square, but in that of a right-angled (cone) neither falls short nor exceeds.

(31) This was his notion because he did not perceive that by a certain single way of having the plane cut the cone in generating the curves, a different one of the curves is produced in each of the cones, and they named it from the property of the cone. For if the cutting plane is drawn parallel to one side of the cone, one only of the three curves is formed, always the same one, which Aristaeus named a section of the (kind of) cone that was cut.

(32) In any event, Apollonius says what the eight books of *Conics* that he wrote contain, placing a summary prospectus in the preface to the first, as follows: "The first contains the generation of the three sections and the opposite branches, and their fundamental *symptomata*, more fully and more thoroughly examined than in the writings of others. The second (has) the properties of the diameters and axes of the sections and opposite branches, the asymptotes, and other things that have pregnant and cogent application in diorisms. From this book you will learn what it is that I call diameters, and what axes. The third (has) many and various useful things,

καὶ ἀμβλυγω|νίου εἶναι, ἣν δὲ ὀρθογωνίου εἶναι δυναμένην |125
ὀξυγωνίου τε καὶ ἀμβλυγωνίου, ἣν δὲ ἀμβλυγωνίου δυναμένην εἶναι ὀξυγωνίου τε καὶ ὀρθογωνίου, μεταθεὶς τὰ ὀνόματα καλεῖ τὴν μὲν ὀξυγωνίου καλουμένην ἔλλειψιν, τὴν δὲ ὀρθογωνίου παραβολήν, τὴν δὲ ἀμβλυγωνίου ὑπερβολήν, ἑκάστην δ' ἀπό τινος ἰδίου συμβεβηκότος. χωρίον γάρ τι παρά τινα γραμμὴν παραβαλλόμενον ἐν μὲν τῆι ὀξυγωνίου κώνου τομῆι ἐλλεῖπον γίνεται τετραγώνωι, ἐν δὲ τῆι ἀμβλυγωνίου ὑπερβάλλον τετραγώνωι, ἐν δὲ τῆι ὀρθογωνίου οὔτε ἐλλεῖπον οὔθ' ὑπερβάλλον.

(31) τοῦτο δ' ἔπαθεν μὴ προσνοήσας ὅτι κατά τινα μίαν πτῶσιν τοῦ τέμνοντος ἐπιπέδου τὸν κῶνον, καὶ γεννῶντος τρεῖς γραμμάς, ἐν ἑκάστωι τῶν κώνων ἄλλη καὶ ἄλλη τῶν γραμμῶν γίνεται, ἣν ὠνόμασαν ἀπὸ τῆς ἰδιότητος τοῦ κώνου. ἐὰν γὰρ τὸ τέμνον ἐπίπεδον ἀχθῆι παράλληλον μιᾶι τοῦ κώνου πλευρᾶι, γίνεται μία μόνη τῶν τριῶν γραμμῶν, ἀεὶ ἡ αὐτή, ἣν ὠνόμασεν ὁ Ἀρισταῖος ἐκείνου τοῦ τμηθέντος κώνου τομήν.

(32) ὁ δ' οὖν Ἀπολλώνιος οἷα περιέχει τὰ ὑπ' αὐτοῦ γραφέντα κωνικῶν η̅ βιβλία λέγει κεφαλαιώδη θεὶς προδήλωσιν ἐν τῶι προοιμίωι τοῦ πρώτου ταύτην· "περιέχει δὲ τὸ μὲν πρῶτον τὰς γενέσεις τῶν τριῶν τομῶν καὶ τὰς ἀντικειμένας καὶ τὰ ἐν αὐταῖς ἀρχικὰ συμπτώματα ἐπὶ πλεῖον <καὶ> καθόλου μᾶλλον ἐξητασμένα παρὰ τὰ ὑπὸ τῶν ἄλλων γεγραμμένα. τὸ δὲ δεύτερον τὰ περὶ τὰς διαμέτρους καὶ τοὺς ἄξονας τῶν τομῶν καὶ τῶν ἀντικειμένων
συμβαίνοντα καὶ τὰς ἀσυμπτώτους καὶ ἄλλα γενικὴν καὶ 676
ἀναγκαίαν χρείαν παρεχόμενα πρὸς τοὺς διορισμούς. τίνας δὲ διαμέτρους ἢ τίνας ἄξονας καλῶ εἰδήσεις ἐκ τούτου τοῦ

‖ 3 ὀξυγωνίου Ha ὀξυγώνιόν A ‖ 6 δ' del Hu (SV) γ' coni. Hu app ‖ 11 τοῦτο — κώνου τομήν secl Hu | προσνοήσας A προνοήσας Ha προσεννοήσας Hu | μίαν] ἰδίαν Hu ‖ 12 ἐπιπέδου τέμνοντος transp Ha | καὶ — κώνων om Ha | post γεννῶντος add τὰς Heiberg$_2$ ‖ 13 ἄλλη καὶ ἄλλη Ha ἄλλην καὶ ἄλλην A ‖ 14 ὠνόμασεν Hu ‖ 16 μόνη Ha μόνηι A ‖ 17 ἐκεῖνος Ha ‖ 18 ὁ γοῦν Hu app ‖ 21 τὰς ἀντικειμένας] τῶν ἀντικειμένων Ha ex Apollonio ‖ 23 καὶ add Ha ex Apoll | ἐξειργασμένα Apoll ‖ 25 καὶ τῶν ἀντικειμένων secl Hu ex Apoll ‖ 26 ἄλλα γενικὴν Ha ex Apoll ἄλλας ενικην A which are both for syntheses of solid loci, and for (their) diorisms; and having found most of them both elegant and novel, we found that the synthesis of the locus on three and four lines was not made by Euclid, but (merely) a fragment of it, nor this felicitously. For one cannot complete the synthesis without the things mentioned above. The fourth (has) the number of times that conic sections intersect each other and an arc of a circle, and in addition in how many points a section of a cone or an arc of a circle meets (opposite branches), and in how many points opposite branches meet opposite branches, neither of these having been put in writing by our predecessors. The remaining four are more in the manner of supplements. Thus the first is on minima and maxima at length, the next on equal and similar sections, the next on theorems pertaining to diorisms, the next on conic problems subjected to diorism."

(33) Thus Apollonius. The locus on three and four lines that he says, in (his account of) the third (book), was not completed by Euclid, neither he nor anyone else would have been capable of; no, he could not have added the slightest thing to what was written by Euclid, using only the conics that had been proved up to Euclid's time, as he himself confesses when he says that it is impossible to complete it without what he was forced to write first. **(34)** But either Euclid, out of respect for Aristaeus as meritorious for the conics he had published already, did not anticipate him, or, because he did not desire to commit to writing the same matter as he (Aristaeus), — for he was the fairest of men, and kindly to everyone who was the slightest bit able to augment knowledge, as one should be, and he was not at all belligerent, and though exacting, not boastful, the way this man

βιβλίου. τὸ δὲ τρίτον πολλὰ καὶ παντοῖα χρήσιμα τὰ πρός τε τὰς συνθέσεις τῶν στερεῶν τόπων καὶ τοὺς διορισμοὺς ὧν τὰ πλείονα καὶ καλὰ καὶ ξένα κατανοήσαντες εὕρομεν μὴ συντιθέμενον ὑπὸ Εὐκλείδου *τὸν ἐπὶ τρεῖς καὶ δ̄ γραμμὰς τόπον ἀλλὰ μόριόν τι αὐτοῦ, καὶ τοῦτο οὐκ εὐτυχῶς· οὐ γὰρ δυνατὸν ἄνευ τῶν προειρημένων τελειωθῆναι τὴν σύνθεσιν. τὸ δὲ δʹ, ποσαχῶς αἱ τῶν κώνων τομαὶ ἀλλήλαις τε καὶ τῆι τοῦ κύκλου περιφέρειαι συμπίπτουσιν, καὶ ἐκ περισσοῦ, |ὧν* |125v
οὐδέτερον ὑπὸ τῶν πρὸ ἡμῶν γέγραπται, κώνου τομὴ <ἢ> κύκλου περιφέρεια κατὰ πόσα σημεῖα συμβάλλει καὶ ἀντικείμεναι ἀντικειμέναις κατὰ πόσα σημεῖα συμβάλλουσιν. τὰ δὲ λοιπὰ δ̄ περιουσιαστικώτερα. ἔστι γὰρ τὸ μὲν περὶ ἐλαχίστων καὶ μεγίστων [τῶν] ἐπὶ πλεῖον, τὸ δὲ περὶ ἴσων καὶ ὁμοίων τομῶν, τὸ δὲ διοριστικῶν θεωρημάτων, τὸ δὲ κωνικῶν προβλημάτων διωρισμένων."

(33) *Ἀπολλώνιος μὲν ταῦτα· ὃν δέ φησιν ἐν τῶι τρίτωι τόπον ἐπὶ γ̄ καὶ δ̄ γραμμὰς μὴ τετελειῶσθαι ὑπὸ* Εὐκλείδου, *οὐδ' ἂν αὐτὸς ἐδυνήθη οὐδ' ἄλλος οὐδεὶς ἀλλ' οὐδὲ μικρόν τι προσθεῖναι τοῖς ὑπὸ* Εὐκλείδου *γραφεῖσιν διά γε μόνων τῶν προδεδειγμένων ἤδη κωνικῶν ἄχρι τῶν κατ'* Εὐκλείδην, *ὡς καὶ αὐτὸς μαρτυρεῖ λέγων ἀδύνατον εἶναι τελειωθῆναι χωρὶς ὧν αὐτὸς προγράφειν ἠναγκάσθη·* (34) *ὁ δὲ Εὐκλείδης ἀποδεχόμενος τὸν Ἀρισταῖον ἀξιωθέντα ἐφ' οἷς ἤδη παρεδεδώκει κωνικοῖς καὶ μὴ φθάσας, ἢ μὴ θελήσας ἐπικαταβάλλεσθαι τούτωι τὴν αὐτὴν πραγματείαν, ἐπιεικέστατος ὢν καὶ πρὸς ἅπαντας εὐμενὴς τοὺς καὶ κατὰ* 678
ποσὸν συναύξειν δυναμένους τὰ μαθήματα ὡς δεῖ καὶ μηδαμῶς προσκρουστικὸς ὑπάρχων, καὶ ἀκριβὴς μὲν οὐκ ἀλαζονικὸς δὲ

‖ 1 *πολλὰ καὶ παράδοξα θεωρήματα χρήσιμα πρός τε* Apoll | post *παντοῖα* add *θεωρήματα* Ha ex Apoll | *τὰ* om Ha ‖ 2 *ὧν τὰ πλεῖστα καλὰ καὶ ξένα. ἃ καὶ κατανοήσαντες συνείδομεν* Apoll ‖ 3 *καὶ* (post *πλείονα*) del Ha ex Apoll | post *ξένα* add *ἃ καὶ* Ha ex Apoll ‖ 5 *τι*] *τὸ τύχον* Apoll ‖ 6 *τῶν προειρημένων*] *τῶν προσευρημένων ἡμῖν* Apoll ‖ 8 *συμβάλλουσι* Apoll | *καὶ ἄλλα ἐκ* Ha ex Apoll ‖ 9 *πρὸ* Ha *πρὸς* A | *ἢ κύκλου περιφέρεια καὶ ἔτι ἀντικείμεναι* Apoll | *ἢ* add Ha ex Apoll ‖ 10 *περιφέρεια*] *περιφερείαι* Ge | *κατὰ* – *συμβάλλει* secl Hu | ante *ἀντικείμεναι* add *ἔτι* Ha ex Apoll ‖ 12 *περιουσιαστικώτερα* Ha ex Apoll *περιους αστικωτερα* A ‖ 13 *τῶν* del Ha ‖ 14 *τομῶν κώνου· τὸ δὲ περὶ διοριστικῶν* Apoll ‖ 15 *προβλημάτων κωνικῶν* Apoll ‖ 17 *τελειωθῆναι* Ha ‖ 18 *οὔτ' ἂν* – *οὔτ'* Ha | *ἐδυνήθη* Ha *ἠδυνήθη* A | *ἀλλ'* – *γραφεῖσιν* secl Hu ‖ 22 *ὁ δὲ εὐκλείδης* – *τοιοῦτος ἐστιν* secl Hu ‖ 23 *ἀρισταῖον* Ha *ἀριστέα* A | *ἀξιωθέντα* Hu p. 1258 *ἄξιον ὄντα* A ‖ 24 *παρεδεδώκει* Ge *παραδεδώκει* A ‖ 25 *τούτωι* Hu app *τούτων* A

(Apollonius) was, – he wrote (only) as far as it was possible to demonstrate the locus by means of the other's *Conics,* without saying that the demonstration was complete. For had he done so, one would have had to convict him, but as things stand, not at all. And in any case, (Apollonius) himself is not castigated for leaving most things incomplete in his *Conics.* **(35)** He was able to add the missing part to the locus because he had Euclid's writings on the locus already before him in his mind, and had studied for a long time in Alexandria under the people who had been taught by Euclid, where he also acquired this so great condition (of mind), which was not without defect.

This locus on three and four lines that he boasts of having augmented instead of acknowledging his indebtedness to the first to have written on it, is like this:

(36) If three straight lines are given in position, and from some single point straight lines are drawn onto the three at given angles, and the ratio of the rectangle contained by two of the (lines) drawn onto (them) to the square of the remaining one is given, the point will touch a solid locus given in position, that is, one of the three conic curves. And if (straight lines) are drawn at given angles onto four straight lines given in positions, and the ratio of the (rectangle contained) by two of the (lines) that were drawn to the (rectangle contained) by the other two that were drawn is given, likewise the point will touch a section of a cone given in position.

(37) Now if (they are drawn) onto only two (lines), the locus has been proved to be plane, but if onto more than four, the point will touch loci that are as yet unknown, but just called 'curves', and whose origins and properties are not yet (known). They have given a synthesis of not one, not even the first and seemingly the most obvious of them, or shown it to be useful. **(38)** The propositions of these (loci) are: If straight lines are drawn from some point at given angles onto five straight lines given in position, and the ratio is given of the rectangular parallelepiped solid contained by three of the (lines) that were drawn to the rectangular parallelepiped solid contained by the remaining two (lines) that were drawn and some given, *καθάπερ οὗτος, ὅσον δυνατὸν ἦν δεῖξαι τοῦ τόπου διὰ τῶν ἐκείνου Κωνικῶν ἔγραψεν, οὐκ εἰπὼν τέλος ἔχειν τὸ δεικνύμενον. τότε γὰρ ἦν ἀναγκαῖον ἐξελέγχειν, νῦν δ' οὐδαμῶς, ἐπείτοι καὶ αὐτὸς ἐν τοῖς Κωνικοῖς ἀτελῆ τὰ πλεῖστα καταλιπὼν οὐκ εὐθύνεται.*

(35) *προσθεῖναι δὲ τῶι τόπωι τὰ λειπόμενα δεδύνηται προφαντασιωθεὶς τοῖς ὑπὸ Εὐκλείδου γεγραμμένοις ἤδη περὶ τοῦ τόπου καὶ σχολάσας τοῖς [ὑπὸ] Εὐκλείδου μαθηταῖς ἐν Ἀλεξανδρείαι πλεῖστον χρόνον ὅθεν ἔσχεν καὶ τὴν τοσαύτην ἕξιν οὐκ ἀπαθῆ. οὗτος δὲ ὁ ἐπὶ γ̄ καὶ δ̄ γραμμὰς τόπος ἐφ' ὧι μέγα φρονεῖ προσθεὶς χάριν ὀφείλων εἰδέναι τῶι πρώτωι γράψαντι τοιοῦτός ἐστιν.*

(36) *ἐὰν γὰρ θέσει δεδομένων τριῶν εὐθειῶν ἀπό τινος τοῦ αὐτοῦ σημείου καταχθῶσιν ἐπὶ τὰς τρεῖς ἐν δεδομέναις γωνίαις εὐθεῖαι, καὶ λόγος ἧι δοθεὶς τοῦ ὑπὸ δύο κατηγμένων περιεχομένου ὀρθογωνίου πρὸς τὸ ἀπὸ τῆς λοιπῆς τετράγωνον, τὸ σημεῖον ἅψεται θέσει δεδομένου |στερεοῦ* |126 *τόπου τουτέστιν μιᾶς τῶν τριῶν κωνικῶν γραμμῶν. καὶ ἐὰν ἐπὶ δ̄ εὐθείας θέσει δεδομένας καταχθῶσιν ἐν δεδομέναις γωνίαις καὶ λόγος ἧι δοθεὶς τοῦ ὑπὸ δύο κατηγμένων πρὸς τὸ ὑπὸ τῶν λοιπῶν δύο κατηγμένων, ὁμοίως τὸ σημεῖον ἅψεται θέσει δεδομένης κώνου τομῆς.*

(37) *ἐὰν μὲν γὰρ ἐπὶ δύο μόνας, ἐπίπεδος ὁ τόπος δέδεικται, ἐὰν δὲ ἐπὶ πλείονας τεσσάρων, ἅψεται τὸ σημεῖον τόπων οὐκέτι γνωρίμων, ἀλλὰ γραμμῶν μόνον λεγομένων, ποδαπῶν δὲ ἢ τινα ἐχουσῶν ἴδια οὐκέτι, ὧν οὐδεμίαν οὐδὲ* 680 *τὴν πρώτην καὶ συμφανεστάτην εἶναι δοκοῦσαν συντεθείκασιν ἀναδείξαντες χρησίμην οὖσαν.*

(38) *αἱ δὲ προτάσεις αὐτῶν εἰσιν· ἐὰν ἀπό τινος σημείου ἐπὶ θέσει δεδομένας εὐθείας πέντε καταχθῶσιν εὐθεῖαι ἐν δεδομέναις γωνίαις καὶ λόγος ἧι δεδομένος τοῦ ὑπὸ τριῶν κατηγμένων περιεχομένου στερεοῦ παραλληλεπιπέδου ὀρθογωνίου πρὸς τὸ ὑπὸ τῶν λοιπῶν δύο κατηγμένων καὶ δοθείσης τινὸς περιεχόμενον παραλληλεπίπεδον ὀρθογώνιον,*

‖ 4 *ἐπείτοι* Ha *ἐπίτοι* A ‖ 8 *συσχολάσας* Hu | *ὑπὸ* ⟨*Εὐκλείδου*⟩ del Heiberg$_1$ *ὑπ' εὐκλείδηι* Hu app ‖ 9 *τοιαύτην* Hu ‖ 10 *οὐκ ἀπαθῆ*] *οὐκανπαθη* A *οὐκ ἀμαθῆ* Friedlein *εἰκαιοπαθῆ* Hu$_1$ ‖ 11 *ὀφείλειν* Hu ‖ 13 *τοῦ αὐτοῦ* secl Hu ‖ 17 *ἅψεται* Ha *ἅπτεται* A ‖ 19 post *καταχθῶσιν* add *εὐθεῖαι* Ha ‖ 21 *ἅψεται* Ha *ἅπτεσθαι* A ‖ 23 *ἐὰν* – *δέδεικται* secl Hu ‖ 26 *ποδαπῶν* – *οὐκέτι* secl Hu | *οὐδεμίαν*] *μίαν* A | *οὐδὲ τὴν πρώτην καὶ*] *οὐδέ τινα* Hu the point will touch a curve given in position. And if onto six, and the ratio of the aforesaid solid contained by the three to that by the remaining three is given, again the point will touch a (curve) given in position. If onto more than six, one can no longer say "the ratio is given of the something contained by four to that by the rest", since there is nothing contained by more than three dimensions.

(39) Our immediate predecessors have allowed themselves to admit meaning to such things, though they express nothing at all coherent when they say "the (thing contained) by these", referring to the square of this (line) or the (rectangle contained) by these. But it was possible to enunciate and generally to prove these things by means of compound ratios, both for the propositions given above, and for the present ones, in this way:

(40) If straight lines are drawn from some point at given angles onto straight lines given in position, and there is given the ratio compounded of that which one drawn line has to one, and another to another, and a different one to a different one, and the remaining one to a given, if there are seven, but if eight, the remaining to the remaining one, the point will touch a curve given in position. And similarly for however many, even or odd in number. As I said, of not one of these that come after the locus on four lines have they made a synthesis so that they know the curve.

(41) They who look at these things are hardly exalted, as were the ancients and all who wrote the finer things. When I see everyone occupied with the rudiments of mathematics and of the material for inquiries that nature sets before us, I am ashamed; I for one have proved things that are much more valuable and offer much application. In order not to end my discourse declaiming this with empty hands, I will give this for the benefit of the readers:

(42) The ratio of solids of complete revolution is compounded of (that) of the revolved figures and (that) of the straight lines similarly drawn to the axes from the centers of gravity in them; that of (solids of) incomplete ἅψεται τὸ σημεῖον θέσει δεδομένης γραμμῆς. ἐὰν δὲ ἐπὶ ϛ̄ καὶ λόγος ἦι δοθεὶς τοῦ ὑπὸ τῶν τριῶν περιεχομένου καὶ εἰρημένου στερεοῦ πρὸς τὸ ὑπὸ τῶν λοιπῶν τριῶν, πάλιν τὸ σημεῖον ἅψεται θέσει δεδομένης. ἐὰν δὲ ἐπὶ πλείονας τῶν ϛ̄ οὐκέτι μὲν ἔχουσι λέγειν λόγος ἦι δοθεὶς τοῦ ὑπὸ τῶν δ̄ περιεχομένου τινὸς πρὸς τὸ ὑπὸ τῶν λοιπῶν, ἐπεὶ οὐκ ἔστιν τι περιεχόμενον ὑπὸ πλειόνων ἢ τριῶν διαστάσεων.

(39) συγκεχωρήκασι δὲ ἑαυτοῖς οἱ βραχὺ πρὸ ἡμῶν ἑρμηνεύειν τὰ τοιαῦτα, μηδὲ ἓν μηδαμῶς διάληπτον σημαίνοντες τὸ ὑπὸ τῶνδε περιεχόμενον λέγοντες ἐπὶ τὸ ἀπὸ τῆσδε τετράγωνον ἢ ἐπὶ τὸ ὑπὸ τῶνδε. παρῆν δὲ διὰ τῶν συνημμένων λόγων ταῦτα καὶ λέγειν καὶ δεικνύναι καθόλου καὶ ἐπὶ τῶν προειρημένων προτάσεων καὶ ἐπὶ τούτων τὸν τρόπον τοῦτον.

(40) ἐὰν ἀπό τινος σημείου ἐπὶ θέσει δεδομένας εὐθείας καταχθῶσιν εὐθεῖαι ἐν δεδομέναις γωνίαις καὶ δεδομένος ἦι λόγος ὁ συνημμένος ἐξ οὗ ἔχει μία κατηγμένη πρὸς μίαν καὶ ἑτέρα πρὸς ἑτέραν καὶ ἄλλη πρὸς ἄλλην καὶ ἡ λοιπὴ πρὸς δοθεῖσαν, ἐὰν ὦσιν ζ̄, ἐὰν δὲ η̄ καὶ ἡ λοιπὴ πρὸς λοιπήν, τὸ σημεῖον ἅψεται θέσει δεδομένης γραμμῆς. καὶ ὁμοίως ὅσαι ἂν |ὦσιν περισσαὶ ἢ ἄρτιαι τὸ πλῆθος. τούτων ὡς ἔφην |126v ἑπομένων τῶι ἐπὶ τέσσαρας τόπωι οὐδὲ ἓν συντεθείκασιν ὥστε τὴν γραμμὴν εἰδέναι.

(41) ταῦθ᾽ οἱ βλέποντες ἥκιστα ἐπαίρονται, καθάπερ οἱ 682 πάλαι καὶ τῶν τὰ κρείττονα γραψάντων ἕκαστοι. ἐγὼ δὲ καὶ πρὸς ἀρχαῖς ἐπὶ τῶν μαθημάτων καὶ τῆς ὑπὸ φύσεως προκειμένης ζητημάτων ὕλης κινουμένους ὁρῶν ἅπαντας, αἰδούμενος ἔχω καὶ δείξας γε πολλῶι κρείσσονα καὶ πολλὴν προσφερόμενα ὠφέλειαν. ἵνα δὲ μὴ κεναῖς χερσὶ τοῦτο φθεγξάμενος ὧδε χωρισθῶ τοῦ λόγου, ταῦτα δώσω τοῖς ἀναγνοῦσιν.

(42) ὁ μὲν τῶν τελείων ἀμφοιστικῶν λόγος συνῆπται ἔκ τε τῶν ἀμφοισμάτων καὶ τῶν ἐπὶ τοὺς ἄξονας ὁμοίως κατηγμένων εὐθειῶν ἀπὸ τῶν ἐν αὐτοῖς κεντροβαρικῶν σημείων, ὁ δὲ τῶν

‖ 1 ἐάν τε Ha ‖ 2 καὶ εἰρημένου] προειρημένου Hu ‖ 4 ἐάν τε Ha ‖ 5 post λέγειν add ἐὰν Hu | λόγον Ge ‖ 8 δ᾽ ἓν Hu app ‖ 10 τῶν δ̄ Ha ‖ 11 τῶν δ̄ Ha ‖ 15 εὐθείας om Ha ‖ 16 δεδομέναις — καὶ bis A del Ha | post ἦι add ὁ Ha ‖ 17 κατηγμένη Ha κατηγμένην A | post μίαν add κατηγμένην Ha ‖ 21 ἄρτιαι Hu (CSV) αἴτιαι A ἄρτιοι Ha ‖ 22 συντεθείκασιν Hu τεθείκασιν A_1 οὖν A_2 supr ‖ 24 ταῦθ᾽ — στοιχείων secl Hu | τοῦθ᾽ Ha | πειρῶνται Hu app ‖ 25 πάλαι καὶ] παλαιοὶ Hu p. 1258 | ἕκαστοι Hu ἕκαστον A ἕκαστος Ha ‖ 26 ἐπὶ] ἔτι Hu | καὶ τῆς — ζητημάτων om A^1 add mg A^2 alia manu ‖ 28 ἔχω] ἐγὼ A | πολλῶι Ha πολλῶν A ‖ 29 προσφερόμενα Hu p. 1258 προφερόμενα A | lacunam post ὠφέλειαν indicavit Ha ‖ 31 ἀγνοοῦσιν Ha ‖ 32 ἀμφοιστικῶν Ha ἀμφοῖν στίχων A (revolution) from (that) of the revolved figures and (that) of the arcs that the centers of gravity in them describe, where the (ratio) of these arcs is, of course, (compounded) of (that) of the (lines) drawn and (that) of the angles of revolution that their extremities contain, if these (lines) are also at <right angles> to the axes. These propositions, which are practically a single one, contain many theorems of all kinds, for curves and surfaces and solids, all at once and by one proof, things not yet and things already demonstrated, such as those in the twelfth book of the *First Elements*.

The eight books of Apollonius' *Conics* contain 487 theorems or diagrams, and there are 70 lemmas, or things assumed in it.

ἀτελῶν ἔκ τε τῶν ἀμφοισμάτων καὶ τῶν περιφερειῶν, ὅσας ἐποίησεν τὰ ἐν τούτοις κεντροβαρικὰ σημεῖα, ὁ δὲ τούτων τῶν περιφερειῶν δῆλον ὡς ἔκ τε τῶν κατηγμένων καὶ ὧν περιέχουσιν αἱ τούτων ἄκραι, εἰ καὶ εἶεν πρὸς <ὀρθὰς> τοῖς ἄξοσιν, ἀμφοιστικῶν γωνιῶν. περιέχουσι δὲ αὗται αἱ προτάσεις, σχεδὸν οὖσαι μία, πλεῖστα ὅσα καὶ παντοῖα θεωρήματα γραμμῶν τε καὶ ἐπιφανειῶν καὶ στερεῶν, πανθ' ἅμα καὶ μιᾶι δείξει καὶ τὰ μήπω δεδειγμένα καὶ τὰ ἤδη ὡς καὶ τὰ ἐν τῶι δωδεκάτωι τῶν Πρώτων Στοιχείων.

ἔχει δὲ τὰ η̄ βιβλία τῶν Ἀπολλωνίου Κωνικῶν θεωρήματα ἤτοι διαγράμματα υπ̄ζ, λήμματα δὲ ἤτοι λαμβανόμενά ἐστιν εἰς αὐτὰ ο̄.

‖ 1 ὅσας Ha ὅσα A ‖ 2 τούτοις] αὐτοῖς Ha ‖ 3 τῶν om Ha | post περιφερειῶν add λόγος συνῆπται Hu | ἔκ Ha εἴς A ‖ 5 περιέχουσι δὲ αὗται Ha περιέχουσαι δε ταύτῃ A ‖ 8 μὴ προδεδειγμένα Ha | ἤδη ὡς Ha ηδεως A ‖ 9 τῶν πρώτων] τῶνδε τῶν A τῶνδε om Ha ‖ 10 η̄ Ha ε̄ A | ἀπολλωνίου Ha ἀπολλωνίωι A ‖ 11 ἤτοι – αὐτὰ secl Hu

(43) (Cutting off of a Ratio, Cutting off of an Area)

1. *(Prop. 1)* To divide a given straight line in a given ratio. Let the given straight line be AB, the given ratio Γ to Δ, and let it be required to cut AB into the ratio Γ to Δ. I inclined AE to line AB at an arbitrary angle, and removed AZ equal to Γ, and ZH equal to Δ. Joining BH, I drew ZΘ parallel to it.

Then since as is AΘ to ΘB, so is AZ to ZH (VI 2),[1] while AZ equals Γ,[2] and ZH equals Δ,[3] therefore as is AΘ to ΘB, so is Γ to Δ.[4] Hence it is divided at point Θ. Q.E.D.

(44) 2. *(Prop. 2)* Given three straight lines AB, BΓ, Δ, to find, as is AB to BΓ, so some other (straight line) to Δ. I again inclined a straight line ΓΘ at an arbitrary angle, and set off ΓZ equal to Δ. I joined BZ and again drew HA parallel to it.

Then once more, as is AB to ΓB, so is HZ to ΓZ (VI 2),[1] that is, (HZ) to Δ.[2] Hence ZH has been found. Similarly too if the third (line) is given, we will find the fourth.

(45) 3. *(Prop. 3)* Let AB have to BΓ a greater ratio than has ΔE to EZ.[1] That also *componendo* AΓ has to ΓB a greater ratio than has ΔZ to ZE.

For as is AB to BΓ, so let some other thing H be made to EZ.[2] Then H has to EZ a greater ratio than has ΔE to EZ.[3] Hence H is greater than ΔE.[4] Let ΘE be made equal to it.[5] Then since as is AB to BΓ, so is ΘE to EZ,[6] therefore *componendo* as is AΓ to BΓ, so is ZΘ to ZE.[7] But ΘZ has to ZE, and hence also AΓ has to ΓB, a greater ratio than has ΔZ to ZE.[8] [9]

(46) 4. *(Prop. 4)* Now let AB have a lesser ratio to BΓ than ΔE has to EZ. That AΓ too has to ΓB a ratio less than ΔZ has to EZ.

For again, since AB has to BΓ a ratio less than has ΔE to EZ,[1] if I make, as AB to BΓ, so something else to EZ, it will be less than ΔE.[3] Let

(43) <α.´> τὴν δοθεῖσαν εὐθεῖαν εἰς τὸν δοθέντα λόγον 684 τεμεῖν. ἔστω ἡ μὲν δοθεῖσα εὐθεῖα ἡ ΑΒ, ὁ δὲ δοθεὶς λόγος ὁ Γ πρὸς Δ, καὶ δέον ἔστω τεμεῖν τὴν ΑΒ εἰς τὸν τῆς Γ πρὸς τὴν Δ λόγον. ἔκλινα πρὸς τὴν ΑΒ εὐθεῖαν ἐν γωνίαι τυχούσηι εὐθεῖαν τὴν ΑΕ, καὶ τῆι μὲν Γ ἴσην ἀφεῖλον τὴν ΑΖ, τῆι δὲ Δ τὴν ΖΗ. καὶ ἐπιζεύξας τὴν ΒΗ ταύτηι παράλληλον ἤγαγον τὴν ΖΘ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΑΘ πρὸς ΘΒ, οὕτως ἡ ΑΖ πρὸς ΖΗ, ἴση δέ ἐστιν ἡ μὲν ΑΖ τῆι Γ, ἡ δὲ ΖΗ τῆι Δ, ἔστιν ἄρα ὡς ἡ ΑΘ πρὸς ΘΒ, οὕτως ἡ Γ πρὸς τὴν Δ. διήιρηται ἄρα κατὰ τὸ Θ σημεῖον, ὅπερ:—

(44) β.´ τριῶν δοθεισῶν εὐθειῶν τῶν ΑΒ, ΒΓ, Δ, εὑρεῖν ὡς τὴν ΑΒ πρὸς |τὴν ΒΓ, οὕτως ἄλλην τινὰ πρὸς τὴν Δ. πάλιν |127 ἔκλινά τινα εὐθεῖαν τὴν ΓΘ ἐν τυχούσηι γωνίαι καὶ τῆι Δ ἴσην ἀπεθέμην τὴν ΓΖ. ἐπέζευξα τὴν ΒΖ ἧι καὶ αὐτῆι παράλληλον ἤγαγον τὴν ΗΑ. γίνεται οὖν πάλιν ὡς ἡ ΑΒ πρὸς τὴν ΓΒ, οὕτως ἡ ΗΖ πρὸς τὴν ΓΖ, τουτέστιν πρὸς τὴν Δ. εὕρηται ἄρα ἡ ΖΗ. ὁμοίως κἂν ἡ τρίτη δοθῆι, τὴν τετάρτην εὑρήσομεν.

(45) <γ.´> ἐχέτω τὸ ΑΒ πρὸς τὸ ΒΓ μείζονα λόγον ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ. ὅτι καὶ κατὰ σύνθεσιν τὸ ΑΓ πρὸς τὸ ΓΒ μείζονα λόγον ἔχει ἤπερ τὸ ΔΖ πρὸς τὸ ΖΕ. πεποιήσθω γὰρ ὡς τὸ ΑΒ πρὸς τὸ ΒΓ, οὕτως ἄλλο τι τὸ Η πρὸς τὸ ΕΖ. καὶ τὸ Η ἄρα πρὸς τὸ ΕΖ μείζονα λόγον ἔχει ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ. μεῖζον ἄρα ἐστὶν τὸ Η τοῦ ΔΕ. κείσθω αὐτῶι ἴσον τὸ ΘΕ. ἐπεὶ οὖν ἐστιν ὡς τὸ ΑΒ πρὸς τὸ ΒΓ, οὕτως τὸ ΘΕ πρὸς ΕΖ, συνθέντι ἄρα ἐστὶν ὡς τὸ ΑΓ πρὸς τὸ ΒΓ, οὕτως τὸ ΖΘ πρὸς τὸ 686 ΖΕ· τὸ δὲ ΘΖ πρὸς τὸ ΖΕ, καὶ τὸ ΑΓ ἄρα πρὸς τὸ ΓΒ μείζονα λόγον ἔχει ἤπερ τὸ ΔΖ πρὸς τὸ ΖΕ.

(46) <δ.´> πάλιν δὴ τὸ ΑΒ πρὸς τὸ ΒΓ ἐλάσσονα λόγον ἐχέτω ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ. ὅτι καὶ τὸ ΑΓ πρὸς τὸ ΓΒ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ΔΖ πρὸς τὸ ΕΖ. πάλιν γὰρ ἐπεὶ τὸ ΑΒ πρὸς τὸ ΒΓ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ, ἐὰν ποιῶ ὡς τὸ ΑΒ πρὸς τὸ ΒΓ οὕτως ἄλλο τι πρὸς τὸ ΕΖ, ἔσται

‖ 1 α´ add Ha ‖ 2 ὁ (Γ)] Ē vel potius Θ̄ A[1] rasendo in O mutatum A[2] | post ὁ add τῆς Ha ‖ 4 ἔκλινα Ha εκλεινα A ‖ 5 ΑΕ] ΑΗ Ha | ἴσην Ha ἴση A ‖ 11 β´ mg A ‖ 13 ἔκλινα Ha εκλεινα A | ΓΘ] ΓΕ Hu ΓΗ Ha ‖ 14 ἧι om Ha | αὐτῆι] ταύτηι Ha ‖ 19 γ´ add Ha ‖ 21 ΖΕ Co (k) ΖΙ A ‖ 25 ante ΕΖ add τὸ Hu (S) ‖ 27 ante καὶ τὸ ΑΓ add μείζονα λόγον ἔχει ἤπερ τὸ ΔΖ πρὸς ΕΖ Ha ‖ 29 δ´ add Ha it be EΘ.[2] Then as is AΓ to ΓB, so too is ΘZ to ZE.[4] But ΘZ has to ZE a lesser ratio than has ΔZ to ZE.[5] Hence AΓ has to ΓB a ratio less than ΔZ has to ZE.[6]

(47) 5. *(Prop. 5)* Now let AB have to BΓ a greater ratio than has ΔE to EZ.[1] That also *alternando* AB has a greater ratio to ΔE than has BΓ to EZ.

For, as is AB to BΓ, so let something else be made to EZ. It will obviously be greater than ΔE.[3] Let it be HE.[2] Then *alternando* as is AB to EH, so is BΓ to EZ.[4] But AB has to ΔE a greater ratio than AB has to EH,[5] that is than BΓ has to EZ. Hence AB has to ΔE a greater ratio than BΓ has to EZ.[6] Likewise if a lesser ratio is given, that also *alternando* (the inequality is valid). For as AB is to BΓ, so too will be something else to EZ. (To show) that it is to something less than ΔE. The rest is the same.

(48) 6. *(Prop. 6)* Let AΓ have a greater ratio to ΓB than has ΔZ to ZE.[1] That *convertendo* ΓA has to AB a ratio less than has ZΔ to ΔE.

For, as is AΓ to ΓB, let ΔZ be made to something else. It will be to something less than ZE.[3] Let it be to ZH.[2] Then *convertendo* as ΓA is to AB, so is ZΔ to ΔH.[4] But ZΔ has to ΔH a lesser ratio than has ZΔ to ΔE.[5] Similarly, let AΓ have to ΓB a ratio less than has ΔZ to ZE.[6] *Convertendo* ΓA has to AB a greater ratio than has ΔZ to ΔE.[7] For as is AB to ΓB, so will be ΔZ to some magnitude greater than ZE. The rest is obvious.

(49) 7. *(Prop. 7)* Now let AB have to BΓ a greater ratio than has ΔE to EZ.[1] That inversely ΓB has to BA a lesser ratio than has ZE to EΔ.

For, as is AB to BΓ, let ΔE be made to something. It will be to something less than EZ.[3] Let it be to EH.[2] Then inversely as is ΓB to BA, so is EH to EΔ.[4] But HE has to EΔ a lesser ratio than has ZE to EΔ.[5]

ἔλασσον τοῦ ΔΕ. ἔστω τὸ ΕΘ. γίνεται ἄρα καὶ ὡς τὸ ΑΓ πρὸς τὸ ΓΒ οὕτως τὸ ΘΖ πρὸς τὸ ΖΕ. τὸ δὲ ΘΖ πρὸς τὸ ΖΕ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ΔΖ πρὸς τὸ ΖΕ. τὸ ΑΓ ἄρα πρὸς τὸ ΓΒ ἐλάσσονα λόγον ἔχει ἤπερ το ΔΖ πρὸς τὸ ΖΕ.

(47) ε.´ ἐχέτω δὴ πάλιν τὸ ΑΒ πρὸς τὸ ΒΓ μείζονα λόγον ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ. ὅτι καὶ ἐναλλὰξ τὸ ΑΒ πρὸς τὸ ΔΕ μείζονα λόγον ἔχει ἤπερ τὸ ΒΓ πρὸς τὸ ΕΖ. πεποιήσθω γὰρ ὡς τὸ ΑΒ πρὸς τὸ ΒΓ οὕτως ἄλλο τι πρὸς τὸ ΕΖ. φανερὸν δὴ ὅτι μεῖ|ζον ἔσται τοῦ ΔΕ. ἔστω τὸ ΗΕ. ἐναλλὰξ ἄρα ἐστὶν ὡς τὸ |127v ΑΒ πρὸς τὸ ΕΗ, οὕτως τὸ ΒΓ πρὸς τὸ ΕΖ. ἀλλὰ τὸ ΑΒ πρὸς τὸ ΔΕ μείζονα λόγον ἔχει ἤπερ τὸ ΑΒ πρὸς τὸ ΕΗ, τουτέστιν ἤπερ τὸ ΒΓ πρὸς ΕΖ. καὶ τὸ ΑΒ ἄρα πρὸς τὸ ΔΕ μείζονα λόγον ἔχει ἤπερ τὸ ΒΓ πρὸς τὸ ΕΖ. τὰ δ' αὐτὰ κἂν ἐλάσσονα λόγον ἔχηι, ὅτι καὶ ἐναλλάξ. ἔσται γὰρ καὶ ὡς τὸ ΑΒ πρὸς τὸ ΒΓ οὕτως ἄλλο τι πρὸς τὸ ΕΖ. ὅτι πρὸς ἐλάσσονα τοῦ ΔΕ. τὰ λοιπὰ τὰ αὐτά.

(48) <ς.´> τὸ ΑΓ πρὸς τὸ ΓΒ μείζονα λόγον ἐχέτω ἤπερ τὸ ΔΖ πρὸς τὸ ΖΕ. ὅτι ἀναστρέψαντι τὸ ΓΑ πρὸς τὸ ΑΒ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ΖΔ πρὸς τὸ ΔΕ. πεποιήσθω γὰρ ὡς τὸ ΑΓ πρὸς τὸ ΓΒ οὕτως τὸ ΔΖ πρὸς ἄλλο τι. ἔσται δὴ πρὸς ἔλασσον τοῦ ΖΕ. ἔστω πρὸς τὸ ΖΗ. ἀναστρέψαντι ἄρα ἐστὶν ὡς τὸ ΓΑ 688 πρὸς τὸ ΑΒ οὕτως τὸ ΖΔ πρὸς τὸ ΔΗ. τὸ δὲ ΖΔ πρὸς τὸ ΔΗ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ΖΔ πρὸς τὸ ΔΕ. ὁμοίως δὴ καὶ τὸ ΑΓ πρὸς τὸ ΓΒ ἐλάσσονα λόγον ἐχέτω ἤπερ τὸ ΔΖ πρὸς τὸ ΖΕ. ἀναστρέψαντι τὸ ΓΑ πρὸς τὸ ΑΒ μείζονα λόγον ἔχει ἤπερ τὸ ΔΖ πρὸς τὸ ΔΕ. ἔσται γὰρ ὡς τὸ ΑΒ πρὸς τὸ ΓΒ οὕτως τὸ ΔΖ πρὸς μεῖζόν τι μέγεθος τοῦ ΖΕ. καὶ τὰ λοιπὰ φανερά.

(49) <ζ.´> ἐχέτω δὴ πάλιν τὸ ΑΒ πρὸς τὸ ΒΓ μείζονα λόγον ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ. ὅτι ἀνάπαλιν τὸ ΓΒ πρὸς τὸ ΒΑ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ΖΕ πρὸς ΕΔ. πεποιήσθω γὰρ ὡς τὸ ΑΒ πρὸς τὸ ΒΓ οὕτως τὸ ΔΕ πρὸς τι. ἔσται δὴ πρὸς ἔλασσον τοῦ ΕΖ. ἔστω πρὸς τὸ ΕΗ. ἀνάπαλιν ἄρα ἐστὶν ὡς το ΓΒ πρὸς τὸ ΒΑ οὕτως τὸ ΕΗ πρὸς τὸ ΕΔ. τὸ δὲ ΗΕ πρὸς ΕΔ ἐλάσσονα

‖ 13 ἔχηι A² supr ἔχει A¹ ‖ 15 ante ὅτι πρὸς add φανερὸν δὴ Ge app | πρὸς ἐλάσσονα] ἔλασσον Co ‖ 17 ς´ add Ha ‖ 18 ΓΑ A¹ ΕΑ (?) A² ‖ 21 ΓΑ Co ΓΔ A ‖ 23 ante ὁμοίως δὴ add καὶ τὸ ΑΓ ἄρα πρὸς τὸ ΑΒ ἐλάσσονα λόγον ἔχει ἤπερ το ΖΔ πρὸς τὸ ΔΕ Co ‖ 25 post ἀναστρέψαντι add ἄρα Ha ‖ 28 ζ´ add Ha ‖ 30 ante ΕΔ add τὸ Ha ‖ 32 ἔστω Hu ὥστε A ὡς Co ἐλάσσονα λόγον ἔχει ἤπερ τὸ ΖΕ πρὸς τὸ ΕΔ Co

Similarly too if AB has a lesser ratio <to BΓ> than has ΔE to EZ,[6] inversely ΓB has to BA a ratio greater than has ZE to EΔ.[7] For, as is AB to BΓ, so will be ΔE to something greater than EZ. The rest is obvious. And from this it is obvious that if AB has to BΓ a greater ratio than has ΔE to EZ, then ZE has to EΔ a greater ratio than has ΓB to BA. But if AΓ has to BΓ a lesser ratio than has ΔE to EZ, then also ZE has to EΔ a lesser ratio than has ΓB to BA.

(50) 8. *(Prop. 8)* Let AB have to ΔE a greater ratio than has BΓ to EZ.[1] That also AB has to ΔE a greater ratio than has AΓ to ΔZ.

For, as is AB to ΔE, let BΓ be made to something. It will be to something less than EZ.[3] Let it be to HE.[2] Then also all AΓ is to all ΔH as is AB to ΔE.[4] But AΓ has to ΔH a greater ratio than to ΔZ.[5] Hence AB has to ΔE a greater ratio than has AΓ to ΔZ.[6] Obviously all AΓ has to all ΔZ a ratio less than has AB to ΔE. And if the part (has) a lesser (ratio to the part than the remainder has to the remainder), the whole (will have) a greater (ratio to the whole than the part has to the part).

(51) 9. *(Prop. 9)* Now let all AΓ have to all ΔZ a greater ratio than AB has to ΔE.[1] That also remainder BΓ has to remainder EZ a greater ratio than has AΓ to ΔZ.

<For, as is AΓ to ΔZ,> so <let> AB <be made> toΔH.[2] Then remainder BΓ is to remainder HZ as is AΓ to ΔZ.[3] But BΓ <has a greater ratio to EZ than> to ZH,[4] and therefore BΓ has to EZ a greater ratio than has AΓ to ΔZ.[5] But if whole to whole (has a) lesser (ratio than the part has to the part), then the remainder (will have) a lesser (ratio to the remainder than the whole has to the whole).

(52) 10. *(Prop. 10)* Let AB be greater than Γ, and Δ equal to E.[1] That AB has to Γ a greater ratio than has Δ to E.

For let BZ be made equal to Γ.[2] Then as is BZ to Γ, so is Δ to E.[3] But AB has to Γ a greater ratio than has BZ to Γ.[4] And so AB has to Γ a λόγον ἔχει ἤπερ τὸ ΖΕ πρὸς τὸ ΕΔ. ὁμοίως δὴ κἂν τὸ ΑΒ <πρὸς τὸ ΒΓ> ἐλάσσονα λόγον ἔχηι ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ, ἀνάπαλιν τὸ ΓΒ πρὸς τὸ ΒΑ μείζονα λόγον ἔχει ἤπερ τὸ ΖΕ πρὸς ΕΔ. ἔσται γὰρ ὡς τὸ ΑΒ πρὸς τὸ ΒΓ οὕτως τὸ ΔΕ πρὸς μεῖζόν τι τοῦ ΕΖ. τὰ δὲ λοιπὰ φανερά. καὶ φανερὸν ἐκ τούτων ὅτι ἐὰν τὸ ΑΒ πρὸς τὸ ΒΓ μείζονα λόγον ἔχηι ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ καὶ τὸ ΖΕ πρὸς τὸ ΕΔ μείζονα λόγον ἔχει ἤπερ τὸ ΓΒ πρὸς τὸ ΒΑ. ἐὰν δὲ τὸ ΑΒ πρὸς τὸ ΒΓ ἐλάσσονα λόγον ἔχηι ἤπερ τὸ ΔΕ πρὸς τὸ ΕΖ, καὶ τὸ ΖΕ πρὸς τὸ ΕΔ |ἐλάσσονα |128 λόγον ἔχει ἤπερ τὸ ΓΒ πρὸς τὸ ΒΑ.

(50) <η.´> ἐχέτω τὸ ΑΒ πρὸς τὸ ΔΕ μείζονα λόγον ἤπερ τὸ ΒΓ πρὸς τὸ ΕΖ. ὅτι καὶ τὸ ΑΒ πρὸς τὸ ΔΕ μείζονα λόγον ἔχει ἤπερ τὸ ΑΓ πρὸς τὸ ΔΖ. πεποιήσθω γὰρ ὡς τὸ ΑΒ πρὸς τὸ ΔΕ οὕτως τὸ ΒΓ πρός τι. ἔσται δὴ πρὸς ἔλασσον τοῦ ΕΖ. ἔστω 690 πρὸς τὸ ΗΕ. καὶ ὅλη ἄρα ἡ ΑΓ πρὸς ὅλην τὴν ΔΗ ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΔΕ. ἡ δὲ ΑΓ πρὸς τὴν ΔΗ μείζονα λόγον ἔχει ἤπερ πρὸς τὴν ΔΖ. καὶ ἡ ΑΒ ἄρα πρὸς τὴν ΔΕ μείζονα λόγον ἔχει ἤπερ ἡ ΑΓ πρὸς τὴν ΔΖ. καὶ φανερὸν ὅτι ὅλη ἡ ΑΓ πρὸς ὅλην τὴν ΔΖ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ΑΒ πρὸς τὸ ΔΕ. κἂν ἐλάσσονα τὸ μέρος, μείζονα ὅλη.

(51) <θ.´> ἐχέτω δὴ πάλιν ὅλη ἡ ΑΓ πρὸς ὅλην τὴν ΔΖ μείζονα λόγον ἤπερ ἡ ΑΒ πρὸς τὴν ΔΕ. ὅτι καὶ λοιπὴ ἡ ΒΓ πρὸς λοιπὴν τὴν ΕΖ μείζονα λόγον ἔχει ἤπερ ἡ ΑΓ πρὸς τὴν ΔΖ. <πεποιήσθω γὰρ ὡς ἡ ΑΓ πρὸς τὴν ΔΖ,> οὕτως ἡ ΑΒ πρὸς τὴν ΔΗ. καὶ λοιπὴ ἄρα ἡ ΒΓ πρὸς λοιπὴν τὴν ΗΖ ἐστιν ὡς ἡ ΑΓ πρὸς τὴν ΔΖ. ἡ δὲ ΒΓ <πρὸς τὴν ΕΖ μείζονα λόγον ἔχει ἤπερ> πρὸς τὴν ΖΗ, καὶ ἡ ΒΓ ἄρα πρὸς τὴν ΕΖ μείζονα λόγον ἔχει ἤπερ ἡ ΑΓ πρὸς τὴν ΔΖ. ἐὰν δὲ ὅλη πρὸς τὴν ὅλην ἐλάσσονα, ἡ λοιπὴ ἐλάσσονα.

(52) <ι.´> ἔστω μεῖζον μὲν τὸ ΑΒ τοῦ Γ, ἴσον δὲ τὸ Δ τῶι Ε. ὅτι τὸ ΑΒ πρὸς τὸ Γ μείζονα λόγον ἔχει ἤπερ τὸ Δ πρὸς τὸ Ε. κείσθω γὰρ τῶι Γ ἴσον τὸ ΒΖ. ἔστιν ἄρα ὡς τὸ ΒΖ πρὸς τὸ Γ οὕτως τὸ Δ πρὸς τὸ Ε. ἀλλὰ τὸ ΑΒ πρὸς τὸ Γ μείζονα λόγον

‖ 1 ante ὁμοίως add καὶ τὸ ΓΒ ἄρα πρὸς τὸ ΒΑ ‖ 2 πρὸς τὸ ΒΓ add Co ‖ 4 ante ΕΔ add τὸ Ge (S) | οὕτως Hu (οὕτω Ha) ουτος A ‖ 6 τούτων] τούτου Ha | ἔχηι Ha ἔχει A ‖ 11 η´ add Ha ‖ 15 ἐστιν A² supr. ‖ 17 ΔΖ Co ΔΗ A | ante ΑΒ add α A¹ del A² ‖ 20 ἐλάσσονα – ὅλη] ἔλασσον τὸ μέρος, μεῖζον ὅλης A ἐλάσσων τοῦ μέρους μείζων ὅλης Co ἐλάσσονα τὰ μέρη, μείζονα ὅλου Hu app ‖ 21 θ´ add Ha ‖ 24 πεποιήσθω – ΔΖ add Co ‖ 26 πρὸς τὴν ΕΖ μείζονα λόγον ἔχει ἤπερ add Co ‖ 28 ὅλη πρὸς τὴν ὅλην ἐλάσσονα, ἡ λοιπὴ ἐλάσσονά Hu ὅλη πρὸς τὴν ὅλην ἔλασσων, ἡ λοιπὴ μείζων A ὅλης πρὸς τὴν ὅλην ἔλασσων, ἡ λοιπὴ μείζων Co ‖ 30 ι´ add Ha greater ratio than has Δ to E.[5] And obviously, if AB is less than Γ, AB has to Γ a lesser ratio than has Δ to E, by inversion.

(53) 11. *(Prop. 11)* But let AB be greater than Γ, and Δ less than E.[1] That AB has to Γ a greater ratio than has Δ to E. This is obvious; and by proof. For if with ΔE equal to Z, AB has to Γ a greater ratio than has ΔE to Z, then with (ΔE) being less, (AB) will have a much greater ratio (to Γ). And by proof, thus.

For since AB is greater than Γ, if I make, as AB to Γ, so something else to Z, it will be greater than Z,[3] and therefore also than ΔE.[4] So let HE be equal to it.[2] Then HE has to Z a greater ratio than has ΔE to Z.[5] But as is HE to Z, so is AB to Γ.[6] Hence AB has a greater ratio to Γ than has ΔE to Z.[7] And obviously where (AB is) less (than Γ), (the ratio is) always less. And that the rectangle contained by AB, Z is greater than the rectangle contained by Γ, ΔE (is obvious). For the rectangle contained by Γ, EH is equal to it; and this is greater than the rectangle contained by Γ, ΔE.

(54) 12. *(Prop. 12)* AB is a straight line; and let it be cut at Γ. That all points between points A, Γ divide AB into ratios less than AΓ to ΓB, but all between Γ, B (divide it) into a greater (ratio).

For let points Δ, E be taken on each side of Γ. Then since ΔA is less than AΓ,[1] and ΔB greater than BΓ,[2] and (hence) ΔA has to AΓ a ratio less than has ΔB to BΓ,[3] *alternando* AΔ has to ΔB a lesser ratio than has AΓ to ΓB.[4] Similarly we will prove that (this is true) for all points between points A, Γ. Again, since EA is greater than AΓ,[5] and EB less than BΓ,[6] therefore EA has to AΓ a greater ratio than has EB to BΓ.[7] *Alternando,* therefore, AE has to EB a ratio greater than has AΓ to ΓB. Similarly for all the remaining points taken between points Γ, B.

ἔχει ἤπερ τὸ ΒΖ πρὸς τὸ Γ. καὶ τὸ ΑΒ ἄρα πρὸς τὸ Γ μείζονα
λόγον ἔχει ἤπερ τὸ Δ πρὸς τὸ Ε. καὶ φανερὸν ὅτι ἂν ἔλασσον
τὸ ΑΒ τοῦ Γ, τὸ ΑΒ πρὸς τὸ Γ ἐλάσσονα λόγον ἔχει ἤπερ τὸ Δ 692
πρὸς τὸ Ε διὰ τὸ ἀνάπαλιν.

(53) <ια´> ἀλλὰ ἔστω μεῖζον μὲν τὸ ΑΒ τοῦ Γ, ἔλασσον δὲ
τὸ Δ τοῦ Ε. ὅτι τὸ ΑΒ πρὸς τὸ Γ μείζονα λόγον ἔχει ἤπερ τὸ
Δ πρὸς τὸ Ε. φανερὸν μὲν οὖν, καὶ διὰ ἀπο|δείξεως. εἰ γάρ, |128v
ὄντος ἴσου τοῦ ΔΕ τῶι Ζ, τὸ ΑΒ πρὸς τὸ Γ μείζονα λόγον ἔχει
ἤπερ τὸ ΔΕ πρὸς τὸ Ζ, ἐλάσσονος ὄντος, πολλῶι μείζονα λόγον
ἕξει. διὰ ἀποδείξεως δὲ οὕτως. ἐπεὶ γὰρ μεῖζόν ἐστιν τὸ ΑΒ
τοῦ Γ, ἐὰν ποιῶ ὡς τὸ ΑΒ πρὸς τὸ Γ οὕτως ἄλλο τι πρὸς τὸ Ζ,
ἔσται μεῖζον τοῦ Ζ, ὥστε καὶ τοῦ ΔΕ. ἔστω οὖν αὐτῶι ἴσον τὸ
ΗΕ. τὸ ΗΕ ἄρα πρὸς τὸ Ζ μείζονα λόγον ἔχει ἤπερ τὸ ΔΕ πρὸς
τὸ Ζ. ἀλλ' ὡς τὸ ΗΕ πρὸς τὸ Ζ, οὕτως τὸ ΑΒ πρὸς τὸ Γ. καὶ τὸ
ΑΒ ἄρα πρὸς τὸ Γ μείζονα λόγον ἔχει ἤπερ τὸ ΔΕ πρὸς τὸ Ζ.
καὶ φανερὸν ὅτι ὅπου τὸ ἔλασσον, ἀεὶ ἐλάττονα. καὶ ὅτι
μεῖζον γίνεται τὸ ὑπὸ τῶν ΑΒ, Ζ τοῦ ὑπὸ τῶν Γ, ΔΕ. ἴσον γὰρ
αὐτῶι ἐστι τὸ ὑπὸ τῶν Γ, ΕΗ, ὅ ἐστιν μεῖζον τοῦ ὑπὸ τῶν Γ,
ΔΕ.

(54) <ιβ´> εὐθεῖα ἡ ΑΒ, καὶ τετμήσθω κατὰ τὸ Γ. ὅτι
πάντα μὲν τὰ μεταξὺ τῶν Α, Γ σημείων εἰς ἐλάσσονας λόγους
διαιρεῖ τὴν ΑΒ τοῦ τῆς ΑΓ πρὸς τὴν ΓΒ, πάντα δὲ τὰ μεταξὺ
τῶν Γ, Β εἰς μείζονας. εἰλήφθω γὰρ σημεῖα ἐφ' ἑκάτερα τοῦ Γ,
τὰ Δ, Ε. ἐπεὶ οὖν ἐλάσσων μὲν ἡ ΔΑ τῆς ΑΓ, μείζων δὲ ἡ ΔΒ τῆς
ΒΓ, ἡ δὲ ΔΑ πρὸς τὴν ΑΓ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΔΒ πρὸς
τὴν ΒΓ, ἐναλλὰξ ἡ ΑΔ πρὸς τὴν ΔΒ ἐλάσσονα λόγον ἔχει ἤπερ ἡ
ΑΓ πρὸς τὴν ΓΒ. ὁμοίως δὴ δείξομεν ὅτι καὶ ἐπὶ πάντων τῶν 694
μεταξὺ τῶν Α, Γ σημείων. πάλιν ἐπεὶ μείζων μέν ἐστιν ἡ ΕΑ
τῆς ΑΓ, ἐλάσσων δὲ ἡ ΕΒ τῆς ΒΓ, ἡ ΕΑ ἄρα πρὸς τὴν ΑΓ μείζονα
λόγον ἔχει ἤπερ ἡ ΕΒ πρὸς τὴν ΒΓ. ἐναλλὰξ ἄρα ἡ ΑΕ πρὸς τὴν
ΕΒ μείζονα λόγον ἔχει ἤπερ ἡ ΑΓ πρὸς τὴν ΓΒ. ὁμοίως καὶ
ἐπὶ τῶν λοιπῶν μεταξὺ [καὶ] τῶν Γ, Β λαμβανομένων σημείων.

‖ 5 ια´ add Ha ‖ 6 Δ τοῦ Ε] ΔΕ τοῦ Ζ Co ‖ 7 Δ πρὸς τὸ Ε] ΔΕ πρὸς τὸ Ζ Co | διὰ] ἄνευ Co ‖ 11 Ζ Co ΖΕ Α ‖ 12 αὐτῶι ἴσον secl Hu ‖ 16 ἀεὶ ἐλάττονα] καὶ ἐλάσσονα Hu app ‖ 18 τοῦ Co τὸ Α ‖ 20 ιβ´ add Ha | post εὐθεῖα add ἔστω Ha ‖ 23 μείζονας Co μείζονα Α | ἑκατέραι Ha ‖ 25 ἡ δὲ ΔΑ] ἡ ΔΑ ἄρα Co ‖ 26 ante ἐναλλὰξ add καὶ Ha | post ἐναλλὰξ add ἄρα Co ‖ 31 ἔχει Ha ἔχειν Α ‖ 32 καὶ (τῶν) del Hu

(55) 13. *(Prop. 13)* If AB is a straight line, and it is bisected at Γ, then of all points taken (on the line), point Γ cuts making the rectangle contained by AΓ, ΓB maximum.

For if a point Δ is taken, the rectangle contained by AΔ, ΔB plus the square of ΓΔ equals the square of AΓ (II 5),[1] that is the rectangle contained by AΓ, ΓB.[2] Hence the rectangle contained by AΓ, ΓB is greater (than the rectangle contained by AΔ, ΔB).[3] The same (is true) for the other side too.

(56) *(Prop. 14)* I also say that the nearer (point) cuts off always a greater area than the further (point). For let yet another point E be taken between A, Δ. One must show that the rectangle contained by AΔ, ΔB is greater than the rectangle contained by AE, EB.

For since the rectangle contained by AΔ, ΔB plus the square of ΔΓ equals the square of AΓ,[1] and the rectangle contained by AE, EB plus the square of ΓE equals the square of AΓ,[2] therefore the rectangle contained by AΔ, ΔB plus the square of ΔΓ equals the rectangle contained by AE, EB plus the square of ΓE.[3] Of these, the square of ΔΓ is less than the square of ΓE.[4] Therefore the remaining rectangle contained by AΔ, ΔB is greater than the rectangle contained by AE, EB.[5]

(57) 14. *(Prop. 15)* For if A plus B equalled Γ plus ΔE,[1] and B were less than ΔE,[2] A would be greater than Γ.

For let ΔZ be made equal to B.[3] A plus ΔZ, therefore, equals ΔE plus Γ.[4] Let the common ΔZ be subtracted. Then remainder A equals Γ plus ZE.[5] Hence A is greater than Γ.[6]

(58) 15. *(Prop. 16)* Let A have a greater ratio to B than has Γ to Δ.[1] That the rectangle contained by A, Δ is greater than the rectangle contained by B, Γ.

For, as is A to B, let Γ be made to E.[2] Then Γ has to E a greater ratio than to Δ.[3] Therefore E is less than Δ.[4] (Make) A a common height. Hence the rectangle contained by E, A is less than the rectangle contained by A, Δ.[5] But the rectangle contained by A, E equals the rectangle contained by B, Γ.[6] Hence the rectangle contained by B, Γ is less than the rectangle contained by A, Δ.[7] Thus the rectangle contained by A, Δ is greater than the rectangle contained by B, Γ. Similarly, if (the ratio is) less, the area will be less than the area.

(55) <ιγʹ> ἐὰν εὐθεῖα ἡ ΑΒ, καὶ τμηθῆι δίχα κατὰ τὸ Γ, πάντων τῶν λαμβανομένων σημείων μέγιστον ἀποτέμνει τὸ ὑπὸ τῶν ΑΓΒ τὸ Γ σημεῖον. ἐὰν γὰρ ληφθῆι σημεῖον τὸ Δ, γίνεται τὸ ὑπὸ τῶν ΑΔΒ μετὰ τοῦ ἀπὸ ΓΔ ἴσον τῶι ἀπὸ ΑΓ, τουτέστιν τῶι ὑπὸ τῶν ΑΓΒ. ὥστε μεῖζόν ἐστιν τὸ ὑπὸ τῶν ΑΓΒ. τὰ δὲ αὐτὰ καὶ ἐπὶ τὰ ἕτερα.

(56) λέγω δ' ὅτι καὶ αἰεὶ τὸ ἔγγιον τοῦ ἀπωτέρου μεῖζον χωρίον ποιεῖ. εἰλήφθω γὰρ καὶ ἕτερον σημεῖον τὸ Ε μεταξὺ τῶν Α, Δ. δεικτέον ὅτι μεῖ|ζόν ἐστιν τὸ ὑπὸ τῶν ΑΔΒ τοῦ ὑπὸ |129 τῶν ΑΕΒ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν ΑΔΒ μετὰ τοῦ ἀπὸ ΔΓ ἴσον ἐστὶν τῶι ἀπὸ τῆς ΑΓ, ἔστιν δὲ καὶ <τὸ> ὑπὸ τῶν ΑΕΒ μετὰ τοῦ ἀπὸ τῆς ΓΕ ἴσον τῶι ἀπὸ τῆς ΑΓ τετραγώνωι, καὶ τὸ ὑπὸ τῶν ΑΔΒ ἄρα μετὰ τοῦ ἀπὸ ΔΓ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΕΒ μετὰ τοῦ ἀπὸ τῆς ΓΕ, ὧν τὸ ἀπὸ ΔΓ ἔλασσόν ἐστιν τοῦ ἀπὸ ΓΕ. λοιπὸν ἄρα τὸ ὑπὸ τῶν ΑΔΒ μεῖζόν ἐστι τοῦ ὑπὸ τῶν ΑΕΒ.

(57) <ιδʹ> εἰ γὰρ εἴη τὸ Α μετὰ τοῦ Β ἴσον τῶι Γ μετὰ τοῦ ΔΕ, καὶ ἔλασσον τὸ Β τοῦ ΔΕ, μεῖζον ἂν γένοιτο τὸ Α τοῦ Γ. κείσθω γὰρ τῶι Β ἴσον τὸ ΔΖ. τὸ Α ἄρα μετὰ τοῦ ΔΖ ἴσον 696 ἐστὶν τῶι ΔΕ μετὰ τοῦ Γ. κοινὸν ἀφῃρήσθω τὸ ΔΖ. λοιπὸν ἄρα τὸ Α ἴσον ἐστὶν τοῖς Γ, ΖΕ. ὥστε μεῖζόν ἐστιν τὸ Α τοῦ Γ.

(58) <ιεʹ> ἡ Α πρὸς τὴν Β μείζονα λόγον ἐχέτω ἤπερ ἡ Γ πρὸς τὴν Δ. ὅτι μεῖζόν ἐστιν τὸ ὑπὸ τῶν Α, Δ τοῦ ὑπὸ τῶν Β, Γ. πεποιήσθω γὰρ ὡς ἡ Α πρὸς τὴν Β οὕτως ἡ Γ πρὸς τὴν Ε. καὶ ἡ Γ ἄρα πρὸς τὴν Ε μείζονα λόγον ἔχει ἤπερ πρὸς τὴν Δ. ὥστε ἐλάσσων ἐστὶν ἡ Ε τῆς Δ. καὶ κοινὸν ὕψος ἡ Α. ἔλασσον ἄρα ἐστὶν τὸ ὑπὸ τῶν Ε, Α τοῦ ὑπὸ τῶν Α, Δ. ἀλλὰ τὸ ὑπὸ τῶν Α, Ε ἴσον ἐστὶν τῶι ὑπὸ τῶν Β, Γ. ἔλασσον ἄρα ἐστὶν τὸ ὑπὸ τῶν Β, Γ τοῦ ὑπὸ τῶν Α, Δ. ὥστε μεῖζόν ἐστιν τὸ ὑπὸ τῶν Α, Δ τοῦ ὑπὸ τῶν Β, Γ. ὁμοίως καὶ ἐὰν ἔλασσον, γίνεται ἔλασσον καὶ τὸ χωρίον τοῦ χωρίου.

‖ 1 ιγʹ add Ha | ἡ] ἦι A ‖ 5 post ΑΓΒ add τοῦ ὑπὸ τῶν ΑΔΒ Hu ‖ 7 ἔγγιον Ha ἔγγειον A | ἀπωτέρου Ha ἀπώτερον A | μεῖζον Ha μείζονα A ‖ 9 ὑπὸ τῶν ΑΔΒ τοῦ bis A corr Co ‖ 11 ἔστιν δὲ καὶ — ἀπὸ τῆς ΑΓ mg A^2 alia manu | τὸ add Ha ‖ 16 ιδʹ add Hu (BS) ‖ 17 post γένοιτο τὸ add ΔΕ τοῦ Β, ὅτι μεῖζον τὸ Co ‖ 22 ιεʹ add Hu (BS) ‖ 30 (ἐὰν) ἔλασσον] ἐλάσσων Hu ἐλάσσων ὁ λόγος Ha | γένηται Ha

(59) But let the rectangle contained by A, Δ be greater than the rectangle contained by B, Γ.[1] That A has a greater ratio to B than has Γ to Δ.

For let the rectangle contained by B, E be made equal the rectangle contained by A, Δ.[2] Then the rectangle contained by B, E is greater than the rectangle contained by B, Γ.[3] Hence also E is greater than Γ.[4] *But as is A to Δ, so is B to E.[5] And B has a greater ratio to E than to Γ.* And thus too Δ has to Γ. And similarly in the converse.

(60) 16. *(Prop. 17)* AB, BΓ are two straight lines, and let BΔ be a mean in ratio between AB, BΓ.[1] Let ΔE be made equal AΔ.[2] That ΓE is the excess by which ABΓ together exceeds the line equal in square to four times the rectangle contained by AB, BΓ.

For since ABΓ together exceeds ABE together by ΓE,[3] ΓE is therefore the excess by which ABΓ together exceeds ABE together. But ABE is two of BΔ.[4] And two of BΔ equal in square four times the rectangle contained by AB, BΓ.[5] ΓE is thus the excess by which ABΓ together exceeds the line equal in square to four times the rectangle contained by AB, BΓ.[6]

(61) 17. *(Prop. 18)* Again let BΔ be mean in ratio between AB, BΓ.[1] Let ΔE be made equal AΔ.[2] That ΓE comprises AB, BΓ together and the line that is equal in square to four times the rectangle contained by AB, BΓ.

For since ΓE comprises ΓΔ, ΔE,[3] while AΔ equals ΔE, ΓE therefore comprises AΔ, ΔΓ,[4] that is AB, BΓ together and two of BΔ. But two of BΔ equal in square four times the rectangle contained by AB, BΓ.[5] Hence ΓE comprises AB, BΓ together and the line equal in square to four times the rectangle contained by AB, BΓ.[6]

(62) 18. *(Prop. 19)* Again let BΔ be mean in ratio between AB, BΓ,[1] and let ΔE be made equal ΓΔ.[2] That AE is the excess by which ABΓ together exceeds the line that is equal in square to four times the rectangle contained by AB, BΓ.

For since ABΓ together exceeds EBΓ together by AE,[3] while EBΓ together is two of BΔ,[4] that is the line equal in square to four times the rectangle contained by AB, BΓ,[5] therefore AE is the excess by which ABΓ together exceeds the line equal in square to four times the rectangle contained by AB, BΓ.[6]

(59) ἀλλὰ δὴ ἔστω πάλιν μεῖζον τὸ ὑπὸ τῶν Α, Δ τοῦ ὑπὸ τῶν Β, Γ. ὅτι ἡ Α πρὸς τὴν Β μείζονα λόγον ἔχει ἤπερ ἡ Γ πρὸς τὴν Δ. κείσθω γὰρ τῶι ὑπὸ τῶν Α, Δ ἴσον τὸ ὑπὸ τῶν Β, Ε. γίνεται ἄρα μεῖζον μὲν τὸ ὑπὸ τῶν Β, Ε τοῦ ὑπὸ τῶν Β, Γ. ὥστε καὶ ἡ Ε τῆς Γ μείζων. ὡς δὲ ἡ Α πρὸς τὴν Δ οὕτως ἡ Β πρὸς τὴν Ε. ἡ δὲ Β πρὸς τὴν Ε μείζονα λόγον ἔχει ἤπερ πρὸς τὴν Γ. καὶ ἡ Δ ἄρα πρὸς τὴν Γ. ὁμοίως καὶ ἀναστρέψαντι.

(60) <ις.´> δύο εὐθεῖαι αἱ ΑΒ, ΒΓ καὶ τῶν ΑΒ, ΒΓ μέση ἀνάλογον ἔστω ἡ ΒΔ, καὶ τῆι ΑΔ ἴση κείσθω ἡ ΔΕ. ὅτι ἡ ΓΕ ὑπεροχή ἐστιν ἧι ὑπερέχει συναμφότερος ἡ ΑΒΓ τῆς δυναμένης τὸ τετράκις |ὑπὸ τῶν ΑΒΓ. ἐπεὶ γὰρ συναμφότερος |129v
ἡ ΑΒΓ συναμφοτέρου τῆς ΑΒΕ ὑπερέχει τῆι ΓΕ, ἡ ΓΕ ἄρα ἐστὶν 698
ἡ ὑπεροχὴ ἧι ὑπερέχει συναμφότερος ἡ ΑΒΓ συναμφοτέρου τῆς ΑΒΕ. συναμφότερος δὲ ἡ ΑΒΕ δύο εἰσὶν αἱ ΒΔ. δύο δὲ αἱ ΒΔ δύνανται τὸ τετράκις ὑπὸ τῶν ΑΒΓ. ἡ ΓΕ ἄρα ἐστὶν ἡ ὑπεροχὴ ἧι ὑπερέχει συναμφότερος ἡ ΑΒΓ τῆς δυναμένης τὸ τετράκις ὑπὸ τῶν ΑΒΓ.

(61) <ιζ.´> ἔστω δὴ πάλιν ἡ τῶν ΑΒ, ΒΓ μέση ἡ ΒΔ, <καὶ> κείσθω τῆι ΑΔ ἴση ἡ ΔΕ. ὅτι ἡ ΓΕ σύγκειται ἔκ τε συναμφοτέρου τῆς ΑΒ, ΒΓ καὶ τῆς δυναμένης τὸ τετράκις ὑπὸ τῶν ΑΒ, ΒΓ. ἐπεὶ γὰρ ἡ ΓΕ ἐστὶν ἡ συγκειμένη ἐκ τῶν ΓΔ, ΔΕ, ἴση δέ ἐστιν ἡ ΑΔ τῆι ΔΕ, ἡ ΓΕ ἄρα ἐστὶν ἡ συγκειμένη ἐκ τῶν ΑΔ, ΔΓ, τουτέστιν ἐκ συναμφοτέρου τῆς ΑΒ, ΒΓ καὶ δύο τῶν ΒΔ. δύο δὲ αἱ ΒΔ δύνανται τὸ τετράκις ὑπὸ τῶν ΑΒΓ. ἡ ΓΕ ἄρα ἐστὶν ἡ συγκειμένη ἔκ τε συναμφοτέρου τῆς ΑΒ, ΒΓ καὶ τῆς δυναμένης τὸ τετράκις ὑπὸ τῶν ΑΒΓ.

(62) <ιη.´> πάλιν τῶν ΑΒ, ΒΓ μέση ἀνάλογον ἡ ΒΔ, καὶ τῆι ΓΔ ἴση κείσθω ἡ ΔΕ. ὅτι ἡ ΑΕ ὑπεροχή ἐστιν ἧι ὑπερέχει συναμφότερος ἡ ΑΒΓ τῆς δυναμένης τὸ τετράκις ὑπὸ ΑΒΓ. ἐπεὶ γὰρ συναμφότερος ἡ ΑΒΓ συναμφοτέρου τῆς ΕΒΓ ὑπερέχει τῆι ΑΕ, συναμφότερος δὲ ἡ ΕΒΓ δύο εἰσὶν αἱ ΒΔ, τουτέστιν ἡ δυναμένη τὸ τετράκις ὑπὸ τῶν ΑΒΓ, ἡ ΑΕ ἄρα ἐστὶν ἡ ὑπεροχὴ ἧι ὑπερέχει συναμφότερος ἡ ΑΒΓ τῆς δυναμένης τὸ τετράκις ὑπὸ τῶν ΑΒΓ.

‖ 4 τὸ Ha τοῦ A ‖ 5 ἡ E Co ἡ B A | ὡς δὲ — ἄρα πρὸς τὴν Γ] ὡς δὲ ἡ Α πρὸς τὴν Β, οὕτως ἡ Ε πρὸς τὴν Δ. ἡ δὲ Ε πρὸς τὴν Δ μείζονα λόγον ἔχει ἤπερ ἡ Γ πρὸς τὴν Δ Co ‖ 8 ις´ add Hu (BS) | post εὐθεῖαι add ἔστωσαν Ha ‖ 15 ΓΕ] γε A ‖ 18 ιζ´ add Hu (BS) | καὶ add Ha ‖ 23 τῆς Hu τῶν A ‖ 27 ιη´ add Hu (BS)

(63) 19. *(Prop. 20)* Again let BΔ be mean in ratio between AB, BΓ,[1] and let ΔE be made equal ΓΔ.[2] That AE comprises ABΓ together and the line that is equal in square to four times the rectangle contained by AB, BΓ.

For since AE comprises AΔ, ΔE,[1] while ΔE equals ΔΓ, AE therefore comprises AΔ, ΔΓ, that is ABΓ together and two of BΔ.[4] But two of BΔ equal in square four times ABΓ.[5] Hence AE comprises ABΓ together and the line equal in square to four times the rectangle contained by AB, BΓ.[6]

These things are assumed in the *Cutting off of a Ratio.* They are also assumed in the *Cutting off of an Area,* only differently.

(64) *(Prop. 21)* Problem for the second (book) of the *Cutting off of a Ratio,* useful for the summation of the fourteenth disposition.

Given two straight lines AB, BΓ, and producing line AΔ, to find a point Δ that makes the ratio BΔ to ΔA the same as that of ΓΔ to the excess by which ABΓ together exceeds the line that is equal in square to four times the rectangle contained by AB, BΓ.

The combination cannot be made in any other way, unless ΔE, AΓ together are equal to the excess EA, and all ΔA to all AB, and furthermore (it is not possible otherwise?) that EA, AB, ΓB have the ratio to one another of a square number to a square number, and that ΓB is twice ΔE.

Let it be accomplished, and let the excess be AE;[1] for we have found this in the foregoing (lemma 7.62). Then as is BΔ to ΔA, so is ΓΔ to AE.[2] And *alternando*[3] and *separando*[4] and area to area, it follows that the rectangle contained by BΓ, EA equals the rectangle contained by ΓΔ, ΔE.[5] But the rectangle contained by BΓ, EA is given;[6] hence the rectangle contained by ΓΔ, ΔE too is given.[7] And it lies along ΓE, given,[8] exceeding by a square. Hence Δ is given (Data 59).[9]

The synthesis will be made thus. Let the excess be EA, and along ΓE let there be applied the rectangle contained by ΓΔ, ΔE, exceeding by a square, and equal to the rectangle contained by BΓ, EA. I say that Δ is the point sought. For since the rectangle contained by BΓ, EA equals the rectangle contained by ΓΔ, ΔE,[10] therefore putting in ratio[11] and *componendo*[12] and *alternando* as is BΔ to ΔA, so is ΓΔ to EA,[13] which is the excess. The same also if we try to take a point making, as BΔ to ΔA, so ΓΔ to the line comprising ABΓ together and the line equal in square to four times the rectangle contained by AB, BΓ. Q.E.D.

(63) <ιθ´> πάλιν τῶν ΑΒ, ΒΓ μέση ἀνάλογον ἔστω ἡ ΒΔ καὶ
τῇ ΓΔ ἴση κείσθω ἡ ΔΕ. ὅτι ἡ ΑΕ ἐστὶν ἡ συγκειμένη ἔκ τε 700
συναμφοτέρου τῆς ΑΒΓ καὶ τῆς δυναμένης τὸ τετράκις ὑπὸ τῶν
ΑΒΓ. ἐπεὶ γὰρ ἡ ΑΕ σύγκειται ἐκ τῶν ΑΔ, ΔΕ, ἴση δέ ἐστιν ἡ
ΔΕ τῇ ΔΓ, ἡ ΑΕ ἄρα σύγκειται ἐκ τῶν ΑΔ, ΔΓ, τουτέστιν
συναμφοτέρου τῆς ΑΒΓ καὶ δύο τῶν ΒΔ. δύο δὲ αἱ ΒΔ δύνανται
τὸ τετράκις ὑπὸ τῶν ΑΒΓ. ἡ ΑΕ ἄρα ἐστὶν ἡ συγκειμένη ἔκ τε
συναμφοτέρου τῆς ΑΒΓ καὶ τῆς δυναμένης τὸ τετράκις ὑπὸ τῶν
ΑΒΓ. |ταῦτα λαμβάνεται εἰς τὴν τοῦ Λόγου Ἀποτομήν. ταῦτα |130
καὶ εἰς τὴν τοῦ Χωρίου Ἀποτομὴν λαμβάνεται, διαφερόντως
μόνον.

(64) Πρόβλημα εἰς τὸ δεύτερον Λόγου Ἀποτομῆς, χρήσιμον
εἰς τὴν τοῦ ιδ´ τόπου ἀνακεφαλαίωσιν.
δύο δοθεισῶν εὐθειῶν τῶν ΑΒ, ΒΓ, λαβεῖν, ἐπεκβαλόντα τὴν ΑΔ
εὐθεῖαν, τὸ Δ ποιοῦν τὸν τῆς ΒΔ πρὸς ΔΑ λόγον τὸν αὐτὸν τῶι
τῆς ΓΔ πρὸς τὴν ὑπεροχὴν ἧι ὑπερέχει συναμφότερος ἡ ΑΒΓ
τῆς δυναμένης τὸ τετράκις ὑπὸ τῶν ΑΒΓ. ἄλλως οὐχ οἷόν τε
συστῆναι εἰ μὴ συναμφότερος μὲν ἡ ΔΕ, ΑΓ ἴση ἧι τῇ ΕΑ
ὑπεροχῇ, ὅλη δὲ ἡ ΔΑ ὅλῃ τῇ ΑΒ καὶ ἔτι τὰς ΕΑ, ΑΒ, ΓΒ πρὸς
ἀλλήλας λόγον ἔχειν ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον
ἀριθμόν, καὶ τὴν ΓΒ τῆς ΔΕ διπλασίαν εἶναι. ἔστω γεγονὸς
καὶ ἡ ὑπεροχὴ ἔστω ἡ ΑΕ (ἐν γὰρ τοῖς ἐπάνω εὕρομεν αὐτήν).
ἔστιν οὖν ὡς ἡ ΒΔ πρὸς τὴν ΔΑ οὕτως ἡ ΓΔ πρὸς τὴν ΑΕ. καὶ
ἐναλλὰξ καὶ διέλοντι χωρίον χωρίωι, τὸ ἄρα ὑπὸ τῶν ΒΓ, ΕΑ
ἴσον τῶι ὑπὸ τῶν ΓΔΕ. δοθὲν δὲ τὸ ὑπὸ τῶν ΒΓ, ΕΑ. δοθὲν ἄρα
καὶ τὸ ὑπὸ τῶν ΓΔΕ. καὶ παρὰ δοθεῖσαν τὴν ΓΕ παράκειται
ὑπερβάλλον τετραγώνωι. δοθὲν ἄρα ἐστὶν τὸ Δ.
συντεθήσεται δὲ οὕτως. ἔστω ἡ ὑπεροχὴ ἡ ΕΑ, καὶ τῶι ὑπὸ τῶν 702
ΒΓ, ΕΑ ἴσον παρὰ τὴν ΓΕ παραβεβλήσθω ὑπερβάλλον τετραγώνωι
τὸ ὑπὸ ΓΔΕ. λέγω ὅτι τὸ ζητούμενον σημεῖόν ἐστιν τὸ Δ.
ἐπεὶ γὰρ ἴσον τὸ ὑπὸ τῶν ΒΓ, ΕΑ τῶι ὑπὸ τῶν ΓΔΕ, ἀνάλογον
καὶ συνθέντι καὶ ἐναλλάξ ἐστιν ἄρα ὡς ἡ ΒΔ πρὸς τὴν ΔΑ
οὕτως ἡ ΓΔ πρὸς ΕΑ, ἥτις ἐστὶν ἡ ὑπεροχή. τὰ δ᾽ αὐτὰ κἂν
ζητῶμεν λαβεῖν σημεῖον ποιοῦν ὡς τὴν ΒΔ πρὸς τὴν ΔΑ οὕτως
τὴν ΓΔ πρὸς τὴν συγκειμένην ἔκ τε συναμφοτέρου τῆς ΑΒΓ καὶ
τῆς δυναμένης τὸ τετράκις ὑπὸ τῶν ΑΒΓ. ὅ(περ): —

‖ 1 ιθ´ add Hu (BS) ‖ 8 τῆς (ΑΒΓ) Hu τῶν A ‖ 9 ταῦτα λαμβάνεται — μόνον secl Hu ‖ 13 ιδ´] ιγ´ A ‖ 15 εὐθεῖαν] δοθὲν A ‖ 17 ἄλλως — εἶναι del Ha | οἷόν τε Hu (S) οιονται A ‖ 18 ΔΕ] ΔΒ A ‖ 19 ΑΒ] ΑΓ A ‖ 20 ἀλλήλας] ἄλληλα A ‖ 24 ante χωρίον add καὶ Ha ‖ 27 τετραγώνωι Ha τετράγωνον A | ante Δ add τὸ Ha ‖ 29 παρὰ τὴν V πάλιν τὴν A πάλιν τῇ Ha

(65) The first (book) of the *Cutting off of a Ratio* contains 7 dispositions, 24 cases, and 5 diorisms, of which three are maxima, two minima. That in the third case of the fifth disposition is maximum, that in the second of the sixth disposition minimum, and that in the same one of the seventh; maxima, those in the fourth of the sixth and seventh. The second (book) of the *Cutting off of a Ratio* <contains 14 dispositions, 63 cases, and for diorisms those from the first, because it refers entirely to the first.>

(66) <The first (book) of the *Cutting off of an Area*> contains 7 dispositions, 24 cases, 7 diorisms, of which 4 are maxima, 3 minima. That in the second of the first disposition is maximum, as is that in the first of <the second disposition, and that in the second> of the fourth, and that in the third of the third, and that in the fourth of the fourth, and that in the first of the sixth. The second (book) of the *Cutting off of an Area* contains 13 dispositions, 60 cases, and the diorisms from the first (book). For it refers to it.

(67) One would like to know why the second (book) of the *Cutting off of a Ratio* contains 14 dispositions, while that of the *Area* only 13. It does so for this reason, that the seventh disposition in the *Cutting off of an Area* is omitted as obvious. For if both parallels fall on the limits, any line drawn through (the point) will cut off a given area. For it is equal to the rectangle contained by the (lines) between the limits and the intersection of both the lines originally given in position. In the *Cutting off of a Ratio* it is not likewise. For this reason, then, it has one disposition in excess of the second (work) in the second (book), the rest being the same.

(65) τὸ πρῶτον Λόγου Ἀποτομῆς ἔχει τόπους ζ, πτώσεις κδ, διορισμοὺς δὲ ε, ὧν τρεῖς μὲν μέγιστοι, δύο δὲ ἐλάχιστοι. καὶ ἔστιν μέγιστος μὲν κατὰ τὴν τρίτην πτῶσιν τοῦ ε΄ τόπου, ἐλάχιστος δὲ κατὰ τὴν β΄ τοῦ ἕκτου τόπου καὶ κατὰ τὴν αὐτὴν |τοῦ ζ΄. μέγιστοι δὲ οἱ κατὰ τὰς τετάρτας τοῦ ἕκτου καὶ τοῦ ἑβδόμου. τὸ δεύτερον Λόγου Ἀποτομῆς <ἔχει 130v τόπους ιδ, πτώσεις δὲ ξγ, διορισμοὺς δὲ τοὺς ἐκ τοῦ πρώτου· ἀπάγεται γὰρ ὅλον εἰς τὸ πρῶτον.>

(66) <τὸ πρῶτον Χωρίου Ἀποτομῆς> ἔχει τόπους ζ, πτώσεις κδ, διορισμοὺς ζ, ὧν δ μὲν μέγιστοι, τρεῖς δὲ ἐλάχιστοι. καὶ ἔστιν μέγιστος μὲν ὁ κατὰ τὴν δευτέραν τοῦ πρώτου τόπου καὶ ὁ κατὰ τὴν πρώτην <τοῦ β΄ τόπου καὶ ὁ κατὰ τὴν β΄> τοῦ δ΄ τόπου καὶ ὁ κατὰ τὴν τρίτην τοῦ τρίτου καὶ ὁ κατὰ τὴν δ΄ τοῦ δ΄ καὶ ὁ κατὰ τὴν πρώτην του ς΄. τὸ δεύτερον Χωρίου Ἀποτομῆς ἔχει τόπους ιγ, πτώσεις ξ, διορισμοὺς δὲ τοὺς ἐκ τοῦ πρώτου. ἀπάγεται γὰρ εἰς αὐτό.

(67) ἐπιστήσειεν ἄν τις διὰ τί ποτε μὲν τὸ Λόγου Ἀποτομῆς δεύτερον ἔχει τόπους ιδ, τὸ δὲ τοῦ Χωρίου ιγ. ἔχει δὲ διὰ τόδε, ὅτι ὁ ζ΄ ἐν τῶι τοῦ Χωρίου Ἀποτομῆς τόπος παραλείπεται ὡς φανερός. ἐὰν γὰρ αἱ παράλληλοι ἀμφότεραι ἐπὶ τὰ πέρατα πίπτωσιν, οἵα ἂν διαχθῆι, δοθὲν 704 ἀποτέμνει χωρίον. ἴσον γὰρ γίνεται τῶι ὑπὸ τῶν μεταξὺ τῶν περάτων καὶ τῆς ἀμφοτέρων τῶν ἐξ ἀρχῆς τῆι θέσει δοθεισῶν εὐθειῶν συμβολῆς. ἐν δὲ τῶι Λόγου Ἀποτομῆς οὐκέτι ὁμοίως. διὰ τοῦτο οὖν προέχει τόπον ἕνα εἰς τὸ δεύτερον τοῦ δευτέρου, καὶ τὰ λοιπὰ ὄντα ταὐτά.

‖ 5 τὴν αὐτὴν Ha τῆς αὐτῆς A ‖ 6 ἔχει – χωρίου ἀποτομῆς add Ha ‖ 12 καὶ ὁ κατὰ τὴν πρώτην τοῦ δ΄ τόπου bis A corr Ha | τοῦ β΄ τόπου καὶ ὁ κατὰ τὴν β΄ add Ha ‖ 15 ξ Ha ζ A ‖ 16 δὲ τοὺς Ha δ τοὺς A ‖ 17 τὸ Ha τοῦ A ‖ 18 δεύτερον Ha δευτέρου A | τοῦ om Ha ‖ 19 τοῦ secl Hu app ‖ 21 ἂν Hu (Co) ἐὰν A ‖ 23 καὶ om Ge ‖ 24 οὐκέτι] οὐκ ἔστι Ge app ‖ 25 τὸ] τὸν Ge | δεύτερον] ἕβδομον Ha ‖ 26 ταὐτά] τὰ ὄντα A

(68) Determinate Section, (Book) 1.

1. *(Prop. 22)* Lemma useful for the first assignment of the fifth problem.

Let there be line AB, and on it three points Γ, Δ, E, and let the rectangle contained by AΔ, ΔΓ be equal to the rectangle contained by BΔ, ΔE. That as is BΔ to ΔE, so is the rectangle contained by AB, BΓ to the rectangle contained by AE, EΓ.

For since the rectangle contained by AΔ, ΔΓ equals the rectangle contained by BΔ, ΔE,[1] therefore in ratio as is AΔ to ΔB, so is EΔ to ΔΓ.[2] Hence all AE to all BΓ is as EΔ to ΔΓ.[3] And also inverting.[4] Again, since the rectangle contained by AΔ, ΔΓ equals the rectangle contained by BΔ, ΔE,[5] therefore in ratio as is AΔ to ΔE, so is BΔ to ΔΓ.[6] Hence all AB is to all ΓE as BΔ to ΔΓ.[7] But as is BΓ to EA, so was ΓΔ to ΔE.[8] Thus the ratio compounded out of AB to ΓE and BΓ to AE is the same as that compounded out of BΔ to ΔΓ and ΓΔ to EΔ.[9] But the (ratio) compounded out of AB to ΓE and BΓ to AE is the rectangle contained by AB, BΓ to (the ratio of) the rectangle contained by AE, EΓ,[10] while the (ratio) compounded out of BΔ to ΔΓ and ΓΔ to ΔE is BΔ to ΔE.[11] And so as is BΔ to ΔE, so is the rectangle contained by AB, BΓ to the rectangle contained by AE, EΓ.[12] Q.E.D.

(69) 2. *(Prop. 22)* The same thing another way.

Since the rectangle contained by AΔ, ΔΓ equals the rectangle contained by BΔ, ΔE,[1] in ratio[2] and taking whole to whole, therefore, as is AE to BΓ, so is AΔ to ΔB.[3] *Componendo,* as AE plus ΓB is to ΓB, so is AB to BΔ.[4] Hence the rectangle contained by AE plus ΓB and BΔ equals the rectangle contained by AB, BΓ.[5] Again, since as is AΔ to ΔB, so is EΔ to ΔΓ,[6] and hence all AE to all ΓB is as EΔ to ΔΓ,[7] therefore inverting[8] and *componendo*[9] (and area to area) the rectangle contained by AE plus ΓB and EΔ equals the rectangle contained by AE, EΓ.[10] But it has been proved that the rectangle contained by AE plus ΓB and BΔ also equals the rectangle contained by AB, BΓ. Hence inverting, as is the rectangle contained by AE plus ΓB and BΔ to the rectangle contained by AE plus ΓB and ΔE, that is, BΔ to ΔE, so is the rectangle contained by AB, BΓ to the rectangle contained by AE, EΓ.[11]

(68) ΔΙΩΡΙΣΜΕΝΗΣ ΤΟΜΗΣ ΠΡΩΤΟΝ

α΄ Λῆμμα χρήσιμον εἰς τὸ πρῶτον ἐπίταγμα τοῦ πέμπτου προβλήματος.

ἔστω εὐθεῖα ἡ ΑΒ *καὶ ἐπ' αὐτῆς τρία σημεῖα τὰ* Γ, Δ, Ε, *καὶ ἔστω τὸ ὑπὸ τῶν* ΑΔΓ *ἴσον τῶι ὑπὸ τῶν* ΒΔΕ. *ὅτι γίνεται ὡς ἡ* ΒΔ *πρὸς* ΔΕ, *οὕτως τὸ ὑπὸ τῶν* ΑΒΓ *πρὸς τὸ ὑπὸ τῶν* ΑΕΓ. *ἐπεὶ γὰρ τὸ ὑπὸ τῶν* ΑΔΓ *ἴσον ἐστὶν τῶι ὑπὸ* ΒΔΕ, *ἀνάλογον ἄρα ὡς ἡ* ΑΔ *πρὸς τὴν* ΔΒ *οὕτως ἡ* ΕΔ *πρὸς τὴν* ΔΓ. *καὶ ὅλη ἄρα ἡ* ΑΕ *πρὸς ὅλην τὴν* ΒΓ *ἐστὶν ὡς ἡ* ΕΔ *πρὸς* ΔΓ. *καὶ ἀνάπαλιν. πάλιν ἐπεὶ τὸ ὑπὸ τῶν* ΑΔΓ *ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΒΔΕ, *ἀνάλογον ἄρα ἐστὶν ὡς ἡ* ΑΔ *πρὸς τὴν* ΔΕ, *οὕτως ἡ* ΒΔ *πρὸς* ΔΓ. *καὶ ὅλη ἄρα ἡ* ΑΒ *πρὸς ὅλην τὴν* ΓΕ *ἐστὶν ὡς ἡ* ΒΔ *πρὸς* ΔΓ. *ἦν δὲ καὶ ὡς ἡ* ΒΓ *πρὸς τὴν* ΕΑ, *οὕτως ἡ* ΓΔ *πρὸς τὴν* ΔΕ. *ὥστε καὶ ὁ συνημμένος λόγος ἔκ τε τοῦ ὃν ἔχει ἡ* ΑΒ *πρὸς* ΓΕ *καὶ ἐξ οὗ ὃν ἔχει ἡ* ΒΓ *πρὸς* ΑΕ *ὁ αὐτός ἐστιν |τῶι ἔκ τε τοῦ ὃν ἔχει ἡ* ΒΔ *πρὸς* ΔΓ *καὶ ἡ* ΓΔ *πρὸς τὴν* ΕΔ. *ἀλλ' ὁ μὲν συνημμένος ἔκ τε* |131 *τοῦ ὃν ἔχει ἡ* ΑΒ *πρὸς* ΓΕ *καὶ ἐξ οὗ ὃν ἔχει ἡ* ΒΓ *πρὸς* ΑΕ *ὁ τοῦ ὑπὸ τῶν* ΑΒΓ *πρὸς τὸ ὑπὸ τῶν* ΑΕΓ *ἐστίν, ὁ δὲ συνημμένος ἔκ τε τοῦ ὃν ἔχει ἡ* ΒΔ *πρὸς* ΔΓ *καὶ ἐξ οὗ ἡ* ΓΔ *πρὸς* ΔΕ *ὁ τῆς* ΒΔ *πρὸς* ΔΕ *ἐστίν. καὶ ὡς ἄρα ἡ* ΒΔ *πρὸς* ΔΕ, *οὕτως τὸ ὑπὸ τῶν* ΑΒΓ *πρὸς τὸ ὑπὸ τῶν* ΑΕΓ. *ὅπερ:—*

(69) *β΄ ἄλλως τὸ αὐτό.*

ἐπεὶ τὸ ὑπὸ τῶν ΑΔΓ *ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΒΔΕ, *ἀνάλογον καὶ ὅλη πρὸς ὅλην, ἔστιν ἄρα ὡς ἡ* ΑΕ *πρὸς* ΒΓ *οὕτως ἡ* ΑΔ *πρὸς* ΔΒ. *συνθέντι ἐστὶν ὡς συναμφότερος ἡ* ΑΕ, ΓΒ *πρὸς* ΓΒ, *οὕτως ἡ* ΑΒ *πρὸς* ΒΔ. *τὸ ἄρα ὑπὸ συναμφοτέρου τῆς* ΑΕ, ΓΒ *καὶ τῆς* ΒΔ 706 *ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΑΒΓ. *πάλιν ἐπεί ἐστιν ὡς ἡ* ΑΔ *πρὸς τὴν* ΔΒ *οὕτως ἡ* ΕΔ *πρὸς τὴν* ΔΓ, *καὶ ὅλη ἄρα ἡ* ΑΕ *πρὸς ὅλην* *<τὴν>* ΓΒ *ἐστὶν ὡς ἡ* ΕΔ *πρὸς* ΔΓ. *ἀνάπαλιν καὶ συνθέντι τὸ ἄρα ὑπὸ συναμφοτέρου τῆς* ΑΕ, ΓΒ *καὶ τῆς* ΕΔ *ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΑΕΓ. *ἐδείχθη δὲ καὶ τὸ ὑπὸ συναμφοτέρου τῆς* ΑΕ, ΓΒ *καὶ τῆς* ΒΔ *ἴσον τῶι ὑπὸ τῶν* ΑΒΓ. *ἐναλλὰξ ἄρα γίνεται ὡς τὸ ὑπὸ συναμφοτέρου τῆς* ΑΕ, ΓΒ *καὶ τῆς* ΒΔ *πρὸς τὸ ὑπὸ συναμφοτέρου τῆς* ΑΕ, ΓΒ *καὶ τῆς* ΔΕ, *τουτέστιν ὡς ἡ* ΒΔ *πρὸς τὴν* ΔΕ, *οὕτως τὸ ὑπὸ τῶν* ΑΒΓ *πρὸς τὸ ὑπὸ τῶν* ΑΕΓ.

‖ 2 *α΄* mg A ‖ 19 *πρὸς* ΔΓ om A[1] *πρὸς* ΔΕΓ add A[2] supr, corr Co ‖ 22 *β΄* mg A ‖ 29 *τὴν* (ΓΒ) add Ge (S) ‖ 33 *τὸ* Co restituens lacunam in k *τοῦ* A

(70) 3. *(Prop. 23)* Another for the first assignment of the fifth problem, after the following two (theorems) have been proved.

Let AB equal ΓΔ, and an arbitrary (point) E on ΓΔ. That the rectangle contained by AΓ, ΓΔ equals the rectangle contained by AE, EΔ plus the rectangle contained by BE, EΓ.

Let BΓ be bisected at point Z.[1] Then the rectangle contained by AΓ, ΓΔ plus the square of ΓZ equals the square of ZΔ.[2] For the same reason, the rectangle contained by AE, EΔ plus the square of ZE equals the square of ZΔ *(II 5)*.[3] Hence the rectangle contained by AΓ, ΓΔ plus the square of ΓZ equals the rectangle contained by AE, EΔ plus the square of EZ,[4] that is (plus) the rectangle contained by BE, EΓ plus the square of ΓZ *(II 6)*.[5] Let the common square of ΓZ be subtracted. Therefore the remaining rectangle contained by AΓ, ΓΔ equals the rectangle contained by AE, EΔ plus the rectangle contained by BE, EΓ.[6]

(71) 4. *(Prop. 24)* *With the same things assumed, let point E be outside AΔ. That again the rectangle contained by BE, EΓ equals the rectangle contained by AΔ, ΔE plus the rectangle contained by BΔ, ΔΓ.

Again let BΓ be bisected at Z.[1] Then the rectangle contained by BE, EΓ plus the square of ΓZ equals the square of ZE *(II 6)*,[2] so that the rectangle contained by BE, EΓ plus the square of ΓZ equals the rectangle contained by AΔ, ΔE plus the square of ΔZ,[3] that is, (plus) the rectangle contained by BΔ, ΔΓ plus the square of ΓZ *(II 6)*.[4] Let the common square of ΓZ be subtracted. Then the remaining rectangle contained by BE, EΓ equals the rectangle contained by AΔ, ΔE plus the rectangle contained by BΔ, ΔΓ.[5] *

(72) 5. *(Prop. 25)* Now that these have been proved, to demonstrate that, if the rectangle contained by AB, BΓ equals the rectangle contained by ΔB, BE, then as is ΔB to BE, so is the rectangle contained by AΔ, ΔΓ to the rectangle contained by AE, EΓ.

For let ZA equal ΓE.[1] Then since the rectangle contained by AB, BΓ equals the rectangle contained by ΔB, BE,[2] add in common the rectangle contained by ZB, BE. Therefore all the rectangle contained by ΔZ, BE equals the rectangle contained by ZB, BE plus the rectangle contained by AB, BΓ.[3] But by the (lemma 7.70) that was written above, this is equal to the rectangle contained by ZΓ, ΓE,[4] that is to the rectangle contained by AE, EΓ.[5] Hence the rectangle contained by ZΔ, BE equals the rectangle contained by AE, EΓ. Introduce the rectangle contained by ZΔ, ΔE. Then, as is the rectangle contained by ZΔ, ΔE to the rectangle contained by ZΔ, BE, that is, as is EΔ to EB, so is the rectangle contained by ZΔ, ΔE to the

(70) <γ´> *ἄλλως εἰς τὸ πρῶτον ἐπίταγμα τοῦ πέμπτου προβλήματος, πρότερον προθεωρηθέντων τῶν ἑξῆς δύο.*

ἔστω ἴση ἡ ΑΒ *τῆι* ΓΔ, *καὶ ἐπὶ τῆς* ΓΔ *τυχὸν τὸ* Ε. *ὅτι τὸ ὑπὸ τῶν* ΑΓΔ *ἴσον ἐστὶν τῶι τε ὑπὸ τῶν* ΑΕΔ *καὶ τῶι ὑπὸ τῶν* ΒΕΓ. *τετμήσθω ἡ* ΒΓ *δίχα κατὰ τὸ* Ζ *σημεῖον. τὸ ἄρα ὑπὸ τῶν* ΑΓΔ *μετὰ τοῦ ἀπὸ τῆς* ΓΖ *ἴσον ἐστὶν τῶι ἀπὸ τῆς* ΖΔ. *διὰ ταὐτὰ δὴ καὶ τὸ ὑπὸ τῶν* ΑΕΔ *μετὰ τοῦ ἀπὸ τῆς* ΖΕ *τετραγώνου ἴσον ἐστὶν τῶι ἀπὸ τῆς* ΖΔ. *καὶ τὸ ὑπὸ τῶν* ΑΓΔ *ἄρα μετὰ τοῦ ἀπὸ τῆς* ΓΖ *ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΑΕΔ *καὶ τῶι ἀπὸ τῆς* ΕΖ *τετραγώνωι, τουτέστιν τῶι τε ὑπὸ τῶν* ΒΕΓ *καὶ τῶι ἀπὸ τῆς* ΓΖ *τετραγώνωι. καὶ κοινὸν ἀφῃρήσθω τὸ ἀπὸ τῆς* ΓΖ *τετράγωνον.* |*λοιπὸν ἄρα τὸ ὑπὸ τῶν* ΑΓΔ *ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΑΕΔ *καὶ* |131v *τῶι ὑπὸ τῶν* ΒΕΓ.

(71) *δ.´ τῶν αὐτῶν ὑποκειμένων, ἔστω τὸ* Ε *σημεῖον ἐκτὸς τῆς* ΑΔ. *ὅτι πάλιν τὸ ὑπὸ τῶν* ΒΕΓ *ἴσον τῶι ὑπὸ τῶν* ΑΔΕ *καὶ τῶι ὑπὸ τῶν* ΒΔΓ. *τετμήσθω πάλιν ἡ* ΒΓ *δίχα κατὰ τὸ* Ζ. *τὸ μὲν ἄρα ὑπὸ τῶν* ΒΕΓ *μετὰ τοῦ ἀπὸ* ΓΖ *ἴσον ἐστὶν τῶι ἀπὸ* ΖΕ, *ὥστε τὸ ὑπὸ* ΒΕΓ *μετὰ τοῦ ἀπὸ* ΓΖ *ἴσον ἐστὶν τῶι ὑπὸ* ΑΔΕ *μετὰ τοῦ ἀπὸ* ΔΖ, *τουτέστιν τοῦ ὑπὸ* ΒΔΓ *καὶ τοῦ ἀπὸ* ΓΖ. *κοινὸν* 708 *ἀφῃρήσθω τὸ ἀπὸ* ΓΖ. *λοιπὸν ἄρα τὸ ὑπὸ* ΒΕΓ *ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΑΔΕ *καὶ τῶι ὑπὸ* ΒΔΓ.

(72) *ε.´ τούτων προτεθεωρημένων, δεῖξαι ὅτι, ἐὰν τὸ ὑπὸ* ΑΒΓ *ἴσον τῶι ὑπὸ* ΔΒΕ, *γίνεται ὡς ἡ* ΔΒ *πρὸς* ΒΕ, *οὕτως τὸ ὑπὸ* ΑΔΓ *πρὸς τὸ ὑπὸ* ΑΕΓ. *κείσθω γὰρ τῆι* ΓΕ *ἴση ἡ* ΖΑ. *ἐπεὶ οὖν τὸ ὑπὸ* ΑΒΓ *ἴσον ἐστὶν τῶι ὑπὸ* ΔΒΕ, *κοινὸν προσκείσθω τὸ ὑπὸ* ΖΒΕ. *ὅλον ἄρα τὸ ὑπὸ* ΔΖ, ΒΕ *ἴσον ἐστὶν τῶι τε ὑπὸ τῶν* ΖΒΕ *καὶ τῶι ὑπὸ τῶν* ΑΒΓ. *ἀλλὰ ταῦτα διὰ τὸ προγεγραμμένον ἴσα ἐστὶν τῶι ὑπὸ τῶν* ΖΓΕ, *τουτέστιν τῶι ὑπὸ τῶν* ΑΕΓ. *καὶ τὸ ὑπὸ τῶν* ΖΔ, ΒΕ *ἄρα ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΑΕΓ. *ἔξωθεν τὸ ὑπὸ τῶν* ΖΔΕ. *ὡς ἄρα τὸ ὑπὸ τῶν* ΖΔΕ *πρὸς τὸ ὑπὸ τῶν* ΖΔ, ΒΕ, *τουτέστιν ὡς ἡ* ΕΔ *πρὸς* ΕΒ, *οὕτως τὸ ὑπὸ τῶν* ΖΔΕ *πρὸς τὸ ὑπὸ τῶν* ΑΕΓ. *συνθέντι ἐστὶν ὡς ἡ* ΔΒ *πρὸς* ΒΕ *οὕτως τὸ ὑπὸ τῶν* <ΖΔΕ *μετὰ τοῦ ὑπὸ τῶν* ΑΕΓ *πρὸς τὸ ὑπὸ τῶν*> ΑΕΓ. *ἀλλὰ τὸ ὑπὸ τῶν* ΖΔΕ *μετὰ τοῦ ὑπὸ τῶν* ΑΕΓ *διὰ τὸ προγεγραμμένον ἴσον ἐστὶν τῶι ὑπὸ τῶν* ΑΔΓ. *ἔστιν ἄρα ὡς ἡ* ΔΒ *πρὸς τὴν* ΒΕ, *οὕτως τὸ ὑπὸ τῶν* ΑΔΓ *πρὸς τὸ* <*ὑπὸ*> ΑΕΓ.

‖ 1 *γ´* add Hu (BS) ‖ 7 ΖΕ Co Ζ A | *τετραγώνου* Ge (S) *τετράγωνον* A ‖ 13 *τῶι* Ge (BS) *τὸ* A ‖ 14 *δ´* mg A ‖ 15 ΑΔΕ] ΑΕΔ Co ‖ 18 ΑΔΕ] ΑΕΔ Co ‖ 19 *τοῦ* Co *τὸ* A ‖ 21 ΑΔΕ] ΑΕΔ Co ‖ 22 *ε´* mg A ‖ 23 ΒΕ Co ΒΓ A ‖ 26 ΔΖ, ΒΕ Co ΔΖΒ A ‖ 30 ante ΖΔΕ add Δ A¹ del A² ‖ 32 ΑΕΓ Co restituens lacunam in k ΔΕΓ A ‖ 33 ΖΔΕ — *τὸ ὑπὸ τῶν* add Co ‖ 35 *τῶι* Ge (BS) *τῶν* A ‖ 36 *ὑπὸ* (ΑΕΓ) add Hu rectangle contained by AE, EΓ.[6] *Componendo,* as is ΔB to BE, so is the rectangle contained by <ZΔ, ΔE plus the rectangle contained by AE, EΓ to the rectangle contained by> AE, EΓ.[7] But by the (lemma 7.71) that was written above, the rectangle contained by ZΔ, ΔE plus the rectangle contained by AE, EΓ equals the rectangle contained by AΔ, ΔΓ.[8] Therefore as is ΔB to BE, so is the rectangle contained by AΔ, ΔΓ to the <rectangle contained by> AE, EΓ.[9]

(73) 6. *(Prop. 26)* If ABΓ is a triangle, and two (lines) AΔ, AE are drawn so that the angles BAΓ, ΔAE equal two right angles, then as is the rectangle contained by BΓ, ΓΔ to the rectangle contained by BE, EΔ, so is the square of ΓA to the square of AE.

For if I circumscribe a circle around triangle ABΔ, and EA and ΓA are produced to Z and H, then the rectangle contained by BΓ, ΓΔ turns into the rectangle contained by HΓ, ΓA,[1] while the rectangle contained by BE, EΔ (turns) into the rectangle contained by ZE, EA[2] *(III 36),* and it will be necessary, *alternando,* to find out whether, as is the rectangle contained by HΓ, ΓA to the square of ΓA, so is the rectangle contained by ZE, EA to the square of EA.[11] This is the same as finding out whether, as is HΓ to ΓA, so is ZE to EA.[10] Hence if it is, then HZ is parallel to BΓ *(VI 2)*; and in fact it is.[9] For since angles BAΓ, ΔAE equal two right angles,[3] angle ΔAE is therefore equal to angle BAH.[4] But angle ΔAE, outside the quadrilateral, equals angle ZBΔ,[5] while angle BAH equals angle BZH.[6] Thus angle ZBΔ equals angle BZH.[7] And they are alternate angles. Hence HZ is (parallel) to BΓ.[8] This is what was sought. Hence (the theorem) is valid.

(74) 7. *(Prop. 27)* The same thing another way.

In triangle ABΓ, let angles BAΓ, ΔAE equal two right angles. That as is the rectangle contained by BΓ, ΓΔ to the rectangle contained by BE, EΔ, so is the square of ΓA to the square of AE.

Let EZ be drawn through E,[1] parallel to AΓ. Then angle ΔAE equals angle AZE.[2] Therefore the rectangle contained by ZE, EH equals the square of AE.[3] Then since, as is AΓ to ZE, so is ΓB to BE,[4] while, as is ΓA to HE, so is ΓΔ to ΔE,[5] therefore the (ratio) compounded out of ΓA to ZE and ΓA to HE is the same as the (ratio) compounded out of ΓB to BE and ΓΔ to ΔE.[6] But the (ratio) compounded out of ΓA to ZE and ΓA to HE is that of the square of ΓA to the rectangle contained by ZE, HE,[7] that is to the square of AE,[8] *while the (ratio) compounded out of ΓB to BE and ΓΔ to ΔE is the same as that of the rectangle contained by BΓ, BE to the rectangle contained by ΓΔ, ΔE.[9] Hence as is the rectangle contained by ΓB, BE to the rectangle contained by ΓΔ, ΔE, so is the square of ΓA to the square of AE.[10] *

(73) <ϛ.΄> ἐὰν ἦι τρίγωνον τὸ ΑΒΓ καὶ δύο διαχθῶσιν αἱ ΑΔ, ΑΕ ὥστε τὰς ὑπὸ ΒΑΓ, ΔΑΕ γωνίας δυσὶν ὀρθαῖς ἴσας εἶναι, γίνεται ὡς τὸ ὑπὸ τῶν ΒΓΔ πρὸς τὸ ὑπὸ τῶν ΒΕΔ, οὕτως τὸ ἀπὸ ΓΑ πρὸς τὸ ἀπὸ ΑΕ. ἐὰν γὰρ περιγράψω κύκλον τῶι ΑΒΔ τριγώνωι καὶ ἐκβληθῶσιν αἱ ΕΑ, ΓΑ ἐπὶ τὰ Ζ, Η, μεταβαίνει τὸ μὲν ὑπὸ τῶν ΒΓΔ εἰς τὸ ὑπὸ τῶν ΗΓΑ, τὸ δὲ ὑπὸ τῶν ΒΕΔ εἰς τὸ
ὑπὸ τῶν ΖΕΑ, καὶ δεήσει ἐναλ|λὰξ ζητῆσαι εἰ ὡς τὸ ὑπὸ τῶν |132
ΗΓΑ πρὸς τὸ ἀπὸ τῆς ΓΑ, οὕτως τὸ ὑπὸ ΖΕΑ πρὸς τὸ ἀπὸ τῆς ΕΑ.
τοῦτο δὲ ταὐτόν ἐστιν τῶι ζητεῖν εἰ ἔστιν ὡς ἡ ΗΓ πρὸς τὴν 710
ΓΑ, οὕτως ἡ ΖΕ πρὸς τὴν ΕΑ. εἰ ἄρα ἐστὶν, ἡ ΗΖ παράλληλός ἐστιν τῆι ΒΓ. ἔστιν δέ. ἐπεὶ γὰρ αἱ ὑπὸ ΒΑΓ, ΔΑΕ γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσίν, ἴση ἐστὶν ἡ ὑπὸ ΔΑΕ γωνία τῆι ὑπὸ ΒΑΗ γωνίαι. ἀλλὰ ἡ μὲν ὑπὸ ΔΑΕ ἴση ἐστὶν τῆι ὑπὸ ΖΒΔ ἐκτὸς τετραπλεύρου. ἡ δὲ ὑπὸ ΒΑΗ γωνία ἴση ἐστὶν τῆι ὑπὸ ΒΖΗ. καὶ ἡ ὑπὸ ΖΒΔ ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ ΒΖΗ γωνίαι. καὶ εἰσὶν ἐναλλὰξ. <παράλληλος> ἄρα ἐστὶν ἡ ΗΖ τῆι ΒΓ. τοῦτο δὲ ἐζητεῖτο. μένει ἄρα:—

(74) <ζ΄> ἄλλως τὸ αὐτό.
ἔστωσαν ἐν τριγώνωι τῶι ΑΒΓ αἱ ὑπὸ ΒΑΓ, ΔΑΕ γωνίαι δυσὶν ὀρθαῖς ἴσαι. ὅτι γίνεται ὡς τὸ ὑπὸ ΒΓΔ πρὸς τὸ ὑπὸ ΒΕΔ, <οὕτως> τὸ ἀπὸ ΓΑ πρὸς τὸ ἀπὸ ΑΕ. ἤχθω διὰ τοῦ Ε τῆι ΑΓ παράλληλος ἡ ΕΖ. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΔΑΕ γωνία τῆι ὑπὸ ΑΖΕ γωνίαι. ἴσον ἄρα ἐστὶν τὸ ὑπὸ τῶν ΖΕΗ τῶι ἀπὸ ΑΕ. ἐπεὶ οὖν ἐστιν ὡς μὲν ἡ ΑΓ πρὸς τὴν ΖΕ, οὕτως ἡ ΓΒ πρὸς ΒΕ, ὡς δὲ ἡ ΓΑ πρὸς ΗΕ, οὕτως ἡ ΓΔ πρὸς ΔΕ, ὁ ἄρα συνημμένος ἔκ τε τοῦ τῆς ΓΑ πρὸς ΖΕ καὶ ἐκ τοῦ τῆς ΓΑ πρὸς ΗΕ ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΓΒ πρὸς ΒΕ καὶ τοῦ τῆς ΓΔ πρὸς ΔΕ. ἀλλ' ὁ μὲν συνημμένος ἔκ τε τοῦ τῆς ΓΑ πρὸς ΖΕ καὶ τοῦ τῆς ΓΑ πρὸς ΗΕ ὁ τοῦ ἀπὸ ΓΑ ἐστὶν πρὸς τὸ ὑπὸ ΖΕ, ΗΕ, τουτέστιν πρὸς τὸ ἀπὸ ΑΕ, ὁ δὲ συνημμένος ἔκ τε τοῦ τῆς ΓΒ πρὸς ΒΕ καὶ
τοῦ τῆς ΓΔ πρὸς ΔΕ ὁ αὐτός ἐστιν τῶι τοῦ ὑπὸ ΒΓ, ΒΕ πρὸς τὸ 712
ὑπὸ ΓΔ, ΔΕ. ἔστιν ἄρα ὡς τὸ ὑπὸ τῶν ΓΒΕ πρὸς τὸ ὑπὸ ΓΔΕ, οὕτως τὸ ἀπὸ ΓΑ πρὸς τὸ ἀπὸ ΑΕ.

‖ 1 ϛ΄ add Hu (BS) | αἱ] ὡς Ge (recc?) ‖ 4 περιγράψω] περιγράψωμεν Ge (recc?) ‖ 8 ΕΑ Co ΘΕΑ A ‖ 9 ταὐτόν Hu τὸ αὐτόν A ‖ 10 παράλληλός — ἔστιν δέ] παράλληλος τῆι ΒΓ, γίνεται ὡς ἡ ΗΓ πρὸς τὴν ΓΑ, οὕτως ἡ ΖΕ πρὸς τὴν ΕΑ. ἔστι δέ Co ‖ 16 παράλληλος add Co ‖ 17 ἐζητεῖτο μένει] εζητεῖ το μεν εἰ A ἐζητοῦμεν εἰ Ge (S) ‖ 18 ζ΄ add Hu (BS) ‖ 19 ἔστωσαν Hu (CV) ἔστω A ‖ 21 οὕτως add Hu οὕτω Ge ‖ 25 συνημμένος Ge (BS) συνημμένης A ‖ 26 ΓΑ Co ΓΔ A ‖ 31 ΒΓ, ΒΕ] ΒΓ, ΓΔ Co ‖ 32 ΓΔ, ΔΕ] ΒΔ, ΔΕ A ΒΕ, ΔΕ Co | ΓΒΕ] ΒΓΔ Co | ΓΔΕ] ΒΕΔ Co ‖ 33 ΓΑ Co ΓΔ A

(75) 8. *(Prop. 28)* Again, let both angles BAE, ΓΑΔ be right. That as is the rectangle contained by ΒΓ, ΓΕ to the rectangle contained by ΒΔ, ΔΕ, so is the square of ΓΑ to the square of ΑΔ.

Let ZH be drawn through Δ, parallel to ΑΓ,[1] and where it meets AE, let point H be. Hence angle ΑΔΖ is right.[2] But angle ZAH too is right.[3] Hence the rectangle contained by ΖΔ, ΔΗ equals the square of ΔΑ.[4] Therefore as is the square of ΓΑ to the square of ΑΔ, so is the square of ΓΑ to the rectangle contained by ΖΔ, ΔΗ.[5] But the ratio of the square of ΑΓ to the rectangle contained by ΖΔ, ΔΗ is compounded out of ΓΑ to ΔΗ, that is ΓΕ to ΕΔ, and ΓΑ to ΖΔ, that is ΓΒ to ΒΔ.[6] But the ratio compounded out of ΓΕ to ΕΔ and ΓΒ to ΒΔ is the same as that of the rectangle contained by ΒΓ, ΓΕ to the rectangle contained by ΒΔ, ΔΕ.[7] Thus as is the rectangle contained by ΒΓ, ΓΕ to the rectangle contained by ΒΔ, ΔΕ, so is the square of ΓΑ to the square of ΑΔ.[8]

(76) 9. *(Prop. 29)* This being so, the lemma written above in another way, namely that as is ΒΔ to ΔΕ, so is the rectangle contained by AB, ΒΓ to the rectangle contained by AE, ΕΓ. From Δ let an arbitrary line be drawn, ΔΖ, and make the square of ΔΖ equal the rectangle contained by ΑΔ, ΔΓ, and join AZ, ΓΖ, EZ, and BZ.

Then since the rectangle contained by ΑΔ, ΔΓ equals the square of ΔΖ,[1] therefore angle ΓΖΔ equals angle A.[2] Again, since the rectangle contained by ΒΔ, ΔΕ equals the square of ΔΖ,[3] therefore angle ΔΖΕ equals angle B.[4] But angle ΓΖΔ equals angle A too. Therefore all angle ΓΖΕ equals angles A and B.[5] But angles A, B plus angle AZB equal two right angles.[6] Hence angles AZB and ΓΖΕ equal two right angles.[7] But by the lemma (7.74) written above, as is the square of BZ to the square of ZE, so is the rectangle contained by AB, ΒΓ to the rectangle contained by AE, ΕΓ.[8] But as is the square of BZ to the square of ZE, so is ΒΔ to ΔΕ,[10] since the rectangle contained by ΒΔ, ΔΕ equals the square of ΔΖ.[9] Therefore as is ΒΔ to ΔΕ, so is the rectangle contained by AB, ΒΓ to the rectangle contained by AE, ΕΓ.[11]

(75) <η´> ἔστω πάλιν ἑκατέρα τῶν ὑπὸ τῶν ΒΑΕ, ΓΑΔ γωνία ὀρθή· ὅτι γίνεται ὡς τὸ ὑπὸ τῶν ΒΓΕ πρὸς τὸ ὑπὸ τῶν ΒΔΕ, οὕτως τὸ ἀπὸ ΓΑ πρὸς τὸ ἀπὸ ΑΔ. ἤχθω διὰ τοῦ Δ τῆι ΑΓ παράλ|ληλος ἡ ΖΗ, καὶ καθ᾽ ὃ συμπίπτει τῆι ΑΕ ἔστω τὸ Η |132v σημεῖον. ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΑΔΖ. ὀρθὴ δὲ καὶ ἡ ὑπὸ ΖΑΗ. τὸ ἄρα ὑπὸ ΖΔΗ ἴσον ἐστὶν τῶι ἀπὸ ΔΑ τετραγώνωι. ἔστιν ἄρα ὡς τὸ ἀπὸ ΓΑ πρὸς τὸ ἀπὸ ΑΔ, <οὕτως> τὸ ἀπὸ ΓΑ πρὸς τὸ ὑπὸ ΖΔΗ. ἀλλὰ ὁ τοῦ ἀπὸ ΑΓ πρὸς τὸ ὑπὸ ΖΔΗ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΓΑ πρὸς ΔΗ, τουτέστιν ἡ ΓΕ πρὸς ΕΔ, καὶ τοῦ ὃν ἔχει ἡ ΓΑ πρὸς ΖΔ, τουτέστιν ἡ ΓΒ πρὸς ΒΔ. ὁ δὲ συνημμένος λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΓΕ πρὸς ΕΔ καὶ ἐκ τοῦ ὃν ἔχει ἡ ΓΒ πρὸς ΒΔ ὁ αὐτός ἐστιν τῶι τοῦ ὑπὸ ΒΓΕ πρὸς τοῦ ὑπὸ ΒΔΕ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΒΓΕ πρὸς τὸ ὑπὸ ΒΔΕ, οὕτως τὸ ἀπὸ ΓΑ τετράγωνον πρὸς τὸ ἀπὸ ΑΔ τετράγωνον.

(76) <θ´> τούτου ὄντος, ἄλλως τὸ προγεγραμμένον λῆμμα, ὅτι γίνεται ὡς ἡ ΒΔ πρὸς τὴν ΔΕ, οὕτως τὸ ὑπὸ τῶν ΑΒΓ πρὸς τὸ ὑπὸ τῶν ΑΕΓ. ἀνήχθω ἀπὸ τοῦ Δ τυχοῦσά τις εὐθεῖα ἡ ΔΖ, καὶ τῶι ὑπὸ τῶν ΑΔΓ ἴσον ὑποκείσθω τὸ ἀπὸ τῆς ΔΖ, καὶ ἐπεζεύχθωσαν αἱ ΑΖ, ΓΖ, ΕΖ, ΒΖ. ἐπεὶ οὖν τὸ ὑπὸ τῶν ΑΔΓ ἴσον ἐστὶν τῶι ἀπὸ τῆς ΔΖ, γωνία ἄρα ἡ ὑπὸ τῶν ΓΖΔ ἴση ἐστὶν τῆι Α γωνίαι. πάλιν ἐπεὶ τὸ ὑπὸ τῶν ΒΔΕ ἴσον ἐστὶν τῶι ἀπὸ τῆς 714 ΔΖ, γωνία ἄρα ἡ ὑπὸ τῶν ΔΖΕ γωνίαι τῆι Β ἴση ἐστίν. ἀλλὰ καὶ ἡ ὑπὸ ΓΖΔ γωνία ἴση ἐστὶν τῆι Α. ὅλη ἄρα ἡ ὑπὸ τῶν ΓΖΕ ἴση ἐστὶν ταῖς Α, Β γωνίαις. ἀλλὰ αἱ Α, Β μετὰ τῆς ὑπὸ ΑΖΒ γωνίας δυσὶν ὀρθαῖς ἴσαι εἰσίν. καὶ αἱ ὑπὸ ΑΖΒ, ΓΖΕ ἄρα γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσί. γίνεται δὴ διὰ τὸ προγεγραμμένον λῆμμα ὡς τὸ ἀπὸ ΒΖ πρὸς τὸ ἀπὸ ΖΕ, οὕτως τὸ ὑπὸ ΑΒΓ πρὸς τὸ ὑπὸ ΑΕΓ. ἀλλ᾽ ὡς τὸ ἀπὸ ΒΖ πρὸς τὸ ἀπὸ ΖΕ, οὕτως ἐστὶν ἡ ΒΔ πρὸς ΔΕ (ἴσον γὰρ ἐστιν τὸ ὑπὸ ΒΔΕ τῶι ἀπὸ ΔΖ), καὶ ὡς ἄρα ἡ ΒΔ πρὸς ΔΕ, οὕτως ἐστὶν τὸ ὑπὸ τῶν ΑΒΓ πρὸς τὸ ὑπὸ τῶν ΑΕΓ.

‖ 1 η´ add Hu (V) | ΒΑΕ Co ΒΔΕ A ‖ 2 ΒΔΕ Co ΔΒΕ A ‖ 7 οὕτως add Hu οὕτω Ge ‖ 8 ἀλλὰ - ὑπὸ bis A^1 uncis secl. A^2 ‖ 10 ΓΑ Co ΓΔ A ‖ 12 ΒΔ Co ΒΑ A ‖ 15 θ´ add Hu (V) ‖ 19 ΑΖ Co ΔΖ A ‖ 27 οὕτως τὸ ὑπὸ ΑΒΓ – πρὸς τὸ ἀπὸ ΖΕ bis A del Co ‖ 29 ἴσον – ΔΖ del Co | γὰρ Simson$_1$ ἄρα A

(77) 10. *(Prop. 30)* Lemma useful for the second assignment of the same problem.

Again, having the rectangle contained by AΔ, ΔE equal to the rectangle contained by BΔ, ΔΓ, to show that, as is BΔ to ΔΓ, so is the rectangle contained by AB, BE to the rectangle contained by EΓ, ΓA.

For since, as is BΔ to ΔE, so is AΔ to ΔΓ,[1] therefore also BA to ΓE is as BΔ to ΔE.[2] Again, since, as is BΔ to ΔA, so is EΔ to ΔΓ,[3] therefore the remainder BE to the remainder AΓ is as EΔ to ΔΓ.[4] But also, as BΔ to ΔE, so was AB to ΓE. Hence the ratio composed out of BΔ to ΔE and EΔ to ΔΓ, which is BΔ to ΔΓ,[6] is the same as the (ratio) compounded out of AB to ΓE and EB to AΓ,[5] which is the same as the (ratio) of the rectangle contained by AB, BE to the rectangle contained by EΓ, ΓA.[7] Therefore, as is BΔ to ΔΓ, so is the rectangle contained by AB, BE to the rectangle contained by EΓ, ΓA.[8] Q.E.D.

(78) 11. *(Prop. 30)* The same thing another way.

Since, as is AΔ to ΔB, so is ΓΔ to ΔE,[1] therefore the remainder AΓ to the remainder EB is as AΔ to ΔB.[2] *Componendo*, as AΓ plus EB is to EB, so is AB to BΔ.[3] Hence the rectangle contained by AΓ plus EB and BΔ equals the rectangle contained by AB, BE.[4] Again, since as is BΔ to ΔA, so is EΔ to ΔΓ,[5] therefore the remainder BE to the remainder ΓA is as one of the ratios, namely as EΔ to ΔΓ.[6] *Componendo*, as EB plus AΓ is to AΓ, so is EΓ to ΓΔ.[7] Therefore the rectangle contained by EB plus AΓ and ΓΔ equals the rectangle contained by EΓ, ΓA.[8] But it has been shown that the rectangle contained by AΓ plus EB and BΔ <equals the rectangle contained by AB, BE. Thus as the rectangle contained by AΓ plus EB and BΔ> is to the rectangle contained by AΓ plus EB and ΓΔ, that is as is BΔ to ΔΓ, so is the rectangle contained by AB, BE to the rectangle contained by EΓ, ΓA.[9] Q.E.D.

(77) <ι.´> λῆμμα χρήσιμον εἰς τὸ β´ ἐπίταγμα τοῦ αὐτοῦ προβλήματος.

πάλιν ὄντος |ἴσου τοῦ ὑπὸ τῶν ΑΔΕ τῶι ὑπὸ ΒΔΓ, δεῖξαι |133 ὅτι γίνεται ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως τὸ ὑπὸ τῶν ΑΒΕ πρὸς τὸ ὑπὸ τῶν ΕΓΑ. ἐπεὶ γάρ ἐστιν ὡς ἡ ΒΔ πρὸς τὴν ΔΕ, οὕτως ἡ ΑΔ πρὸς ΔΓ, καὶ ὅλη ἄρα ἡ ΒΑ πρὸς ὅλην τὴν ΓΕ ἐστὶν ὡς ἡ ΒΔ πρὸς τὴν ΔΕ. πάλιν ἐπεί ἐστιν ὡς ἡ ΒΔ πρὸς τὴν ΔΑ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΓ, λοιπὴ ἄρα ἡ ΒΕ πρὸς λοιπὴν τὴν ΑΓ ἐστὶν ὡς ἡ ΕΔ πρὸς τὴν ΔΓ. ἦν δὲ καὶ ὡς ἡ ΒΔ πρὸς τὴν ΔΕ, οὕτως ἡ ΑΒ πρὸς τὴν ΓΕ. καὶ ὁ συγκείμενος ἄρα λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΒΔ πρὸς τὴν ΔΕ καὶ ἐξ οὗ ὃν ἔχει ἡ ΕΔ πρὸς τὴν ΔΓ, ὅς ἐστιν ὁ τῆς ΒΔ πρὸς τὴν ΔΓ, ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΑΒ πρὸς τὴν ΓΕ καὶ τοῦ τῆς ΕΒ πρὸς τὴν ΑΓ, ὅς ἐστιν ὁ αὐτὸς τῶι τοῦ ὑπὸ τῶν ΑΒΕ πρὸς τὸ ὑπὸ τῶν ΕΓΑ. ἔστιν <ἄρα> ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως τὸ ὑπὸ τῶν ΑΒΕ πρὸς τὸ ὑπὸ τῶν ΕΓΑ. ὅ(περ): —

(78) <ια.´> ἄλλως τὸ αὐτό. ἐπεί ἐστιν ὡς ἡ ΑΔ πρὸς τὴν 716 ΔΒ, οὕτως ἡ ΓΔ πρὸς τὴν ΔΕ, λοιπὴ ἄρα ἡ ΑΓ πρὸς λοιπὴν τὴν ΕΒ ἐστὶν ὡς ἡ ΑΔ πρὸς τὴν ΔΒ. καὶ συνθέντι ἐστὶν ὡς συναμφότερος ἡ ΑΓ, ΕΒ πρὸς τὴν ΕΒ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΔ. τὸ ἄρα ὑπὸ συναμφοτέρου τῆς ΑΓ, ΕΒ καὶ τῆς ΒΔ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΒΕ. πάλιν ἐπεί ἐστιν ὡς ἡ ΒΔ πρὸς τὴν ΔΑ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΓ, λοιπὴ ἄρα ἡ ΒΕ πρὸς λοιπὴν τὴν ΓΑ ἐστὶν ὡς εἷς τῶν λόγων, ὡς ἡ ΕΔ πρὸς τὴν ΔΓ. καὶ συνθέντι ἐστὶν ὡς συναμφότερος ἡ ΕΒ, ΑΓ πρὸς τὴν ΑΓ, οὕτως ἡ ΕΓ πρὸς τὴν ΓΔ. τὸ ἄρα ὑπὸ συναμφοτέρου τῆς ΕΒ, ΑΓ καὶ τῆς ΓΔ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΕΓΑ. ἐδείχθη δὲ καὶ τὸ ὑπὸ συναμφοτέρου τῆς ΑΓ, ΕΒ καὶ τῆς ΒΔ <ἴσον τῶι ὑπὸ τῶν ΑΒΕ, καὶ ὡς ἄρα τὸ ὑπὸ συναμφοτέρου τῆς ΑΓ, ΕΒ καὶ τῆς ΒΔ> πρὸς τὸ ὑπὸ συναμφοτέρου τῆς ΑΓ, ΕΒ καὶ τῆς ΓΔ, τουτέστιν ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως τὸ ὑπὸ τῶν ΑΒΕ πρὸς τὸ ὑπὸ τῶν ΕΓΑ. ὅπερ: —

‖ 1 ι´ add Hu (V) ‖ 15 ἄρα add Co | ΔΓ Co ΔΕ A ‖ 16 ΕΓΑ ὅπερ Ge ΕΓΑΟ A ‖ 17 ια´ add Hu (BS) ‖ 18 τὴν (ΕΒ) Ge (BS) τῆς A ‖ 24 ὡς εἷς τῶν λόγων om Co ‖ 28 ἴσον — καὶ τῆς ΒΔ add Co ‖ 31 ΕΓΑ Co ΕΓΔ A

(79) 12. *(Prop. 31)* The same thing another way, after the following has first been proved.

With AB equal to ΓΔ, if some point E is taken, to prove that the rectangle contained by AE, EΔ equals the rectangle contained by AΓ, ΓΔ plus the rectangle contained by BE, EΓ.

Let BΓ be bisected at point Z. Then the rectangle contained by AE, EΔ plus the square of EZ equals the square of ΔZ.[2] But the rectangle contained by AΓ, ΓΔ plus the square of ΓZ equals the square of ΔZ.[3] Hence the rectangle contained by AE, EΔ plus the square of EZ equals the rectangle contained by AΓ, ΓΔ plus the square of ΓZ,[4] that is, (plus) the rectangle contained by BE, EΓ plus the square of EZ.[5] Let the common square of EZ be subtracted. Then the remaining rectangle contained by AE, EΔ equals the rectangle contained by AΓ, ΓΔ plus the rectangle contained by BE, EΓ.[6]

(80) 13. *(Prop. 32)* Now that this has been demonstrated beforehand, let the rectangle contained by AB, BΓ be equal to the rectangle contained by ΔB, BE. That, as is ΔB to BE, so is the rectangle contained by <AΔ, ΔΓ to the rectangle contained by> AE, EΓ.

Let AZ be made equal to ΓΔ.[1] But according to the (lemma 7.79) that was written above, the rectangle contained by ZB, BΔ equals the rectangle contained by ZΓ, ΓΔ plus the rectangle contained by AB, BΓ.[2] But since the rectangle contained by AB, BΓ equals the rectangle contained by ΔB, BE,[3] let each be subtracted from the rectangle contained by ZB, BΔ. Then the remaining rectangle contained by ZΓ, ΓΔ, which is the rectangle contained by AΔ, ΔΓ,[5] equals the rectangle contained by ΔB, ZE.[4] Again, since the rectangle contained by AB, BΓ equals the rectangle contained by ΔB, BE,[6] in ratio[7] and *separando*, as is AE to EB, so is ΔΓ to ΓB,[8] that is ZA to BΓ.[9] Hence all ZE is to all EΓ as is AE to EB.[10] Thus the rectangle contained by ZE, EB equals the rectangle contained by ΓE, EA.[11] But it was shown that the rectangle contained by ZE, BΔ is equal to the rectangle contained by AΔ, ΔΓ.[12] Therefore *alternando*, as is the rectangle contained by ZE, BΔ to the rectangle contained by ZE, EB, that is, as ΔB to BE, so is the rectangle contained by AΔ, ΔΓ to the rectangle contained by AE, EΓ.[13]

(81) 14. *(Prop. 33)* After the following has first been proved, the same thing will be proved in another way.

Let ABΓ be a triangle, and let there be drawn inside it AΔ, AE making both angles BAE, ΓAΔ right angles. That, as is the rectangle contained by BΓ, ΓE to the rectangle contained by BΔ, ΔE, so is the square of ΓA to the square of AΔ.

(79) <ιβ΄> ἄλλως τὸ αὐτό, προθεωρηθέντος τοῦδε.

οὔσης ἴσης τῆς ΑΒ τῆι ΓΔ, ἐὰν ληφθῆι τι σημεῖον τὸ Ε, δεῖξαι ὅτι ἴσον ἐστὶ τὸ ὑπὸ τῶν ΑΕΔ τῶι <τε> ὑπὸ τῶν ΑΓΔ καὶ τῶι ὑπὸ ΒΕΓ. τετμήσθω ἡ ΒΓ δίχα κατὰ τὸ Ζ σημεῖον. τὸ μὲν ἄρα ὑπὸ ΑΕΔ μετὰ |τοῦ ἀπὸ ΕΖ ἴσον ἐστὶν τῶι ἀπὸ ΔΖ. τὸ δ' ὑπὸ ΑΓΔ μετὰ τοῦ ἀπὸ ΓΖ ἴσον ἐστὶν τῶι ἀπὸ ΔΖ. ὥστε καὶ |133v τὸ ὑπὸ ΑΕΔ μετὰ τοῦ ἀπὸ ΕΖ τετραγώνου ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΓΔ μετὰ τοῦ ἀπὸ ΓΖ, τουτέστιν τοῦ ὑπὸ ΒΕΓ μετὰ τοῦ ἀπὸ ΕΖ. κοινὸν ἀφῃρήσθω τὸ ἀπὸ ΕΖ τετράγωνον. λοιπὸν ἄρα τὸ ὑπὸ ΑΕΔ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΓΔ καὶ τῶι ὑπὸ ΒΕΓ.

(80) <ιγ΄> τούτου προτεθεωρημένου, ἔστω τὸ ὑπὸ τῶν ΑΒΓ ἴσον τῶι ὑπὸ τῶν ΔΒΕ. ὅτι ἐστὶν ὡς ἡ ΔΒ πρὸς τὴν ΒΕ, οὕτως τὸ ὑπὸ <τῶν ΑΔΓ πρὸς τὸ ὑπὸ τῶν> ΑΕΓ. κείσθω τῆι ΓΔ ἴση ἡ ΑΖ. διὰ δὴ τὸ προγεγραμμένον γίνεται τὸ ὑπὸ τῶν ΖΒΔ ἴσον 718 τῶι τε ὑπὸ ΖΓΔ καὶ τῶι ὑπὸ ΑΒΓ. ἐπεὶ δὲ τὸ ὑπὸ τῶν ΑΒΓ ἴσον ἐστὶν τῶι [τὸ] ὑπὸ τῶν ΔΒΕ, ὁπότερα ἀφῃρήσθω ἀπὸ τοῦ ὑπὸ τῶν ΖΒΔ. λοιπὸν ἄρα τὸ ὑπὸ τῶν ΖΓΔ, ὅ ἐστιν τὸ ὑπὸ ΑΔΓ, ἴσον ἐστὶν τῶι ὑπὸ τῶν ΔΒ, ΖΕ. πάλιν ἐπεὶ τὸ ὑπὸ τῶν ΑΒΓ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΔΒΕ, ἀνάλογον καὶ διελόντι ὡς ἡ ΑΕ πρὸς τὴν ΕΒ, οὕτως ἡ ΔΓ πρὸς ΓΒ ἐστίν, τουτέστιν ἡ ΖΑ πρὸς τὴν ΒΓ. καὶ ὅλη ἄρα ἡ ΖΕ πρὸς ὅλην τὴν ΕΓ ἐστὶν ὡς ἡ ΑΕ πρὸς τὴν ΕΒ. τὸ ἄρα ὑπὸ τῶν ΖΕΒ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΓΕΑ. ἐδείχθη δὲ καὶ τὸ ὑπὸ τῶν ΖΕ, ΒΔ ἴσον τῶι ὑπὸ τῶν ΑΔΓ. ἐναλλὰξ ἄρα ἐστὶν ὡς τὸ ὑπὸ τῶν ΖΕ, ΒΔ πρὸς τὸ ὑπὸ τῶν ΖΕΒ, τουτέστιν ὡς ἡ ΔΒ πρὸς ΒΕ, οὕτως τὸ ὑπὸ τῶν ΑΔΓ πρὸς τὸ ὑπὸ τῶν ΑΕΓ.

(81) <ιδ΄> προθεωρηθέντος καὶ τοῦδε, ἄλλως τὸ αὐτὸ δειχθήσεται. [ἔσται] ἔστω τρίγωνον τὸ ΑΒΓ καὶ διήχθωσαν ἐντὸς αἱ ΑΔ, ΑΕ ποιοῦσαι ἑκατέραν τῶν ὑπὸ ΒΑΕ, ΓΑΔ γωνιῶν ὀρθήν. ὅτι γίνεται ὡς τὸ ὑπὸ τῶν ΒΓΕ πρὸς τὸ ὑπὸ τῶν ΒΔΕ, οὕτως τὸ ἀπὸ ΓΑ τετράγωνον πρὸς τὸ ἀπὸ ΑΔ τετράγωνον. περιγεγράφθω περὶ τὸ ΑΒΕ τρίγωνον κύκλος ὁ ΑΒΖΗ καὶ ἐπεζεύχθω ἡ ΖΗ. ἐπεὶ οὖν ὀρθή ἐστιν ἑκατέρα τῶν ὑπὸ ΒΑΕ, ΓΑΔ γωνιῶν, διάμετρός ἐστιν ἑκατέρα τῶν ΒΕ, ΖΗ τοῦ κύκλου. ὥστε κέντρον ἐστὶν τὸ Θ. ἐπεὶ οὖν ἴση ἐστὶν ἡ ΖΘ τῆι ΘΗ,

‖ 1 ιβ΄ add Hu (BS) | τοῦδε Ge (BS) τοῦ ΔΕ A ‖ 2 ἐν A^1 α supr A^2 ‖ 3 τὸ Ge (BS) τῶι A ‖ 11 ιγ΄ add Hu (BS) ‖ 12 τῶι ὑπὸ τῶν ΔΒΕ Ge (Co) τῶν ὑπὸ τῶν ΔΒ A ‖ 13 τῶν ΑΔΓ πρὸς τὸ ὑπὸ τῶν add Co ‖ 16 τὸ del Ge (BS) | ὁπότερα] ἑκάτερον Hu app ‖ 25 ΑΔΓ Co ΑΛΓ A ‖ 26 ιδ΄ add Hu (BS) | προθεωρηθέντος] -ρη- A^2 (alia manu?) in ras. | αὐτὸ] προγεγραμμένον Hu app ‖ 27 ἔσται om Ge (BS) ‖ 28 γωνιῶν] γωνίαν A ‖ 31 περιγεγράφθω Ge (BS) περγεγράφθω A ‖ 33 γωνιῶν] γωνία A

Let there be circumscribed around triangle ΑΒΕ a circle ΑΒΖΗ, and let ΖΗ be joined. Then since each of angles ΒΑΕ, ΓΑΔ is right,[1] therefore ΒΕ and ΖΗ are both diameters of the circle.[2] Hence Θ is the center.[3] Then since ΖΘ equals ΘΗ,[4] therefore as is ΑΓ to ΓΗ, so is ΑΔ to ΔΖ (lemma 81.1);[5] and by inversion (ΖΔ to ΔΑ is as ΗΓ to ΓΑ).[6] But as is ΓΗ to ΓΑ, so is the rectangle contained by ΑΓ, ΓΗ to the square of ΓΑ, that is the rectangle contained by ΒΓ, ΓΕ to the square of ΓΑ;[7] while as is ΖΔ to ΔΑ, so is the rectangle contained by ΖΔ, ΔΑ to the square of ΔΑ, that is, the rectangle contained by ΒΔ, ΔΕ to the square of ΔΑ.[8] Hence *alternando*, as is the rectangle contained by ΒΓ, ΓΕ to the rectangle contained by ΒΔ, ΔΕ, so is the square of ΓΑ to the square of ΑΔ.[9] Q.E.D.

(82) 15. *(Prop. 34)* This being true, the (lemma) that was written above, in another way, namely that, as is ΒΔ to ΔΓ, so is the rectangle contained by ΑΒ, ΒΕ to the rectangle contained by ΑΓ, ΓΕ.

Let ΔΖ be erected from Δ, at right angles to ΑΒ, and let the square of ΔΖ be made equal to either of the rectangles contained by ΑΔ, ΔΕ, or by ΒΔ, ΔΓ,[1] and let ΑΖ, ΖΓ, ΖΕ, ΖΒ be joined. Then angles ΑΖΕ, ΓΖΒ are both right.[2] But according to the (lemma 7.81) that was written above, as is the rectangle contained by ΑΒ, ΒΕ to the rectangle contained by ΑΓ, ΓΕ, that is to the rectangle contained by ΕΓ, ΓΑ, so is the square of ΒΖ to the square of ΖΓ.[3] But as is the square of ΒΖ to the square of ΖΓ, so is ΒΔ to ΔΓ.[4] Hence as is ΒΔ to ΔΓ, so is the rectangle contained by ΑΒ, ΒΕ to the rectangle contained by ΑΓ, ΓΕ.[5]

(83) 16. *(Prop. 35a)* For the first assignment of the sixth problem. (Let) ΑΒ be a straight line, and on it (let there be) three points Γ, Δ, Ε, and let the rectangle contained by ΑΒ, ΒΕ equal the rectangle contained by ΓΒ, ΒΔ. That, as is ΑΒ to ΒΕ, so is the rectangle contained by ΔΑ, ΑΓ to the rectangle contained by ΓΕ, ΕΔ.

For since the rectangle contained by ΑΒ, ΒΕ equals the rectangle contained by ΓΒ, ΒΔ,[1] therefore in ratio[2] and remainder to remainder[3] and *convertendo*[4] (and inverting) as is the excess of ΑΓ over ΕΔ to ΑΓ, so is ΒΑ to ΑΔ.[5] Hence the rectangle contained by the excess of ΑΓ over ΕΔ and ΑΒ equals the rectangle contained by ΔΑ, ΑΓ.[6] Again, since as is ΑΕ to ΕΔ, so is ΓΒ:ΒΕ,[7] therefore remainder ΑΓ to remainder ΔΕ is as ΓΒ to ΒΕ.[8] *Separando*, as is the excess of ΑΓ over ΕΔ to ΔΕ, so is ΓΕ to ΕΒ.[9] Thus the rectangle contained by the excess of ΑΓ over ΔΕ and ΕΒ equals the rectangle contained by ΓΕ, ΕΔ.[10] But it was shown also that the (rectangle contained by) the (excess) of ΑΓ over ΕΔ and ΑΒ equals the rectangle contained by ΔΑ, ΑΓ.[11] Hence *alternando*, as is the rectangle contained by the excess of ΑΓ over ΔΕ and ΑΒ to the rectangle contained by the excess of ΑΓ over ΔΕ and ΒΕ, that is, as is ΑΒ to ΒΕ, so is the rectangle contained by ΔΑ to ΑΓ to the rectangle contained by ΓΕ to ΕΔ.[12]

ἔστιν ἄρα ὡς ἡ ΑΓ πρὸς τὴν ΓΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΖ. καὶ 720
|ἀνάπαλιν. ἀλλ' ὡς μὲν ἡ ΓΗ πρὸς τὴν ΓΑ, οὕτως ἐστὶν τὸ ὑπὸ |134
τῶν ΑΓΗ πρὸς τὸ ἀπὸ τῆς ΓΑ, τουτέστιν τὸ ὑπὸ ΒΓΕ πρὸς τὸ
ἀπὸ ΓΑ. ὡς δὲ ἡ ΖΔ πρὸς τὴν ΔΑ, οὕτως ἐστὶν τὸ ὑπὸ τῶν ΖΔΑ
πρὸς τὸ ἀπὸ ΔΑ, τουτέστιν τὸ ὑπὸ ΒΔΕ πρὸς τὸ ἀπὸ ΔΑ.
ἐναλλὰξ ἄρα γίνεται ὡς τὸ ὑπὸ ΒΓΕ πρὸς τὸ ὑπὸ ΒΔΕ, οὕτως τὸ
ἀπὸ ΓΑ τετράγωνον πρὸς τὸ ἀπὸ ΑΔ τετράγωνον. ὅ(περ): —

(82) <ιε´> τούτου ὄντος, ἄλλως τὸ προγεγραμμένον, ὅτι γίνεται ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως τὸ ὑπὸ ΑΒΕ πρὸς τὸ ὑπὸ ΑΓΕ. ἀνήχθω ἀπὸ τοῦ Δ τῆι ΑΒ ὀρθὴ ἡ ΔΖ, καὶ ὁποτέρωι τῶν ὑπὸ ΑΔΕ, ΒΔΓ ἴσον κείσθω τὸ ἀπὸ ΔΖ τετράγωνον, καὶ ἐπεζεύχθωσαν αἱ ΑΖ, ΖΓ, ΖΕ, ΖΒ. ὀρθὴ ἄρα ἐστὶν ἑκατέρα τῶν ὑπὸ τῶν ΑΖΕ, ΓΖΒ γωνιῶν. διὰ δὴ τὸ προγεγραμμένον γίνεται ὡς τὸ ὑπὸ τῶν ΑΒΕ πρὸς τὸ ὑπὸ τῶν ΑΓΕ, τουτέστιν πρὸς τὸ ὑπὸ τῶν ΕΓΑ, οὕτως τὸ ἀπὸ ΒΖ πρὸς τὸ ἀπὸ ΖΓ. ὡς δὲ τὸ ἀπὸ ΒΖ πρὸς τὸ ἀπὸ ΖΓ, οὕτως ἐστὶν ἡ ΒΔ πρὸς τὴν ΔΓ. καὶ ὡς ἄρα ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως ἐστὶν τὸ ὑπὸ τῶν ΑΒΕ πρὸς τὸ ὑπὸ ΑΓΕ.

(83) <ις´> εἰς τὸ πρῶτον ἐπίταγμα τοῦ ς´ προβλήματος.
εὐθεῖα ἡ ΑΒ καὶ ἐπ' αὐτῆς τρία σημεῖα τὰ Γ, Δ, Ε, καὶ ἔστω τὸ
ὑπὸ τῶν ΑΒΕ ἴσον τῶι ὑπὸ τῶν ΓΒΔ. ὅτι γίνεται ὡς ἡ ΑΒ πρὸς
τὴν ΒΕ, οὕτως τὸ ὑπὸ ΔΑΓ πρὸς τὸ ὑπὸ τῶν ΓΕΔ. ἐπεὶ γὰρ τὸ
ὑπὸ τῶν ΑΒΕ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΓΒΔ, ἀνάλογον καὶ λοιπὸν
πρὸς λοιπόν, καὶ ἀναστρέψαντι, ἔστιν ἄρα ὡς ἡ τῶν ΑΓ, ΕΔ 722
ὑπεροχὴ πρὸς τὴν ΑΓ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΔ. τὸ ἄρα ὑπὸ
<τῆς> τῶν ΑΓ, ΕΔ ὑπεροχῆς καὶ τῆς ΑΒ ἴσον ἐστὶν τῶι ὑπὸ
τῶν ΔΑΓ. πάλιν ἐπεί ἐστιν ὡς ἡ ΑΕ πρὸς τὴν ΕΔ, οὕτως ἡ ΓΒ
πρὸς τὴν ΒΕ, λοιπὴ ἄρα ἡ ΑΓ πρὸς λοιπὴν τὴν ΔΕ ἐστὶν ὡς ἡ ΓΒ
πρὸς τὴν ΒΕ. διελόντι ἐστὶν ὡς ἡ τῶν |ΑΓ, ΕΔ ὑπεροχὴ πρὸς |134v
τὴν ΔΕ, οὕτως ἡ ΓΕ πρὸς τὴν ΕΒ. τὸ ἄρα ὑπὸ τῆς τῶν ΑΓ, ΔΕ
ὑπεροχῆς καὶ τῆς ΕΒ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΓΕΔ. ἐδείχθη δὲ
καὶ <τὸ ὑπὸ> τῆς τῶν ΑΓ, ΕΔ <ὑπεροχῆς> καὶ τῆς ΑΒ ἴσον τῶι
ὑπὸ τῶν ΔΑΓ. ἐναλλὰξ ἄρα ἐστὶν ὡς τὸ ὑπὸ τῆς τῶν ΑΓ, ΔΕ
ὑπεροχῆς καὶ τῆς ΑΒ πρὸς τὸ ὑπὸ τῆς τῶν ΑΓ, ΔΕ ὑπεροχῆς καὶ
τῆς ΒΕ, τουτέστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΕ, οὕτως τὸ ὑπὸ ΔΑΓ πρὸς
τὸ ὑπὸ ΓΕΔ.

‖ 4 τὸ ὑπὸ τῶν ΖΔΑ — τουτέστιν bis A del Co ‖ 5 ΒΔΕ Co ΒΑΕ A ‖ 6 τὸ ὑπὸ ΒΓΕ πρὸς bis A del Co ‖ 8 ιε´ add Hu (BS) ‖ 9 ΑΒΕ Co ΑΒΓ A ‖ 10 ὁποτέρωι] ἑκατέρωι Hu app ‖ 12 post ΖΓ add. ζδ supr A² alia manu ‖ 13 γωνιῶν] γωνία A ‖ 14 τουτέστιν — ΕΓΑ del Hu ‖ 18 ις´ add Hu (BS) ‖ 23 ΕΔ Co ΕΒ A ‖ 24 ΒΑ πρὸς τὴν ΑΔ] ΔΑ πρὸς τὴν ΑΒ Co | post τὸ ἄρα add τῆς Ge ‖ 25 τῆς add Hu | ΕΔ Co ΕΒ A ‖ 26 ΑΕ πρὸς τὴν ΕΔ] ΑΓ πρὸς τὴν ΕΔ Co ΑΒ πρὸς τὴν ΒΔ Hu (V) ‖ 31 τὸ ὑπὸ add Ge (V) | ὑπεροχῆς add Ge

(84) 17. *(Prop. 35a)* The same thing another way, by means of compounded ratio.

For since, as is AB to BΓ, so is ΔB to BE,[1] therefore remainder AΔ to remainder ΓE is as AB to BΓ.[2] Again, since, as is AB to BΔ, so is ΓB to BE,[3] therefore remainder AΓ to remainder ΔE is as ΓB to BE.[4] Hence the (ratio) compounded out of AB to BΓ and ΓB to BE, which is AB to BE,[6] is the same as the (ratio) compounded out of AΔ to ΓE and AΓ to ΔE,[5] which is the same as the (ratio) of the rectangle contained by ΔA, AΓ to the rectangle contained by ΓE, EΔ.[7]

(85) 18. *(Prop. 35b)* Another way. Let there be described on AE a semicircle AZE, and let BZ be drawn tangent, and let AZ, <ΓZ>, ΔZ, EZ be joined.

Then since BZ is tangent, and BΔ cuts (the circle), the rectangle contained by AB, BE equals the square of BZ *(III 36)*.[1] But the rectangle contained by AB, BE is assumed to be equal to the rectangle contained by ΓB, BΔ.[2] Hence the rectangle contained by ΓB, BΔ equals the square of BZ.[3] Thus angle BZΔ equals angle BΓZ.[4] But out of these, angle BZE equals angle ZAΓ.[5] Therefore remaining angle ΔZE equals remaining angle AZΓ.[6] Thus, as is the rectangle contained by ΔA, AΓ to the rectangle contained by ΓE, EΔ, so is the square of AZ to the square of ZE (lemma 85.1).[7] But as is the square of AZ to the square of ZE, so is AB to BE.[8] Hence, as is AB to BE, so is the rectangle contained by ΔA, AΓ to the rectangle contained by ΓE, EΔ.[9]

(86) 19. *(Prop. 36a)* Lemma for the third assignment of the sixth problem. Again, with the rectangle contained by AB, BE equal to the rectangle contained by ΓB, BΔ, to prove that, as is ΓB to BΔ, so is the rectangle contained by AΓ, ΓE to the rectangle contained by AΔ, ΔE.

For since, as is AB to BΔ, so is ΓB to BE,[1] therefore remainder AΓ to remainder ΔE is as one of the other (ratios), as ΓB to BE.[2] For the same reasons, also remainder AΔ to remainder ΓE is as ΔB to BE;[3] and also by inversion (as is BE to BΔ, so is ΓE to ΔA).[4] Hence the ratio compounded out of ΓB to BE and EB to BΔ, which is the same as ΓB to BΔ, is the same as the (ratio) compounded out of AΓ to ΔE and EΓ to ΔA,[5] which is the (ratio) of the rectangle contained by AΓ, ΓE to the rectangle contained by AΔ, ΔE. Thus, as is ΓB to BΔ, so is the rectangle contained by AΓ, ΓE to the rectangle contained by AΔ, ΔE.[6]

(84) <ιζ.´> ἄλλως τὸ αὐ<τὸ διὰ> τοῦ συνημμένου.

ἐπεί ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΒ πρὸς τὴν ΒΕ, λοιπὴ ἄρα ἡ ΑΔ πρὸς λοιπὴν τὴν ΓΕ ἐστὶν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ. πάλιν ἐπεί ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΔ, οὕτως ἡ ΓΒ πρὸς τὴν ΒΕ, λοιπὴ ἄρα ἡ ΑΓ πρὸς λοιπὴν τὴν ΔΕ ἐστὶν ὡς ἡ ΓΒ πρὸς τὴν ΒΕ. ὥστε ὁ συνημμένος ἔκ τε τοῦ τῆς ΑΒ πρὸς ΒΓ καὶ τοῦ τῆς ΓΒ πρὸς ΒΕ, ὅς ἐστιν ὁ τῆς ΑΒ πρὸς ΒΕ, ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΑΔ πρὸς ΓΕ καὶ τοῦ τῆς ΑΓ πρὸς ΔΕ, ὅς ἐστιν ὁ αὐτὸς τῶι τοῦ ὑπὸ ΔΑΓ πρὸς τὸ ὑπὸ ΓΕΔ.

(85) <ιη.´> ἄλλως.

γεγράφθω ἐπὶ τῆς ΑΕ ἡμικύκλιον τὸ ΑΖΕ, καὶ ἤχθω ἐφαπτομένη ἡ ΒΖ, καὶ ἐπεζεύχθωσαν αἱ ΑΖ, <ΓΖ> ΔΖ, ΕΖ. ἐπεὶ οὖν ἐφάπτεται μὲν ἡ ΒΖ, τέμνει δὲ ἡ ΒΔ, τὸ ὑπὸ τῶν ΑΒΕ ἴσον ἐστὶν τῶι ἀπὸ ΒΖ. ἀλλὰ τὸ ὑπὸ ΑΒΕ τῶι ὑπὸ ΓΒΔ ἴσον 724 ὑπόκειται. καὶ τὸ ὑπὸ ΓΒΔ ἄρα ἴσον ἐστὶν τῶι ἀπὸ ΒΖ τετραγώνωι. ὥστε ἴση ἐστὶν ἡ ὑπὸ τῶν ΒΖΔ γωνία τῆι ὑπὸ ΒΓΖ γωνίαι. ὧν ἡ ὑπὸ ΒΖΕ γωνία ἴση ἐστὶν τῆι ὑπὸ ΖΑΓ γωνίαι. λοιπὴ ἄρα ἡ ὑπὸ ΔΖΕ γωνία λοιπῆι τῆι ὑπὸ ΑΖΓ γωνίαι ἴση ἐστίν. ὡς ἄρα τὸ ὑπὸ τῶν ΔΑΓ πρὸς τὸ ὑπὸ τῶν ΓΕΔ, οὕτως ἐστὶν τὸ ἀπὸ ΑΖ πρὸς τὸ ἀπὸ ΖΕ. ὡς δὲ τὸ ἀπὸ ΑΖ πρὸς τὸ ἀπὸ ΖΕ, οὕτως ἐστὶν ἡ ΑΒ πρὸς τὴν ΒΕ. ὡς ἄρα ἡ ΑΒ πρὸς τὴν ΒΕ, οὕτως ἐστὶν τὸ ὑπὸ ΔΑΓ πρὸς τὸ ὑπὸ ΓΕΔ.

(86) <ιθ.´> λῆμμα εἰς τὸ τρίτον ἐπίταγμα τοῦ ἕκτου προβλήματος.

ὄντος πάλιν ἴσου τοῦ ὑπὸ τῶν ΑΒΕ τῶι ὑπὸ τῶν ΓΒΔ, δεῖξαι ὅτι γίνεται ὡς ἡ ΓΒ πρὸς ΒΔ, οὕτως τὸ ὑπὸ τῶν ΑΓΕ πρὸς τὸ ὑπὸ τῶν ΑΔΕ. ἐπεὶ |γάρ ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΔ, οὕτως ἡ ΓΒ |135 πρὸς τὴν ΒΕ, λοιπὴ ἄρα ἡ ΑΓ πρὸς λοιπὴν τὴν ΔΕ ἐστιν ὡς εἷς τῶν λοιπῶν, ὡς ἡ ΓΒ πρὸς τὴν ΒΕ. διὰ ταὐτὰ καὶ λοιπὴ ἡ ΑΔ πρὸς λοιπὴν τὴν ΓΕ ἐστιν ὡς ἡ ΔΒ πρὸς τὴν ΒΕ. καὶ ἀνάπαλιν. ὥστε ὁ συνημμένος λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΓΒ πρὸς τὴν ΒΕ καὶ ἐξ οὗ ὃν ἔχει ἡ ΕΒ πρὸς τὴν ΒΔ, ὅς ἐστιν ὁ αὐτὸς τῶι τῆς ΓΒ πρὸς τὴν ΒΔ, ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ ὃν ἔχει ἡ ΑΓ πρὸς τὴν ΔΕ καὶ ἡ ΕΓ πρὸς τὴν ΔΑ, ὅς ἐστιν τοῦ ὑπὸ τῶν ΑΓΕ πρὸς τὸ ὑπὸ ΑΔΕ. ἔστιν ἄρα ὡς ἡ ΓΒ πρὸς τὴν ΒΔ, οὕτως τὸ ὑπὸ τῶν ΑΓΕ πρὸς τὸ ὑπὸ τῶν ΑΔΕ.

‖ 1 ιζ´ add Hu (BS) | (αὐ)τὸ διὰ add Co ‖ 7 ὅς Ge (S) ὁ A ‖ 9 ὅς] ὁ A^1 σ add supr A^2 alia manu ‖ 10 ιη´ add Hu (BS) ‖ 12 ΓΖ add Co ‖ 13 ἐφάπτεται Ge (BS) ἐφάπτηται A | ΒΔ] ΒΑ Co ‖ 18 ΑΖΓ A^1 ut uidetur, Co ΔΖΓ A^2 ut uidetur ‖ 23 ιθ´ add Hu (BS) | τρίτον... ἕκτου $Simson_1$ πρῶτον... πρώτου A ‖ 28 ὡς εἷς τῶν λοιπῶν del Hu ‖ 31 λόγος Ge (recc?) λοιπὸς A | ὃν ἔχει ἡ ΓΒ πρὸς τὴν ΒΕ καὶ ἐξ οὗ bis A del Co ‖ 33 ΒΔ Co ΒΕ A ‖ 34 ΕΓ... ΔΑ] ΒΓ... ΒΔ A ΓΕ... ΑΔ Co

(87) 20. *(Prop. 36a)* The same thing another way.

Since, as is AB to BΔ, so is ΓB to BE,[1] remainder AΓ to remainder ΔE is as ΓB to BE.[2] *Convertendo,* as is AΓ to the excess of AΓ over ΔE, so is ΓB to ΓE.[3] Therefore the rectangle contained by AΓ, ΓE equals the rectangle contained by the excess of AΓ over ΔE and BΓ.[4] Again, since remainder AΓ to remainder ΔE is as AB to BΔ,[5] *separando,* as is the excess of AΓ over ΔE to ΔE, so is ΔA to ΔB.[6] Hence the rectangle contained by AΔ, ΔE equals the rectangle contained by the excess of AΓ over ΔE and ΔB.[7] Thus, as is the rectangle contained by the excess of AΓ over ΔE and <BΓ to the rectangle contained by the excess of AΓ over ΔE and> ΔB, that is, as is ΓB to BΔ, so is the rectangle contained by AΓ, ΓE to the rectangle contained by AΔ, ΔE.[8] Q.E.D.

(88) 21. *(Prop. 36b)* The same thing another way. Let there be described on ΓB a semicircle ΓZΔ, let BZ be drawn tangent, and let AZ, <ΓZ>, ΔZ, <EZ> be joined.

Then since the rectangle contained by AB, BE equals the rectangle contained by ΓB, BΔ,[1] but the rectangle contained by ΓB, BΔ equals the square of the tangent BZ,[2] therefore the rectangle contained by AB, BE too equals the square of BZ.[3] Thus angle BZE equals angle A.[4] But also all angle BZΔ equals angle ZΓB.[5] Therefore remaining angle EZΔ equals remaining angle AZΓ.[6] Hence, as is the square of ΓZ to the square of ZΔ, so is the rectangle contained by AΓ, ΓE to the rectangle contained by AΔ, ΔE (lemma 85.1).[7] But as is the square of ΓZ to the square of ZΔ, so is ΓB to BΔ.[8] And therefore, as is ΓB to BΔ, so is the rectangle contained by AΓ, ΓE to the rectangle contained by AΔ, ΔE.[9]

(89) 22. *(Prop. 37)* (Let) AB be a straight line, and on it (let there be) two points Γ, Δ, and, as is AB to BΓ, so let the square of AΔ be to the square of ΔΓ. That the rectangle contained by AB, BΓ equals the square of BΔ.

Let ΔE be made equal to ΓΔ.[1] Then *separando,* as is AΓ to ΓB, so is the rectangle contained by ΓA, AE to the square of ΓΔ,[2] that is to the rectangle contained by EΔ, ΔΓ.[3] But as is AΓ to ΓB, so, when AE is taken as a common height, is the rectangle contained by ΓA, AE to the rectangle contained by AE, ΓB.[4] Hence, as is the rectangle contained by ΓA, AE to the rectangle contained by EΔ, ΔΓ, <so is the rectangle contained by ΓA, AE to the rectangle contained by AE, ΓB.>[5] Thus the rectangle contained by AE, ΓB equals the rectangle contained by EΔ, ΔΓ.[6] In ratio[7] and

(87) <κ.´> ἄλλως τὸ αὐτό.

ἐπεί ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΔ, οὕτως ἡ ΓΒ πρὸς τὴν ΒΕ, λοιπὴ ἡ ΑΓ πρὸς λοιπὴν τὴν ΔΕ ἐστὶν ὡς ἡ ΓΒ πρὸς τὴν ΒΕ. ἀναστρέψαντί ἐστιν ὡς ἡ ΑΓ πρὸς τὴν <τῶν> ΑΓ, ΔΕ ὑπεροχήν, οὕτως [ἐστὶν] ἡ ΓΒ πρὸς τὴν ΓΕ. τὸ ἄρα ὑπὸ τῶν ΑΓΕ ἴσον ἐστὶν τῶι ὑπὸ τῆς τῶν ΑΓ, ΔΕ ὑπεροχῆς καὶ τῆς ΒΓ. πάλιν 726
ἐπεὶ λοιπὴ ἡ ΑΓ πρὸς λοιπὴν τὴν ΔΕ γίνεται ὡς ἡ ΑΒ πρὸς τὴν ΒΔ, διελόντι ὡς ἡ τῶν ΑΓ, ΔΕ ὑπεροχὴ πρὸς τὴν ΔΕ, οὕτως ἡ ΔΑ πρὸς τὴν ΔΒ. τὸ ἄρα ὑπὸ τῶν ΑΔΕ ἴσον ἐστὶν τῶι ὑπὸ τῆς τῶν ΑΓ, ΔΕ ὑπεροχῆς καὶ τῆς ΔΒ. ὡς ἄρα τὸ ὑπὸ τῆς τῶν ΑΓ, ΔΕ ὑπεροχῆς καὶ τῆς <ΒΓ πρὸς τὸ ὑπὸ τῶν ΑΓ, ΔΕ ὑπεροχῆς καὶ τῆς> ΔΒ, τουτέστιν ὡς ἡ ΓΒ πρὸς τὴν ΒΔ, οὕτως τὸ ὑπὸ ΑΓΕ πρὸς τὸ ὑπὸ ΑΔΕ. ὅ(περ): —

(88) <κα.´> ἄλλως τὸ αὐτό.

γεγράφθω ἐπὶ τῆς ΓΔ ἡμικύκλιον τὸ ΓΖΔ, ἐφαπτομένη ἤχθω ἡ ΒΖ, καὶ ἐπεζεύχθωσαν αἱ ΑΖ, <ΓΖ>, ΔΖ, <ΕΖ>. ἐπεὶ οὖν τὸ ὑπὸ ΑΒΕ ἴσον ἐστὶν τῶι ὑπὸ ΓΒΔ, ἀλλὰ τὸ ὑπὸ ΓΒΔ ἴσον ἐστὶ τῶι ἀπὸ τῆς ἐφαπτομένης τῆς ΒΖ, καὶ τὸ ὑπὸ τῶν ΑΒΕ ἄρα ἴσον ἐστὶν τῶι ἀπὸ τῆς ΒΖ. γωνία ἄρα ἡ ὑπὸ ΒΖΕ γωνίαι τῆι Α ἴση ἐστίν. ἀλλὰ καὶ ὅλη ἡ ὑπὸ ΒΖΔ τῆι ὑπὸ ΖΓΒ ἴση ἐστίν. λοιπὴ ἄρα ἡ ὑπὸ ΕΖΔ γωνία λοιπῆι τῆι ὑπὸ τῶν ΑΖΓ ἴση ἐστίν. ὡς ἄρα τὸ ἀπὸ ΓΖ πρὸς τὸ ἀπὸ ΖΔ, οὕτως ἐστὶν τὸ ὑπὸ ΑΓΕ πρὸς τὸ ὑπὸ ΑΔΕ. ὡς δὲ τὸ ἀπὸ ΓΖ πρὸς τὸ ἀπὸ ΖΔ, οὕτως ἐστὶν ἡ ΓΒ πρὸς τὴν ΒΔ. καὶ ὡς ἄρα ἡ ΓΒ πρὸς τὴν ΒΔ, οὕτως ἐστὶν τὸ ὑπὸ ΑΓΕ πρὸς τὸ ὑπὸ ΑΔΕ.

(89) <κβ.´> εὐθεῖα ἡ ΑΒ καὶ ἐπ' αὐτῆς δύο σημεῖα τὸ Γ, Δ, ἔστω δὲ |ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΔΓ. |135v
ὅτι τὸ ὑπὸ τῶν ΑΒΓ ἴσον ἐστὶν τῶι ἀπὸ τῆς ΒΔ. κείσθω τῆι ΓΔ ἴση ἡ ΔΕ. διελόντι ἄρα γίνεται ὡς ἡ ΑΓ πρὸς τὴν ΓΒ, οὕτως τὸ ὑπὸ ΓΑΕ πρὸς τὸ ἀπὸ ΓΔ, τουτέστιν πρὸς τὸ ὑπὸ ΕΔΓ. ὡς δὲ ἡ ΑΓ πρὸς τὴν ΓΒ, οὕτως ἐστί, κοινοῦ ὕψους παραληφθείσης 728
τῆς ΑΕ, τὸ ὑπὸ τῶν ΓΑΕ πρὸς τὸ ὑπὸ τῶν ΑΕ, ΓΒ. ἔστιν ἄρα ὡς τὸ ὑπὸ τῶν ΓΑΕ πρὸς τὸ ὑπὸ τῶν ΕΔΓ, <οὕτως τὸ ὑπὸ τῶν ΓΑΕ πρὸς τὸ ὑπὸ τῶν ΑΕ, ΓΒ.> ἴσον ἄρα ἐστὶν τὸ ὑπὸ τῶν ΑΕ, ΓΒ τῶι

‖ 1 κ´ add Hu (BS) ‖ 4 τῶν add Ge (V) | ΑΓ, ΔΕ Co ΑΔ, ΓΕ A ‖ 5 ἐστιν secl Hu ‖ 10 ΑΓ, ΔΕ Co ΑΒΓΔΕ A ‖ 11 ΒΓ — ὑπεροχῆς καὶ τῆς add Co ‖ 13 Α̅Δ̅Ε̅Ο̅ A ΑΔΕ ὅπερ Ge (V) ‖ 14 κα´ add Hu (BS) ‖ 16 καὶ om A^1 signum supr A^2 | ΓΖ et ΕΖ add Co ‖ 26 κβ´ add Hu (BS) ‖ 27 ΒΓ Co ΒΔ A ‖ 30 ΓΑΕ Co ΓΔ A ‖ 31 κοινοῦ ὕψους Co κοινὸν ὕψος A ‖ 33 ΕΔΓ οὕτως — ΑΕ, ΓΒ] οὕτω τὸ ὑπὸ τῶν ΓΑΕ πρὸς τὸ ὑπὸ τῶν ΑΕ, ΓΒ add Co. ante ΕΔΓ add οὕτως τὸ ὑπὸ τῶν ΓΑΕ πρὸς τὸ ὑπὸ τῶν V^2 (Co) *componendo*, as is AΔ to ΔE, that is to ΔΓ, so is ΔB to BΓ.[8] [9] Therefore all AB to all BΔ is as ΔB to BΓ.[10] Thus the rectangle contained by AB, BΓ equals the <square of> BΔ.[11] Q.E.D.

(90) 23. *(Prop. 38)* Again, as is AB to BΓ, so let the square of AΔ be to the square of ΔΓ. That the rectangle contained by AB, BΓ equals the square of BΔ.

Let ΔE be made equal to ΓΔ.[1] Then *separando*, as AΓ is to ΓB, that is, as the rectangle contained by EA, AΓ is to the rectangle contained by EA, BΓ, so is the rectangle contained by EA, AΓ to the rectangle contained by ΓΔ, ΔE.[2] [3] Hence the rectangle contained by AE, BΓ equals the rectangle contained by ΓΔ, ΔE.[4] In ratio[5] and *separando*, *as AΔ is to ΔE, that is to ΔΓ, so is AΓ to ΓB.[6] And thus remainder ΓB is to remainder ΔB as AΓ to ΓB.*[7] Therefore the rectangle contained by AB, BΓ equals the square of BΔ.[8]

(91) 24. *(Prop. 39a)* (Let) AB be a straight line, and on it (let there be) three points Γ, Δ, E, and, as is the rectangle contained by BA, AE to the rectangle contained by BΔ, ΔE, so let the square of AΓ be to the square of ΓΔ. That, as is the rectangle contained by AB, BΔ to the rectangle contained by AE, EΔ, so is the square of BΓ to the square of ΓE.

Let the point of equation Z be taken, so that the rectangle contained by AZ, ZΔ equals the rectangle contained by BZ, ZE.[1] Then, as is AZ to ΔZ, so is the rectangle contained by BA, AE to the rectangle contained by BΔ, ΔE;[2] for this is a lemma in the *Determinate (Section*, cf. 7.68). But as is the rectangle contained by BA, AE to the rectangle contained by BΔ, ΔE, so is the square of AΓ to the square of ΓΔ.[3] Therefore, as is AZ to ZΔ, so is the square of AΓ to the square of ΓΔ.[4] Hence the rectangle contained by AZ, ZΔ, that is the rectangle contained by BZ, ZE, equals the square of ZΓ.[5] [6] Thus, as is BZ to ZE, so is the square of BΓ to the square of ΓE.[7] But as is BZ to ZE, so is the rectangle contained by AB, BΔ to the rectangle contained by AE, EΔ.[8] And thus, as is the rectangle contained by AB, BΔ to the rectangle contained by AE, EΔ, so is the square of BΓ to the square of ΓE.[9]

ὑπὸ τῶν ΕΔΓ. ἀνάλογον καὶ συνθέντι ἐστὶν ὡς ἡ ΑΔ πρὸς τὴν ΔΕ, τουτέστιν πρὸς τὴν ΔΓ, οὕτως ἡ ΔΒ πρὸς τὴν ΒΓ. καὶ ὅλη <ἄρα> ἡ ΑΒ πρὸς ὅλην τὴν ΒΔ ἐστὶν ὡς ἡ ΔΒ πρὸς τὴν ΒΓ. τὸ ἄρα ὑπὸ τῶν ΑΒΓ ἴσον ἐστὶν τῶι <ἀπὸ> ΒΔ. ὅπερ:—

(90) <κγ.´> ἔστω δὴ πάλιν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΔΓ. ὅτι γίνεται ἴσον τὸ ὑπὸ ΑΒΓ τῶι ἀπὸ ΒΔ τετραγώνωι. κείσθω τῆι ΓΔ ἴση ἡ ΔΕ. κατὰ διαίρεσιν ἄρα γίνεται ὡς ἡ ΑΓ πρὸς τὴν ΓΒ, τουτέστιν ὡς τὸ ὑπὸ τῶν ΕΑΓ πρὸς τὸ ὑπὸ τῶν ΕΑ, ΒΓ, οὕτως τὸ ὑπὸ τῶν ΕΑΓ πρὸς τὸ ὑπὸ τῶν ΓΔΕ. ἴσον ἄρα ἐστὶν τὸ ὑπὸ τῶν ΑΕ, ΒΓ τῶι ὑπὸ τῶν ΓΔΕ. ἀνάλογον καὶ διελόντι ἐστὶν ὡς ἡ ΑΔ πρὸς τὴν ΔΕ, τουτέστιν πρὸς τὴν ΔΓ, οὕτως ἡ ΑΓ πρὸς τὴν ΓΒ. καὶ λοιπὴ ἄρα ἡ ΓΒ πρὸς λοιπὴν τὴν ΔΒ ἐστὶν ὡς ἡ ΑΓ πρὸς τὴν ΓΒ. τὸ ἄρα ὑπὸ ΑΒΓ ἴσον ἐστὶν τῶι ἀπὸ ΒΔ τετραγώνωι.

(91) <κδ.´> εὐθεῖα ἡ ΑΒ καὶ ἐπ' αὐτῆς τρία σημεῖα τὰ Γ, Δ, Ε, ἔστω δὲ ὡς τὸ ὑπὸ ΒΑΕ πρὸς τὸ ὑπὸ ΒΔΕ, οὕτως τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΓΔ. ὅτι γίνεται καὶ ὡς τὸ ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΕΔ, οὕτως τὸ ἀπὸ ΒΓ πρὸς τὸ ἀπὸ ΓΕ. εἰλήφθω γὰρ ἰσότητος σημεῖον τὸ Ζ, ὥστε ἴσον εἶναι τὸ ὑπὸ τῶν ΑΖΔ τῶι ὑπὸ ΒΖΕ. ἔστιν ἄρα ὡς ἡ ΑΖ πρὸς τὴν ΔΖ, οὕτως τὸ ὑπὸ ΒΑΕ πρὸς τὸ ὑπὸ ΒΔΕ (λῆμμα γὰρ ἐν Διωρισμένηι). ὡς δὲ τὸ ὑπὸ ΒΑΕ πρὸς τὸ 730
ὑπὸ ΒΔΕ, οὕτως ἐστὶν τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΓΔ. καὶ ὡς ἄρα ἡ ΑΖ πρὸς τὴν ΖΔ, οὕτως τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΓΔ. τὸ ἄρα ὑπὸ ΑΖΔ, τουτέστιν τὸ ὑπὸ ΒΖΕ, ἴσον ἐστὶν τῶι ἀπὸ ΖΓ. ἔστιν ἄρα ὡς ἡ ΒΖ πρὸς τὴν ΖΕ, οὕτως τὸ ἀπὸ ΒΓ πρὸς τὸ ἀπὸ ΓΕ. ὡς δὲ ἐστιν ἡ ΒΖ πρὸς τὴν ΖΕ, οὕτως ἐστὶν τὸ ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΕΔ. καὶ ὡς ἄρα τὸ ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΕΔ, οὕτως |ἐστιν τὸ |136
ἀπὸ ΒΓ πρὸς τὸ ἀπὸ ΓΕ.

‖ 3 ἄρα add Hu ‖ 4 ἀπὸ add Co ἀπὸ τῆς add V² (Co) ‖ 5 κγ´ add Hu (BS) ‖ 12 ΑΓ πρὸς τὴν ΓΒ] ΔΒ πρὸς τὴν ΒΓ Co | ΓΒ – ΓΒ] ΑΒ πρὸς λοιπὴν τὴν ΒΔ ἐστὶν ὡς ἡ ΔΒ πρὸς τὴν ΒΓ Co ‖ 15 κδ´ add Hu (BS) ‖ 22 ΒΔΕ Co ΒΑΕ A

(92) 25. *(Prop. 39b)* The same thing another way.

On straight lines AE, ΔB, let semicircles AZE, ΔZB be described, and let AZ, ZΓ, ZΔ, ZE, ZB be joined. Then, since angles AZB, ΔZE equal two right angles,[1] therefore as is the rectangle contained by BA, AE to the rectangle contained by BΔ, ΔE, so is the square of AZ to the square of ZΔ.[2] But as is the rectangle contained by BA, AE to the rectangle contained by BΔ, ΔE, so was the square of AΓ to the square of ΓΔ.[3] Hence, as is the square of AΓ to the square of ΓΔ, so is the square of AZ to the square of ZΔ.[4] Thus too, as is AΓ to ΓΔ, so is AZ to ZΔ.[5] Hence angle AZΔ is bisected by straight line ZΓ.[6] But also if BZ is produced to H, angle ΔZE equals angle HZA.[7] Hence all angle EZΓ equals all angle ΓZH.[8] Therefore, as is BΓ to ΓE, so is BZ to ZE;[9] and as the (square of BΓ) is to the (square of ΓE, so is the square of BZ to the square of ZE).[10] But, as is the square of BZ to the square of ZE, so is the rectangle contained by AB, BΔ to the rectangle contained by AE, EΔ.[11] And thus, as is the rectangle contained by AB, BΔ to the rectangle contained by AE, EΔ, so is the square of BΓ to the square of ΓE.[12] Q.E.D.

(93) 26. *(Prop. 40a)* Again, as is the rectangle contained by AΓ, ΓB to the rectangle contained by AE, EB, so let the square of ΓΔ be to the square of ΔE. That, as is the rectangle contained by EA, AΓ to the rectangle contained by ΓB, BE, so is the square of AΔ to the square of ΔB.

Again, let the point of equation Z be taken, so that the rectangle contained by AZ, ZB <equals> the rectangle contained by ΓZ, ZE.[1] Then, as is ΓZ to ZE, so is the rectangle contained by AΓ, ΓB to the rectangle contained by AE, EB.[2] But, as is the rectangle contained by AΓ, ΓB to the rectangle contained by AE, EB, so is the square of ΓΔ to the square of ΔE.[3] And so, as is ΓZ to ZE, so is the square of ΓΔ to the square of ΔE.[4] Hence the rectangle contained by ΓZ, ZE, that is the rectangle contained by AZ, ZB, equals the square of ZΔ.[5] [6] Therefore, as is AZ to ZB, so is the square of ΔA to the square of ΔB.[7] But as is AZ to ZB, so is the rectangle contained by EA, AΓ to the rectangle contained by ΓB, BE.[8] Therefore, as is the rectangle contained by EA, AΓ to the rectangle contained by ΓB, BE, so is the square of AΔ to the square of ΔB.[9] Q.E.D.

(92) <κε.´> ἄλλως τὸ αὐτό.

γεγράφθω ἐπὶ τῶν ΑΕ, ΔΒ εὐθειῶν ἡμικύκλια τὰ ΑΖΕ ΔΖΒ, καὶ ἐπεζεύχθωσαν αἱ ΑΖ, ΖΓ, ΖΔ, ΖΕ, ΖΒ. ἐπεὶ οὖν αἱ ὑπὸ ΑΖΒ, ΔΖΕ γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσίν, ἔστιν ἄρα ὡς τὸ ὑπὸ ΒΑΕ πρὸς τὸ ὑπὸ ΒΔΕ, οὕτως τὸ ἀπὸ ΑΖ πρὸς τὸ ἀπὸ ΖΔ. ὡς δὲ τὸ ὑπὸ ΒΑΕ πρὸς τὸ ὑπὸ ΒΔΕ, οὕτως ἦν τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΓΔ. ὡς ἄρα τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΓΔ, οὕτως τὸ ἀπὸ ΑΖ πρὸς τὸ ἀπὸ ΖΔ. ὥστε καὶ ὡς ἡ ΑΓ πρὸς τὴν ΓΔ, οὕτως ἡ ΑΖ πρὸς τὴν ΖΔ. δίχα ἄρα τέτμηται ἡ ὑπὸ ΑΖΔ γωνία τῆι ΖΓ εὐθείαι. ἀλλὰ καὶ ἐκβληθείσης τῆς ΒΖ ἐπὶ τὸ Η, ἴση ἐστὶν ἡ ὑπὸ ΔΖΕ γωνία τῆι ὑπὸ ΗΖΑ γωνίαι. ὅλη ἄρα ἡ ὑπὸ τῶν ΕΖΓ ὅληι τῆι ὑπὸ τῶν ΓΖΗ γωνίαι ἴση ἐστίν. <ἔστιν ἄρα> ὡς ἡ ΒΓ πρὸς τὴν ΓΕ, οὕτως ἡ ΒΖ πρὸς τὴν ΖΕ. καὶ ὡς τὸ ἀπὸ πρὸς τὸ ἀπό. ἀλλ' ὡς τὸ ἀπὸ ΒΖ [732] πρὸς τὸ ἀπὸ ΖΕ, οὕτως ἐστὶν τὸ ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΕΔ. καὶ ὡς ἄρα τὸ ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΕΔ, οὕτως τὸ ἀπὸ ΒΓ πρὸς τὸ ἀπὸ ΓΕ. ὅ(περ): –

(93) <κς.´> ἔστω πάλιν ὡς τὸ ὑπὸ ΑΓΒ πρὸς τὸ ὑπὸ ΑΕΒ, οὕτως τὸ ἀπὸ ΓΔ πρὸς τὸ ἀπὸ ΔΕ. ὅτι γίνεται ὡς τὸ ὑπὸ ΕΑΓ πρὸς τὸ ὑπὸ ΓΒΕ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΔΒ. εἰλήφθω πάλιν ἰσότητος σημεῖον τὸ Ζ, ὥστε <ἴσον εἶναι> τὸ ὑπὸ τῶν ΑΖΒ τῶι ὑπὸ τῶν ΓΖΕ. ἔστιν ἄρα ὡς ἡ ΓΖ πρὸς τὴν ΖΕ, οὕτως τὸ ὑπὸ τῶν ΑΓΒ πρὸς τὸ ὑπὸ τῶν ΑΕΒ. ὡς δὲ τὸ ὑπὸ τῶν ΑΓΒ πρὸς τὸ ὑπὸ τῶν ΑΕΒ, οὕτως τὸ ἀπὸ ΓΔ πρὸς τὸ ἀπὸ ΔΕ. καὶ ὡς ἄρα ἡ ΓΖ πρὸς τὴν ΖΕ, οὕτως ἐστὶν τὸ ἀπὸ ΓΔ πρὸς τὸ ἀπὸ ΔΕ. ἴσον ἄρα ἐστὶν τὸ ὑπὸ ΓΖΕ, τουτέστιν τὸ ὑπὸ ΑΖΒ, τῶι ἀπὸ ΖΔ. ἔστιν ἄρα ὡς ἡ ΑΖ πρὸς τὴν ΖΒ, οὕτως τὸ ἀπὸ ΔΑ πρὸς τὸ ἀπὸ ΔΒ. ὡς δὲ ἡ ΑΖ πρὸς τὴν ΖΒ, οὕτως ἐστὶν τὸ ὑπὸ τῶν ΕΑΓ πρὸς τὸ ὑπὸ ΓΒΕ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΕΑΓ πρὸς τὸ ὑπὸ ΓΒΕ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΔΒ. ὅ(περ): –

‖ 1 κε´ add Hu (BS) ‖ 12 ἔστιν ἄρα add Ge (BS) ‖ 13 ἀπὸ πρὸς τὸ ἀπό] ἀπὸ ΒΓ πρὸς τὸ ἀπὸ ΓΕ Co ‖ 17 κς´ add Hu (BS) ‖ 20 ἴσον εἶναι add Hu (Co) ‖ 28 ΓΒΕ Co ΓΒ A

(94) 27. *(Prop. 40b)* The same thing another way.

About AE, ΓB let semicircles AZE, ΓZB be described, and let AZ, ΓZ, ΔZ, EZ, BZ be joined. Then angle AZΓ equals angle EZB.[1] Hence as is the rectangle contained by AΓ, ΓB to the rectangle contained by AE, EB, so is the square of ΓZ to the square of ZE.[2] But, as is the rectangle contained by AΓ, ΓB to the rectangle contained by AE, EB, so was the square of ΓΔ to the square of ΔE.[3] And therefore, as is the square of ΓΔ to the square of ΔE, so is the square of ΓZ to the square of ZE.[4] And hence, as is ΓΔ to ΔE, so is ΓZ to ZE.[5] Thus angle ΓZΔ equals angle ΔZE.[6] But angle AZΓ equals angle BZE.[7] Therefore all angle AZΔ equals all angle BZΔ.[8] Hence as is the square of AZ to the square of ZB, so is the square of AΔ to the square of ΔB.[9] But, as is the square of AZ to the square of ZB, so is the rectangle contained by EA, AΓ to the rectangle contained by ΓB, BE.[10] <Thus, as is the rectangle contained by EA, AΓ to the rectangle contained by ΓB, BE,> so is the square of AΔ to the square of ΔB.[11] Q.E.D.

(94) <κζ.´> ἄλλως τὸ αὐτό.

γεγράφθω περὶ τὰς ΑΕ, ΓΒ ἡμικύκλια |τὰ ΑΖΕ, ΓΖΒ, καὶ |136v
ἐπεζεύχθωσαν αἱ ΑΖ, ΓΖ, ΔΖ, ΕΖ, ΒΖ. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΑΖΓ
γωνία τῆι ὑπὸ ΕΖΒ γωνίαι. ἔστιν ἄρα ὡς τὸ ὑπὸ ΑΓΒ πρὸς τὸ
ὑπὸ ΑΕΒ, οὕτως τὸ ἀπὸ ΓΖ πρὸς τὸ ἀπὸ ΖΕ. ὡς δὲ τὸ ὑπὸ ΑΓΒ
πρὸς τὸ ὑπὸ ΑΕΒ, οὕτως ἦν τὸ ἀπὸ ΓΔ πρὸς τὸ ἀπὸ ΔΕ. καὶ ὡς
ἄρα τὸ ἀπὸ ΓΔ πρὸς τὸ ἀπὸ ΔΕ, οὕτως τὸ ἀπὸ ΓΖ πρὸς τὸ ἀπὸ ΖΕ.
ὥστε καὶ ὡς ἡ ΓΔ πρὸς τὴν ΔΕ, οὕτως ἡ ΓΖ πρὸς τὴν ΖΕ. ἴση
ἄρα ἐστὶν ἡ ὑπὸ ΓΖΔ γωνία τῆι ὑπὸ ΔΖΕ γωνίαι. ἔστιν δὲ καὶ 734
ἡ ὑπὸ ΑΖΓ γωνία τῆι ὑπὸ ΒΖΕ γωνίαι. ὅλη ἄρα ἡ ὑπὸ ΑΖΔ γωνία
ὅλῃι τῆι ὑπὸ ΒΖΔ γωνίαι ἴση ἐστίν. ὡς ἄρα τὸ ἀπὸ ΑΖ πρὸς τὸ
ἀπὸ ΖΒ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΔΒ. ὡς δὲ τὸ ἀπὸ ΑΖ πρὸς
τὸ ἀπὸ ΖΒ, οὕτως ἐστὶν τὸ ὑπὸ ΕΑΓ πρὸς τὸ ὑπὸ ΓΒΕ. <ἔστιν
ἄρα ὡς τὸ ὑπὸ ΕΑΓ πρὸς τὸ ὑπὸ ΓΒΕ>, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ
ἀπὸ ΔΒ. ὅπερ:–

‖ 1 κζ´ add Hu (BS) ‖ 2 ΑΕΖ A^1 Ε ante Ζ in ras., add post Ζ A^2 ‖ 6 ΑΕΒ Co ΔΕΒ A ‖ 10 ΑΖΔ Co ΓΖΔ A ‖ 11 ΑΖ Co ΔΖ A ‖ 13 ΕΑΓ Co ΑΕΓ A | ἔστιν – ΓΒΕ add Ge (Co)

(95) Lemmas useful for the Determinate Section, (Book) 2.

1. *(Prop. 41)* Let AB be a straight line, and (on it) three points Γ, Δ, E so that the rectangle contained by AΔ, ΔΓ equals the rectangle contained by BΔ, ΔE, and let (line) Z be made equal to AE plus ΓB. That the rectangle contained by Z, AΔ equals the rectangle contained by BA, AE, and the rectangle contained by Z, ΓΔ equals the rectangle contained by BΓ, ΓE, and the rectangle contained by Z, BΔ equals the rectangle contained by AB, BΓ, and the rectangle contained by Z, ΔE equals the rectangle contained by AE, EΓ.

For since the rectangle contained by AΔ, ΔΓ equals the rectangle contained by BΔ, ΔE,[1] in ratio[2] and inverting[3] and sum to sum[4] and *componendo*, as is BΓ plus AE, that is Z, to AE, so is BA to AΔ.[5] Hence the rectangle contained by Z, AΔ equals the rectangle contained by BA, AE.[6] Again, since all AE is to all ΓB as is EΔ to ΔΓ,[7] *componendo*, as is AE plus ΓB to ΓB, that is, as is Z to ΓB, so is ΓE to ΓΔ.[8] [9] Hence the rectangle contained by Z, ΓΔ equals the rectangle contained by BΓ, ΓE.[10] The same (will be proved) also for the remaining (ratios). Hence the four result.

(96) 2. *(Prop. 42)* Again, let the rectangle contained by AΔ, ΔΓ equal the rectangle contained by BΔ, ΔE, and let Z be made equal to AE plus ΓB. That again four things result, namely that the rectangle contained by Z, AΔ equals the rectangle contained by <BA, AE, and the rectangle contained by Z, ΓΔ equals the rectangle contained by> BΓ, ΓE, and the rectangle contained by Z, BΔ equals the rectangle contained by AB, BΓ, and the rectangle contained by Z, ΔE equals the rectangle contained by AE, EΓ.

For since the rectangle contained by AΔ, ΔΓ equals the rectangle contained by BΔ, ΔE,[1] in ratio[2] and inverting[3] and remainder to remainder[4] and *componendo*, then, as is AE plus ΓB to AE, so is BA to AΔ.[5] But AE plus ΓB equals Z.[6] Hence, as is Z to AE, so is BA to AΔ.[7] Therefore the rectangle contained by Z, AΔ equals the rectangle contained by BA, AE.[8] Again, since, as is AΔ to ΔB, so is EΔ to ΔΓ,[9] therefore remainder AE to remainder ΓB is as EΔ to ΔΓ.[10] *Componendo*, as is AE plus ΓB, that is, as is Z, to ΓB, so is EΓ to ΓΔ.[11] [12] Therefore the rectangle contained by Z, ΓΔ equals the rectangle contained by BΓ, ΓE.[13] We shall prove the same also for the remaining two. Hence the four result.

(95) ΛΗΜΜΑΤΑ ΧΡΗΣΙΜΑ ΕΙΣ ΤΟ ΔΕΥΤΕΡΟΝ ΔΙΩΡΙΣΜΕΝΗΣ ΤΟΜΗΣ

<α´> ἔστω εὐθεῖα ἡ ΑΒ, καὶ τρία σημεῖα τὰ Γ, Δ, Ε, ὥστε τὸ ὑπὸ τῶν ΑΔΓ ἴσον εἶναι τῶι ὑπὸ τῶν ΒΔΕ, καὶ συναμφοτέρωι τῆι ΑΕ, ΓΒ ἴση κείσθω ἡ Ζ· ὅτι γίνεται τὸ μὲν ὑπὸ τῶν Ζ, ΑΔ ἴσον τῶι ὑπὸ τῶν ΒΑΕ, τὸ δὲ ὑπὸ τῶν Ζ, ΓΔ ἴσον τῶι ὑπὸ τῶν ΒΓΕ, τὸ δὲ ὑπὸ τῶν Ζ, ΒΔ ἴσον τῶι ὑπὸ τῶν ΑΒΓ, τὸ δὲ ὑπὸ Ζ, ΔΕ τῶι ὑπὸ ΑΕΓ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν ΑΔΓ ἴσον τῶι ὑπὸ τῶν ΒΔΕ, ἀνάλογον καὶ ἀνάπαλιν καὶ ὅλη πρὸς ὅλην καὶ συνθέντι, ὡς συναμφότερος ἡ ΒΓ, ΑΕ, τουτέστιν ἡ Ζ, πρὸς τὴν ΑΕ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΔ. τὸ ἄρα ὑπὸ τῶν Ζ, ΑΔ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΒΑΕ. πάλιν ἐπεὶ ὅλη ἡ ΑΕ πρὸς ὅλην τὴν ΓΒ ἐστὶν ὡς ἡ ΕΔ πρὸς τὴν ΔΓ, συνθέντι ἐστὶν ὡς συναμφότερος ἡ ΑΕ, ΓΒ πρὸς τὴν ΓΒ, τουτέστιν ὡς ἡ Ζ πρὸς τὴν ΓΒ, οὕτως ἡ ΓΕ πρὸς τὴν ΓΔ. τὸ ἄρα ὑπὸ τῶν Ζ, ΓΔ ἴσον τῶι ὑπὸ τῶν ΒΓΕ. τὰ αὐτὰ καὶ |ἐπὶ |137 τῶν λοιπῶν. γίνεται ἄρα τέσσαρα.

(96) <β´> ἔστω νῦν πάλιν τὸ ὑπὸ τῶν ΑΔΓ ἴσον τῶι ὑπὸ τῶν ΒΔΕ. καὶ συναμφοτέρωι τῆι ΑΕ, ΓΒ ἴση κείσθω ἡ Ζ. ὅτι πάλιν γίνεται τέσσαρα, τὸ μὲν ὑπὸ τῶν Ζ, ΑΔ ἴσον τῶι ὑπὸ τῶν <ΒΑΕ, τὸ δὲ ὑπὸ τῶν Ζ, ΓΔ ἴσον τῶι ὑπὸ τῶν> ΒΓΕ, τὸ δὲ ὑπὸ τῶν Ζ, 736 ΒΔ ἴσον τῶι ὑπὸ τῶν ΑΒΓ, τὸ δὲ ὑπὸ τῶν Ζ, ΔΕ τῶι ὑπὸ ΑΕΓ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν ΑΔΓ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΒΔΕ, ἀνάλογον καὶ ἀνάπαλιν καὶ λοιπὴ πρὸς λοιπὴν καὶ συνθέντι, ἔστιν ἄρα ὡς συναμφότερος ἡ ΑΕ, ΓΒ πρὸς τὴν ΑΕ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΔ. συναμφότερος δὲ ἡ ΑΕ, ΓΒ ἴση ἐστὶν τῆι Ζ. ἔστιν ἄρα ὡς ἡ Ζ πρὸς τὴν ΑΕ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΔ. τὸ ἄρα ὑπὸ τῶν Ζ, ΑΔ ἴσον τῶι ὑπὸ τῶν ΒΑΕ. πάλιν ἐπεί ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΒ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΓ, λοιπὴ ἄρα ἡ ΑΕ πρὸς λοιπὴν τὴν ΓΒ ἐστὶν ὡς ἡ ΕΔ πρὸς τὴν ΔΓ. συνθέντι ὡς συναμφότερος ἡ ΑΕ, ΓΒ, τουτέστιν ὡς ἡ Ζ, πρὸς τὴν ΓΒ, οὕτως ἡ ΕΓ πρὸς τὴν ΓΔ. τὸ ἄρα ὑπὸ τῶν Ζ, ΓΔ ἴσον τῶι ὑπὸ τῶν ΒΓΕ. τὰ δ᾽ αὐτὰ καὶ ἐπὶ τῶν λοιπῶν δύο δείξομεν. γίνεται ἄρα τέσσαρα.

‖ 2 α´ add Hu (BS) ‖ 6 ΒΓΕ Co ΑΓΒ A ‖ 7 ante ΑΕΓ add τῶν Ge ‖ 8 καὶ ἀνάπαλιν del Simson$_1$ ἄρα Hu app ‖ 11 ΒΑΕ Co ΒΔΕ A ‖ 12 ΔΓ Co ΑΓ A | πρὸς τὴν om A^1 add mg A^2 alia manu ‖ 16 β´ add Hu (BS) ‖ 17 συναμφοτέρωι Ge (BS) συναμφότερα A ‖ 18 ΒΑΕ – ἴσον τῶι ὑπὸ τῶν add Co ‖ 20 ΒΔ in ras. A ‖ 22 καὶ ἀνάπαλιν del Simson$_1$ | λοιπὴ πρὸς λοιπὴν Hu λοιπὰ πρὸς λοιπά A | συνθέντι Hu συνθέσεις A ‖ 30 ΒΓΕ Co ΒΕΓ A

(97) 3. *(Prop. 43)* Let the point (Δ) be outside the whole (line), and let the rectangle contained by AΔ, ΔΓ equal the rectangle contained by BΔ, ΔE. That, again, if Z is made equal to the excess of AE over ΓB, then four things result, namely that the rectangle contained by Z, AΔ equals the rectangle contained by BA, AE, and the rectangle <contained by Z, ΓΔ equals the rectangle> contained by BΓ, ΓE, and the rectangle contained by Z, BΔ equals the rectangle contained by AB, BΓ, and the rectangle contained by Z, ΔE equals the rectangle contained by AE, EΓ.

For since the rectangle contained by AΔ, ΔΓ equals the rectangle contained by BΔ, ΔE,[1] in ratio[2] and remainder to remainder[3] and *convertendo*, then, as is AE to the excess of AE over ΓB, so is ΔA to AB.[4] But the excess of AE over ΓB is Z.[5] Hence the rectangle contained by Z, AΔ equals the rectangle contained by BA, AE.[6] Again, since remainder AE is to remainder BΓ as is EΔ to ΔΓ,[7] *separando*, as is the excess of AE over BΓ to BΓ, so is EΓ to ΓΔ.[8] Hence the rectangle contained by the excess of AE over BΓ, that is Z, and ΓΔ equals the rectangle contained by BΓ, ΓE.[9] [10] We shall prove the same also for the remaining two. Thus the four result.

(98) 4. *(Prop. 44)* Now that this has been proved, the (lemmas) for the *Determinate (Section)*, Book 1, can easily be found, namely that, under the same assumptions, as is BΔ to ΔE, so is the rectangle contained by AB, BΓ to the rectangle contained by AE, EΓ.

For since it has been proved that the rectangle contained by Z, BΔ equals the rectangle contained by AB, BΓ,[1] while the rectangle contained by Z, ΔE equals the rectangle contained by AE, EΓ,[2] therefore as is the rectangle contained by Z, ΔB to the rectangle contained by Z, ΔE, that is, as is BΔ to ΔE, so is the rectangle contained by AB, BΓ to the rectangle contained by AE, EΓ.[3] [4]

(97) <γ.´> ἔστω δὲ ἐκτὸς τῆς ὅλης τὸ σημεῖον, καὶ ἔστω τὸ ὑπὸ τῶν ΑΔΓ τῶι ὑπὸ τῶν ΒΔΕ. ὅτι πάλιν, ἐὰν τῆι τῶν ΑΕ, ΓΒ ὑπεροχῆι ἴση τεθῆι ἡ Ζ, γίνεται τέσσαρα, τὸ μὲν ὑπὸ τῶν Ζ, ΑΔ ἴσον τῶι ὑπὸ τῶν ΒΑΕ, τὸ δὲ <ὑπὸ τῶν Ζ, ΓΔ τῶι> ὑπὸ ΒΓΕ, τὸ δὲ ὑπὸ Ζ, ΒΔ τῶι ὑπὸ ΑΒΓ, τὸ δὲ ὑπὸ Ζ, ΔΕ τῶι ὑπὸ ΑΕΓ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν ΑΔΓ ἴσον τῶι ὑπὸ τῶν ΒΔΕ, ἀνάλογον καὶ λοιπὴ πρὸς λοιπὴν καὶ ἀναστρέψαντι, ἔστιν ἄρα ὡς ἡ ΑΕ πρὸς τὴν τῶν ΑΕ, ΓΒ ὑπεροχήν, οὕτως ἡ ΔΑ πρὸς τὴν ΑΒ. ἡ δὲ τῶν ΑΕ, ΓΒ ὑπεροχή ἐστιν ἡ Ζ. τὸ ἄρα ὑπὸ Ζ, ΑΔ ἴσον τῶι ὑπὸ ΒΑΕ. πάλιν ἐπεὶ λοιπὴ ἡ ΑΕ πρὸς λοιπὴν τὴν ΒΓ ἐστὶν ὡς ἡ ΕΔ πρὸς τὴν ΔΓ, διελόντι ἐστὶν ὡς ἡ τῶν ΑΕ, ΒΓ ὑπεροχὴ πρὸς τὴν ΒΓ, οὕτως ἡ ΕΓ πρὸς τὴν ΓΔ. τὸ ἄρα ὑπὸ τῆς τῶν ΑΕ, ΒΓ ὑπεροχῆς, τουτέστιν τῆς Ζ καὶ τῆς ΓΔ, ἴσον τῶι ὑπὸ τῶν ΒΓΕ. τὰ δὲ 738
αὐτὰ καὶ ἐπὶ τῶν λοιπῶν δύο δείξομεν. γίνεται ἄρα τέσσαρα.

(98) <δ.´> |τούτου δ' ἂν δειχθέντος, ῥαιδίως εὑρεθείη τὰ εἰς τὸ πρῶτον Διωρισμένης, τῶν αὐτῶν ὑποκειμένων, ὅτι |137v
γίνεται ὡς ἡ ΒΔ πρὸς τὴν ΔΕ, οὕτως τὸ ὑπὸ τῶν ΑΒΓ πρὸς τὸ ὑπὸ τῶν ΑΕΓ. ἐπεὶ γὰρ δέδεικται τὸ μὲν ὑπὸ τῶν Ζ, ΒΔ ἴσον τῶι ὑπὸ τῶν ΑΒΓ, τὸ δὲ ὑπὸ τῶν Ζ, ΔΕ τῶι ὑπὸ ΑΕΓ, ἔστιν ἄρα ὡς τὸ ὑπὸ Ζ, ΔΒ πρὸς τὸ ὑπὸ Ζ, ΔΕ, τουτέστιν ὡς ἡ ΒΔ πρὸς τὴν ΔΕ, οὕτως τὸ ὑπὸ τῶν ΑΒΓ πρὸς τὸ ὑπὸ τῶν ΑΕΓ.

‖ 1 γ´ add Hu (BS) | τὸ σημεῖον] τὰ σημεῖα Co ‖ 2 ΑΔΓ Co ΔΑΓ A | post ΑΔΓ add ἴσον Co | (Β)ΔΕ in ras. A | ΑΕ Co ΛΕ A ‖ 4 τῶι Ge τὸ A | ὑπὸ τῶν Ζ, ΓΔ τῶι add Co ‖ 7 λοιπὴ πρὸς λοιπὴν Hu λοιπὰ πρὸς λοιπά A ‖ 10 ἐπεὶ λοιπὴ Ge (S) ἐπιλοιπὴν A ‖ 11 ΒΓ (post ΑΕ) Co ΔΓ A ‖ 15 δ´ add Hu (BS) ‖ 16 ὅτι Co οὕτως A

(99) 5. *(Prop. 45)* For the first assignment of the first problem.

Now let the rectangle contained by <AΔ, ΔΓ equal the rectangle contained by> BΔ, ΔE, and let Z be an arbitrary point. That, if H is made equal to AE plus ΓB, the rectangle contained by AZ, ZΓ exceeds the rectangle contained by BZ, ZE by the rectangle contained by H, ΔZ.

For since it has already been proved that the rectangle contained by H, ΔE equals the rectangle contained by AE, EΓ,[1] let the rectangle contained by H, ZE be subtracted in common. Then the remaining rectangle contained by H, ΔZ is the excess by which the rectangle contained by AE, EΓ exceeds the rectangle contained by H, EZ.[2] But the amount by which the rectangle contained by AE, EΓ exceeds the rectangle contained by H, EZ, when the rectangle contained by AE, EZ has been subtracted in common, is the amount by which the rectangle contained by AE, ΓZ exceeds the rectangle contained by BΓ, ZE;[3] and the amount by which the rectangle contained by AE, ΓZ exceeds the rectangle contained by ΓB, ZE, when the rectangle contained by ΓZ, ZE has been subtracted in common, is the amount by which the rectangle contained by AZ, ZΓ exceeds the rectangle contained by BZ, ZE.[4] Thus the rectangle contained by AZ, ZΓ exceeds the rectangle contained by BZ, ZE by the rectangle contained by H, ΔZ.[5] Q.E.D.

(100) 6. *(Prop. 46)* Another (lemma) for the third (assignment) of the second (problem).

Let (point) Z be between points E, B. That the rectangle contained by AZ, ZΓ plus the rectangle contained by EZ, ZB equals the rectangle contained by H, ΔZ.

For since it has already been proved that the rectangle contained by H, ΔE equals the rectangle contained by AE, EΓ,[1] let the rectangle contained by H, EZ be added in common. Then all the rectangle contained by H, ΔZ equals the rectangle contained by AE, EΓ plus the rectangle contained by AE, EZ plus the rectangle contained by BΓ, EZ.[2] But in addition the rectangle contained by AE, EΓ plus the rectangle contained by AE, EZ is the whole rectangle contained by AE, ΓZ.[3] Thus it has resulted that the rectangle contained by H, ΔZ equals the rectangle contained by AE, ΓZ plus the rectangle contained by ΓB, EZ.[4] But again, the rectangle contained by ΓB, EZ equals the rectangle contained by ΓZ, ZE plus the rectangle contained by EZ, ZB;[5] while the rectangle contained by AE, ΓZ plus the rectangle contained by ΓZ, ZE is the whole rectangle contained by AZ, ZΓ.[6] And we also had the rectangle contained by EZ, ZB. Thus the rectangle contained by H, ΔZ equals the rectangle contained by AZ, ZΓ plus the rectangle contained by EZ, ZB.[7]

(99) <ε.´> εἰς τὸ πρῶτον ἐπίταγμα τοῦ πρώτου προβλήματος.

ἔστω πάλιν ἴσον τὸ ὑπὸ τῶν <ΑΔΓ τῶι ὑπὸ τῶν> ΒΔΕ, καὶ τυχὸν σημεῖον ἔστω τὸ Ζ. ὅτι, ἐὰν συναμφοτέρωι τῆι ΑΕ, ΓΒ ἴση τεθῆι ἡ Η, τὸ ὑπὸ τῶν ΑΖΓ τοῦ ὑπὸ τῶν ΒΖΕ ὑπερέχει τῶι ὑπὸ τῶν Η, ΔΖ. ἐπεὶ γὰρ προδέδεικται τὸ ὑπὸ τῶν Η, ΔΕ ἴσον τῶι ὑπὸ τῶν ΑΕΓ, κοινὸν ἀφηιρήσθω τὸ ὑπὸ τῶν Η, ΖΕ. λοιπὸν ἄρα τὸ ὑπὸ τῶν Η, ΔΖ ἡ ὑπεροχή ἐστιν ἧι ὑπερέχει τὸ ὑπὸ τῶν ΑΕΓ τοῦ ὑπὸ τῶν Η, ΕΖ. ὧι δὲ ὑπερέχει τὸ ὑπὸ τῶν ΑΕΓ τοῦ ὑπὸ τῶν Η, ΕΖ, κοινοῦ ἀφαιρεθέντος τοῦ ὑπὸ τῶν ΑΕΖ, τούτωι ὑπερέχει τὸ ὑπὸ τῶν ΑΕ, ΓΖ τοῦ ὑπὸ τῶν ΒΓ, ΖΕ. ὧι δὲ ὑπερέχει τὸ ὑπὸ τῶν ΑΕ, ΓΖ τοῦ ὑπὸ τῶν ΓΒ, ΖΕ, κοινοῦ ἀφαιρεθέντος τοῦ ὑπὸ τῶν ΓΖΕ, τούτωι ὑπερέχει τὸ ὑπὸ τῶν ΑΖΓ τοῦ ὑπὸ τῶν ΒΖΕ. τὸ ἄρα ὑπὸ τῶν ΑΖΓ τοῦ ὑπὸ τῶν ΒΖΕ ὑπερέχει <τῶι> ὑπὸ τῶν Η, ΔΖ. ὅπερ:–

(100) <ς.´> ἄλλο εἰς τὸ τρίτον τοῦ δευτέρου

ἔστω μεταξὺ τῶν σημείων τῶν Ε, Β τὸ Ζ. ὅτι τὸ ὑπὸ τῶν ΑΖΓ μετὰ τοῦ ὑπὸ ΕΖΒ ἴσον τῶι ὑπὸ τῶν Η, ΔΖ. ἐπεὶ γὰρ προαποδέδεικται τὸ ὑπὸ τῶν Η, ΔΕ ἴσον τῶι ὑπὸ τῶν ΑΕΓ, κοινὸν προσκείσθω τὸ ὑπὸ Η, ΕΖ. ὅλον ἄρα τὸ ὑπὸ τῶν Η, ΔΖ ἴσον τῶι τε ὑπὸ τῶν ΑΕΓ καὶ τῶι ὑπὸ τῶν ΑΕΖ καὶ τῶι ὑπὸ τῶν ΒΓ, ΕΖ. ἀλλὰ καὶ τὸ ὑπὸ ΑΕΓ μετὰ τοῦ ὑπὸ ΑΕΖ ὅλον ἐστὶν τὸ ὑπὸ ΑΕ, ΓΖ. γέγονεν οὖν τὸ ὑπὸ Η, ΔΖ ἴσον τῶι τε ὑπὸ ΑΕ, ΓΖ καὶ τῶι ὑπὸ ΓΒ, ΕΖ. ἀλλὰ πάλιν τὸ ὑπὸ ΓΒ, ΕΖ |ἴσον τῶι τε ὑπὸ |138
ΓΖΕ καὶ τῶι ὑπὸ ΕΖΒ. τὸ δὲ ὑπὸ ΑΕ, ΓΖ μετὰ τοῦ ὑπὸ ΓΖΕ ὅλον [ἄρα] ἐστὶν τὸ ὑπὸ ΑΖΓ. εἴχομεν δὲ καὶ τὸ ὑπὸ ΕΖΒ. τὸ ἄρα ὑπὸ τῶν Η, ΔΖ ἴσον τῶι τε ὑπὸ ΑΖΓ καὶ τῶι ὑπὸ ΕΖΒ.

‖ 1 ε´ add Hu (BS) ‖ 3 ΑΔΓ τῶι ὑπὸ τῶν add Co ‖ 4 συναμφοτέρωι Ge (S) συναμφότερος A ‖ 9 ὧι Co ὡς A ‖ 11 τὸ (ὑπὸ τῶν ΑΕ, ΓΖ) Co τοῦ A | ὧι Ge ὡς A ‖ 13 ΓΖΕ Co ΒΖΕ A ‖ 14 ΑΖΓ Co ΑΖΕ A ‖ 15 τῶι add Ge (S) ‖ 16 ς´ Hu (BS) | δευτέρου Simson₁ τρίτου A ‖ 17 μεταξὺ τῶν σημείων] μετὰ τὸ σημεῖον A τὸ σημεῖον μεταξὺ Ge (Co) ‖ 18 ΑΖΓ Co ΑΖΔ A ‖ 20 ΔΖ Co ΒΖ A ‖ 25 ὑπὸ ΑΕ, ΓΖ μετὰ τοῦ bis A corr Co ‖ 26 ἄρα secl Hu ‖ 27 ΑΖΓ Co ΑΓΖ A

(101) 7. *(Prop. 47)* For the first assignment of the third problem.

Now let the point Z be outside (line) AB. To prove that the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB by the rectangle contained by H, ΔZ.

For since the rectangle contained by H, ΔB equals the rectangle contained by AB, BΓ,[1] let the rectangle contained by H, BZ be added in common. Then the whole rectangle contained by H, ΔZ equals the rectangle contained by AB, BΓ plus the rectangle contained by H, BZ,[2] that is (plus) the rectangle contained by AE, BZ plus the rectangle contained by ΓB, ΓZ.[3] But the rectangle contained by AB, BΓ plus the rectangle contained by ΓB, BZ is the whole rectangle contained by AZ, ΓB.[4] Hence the rectangle contained by H, ΔZ equals the rectangle contained by AZ, ΓB plus the rectangle contained by AE, BZ.[5] But the rectangle contained by AZ, ΓB plus the rectangle contained by AE, BZ is the excess by which the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB.[6] Thus the rectangle contained by H, ΔZ too is the excess by which the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB.[7]

(102) 8. *(Prop. 48)* For the second assignment of the first problem.

Let the rectangle contained by AΔ, ΔΓ equal the rectangle contained by EΔ, ΔB, and let point Z be between Δ, Γ, and let (line) H be made equal to AE plus ΓB. That the rectangle contained by EZ, ZB exceeds the rectangle contained by AZ, ZΓ by the rectangle contained by H, ΔZ.

For since the rectangle contained by H, ΔΓ equals the rectangle contained by BΓ, ΓE,[1] let the rectangle contained by H, ZΓ be subtracted in common. Then the remaining rectangle contained by H, ΔZ is the excess by which the rectangle contained by EΓ, ΓB exceeds the rectangle contained by H, ΓZ.[2] But the amount by which the rectangle contained by EΓ, ΓB exceeds the rectangle contained by H, ZΓ, when the rectangle contained by BΓ, ΓZ has been subtracted in common, is the amount by which the rectangle contained by EZ, ΓB exceeds the rectangle contained by AE, ZΓ;[3] while the amount by which the rectangle contained by EZ, ΓB exceeds the rectangle contained by AE, ZΓ, when the rectangle contained by EZ, ZΓ has been *subtracted* in common, is the amount by which the rectangle contained by EZ, ZB exceeds the rectangle contained by AZ, ZΓ.[4] Thus the rectangle contained by EZ, ZB exceeds the rectangle contained by AZ, ZΓ by the rectangle contained by H, ΔZ.[5]

(101) <ζ.´> εἰς τὸ πρῶτον ἐπίταγμα τοῦ τρίτου προβλήματος.

ἔστω πάλιν τὸ σημεῖον ἐκτὸς τῆς ΑΒ τὸ Ζ. δεῖξαι ὅτι τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ὑπερέχει τῶι ὑπὸ Η, ΔΖ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν Η, ΔΒ ἴσον τῶι ὑπὸ ΑΒΓ, κοινὸν προσκείσθω τὸ ὑπὸ τῶν Η, ΒΖ. ὅλον ἄρα τὸ ὑπὸ τῶν Η, ΔΖ ἴσον τῶι τε ὑπὸ τῶν ΑΒΓ καὶ τῶι ὑπὸ Η, ΒΖ, τουτέστιν τῶι τε ὑπὸ ΑΕ, ΒΖ καὶ τῶι ὑπὸ ΓΒΖ. τὸ δὲ ὑπὸ ΑΒΓ μετὰ τοῦ ὑπὸ ΓΒΖ ὅλον [ἄρα] ἐστὶν τὸ ὑπὸ ΑΖ, ΓΒ. τὸ ἄρα ὑπὸ Η, ΔΖ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΖ, ΓΒ καὶ τῶι ὑπὸ ΑΕ, ΒΖ. τὸ δὲ ὑπὸ ΑΖ, ΓΒ μετὰ τοῦ ὑπὸ ΑΕ, ΒΖ ὑπεροχή ἐστιν ἧι ὑπερέχει τὸ ὑπὸ τῶν ΑΖΓ τοῦ ὑπὸ τῶν ΕΖΒ. καὶ τὸ ὑπὸ τῶν Η, 742 ΔΖ ἄρα ἡ ὑπεροχὴ ἧι ὑπερέχει τὸ ὑπὸ τῶν ΑΖΓ τοῦ ὑπὸ τῶν ΕΖΒ.

(102) <η.´> εἰς τὸ δεύτερον ἐπίταγμα τοῦ πρώτου προβλήματος.

ἔστω τὸ ὑπὸ τῶν ΑΔΓ ἴσον τῶι ὑπὸ τῶν ΕΔΒ, σημεῖον ἔστω τὸ Ζ μεταξὺ τῶν Δ, Γ, καὶ συναμφοτέρωι τῆι ΑΕ, ΓΒ ἴση κείσθω ἡ Η. ὅτι τὸ ὑπὸ τῶν ΕΖΒ τοῦ ὑπὸ ΑΖΓ ὑπερέχει τῶι ὑπὸ τῶν Η, ΔΖ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν Η, ΔΓ ἴσον τῶι ὑπὸ τῶν ΒΓΕ, κοινὸν ἀφῃρήσθω τὸ ὑπὸ τῶν Η, ΖΓ. λοιπὸν ἄρα τὸ ὑπὸ Η, ΔΖ ὑπεροχή ἐστιν ἧι ὑπερέχει τὸ ὑπὸ τῶν ΕΓΒ τοῦ ὑπὸ τῶν Η, ΖΓ. ὧι δὲ ὑπερέχει τὸ ὑπὸ τῶν ΕΓΒ τοῦ ὑπὸ τῶν Η, ΖΓ, κοινοῦ ἀφαιρεθέντος τοῦ ὑπὸ ΒΓΖ, τούτωι ὑπερέχει τὸ ὑπὸ τῶν ΕΖ, ΓΒ τοῦ ὑπὸ ΑΕ, ΖΓ. ὧι δὲ ὑπερέχει τὸ ὑπὸ ΕΖ, ΓΒ τοῦ ὑπὸ ΑΕ, ΖΓ, κοινοῦ ἀφαιρεθέντος τοῦ ὑπὸ ΕΖΓ, τούτωι ὑπερέχει τὸ ὑπὸ ΕΖΒ τοῦ ὑπὸ ΑΖΓ. καὶ τὸ ὑπὸ ΕΖΒ ἄρα τοῦ ὑπὸ ΑΖΓ ὑπερέχει τῶι ὑπὸ Η, ΔΖ.

‖ 1 ζ´ add Hu (BS) ‖ 3 ἐκτὸς Co ἐπὶ A | τὸ Ζ secl Hu app ‖ 7 Η om A^1 add supr A^2 | ΓΒΖ Co ΒΖ A ‖ 8 ἄρα secl Hu ‖ 11 ΑΖΓ Co ΔΖΓ A | ΕΖΒ Co ΕΒΖ A ‖ 12 ΔΖ Co ΒΖ A ‖ 14 η´ add Hu (BS) | πρώτου Simson$_1$ αὐτοῦ A ‖ 17 συναμφοτέρωι Ge (S) συναμφότερος A ‖ 18 (Ε)ΖΒ in ras. A ‖ 24 ΑΕ Co ΑΒ A ‖ 25 ἀφαιρεθέντος] προστεθέντος Co ‖ 26 ΕΖΒ Co ΕΖΔ A | τοῦ (ὑπὸ ΑΖΓ) Co τὸ A

(103) 9. *(Prop. 49)* For the second assignment of the second problem.

But let the point Z be between Γ, B. That the rectangle contained by AZ, ZΓ plus the rectangle contained by BZ, ZE equals the rectangle contained by H, ΔZ.

For since the rectangle contained by H, ΔΓ equals the rectangle contained by BΓ, ΓE,[1] let the rectangle contained by H, ΓZ be added in common. Then the whole rectangle contained by H, ΔZ equals the rectangle contained by BΓ, ΓE plus the rectangle contained by H, ΓZ,[2] that is (plus) the rectangle contained by AE, ΓZ plus the rectangle contained by BΓ, ΓZ.[3] But the rectangle contained by EΓ, ΓB plus the rectangle contained by BΓ, ΓZ is the whole rectangle contained by EZ, ΓB.[4] Hence it has resulted that the rectangle contained by EZ, ΓB plus the rectangle contained by AE, ΓZ equals the rectangle contained by H, ΔZ.[5] But the rectangle contained by EZ, ΓB equals the rectangle contained by EZ, ZΓ plus the rectangle contained by BZ, ZE;[6] while the rectangle contained by EZ, ZΓ plus the rectangle contained by AE, ΓZ is the whole rectangle contained by AZ, ZΓ.[7] Thus the rectangle contained by AZ, ZΓ plus the rectangle contained by BZ, ZE equals the rectangle contained by H, ΔZ.[8]

(104) 10. *(Prop. 50)* For the second assignment of the third problem.

But let the point Z be outside (line) AB. That the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB by the rectangle contained by H, ZΔ.

For since the rectangle contained by H, ΔB equals the rectangle contained by AB, BΓ,[1] let the rectangle contained by <H, BZ> be added in common. <Then the whole rectangle contained by> H, ΔZ equals the rectangle contained by AB, BΓ plus the rectangle contained by H, BZ,[2] that is (plus) the rectangle contained by AE, ZB plus the rectangle contained by ΓB, BZ.[3] But the rectangle contained by AB, BΓ plus the rectangle contained by ΓB, BZ is the whole rectangle contained by AZ, ΓB.[4] Hence the rectangle contained by AZ, ΓB plus the rectangle contained by AE, ZB equals the rectangle contained by H, ΔZ.[5] But the rectangle contained by AZ, BΓ plus the rectangle contained by AE, ZB is the excess by which the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB.[6] Hence the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB by the rectangle contained by H, ΔZ.[7] Q.E.D.

(103) <θ´> εἰς τὸ δεύτερον ἐπίταγμα τοῦ δευτέρου προβλήματος.

|ἀλλὰ ἔστω τὸ σημεῖον μεταξὺ τῶν Γ, Β τὸ Ζ. ὅτι γίνεται |138v τὸ ὑπὸ τῶν ΑΖΓ μετὰ τοῦ ὑπὸ ΒΖΕ ἴσον τῶι ὑπὸ Η, ΔΖ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν Η, ΔΓ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΒΓΕ, κοινὸν προσκείσθω τὸ ὑπὸ Η, ΓΖ. ὅλον ἄρα τὸ ὑπὸ τῶν Η, ΔΖ ἴσον ἐστὶν τῶι τε ὑπὸ ΒΓΕ καὶ τῶι ὑπὸ Η, ΓΖ, ὅ ἐστιν τῶι τε ὑπὸ ΑΕ, ΓΖ καὶ τῶι ὑπὸ ΒΓΖ. ἀλλὰ τὸ ὑπὸ ΕΓΒ μετὰ τοῦ ὑπὸ ΒΓΖ ὅλον ἐστὶν τὸ ὑπὸ ΕΖ, ΓΒ. γέγονεν οὖν τὸ ὑπὸ ΕΖ, ΓΒ μετὰ τοῦ 744 ὑπὸ ΑΕ, ΓΖ ἴσον τῶι ὑπὸ Η, ΔΖ. ἀλλὰ τὸ μὲν ὑπὸ ΕΖ, ΓΒ ἴσον τῶι τε ὑπὸ ΕΖΓ καὶ τῶι ὑπὸ ΒΖΕ. τὸ δὲ ὑπὸ ΕΖΓ μετὰ τοῦ ὑπὸ ΑΕ, ΓΖ ὅλον ἐστὶν τὸ ὑπὸ ΑΖΓ. τὸ ἄρα ὑπὸ ΑΖΓ μετὰ τοῦ ὑπὸ ΒΖΕ ἴσον ἐστὶν τῶι ὑπὸ Η, ΔΖ.

(104) <ι´> εἰς τὸ δεύτερον ἐπίταγμα τοῦ τρίτου προβλήματος.

ἔστω δὴ τὸ σημεῖον ἐκτὸς τῆς ΑΒ τὸ Ζ. ὅτι τὸ ὑπὸ τῶν ΑΖΓ τοῦ ὑπὸ τῶν ΕΖΒ ὑπερέχει τῶι ὑπὸ τῶν Η, ΖΔ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν Η, ΔΒ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΒΓ, κοινὸν προσκείσθω τὸ ὑπὸ <Η, ΒΖ. ὅλον ἄρα τὸ ὑπὸ> τῶν Η, ΔΖ ἴσον ἐστὶν τῶι τε ὑπὸ τῶν ΑΒΓ καὶ τῶι ὑπὸ Η, ΒΖ, ὅ ἐστιν τῶι τε ὑπὸ ΑΕ, ΖΒ καὶ τῶι ὑπὸ ΓΒΖ. τὸ δὲ ὑπὸ ΑΒΓ μετὰ τοῦ ὑπὸ ΓΒΖ ὅλον ἐστὶν τὸ ὑπὸ ΑΖ, ΓΒ. τὸ ἄρα ὑπὸ ΑΖ, ΓΒ μετὰ τοῦ ὑπὸ ΑΕ, ΖΒ ἴσον ἐστὶν τῶι ὑπὸ Η, ΔΖ. ἀλλὰ τὸ ὑπὸ ΑΖ, ΒΓ μετὰ τοῦ ὑπὸ ΑΕ, ΖΒ ὑπεροχή ἐστιν <ἧι> ὑπερέχει τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ. καὶ τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ὑπερέχει τῶι ὑπὸ Η, ΔΖ. ὅ(περ): —

‖ 1 θ´ add Hu (BS) ‖ 3 τὸ Ζ secl Hu app ‖ 4 ΒΖΕ Co ΑΖΕ A ‖ 7 τῶι (τε) Ge (recc?) τὸ A ‖ 10 ΑΕ Co ΑΒ A ‖ 14 ι´ add Hu (BS) ‖ 19 Η, ΒΖ. ὅλον ἄρα τὸ ὑπὸ add Co ‖ 20 τῶι (τε) Ge (BS) τὸ A ‖ 21 τῶι (ὑπὸ ΓΒΖ) Ge (S) τὸ A ‖ 22 ΑΖ Co ΑΗ A ‖ 23 ΔΖ Co ΑΖ A | ΒΓ] ΓΒ Co ΔΓ A ‖ 24 ἧι add Ge (BS) | post ΑΖΓ add ἄρα Hu

(105) 11. *(Prop. 51)* For the third assignment of the first problem.

Let the rectangle contained by AΔ, ΔΓ equal the rectangle contained by BΔ, ΔE, and let H be made equal to the excess of AE over BΓ, and let some point Z be taken between E, B. That the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB by the rectangle contained by H, ZΔ.

For since the rectangle contained by H, BΔ equals the rectangle contained by AB, BΓ,[1] let the rectangle contained by H, BZ be added in common. Then the whole rectangle contained by H, ZΔ equals the rectangle contained by AB·BΓ plus the rectangle contained by H, BZ,[2] that is (plus) the rectangle contained by the excess of AE over BΓ and BZ.[3] But the rectangle contained by AB, BΓ is the rectangle contained by AZ, BΓ plus the rectangle contained by ZB, BΓ.[4] Hence it has resulted that the rectangle contained by H, ZΔ equals the rectangle contained by AZ, BΓ plus the rectangle contained by ΓB, BZ plus the rectangle contained by the excess of AE over ΓB and BZ.[5] But the rectangle contained by ΓB, BZ plus the rectangle contained by the excess of AE over ΓB and BZ is the whole rectangle contained by AE, ZB.[6] Hence the rectangle contained by H, ZΔ equals the rectangle contained by AZ, ΓB plus the rectangle contained by AE, ZB.[7] But the rectangle contained by AZ, BΓ plus the rectangle contained by AE, ZB is the excess by which the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB.[8] Hence the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB by the rectangle contained by H, ZΔ.[9] Q.E.D.

(106) 12. *(Prop. 52)* For the first assignment of the second problem.

Under the same assumptions, let point Z be between B, Γ. That the rectangle contained by AZ, ZΓ plus the rectangle contained by EZ, ZB equals the rectangle contained by H, ZΔ.

For since the rectangle contained by H, ΓΔ equals the rectangle contained by EΓ, ΓB,[1] let the rectangle contained by H, ZΓ be added in common. Then the whole rectangle contained by H, ZΔ equals the rectangle contained by EΓ, ΓB plus the rectangle contained by H, ZΓ.[2] But the rectangle contained by H, ZΓ is the rectangle contained by the excess of AE over BΓ and ZΓ,[3] while the rectangle contained by EΓ, ΓB is the rectangle contained by BΓ, ΓZ plus the rectangle contained by EZ, BΓ.[4] Hence it has resulted that the rectangle contained by H, ZΔ equals the rectangle contained by EZ, BΓ plus the rectangle contained by BΓ, ΓZ plus the rectangle contained by the excess of AE over BΓ and ZΓ.[5] <But the rectangle contained by the excess of AE over BΓ and ΓZ plus the rectangle contained by BΓ, ΓZ is the whole rectangle contained by AE, ΓZ.[6] Hence the rectangle contained by H, ZΔ equals the rectangle contained by AE, ΓZ plus the rectangle contained by EZ, BΓ.[7] But the rectangle contained by EZ, BΓ is the rectangle contained by EZ, ZΓ plus the rectangle contained by EZ, ZB,[8] while the rectangle contained by EZ, ZΓ plus the rectangle contained by AE, ZΓ is the whole rectangle contained

(105) <ια´> εἰς τὸ τρίτον ἐπίταγμα τοῦ πρώτου προβλήματος.

ἔστω τὸ ὑπὸ τῶν ΑΔ, ΔΓ ἴσον τῶι ὑπὸ τῶν ΒΔ, ΔΕ, καὶ τῆι τῶν ΑΕ, ΒΓ ὑπεροχῆι ἴση κείσθω ἡ Η, καὶ εἰλήφθω τι σημεῖον τὸ Ζ μεταξὺ τῶν Ε, Β. ὅτι τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ὑπερέχει τῶι ὑπὸ τῆς Η καὶ τῆς ΖΔ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν Η, ΒΔ ἴσον ἐστὶν τῶι ὑπὸ ΑΒΓ, κοινὸν προσκείσθω τὸ ὑπὸ Η, ΒΖ. ὅλον ἄρα τὸ ὑπὸ Η, ΖΔ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΒΓ καὶ τῶι ὑπὸ Η, ΒΖ, ὅ 746 ἐστιν τῶι ὑπὸ τῆς τῶν ΑΕ, ΒΓ ὑπεροχῆς καὶ τῆς ΒΖ. |ἀλλὰ τὸ |139 ὑπὸ ΑΒΓ τὸ ὑπὸ ΑΖ, ΒΓ ἐστὶν καὶ τὸ ὑπὸ ΖΒ, ΒΓ. γέγονεν οὖν τὸ ὑπὸ Η, ΖΔ ἴσον τῶι τε ὑπὸ τῶν ΑΖ, ΒΓ καὶ τῶι ὑπὸ ΓΒ, ΒΖ καὶ τῶι ὑπὸ τῆς τῶν ΑΕ, ΓΒ ὑπεροχῆς καὶ τῆς ΒΖ. τὸ δὲ ὑπὸ ΓΒ, ΒΖ μετὰ τοῦ ὑπὸ τῆς τῶν ΑΕ, ΓΒ ὑπεροχῆς καὶ τῆς ΒΖ ὅλον ἐστὶν τὸ ὑπὸ ΑΕ, ΖΒ. τὸ οὖν ὑπὸ Η, ΖΔ ἴσον ἐστὶν τῶι τε ὑπὸ τῶν ΑΖ, ΓΒ καὶ τῶι ὑπὸ ΑΕ, ΖΒ. ἀλλὰ τὸ ὑπὸ ΑΖ, ΒΓ μετὰ τοῦ ὑπὸ ΑΕ, ΖΒ ὑπεροχή ἐστιν ἧι ὑπερέχει τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ. τὸ ἄρα ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ὑπερέχει τῶι ὑπὸ Η, ΖΔ. ὅ(περ): —

(106) ιβ´ εἰς τὸ πρῶτον ἐπίταγμα τοῦ δευτέρου προβλήματος.

τῶν αὐτῶν ὑποκειμένων ἔστω τὸ Ζ σημεῖον μεταξὺ τῶν Β, Γ. ὅτι τὸ ὑπὸ ΑΖΓ μετὰ τοῦ ὑπὸ ΕΖΒ ἴσον ἐστὶ τῶι ὑπὸ τῆς Η καὶ τῆς ΖΔ. ἐπεὶ γὰρ τὸ ὑπὸ Η, ΓΔ ἴσον ἐστὶν τῶι ὑπὸ ΕΓΒ, κοινὸν προσκείσθω τὸ ὑπὸ Η, ΖΓ. ὅλον ἄρα τὸ ὑπὸ Η, ΖΔ τῶι ὑπὸ ΕΓΒ καὶ τῶι ὑπὸ Η, ΖΓ ἐστὶν ἴσον. ἀλλὰ τὸ μὲν ὑπὸ Η, ΖΓ τὸ ὑπὸ τῆς τῶν ΑΕ, ΒΓ ἐστὶν ὑπεροχῆς καὶ τῆς ΖΓ, τὸ δὲ ὑπὸ ΕΓΒ τὸ ὑπὸ ΒΓΖ ἐστὶν καὶ τὸ ὑπὸ ΕΖ, ΒΓ. γέγονεν οὖν τὸ ὑπὸ Η, ΖΔ ἴσον τῶι ὑπὸ ΕΖ, ΒΓ καὶ τῶι ὑπὸ ΒΓΖ καὶ τῶι ὑπὸ τῆς τῶν ΑΕ, ΒΓ ὑπεροχῆς καὶ τῆς ΖΓ. <τὸ δὲ ὑπὸ τῆς τῶν ΑΕ, ΒΓ ὑπεροχῆς καὶ τῆς ΓΖ> μετὰ τοῦ ὑπὸ ΒΓΖ ὅλον ἐστὶν τὸ ὑπὸ ΑΕ, ΓΖ. τὸ ἄρα ὑπὸ Η, ΖΔ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΕ, ΓΖ καὶ τῶι ὑπὸ ΕΖ, ΒΓ. ἀλλὰ τὸ μὲν ὑπὸ ΕΖ, ΒΓ τό τε ὑπὸ ΕΖ, ΖΓ ἐστὶν καὶ τὸ ὑπὸ ΕΖ, ΖΒ, τὸ δὲ ὑπὸ ΕΖΓ μετὰ τοῦ ὑπὸ ΑΕ, ΖΓ ὅλον ἐστὶν τὸ ὑπὸ ΑΖΓ. εἴχομεν δὲ καὶ τὸ ὑπὸ ΕΖΒ. τὸ ἄρα ὑπὸ ΑΖΓ μετὰ τοῦ ὑπὸ ΕΖΒ ἴσον ἐστὶν τῶι ὑπὸ Η, ΖΔ. ὅπερ: —

‖ 1 ια´ add Hu (BS) ‖ 3 τῆι ... ὑπεροχῆι Ge (BS) τὴν ... ὑπεροχὴ A ‖ 7 ΑΒΓ Co ΑΓΒ A ‖ 11 Η, ΖΔ Co ΗΖ ΖΔ A ‖ 13 ΑΕ Co ΑΓ A ‖ 14 ΑΕ, ΖΒ Co ΑΕΖ A ‖ 15 (Β)Γ om A¹ add A² | ὑπὸ ΑΕ, ΖΒ Co ΑΕΖ A ‖ 18 ιβ´ mg A ‖ 21 τῆς (Η) Hu τῶν A ‖ 23 ΖΓ. ὅλον Co Ζ λόγον A ‖ 27 ΕΖ, ΒΓ Co ΕΖΒ A ‖ 28 τὸ δὲ ὑπὸ τῆς τῶν ΑΕ, ΒΓ ὑπεροχῆς καὶ τῆς ΓΖ add Co ‖ 31 ΕΖ, ΖΒ Co ΒΓ, ΓΖ A ΒΓ, ΒΖ Ge ‖ 32 ΕΖΓ Co ΒΖΓ A | ΑΖΓ Co ΑΓΖ A by AZ, ZΓ.[9] And we also had the rectangle contained by EZ, ZB. Thus the rectangle contained by AZ, ZΓ plus the rectangle contained by EZ, ZB equals the rectangle contained by H, ZΔ.[10] Q.E.D.

(107) 13. *(Prop. 53)* For the third assignment of the third problem.

Now let the point be between Γ, Δ. That the rectangle contained by AZ, ZΓ falls short of the rectangle contained by EZ, ZB by the rectangle contained by H, ZΔ.

For since the rectangle contained by H, ΓΔ equals the rectangle contained by EΓ·ΓB,[1] let the rectangle contained by H, ΓZ be subtracted in common. Then the remaining rectangle contained by H, ZΔ is the excess by which the rectangle contained by EΓ, ΓB exceeds the rectangle contained by H, ΓZ,[2] that is the rectangle contained by the excess of AE over ΓB and ΓZ.[3] But the amount by which the rectangle contained by EΓ, ΓB exceeds the rectangle contained by the excess of AE over ΓB and ΓZ, when the rectangle contained by ZΓ, ΓB has been added in common, is the amount by which the rectangle contained by EZ, BΓ exceeds the rectangle contained by AE, ΓZ,[4] <while the amount by which the rectangle contained by EZ, BZ exceeds the rectangle contained by AE, ΓZ,> when the rectangle contained by EZ, ZΓ has been added in common, is the amount by which the rectangle contained by EZ, ZB exceeds the rectangle contained by AZ, ZΓ.[5] And so the rectangle contained by AZ, ZΓ falls short of the rectangle contained by EZ, ZB by the rectangle contained by H, ZΔ.[6]

(108) 14. *(Prop. 54)* For the third assignment of the third problem.

But let point Z be outside (ΓΔ). That now the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB by the rectangle contained by H, ΔZ.

For since the rectangle contained by H, ΓΔ equals the rectangle contained by EΓ, ΓB,[1] let both be subtracted from the rectangle contained by H, ΓZ. Then the remaining rectangle contained by H, ΔZ is the excess by which the rectangle contained by H, ΓZ exceeds the rectangle contained by EΓ, ΓB.[2] <But the amount by which the rectangle contained by H, ΓZ exceeds the rectangle contained by EΓ, ΓB,> when the rectangle contained by BΓ, ΓZ has been added in common, is the amount by which the rectangle contained by AE, ΓZ exceeds the rectangle contained by EZ, BΓ,[3] since the excess of AE over BΓ plus BΓ is AE. Again, the amount by which the rectangle contained by AE, ΓZ exceeds the rectangle contained by EZ, BΓ, when the rectangle contained by EZ, ZΓ has been added in common, is the amount by which the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB.[4] Thus the rectangle contained by AZ, ZΓ exceeds the rectangle contained by EZ, ZB by the rectangle contained by H, ΔZ.[5]

(107) <ιγ.΄> εἰς τὸ τρίτον ἐπίταγμα τοῦ τρίτου προβλήματος.

ἔστω πάλιν τὸ σημεῖον μεταξὺ τῶν Γ, Δ. ὅτι τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ἐλλείπει τῶι ὑπὸ Η, ΖΔ. ἐπεὶ γὰρ τὸ ὑπὸ Η, ΓΔ ἴσον ἐστὶν τῶι ὑπὸ ΕΓΒ, κοινὸν ἀφῃρήσθω τὸ ὑπὸ Η, ΓΖ. λοιπὸν ἄρα τὸ ὑπὸ Η, ΖΔ ὑπεροχή ἐστιν ἧι ὑπερέχει τὸ ὑπὸ ΕΓΒ τοῦ ὑπὸ Η, ΓΖ, τουτέστιν τοῦ ὑπὸ τῆς τῶν ΑΕ, ΓΒ ὑπεροχῆς καὶ τῆς ΓΖ. ᾧι δὲ ὑπερέχει τὸ ὑπὸ ΕΓΒ τοῦ ὑπὸ τῆς τῶν ΑΕ, ΓΒ ὑπεροχῆς καὶ τῆς ΓΖ, κοινοῦ προστεθέντος |τοῦ ὑπὸ ΖΓΒ, |139v τούτωι ὑπερέχει τὸ ὑπὸ ΕΖ, ΒΓ τοῦ ὑπὸ ΑΕ, ΓΖ, <ᾧι δὲ ὑπερέχει τὸ ὑπὸ ΕΖ, ΒΖ τοῦ ὑπὸ ΑΕ, ΓΖ,> κοινοῦ προστεθέντος τοῦ ὑπὸ ΕΖΓ, τούτωι ὑπερέχει τὸ ὑπὸ ΕΖΒ τοῦ ὑπὸ ΑΖΓ. ὥστε τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ἐλλείπει τῶι ὑπὸ τῆς Η καὶ τῆς ΖΔ.

(108) <ιδ.΄> εἰς τὸ τρίτον ἐπίταγμα τοῦ τρίτου προβλήματος.

ἀλλὰ ἔστω ἐκτὸς τὸ Ζ σημεῖον. ὅτι πάλιν τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ὑπερέχει τῶι ὑπὸ Η, ΔΖ. ἐπεὶ γὰρ τὸ ὑπὸ Η, ΓΔ ἴσον ἐστὶν τῶι ὑπὸ ΕΓΒ, ἀμφότερα ἀφῃρήσθω ἀπὸ τοῦ ὑπὸ Η, ΓΖ. λοιπὸν ἄρα τὸ ὑπὸ Η, ΔΖ ἡ ὑπεροχή ἐστιν ἧι ὑπερέχει τὸ ὑπὸ Η, ΓΖ τοῦ ὑπὸ ΕΓΒ. <ᾧι δὲ ὑπερέχει τὸ ὑπὸ Η, ΓΖ τοῦ ὑπὸ ΕΓΒ,> κοινοῦ προστεθέντος τοῦ ὑπὸ ΒΓΖ, τούτωι ὑπερέχει τὸ ὑπὸ ΑΕ, ΓΖ τοῦ ὑπὸ ΕΖ, ΒΓ. ἡ γὰρ τῶν ΑΕ, ΒΓ ὑπεροχὴ μετὰ τῆς ΒΓ ἡ ΑΕ ἐστίν. ᾧι δὲ πάλιν ὑπερέχει τὸ ὑπὸ ΑΕ, ΓΖ τοῦ ὑπὸ 750 ΕΖ, ΒΓ, κοινοῦ προστεθέντος τοῦ ὑπὸ ΕΖΓ, τούτωι ὑπερέχει τὸ ὑπὸ ΑΖ, ΖΓ τοῦ ὑπὸ ΕΖΒ. τὸ ἄρα ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ὑπερέχει τῶι ὑπὸ Η, ΔΖ.

‖ 1 ιγ΄ add Hu (BS) ‖ 7 τῆς Co καὶ A ‖ 8 ᾧι Ge (BS) ὡς A | post ὑπερέχει litterae fere quattuor in ras. | ΕΓΒ Co ΑΓΒ A ‖ 10 τοῦ (ὑπὸ ΑΕ, ΓΖ) Ge (V) τὸ A | ᾧι δὲ – ΑΕ, ΓΖ add Hu ‖ 14 ιδ΄ add Hu (BS) ‖ 17 Δ(Ζ) in ras. A ‖ 19 λοιπὸν Co ὅλον A ‖ 20 ᾧι δὲ – ΕΓΒ add Hu ‖ 22 γὰρ Co ἄρα A ‖ 23 τοῦ Ge τὸ A ‖ 25 ΕΖΒ Co ΕΖ A

(109) 15. *(Prop. 55)* For the first assignment of the second problem.

Now let point Z be between A, E. That the rectangle contained by AZ, ZΓ plus the rectangle contained by EZ, ZB equals the rectangle contained by H, ZΔ.

For since the rectangle contained by H, BΔ equals the rectangle contained by AB, BΓ,[1] let the rectangle contained by H, BZ be added in common. Then the whole rectangle contained by H, ZΔ equals the rectangle contained by AB, BΓ plus the rectangle contained by H, ZB.[2] But the rectangle contained by AB, BΓ equals the rectangle contained by AZ, BΓ plus the rectangle contained by ZB, BΓ,[3] while the rectangle contained by the excess of AE over BΓ and ZB plus the rectangle contained by ΓB, BZ equals the rectangle contained by AE, BZ,[4] that is the rectangle contained by BZ, ZE plus the rectangle contained by AZ, ZB,[5] which (the rectangle contained by AZ, ZB) plus the rectangle contained by AZ, BΓ is the rectangle contained by AZ, ZΓ.[6] Therefore the rectangle contained by AZ, ZΓ plus the rectangle contained by BZ, ZE equals the rectangle contained by H, ZΔ.[7] Q.E.D.

(110) 16. *(Prop. 56)* For the third assignment of the third problem.

But now let point Z be outside (EA produced past A). That the rectangle contained by AZ, ZΓ falls short of the rectangle contained by EZ, ZB by the rectangle contained by H, ZΔ.

For since the rectangle contained by H, AΔ equals the rectangle contained by BA, AE,[1] let the rectangle contained by H, AZ be added in common. Then the whole rectangle contained by H, ΔZ equals the rectangle contained by BA, AE plus the rectangle contained by the excess of AE over ΓB and AZ.[2] <But the rectangle contained by BA, AE plus the rectangle contained by the excess of AE over ΓB and AZ is> the rectangle contained by ZB, AE diminished by the rectangle contained by ZA, BΓ.[3] Hence too the rectangle contained by H, ZΔ is the excess by which the rectangle contained by BZ, AE exceeds the rectangle contained by ZA, BΓ.[4] But the rectangle contained by ZB, AE exceeds the rectangle contained by ZA, BΓ, when the rectangle contained by BZ, ZA has been added, by the same amount as the rectangle contained by BZ, ZE exceeds the rectangle contained by ΓZ, ZA.[5] Hence the rectangle contained by BZ, ZE exceeds the rectangle contained by ΓZ, ZA by the rectangle contained by H, ZΔ.[6] Therefore the rectangle contained by ΓZ, ZA falls short of the rectangle contained by BZ, ZE by the rectangle contained by H, ZΔ. Q.E.D.

(109) <ιε΄> εἰς τὸ πρῶτον ἐπίταγμα τοῦ δευτέρου προβλήματος.

πάλιν ἔστω τὸ Ζ σημεῖον μεταξὺ τῶν Α, Ε. ὅτι τὸ ὑπὸ ΑΖΓ μετὰ τοῦ ὑπὸ ΕΖΒ ἴσον ἐστὶν τῶι ὑπὸ Η, ΖΔ. ἐπεὶ τὸ ὑπὸ Η, ΒΔ ἴσον ἐστὶν τῶι ὑπὸ ΑΒΓ, κοινὸν προσκείσθω τὸ ὑπὸ Η, ΒΖ. ὅλον ἄρα τὸ ὑπὸ Η, ΖΔ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΒΓ καὶ τῶι ὑπὸ Η, ΖΒ. ἀλλὰ τὸ μὲν ὑπὸ ΑΒΓ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΖ, ΒΓ καὶ τῶι ὑπὸ ΖΒΓ, τὸ δὲ ὑπὸ τῆς τῶν ΑΕ, ΒΓ ὑπεροχῆς καὶ τῆς ΖΒ μετὰ τοῦ ὑπὸ ΓΒΖ ἴσον ἐστὶν τῶι ὑπὸ ΑΕ, ΒΖ. ὅ ἐστιν τό τε ὑπὸ ΒΖΕ καὶ τὸ ὑπὸ ΑΖΒ, ὃ μετὰ τοῦ ὑπὸ ΑΖ, ΒΓ ἐστὶν τὸ ὑπὸ ΑΖΓ. τὸ οὖν ὑπὸ ΑΖΓ μετὰ τοῦ ὑπὸ ΒΖΕ ἴσον ἐστὶν τῶι ὑπὸ Η, ΖΔ. ὅ(περ): —

(110) <ις΄> εἰς τὸ τρίτον ἐπίταγμα τοῦ τρίτου προβλήματος.

ἔστω δὴ πάλιν ἐκτὸς τὸ Ζ σημεῖον. ὅτι τὸ ὑπὸ ΑΖΓ τοῦ ὑπὸ ΕΖΒ ἐλλείπει τῶι ὑπὸ Η, ΖΔ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν Η, ΑΔ ἴσον ἐστὶν τῶι ὑπὸ ΒΑΕ, κοινὸν προσκείσθω τὸ ὑπὸ Η, ΑΖ. ὅλον ἄρα τὸ ὑπὸ Η, ΔΖ ἴσον ἐστὶν τῶι τε ὑπὸ ΒΑΕ καὶ τῶι ὑπὸ τῆς τῶν ΑΕ, ΓΒ ὑπεροχῆς καὶ τῆς ΑΖ. <ἀλλ' ἔστιν τὸ ὑπὸ ΒΑΕ 752
καὶ τὸ ὑπὸ τῆς τῶν ΑΕ, ΓΒ ὑπεροχῆς καὶ τῆς ΑΖ> τὸ ὑπὸ ΖΒ, ΑΕ λειπὸν τῶι ὑπὸ ΖΑ, ΒΓ. ὥστε καὶ τὸ ὑπὸ Η, ΖΔ ἡ ὑπερο|χή ἐστιν |140
ἧι ὑπερέχει τὸ ὑπὸ ΒΖ, ΑΕ τοῦ ὑπὸ ΖΑ, ΒΓ. ἀλλὰ τὸ ὑπὸ ΖΒ, ΑΕ τοῦ ὑπὸ ΖΑ, ΒΓ ὑπερέχει, κοινοῦ προστεθέντος τοῦ ὑπὸ ΒΖΑ τούτωι ὑπερέχει, ὧι καὶ τὸ ὑπὸ ΒΖΕ τοῦ ὑπὸ ΓΖΑ. τὸ οὖν ὑπὸ ΒΖΕ τοῦ ὑπὸ ΓΖΑ ὑπερέχει τῶι ὑπὸ Η, ΖΔ. ὥστε τὸ ὑπὸ ΓΖΑ τοῦ ὑπὸ ΒΖΕ ἐλλείπει τῶι ὑπὸ Η, ΖΔ. ὅ(περ): —

‖ 1 ιε΄ add Hu (BS) | πρῶτον … δευτέρου Simson$_1$ τρίτον … τρίτου A | ἐπι A^1 -ταγμα add supr A^2 ‖ 9 ὅ ἐστιν] ἄρα ἐστιν A ante quae τὸ ὑπὸ ΑΕ, ΒΖ add Co ‖ 10 ΒΖΕ Co ΒΖΓ A | ΑΖ, ΒΓ Co ΑΔΒ A^1 Δ in Ζ mut. A^2 ‖ 13 ις΄ add Hu (BS) ‖ 17 ΑΖ Co ΑΔ A ‖ 19 ἀλλ' ἔστιν — τῆς ΑΖ] ἀλλὰ τὸ ὑπὸ ΒΑΕ μετὰ τοῦ ὑπὸ τῆς τῶν ΑΕ, ΓΒ ὑπεροχῆς καὶ τῆς ΑΖ ὅλον ἐστὶν add Hu τουτέστιν add Simson$_1$ πάλιν κοινὸν προσκείσθω τὸ ὑπὸ ΖΑ, ΒΓ. ἀλλὰ τὸ μὲν ὑπὸ τῆς τῶν ΑΕ, ΒΓ ὑπεροχῆς καὶ τῆς ΑΖ μετὰ τοῦ ὑπὸ ΖΑ, ΒΓ ἴσον ἐστὶν τῶι ὑπὸ ΖΑΕ, τὸ δὲ ὑπὸ ΒΑΕ μετὰ τοῦ ὑπὸ ΖΑΕ ὅλον ἐστὶ τὸ ὑπὸ ΖΒ, ΑΕ. τὸ ἄρα ὑπὸ ΖΒ, ΑΕ ἴσον ἐστὶ τῶι τε ὑπὸ Η, ΔΖ καὶ τῶι ὑπὸ ΖΑ, ΒΓ, ὥστε καὶ add Co ‖ 21 λειπὸν τῶι Hu λοιπὸν τὸ A | ὥστε — ΖΑ, ΒΓ bis (om. καὶ) A del Co ‖ 22 post ἀλλὰ add ὧι Co ‖ 24 ὧι del Co | ΓΖΑ Co ΓΒΑ A ‖ 25 ΓΖΑ Co ΓΖ ἀπὸ A

(111) 17. *(Prop. 57)* For the third assignment of the first problem.

Let AB be equal to ΓΔ, and let E be an arbitrary point between points B, Γ. That the rectangle contained by AE, EΔ exceeds the rectangle contained by BE, EΓ by the rectangle contained by AΓ, ΓΔ.

For since the rectangle contained by AE, EΔ equals the rectangle contained by AE, EΓ – that is the rectangle contained by BE, EΓ plus the rectangle contained by AB, EΓ – and in addition the rectangle contained by AE, ΓΔ,[1] [2] therefore the rectangle contained by AE, EΔ exceeds the rectangle contained by BE, EΓ by the rectangle contained by EΓ, AB, that is the rectangle contained by EΓ, ΓΔ – for AB and ΓΔ are equal, – plus the rectangle contained by AE, ΓΔ. These make up the whole rectangle contained by AΓ, ΓΔ. Thus the rectangle contained by AE, EΔ exceeds the rectangle contained by BE, EΓ by the rectangle contained by AΓ, ΓΔ.[3]

(112) 18. *(Prop. 58)* For the first assignment of the second problem.

Let AB be equal to ΓΔ, and let some point E be taken between Γ, Δ. That the rectangle contained by AE, EΔ plus the rectangle contained by BE, EΓ equals the <rectangle contained by AΓ, ΓΔ.

For since the rectangle contained by AE, EΔ equals the> rectangle contained by AΓ, EΔ plus the rectangle contained by ΓE, EΔ,[1] let the rectangle contained by BE, EΓ be added in common. Then the rectangle contained by AE, EΔ plus the rectangle contained by BE, EΓ equals the rectangle contained by AΓ, EΔ plus the rectangle contained by ΓE, EΔ and in addition the rectangle contained by BE, EΓ.[2] But the rectangle contained by ΓE, EΔ plus the rectangle contained by BE, EΓ is the whole rectangle contained by BΔ, ΓE,[3] that is the rectangle contained by AΓ, ΓE[4] – for all AΓ and all BΔ are equal, – while the rectangle contained by AΓ, EΔ plus the rectangle contained by AΓ, ΓE is the whole rectangle contained by AΓ, ΓΔ.[5] Thus the rectangle contained by AE, EΔ plus the rectangle contained by BE, EΓ equals the rectangle contained by AΓ, ΓΔ.[6]

(111) <ιζ΄> εἰς τὸ τρίτον ἐπίταγμα τοῦ πρώτου προβλήματος.

ἔστω ἡ ΑΒ <ἴση> τῆι ΓΔ, καὶ τυχὸν σημεῖον τὸ Ε μεταξὺ τῶν Β, Γ σημείων. ὅτι τὸ ὑπὸ ΑΕ, ΕΔ τοῦ ὑπὸ ΒΕ, ΕΓ ὑπερέχει τῶι ὑπὸ ΑΓΔ. ἐπεὶ γὰρ τὸ ὑπὸ ΑΕ, ΕΔ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΕ, ΕΓ, τουτέστιν τῶι τε ὑπὸ ΒΕ, ΕΓ καὶ τῶι ὑπὸ ΑΒ, ΕΓ καὶ ἔτι τῶι ὑπὸ ΑΕ, ΓΔ, τὸ ἄρα ὑπὸ ΑΕΔ τοῦ ὑπὸ ΒΕ, ΕΓ ὑπερέχει τῶι τε ὑπὸ ΕΓ, ΑΒ, τουτέστιν τῶι ὑπὸ ΕΓ, ΓΔ (ἴσαι γάρ εἰσιν αἱ ΑΒ, ΓΔ) καὶ τῶι ὑπὸ ΑΕ, ΓΔ. ἃ γίνεται ὅλον τὸ ὑπὸ ΑΓ, ΓΔ. τὸ ἄρα ὑπὸ ΑΕ, ΕΔ τοῦ ὑπὸ ΒΕ, ΕΓ ὑπερέχει τῶι ὑπὸ ΑΓ, ΓΔ.

(112) <ιη΄> εἰς τὸ πρῶτον ἐπίταγμα τοῦ δευτέρου προβλήματος.

ἔστω ἡ ΑΒ <ἴση> τῆι ΓΔ, καὶ εἰλήφθω τι σημεῖον μεταξὺ τῶν Γ, Δ τὸ Ε. ὅτι τὸ ὑπὸ ΑΕ, ΕΔ μετὰ τοῦ ὑπὸ ΒΕ, ΕΓ ἴσον ἐστὶν τῶι <ὑπὸ ΑΓΔ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν ΑΕ, ΕΔ ἴσον ἐστὶν τῶι> τε ὑπὸ τῶν ΑΓ, ΕΔ καὶ τῶι ὑπὸ ΓΕ, ΕΔ, [καὶ] κοινὸν προσκείσθω τὸ ὑπὸ ΒΕ, ΕΓ. τὸ ἄρα ὑπὸ ΑΕΔ μετὰ τοῦ ὑπὸ ΒΕΓ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΓ, ΕΔ καὶ τῶι ὑπὸ ΓΕ, ΕΔ καὶ ἔτι τῶι ὑπὸ ΒΕ, ΕΓ. ἀλλὰ τὸ μὲν ὑπὸ ΓΕ, ΕΔ μετὰ τοῦ ὑπὸ ΒΕ, ΕΓ ὅλον ἐστὶν τὸ ὑπὸ ΒΔ, ΓΕ, τουτέστιν τὸ ὑπὸ ΑΓΕ (ἴσαι γάρ εἰσιν και ὅλαι αἱ ΑΓ, ΒΔ), τὸ δὲ ὑπὸ τῶν ΑΓ, ΕΔ μετὰ τοῦ ὑπὸ ΑΓΕ ὅλον ἐστὶν τὸ ὑπὸ ΑΓ, ΓΔ. τὸ ἄρα ὑπὸ ΑΕ, ΕΔ μετὰ τοῦ ὑπὸ ΒΕ, ΕΓ ἴσον ἐστὶν τῶι ὑπὸ ΑΓΔ.

‖ 1 ιζ΄ add Hu (BS) ‖ 3 ἴση add Co ‖ 7 ΑΕΔ] ΑΕ, ΕΔ Co ΑΓΔ A ‖ 9 ΑΕ Co ΑΓ A | ΓΔ. ἃ A[2] Γ̄Δ̄Ā A[1] ΓΔ. αλλὰ τό τε ὑπὸ ΕΓ, ΓΔ καὶ τὸ ὑπὸ ΑΕ, ΓΔ Co ‖ 11 ιη΄ add Hu (BS) | δευτέρου Simson, πρώτου A ‖ 13 ἴση add Co ‖ 15 τῶι Ge (BS) τῶν A | ὑπὸ ΑΓΔ – ἐστὶν τῶι add Co ‖ 16 καὶ (προσκείσθω) del Co ‖ 19 ΕΔ Co ΕΒ A

(113) 19. *(Prop. 59)* Lemma useful for the singularities <of the third assignment> of the first, second, and third problems.

With AEB being a semicircle on (line) BA, and (lines) ΓE and ΔZ at right angles, and straight line EZH drawn, and a perpendicular BH (drawn) to it, three things result, namely that the rectangle contained by ΓB, BΔ (equals) the square of BH, and the rectangle contained by AΓ, ΔB (equals) the square of ZH, and the rectangle contained by AΔ, ΓB (equals) the square EH.

For let HΓ, HΔ, AZ, EΔ, AH, ZB be joined. Then since angle Z is right and ZΔ is a perpendicular,[1] angle ΔZB equals angle BAZ *(VI 8)*.[2] But angle ΔZB equals angle ΔHB *(III 22 & 21)*,[3] while angle BAZ, if EB is joined, equals angle BEZ,[4] that is angle BΓH.[5] Hence angle ΔHB equals angle BΓH.[6] Therefore the rectangle contained by ΓB, BΔ equals the square of BH.[7] But also the whole rectangle contained by AB, BΔ equals the square of BZ.[8] Therefore the remaining rectangle contained by AΓ, ΔB equals the square of ZH.[9] Again, since the rectangle contained by AB, BΓ equals the square of BE,[10] and out of these the rectangle contained by ΓB, BΔ equals the square of BH,[11] therefore the remaining rectangle contained by AΔ, ΓB equals the square of EH.[12] Thus the three things result.

(114) 20. *(Prop. 60)* For the singularity of the third <assignment of the second> problem.

(Let) ABΓ be a triangle, and let AΔ, BE, ΓZ be joined; and let AΔ be a perpendicular to BΓ, and let points A, Z, E, H be on a circle. That angles Z, E are right.

Let AΔ be produced, and let ΔΘ be made equal to HΔ,[1] and let BΘ, ΘΓ be joined. Then angle Θ equals angle BHΓ,[2] that is angle ZHE.[3] But angle ZHE plus angle A equalled two right angles.[4] Hence angle BΘΓ plus angle A too equals two right angles.[5] Therefore points A, B, Θ, Γ are on a circle.[6] Hence angle BAH equals angle BΓΘ,[7] that is angle HΓΔ.[8] But the vertical angles at H too are equal to one another.[9] Therefore the remaining angle Δ equals the remaining angle Z.[10] But Δ is right, and so the angle at point Z too is right.[11] For the same reasons angle E is also right. Thus the angles at points Z, E are right. Q.E.D.

(113) <ιθ.´> λῆμμα χρήσιμον εἰς τοὺς μοναχοὺς <τοῦ τρίτου ἐπιτάγματος> τοῦ τε πρώτου καὶ δευτέρου καὶ τοῦ τρίτου προβλήματος.

ἡμικυκλίου ὄντος [τοῦ τρίτου ἐπι] τῆς ΒΑ, καὶ ὀρθῶν τῶν ΓΕ, ΔΖ, καὶ ἀχθείσης εὐθείας τῆς ΕΖΗ, καὶ ἐπ' αὐτῆς καθέτου τῆς ΒΗ, γίνεται τρία· τὸ μὲν ὑπὸ ΓΒ, ΒΔ τῶι ἀπὸ ΒΗ, τὸ δὲ ὑπὸ ΑΓ, ΔΒ τῶι ἀπὸ ΖΗ, |τὸ δὲ ὑπὸ ΑΔ, ΓΒ τῶι ἀπὸ ΕΗ. ἐπεζεύχθωσαν |140v γὰρ αἱ ΗΓ, ΗΔ, ΑΖ, ΕΔ, [ΔΗ], ΑΗ, ΖΒ. ἐπεὶ οὖν ὀρθὴ ἡ πρὸς τῶι Ζ καὶ κάθετος ἡ ΖΔ, ἴση ἐστὶν ἡ ὑπὸ ΔΖΒ γωνία τῆι ὑπὸ ΒΑΖ γωνίαι. ἀλλὰ ἡ μὲν ὑπὸ ΔΖΒ ἴση ἐστὶν τῆι ὑπὸ ΔΗΒ, ἡ δὲ ὑπὸ ΒΑΖ, ἐὰν ἐπιζευχθῆι ἡ ΕΒ, τῆι ὑπὸ ΒΕΖ, τουτέστιν τῆι ὑπὸ ΒΓΗ. καὶ ἡ ὑπὸ ΔΗΒ ἄρα ἴση τῆι ὑπὸ ΒΓΗ. ὥστε τὸ ὑπὸ ΓΒΔ ἴσον ἐστὶν τῶι ἀπὸ ΒΗ. ἔστιν δὲ καὶ ὅλον τὸ ὑπὸ ΑΒΔ ἴσον τῶι ἀπὸ ΒΖ. λοιπὸν ἄρα τὸ ὑπὸ ΑΓ, ΔΒ ἴσον ἐστὶν τῶι ἀπὸ ΖΗ. 756 πάλιν ἐπεὶ τὸ ὑπὸ ΑΒΓ ἴσον ἐστὶν τῶι ἀπὸ ΒΕ τετραγώνωι, ὧν τὸ ὑπὸ ΓΒΔ ἴσον ἐστὶν τῶι ἀπὸ ΒΗ, λοιπὸν ἄρα τὸ ὑπὸ ΑΔ, ΓΒ ἴσον ἐστὶν τῶι ἀπὸ ΕΗ τετραγώνωι. γίνεται ἄρα τρία.

(114) <κ.´> εἰς τὸν μοναχὸν τοῦ τρίτου <ἐπιτάγματος τοῦ δευτέρου> προβλήματος.

τρίγωνον τὸ ΑΒΓ, καὶ διήχθωσαν αἱ ΑΔ, ΒΕ, ΓΖ, ἔστω δὲ ἡ μὲν ΑΔ ἐπὶ τῆς ΒΓ κάθετος, ἐν κύκλωι δὲ τὰ Α, Ζ, Ε, Η σημεῖα. ὅτι ὀρθαί εἰσιν αἱ πρὸς τοῖς Ζ, Ε γωνίαι. ἐκβεβλήσθω ἡ ΑΔ, καὶ τῆι ΗΔ ἴση κείσθω ἡ ΔΘ, καὶ ἐπεζεύχθωσαν αἱ ΒΘ, ΘΓ. ἴση ἄρα ἐστὶν ἡ Θ γωνία τῆι ὑπὸ ΒΗΓ, τουτέστιν τῆι ὑπὸ ΖΗΕ. ἀλλ' ἦν ἡ ὑπὸ ΖΗΕ μετὰ τῆς Α δυσὶν ὀρθαῖς ἴση. καὶ ἡ ὑπὸ ΒΘΓ ἄρα μετὰ τῆς Α δυσὶν ὀρθαῖς ἴση ἐστίν. ἐν κύκλωι ἄρα ἐστὶν τὰ Α, Β, Θ, Γ σημεῖα. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΒΑΗ γωνία τῆι ὑπὸ ΒΓΘ, τουτέστιν τῆι ὑπὸ ΗΓΔ. εἰσὶν δὲ καὶ αἱ πρὸς τῶι Η κατὰ κορυφὴν ἴσαι ἀλλήλαις. λοιπὴ ἄρα ἡ Δ ἴση τῆι πρὸς τῶι Ζ. ὀρθὴ δέ ἐστιν ἡ Δ. ὀρθὴ ἄρα ἐστὶν καὶ ἡ πρὸς τῶι Ζ σημείωι. διὰ ταὐτὰ δὴ καὶ ἡ πρὸς τῶι Ε γωνία ὀρθή ἐστιν. ὀρθαὶ ἄρα εἰσὶν αἱ πρὸς τοῖς Ζ, Ε σημείοις. ὅπερ:—

‖ 1 ιθ´ add Hu (BS) ‖ 3 προβλήματος correxi ex Simson₁ ἐπιτάγματος A ‖ 4 τοῦ τρίτου ἐπι seclusi τοῦ ΑΕΒ ἐπὶ διαμέτρου Ge (S) | ὀρθῶν] αὐτῆι πρὸς ὀρθὰς Hu app ‖ 6 post ΓΒ, ΒΔ add ἴσον Hu ‖ 8 ΕΔ, ΔΗ, ΑΗ del Hu ‖ 18 κ´ add Hu (BS) | τὸν Ge τὸ A | ἐπιτάγματος τοῦ δευτέρου add Hu (Simson₁) ‖ 20 ΒΕ, ΓΖ] ΒΖ, ΓΕ Hu (V²) ‖ 21 σημεῖα Ge (BS) σημεῖον A ‖ 23 ἐπεζεύχθωσαν αἱ Hu ἐπεζεύχθω ἡ A ‖ 24 ΒΗΓ Ge (S) ΒΗΗΓ A | ἀλλ' ἦν ἡ Hu ἀλλὰ μὴ A ‖ 27 Θ Co Β A ‖ 29 λοιπὴ Co λοιπὸν A | Ζ] Ε Ge (V²) ‖ 30 Ζ] Ε Ge (V²) om Co | σημείωι Ge (BS) σημεῖον A ‖ 31 ἡ Ge (BS) μὴ A | Ε] Ζ Ge (V²)

(115) 21. *(Prop. 61)* The singularity of the first problem of the third assignment.

Given three straight lines AB, BΓ, ΓΔ, if, as is the rectangle contained by AB, BΔ to the rectangle contained by AΓ, ΓΔ, so is the square of BE to the square of EΓ, then the singular and least ratio is that of the rectangle contained by AE, EΔ to the rectangle contained by BE, EΓ. I say that it is the same as that of the square of AΔ to the square of the excess by which the (line) equal in square to the rectangle contained by AΓ, BΔ exceeds the (line) equal in square to the rectangle contained by AB, ΓΔ.

Let a circle be described around AΔ, and let BZ, ΓH be drawn at right angles (to AΔ). Then since, as is the rectangle contained by AB, BΔ to the rectangle contained by AΓ, ΓΔ, that is as is the square of BZ to the square of ΓH, so is the square of BE to the square of EΓ,[1] [2] therefore in breadth (alone) as is BZ to ΓH, so is BE to EΓ.[3] Hence the line through Z, E, H is straight.[4] Let it be ZEH, and let HΓ be produced to Θ, and let ZΘ be joined and produced to K, and let ΔK be drawn as a perpendicular to it. According to the lemma (7.113) written above, the rectangle contained by AΓ, BΔ equals the square of ZK,[5] and the rectangle contained by AB, ΓΔ equals the square of ΘK.[6] Therefore the remainder ZΘ is the excess by which the (line) equal in square to the rectangle contained by AΓ, BΔ exceeds the (line) equal in square to the rectangle contained by AB, ΓΔ.[7] Let ZΛ therefore be drawn through the center, and let ΘΛ be joined. Then since right angle ZΘΛ equals right angle EΓH,[8] and angle Λ equals angle H,[9] therefore the triangles (ΘZΛ, ΓEH) are equiangular.[10] Hence as is ΛZ to ΘZ, that is as is AΔ to ZΘ, so is EH to EΓ.[11] [12] Therefore as is the square of AΔ to the square <of ΘZ, so is the square> of EH to the square of EΓ, and so also is the rectangle contained by HE, EZ, that is the rectangle contained by AE, EΔ, to the rectangle contained by BE, EΓ.[13] And the ratio of the rectangle contained by AE, EΔ to the rectangle contained by BE, EΓ is singular and least, while ZΘ is the excess by which the (line) equal in square to the rectangle contained by AΓ, BΔ exceeds the (line) equal in square to the rectangle contained by AB, ΓΔ, that is (that by which) the square of ZK (exceeds) the square of ΘK. Thus the singular and lesser ratio is the same as that of the square of AΔ to the square of the excess by which the (line) equal in square to the rectangle contained by AΓ, BΔ exceeds the (line) equal in square to the rectangle contained by AB, ΓΔ. Q.E.D.

(115) <κα´> ὁ μοναχὸς <τοῦ> πρώτου προβλήματος τοῦ τρίτου ἐπιτάγματος·

τριῶν δοθεισῶν εὐθειῶν τῶν ΑΒ, ΒΓ, ΓΔ, ἐὰν γένηται ὡς τὸ
ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΓΔ, οὕτως τὸ ἀπὸ ΒΕ |πρὸς τὸ ἀπὸ ΕΓ, ὁ |141 758
μοναχὸς λόγος καὶ ἐλάχιστός ἐστιν ὁ τοῦ ὑπὸ ΑΕΔ πρὸς τὸ ὑπὸ ΒΕΓ. λέγω δὴ ὅτι ὁ αὐτός ἐστιν τῶι τοῦ ἀπὸ τῆς ΑΔ πρὸς τὸ ἀπὸ τῆς ὑπεροχῆς ἧι ὑπερέχει ἡ δυναμένη τὸ ὑπὸ ΑΓ, ΒΔ τῆς δυναμένης τὸ ὑπὸ ΑΒ, ΓΔ. γεγράφθω περὶ τὴν ΑΔ κύκλος, καὶ ἤχθωσαν ὀρθαὶ αἱ ΒΖ, ΓΗ. ἐπεὶ οὖν ἐστιν ὡς τὸ ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΓΔ, τουτέστιν ὡς τὸ ἀπὸ ΒΖ πρὸς τὸ ἀπὸ ΓΗ, οὕτως τὸ ἀπὸ ΒΕ πρὸς τὸ ἀπὸ ΕΓ, καὶ μήκει ἄρα ἐστὶν ὡς ἡ ΒΖ πρὸς τὴν ΓΗ, οὕτως ἡ ΒΕ πρὸς τὴν ΕΓ. εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Ζ, Ε, Η. ἔστω ἡ ΖΕΗ, καὶ ἐκβεβλήσθω ἡ μὲν ΗΓ ἐπὶ τὸ Θ, ἐπιζευχθεῖσα δὲ ἡ ΖΘ ἐκβεβλήσθω ἐπὶ τὸ Κ καὶ ἐπ' αὐτὴν κάθετος ἤχθω ἡ ΔΚ, καὶ διὰ δὴ τὸ προγεγραμμένον λῆμμα γίνεται <τὸ> μὲν ὑπὸ ΑΓ, ΒΔ ἴσον τῶι ἀπὸ ΖΚ, τὸ δὲ ὑπὸ ΑΒ, ΓΔ τῶι ἀπὸ ΘΚ. λοιπὴ ἄρα ἡ ΖΘ ἐστὶν ἡ ὑπεροχὴ ἧι ὑπερέχει ἡ δυναμένη τὸ ὑπὸ ΑΓ, ΒΔ τῆς δυναμένης τὸ ὑπὸ ΑΒ, ΓΔ. ἤχθω οὖν διὰ τοῦ κέντρου ἡ ΖΛ, καὶ ἐπεζεύχθω ἡ ΘΛ. ἐπεὶ οὖν ὀρθὴ ἡ ὑπὸ ΖΘΛ ὀρθῆι τῆι ὑπὸ ΕΓΗ ἐστὶν ἴση, ἔστιν δὲ καὶ ἡ πρὸς τῶι Λ τῆι πρὸς τῶι Η γωνία ἴση, ἰσογώνια ἄρα τὰ τρίγωνα. ἔστιν ἄρα ὡς ἡ ΛΖ πρὸς τὴν ΘΖ, τουτέστιν ὡς ἡ ΑΔ πρὸς τὴν ΖΘ, οὕτως ἡ ΕΗ πρὸς τὴν ΕΓ. καὶ ὡς ἄρα τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ <ΘΖ, οὕτως τὸ ἀπὸ> ΕΗ πρὸς τὸ ἀπὸ ΕΓ, καὶ τὸ ὑπὸ ΗΕ, ΕΖ, τουτέστιν τὸ ὑπὸ ΑΕ, ΕΔ, πρὸς τὸ ὑπὸ ΒΕ, ΕΓ. καὶ ἔστιν ὁ μὲν τοῦ ὑπὸ ΑΕ, ΕΔ πρὸς τὸ ὑπὸ ΒΕ, ΕΓ μοναχὸς <καὶ> ἐλάσσων ὁ λόγος, ἡ δὲ ΖΘ ἡ ὑπεροχὴ ἧι ὑπερέχει ἡ δυναμένη τὸ ὑπὸ τῶν
ΑΓ, ΒΔ τῆς δυναμένης τὸ ὑπὸ ΑΒ, ΓΔ, τουτέστιν τὸ ἀπὸ τῆς ΖΚ 760
τοῦ ἀπὸ τῆς ΘΚ. ὥστε ὁ μοναχὸς καὶ ἐλάσσων λόγος ὁ αὐτός ἐστιν τῶι ἀπὸ τῆς ΑΔ πρὸς τὸ ἀπὸ τῆς ὑπεροχῆς ἧι ὑπερέχει ἡ δυναμένη τὸ ὑπὸ ΑΓ, ΒΔ τῆς δυναμένης τὸ ὑπὸ ΑΒ, ΓΔ. ὅ(περ): —

‖ 1 κα´ Hu (BS) | τοῦ add Ge | πρώτου Ge (BS) πρωτον A ‖ 2 πρώτου — ἐπιτάγματος] τρίτου ἐπιτάγματος τοῦ πρώτου προβλήματος Hu app ‖ 4 ΒΕ πρὸς τὸ ἀπὸ bis A corr Co | ὁ secl Hu ‖ 14 post ΖΘ asteriscuv in A ‖ 16 τὸ add Hu (V²) ‖ 22 ΛΖ Co ΑΖ A ‖ 24 ΘΖ, οὕτως τὸ ἀπὸ add Co ‖ 26 ΑΕ Co ΔΕ A | καὶ (post μοναχὸς) add Co | ὁ secl Hu ‖ 28 τουτέστιν — ΘΚ secl Hu ‖ 31 τὸ ὑπὸ ΑΓ, ΒΔ τῆς δυναμένης bis A corr Co

(116) 22. *(Prop. 62)* The singularity of the third assignment of the second problem.

Again given three straight lines AB, BΓ, ΓΔ, if, as is the rectangle contained by AΔ, ΔB to the rectangle contained by AΓ, ΓB, so is the square of ΔE to the square of EΓ, then the singular and lesser ratio is the same as that of the square of the (line) composed of the (line) equal in square to the rectangle contained by AΓ, BΔ and of the (line) equal in square to the rectangle contained by AΔ, BΓ to the square of ΔΓ.

From E let EZ be drawn at right angles to AΔ, and let it be produced, and (let) the square of ZΔ be equal to the rectangle contained by AΔ, ΔB, and let HΓ be drawn parallel to line ZΔ. Then since, as is the rectangle contained by AΔ, ΔB to the rectangle contained by AΓ, ΓB, so is the square of ΔE to the square of EΓ,[1] that is the square of ΔZ to the square of ΓH,[2] and the rectangle contained by AΔ, ΔB equals the square of ZΔ,[3] therefore the rectangle contained by AΓ, ΓB equals the square of ΓH.[4] Now let AZ, ZB, AH, HB be joined. Then since the rectangle contained by AΔ, ΔB equals the square of ΔZ,[5] angle BZΔ equals angle ZAB.[6] And angle BHΓ also equals angle BAH.[7] But also angle BZΔ equals angle BΘH.[8] Hence angle BΘH plus angle BHΘ, that is, if BK is produced, angle KBZ, equals angle ΛAK.[9] [10] Hence points A, Λ, B, K are on a circle.[11] Therefore by the lemma (7.114) written above, the angles at points K, Λ are right.[12] Now let BM be drawn as a perpendicular to ZΔ,[13] and let ΔN be joined, and let it be produced to Ξ. Then this is a perpendicular to ZΛ, and parallel to HΛ.[14] Again, let HΓ be joined and produced to O. Then this is a perpendicular to BN;[16] for ZΔ too is (perpendicular) to MB.[15] Then since the rectangle contained by AΓ, ΓB equals the square of ΓH,[17] therefore angle BHΓ equals angle HAΓ.[18] But angle BHΓ equals angle ΓNB in the circle (see commentary);[19] and angle HAB equals angle BΔN in parallels.[20] Therefore angle BNΓ equals angle BΔN.[21] Thus the rectangle contained by ΔB, BΓ equals the square of BN.[22] And since in triangle BΔZ a perpendicular ΔNΞ has been drawn, and ZN and NB have made an inflection on (ΔNΞ), therefore the excess of the square of ZΔ over the square of ΔB equals the (excess) of the square of ZN over the square of NB.[23] But the excess of the square of ZΔ over the square of ΔB is the rectangle contained by AB, BΔ.[24] Hence the excess of the square of ZN over the square of NB is the rectangle contained by AB, BΔ too.[25] And the rectangle contained by ΔB, BΓ equals the square of BN.[26] Therefore NZ is the (line) equal in square to the rectangle contained by AΓ, BΔ.[27] Again, since the excess of the square of HN over the square of NB equals the excess of the square of HΓ over the square of ΓB,[28] whereas the excess of

(116) <κβ´> ὁ μοναχὸς τοῦ τρίτου ἐπιτάγματος τοῦ δευτέρου προβλήματος.

πάλιν τριῶν δοθεισῶν εὐθειῶν τῶν ΑΒ, ΒΓ, ΓΔ, ἐὰν γένηται ὡς τὸ ὑπὸ ΑΔΒ πρὸς τὸ |ὑπὸ ΑΓΒ, οὕτως τὸ ἀπὸ ΔΕ πρὸς τὸ ἀπὸ |141v
ΕΓ, μοναχὸς καὶ ἐλάσσων λόγος ὁ αὐτός ἐστιν τῶι ἀπὸ τῆς συγκειμένης ἔκ τε τῆς δυναμένης τὸ ὑπὸ τῶν ΑΓ, ΒΔ καὶ τῆς δυναμένης τὸ ὑπὸ τῶν ΑΔ, ΒΓ πρὸς τὸ ἀπὸ τῆς ΔΓ. ἤχθω ἀπὸ τοῦ Ε τῆι ΑΔ ὀρθὴ ἡ ΕΖ, καὶ ἐκβεβλήσθω, καὶ τῶι ὑπὸ ΑΔΒ ἴσον τὸ ἀπὸ ΖΔ, καὶ τῆι ΖΔ εὐθείαι παράλληλος ἤχθω ἡ ΗΓ. ἐπεὶ οὖν ἐστιν ὡς τὸ ὑπὸ ΑΔΒ πρὸς τὸ ὑπὸ ΑΓΒ, οὕτως τὸ ἀπὸ ΔΕ πρὸς τὸ ἀπὸ ΕΓ, τουτέστιν τὸ ἀπὸ ΔΖ πρὸς τὸ ἀπὸ ΓΗ, καὶ ἔστιν ἴσον τὸ ὑπὸ τῶν ΑΔΒ τῶι ἀπὸ ΖΔ, καὶ τὸ ὑπὸ ΑΓΒ ἄρα ἴσον τῶι ἀπὸ ΓΗ. ἐπεζεύχθωσαν δὴ αἱ ΑΖ, ΖΒ, ΑΗ, ΗΒ. ἐπεὶ οὖν τὸ ὑπὸ ΑΔΒ ἴσον ἐστὶν τῶι ἀπὸ ΔΖ, ἴση ἐστὶν ἡ ὑπὸ ΒΖΔ γωνία τῆι ὑπὸ ΖΑΒ γωνίαι. ἔστιν δὲ καὶ ἡ ὑπὸ ΒΗΓ ἴση τῆι ὑπὸ ΒΑΗ. ἀλλὰ καὶ ἡ ὑπὸ ΒΖΔ ἴση ἐστὶν τῆι ὑπὸ ΒΘΗ. αἱ ἄρα ὑπὸ ΒΘΗ, 762
ΒΗΘ γωνίαι, τουτέστιν ἐὰν ἐκβληθῆι ἡ ΒΚ, ἡ ὑπὸ ΚΒΖ γωνία ἴση ἐστὶν τῆι ὑπὸ ΛΑΚ γωνίαι· ὥστε ἐν κύκλωι ἐστὶν τὰ Α, Λ, Β, Κ σημεῖα. διὰ ἄρα τὸ προγεγραμμένον γίνονται ὀρθαὶ αἱ πρὸς τοῖς Κ, Λ σημείοις γωνίαι. ἤχθω δὴ κάθετος ἐπὶ τὴν ΖΔ ἡ ΒΜ, καὶ ἐπεζεύχθω ἡ ΔΝ, καὶ ἐκβεβλήσθω ἐπὶ τὸ Ξ. κάθετος ἄρα ἐστὶν ἐπὶ τῆς ΖΛ, καὶ παράλληλος τῆι ΗΛ. πάλιν δὲ ἐπιζευχθεῖσα ἡ ΗΓ ἐκβεβλήσθω ἐπὶ τὸ Ο. κάθετος ἄρα ἐστὶν ἐπὶ τῆς ΒΝ. καὶ γὰρ ἡ ΖΔ ἐπὶ τῆς ΜΒ. ἐπεὶ οὖν τὸ ὑπὸ ΑΓΒ ἴσον ἐστὶν τῶι ἀπὸ τῆς ΓΗ, γωνία ἄρα ἡ ὑπὸ ΒΗΓ γωνίαι τῆι ΗΑΓ ἴση ἐστίν. ἀλλὰ ἡ μὲν ὑπὸ ΒΗΓ ἴση ἐστὶν τῆι ὑπὸ ΓΝΒ ἐν κύκλωι· ἡ δὲ ὑπὸ ΗΑΒ ἴση ἐστὶν τῆι ὑπὸ ΒΔΝ ἐν παραλλήλωι. καὶ ἡ ὑπὸ ΒΝΓ ἄρα ἴση ἐστὶν τῆι ὑπὸ ΒΔΝ. τὸ ἄρα ὑπὸ ΔΒΓ 764
ἴσον ἐστὶν τῶι ἀπὸ ΒΝ τετραγώνωι. ἐπεὶ δὲ ἐν τριγώνωι τῶι ΒΔΖ κάθετος ἦκται ἡ ΔΝΞ, καὶ κεκλασμέναι πρὸς αὐτῆι εἰσιν αἱ ΖΝ, ΝΒ, ἡ ἄρα τῶν ἀπὸ ΖΔ, ΔΒ ὑπεροχὴ ἴση τῆι τῶν ἀπὸ ΖΝ, ΝΒ ὑπεροχῆι· ἀλλὰ ἡ τῶν ἀπὸ ΖΔ, ΔΒ ὑπεροχή ἐστιν τὸ ὑπὸ ΑΒΔ. καὶ ἡ τῶν ἀπὸ τῶν ΖΝ, ΝΒ ἄρα ὑπεροχή ἐστιν τὸ ΑΒΔ. ἔστιν δὲ καὶ τὸ ὑπὸ ΔΒΓ ἴσον τῶι ἀπὸ ΒΝ. ἡ ΝΖ ἄρα ἐστὶν ἡ δυναμένη τὸ ὑπὸ τῶν ΑΓ, ΒΔ. πάλιν ἐπεὶ ἡ τῶν ἀπὸ τῶν ΗΝ, ΝΒ ὑπεροχὴ ἴση ἐστὶν τῆι τῶν ἀπὸ τῶν ΗΓ, ΓΒ ὑπεροχῆι, ἀλλὰ ἡ τῶν ἀπὸ τῶν

‖ 1 κβ´ add Hu (BS) | ἐπιτάγματος ... προβλήματος Simson₁ προβλήματος ... ἐπιτάγματος A ‖ 4 (Α)ΔΒ in ras. A | ΑΓΒ Co ΑΕΓ A ‖ 5 post λόγος add ἐστιν ὁ τοῦ ὑπὸ ΑΕΒ πρὸς τὸ ὑπὸ ΓΕΔ. λέγω δὴ ὅτι add Hu ‖ 10 ΑΓΒ Co ΑΒΓ A ‖ 13 δὴ] ἂν A del Hu ‖ 16 ΒΖΔ Co ΑΖΔ A ‖ 20 Κ Co (k) Χ A ‖ 21 κάθετος – ἐπὶ τὸ Ο om A¹ add mg A² alia manu ‖ 24 ΒΝ Co ΒΗ A | ΖΔ Co ΗΘ A | ΜΒ Co ΝΒ A ‖ 26 ΓΝΒ Co ΓΗΒ A ‖ 28 ΔΒΓ Co ΒΔΓ A ‖ 29 ἴσον Ge (recc?) ἴσω A | ἐπεὶ Ge (BS) ἐπὶ A ‖ 31 ΖΝ Co ΖΜ A ‖ 33 καὶ ἡ τῶν – τὸ ΑΒΔ om A¹ add mg A² alia manu | ΖΝ, ΝΒ Co ΖΗ, ΗΒ A ‖ 35 τὸ Hu (S) τοῦ A | ΗΝ, ΝΒ Simson₁ ΝΗ ΗΒ A the square of HΓ over the square of ΓB is the rectangle contained by AB, BΓ,[29] therefore the excess of the square of HN over the square of NB is the rectangle contained by AB, BΓ.[30] And the rectangle contained by ΔB, BΓ equals the square of BN.[31] Therefore NH is the (line) equal in square to the whole rectangle contained by AΔ, BΓ.[32] But also ZN is the (line) equal in square to the rectangle contained by AΓ, BΔ.[33] <Hence all ZH equals the (line) equal in square to the rectangle contained by AΔ, BΓ> plus the (line) equal in square to the rectangle contained by AΓ, BΔ.[34] Since angle ZKH is right, and AE is a perpendicular (to ZH),[35] therefore the rectangle contained by AE, EB equals the rectangle contained by ZE, EH.[36] Therefore, as is the rectangle contained by AE, EB to the rectangle contained by ΓE, EΔ, so is the rectangle contained by ZE, EH to the rectangle contained by ΓE, EΔ.[37] But as is the rectangle contained by ZE, EH to the rectangle contained by ΓE, EΔ, so is the square <of ZH to the square> of ΓΔ.[38] And therefore as is the rectangle contained by AE, EB to the rectangle contained by ΓE, EΔ, so is the square of ZH to the square of ΓΔ.[39] And the ratio of the rectangle contained by AE, EB to the rectangle contained by ΓE, EΔ is the singular and lesser ratio, while ZH is the (line) composed of the (line) equal in square to the rectangle contained by AΓ, BΔ <and the (line) equal in square to the rectangle contained by AΔ, BΓ.> Thus the singular and lesser ratio is the same as that of the square of the (line) composed of the (line) equal in square to the rectangle contained by AΓ, BΔ and the (line) equal in square to the rectangle contained by AΔ, ΓB to the square of ΓΔ.

ΗΓ, ΓΒ ὑπεροχή ἐστιν τὸ ὑπὸ τῶν ΑΒ, ΒΓ, καὶ ἡ τῶν ἀπὸ τῶν ΗΝ ΝΒ ἄρα ὑπεροχή ἐστιν τὸ ὑπὸ τῶν ΑΒ, ΒΓ. ἔστιν δὲ καὶ τὸ ὑπὸ ΔΒΓ ἴσον τῶι ἀπὸ ΒΝ. ἡ ΝΗ ἄρα ἐστὶν ἡ δυναμένη ὅλον τὸ ὑπὸ ΑΔ, ΒΓ. ἀλλὰ καὶ ἡ ΖΝ ἐστὶν ἡ δυναμένη τὸ ὑπὸ τῶν ΑΓ, ΒΔ. <ὅλη ἄρα ἡ ΖΗ ἴση ἐστὶν τῆι τε δυναμένηι τὸ ὑπὸ ΑΔ, ΒΓ> καὶ τῆι δυναμένηι τὸ ὑπὸ τῶν ΑΓ, ΒΔ |ἐπειδὴ ὀρθή ἐστιν ἡ ὑπὸ ΖΚΗ |142 766 γωνία, καὶ κάθετος ἡ ΑΕ, τὸ ἄρα ὑπὸ ΑΕΒ ἴσον ἐστὶν τῶι ὑπὸ ΖΕΗ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΑΕΒ πρὸς τὸ ὑπὸ ΓΕΔ, οὕτως τὸ ὑπὸ ΖΕΗ πρὸς τὸ ὑπὸ ΓΕΔ. ὡς δὲ τὸ ὑπὸ ΖΕΗ πρὸς τὸ ὑπὸ ΓΕΔ, οὕτως τὸ ἀπὸ <ΖΗ πρὸς τὸ ἀπὸ> τῆς ΓΔ. καὶ ὡς ἄρα τὸ ὑπὸ ΑΕΒ πρὸς τὸ ὑπὸ ΓΕΔ, οὕτως τὸ ἀπὸ τῆς ΖΗ πρὸς τὸ ἀπὸ τῆς ΓΔ. καὶ ἔστιν ὁ μὲν τοῦ ὑπὸ ΑΕΒ πρὸς τὸ ὑπὸ ΓΕΔ λόγος ὁ μοναχὸς καὶ ἐλάσσων, ἡ δὲ ΖΗ ἡ συγκειμένη ἔκ τε τῆς δυναμένης τὸ ὑπὸ ΑΓ, ΒΔ <καὶ τῆς δυναμένης τὸ ὑπὸ ΑΔ, ΒΓ.> ὁ ἄρα [ἐστὶν] μοναχὸς καὶ ἐλάσσων λόγος ὁ αὐτός ἐστιν τῶι ἀπὸ τῆς συγκειμένης ἔκ τε τῆς δυναμένης τὸ ὑπὸ ΑΓ, ΒΔ καὶ τῆς δυναμένης τὸ ὑπὸ ΑΔ, ΓΒ πρὸς τὸ ἀπὸ τῆς ΓΔ.

‖ 1 ΗΓ, ΓΒ Hu (Simson$_1$) ΝΓ ἡ Α | ΑΒ, ΒΓ] ΕΓΒ Co | ΗΝ, ΝΒ Simson$_1$ ΝΗ, ΗΒ Α ‖ 3 ΔΒΓ Simson$_1$ ΑΒΓ Α | ΒΝ Simson$_1$ ΒΗ Α ‖ 4 ΒΓ Co ΔΓ Α ‖ 5 ὅλη – ΑΔ, ΒΓ add Hu | καὶ – ΑΓ, ΒΔ del Co ‖ 6 ΑΓ, ΒΔ Co ΑΒΓΔ Α ‖ 7 ΑΕ Co ΚΕ Α | ΑΕΒ Co ΚΕΒ Α ‖ 10 ΖΗ πρὸς τὸ ἀπὸ add Co ‖ 12 τοῦ om A^1 add supr A^2 | ὁ secl Hu ‖ 13 ΖΗ Hu ΕΖΗ Α ΖΕΗ Co | ΑΓ, ΒΔ Co ΑΒ, ΓΔ Α ‖ 14 καὶ τῆς – ΑΔ, ΒΓ add Co | ἐστὶν del Hu ‖ 16 ΑΔ, ΓΒ] ΑΒ, ΓΔ Α ΑΔ, ΒΓ Co ‖ 17 ΓΔ Co ΚΔ Α

(117) 23. *(Prop. 63)* For the third assignment of the third problem.

Let AB be equal to ΓΔ, and the rectangle contained by BE, EΓ be greater than the rectangle contained by AB, BΔ. That the rectangle contained by BE, EΓ exceeds the rectangle contained by AE, EΔ by the rectangle contained by BΔ, ΔΓ.

For since the rectangle contained by BE, EΓ equals the rectangle contained by BΓ, ΓE plus the square of EΓ,[1] that is plus the rectangle contained by ΓE, EΔ plus the rectangle contained by EΓ, ΓΔ,[2] but the rectangle contained by BΓ, ΓE plus the rectangle contained by EΓ, ΓΔ is the whole rectangle contained by BΔ, ΓE,[3] that is the rectangle contained by AΓ, ΓE,[4] therefore the rectangle contained by BE, EΓ equals the rectangle contained by AΓ, ΓE plus the rectangle contained by ΓE, EΔ.[5] But the rectangle contained by AΓ, ΓE equals the rectangle contained by AΓ, EΔ plus the rectangle contained by AΓ, ΓΔ,[6] while the rectangle contained by AΓ, EΔ plus the rectangle contained by ΓE, EΔ is the whole rectangle contained by AE, EΔ.[7] Hence it follows that the rectangle contained by BE, EΓ equals the rectangle contained by AE, EΔ plus the rectangle contained by AΓ, ΓΔ,[8] which is the rectangle contained by BΔ, ΔΓ.[9] Thus the rectangle contained by BE, EΓ exceeds the rectangle contained by AE, EΔ by the rectangle contained by BΔ, ΔΓ.[10] Q.E.D.

(118) 24. *(Prop. 64)* Singularity of the third <assignment of the third> problem.

Given three straight lines AB, <BΓ>, ΓΔ, and some (line ΔE) added on, if, as is the rectangle contained by AB, BΔ to the rectangle contained by AΓ, ΓΔ, so is the square of BE to the square of EΓ, then the ratio of the rectangle contained by AE, EΔ to the rectangle contained by BE, EΓ is the singular and greatest ratio. I say that it is the same as that of the square of AΔ to the square of the (line) composed of the (line) equal in square to the rectangle contained by AΓ, BΔ and the (line) equal in square to the rectangle contained by AB, ΓΔ.

Let there be described around AΔ a semicircle AZHΔ, and let BZ, ΓH be drawn at right angles to AΔ. Then since, as is the rectangle contained by AB, BΔ to the rectangle contained by AΓ, ΓΔ, so is the square of <BE> to the square of EΓ,[1] whereas the rectangle contained by AB, BΔ equals the square of BZ[2] in the semicircle, and the square ΓH equals the rectangle contained by AΓ, ΓΔ,[3] therefore as is the square of BZ to the square of ΓH, so is the square of BE to the square of EΓ.[4] And also in breadth (alone as is BZ to ΓH, so is BE to EΓ).[5] And BZ and ΓH are parallel.[6] Therefore the line through Z, H, <E> is straight.[7] Let it be ZHE, and let it be produced, and let AΘ and ΔK be drawn as perpendiculars to it.[8] Then since the ratio of the rectangle contained by AE, EΔ to the rectangle contained by BE, EΓ is singular and greatest,[9] whereas the rectangle contained by ZE, EH <equals the rectangle contained by AE, EΔ,>[10] therefore the singular and greatest ratio is the same as that of the rectangle contained by ZE, EH to the rectangle contained by BE, EΓ.[11]

(117) <κγ.´> εἰς τὸ τρίτον ἐπίταγμα τοῦ τρίτου προβλήματος.

ἔστω ἴση ἡ μὲν ΑΒ τῆι ΓΔ, μεῖζον δὲ τὸ ὑπὸ ΒΕΓ τοῦ ὑπὸ ΑΒΔ. ὅτι τὸ ὑπὸ ΒΕΓ τοῦ ὑπὸ ΑΕΔ ὑπερέχει τῶι ὑπὸ ΒΔΓ. ἐπεὶ γὰρ τὸ ὑπὸ ΒΕΓ ἴσον τῶι τε ὑπὸ ΒΓΕ καὶ τῶι ἀπὸ ΕΓ, τουτέστιν καὶ τῶι ὑπὸ ΓΕΔ μετὰ τοῦ ὑπὸ ΕΓΔ, ἀλλὰ τὸ ὑπὸ ΒΓΕ μετὰ τοῦ ὑπὸ ΕΓΔ ὅλον ἐστὶν τὸ ὑπὸ ΒΔ, ΓΕ, τουτέστιν τὸ ὑπὸ ΑΓ, ΓΕ, τὸ ἄρα ὑπὸ ΒΕΓ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΓΕ καὶ τῶι ὑπὸ ΓΕΔ. ἀλλὰ τὸ μὲν ὑπὸ ΑΓΕ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΓ, ΕΔ καὶ τῶι ὑπὸ ΑΓ, ΓΔ. τὸ δὲ ὑπὸ ΑΓ, ΕΔ μετὰ τοῦ ὑπὸ ΓΕΔ ὅλον ἐστὶν τὸ ὑπὸ ΑΕΔ. γέγονεν οὖν τὸ ὑπὸ ΒΕΓ ἴσον τῶι τε ὑπὸ ΑΕΔ καὶ τῶι ὑπὸ ΑΓΔ, ὅ ἐστιν τὸ ὑπὸ ΒΔ, ΔΓ. ὥστε τὸ ὑπὸ ΒΕΓ τοῦ ὑπὸ ΑΕΔ ὑπερέχει τῶι ὑπὸ ΒΔΓ. ὅπερ:–

(118) <κδ.´> μοναχὸς τοῦ τρίτου <ἐπιτάγματος τοῦ τρίτου> προβλήματος. 768

τριῶν δοθεισῶν εὐθειῶν τῶν ΑΒ, <ΒΓ>, ΓΔ, <καὶ> προστιθεμένης τινός, ἐὰν γένηται ὡς τὸ ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΓΔ, οὕτως τὸ ἀπὸ ΒΕ πρὸς τὸ ἀπὸ ΕΓ, μοναχὸς καὶ μέγιστος λόγος ἐστιν ὁ τοῦ ὑπὸ ΑΕΔ πρὸς τὸ <ὑπὸ> ΒΕΓ. λέγω δὴ ὅτι ὁ αὐτός ἐστιν τῶι ἀπὸ τῆς ΑΔ πρὸς τὸ ἀπὸ τῆς συγκειμένης ἔκ τε τῆς δυναμένης τὸ ὑπὸ τῶν ΑΓ, ΒΔ καὶ τῆς δυναμένης τὸ ὑπὸ ΑΒ, ΓΔ. γεγράφθω ἐπὶ τῆς ΑΔ |ἡμικύκλιον τὸ ΑΖΗΔ, καὶ τῆι ΑΔ ὀρθαὶ ἤχθωσαν αἱ ΒΖ, ΓΗ. ἐπεὶ οὖν γεγένηται ὡς τὸ ὑπὸ ΑΒΔ πρὸς τὸ ὑπὸ ΑΓΔ, οὕτως τὸ ἀπὸ <ΒΕ> πρὸς τὸ ἀπὸ ΕΓ, ἀλλὰ τὸ μὲν ὑπὸ ΑΒΔ ἴσον ἐστὶν ἐν ἡμικυκλίωι τῶι ἀπὸ ΒΖ, τῶι δὲ ὑπὸ ΑΓΔ ἴσον τὸ ἀπὸ ΓΗ, ἔστιν ἄρα ὡς τὸ ἀπὸ ΒΖ πρὸς τὸ ἀπὸ ΓΗ, οὕτως τὸ ἀπὸ ΒΕ πρὸς τὸ ἀπὸ ΕΓ. καὶ μήκει. καὶ εἰσὶν παράλληλοι αἱ ΒΖ, ΓΗ. εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Ζ, Η, <Ε>. ἔστω ἡ ΖΗΕ, καὶ ἐκβεβλήσθω, καὶ ἐπ' αὐτὴν κάθετοι ἤχθωσαν αἱ ΑΘ, ΔΚ. ἐπεὶ οὖν μοναχὸς καὶ μέγιστος λόγος ἐστιν ὁ τοῦ ὑπὸ ΑΕΔ πρὸς τὸ ὑπὸ ΒΕΓ, ἀλλὰ τὸ ὑπὸ ΖΕΗ <ἴσον ἐστὶν τῶι ὑπὸ ΑΕΔ,> ὁ ἄρα μοναχὸς καὶ μέγιστος λόγος ὁ αὐτός ἐστιν τῶι τοῦ ὑπὸ ΖΕΗ πρὸς τὸ ὑπὸ ΒΕΓ. <ὡς δὲ τὸ ὑπὸ ΖΕΗ πρὸς τὸ ὑπὸ |142v

‖ 1 κγ´ add Hu (BS) ‖ 3 μεῖζον Ge (recc?) μείζων A ‖ 4 ΑΒΔ Ge (S) ΑΕΔ A ‖ 5 τὸ Ge (BS) τοῦ A ‖ 8 ΑΓΕ Co ΓΑΕ A ‖ 9 ΑΓΕ Co ΒΓΕ A ‖ 10 ΓΔ Co ΓΕ A ‖ 12 ΒΔ Co ΒΑ A | ΒΕΓ Co ΒΕ A ‖ 14 κδ´ add Hu (BS) | ἐπιτάγματος τοῦ τρίτου add Simson₁ ‖ 16 ΒΓ add Simson₁ | post ΓΔ add ΕΖ A del Simson₁ | καὶ add Hu ‖ 17 post τινός add ΔΕ Hu (Simson₁) ‖ 18 ΕΓ Co ΕΔΑ A ‖ 19 ὑπὸ (ΒΕΓ) add Hu (recc?) ‖ 21 ΑΓ Co ΑΕ A ‖ 22 ἡμικύκλιον Ge (S) ἡμικύκλια A | τῆι ΑΔ ὀρθαὶ Ge (S) τῆς ΑΔ ὀρθῆς A ‖ 24 ΒΕ add Co, spatium litterarum fere quinque A ‖ 28 Ε add Co ‖ 30 post μέγιστος λόγος add ὁ αὐτός A¹ del A² ‖ 31 ΖΕΗ A² ex ΖΕΝ | ἴσον – ΑΕΔ add Co ‖ 33 ὡς δὲ – ΒΕΓ add Co

<But as is the rectangle contained by ZE, EH to the rectangle contained by BE, EΓ,> so is the square of HE to the square of EΓ[12] in parallels, that is the square of AE to the square of EΘ;[15] for points Θ, A, Γ, H are on a circle,[14] since the angles at points Θ, Γ are right.[13] But as is the square of EA to the square of EΘ, so is the square of AΔ to the square of ΘK in parallels.[16] Therefore the singular and greatest ratio is that of the square of ΔA to the square of ΘK.[17] But ΘK is the (line) equal in square to the rectangle contained by AΓ, BΔ plus the (line equal in square to) the rectangle contained by AB, ΓΔ.[18] Thus the singular and greatest ratio is the same as that of the square of AΔ to the (line) composed of the (line) equal in square to the rectangle contained by AΓ, BΔ and the (line) equal in square to the rectangle contained by AB, ΓΔ.[19]

(119) The first (book) of the *Determinate Section* contains six problems, sixteen assignments, and five diorisms, of which four are maxima, one minimum. The maxima are the one in the second assignment of the second problem, and that in the third of the fourth problem, and that in the third of the fifth, and that in the third of the sixth; the one in the third assignment of the third problem is a minimum. The second (book) of the *Determinate (Section)* contains three problems, nine assignments, and three diorisms, of which two are minima, one maximum. The minima are the ones in the third (assignment) of the first (problem) and in the third of the second; the one in the third of the third problem is a maximum.

ΒΕΓ,> οὕτως ἐστὶν ἐν παραλλήλωι τὸ ἀπὸ ΗΕ πρὸς τὸ ἀπὸ ΕΓ, τουτέστιν τὸ ἀπὸ ΑΕ πρὸς τὸ ἀπὸ ΕΘ· ἐν κύκλωι γὰρ τὰ Θ, Α, Γ, 770 Η σημεῖα, ἐπειδήπερ ὀρθαί εἰσιν αἱ πρὸς τοῖς Θ, Γ σημείοις γωνίαι. ὡς δὲ τὸ ἀπὸ ΕΑ πρὸς τὸ <ἀπὸ> ΕΘ, οὕτως ἐστὶν τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΘΚ ἐν παραλλήλωι. ὁ ἄρα μοναχὸς καὶ μέγιστος λόγος ἐστὶν ὁ τοῦ ἀπὸ ΔΑ πρὸς τὸ ἀπὸ ΘΚ. ἡ δὲ ΘΚ ἐστὶν ἡ δυναμένη τε τὸ ὑπὸ τῶν ΑΓ, ΒΔ καὶ <ἡ> τὸ ὑπὸ ΑΒ, ΓΔ. ὥστε ὁ μοναχὸς καὶ μέγιστος λόγος ὁ αὐτός ἐστιν τῶι τοῦ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ τῆς συγκειμένης ἔκ τε τῆς δυναμένης τὸ ὑπὸ τῶν ΑΓ, ΒΔ καὶ τῆς δυναμένης τὸ ὑπὸ τῶν ΑΒ, ΓΔ.

(119) τὸ πρῶτον Διωρισμένης Τομῆς ἔχει προβλήματα ϛ̄, ἐπιτάγματα ιϛ̄, διορισμοὺς δὲ ε̄, ὧν μέγιστοι μὲν δ̄, ἐλάχιστος δὲ ᾱ. καὶ εἰσὶν μέγιστοι μὲν ὅ τε κατὰ τὸ β´ ἐπίταγμα τοῦ β´ προβλήματος, καὶ ὁ κατὰ τὸ τρίτον τοῦ τετάρτου προβλήματος, καὶ ὁ κατὰ τὸ τρίτον τοῦ πέμπτου, καὶ ὁ κατὰ τὸ τρίτον τοῦ ἕκτου· ἐλάχιστος δὲ ὁ κατὰ τὸ τρίτον ἐπίταγμα τοῦ τρίτου προβλήματος. τὸ δὲ δεύτερον Διωρισμένης ἔχει προβλήματα τρία, ἐπιτάγματα θ̄, διορισμοὺς γ̄, ὧν ἐλάχιστοι μὲν δύο, μέγιστος δὲ ᾱ. καὶ εἰσὶν ἐλάχιστοι μὲν ὅ τε κατὰ τὸ τρίτον τοῦ πρώτου καὶ ὁ κατὰ τὸ τρίτον τοῦ δευτέρου· μέγιστος δὲ ὁ κατὰ τὸ γ´ τοῦ γ´ προβλήματος.

‖ 1 ΕΓ Co ΗΓ A ‖ 2 ΕΘ Co ΕΔ A ‖ 4 ἀπὸ (ΕΘ) add Hu ‖ 7 τε τὸ Hu τό τε A | ἡ add Hu | ΑΒ, ΓΔ Co ΑΒΓ A ‖ 10 ΑΓ, ΒΔ Co ΑΒ, ΓΔ A ‖ 11 προβλήματα Ge (recc?) πρόβλημα τὸ A ‖ 13 ἐλάχιστος BS ἐλάχιστοι A ‖ 16 ἐλάχιστος ὁ Ge (Co) ἐλαχιστοι οἱ A ‖ 18 (τρι)α in ras. A ‖ 21 τὸ γ´ Hu τὸν A

(120) Neuses, (Book) 1.

1. *(Prop. 65)* Lemma useful for the first problem.

Let AB be greater than ΓΔ, and let the rectangle contained by AE, EB be equal to the rectangle contained by ΓZ, ZΔ. That AE is greater than ΓZ.

Also bisect both AB and ΓΔ at points <H,> Θ. Evidently HB is greater than ΘΔ.[1] Then since the rectangle contained by AE, EB equals the rectangle contained by ΓZ, ZΔ,[2] while the square of HB is greater than the square of ΘΔ,[3] therefore the rectangle contained by AE, EB plus the square <of HB is (greater than) the rectangle contained by ΓZ, ZΔ plus the square> of ΘΔ.[4] But the rectangle contained by AE, EB plus the square of HB equals the square of HE,[5] while the rectangle contained by ΓZ, ZΔ plus the square of ΘΔ equals the square of ZΘ.[6] Hence the square of HE is greater than the square of ΘZ.[7] Therefore HE is greater than ΘZ.[8] But also AH is greater than ΓΘ.[9] Therefore all AE is greater than all ΓZ.[10] Similarly, if AB is less than ΓΔ, and the rectangle contained by AE, EB is equal to the rectangle contained by ΓZ, ZΔ, all AE will be less than all ΓZ.

(121) 2. *(Prop. 66)* Let AB be greater than ΓΔ, and let ΓΔ be bisected at E. Then it is obviously possible to apply to AB a (rectangle) equal to the rectangle contained by ΓE, EΔ, (and deficient by a square). For the rectangle contained by ΓE, EΔ equals the square of ΓE, while the square of ΓE is less than the square of half AB.

Let it be applied, and let it be the rectangle contained by AZ, ZB, and let AZ be greater than ZB. Again it is evident that AZ is greater than ΓE, while BZ is less than EΔ. For AZ is <greater than> half the greater, while ΓE is half the lesser. But as is AZ to ΓE, so is EΔ to ZB. Q.E.D.

(122) 3. *(Prop. 67)* Again let the rectangle contained by AZ, ZB be equal to the rectangle contained by ΓE, EΔ, and let AB be less than ΓΔ, and furthermore let ΔE be less than EΓ, and BZ than ZA. That also AZ is less than ΓE.

Let ΓΔ, AB be bisected at points H, Θ. Then AΘ is less than ΓH,[1] so that also the square of AΘ is less than the square of ΓH.[2] But the square

(120) |ΝΕΥΣΕΩΝ ΠΡΩΤΟΝ |143

α.΄ λῆμμα χρήσιμον εἰς τὸ πρῶτον πρόβλημα.

ἔστω μείζων ἡ ΑΒ *τῆς* ΓΔ, *καὶ ἴσον τὸ ὑπὸ* ΑΕΒ *τῶι ὑπὸ* ΓΖΔ. *ὅτι μείζων ἐστὶν ἡ* ΑΕ *τῆς* ΓΖ. *καὶ τετμήσθω ἑκατέρα τῶν* ΑΒ, ΓΔ *δίχα καθ' ἑκάτερα τῶν* <Η,> Θ *σημείων. φανερὸν δὴ ὅτι μείζων ἐστὶν ἡ* ΗΒ *τῆς* ΘΔ. *ἐπεὶ οὖν ἴσον μέν ἐστιν τὸ ὑπὸ* ΑΕΒ *τῶι ὑπὸ* ΓΖΔ, *μεῖζον δὲ τὸ ἀπὸ* ΗΒ *τοῦ ἀπὸ* ΘΔ, *μεῖζον ἄρα* 772 *καὶ τὸ ὑπὸ* ΑΕΒ *μετὰ τοῦ ἀπὸ* <ΗΒ, *τοῦ ὑπὸ* ΓΖΔ *μετὰ τοῦ ἀπὸ*> ΘΔ. *ἀλλὰ τὸ μὲν ὑπὸ* ΑΕΒ *μετὰ τοῦ ἀπὸ* ΗΒ *ἴσον ἐστὶν τῶι ἀπὸ* ΗΕ, *τὸ δὲ ὑπὸ* ΓΖΔ *μετὰ τοῦ ἀπὸ* ΘΔ *ἴσον ἐστὶ τῶι ἀπὸ* ΖΘ. *μεῖζον ἄρα ἐστὶν καὶ τὸ ἀπὸ* ΗΕ *τοῦ ἀπὸ* ΘΖ. *ὥστε μείζων ἐστὶν ἡ* ΗΕ *τῆς* ΘΖ. *ἔστιν δὲ καὶ ἡ* ΑΗ *τῆς* ΓΘ *μείζων. ὅλη ἄρα ἡ* ΑΕ *ὅλης τῆς* ΓΖ *ἐστὶν μείζων. ὁμοίως δὲ καί, ἐὰν ἐλάσσων ἦι ἡ* ΑΒ *τῆς* ΓΔ, *καὶ ἴσον τὸ ὑπὸ* ΑΕΒ *τῶι ὑπὸ* ΓΖΔ, *ἐλάσσων ἔσται ὅλη ἡ* ΑΕ [*τῆς*] *ὅλης τῆς* ΓΖ.

(121) <*β.΄*> *ἔστω μείζων ἡ* ΑΒ *τῆς* ΓΔ, *καὶ τετμήσθω δίχα ἡ* ΓΔ *κατὰ τὸ* Ε. *φανερὸν μὲν οὖν ὅτι δυνατόν ἐστιν τῶι ὑπὸ τῶν* ΓΕ, ΕΔ *ἴσον παρὰ τὴν* ΑΒ *παραβαλεῖν. τὸ μὲν γὰρ ὑπὸ* ΓΕΔ *ἴσον τῶι ἀπὸ* ΓΕ, *τὸ δὲ ἀπὸ* ΓΕ *ἔλασσόν ἐστιν τοῦ ἀπὸ τῆς ἡμισείας τῆς* ΑΒ. *παραβεβλήσθω, καὶ ἔστω τὸ ὑπὸ τῶν* ΑΖΒ, *καὶ ἔστω μείζων ἡ* ΑΖ *τῆς* ΖΒ. *πάλιν δὴ φανερὸν ὅτι μείζων ἐστὶν ἡ* ΑΖ *τῆς* ΓΕ, *ἐλάσσων δὲ ἡ* ΒΖ *τῆς* ΕΔ. *ἡ μὲν γὰρ* ΑΖ *τῆς μείζονος* <*μείζων*> *ἐστὶν* <*ἢ*> *ἡμίσεια, ἡ δὲ* ΓΕ *τῆς ἐλάσσονός ἐστιν ἡμίσεια. ὡς δὲ ἡ* ΑΖ *πρὸς τὴν* ΓΕ, *οὕτως ἡ* ΕΔ *πρὸς τὴν* ΖΒ. *ὅ(περ)*: —

(122) <*γ.΄*> *ἔστω δὴ πάλιν ἴσον τὸ ὑπὸ* ΑΖΒ *τῶι ὑπὸ* ΓΕΔ, *καὶ ἐλάσσων* [*ἦι*] *ἡ* ΑΒ *τῆς* ΓΔ, *καὶ ἔτι ἐλάσσων μὲν ἡ* ΔΕ *τῆς* ΕΓ, *ἔτι δὲ ἡ* ΒΖ *τῆς* ΖΑ. *ὅτι καὶ ἡ* ΑΖ *τῆς* ΓΕ *ἐλάσσων ἐστίν. τετμήσθωσαν δὲ δίχα αἱ* ΓΔ, ΑΒ *κατὰ τὰ* Η, Θ *σημεῖα. ἐλάσσων ἄρα ἐστὶν καὶ ἡ* ΑΘ *τῆς* ΓΗ, *ὥστε καὶ τὸ ἀπὸ* ΑΘ *τοῦ ἀπὸ* ΓΗ

‖ 2 *α΄* mg A ‖ 3 ΑΒ Co ΑΓ A ‖ 5 Η add Co | *ὅτι*] *ὅ* (spatium quattuor litt.) *τι* A ‖ 8 ΗΒ — *ἀπὸ* add Co ‖ 10 ΓΖΔ Co ΖΔ A ‖ 11 *μείζων* Ge (BS) *μεῖζον* A ‖ 13 *ἐλάσσων* Ge (recc?) *ἔλασσον* A ‖ 14 *ἐλάσσων* Ge (recc?) *ἐλάσσονι* A ‖ 15 *τῆς* (*ὅλης*) del Ge (BS) ‖ 16 *β΄* add Hu (BS) ‖ 22 *ἐλάσσων* Ge (BS) *ἔλασσον* A ‖ 23 *μείζων… ἢ* add Co ‖ 24 *ἡμίσεια* Co *ἡμισείας* A | *ὡς* — ΖΒ secl Hu ‖ 25 post ΖΒ add *μείζων ἄρα ἐστὶν ἡ* ΑΖ *τῆς* ΓΕ Hu (Co) ‖ 26 *γ΄* add Hu (BS) ‖ 27 *ἦι* del Ge (S) | *ἔτι* Co *ὅτι* A ‖ 28 *ἔτι δὲ ἡ*] *ἔστιν δὲ ἡ* A *ἡ δὲ* Co | *ἐλάσσων* Ge (BS) *ἔλασσόν* A | post *ἐστὶν* add *ἡ δὲ* ΖΒ *τῆς* ΕΔ *μείζων* Co ‖ 29 *ἐλάσσων* Ge (BS) *ἔλασσον* A ‖ 30 *ὥστε* Hu *ἔστω* A of AΘ is equal to the rectangle contained by AZ, ZB plus the square of ZΘ,[3] while the square of ΓH equals the rectangle contained by ΓE, EΔ plus the square of HE.[4] Therefore the rectangle contained by AZ, ZB plus the square of ZΘ is less than the rectangle contained by ΓE, EΔ plus the square of HE.[5] Out of these the rectangle contained by AZ, ZB is assumed to be (equal) to ΓE, EΔ.[6] Therefore the remaining square of ΘZ is less than the square of HE.[7] Hence ΘZ is less than HE.[8] But also AΘ was less than ΓH.[9] Therefore all AZ is less than all ΓE,[10] and the remainder (ZB) is greater than the remainder (EΔ).[11]

(123) 4. *(Prop. 68)* Again let AB be greater than ΓΔ, and let ΓΔ be divided at E so that ΔE is not less than EΓ. Now it is obviously possible to apply to AB a (rectangle) equal to the rectangle contained by ΓE, EΔ and deficient by a square. For since ΔE is not less than EΓ, it is either equal to it or greater (than it). And if it is <equal>, then the rectangle contained by ΓE, EΔ equals the square of half ΓΔ, so that it is less than the square of half AB; while if it is greater, the rectangle contained by ΓE, EΔ is much less than the square of half AB, since it is less than the square of half ΓΔ. Hence it is possible to apply to AB a (rectangle) equal to the rectangle contained by ΓE, EΔ, and deficient by a square. Let it be applied, and let it be the rectangle contained by AZ, ZB, and let the greater part be AZ. That ZB is less than ΓE.

For since ΔE is not less than EΓ,[1] it is therefore either equal or greater. First let ΔE equal EΓ. Then since AB is greater than ΓΔ,[2] and AZ is greater than half AB,[3] but ΔE is half ΓΔ,[4] therefore AZ is greater than ΔE.[5] And as is AZ to ΓE, so is ΔE to ZB.[6] Hence ΓE too is greater than ZB.[7] Thus ZB is less than ΓE.

ἐστὶν ἔλασσον. ἀλλὰ τὸ μὲν ἀπὸ ΑΘ ἴσον ἐστὶν τῶι τε ὑπὸ 774
τῶν ΑΖΒ καὶ τῶι ἀπὸ ΖΘ. τὸ δὲ ἀπὸ ΓΗ ἴσον ἐστὶν τῶι τε ὑπὸ
ΓΕΔ καὶ τῶι ἀπὸ ΗΕ. καὶ τὸ ὑπὸ ΑΖΒ ἄρα μετὰ τοῦ ἀπὸ ΖΘ
ἔλασσόν ἐστιν τοῦ ὑπὸ ΓΕΔ μετὰ τοῦ ἀπὸ ΗΕ. ὧν τὸ ὑπὸ ΑΖΒ
ὑπόκειται τῶι ὑπὸ ΓΕΔ. λοιπὸν ἄρα τὸ |ἀπὸ ΘΖ ἔλασσόν ἐστιν
τοῦ ἀπὸ ΗΕ. ἐλάσσων ἄρα ἐστὶν ἡ ΘΖ τῆς ΗΕ. ἦν δὲ καὶ ἡ ΑΘ |143v
τῆς ΓΗ ἐλάσσων. ὅλη ἄρα ἡ ΑΖ ὅλης τῆς ΓΕ ἐστὶν ἐλάσσων, ἡ
δὲ λοιπὴ τῆς λοιπῆς μείζων.

(123) <δʹ> ἔστω δὴ πάλιν μείζων ἡ ΑΒ τῆς ΓΔ, καὶ
τετμήσθω ἡ ΓΔ κατὰ τὸ Ε ὥστε τὴν ΔΕ τῆς ΕΓ μὴ εἶναι
ἐλάσσονα. φανερὸν μὲν οὖν ὅτι ἔστιν τῶι ὑπὸ τῶν ΓΕΔ ἴσον
παρὰ τὴν ΑΒ παραβαλεῖν ἐλλεῖπον τετραγώνωι. ἐπεὶ γὰρ μή
ἐστιν ἐλάσσων ἡ ΔΕ τῆς ΕΓ, ἤτοι ἴση ἐστὶν αὐτῆι ἢ μείζων.
καὶ εἰ μὲν <ἴση,> ἴσον τὸ ὑπὸ ΓΕΔ τῶι ἀπὸ τῆς ἡμισείας τῆς
ΓΔ, ὥστε ἔλασσον τοῦ ἀπὸ τῆς ἡμισείας τῆς ΑΒ. εἰ δὲ μείζων,
πόλλωι ἔλασσον ἐστιν τὸ ὑπὸ ΓΕΔ τοῦ ἀπὸ τῆς ἡμισείας τῆς
ΑΒ· καὶ γὰρ τοῦ ἀπὸ τῆς ἡμισείας τῆς ΓΔ ἐστὶν ἔλασσον.
δυνατὸν ἄρα ἐστὶν τῶι ὑπὸ τῶν ΓΕΔ ἴσον παρὰ τὴν ΑΒ
παραβαλεῖν, ἐλλεῖπον τετραγώνωι. παραβεβλήσθω, καὶ ἔστω τὸ
ὑπὸ τῶν ΑΖΒ, καὶ τὸ μεῖζον τμῆμα ἔστω ἡ ΑΖ. ὅτι δὴ ἐλάσσων
ἐστὶν ἡ ΖΒ τῆς ΓΕ. ἐπεὶ γὰρ ἡ ΔΕ τῆς ΕΓ οὐκ ἔστιν ἐλάσσων,
ἤτοι ἄρα ἴση ἐστὶν ἢ μείζων. ἔστω πρότερον ἴση ἡ ΔΕ τῆι ΕΓ.
ἐπεὶ οὖν μείζων ἐστὶν ἡ ΑΒ τῆς ΓΔ, καὶ ἔστι τῆς μὲν ΑΒ 776
μείζων <ἡ> ἡμίσεια ἡ ΑΖ, τῆς δὲ ΓΔ ἡμίσεια ἡ ΔΕ, μείζων ἄρα
ἐστὶν ἡ ΑΖ τῆς ΔΕ. καὶ ἔστιν ὡς ἡ ΑΖ πρὸς τὴν ΓΕ, οὕτως ἡ ΔΕ
πρὸς τὴν ΖΒ. μείζων ἄρα καὶ ἡ ΓΕ τῆς ΖΒ. ὥστε ἐλάσσων
ἐστὶν ἡ ΖΒ τῆς ΓΕ.

‖ 1 ΑΘ Co ΑΖ ἄρα A | τε Ge (recc?) τὸ A ‖ 4 ΗΕ Co ΗΘ A | post ΑΖΒ add ἴσον Hu ‖ 6 ἐλάσσων Ge (S) ἐλάσσονος A ‖ 9 δʹ add Ge (BS) ‖ 11 ἐλάσσονα Hu (BS) ἔλασσον A | ante ἐστιν add δυνατόν Co ‖ 12 τετραγώνωι Co τετράγωνον A | μὴ] οὐκ Ge ‖ 13 ἐλάσσων Ge (S) ελασσον A ‖ 14 ἴση add Co | post ἴσον add ἐστὶ Co | τὸ Co τοῦ A ‖ 15 ἔλασσον Co restituens lacunam in k ἐλάσσονος A | εἰ Co ἡ A ‖ 16 ἔλασσον Co ἐλάσσονος A ‖ 17 γὰρ Ge (B) γ̄ A ‖ 18 δυνατὸν Co δὲ A | τῶν Ge (BS) τῶι A | ἴσον Co ἴση A | τὴν Co τῆς A ‖ 19 τετραγώνωι Co τετράγωνον A ‖ 20 δὴ Co δὲ A ‖ 21 τῆς ΓΕ bis A corr Co | ἐλάσσων Ge (BS) ἔλασσον A ‖ 24 ἡ add Ge (S) | δὲ Ge (recc?) Δ̄Ε̄ A ‖ 25 ΓΕ Hu ΔΕ A | ΔΕ Hu ΓΕ A ‖ 27 ΖΒ Co ΖΘ A

(124) But let ΔE be greater than EΓ, and let ΓΔ be bisected at point H, and AB at point Θ. Then since AB is greater than ΓΔ,[8] and ΘB is half AB,[9] and ΓH half of ΓΔ,[10] therefore ΘB is greater than ΓH.[11] Hence also the square of ΘB is greater than the square of ΓH.[12] But the square of ΘB equals the rectangle contained by AZ, ZB plus the square of ZΘ,[13] while the rectangle contained by ΓH equals the <rectangle contained by> ΓE, EΔ plus the square of EH.[14] Therefore the rectangle contained by AZ, ZB plus the square of ZΘ is (greater) than the rectangle contained by ΓE, EΔ plus the square of EH.[15] Out of these, the rectangle contained by AZ, ZB equals the rectangle contained by ΓE, EΔ.[16] Therefore the remaining square of ΘZ is greater than the square of EH.[17] Hence ΘZ is greater than EH.[18] But also AΘ is greater than ΔH.[19] Therefore all AZ is greater than all ΔE.[20] And as is AZ to ΓE, so is ΔE to ZB.[21] Therefore ΓE too is greater than ZB.[22] Thus ZB is less than ΓE. Q.E.D.

(125) 5. *(Prop. 69)* For the sixth problem.

Let AB be less than ΓΔ, and the rectangle contained by AE, EB equal to the rectangle contained by ΓZ, ZΔ. That AE is less than ΓZ.

Let AB, ΓΔ be bisected at points Θ, H. Then ΘB is less than HΔ.[1] So since the rectangle contained by ΓZ, ZΔ equals the rectangle contained by AE, EB,[2] while the square of ΘB is less than the square of HΔ,[3] therefore the rectangle contained by AE, EB plus the square of ΘB, that is the square of ΘE,[5] is less than the rectangle contained by ΓZ, ZΔ plus the square of HΔ,[4] that is the square of HZ.[6] Hence EΘ is less than HZ.[7] But also AΘ is less than ΓH.[8] Therefore all AE is less than all ΓZ.[9] Similarly, if (AB) is greater (than ΓΔ), all (AE will be greater) than all (ΓZ).

(124) ἔστω δὲ μείζων ἡ ΔΕ τῆς ΕΓ, καὶ τετμήσθω δίχα ἡ ΓΔ κατὰ τὸ Η σημεῖον, ἡ δὲ ΑΒ δίχα κατὰ τὸ Θ σημεῖον. ἐπεὶ οὖν μείζων ἐστὶν ἡ ΑΒ τῆς ΓΔ, καὶ ἔστι τῆς μὲν ΑΒ ἡμίσεια ἡ ΘΒ, τῆς δὲ ΓΔ ἡμίσεια ἡ ΓΗ, μείζων ἄρα ἡ ΘΒ τῆς ΓΗ. ὥστε καὶ τὸ ἀπὸ ΘΒ τοῦ ἀπὸ ΓΗ μεῖζόν ἐστιν. ἀλλὰ τὸ μὲν ἀπὸ ΘΒ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΖΒ καὶ τῶι ἀπὸ ΖΘ, τὸ δὲ ἀπὸ ΓΗ ἴσον ἐστὶν τῶι τε <ὑπὸ> τῶν ΓΕΔ καὶ τῶι ἀπὸ τῆς ΕΗ. μεῖζον ἄρα ἐστὶν τὸ ὑπὸ ΑΖΒ μετὰ τοῦ ἀπὸ ΖΘ τοῦ ὑπὸ ΓΕΔ μετὰ τοῦ ἀπὸ ΕΗ. ὧν τὸ ὑπὸ ΑΖΒ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΓΕΔ. λοιπὸν ἄρα τὸ ἀπὸ ΘΖ μεῖζόν ἐστιν τοῦ ἀπὸ ΕΗ. ὥστε μείζων ἐστὶν ἡ ΘΖ τῆς ΕΗ. ἔστιν δὲ καὶ ἡ ΑΘ τῆς ΔΗ μείζων. ὅλη ἄρα ἡ ΑΖ ὅλης τῆς ΔΕ μείζων ἐστίν. καὶ |ἔστιν ὡς ἡ ΑΖ πρὸς τὴν ΓΕ, οὕτως ἡ ΔΕ |144
πρὸς τὴν ΖΒ. μείζων ἄρα καὶ ἡ ΓΕ τῆς ΖΒ. ὥστε ἐλάσσων ἐστὶν ἡ ΖΒ τῆς ΓΕ. ὅπερ: —

(125) <ε.´> εἰς τὸ ς´ πρόβλημα.

ἔστω ἐλάσσων μὲν ἡ ΑΒ τῆς ΓΔ, ἴσον δὲ τὸ ὑπὸ τῶν ΑΕΒ τῶι ὑπὸ ΓΖΔ. ὅτι ἐλάσσων ἐστὶν ἡ ΑΕ τῆς ΓΖ. τετμήσθωσαν δίχα αἱ ΑΒ, ΓΔ κατὰ τὰ Θ, Η σημεῖα. ἐλάσσων ἄρα ἐστὶν ἡ ΘΒ τῆς ΗΔ. ἐπεὶ οὖν τὸ μὲν ὑπὸ ΓΖΔ ἴσον ἐστὶν τῶι ὑπὸ ΑΕΒ, τὸ δὲ ἀπὸ ΘΒ ἔλασσόν ἐστιν τοῦ ἀπὸ ΗΔ, τὸ ἄρα ὑπὸ ΑΕΒ μετὰ τοῦ ἀπὸ ΘΒ, ὅ ἐστιν τὸ ἀπὸ ΘΕ, ἔλασσόν ἐστιν τοῦ ὑπὸ ΓΖΔ μετὰ τοῦ ἀπὸ ΗΔ, 778
τουτέστιν τοῦ ἀπὸ ΗΖ. ὥστε ἐλάσσων ἐστὶν ἡ ΕΘ τῆς ΗΖ. ἔστιν δὲ καὶ ἡ ΑΘ τῆς ΓΗ ἐλάσσων. ὅλη ἄρα ἡ ΑΕ ὅλης τῆς ΓΖ ἐστὶν ἐλάσσων. ὁμοίως κἂν μείζων, ἡ ὅλη τῆς ὅλης.

‖ 3 ἡμίσεια Co ἄρα A ‖ 4 ἡμίσεια Co ἄρα A ‖ 6 ΖΘ τὸ δὲ ἀπὸ bis A corr Co ‖ 7 ὑπὸ add Ge | ἄρα ex δρα A² ‖ 8 τοῦ ὑπὸ ΓΕΔ Co τὸ ὑπὸ ΑΕΔ A ‖ 12 ΓΕ Hu ΔΕ A | ΔΕ Hu ΑΒ A ΓΕ Co ‖ 13 ΓΕ Co ΑΕ A ‖ 14 ΓΕ Co ΑΕ A ‖ 15 ε´ add Hu (BS) ‖ 17 ΓΖΔ Ge (S) ΓΖ A ‖ 18 ἐλάσσων Ge (BS) ἔλασσον A ‖ 21 ΓΖΔ Co ΑΖΔ A ‖ 24 ἡ] ἧι ἡ Ge

(126) 6. *(Prop. 70)* Overlooked in the eighth problem.

AΔ being a rhombus whose diameter (produced) is BΓE, if EZ is taken as mean proportional between BE and EΓ, and with center E and radius EZ a circle ZHΘ is described, and ΛΓH is produced, then the line through H, K, B will be straight.

For let ΛE, EK, BK, KH, <EH> be joined. Then since angle ΛΓZ equals angle ZΓK,[1] and they are on either side of the circle's diameter, ΛΓ and ΓK are equal;[2] for this is a lemma. But also ΛE equals EK.[3] Therefore angle ΓΛE equals angle ΓKE.[4] But angle ΓΛE equals angle ΓHE.[5] Therefore angle ΓHE equals angle ΓKE.[6] But also angle ΓKE (equals) angle ΓBK.[7] Therefore also angle ΓBK equals angle ΓHE.[8] But also angle HΓE equals angle BΓK.[9] Therefore the remaining angle ΓEH (in triangle ΓEH) equals the remaining angle ΓKB (in triangle ΓKB).[10] But angle ΓEH plus angle ΓKH equals two right angles.[11] Therefore also angle ΓKB plus angle ΓKH equals two right angles.[12] Thus the line through points B, K, H is straight.[13]

(127) 7. *(Prop. 71)* Lemma useful for the problem on a square, that does the same thing as for the rhombus.

Let AΔ be a square, and let BHE be drawn, and let EZ be drawn at right angles to it. That the squares of ΓΔ and HE equal the square of ΔZ.

Through E draw EΘ parallel to ΓΔ.[1] Then angle ΓEΘ is right.[2] But also angle ZEH is right.[3] Therefore angle ΓEH, that is angle ΔBH, equals angle ZEΘ as well.[4] But also angle ZΘE equals right angle BΔH.[5] And EΘ equals BΔ.[6] Therefore also EZ equals HB.[7] And since the square of BZ equals the squares of BE and EZ,[8] and out of these the rectangle contained by ZB, BΔ equals the rectangle contained by EB, BH[10] – for points Δ, Z, E, H are on a circle[9] – therefore the remaining rectangle contained by BZ, ZΔ equals the rectangle contained by BE, EH plus the square of EZ,[11] that is plus the square of BH.[12] But the rectangle contained by BE, EH plus the square of BH is the rectangle contained by EB, BH plus the square of EH.[13] Therefore the rectangle contained by BZ, ZΔ equals the rectangle contained by EB, BH, that is (the rectangle contained by) ZB, BΔ, plus the square of HE.[14] [15] Let the rectangle contained by BΔ, ΔZ be subtracted in common. Then the remaining square of ZΔ equals the squares of BΔ and HE, that is the squares of ΓΔ and HE.[16]

(128) 8. *(Prop. 72)* Problem, as Heraclitus.

AΔ being a square (given) in position, to place a given (line) EZ, making a neusis on B. Let it be accomplished, and from point E let EH be drawn at right angles to BE; for (BZE) is a straight line.

Then since the squares of ΓΔ and ZE equal the square of ΔH (lemma 7.127),[1] while the squares of ΓΔ and ZE are given,[3] because both (ΓΔ, ZE) are given in magnitude,[2] therefore also the square of ΔH is given.[4] Therefore ΔH is given in magnitude.[5] And therefore all BH is given in magnitude.[6] But it is also (given) in position.[7] Therefore the semicircle on

(126) <ϛ.´> παραθεωρούμενον ἐν τῶι η´ προβλήματι.

ῥόμβου ὄντος τοῦ ΑΔ, οὗ διάμετρος ἡ ΒΓΕ, ἐὰν τῶν ΒΕ, ΕΓ μέση ἀνάλογον ληφθῆι ἡ ΕΖ, καὶ κέντρωι μὲν τῶι Ε, διαστήματι δὲ τῶι ΕΖ, κύκλος γραφῆι ὁ ΖΗΘ, καὶ ἐκβληθῆι ἡ ΛΓΗ, ἔσται εὐθεῖα ἡ διὰ τῶν Η, Κ, Β. ἐπεζεύχθωσαν γὰρ αἱ ΛΕ, ΕΚ, ΒΚ, ΚΗ <ΕΗ>. ἐπεὶ οὖν ἴση ἐστὶν ἡ ὑπὸ ΛΓΖ γωνία τῆι ὑπὸ ΖΓΚ γωνίαι, καὶ ἐφ' ἑκάτερα τῆς τοῦ κύκλου διαμέτρου εἰσίν, αἱ ΛΓ, ΓΚ ἴσαι εἰσίν (λῆμμα γάρ). ἀλλὰ καὶ ἡ ΛΕ τῆι ΕΚ ἴση ἐστίν. γωνία ἄρα ἡ ὑπὸ ΓΛΕ γωνίαι τῆι ὑπὸ ΓΚΕ ἴση ἐστίν. ἀλλὰ ἡ ὑπὸ ΓΛΕ ἴση ἐστὶν τῆι ὑπὸ ΓΗΕ. καὶ ἡ ὑπὸ ΓΗΕ ἄρα ἴση ἐστὶν τῆι ὑπὸ ΓΚΕ. ἔστιν δὲ καὶ ἡ ὑπὸ ΓΚΕ τῆι ὑπὸ ΓΒΚ. καὶ ἡ ὑπὸ ΓΒΚ ἄρα ἴση ἐστὶν τῆι ὑπὸ ΓΗΕ. ἀλλὰ καὶ ἡ ὑπὸ ΗΓΕ τῆι 780 ὑπὸ ΒΓΚ ἴση ἐστίν. λοιπὴ ἄρα ἡ ὑπὸ ΓΕΗ λοιπῆι τῆι ὑπὸ ΓΚΒ ἴση ἐστίν. ἀλλὰ ἡ ὑπὸ ΓΕΗ μετὰ τῆς ὑπὸ ΓΚΗ δυσὶν ὀρθαῖς ἴσαι εἰσίν. καὶ ἡ ὑπὸ ΓΚΒ ἄρα μετὰ τῆς ὑπὸ ΓΚΗ γωνίας δυσὶν ὀρθαῖς ἴσαι εἰσίν. ὥστε εὐθεῖά ἐστιν ἡ διὰ τῶν Β, Κ, Η σημείων.

(127) <ζ.´> λῆμμα χρήσιμον εἰς τὸ πρόβλημα ἐπὶ τετραγώνου ποιοῦντος τὰ αὐτὰ τῶι ῥόμβωι.

ἔστω τετράγωνον τὸ ΑΔ, |καὶ ἤχθω ἡ ΒΗΕ, καὶ αὐτῆι ὀρθὴ ἤχθω ἡ ΕΖ. ὅτι τὰ ἀπὸ τῶν ΓΔ, ΗΕ τετράγωνα ἴσα ἐστὶν τῶι |144v ἀπὸ τῆς ΔΖ τετραγώνωι. ἤχθω διὰ τοῦ Ε τῆι ΓΔ παράλληλος ἡ ΕΘ. ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΓΕΘ γωνία. ἔστιν δὲ καὶ ἡ ὑπὸ ΖΕΗ γωνία ὀρθή. ἴση ἄρα ἐστὶν καὶ ἡ ὑπὸ ΓΕΗ γωνία, τουτέστιν ἡ ὑπὸ ΔΒΗ γωνία, τῆι ὑπὸ ΖΕΘ γωνίαι. ἔστιν δὲ καὶ ἡ ὑπὸ ΖΘΕ γωνία ὀρθῆι τῆι ὑπὸ ΒΔΗ ἴση. καὶ ἔστιν ἴση ἡ ΕΘ τῆι ΒΔ. ἴση ἄρα ἐστὶν καὶ ἡ ΕΖ τῆι ΗΒ. ἐπεὶ δὲ τὸ ἀπὸ τῆς ΒΖ ἴσον τοῖς ἀπὸ τῶν ΒΕ, ΕΖ τετραγώνοις, ὧν τὸ ὑπὸ ΖΒΔ ἴσον ἐστὶν τῶι ὑπὸ ΕΒΗ (ἐν κύκλωι γάρ ἐστιν τὰ Δ, Ζ, Ε, Η σημεῖα), λοιπὸν ἄρα τὸ ὑπὸ ΒΖΔ ἴσον ἐστὶν τῶι τε ὑπὸ ΒΕΗ καὶ τῶι ἀπὸ ΕΖ τετραγώνωι, τουτέστιν καὶ τῶι ἀπὸ ΒΗ τετραγώνωι. ἀλλὰ τὸ ὑπὸ ΒΕΗ μετὰ τοῦ ἀπὸ ΒΗ τετραγώνου τὸ ὑπὸ ΕΒΗ ἐστὶν μετὰ τοῦ ἀπὸ ΕΗ. τὸ ἄρα ὑπὸ ΒΖΔ ἴσον ἐστὶν τῶι τε ὑπὸ ΕΒΗ, τουτέστιν ὑπὸ ΖΒΔ, καὶ τῶι ἀπὸ ΗΕ. κοινὸν ἀφῃρήσθω τὸ ὑπὸ 782 ΒΔΖ. λοιπὸν ἄρα τὸ ἀπὸ ΖΔ ἴσον ἐστὶν τοῖς ἀπὸ τῶν ΒΔ, ΗΕ, τουτέστιν τοῖς ἀπὸ τῶν ΓΔ, ΗΕ τετραγώνοις.

‖ 1 ϛ´ add Hu (BS) ‖ 3 ΕΖ Co ΒΖ A ‖ 5 ἐπεζεύχθωσαν Ge (BS) ἐπεζεύχθω A | ΛΕ, ΕΚ, ΒΚ, ΚΗ, ΕΗ Horsley ΛΕ, ΕΚ, ΒΚ, ΚΗ A ΕΗ, ΒΚ, ΚΗ Co ‖ 7 διαμέτρου Ge (S) διάμετροι A ‖ 8 τῆι] τῆς A¹ ς del A² ‖ 9 ΓΚΕ Ge (S) ΓΚ A ‖ 10 ΓΗΕ (καὶ) Co ΓΗΘ A ‖ 11 (Γ)ΒΚ in ras. A ‖ 15 γωνίας Ge γωνία A ‖ 18 ζ´ add Hu (BS) | πρόβλημα om A¹ add subt. alia manu A² ‖ 19 τετραγώνου ποιοῦντος] τετράγωνον ποιοῦν A τετραγώνων ποιούντων Hu | ποιοῦν] ποιούντων Hu ‖ 20 ὀρθὴ Ge (BS) ὀρθῆι A ‖ 29 Ε Co Θ A ‖ 32 post ἐστὶν add ἄρα A¹ expunctum A² ‖ 35 ΒΔΖ Co ΒΖΔ A

BH is given in position.[8] And it passes through E,[9] and hence E is on a (circular) arc (given) in position. But (it is) also (on) AE (which is given) in position.[10] Hence it is given.[11] But B too is given.[12] Therefore BE is (given) in position.[13]

(129) The synthesis of the problem will be made thus. Let the square be AΔ, the given straight line Θ, and let the square of ΔH be equal to the squares of ΓΔ and Θ.

Then HΔ is greater than ΔΓ.[1] Hence the rectangle contained by HΔ, ΔB is greater than the square of ΔΓ.[2] Therefore the semicircle on BH when drawn will fall beyond point Γ.[3] Let it be drawn, and let it be BKEH, and let AΓ be produced to E, and let BE, EH be joined. Then the squares of ΓΔ and EZ equal the square of HΔ (lemma 7.127).[4] But the squares of ΓΔ and Θ were set equal to the square of ΔH.[5] Therefore the squares of ΓΔ and Θ equal the squares of ΓΔ and EZ.[6] Hence the square of Θ equals the square of EZ.[7] Therefore Θ equals EZ.[8] And EZ is given. Thus EZ solves the problem.

I say that it alone (solves the problem). For let some other (line) BΛ be drawn.

Now if BΛ too solves the problem, then NΛ will equal EZ,[1] but ZB will be greater than NB.[2] Therefore all BΛ is less then BE;[3] which is absurd, since it is also greater. Hence BΛ does not solve the problem. Thus BE alone (solves it).

In order to find out which of them is greater, we will make the demonstration as follows.

Since ΛB is greater than BE,[1] and BZ than BN,[2] therefore remainder NΛ is greater than ZE.[3] And it is evident that the (line) nearest point Γ is always less than the farther one.

(128) <η.´> πρόβλημα ὡς Ἡράκλειτος.

τετραγώνου ὄντος θέσει τοῦ ΑΔ, θεῖναι δοθεῖσαν τὴν ΕΖ, νεύουσαν ἐπὶ τὸ Β. γεγονέτω, καὶ ἀπὸ τοῦ Ε σημείου τῆι ΒΕ ὀρθογώνιος (εὐθεῖα γὰρ) ἤχθω ἡ ΕΗ. ἐπεὶ οὖν τὰ ἀπὸ τῶν ΓΔ, ΖΕ τετράγωνά ἐστιν τῶι ἀπὸ ΔΗ τετραγώνωι, δοθέντα δὲ τὰ ἀπὸ τῶν ΓΔ, ΖΕ (δοθεῖσα γὰρ ἑκατέρα τῶι μεγέθει), δοθὲν ἄρα καὶ τὸ ἀπὸ ΔΗ. δοθεῖσα ἄρα ἐστὶν ἡ ΔΗ τῶι μεγέθει. καὶ ὅλη ἄρα ἡ ΒΗ δέδοται τῶι μεγέθει· ἀλλὰ καὶ τῆι θέσει. δέδοται ἄρα τῆι θέσει τὸ ἐπὶ τῆς ΒΗ ἡμικύκλιον. καὶ ἔρχεται διὰ τοῦ Ε. τὸ Ε ἄρα θέσει περιφερείας ἅπτεται· ἀλλὰ καὶ θέσει εὐθείας τῆς ΑΕ. δοθεῖσα ἄρα ἐστίν. ἀλλὰ καὶ τὸ Β ἐστὶν δοθέν. θέσει ἄρα ἐστὶν ἡ ΒΕ.

(129) συντεθήσεται δὴ τὸ πρόβλημα οὕτως. ἔστω τὸ μὲν τετράγωνον τὸ ΑΔ, ἡ δὲ δοθεῖσα εὐθεῖα ἡ Θ, καὶ τοῖς ἀπὸ τῶν ΓΔ, Θ ἴσον ἔστω τὸ ἀπὸ τῆς ΔΗ τετράγωνον. μείζων ἄρα ἐστὶν ἡ ΗΔ τῆς ΔΓ. ὥστε καὶ τὸ ὑπὸ ΗΔ, ΔΒ μεῖζόν ἐστιν τοῦ ἀπὸ ΔΓ. τὸ ἄρα ἐπὶ τῆς ΒΗ ἡμικύκλιον γραφόμενον ὑπερπεσεῖται τὸ Γ σημεῖον. γεγράφθω, καὶ ἔστω τὸ ΒΚΕΗ, καὶ ἐκβεβλή|σθω ἡ ΑΓ 784
ἐπὶ τὸ Ε, καὶ ἐπεζεύχθωσαν αἱ ΒΕ, ΕΗ. τὰ ἄρα ἀπὸ τῶν ΓΔ, ΕΖ |145
τετράγωνα ἴσα ἐστὶν τῶι ἀπὸ ΗΔ τετραγώνωι. τῶι δὲ ἀπὸ ΔΗ ἴσα ἐτέθη τὰ ἀπὸ τῶν ΓΔ, Θ τετράγωνα. ἴσα ἄρα ἐστὶν τὰ ἀπὸ τῶν ΓΔ, Θ τετράγωνα τοῖς ἀπὸ τῶν ΓΔ, ΕΖ. ὥστε ἴσον ἐστὶν τὸ ἀπὸ Θ τῶι ἀπὸ ΕΖ τετραγώνωι. ἴση ἄρα ἐστὶν ἡ Θ τῆι ΕΖ. καὶ ἔστιν δοθεῖσα ἡ ΕΖ· ἡ ΕΖ ἄρα ποιεῖ τὸ πρόβλημα.

λέγω δὴ ὅτι καὶ μόνη. διήχθω γάρ τις καὶ ἑτέρα ἡ ΒΛ. εἰ δὴ καὶ ἡ ΒΛ ποιεῖ τὸ πρόβλημα, ἔσται ἴση ἡ ΝΛ τῆι ΕΖ, μείζων δὲ ἡ ΖΒ τῆς ΝΒ. ὅλη ἄρα ἡ ΒΛ ἐλάσσων ἐστὶν τῆς ΒΕ. ὅπερ ἄτοπον. ἔστιν γὰρ καὶ μείζων. οὐκ ἄρα ἡ ΒΛ ποιεῖ τὸ πρόβλημα. ἡ ΒΕ ἄρα μόνη.

ἵνα δὲ καὶ ἐπιγνῶμεν ποτέρα αὐτῶν μείζων, δείξομεν οὕτως. ἐπεὶ μείζων ἐστὶν ἡ μὲν ΛΒ τῆς ΒΕ, ἡ δὲ ΒΖ τῆς ΒΝ, λοιπὴ ἄρα ἡ ΝΛ τῆς ΖΕ μείζων ἐστί. καὶ φανερὸν ὅτι αἰεὶ ἡ ἔγγιστα τοῦ Γ σημείου τῆς ἀπώτερον ἐλάσσων.

‖ 1 η´ add Hu (BS) ‖ 2 θέσει del Co | θεῖναι] θ̄ εἶναι A ποιεῖν Hu ‖ 4 ὀρθογώνιος εὐθεῖα γὰρ] ὀρθογώνιον εὐθεῖα γὰρ A ὀρθογωνιος Hu ὀρθογώνιος εὐθεῖα Ge ὀρθὴ Co ‖ 5 post τετράγωνα add ἴσα Co ‖ 6 δοθεῖσα γὰρ ἑκατέρα Hu app δοθέντα γὰρ ἑκάτερα A | post ἑκατέρα add τῶν ΓΔ, ΖΕ Hu app ‖ 10 περιφερείας Co περιφέρεια A ‖ 11 εὐθείας Co εὐθεῖα A ‖ 12 δοθὲν Co δοθεῖσα A ‖ 19 ἐπεζεύχθωσαν Ge (BS) ἐπεζεύχθω A ‖ 20 τετράγωνα ἴσα Co τετράγωνον ἴσον ἄρα A ‖ 24 ΕΖ Co ΕΖΒ A ‖ 26 ΝΛ Co ΗΛ A ‖ 27 ἐστὶν] ἔσται Hu ‖ 28 καὶ del Ge (recc?) ‖ 31 ΛΒ Co ΑΒ A ‖ 32 ΖΕ Co ΖΗ A

(130) 9. *(Prop. 73)* Lemma useful for the diorism of the ninth theorem, as among the ancients.

Let BA be equal to AΓ, and let BΓ be bisected at point Δ. That BΓ is the least of all the lines drawn through point Δ. For let some other (line) EZ be drawn, and let AB be produced to Z. That EZ is greater than ΓB.

Since angle ABΓ, that is angle Γ, is greater than angle BZE,[1] it is possible to take away from angle Γ an (angle) equal to angle BZE. Let angle ΔΓH be equal to it.[2] Then as is ZΔ to ΔB, so is ΓΔ to ΔH,[3] while ZΔ is greater than ΔB.[4] Therefore ΓΔ too is greater than ΔH.[5] Then since ZΔ is greater than ΔB, that is than ΔΓ,[6] but ΔΓ is greater than ΔH,[7] <therefore ZΔ is greatest, ΔH least.>[8] So since there are four straight lines ZΔ, ΔB, ΔΓ, ΔH that are in ratio,[9] and ZΔ is greatest, ΔH least, therefore ZH is greater than BΓ (V, 25).[10] Thus BΓ is less than ZH. Hence it is much less than EZ.[11] Similarly we shall prove that BΓ is less than all the straight lines drawn through Δ.

Thus BΓ is less than all the straight lines drawn through Δ. I also say that the nearest (line) to it is less than the farther (line). For let some other (line) ΘK be drawn, and let angle ΔEΛ be made equal to angle K;[12] for this is possible. Again, KΔ is greater than ZΔ,[13] and EΔ than ΔΛ.[14] Therefore all KΛ is greater than EZ.[15] Therefore ΘK is much greater than EZ.[16] Hence EZ is less than ΘK. Thus BΓ is less than all the straight lines drawn through Δ, and the nearest to it is always less than the farther one.

(131) 10. *(Prop. 74)* This being so, the diorism is obvious. For if we set out the rhombus ABΓΔ, and if I join AΔ and draw EZ at right angles to it and intersecting AΓ and AB at E, Z, I have to make the distinction of whether it is greatest or least of all the straight lines drawn through Δ.

And since AΔ is a diagonal,[1] and EZ is at right angles to AΔ,[2] I have obtained an isosceles triangle EAZ,[3] having EA equal to AZ. But by the foregoing lemma (7.130), EZ is less than all the straight lines drawn through Δ, and the nearer to it is always less than the farther (line).[4]

(130) <θ.´> λῆμμα χρήσιμον εἰς τὸν τοῦ θ´ θεωρήματος διορισμόν, ὡς ἐν τοῖς ἀρχαίοις.

ἔστω ἴση ἡ ΒΑ τῆι ΑΓ, καὶ τετμήσθω ἡ ΒΓ δίχα κατὰ τὸ Δ σημεῖον. ὅτι ἐλαχίστη ἐστὶν ἡ ΒΓ πασῶν τῶν διὰ τοῦ Δ σημείου διαγομένων εὐθειῶν. διήχθω γάρ τις καὶ ἑτέρα ἡ ΕΖ, καὶ ἐκβεβλήσθω ἡ ΑΒ ἐπὶ τὸ Ζ. ὅτι μείζων ἐστὶν ἡ ΕΖ τῆς ΓΒ. ἐπεὶ μείζων ἐστὶν ἡ ὑπὸ ΑΒΓ γωνία, τουτέστιν ἡ Γ, τῆς ὑπὸ ΒΖΕ, δυνατόν ἐστιν τῆι ὑπὸ ΒΖΕ ἴσην ἀπὸ τῆς Γ ἀφελεῖν. ἔστω αὐτῆι ἴση ἡ ὑπὸ ΔΓΗ γωνία. ἔστιν ἄρα ὡς ἡ ΖΔ πρὸς τὴν ΔΒ, 786 οὕτως ἡ ΓΔ πρὸς τὴν ΔΗ, μείζων δὲ ἡ ΖΔ τῆς ΔΒ. μείζων ἄρα καὶ ἡ ΓΔ τῆς ΔΗ. ἐπεὶ οὖν μείζων ἐστὶν ἡ ΖΔ τῆς ΔΒ, τουτέστιν τῆς ΔΓ, ἀλλὰ ἡ ΔΓ τῆς ΔΗ μείζων ἐστίν, <μεγίστη ἄρα ἐστὶν ἡ ΖΔ, ἐλαχίστη δὲ ἡ ΔΗ.> ἐπεὶ οὖν τέσσαρες εὐθεῖαι αἱ ἀνάλογόν εἰσιν, αἱ ΖΔ, ΔΒ, ΔΓ, ΔΗ, καὶ ἔστι μεγίστη μὲν ἡ ΖΔ, ἐλαχίστη δὲ ἡ ΔΗ, μείζων ἄρα ἐστὶν ἡ ΖΗ τῆς ΒΓ. ὥστε ἡ ΒΓ ἐλάσσων ἐστὶν τῆς ΖΗ. πολλῶι ἐλάσσων <ἄρα> ἐστὶν τῆς ΕΖ. ὁμοίως δείξομεν ὅτι καὶ πασῶν τῶν διὰ τοῦ Δ διαγομένων εὐθειῶν ἐλάσσων ἐστὶν ἡ ΒΓ. ἡ ΒΓ ἄρα ἐλάσσων ἐστὶν πασῶν τῶν διὰ τοῦ Δ διαγομένων εὐθειῶν. λέγω δὴ ὅτι καὶ ἡ ἔγγιστα αὐτῆς τῆς ἀπώτερον ἐλάσσων ἐστίν. διήχθω |γάρ τις καὶ ἑτέρα, ἡ ΘΚ, καὶ τῆι Κ γωνίαι ἴση |145v συνεστάτω ἡ ὑπὸ ΔΕΛ· δυνατὸν γάρ. πάλιν δὴ μείζων ἡ μὲν ΚΔ τῆς ΖΔ, ἡ δὲ ΕΔ τῆς ΔΛ. ὥστε ὅλη ἡ ΚΛ μείζων ἐστὶν τῆς ΕΖ. πολλῶι ἄρα μείζων ἡ ΘΚ τῆς ΕΖ. ὥστε ἐλάσσων ἐστὶν ἡ ΕΖ τῆς ΘΚ. ἐλάσσων μὲν ἄρα ἐστὶν ἡ ΒΓ πασῶν τῶν διὰ τοῦ Δ διαγομένων εὐθειῶν, αἰεὶ δὲ ἡ ἔγγιστα αὐτῆς τῆς ἀπώτερον ἐλάσσων.

(131) <ι.´> τούτου ὄντος, φανερὸς ὁ διορισμός. ἐὰν γὰρ ἐκθώμεθα τὸν ῥόμβον τὸν ΑΒΓΔ, καὶ ἐπιζεύξας τὴν ΑΔ ἀγάγω αὐτῆι ὀρθὴν τὴν ΕΖ συμπίπτουσαν ταῖς ΑΓ, ΑΒ κατὰ τὰ Ε, Ζ, δεῖ με διορίζεσθαι πότερον μεγίστη ἐστὶν ἢ ἐλάσσων πασῶν τῶν διὰ τοῦ Δ διαγομένων εὐθειῶν. καὶ ἐπεὶ διαγώνιός ἐστιν ἡ ΑΔ, καὶ τῆι ΑΔ ὀρθὴ ἡ ΕΖ, γέγονέ μοι ἰσοσκελὲς 788 τρίγωνον τὸ ΕΑΖ, ἴσην ἔχον τὴν ΕΑ τῆι ΑΖ. διὰ δὴ τὸ προγεγραμμένον λῆμμα, γίνεται ἡ ΕΖ ἐλάσσων πασῶν τῶν διὰ τοῦ Δ διαγομένων εὐθειῶν, καὶ αἰεὶ <ἡ> ἔγγιον αὐτῆς τῆς ἀπώτερον ἐλάσσων.

‖ 1 θ´ add Hu (BS) | θεωρήματος] προβλήματος Hu (Horsley) ‖ 5 διαγομένων Ge (S) αἱ ἀγομένων A ‖ 8 ΒΖΕ (δυνατόν) Co ΒΕΖΕ A ‖ 12 μεγίστη – ΔΗ add Co ‖ 14 αἱ om Ge (S) ‖ 15 μείζων Ge (BS) μεῖζον A ‖ 16 ἐστὶν] οὖσα Co ‖ 17 ἄρα addidi ex S | διὰ Ge (Co) ἀπὸ A ‖ 18 ἡ ΒΓ – εὐθειῶν del Co ‖ 23 ΔΛ Co ΑΛ A ‖ 26 αἰεὶ δὲ ἡ Hu app αἱ δὲ A ἡ δὲ Ge (S) ‖ 28 ι´ add Hu (BS) ‖ 29 ἐκθώμεθα] ἐκθῶμαι Hu app ‖ 31 μεγίστη Ge (V) μεγίστης A ‖ 35 προγεγραμμένον Ge (BS) προσγεγραμμένον A ‖ 36 ἡ ἔγγιον Ge (BS) ἔγγειον A

(132) Neuses, (Book) 2.

1. *(Prop. 75)* (Given) the semicircle on AB, let an arbitrary (line) ΔE be drawn through it, and perpendiculars to it AΔ, BE. That ΔZ equals HE.

Let the <center of the semicircle Θ be taken, and from> Θ let a perpendicular ΘK be drawn to ΔE. Hence it is parallel to AΔ and BE,[1] and ZK equals KH (III, 3).[2] Since AΔ, ΘK, BE are three parallels,[3] and AΘ equals ΘB,[4] therefore ΔK equals KE.[5] But out of these ZK equals KH.[6] Therefore remainder ΔZ equals remainder HE.[7] And clearly ΔH too equals EZ.[8]

(133) 2. *(Prop. 76)* Again, let there be the semicircle on AB, and let ΓΔ be drawn tangent, and let it be produced and let AE and BZ be perpendiculars to it. That again EΔ equals ΔZ.

Let the center be H, and let ΔH be joined. Then it is parallel to AE, BZ;[2] for the angles at Δ are right.[1] Hence since AE, HΔ, BZ are three parallels, and AH equals HB,[3] therefore EΔ equals ΔZ.[4] Q.E.D.

(134) 3. *(Prop. 77)* For the fifth problem.

Let ABΓ, ΔEZ be two semicircles on AΓ, and let AΔ equal ΓZ, and from Γ let ΓB be drawn through (the semicircles). That as well BE equals HΓ.

For since AΔ equals ΓZ,[1] the semicircles are around the same center.[2] Then let the center Θ of the semicircles be taken, and from Θ let perpendicular ΘK be drawn to EH.[3] Then EK equals KH.[4] So let AB be joined. And since AB, ΘK are parallel,[5] and AΘ equals ΘΓ,[6] therefore BK equals KΓ as well.[7] Out of these EK equals KH.[8] Therefore remainder BE equals remainder HΓ.[9] And it is obvious that also BH equals EΓ.[10] Q.E.D.

(135) 4. *(Prop. 78)* Again, let ABΓ, ΔEZ be semicircles, and from Γ let ΓE be drawn tangent to (semicircle) ΔEZ, and let it be produced to B. That BE equals EΓ, given that AΔ equals ZΓ.

Obviously the semicircles are around the same center. Again let the center of the semicircles H be taken, and let HE, AB be joined. Then angle

(132) ΝΕΥΣΕΩΝ ΔΕΥΤΕΡΟΝ

α.´ ἡμικύκλιον τὸ ἐπὶ τῆς ΑΒ, διήχθω τυχοῦσα ἡ ΔΕ, καὶ ἐπ' αὐτὴν κάθετοι αἱ ΑΔ, ΒΕ. ὅτι ἴση ἐστὶν ἡ ΔΖ τῆι ΗΕ. εἰλήφθω τὸ <κέντρον τοῦ ἡμικυκλίου τὸ Θ, καὶ ἀπὸ> τοῦ Θ ἐπὶ τὴν ΔΕ κάθετος ἤχθω ἡ ΘΚ. παράλληλος ἄρα ἐστὶν ταῖς ΑΔ, ΒΕ, καὶ ἴση ἐστὶν ἡ ΖΚ τῆι ΚΗ. ἐπεὶ δὲ τρεῖς εἰσιν παράλληλοι αἱ ΑΔ, ΘΚ, ΒΕ, καὶ ἔστιν ἴση ἡ ΑΘ τῆι ΘΒ, ἴση ἄρα καὶ ἡ ΔΚ τῆι ΚΕ. ὧν ἡ ΖΚ τῆι ΚΗ ἐστὶν ἴση. λοιπὴ ἄρα ἡ ΔΖ λοιπῆι τῆι ΗΕ ἐστὶν ἴση. καὶ φανερὸν ὅτι καὶ ἡ ΔΗ τῆι ΕΖ ἴση ἐστίν.

(133) <β.´> ἔστω πάλιν ἡμικύκλιον τὸ ἐπὶ τῆς ΑΒ, καὶ ἐφαπτομένη ἤχθω ἡ ΓΔ, καὶ ἐκβεβλήσθω καὶ κάθετοι ἐπ' αὐτὴν αἱ ΑΕ, ΒΖ. ὅτι πάλιν ἴση ἡ ΕΔ τῆι ΔΖ. ἔστω τὸ κέντρον τὸ Η, καὶ ἐπεζεύχθω ἡ ΔΗ. παράλληλος ἄρα ἐστὶν ταῖς ΑΕ, ΒΖ. |γίνονται γὰρ ὀρθαὶ αἱ πρὸς τῶι Δ γωνίαι. ἐπεὶ οὖν τρεῖς |146 παράλληλοι αἱ ΑΕ, ΗΔ, ΒΖ, καὶ ἴση ἐστὶν ἡ ΑΗ τῆι ΗΒ, ἴση ἄρα ἐστὶν καὶ ἡ ΕΔ τῆι ΔΖ. ὅπερ:—

(134) <γ.´> εἰς τὸ ε´ πρόβλημα.

ἔστω δύο ἡμικύκλια ἐπὶ τῆς ΑΓ τὰ ΑΒΓ, ΔΕΖ, καὶ ἔστω ἴση ἡ ΑΔ τῆι ΓΖ, καὶ ἀπὸ τοῦ Γ διήχθω ἡ ΓΒ. ὅτι ἴση ἐστὶν καὶ ἡ ΒΕ τῆι ΗΓ. ἐπεὶ γὰρ ἴση ἐστὶν ἡ ΑΔ τῆι ΓΖ, περὶ τὸ αὐτὸ κέντρον ἐστὶν τὰ ἡμικύκλια. εἰλήφθω ἄρα τὸ κέντρον τῶν 790 ἡμικυκλίων τὸ Θ, καὶ ἀπὸ τοῦ Θ ἐπὶ τὴν ΕΗ κάθετος ἤχθω ἡ ΘΚ. ἴση ἄρα ἐστὶν ἡ ΕΚ τῆι ΚΗ. ἐπεζεύχθω οὖν ἡ ΑΒ. καὶ ἐπεὶ παράλληλοί εἰσιν αἱ ΑΒ, ΘΚ, καὶ ἔστιν ἴση ἡ ΑΘ τῆι ΘΓ, ἴση ἄρα ἐστὶν καὶ ἡ ΒΚ τῆι ΚΓ. ὧν ἡ ΕΚ τῆι ΚΗ ἴση ἐστίν. λοιπὴ ἄρα ἡ ΒΕ λοιπῆι τῆι ΗΓ ἐστὶν ἴση. φανερὸν δὴ ὅτι καὶ ἡ ΒΗ τῆι ΕΓ ἐστὶν ἴση. ὅπερ:—

(135) <δ.´> ἔστω δὴ πάλιν τὰ ΑΒΓ, ΔΕΖ ἡμικύκλια, καὶ ἀπὸ τοῦ Γ ἤχθω ἐφαπτομένη τοῦ ΔΕΖ ἡ ΓΕ, καὶ ἐκβεβλήσθω ἐπὶ τὸ Β. ὅτι ἴση ἐστὶν ἡ ΒΕ τῆι ΕΓ, ἴσης οὔσης τῆς ΑΔ τῆι ΖΓ. φανερὸν ὅτι περὶ τὸ αὐτὸ κέντρον εἰσὶν τὰ ἡμικύκλια. εἰλήφθω πάλιν τὸ κέντρον τῶν ἡμικυκλίων τὸ Η, καὶ

‖ 2 α´ mg A ‖ 3 ὅτι Ge ΘΗ A ‖ 4 κέντρον — ἀπὸ τοῦ] τοῦ ἡμικυκλίου κέντρον τὸ Hu τὸ κέντρον τὸ Co | ante ἐπὶ add καὶ Ge (Co) ‖ 8 ΗΕ Co ΗΘ A ‖ 10 β´ add Hu (BS) ‖ 13 ΑΕ, ΒΖ Co ΔΕ, ΕΖ A ‖ 17 γ´ add Hu (BS) ‖ 21 ἄρα] γὰρ A ‖ 22 τὴν Ge τῶν A ‖ 27 ὅπερ ante φανερὸν transp Hu ‖ 28 δ´ add Hu (BS)

E is right.[1] But angle B too (is right).[2] Therefore AB is parallel to EH.[3] And AH equals ΓH.[4] Thus also BE equals EΓ.[5] Q.E.D.

(136) 5. *(Prop. 79)* For the seventh.

Again let ABΓ, ΔEZ be semicircles, and let AΔ equal ZΓ, and let the greater circle be filled out, and through let some (line) BH be drawn through Z. That BE equals ZH.

Let the center be Θ, and from Θ let ΘK be drawn perpendicular to BH.[1] Then BK equals KH.[2] Now let EΔ be joined. Then since ΔE, ΘK are parallel,[3] and ΔΘ equals ΘZ,[5] because all <AΘ (equals) all ΘΓ,[4] therefore> EK <equals> KZ.[6] But also all BK equals all KH.[7] Thus remainder BE equals remainder ZH.[8] Q.E.D.

Obviously also BZ equals EH.[9] Q.E.D.

(137) 6. *(Prop. 80)* For the ninth.

Let ABΓ, ΔEZ be two semicircles, and let ZH be made equal to AΔ, and with BΓ drawn through, from H let HΘ be drawn perpendicular to it. That BE equals KΘ.

Let the center Λ of semicircle ΔEZ be taken, and from Λ let ΛM be drawn perpendicular to KE.[1] Then EM equals MK.[2] But since AΔ equals ZH,[3] and ΔΛ equals ΛZ,[4] therefore all AΛ equals all ΛH.[5] And AB, MΛ, ΘH are three parallels.[6] Therefore BM too equals MΘ.[7] Out of these EM equals MK.[8] Therefore remainder BE equals remainder KΘ.[9] And obviously also BK equals EΘ.[10]

(138) 7. *(Prop. 81)* With the same things assumed, let BΓ be tangent to semicircle ΔEZ. That again BE equals EΘ.

Again let the center Λ of semicircle ΔEZ be taken, and let ΛE be joined. Then it is a perpendicular to BΓ.[1] And there have resulted three parallels, AB, EΛ, HΘ.[2] But AΛ equals ΛH.[3] Therefore BE too equals EΘ.[4] Q.E.D.

ἐπεζεύχθωσαν αἱ ΗΕ, ΑΒ. ὀρθὴ ἄρα ἐστὶν ἡ πρὸς τῶι Ε γωνία. ἀλλὰ καὶ ἡ πρὸς τῶι Β. παράλληλος ἄρα ἐστὶν ἡ ΑΒ τῆι ΕΗ. καὶ ἴση ἐστὶν ἡ ΑΗ τῆι ΓΗ. ἴση ἄρα ἐστὶν καὶ ἡ ΒΕ τῆι ΕΓ. ὅπερ: —

(136) <ε.´> εἰς τὸ ἕβδομον.
ἔστω πάλιν τὰ ΑΒΓ, ΔΕΖ ἡμικύκλια, καὶ ἔστω ἴση ἡ ΑΔ τῆι
ΖΓ, καὶ προσαναγεγραμμένος ὁ μείζων κύκλος, καὶ διὰ τοῦ Ζ
διήχθω τις ἡ ΒΗ. ὅτι ἴση ἐστὶν ἡ ΒΕ τῆι ΖΗ. ἔστω τὸ κέντρον
τὸ Θ, καὶ ἀπὸ τοῦ Θ ἐπὶ τὴν ΒΗ κάθετος |ἤχθω ἡ ΘΚ. ἴση ἄρα 146v
ἐστὶν ἡ ΒΚ τῆι ΚΗ. ἐπεζεύχθω δὴ ἡ ΕΔ. ἐπεὶ οὖν παράλληλοί
εἰσιν αἱ ΔΕ, ΘΚ, καὶ ἔστιν ἴση ἡ ΔΘ τῆι ΘΖ (ὅλη γάρ ἐστιν καὶ 792
ἡ <ΑΘ ὅληι τῆι ΘΓ), ἴση ἄρα ἐστὶν ἡ> ΕΚ τῆι ΚΖ. ἔστιν δὲ
καὶ ὅλη ἡ ΒΚ ὅληι τῆι ΚΗ ἴση. λοιπὴ ἄρα ἡ ΒΕ λοιπῆι τῆι ΖΗ
ἴση ἐστίν. ὅπερ: —

φανερὸν ὅτι καὶ ἡ ΒΖ [τῆι ΕΖ ἐστὶν] τῆι ΕΗ ἴση ἐστίν. ὅπερ: —

(137) ϛ.´ εἰς τὸ θ.´
ἔστω δύο ἡμικύκλια τὰ ΑΒΓ, ΔΕΖ, καὶ τῆι ΑΔ ἴση κείσθω ἡ ΖΗ, καὶ διαχθείσης τῆς ΒΓ, ἀπὸ τοῦ Η ἐπ᾽ αὐτὴν κάθετος ἤχθω ἡ ΗΘ. ὅτι ἴση ἐστὶν ἡ ΒΕ τῆι ΚΘ. εἰλήφθω τὸ κέντρον τοῦ ΔΕΖ ἡμικυκλίου τὸ Λ, καὶ ἀπὸ τοῦ Λ ἐπὶ τὴν ΚΕ κάθετος ἤχθω ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΕΜ τῆι ΜΚ. ἐπεὶ δὲ ἴση ἐστὶν ἡ μὲν ΑΔ τῆι ΖΗ, ἡ δὲ ΔΛ τῆι ΛΖ, ὅλη ἄρα ἡ ΑΛ ὅληι τῆι ΛΗ ἴση ἐστίν. καὶ εἰσὶν τρεῖς παράλληλοι αἱ ΑΒ, ΜΛ, ΘΗ. ἴση ἄρα καὶ ἡ ΒΜ τῆι ΜΘ. ὧν ἡ ΕΜ τῆι ΜΚ ἴση ἐστίν. λοιπὴ ἄρα ἡ ΒΕ λοιπῆι τῆι ΚΘ ἴση ἐστίν. φανερὸν δὴ ὅτι καὶ ἡ ΒΚ τῆι ΕΘ ἴση ἐστίν.

(138) <ζ.´> τῶν αὐτῶν ὑποκειμένων, ἐφαπτέσθω ἡ ΒΓ τοῦ ΔΕΖ ἡμικυκλίου. ὅτι πάλιν ἡ ΒΕ τῆι ΕΘ ἴση ἐστίν. πάλιν εἰλήφθω τὸ κέντρον τοῦ ΔΕΖ ἡμικυκλίου τὸ Λ, καὶ ἐπεζεύχθω ἡ ΛΕ. κάθετος ἄρα ἐστὶν ἐπὶ τὴν ΒΓ. καὶ γεγόνασιν τρεῖς παράλληλοι αἱ ΑΒ, ΕΛ, ΗΘ. καὶ ἔστιν ἴση ἡ ΑΛ τῆι ΛΗ. ἴση ἄρα καὶ ἡ ΒΕ τῆι ΕΘ. ὅ(περ): —

‖ 5 ε´ add Hu (BS) ‖ 7 προσαναγεγραμμένος] προσαναγεγράφθω Hu προσαναπεπληρώσθω Ge προσαναγεγραμμένος ἔστω Friedlein | Ζ διήχθω] ΖΔ ἤχθω ΑΖ ἤχθω Co ‖ 11 ὅλη — ΕΚ] ἴση ἄρα ἐστὶν καὶ ἡ ΕΚ Co ‖ 15 τῆι ΕΖ ἐστὶν del Co ‖ 16 ὅπερ secl Hu ‖ 17 ϛ´ mg A ‖ 19 ΖΗ Co ΖΕ A ‖ 21 ΛΜ Co ΛΗ A ‖ 23 ΛΗ Co ΑΗ A ‖ 25 ΜΚ Co ΜΑ A ‖ 27 ζ´ add Hu (BS) | ἐφαπτέσθω Co ἐφάπτεται A ‖ 31 ΕΛ Co ΕΚ A ‖ 32 ΒΕ Co ΔΕ A

(139) 8. *(Prop. 82)* For the eighth.

Let ABΓ, ΔEZ be two semicircles, and let AΔ be less than ZΓ, and let ΓH be made equal to AΔ. and let the circle BAKΓ be filled out, and let an arbitrary (line) BK be drawn through (the semicircles), and from H let HΘ be drawn perpendicular to it. That BE equals ΘK as well.

Let the center Λ be taken of circle ABΓ, and from Λ let ΛM be drawn perpendicular to EZ.[1] Then BM equals MK.[2] But since AΛ equals ΛΓ,[3] and AΔ equals HΓ,[4] therefore remainder ΔΛ equals remainder ΛH.[5] And ΔE, ΛM, HΘ are three parallels.[6] Therefore also EM equals MΘ.[7] But also all BM equals all MK.[8] Therefore remainder BE equals remainder ΘK.[9] And obviously also ΘB equals EK.[10]

(140) 9. *(Prop. 83)* For the seventeenth.

With the same things assumed, let AΔ be greater than ZΓ, and let ZH be made equal to (AΔ), and, with BΓΘ drawn through, let HΘ be drawn perpendicular to it. That BE equals KΘ.

Let the center Λ of semicircle ΔEZ be taken, and from it let ΛM be drawn perpendicular to EK.[1] Then EM equals MK.[2] But since AΔ equals ZH,[3] and ΔΛ equals ΛZ,[4] therefore all AΛ equals all ΛH.[5] And again BA, MΛ, HΘ are three parallels.[6] Therefore also BM equals MΘ.[7] Out of these, EM equals MK.[8] Therefore remainder BE equals remainder KΘ.[9] And obviously also BK equals EΘ.[10] Q.E.D.

(141) 10. *(Prop. 84)* With the same things assumed, let BΓ be tangent to semicircle ΔEZ. That BE equals EΘ.

Again let the center Λ be taken of semicircle ΔEZ, and let ΛE be joined. Then it is a perpendicular to BΘ.[1] Thus AB, ΛE, HΘ are three parallels.[2] And AΛ equals ΛH.[3] Therefore BE equals EΘ as well.[4]

(142) 11. *(Prop. 85 a)* Problem useful for the synthesis of the seventeenth.

With ABΓ being a semicircle (given) in position, and Δ given, to draw through Δ a semicircle, such as ΔEZ, so that if BΓ is drawn tangent, AΔ will be equal to BE.

(139) <η.´> εἰς τὸ η.´
ἔστω δύο ἡμικύκλια τὰ ΑΒΓ, ΔΕΖ, καὶ ἔστω ἐλάσσων ἡ ΑΔ 794 τῆς ΖΓ, καὶ τῆι ΑΔ ἴση κείσθω ἡ ΓΗ, καὶ προσαναπεπληρώσθω ὁ ΒΑΚΓ κύκλος, καὶ διήχθω [ἡ] τυχοῦσα ἡ ΒΚ, καὶ ἀπὸ τοῦ Η ἐπ' αὐτὴν κάθετος ἡ ΗΘ· ὅτι ἴση ἐστὶν καὶ ἡ ΒΕ τῆι ΘΚ. εἰλήφθω τὸ κέντρον τοῦ ΑΒΓ κύκλου τὸ Λ, καὶ ἀπὸ τοῦ Λ ἐπὶ τὴν ΕΖ κάθετος ἤχθω ἡ ΛΜ· ἴση ἄρα ἐστὶν ἡ ΒΜ τῆι ΜΚ. ἐπεὶ δὲ ἴση ἐστὶν ἡ μὲν ΑΛ τῆι ΛΓ, |ἡ δὲ ΑΔ τῆι ΗΓ, λοιπὴ ἄρα ἡ ΔΛ λοιπῆι |147 τῆι ΛΗ ἐστὶν ἴση. καὶ εἰσὶ τρεῖς παράλληλοι αἱ ΔΕ, ΛΜ, ΗΘ. ἴση ἄρα ἐστὶν καὶ ἡ ΕΜ τῆι ΜΘ. ἔστιν δὲ καὶ ὅλη ἡ ΒΜ ὅληι τῆι ΜΚ ἴση. λοιπὴ ἄρα ἡ ΒΕ λοιπῆι τῆι ΘΚ ἐστὶν ἴση. φανερὸν δὲ ὅτι καὶ ἡ ΘΒ τῆι ΕΚ ἴση ἐστίν.

(140) <θ.´> εἰς τὸ ιζ.´
τῶν αὐτῶν ὑποκειμένων, ἔστω μείζων ἡ ΑΔ τῆς ΖΓ, καὶ αὐτῆι ἴση κείσθω ἡ ΖΗ, καὶ διαχθείσης τῆς ΒΓΘ, ἐπ' αὐτὴν κάθετος ἤχθω ἡ ΗΘ· ὅτι ἴση ἐστὶν ἡ ΒΕ τῆι ΚΘ. εἰλήφθω τὸ κέντρον τοῦ ΔΕΖ ἡμικυκλίου τὸ Λ, καὶ ἀπ' αὐτοῦ ἐπὶ τὴν ΕΚ κάθετος ἡ ΛΜ· ἴση ἄρα ἐστὶν ἡ ΕΜ τῆι ΜΚ. ἐπεὶ δὲ ἴση ἐστὶν ἡ μὲν ΑΔ τῆι ΖΗ, ἡ δὲ ΔΛ τῆι ΛΖ, ὅλη ἄρα ἡ ΑΛ ὅληι τῆι ΛΗ ἐστὶν ἴση. καὶ εἰσὶν πάλιν τρεῖς παράλληλοι αἱ ΒΑ, ΜΛ, ΗΘ. ἴση ἄρα ἐστὶν καὶ ἡ ΒΜ τῆι ΜΘ. ὧν ἡ ΕΜ τῆι ΜΚ ἐστὶν ἴση. λοιπὴ ἄρα ἡ ΒΕ λοιπῆι τῆι ΚΘ ἐστὶν ἴση. φανερὸν δὲ ὅτι καὶ ἡ ΒΚ τῆι ΕΘ ἐστὶν ἴση. ὅπερ:—

(141) <ι.´> τῶν αὐτῶν ὑποκειμένων, ἐφαπτέσθω ἡ ΒΓ τοῦ ΔΕΖ ἡμικυκλίου· ὅτι ἴση ἐστὶν ἡ ΒΕ τῆι ΕΘ. εἰλήφθω πάλιν τὸ κέντρον τοῦ ΔΕΖ ἡμικυκλίου τὸ Λ, καὶ ἐπεζεύχθω ἡ ΛΕ. κάθετος ἄρα ἐστὶν ἐπὶ τὴν ΒΘ. ὥστε τρεῖς εἰσιν παράλληλοι αἱ ΑΒ, ΛΕ, ΗΘ. καὶ ἔστιν ἴση ἡ ΑΛ τῆι ΛΗ. ἴση ἄρα ἐστὶν καὶ ἡ ΒΕ τῆι ΕΘ.

(142) <ια.´> πρόβλημα χρήσιμον εἰς τὴν σύνθεσιν τοῦ ιζ.´
θέσει ἡμικυκλίου ὄντος τοῦ ΑΒΓ, καὶ δοθέντος τοῦ Δ, γράψαι διὰ τοῦ Δ ἡμικύκλιον ὡς τὸ ΔΕΖ, ἵνα ἐὰν ἐφαπτομένη ἀχθῆι ἡ ΒΓ, ἴση γένηται ἡ ΑΔ τῆι ΒΕ. γεγονέτω. ἔστιν ἄρα ὡς

‖ 1 η´ (εἰς) add Hu (V) ‖ 3 τῆς Ge (recc?) τῆι A ‖ 4 ἡ secl Hu ‖ 5 καὶ del Ge (S) ‖ 8 ΔΛ Co ΑΛ A ‖ 12 ΘΒ Co ΕΒ A ‖ 13 θ´ add Hu (BS) ‖ 16 ὅτι Co ἐπεὶ A ‖ 17 ΔΕΖΗ A¹ Η del A² ‖ 23 ὅπερ ante φανερὸν transp Hu ‖ 24 ι´ add Hu (BS) ‖ 28 ΛΗ Co ΛΘ A ‖ 30 ια´ add Hu (V) ‖ 32 ἡμικύκλιον Ge (recc?) ἡμικυκλίου A

Let it be accomplished. Then as is AΔ to EΓ, so is EB to EΓ.[1] And so as is the square of EB to the square of EΓ, so is the square of AΔ to the square of EΓ.[2] But as is the square of BE to the square of EΓ, so, if center H of semicircle ΔEZ is taken and HE is joined, is the square of AH to the square of HΓ.[3] But the square of EΓ is the excess of the squares of EH, HΓ.[4] Therefore as is the square of AΔ to the excess of the squares of ΔH, HΓ, so is the square of AH to the square of HΓ.[5] Let AΘ be made equal to ΔA,[6] and let ΔΓ be bisected at point K.[7] Then since as is the square of AH to the square of HΓ, so is the square of AΔ to the excess of the squares of ΔH, HΓ,[8] therefore the remaining rectangle contained by ΔH, HΘ to the remaining square of HΔ, that is ΘH to HΔ,[10] is as one of the ratios, as the square of AΔ to the excess of the squares of ΔH, HΓ,[9] that is to twice the rectangle contained by ΔΓ, HK.[11] [12] Then let twice the rectangle contained by ΔΓ, Λ be made equal to the square of AΔ.[13] But the square of AΔ is given.[14] Therefore also twice the rectangle contained by ΔΓ, Λ is given;[15] and hence also once (the rectangle contained by ΔΓ, Λ).[16] And ΔΓ is given.[17] Therefore Λ too is given.[18] But since as is HΘ to HΔ, so is the square of AΔ, that is twice the rectangle contained by Λ, ΔΓ, to twice the rectangle contained by ΔΓ, HK, that is Λ to HK,[19] [20] therefore the rectangle contained by ΘH, HK equals the rectangle contained by Λ, HΔ.[21] And the three ΘΔ, ΔK, Λ are given.[22] It has been reduced to, in the *Determinate (Section)*, the "given three straight lines ΘΔ, ΔK, Λ, to divide ΔK at H, making the ratio of the rectangle contained by ΘH, HK to the rectangle contained by Λ, HΔ, that of equal to equal." But this is obvious, and it is without diorism. Therefore H is given,[23] and it is the center of semicircle ΔEZ. Therefore the semicircle is (given) in position.[24] And from a given (point) Γ, BΓ has been drawn tangent.[25] Thus BΓ is (given) in position.[26] The same (argument) will be applicable if the point <is given> at <Z>. Q.E.D.

(143) 12. *(Prop. 85 a)* The synthesis of the problem will be made as follows. Let the semicircle be ABΓ, the given (point) Δ, and let it be required to solve the problem.

Let twice the rectangle contained by ΔΓ, Λ be made equal to the square of AΔ,[1] and let AΘ be made equal to ΔA.[2] Let ΔΓ be bisected at point K.[3] And given three straight lines ΘΔ, ΔK, Λ, let ΔK be divided at H to make the ratio of the rectangle contained by Λ, HΔ to the rectangle contained by ΘH, HK that of equal to equal. And around center H let semicircle ΔEZ be described. I say that ΔEZ solves the problem.

ἡ ΑΔ πρὸς τὴν ΕΓ, οὕτως ἡ ΕΒ πρὸς τὴν ΕΓ. καὶ ὡς ἄρα τὸ ἀπὸ ΕΒ πρὸς τὸ ἀπὸ ΕΓ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΕΓ. ἀλλ' ὡς τὸ ἀπὸ ΒΕ πρὸς τὸ ἀπὸ ΕΓ, οὕτως ἐστίν, ἐὰν κέντρον τοῦ ΔΕΖ ἡμικυκλίου ληφθῆι τὸ Η καὶ ἐπιζευχθῆι <ἡ> ΗΕ, τὸ ἀπὸ ΑΗ πρὸς τὸ ἀπὸ ΗΓ. ἀλλὰ τὸ ἀπὸ ΕΓ ἡ τῶν |ἀπὸ ΕΗ, ΗΓ ἐστὶν ὑπεροχή. ἔστιν ἄρα ὡς τὸ ἀπὸ ΑΔ πρὸς τὴν τῶν ἀπὸ ΔΗ, ΗΓ |147v ὑπεροχήν, <οὕτως> τὸ ἀπὸ ΑΗ πρὸς τὸ ἀπὸ ΗΓ. κείσθω τῆι ΔΑ ἴση ἡ ΑΘ, καὶ τετμήσθω ἡ ΔΓ δίχα κατὰ τὸ Κ σημεῖον. ἐπεὶ οὖν ἐστιν ὡς τὸ ἀπὸ ΑΗ πρὸς τὸ ἀπὸ ΗΓ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὴν τῶν ἀπὸ ΔΗ, ΗΓ ὑπεροχήν, [λοιπὴ πρὸς] λοιπὸν ἄρα τὸ ὑπὸ ΔΗΘ πρὸς λοιπὸν τὸ ἀπὸ ΗΔ, τουτέστιν ἡ ΘΗ πρὸς ΗΔ, ἔστιν ὡς 798 εἷς τῶν λόγων, ὡς τὸ ἀπὸ ΑΔ πρὸς τὴν τῶν ἀπὸ ΔΗ, ΗΓ ὑπεροχήν, τουτέστιν πρὸς τὸ δὶς ὑπὸ ΔΓ, ΗΚ. κείσθω οὖν τῶι ἀπὸ ΑΔ τετραγώνωι ἴσον τὸ δὶς ὑπὸ ΔΓ, Λ. δοθὲν δὲ τὸ ἀπὸ ΑΔ. δοθὲν ἄρα καὶ τὸ δὶς ὑπὸ ΔΓ, Λ. ὥστε καὶ τὸ ἅπαξ. καὶ ἔστιν δοθεῖσα ἡ ΔΓ. δοθεῖσα ἄρα ἐστὶ καὶ ἡ Λ. ἐπεὶ δέ ἐστιν ὡς ἡ ΗΘ πρὸς τὴν ΗΔ, οὕτως τὸ ἀπὸ ΑΔ, τουτέστιν τὸ δὶς ὑπὸ Λ, ΔΓ, πρὸς τὸ δὶς ὑπὸ ΔΓ, ΗΚ, τουτέστιν ἡ Λ πρὸς ΗΚ, τὸ ἄρα ὑπὸ ΘΗΚ ἴσον τῶι ὑπὸ Λ, ΗΔ. καὶ εἰσὶν αἱ τρεῖς αἱ ΘΔ, ΔΚ, Λ δοθεῖσαι. ἀπῆκται εἰς Διωρισμένης "δεδομένων τριῶν εὐθειῶν τῶν ΘΔ, ΔΚ, Λ, τεμεῖν τὴν ΔΚ κατὰ τὸ Η, καὶ ποιεῖν λόγον τοῦ ὑπὸ ΘΗΚ πρὸς τὸ ὑπὸ Λ, ΗΔ, ἴσου πρὸς ἴσον". τοῦτο δὲ φανερόν, καὶ ἔστιν ἀδιόριστον. δοθὲν ἄρα τὸ Η, καὶ κέντρον τοῦ ΔΕΖ ἡμικυκλίου. θέσει ἄρα τὸ ἡμικύκλιον. καὶ ἀπὸ δοθέντος τοῦ Γ ἦκται ἐφαπτομένη ἡ ΒΓ. θέσει ἄρα ἡ ΒΓ. τὸ δ' αὐτὸ ἁρμόσει τοῦ σημείου κατὰ <τὸ Ζ δοθέντος.> ὅπερ:—

(143) <ιβ´> συντεθήσεται δὴ τὸ πρόβλημα οὕτως. ἔστω τὸ μὲν ἡμικύκλιον τὸ ΑΒΓ, τὸ δὲ δοθὲν τὸ Δ, καὶ δέον ἔστω ποιεῖν τὸ πρόβλημα. κείσθω τῶι ἀπὸ ΑΔ τετραγώνωι ἴσον τὸ δὶς ὑπὸ ΔΓ, Λ, καὶ τῆι μὲν ΔΑ ἴση κείσθω ἡ ΑΘ. ἡ δὲ ΔΓ δίχα τετμήσθω κατὰ τὸ Κ σημεῖον. καὶ τριῶν δοθεισῶν εὐθειῶν 800 τῶν ΘΔ, ΔΚ, Λ, τετμήσθω ἡ ΔΚ κατὰ τὸ Η, καὶ ποιείτω λόγον τοῦ ὑπὸ Λ, ΗΔ πρὸς τὸ ὑπὸ ΘΗΚ ἴσου πρὸς ἴσον. καὶ περὶ κέντρον τὸ Η ἡμικύκλιον γεγράφθω τὸ ΔΕΖ. λέγω ὅτι τὸ ΔΕΖ ποιεῖ τὸ πρόβλημα. ἤχθω γὰρ ἐφαπτομένη τοῦ ἡμικυκλίου ἡ ΒΓ. ὅτι

‖ 4 ἐπιζευχθῆι Ge ἐπεζεῦχθαι A | ἡ add Ge | ΑΗ Co ΔΗ A ‖ 7 οὕτως add Hu οὕτω Ge ‖ 10 λοιπὴ πρὸς del Co ‖ 11 ΘΗ Co ΘΝ A | ὡς εἷς τῶν λόγων secl Hu ‖ 13 post τουτέστιν add τὸ δὶς ὑπὸ ΔΓ, Λ Co | ΔΓ, ΗΚ Co ΔΓΗ A ‖ 18 ἡ Λ Ge (Co) ΗΛ A ‖ 20 post ἀπῆκται add ἄρα Ge | Διωρισμένης] Διωρισμένην Co Διωρισμένης α´ Hu ‖ 23 ἀδιόριστον Hu (Co) ἀδιόριστος A ‖ 25 ἐφαπτομένη Ge (S) ἐφάπτεται A | τὸ — κάτω secl Hu ‖ 26 κατὰ] κάτω Hu (S) κάτω ληφθέντος Hu app (Co) ‖ 27 ιβ´ add Hu (BS) ‖ 32 ΔΚ Co ΔΗ A ‖ 33 τὸ ὑπὸ Ge (S) τοῦ A

For let ΒΓ be drawn tangent to the semicircle. That ΑΔ equals ΒΕ. For since the rectangle contained by ΘΗ, ΗΚ equals the rectangle contained by Λ, ΗΔ,[4] in ratio, as is ΘΗ to ΗΔ, so is <Λ to> ΗΚ.[5] But as is ΘΗ to ΗΔ, so is the rectangle contained by ΘΗ, ΗΔ to the square of ΗΔ, that is the excess of the squares of ΗΑ, ΑΔ to the square of ΗΔ,[6] while as is Λ to ΗΚ, so is twice the rectangle contained by Λ, ΔΓ to twice the rectangle contained by ΔΓ, ΗΚ, that is the square of ΑΔ to the excess of the squares of ΔΗ, ΗΓ.[7] And so as is the excess of the squares of ΗΑ, ΑΔ to the square of ΗΔ, so is the square of ΑΔ to the excess of the squares of ΓΗ, ΗΔ.[8] Thus as is the square of ΑΗ to the square of ΗΓ, so is the square of ΑΔ to the excess of the squares of ΔΗ, ΗΓ, that is to the excess of the squares of ΓΗ, ΗΕ, that is to the square of ΕΓ.[9] And so as is the square of ΑΗ to the square of ΗΓ, so is the square of ΑΔ to the square of ΓΕ. But as is the square of ΑΗ to the square of ΗΓ, so is the square of ΒΕ to the square of ΕΓ.[10] Therefore as is the square of ΒΕ to the square of ΕΓ, so is the square of ΑΔ to the square of ΕΓ.[11] Therefore the square of ΑΔ equals the square of ΒΕ,[12] so that ΑΔ equals ΒΕ.[13] And it is apparent that ΒΕ is greater than ΕΓ. For we had, as ΘΗ to ΗΔ, so the square of ΑΔ to the square of ΕΓ;[13] it goes back to things that have been observed. ΘΗ is greater than ΗΔ,[14] hence the square of ΑΔ is greater than the square of ΕΓ,[15] and so ΑΔ is greater than ΕΓ.[16] Therefore it is much greater than ΖΓ.[17] Thus semicircle ΔΕΖ solves the problem.

(Prop. 85 b) Now I say also that it alone (solves the problem). For let some other (semicircle) ΔΜΝ be described, and let ΓΜΞ be drawn tangent. Now if ΔΜΝ too solves the problem, then ΑΔ will equal ΜΞ. And let the center Ο of semicircle ΔΜΝ be taken, and let ΟΜ be joined. Then in accordance with the analysis, the rectangle contained by ΘΟ, ΟΚ will equal the rectangle contained by Λ, ΔΟ. But this is absurd, for in the *Determinate (Section)* it was proved to be greater. Therefore semicircle ΔΜΝ does not solve the problem. Similarly we shall prove that no other but ΔΕΖ (solves it). Thus ΔΖΕ alone solves the problem.

(144) *(Prop. 85 b)* But to find out which of them cuts off a greater (line), we shall make the demonstration as follows.

Since in the *Determinate (Section)* it was proved that the rectangle contained by Λ, ΔΟ is less than the rectangle contained by ΘΟ, ΟΚ,[1] in ratio Λ has to ΟΚ a lesser ratio than has ΘΟ to ΟΔ.[2] But as is Λ to ΚΟ, so

ἴση ἐστὶν ἡ ΑΔ τῆι ΒΕ. ἐπεὶ γὰρ τὸ ὑπὸ ΘΗΚ ἴσον ἐστὶν τῶι ὑπὸ Λ, ΗΔ, ἀνάλογόν ἐστιν ὡς ἡ ΘΗ πρὸς τὴν ΗΔ, |οὕτως ἐστὶν |148
<ἡ Λ πρὸς> τὴν ΗΚ. ἀλλ' ὡς μὲν ἡ ΘΗ πρὸς τὴν ΗΔ, οὕτως ἐστὶν τὸ ὑπὸ ΘΗΔ πρὸς τὸ ἀπὸ ΗΔ, τουτέστιν ἡ τῶν ἀπὸ ΗΑ, ΑΔ ὑπεροχὴ πρὸς τὸ ἀπὸ ΗΔ. ὡς δὲ ἡ Λ πρὸς τὴν ΗΚ, οὕτως ἐστὶν τὸ δὶς ὑπὸ Λ, ΔΓ πρὸς τὸ δὶς ὑπὸ ΔΓ, ΗΚ, τουτέστιν τὸ ἀπὸ ΑΔ πρὸς τὴν τῶν ἀπὸ ΔΗ, ΗΓ ὑπεροχήν. καὶ ὡς ἄρα ἡ τῶν ἀπὸ ΗΑ, ΑΔ ὑπεροχὴ πρὸς τὸ ἀπὸ ΗΔ, οὕτως ἐστὶν τὸ ἀπὸ ΑΔ πρὸς τὴν τῶν ἀπὸ ΓΗ, ΗΔ ὑπεροχήν. ἔστιν ἄρα ὡς τὸ ἀπὸ ΑΗ πρὸς τὸ ἀπὸ ΗΓ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὴν τῶν ἀπὸ ΔΗ, ΗΓ ὑπεροχήν, τουτέστιν πρὸς τὴν τῶν ἀπὸ ΓΗ, ΗΕ ὑπεροχήν, τουτέστιν πρὸς τὸ ἀπὸ ΕΓ. καὶ ὡς ἄρα τὸ ἀπὸ ΑΗ τετράγωνον πρὸς τὸ ἀπὸ ΗΓ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΓΕ. ὡς δὲ τὸ ἀπὸ ΑΗ πρὸς τὸ ἀπὸ ΗΓ, οὕτως ἐστὶν τὸ ἀπὸ ΒΕ πρὸς τὸ ἀπὸ ΕΓ. ὡς ἄρα τὸ ἀπὸ ΒΕ πρὸς τὸ ἀπὸ ΕΓ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΕΓ. ἴσον ἄρα ἐστὶν τὸ ἀπὸ ΑΔ τῶι ἀπὸ ΒΕ. ὥστε ἴση ἐστὶν ἡ ΑΔ τῆι ΒΕ. καὶ φανερὸν ὅτι μείζων ἐστὶν ἡ ΒΕ τῆς ΕΓ. εἴχομεν γὰρ ὡς τὴν ΘΗ πρὸς τὴν ΗΔ, οὕτως τὸ ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΕΓ. ἀναβαίνει δὲ ἐπὶ ἐπισκεπτομένων· μείζων δὲ ἡ ΘΗ τῆς ΗΔ. μείζων <ἄρα> τὸ ἀπὸ ΑΔ τοῦ ἀπὸ ΕΓ. ὥστε μείζων ἐστὶν ἡ ΑΔ τῆς ΕΓ. πολλῶι ἄρα τῆς ΖΓ μείζων ἐστίν. τὸ ΔΕΖ ἄρα ἡμικύκλιον ποιεῖ τὸ 802
πρόβλημα.

λέγω δὴ ὅτι καὶ μόνον. γεγράφθω γάρ τι καὶ ἕτερον ΔΜΝ, καὶ ἤχθω ἐφαπτομένη ἡ ΓΜΞ. εἰ δὴ καὶ τὸ ΔΜΝ ποιεῖ τὸ πρόβλημα, ἔσται ἴση ἡ ΑΔ τῆι ΜΞ. καὶ εἰλήφθω τὸ κέντρον τοῦ ΔΜΝ ἡμικυκλίου τὸ Ο, καὶ ἐπεζεύχθω ἡ ΟΜ. ἔσται ἀκολούθως τῆι ἀναλύσει τὸ ὑπὸ τῶν ΘΟΚ ἴσον τῶι ὑπὸ τῶν Λ, ΔΟ. ὅπερ ἐστὶν ἄτοπον· ἐν γὰρ τῆι Διωρισμένηι δέδεικται μεῖζον. οὐκ ἄρα τὸ ΔΜΝ ἡμικύκλιον ποιεῖ τὸ πρόβλημα. ὁμοίως δὴ δείξομεν ὅτι οὐδὲ ἄλλο τι πλὴν τοῦ ΔΕΖ. τὸ ΔΖΕ ἄρα μόνον ποιεῖ τὸ πρόβλημα.

(144) ἵνα δὲ καὶ ἐπιγνῶμεν πότερον αὐτῶν μεῖζον ἀποτέμνει, δείξομεν οὕτως. ἐπεὶ ἐν τῆι Διωρισμένηι δέδεικται ἔλασσον τὸ ὑπὸ τῶν Λ, ΔΟ τοῦ ὑπὸ τῶν ΘΟΚ, ἀνάλογον ἡ Λ πρὸς ΟΚ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΘΟ πρὸς ΟΔ.

‖ 1 ΑΔ A² ex ΑΔΓ ‖ 3 ἡ Λ πρὸς add Ge (S), eadem pro ἐστὶν Hu ‖ 5 Λ Co ΗΛ A ‖ 6 ΔΓ (πρὸς) Ge (CSV) ΑΓ A ‖ 9 ΓΗ, ΗΔ] ΓΗΔ A ΔΓ, ΗΓ Co ‖ 17 εἴχομεν] ἔχομεν A ‖ 18 ἀναβαίνει δὲ ἐπὶ ἐπισκεπτομένων del Co ‖ 19 ἄρα add Co ‖ 23 δὴ] δὲ A ‖ 24 ἡ ΓΜΞ Ge (Co) ΗΓΜΞ A ‖ 27 Λ Co Α A ‖ 28 μεῖζον Ge (S) μείζων A ‖ 30 ἄρα Co ἐστιν A ‖ 35 ἀνάλογον Hu ἀνάλογος A is the square of AΔ to the excess of the squares of ΔO, OΓ;[3] for this has been proved. But as is ΘO to OΔ, so is the excess of the squares of OA, AΔ to the square of OΔ.[4] And the square of AΔ therefore has to the excess of the squares of ΔO, OΓ a ratio less than has the excess of the squares of OA, AΔ to the square of OΔ.[5] And all to all,[6] [as the square of AΔ is to the excess of the squares of ΓO, OΔ, that is to the square of ΓM], therefore, the square of AΔ has to the square of ΓM a lesser ratio than has the square of AO to the square of OΓ,[7] that is the square of ΞM to the square of MΓ.[8] Thus ΞM is greater than AΔ.[9]

Similarly we shall prove that all the straight lines that are between points A, B are greater than AΔ, but those between B, Γ are less. For if we again describe semicircle ΔΠP, and ΣΠΓ is drawn tangent, and the same construction is made as before, then the center T of semicircle ΔΠP will be on the other side of H (from O). But in the *Determinate (Section)* the rectangle contained by Λ, AT will be greater than the rectangle contained by ΘT, TK. By the same argument again AΔ will be greater than ΣΠ. Thus the (points) nearest A make the tangents greater than AΔ, while the farther ones (make them) less. Hence it is possible to describe through Δ semicircles so that the tangent to each of them, produced to the <arc> of the greater semicircle, makes the (line) between the point of tangency and the <arc> of the greater semicircle equal to AΔ, and, in turn, greater and less.

ἀλλ᾽ ὡς μὲν ἡ Λ πρὸς ΚΟ, οὕτως ἐστὶν τὸ ἀπὸ ΑΔ πρὸς τὴν τῶν ἀπὸ ΔΟ, ΟΓ ὑπεροχήν· δέδεικται γάρ· ὡς δὲ ἡ ΘΟ πρὸς ΟΔ, οὕτως ἐστὶν ἡ τῶν ἀπὸ ΟΑ, ΑΔ πρὸς τὸ ἀπὸ ΟΔ. καὶ τὸ ἀπὸ ΑΔ ἄρα πρὸς τὴν τῶν ἀπὸ ΔΟ, ΟΓ ὑπεροχὴν ἐλάσσονα λόγον ἔχει ἤπερ ἡ τῶν ἀπὸ ΟΑ, |ΑΔ ὑπεροχὴ πρὸς τὸ ἀπὸ ΟΔ. καὶ πάντα πρὸς πάντα, [ὡς τὸ ἀπὸ ΑΔ πρὸς τὴν τῶν ἀπὸ ΓΟ, ΟΔ ὑπεροχήν, |148v 804 τουτέστιν πρὸς τὸ ἀπὸ ΓΜ] τὸ ἄρα ἀπὸ ΑΔ πρὸς τὸ ἀπὸ ΓΜ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ἀπὸ ΑΟ πρὸς τὸ ἀπὸ ΟΓ, τουτέστιν τὸ ἀπὸ ΞΜ πρὸς τὸ ἀπὸ ΜΓ. μείζων ἄρα ἐστὶν ἡ ΞΜ τῆς ΑΔ.

ὁμοίως δὴ δείξομεν ὅτι καὶ πᾶσαι αἱ μεταξὺ τῶν Α, Β σημείων γινόμεναι εὐθεῖαι μείζονές εἰσιν τῆς ΑΔ, αἱ δὲ μεταξὺ τῶν Β, Γ ἐλάσσονες. ἐὰν γὰρ πάλιν γράψωμεν ἡμικύκλιον τὸ ΔΠΡ, καὶ ἐφαπτομένη ἀχθῆι ἡ ΣΠΓ, καὶ τὰ αὐτὰ τοῖς πρότερον κατασκευασθῆι, τὸ μὲν κέντρον ἔσται τοῦ ΔΠΡ ἡμικυκλίου τὸ Τ ἐπὶ τὰ ἕτερα μέρη τοῦ Η. ἐν δὲ τῆι Διωρισμένηι μεῖζον ἔσται τὸ ὑπὸ Λ, ΑΤ τοῦ ὑπὸ ΘΤΚ. κατὰ τὰ αὐτὰ μείζων ἔσται πάλιν ἡ ΑΔ τῆς ΣΠ. ὥστε τὰ μὲν ἔγγιστα τοῦ Α τὰς ἐφαπτομένας ἔχοντα μείζω ποιεῖ τῆς ΑΔ, τὰ δὲ ἀπώτερον ἐλάσσω. δυνατὸν ἄρα ἐστὶν γράψαι διὰ τοῦ Δ ἡμικύκλια ἵνα ἡ ἐφαπτομένη ἑκάστου αὐτῶν προσεκβαλλομένη ἐπὶ τὴν τοῦ μείζονος ἡμικυκλίου <περιφέρειαν> τὴν μεταξὺ τῆς ἀφῆς καὶ τῆς τοῦ μείζονος ἡμικυκλίου <περιφερείας> ἴσην ποιῆι τῆι ΑΔ. καὶ πάλιν μείζω καὶ ἐλάσσω.

‖ 1 Λ... ΚΟ Co ΚΟ... Λ A ‖ 2 ΔΟ Co ΔΘ A ‖ 3 ΟΑ, ΑΔ Co ΟΑΔ A | ΟΔ Co ΟΑ A ‖ 4 ΔΟ, ΟΓ Co ΟΔ, ΔΓ A ‖ 6 post πρὸς πάντα add τουτέστιν τὸ ἀπὸ ΑΟ πρὸς τὸ ἀπὸ ΟΓ add Co, post quae add μείζονα λόγον ἔχει ἤπερ (pro ὡς) Hu ‖ 8 ΑΟ Co ΑΘ A | ΟΓ A² ex ΘΓ ‖ 9 μείζων Ge (BS) μεῖζον A ‖ 13 Γ Co Ε A ‖ 15 τοῦ Co τὸ ὑπὸ A ‖ 16 ἡμικυκλίου Co ἡμικύκλιον A ‖ 17 Λ, ΑΤ] ΘΗΚ Co | ΘΤΚ Co ΟΤΚ A | post ΘΤΚ add καὶ Hu ‖ 19 μείζω Hu μεῖζον A μείζονα BS | τῆς Co τὴν A ‖ 21 ἡμικύκλια Ge (recc?) ἡμικυκλίου A | προσεκβαλλομένη Ge (recc?) —ενης A ‖ 22 περιφέρειαν add Hu (Co) | τὴν μεταξὺ — μείζονος ἡμικυκλίου om A¹ τῆς μεταξὺ — μείζονος add mg A² alia manu, τὴν pro τῆς Ge ‖ 23 περιφερείας add Hu ‖ 24 ἐλάσσω Ge (S) ἐλάσσων A

(145) 13. *(Prop. 86)* For the nineteenth.

Again let there be the semicircles, and AΔ greater than ΓZ, and let ΓH be made equal to AΔ, and, with BK drawn through, from H let HΘ be drawn perpendicular to it, and let semicircle ABΓ be filled out, and let BZ be produced to K. That BΘ equals EK.

Let the center Λ of circle ABΓ be taken, and from Λ let ΛM be drawn perpendicular to BK.[1] Then MB equals MK.[2] So since AΛ equals ΛΓ,[3] and AΔ equals HΓ,[4] therefore remainder ΔΛ equals remainder ΛH.[5] And ΔE, ΛM, HΘ are three parallels.[6] Therefore EM too equals MΘ.[7] But also all BM equals all MK.[8] Therefore remainder BE equals remainder ΘK.[9] So it is obvious that also BΘ equals EK.[10] Q.E.D.

(146) 14. *(Prop. 87)* Problem for the same (problem).

With ABΓ being a semicircle, and Δ a point, to describe on AΓ and through Δ a semicircle so that, if ZB is drawn tangent, AΔ equals ZB.

Let it be accomplished. Then since AΔ equals ZB,[1] also the square of AΔ equals the square of ZB,[2] that is the rectangle contained by AZ, ZΓ.[3]

So if we apply to AΓ a (rectangle) equal to the square of AΔ, and deficient by a square, as AZΓ,[4] and if I draw ZB at right angles,[5] and describe on ΔZ a semicircle ΔEZ, BZ will be tangent to the semicircle, and will be equal to AΔ.[6]

This occurs whenever AΔ is less than half AΓ. With this found, if I draw through Δ other semicircles, such as ΔHΘ, ΔKΛ, and ΘM and ΛN are drawn tangent, ΘM will be greater than AΔ, and ΛN less. *For since AΔ (ΔΘ!) is less than ΔΓ, therefore ΘM will be between Δ, Γ. Now it will not fall on Z, since (in that case) it will result that AΔ (ΔΘ!) equals ZΓ (ZΔ!), which is absurd; much more is it impossible (for it to be) between Γ, Z, since again it results that AΔ (ΔΘ!) is less than ZΓ (ZΔ!), which is absurd. For it is also greater, as was assumed in the original problem.* Hence Θ will be between Z, Δ. But the rectangle contained by AΘ, ΘΓ, that is the square of MΘ, is greater than the rectangle contained by AZ, ZΓ, that is the square of ZB. Hence it is also greater than the square of AΔ, and so ΘM is greater than AΔ. But <ΛN> is between Γ, Z. Since the rectangle

(145) <ιγʹ> εἰς τὸ ιθʹ. 806

ἔστω πάλιν τὰ ἡμικύκλια, μείζων δὲ ἡ ΑΔ τῆς ΓΖ, καὶ τῆι ΑΔ ἴση κείσθω ἡ ΓΗ, καὶ διαχθείσης τῆς ΒΖ, ἀπὸ τοῦ Η ἐπ' αὐτὴν κάθετος ἤχθω ἡ ΗΘ, καὶ προσαναπεπληρώσθω τὸ ΑΒΓ ἡμικύκλιον, καὶ ἐκβεβλήσθω ἡ ΒΖ ἐπὶ τὸ Κ. ὅτι ἴση ἐστὶν ἡ ΒΘ τῆι ΕΚ. εἰλήφθω τὸ κέντρον τοῦ ΑΒΓ κύκλου τὸ Λ, καὶ ἀπὸ τοῦ Λ ἐπὶ τὴν ΒΚ κάθετος ἤχθω ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΜΒ τῆι ΜΚ. ἐπεὶ οὖν ἴση ἐστὶν ἡ μὲν ΑΛ τῆι ΛΓ, ἡ δὲ ΑΔ τῆι ΗΓ, λοιπὴ ἄρα ἡ ΔΛ λοιπῆι τῆι ΛΗ ἐστὶν ἴση. καὶ εἰσὶν τρεῖς παράλληλοι αἱ ΔΕ, ΛΜ, ΗΘ. ἴση ἄρα καὶ ἡ ΕΜ τῆι ΜΘ. ἔστιν δὲ καὶ ὅλη ἡ ΒΜ ὅληι τῆι ΜΚ ἴση. λοιπὴ ἄρα ἡ ΒΕ λοιπῆι τῆι ΘΚ ἐστὶν ἴση. φανερὸν οὖν ὅτι καὶ ἡ ΒΘ τῆι ΕΚ. ὅπερ:—

(146) <ιδʹ> πρόβλημα εἰς τὸ αὐτό.

ἡμικυκλίου ὄντος τοῦ ΑΒΓ, καὶ σημείου τοῦ Δ, γράψαι ἐπὶ τῆς ΑΓ διὰ τοῦ Δ ἡμι|κύκλιον ἵνα ἴση ἡ ΑΔ, ἐὰν ἐφαπτομένη ἀχθῆι ἡ ΖΒ, τῆι ΖΒ. γεγονέτω. ἐπεὶ οὖν ἴση ἐστὶν ἡ ΑΔ τῆι |149 ΖΒ, ἴσον καὶ τὸ ἀπὸ ΑΔ τῶι ἀπὸ ΖΒ, τουτέστι τῶι ὑπὸ ΑΖΓ. ἐὰν ἄρα τῶι ἀπὸ ΑΔ ἴσον παρὰ τὴν ΑΓ παραβάλωμεν, ἐλλεῖπον τετραγώνωι, ὡς τὸ ὑπὸ ΑΖΓ, καὶ ἀγάγω ὀρθὴν τὴν ΖΒ, καὶ ἐπὶ 808 τῆς ΔΖ ἡμικύκλιον γράψω τὸ ΔΕΖ, ἐφάψεται ἡ ΒΖ τοῦ ἡμικυκλίου, καὶ ἔσται ἴση τῆι ΑΔ. τοῦτο δὲ γίνεται ὁπόταν ἡ ΑΔ ἐλάσσων <ἦι> ἢ ἡμίσεια τῆς ΑΓ. εὑρημένου δὴ τούτου, ἐὰν διὰ τοῦ Δ ἕτερα ἡμικύκλια γράψω ὡς τὰ ΔΗΘ, ΔΚΛ, καὶ ἐφαπτόμεναι ἀχθῶσιν αἱ ΘΜ, ΛΝ, ἔσται ἡ μὲν ΘΜ μείζων τῆς ΑΔ, ἡ δὲ ΛΝ ἐλάσσων. ἐπεὶ γὰρ ἡ ΑΔ τῆς ΔΓ ἐλάσσων ἐστίν, ἡ ΘΜ ἄρα ἔσται μεταξὺ τῶν Δ, Γ. ἐπὶ μὲν οὖν τὸ Ζ οὐ πεσεῖται, ἐπεὶ συμβήσεται ἴσην γίνεσθαι τὴν ΑΔ τῆι ΖΓ, ὅπερ ἄτοπον, μεταξὺ δὲ τῶν Γ, Ζ πολλῶι μᾶλλον οὐκ ἔστιν, ἐπεὶ πάλιν συμβαίνει ἐλάσσονα εἶναι τῆς ΑΔ τὴν ΖΓ, ὅπερ ἄτοπον. ἔστιν γὰρ καὶ μείζων ὡς ἐν τῶι ἐξ ἀρχῆς ὑπόκειται προβλήματι. ἔσται ἄρα μεταξὺ τῶν Ζ, Δ τὸ Θ. μεῖζον δὲ τὸ ὑπὸ ΑΘΓ, τουτέστιν τὸ ἀπὸ ΜΘ, τοῦ ὑπὸ ΑΖΓ, τουτέστιν τοῦ ἀπὸ ΖΒ. μεῖζον ἄρα καὶ τοῦ ἀπὸ ΑΔ, ὥστε μείζων ἡ ΘΜ τῆς ΑΔ. ἡ δὲ

‖ 1 ιγʹ add Hu (V) ‖ 2 μείζων δ' Hu μείζονα A μείζων δὲ Ge (S) ‖ 3 ΒΖ] ΒΗΚ A ΒΕΖ Co ‖ 4 τὸ ΑΒΓ ἡμικύκλιον] τὰ ΒΓ ἡμικύκλια A ὁ ΑΒΓ κύκλος Co ‖ 6 ΑΒΓ in ras. A | Λ Co A A ‖ 7 Λ Co A A | τὴν Ge (S) τῶν A | ΛΜ Co ΑΜ A ‖ 9 ΔΛ Co ΑΛ A ‖ 12 post ΕΚ add ἴση ἐστὶν Ge ‖ 13 ιδʹ add Hu (V) ‖ 15 ἴση <ἦι> ἡ ΑΔ post ἡ ΖΒ transp Hu ‖ 20 ἡμικύκλιον Ge (recc?) ἡμικύκλια A | ἐφάψεται Hu app ἐφάπτεται A ‖ 22 ἦι ἢ Hu (S) ἦι ἢ ἡ Ge ἦι ἤπερ ἡ Hu app ‖ 25 ΛΝ (ἐλάσσων) Co ΛΗ A | ἐπεὶ γὰρ — πρὸς τὸ Γ μέρη εὐθεῖαι secl Hu | ΑΔ] ΔΘ Hu app ‖ 26 ἐπὶ Ge (S) ἐπεὶ A ‖ 27 ἐπεὶ Ge (S) ἐπὶ A | ΑΔ] ΔΘ Hu app ΘΓ Co | ΖΓ] ΔΖ Hu app ‖ 29 συμβαίνει] συνέβαινεν ἂν Hu app | τῆς ΑΔ τὴν ΖΓ] τὴν ΑΔ τῆς ΖΓ A τῆς ΘΔ τὴν ΔΖ Hu app ‖ 30 προβλήματι] προβληθέντι Hu app ‖ 32 τὸ (ἀπὸ ΜΘ) Ge τὰ A contained by AΛ, ΛΓ is less than the square of AΔ, because (it is less) also than the rectangle contained by AZ, ZΓ, therefore also the square of ΛN is less than the square of AΔ. Hence ΛN is less than AΔ. Similarly, all the straight lines in the direction of Γ (are less then AΔ). And generally, as the semicircles approach Γ, the tangent is less than AΔ, but as they move away, it is always greater. Thus it is possible to describe on AΓ and through Δ semicircles so that sometimes the tangents to them equal AΔ, sometimes they are greater, sometimes less.

(147) 15. *(Prop. 88)* For the twenty-first.

Let ABΓ, ΔEZ be semicircles, and let AH be made equal to ΓΔ, and, with ZB drawn through, let HΘ be drawn perpendicular to it. That ΘB equals KE.

Let the center Λ of semicircle ABΓ be taken, and from Λ let ΛM be drawn perpendicular to BZ.[1] Then BM equals MK.[2] But since HA equals ΓΔ,[3] and AΛ equals ΛΓ,[4] therefore all HΛ equals all ΛΔ.[5] And HΘ, ΛM, ΔE are three parallels.[6] Therefore ΘM too equals ME.[7] Out of these BM equals MK.[8] Therefore remainder ΘB equals KE.[9] And it is obvious that also ΘK equals BE.[10] Q.E.D.

(148) 16. *(Prop. 89)* With the same things (assumed), let BZ be tangent at B. That again ΘB equals BE.

For again let the center K of semicircle ABΓ be taken, and from K to B let KB be joined. Then it is a perpendicular to BZ.[1] Then since HK equals KΔ[3] in three parallels HΘ, BK, ΔE,[2] therefore ΘB too equals BE.[4] Q.E.D.

(149) 17. *(Prop. 90)* For the twenty-third.

Let there be the semicircles ABΓ, ΔEZ, and let AH be made equal to ΓZ, and, with EΘ drawn through, let HΘ be drawn perpendicular to it. That ΘB equals KE.

Let the center Λ of semicircle ABΓ be taken, and let ΛM be a perpendicular.[1] Then BM equals MK.[2] Since HA equals ΓZ,[3] and AΛ equals ΛΓ,[4] therefore all HΛ equals all ΛZ.[5] And HΘ, ΛM, EZ are three parallels.[6] Therefore also ΘM equals ME.[7] Out of these, BM equals MK.[8] Therefore remainder ΘB equals remainder KE.[9] And if it is tangent, (the

<ΛΝ> μεταξὺ τῶν Γ, Ζ. ἐπειδὴ ἔλασσόν ἐστιν τὸ ὑπὸ ΑΛΓ τοῦ ἀπὸ ΑΔ (ἐπεὶ καὶ τοῦ ὑπὸ ΑΖΓ), ἔλασσον ἄρα καὶ τὸ ἀπὸ ΛΝ τοῦ ἀπὸ ΑΔ. ὥστε ἐλάσσων ἐστὶν ἡ ΛΝ τῆς ΑΔ. ὁμοίως καὶ πᾶσαι αἱ ἐπὶ τὰ ὡς πρὸς τὸ Γ μέρη εὐθεῖαι. καὶ καθόλου προσιόντων μὲν τῶν ἡμικυκλίων τῶι Γ σημείωι ἡ ἐφαπτομένη ἐλάσσων ἐστὶν τῆς ΑΔ, ἀποχωρούντων δὲ ἀεὶ μείζων. δυνατὸν ἄρα ἐστὶν ἐπὶ [μὲν] τῆς ΑΓ διὰ τοῦ Δ ἡμικύκλια γράψαι ἵνα ὅτε μὲν αἱ ἐφαπτόμεναι αὐτῶν ἴσαι ὦσιν τῆι ΑΔ, ὅτε δὲ μείζονες, ὅτε δὲ ἐλάσσονες.

(147) <ιε.´> εἰς τὸ κα.´
ἔστω ἡμικύκλια τὰ ΑΒΓ, ΔΕΖ, τῆι ΓΔ ἴση κείσθω ἡ ΑΗ, καί, 810
διαχθείσης τῆς ΖΒ, ἐπ' αὐτὴν κάθετος ἤχθω ἡ ΗΘ. ὅτι ἴση ἐστὶν ἡ ΘΒ τῆι ΚΕ. εἰλήφθω τὸ κέντρον τοῦ ΑΒΓ ἡμικυκλίου τὸ Λ, καὶ ἀπὸ τοῦ Λ ἐπὶ τὴν ΒΖ κάθετος ἤχθω ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΒΜ τῆι ΜΚ. ἐπεὶ δὲ ἴση ἐστὶν ἡ μὲν ΗΑ τῆι ΓΔ, ἡ δὲ ΑΛ τῆι ΛΓ, ὅλη ἄρα ἡ ΗΛ ὅλῃ τῆι ΛΔ ἴση ἐστίν. καὶ |εἰσὶν |149v
τρεῖς παράλληλοι αἱ ΗΘ, ΛΜ, ΔΕ. ἴση ἄρα ἐστὶν καὶ ἡ ΘΜ τῆι ΜΕ. ὧν ἡ ΒΜ τῆι ΜΚ ἐστὶν ἴση. λοιπὴ ἄρα ἡ ΘΒ τῆι ΚΕ ἴση ἐστίν. φανερὸν δὲ ὅτι καὶ ἡ ΘΚ τῆι ΒΕ ἴση ἐστίν. ὅπερ:—

(148) <ις.´> τῶν αὐτῶν ὄντων, ἐφαπτέσθω ἡ ΒΖ κατὰ τὸ Β. ὅτι πάλιν ἴση ἐστὶν ἡ ΘΒ τῆι ΒΕ. εἰλήφθω γὰρ πάλιν τὸ κέντρον τοῦ ΑΒΓ ἡμικυκλίου τὸ Κ, καὶ ἀπὸ τοῦ Κ ἐπὶ τὸ Β ἐπεζεύχθω ἡ ΚΒ. κάθετος <ἄρα> ἐστὶν ἐπὶ τὴν ΒΖ. ἐπεὶ οὖν ἐν τρισὶν παραλλήλοις ταῖς ΗΘ, ΒΚ, ΔΕ, ἴση ἐστὶν ἡ ΗΚ τῆι ΚΔ, ἴση ἄρα ἐστὶν καὶ ἡ ΘΒ τῆι ΒΕ. ὅπερ:—

(149) <ιζ.´> εἰς τὸ κγ.´
ἔστω τὰ ἡμικύκλια τὰ ΑΒΓ, ΔΕΖ, καὶ τῆι ΓΖ ἴση κείσθω ἡ ΑΗ, καί, διαχθείσης τῆς ΕΘ, ἐπ' αὐτὴν κάθετος ἤχθω ἡ ΗΘ. ὅτι ἴση ἐστὶν ἡ ΘΒ τῆι ΚΕ. εἰλήφθω τὸ τοῦ ΑΒΓ ἡμικυκλίου 812
κέντρον τὸ Λ, καὶ κάθετος ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΒΜ τῆι ΜΚ. ἐπεὶ ἴση ἐστὶν ἡ μὲν ΗΑ τῆι ΓΖ, ἡ δὲ ΑΛ τῆι ΛΓ, ὅλη ἄρα ἡ ΗΛ ὅλῃ τῆι ΛΖ ἐστιν ἴση. καὶ εἰσὶν τρεῖς παράλληλοι αἱ ΗΘ, ΛΜ, ΕΖ. ἴση ἄρα ἐστὶν καὶ ἡ ΘΜ τῆι ΜΕ. ὧν ἡ ΒΜ τῆι ΜΚ ἴση.

‖ 1 ΛΝ add Co ‖ 2 τοῦ (ὑπὸ)] τὸ A | ἔλασσον Hu (BS) ἐλάσσων A ‖ 3 ΛΝ Co ΑΝ A ‖ 4 ἐπὶ τὰ ὡς πρὸς τὸ Γ μέρη ἐπὶ ταύτηι ὡς πρὸς τὸ Γ ἡγμέναι Hu ἔπειτα ὡς πρὸς τὸ Γ ἡγμέναι Hu app ‖ 5 ἡ ἐφαπτομένη Hu οὐ ἐφάπτεται A ‖ 7 μὲν secl Hu | διὰ] μένοντος Hu | ἡμικύκλια Ge ἡμικυκλίου A ‖ 8 αἱ A² ex εἰ ‖ 10 ιε´ add Hu (V) ‖ 14 τὴν Co τῶν A ‖ 18 λοιπὴ Ge (Co) λοιπὸν A ‖ 19 ὅπερ ante φανερὸν transp Hu ‖ 20 ις´ add Hu (V) ‖ 21 ΘΒ Co ΕΒ A ‖ 23 ἄρα add Hu (Co) ‖ 26 ιζ´ add Hu (V) ‖ 28 αὐτὴν Ge αὐτῆς A ‖ 29 ΘΒ... ΚΕ] ΘΚ... ΚΕ A ΘΒ... ΚΔ Co ‖ 31 ΓΖ] ΓΖΗ A ΓΔ Co ‖ 32 ΛΖ] ΑΖ A ΛΔ Co ‖ 33 ΜΕ] ΜΔ Co proposition) is obvious; for the (line) drawn from the center to the point of tangency (is perpendicular to ΘE, hence in three parallels ΘB equals KE). Q.E.D.

(150) 18. *(Prop. 91)* For the twenty-fourth.

Let there be two semicircles, as ABΓ, ΔEZ, and let AΔ equal ΔΓ, and let ZB be drawn through. That also BE equals EH. But it is obvious. For if ΔE is joined, then angle ΔEZ is right because it is in a semicircle. And ΔE is from the center in semicircle ABΓ. Thus BE equals EH. Q.E.D.

(151) 19. *(Prop. 92)*

For the twenty-fifth.

With the same things (assumed), let AΔ be greater than ΔΓ, and let AH be made equal to ΔΓ, and let HΘ be perpendicular to BZ. That BΘ equals EK.

Since AΔ is greater than ΔΓ,[1] therefore the center of semicircle ABΓ is between A, Δ. Let it be Λ,[2] and again let ΛM be a perpendicular.[3] Therefore MB equals MK.[4] But since AH equals ΔΓ,[5] and AΛ equals ΛΓ,[6] therefore remainder HΛ equals ΛΔ.[7] And HΘ, ΛM, ΔE are three parallels.[8] Therefore ΘM too equals ME.[9] But also all BM equalled all MK.[10] Therefore remainder BΘ equals remainder EK.[11] Q.E.D.

λοιπὴ ἄρα ἡ ΘΒ λοιπῆι τῆι ΚΕ ἐστὶν ἴση. κἂν ἐφάπτηται, φανερόν. ἡ γὰρ ἀπὸ τοῦ κέντρου ἐπιζευχθεῖσα ἐπὶ τὴν ἀφήν. ὅπερ:—

(150) <ιη.´> εἰς τὸ κδ.´
ἔστω δύο ἡμικύκλια ὡς τὰ ΑΒΓ, ΔΕΖ, καὶ ἔστω ἴση ἡ ΑΔ τῆι ΔΓ, καὶ διήχθω ἡ ΖΒ. ὅτι γίνεται ἴση καὶ ἡ ΒΕ τῆι ΕΗ. ἔστιν δὲ φανερόν. ἐὰν γὰρ ἐπιζευχθῆι ἡ ΔΕ, γίνεται ὀρθὴ ἡ ὑπὸ ΔΕΖ γωνία διὰ τὸ <ἐν> ἡμικυκλίωι εἶναι. καὶ ἔστιν ἀπὸ τοῦ κέντρου ἐν ἡμικυκλίωι τῶι ΑΒΓ ἡ ΔΕ. ἴση ἄρα ἐστὶν ἡ ΒΕ τῆι ΕΗ. ὅπερ:—

(151) <ιθ.´> |εἰς τὸ κε.´ |150
τῶν αὐτῶν ὄντων, ἔστω μείζων ἡ ΑΔ τῆς ΔΓ, καὶ τῆι ΔΓ ἴση κείσθω ἡ ΑΗ, καὶ κάθετος ἐπὶ τὴν ΒΖ ἡ ΗΘ. ὅτι ἴση ἐστὶν ἡ ΒΘ τῆι ΕΚ. ἐπεὶ μείζων ἐστὶν ἡ ΑΔ τῆς ΔΓ, τὸ ἄρα κέντρον τοῦ ΑΒΓ ἡμικυκλίου ἐστὶ μεταξὺ τῶν Α, Δ. ἔστω τὸ Λ. καὶ πάλιν κάθετος ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΜΒ τῆι ΜΚ. ἐπεὶ δὲ ἴση ἐστὶν 814
ἡ μὲν ΑΗ τῆι ΔΓ, ἡ δὲ ΑΛ τῆι ΛΓ, λοιπὴ ἄρα ἡ ΗΛ τῆι ΛΔ ἴση ἐστίν. καὶ εἰσὶν τρεῖς παράλληλοι αἱ ΗΘ, ΛΜ, ΔΕ. ἴση ἄρα καὶ ἡ ΘΜ τῆι ΜΕ. ἦν δὲ καὶ ὅλη ἡ ΒΜ ὅληι τῆι ΜΚ ἴση. λοιπὴ ἄρα ἡ ΒΘ λοιπῆι τῆι ΕΚ ἐστὶν ἴση. ὅπερ:— (152) <κ.´> εἰς τὸ κϛ.´
ἔστω ἡ ΑΔ ἐλάσσων τῆς ΔΓ, καὶ τῆι ΑΔ ἴση κείσθω ἡ ΓΗ, καὶ κάθετος ἡ ΗΘ. ὅτι ἴση ἐστὶν ἡ ΒΕ τῆι ΚΘ. ἐπεὶ γὰρ ἐλάσσων ἐστὶν ἡ ΑΔ τῆς ΓΔ, τοῦ ΑΒΓ ἡμικυκλίου τὸ κέντρον ἐστὶ μεταξὺ τῶν Δ, Η, ἔστω τὸ Λ, καὶ ἀπὸ τοῦ Λ ἐπὶ τὴν ΖΒ κάθετος ἤχθω ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΒΜ τῆι ΜΚ. ἐπεὶ δὲ ἴση ἐστὶν ἡ ΑΔ <τῆι ΓΗ, ἡ δὲ ΑΛ τῆι ΛΓ, ἴσον ἄρα ἐστὶν ἡ ΔΛ> τῆι ΛΗ. καὶ τρεῖς παράλληλοι αἱ ΔΕ, ΛΜ, ΗΘ. ἴση ἄρα ἐστὶν καὶ ἡ ΕΜ τῆι ΜΘ. ἔστιν δὲ καὶ ὅλη ἡ ΒΜ ὅληι τῆι ΜΚ ἴση. λοιπὴ ἄρα ἡ ΒΕ λοιπῆι τῆι ΚΘ ἐστὶν ἴση. ὅπερ:—

‖ 1 λοιπὴ... λοιπῆι Co (k) λοιπὸν... λοιπὸν A | ΚΕ] Ε om A[1] add supr A[2] ΚΔ Co | κἂν — ἀφήν secl Co | ἐφάπτηται Ge (recc?) ἐφάπτεται A ‖ 4 ιη´ add Hu (V) ‖ 7 ΔΕΖ Co ΔΕΓ A ‖ 8 ἐν add Ge (recc?) ‖ 9 ἡ ΔΕ Ge (S) ἡ in ras., sequitur ΗΔΕ A ‖ 11 ιθ´ add Hu (V) ‖ 15 ἔστω] compendium A ‖ 17 post ΗΛ add λοιπῆι Ge (V) | ΛΔ Co ΑΔ A ‖ 20 κ´ add Hu (V) ‖ 27 τῆι ΓΗ — τῆι ΛΗ] τῆι ΓΗ, ἡ δὲ ΑΛ τῆι ΛΓ, λοιπὴ ἄρα ἡ ΔΛ λοιπὴ τῆι ΛΗ ἴση ἐστίν Co | post καὶ add εἰσὶν Hu ‖ 29 ΒΜ Co ΕΜ A

(152) 20. *(Prop. 93)* For the twenty-sixth.

Let AΔ be less than ΔΓ, and let ΓH be made equal to AΔ, and HΘ a perpendicular. That BE equals KΘ.

For since AΔ is less than ΓΔ,[1] the center of semicircle ABΓ is between Δ, H. Let it be Λ,[2] and from Λ let ΛM be drawn perpendicular to ZB.[3] Then BM equals MK.[4] But since AΔ equals <ΓH,[5] and AΛ equals ΛΓ,[6] therefore ΔΛ> equals ΛH.[7] And ΔE, ΛM, HΘ are three parallels.[8] Therefore EM too equals MΘ.[9] But also all BM equals all MK.[10] Therefore remainder BE equals remainder KΘ.[11] Q.E.D.

(153) 21. *(Prop. 93bis)* For the twenty-ninth.

With two semicircles ABΓ, ΔEZ, and AΔ being greater than ΔΓ, if AH is made equal to ΔΓ, and ZB drawn through, and HΘ is drawn perpendicular to it, that ΘB equals KE.

Let the center Λ of semicircle ABΓ be taken, and from Λ let ΛM be drawn perpendicular to BZ.[1] Then BM is equal to MK.[2] But since AΛ equals ΛΓ,[3] and AH equals ΔΓ,[4] therefore remainder HΛ equals remainder ΛΔ.[5] And HΘ, ΛM, ΔE are three parallels.[6] Therefore ΘM equals ME.[7] Out of these BM equals MK.[8] Therefore remainder ΘB equals remainder KE.[9] Obviously also ΘK equals BE.[10] Q.E.D.

(154) 22. *(Prop. 93tris)* For the thirty-first.

Let there be semicircles ABΓ, ΔEZ, and again let AΔ be less than ΔΓ, and let ZEB be drawn through, and let ΓH be made equal to AΔ, and let HΘ be drawn perpendicular to ZB; for it is apparent that it falls neither on K nor between Z, K.

For if the center Λ is taken, and from Λ ΛM is drawn perpendicular to BZ,[1] then BM will be equal to MK.[2] But also, because ΔE, ΛM, HΘ are three parallels,[3] EM is equal to MK,[5] because ΔΛ equals ΛH.[4] And BM would equal ME,[6] the greater to the less; which is impossible. Hence it does not fall on K. Much less does it fall between Z, K. Hence (it falls) outside. AΛ (equals) ΛΓ,[7] and AΔ (equals) HΓ.[8] Therefore remainder ΔΛ equals remainder ΛH.[9] And ΔE, ΛM, HΘ are three parallels.[10] Therefore also EM equals MΘ.[11] Out of these BM equals MK.[12] Thus remainder EB equals remainder KΘ.[13] And obviously EK equals BΘ.[14] Q.E.D.

(155) 23. *(Prop. 94)* For the thirty-fourth.

Let there be semicircles ABΓ, ΔEZ, let ΔΓ be greater than ΓZ, let ZH be made equal to AΔ, and let the circle (ΔEZ) be filled out. Let BΓΘ be drawn through, and from H let (HΘ) be drawn perpendicular to BΓ. Obviously it falls outside the circle; for it is parallel to AB, and AB falls away, so HΘ too falls away. Let it be HΘ. That BE equals ΘK.

(153) <κα.´> εἰς τὸ κθ.´
ὄντων δύο ἡμικυκλίων τῶν ΑΒΓ, ΔΕΖ, καὶ μείζονος οὔσης
τῆς ΑΔ τῆς ΔΓ, ἐὰν τῆι ΔΓ ἴση τεθῆι ἡ ΑΗ, καὶ διαχθείσης τῆς 816
ΖΒ, [καὶ] ἐπ' αὐτὴν κάθετος ἀχθῆι <ἡ> ΗΘ, ὅτι ἴση ἐστὶν ἡ ΘΒ
τῆι ΚΕ. εἰλήφθω τὸ κέντρον τοῦ ΑΒΓ ἡμικυκλίου τὸ Λ, καὶ ἀπὸ
τοῦ Λ ἐπὶ τὴν ΒΖ κάθετος ἤχθω ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΒΜ τῆι
ΜΚ. ἐπεὶ δὲ ἴση ἐστὶν ἡ μὲν ΑΛ τῆι ΛΓ, ἡ δὲ ΑΗ τῆι ΔΓ, λοιπὴ
ἄρα ἡ ΗΛ λοιπῆι τῆι ΛΔ ἐστὶν ἴση. καὶ εἰσὶν τρεῖς
παράλληλοι αἱ ΗΘ, ΛΜ, ΔΕ. ἴση ἄρα ἐστὶν ἡ ΘΜ τῆι ΜΕ. ὧν ἡ ΒΜ
τῆι ΜΚ ἐστὶν ἴση. λοιπὴ ἄρα ἡ ΘΒ λοιπῆι τῆι ΚΕ ἐστὶν ἴση.
φανερὸν |ὡς καὶ ἡ ΘΚ τῆι ΒΕ ἐστὶν ἴση. ὅπερ:— |150v

(154) <κβ.´> εἰς <τὸ> λα.´
ἔστω τὰ ΑΒΓ, ΔΕΖ ἡμικύκλια, καὶ πάλιν ἔστω ἐλάσσων ἡ ΑΔ
τῆς ΔΓ, καὶ διήχθω ἡ ΖΕΒ, καὶ τῆι ΑΔ ἴση κείσθω ἡ ΓΗ, καὶ ἐπὶ
τὴν ΖΒ κάθετος ἤχθω ἡ ΗΘ. φανερὸν γὰρ ὅτι οὔτε ἐπὶ τὸ Κ
πίπτει οὔτε μεταξὺ τῶν Ζ, Κ. ἐὰν <γὰρ> τὸ κέντρον ληφθῆι
τὸ Λ, καὶ ἀπὸ τοῦ Λ ἐπὶ τὴν ΒΖ κάθετος ἀχθῆι ἡ ΛΜ, ἔσται ἴση
ἡ ΒΜ τῆι ΜΚ. ἀλλὰ καὶ διὰ τὸ τρεῖς εἶναι παραλλήλους τὰς
ΔΕ, ΛΜ, ΗΘ, ἴση γίνεται ἡ ΕΜ τῆι ΜΚ (ἴση γὰρ ἡ ΔΛ τῆι ΛΗ). εἴη
ἂν καὶ ἡ ΒΜ τῆι ΜΕ ἴση, ἡ μείζων τῆι ἐλάσσονι· ὅπερ
ἀδύνατον. οὐκ ἄρα ἐπὶ τὸ Κ πίπτει. πολλῶι δὲ μᾶλλον ὅτι 818
οὐδὲ μεταξὺ τῶν Ζ, Κ. [τῶν] ἐκτὸς ἄρα. ἔστιν ἡ μὲν ΑΛ τῆι ΛΓ,
ἡ δε ΑΔ τῆι ΗΓ. λοιπὴ ἄρα ἡ ΔΛ λοιπῆι τῆι ΛΗ ἴση ἐστίν. καὶ
εἰσὶν τρεῖς παράλληλοι αἱ ΔΕ, ΛΜ, ΗΘ. ἴση ἄρα καὶ ἡ ΕΜ τῆι
ΜΘ. ὧν ἡ ΒΜ τῆι ΜΚ ἐστὶν ἴση. λοιπὴ ἄρα ἡ ΕΒ λοιπῆι τῆι ΚΘ
ἐστὶν ἴση. φανερὸν δὲ καὶ ὡς ἡ ΕΚ τῆι ΒΘ ἐστὶν ἴση. ὅπερ:—

(155) <κγ.´> εἰς τὸ λδ.´
ἔστω τὰ ΑΒΓ, ΔΕΖ ἡμικύκλια, μείζων ἔστω ἡ ΔΓ τῆς ΓΖ, καὶ
τῆι ΑΔ ἴση κείσθω ἡ ΖΗ, καὶ προσαναπεπληρώσθω ὁ κύκλος.
διήχθω ἡ ΒΓΘ, καὶ ἀπὸ τοῦ Η ἐπὶ τὴν ΒΓ κάθετος ἤχθω.
φανερὸν ὅτι ἐκτὸς πίπτει τοῦ κύκλου. παράλληλος γὰρ
γίνεται τῆι ΑΒ, ἡ δὲ ΑΒ ὑποπίπτει. καὶ ἡ ΗΘ ἄρα ὑποπίπτει.
ἔστω ἡ ΗΘ. ὅτι ἴση ἐστὶν ἡ ΒΕ τῆι ΘΚ. ἐπεὶ μείζων ἐστὶν ἡ
ΔΓ τῆς ΓΖ, τὸ τοῦ ΔΕΖ ἡμικυκλίου κέντρον μεταξύ ἐστιν τῶν

‖ 1 κα´ add Hu (V) ‖ 3 τῆς Hu (V) τῆι A | ἐὰν τῆι ΔΓ om A¹ add supr A² ‖ 4 καὶ secl Ge | ἡ add Ge (recc?) ‖ 6 ΒΖ Ge ΕΖ A ‖ 7 ΑΗ Co ΑΝ A ‖ 8 ΛΔ Ge ΑΔ A ‖ 11 ὅπερ ante φανερὸν transp Hu ‖ 12 κβ´ add Hu (BS) | τὸ add Ge (BS) ‖ 14 τῆς Ge (BS) τῆι A ‖ 15 τὴν Ge τῆς A | post ΗΘ add ὅτι ἴση ἐστὶν ἡ ΕΒ τῆι ΚΘ Hu ὅτι ἡ ΕΒ τῆι ΚΘ ἴση ἐστίν Ge | post ὅτι add ἡ ΗΘ Horsley ‖ 19 ΔΛ Ge ΑΛ A ‖ 21 ἄρα Ge (S) ἔστιν A ‖ 22 τῶν secl Hu πίπτει Hu app | ante ἔστιν add ἐπεὶ δὲ Ge ‖ 26 ὅπερ ante φανερὸν transp Hu ‖ 27 κγ´ add Hu (BS) ‖ 29 post ὁ add ΔΕΖΚ Co ‖ 30 ΒΓ] ΒΘ Hu ‖ 31 φανερὸν — ἔστω secl Hu ‖ 32 ὑποπίπτει] ἐκτὸς πίπτει Co | ὑποπίπτει] ἐκτὸς πίπτει Co ‖ 33 ἔστω ἡ ΗΘ del Co ‖ 34 ἡμικυκλίου] κύκλου Co

Since ΔΓ is greater then ΓΖ,[1] the center of semicircle ΔΕΖ is between Δ, Γ. Let it be Λ,[2] and ΛΜ a perpendicular.[3] Then since ΑΔ equals ΖΗ,[4] and ΔΛ equals ΛΖ,[5] therefore all ΑΛ equals all ΛΗ.[6] And ΑΒ, ΛΜ, ΗΘ are three parallels.[7] Therefore also ΒΜ equals ΜΘ.[8] Out of these ΕΜ equals ΜΚ.[9] Therefore remainder ΒΕ equals remainder ΚΘ.[10] Obviously also ΒΚ equals ΕΘ.[11] Q.E.D.

(156) 24. *(Prop. 95)* Again let the semicircles be ΑΒΓ, ΔΕΖ, and ΔΓ greater than ΓΖ, and let ΖΗ be made equal to ΑΔ, and let circle ΔΕΖΚ be filled out, and let ΕΒΚ be drawn through, and from Η let ΗΘ be drawn perpendicular to it. Clearly it falls inside the circle, since ΑΒ which is parallel to it is inside. To prove that ΕΒ equals ΘΚ.

Let the center be Λ, and again ΛΜ a perpendicular.[1] Then ΕΜ equals ΜΚ.[2] But since ΑΛ equals ΛΗ[4] in three parallels, ΑΒ, ΛΜ, ΗΘ,[3] therefore also ΒΜ equals ΜΘ.[5] But also all ΕΜ equals all ΜΚ.[6] Thus remainder ΕΒ equals remainder ΚΘ.[7] Q.E.D.

(157) The first (book) of the *Neuses* contains nine problems, and three diorisms. The three are minima, that in the fifth and that in the seventh, and that in the ninth. The second (book) of *Neuses* contains forty-five problems, and three diorisms, that in the seventeenth problem, and that in the nineteenth, and that in the twenty-third; and the three are minima.

Δ, Γ. ἔστω τὸ Λ, καὶ κάθετος ἡ ΛΜ. ἐπεὶ οὖν ἴση ἐστὶν ἡ μὲν
ΑΔ τῆι ΖΗ, ἡ δὲ ΔΛ τῆι ΛΖ, ὅλη ἄρα ἡ ΑΛ ὅληι τῆι ΛΗ ἐστὶν ἴση.
καὶ εἰσὶν τρεῖς παράλληλοι αἱ ΑΒ, ΛΜ, ΗΘ. ἴση ἄρα ἐστὶν καὶ
ἡ ΒΜ τῆι ΜΘ. ὧν ἡ ΕΜ τῆι ΜΚ ἐστὶν ἴση. λοιπὴ ἄρα |ἡ ΒΕ |151
λοιπῆι τῆι ΚΘ ἐστὶν ἴση. φανερὸν ὡς καὶ ἡ ΒΚ τῆι ΕΘ ἐστὶν
ἴση. ὅ(περ): —

(156) <κδ.´> ἔστω πάλιν τὰ ἡμικύκλια τὰ ΑΒΓ, ΔΕΖ, καὶ
μείζων ἡ ΔΓ τῆς ΓΖ, καὶ τῆι ΑΔ ἴση κείσθω ἡ ΖΗ, καὶ
προσαναπεπληρώσθω ὁ ΔΕΖΚ κύκλος, καὶ διήχθω ἡ ΕΒΚ, καὶ ἀπὸ
τοῦ Η ἐπ' αὐτὴν κάθετος ἤχθω ἡ ΗΘ. φανερὸν δὲ ὅτι ἐντὸς
πίπτει τοῦ κύκλου, ἐπεὶ καὶ ἡ παράλληλος αὐτῆι ἡ ΑΒ ἐντός. 820
δεῖξαι ὅτι ἴση ἐστὶν ἡ ΕΒ τῆι ΘΚ. ἔστω τὸ κέντρον τὸ Λ, καὶ
πάλιν κάθετος ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΕΜ τῆι ΜΚ. ἐπεὶ δὲ ἐν
τρισὶ παραλλήλοις ταῖς ΑΒ, ΛΜ, ΗΘ ἴση ἐστὶν ἡ ΑΛ τῆι ΛΗ, ἴση
ἄρα ἐστὶν καὶ ἡ ΒΜ τῆι ΜΘ. ἔστιν δὲ καὶ ὅλη ἡ ΕΜ ὅληι τῆι
ΜΚ ἴση. λοιπὴ ἄρα ἡ ΕΒ λοιπῆι τῆι ΚΘ ἐστὶν ἴση. ὅπερ: —

(157) τὸ πρῶτον τῶν Νεύσεων ἔχει προβλήματα $\bar{\theta}$,
διορισμοὺς τρεῖς. καὶ εἰσὶν οἱ τρεῖς ἐλάσσονες, ὅ τε κατὰ
τὸ πέμπτον καὶ ὁ κατὰ τὸ ζ´ πρόβλημα, καὶ ὁ κατὰ τὸ θ´. τὸ
δεύτερον Νεύσεων ἔχει προβλήματα με̄, διορισμοὺς τρεῖς, τόν
τε κατὰ τὸ ιζ´ πρόβλημα, καὶ τὸν κατὰ τὸ ιθ´, καὶ τὸν κατὰ
τὸ κγ´ καὶ εἰσὶν οἱ τρεῖς ἐλάσσονες.

‖ 4 ἴση Ge (BS) ἴσαι A ‖ 6 ὅπερ ante φανερὸν transp Hu ‖ 7 κδ´ add Hu (BS) | ΔΕΖ Co ΕΔΖ A ‖ 10 φανερὸν — ἡ ΑΒ ἐντός secl Hu ‖ 16 τῆι Ge (recc?) τῆς A ‖ 18 διορισμοὺς Ge (BS) διωρισμένους A | ὅ τε Hu ὄντες A

(158) Tangencies, (Book) 1.

1. *(Prop. 96)* For the fifth problem.

(Let) AB, ΓΔ be two parallel lines, and let circle EZ be tangent (to them) at points E, Z, and let EZ be joined. That (EZ) is a diameter of circle EZ.

Let points H, Θ be taken on the circumference of the circle, and let EH, HZ, EΘ, ΘZ be joined. Then since AE is tangent to, and EZ cuts (the circle),[1] therefore angle AEZ equals the angle in the alternate segment, EΘZ (III 32).[2] For the same reasons, also angle ΔZE equals angle <ZHE.[3] But angle> ΔZE <is equal to angle AEZ> as alternate angles.[4] Hence too angle EΘZ equals angle EHZ.[5] And they equal two right angles (III 22).[6] Hence each of them is right,[7] so that each of EΘZ, EHZ is a semicircle.[8] Thus EZ is a diameter of circle EZ.[9] Q.E.D.

(159) 2. *(Prop. 97)* Let there be circle ABΔ, and let BΓ, ΓA be tangent to it, and let angle Γ be bisected by straight line ΓΔ. That the center of circle ABΓ is on ΓΔ.

Let ΔA, AE, ΔB, BE be joined. Then since AΓ is tangent to, and ΓΔ cuts (the circle),[1] the rectangle contained by ΔΓ, ΓE equals the square of ΓA (III 36).[2] Hence angle ΔAΓ equals angle AEΓ.[3] For the same reasons too angle ΔBΓ equals angle BEΓ.[4] But angle BΓΔ is equal to angle AΓΔ.[5] Hence angle ΔAE equals angle ΔBE,[6] so that each of them is right.[7] Hence ΔE is a diameter of circle ABΔ.[8] Thus the center of circle ABΔ is on ΓΔ.

(160) 3. *(Prop. 98)* Let there be two circles, AB, BΓ, tangent to each other at point B, and let ABΓ be drawn through, and let the center of circle AB be on it. That also the center of circle BΓ is on ABΓ.

For let ΔBE be drawn tangent to both circles.[1] Then angle ABΔ is right,[2] and so the complementary angle ΔBΓ is also right.[3] And ΔE is tangent to circle BΓ.[4] Thus the center of circle BΓ is on BΓ,[5] as is also that of (circle) AB.

ΕΠΑΦΩΝ ΠΡΩΤΟΝ

(158) <α.´> εἰς τὸ ε´ πρόβλημα.

δύο παράλληλοι αἱ ΑΒ, ΓΔ, καὶ κύκλος ἐφαπτέσθω ὁ ΕΖ κατὰ τὰ Ε, Ζ σημεῖα, καὶ ἐπεζεύχθω ἡ ΕΖ. ὅτι διάμετρός ἐστιν τοῦ ΕΖ κύκλου. εἰλήφθω σημεῖα ἐπὶ τῆς τοῦ κύκλου περιφερείας τὰ Η, Θ, καὶ ἐπεζεύχθωσαν αἱ ΕΗ, ΗΖ, ΕΘ, ΘΖ. ἐπεὶ οὖν ἐφάπτεται μὲν ἡ ΑΕ, τέμνει δὲ ἡ ΕΖ, ἴση ἄρα ἐστὶν ἡ ὑπὸ ΑΕΖ γωνία τῆι ἐν τῶι [ε] ἐναλλὰξ τμήματι γωνίαι τῆι ὑπὸ ΕΘΖ. διὰ ταὐτὰ καὶ ἡ ὑπὸ ΔΖΕ ἴση ἐστὶν τῆι ὑπὸ <ΖΗΕ. ἀλλὰ τῆι ὑπὸ ΑΕΖ ἴση ἐστὶν ἡ ὑπὸ> ΔΖΕ ἐναλλάξ. καὶ ἡ ὑπὸ ΕΘΖ ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ ΕΗΖ γωνίαι. καὶ εἰσὶν δυσὶν ὀρθαῖς 822
ἴσαι. ὀρθὴ ἄρα ἐστὶν ἑκατέρα αὐτῶν. ὥστε ἡμικύκλιόν ἐστιν ἑκάτερον <τῶν> ΕΘΖ, ΕΗΖ. διάμετρος ἄρα ἐστὶν |ἡ ΕΖ τοῦ ΕΖ |151v
κύκλου. ὅπερ: –

(159) <β.´> ἔστω κύκλος ὁ ΑΒΔ, καὶ ἐφαπτέσθωσαν αὐτοῦ αἱ ΒΓ, ΓΑ, καὶ τετμήσθω ἡ Γ γωνία δίχα τῆι ΓΔ εὐθείαι. ὅτι ἐπὶ τῆς ΓΔ τὸ κέντρον ἐστὶν τοῦ ΑΒΓ κύκλου. ἐπεζεύχθωσαν αἱ ΔΑ, ΑΕ, ΔΒ, ΒΕ. ἐπεὶ οὖν ἐφάπτεται μὲν ἡ ΑΓ, τέμνει δὲ ἡ ΓΔ, τὸ ὑπὸ ΔΓΕ ἴσον ἐστὶν τῶι ἀπὸ ΓΑ. ἴση ἄρα ἐστὶν καὶ ἡ ὑπὸ ΔΑΓ γωνία τῆι ὑπὸ ΑΕΓ γωνίαι. διὰ ταὐτὰ καὶ ἡ ὑπὸ ΔΒΓ γωνία ἴση ἐστὶν τῆι ὑπὸ ΒΕΓ γωνίαι. ἀλλὰ τῆι ὑπὸ ΑΓΔ ἴση ἐστὶν ἡ ὑπὸ ΒΓΔ γωνία. καὶ ἡ ὑπὸ ΔΑΕ ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ ΔΒΕ γωνίαι. ὥστε ὀρθή ἐστιν ἑκατέρα αὐτῶν. διάμετρος ἄρα ἐστὶν ἡ ΔΕ τοῦ ΑΒΔ κύκλου. ἐπὶ τῆς ΓΔ ἄρα τὸ κέντρον ἐστὶν τοῦ ΑΒΔ κύκλου.

(160) <γ.´> εἰς τὸ ιβ.´

ἔστωσαν δύο κύκλοι ἐφαπτόμενοι ἀλλήλων οἱ ΑΒ, ΒΓ κατὰ τὸ Β σημεῖον, καὶ διήχθω ἡ ΑΒΓ. ἔστω δὲ ἐπ᾽ αὐτῆς τὸ τοῦ ΑΒ κύκλου κέντρον. ὅτι καὶ τὸ τοῦ ΒΓ κύκλου κέντρον ἐστὶν ἐπὶ τῆς ΑΒΓ. ἤχθω γὰρ ἀμφοτέρων τῶν κύκλων ἐφαπτομένη ἡ ΔΒΕ. ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΑΒΔ γωνία. καὶ ἡ ἐφεξῆς ἄρα ἡ ὑπὸ ΔΒΓ
ἐστὶν ὀρθή. καὶ ἐφάπτεται ἡ ΔΕ τοῦ ΒΓ κύκλου. τὸ ἄρα 824
κέντρον τοῦ ΒΓ κύκλου ἐστὶν ἐπὶ τῆς ΒΓ, ὁμοίως καὶ τοῦ ΑΒ.

‖ 2 α´ add Camer$_1$ (BS) ‖ 8 ε del Co ‖ 10 ΔΖΕ] ΕΗΖ Co ‖ 12 ἡμικύκλιόν Hu ‖ 13 τῶν add Hu (V) ‖ 15 β´ add Camer$_1$ (BS) ‖ 17 κύκλου Hu (Co) καὶ A ‖ 20 ΑΕΓ Co ΑΓΔ A | ΔΒΓ Co ΔΑΕ A ‖ 21 ΒΕΓ Co ΒΓΔ A | ΑΓΔ] ΕΑΓ Co ‖ 22 ἡ Camer$_1$ (S) τῆι A | ΒΓΔ] ΕΒΓ Co | ΔΑΕ Co ΔΑΓ A ‖ 26 γ´ add Camer$_1$ (BS) ‖ 31 ΑΒΔ Co ΑΒ, ΓΔ A ‖ 33 ΒΓ] ΑΒΓ Hu | post ὁμοίως add ὡς Hu

(161) 4. *(Prop. 99)* Another way. Again let AB, BΓ be <diameters> of circles. That circles AB, BΓ are tangent to each other.

Again let ΔE be drawn tangent to circle AB.[1] Then angle ABΔ is right,[2] and complementary angle ΔBΓ is right.[3] And BΓ is from the other center.[4] Hence ΔE is tangent to circle BΓ.[5] But it is also to (circle) AB at (point) B itself.[6] Thus (circle) AB is tangent also to circle BΓ at point B.[7] *On the same figure.*

(162) 5. *(Prop. 100)* (Let there be) two circles, AB, BΓ, tangent to each other (internally) at point B, and let AΓB be drawn through, and let the center of circle AB be on it. That the center of (circle) BΓ too is on BΓ.

Let ΔE be drawn tangent to the circles. Then since ΔE is tangent to circle AB[1] and AB is through the center (of circle AB),[2] angle ΔBΓ is right.[3] And it was drawn from the point of tangency B. Thus the center of circle BΓ is on BΓ.[4]

It is also apparent in the following way. For if BZH were drawn through, and ΓZ, AH joined, then angle ABΔ would be equal to each of angles BZΓ, AHB.[5] And angle AHB is right.[6] Therefore angle BZΓ too is right.[7] Therefore the center of (circle) BΓ is on BΓ.[8] And similarly if (the center) of (circle) AB is given on AB, we shall prove that (the center) of (circle) AB too is (on it).

(163) 6. *(Prop. 100)* But again let there be diameters AB, BΓ. That the circles are tangent to each other.

Let straight line ΔBE be drawn tangent to circle AB.[1] Then angle ABE is right.[2] And BΓ is diameter.[3] Therefore ΔE is tangent to circle BΓ at point B;[4] for if ΓZ were produced to Δ, then the rectangle contained by ΓΔ, ΔZ would equal the square of ΔB,[5] because angle Z is right while angle B is right.[6] But (ΔE) is also tangent to circle AB at B.[7] Thus circle AB too is tangent to circle BΓ at B.[8] *On the same figure.*

(161) <δ.´> ἄλλως. ἔστωσαν πάλιν αἱ ΑΒ, ΒΓ <διάμετροι> κύκλων. ὅτι οἱ ΑΒ, ΒΓ κύκλοι ἐφάπτονται ἀλλήλων. ἤχθω πάλιν ἐφαπτομένη [ἡ] τοῦ ΑΒ κύκλου ἡ ΔΕ. ὀρθὴ <ἄρα> ἐστὶν ἡ ὑπὸ ΑΒΔ γωνία. καὶ ἐφεξῆς ἡ ὑπὸ ΔΒΓ γωνία ὀρθή ἐστιν. καὶ ἔστιν ἐκ θατέρου κέντρου ἡ ΒΓ. ἡ ΔΕ ἄρα ἐφάπτεται τοῦ ΒΓ κύκλου. ἀλλὰ καὶ τοῦ ΑΒ κατ' αὐτὸ τὸ Β. καὶ ὁ ΑΒ |ἄρα τοῦ |152 ΒΓ κύκλου ἐφάπτεται κατὰ τὸ Β σημεῖον. (ἐπὶ τῆς αὐτῆς καταγραφῆς.)

(162) <ε.´> δύο κύκλοι ἐφαπτόμενοι ἀλλήλων οἱ ΑΒ, ΒΓ κατὰ τὸ Β σημεῖον, καὶ διήχθω ἡ ΑΓΒ. ἔστω δὲ ἐπ' αὐτῆς τὸ κέντρον τοῦ ΑΒ κύκλου. ὅτι καὶ τοῦ ΒΓ τὸ κέντρον ἐστὶν ἐπὶ τῆς ΒΓ. ἤχθω ἐφαπτομένη τῶν κύκλων ἡ ΔΕ. ἐπεὶ οὖν ἐφάπτεται ἡ ΔΕ τοῦ ΑΒ κύκλου <καὶ> διὰ τοῦ κέντρου ἡ ΑΒ, ὀρθή ἐστιν ἡ ὑπὸ ΔΒΓ γωνία. καὶ ἦκται ἀπὸ τῆς ἁφῆς τῆς Β, ἐπὶ τῆς ΒΓ ἄρα τὸ κέντρον ἐστὶν τοῦ ΒΓ κύκλου.

φανερὸν δὲ καὶ οὕτως. εἰ γὰρ διαχθείη ἡ ΒΖΗ, καὶ ἐπεζευχθείησαν αἱ ΓΖ, ΑΗ, γένοιτο ἂν ἴση ἡ ὑπὸ ΑΒΔ γωνία ἑκατέραι τῶν ὑπὸ τῶν ΒΖΓ, ΑΗΒ γωνίαι. καὶ ἔστιν ὀρθὴ ἡ ὑπὸ 826 ΑΗΒ γωνία. ὀρθὴ ἄρα ἐστὶν καὶ ἡ ὑπὸ ΒΖΓ γωνία. ὥστε ἐπὶ τῆς ΒΓ τὸ κέντρον ἐστὶν τοῦ ΒΓ. καὶ ὁμοίως κἂν τοῦ ΒΓ δοθῆι ἐπὶ τῆς ΑΒ, δείξομεν ὅτι καὶ τοῦ ΑΒ.

(163) <ϛ.´> ἀλλὰ δὴ πάλιν ἔστωσαν διάμετροι αἱ ΑΒ, ΒΓ. ὅτι οἱ κύκλοι ἐφάπτονται ἀλλήλων. ἤχθω τοῦ ΑΒ κύκλου ἐφαπτομένη εὐθεῖα ἡ ΔΒΕ. ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΑΒΕ γωνία. καὶ ἔστιν διάμετρος ἡ ΒΓ. ἡ ΔΕ ἄρα ἐφάπτεται τοῦ ΒΓ κύκλου κατὰ τὸ Β σημεῖον· εἰ γὰρ ἐκβληθείη ἡ ΓΖ ἐπὶ τὸ Δ, γένοιτο ἂν τὸ ὑπὸ ΓΔΖ ἴσον τῶι ἀπὸ ΔΒ, διὰ τὸ ὀρθὴν γίνεσθαι τὴν πρὸς τῶι Ζ γωνίαν, οὔσης τῆς πρὸς τῶι Β ὀρθῆς. ἀλλὰ γὰρ καὶ τοῦ ΑΒ κύκλου ἐφάπτεται κατὰ τὸ Β. καὶ ὁ ΑΒ ἄρα κύκλος τοῦ ΒΓ κύκλου ἐφάπτεται κατὰ τὸ Β. (ἐπὶ τῆς αὐτῆς καταγραφῆς.)

‖ 1 δ´ add Camer$_1$ (BS) | διάμετροι κύκλων] κύκλων A κύκλων διάμετροι Co ‖ 3 ἡ (τοῦ ΑΒ) secl Hu | ἄρα add Hu ‖ 5 ἐκ θατέρου κέντρου ἡ ΒΓ] εκατερα κέντρον ἡ ΒΓ A τοῦ ΒΓ κύκλου κέντρον ἐπὶ τῆς ΒΓ Hu ἑκάτερον κέντρον τῶν ΑΒ, ΒΓ κύκλων ἐπὶ τῆς ΑΒΓ Camer$_1$ ἐν ἑκατέραι κέντρον τῶν ΑΒ, ΒΓ Haumann ‖ 7 ἐπὶ – καταγραφῆς secl Hu ‖ 9 ε´ add Camer$_1$ (BS) ‖ 13 ἐφάπτεται] ἐφαπτομένη A | καὶ add Camer$_1$ (S) ‖ 14 ΔΒΓ] ΔΒΑ Hu | τῆς Β] τῆς ΒΕ A ἡ ΒΓ Co τῆς Β ἡ ΒΓ Camer$_1$ ‖ 15 τῆς Co τὴν A | ἄρα post κέντρον A transp Co ‖ 16 διαχθείη Hu διαχθῇ A | ΒΖΗ Co ΒΖ A ‖ 17 ἐπεζευχθείησαν Hu ἐπεζεύχθωσαν A | ΑΒΔ] ΔΒΖ A ΕΒΓ Camer$_1$ ‖ 18 ΒΖΓ Camer$_1$ ΕΖΓ A ‖ 22 ϛ´ add Camer$_1$ (BS) ‖ 25 ἐφάπτεται] ἐφαπτομένη A ‖ 26 εἰ γὰρ – τῶι Β ὀρθῆς del Co ‖ 27 ΔΒ Camer$_1$ ΑΒ A ‖ 30 ἐπὶ – καταγραφῆς secl Hu

(164) 7. *(Prop. 102)* For the sixteenth.

Let there be two circles ABΓ, ΔEB, tangent to each other at point B, and let ΓBΔ, ABE be drawn through B, and let AΓ, ΔE be joined. That AΓ and ΔE are parallel.

For let straight line ZH be drawn tangent to the circles at point B.[1] Then since BZ is tangent to, and BA cuts (circle AB),[2] angle ABZ equals angle AΓB.[3] For the same reasons also angle HBE equals angle BΔE.[4] But angle ABZ equals angle EBH.[5] Hence angle AΓB too equals angle EΔB.[6] And they are alternate angles.[7] Therefore AΓ is parallel to ΔE.[8] Q.E.D.

(165) 8. *(Prop. 103)* (Let there be) circle ABΓ, and let AB, BΓ, <AΓ> be joined, and let some line ΔE be drawn through A so that angle B equals angle EAΓ. That ΔE is tangent to circle AB at point A.

Now if AΓ is through the center, then it is obvious. For angle EAΓ turns out to be right,[1] since also angle B is right.[2] This was proved before. But if not, then let the center be Z,[3] and let AZ be joined, and let it be produced to H, and let BH be joined. Then angle ABH is right.[4] So since angle EAΓ equals angle ABΓ,[5] while angle HAΓ equals HBΓ in the same segment,[6] therefore all angle EAH equals angle ABH.[7] But angle ABH is right.[8] Therefore angle EAH too is right.[9] And AZ is from the center.[10] Therefore ΔE is tangent to circle ABΓ;[11] for this was proved before.

(166) 9. *(Prop. 104)* This being so, the converse of the foregoing (lemma), namely, with AΓ being parallel to ΔE, to prove that ABΓ, ΔEB are tangent to each other at point B.

Again let straight line ZH be drawn tangent to circle ABΓ.[1] Then angle ABZ equals angle Γ.[2] But angle ABZ equals angle EBH,[3] while angle Γ equals alternate angle Δ,[4] so that also angle HBE equals angle Δ.[5] But according to the (lemma) written above, ZH is tangent to circle ΔBE.[6] But it is also (tangent) to (circle) ABΓ at B.[7] Thus circle ABΓ is tangent to circle BΔE at point B.[8]

(164) <ϛ.´> εἰς τὸ ιϛ.´
ἔστωσαν δύο κύκλοι ἐφαπτόμενοι ἀλλήλων οἱ ΑΒΓ, ΔΕΒ κατὰ τὸ Β σημεῖον, καὶ διὰ τοῦ Β διήχθωσαν αἱ ΓΒΔ, ΑΒΕ, καὶ ἐπεζεύχθωσαν αἱ ΑΓ, ΔΕ. ὅτι παράλληλοι αἱ ΑΓ, ΔΕ. ἤχθω γὰρ τῶν κύκλων ἐφαπτομένη εὐθεῖα ἡ ΖΗ κατὰ τὸ Β σημεῖον. ἐπεὶ οὖν ἐφάπτεται μὲν ἡ ΒΖ, τέμνει δὲ ἡ ΒΑ, ἴση ἐστὶν ἡ ὑπὸ ΑΒΖ γωνία τῆι ὑπὸ ΑΓΒ. διὰ ταὐτὰ δὴ καὶ ἡ ὑπὸ ΗΒΕ γωνία ἴση ἐστὶν τῆι ὑπὸ ΒΔΕ γωνίαι. ἀλλὰ ἡ ὑπὸ ΑΒΖ γωνία ἴση ἐστὶν τῆι ὑπὸ ΕΒΗ γωνίαι. καὶ ἡ ὑπὸ ΑΓΒ ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ ΕΔΒ γωνίαι. καὶ εἰσὶν ἐναλλάξ. παράλληλος ἄρα ἐστὶν ἡ ΑΓ τῆι ΔΕ. ὅπερ:—

(165) <η.´> |κύκλος ὁ ΑΒΓ, καὶ ἐπεζεύχθωσαν αἱ ΑΒ, ΒΓ, 152v 828
<ΑΓ>, καὶ διὰ τοῦ Α διήχθω τις εὐθεῖα ἡ ΔΕ ὥστε ἴσην εἶναι τὴν Β γωνίαν τῆι ὑπὸ ΕΑΓ γωνίαι. ὅτι ἐφάπτεται ἡ ΔΕ τοῦ ΑΒ κύκλου κατὰ τὸ Α σημεῖον. εἰ μὲν οὖν ἡ ΑΓ διὰ τοῦ κέντρου ἐστίν, φανερὸν ἔσται. γίνεται γὰρ ὀρθὴ ἡ ὑπὸ ΕΑΓ γωνία, διὰ τὸ καὶ τὴν Β γωνίαν εἶναι ὀρθήν. τοῦτο δὲ προδέδεικται. εἰ δὲ μή, ἔστω τὸ κέντρον τὸ Ζ, καὶ ἐπεζεύχθω ἡ ΑΖ, καὶ ἐκβεβλήσθω ἐπὶ τὸ Η, καὶ ἐπεζεύχθω ἡ ΒΗ. ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΑΒΗ γωνία. ἐπεὶ οὖν ἴση ἐστὶν ἡ μὲν ὑπὸ ΕΑΓ γωνία τῆι ὑπὸ ΑΒΓ, ἡ δὲ ὑπὸ ΗΑΓ γωνία ἐν τῶι αὐτῶι τμήματι τῆι ὑπὸ ΗΒΓ, ὅλη ἄρα ἡ ὑπὸ ΕΑΗ γωνία τῆι ὑπὸ ΑΒΗ γωνίαι ἴση ἐστίν. ὀρθὴ δὲ ἡ ὑπὸ ΑΒΗ. ὀρθὴ ἄρα καὶ ἡ ὑπὸ ΕΑΗ. καὶ ἔστιν ἐκ τοῦ κέντρου ἡ ΑΖ. ἐφάπτεται ἄρα ἡ ΔΕ τοῦ ΑΒΓ κύκλου. τοῦτο γὰρ προγέγραπται.

(166) <θ.´> τούτου ὄντος, ἀναστρόφιον τοῦ πρὸ αὐτοῦ. παραλλήλου οὔσης τῆς ΑΓ τῆι ΔΕ, δεῖξαι ὅτι ἐφάπτονται οἱ ΑΒΓ, ΔΕΒ ἀλλήλων κατὰ τὸ Β σημεῖον. ἤχθω πάλιν τοῦ ΑΒΓ κύκλου ἐφαπτομένη εὐθεῖα ἡ ΖΗ. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΑΒΖ γωνία τῆι Γ. ἀλλὰ ἡ ὑπὸ ΑΒΖ γωνία ἴση ἐστὶν τῆι ὑπὸ ΕΒΗ, ἡ δὲ Γ τῆι Δ ἐναλλὰξ ἴση ἐστίν, <ὥστε> καὶ ἡ ὑπὸ ΗΒΕ γωνία τῆι Δ. διὰ δὴ τὸ προγεγραμμένον, ἐφάπτεται ἡ ΖΗ τοῦ ΔΒΕ κύκλου. ἀλλὰ καὶ τοῦ ΑΒΓ κατὰ τὸ Β. καὶ ὁ ΑΒΓ ἄρα κύκλος τοῦ ΒΔΕ κύκλου ἐφάπτεται κατὰ τὸ Β σημεῖον.

‖ 1 ϛ´ add Camer₁ (BS) ‖ 9 ΑΓΒ Co ΑΒΓ A ‖ 12 η´ add Camer₁ (BS) | ἐπεζεύχθωσαν αἱ Camer₁ ἐπεζεύχθω ἡ A ‖ 13 ΑΓ add Co | διὰ Hu ἀπὸ A | διήχθω Camer₁ (BS) διήχθη A | ΔΕ Hu ΑΕ A ‖ 14 ΑΒ] ΑΒΓ Co ‖ 24 ἐφάπτεται Co ἐφαπτομένη A ‖ 26 θ´ add Camer₁ (BS) | ἀναστρόφιον] ἀνάστροφον Ge (B) ἀντίστροφον Camer₁ ‖ 27 ἐφάπτονται] ἐφαπτόμενοι A ‖ 30 ἡ ὑπὸ] ἡ πὸ A¹ corr A² ‖ 31 ὥστε add Hu | ΗΒΕ Co ΑΒΕ A ‖ 32 ΔΒΕ Co ΑΒΕ A

(167) 10. *(Prop. 105)* Problem for the same (problem).

Given circle ABΓ in position, and two points Δ, E given, to inflect a straight line ΔBE and, with it produced, to make AΓ parallel to ΔE.

Let it be accomplished, and let ZA be drawn tangent.[1] Then since AΓ is parallel to ΔE,[2] angle Γ equals angle ΓΔE.[3] But angle Γ equals angle ZAE,[5] because (ZA) is tangent to, and (AΓ) cuts (the circle).[4] And hence angle ZAE equals angle ΓΔE.[6] Thus points A, B, Δ, Z are on a circle.[7] Hence the rectangle contained by AE, EB equals the rectangle contained by ZE, EΔ.[8] But the rectangle contained by AE, EB is given,[9] because it equals the square of the tangent (from E to circle ABΓ). Therefore also the rectangle contained by ΔE, EZ is given.[10] And ΔE is given.[11] Hence EZ too is given.[12] But it is also (given) in position;[13] and E is given.[14] Hence Z too is given.[15] But from a given point Z a straight line ZA has been drawn tangent to a circle ABΓ given in position.[16] Hence ZA is given in position and magnitude.[17] And Z is given.[18] Therefore A too (is given).[19] But E too is given.[20] Therefore AE is (given) in position.[21] But the circle too is (given) in position.[22] Therefore point B is given.[23] But each of Δ, E is given.[24] Hence each of ΔB, BE is given in position.[25]

(168) *(Prop. 105)* The synthesis of the problem will be made as follows. Let the circle be ABΓ, and the given two points Δ, E, and let the rectangle contained by ΔE and some other (line) EZ be made equal to the square of the tangent (from E), and from Z let a straight line ZA be drawn tangent to circle ABΓ, and let AE be joined, and let ΔB be joined and produced to Γ, and let AΓ be joined. I say that AΓ is parallel to ΔE.

For since the rectangle contained by ZE, EΔ equals the square of the tangent (from E),[1] while the rectangle contained by AE, EB too equals the square of the tangent,[2] therefore the rectangle contained by AE, EB equals the rectangle contained by ZE, EΔ.[3] Hence <points A, B, Δ, Z> are on a circle.[4] <Therefore> angle ZAE <equals> angle BΔE.[5] But angle ZAE also equals angle AΓB in the alternate segment.[6] Hence angle AΓB equals angle BΔE.[7] And they are alternate angles.[8] Thus AΓ is parallel to ΔE.[9]

(167) <ι.´> πρόβλημα εἰς τὸ αὐτό. 830

θέσει δοθέντος κύκλου τοῦ ΑΒΓ, καὶ δύο δοθέντων τῶν Δ, Ε, ἀπὸ τῶν Δ, Ε κλᾶν εὐθεῖαν τὴν ΔΒΕ, καί, ἐκβληθείσης, ποιεῖν παράλληλον τὴν ΑΓ τῆι ΔΕ. γεγονέτω, καὶ ἤχθω ἐφαπτομένη ἡ ΖΑ. ἐπεὶ οὖν |παράλληλος ἡ ΑΓ τῆι ΔΕ, ἴση ἐστὶν ἡ Γ γωνία τῆι ὑπὸ ΓΔΕ γωνίαι. ἀλλὰ ἡ Γ ἴση ἐστὶν τῆι ὑπὸ ΖΑΕ |153 (ἐφάπτεται γάρ, καὶ τέμνει). καὶ ἡ ὑπὸ ΖΑΕ ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ ΓΔΕ. ἐν κύκλωι ἄρα ἐστὶν τὰ Α, Β, Δ, Ζ σημεῖα. ἴσον ἄρα ἐστὶν τὸ ὑπὸ ΑΕΒ τῶι ὑπὸ ΖΕΔ. δοθὲν δὲ τὸ ὑπὸ ΑΕΒ (ἴσον γάρ ἐστιν τῶι ἀπὸ τῆς ἐφαπτομένης). δοθὲν ἄρα καὶ τὸ ὑπὸ τῶν ΔΕΖ. καὶ δοθεῖσα ἡ ΔΕ. δοθεῖσα ἄρα καὶ ἡ ΕΖ. ἀλλὰ καὶ τῆι θέσει. καὶ ἔστιν δοθὲν τὸ Ε. δοθὲν ἄρα καὶ τὸ Ζ. ἀπὸ <δὴ> δεδομένου σημείου τοῦ Ζ θέσει δεδομένου κύκλου τοῦ ΑΒΓ ἐφαπτομένη εὐθεῖα ἦκται ἡ ΖΑ. δέδοται ἄρα ἡ ΖΑ τῆι θέσει καὶ τῶι μεγέθει. καὶ ἔστιν δοθὲν τὸ Ζ. δοθὲν ἄρα καὶ τὸ Α. ἀλλὰ καὶ τὸ Ε δοθέν. θέσει ἄρα ἐστὶν ἡ ΑΕ. θέσει δὲ καὶ ὁ κύκλος. δοθὲν ἄρα τὸ Β σημεῖον. ἔστιν δὲ καὶ ἑκάτερον τῶν Δ, Ε δοθέν. δοθεῖσα ἄρα ἐστὶν ἑκατέρα τῶν ΔΒ, ΒΕ τῆι θέσει.

(168) συντεθήσεται δὴ τὸ πρόβλημα οὕτως. ἔστω ὁ μὲν κύκλος ὁ ΑΒΓ, τὰ δὲ δοθέντα δύο σημεῖα τὰ Δ, Ε, <καὶ> κείσθω τῶι ἀπὸ τῆς ἐφαπτομένης ἴσον τὸ ὑπὸ τῆς ΔΕ, καὶ ἄλλης τινὸς τῆς ΕΖ, καὶ ἀπὸ τοῦ Ζ τοῦ ΑΒΓ κύκλου ἐφαπτομένη εὐθεῖα γραμμὴ ἤχθω ἡ ΖΑ, καὶ ἐπεζεύχθω ἡ ΑΕ, καὶ ἐπιζευχθεῖσα ἡ ΔΒ ἐκβεβλήσθω ἐπὶ τὸ Γ, καὶ ἐπεζεύχθω ἡ ΑΓ. λέγω ὅτι παράλληλός ἐστιν ἡ ΑΓ τῆι ΔΕ. ἐπεὶ γὰρ τὸ ὑπὸ ΖΕΔ ἴσον ἐστὶν τῶι ἀπὸ τῆς ἐφαπτομένης, ἀλλὰ καὶ τὸ ὑπὸ ΑΕΒ ἴσον ἐστὶν τῶι ἀπὸ τῆς ἐφαπτομένης, ἴσον ἄρα ἐστὶν τὸ ὑπὸ ΑΕΒ τῶι ὑπὸ ΖΕΔ. ἐν κύκλωι ἄρα ἐστὶν <τὰ Α, Β, Δ, Ζ σημεῖα. ἴση ἄρα ἐστὶν> ἡ ὑπὸ ΖΑΕ γωνία τῆι ὑπὸ ΒΔΕ γωνίαι. ἀλλὰ καὶ ἡ ὑπὸ ΖΑΕ γωνία ἴση ἐστὶν τῆι ἐν τῶι ἐναλλὰξ τμήματι τῆι ὑπὸ ΑΓΒ. καὶ ἡ ὑπὸ ΑΓΒ ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ ΒΔΕ γωνία. καὶ εἰσὶν ἐναλλάξ. παράλληλος ἄρα ἐστὶν ἡ ΑΓ τῆι ΔΕ.

‖ 1 ι´ add Camer$_1$ (BS) ‖ 3 κλᾶν εὐθεῖαν τὴν] ἂν δοθῆι ἡ Α ἂν κλασθῆι ἡ Co | ἐκβληθείσης] ἐκβληθῆι A ‖ 4 τὴν Camer$_1$ (recc?) τη A | ἡ ΖΑ Co ΖΗ A ‖ 11 ΕΖ Co ΒΖ A ‖ 13 δὴ add Co ‖ 14 τοῦ ΑΒΓ ἐφαπτομένη εὐθεῖα Co τὸ ΑΒΓ ἐφάπτεται πρὸς εὐθεῖαν A | ΖΑ Co ΖΑΝ A ‖ 21 καὶ add Ge ‖ 22 τὸ Camer$_1$ (BCV) τοῦ A ‖ 27 ἀλλὰ — ἐφαπτομένης bis A corr Co ‖ 29 τὰ — ἐστὶν add Co ‖ 34 ΔΕ Co ΔΖ A

(169) 11. *(Prop. 106)* For the seventeenth.

Let there be two circles ABΓ, AEΔ, tangent to each other (internally) at point A, and let straight lines AΔB, AEΓ be drawn through (the circles) from A, and let ΔE, BΓ be joined. That ΔE and BΓ are parallel.

Through A let ZH be drawn tangent.[1] Then angle ZAB equals each of angles AΓB, AEΔ, so that also angle AΓB equals angle AEΔ.[2] Thus ΔE is parallel to BΓ.[3]

<But let ΔE be parallel to BΓ.> That circles ABΓ, AΔE are tangent to each other.

For let ZH be drawn tangent to circle ABΓ.[1] Then angle ZAΔ is equal to angle Γ.[2] But angle Γ equals angle E.[3] Therefore angle ZAΔ too equals angle E.[4] Thus ZH is tangent to circle AΔE;[5] for this was proved before.

(170) 12. *(Prop. 107)* Problem for the same (problem).

With circle ABΓ (given) in position, and two (points) Δ, E given, to inflect a straight line ΔAE, making BΓ parallel to ΔE.

Let it be accomplished, and from B let BZ be drawn tangent. Then since BZ is tangent to, and BΓ cuts (the circle),[1] angle ZBΓ, that is angle ΔZB, equals angle A.[2] Hence points A, B, E, Z are on a circle.[3] Therefore the rectangle contained by AΔ, ΔB equals the rectangle contained by EΔ, ΔZ.[4] But the rectangle contained by AΔ, ΔB is given,[5] because the rectangle contained by BΔ, ΔA equals a given. Hence also the rectangle contained by EΔ, ΔZ is given.[6] And ΔE is given.[7] Therefore also ΔZ is given.[8] But it is also (given) in position;[9] and Δ is given.[10] Hence Z too is given.[11] But from a given point Z, ZB has been drawn tangent to a circle given in position.[12] Therefore ZB is given in position.[13] But also circle ABΓ is (given) in position.[14] Therefore point B is given.[15] But Δ too is given.[16] Hence AΔ is (given) in position.[17] But the circle too is (given) in position.[18] Therefore A is given.[19] But E too is given.[20] Thus each of ΔA, AE is given in position.[21]

(171) *(Prop. 107)* The synthesis of the problem will be made as follows. Let the circle be ABΓ, and the given points Δ, E, and let the rectangle contained by EΔ, ΔZ be made equal to the square of the tangent (from Δ), and from Z let straight line ZB be drawn <tangent> to circle ABΓ, and let ΔB be joined and produced to A, and let AE, BΓ be joined. I

(169) <ια.´> εἰς τὸ ιζ.´

ἔστωσαν δύο κύκλοι οἱ ΑΒΓ, ΑΕΔ ἐφαπτόμενοι ἀλλήλων κατὰ τὸ Α σημεῖον, καὶ διήχθωσαν διὰ τοῦ Α εὐθεῖαι αἱ ΑΔΒ, ΑΕΓ, καὶ ἐπεζεύχθωσαν αἱ ΔΕ, ΒΓ. ὅτι παράλ|ληλοί εἰσιν αἱ ΔΕ, ΒΓ. |153v ἤχθω ἀπὸ τοῦ Α ἐφαπτομένη εὐθεῖα ἡ ΖΗ· ἴση ἄρα ἐστὶν ἡ ὑπὸ ΖΑΒ γωνία ἑκατέραι τῶν ὑπὸ ΑΓΒ, ΑΕΔ· ὥστε καὶ ἡ ὑπὸ ΑΓΒ γωνία ἴση ἐστὶν τῆι ὑπὸ ΑΕΔ· παράλληλος ἄρα ἐστὶν ἡ ΔΕ τῆι ΒΓ.

<ἀλλὰ παράλληλος ἔστω ἡ ΔΕ τῆι ΒΓ.> ὅτι ἐφάπτονται οἱ ΑΒΓ, ΑΔΕ κύκλοι ἀλλήλων. ἤχθω γὰρ τοῦ ΑΒΓ κύκλου ἐφαπτομένη ἡ ΖΗ· ἴση ἄρα ἐστὶν ἡ ὑπὸ ΖΑΔ γωνία τῆι Γ 834 γωνίαι. ἀλλὰ ἡ Γ γωνία ἴση ἐστὶν τῆι Ε. καὶ ἡ ὑπὸ ΖΑΔ ἄρα γωνία ἴση ἐστὶν τῆι Ε γωνίαι· ὥστε ἐφάπτεται ἡ ΖΗ τοῦ ΑΔΕ κύκλου. τοῦτο γὰρ προδέδεικται.

(170) <ιβ.´> πρόβλημα εἰς τὸ αὐτό.

θέσει ὄντος κύκλου τοῦ ΑΒΓ, καὶ δύο δοθέντων τῶν Δ, Ε, κλᾶν εὐθεῖαν τὴν ΔΑΕ, καὶ ποιεῖν παράλληλον τὴν ΒΓ τῆι ΔΕ. γεγονέτω, καὶ ἀπὸ τοῦ Β ἐφαπτομένη ἤχθω ἡ ΒΖ. ἐπεὶ οὖν ἐφάπτεται μὲν ἡ ΒΖ, τέμνει δὲ ἡ ΒΓ, ἴση ἐστὶν ἡ ὑπὸ ΖΒΓ γωνία, τουτέστιν ἡ ὑπὸ ΔΖΒ, τῆι Α. ἐν κύκλωι ἄρα ἐστὶν τὰ Α, Β, Ε, Ζ σημεῖα. ἴσον ἄρα ἐστὶν τὸ ὑπὸ ΑΔΒ τῶι ὑπὸ ΕΔΖ. δοθὲν δὲ τὸ ὑπὸ ΑΔΒ (ἴσον γὰρ τὸ ὑπὸ τῶν ΒΔΑ δοθέντι). δοθὲν ἄρα καὶ τὸ ὑπὸ ΕΔΖ. καὶ ἔστιν δοθεῖσα ἡ ΔΕ. δοθεῖσα ἄρα καὶ ἡ ΔΖ. ἀλλὰ καὶ τῆι θέσει. καὶ ἔστιν δοθὲν τὸ Δ. δοθὲν ἄρα καὶ τὸ Ζ. ἀπὸ δὴ δοθέντος σημείου τοῦ Ζ [τῆι] θέσει [δὲ] δοθέντος κύκλου ἐφαπτομένη ἧκται ἡ ΖΒ· δέδοται ἄρα ἡ ΖΒ τῆι θέσει. ἀλλὰ καὶ ὁ ΑΒΓ κύκλος θέσει. δοθὲν ἄρα ἐστὶν τὸ Β σημεῖον. ἔστιν δὲ καὶ τὸ Δ δοθέν. θέσει ἄρα ἐστὶν ἡ ΑΔ. 836 θέσει δὲ καὶ ὁ κύκλος. δοθὲν ἄρα ἐστὶ τὸ Α. ἔστιν δὲ καὶ τὸ Ε δοθέν. δοθεῖσα ἄρα ἐστὶν ἑκατέρα τῶν ΔΑ, ΑΕ τῆι θέσει.

(171) συντεθήσεται δὲ τὸ πρόβλημα οὕτως. ἔστω ὁ μὲν κύκλος ὁ ΑΒΓ, τὰ δὲ δοθέντα σημεῖα τὰ Δ, Ε, καὶ τῶι ἀπὸ τῆς ἐφαπτομένης ἴσον κείσθω τὸ ὑπὸ ΕΔΖ, καὶ ἀπὸ τοῦ Ζ τοῦ ΑΒΓ κύκλου <ἐφαπτομένη> εὐθεῖα γραμμὴ ἤχθω ἡ ΖΒ, καὶ

‖ 1 ια´ add Camer$_1$ (BS) | ιζ´] ι´δ´ A^1 ζ supr A^2 ‖ 3 διὰ] ἀπὸ A ‖ 9 ἀλλὰ – ΒΓ add Co ‖ 13 ἐφάπτεται] ἐφαπτομένη A ‖ 14 post κύκλου add οἱ ΑΒΓ, ΑΔΕ ἄρα κύκλοι ἐφάπτονται ἀλλήλων κατὰ τὸ Α σημεῖον Hu (Co) ‖ 15 ιβ´ add Camer$_1$ (BS) ‖ 17 κλᾶν Camer$_1$ ΚΛ̄ ἂν A κλάσαι Co | εὐθεῖαν] δοθεῖσαν A del Co ‖ 19 ἡ Camer$_1$ τῆι A ‖ 22 ΑΔΒ Co ΛΑΜΒ A | τὸ ὑπὸ τῶν ΒΔΑ δοθέντι] τὸ ἀπὸ τῆς ΒΖ δοθέντι A τὸ ἀπὸ τῆς ἐφαπτομένης δοθέντι Co τὸ ἀπὸ τῆς ἐφαπτομένης, τουτέστιν δοθέντι Hu app τὸ ἀπὸ τῆς ἐφαπτομένης ἀπὸ τοῦ Δ Camer$_1$ τὸ ἀπὸ τῆς ΒΖ δοθείσης Ge ‖ 25 τῆι ... δὲ secl Hu ‖ 28 ΑΔ] ΒΔ Co ‖ 29 Α Co Λ A ‖ 30 δοθεῖσα Co δοθὲν A ‖ 34 ἐφαπτομένη add Camer$_1$

say that BΓ is parallel to ΔE.

For since the rectangle contained by EΔ, ΔZ equals the square of the tangent (from Δ),[1] <but the rectangle contained by AΔ, ΔB too equals the square of the tangent,>[2] therefore angle A, that is angle ΓBZ, – for BZ is tangent to, and BΓ cuts (the circle)[3] – equals angle BZΔ.[4] And they are alternate angles.[5] Hence BΓ is <parallel> to ΔE.[6]

(172) 13. *(Prop. 108)* Problem for the eighteenth.

Given circle ABΓ in position, and given two points Δ, E, to inflect a straight line AΔE from Δ, making ΔE parallel to BΓ.

Let it be accomplished, and let straight line BZ be drawn from B, tangent to circle ABΓ.[1] Then angle ZBA equals angle Γ, that is, angle E.[2] Hence points B, Z, A, E are on a circle.[3] Therefore the rectangle contained by BΔ, ΔA equals the rectangle contained by ZΔ, ΔE.[4] But the rectangle contained by BΔ, ΔA is given,[5] since AΔB has been drawn from a given point Δ through to a circle given in position. Hence also the rectangle contained by ZΔ, ΔE is given.[6] And ΔE is given.[7] Therefore ZΔ too is given.[8] And Δ is given.[9] <Therefore Z too is given.[10] But from a given point Z,> ZB has been drawn tangent to a circle <given in position.>[11] Hence ZB is (given) in position.[12] But the circle too is (given) in position.[13] Therefore point B is given.[14] But also Δ is given.[15] Hence BΔ is (given) in position.[16] But the circle too is (given) in position.[17] Therefore point A is given.[18] But also each of Δ, E is given.[19] Thus each of ΔA, AE is given in position.[20]

(173) *(Prop. 108)* The synthesis of the problem will be made as follows. Let the circle given in position be ABΓ, the given two points Δ, E, and let an arbitrary (line) AΔB be drawn through, and let the rectangle contained by EΔ, ΔZ be made equal to the rectangle contained by AΔ, ΔB, <and from> Z let BZ be drawn tangent to circle ABΓ, and let ΓEA be joined.

Then since angle ZBA equals angle E,[2] because points A, B, E, Z are on a circle,[1] but also angle ZBA equals angle Γ,[4] because (ZB) is tangent to, and (BA) cuts (the circle),[3] therefore angle Γ too equals angle E.[5] Thus BΓ is parallel to ΔE.[6] Q.E.D.

ἐπεζεύχθω ἡ ΔΒ καὶ ἐκβεβλήσθω ἐπὶ τὸ Α, καὶ ἐπεζεύχθωσαν αἱ ΑΕ, ΒΓ. λέγω ὅτι παράλληλός ἐστιν ἡ ΒΓ τῆι ΔΕ. ἐπεὶ γὰρ τὸ ὑπὸ ΕΔΖ ἴσον ἐστὶν τῶι ἀπὸ τῆς ἐφαπτομένης, <ἀλλὰ καὶ τὸ ὑπὸ ΑΔΒ ἴσον ἐστὶν τῶι ἀπὸ τῆς ἐφαπτομένης,> ἴση ἄρα ἐστὶν ἡ Α γωνία, τουτέστιν ἡ ὑπο ΓΒΖ (ἐφάπτεται γὰρ ἡ ΒΖ, τέμνει δὲ ἡ ΒΓ) τῆι ὑπὸ ΒΖΔ. καί εἰσιν ἐναλλάξ. <παράλληλος> ἄρα ἐστὶν ἡ ΒΓ τῆι ΔΕ.

(172) <ιγʹ> |πρόβλημα εἰς τὸ ιηʹ. |154

θέσει δοθέντος κύκλου τοῦ ΑΒΓ, καὶ δύο δοθέντων σημείων τῶν Δ, Ε, ἀπὸ τῶν Δ, Ε κλᾶν εὐθεῖαν τὴν ΑΔΕ καὶ ποιεῖν τῆι ΔΕ παράλληλον τὴν ΒΓ. γεγονέτω, καὶ ἤχθω ἀπὸ τοῦ Β τοῦ ΑΒΓ 838 κύκλου ἐφαπτομένη εὐθεῖα γραμμὴ ἡ ΒΖ. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΖΒΑ γωνία τῆι Γ, τουτέστιν τῆι Ε. ἐν κύκλωι ἄρα ἐστὶν τὰ Β, Ζ, Α, Ε σημεῖα. τὸ ἄρα ὑπὸ ΒΔΑ ἴσον ἐστὶν τῶι ὑπὸ ΖΔΕ. δοθὲν δὲ τὸ ὑπὸ ΒΔΑ (ἀπὸ γὰρ δοθέντος τοῦ Δ εἰς θέσει δεδομένον κύκλον διῆκται ἡ ΑΔΒ). δοθὲν ἄρα καὶ τὸ ὑπὸ ΖΔΕ. καί ἐστι δοθεῖσα ἡ ΔΕ. δοθεῖσα ἄρα καὶ ἡ ΖΔ. καί ἐστιν δοθὲν τὸ Δ. <δοθὲν ἄρα καὶ τὸ Ζ. ἀπὸ δὴ δεδομένου σημείου τοῦ Ζ θέσει δεδομένου> κύκλου ἐφαπτομένη ἦκται ἡ ΖΒ. θέσει ἄρα ἐστὶν ἡ ΖΒ. θέσει δὲ καὶ ὁ κύκλος. δοθὲν ἄρα ἐστὶν τὸ Β σημεῖον. ἀλλὰ καὶ τὸ Δ δοθέν. θέσει ἄρα ἐστὶν ἡ ΒΔ. θέσει δὲ καὶ ὁ κύκλος. δοθὲν ἄρα ἐστὶν τὸ Α σημεῖον. ἔστιν δὲ καὶ ἑκάτερον τῶν Δ, Ε δοθέν. δοθεῖσα ἄρα ἐστὶν ἑκατέρα τῶν ΔΑ, ΑΕ τῆι θέσει.

(173) συντεθήσεται δὴ τὸ πρόβλημα οὕτως. ἔστω ὁ μὲν τῆι θέσει δεδομένος κύκλος ὁ ΑΒΓ, τὰ δὲ δοθέντα δύο σημεῖα τὰ Δ, Ε, καὶ διήχθω τυχοῦσα ἡ ΑΔΒ, καὶ τῶι ὑπὸ ΑΔΒ ἴσον κείσθω τὸ ὑπὸ ΕΔΖ, <καὶ ἀπὸ> τοῦ Ζ τοῦ ΑΒΓ κύκλου ἐφαπτομένη ἤχθω ἡ ΒΖ, καὶ ἐπεζεύχθω ἡ ΓΕΑ. ἐπεὶ οὖν ἴση ἐστὶν ἡ ὑπὸ ΖΒΑ γωνία τῆι πρὸς τῶι Ε (ἐν κύκλωι γάρ ἐστιν τὰ Α, Β, Ε, Ζ σημεῖα), ἀλλὰ καὶ ἡ ὑπὸ ΖΒΑ ἴση ἐστὶν τῆι Γ (ἐφάπτεται γάρ, καὶ τέμνει), καὶ ἡ Γ ἄρα γωνία ἴση ἐστὶν τῆι Ε. παράλληλος ἄρα ἐστὶν ἡ ΒΓ τῆι ΔΕ. ὅ(περ): —

‖ 1 ΔΒ καὶ Co ΔΒΚ A ‖ 3 ἀλλὰ — ἐφαπτομένης add Co, post quae add ἴσον ἄρα ἐστὶ τὸ ὑπὸ ΑΔΒ τῶι ὑπὸ ΕΔΖ. ἐν κύκλωι ἄρα ἐστὶν τὰ Α, Β, Ζ, Ε σημεῖα Co. pro haec add τουτέστιν τῶι ὑπὸ ΑΔΒ. ἐν κύκλωι ἄρα ἐστὶν τὰ Α, Β, Ζ, Ε σημεῖα Hu ‖ 6 τῆι Camer₁ τὴν A ‖ 7 παράλληλος add Co ‖ 8 ιγʹ add Camer₁ (BS) ‖ 10 κλᾶν Camer₁ ΚΛ ἂν A κλάσαι Co | εὐθεῖαν] δοθ(εῖσαν) (compendium) A del Co | ΔΑΕ Co ΑΛΕ A | τῆι ΔΕ... τὴν ΒΓ Hu τὴν ΔΕ... τῆι ΒΓ A τὴν ΒΓ... τῆι ΔΕ Co ‖ 13 ΖΒΑ Hu ΖΒΔ A ‖ 15 Δ Camer₁ Α A | δεδομένον κύκλον Camer₁ δεδομένην γωνίαν A ‖ 18 δοθὲν — δεδομένου add Co ‖ 21 ΒΔ Co ΒΑ A ‖ 23 ἑκάτερον... δοθέν. δοθεῖσα Co ἑκατέρα... δοθέντων δοθὲν A ‖ 28 καὶ ἀπὸ τοῦ Ζ Hu τουτέστιν A ‖ 29 ΖΒΑ Hu ΖΒΔ A ‖ 31 ΖΒΑ Hu ΖΒΔ A

(174) 14. *(Prop. 109)* Problem for the nineteenth.

With circle ABΓ (given) in position, and two (points) Δ, E given, to inflect a straight line ΔAE so that BΓ is parallel to ΔE.

Let it be accomplished, and let BZ be drawn tangent.[1] Then again points A, Z, B, E are on a circle,[2] and the rectangle contained by AΔ, ΔB equals the rectangle contained by EΔ, ΔZ.[3] But the rectangle contained by AΔ, ΔB is given.[4] Therefore the rectangle contained by EΔ, ΔZ is also given.[5] And ΔE is given.[6] Therefore ΔZ too is given.[7] But it is also (given) in position.[8] And Δ is given.[9] Therefore Z is also given,[10] and hence BZ is (given) in position.[11] But the circle too (is given in position).[12] Hence B is given.[13] But also Δ, E (are given).[14] Therefore each of ΔA, AE (is given).[15] For we shall prove it just as for the foregoing (lemmas); and the synthesis similarly to the one before.

(175) 15. *(Prop. 110)* For the twenty-fourth.

Let two circles AB, BΓ be tangent to each other at point B, and let their centers Δ, E be taken, and let AΔ, ΔB, ΓE, EB be joined. Let AΔ be parallel to ΓE. That the lines through Δ, B, E and through A, B, Γ are straight.

For let straight line ZH be drawn tangent to circles AB, BΓ.[1] Then since ZH is tangent, and ΔB from the center, therefore angle ΔBZ is right.[2] For the same reasons angle ZBE too is right.[2] Hence the line through Δ, B, E is straight.[3] But since AΔ equals ΔB,[4] and EΓ equals EB,[5] as is AΔ to ΔB, so is EΓ to EB.[6] And the sides around equal angles Δ, E are in ratio,[7] and so angle ΔBA equals angle ΓBE.[8] And ΔBE is a straight line.[9] Thus the line through A, B, Γ is straight.[10] Q.E.D.

(176) 16. *(Prop. 111)* With AB being equal to BΓ, and AΔ to ΔE, and ΔE being parallel to BΓ, to prove that the line through points A, E, Γ is straight.

Let AE, EΓ be joined, and let BZ be drawn parallel to AE,[1] and let EΔ be produced to Z. Then ΔZ equals ΔB.[2] But also AΔ equals ΔE.[3] Hence all AB equals all ZE.[4] But AB equals BΓ.[5] Therefore BΓ equals ZE.[6] But it is also parallel (to it).[7] Hence ΓE is (parallel) to BZ.[8] But also AE is parallel to BZ.[9] Therefore the <line through> A, E, Γ is straight;[10] for this is obvious.

(174) <ιδ´> πρόβλημα εἰς τὸ ιθ´. 840

θέσει ὄντος τοῦ ΑΒΓ κύκλου, <καὶ> δύο δοθέντων τῶν Δ, Ε, κλᾶν εὐθεῖαν τὴν ΔΑΕ ὥστε παράλληλον εἶναι τὴν ΒΓ τῆι ΔΕ. γεγονέτω, καὶ ἤχθω ἐφαπτομένη ἡ ΒΖ. γίνεται οὖν πάλιν ἐν κύκλωι τὰ Α, Ζ, Β, Ε σημεῖα, καὶ ἴσον τὸ ὑπὸ ΑΔΒ τῶι ὑπὸ ΕΔΖ. δοθὲν δὲ τὸ ὑπὸ ΑΔΒ. δοθὲν ἄρα καὶ τὸ ὑπὸ ΕΔΖ. καὶ ἔστιν δοθεῖσα ἡ ΔΕ. δοθεῖσα ἄρα καὶ ἡ ΔΖ. ἀλλὰ καὶ τῆι θέσει. καὶ ἔστιν δοθὲν τὸ Δ. δοθὲν ἄρα καὶ τὸ Ζ. ὥστε θέσει ἡ ΒΖ. ἀλλὰ καὶ ὁ κύκλος. δοθὲν ἄρα ἐστὶν τὸ Β. ἀλλὰ καὶ τὰ Δ, Ε. δοθεῖσα ἄρα |ἐστὶν ἑκατέρα τῶν ΔΑ, ΑΕ. ὁμοίως γὰρ τοῖς πρότερον δείξομεν, καὶ ὁμοίως ἡ σύνθεσις τῶι πρὸ αὐτοῦ. |154v

(175) <ιε´> εἰς τὸ κδ´.

ἀπτέσθωσαν δύο κύκλοι ἀλλήλων οἱ ΑΒ, ΒΓ κατὰ τὸ Β σημεῖον, καὶ εἰλήφθω τὰ κέντρα αὐτῶν τὰ Δ, Ε, καὶ ἐπεζεύχθωσαν αἱ ΑΔ, ΔΒ, ΓΕ, ΕΒ. ἔστω δὲ παράλληλος ἡ ΑΔ τῆι ΓΕ. ὅτι εὐθεῖαί εἰσιν αἱ διὰ τῶν Δ Β Ε, Α Β Γ. ἤχθω γὰρ τῶν ΑΒ, ΒΓ κύκλων ἐφαπτομένη εὐθεῖα ἡ ΖΗ. ἐπεὶ οὖν ἐφάπτεται μὲν ἡ ΖΗ, ἐκ δὲ τοῦ κέντρου ἐστὶν ἡ ΔΒ, ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ τῶν ΔΒΖ γωνία. διὰ ταὐτὰ καὶ ἡ ὑπὸ ΖΒΕ γωνία ἐστὶν ὀρθή. εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Δ, Β, Ε. ἐπεὶ δὲ ἴση ἐστὶν ἡ μὲν
ΑΔ τῆι ΔΒ, ἡ δὲ ΕΓ τῆι ΕΒ, ἔστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΒ, οὕτως ἡ 842
ΕΓ πρὸς τὴν ΕΒ. καὶ περὶ ἴσας γωνίας τὰς Δ, Ε αἱ πλευραὶ ἀνάλογόν εἰσιν, ἴση ἄρα ἐστὶν ἡ ὑπὸ τῶν ΔΒΑ γωνία τῆι ὑπὸ ΓΒΕ. καὶ ἔστιν εὐθεῖα ἡ ΔΒΕ. εὐθεῖα ἄρα ἐστὶν καὶ ἡ διὰ τῶν Α, Β, Γ. ὅπερ:—

(176) <ις´> εἰς τὸ κε´.

ἴσης οὔσης τῆς μὲν ΑΒ τῆι ΒΓ, τῆς δὲ ΑΔ τῆι ΔΕ, καὶ παραλλήλου οὔσης τῆς ΔΕ τῆι ΒΓ, δεῖξαι ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Ε, Γ σημείων. ἐπεζεύχθωσαν αἱ ΑΕ, ΕΓ, καὶ τῆι ΑΕ παράλληλος ἤχθω ἡ ΒΖ, καὶ ἐκβεβλήσθω ἡ ΕΔ ἐπὶ τὸ Ζ. ἴση ἄρα ἐστὶν ἡ ΔΖ τῆι ΔΒ. ἔστι δὲ καὶ ἡ ΑΔ τῆι ΔΕ. ὅλη ἄρα ἡ ΑΒ ὅληι τῆι ΖΕ ἐστὶν ἴση. ἀλλὰ ἡ ΑΒ τῆι ΒΓ ἴση ἐστίν. καὶ ἡ ΒΓ ἄρα τῆι ΖΕ ἐστὶν ἴση. ἀλλὰ καὶ παράλληλος. καὶ ἡ ΓΕ ἄρα τῆι ΒΖ. ἀλλὰ καὶ ἡ ΑΕ τῆι ΒΖ παράλληλός ἐστιν. εὐθεῖα ἄρα ἐστὶν <ἡ διὰ τῶν> Α, Ε, Γ. τοῦτο γὰρ φανερόν.

‖ 1 ιδ´ add Camer₁ (BS) ‖ 2 ὄντος] δοθέντος Camer₁ (Co) | καὶ add Camer₁ ‖ 3 κλᾶν Camer₁ ΚΛ̄ ἂν A | εὐθεῖαν] δοθέντων A ἀπ' αὐτῶν Hu ἀπὸ τῶν Δ, Ε Hu app ἀπὸ τῶν δοθέντων Camer₁ ‖ 12 ιε´ add Camer₁ (BS) ‖ 15 ΕΒ Co ΕΒΑ A ‖ 16 ἤχθω Camer₁ (S) ἤχθωσαν A ‖ 17 εὐθεῖα Ge (recc?) εὐθεῖαι A om Camer₁ | ΖΒΗ] ΖΗΝ A ΖΗ Camer₁ ‖ 18 ΔΒ Co ΑΒ A ‖ 22 γωνίας bis A corr Co ‖ 26 ις´ add Camer₁ (V) ‖ 28 ΔΕ Co ΖΕ A ‖ 31 ὅλη] ἴση A ἴση ὅλη Camer₁ (S) ‖ 35 ἡ διὰ τῶν] ἡ add Camer₁ (BS), reliqua supplevi

(177) 17. *(Prop. 112)* For the twenty-first.

If there is a circle ABΓ, and two equal (lines) BΔ, ΔΓ are drawn to it, and BΔ is tangent, that also ΔΓ is tangent.

This is obvious. For if ΔA is drawn through, the rectangle contained by AΔ, ΔE equals the square of ΔB.[1] But the square of ΔB equals the square of ΔΓ.[2] Hence the rectangle contained by AΔ, ΔE equals the square of ΔΓ.[3] Thus ΔΓ is tangent to circle ABΓ.[4]

(178) 18. *(Prop. 113)* (Let there be) two circles AB, BΓ, and through B let some (straight line) ABΓ be drawn, and two parallel (lines) AΔ, EΓ, pointing toward the centers of the circles. That circles AB, BΓ are tangent to each other at point B.

Let the centers of the circles Δ, E be taken, and let ΔB, BE be joined. Then the line through Δ, B, E is straight. For AΔ is parallel to ΓE,[1] and as is AΔ to ΔB, so is ΓE to EB;[2] and there result two triangles that have one angle equal to one angle, A to Γ, and having the sides around other angles Δ, E in ratio. Hence the triangles are equiangular,[3] and so angle ABΔ equals ΓBE.[4] And line ABΓ is straight;[5] therefore line ΔBE too is straight.[6] But since the line through the centers and the point of tangency is straight, therefore circles AB, BΓ are tangent to each other at point B.[7]

(179) 19. *(Prop. 114)* For the fifty-second.

Let AB be parallel to ΓΔ, and AΓ equal to BΔ, with angle AΓΔ obtuse, angle BΔΓ acute. That AΔ is a parallelogram.

For since angle AΓΔ is obtuse, while angle BΔΓ is acute, the perpendiculars drawn from A, B to ΓΔ <fall>, that from A outside Γ, that from B inside Δ. Let them be dropped, and let them be AE, BZ.[1] Then AE is parallel to BZ.[2] But AB is also parallel to ΓΔ.[3] And the angles at points E, Z are right.[4] Therefore <ZΔ> equals <EΓ,[6] because also> BΔ <equals> AΓ.[5] Hence also all EZ equals ΓΔ.[7] And thus AB equals ΓΔ.[8]

(177) <ιζ.´> εἰς τὸ λα.´
ἐὰν ᾖι κύκλος ὁ ΑΒΓ, καὶ δύο προσβληθῶσιν αἱ ΒΔ, ΔΓ, ἴσαι οὖσαι, ἡ δὲ ΒΔ ἐφάπτηται, ὅτι καὶ ἡ ΔΓ ἐφάπτεται. τοῦτο δὲ φανερόν. ἂν γὰρ διαχθῆι ἡ ΔΑ, τὸ ὑπὸ ΑΔΕ ἴσον ἐστὶν τῶι ἀπὸ ΔΒ. ἀλλὰ τὸ ἀπὸ ΔΒ τῶι ἀπὸ ΔΓ ἴσον ἐστίν. καὶ τὸ ὑπὸ τῶν ΑΔΕ ἄρα ἴσον ἐστὶν τῶι ἀπὸ ΔΓ. ἐφάπτεται ἄρα ἡ ΔΓ τοῦ ΑΒΓ 844
κύκλου.

(178) <ιη.´> |δύο κύκλοι οἱ ΑΒ, ΒΓ, καὶ διὰ τοῦ Β διήχθω |155
τις ἡ ΑΒΓ, καὶ δύο παράλληλοι αἱ ΑΔ, ΕΓ, νεύουσαι ἐπὶ τὰ κέντρα τῶν κύκλων. ὅτι οἱ ΑΒ, ΒΓ κύκλοι ἐφάπτονται ἀλλήλων κατὰ τὸ Β σημεῖον. εἰλήφθω τὰ κέντρα τῶν κύκλων τὰ Δ, Ε, καὶ ἐπεζεύχθωσαν αἱ ΔΒ, ΒΕ. εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Δ, Β, Ε· παράλληλος γάρ ἐστιν ἡ ΑΔ τῆι ΓΕ. καὶ ἔστιν ὡς ἡ ΑΔ πρὸς ΔΒ, οὕτως ἡ ΓΕ πρὸς ΕΒ. καὶ γίνεται δύο τρίγωνα μίαν γωνίαν μίαι γωνίαι ἴσην ἔχοντα τὴν Α τῆι Γ, περὶ δὲ ἄλλας γωνίας τὰς Δ, Ε τὰς πλευρὰς ἀνάλογον. ἰσογώνια ἄρα ἐστὶν τὰ τρίγωνα. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΑΒΔ γωνία τῆι ὑπὸ ΓΒΕ. καὶ ἔστιν εὐθεῖα ἡ ΑΒΓ. εὐθεῖα ἄρα ἐστὶν καὶ ἡ ΔΒΕ. ἐπεὶ δὲ εὐθεῖά ἐστιν ἡ διὰ τῶν κέντρων καὶ τῆς ἁφῆς, ἐφάπτονται ἄρα οἱ ΑΒ, ΒΓ κύκλοι ἀλλήλων κατὰ τὸ Β σημεῖον.

(179) <ιθ.´> εἰς τὸ νβ.´
ἔστω ἡ μὲν ΑΒ τῆι ΓΔ παράλληλος, ἴση δὲ <ἡ> ΑΓ τῆι ΒΔ, οὔσης ἀμβλείας μὲν τῆς ὑπὸ τῶν ΑΓΔ, ὀξείας δὲ τῆς ὑπὸ ΒΔΓ. ὅτι παραλληλόγραμμόν ἐστιν τὸ ΑΔ. ἐπεὶ γὰρ ἀμβλεῖα μέν ἐστιν ἡ ὑπὸ ΑΓΔ, ὀξεῖα δὲ ἡ ὑπὸ ΒΔΓ, αἱ ἀπὸ τῶν Α, Β ἐπὶ τὴν ΓΔ κάθετοι ἀγόμεναι, ἡ μὲν ἀπὸ τοῦ Α ἐκτὸς τοῦ Γ, ἡ δὲ ἀπὸ τοῦ Β ἐντὸς τοῦ Δ <πίπτουσιν>. πιπτέτωσαν, καὶ ἔστωσαν αἱ ΑΕ, ΒΖ. παράλληλος ἄρα ἐστὶν ἡ ΑΕ τῆι ΒΖ. ἔστιν δὲ καὶ ἡ ΑΒ 846
τῆι ΓΔ παράλληλος. καὶ εἰσὶν ὀρθαὶ <αἱ> πρὸς τοῖς Ε, Ζ σημείοις γωνίαι. ἴση ἄρα ἐστὶν <ἡ ΖΔ τῆι ΕΓ (ἴση γάρ ἐστιν> καὶ ἡ ΒΔ τῆι ΑΓ). ὥστε καὶ ὅλη ἡ ΕΖ τῆι ΓΔ ἐστὶν ἴση. καὶ ἡ ΑΒ ἄρα τῆι ΓΔ ἐστὶν ἴση.

‖ 1 titulum ἐπαφῶν δεύτερον add Hu | ιζ´ add Camer₁ (BS) ‖ 2 προσβληθῶσιν Hu (index, s.v. προσβάλλω) προβληθῶσιν A ‖ 8 ιη´ add Camer₁ (BS) ‖ 13 γάρ Co ἄρα A ‖ 14 ΔΒ Co ΑΒ A ‖ 21 ιθ´ add Camer₁ (BS) ‖ 22 ἡ add Hu (S) ‖ 27 πίπτουσιν Hu (Co), pro πιπτέτωσαν ‖ 29 αἱ add Camer₁ (S) ‖ 30 ἴση – ΑΓ] ἴση ἄρα ἐστὶν καὶ ἡ ΖΔ τῆι ΕΓ Co

(180) 20. *(Prop. 115)* (Let there be) two equal circles AB, ΓΔ, and through the centers (let there be) AΔ, and EZ parallel to ΓΔ. I say that if produced, it cuts circle AB too.

Let the centers H, Θ of the circles be taken, and from points H, Θ let HK, <ΘΛ> be drawn at right angles to AΔ.[1] <And let KΛ be joined. Then HK equals> ΘΛ.[2] But it is also parallel.[3] Hence KΛ too is equal and parallel to HΘ.[4] Therefore angles K. Λ are right.[5] And HK, ΘΛ are from the centers.[6] Hence KΛ is tangent to the circles.[7] Accordingly it is obvious that the (line) tangent to ΓΔ is tangent also to AB.[8] Therefore the (line) cutting ΓΔ, namely EZ, also cuts AB when produced,[9] and will be between B, Λ, as EZ is between Γ, K. [EZ greater]

(181) 21. *(Prop. 116)* Let ΔA be equal to AE, and BΔ greater than ΓE, and let ΔE be joined. That ΔE produced intersects BΓ.

Let ΔZ be made equal to ΓE,[1] and let ΓZ be joined. Then it is parallel to ΔE;[2] and it intersects BΓ.[3] Therefore ΔE too intersects BΓ.[4]

(182) 22. *(Prop. 117)* Problem for the same.

With circle ABΓ (given) in position, and three given points Δ, E, Z on a straight line, to inflect a straight line ΔAE, making BΓ in a straight line with ΓZ.

Let it be accomplished, and through B let BH be drawn parallel to ΔZ,[1] and let HΓ be joined and produced to Θ. Then angle BHΓ, that is angle A, equals angle ΓΘZ.[2] Hence the rectangle contained by AE, EΓ equals the rectangle contained by ΔE, EΘ.[3] But the rectangle contained by AE, EΓ is given,[4] since it equals the square of the tangent from E. Therefore also the rectangle contained by ΔE, EΘ is given.[5] And ΔE is given.[6] Hence EΘ too is given.[7] But it is also (given) in position;[8] and E is given.[9] Hence Θ is given too.[10] But Z is also given.[11] My problem has become to make an inflection from two given points Θ, Z, making BH parallel to ΘEZ; but this was written above (lemmas 7.167, .170, .172). Hence Γ is given.[12] But E too is given.[13] Therefore ΓE is (given) in position.[14] But the circle too is given.[15] Hence A is given.[16] But Δ is also given.[17] Thus ΔA too is (given) in position.[18] Q.E.D.

(180) <κ.´> δύο ἴσοι κύκλοι οἱ ΑΒ, ΓΔ, καὶ διὰ τῶν κέντρων ἡ ΑΔ, καὶ τῆι ΓΔ παράλληλος ἡ ΕΖ. λέγω ὅτι ἐκβληθεῖσα τέμνει καὶ τὸν ΑΒ κύκλον. εἰλήφθω τὰ κέντρα τῶν κύκλων τὰ Η, Θ, καὶ ἀπὸ τῶν Η, Θ σημείων τῆι ΑΔ ὀρθαὶ ἤχθωσαν αἱ ΗΚ, <ΘΛ. καὶ ἐπεζεύχθω ἡ ΚΛ. ἴση ἄρα ἐστὶν ἡ ΗΚ> τῆι ΘΛ. ἀλλὰ καὶ παράλληλος. καὶ ἡ ΚΛ <ἄρα> τῆι ΗΘ ἴση ἐστὶν καὶ παράλληλος. ὥστε |ὀρθαί εἰσιν αἱ πρὸς τοῖς Κ, Λ γωνίαι. καὶ |155v εἰσὶν ἐκ τῶν κέντρων αἱ ΗΚ, ΘΛ. ἡ ΚΛ ἄρα ἐφάπτεται τῶν κύκλων. φανερὸν οὖν ὅτι ἡ τοῦ ΓΔ ἐφαπτομένη καὶ τοῦ ΑΒ ἐφάπτεται. ἡ ἄρα τὸν ΓΔ τέμνουσα ἡ ΕΖ καὶ τὸν ΑΒ τέμνει ἐκβληθεῖσα [δὲ], καὶ μεταξὺ τῶν Β, Λ ἔσται, ὡς ἡ ΕΖ τῶν Γ, Κ ἐστὶν μεταξύ. [ἡ ΕΖ μείζων.]

(181) <κα.´> ἔστω ἴση ἡ μὲν ΔΑ τῆι ΑΕ, μείζων δὲ ἡ ΒΔ τῆς ΓΕ, καὶ ἐπεζεύχθω ἡ ΔΕ. ὅτι ἐκβληθεῖσα ἡ ΔΕ συμπίπτει τῆι ΒΓ. κείσθω τῆι ΓΕ ἴση ἡ ΔΖ, καὶ ἐπεζεύχθω ἡ ΓΖ. παράλληλος ἄρα ἐστὶν τῆι ΔΕ, καὶ συμπίπτει τῆι ΒΓ. καὶ ἡ ΔΕ ἄρα 848 συμπίπτει τῆι ΒΓ.

(182) <κβ.´> πρόβλημα εἰς τὸ αὐτό.

θέσει ὄντος κύκλου τοῦ ΑΒΓ, <καὶ> τριῶν δοθέντων σημείων τῶν Δ, Ε, Ζ ἐπ' εὐθείας, κλᾶν εὐθεῖαν τὴν ΔΑΕ, καὶ ποιεῖν ἐπ' εὐθείας τὴν ΒΓ τῆι ΓΖ. γεγονέτω, καὶ διὰ τοῦ Β τῆι ΔΖ παράλληλος ἤχθω ἡ ΒΗ, καὶ ἐπεζευχθεῖσα ἡ ΗΓ ἐκβεβλήσθω ἐπὶ τὸ Θ. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΒΗΓ γωνία, τουτέστιν ἡ Α, τῆι ὑπὸ ΓΘΖ γωνίαι. τὸ ἄρα ὑπὸ ΑΕΓ ἴσον ἐστὶν τῶι ὑπὸ ΔΕΘ. δοθὲν δὲ τὸ ὑπὸ ΑΕΓ (ἴσον γὰρ τῶι ἀπὸ τῆς ἀπὸ τοῦ Ε ἐφαπτομένης). δοθὲν ἄρα καὶ τὸ ὑπὸ τῶν ΔΕΘ. καὶ ἔστιν δοθεῖσα ἡ ΔΕ. δοθεῖσα ἄρα καὶ ἡ ΕΘ. ἀλλὰ καὶ τῆι θέσει. καὶ ἔστιν δοθὲν τὸ Ε. δοθὲν ἄρα καὶ τὸ Θ. ἔστιν δὲ καὶ τὸ Ζ δοθέν. γέγονεν δή μοι ἀπὸ δύο δοθέντων τῶν Θ, Ζ κλᾶν καὶ ποιεῖν παράλληλον τὴν ΒΗ τῆι ΘΕΖ. τοῦτο δὲ προγέγραπται. δοθὲν ἄρα τὸ Γ. ἀλλὰ καὶ τὸ Ε δοθέν. θέσει ἄρα ἡ ΓΕ. ἀλλὰ καὶ ὁ κύκλος δοθείς. δοθὲν ἄρα τὸ Α. ἔστιν δὲ καὶ τὸ Δ δοθέν. θέσει ἄρα καὶ ἡ ΔΑ. ὅπερ:—

‖ 1 κ´ add Camer₁ (BS) | ΓΔ Hu ΒΓ A ‖ 5 ΘΛ add Co καὶ ἐπεζεύχθω ἡ ΚΛ add Camer₁ ἴση — ΗΚ add Co ‖ 6 ἄρα add Co ‖ 9 ΓΔ ἐφαπτομένη Co ΔΕ ἐφάπτεται A | ἐφάπτεται καὶ τοῦ ΑΒ bis A corr Co ‖ 11 δὲ] ἐπεὶ Hu ‖ 12 ἡ ΕΖ μείζων del Co ‖ 13 κα´ add Camer₁ (BS) ‖ 18 κβ´ add Camer₁ (BS) ‖ 19 ὄντος] δοθέντος Camer₁ | καὶ add Co ‖ 20 κλᾶν Camer₁ καν A κλάσαι Co | εὐθεῖαν] δοθεῖσαν A del Co ‖ 22 ἐπεζευχθεῖσα ἡ ΗΓ Hu app ἐπεζεύχθω ἡ ΗΓ A post quae add καὶ Ge ‖ 24 post γωνίαι add ἐν κύκλωι ἄρα ἐστὶ τὰ Α, Γ, Β, Δ σημεῖα Co ‖ 27 ΕΘ Co ΖΘ A ‖ 30 κλᾶν Camer₁ ΚΛᾶν A κλάσαι Co | ante καὶ ποιεῖν add τὴν ΘΓΖ Co | ΘΕΖ Co ΘΚΖ A ΘΖ Camer₁

(183) *(Prop. 117)* The synthesis of the problem will be made as follows. Let the circle ABΓ be given, and the three points given on a straight line Δ, E, Z, and let the rectangle contained by ΔE, EΘ be made equal to the square of the tangent (from E). And from two given points Θ, Z, let a straight line make an inflection on the circle so that BH is parallel to ΘZ. I say that the line through A, B, Δ is straight.

For since each of the rectangles contained by AE, EΓ and by ΔE, EΘ equals the square of the tangent from E, the rectangle contained by AE, EΓ equals the rectangle contained by ΔE, EΘ.[1] Hence points Δ, Θ, Γ, A are on a circle.[2] And since angle BHΓ equals ΓΘZ,[3] but angle BHΓ equals angle BAΓ in the circle,[4] therefore angle BAΓ equals angle ΓΘE.[5] And points A, Γ, Δ, Θ are on a circle.[6] Hence AB is in a straight line with BΔ.[7] Q.E.D.

The circumstances of the cases remain the same; for they refer to the circumstances of the cases for the (problem) to which this (problem) refers.

(184) 23. *(Prop. 118)* Let there be two circles AB, ΓΔ, and let AΔ be produced, and let the radius of circle AB be made to the radius of circle ΓΔ as is EH to HZ. That the (line) drawn through from H and cutting circle ΓΔ, when produced, also cuts (circle) AB.

For let the centers of the circles, points E, Z, be taken, and from H let HΘ be drawn tangent to circle ΓΔ,[1] and let <ZΘ> be joined. And let EK be drawn parallel to ZΘ.[2] Then since as is EH to HZ, so is EK to ZΘ,[3] therefore the line through H, Θ, K is straight.[4] And angle Θ is right.[5] Hence angle K too is right.[6] Hence if the (line) from H is tangent to (circle) ΓΔ, produced it will also be tangent to (circle) AB.[7] But the (lines) that cut (circle) ΓΔ are between Δ, Θ.[8] Hence produced they will be between K, B.[9] And HK is tangent.[10] Therefore the (line) between B, K and Δ, Θ will cut (circle ΓΔ).[11] But the same (line) also cuts (circle) AB.[12] Hence the (line) drawn from point H that cuts (circle) ΓΔ, also cuts (circle) AB.[13]

The first (book) of *Tangencies* <has> seven problems; the second, four problems.

(183) συντεθήσεται δὴ τὸ πρόβλημα οὕτως. ἔστω ὁ μὲν κύκλος ὁ ΑΒΓ δοθείς, τὰ δὲ δοθέντα ἐπ' εὐθείας τρία σημεῖα τὰ Δ, Ε, Ζ, καὶ τῶι ἀπὸ τῆς ἐφαπτομένης ἴσον κείσθω τὸ ὑπὸ 850 ΔΕΘ. καὶ δύο δοθέντων σημείων τῶν Θ, Ζ, εἰς τὸν κύκλον ἀπὸ τῶν Θ, Ζ κεκλάσθω εὐθεῖα ὥστε παράλληλον εἶναι τὴν ΒΗ τῆι ΘΖ. λέγω ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Β, Δ. ἐπεὶ γὰρ ἑκάτερον τῶν ὑπὸ ΑΕΓ, ΔΕΘ ἴσον ἐστὶν τῶι ἀπὸ τῆς ἀπὸ τοῦ Ε ἐφαπτομένης, ἴσον ἐστὶν τὸ ὑπὸ ΑΕΓ τῶι ὑπὸ ΔΕΘ. ἐν κύκλωι ἄρα ἐστὶν τὰ Δ, Θ, Γ, Α σημεῖα. καὶ ἐπεὶ ἴση ἐστὶν ἡ ὑπὸ ΒΗΓ γωνία |τῆι ὑπὸ ΓΘΖ, ἀλλὰ ἡ ὑπὸ ΒΗΓ ἴση ἐστὶν τῆι ὑπὸ ΒΑΓ ἐν κύκλωι [ἀλ], ἡ ὑπὸ ΒΑΓ ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ ΓΘΕ |156 γωνίαι. καὶ ἔστιν ἐν κύκλωι τὰ Α, Γ, Δ, Θ σημεῖα. ἐπ' εὐθείας ἄρα ἐστὶν ἡ ΑΒ τῆι ΒΔ. ὅπερ: —

μένει δ' αὐτοῦ καὶ τὰ πτωτικά. ἀπάγεται γὰρ εἰς τὰ πτωτικὰ τοῦ εἰς <ὃ> τοῦτο ἀπάγεται.

(184) <κγ.´> ἔστωσαν δύο κύκλοι οἱ ΑΒ, ΓΔ, καὶ ἐκβεβλήσθω ἡ ΑΔ, καὶ πεποιήσθω ὡς ἡ ΕΗ πρὸς τὴν ΗΖ, οὕτως ἡ ἐκ τοῦ κέντρου τοῦ ΑΒ κύκλου πρὸς τὴν ἐκ <τοῦ> κέντρου τοῦ ΓΔ κύκλου. ὅτι ἡ ἀπὸ τοῦ Η διαγομένη τέμνουσα τὸν ΓΔ κύκλον ἐκβληθεῖσα καὶ τὸν ΑΒ τέμνει. εἰλήφθω γὰρ τὰ κέντρα τῶν κύκλων τὰ Ε, Ζ σημεῖα, καὶ ἀπὸ τοῦ Η τοῦ ΓΔ κύκλου 852 ἐφαπτομένη ἤχθω ἡ ΗΘ, καὶ ἐπεζεύχθω ἡ <ΖΘ. καὶ τῆι> ΖΘ παράλληλος ἤχθω ἡ ΕΚ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΕΗ πρὸς τὴν ΗΖ, οὕτως ἡ ΕΚ πρὸς τὴν ΖΘ, εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Η, Θ, Κ. καὶ ἔστιν ὀρθὴ ἡ Θ γωνία. ὀρθὴ ἄρα καὶ ἡ Κ γωνία. ὥστε εἰ τοῦ ΓΔ ἐφάπτεται ἡ ἀπὸ τοῦ Η, ἐκβληθεῖσα καὶ τοῦ ΑΒ ἐφάψεται. ἀλλὰ αἱ τέμνουσαι τὸν ΓΔ μεταξὺ τῶν Δ, Θ εἰσίν. ἐκβαλλόμεναι ἄρα μεταξὺ τῶν Κ, Β ἔσονται. καὶ ἔστιν ἐφαπτομένη ἡ ΗΚ. τέμνει ἄρα ἡ μεταξὺ τῶν Β Κ, Δ Θ. ἀλλὰ ἡ αὐτὴ καὶ τὸν ΑΒ τέμνει. ἡ ἄρα τὸν ΓΔ τέμνουσα καὶ τὸν ΑΒ τέμνει ἀγομένη ἀπὸ τοῦ Η σημείου.

τὸ πρῶτον τῶν ἐπαφῶν <ἔχει> προβλήματα ἑπτά, τὸ δεύτερον προβλήματα δ.

‖ 2 ΑΒΓ δοθείς] ΑΒΓΔ Α ΑΒΓ Co ‖ 5 εὐθεῖα] ἡ ΘΓΖ Co ‖ 6 ante λέγω add καὶ ἐπεζεύχθω ἡ ΕΓ καὶ ἐκβεβλήσθω ἐπὶ τὸ Α Co ‖ 8 τὸ A² ex τῶι ‖ 11 ἀλ in fine vers. A del Camer₁ τμήματι Hu app | ΓΘΕ] ΓΘΖ Hu ‖ 15 τοῦ — ἀπάγεται] τοῦ ἑπτακαιδεκάτου Hu τοῦ εἰς τὸ ις´ Camer₁ ‖ 16 κγ´ add Camer₁ (BS) | ἔστωσαν Camer₁ (BS) ἔστω A ‖ 17 ΗΖ Co ΗΔ A ‖ 22 ΗΘ] ΗΖ A¹ Θ add supr A² | ΖΘ καὶ τῆι add Hu ‖ 25 ἡ Θ Camer₁ (Co) ΗΘ A ‖ 27 αἱ Hu καὶ A ‖ 29 Δ Co Ζ A ‖ 30 ΑΒ] ΓΔ Hu ‖ 32 ἔχει add Hu

(185) Plane Loci, (Books) 1, 2.

1. *(Prop. 119)* For the first locus of the second (book).

(Let there be) triangle ABΓ, and let straight line AΔ be drawn, and, as is BΔ to ΔΓ, so let the square of BA be to the square of AΓ. That the rectangle contained by BΔ, ΔΓ equals the square of AΔ.

Through Γ draw ΓE parallel to AB.[1] Then as is BΔ to ΔΓ, so is AB to ΓE, and the square of AB to the rectangle contained by AB, ΓE.[2] But as is BΔ to ΔΓ, so was the square of BA to the square of AΓ.[3] Hence the rectangle contained by BA, ΓE equals the square of ΓA.[4] Therefore the (sides) around equal alternate angles are in ratio. Hence angle ΓAΔ equals angle B.[5] Thus the rectangle contained by BΔ, ΔΓ equals the square of ΔA.[6] The converse is obvious.

(186) 2. *(Prop. 120)* For the second locus.

(Let there be) triangle ABΓ, and ΔA a perpendicular. That the excess of the squares of BA, AΓ equals the excess of the squares of BΔ, ΔΓ, and if BΓ is bisected at E, the <excess> of the squares of BΔ, <ΔΓ> is twice the rectangle contained by BΓ, EΔ.

Now it is obvious that the excess of the squares of BA, AΓ is equal to the excess of the squares of ΔΓ, ΔΓ. For the square of AB equals the squares of BΔ, <AΔ>,[1] while the square of AΓ equals the squares of AΔ, ΔΓ.[2] Hence the amount by which the square of AB exceeds the square of AΓ is the amount by which the squares of AΔ, ΔB exceed the squares of AΔ, ΔΓ.[3] And let the square of AΔ be subtracted. Then the remainder, that by which the square of BΔ exceeds the square of ΔΓ, is the amount by which the square of AB exceeds the square of AΓ.[4] But (the excess) of the squares of BΔ, ΔΓ is twice the rectangle contained by BΓ, EΔ.[5] Thus also (the excess) of the squares of AB, AΓ.[6]

But that also the excess of the squares of BΔ, ΔΓ is twice the rectangle contained by BΓ, ΔE (is proved) as follows. For since BE equals EΓ,[7] therefore BΔ equals ΓE plus EΔ.[8] And the square of BΔ therefore equals the square of ΓE plus EΔ.[9] But the square of ΓE plus EΔ exceeds the square of ΓΔ by four times the rectangle contained by ΓE, EΔ, that is twice the rectangle contained by BΓ, ΔE.[10] Thus the excess of the squares of BΔ, ΔΓ is twice the rectangle contained by BΓ, ΔE.[11]

(185) ΕΠΙΠΕΔΩΝ ΤΟΠΩΝ Α´ Β´

<α.´> εἰς τὸν τοῦ δευτέρου πρῶτον τόπον.

τρίγωνον τὸ ΑΒΓ, καὶ διήχθω εὐθεῖα ἡ ΑΔ, καὶ ἔστω ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως τὸ ἀπὸ ΒΑ πρὸς τὸ ἀπὸ ΑΓ. ὅτι γίνεται ἴσον τὸ ὑπὸ τῶν ΒΔΓ τῶι ἀπὸ ΑΔ. ἤχθω διὰ τοῦ Γ τῆι ΑΒ παράλληλος ἡ ΓΕ. ἔστιν ἄρα ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΓΕ, καὶ τὸ ἀπὸ ΑΒ πρὸς τὸ ὑπὸ ΑΒ, ΓΕ. ὡς δὲ ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως ἦν τὸ ἀπὸ ΒΑ πρὸς τὸ ἀπὸ ΑΓ. ἴσον ἄρα ἐστὶν τὸ ὑπὸ ΒΑ, ΓΕ τῶι ἀπὸ ΓΑ. ἀνάλογον ἄρα αἱ περὶ ἴσας 854
γωνίας τὰς ἐναλλάξ. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΓΑΔ τῆι Β. ὥστε ἴσον ἐστὶν τὸ ὑπὸ ΒΔΓ τῶι ἀπὸ ΔΑ. τὸ δὲ ἀναστρεφόμενον φανερόν.

(186) <β.´> εἰς τὸν δεύτερον τόπον.

τρίγωνον τὸ ΑΒΓ, καὶ κάθετος ἡ ΔΑ. ὅτι <ἡ> μὲν τῶν ἀπὸ ΒΑ, ΑΓ ὑπεροχὴ ἴση ἐστὶν τῆι τῶν ἀπὸ ΒΔ, ΔΓ ὑπεροχῆι, ἐὰν δὲ ἡ ΒΓ δίχα τμηθῆι <κατὰ> τὸ Ε, ἡ τῶν ἀπὸ ΒΔ, <ΔΓ ὑπεροχὴ> ἐστιν τὸ δὶς ὑπὸ ΒΓ, ΕΔ. ὅτι μὲν οὖν ἡ τῶν ἀπὸ ΒΑ, ΑΓ ὑπεροχὴ ἴση ἐστὶν τῆι τῶν ἀπὸ ΔΒ, ΔΓ ὑπεροχῆι, φανερόν. ἔστιν γὰρ τὸ μὲν ἀπὸ [τῶν] ΑΒ ἴσον τοῖς ἀπὸ τῶν ΒΔ, <ΑΔ>, τὸ δὲ ἀπὸ ΑΓ τοῖς ἀπὸ τῶν ΑΔ, ΔΓ. ὧι ἄρα ὑπερέχει τὸ ἀπὸ ΑΒ τοῦ ἀπὸ ΑΓ, τούτωι ὑπερέχει τὰ ἀπὸ ΑΔ, ΔΒ τῶν ἀπὸ ΑΔ, ΔΓ. κἀφηιρήσθω τὸ ἀπὸ ΑΔ. λοιπὸν ἄρα ὧι ὑπερέχει τὸ ἀπὸ ΒΔ τοῦ ἀπὸ ΔΓ, τούτωι ὑπερέχει τὸ ἀπὸ ΑΒ τοῦ ἀπὸ ΑΓ. τῶν δὲ ἀπὸ ΒΔ, ΔΓ τὸ δὶς ὑπὸ ΒΓ, ΕΔ. ὥστε καὶ τῶν ἀπὸ ΑΒ, ΑΓ.

ὅτι δὴ ἡ τῶν ἀπὸ ΒΔ, ΔΓ ὑπεροχή ἐστιν τὸ δὶς ὑπὸ τῶν ΒΓ, ΔΕ, οὕτως. ἐπεὶ γὰρ ἴση ἐστὶν ἡ ΒΕ τῆι ΕΓ, ἡ ΒΔ ἄρα ἴση ἐστὶν συναμφοτέρωι τῆι ΓΕΔ. καὶ τὸ ἀπὸ ΒΔ ἄρα ἴσον ἐστὶν τῶι ἀπὸ συναμφοτέρου τῆς ΓΕΔ. ἀλλὰ τὸ ἀπὸ συναμφοτέρου τῆς ΓΕΔ τοῦ ἀπὸ ΓΔ ὑπερέχει τῶι τετράκις ὑπὸ ΓΕΔ, τουτέστιν τῶι δὶς ὑπὸ τῶν ΒΓ, ΔΕ. ἡ ἄρα τῶν ἀπὸ ΒΔ, ΔΓ ὑπεροχή ἐστιν τὸ δὶς ὑπὸ τῶν ΒΓ, ΔΕ.

‖ 2 α´ add Camer$_2$ (BS) ‖ 3 εὐθεῖα] τυχοῦσα A, secl Hu (Simson$_2$) ‖ 9 αἱ Hu καὶ A | ἴσας γωνίας Camer$_2$ (S) ἴσην γωνίαν A ‖ 11 ΒΔΓ Co ΒΑΓ A | ἀναστρεφόμενον Camer$_2$ (B) ἀναγραφόμενον A ‖ 13 β´ add Camer$_2$ (BS) ‖ 14 ἡ add Camer$_2$ (BS) post μὲν ‖ 15 ὑπεροχῆι Camer$_2$ (S) ὑπεροχῆς A ‖ 16 κατὰ add Ge | (ΒΔ) ΔΓ ὑπεροχὴ add Co ΒΑ ΑΓ ὑπεροχὴ coni. Co. ‖ 17 ἐστιν] ἔσται Hu app ‖ 19 τῶν del Co | ΑΔ add Co ‖ 20 ὧι Camer$_2$ (S) ὡς A ‖ 23 ΔΓ Camer$_2$ (Co) ΔΒ A | τῶν – ΑΒ, ΑΓ del Hu ‖ 25 δὴ] καὶ A δὲ καὶ coni. Hu app ‖ 27 συναμφοτέρωι Camer$_2$ (S) συναμφότερος A ‖ 30 τῶι (δὶς) Camer$_2$ (BS) τὸ A

(187) 3. *(Prop. 121)* For the same (locus), if the ratio is not that of equal to equal.

(Let there be) triangle ABΓ, and let the square of BA be greater than the square of AΓ by a given than in ratio, namely given E, ratio BΔ to ΔΓ.[1] That the rectangle contained by ΔB, BΓ is greater than area E.

For let the given area, (E, namely) ABH, be subtracted. Then the ratio of the remaining rectangle contained by BA, AH to the square of AΓ is the given ratio, the same as that of BΔ to ΔΓ.[2] Let the rectangle contained by ZA, AΓ be made equal to the rectangle contained by BA, AH.[3] Then the ratio of the rectangle contained by ZA, AΓ to the square of AΓ, that is of ZA to AΓ, is the same as that of BΔ to ΔΓ.[4] Hence AΔ is parallel to ZB.[5] Therefore angle Z equals angle ΓAΔ.[6] But angle Z equals angle AHΓ,[7] and so angle AHΓ equals angle ΓAΔ.[8] But angle AΔΘ is greater than angle ΓAΔ.[9] And so angle ΓHA too is greater than angle AΔΘ.[10] Thus the rectangle contained by ΔB, BΓ is greater than the rectangle contained by AB, BH, that is than E, the given area.[11]

(188) 4. *(Prop. 122)* For the third locus. If ABΓ is a triangle, and some (line) AΔ is drawn through, cutting BΓ, that the squares of BA, AΓ are twice the squares of AΔ, ΔΓ.

Let perpendicular AE be drawn.[1] The squares of BE, EΓ are twice the squares of BΔ, ΔE.[2] But also twice the square of AE plus twice the square of ΔE is twice the square of AΔ;[3] and the squares of BE, EΓ plus twice the square of AE is equal to the squares of BA, AΓ.[4] Hence the squares of BA, AΓ are twice the squares of AΔ, ΔB,[5] that is (twice) the squares of ΓΔ, ΔA.[6]

(189) 5. *(Prop. 123)* (Given) ratio AB to BΓ, and area the rectangle contained by ΓA, AΔ, if the mean proportional BE is taken of ΔB, BΓ, to prove that the square of AE is greater than the square of EΓ by the rectangle contained by ΓA, AΔ than in the ratio of AB to BΓ.

For, as is AB to BΓ, so let some other (line) ZE be made to EΓ.[1] Then in ratio and *separando* as is AΓ to ΓB, so is ZΓ to ΓE.[2] And hence all AZ is to all BE as is AΓ to BΓ.[3] Thus *alternando* as is ZA to AΓ, so is EB to BΓ.[4] But as is EB to BΓ, so is ΔE to EΓ,[5] because it is mean proportional. Hence as is ZA to AΓ, so is EΔ to ΓE.[6] Area to area, therefore, the rectangle contained by AZ, EΓ is equal to the rectangle contained by AΓ, ΔE.[7] But [four times] the rectangle contained by AZ, ΓE exceeds the rectangle contained by AE, EΓ by the rectangle contained by ZE, EΓ.[8] And the amount by which the rectangle contained <by AΓ, ΔE exceeds the rectangle contained by AE, EΓ is the amount by which also the rectangle contained by AZ, ΓE exceeds the rectangle contained by AE, EΓ.[9] Hence the> rectangle contained by AΓ, ΔE is greater than the rectangle contained by AE, EΓ by the rectangle contained by ZE, EΓ.[10] But the amount by which the rectangle contained by AΓ, ΔE exceeds the rectangle contained by AE, EΓ is the amount by which also the square of

(187) <γ.´> *εἰς τὸν αὐτόν, ἐὰν μὴ ὁ λόγος ἴσου πρὸς ἴσον.* 856
τρίγωνον τὸ ΑΒΓ, *καὶ τὸ ἀπὸ* ΒΑ *τοῦ ἀπὸ* ΑΓ *δοθέντι μείζων ἔστω ἢ ἐν λόγωι, δοθέντι μὲν τῶι* Ε, *λόγωι δὲ τῶι τῆς* ΒΔ *πρὸς τὴν* ΔΓ. *ὅτι μεῖζόν ἐστιν τὸ ὑπὸ* ΔΒΓ *τοῦ* Ε *χωρίου. ἀφῃρήσθω γὰρ τὸ δοθὲν χωρίον, τὸ ὑπὸ* ΑΒΗ. *λοιποῦ ἄρα τοῦ ὑπὸ* ΒΑΗ *πρὸς τὸ ἀπὸ* ΑΓ *λόγος ἐστὶν δοθείς, ὁ αὐτὸς τῶι τῆς* ΒΔ *πρὸς τὴν* ΔΓ. *κείσθω τῶι ὑπὸ* ΒΑΗ *ἴσον τὸ ὑπὸ* ΖΑΓ. *λόγος ἄρα τοῦ ὑπὸ* ΖΑΓ *πρὸς τὸ ἀπὸ* ΑΓ, *τουτέστιν τῆς* ΖΑ *πρὸς τὴν* ΑΓ *ὁ αὐτὸς τῶι τῆς* ΒΔ *πρὸς τὴν* ΔΓ. *παράλληλος ἄρα ἐστὶν ἡ* ΑΔ *τῆι* ΖΒ. *ἴση ἄρα ἐστὶν ἡ* Ζ *γωνία τῆι ὑπὸ* ΓΑΔ *γωνίαι. ἀλλὰ ἡ* Ζ *ἴση ἐστὶν τῆι ὑπὸ* ΑΗΓ *γωνίαι. καὶ ἡ ὑπὸ* ΑΗΓ *ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ* ΓΑΔ *γωνίαι.* |*μείζων* <δ'> *ἐστὶν ἡ ὑπὸ* ΑΔΘ |157 *τῆς ὑπὸ* ΓΑΔ. *καὶ τῆς ὑπὸ* ΓΗΑ *ἄρα μείζων ἐστὶν ἡ ὑπὸ* ΑΔΘ *γωνία. ὥστε μεῖζόν ἐστιν τὸ ὑπὸ* ΔΒΓ *τοῦ ὑπὸ* ΑΒΗ, *τουτέστιν τοῦ* Ε, *τοῦ δοθέντος χωρίου.*

(188) <δ.´> *εἰς τὸν τρίτον τόπον.*
ἐὰν ᾖι τρίγωνον τὸ ΑΒΓ, *καὶ διαχθῆι τις ἡ* ΑΔ *δίχα τέμνουσα τὴν* ΒΓ, *ὅτι τὰ ἀπὸ τῶν* ΒΑ, ΑΓ *τετράγωνα διπλάσιά ἐστιν τῶν ἀπὸ τῶν* ΑΔ, ΔΓ *τετραγώνων. ἤχθω κάθετος ἡ* ΑΕ. *τὰ δὲ ἀπὸ τῶν* ΒΕ, ΕΓ *τετράγωνα διπλάσιά ἐστιν τῶν ἀπὸ τῶν* ΒΔ, ΕΔ *τετραγώνων. ἔστιν δὲ καὶ τὸ δὶς ἀπὸ* ΑΕ *μετὰ τοῦ δὶς ἀπὸ* 858 ΔΕ *διπλάσιον τοῦ ἀπὸ* ΑΔ. *τὰ δὲ ἀπὸ τῶν* ΒΕ, ΕΓ *μετὰ τοῦ δὶς ἀπὸ* ΑΕ *ἴσα ἐστὶν τοῖς ἀπὸ τῶν* ΒΑ, ΑΓ. *τὰ ἄρα ἀπὸ* ΒΑ, ΑΓ *διπλάσιά ἐστιν τῶν ἀπὸ* ΑΔ, ΔΒ *τετραγώνων, τουτέστιν τῶν ἀπὸ* ΓΔ, ΔΑ *τετραγώνων.*

(189) <ε.´> *λόγου ὄντος τοῦ τῆς* ΑΒ *πρὸς τὴν* ΒΓ, *καὶ χωρίου τοῦ ὑπὸ τῶν* ΓΑ, ΑΔ, *ἐὰν τῶν* ΔΒ, ΒΓ *μέση ἀνάλογον ληφθῆι ἡ* ΒΕ, *δεῖξαι ὅτι τὸ ἀπὸ* ΑΕ *τοῦ ἀπὸ* ΕΓ *μεῖζόν ἐστιν τῶι ὑπὸ* ΓΑ, ΑΔ *ἢ ἐν λόγωι τῶι τῆς* ΑΒ *πρὸς τὴν* ΒΓ. *πεποιήσθω γὰρ ὡς ἡ* ΑΒ *πρὸς τὴν* ΒΓ, *οὕτως ἄλλη τις ἡ* ΖΕ *πρὸς τὴν* ΕΓ. *ἀνάλογον ἄρα ἐστὶν κατὰ διαίρεσιν ὡς ἡ* ΑΓ *πρὸς τὴν* ΓΒ, *οὕτως ἡ* ΖΓ *πρὸς τὴν* ΓΕ. *καὶ ὅλη ἄρα ἡ* ΑΖ *πρὸς ὅλην τὴν* ΒΕ *ἐστὶν ὡς ἡ* ΑΓ *πρὸς τὴν* ΒΓ. *ἐναλλὰξ ἄρα ἐστὶν ὡς ἡ* ΖΑ *πρὸς τὴν* ΑΓ, *οὕτως ἡ* ΕΒ *πρὸς τὴν* ΒΓ. *ὡς δὲ ἡ* ΕΒ *πρὸς τὴν* ΒΓ, *οὕτως*

‖ 1 *γ´* add Camer₂ (BS) ‖ 2 *δοθέντι* Ge (Co) *δοθέντος* A ‖ 3 *δοθέντι* Ge *δοθὲν* A | ante *λόγωι* add *ἐν* Hu ‖ 4 ΔΒΓ Co ΒΔΓ A ‖ 5 *τὸ δοθὲν χωρίον*] *τῶι δοθέντι χωρίωι ἴσον* Hu app | *λοιποῦ* Co *λοιπὸν* A ‖ 7 ΔΓ Co ΑΓ A | *λόγος*] *λοιπὸν* A *λοιποῦ* Co ‖ 10 Ζ Co ΗΖ A ‖ 12 *δ'* add Hu app ‖ 13 ΓΗΑ Co ΓΗΛ A ‖ 15 *τοῦ δοθέντος* secl Hu ‖ 16 *δ´* add Camer₂ (BS) ‖ 20 ΒΕ Co ΑΕ A ‖ 21 *ἀπὸ* (ΑΕ) Ge (Co) *ὑπὸ* A ‖ 24 ΔΒ Co ΔΑ A ‖ 25 ΔΑ Co ΕΑ A ‖ 26 *ε´* add Camer₂ (BS) ‖ 27 *τοῦ* (*ὑπὸ*) Camer₂ (BS) *τὸ* A ‖ 31 *ἀνάλογον — διαίρεσιν*] *διελόντι ἄρα ἐστὶν καὶ* Hu

AE exceeds the rectangle contained by ΔA, AΓ.[11] Hence the square of AE is greater than the rectangle contained by ΓA, AΔ by the rectangle contained by ZE, EΓ.[12] <But the rectangle contained by ZE, EΓ> has to the square of EΓ the same ratio as that of AB to BΓ.[13] Thus the square of AE is greater than the square of EΓ by the rectangle contained by ΓA, AΔ than in the ratio of AB to BΓ.[14]

(190) 6. *(Prop. 124)* (Given) ratio AB to BΓ, and area the rectangle contained by ΓA, AΔ. If the mean proportional BE is taken of ΔB, BΓ, that the square of AE is greater than the square of EΓ by the rectangle contained by ΓA, AΔ than in the ratio of AB to BΓ.

For, as is AB to BΓ, so let some other (line) EZ be to ΓE.[1] Then *separando* and remainder to remainder, as is ZA to BE, so is AΓ to BΓ.[2] *Alternando*, as is ZA to AΓ, so is EB to BΓ.[3] But as is EB to BΓ, so is ΔE to EΓ.[4] And so as is ZA to AΓ, so is ΔE to ΓE.[5] Area to area, therefore, the rectangle contained by ZA, ΓE equals the rectangle contained by EΔ, AΓ.[6] Let the rectangle contained by AE, EΓ plus the rectangle contained by ΓA, AΔ be added in common. Then the whole square of AE equals the whole of the rectangle contained by ZE, EΓ and as well the rectangle contained by ΓA, AΔ.[7] Hence the square of AE is greater than the square of EΓ by the rectangle contained by ΓA, AΔ than in the ratio of AB to BΓ.[8] For the rectangle contained by ZE, EΓ has this ratio to the square of EΓ.[9]

ἐστὶν ἡ ΔΕ πρὸς τὴν ΕΓ, ἐκ τοῦ εἶναι μέσην ἀνάλογον. καὶ ὡς ἄρα ἡ ΖΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΕΔ πρὸς τὴν ΓΕ. χωρίον χωρίωι, τὸ ἄρα ὑπὸ τῶν ΑΖ, ΕΓ ἴσον ἐστὶν τῶι ὑπὸ ΑΓ, ΔΕ. τὸ δὲ [τετράκις] ὑπὸ ΑΖ, ΓΕ τοῦ ὑπὸ ΑΕΓ ὑπερέχει τῶι ὑπὸ ΖΕΓ. ὧι δὲ ὑπερέχει τὸ <ὑπὸ ΑΓ, ΔΕ τοῦ ὑπὸ ΑΕΓ, τούτωι ὑπερέχει καὶ τὸ ὑπὸ ΑΖ, ΓΕ τοῦ ὑπὸ ΑΕΓ. τὸ ἄρα> ὑπὸ ΑΓ, ΔΕ τοῦ ὑπὸ ΑΕΓ μεῖζόν ἐστιν τῶι ὑπὸ ΖΕΓ. ὧι δὲ ὑπερέχει τὸ ὑπὸ ΑΓ, ΔΕ τοῦ 860 ὑπὸ ΑΕΓ, τούτωι ὑπερέχει καὶ τὸ ἀπὸ ΑΕ τοῦ ὑπὸ ΔΑΓ. τὸ ἄρα ἀπὸ ΑΕ τετράγωνον τοῦ ὑπὸ ΓΑΔ μεῖζόν ἐστιν τῶι ὑπὸ ΖΕΓ. <τὸ δὲ ὑπὸ ΖΕΓ> λόγον ἔχει <πρὸς> τὸ ἀπὸ ΕΓ τὸν αὐτὸν τῶι τῆς ΑΒ πρὸς τὴν ΒΓ. ὥστε τὸ ἀπὸ ΑΕ τοῦ ἀπὸ ΕΓ μεῖζόν ἐστιν τῶι ὑπὸ ΓΑΔ ἢ ἐν λόγωι τῶι τῆς ΑΒ πρὸς τὴν ΒΓ.

(190) <ϛʹ> |λόγος τῆς ΑΒ πρὸς τὴν ΒΓ, χωρίον τὸ ὑπὸ ΓΑΔ. |157v ἐὰν τῶν ΔΒ, ΒΓ μέση ἀνάλογον ληφθῆι ἡ ΒΕ, ὅτι τὸ ἀπὸ τῆς ΑΕ τοῦ ἀπὸ τῆς ΕΓ μεῖζόν ἐστιν τῶι ὑπὸ ΓΑΔ ἢ ἐν λόγωι τῶι τῆς ΑΒ πρὸς τὴν ΒΓ. πεποιήσθω γὰρ ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἄλλη τις ἡ ΕΖ πρὸς τὴν ΓΕ. διελόντι ἄρα καὶ λοιπὴ πρὸς λοιπήν ἐστιν ὡς ἡ ΖΑ πρὸς τὴν ΒΕ, οὕτως ἡ ΑΓ πρὸς τὴν ΒΓ. ἐναλλάξ ἐστιν ὡς ἡ ΖΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΕΒ πρὸς τὴν ΒΓ. ὡς δὲ ἡ ΕΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΕ πρὸς τὴν ΕΓ. καὶ ὡς ἄρα ἡ ΖΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΔΕ πρὸς τὴν ΓΕ. χωρίον χωρίωι, τὸ ἄρα ὑπὸ τῶν ΖΑ, ΓΕ ἴσον ἐστὶν τῶι ὑπὸ ΕΔ, ΑΓ. κοινὸν προσκείσθω τὸ ὑπὸ ΑΕΓ μετὰ τοῦ ὑπὸ ΓΑΔ. ὅλον ἄρα τὸ ἀπὸ ΑΕ ἴσον ἐστὶν ὅλωι τῶι τε ὑπὸ ΖΕΓ καὶ ἔτι τῶι ὑπὸ ΓΑΔ. ὥστε τὸ ἀπὸ ΑΕ τοῦ ἀπὸ ΕΓ μείζων τῶι ὑπὸ ΓΑΔ ἢ ἐν λόγωι τῶι τῆς ΑΒ πρὸς τὴν ΒΓ. τὸ γὰρ ὑπὸ ΖΕΓ πρὸς τὸ ἀπὸ ΕΓ τοῦτον ἔχει τὸν λόγον.

‖ 2 χωρίον χωρίωι τὸ ἄρα] χωρίωι ἄρα τὸ coni. Hu app ‖ 4 τετράκις del Co ‖ 5 ὑπὸ ΑΓ, ΔΕ... ΑΖ, ΓΕ... τὸ ἄρα] ὑπὸ ΑΖ, ΕΓ... ΑΓ, ΔΕ... τὸ ἄρα add Co ‖ 7 μεῖζόν – ΑΕΓ del Camer$_2$ ‖ 8 ΔΑΓ] ΔΕΓ A ΓΑΔ Co ‖ 9 μεῖζον Camer$_2$ (recc?) μείζων A | ΖΕΓ Co ΖΕ A ‖ 10 τὸ – ΖΕΓ add Co | πρὸς add Co ‖ 11 μεῖζόν Hu (recc?) μείζων A ‖ 13 ϛʹ add Camer$_2$ (BS) ‖ 14 ΔΒ, ΒΓ Co ΔΑ, ΑΒ A ‖ 15 ΓΑΔ Co ΒΑΔ A ‖ 17 ΕΖ Hu ΕΓ A ΖΕ Co | ΓΕ Hu ΓΒ A ΕΓ Co ‖ 18 ΖΑ Co ΖΓ A | ΒΕ Co ΓΕ A ‖ 20 ΔΕ Co ΕΔΕ A ‖ 21 ΔΕ Co ΔΓ A ‖ 22 ΕΔ, ΑΓ Co ΕΔΓ A ‖ 23 ΑΕΓ Co ΔΕΓ A | ΑΕ Co ΔΕ A ‖ 25 ΑΕ Co ΔΕ A | τοῦ Camer$_2$ (BS) τούτου A

(191) 7. *(Prop. 125)* (Given) straight line AB, and two points Γ, Δ. That <if> the square of AΔ is put together with that which has the same ratio to the square of ΔB as that of AΓ to ΓB, then there results the square of AΓ plus that which has the same ratio to the square of ΓB as that of AΓ to ΓB and as well that which has the same ratio to the square of ΓΔ as that of AB to BΓ.

For let the (ratio) of ZΔ to ΔB be the same as that of AΓ to ΓB.[1] And so *componendo* (and remainder to remainder) also remainder AZ is to remainder ΓΔ, that is the rectangle contained by AZ, ΓΔ, is to the square of ΓΔ, as is AB to BΓ.[2] Hence that which has the same ratio to the square of ΔB as that of AΓ to ΓB is the rectangle contained by ZΔ, ΔB,[3] and that which has <the same ratio> to the square of ΓB <as that of AΓ to ΓB> is the rectangle contained by AΓ, ΓB,[4] and that which has the same ratio to the square of ΓΔ as that of AB to BΓ is the rectangle contained by AZ, ΔΓ.[5] Hence (to prove) that the square of AΔ plus the rectangle contained by BΔ, ΔZ equals the rectangle contained by BA, AΓ plus the rectangle contained by AZ, ΓΔ.[6] And let the rectangle contained by ΔA, AΓ be subtracted in common. That the remaining rectangle contained by AΔ, ΔΓ plus the rectangle contained by ZΔ, ΔB equals the rectangle contained by AΓ, ΔB plus the rectangle contained by AZ, ΓΔ.[7] Let the rectangle contained by AZ, ΓΔ be subtracted in common. Then (to prove) that the rectangle contained by ZΔ, ΔΓ plus the rectangle contained by ZΔ, ΔB — this turns out to be the whole rectangle contained by ZΔ, ΓB — equals the rectangle contained by AΓ, ΔB.[8] But it is; for straight lines AΓ, ΓB, ZΔ, ΔB are in ratio.[9]

(192) 8. *(Prop. 126)* (Given) straight line AB in position, and Γ arbitrary. That there is a given (point) on AB, so that the square of AΓ plus that which has a given ratio to the square of ΓB equals a given plus that which has a given ratio to the square of the (line) between the given (point) and Γ.

For let AΔ be made to ΔB as the (first) given ratio.[1] Then the ratio of AΔ to ΔB is given; and so point Δ is given.[2] But since AB is a straight line, and Δ, Γ are two points (on it), therefore the square of AΓ plus that which has the same ratio to the square of ΓB as that of AΔ to ΔB equals the square of AΔ plus that which has the same ratio to the square of ΔB as that of AΔ of ΔB plus as well that which has the same ratio to the square of ΔΓ as that of AB to BΔ (lemma 7.191).[4] And that which has the same ratio to the square of ΔB as that of AΔ to BΔ is the rectangle contained by

(191) <ζ.´> εὐθεῖα ἡ ΑΒ, καὶ δύο σημεῖα τὰ Γ, Δ. ὅτι 862 <ἐὰν> τὸ ἀπὸ ΑΔ καὶ τὸ λόγον ἔχον πρὸς τὸ ἀπὸ ΔΒ τὸν αὐτὸν τῶι τῆς ΑΓ πρὸς τὴν ΓΒ συντεθῆι, γίνεται τό τε ἀπὸ ΑΓ καὶ τὸ λόγον ἔχον πρὸς τὸ ἀπὸ ΓΒ τὸν αὐτὸν τῶι τῆς ΑΓ πρὸς τὴν ΓΒ, καὶ ἔτι τὸ λόγον ἔχον πρὸς τὸ ἀπὸ ΓΔ τὸν αὐτὸν τῶι τῆς ΑΒ πρὸς τὴν ΒΓ. τῶι γὰρ τῆς ΑΓ πρὸς τὴν ΓΒ λόγωι ὁ αὐτὸς γεγονέτω ὁ τῆς ΖΔ πρὸς τὴν ΔΒ. καὶ συνθέντι ἄρα καὶ λοιπὴ ἡ ΑΖ πρὸς λοιπὴν τὴν ΓΔ, τουτέστιν τὸ ὑπὸ ΑΖ, ΓΔ πρὸς τὸ ἀπὸ ΓΔ, ἔστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ. ὥστε τὸ μὲν λόγον ἔχον πρὸς τὸ ἀπὸ ΔΒ τὸν αὐτὸν τῶι τῆς ΑΓ πρὸς τὴν ΓΒ ἐστὶν τὸ ὑπὸ ΖΔΒ, τὸ δὲ λόγον ἔχον πρὸς τὸ ἀπὸ ΓΒ <τὸν αὐτὸν τῶι τῆς ΑΓ πρὸς τὴν ΓΒ> ἐστὶν τὸ ὑπὸ ΑΓΒ, τὸ δὲ λόγον ἔχον πρὸς τὸ ἀπὸ ΓΔ τὸν αὐτὸν τῶι τῆς αὐτῆς ΑΒ πρὸς τὴν ΒΓ ἐστὶν τὸ ὑπὸ ΑΖ, ΔΓ. ὅτι οὖν τὸ ἀπὸ ΑΔ μετὰ τοῦ ὑπὸ ΒΔΖ ἴσον ἐστὶν τῶι τε ὑπὸ ΒΑΓ καὶ τῶι ὑπὸ ΑΖ, ΓΔ. καὶ κοινὸν ἀφῃρήσθω τοῦ ὑπὸ ΔΑΓ. ὅτι λοιπὸν τὸ ὑπὸ ΑΔΓ μετὰ τοῦ ὑπὸ ΖΔΒ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΓ, ΔΒ καὶ τῶι ὑπὸ ΑΖ, ΓΔ. κοινὸν ἀφῃρήσθω τὸ ὑπὸ ΑΖ, ΓΔ. ὅτι ἄρα τὸ ὑπὸ ΖΔΓ μετὰ τοῦ ὑπὸ ΖΔΒ (γίνεται ὅλον τὸ ὑπὸ ΖΔ, ΓΒ) ἴσον ἐστὶν τῶι ὑπὸ ΑΓ, ΔΒ. ἔστιν δέ. ἀνάλογον 864 γὰρ αἱ ΑΓ, ΓΒ, ΖΔ, ΔΒ εἰσὶν εὐθεῖαι.

(192) <η.´> θέσει εὐθεῖα ἡ ΑΒ, καὶ τυχὸν |τὸ Γ. ὅτι ἐστὶν |158 δοθὲν ἐπὶ τῆς ΑΒ, ὥστε τὸ ἀπὸ ΑΓ καὶ τὸ λόγον ἔχον πρὸς τὸ ἀπὸ τῆς ΓΒ δοθέντα ἴσον ἐστὶν δοθέντι καὶ τῶι λόγον ἔχοντι πρὸς τὸ ἀπὸ τῆς μεταξὺ τοῦ τε δοθέντος καὶ τοῦ Γ δοθέντα. πεποιήσθω γὰρ ὡς ὁ δοθεὶς λόγος, οὕτως ἡ ΑΔ πρὸς τὴν ΔΒ. λόγος ἄρα καὶ τῆς ΑΔ πρὸς τὴν ΔΒ δοθείς. ὥστε δοθέν ἐστιν τὸ Δ σημεῖον. ἐπεὶ δὲ εὐθεῖά ἐστιν ἡ ΑΒ, καὶ δύο σημεῖα τὰ Δ, Γ, τὸ ἄρα ἀπὸ ΑΓ καὶ τὸ λόγον ἔχον πρὸς τὸ ἀπὸ ΓΒ τὸν αὐτὸν τῶι τῆς ΑΔ πρὸς τὴν ΔΒ ἴσον ἐστὶν τῶι τε ἀπὸ ΑΔ καὶ τῶι λόγον ἔχοντι πρὸς τὸ ἀπὸ ΔΒ τὸν αὐτὸν τῶι τῆς ΑΔ πρὸς τὴν ΔΒ καὶ ἔτι τῶι λόγον ἔχοντι πρὸς τὸ ἀπὸ ΔΓ τὸν αὐτὸν τῶι τῆς ΑΒ πρὸς τὴν ΒΔ. καὶ τὸ λόγον ἔχον πρὸς τὸ ἀπὸ ΔΒ

‖ 1 ζ´ add Camer$_2$ (BS) ‖ 2 ἐὰν add Co ‖ 3 ΓΒ Co ΓΔ A | συντεθῆι Hu συντεθέσεται A ‖ 5 τὸ (λόγον) Camer$_2$ (BS) τὸν A ‖ 6 τῶι ... λόγωι Camer$_2$ τῶι ... λόγον ἔχον A τῶι γὰρ λόγον ἔχοντι Co ‖ 7 συνθέντι Co συντεθήσεται A | λοιπὴ Hu app τὰ λοιπὰ A ‖ 11 τὸν — ΓΒ add Co ‖ 13 αὐτῆς secl Hu (Co) ‖ 14 ΒΔΖ Hu ΔΓΖ A ΖΔΒ Co ‖ 18 ΖΔΓ Co ΖΑΓ A | ΖΔΒ Co ΖΔ ΒΑ A | γίνεται] τουτέστιν Hu ‖ 20 ΔΒ Co ΑΒ A ‖ 21 η´ add Camer$_2$ (BS) | post θέσει add καὶ μεγέθει Camer$_2$ (Simson$_2$) | τὸ Γ — ΑΓ] τὸ Γ ἐπὶ τῆς ΑΒ. ὅτι τὸ ἀπὸ ΑΓ Co ‖ 23 δοθέντα Co δοθὲν A | δοθέντι Co δοθὲν A | λόγον Camer$_2$ (S) λόγωι A ‖ 24 τοῦ τε Hu τοῦτο A | Γ δοθέντα Hu ὑπὸ ΓΔοθέντος A Γ δοθέντος Camer$_2$ ‖ 30 ΔΒ (τὸν) Co ΑΒ A ‖ 31 ἔτι Co ἐν A | λόγον Camer$_2$ (BS) λόγωι A | ΔΓ Co ΑΒ A ‖ 32 ΒΔ — ΑΔΒ del Co | καὶ τὸ Ge καὶ τῶι A τὸ δὲ Camer$_2$ | ἔχον Camer$_2$ ἔχοντι A | ΔΒ Camer$_2$ ΑΓ A

AΔ, ΔB.[5] Therefore the square of AΓ plus that which has the same ratio to the square of ΓB as that of AΔ to ΔB, that is <a given (ratio), equals> the rectangle contained by BA, AΔ, that is a given, plus that which has the same ratio to the square of ΔΓ as that of AB to BΔ, that is a given (ratio).[6] Similarly, if the given (point) Γ is outside straight line AB, we shall prove by the same course.

τὸν αὐτὸν τῶι τῆς ΑΔ πρὸς τὴν ΒΔ, τὸ ὑπὸ ΑΔΒ. τὸ ἄρα ἀπὸ ΑΓ καὶ τὸ λόγον ἔχον πρὸς τὸ ἀπὸ ΓΒ τὸν αὐτὸν τῶι τῆς ΑΔ πρὸς τὴν ΔΒ, τουτέστιν <δοθέντα, ἴσον ἐστὶν> τῶι τε ὑπὸ ΒΑΔ, τουτέστιν δοθέντι, καὶ τῶι λόγον ἔχοντι πρὸς τὸ ἀπὸ ΔΓ τὸν αὐτὸν τῶι τῆς ΑΒ πρὸς τὴν ΒΔ, τουτέστιν δοθέντα. ὁμοίως καὶ ἐὰν τὸ δοθὲν τὸ Γ ἐκτὸς ἦι τῆς ΑΒ εὐθείας, τῆι αὐτῆι ἀκολουθίαι δείξομεν.

‖ 1 τῶι Camer$_2$ τῶν A | ΑΔ Camer$_2$ ΑΒ A ‖ 3 δοθέντα ἴσον ἐστὶν add Camer$_2$ δοθέντι ἴσον ἐστὶν Co ‖ 4 λόγον Camer$_2$ (BS) λόγωι A | ΔΓ Co ΑΓ A ‖ 5 δοθέντα Camer$_2$ (Simson$_2$) δοθέν A δοθέντι Co ‖ 6 τὸ δοθὲν secl Hu τὸ τυχὸν Hu app

(193) Porisms, (Books) 1, 2, 3.

From Book 1:

1. *(Prop. 127 a– e)* For the first porism.

Let there be figure ΑΒΓΔΕΖΗ, and, as is ΑΖ to ΖΗ, so let ΑΔ be to ΔΓ, and let ΘΚ be joined. That ΘΚ is parallel to ΑΓ.

Let ΖΛ be drawn through Ζ parallel to ΒΔ.[1] Then since, as is ΑΖ to ΖΗ, so is ΑΔ to ΔΓ,[2] by inversion and *componendo* and *alternando* as is ΔΑ to ΑΖ, that is, in parallels, as is ΒΑ to ΑΛ,[4] so is ΓΑ to ΑΗ.[3] Hence ΛΗ is parallel to ΒΓ.[5] Therefore as is ΕΒ to ΒΛ, so is <ΕΘ to ΘΗ.[6] But also as is ΕΒ to ΒΛ, so>, in parallels, is ΕΚ to ΚΖ.[7] Thus as is ΕΚ to ΚΖ, so is ΕΘ to ΘΗ.[8] ΘΚ is therefore parallel to ΑΓ.[9]

(194) *(Prop. 127 a– e)* By compound ratios, as follows:

Since, as is ΑΖ to ΖΗ, so is ΑΔ to ΔΓ,[1] by inversion, as is ΗΖ to ΖΑ, so is ΓΔ to ΔΑ.[2] *Componendo* and *alternando* and *convertendo*, as is ΑΔ to ΔΖ, so is ΑΓ to ΓΗ.[3] But the (ratio) of ΑΔ to ΔΖ is compounded out of that of <ΑΒ to ΒΕ and that of ΕΚ to ΚΖ[4] *(see commentary)*, while that of ΑΓ to ΓΗ (is compounded) out of that of> ΑΒ to ΒΕ and that of ΕΘ to ΘΗ[5] *(see commentary)*. Therefore the ratio compounded out of that which ΑΒ has to ΒΕ and ΕΚ has to ΚΖ is the same as the (ratio) compounded out of that which ΑΒ has to ΒΕ and ΕΘ has to ΘΗ.[6] And let the ratio of ΑΒ to ΒΕ be removed in common. Then there remains the ratio of ΕΚ to ΚΖ equal to the ratio of ΕΘ to ΘΗ.[7] Thus ΘΚ is parallel to ΑΓ.[8]

(195) *(Prop. 128)* For the second porism.

Figure ΑΒΓΔΕΖΗ. Let ΑΖ be parallel to ΔΒ, and as is ΑΕ to ΕΖ, so let ΓΗ be to ΗΖ. That the (line) through Θ, Κ, Ζ is straight.

Let ΗΛ be drawn through Η parallel to ΔΕ,[1] and let ΘΚ be joined and produced to Λ. Then since, as is ΑΕ to ΕΖ, so is ΓΗ to ΗΖ,[2] *alternando* as

(193) ΠΟΡΙΣΜΑΤΩΝ Α Β Γ

τοῦ πρώτου.

α´. εἰς τὸ πρῶτον πόρισμα.

ἔστω καταγραφὴ ἡ ΑΒΓΔΕΖΗ, καὶ ἔστω ὡς ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, καὶ ἐπεζεύχθω ἡ ΘΚ. ὅτι παράλληλός ἐστιν ἡ ΘΚ τῆι ΑΓ. ἤχθω διὰ τοῦ Ζ τῆι ΒΔ παράλληλος ἡ ΖΛ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, ἀνάπαλιν καὶ συνθέντι καὶ ἐναλλὰξ ἐστιν ὡς ἡ ΔΑ πρὸς τὴν ΑΖ, τουτέστιν ἐν παραλλήλωι, ὡς ἡ ΒΑ πρὸς τὴν ΑΛ, οὕτως ἡ ΓΑ πρὸς τὴν ΑΗ. παράλληλος ἄρα ἐστὶν ἡ ΛΗ τῆι ΒΓ. ἔστιν ἄρα ὡς ἡ ΕΒ πρὸς τὴν ΒΛ, οὕτως ἡ <ΕΘ πρὸς τὴν ΘΗ. ἔστιν δὲ καὶ ὡς ἡ ΕΒ πρὸς τὴν ΒΛ, οὕτως> ἐν παραλλήλωι ἡ ΕΚ πρὸς τὴν ΚΖ. καὶ ὡς ἄρα ἡ ΕΚ πρὸς τὴν ΚΖ, οὕτως ἐστὶν ἡ ΕΘ πρὸς τὴν ΘΗ. παράλληλος ἄρα ἐστὶν ἡ ΘΚ τῆι ΑΓ.

(194) διὰ δὲ τοῦ συνημμένου οὕτως. ἐπεί ἐστιν ὡς ἡ ΑΖ 868
πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, ἀνάπαλίν ἐστιν ὡς ἡ ΗΖ πρὸς τὴν ΖΑ, οὕτως ἡ ΓΔ πρὸς τὴν ΔΑ. συνθέντι καὶ ἐναλλὰξ καὶ ἀναστρέψαντί ἐστιν ὡς ΑΔ πρὸς τὴν ΔΖ, |οὕτως ἡ ΑΓ πρὸς |158v
τὴν ΓΗ. ἀλλ' ὁ μὲν τῆς ΑΔ πρὸς τὴν ΔΖ συνῆπται ἔκ τε τοῦ τῆς <ΑΒ πρὸς τὴν ΒΕ καὶ τοῦ τῆς ΕΚ πρὸς τὴν ΚΖ, ὁ δὲ τῆς ΑΓ πρὸς τὴν ΓΗ ἔκ τε τοῦ τῆς> ΑΒ πρὸς τὴν ΒΕ καὶ τοῦ τῆς ΕΘ πρὸς τὴν ΘΗ. ὁ ἄρα συνημμένος λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΑΒ πρὸς τὴν ΒΕ καὶ ἡ ΕΚ πρὸς τὴν ΚΖ ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ ὃν ἔχει ἡ ΑΒ πρὸς τὴν ΒΕ καὶ ἡ ΕΘ πρὸς τὴν ΘΗ. καὶ κοινὸς ἐκκεκρούσθω ὁ τῆς ΑΒ <πρὸς> τὴν ΒΕ λόγος. λοιπὸν ἄρα ὁ τῆς ΕΚ πρὸς τὴν ΚΖ λόγος <ὁ αὐτός> ἐστιν τῶι τῆς ΕΘ πρὸς τὴν ΘΗ λόγωι. <παράλληλος> ἄρα ἐστὶν ἡ ΘΚ τῆι ΑΓ.

(195) εἰς τὸ δεύτερον πόρισμα.

καταγραφὴ ἡ ΑΒΓΔΕΖΗΘ. ἔστω δὲ παράλληλος ἡ ΑΖ τῆι ΔΒ, ὡς δὲ ἡ ΑΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΖ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Θ, Κ, Ζ. ἤχθω διὰ τοῦ Η παράλληλος τῆι ΔΕ ἡ ΗΛ, καὶ ἐπιζευχθεῖσα ἡ ΘΚ ἐκβεβλήσθω ἐπὶ τὸ Λ. ἐπεὶ οὖν

‖ 4 α´ mg A ‖ 5 post ὡς add ἡ Ge (BS) ‖ 11 ΛΗ] ΑΗ A^1 Λ supra A^2 ‖ 12 ἡ (ΕΘ) del Hu ἐν παραλλήλωι ἡ Heiberg$_3$ | ΕΘ – οὕτως add Heiberg$_3$ ‖ 13 ΕΚ πρὸς τὴν ΚΖ A, post quae add καὶ ἡ ΕΘ πρὸς τὴν ΘΗ Co ‖ 17 ΗΖ Co ΝΖ A ‖ 19 post ὡς add ἡ Ge (BS) | ΔΖ Co ΑΖ A ‖ 21 ΑΒ – τῆς (ΑΒ) add Heiberg$_3$ ‖ 26 κοινὸς] signum quasi κ° A | πρὸς add Ge (BS) ‖ 27 ὁ αὐτός add Co ‖ 28 λόγωι. παράλληλος] λόγος A παράλληλος Co ‖ 32 παράλληλος τῆι] παρὰ τὴν A ‖ 33 ἐπιζευχθεῖσα Hu ἐπεζεύχθω A | post τὸ Λ spatium litterarum fere septem relictum A is AE to ΓH, so is EZ to ZH.[3] But as is AE to ΓH, so is EΘ to HΛ,[4] and *alternando*, because there are two by two (parallel lines). Therefore as is EZ to ZH, so is EΘ to HΛ.[5] And EΘ is parallel to HΛ.[6] Thus (VI, 32) the (line) through Θ, Λ, Z is straight.[7] Q.E.D.

(196) *(Prop. 129 a– h)* Let two straight lines ΘE, ΘΔ be drawn onto three straight lines AB, ΓA, ΔA. That, as is the rectangle contained by ΘE, HZ to the rectangle contained by ΘH, ZE, so is the rectangle contained by ΘB, ΔΓ to the rectangle contained by ΘΔ, BΓ.

Let KΛ be drawn through Θ parallel to ZΓA,[1] and let ΔA and AB intersect it at points K and Λ; and (let there be drawn) ΛM through Λ parallel to ΔA,[2] and let it intersect EΘ at M. Then since, as is EZ to ZA, so is EΘ to ΘΛ,[3] while as is AZ to ZH, so is ΘΛ to ΘM,[5] because ΘK is to ΘH also (as is ΘΛ to ΘM) in parallels,[4] therefore *ex aequali* as is EZ to ZH, so is EΘ to ΘM.[6] Therefore the rectangle contained by ΘE, HZ equals the rectangle contained by EZ, ΘM.[7] But (let) the rectangle contained by EZ, ΘH (be) another arbitrary quantity. Then as is the rectangle contained by EΘ, HZ to the rectangle contained by EZ, HΘ, so is the rectangle contained by EZ, ΘM to the rectangle contained by EZ, HΘ,[8] that is ΘM to ΘH,[9] that is ΛΘ to ΘK.[10] By the same argument also as is KΘ to ΘΛ, so is the rectangle contained by ΘΔ, BΓ to the rectangle contained by ΘB, ΓΔ.[11] By inversion, therefore, as is ΛΘ to ΘK, so is the rectangle contained by ΘB, ΓΔ to the rectangle contained by ΘΔ, BΓ.[12] But as is ΛΘ to ΘK, so the rectangle contained by EΘ, HZ was shown to be to the rectangle contained by EZ, HΘ. And thus as is the rectangle contained by EΘ, HZ to the rectangle contained by EZ, HΘ, so is the rectangle contained by ΘB, ΓΔ to the rectangle contained by ΘΔ, BΓ.[13]

(197) *(Prop. 129 a– h)* By means of compounded ratios, as follows:

Since the ratio of the rectangle contained by ΘE, HZ to the rectangle contained by ΘH, ZE is compounded out of that which ΘE has to EZ and that which ZH has to HΘ,[1] and as is ΘE to EZ, so is ΘΛ to ZA,[2] while as is ZH to HΘ, so is ZA to ΘK,[3] therefore the (ratio of the) rectangle contained by ΘE, HZ to the rectangle contained by ΘH, EZ is compounded out of that which ΘΛ has to ZA and that which ZA has to ΘK.[4] But the (ratio) compounded out of that which ΘΛ has to ZA and that which ZA has to ΘK is the same as that of ΘΛ to ΘK.[5] Hence as is the rectangle contained by ΘE, HZ to the rectangle contained by ΘH, ZE, so is ΘΛ to ΘK.[6] For the same reasons also as is the rectangle contained by ΘΔ, BΓ to ἔστιν ὡς ἡ ΑΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΖ, ἐναλλάξ ἐστιν ὡς ἡ ΑΕ πρὸς τὴν ΓΗ, οὕτως ἡ ΕΖ πρὸς τὴν ΖΗ. ὡς δὲ ἡ ΑΕ πρὸς τὴν ΓΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΗΛ, καὶ ἐναλλάξ, διὰ τὸ εἶναι δύο παρὰ δύο· καὶ ὡς ἄρα ἡ ΕΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΗΛ. καὶ ἔστιν παράλληλος ἡ ΕΘ τῆι ΗΛ. εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Θ, Λ, Ζ. ὅ(περ): — 870

(196) εἰς τρεῖς εὐθείας τὰς ΑΒ, ΓΑ, ΔΑ διήχθωσαν δύο εὐθεῖαι αἱ ΘΕ, ΘΔ· ὅτι ἐστὶν ὡς τὸ ὑπὸ ΘΕ, ΗΖ πρὸς τὸ ὑπὸ ΘΗ, ΖΕ, οὕτως τὸ ὑπὸ ΘΒ, ΔΓ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ. ἤχθω διὰ μὲν τοῦ Θ τῆι ΖΓΑ παράλληλος ἡ ΚΛ, καὶ αἱ ΔΑ, ΑΒ συμπιπτέτωσαν αὐτῆι κατὰ τὰ Κ, Λ σημεῖα· διὰ δὲ τοῦ Λ τῆι ΔΑ παράλληλος ἡ ΛΜ, καὶ συμπιπτέτω τῆι ΕΘ ἐπὶ τὸ Μ. ἐπεὶ οὖν ἐστιν ὡς μὲν ἡ ΕΖ πρὸς τὴν ΖΑ, οὕτως ἡ ΕΘ πρὸς τὴν ΘΛ, ὡς δὲ ἡ ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΘΛ πρὸς τὴν ΘΜ (καὶ γὰρ ἡ ΘΚ πρὸς τὴν ΘΗ ἐν παραλλήλωι) δι' ἴσου ἄρα ἐστὶν ὡς ἡ ΕΖ πρὸς ΖΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΘΜ. τὸ ἄρα ὑπὸ |τῶν ΘΕ, ΗΖ ἴσον ἐστὶν τῶι ὑπὸ τῶν |159 ΕΖ, ΘΜ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ τῶν ΕΖ, ΘΗ. ἔστιν ἄρα ὡς τὸ ὑπὸ τῶν ΕΘ, ΗΖ πρὸς τὸ ὑπὸ τῶν ΕΖ, ΗΘ, οὕτως τὸ ὑπὸ ΕΖ, ΘΜ πρὸς τὸ ὑπὸ ΕΖ, ΗΘ, τουτέστιν ἡ ΘΜ πρὸς ΘΗ, τουτέστιν ἡ ΛΘ πρὸς τὴν ΘΚ. κατὰ τὰ αὐτὰ καὶ ὡς ἡ ΚΘ πρὸς τὴν ΘΛ, οὕτως τὸ ὑπὸ ΘΔ, ΒΓ πρὸς τὸ ὑπὸ ΘΒ, ΓΔ. ἀνάπαλιν ἄρα γίνεται ὡς ἡ ΛΘ πρὸς τὴν ΘΚ, οὕτως τὸ ὑπὸ ΘΒ, ΓΔ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ. ὡς δὲ ἡ 872 ΛΘ πρὸς τὴν ΘΚ, οὕτως ἐδείχθη τὸ ὑπὸ ΕΘ, ΗΖ πρὸς τὸ ὑπὸ ΕΖ, ΗΘ. καὶ ὡς ἄρα τὸ ὑπὸ ΕΘ, ΗΖ πρὸς τὸ <ὑπὸ> ΕΖ, ΗΘ, οὕτως τὸ ὑπὸ ΘΒ, ΓΔ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ.

(197) διὰ δὲ τοῦ συνημμένου οὕτως. ἐπεὶ <ὁ> τοῦ ὑπὸ ΘΕ, ΗΖ πρὸς τὸ ὑπὸ ΘΗ, ΖΕ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΘΕ πρὸς τὴν ΕΖ καὶ τοῦ ὃν ἔχει ἡ ΖΗ πρὸς τὴν ΗΘ, καὶ ἔστιν ὡς μὲν ἡ ΘΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΘΛ πρὸς τὴν ΖΑ, ὡς δὲ ἡ ΖΗ πρὸς τὴν ΗΘ, οὕτως ἡ ΖΑ πρὸς τὴν ΘΚ, τὸ ἄρα ὑπὸ ΘΕ, ΗΖ πρὸς τὸ ὑπὸ ΘΗ, ΕΖ συνῆπται ἔκ τε τοῦ ὃν ἔχει ἡ ΘΛ πρὸς τὴν ΖΑ καὶ τοῦ ὃν ἔχει ἡ ΖΑ πρὸς τὴν ΘΚ. ὁ δὲ συνημμένος ἔκ τε τοῦ τῆς ΘΛ πρὸς τὴν ΖΑ καὶ τοῦ τῆς ΖΑ πρὸς τὴν ΘΚ ὁ αὐτός ἐστιν τῶι τῆς ΘΛ πρὸς τὴν ΘΚ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΘΕ, ΗΖ πρὸς τὸ ὑπὸ

‖ 3 καὶ ἐναλλὰξ in ras. A, post διὰ τὸ εἶναι δύο παρὰ δύο transp. Hu, quae omnia del Heiberg$_3$ ‖ 6 post Θ add Κ Ge (S) | post Ζ add τουτέστιν ἡ διὰ τῶν Θ, Κ, Ζ Hu ‖ 11 Δ(Α) in ras. A | pro ἡ ΛΜ καὶ coni. διαχθεῖσα ἡ ΛΜ Hu app ‖ 17 pro τυχὸν coni. ἔχομεν Hu app ‖ 21 ἀνάπαλιν Co ἀνάλογον A ‖ 24 ὑπὸ (ΕΖ) add Ge (S) ‖ 26 ὁ add Heiberg$_3$ ‖ 28 πρὸς τὴν ΕΖ — ΖΗ bis A corr Co the rectangle contained by ΘB, ΓΔ, so is ΘK to ΘΛ.[7] And by inversion, as is the rectangle contained by ΘB, ΓΔ to the rectangle contained by ΘΔ, BΓ, so is ΛΘ to ΘK.[8] But as is the rectangle contained by ΘE, ZH to the rectangle contained by ΘH, ZE, <so was ΘΛ to ΘK. Thus, as is the rectangle contained by ΘE, ZH to the rectangle contained by ΘH, ZE,> so is the rectangle contained by ΘB, ΓΔ to the rectangle contained by ΘΔ, BΓ.[9]

(198) *(Prop. 130 a – h)* Figure ABΓΔEZHΘKΛ. As is the rectangle contained by AZ, BΓ to the rectangle contained by AB, ΓZ, so let the rectangle contained by AZ, ΔE be to the rectangle contained by AΔ, EZ. That the (line) through points Θ, H, Z is straight.

Since, as is the rectangle contained by AZ, BΓ to the rectangle contained by AB, ΓZ, so is the rectangle contained by AZ, ΔE to the rectangle contained by AΔ, EZ,[1] *alternando* as is the rectangle contained by AZ, BΓ to the rectangle contained by AZ, ΔE, that is as is BΓ to ΔE,[3] so is the rectangle contained by AB, ΓZ to the rectangle contained by AΔ, EZ.[2] But the ratio of BΓ to ΔE is compounded, if KM is drawn through K parallel to AZ,[4] out of that which BΓ has to KN and that which KN has to KM, and as well that which KM has to ΔE.[5] But the (ratio) of the rectangle contained by AB, ΓZ to the rectangle contained by AΔ, EZ is compounded out of that of BA to AΔ and that of ΓZ to ZE.[6] Let the (ratio) of BA to AΔ be removed in common, this being the same as that of NK to KM.[7] Then the remaining (ratio) of ΓZ to ZE is compounded out of that of BΓ to KN, that is that of ΘΓ to KΘ,[9] and that of KM to ΔE, that is that of KH to HE.[10] [8] Thus the (line) through Θ, H, Z is straight.

For if I draw EΞ through E parallel to ΘΓ,[11] and ΘH is joined and produced to Ξ, the ratio of KH to HE is the same as that of KΘ to EΞ,[12] while the (ratio) compounded out of that of ΓΘ to ΘK and that of ΘK to EΞ is converted into the ratio of ΘΓ to EΞ,[13] and the ratio of ΓZ to ZE is the same as that of ΓΘ to EΞ.[14] Because ΓΘ is (therefore) parallel to EΞ,[15] the (line) through Θ, Ξ, Z is straight;[16] for that is obvious. Therefore the (line) through Θ, H, Z is also straight.[17]

(199) *(Prop. 131)* If there is figure ABΓΔEZHΘ, then as AΔ is to ΔΓ, so is AB to BΓ. So let AB be to BΓ as is AΔ to ΔΓ. That the (line) through A, H, Θ is straight.

ΘΗ, ΖΕ, οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ. διὰ ταὐτὰ καὶ ὡς τὸ ὑπὸ ΘΔ, ΒΓ πρὸς τὸ ὑπὸ ΘΒ, ΓΔ, οὕτως ἐστὶν ἡ ΘΚ πρὸς τὴν ΘΛ. καὶ ἀνάπαλίν ἐστιν ὡς τὸ ὑπὸ ΘΒ, ΓΔ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ, οὕτως ἡ ΛΘ πρὸς τὴν ΘΚ. ἦν δὲ καὶ ὡς τὸ ὑπὸ τῶν ΘΕ, ΖΗ πρὸς τὸ ὑπὸ ΘΗ, ΖΕ, <οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ. καὶ ὡς ἄρα τὸ ὑπὸ τῶν ΘΕ, ΖΗ πρὸς τὸ ὑπὸ ΘΗ, ΖΕ,> οὕτως τὸ ὑπὸ ΘΒ, ΓΔ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ.

(198) |καταγραφὴ ἡ ΑΒΓΔΕΖΗΘΚΛ. ἔστω δὲ ὡς τὸ ὑπὸ ΑΖ, ΒΓ |159v
πρὸς τὸ ὑπὸ ΑΒ, ΓΖ, οὕτως τὸ ὑπὸ ΑΖ, ΔΕ πρὸς τὸ ὑπὸ ΑΔ, ΕΖ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Θ, Η, Ζ σημείων. ἐπεί ἐστιν ὡς τὸ ὑπὸ ΑΖ, ΒΓ πρὸς τὸ ὑπὸ ΑΒ, ΓΖ, οὕτως τὸ ὑπὸ ΑΖ, ΔΕ πρὸς τὸ ὑπὸ ΑΔ, ΕΖ, ἐναλλάξ ἐστιν ὡς τὸ ὑπὸ ΑΖ, ΒΓ πρὸς τὸ ὑπὸ ΑΖ, ΔΕ, 874
τουτέστιν ὡς ἡ ΒΓ πρὸς τὴν ΔΕ, οὕτως τὸ ὑπὸ ΑΒ, ΓΖ πρὸς τὸ ὑπὸ ΑΔ, ΕΖ. ἀλλ' ὁ μὲν τῆς ΒΓ πρὸς τὴν ΔΕ συνῆπται λόγος, ἐὰν διὰ τοῦ Κ τῆι ΑΖ παράλληλος ἀχθῆι ἡ ΚΜ, ἔκ τε τοῦ τῆς ΒΓ πρὸς ΚΝ καὶ <τοῦ> τῆς ΚΝ πρὸς ΚΜ, καὶ ἔτι τοῦ τῆς ΚΜ πρὸς ΔΕ. ὁ δὲ τοῦ ὑπὸ ΑΒ, ΓΖ πρὸς τὸ ὑπὸ ΑΔ, ΕΖ συνῆπται ἔκ τε τοῦ τῆς ΒΑ πρὸς ΑΔ καὶ τοῦ τῆς ΓΖ πρὸς τὴν ΖΕ. κοινὸς ἐκκεκρούσθω ὁ τῆς ΒΑ πρὸς ΑΔ, ὁ αὐτὸς ὢν τῶι τῆς ΝΚ πρὸς ΚΜ. λοιπὸς ἄρα ὁ τῆς ΓΖ πρὸς τὴν ΖΕ συνῆπται ἔκ τε τοῦ τῆς ΒΓ πρὸς τὴν ΚΝ, τουτέστιν τοῦ τῆς ΘΓ πρὸς τὴν ΚΘ, καὶ τοῦ τῆς ΚΜ πρὸς τὴν ΔΕ, τουτέστιν <τοῦ> τῆς ΚΗ πρὸς τὴν ΗΕ. εὐθεῖα ἄρα ἡ διὰ τῶν Θ, Η, Ζ. ἐὰν γὰρ διὰ τοῦ Ε τῆι ΘΓ παράλληλον ἀγάγω τὴν ΕΞ, καὶ ἐπιζευχθεῖσα ἡ ΘΗ ἐκβληθῆι ἐπὶ τὸ Ξ, ὁ μὲν τῆς ΚΗ πρὸς τὴν ΗΕ λόγος ὁ αὐτός ἐστιν τῶι τῆς ΚΘ πρὸς τὴν ΕΞ, ὁ δὲ συνημμένος ἔκ τε τοῦ τῆς ΓΘ πρὸς τὴν ΘΚ καὶ τοῦ τῆς ΘΚ πρὸς τὴν ΕΞ μεταβάλλεται εἰς τὸν τῆς ΘΓ πρὸς ΕΞ λόγον, καὶ ὁ τῆς ΓΖ πρὸς ΖΕ λόγος ὁ αὐτὸς τῶι τῆς ΓΘ πρὸς τὴν ΕΞ. παραλλήλου οὔσης τῆς ΓΘ τῆι ΕΞ, εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Θ, Ξ, Ζ· τοῦτο γὰρ φανερόν. ὥστε καὶ ἡ διὰ τῶν Θ, Η, Ζ εὐθεῖά ἐστιν.

(199) |ἐὰν ἦι καταγραφὴ ἡ ΑΒΓΔΕΖΗΘ, γίνεται ὡς ἡ ΑΔ πρὸς |160
τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΓ. ἔστω οὖν ὡς ἡ ΑΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΓ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Η, Θ. ἤχθω διὰ τοῦ Η τῆι ΑΔ παράλληλος ἡ ΚΛ. ἐπεὶ οὖν ἐστιν ὡς ἡ

‖ 4 ἦν Ge (S) τὴν A ‖ 5 οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ add Ge | καὶ ὡς ἄρα — (τῶν) ΘΗ, ΖΕ add Co (τῶν) del Ge ‖ 7 ΑΒΓΔΕΖΗΘΚΛ Co ΑΒΓΔΕΖΗΘΙΚΛ A ‖ 15 ΚΝ (καὶ) Co ΚΗ A | (ἔτι) τοῦ Co τὸ A ‖ 17 ΖΕ Co ΔΕ A | κοινὸς] κ° A ‖ 19 λοιπὸς Ge λοιπὸν A ‖ 21 τοῦ add Hu ‖ 22 Ζ Co Κ A | ΘΓ Co ΒΓ A ‖ 23 ΕΞ Co ΕΖ A | ἐπιζευχθεῖσα ἡ Hu ἐπιζευχθείσης τῆς A ‖ 26 μεταβάλλεται Hu μεταβαλλόμενος A | τὸν Ge (S) τὸ A ‖ 27 ΕΞ Co ΘΖ A

Let KΛ be drawn through H parallel to AΔ.[1] Then since as is AΔ to ΔΓ, so is AB to BΓ,[2] while as is AΔ to ΔΓ, so is KΛ to ΛH,[3] and as is AB to BΓ, so is KH to HM,[4] therefore as is KΛ to ΛH, so is KH to HM.[5] And remainder HΛ is to remainder ΛM as is KΛ to ΛH,[6] that is as AΔ is to ΔΓ.[7] *Alternando* as is AΔ to HΛ, so is ΓΔ to ΛM,[8] that is ΔΘ to ΘΛ.[9] And HΛ is parallel to AB.[10] Hence the (line) through points A, H, Θ is straight;[11] for this is obvious.

(200) *(Prop. 132)* Again if there is a figure (ABΓΔEZH), and ΔZ is parallel to BΓ, then AB equals BΓ. So let it be equal. That (ΔZ) is parallel (to BΓ).

But it is. For if, with EB drawn through, I make BΘ equal to HB,[1] and I join AΘ and ΘΓ, then there results a parallelogram AΘΓH,[2] and because of this, as is AΔ to ΔE, so is ΓZ to ZE.[4] For each of the foregoing (ratios) is the same as the ratio of ΘH to HE.[3] Thus (VI, 2) ΔZ is parallel to AΓ.[5]

(201) *(Prop. 133)* Let there be a figure (ABΓΔEZHΘ), and let BA be a mean proportional between ΔB and BΓ. That ZH is parallel to AΓ.

Let EB be produced, and let AK be drawn through A parallel to straight line ΔZ,[1] and let ΓK be joined. Then since as is ΓB to BA, so is AB to BΔ,[2] while as is AB to BΔ, so is KB to BΘ,[3] therefore as is ΓB to BA, so is KB to BΘ.[4] Hence AΘ is parallel to KΓ.[5] Therefore again, as is AZ to ZE, so is ΓH to HE;[7] for either of the foregoing ratios is the same as that of KΘ to EΘ.[6] Thus ZH is parallel to AΔ.[8]

(202) *(Prop. 134)* Let there be an "altar" ABΓΔEZH, and let ΔE be parallel to BΓ, and EH to BZ. That ΔZ too is parallel to ΓH.

Let BE, ΔΓ, and ZH be joined. Then triangle ΔBE equals triangle ΔΓE.[1] Let triangle ΔAE be added in common. Then all triangle ABE equals all triangle ΓΔA.[2] Again, since BZ is parallel to EH,[3] triangle BZE equals triangle BZH.[4] Let triangle ABZ be subtracted in common. Then the remaining triangle ABE equals the remaining triangle AHZ.[5] But

ΑΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΓ, ἀλλ' ὡς μὲν ἡ ΑΔ πρὸς 876 τὴν ΔΓ, οὕτως ἡ ΚΛ πρὸς τὴν ΛΗ, ὡς δὲ ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΚΗ πρὸς τὴν ΗΜ, καὶ ὡς ἄρα ἡ ΚΛ πρὸς τὴν ΛΗ, οὕτως ἡ ΚΗ πρὸς τὴν ΗΜ. καὶ λοιπὴ ἡ ΗΛ πρὸς λοιπὴν τὴν ΛΜ ἐστὶν ὡς ἡ ΚΛ πρὸς τὴν ΛΗ, τουτέστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΓ. ἐναλλάξ ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΗΛ, οὕτως ἡ ΓΔ πρὸς τὴν ΛΜ, τουτέστιν ἡ ΔΘ πρὸς ΘΛ. καὶ ἔστι παράλληλος ἡ ΗΛ τῆι ΑΒ. εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α, Η, Θ σημείων· τοῦτο γὰρ φανερόν.

(200) πάλιν ἐὰν ἦι καταγραφή, καὶ παράλληλος ἡ ΔΖ τῆι ΒΓ, γίνεται ἴση ἡ ΑΒ τῆι ΒΓ. ἔστω οὖν ἴση. ὅτι παράλληλος. ἔστιν δέ. ἐὰν γὰρ διαχθείσης τῆς ΕΒ θῶ τῆι ΗΒ ἴσην τὴν ΒΘ, καὶ ἐπιζεύξω τὰς ΑΘ, ΘΓ, γίνεται παραλληλόγραμμον τὸ ΑΘΓΗ, καὶ διὰ τοῦτό ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΕ, οὕτως ἡ ΓΖ πρὸς τὴν ΖΕ. ἑκάτερος γὰρ τῶν εἰρημένων ὁ αὐτός ἐστιν τῶι τῆς ΘΗ πρὸς τὴν ΗΕ λόγωι. ὥστε παράλληλός ἐστιν ἡ ΔΖ τῆι ΑΓ.

(201) ἔστω καταγραφή, καὶ τῶν ΔΒ, ΒΓ μέση ἀνάλογον ἔστω ἡ ΒΑ. ὅτι παράλληλός ἐστιν ἡ ΖΗ τῆι ΑΓ. ἐκβεβλήσθω ἡ ΕΒ, καὶ διὰ τοῦ Α τῆι ΔΖ εὐθείαι παράλληλος ἤχθω ἡ ΑΚ, καὶ ἐπεζεύχθω ἡ ΓΚ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΒ πρὸς τὴν ΒΑ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΔ, ὡς δὲ ἡ ΑΒ πρὸς τὴν ΒΔ, οὕτως ἡ ΚΒ πρὸς τὴν ΒΘ, καὶ ὡς ἄρα ἡ ΓΒ πρὸς τὴν ΒΑ, οὕτως ἡ ΚΒ πρὸς τὴν ΒΘ. 878 παράλληλος ἄρα ἐστὶν ἡ ΑΘ τῆι ΚΓ. ἔστιν οὖν πάλιν ὡς ἡ ΑΖ πρὸς τὴν ΖΕ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΕ. ἑκάτερος γὰρ τῶν εἰρημένων λόγος ὁ αὐτός ἐστιν τῶι τῆς ΚΘ πρὸς τὴν ΕΘ. ὥστε παράλληλός ἐστιν ἡ ΖΗ τῆι ΑΔ.

(202) |ἔστω βωμίσκος ὁ ΑΒΓΔΕΖΗ, καὶ ἔστω παράλληλος ἡ μὲν |160v ΔΕ τῆι ΒΓ, ἡ δὲ ΕΗ τῆι ΒΖ. ὅτι καὶ ἡ ΔΖ τῆι ΓΗ παράλληλός ἐστιν. ἐπεζεύχθωσαν αἱ ΒΕ, ΔΓ, ΖΗ. ἴσον ἄρα ἐστὶν τὸ ΔΒΕ τρίγωνον τῶι ΔΓΕ τριγώνωι. κοινὸν προσκείσθω τὸ ΔΑΕ τρίγωνον. ὅλον ἄρα τὸ ΑΒΕ τρίγωνον ὅλωι τῶι ΓΔΑ τριγώνωι ἴσον ἐστίν. πάλιν ἐπεὶ παράλληλός ἐστιν ἡ ΒΖ τῆι ΕΗ, ἴσον ἐστὶν τὸ ΒΖΕ τρίγωνον τῶι ΒΖΗ τριγώνωι. κοινὸν ἀφῃρήσθω τὸ ΑΒΖ τρίγωνον. λοιπὸν ἄρα τὸ ΑΒΕ τρίγωνον λοιπῶι τῶι ΑΗΖ

‖ 2 ΚΛ Co ΚΑ A | ΛΗ Co ΛΜ A | ΑΒ Co ΑΕ A ‖ 3 ΚΛ... ΛΗ Ge ΗΛ... ΛΜ A ‖ 4 καὶ λοιπη – τὴν ΛΗ del Co ‖ 5 ΚΛ... ΛΗ Ge ΚΜ... ΛΜ A | ΔΓ Co ΑΓ A | ἐναλλάξ Co ἀνάλογον A ‖ 6 ΗΛ Co ΗΔ A ‖ 7 ΑΒ] ΔΘ A ΑΔ Co ‖ 11 διαχθείσης τῆς ΕΒ θῶ] διὰ τὴν ΕΒ θῶ A ἐπὶ τῆς ΕΒ θῶ Hu τῆι ΕΒ προσθῶ Heiberg$_3$, del Co ‖ 14 ἑκάτερος Heiberg$_3$ ἑκάτερα A ἑκατέρων Hu ‖ 15 λόγωι Heiberg$_3$ λόγον A λόγος Ge (BS) ‖ 16 καὶ Co κατὰ A | ΔΒ, ΒΓ μέση Hu ΑΒ, ΒΓ μέση A ΑΒ, ΒΓ τρίτη Co ΓΒ, ΑΒ τρίτη Breton ‖ 17 ΒΑ Hu ΒΔ A | ἐκβεβλήσθω Co ἐκβληθεῖσα A | ΕΒ Co ΑΒ A ‖ 21 ΒΑ Co ΒΛ A ‖ 22 ΑΘ Co ΛΘ A ‖ 23 ΖΕ Co ΖΓ A | ἑκάτερος Heiberg$_3$ ἑκάτερα A ἑκατέρων Hu ‖ 24 ΕΘ] ΒΘ A ΘΕ Co ‖ 25 ΑΔ] ΑΓ Breton ‖ 26 ὁ] ἡ Ge ‖ 31 ἡ... τῆι] τῆι... ἡ coni Hu app ‖ 32 ἀφῃρήσθω Ge (BS) ἀφαιρησθω A triangle ABE equals triangle ΑΓΔ. Therefore triangle ΑΓΔ too equals triangle AZH.[6] Let triangle ΑΓΗ be added in common. Then all triangle ΓΔΗ equals all triangle ΓΖΗ.[7] And they are on the same base, ΓΗ. Hence (I, 39) ΓΗ is parallel to ΔΖ.[8]

(203) *(Prop. 135)* Let there be triangle ΑΒΓ, and let ΑΔ and AE be drawn through it, and let ZH be drawn parallel to ΒΓ, and let ΖΘΗ be inflected. Let ΔΘ be to ΘΕ as is ΒΘ to ΘΓ. That ΚΛ is parallel to ΒΓ.

For since ΔΘ is to ΘΕ as is ΒΘ to ΘΓ,[1] therefore remainder ΒΔ is to remainder ΓΕ as is ΔΘ to ΘΕ.[2] But as is ΒΔ to ΕΓ, so is ZM to NH.[3] <Hence as is ZM to NH,> so is ΔΘ to ΘΕ.[4] *Alternando* as is ZM to ΔΘ, so is NH to ΘΕ.[5] But as is ZM to ΔΘ, so is ZK to ΚΘ in parallels;[6] while as HN is to ΘΕ, so is ΗΛ to ΛΘ.[7] Therefore as is ZK to ΚΘ, so is ΗΛ to ΛΘ.[8] Thus ΚΛ is parallel to HZ,[9] and therefore also to ΓΒ.[10]

(204) *(Prop. 136)* Let two straight lines ΔΘ, ΘΕ be drawn onto two straight lines BAE, ΔΑΗ from point Θ. Let the rectangle contained by ΘΗ, ZE be to the rectangle contained by ΘΕ, ZH as is the rectangle contained by ΔΘ, ΒΓ to the rectangle contained by ΔΓ, ΒΘ. That the (line) through Γ, A, Z is straight.

Let ΚΛ be drawn through Θ parallel to ΓΑ,[1] and let it intersect AB and ΑΔ at points K and Λ. And let ΛΜ be drawn through Λ parallel to ΑΔ,[2] and let ΕΘ be produced to M. And let KN be drawn through K parallel to AB,[3] and let ΔΘ be produced to N.

Then since because of the parallels ΔΓ is to ΓΒ as is ΔΘ to ΘΝ,[4] therefore the rectangle contained by ΔΘ, ΓΒ equals the rectangle contained by ΔΓ, ΘΝ.[5] (Let) the rectangle contained by ΔΓ, ΒΘ (be) some other arbitrary quantity. Then as is the rectangle contained by ΔΘ, ΒΓ to the rectangle contained by ΔΓ, ΒΘ, so is the rectangle contained by ΓΔ, ΘΝ to the rectangle contained by ΔΓ, ΒΘ,[6] that is ΘΝ to ΘΒ.[7] But as is the rectangle contained by ΘΔ, ΒΓ to the rectangle contained by ΔΓ, ΒΘ, so was the rectangle contained by ΘΗ, ZE assumed to be to the rectangle contained by ΘΕ, ZH,[8] while as is ΘΝ to ΘΒ, so is ΚΘ to ΘΛ,[9] that is in parallels ΗΘ to ΘΜ,[10] that is the rectangle contained by ΘΗ, ZE to the rectangle contained by ΘΜ, ZE.[11] Hence as is the rectangle contained by ΘΗ, ZE to the rectangle contained by ΘΕ, ZH, so is the rectangle contained by ΘΗ, ZE to the rectangle contained by ΘΜ, ZE.[12] Therefore <the rectangle contained by ΘΕ, ZH> equals <the rectangle contained by ΘΜ, ZE.[13] In ratio, therefore,> as is ΜΘ to ΘΕ, so is HZ to ZE.[14] *Componendo*[15] and *alternando* as is ME to EH, so is ΘΕ to EZ.[16] But ΛΕ is to EA as is ME to EH.[17] Therefore as is ΛΕ to EA, so is ΘΕ to EZ.[18] Hence AZ is parallel to ΚΛ.[19] But ΓΑ is also (parallel) to (ΚΛ).[20] Thus ΓΑΖ is straight.[21] Q.E.D.

τριγώνωι ἴσον ἐστίν. ἀλλὰ τὸ ΑΒΕ τρίγωνον τῶι ΑΓΔ τριγώνωι ἐστὶν ἴσον. καὶ τὸ ΑΓΔ ἄρα τρίγωνον τῶι ΑΖΗ τριγώνωι ἴσον ἐστίν. κοινὸν προσκείσθω τὸ ΑΓΗ τρίγωνον. ὅλον ἄρα τὸ ΓΔΗ τρίγωνον ὅλωι τῶι ΓΖΗ τριγώνωι ἴσον ἐστίν. καὶ ἔστιν ἐπὶ τῆς αὐτῆς βάσεως τῆς ΓΗ. παράλληλος ἄρα ἐστὶν ἡ ΓΗ τῆι ΔΖ.

(203) ἔστω τρίγωνον τὸ ΑΒΓ, καὶ ἐν αὐτῶι διήχθωσαν αἱ ΑΔ, ΑΕ, καὶ τῆι ΒΓ παράλληλος ἤχθω ἡ ΖΗ, καὶ κεκλάσθω ἡ ΖΘΗ. ἔστω δὲ ὡς ἡ ΒΘ πρὸς τὴν ΘΓ, οὕτως ἡ ΔΘ πρὸς τὴν ΘΕ. ὅτι παράλληλός ἐστιν ἡ ΚΛ τῆι ΒΓ. ἐπεὶ γάρ ἐστιν ὡς ἡ ΒΘ πρὸς τὴν ΘΓ, οὕτως ἡ ΔΘ πρὸς τὴν ΘΕ, λοιπὴ ἄρα ἡ ΒΔ πρὸς λοιπὴν τὴν ΓΕ ἐστὶν ὡς ἡ ΔΘ πρὸς τὴν ΘΕ. ὡς δὲ ἡ ΒΔ πρὸς τὴν ΕΓ, οὕτως ἐστὶν ἡ ΖΜ πρὸς τὴν ΝΗ. <καὶ ὡς ἄρα ἡ ΖΜ πρὸς ΝΗ,> 880 οὕτως ἐστὶν ἡ ΔΘ πρὸς τὴν ΘΕ. ἐναλλάξ ἐστιν ὡς ἡ ΖΜ πρὸς τὴν ΔΘ, οὕτως ἡ ΝΗ πρὸς τὴν ΘΕ. ἀλλ' ὡς μὲν ἡ ΖΜ πρὸς τὴν ΔΘ, οὕτως ἐστὶν ἐν παραλλήλωι ἡ ΖΚ πρὸς τὴν ΚΘ· ὡς δὲ ἡ ΗΝ πρὸς τὴν ΘΕ, οὕτως ἐστὶν ἡ ΗΛ πρὸς τὴν ΛΘ. καὶ ὡς ἄρα ἡ ΖΚ πρὸς τὴν ΚΘ, οὕτως ἐστὶν ἡ ΗΛ πρὸς τὴν ΛΘ. παράλληλος ἄρα ἐστὶν ἡ ΚΛ τῆι ΗΖ. ὥστε καὶ τῆι ΓΒ.

(204) |εἰς δύο εὐθείας τὰς ΒΑΕ, ΔΑΗ ἀπὸ τοῦ Θ σημείου δύο διήχθωσαν εὐθεῖαι αἱ ΔΘ, ΘΕ. ἔστω δὲ ὡς τὸ ὑπὸ τῶν ΔΘ, ΒΓ |161 πρὸς τὸ ὑπὸ ΔΓ, ΒΘ, οὕτως τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΕ, ΖΗ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Γ, Α, Ζ. ἤχθω διὰ τοῦ Θ τῆι ΓΑ παράλληλος ἡ ΚΛ, καὶ συμπιπτέτω ταῖς ΑΒ, ΑΔ κατὰ τὰ Κ, Λ σημεῖα. καὶ διὰ τοῦ Λ τῆι ΑΔ παράλληλος ἤχθω ἡ ΛΜ, καὶ ἐκβεβλήσθω ἡ ΕΘ ἐπὶ τὸ Μ. διὰ δὲ τοῦ Κ τῆι ΑΒ παράλληλος ἤχθω ἡ ΚΝ, καὶ ἐκβεβλήσθω ἡ ΔΘ ἐπὶ τὸ Ν. ἐπεὶ οὖν διὰ τὰς παραλλήλους γίνεται ὡς ἡ ΔΘ πρὸς τὴν ΘΝ, οὕτως ἡ ΔΓ πρὸς τὴν ΓΒ, τὸ ἄρα ὑπὸ τῶν ΔΘ, ΓΒ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΔΓ, ΘΝ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ ΔΓ, ΒΘ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΔΘ, ΒΓ πρὸς τὸ ὑπὸ ΔΓ, ΒΘ, οὕτως τὸ ὑπὸ ΓΔ, ΘΝ πρὸς τὸ ὑπὸ ΔΓ, ΒΘ, τουτέστιν ἡ ΘΝ πρὸς ΘΒ. ἀλλ' ὡς μὲν τὸ ὑπὸ ΘΔ, ΒΓ πρὸς τὸ 882 ὑπὸ ΔΓ, ΒΘ ὑπόκειται τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΕ, ΖΗ. ὡς δὲ ἡ ΘΝ πρὸς ΘΒ, οὕτως ἡ ΚΘ πρὸς ΘΛ, τουτέστιν ἐν παραλλήλωι ἡ

‖ 2 ἐστὶν –ΑΖΗ τριγώνωι om A¹ add mg A² ‖ 6 ἡ ... τῆι] τῆι ... ἡ coni. Hu app ‖ 8 ΖΘΗ Co ΖΗ A ‖ 11 λοιπὴ Ge (BS) λοιπὸν A ‖ 13 τὴν (ΝΗ) om Hu | καὶ – ΝΗ add Co ‖ 17 καὶ ὡς ἄρα – ΛΘ tris A corr Co ‖ 21 διήχθωσαν Ge (BS) διήχθω A ‖ 27 ἐκβεβλήσθω Hu ἐκβληθῆι A ‖ 28 παραλλήλους Ge (S) παραλληλα A ‖ 29 ΘΝ Co ΘΗ A

The characteristics of the cases of this (proposition are) as the foregoing ones, of which it is the converse.

(205) *(Prop. 137)* Triangle ABΓ, and AΔ parallel to BΓ, and let ΔE be drawn through and intersect BΓ at point E. That ΓB is to BE as is the rectangle contained by ΔE, ZH to the rectangle contained by EZ, HΔ.

Let ΓΘ be drawn through Γ parallel to ΔE,[1] and let AB be produced to Θ. Then since ΓΘ is to ZH as is ΓA to AH,[2] while EΔ is to ΔH as is ΓA to AH,[3] therefore ΘΓ is to ZH as is EΔ to ΔH.[4] Hence the rectangle contained by ΓΘ, ΔH equals the rectangle contained by EΔ, ZH.[5] (Let) the rectangle contained by EZ, HΔ (be) some other arbitrary quantity. Then as is the rectangle contained by ΔE, ZH to the rectangle contained by ΔH, EZ, so is the rectangle contained by ΓΘ, ΔH to the rectangle contained by ΔH, EZ,[6] that is ΓΘ to EZ,[7] that is ΓB to BE.[8] Thus as is the rectangle contained by ΔE, ZH to the rectangle contained by EZ, HΔ, so is ΓB to BE. The same if parallel AΔ is drawn on the other side, and the straight line (ΔE) is drawn through from Δ outside (the triangle) in the direction of Γ.

(206) *(Prop. 138)* Now that these things have been proved, let it be required to prove that, if AB and ΓΔ are parallel, and some straight lines AΔ, AZ, BΓ, BZ intersect them, and EΔ and EΓ are joined, it results that the (line) through H, M, and K is straight.

For since ΔAZ is a triangle, and AE is parallel to ΔZ,[1] and EΓ has been drawn through intersecting ΔZ at Γ, by the foregoing (lemma) it turns out that as ΔZ is to ZΓ, so is the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE.[2] Again, since ΓBZ is a triangle, and BE

ΗΘ πρὸς τὴν ΘΜ, τουτέστιν τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΜ, ΖΕ. καὶ ὡς ἄρα τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΕ, ΖΗ, οὕτως ἐστὶν τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΜ, ΖΕ. ἴσον ἄρα ἐστὶν <τὸ ὑπὸ ΘΕ, ΖΗ τῶι ὑπὸ ΘΜ, ΖΕ. ἀνάλογον ἄρα ἐστὶν> ὡς ἡ ΜΘ πρὸς τὴν ΘΕ, οὕτως ἡ ΗΖ πρὸς τὴν ΖΕ. συνθέντι καὶ ἐναλλάξ ἐστιν ὡς ἡ ΜΕ πρὸς τὴν ΕΗ, οὕτως ἡ ΘΕ πρὸς τὴν ΕΖ. ἀλλ' ὡς ἡ ΜΕ πρὸς τὴν ΕΗ, οὕτως ἐστὶν ἡ ΛΕ πρὸς τὴν ΕΑ. καὶ ὡς ἄρα ἡ ΛΕ πρὸς τὴν ΕΑ, οὕτως ἡ ΘΕ πρὸς τὴν ΕΖ. παράλληλος ἄρα ἐστὶν ἡ ΑΖ τῆι ΚΛ. ἀλλὰ καὶ ἡ ΓΑ. εὐθεῖα ἄρα ἐστὶν ἡ ΓΑΖ. ὅ(περ):—

τὰ δὲ πτωτικὰ αὐτοῦ ὁμοίως τοῖς προγεγραμμένοις, ὧν ἐστιν ἀναστρόφιον.

(205) τρίγωνον τὸ ΑΒΓ, καὶ τῆι ΒΓ παράλληλος ἡ ΑΔ, καὶ διαχθεῖσα ἡ ΔΕ τῆι ΒΓ συμπιπτέτω κατὰ τὸ Ε σημεῖον. ὅτι ἐστὶν ὡς τὸ ὑπὸ ΔΕ, ΖΗ πρὸς τὸ ὑπὸ ΕΖ, ΗΔ, οὕτως ἡ ΓΒ πρὸς τὴν ΒΕ. ἤχθω διὰ τοῦ Γ τῆι ΔΕ παράλληλος ἡ ΓΘ, καὶ ἐκβεβλήσθω ἡ ΑΒ ἐπὶ τὸ Θ. ἐπεὶ οὖν ἐστιν |ὡς ἡ ΓΑ πρὸς τὴν |161v ΑΗ, οὕτως ἡ ΓΘ πρὸς τὴν ΖΗ, ὡς δὲ ἡ ΓΑ πρὸς τὴν ΑΗ, οὕτως ἐστὶν ἡ ΕΔ πρὸς τὴν ΔΗ, καὶ ὡς ἄρα ἡ ΕΔ πρὸς τὴν ΔΗ, οὕτως ἐστὶν ἡ ΘΓ πρὸς τὴν ΖΗ. τὸ ἄρα ὑπὸ τῶν ΓΘ, ΔΗ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΕΔ, ΖΗ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ ΕΖ, ΗΔ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΔΕ, ΖΗ πρὸς τὸ ὑπὸ ΔΗ, ΕΖ, οὕτως τὸ ὑπὸ ΓΘ, ΔΗ πρὸς τὸ 884 ὑπὸ ΔΗ, ΕΖ, τουτέστιν ἡ ΓΘ πρὸς ΕΖ, τουτέστιν ἡ ΓΒ πρὸς ΒΕ. ἔστιν οὖν ὡς τὸ ὑπὸ ΔΕ, ΖΗ πρὸς τὸ ὑπὸ ΕΖ, ΗΔ, οὕτως ἡ ΓΒ πρὸς ΒΕ. τὰ δ' αὐτὰ κἂν ἐπὶ τὰ ἕτερα μέρη ἀχθῆι ἡ ΑΔ παράλληλος, καὶ ἀπὸ τοῦ Δ ἐκτὸς ὡς ἐπὶ τὸ Γ διαχθῆι ἡ εὐθεῖα.

(206) ἀποδεδειγμένων νῦν τούτων, ἔστω δεῖξαι ὅτι ἐὰν παράλληλοι ὦσιν αἱ ΑΒ, ΓΔ, καὶ εἰς αὐτὰς ἐμπίπτωσιν εὐθεῖαί τινες αἱ ΑΔ, ΑΖ, ΒΓ, ΒΖ, καὶ ἐπιζευχθῶσιν αἱ ΕΔ, ΕΓ, ὅτι γίνεται εὐθεῖα ἡ διὰ τῶν Η, Μ, Κ. ἐπεὶ γὰρ τρίγωνον τὸ ΔΑΖ, καὶ τῆι ΔΖ παράλληλος ἡ ΑΕ, καὶ διῆκται ἡ ΕΓ συμπίπτουσα τῆι ΔΖ κατὰ τὸ Γ, διὰ τὸ προγεγραμμένον γίνεται ὡς ἡ ΔΖ πρὸς τὴν ΖΓ, οὕτως τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ. πάλιν

‖ 3 τὸ ὑπὸ ΘΕ, ΖΗ — ἄρα ἐστὶν ὡς] τὸ ὑπὸ ΘΕ, ΖΗ τῶι ὑπὸ ΘΜ, ΘΕ. καὶ ὡς ἄρα add Co ‖ 9 ΓΑ Co ΓΔ A | ΓΑΖ ὅ(περ) Ge (V) ΓΑΖΟ ο: A ‖ 25 ἐκτὸς — εὐθεῖα Heiberg₃ ἐκτὸς ὡς ἐπὶ τὸ Γ διὰ τὴν εὐθεῖαν A ἐκτὸς τοῦ Γ ὡς ἐπὶ τὸ Ε ἀχθῆι ἡ ΔΕ Co, quorum ὡς ἐπὶ τὸ Ε del Hu ‖ 27 νῦν] οὖν coni. Hu app | ἔστω] ἔσται A | ὅτι del Ge ‖ 29 ὅτι secl Hu has been drawn parallel to ΓΔ,[3] and ΔE has been drawn through intersecting ΓZΔ at Δ, it turns out that as ΓZ is to ZΔ, so is the rectangle contained by ΔE, ΛK to the rectangle contained by ΔK, ΛE.[4] By inversion, therefore, as ΔZ is to ZΓ, so is the rectangle contained by ΔK, ΛE to the rectangle contained by ΔE, ΛK.[5] But also as ΔZ is to ZΓ, so was the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE. Therefore as the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE, so is the rectangle contained by ΔK, ΛE to the rectangle contained by ΔE, KΛ.[6] This has been reduced to the (lemma) before last. Then since two straight lines EΓ, EΔ have been drawn onto two straight lines ΓMΛ, ΔMΘ, and as the rectangle contained by ΓE, HΘ is to the rectangle contained by ΓH, ΘE, so is the rectangle contained by ΔK, EΛ to the rectangle contained by ΔE, ΛK, therefore the (line) through H, M, K is straight;[7] for this was proved before (lemma 7.204).

(207) *(Prop. 139)* But now let AB and ΓΔ not be parallel, but let them intersect at N. That again the (line) through H, M, and K is straight.

Since two (straight lines) ΓE and ΓΔ have been drawn through from the same point Γ onto three straight lines AN, AZ, AΔ, it turns out that as is the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE, so is the rectangle contained by ΓN, ZΔ to the rectangle contained by NΔ, ΓZ (lemma 7.196).[1] Again, since two (straight lines) ΔE, ΔN have been drawn through from the same point Δ onto three straight lines BN, BΓ, ΓZ, as is the rectangle contained by NΓ, ZΔ to the rectangle contained by NΔ, ZΓ, so is the rectangle contained by ΔK, EΛ to the rectangle contained by ΔE, KΛ.[2] But as is the rectangle contained by NΓ, ZΔ to the rectangle contained by NΔ, ΓZ, so the rectangle contained by ΓE, HΘ was proved to be to the rectangle contained by ΓH, ΘE. Therefore as is the rectangle contained by ΓE, ΘH to the rectangle contained by ΓH, ΘE, so is the rectangle contained by ΔK, EΛ to the rectangle contained by ΔE, KΛ.[3] It has been reduced to the (lemma) which (it was reduced to) also in the case of the parallels. Because of the foregoing (lemma 7.204) the (line) through H, M, K is straight.[4]

(208) *(Prop. 140)* Let AB be parallel to ΓΔ, and let AE and ΓB be drawn through, and (let) Z (be) a point on BH, so that as is ΔE to EΓ, so will the rectangle contained by ΓB, HZ be to the rectangle contained by ZB, ΓH. That the (line) through A, Z, Δ is straight.

Let ΔΘ be drawn through Δ parallel to BΓ,[1] and let AE be produced to Θ; and let ΘK be drawn through Θ parallel to ΓΔ,[2] and let BΓ be produced to K. Then since as is ΔE to EΓ, so is the rectangle contained by ΓB, ZH to the rectangle contained by BZ, ΓH (lemma 7.205),[4] while as is ΔE to EΓ, so are ΔΘ to ΓH and (consequently) the rectangle contained by ΔΘ, BZ to the rectangle contained by ΓH, BZ,[3] therefore the rectangle contained by BΓ, ZH equals the rectangle contained by ΔΘ, BZ.[5] Hence in ἐπεὶ τρίγωνόν ἐστιν τὸ ΓΒΖ, καὶ τῆι ΓΔ παράλληλος ἦκται ἡ ΒΕ, καὶ διῆκται ἡ ΔΕ συμπίπτουσα τῆι ΓΖΔ κατὰ τὸ Δ, γίνεται ὡς ἡ ΓΖ πρὸς τὴν ΖΔ, οὕτως τὸ ὑπὸ ΔΕ, ΛΚ πρὸς τὸ ὑπὸ ΔΚ, ΛΕ. ἀνάπαλιν ἄρα γίνεται ὡς ἡ ΔΖ πρὸς τὴν ΖΓ, οὕτως τὸ ὑπὸ ΔΚ, ΛΕ πρὸς τὸ ὑπὸ ΔΕ, ΛΚ. ἦν δὲ καὶ ὡς ἡ ΔΖ πρὸς τὴν ΖΓ, οὕτως τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ. καὶ ὡς ἄρα τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ, οὕτως ἐστὶν τὸ ὑπὸ ΔΚ, ΛΕ πρὸς τὸ ὑπὸ ΔΕ, ΚΛ. ἀπῆκται εἰς τὸ πρὸ ἑνός. ἐπεὶ οὖν εἰς δύο εὐθείας τὰς ΓΜΛ, ΔΜΘ, δύο εὐθεῖαι διηγμέναι εἰσὶν αἱ ΕΓ, ΕΔ, καί ἐστιν ὡς τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ, οὕτως τὸ ὑπὸ ΔΚ, ΕΛ πρὸς τὸ ὑπὸ ΔΕ, ΛΚ, εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Η, Μ, Κ. τοῦτο γὰρ προδέδεικται.

(207) ἀλλὰ δὴ μὴ ἔστωσαν αἱ ΑΒ, ΓΔ παράλληλοι, ἀλλὰ συμπιπτέτωσαν κατὰ τὸ Ν. ὅτι πάλιν εὐθεῖά ἐστιν ἡ διὰ τῶν Η, Μ, Κ. ἐπεὶ εἰς τρεῖς εὐθείας τὰς ΑΝ, ΑΖ, ΑΔ ἀπὸ τοῦ αὐτοῦ |σημείου τοῦ Γ, δύο διηγμέναι εἰσὶν αἱ ΓΕ, ΓΔ, γίνεται ὡς τὸ |162 ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ, οὕτως τὸ ὑπὸ τῶν ΓΝ, ΖΔ πρὸς τὸ ὑπὸ τῶν ΝΔ, ΓΖ. πάλιν ἐπεὶ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ εἰς τρεῖς εὐθείας τὰς ΒΝ, ΒΓ, ΓΖ δύο εἰσὶν διηγμέναι αἱ ΔΕ, ΔΝ, ἔστιν ὡς τὸ ὑπὸ ΝΓ, ΖΔ πρὸς τὸ ὑπὸ ΝΔ, ΖΓ, οὕτως τὸ ὑπὸ ΔΚ, ΕΛ πρὸς τὸ ὑπὸ ΔΕ, ΚΛ. ἀλλ' ὡς τὸ ὑπὸ ΝΓ, ΖΔ πρὸς τὸ ὑπὸ ΝΔ, ΓΖ, οὕτως ἐδείχθη τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ. καὶ ὡς ἄρα τὸ ὑπὸ ΓΕ, ΘΗ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ, οὕτως ἐστὶν τὸ ὑπὸ ΔΚ, ΕΛ πρὸς τὸ ὑπὸ ΔΕ, ΚΛ. ἀπῆκται εἰς ὃ καὶ ἐπὶ τῶν παραλλήλων. διὰ δὴ τὸ προγεγραμμένον εὐθεῖά ἐστιν ἡ διὰ τῶν Η, Μ, Κ.

(208) ἔστω παράλληλος ἡ ΑΒ τῆι ΓΔ, καὶ διήχθωσαν αἱ ΑΕ, ΓΒ, καὶ σημεῖον ἐπὶ τῆς ΒΗ τὸ Ζ, ὥστε εἶναι ὡς τὴν ΔΕ πρὸς τὴν ΕΓ, οὕτως τὸ ὑπὸ ΓΒ, ΗΖ πρὸς τὸ ὑπὸ ΖΒ, ΓΗ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Ζ, Δ. ἤχθω διὰ μὲν τοῦ Δ τῆι ΒΓ παράλληλος ἡ ΔΘ, καὶ ἐκβεβλήσθω ἡ ΑΕ ἐπὶ τὸ Θ, διὰ δὲ τοῦ Θ τῆι ΓΔ παράλληλος ἡ ΘΚ, καὶ ἐκβεβλήσθω ἡ ΒΓ ἐπὶ τὸ Κ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΔΕ πρὸς τὴν ΕΓ, οὕτως τὸ ὑπὸ ΓΒ, ΖΗ πρὸς τὸ ὑπὸ ΒΖ, ΓΗ, ὡς δὲ ἡ ΔΕ πρὸς τὴν ΕΓ, οὕτως ἐστὶν ἥ τε ΔΘ πρὸς τὴν ΓΗ καὶ τὸ ὑπὸ ΔΘ, ΒΖ πρὸς τὸ ὑπὸ τῶν ΓΗ, ΒΖ, ἴσον ἄρα

‖ 1 ΓΔ] ΓΖ coni. Hu app ‖ 8 ἀπῆκται Hu p. 1263 ἀνῆκται A | ἀπῆκται — ἑνος secl Hu ‖ 9 ΓΜΛ Co ΓΜΔ A ‖ 10 ΘΗ Co ΓΕ A | ΕΛ A² ex ΔΛ ‖ 14 Ν Co Η A ‖ 16 Γ Co Κ A | ΓΔ Co ΝΔ A ‖ 17 ΓΝ Co ΓΗ A ‖ 24 ἀπῆκται] ἀνῆκται Ge | ἀπῆκται — παραλλήλων secl Hu | ὃ καὶ] τὸ δέκατον coni. Hu app ‖ 27 ἐπὶ Ge (BS) ἐπεὶ A | ΒΗ Co ΖΗ A ‖ 30 ἐκβεβλήσθω Ge ἐκβληθῆι A ‖ 33 ΒΖ, ΓΗ Heiberg₃ ΒΓ, ΖΗ A ΖΒ, ΓΗ Co | ἐστὶν del coni. Hu app ratio as ΓB is to BZ, so is ΔΘ, that is ΓK,[7] to HZ.[6] Hence the sum KB is to the sum BH as KΓ is to ZH,[8] that is as ΔΘ is to ZH.[9] But as is KB to BH, so in parallels are ΘA to AH, and ΔΘ to ZH.[10] And ΔΘ and ZH are parallel.[11] Thus the (line) through points A, Z, Δ is straight.[12]

(209) *(Prop. 141)* Now that this has been proved, let AB be parallel to ΓΔ, and let straight lines AZ, ZB, ΓE, EΔ intersect them, and let BΓ and HK be joined. That the (line) through A, M, Δ is straight.

Let ΔM be joined and produced to Θ. Then since, having a triangle BΓZ, BE has been drawn parallel to ΓΔ from the apex point B (and falling) outside (the triangle), and ΔE has been drawn through, it turns out (lemma 7.205) that as ΓZ is to ZΔ, so is the rectangle contained by ΔE, KΛ to the rectangle contained by EΛ, KΔ.[1] Thus as the rectangle contained by ΔE, KΛ is to the rectangle contained by ΔK, ΛE, so is the rectangle contained by ΓH, ΘE to the rectangle contained by ΓE, HΘ (lemma 7.196);[2] for two (straight lines) EΓ, EΔ have been drawn through from the same point E onto three straight lines ΓΛ, ΔΘ, HK. And so as is ΔZ to ZΓ, so is the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE.[3] And the (line) through H, M, K is straight.[4] Hence by the foregoing (lemma 7.208) the (line) through A, M, Δ is also straight.[5]

(210) *(Prop. 142 a – b)* Let two (straight lines) ΔB, ΔE be drawn across two straight lines AB, AΓ from the same point Δ, and let points H, Θ be chosen on them. And as is the rectangle contained by EH, ZΔ to the rectangle contained by ΔE, HZ, so let the rectangle contained by BΘ, ΓΔ be to the rectangle contained by BΔ, ΓΘ. That the (line) through A, H, Θ is straight.

Let KΛ be drawn through H parallel to BΔ.[1] Then since as the rectangle contained by EH, ZΔ is to the rectangle contained by ΔE, ZH, so is the rectangle contained by BΘ, ΓΔ to the rectangle contained by BΔ, ΓΘ,[2] while the ratio of the rectangle contained by EH, ZΔ to the rectangle contained by ΔE, HZ is compounded out of that which HE has to EΔ, that is KH to BΔ,[4] and that which ΔZ has to ZH, that is ΔΓ to HΛ;[5] [3] and the ratio of the rectangle contained by BΘ, ΓΔ to the rectangle contained by BΔ, ΓΘ is compounded out of that which ΘB has to BΔ and that which ΔΓ has to ΓΘ,[6] therefore the (ratio compounded) out of that of KH to BΔ and that of ΔΓ to HΛ is the same as that compounded out of that of BΘ to BΔ and that of ΔΓ to ΓΘ.[7] But the (ratio) of KH to BΔ is compounded out of that of KH to BΘ and that of BΘ to BΔ.[8] Therefore the (ratio) compounded

ἐστὶν τὸ ὑπὸ τῶν ΒΓ, ΖΗ τῶι ὑπὸ ΔΘ, ΒΖ. ἀνάλογον ἄρα ἐστὶν 888
ὡς ἡ ΓΒ πρὸς τὴν ΒΖ, οὕτως ἡ ΔΘ, τουτέστιν ὡς ἡ ΓΚ, πρὸς τὴν ΗΖ. καὶ ὅλη ἄρα ἡ ΚΒ πρὸς ὅλην τὴν ΒΗ ἐστὶν ὡς ἡ ΚΓ πρὸς ΖΗ, τουτέστιν ὡς ἡ ΔΘ πρὸς ΖΗ. ἀλλ' ὡς ἡ ΚΒ πρὸς ΒΗ, ἐν παραλλήλωι οὕτως ἐστὶν ἡ ΘΑ πρὸς ΑΗ, καὶ ἡ ΔΘ πρὸς ΖΗ. καὶ εἰσὶν παράλληλοι αἱ ΔΘ, ΖΗ. εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α, Ζ, Δ σημείων.

(209) τούτου προτεθεωρημένου ἔστω παράλληλος ἡ ΑΒ τῆι ΓΔ, καὶ εἰς αὐτὰς ἐμπιπτέτωσαν εὐθεῖαι ΑΖ, ΖΒ, ΓΕ, ΕΔ, καὶ ἐπεζεύχθωσαν αἱ ΒΓ, ΗΚ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Μ, Δ. ἐπιζευχθεῖσα ἡ ΔΜ ἐκβεβλήσθω ἐπὶ τὸ Θ. ἐπεὶ οὖν τριγώνου τοῦ ΒΓΖ ἐκτὸς ἀπὸ τῆς κορυφῆς τοῦ Β |σημείου τῆι ΓΔ |162v
παράλληλος ἧκται ἡ ΒΕ, καὶ διῆκται ἡ ΔΕ, γίνεται ὡς ἡ ΓΖ πρὸς ΖΔ, οὕτως τὸ ὑπὸ ΔΕ, ΚΛ πρὸς τὸ ὑπὸ ΕΛ, ΚΔ. ὡς ἄρα τὸ ὑπὸ ΔΕ, ΚΛ πρὸς τὸ ὑπὸ ΔΚ, ΛΕ, οὕτως ἐστὶν τὸ ὑπὸ ΓΗ, ΘΕ πρὸς τὸ ὑπὸ ΓΕ, ΗΘ· εἰς τρεῖς <γὰρ> εὐθείας τὰς ΓΛ, ΔΘ, ΗΚ δύο εἰσὶν διηγμέναι ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Ε αἱ ΕΓ, ΕΔ. καὶ ὡς ἄρα ἡ ΔΖ πρὸς ΖΓ, οὕτως ἐστὶν τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ. καὶ ἔστιν εὐθεῖα ἡ διὰ τῶν Η, Μ, Κ. διὰ τὸ προγεγραμμένον ἄρα καὶ ἡ διὰ τῶν Α, Μ, Δ ἐστὶν εὐθεῖα.

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(210) εἰς δύο εὐθείας τὰς ΑΒ, ΑΓ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ δύο διήχθωσαν αἱ ΔΒ, ΔΕ, καὶ ἐπ' αὐτῶν εἰλήφθω σημεῖα τὰ Η, Θ. ἔστω δὲ ὡς τὸ ὑπὸ ΕΗ, ΖΔ πρὸς τὸ ὑπὸ ΔΕ, ΗΖ, οὕτως τὸ ὑπὸ ΒΘ, ΓΔ πρὸς τὸ ὑπὸ ΒΔ, ΓΘ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Η, Θ. ἤχθω διὰ τοῦ Η τῆι ΒΔ παράλληλος ἡ ΚΛ. ἐπεὶ οὖν ἐστιν ὡς τὸ ὑπὸ ΕΗ, ΖΔ πρὸς τὸ ὑπὸ ΔΕ, ΖΗ, οὕτως τὸ ὑπὸ ΒΘ, ΓΔ πρὸς τὸ ὑπὸ ΒΔ, ΓΘ, ἀλλ' <ὁ τοῦ> ὑπὸ ΕΗ, ΖΔ πρὸς τὸ ὑπὸ ΔΕ, ΗΖ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΗΕ πρὸς ΕΔ, τουτέστιν ἡ ΚΗ πρὸς ΒΔ, καὶ ἐξ οὗ ὃν ἔχει ἡ ΔΖ πρὸς ΖΗ, τουτέστιν ἡ ΔΓ πρὸς τὴν ΗΛ, ὁ δὲ τοῦ ὑπὸ ΒΘ, ΓΔ πρὸς τὸ ὑπὸ ΒΔ, ΓΘ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΘΒ πρὸς ΒΔ καὶ ἐξ οὗ ὃν ἔχει ἡ ΔΓ πρὸς ΓΘ, καὶ ὁ <ἔκ τε τοῦ> τῆς ΚΗ ἄρα πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΗΛ ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΒΘ πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΓΘ. ὁ δὲ τῆς ΚΗ πρὸς ΒΔ

‖ 2 post ΔΘ add πρὸς τὴν ΗΖ Hu ‖ 3 ὅλη Ge (BS) ὅλῃι A ‖ 6 εὐθεῖα Ge (S) εὐθειαι A ‖ 10 ἐπεζεύχθωσαν Ge (BS) ἐπεζεύχθω A | Α, Μ, Δ Co ΗΜΚ A ‖ 11 ἐπιζευχθεῖσα ἡ ΔΜ] ἐπεζεύχθω ἡ ΔΜ A post quae add καὶ Co | Θ Co Κ A ‖ 12 ἐκτὸς secl Hu (Simson$_1$) ‖ 13 ΔΕ Co ΔΒ A ‖ 14 ΖΔ Co ΖΓ A | ἄρα] δὲ A ‖ 15 ΛΕ Co ΛΒ A ‖ 16 γὰρ add Hu ‖ 19 καὶ – Η, Μ, Κ del Heiberg$_3$ | Η, Μ, Κ] Δ, Μ, Θ Co Θ, Μ, Δ Hu ‖ 20 καὶ del Heiberg$_3$ ‖ 22 διήχθωσαν Ge (BS) διήχθη A ‖ 23 δὲ Hu δὴ A ‖ 27 ἀλλ'] ἀλλὰ A ὁ τοῦ add Ge (BS) ‖ 32 ΓΘ Co ΓΕ A | ἔκ τε τοῦ add Hu ‖ 34 ΒΔ Co ΘΔ A | ΔΓ Co ΑΓ A out of that of KH to BΘ and that of BΘ to BΔ and furthermore of that of ΔΓ to HΛ is the same as the (ratio) compounded out of that of BΘ to BΔ and that of ΔΓ to ΓΘ.[9] Let the ratio of ΘB to BΔ be removed in common. Then the remaining (ratio) compounded out of that of KH to BΘ and that of ΔΓ to HΛ is the same as that of ΔΓ to ΓΘ,[10] that is the (ratio) compounded out of that of ΔΓ to HΛ and that of HΛ to ΘΓ.[11] And again, let the ratio of ΔΓ to HΛ be removed in common. Then the remaining ratio of KH to BΘ is the same as that of HΛ to ΘΓ.[12] And *alternando,* as is KH to HΛ, so is BΘ to ΘΓ.[13] And KΛ and BΓ are parallel.[14] Therefore the (line) through points A, H, Θ is straight.[15]

(211) 18. *(Prop. 143)* But now let AB not be parallel to ΓΔ, but let it intersect it at N.

Then since two straight lines ΔE, ΔN have been drawn from the same point Δ across three straight lines BN, BΓ, BZ, as the rectangle contained by NΔ, ΓZ is to the rectangle contained by NΓ, ΔZ, so is the rectangle contained by ΔE, KΛ to the rectangle contained by EΛ, KΔ (lemma 7.196).[1] But as is the rectangle contained by EΔ, KΛ to the rectangle contained by EΛ, KΔ, so is the rectangle contained by EΘ, ΓH to the rectangle contained by EΓ, ΘH;[2] for again two (straight lines) EΓ, EΔ have been drawn from the same point E across three (straight lines) ΓΛ, ΔΘ, HK. Therefore as is the rectangle contained by EΘ, ΓH to the rectangle contained by EΓ, ΘH, so is the rectangle contained by NΔ, ΓZ to the rectangle contained by NΓ, ZΔ.[3] By the foregoing (lemma) the (line) through A, Θ, Δ is straight.[4] Thus the (line) through A, M, Δ too is straight.[5]

(212) *(Prop. 144)* (Let there be) triangle ABΓ, and let AΔ be drawn parallel to BΓ, and let ΔE, ZH be drawn across. And as the square of EB is to the rectangle contained by EΓ, ΓB, so let BH be to HΓ. That, if BΔ is joined, the (line) through Θ, K, Γ is straight.

Since, as is the square of EB to the rectangle contained by EΓ, ΓB, so is BH to HΓ,[1] let the ratio of ΓE to EB be applied in common, this being the same as that of the rectangle contained by EΓ, ΓB to the rectangle contained by EB, BΓ.[2] Then *ex aequali* the ratio of the square of EB to the rectangle contained by EB, BΓ, that is the (ratio) of EB to BΓ, is the same as the (ratio) compounded out of that of BH to HΓ and that of the rectangle contained by EΓ, ΓB to the rectangle contained by EB, BΓ,[3] which is the same as that of EΓ to EB.[4] Therefore the (ratio) of the square of EB to the συνῆπται ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΒΘ πρὸς ΒΔ. ὁ ἄρα συνημμένος ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΒΘ πρὸς ΒΔ, καὶ ἔτι τοῦ τῆς ΔΓ πρὸς ΗΛ ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΒΘ πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΓΘ. κοινὸς ἐκκεκρούσθω ὁ τῆς ΘΒ πρὸς ΒΔ λόγος. λοιπὸς ἄρα ὁ συνημμένος ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΔΓ πρὸς ΗΛ ὁ αὐτός ἐστιν τῶι τῆς ΔΓ πρὸς τὴν ΓΘ, τουτέστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΔΓ πρὸς τὴν ΗΛ καὶ τοῦ τῆς ΗΛ πρὸς τὴν ΘΓ. καὶ πάλιν κοινὸς ἐκκεκρούσθω ὁ τῆς ΔΓ πρὸς τὴν ΗΛ λόγος. λοιπὸς ἄρα ὁ τῆς ΚΗ πρὸς τὴν ΒΘ λόγος ὁ αὐτός ἐστιν τῶι τῆς ΗΛ πρὸς τὴν ΘΓ. |καὶ ἐναλλάξ ἐστιν ὡς ἡ ΚΗ πρὸς τὴν 892 ΗΛ, οὕτως ἡ ΒΘ πρὸς τὴν ΘΓ. καὶ εἰσὶν αἱ ΚΛ, ΒΓ παράλληλοι. |163 εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α, Η, Θ σημείων.

(211) ιη΄. ἀλλὰ δὴ μὴ ἔστω παράλληλος ἡ ΑΒ τῆι ΓΔ, ἀλλὰ συμπιπτέτω κατὰ τὸ Ν. ἐπεὶ οὖν ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ εἰς τρεῖς εὐθείας τὰς ΒΝ, ΒΓ, ΒΖ δύο εὐθεῖαι διηγμέναι εἰσὶν αἱ ΔΕ, ΔΝ, ἔστιν ὡς τὸ ὑπὸ ΝΔ, ΓΖ πρὸς τὸ ὑπὸ ΝΓ, ΔΖ, οὕτως τὸ ὑπὸ ΔΕ, ΚΛ πρὸς τὸ ὑπὸ ΕΛ, ΚΔ. ὡς δὲ τὸ ὑπὸ ΕΔ, ΚΛ πρὸς τὸ ὑπὸ ΕΛ, ΚΔ, οὕτως ἐστὶν τὸ ὑπὸ ΕΘ, ΓΗ πρὸς τὸ ὑπὸ ΕΓ, ΘΗ. πάλιν γὰρ εἰς τρεῖς τὰς ΓΛ, ΔΘ, ΗΚ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Ε δύο ἠγμέναι εἰσὶν αἱ ΕΓ, ΕΔ. καὶ ὡς ἄρα τὸ ὑπὸ ΕΘ, ΓΗ πρὸς τὸ ὑπὸ ΕΓ, ΘΗ, οὕτως τὸ ὑπὸ ΝΔ, ΓΖ πρὸς τὸ ὑπὸ ΝΓ, ΖΔ. διὰ τὸ προγεγραμμένον εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Θ, Δ. καὶ ἡ διὰ τῶν Α, Μ, Δ ἄρα εὐθεῖά ἐστιν.

(212) τρίγωνον τὸ ΑΒΓ, καὶ τῆι ΒΓ παράλληλος ἤχθω ἡ ΑΔ, καὶ διήχθωσαν αἱ ΔΕ, ΖΗ. ἔστω δὲ ὡς τὸ ἀπὸ ΕΒ πρὸς τὸ ὑπὸ ΕΓΒ, οὕτως ἡ ΒΗ πρὸς τὴν ΗΓ. ὅτι ἐὰν ἐπιζευχθῆι ἡ ΒΔ, γίνεται εὐθεῖα ἡ διὰ τῶν Θ, Κ, Γ. ἐπεί ἐστιν ὡς τὸ ἀπὸ τῆς ΕΒ πρὸς τὸ ὑπὸ ΕΓΒ, οὕτως ἡ ΒΗ πρὸς ΗΓ, κοινὸς ἄρα προσκείσθω ὁ τῆς ΓΕ πρὸς ΕΒ λόγος, ὁ αὐτὸς ὢν τῶι τοῦ ὑπὸ ΕΓΒ πρὸς τὸ ὑπὸ ΕΒΓ. δι' ἴσου ἄρα ὁ τοῦ ἀπὸ ΕΒ πρὸς τὸ ὑπὸ 894 ΕΒΓ λόγος, τουτέστιν ὁ τῆς ΕΒ πρὸς τὴν ΒΓ, ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΒΗ πρὸς ΗΓ καὶ τοῦ τοῦ ὑπὸ ΕΓΒ πρὸς τὸ ὑπὸ ΕΒΓ, ὅς ἐστιν ὁ αὐτὸς τῶι τῆς ΕΓ πρὸς ΕΒ. ὥστε ὁ

‖ 4 κοινὸς] κ° Α ‖ 9 κοινὸς] κ° Α ‖ 14 ιη΄ mg Α ‖ 17 ΔΝ Co ΔΗ Α | ΝΔ Co ΝΛ Α ‖ 21 ΕΔ Co ΕΑ Α ‖ 22 τὸ ὑπὸ ΝΔ, ΓΖ bis Α corr Co ‖ 23 post διὰ add δὴ Ge ‖ 26 τὸ (ἀπὸ ΕΒ) Ge (S) τὰ Α ‖ 29 κοινὸς Ge (S) κοινὸν Α | ἄρα secl Hu ‖ 33 τοῦ Hu τῶι Α del Ge rectangle contained by EB, BΓ is compounded out of that which BH has to HΓ and that which EΓ has to EB,[5] which is the same as that of the rectangle contained by EΓ, BH to the rectangle contained by EB, ΓH.[6] But as is EB to BΓ, so, by the foregoing lemma (7.205), is *the rectangle contained by ΔE, ZΘ to the rectangle contained by ΔZ, ΘE.[7] And therefore as is the rectangle contained by ΓE, BH to the rectangle contained by ΓH, EB, so is the rectangle contained by ΔE, ZΘ to the rectangle contained by ΔZ, ΘE.[8] * Therefore the (line) through Θ, K, Γ is straight;[9] for that is in the case-variants of the converses.

(213) *(Prop. 145)* Let two (straight lines) EZ, EB be drawn from some point E across three straight lines AB, AΓ, AΔ, and, as EZ is to ZH, so let ΘE be to ΘH. That also as BE is to BΓ, so is EΔ to ΔΓ.

Let ΛK be drawn through H parallel to BE.[1] Then since as is EZ to ZH, so is EΘ to ΘH,[2] but as is EZ to ZH, so is EB to HK,[3] while as is EΘ to ΘH, so is ΔE to HΛ,[4] therefore as is BE to HK, so is ΔE to HΛ.[5] *Alternando*, as is EB to EΔ, so is KH to HΛ.[6] But as is KH to HΛ, so is BΓ to ΓΔ.[7] Therefore as is BE to EΔ, so is BΓ to ΓΔ.[8] *Alternando*, as is EB to BΓ, so is EΔ to ΔΓ.[9] The case-variants likewise.

(214) *(Prop. 146)* Let there be two triangles ABΓ, ΔEZ that have angles A, Δ equal. That, as is the rectangle contained by BA, AΓ to the rectangle contained by EΔ, ΔZ, so is triangle ABΓ to triangle EΔZ.

Let perpendiculars BH, EΘ be drawn.[1] Then since angle A equals Δ, and H (equals) Θ,[2] therefore as is AB to BH, so is ΔE to EΘ.[3] But as AB is to BH, so is the rectangle contained by BA, AΓ to the rectangle contained by BH, AΓ,[4] while as is ΔE to EΘ, so is the rectangle contained by EΔ, ΔZ to the rectangle contained by EΘ, ΔZ.[5] Therefore as is the rectangle contained by BA, AΓ to the rectangle contained by BH, AΓ, so is the rectangle contained by EΔ, ΔZ to the rectangle contained by EΘ, ΔZ;[6] and *alternando*.[7] But as is the rectangle contained by BH, AΓ to the rectangle contained by EΘ, ΔZ, so is triangle ABΓ to triangle ΔEZ;[8] for each of BH and EΘ is a perpendicular of each of the triangles named. Therefore as is the rectangle contained by BA, AΓ to the rectangle contained by EΔ, ΔZ, so is triangle ABΓ to triangle ΔEZ.[9]

τοῦ ἀπὸ ΕΒ *πρὸς τὸ ὑπὸ* ΕΒΓ *συνῆπται ἔκ τε τοῦ ὃν ἔχει ἡ* ΒΗ *πρὸς* ΗΓ *καὶ τοῦ ὃν ἔχει ἡ* ΕΓ *πρὸς* ΕΒ, *ὅς ἐστιν ὁ αὐτὸς τῶι τοῦ ὑπὸ* ΕΓ, ΒΗ *πρὸς τὸ ὑπὸ* ΕΒ, ΓΗ. *ὡς δὲ ἡ* ΕΒ *πρὸς τὴν* ΒΓ, *οὕτως ἐστὶν διὰ τὸ προγεγραμμένον λῆμμα τὸ ὑπὸ* ΔΕ, ΖΘ *πρὸς τὸ ὑπὸ* ΔΖ, ΘΕ. *καὶ ὡς ἄρα τὸ ὑπὸ* ΓΕ, ΒΗ *πρὸς τὸ ὑπὸ* ΓΗ, ΕΒ, *οὕτως ἐστὶν τὸ ὑπὸ* ΔΕ, ΖΘ *πρὸς τὸ ὑπὸ* ΔΖ, ΘΕ. *εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν* Θ, Κ, Γ· *τοῦτο γὰρ ἐν τοῖς πτωτικοῖς τῶν ἀναστροφίων.*

(213) |*εἰς τρεῖς εὐθείας τὰς* ΑΒ, ΑΓ, ΑΔ *ἀπό τινος σημείου* |163v *τοῦ* Ε *δύο διήχθωσαν αἱ* ΕΖ, ΕΒ. *ἔστω δὲ ὡς ἡ* ΕΖ *πρὸς τὴν* ΖΗ, *οὕτως ἡ* ΘΕ *πρὸς τὴν* ΘΗ. *ὅτι γίνεται καὶ ὡς ἡ* ΒΕ *πρὸς τὴν* ΒΓ, *οὕτως ἡ* ΕΔ *πρὸς τὴν* ΔΓ. *ἤχθω διὰ τοῦ* Η *τῆι* ΒΕ *παράλληλος ἡ* ΛΚ. *ἐπεὶ οὖν ἐστιν ὡς ἡ* ΕΖ *πρὸς τὴν* ΖΗ, *οὕτως ἡ* ΕΘ *πρὸς τὴν* ΘΗ, *ἀλλ' ὡς μὲν ἡ* ΕΖ *πρὸς τὴν* ΖΗ, *οὕτως ἡ* ΕΒ *πρὸς τὴν* ΗΚ, *ὡς δὲ ἡ* ΕΘ *πρὸς τὴν* ΘΗ, *οὕτως ἐστὶν ἡ* ΔΕ *πρὸς τὴν* ΗΛ, *καὶ ὡς ἄρα ἡ* ΒΕ *πρὸς τὴν* ΗΚ, *οὕτως ἐστὶν ἡ* ΔΕ *πρὸς τὴν* ΗΛ. *ἐναλλάξ ἐστιν ὡς ἡ* ΕΒ *πρὸς τὴν* ΕΔ, *οὕτως ἡ* ΚΗ *πρὸς τὴν* ΗΛ. *ὡς δὲ ἡ* ΚΗ *πρὸς τὴν* ΗΛ, *οὕτως ἐστὶν ἡ* ΒΓ *πρὸς τὴν* ΓΔ. *καὶ ὡς ἄρα ἡ* ΒΕ *πρὸς τὴν* ΕΔ, *οὕτως ἡ* ΒΓ *πρὸς τὴν* ΓΔ. *ἐναλλάξ ἐστιν ὡς ἡ* ΕΒ *πρὸς τὴν* ΒΓ, *οὕτως ἡ* ΕΔ *πρὸς τὴν* ΔΓ. *τὰ δὲ πτωτικὰ ὁμοίως.*

(214) *ἔστω δύο τρίγωνα τὰ* ΑΒΓ, ΔΕΖ *ἴσας ἔχοντα τὰς* Α, Δ *γωνίας. ὅτι ἐστὶν ὡς τὸ ὑπὸ* ΒΑΓ *πρὸς τὸ ὑπὸ* ΕΔΖ, *οὕτως τὸ* ΑΒΓ *τρίγωνον πρὸς τὸ* ΕΔΖ *τρίγωνον. ἤχθωσαν κάθετοι αἱ* ΒΗ, 896 ΕΘ. *ἐπεὶ οὖν ἴση ἐστὶν ἡ μὲν* Α *γωνία τῆι* Δ, *ἡ δὲ* Η *τῆι* Θ, *ἔστιν ἄρα ὡς ἡ* ΑΒ *πρὸς τὴν* ΒΗ, *οὕτως ἡ* ΔΕ *πρὸς τὴν* ΕΘ. *ἀλλ' ὡς μὲν ἡ* ΑΒ *πρὸς τὴν* ΒΗ, *οὕτως ἐστὶν τὸ ὑπὸ* ΒΑΓ *πρὸς τὸ ὑπὸ* ΒΗ, ΑΓ, *ὡς δὲ ἡ* ΔΕ *πρὸς τὴν* ΕΘ, *οὕτως ἐστὶν τὸ ὑπὸ* ΕΔΖ *πρὸς τὸ ὑπὸ* ΕΘ, ΔΖ. *ἔστιν ἄρα ὡς τὸ ὑπὸ* ΒΑΓ *πρὸς τὸ ὑπὸ* ΒΗ, ΑΓ, *οὕτως τὸ ὑπὸ* ΕΔΖ *πρὸς τὸ ὑπὸ* ΕΘ, ΔΖ· *καὶ ἐναλλάξ. ἀλλ' ὡς τὸ ὑπὸ* ΒΗ, ΑΓ *πρὸς τὸ ὑπὸ* ΕΘ, ΔΖ, *οὕτως ἐστὶν τὸ* ΑΒΓ *τρίγωνον πρὸς τὸ* ΔΕΖ *τρίγωνον· ἑκατέρα γὰρ τῶν* ΒΗ, ΕΘ *κάθετός ἐστιν ἑκατέρου τῶν εἰρημένων τριγώνων. καὶ ὡς ἄρα τὸ ὑπὸ* ΒΑΓ *πρὸς τὸ ὑπὸ* ΕΔΖ, *οὕτως ἐστὶν τὸ* ΑΒΓ *τρίγωνον πρὸς τὸ* ΔΕΖ *τρίγωνον.*

‖ 1 *τοῦ ἀπὸ* Hu *ἀπὸ τοῦ* A | *συνῆπται* Ge (BS) *συνῆκται* A | ΒΗ Co ΒΝ A ‖ 4 ΔΕ, ΖΘ... ΔΖ, ΘΕ] ΔΖ, ΘΕ... ΔΕ, ΖΘ Simson₁ ‖ 5 ΕΒ Co ΘΒ A ‖ 6 ΔΕ, ΖΘ... ΔΖ, ΘΕ] ΔΖ, ΘΕ... ΔΕ, ΖΘ Simson₁ ‖ 12 *ἤχθω* Ge (S) *ἤχθη* A ‖ 15 *ἐστὶν* secl Hu ‖ 25 ΕΘ Co ΗΘ A ‖ 26 ΒΗ Co ΒΕ A

(215) *(Prop. 147)* Now let (angles) A, Δ equal two right angles. That again, as is the rectangle contained by BA, AΓ to the rectangle contained by EΔ, ΔZ, so is triangle ABΓ to triangle ΔEZ.

Let BA be produced, and let AH be made equal to BA,[1] and let ΓH be joined. Then since angles A, Δ equal two right angles,[2] but also angles BAΓ, ΓAH (equal) two right angles,[3] therefore angle ΓAH equals (angle) Δ.[4] Thus as is the rectangle contained by HA, AΓ to the rectangle contained by EΔ, ΔZ, so is triangle AHΓ to triangle ΔEZ.[5] But HA equals AB,[6] and triangle HAΓ (equals) triangle ABΓ.[7] Therefore as is the rectangle contained by BA, AΓ to the rectangle contained by EΔ, ΔZ, so is triangle ABΓ to triangle ΔEZ.[8]

(216) *(Prop. 148)* (Let there be) straight line AB, and on it two points Γ, Δ, and let twice the rectangle contained by AB, ΓΔ equal the square of ΓB. That as well the square of AΔ equals the squares of AΓ and ΔB.

For since twice the rectangle contained by AB, ΓΔ equals the square of ΓB,[1] let twice the rectangle contained by BΔ, ΔΓ be subtracted in common. Then the remaining twice the rectangle contained by AΔ, ΔΓ equals the squares of ΓΔ and ΔB.[2] Let the square of ΓΔ be subtracted in common. Then the remaining twice the rectangle contained by AΓ, ΓΔ plus the square of ΓΔ equals the square of ΔB.[3] Let the square of AΓ be added in common. Then the sum, the square of AΔ, equals the squares of AΓ and ΔB.[4]

(217) *(Prop. 149)* Let the rectangle contained by AB, BΓ equal the square of BΔ. That three things result: that the rectangle contained by AΔ and ΔΓ taken together and BΔ (that is, (AΔ + ΔΓ)·BΔ) equals the rectangle contained by AΔ, ΔΓ; that the rectangle contained by AΔ, ΔΓ taken together and BΓ equals the square of ΔΓ; and that the rectangle contained by AΔ, ΔΓ taken together and BA equals the square of AΔ.

For since the rectangle contained by AB, BΓ equals the square of BΔ,[1] in ratio[2] and whole to whole[3] and inverting[4] and *componendo*, as is ΓΔ, ΔA taken together to ΔA, so is ΓΔ to ΔB.[5] Therefore the rectangle contained by AΔ, ΔΓ taken together and BΔ equals the rectangle contained by AΔ, ΔΓ.[6] Again, since all AΔ is to all ΔΓ as ΔB is to BΓ,[7] *componendo*, as is AΔ, ΔΓ taken together to ΔΓ, so is ΔΓ to ΓB.[8] Therefore the rectangle contained by AΔ, ΔΓ taken together and ΓB equals the square of ΔΓ.[9] Again, since all AΔ is to all ΔΓ as AB is to BΔ,[10] by inversion[11]

(215) ἔστωσαν δὴ αἱ Α, Δ δυσὶν ὀρθαῖς ἴσαι. ὅτι πάλιν γίνεται ὡς τὸ ὑπὸ ΒΑΓ πρὸς τὸ ὑπὸ ΕΔΖ, οὕτως τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΔΕΖ τρίγωνον. |ἐκβεβλήσθω ἡ ΒΑ, καὶ κείσθω |164 τῆι ΒΑ ἴση ἡ ΑΗ, καὶ ἐπεζεύχθω ἡ ΓΗ. ἐπεὶ οὖν αἱ Α, Δ γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσίν, ἀλλὰ καὶ <αἱ> ὑπὸ ΒΑΓ, ΓΑΗ γωνίαι δυσὶν ὀρθαῖς, ἴση ἄρα ἐστὶν ἡ ὑπὸ ΓΑΗ γωνία τῆι Δ. ἔστιν οὖν ὡς τὸ ὑπὸ ΗΑΓ πρὸς τὸ ὑπὸ ΕΔΖ, οὕτως τὸ ΑΗΓ τρίγωνον πρὸς τὸ ΔΕΖ τρίγωνον. ἴση δέ ἐστιν ἡ μὲν ΗΑ τῆι ΑΒ, τὸ δὲ ΗΑΓ τρίγωνον τῶι ΑΒΓ τριγώνωι. ἔστιν ἄρα ὡς τὸ ὑπὸ ΒΑΓ πρὸς τὸ ὑπὸ ΕΔΖ, οὕτως τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΔΕΖ τρίγωνον.

(216) εὐθεῖα ἡ ΑΒ, καὶ ἐπ' αὐτῆς δύο σημεῖα τὰ Γ, Δ, ἔστω δὲ τὸ δὶς ὑπὸ ΑΒ, ΓΔ ἴσον τῶι ἀπὸ ΓΒ. ὅτι καὶ τὸ ἀπὸ ΑΔ ἴσον ἐστὶν τοῖς ἀπὸ τῶν ΑΓ, ΔΒ τετραγώνοις. ἐπεὶ γὰρ τὸ δὶς ὑπὸ 898 ΑΒ, ΓΔ ἴσον ἐστὶν τῶι ἀπὸ ΓΒ, κοινὸν ἀφηιρήσθω τὸ δὶς ὑπὸ ΒΔΓ. λοιπὸν ἄρα τὸ δὶς ὑπὸ ΑΔΓ ἴσον ἐστὶν τοῖς ἀπὸ τῶν ΓΔ, ΔΒ τετραγώνοις. κοινὸν ἀφηιρήσθω τὸ ἀπὸ ΓΔ τετράγωνον. λοιπὸν ἄρα τὸ δὶς ὑπὸ ΑΓΔ μετὰ τοῦ ἀπὸ ΓΔ ἴσον ἐστὶν τῶι ἀπὸ ΔΒ τετραγώνωι. κοινὸν προσκείσθω τὸ ἀπὸ ΑΓ τετράγωνον. ὅλον ἄρα τὸ ἀπὸ ΑΔ τετράγωνον ἴσον ἐστὶν τοῖς ἀπὸ τῶν ΑΓ, ΔΒ τετραγώνοις.

(217) ἔστω τὸ ὑπὸ ΑΒΓ ἴσον τῶι ἀπὸ ΒΔ τετραγώνωι. ὅτι γίνεται γ̅, τὸ μὲν ὑπὸ συναμφοτέρου τῆς ΑΔ, ΔΓ, καὶ τῆς ΒΔ ἴσον τῶι ὑπὸ ΑΔ, ΔΓ, τὸ δὲ ὑπὸ συναμφοτέρου τῆς ΑΔΓ καὶ τῆς ΒΓ ἴσον τῶι ἀπὸ ΔΓ τετραγώνωι, τὸ δὲ ὑπὸ συναμφοτέρου τῆς ΑΔΓ καὶ τῆς ΒΑ ἴσον τῶι ἀπὸ ΑΔ τετραγώνωι. ἐπεὶ γὰρ τὸ ὑπὸ ΑΒΓ ἴσον ἐστὶν τῶι ἀπὸ ΒΔ, ἀνάλογον καὶ ὅλη πρὸς ὅλην, καὶ ἀνάπαλιν καὶ συνθέντι, ἔστιν ἄρα ὡς συναμφότερος ἡ ΓΔ, ΔΑ πρὸς τὴν ΔΑ, οὕτως ἡ ΓΔ πρὸς τὴν ΔΒ. τὸ ἄρα ὑπὸ συναμφοτέρου τῆς ΑΔ, ΔΓ καὶ τῆς ΒΔ ἴσον ἐστὶ τῶι ὑπὸ τῶν ΑΔΓ. πάλιν ἐπεὶ ὅλη ἡ ΑΔ πρὸς ὅλην τὴν ΔΓ ἐστιν ὡς ἡ ΔΒ πρὸς τὴν ΒΓ, συνθέντι ἐστὶν ὡς συναμφότερος ἡ ΑΔΓ πρὸς τὴν ΔΓ, οὕτως ἡ ΔΓ πρὸς τὴν ΓΒ. τὸ ἄρα ὑπὸ συναμφοτέρου τῆς ΑΔΓ καὶ τῆς ΓΒ ἴσον ἐστὶν τῶι ἀπὸ ΔΓ. πάλιν ἐπεὶ ὅλη ἡ ΑΔ πρὸς

‖ 3 ἐκβεβλήσθω Hu ἐκβληθῆι A ‖ 4 ΑΗ Co ΑΒ A ‖ 5 αἱ add Ge (recc?) | γωνίαι Ge (S) γωνία A ‖ 6 post ὀρθαῖς add ἴσαι Hu app | ΓΑΗ Co ΓΔΗ A | (ΓΑΗ) γωνία Ge (BS) γωνίαι A ‖ 9 ΑΒΓ Co ΑΘΓ A ‖ 15 τῶι Ge (BS) τοῖς A ‖ 20 ὅλον – τετράγωνον bis (sed ΑΛ habet pro altero ΑΔ) A corr Co ‖ 23 ΑΔ, ΔΓ Co ΑΔ, ΕΓ A ΑΔΓ Hu ‖ 24 ΔΓ Co ΔΤ A | ΑΔΓ] ΑΔ, ΔΓ Co ‖ 26 ΑΔΓ] ΑΔ, ΔΓ Co ‖ 27 ἀνάλογον Co ἀνάπαλιν A | ὅλη Ge (recc?) ὅληι A ‖ 30 τῶι Ge (S) τὸ A ‖ 31 ὅλη Ge (BS) ὅληι A ‖ 32 ΑΔΓ] ΑΔ, ΔΓ Co ‖ 33 ΑΔΓ] ΑΔ, ΔΓ Co and *componendo*, as is ΓΔ, ΔA taken together to ΔA, so is ΔA to AB.[12] Therefore the rectangle contained by AΔ, ΔΓ taken together and AB equals the square of AΔ.[13]

(218) *(Prop. 150)* (Let there be) straight line AB, and on it two points Γ, Δ, and let the square of ΓΔ equal twice the rectangle contained by AΓ, BΔ. That as well the square of AB equals the squares of AΔ and ΓB.

For since the square of ΓΔ equals twice the rectangle contained by AΓ, ΔB,[1] <let twice the rectangle contained by AΓΔ be added in common. Then the sum, twice the rectangle contained by AΓ, ΓB>, equals the square of ΓΔ and twice the rectangle contained by AΓ, ΓΔ.[2] Let the square of AΓ be added in common. Then twice the rectangle contained by AΓ, ΓB plus the square of AΓ equals the square of AΔ.[3] Let the square of BΓ be added in common. Then the sum, the square of AB, equals the squares of AΔ and ΓB.[4]

(219) *(Prop. 151)* Let the rectangle contained by AB, BΓ equal the square of BΔ. That three things result: that the rectangle contained by the difference of AΔ and ΔΓ, and BΔ equals the rectangle contained by AΔ, ΔΓ; that the rectangle contained by the difference of AΔ and ΔΓ, and BΓ equals the square of ΔΓ; and that the rectangle contained by the difference of AΔ and ΔΓ, and BA equals the square of AΔ.

For since as is AB to BΔ, so is BΔ to BΓ,[1] remainder to remainder[2] and *separando*, then, as is the difference of AΔ and ΔΓ to ΔΓ, so is AΔ to ΔB.[3] Therefore the rectangle contained by the difference of AΔ and ΔΓ, and BΔ equals the rectangle contained by AΔ, ΔΓ.[4] Again, since remainder AΔ is to remainder ΔΓ as ΔB is to BΓ,[5] *separando*, as is the difference of AΔ and ΔΓ to ΔΓ, so is ΔΓ to ΓB.[6] Therefore the rectangle contained by the difference of AΔ and ΔΓ, and BΓ equals the square of ΔΓ.[7] Again, since AΔ is to ΔΓ as AB is to BΔ,[8] by inversion[9] and *separando*, as is the difference of AΔ and ΔΓ to ΔA, so is ΔA to AB.[10] Therefore the rectangle contained by the difference of AΔ and ΔΓ, and AB equals the square of AΔ.[11]

ὅλην τὴν ΔΓ ἐστὶν ὡς ἡ ΑΒ πρὸς τὴν ΒΔ, ἀνάπαλιν καὶ συνθέντι ἐστὶν ὡς συναμφότερος ἡ ΓΔ, ΔΑ πρὸς τὴν ΔΑ, οὕτως ἡ ΔΑ πρὸς τὴν ΑΒ· τὸ ἄρα ὑπὸ συναμφοτέρου τῆς ΑΔΓ καὶ τῆς ΑΒ ἴσον ἐστὶν τῶι ἀπὸ ΑΔ τετραγώνωι.

(218) |εὐθεῖα ἡ ΑΒ, καὶ δύο σημεῖα τὰ Γ, Δ, καὶ ἔστω τὸ ἀπὸ ΓΔ τετράγωνον ἴσον τῶι δὶς ὑπὸ ΑΓ, ΒΔ· ὅτι καὶ τὸ ἀπὸ ΑΒ [900] τετράγωνον ἴσον ἐστὶν τοῖς ἀπὸ τῶν ΑΔ, ΓΒ τετραγώνοις. [|164v] ἐπεὶ γὰρ τὸ ἀπὸ ΓΔ ἴσον ἐστὶν τῶι δὶς ὑπὸ ΑΓ, ΔΒ, <κοινὸν προσκείσθω τὸ δὶς ὑπὸ ΑΓΔ· ὅλον ἄρα τὸ δὶς ὑπὸ ΑΓΒ> ἴσον ἐστὶν τῶι τε ἀπὸ τῆς ΓΔ καὶ τῶι δὶς ὑπὸ τῶν ΑΓΔ. κοινὸν προσκείσθω τὸ ἀπὸ ΑΓ· τὸ ἄρα δὶς ὑπὸ ΑΓΒ μετὰ τοῦ ἀπὸ ΑΓ ἴσον ἐστὶν τῶι ἀπὸ ΑΔ. κοινὸν προσκείσθω τὸ ἀπὸ ΒΓ· ὅλον ἄρα τὸ ἀπὸ ΑΒ τετράγωνον ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΔ, ΓΒ τετραγώνοις.

(219) ἔστω τὸ ὑπὸ τῶν ΑΒΓ ἴσον τῶι ἀπὸ τῆς ΒΔ· ὅτι γίνεται γ̄, τὸ μὲν ὑπὸ τῆς τῶν ΑΔ, ΔΓ ὑπεροχῆς καὶ τῆς ΒΔ ἴσον τῶι ὑπὸ ΑΔΓ, τὸ δὲ ὑπὸ τῆς τῶν ΑΔΓ ὑπεροχῆς καὶ τῆς ΒΓ ἴσον τῶι ἀπὸ τῆς ΔΓ τετραγώνωι, τὸ δὲ ὑπὸ τῆς τῶν ΑΔ, ΔΓ ὑπεροχῆς καὶ τῆς ΒΑ ἴσον τῶι ἀπὸ τῆς ΑΔ τετραγώνωι. ἐπεὶ γάρ ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΔ, οὕτως ἡ ΒΔ πρὸς τὴν ΒΓ, λοιπὴ πρὸς λοιπὴν καὶ διελόντι, ἔστιν οὖν ὡς ἡ τῶν ΑΔ, ΔΓ ὑπεροχὴ πρὸς τὴν ΔΓ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΒ. τὸ ἄρα ὑπὸ τῆς τῶν ΑΔ, ΔΓ ὑπεροχῆς καὶ τῆς ΔΒ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΔ, ΔΓ. πάλιν ἐπεὶ λοιπὴ ἡ ΑΔ πρὸς λοιπὴν <τὴν> ΔΓ ἐστὶν ὡς ἡ ΔΒ πρὸς τὴν ΒΓ, διελόντι ἐστὶν ὡς ἡ τῶν ΑΔΓ ὑπεροχὴ πρὸς τὴν ΔΓ, οὕτως ἡ ΔΓ πρὸς τὴν ΓΒ. τὸ ἄρα ὑπὸ τῆς τῶν ΑΔ, ΔΓ ὑπεροχῆς καὶ τῆς ΒΓ ἴσον ἐστὶν τῶι ἀπὸ τῆς ΔΓ τετραγώνωι. πάλιν ἐπεί ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΔ, [902] ἀνάπαλιν καὶ διελόντι ἐστὶν ὡς ἡ τῶν ΑΔ, ΔΓ ὑπεροχὴ πρὸς τὴν ΔΑ, οὕτως ἡ ΔΑ πρὸς τὴν ΑΒ· τὸ ἄρα ὑπὸ τῆς τῶν ΑΔ, ΔΓ ὑπεροχῆς καὶ τῆς ΑΒ ἴσον ἐστὶν τῶι ἀπὸ τῆς ΑΔ τετραγώνωι.

‖ 2 ΓΔ, ΔΑ Co ΓΔ, ΛΑ A ΓΔΑ Hu ‖ 3 ὑπὸ Ge ἀπὸ A | ΑΔΓ] ΑΔ, ΔΓ Co ‖ 6 ΑΓ ΒΔ ὅτι Heiberg$_3$ $\overline{\text{ΑΓΒ}}$ διότι A ΑΓ ΔΒ ὅτι Co ‖ 8 ΔΓ Co ΑΒ A | κοινὸν – ΑΓΒ] τὸ ἄρα δὶς ὑπὸ ΑΓΒ add Co ‖ 17 (τῶν) ΑΔΓ] ΑΔ, ΔΓ Co | ΒΓ Co ΒΔ A ‖ 18 ΔΓ (τετραγώνωι) Co ΑΓ A | ΔΓ (ὑπεροχῆς) Co ΑΓ A ‖ 20 ΒΓ Co ΔΓ A | λοιπὴ Ge (BS) λοιπηι A ‖ 21 οὖν] ἄρα Hu ‖ 24 λοιπὴ Ge (BS) λοιπηι A | τὴν add Ge (BS) ‖ 25 ΑΔΓ] ΑΔ, ΔΓ Co

(220) *(Prop. 152)* Let the square of ΑΔ be to the square of ΔΓ as ΑΒ is to ΒΓ. That the rectangle contained by ΑΒ, ΒΓ equals the square of ΒΔ.

Let ΔΕ be made equal to ΓΔ.[1] Then the rectangle contained by ΕΑ, ΑΓ plus the square of ΓΔ, that is the rectangle contained by ΓΔ, ΔΕ,[3] equals the square of ΑΔ.[2] Then since, as is ΑΒ to ΒΓ, so is the square of ΑΔ to the square of ΔΓ,[4] *separando*, as is ΑΓ to ΓΒ, that is as is the rectangle contained by ΕΑ, ΑΓ to the rectangle contained by ΕΑ, ΒΓ,[6] so is the rectangle contained by ΕΑ, ΑΓ to the rectangle contained by ΓΔ, ΔΕ.[5] Therefore the rectangle contained by ΑΕ, ΒΓ equals the rectangle contained by ΓΔ, ΔΕ.[7] In ratio[8] and *separando*, as is ΑΔ to ΔΕ, that is to ΔΓ,[10] so is ΔΒ to ΒΓ.[9] Therefore remainder ΑΒ is to remainder ΒΔ as ΒΔ is to ΒΓ.[11] Thus the rectangle contained by ΑΒ, ΒΓ equals the square of ΒΔ.[12]

(221) *(Prop. 153)* Again, let the square of ΑΔ be to the square of ΔΓ as ΑΒ is to ΒΓ. That the rectangle contained by ΑΒ, ΒΓ equals the square of ΒΔ.

For in the same way (as in 7.220) let ΔΕ be made equal to ΓΔ.[1] Then the rectangle contained by ΓΑ, ΑΕ plus the square of ΓΔ, that is the rectangle contained by ΕΔ, ΔΓ,[3] equals the square of ΑΔ.[2] It results that *separando*, as is ΑΓ to ΓΒ, that is as is <the rectangle contained by ΕΑ, ΑΓ to the rectangle contained by ΕΑ, ΓΒ,[5] so> is the rectangle contained by ΓΑ, ΑΕ to the rectangle contained by ΕΔ, ΔΓ.[4] Therefore the rectangle contained by ΑΕ, ΓΒ equals the rectangle contained by ΕΔ, ΔΓ.[6] In ratio[7] and *componendo*, as is ΑΔ to ΔΕ, that is to ΔΓ,[9] so is ΔΒ to ΒΓ.[8] Therefore the sum ΑΒ is to the sum ΒΔ as ΒΔ is to ΒΓ.[10] Thus the rectangle contained by ΑΒ, ΒΓ equals the square of ΒΔ.[11]

(222) *(Prop. 154)* Let ΑΔ, ΔΓ be tangent to circle ΑΒΓ, and let ΑΓ be joined, and let an arbitrary (line) ΔΒ be drawn across. That, as ΒΔ is to ΔΕ, so is ΒΖ to ΖΕ.

For since ΑΔ equals ΔΓ,[1] therefore the rectangle contained by ΑΖ, ΖΓ plus the square of ΖΔ equals the square of ΔΑ (lemma 222.1).[2] But the rectangle contained by ΑΖ, ΖΓ equals the rectangle contained by ΒΖ, ΖΕ (III 35),[3] while the square of ΔΑ equals the rectangle contained by ΒΔ, ΔΕ (III 36).[4] Therefore the rectangle contained by ΒΖ, ΖΕ plus the square of ΔΖ equals the rectangle contained by ΒΔ, ΔΕ.[5] But if this is so, then ΒΖ is to ΖΕ as ΒΔ is to ΔΕ (lemma 222.2).[6]

(220) *ἔστω ὡς ἡ* ΑΒ *πρὸς τὴν* ΒΓ, *οὕτως τὸ ἀπὸ* ΑΔ *πρὸς τὸ ἀπὸ* ΔΓ. *ὅτι τὸ ὑπὸ τῶν* ΑΒΓ *ἴσον ἐστὶν τῶι ἀπὸ τῆς* ΒΔ *τετραγώνωι. κείσθω τῆι* ΓΔ *ἴση ἡ* ΔΕ. *τὸ ἄρα ὑπὸ* ΕΑΓ *μετὰ τοῦ ἀπὸ* ΓΔ, *τουτέστιν τοῦ ὑπὸ* ΓΔΕ, *ἴσον τῶι ἀπὸ* ΑΔ. *ἐπεὶ οὖν ἐστιν ὡς ἡ* ΑΒ *πρὸς τὴν* ΒΓ, *οὕτως τὸ ἀπὸ* ΑΔ *πρὸς τὸ ἀπὸ* ΔΓ, *διελόντι ἐστὶν ὡς ἡ* ΑΓ *πρὸς τὴν* ΓΒ, *τουτέστιν ὡς τὸ ὑπὸ* ΕΑΓ *πρὸς τὸ ὑπὸ* ΕΑ, ΒΓ, *οὕτως τὸ ὑπὸ* ΕΑΓ *πρὸς τὸ ὑπὸ* ΓΔΕ. *ἴσον ἄρα ἐστὶν τὸ ὑπὸ* ΑΕ, ΒΓ *τῶι ὑπὸ* ΓΔΕ. *ἀνάλογον καὶ διελόντι ἐστὶν ὡς ἡ* ΑΔ *πρὸς τὴν* ΔΕ, *τουτέστιν πρὸς τὴν* ΔΓ, *οὕτως ἡ* ΔΒ *πρὸς* |*τὴν* ΒΓ. *καὶ λοιπὴ ἄρα ἡ* ΑΒ *πρὸς λοιπὴν τὴν* ΒΔ *ἐστὶν ὡς ἡ* ΒΔ *πρὸς τὴν* ΒΓ. *τὸ ἄρα ὑπὸ* ΑΒΓ *ἴσον ἐστὶν τῶι ἀπὸ τῆς* |165
ΒΔ *τετραγώνωι.*

(221) *ἔστω δὲ πάλιν ὡς ἡ* ΑΒ *πρὸς τὴν* ΒΓ, *οὕτως τὸ ἀπὸ* ΑΔ *τετράγωνον πρὸς τὸ ἀπὸ* ΔΓ *τετράγωνον. ὅτι τὸ ὑπὸ* ΑΒΓ *ἴσον ἐστὶν τῶι ἀπὸ τῆς* ΒΔ *τετραγώνωι. κείσθω γὰρ ὁμοίως τῆι* ΓΔ *ἴση ἡ* ΔΕ. *τὸ ἄρα ὑπὸ* ΓΑΕ *μετὰ τοῦ ἀπὸ* ΓΔ, *τουτέστιν τοῦ ὑπὸ* ΕΔΓ, *ἴσον τῶι ἀπὸ* ΑΔ. *καὶ γίνεται κατὰ διαίρεσιν ὡς ἡ* ΑΓ *πρὸς τὴν* ΓΒ, *τουτέστιν ὡς* <*τὸ ὑπὸ* ΕΑΓ *πρὸς τὸ ὑπὸ* ΕΑ, ΓΒ, *οὕτως*> *τὸ ὑπὸ* ΓΑΕ *πρὸς τὸ ὑπὸ* ΕΔΓ. *ἴσον ἄρα ἐστὶν τὸ ὑπὸ* ΑΕ, ΓΒ *τῶι ὑπὸ* ΕΔΓ. *ἀνάλογον καὶ συνθέντι ἐστὶν ὡς ἡ* ΑΔ *πρὸς τὴν* ΔΕ, *τουτέστιν πρὸς τὴν* ΔΓ, *οὕτως ἡ* ΔΒ *πρὸς τὴν* ΒΓ. *καὶ ὅλη ἄρα ἡ* ΑΒ *πρὸς ὅλην τὴν* ΒΔ *ἐστὶν ὡς ἡ* ΒΔ *πρὸς τὴν* ΒΓ. *τὸ ἄρα ὑπὸ τῶν* ΑΒΓ *ἴσον ἐστὶν τῶι ἀπὸ τῆς* ΒΔ *τετραγώνωι.*

(222) *κύκλου τοῦ* ΑΒΓ *ἐφαπτέσθωσαν αἱ* ΑΔ, ΔΓ, *καὶ* 904
ἐπεζεύχθω ἡ ΑΓ, *καὶ διήχθω τυχοῦσα ἡ* ΔΒ. *ὅτι γίνεται ὡς ἡ* ΒΔ *πρὸς τὴν* ΔΕ, *οὕτως ἡ* ΒΖ *πρὸς τὴν* ΖΕ. *ἐπεὶ γὰρ ἴση ἐστὶν ἡ* ΑΔ *τῆι* ΔΓ, *τὸ ἄρα ὑπὸ* ΑΖΓ *μετὰ τοῦ ἀπὸ* ΖΔ *ἴσον ἐστὶν τῶι ἀπὸ* ΔΑ. *ἀλλὰ τὸ μὲν ὑπὸ* ΑΖΓ *ἴσον ἐστὶν τῶι ὑπὸ* ΒΖΕ, *τὸ δὲ ἀπὸ* ΔΑ <*ἴσον*> *ἐστὶν τῶι ὑπὸ* ΒΔΕ. *τὸ ἄρα* [*τὸ*] *ὑπὸ* ΒΖΕ *μετὰ τοῦ ἀπὸ* ΔΖ *ἴσον ἐστὶν τῶι ὑπὸ* ΒΔΕ. *ἐὰν δὲ ἦι τοῦτο, γίνεται ὡς ἡ* ΒΔ *πρὸς τὴν* ΔΕ, *οὕτως ἡ* ΒΖ *πρὸς τὴν* ΖΕ.

‖ 4 *τοῦ* (*ὑπὸ* ΓΔΕ) Co *τὸ* A ‖ 7 ΒΓ Co ΘΓ A ‖ 8 *ἀνάλογον* Co *ἀνάπαλιν* A ‖ 11 ΒΓ Co ΔΓ A ‖ 12 ΒΔ Co ΒΛ A ‖ 14 *ἴσον* Ge (BS) *ἴση* A ‖ 15 *τῆι* Ge (S) *τῆς* A ‖ 16 *τοῦ* (*ὑπὸ* ΕΔΓ) Hu *τὸ* A ‖ 18 *τὸ* — *οὕτως* add Co ‖ 20 *ἀνάλογον* Co *ἀνάπαλιν* A ‖ 24 *ἐφαπτέσθωσαν* Co *ἐφάπτεται* A ‖ 25 ΔΒ Simson₁ ΑΒ A ‖ 29 *ἴσον ἐστὶν τῶι*] *ἐστιν τὸ* A *ἴσον τῶι* Hu | *τὸ* (*ὑπὸ*) del Ge (S)

(223) *(Prop. 155)* Given a segment (of a circle) on AB, to inflect a straight line AΓB in a given ratio.

Let it have been done,[1] and let ΓΔ be drawn tangent from Γ.[2] Then as is the square of AΓ to the square of BΓ, so is AΔ to ΔB (lemma 223.1).[3] But the ratio of the square of AΓ to <the square of ΓB is given,[4] so that also the (ratio) of AΔ to> BΔ is given.[5] And <A and B> are given.[6] Therefore Δ is given[7], and thus Γ too is given.[8]

The synthesis of the problem will be made thus. Let the segment be ABΓ, and the ratio that of E to Z, and, as is the square of E to the square of Z, so let AΔ be made to ΔB,[9] and let ΔΓ be drawn tangent,[10] and let AΓ, ΓB be joined. I say that AΓ, ΓB solve the problem.

For since as is the square of E to the square of Z, so is AΔ to ΔB, while as is AΔ to ΔB, so is the square of AΓ to the square of ΓB,[11] because ΓΔ is tangent, therefore as is the square of E to the square of Z, so is the square of AΓ to the square of ΓB.[12] Therefore also as is E to Z, so is AΓ to ΓB.[13] Thus AΓB solves the problem.

(224) *(Prop. 156)* (Let there be) a circle whose diameter is AB, and from an arbitrary (point) a perpendicular ΔE onto (AB). Let ΔZ be drawn across. Let EZ be joined and produced, and where it intersects the diameter, let (the point) be H. That, as is AH to HB, so is AΘ to ΘB.

Let ΔA, AE, AZ be joined. Then since ΔE is a perpendicular to the diameter,[1] angle ΔAB equals <angle BAE (III 3, I 4).[2] But angle ΔAB equals> the (angle) in the same segment, angle ΘZB.[3] Angle BAE equals angle BZH outside the quadrilateral (III 22)[4]. Therefore angle BZH equals angle ΘZB.[5] And angle AZB is right.[6] Because of the lemma (224.1), as is AH to HB, so is AΘ to BΘ.[7]

(225) *(Prop. 157)* (Let there be) a semicircle on AB, and from points A, B let straight lines BΔ, AE be drawn at right angles to AB, and let an arbitrary (line) ΔE be drawn, and let a straight line ZH from Z and at right angles to ΔE intersect AB at H. That the rectangle contained by AE, BΔ equals the rectangle contained by AH, HB.[1]

Hence that as is EA to AH, so is HB to BΔ.[2] The sides around equal angles are in ratio. Hence that angle AHE equals angle BΔH.[3] But angle

(223) τμήματος δοθέντος τοῦ ἐπὶ τῆς ΑΒ, κλάσαι εὐθεῖαν τὴν ΑΓΒ ἐν λόγωι τῶι δοθέντι. γεγονέτω, καὶ διήχθω ἀπὸ τοῦ Γ ἐφαπτομένη ἡ ΓΔ. ὡς ἄρα τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΒΓ, οὕτως ἡ ΑΔ πρὸς ΔΒ. λόγος δὲ τοῦ ἀπὸ ΑΓ πρὸς <τὸ ἀπὸ ΓΒ δοθείς, ὥστε καὶ ὁ τῆς ΑΔ πρὸς> τὴν ΒΔ δοθείς. καὶ ἔστιν δοθέντα <τὰ Α Β.> δοθὲν ἄρα ἐστὶν τὸ Δ. ὥστε καὶ τὸ Γ δοθέν.

συντεθήσεται δὴ τὸ πρόβλημα οὕτως. ἔστω τὸ μὲν τμῆμα 906 τὸ ΑΒΓ, ὁ δὲ λόγος ὁ τῆς Ε πρὸς τὴν Ζ, καὶ πεποιήσθω ὡς τὸ ἀπὸ Ε πρὸς τὸ ἀπὸ Ζ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΒ, καὶ ἤχθω ἐφαπτομένη ἡ ΔΓ, καὶ ἐπεζεύχθωσαν αἱ ΑΓ, ΓΒ. λέγω ὅτι αἱ ΑΓ, ΓΒ ποιοῦσι τὸ πρόβλημα. ἐπεὶ γάρ ἐστιν ὡς τὸ ἀπὸ Ε πρὸς τὸ ἀπὸ Ζ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΒ, ὡς δὲ ἡ ΑΔ πρὸς τὴν ΔΒ, οὕτως τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΓΒ, διὰ τὸ ἐφάπτεσθαι τὴν ΓΔ, καὶ ὡς ἄρα τὸ ἀπὸ Ε πρὸς τὸ ἀπὸ Ζ, οὕτως τὸ ἀπὸ ΑΓ πρὸς τὸ |ἀπὸ ΓΒ. |165v ὥστε καὶ ὡς ἡ Ε πρὸς τὴν Ζ, οὕτως ἡ ΑΓ πρὸς τὴν ΓΒ. ἡ ΑΓΒ ἄρα ποιεῖ τὸ πρόβλημα.

(224) κύκλος οὗ διάμετρος ἡ ΑΒ, καὶ ἀπὸ τυχόντος ἐπ' αὐτὴν κάθετος ἡ ΔΕ. διήχθω ἡ ΔΖ. ἐπεζεύχθω ἡ ΕΖ καὶ ἐκβεβλήσθω, καὶ καθ' ὃ συμπίπτει τῆι διαμέτρωι, ἔστω τὸ Η. ὅτι ἐστὶν ὡς ἡ ΑΗ πρὸς τὴν ΗΒ, οὕτως ἡ ΑΘ πρὸς τὴν ΘΒ. ἐπεζεύχθωσαν αἱ ΔΑ, ΑΕ, ΑΖ. ἐπεὶ οὖν ἐπὶ διάμετρον κάθετος ἡ ΔΕ, ἴση ἐστὶν ἡ ὑπὸ ΔΑΒ τῆι <ὑπὸ ΒΑΕ, ἀλλ' ἡ ὑπὸ ΔΑΒ τῆι> ἐν τῶι αὐτῶι τμήματι ἴση ἐστὶν τῆι ὑπὸ ΘΖΒ. ἡ δὲ ὑπὸ ΒΑΕ ἴση ἐστὶν τῆι ἐκτὸς τετραπλεύρου τῆι ὑπὸ ΒΖΗ. καὶ τῆι ὑπὸ ΘΖΒ ἄρα γωνίαι ἴση ἐστὶν ἡ ὑπὸ ΒΖΗ. καὶ ἔστιν ὀρθὴ ἡ ὑπὸ ΑΖΒ γωνία. διὰ δὴ τὸ λῆμμα γίνεται ὡς ἡ ΑΗ πρὸς τὴν ΗΒ, οὕτως ἡ ΑΘ πρὸς τὴν ΒΘ.

(225) ἡμικύκλιον τὸ ἐπὶ τῆς ΑΒ, καὶ ἀπὸ τῶν Α, Β σημείων τῆι ΑΒ πρὸς ὀρθὰς γωνίας εὐθεῖαι γραμμαὶ ἤχθωσαν αἱ ΒΔ, ΑΕ, 908 καὶ ἤχθω τυχοῦσα ἡ ΔΕ, καὶ ἀπὸ τοῦ Ζ τῆι ΔΕ πρὸς ὀρθὰς γωνίας εὐθεῖα γραμμὴ ἡ ΖΗ συμπιπτέτω τῆι ΑΒ κατὰ τὸ Η. ὅτι τὸ ὑπὸ τῶν ΑΕ, ΒΔ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΗΒ. ὅτι ἄρα ἐστὶν ὡς ἡ ΕΑ πρὸς τὴν ΑΗ, οὕτως ἡ ΗΒ πρὸς τὴν ΒΔ. περὶ ἴσας γωνίας ἀνάλογόν εἰσιν αἱ πλευραί. ὅτι ἄρα ἴση ἐστὶν ἡ ὑπὸ

‖ 4 τὸ ἀπὸ – πρὸς add Co ‖ 5 δοθείς Hu (S^2) δοθέν A | δοθέντα τὰ Α, Β Hu (Simson$_1$) δύο A ‖ 6 Γ δοθέν Hu (Simson$_1$) ΒΔ ὅθεν A ‖ 8 ὁ (τῆς) A^2 supr | Ε Co Θ A ‖ 10 ἐπεζεύχθω A^1 σαν supr A^2 ‖ 14 ἀπὸ (ante ΑΓ) om A^1 add supr A^2 ‖ 18 ante διήχθω add καὶ Ge ‖ 21 διάμετρον Heiberg$_3$ διαμέτρου A ‖ 22 ὑπὸ ΒΑΕ – ΔΑΒ τῆι add Co ‖ 23 ἴση Ge (BS) ἴσηι A ‖ 25 ἡ (ὑπὸ ΒΖΗ) Hu (S^2) τῶι A | δὴ τὸ] δή τι coni. Hu app ‖ 29 ΑΒ Hu ΑΓΒ A ‖ 31 συμπιπτέτω Hu συμπίπτει A

AHE equals angle AZE in the same segment,[4] while again angle BΔH equals angle BZH in the same segment.[5] Hence that angle AZE equals angle BZH.[6] But it does.[8] For both angles AZB, EZH are right.[7]

(226) *(Prop. 158)* (Let there be) a triangle ABΓ that has (side) AB equal to AΓ, and let AB be produced to Δ, and from Δ let ΔE be drawn across making triangle BΔE equal to triangle ABΓ. That if one of the equal sides, the one near the equal triangle, is bisected by BZ, then as is ZBH taken together to ZH, so is the square of AZ to the square of ZΘ.[1]

Let BK be drawn through B parallel to ΔE,[2] and let AΓ be produced to K. Hence that as is ZK, KΘ taken together to ZΘ, that is (as is) the rectangle contained by ZK, KΘ taken together and ZΘ to the square of ZΘ,[4] so is the <square of AZ to the> square of ZΘ.[3] *But the rectangle contained by ZKΘ taken together and ZΘ, that is the difference of the squares of ZK, KΘ,[6] equals the square of AZ.[5] Hence the difference of the squares of KZ, ZA is the square of KΘ.[7] * But the difference of the squares of KZ, ZA is the rectangle contained by ΓK, KA.[8] Hence that the rectangle contained by ΓK, KA equals the square of ΘK.[9] Hence that as is ΓK to KΘ, that is as is ΓB to BE,[11] so is KΘ to KA, that is ΔB to BA.[12] [10] But it is.[16] For AE is parallel to ΔΓ,[15] since triangle ΔBE equals triangle ABΓ.[13] and (therefore), when (triangle) ABE has been subtracted in common, remainder (triangle) ΔAE equals remainder (triangle) AΓE.[14] And they are on the same base.

(227) *(Prop. 159)* (Let there be) a circle about diameter AB, and let AB be produced, and let it be a perpendicular to an arbitrary (line) ΔE, and let the square of ZH be made equal to the rectangle contained by AZ, ZB. That, if some point such as E were chosen, and the (line) from it to H were joined and produced to Θ, then also the rectangle contained by ΘE, EK equals the square of EH.

Let AE, BΛ be joined. Then angle Λ is right.[1] But (angle) Z is right too.[2] Therefore the rectangle contained by AE, EΛ equals the rectangle contained by AZ, ZB plus the square of ZE (lemma 227.1).[3] But the rectangle contained by AE, EΛ equals the rectangle contained by ΘE, EK,[4] while the rectangle contained by AZ, ZB equals the square of ZH.[5] Therefore the rectangle contained by ΘE, EK equals the squares of EZ, τῶν ΑΗΕ γωνία τῆι ὑπὸ τῶν ΒΔΗ γωνίαι. ἀλλὰ ἡ μὲν ὑπὸ ΑΗΕ ἴση ἐστὶν ἐν τῶι αὐτῶι τμήματι τῆι ὑπὸ ΑΖΕ· ἡ δὲ ὑπὸ ΒΔΗ πάλιν ἐν τῶι αὐτῶι τμήματι τῆι ὑπὸ ΒΖΗ. ὅτι ἄρα ἴση ἐστὶν ἡ ὑπὸ ΑΖΕ γωνία τῆι ὑπὸ ΒΖΗ γωνίαι. ἔστιν δέ. ὀρθὴ γάρ ἐστιν ἑκατέρα τῶν ὑπὸ ΑΖΒ, ΕΖΗ γωνιῶν.

(226) τρίγωνον τὸ ΑΒΓ ἴσην ἔχον τὴν ΑΒ τῆι ΑΓ, καὶ ἐκβεβλήσθω ἡ ΑΒ ἐπὶ τὸ Δ, καὶ ἀπὸ τοῦ Δ διήχθω ἡ ΔΕ ποιοῦσα
ἴσον τὸ ΒΔΕ τρίγωνον τῶι ΑΒΓ τριγώ|νωι. ὅτι ἐὰν δίχα |166
τμηθῆι μία τῶν ἴσων πλευρῶν, ἡ πρὸς τῶι ἴσωι τριγώνωι, τῆι ΒΖ, γίνεται ὡς συναμφότερος ἡ ΖΒΗ πρὸς τὴν ΖΗ, οὕτως τὸ ἀπὸ ΑΖ τετράγωνον πρὸς τὸ ἀπὸ ΖΘ τετράγωνον. ἤχθω διὰ τοῦ Β τῆι ΔΕ παράλληλος ἡ ΒΚ, καὶ ἐκβεβλήσθω ἡ ΑΓ ἐπὶ τὸ Κ. ὅτι
ἄρα ἐστὶν ὡς συναμφότερος ἡ ΖΚ, ΚΘ πρὸς τὴν ΖΘ, τουτέστιν 910
τὸ ὑπὸ συναμφοτέρου τῆς ΖΚ, ΚΘ καὶ τῆς ΖΘ, πρὸς τὸ ἀπὸ ΖΘ, οὕτως τὸ ἀπὸ <ΑΖ τετράγωνον πρὸς τὸ ἀπὸ> ΖΘ τετράγωνον. τὸ δὴ ὑπὸ συναμφοτέρου τῆς ΖΚΘ καὶ τῆς ΖΘ, τουτέστιν ἡ τῶν ἀπὸ ΖΚ, ΚΘ ὑπεροχή, ἴση ἐστὶν τῶι ἀπὸ ΑΖ. ἡ ἄρα τῶν ἀπὸ ΚΖ, ΖΑ ὑπεροχή ἐστιν τὸ ἀπὸ ΚΘ. ἀλλὰ ἡ τῶν ἀπὸ ΚΖ, ΖΑ ὑπεροχή ἐστιν τὸ ὑπὸ ΓΚΑ. ὅτι ἄρα τὸ ὑπὸ ΓΚΑ ἴσον ἐστὶν τῶι ἀπὸ ΘΚ. ὅτι ἄρα ἐστὶν ὡς ἡ ΓΚ πρὸς τὴν ΚΘ, τουτέστιν ὡς ἡ ΓΒ πρὸς τὴν ΒΕ, οὕτως ἡ ΚΘ πρὸς τὴν ΚΑ, τουτέστιν ἡ ΔΒ πρὸς τὴν ΒΑ. ἔστιν δέ. παράλληλος γάρ ἐστιν ἡ ΑΕ τῆι ΔΓ, ἐπειδὴ τὸ ΔΒΕ τρίγωνον ἴσον ἐστὶν τῶι ΑΒΓ τριγώνωι· κοινοῦ <δ'> ἀφαιρουμένου τοῦ ΑΒΕ, λοιπὸν τὸ ΔΑΕ λοιπῶι τῶι ΑΓΕ ἐστὶν ἴσον. καὶ ἔστιν ἐπὶ τῆς αὐτῆς βάσεως.

(227) κύκλος περὶ διάμετρον τὴν ΑΒ, καὶ ἐκβεβλήσθω ἡ ΑΒ, καὶ ἔστω ἐπὶ τυχοῦσαν τὴν ΔΕ κάθετος, καὶ τῶι ὑπὸ ΑΖΒ ἴσον κείσθω τὸ ἀπὸ ΖΗ τετράγωνον. ὅτι, οἷον ἐὰν ληφθῆι σημεῖον ὡς τὸ Ε, καὶ ἀπ' αὐτοῦ ἐπὶ τὸ Η ἐπιζευχθεῖσα ἐκβληθῆι ἐπὶ τὸ Θ, γίνεται καὶ τὸ ὑπὸ ΘΕΚ ἴσον τῶι ἀπὸ ΕΗ τετραγώνωι. ἐπεζεύχθωσαν αἱ ΑΕ, ΒΛ. ὀρθὴ ἄρα ἐστὶν ἡ Λ γωνία. ἔστιν δὲ καὶ ἡ Ζ ὀρθή. τὸ ἄρα ὑπὸ ΑΕΛ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΖΒ καὶ
τῶι ἀπὸ ΖΕ τετραγώνωι. ἀλλὰ τὸ μὲν ὑπὸ ΑΕΛ ἴσον ἐστὶν τῶι 912
ὑπὸ ΘΕΚ· τὸ δὲ ὑπὸ ΑΖΒ ἴσον ἐστὶν τῶι ἀπὸ ΖΗ τετραγώνωι.

‖ 1 ΑΗΕ Co ΑΗΘ A ‖ 6 ΑΓ Co ΒΓ A ‖ 9 ἡ Ge (recc?) ηι A ‖ 10 ΖΒΗ] Β´´ Ζ´ Η´´´ A ΖΒ, ΒΗ Co ‖ 13 συναμφότερος Ge (S) συναμφοτερα A ‖ 15 ΑΖ – τὸ ἀπὸ add Co ‖ 16 δὴ Heiberg, ΔΕ A (δὲ BS) ἄρα Co | ΖΚΘ] ΖΚ, ΚΘ Co ‖ 23 δ' add Hu

ZH,[6] that is the square of EH.[7]

(228) *(Prop. 160)* As is AB to BΓ, let AΔ be to ΔΓ, and let AΓ be bisected at point E. That three things result: the rectangle contained by BE, EΔ equals the square of EΓ, the rectangle contained by BΔ, ΔE equals the rectangle contained by AΔ, ΔΓ, and the rectangle contained by AB, BΓ equals the rectangle contained by EB, BΔ.

For since as is AB to BΓ, so is AΔ to ΔΓ,[1] *componendo*[2] and (taking) halves of the leading (members)[3] and *convertendo* therefore, as is BE to EΓ, so is EΓ to EΔ.[4] Therefore the rectangle contained by BE, EΔ equals the square of EΓ.[5] Let the square of ΔE be subtracted in common. Then the remaining rectangle contained by BΔ, ΔE equals the rectangle contained by AΔ, ΔΓ.[6] Again, the rectangle contained by BE, EΔ equals the square of EΓ.[7] Let both be subtracted from the square of BE.[8] Then the remaining rectangle contained by AB, BΓ equals the rectangle contained by EB, BΔ.

But now let the rectangle contained by BΔ, ΔE be equal to the rectangle contained by AΔ, ΔΓ, and let ΓA be bisected at E. That, as is AB to BΓ, so is AΔ to ΔΓ.

For since the rectangle contained by BΔ, ΔE equals the rectangle contained by AΔ, ΔΓ,[1] let the square of ΔE be added in common. Then the sum, the rectangle contained by BE, EΔ, equals the square of ΓE.[2] In ratio[3] <and *convertendo*>[4] and (taking) twice the leading (members)[5] and *separando*, therefore, as is AB to BΓ, so is AΔ to ΔΓ.[6]

(229) *(Prop. 161)* These things being so, let there be the circle about diameter AB, and let AB be produced. Let it be a perpendicular to an arbitrary (line) ΔE, and as AZ is to ZB, so let AH be made to HB. That again (as in 7.227) if some point such as E should be chosen on EΔ, and EH joined and produced to Θ, then as ΘE is to EK, so is ΘH to HK.

Let the center Λ of the circle be taken, and from Λ let ΛM be drawn as a perpendicular to EΘ.[1] Then KM equals MΘ.[2] Since both angles M, Z are right,[3] points E, Z, Λ, M are on a circle.[4] Therefore the rectangle contained by ZH, HΛ equals the rectangle contained by EH, HM.[5] But the rectangle contained by ZH, HΛ equals the rectangle contained by AH, HB[8], because as is AZ to ZB, so is AH to HB,[6] and AB has been bisected at Λ (7.228).[7] And therefore the rectangle contained by EH, HM equals the rectangle contained by AH, HB,[9] that is, since they are in a circle, the τὸ ἄρα ὑπὸ ΘΕΚ ἴσον ἐστὶν τοῖς ἀπὸ τῶν ΕΖΗ τετραγώνοις, τουτέστιν τῶι ἀπὸ ΕΗ τετραγώνωι.

(228) ἔστω ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, καὶ τετμήσθω ἡ ΑΓ δίχα κατὰ τὸ Ε σημεῖον. ὅτι γίνεται τρία, τὸ μὲν ὑπὸ ΒΕΔ ἴσον τῶι ἀπὸ ΕΓ τετραγώνωι, τὸ δὲ ὑπὸ ΒΔΕ τῶι ὑπὸ ΑΔΓ, τὸ δὲ ὑπὸ ΑΒΓ τῶι ὑπὸ ΕΒΔ. ἐπεὶ γὰρ ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, συνθέντι καὶ τὰ ἡμίση τῶν ἡγουμένων, |καὶ ἀναστρέψαντι ἄρα ἐστὶν ὡς ἡ ΒΕ πρὸς τὴν ΕΓ, |166v ἡ ΕΓ πρὸς τὴν ΕΔ. τὸ ἄρα ὑπὸ ΒΕΔ ἴσον ἐστὶν τῶι ἀπὸ ΕΓ. κοινὸν ἀφῃρήσθω τὸ ἀπὸ ΔΕ τετράγωνον. λοιπὸν ἄρα τὸ ὑπὸ ΒΔΕ ἴσον ἐστὶν τῶι ὑπὸ ΑΔΓ. πάλιν τὸ ὑπὸ ΒΕΔ ἴσον ἐστὶν τῶι ἀπὸ ΕΓ τετραγώνωι. ἀμφότερα ἀφῃρήσθω ἀπὸ τοῦ ἀπὸ τῆς ΒΕ τετραγώνου. λοιπὸν ἄρα τὸ ὑπὸ τῶν ΑΒΓ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΕΒΔ.

ἀλλὰ ἔστω νῦν τὸ ὑπὸ τῶν ΒΔΕ ἴσον τῶι ὑπὸ τῶν ΑΔΓ, καὶ τετμήσθω δίχα ἡ ΓΑ κατὰ τὸ Ε. ὅτι ἐστὶν <ὡς> ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ. ἐπεὶ γὰρ τὸ ὑπὸ τῶν ΒΔΕ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΔΓ, κοινὸν προσκείσθω τὸ ἀπὸ ΔΕ τετράγωνον. ὅλον ἄρα τὸ ὑπὸ ΒΕΔ ἴσον τῶι ἀπὸ ΓΕ τετραγώνωι. ἀνάλογον <καὶ ἀναστρέψαντι> καὶ δὶς τὰ ἡγούμενα, καὶ διελόντι ἄρα ἐστὶν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ.

(229) τούτων ὄντων, ἔστω κύκλος ὁ περὶ διάμετρον τὴν ΑΒ, 914 καὶ ἐκβεβλήσθω ἡ ΑΒ. ἔστω δὲ ἐπὶ τυχοῦσαν τὴν ΔΕ κάθετος, καὶ πεποιήσθω ὡς ἡ ΑΖ πρὸς τὴν ΖΒ, οὕτως ἡ ΑΗ πρὸς τὴν ΗΒ. ὅτι πάλιν οἷον ἐὰν ἐπὶ τῆς ΕΔ σημεῖον ληφθῆι ὡς τὸ Ε, καὶ ἐπιζευχθεῖσα ἡ ΕΗ ἐκβληθῆι ἐπὶ τὸ Θ, γίνεται ὡς ἡ ΘΕ πρὸς τὴν ΕΚ, οὕτως ἡ ΘΗ πρὸς τὴν ΗΚ. εἰλήφθω τὸ κέντρον τοῦ κύκλου τὸ Λ, καὶ ἀπὸ τοῦ Λ ἐπὶ τὴν ΕΘ κάθετος ἤχθω ἡ ΛΜ. ἴση ἄρα ἐστὶν ἡ ΚΜ τῆι ΜΘ. ἐπεὶ δὲ ὀρθή ἐστιν ἑκατέρα τῶν Μ, Ζ γωνιῶν, ἐν κύκλωι ἐστὶν τὰ Ε, Ζ, Λ, Μ σημεῖα. τὸ ἄρα ὑπὸ ΖΗΛ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΕΗΜ. ἀλλὰ τὸ ὑπὸ τῶν ΖΗΛ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΗΒ (διὰ τὸ εἶναι ὡς τὴν ΑΖ πρὸς τὴν ΖΒ, οὕτως τὴν ΑΗ πρὸς τὴν ΗΒ· καὶ τέτμηται ἡ ΑΒ δίχα κατὰ τὸ Λ). καὶ

‖ 1 ΕΖΗ] ΕΖ, ΖΗ Co ‖ 8 ΒΕ Co ΑΕ A ‖ 9 ante ἡ add οὕτως Ge | ΒΕΔ Co ΑΕΔ A ‖ 10 ΔΕ Co ΓΕ A ‖ 11 ΒΕΔ Co ΑΕΔ A ‖ 12 ΕΓ Co ΑΓ A ‖ 14 ΕΒΔ Co ΕΒΓ A ‖ 16 ὡς add Ge (S) ‖ 20 καὶ ἀναστρέψαντι add Co ‖ 26 ΕΔ Co ΓΔ A ‖ 27 ἐπιζευχθεῖσα Hu ἐπιζευχθῆι A | ἐκβληθῆι Hu ἐκβεβλήσθω A καὶ ἐκβληθῆι Ge ‖ 32 ἴσον (post ΖΗΛ) Ge (BS) ἴση A ‖ 34 τέτμηται ἡ] τετμῆσθι τὴν coni. Hu app rectangle contained by ΘH, HK.[10] And ΘK has been bisected at M.[11] Because of the foregoing (lemma 7.228), as is ΘE to EK, so is ΘH to HK.[12]

(230) *(Prop. 162)* (Let there be) the semicircle on AB, and AB parallel to ΓΔ, and let perpendiculars ΓE, ΔH be drawn. That AE equals HB.

Let the center Z of the circle be taken, and let ΓZ and ZΔ be joined. Then ΓZ equals ZΔ.[1] Hence too the square of ΓZ equals the square of ZΔ.[2] But the squares of ΓE, EZ equal the square of ΓZ,[3] while the squares of ΔH, HZ equal the square of ΔZ.[4] Therefore the squares of ΓE, EZ equal the squares of ZH, HΔ.[5] Of these the square of ΓE equals the square of ΔH.[6] Therefore the remaining square of EZ equals the remaining square of ZH.[7] Thus EZ equals ZH.[8] But also all AZ equals all ZB.[9] Therefore remainder AE equals remainder HB.[10] Q.E.D.

(231) *(Prop. 163)* (Let there be) the semicircle on AB, and from an arbitrary (point) Γ let ΓΔ be drawn across, and let perpendicular ΔE be drawn. That the square of AΓ exceeds the square of ΓΔ by the rectangle contained by AΓ, ΓB taken together and AE.[1]

Hence that the square of AΓ equals the square of ΔΓ, that is the squares of ΔE and EΓ,[3] and the rectangle contained by AΓB taken together and AE.[2] Hence that, with the rectangle contained by ΓA, AE subtracted in common, the remaining rectangle contained by AΓ, ΓE equals the square of ΔE, that is the rectangle contained by AE, EB,[5] plus the square of ΓE plus the rectangle contained by AE, ΓB.[4] With the square of ΓE subtracted in common, that the remaining rectangle contained by AE, EΓ equals the rectangle contained by AE, EB plus the rectangle contained by AE, BΓ.[6] But it does.[7]

(232) For the <...> porism of the first (book).

(Prop. 164) AΔ being a parallelogram (given) in position, to draw EZ across from a given (point) E, making triangle ZΓH equal to parallelogram AΔ.

τὸ ὑπὸ τῶν ΕΗΜ ἄρα ἴσον ἐστὶν τῶι ὑπὸ τῶν ΑΗΒ, τουτέστιν (ἐν κύκλωι γάρ) τῶι ὑπὸ τῶν ΘΗΚ. καὶ τέτμηται δίχα ἡ ΘΚ κατὰ τὸ Μ. διὰ <δὴ> τὸ προγεγραμμένον γίνεται ὡς ἡ ΘΕ πρὸς τὴν ΕΚ, οὕτως ἡ ΘΗ πρὸς τὴν ΗΚ.

(230) ἡμικύκλιον τὸ ἐπὶ τῆς ΑΒ, καὶ παράλληλος τῆι ΑΒ ἡ ΓΔ, καὶ κάθετοι ἤχθωσαν αἱ ΓΕ, ΔΗ. ὅτι ἴση ἐστὶν ἡ ΑΕ τῆι ΗΒ. εἰλήφθω τὸ κέντρον τοῦ κύκλου τὸ Ζ, καὶ ἐπεζεύχθωσαν αἱ ΓΖ, ΖΔ. ἴση ἄρα ἐστὶν ἡ ΓΖ τῆι ΖΔ. ὥστε καὶ τὸ ἀπὸ τῆς ΓΖ ἴσον τῶι ἀπὸ τῆς ΖΔ τετραγώνωι. ἀλλὰ τῶι μὲν ἀπὸ ΓΖ 916 τετραγώνωι ἴσα ἐστὶν τὰ ἀπὸ τῶν ΓΕ, ΕΖ τετράγωνα. τῶι δὲ ἀπὸ ΔΖ τετρα|γώνωι ἴσα ἐστὶν τὰ ἀπὸ τῶν ΔΗ, ΗΖ τετράγωνα. |167 καὶ τὰ ἀπὸ τῶν ΓΕ, ΕΖ ἄρα τετράγωνα ἴσα ἐστὶν τοῖς ἀπὸ τῶν ΖΗ, ΗΔ τετραγώνοις. ὧν τὸ ἀπὸ ΓΕ τετράγωνον ἴσον ἐστὶν τῶι ἀπὸ τῆς ΔΗ τετραγώνωι. λοιπὸν ἄρα τὸ ἀπὸ τῆς ΕΖ τετράγωνον λοιπῶι τῶι ἀπὸ ΖΗ τετραγώνωι ἐστὶν ἴσον. ἴση ἄρα ἐστὶν ἡ ΕΖ τῆι ΖΗ. ἔστιν δὲ καὶ ὅλη ἡ ΑΖ ὅλῃ τῆι ΖΒ ἴση. λοιπὴ ἄρα ἡ ΑΕ λοιπῆι τῆι ΗΒ ἐστὶν ἴση. ὅ(περ): —

(231) ἡμικύκλιον τὸ ἐπὶ τῆς ΑΒ, καὶ ἀπὸ τυχόντος τοῦ Γ διήχθω ἡ ΓΔ, καὶ κάθετος ἤχθω ἡ ΔΕ. ὅτι τὸ ἀπὸ ΑΓ τοῦ ἀπὸ ΓΔ ὑπερέχει τῶι ὑπὸ συναμφοτέρου τῆς ΑΓ, ΓΒ καὶ τῆς ΑΕ. ὅτι ἄρα τὸ ἀπὸ ΑΓ ἴσον ἐστὶν τῶι τε ἀπὸ ΔΓ, τουτέστιν τοῖς ἀπὸ ΔΕ, ΕΓ, καὶ τῶι ὑπὸ συναμφοτέρου τῆς ΑΓΒ καὶ τῆς ΑΕ. ὅτι ἄρα, κοινοῦ ἀφαιρεθέντος τοῦ ὑπὸ ΓΑΕ, λοιπὸν τὸ ὑπὸ ΑΓΕ ἴσον ἐστὶν τῶι τε ἀπὸ ΔΕ, τουτέστιν τῶι ὑπὸ ΑΕΒ, καὶ τῶι ἀπὸ ΓΕ καὶ τῶι ὑπὸ ΑΕ, ΓΒ. κοινὸν ἀφαιρεθέντος τοῦ ἀπὸ ΓΕ, ὅτι λοιπὸν τὸ ὑπὸ ΑΕΓ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΕΒ καὶ τῶι ὑπὸ ΑΕ, ΒΓ. ἔστιν δέ.

(232) ΕΙΣ ΤΟ <...> ΠΟΡΙΣΜΑ Α΄ ΒΙΒΛΙΟΥ

θέσει ὄντος παραλληλογράμμου τοῦ ΑΔ, ἀπὸ δοθέντος τοῦ Ε διαγαγεῖν τὴν ΕΖ καὶ ποιεῖν ἴσον τὸ ΖΓΗ τρίγωνον τῶι ΑΔ παραλληλογράμμωι. γεγονέτω. ἐπεὶ οὖν ἴσον ἐστὶν τὸ ΖΓΗ 918 τρίγωνον τῶι ΑΔ παραλληλογράμμωι, τὸ δὲ ΑΔ

‖ 1 ἐν κύκλωι γὰρ post ΘΗΚ transp. Ge ‖ 3 δὴ add Ge ‖ 18 ΑΒ Co ΑΒΓ Α ‖ 19 ΔΕ Co ΔΘ Α ‖ 22 ΑΓΒ] ΑΓ, ΓΒ Co ‖ 23 κοινοῦ Hu καὶ Α ‖ 24 ΔΕ Co ΑΕ Α ‖ 28 ΕΙΣ] ΕΙ $Α^1$ Σ add $Α^2$ | ante Α΄ add ΤΟΥ Hu

Let it have been done. Then since triangle ZΓH equals parallelogram AΔ,[1] while parallelogram AΔ is twice triangle AΓΔ,[2] therefore triangle ZΓH is twice triangle AΓΔ.[3] But as is the triangle to the triangle, because they are about the same angle Γ, so is the rectangle contained by ZΓ, ΓH to the rectangle contained by AΓ, ΓΔ[4] (Lemma 7.214). But the rectangle contained by AΓ, ΓΔ is given.[5] Therefore also the rectangle contained by ZΓ, ΓH is given.[6] And with (point) E given, <line EZ> has been drawn across (lines) AΓ, ΓΔ (given) in position, <and cutting off an area, the rectangle contained by ZΓ, ΓH, equal to a given (area). It has been brought to a reference> to the *Cutting off of an Area.* Hence EZ is (given) in position.[7]

The synthesis will be made thus. Let the parallelogram (given) in position be AΔ, the given (point) E. Let straight line EZ be drawn from E across (straight lines) ZΓ, ΓH (given) in position, cutting off an area, the rectangle contained by ZΓ, ΓH, equal to a given area, twice the rectangle contained by AΓ, ΓΔ. And following the analysis, we shall prove that triangle ZΓH equals parallelogram AΔ. Thus EZ solves the problem. Hence it is clearly unique (in solving it), since that (line, the one found in the *Cutting off of an Area*), is unique.

παραλληλόγραμμον διπλάσιόν ἐστιν τοῦ ΑΓΔ τριγώνου, καὶ τὸ ΖΓΗ ἄρα τρίγωνον διπλάσιόν ἐστιν τοῦ ΑΓΔ τριγώνου. ὡς δὲ τὸ τρίγωνον πρὸς τὸ τρίγωνον, διὰ τὸ <εἶναι> περὶ τὴν αὐτὴν γωνίαν τὴν Γ, οὕτως ἐστὶν τὸ ὑπὸ ΖΓΗ πρὸς τὸ ὑπὸ ΑΓΔ. δοθὲν δὲ τὸ ὑπὸ ΑΓΔ. δοθὲν ἄρα καὶ τὸ ὑπὸ ΖΓΗ. καὶ δοθέντος τοῦ Ε εἰς θέσει τὰς ΑΓ, ΓΔ διῆκται <ἡ ΕΖ ἀποτέμνουσα χωρίον τὸ ὑπὸ ΖΓΗ ἴσον δοθέντι· ἀπῆκται> εἰς Χωρίου Ἀποτομήν. θέσει ἄρα ἐστὶν ἡ ΕΖ.

συντεθήσεται δὲ οὕτως. ἔστω τὸ μὲν τῆι θέσει παραλληλόγραμμον τὸ ΑΔ, τὸ δὲ δοθὲν τὸ Ε. διήχθω ἀπὸ τοῦ Ε εἰς θέσει τὰς ΖΓ, ΓΗ εὐθεῖα ἡ ΕΖ ἀποτέμνουσα χωρίον τὸ ὑπὸ ΖΓΗ ἴσον δοθέντι χωρίωι τῶι διπλασίονι τοῦ ὑπὸ ΑΓΔ. καὶ κατὰ τὰ αὐτὰ τῆι ἀναλύσει, δείξομεν ἴσον τὸ ΖΓΗ τρίγωνον τῶι ΑΔ παραλληλογράμμωι. ἡ ΕΖ ἄρα ποιεῖ τὸ πρόβλημα. φανερὸν οὖν ὅτι μόνη, ἐπεὶ κἀκείνη μόνη.

‖ 1 καὶ τὸ – ΑΓΔ τριγώνου om A^1 add mg A^2 alia manu ‖ 3 εἶναι add Hu ‖ 5 post ΖΓΗ καὶ add ἀπὸ Hu (Co) ‖ 6 ἡ ΕΖ add Hu (Co) | ἀποτέμνουσα – ἀπῆκται addidi (idem fere coni. Hu app) ‖ 11 ΖΓ, ΓΗ Co ΖΓΗ A | ὑπὸ del Heiberg$_3$ ‖ 12 δοθέντι Co δοθεντινι A | διπλασίονι Co διπλάσιον A | ὑπὸ del Heiberg$_3$ ‖ 13 τῆι ἀναλύσει Co τὴν ἀνάλυσιν A ‖ 15 οὖν] compendium A

(233) (Conics, Book 1)

(Prop. 165) Let there be a cone, the base of which is circle AB, and whose apex is point Γ. Now if the cone is isosceles, clearly all the straight lines falling from Γ onto circle AΓ are equal to each other. But if it is scalene, let it be (required) to find which (line) is greatest, and which least.

For let a perpendicular be drawn from point Γ onto the plane of circle AB, and first let it fall inside circle AB, and let it be ΓΔ; and let the center of the circle, E, be taken, and let ΔE be joined and produced in both directions to points A, B, and let AΓ, ΓB be joined. I say that BΓ is the greatest, AΓ the least of all the (lines) falling from Γ onto (circle) AB.

For let some other (line) ΓZ be dropped, and let ΔZ be joined. Then BΔ is greater than ΔZ.[1] But ΓΔ is common; and the angles at Δ are right.[2] Therefore BΓ is greater than ΓZ.[3] By the same argument too ΓZ is greater than ΓA.[4] Thus ΓB is the greatest, ΓA the least.

(234) *(Prop. 166)* But now let the perpendicular drawn from Γ fall on the circumference of circle AB, and let it be ΓA, and again let AΔ be joined to the center Δ of the circle, and let it be produced to B, and let BΓ be joined. I say that BΓ is the greatest, AΓ the least.

That ΓB is greater than ΓA is obvious. Let some other (line) ΓE be drawn, and let AE be joined. Since AB is a diameter, it is greater than AE.[1] And AΓ is at right angles to them.[2] Therefore BΓ is greater than ΓE.[3] Similarly (it is greater) than all. And by the same argument, it will be proved that EΓ is greater than ΓA.[4] Thus BΓ is greatest, ΓA least of the straight lines that fall from point Γ onto circle AB.

(235) *(Prop. 167)* With the same things assumed, let the perpendicular fall outside the circle, and let it be ΓΔ, and let ΔE be joined to the center E of the circle and produced, and let AΓ and BΓ be joined. I say that BΓ is the greatest, AΓ the least of all the straight lines that fall from Γ onto circle AB.

That BΓ is greater than ΓA is obvious. I say that (it is greater) also than all the (lines) that fall from Γ onto the circumference of circle AB. For

(233) ἔστω κῶνος οὗ βάσις μὲν ὁ ΑΒ κύκλος, κορυφὴ δὲ τὸ Γ σημεῖον. εἰ μὲν οὖν ἰσο|σκελής ἐστιν ὁ κῶνος, φανερὸν ὅτι |167v πᾶσαι αἱ ἀπὸ τοῦ Γ πρὸς τὸν ΑΒ κύκλον προσπίπτουσαι εὐθεῖαι ἴσαι ἀλλήλαις εἰσίν. εἰ δὲ σκαληνός, ἔστω εὑρεῖν τίς μεγίστη καὶ τίς ἐλαχίστη. ἤχθω γὰρ ἀπὸ τοῦ Γ σημείου ἐπὶ τὸ τοῦ ΑΒ κύκλου ἐπίπεδον κάθετος, καὶ πιπτέτω 920 πρότερον ἐντὸς τοῦ ΑΒ κύκλου, καὶ ἔστω ἡ ΓΔ, καὶ εἰλήφθω τὸ κέντρον τοῦ κύκλου τὸ Ε, καὶ ἐπιζευχθεῖσα ἡ ΔΕ ἐκβεβλήσθω ἐφ' ἑκάτερα τὰ μέρη ἐπὶ τὰ Α, Β σημεῖα, καὶ ἐπεζεύχθωσαν αἱ ΑΓ, ΓΒ. λέγω ὅτι μεγίστη μέν ἐστιν ἡ ΒΓ, ἐλαχίστη δὲ ἡ ΑΓ πασῶν τῶν ἀπὸ τοῦ Γ πρὸς τὸν ΑΒ προσπιπτουσῶν. προσβεβλήσθω γάρ τις καὶ ἑτέρα ἡ ΓΖ, καὶ ἐπεζεύχθω ἡ ΔΖ. μείζων ἄρα ἐστὶν ἡ ΒΔ τῆς ΔΖ· κοινὴ δὲ ἡ ΓΔ. καὶ εἰσὶν αἱ πρὸς τῶι Δ γωνίαι ὀρθαί. μείζων ἄρα ἐστὶν ἡ ΒΓ τῆς ΓΖ. κατὰ τὰ αὐτὰ καὶ ἡ ΓΖ τῆς ΓΑ μείζων ἐστίν. ὥστε μεγίστη μέν ἐστιν ἡ ΓΒ, ἐλαχίστη δὲ ἡ ΓΑ.

(234) ἀλλὰ δὴ πάλιν ἡ ἀπὸ τοῦ Γ κάθετος ἀγομένη πιπτέτω ἐπὶ τῆς περιφερείας τοῦ ΑΒ κύκλου, καὶ ἔστω <ἡ> ΓΑ, καὶ πάλιν ἐπὶ τὸ κέντρον τοῦ κύκλου τὸ Δ ἐπεζεύχθω ἡ ΑΔ, καὶ ἐκβεβλήσθω ἐπὶ τὸ Β, καὶ ἐπεζεύχθω ἡ ΒΓ. λέγω ὅτι μεγίστη μέν ἐστιν ἡ ΒΓ, ἐλαχίστη δὲ ἡ ΑΓ. ὅτι μὲν οὖν μείζων ἡ ΓΒ τῆς ΓΑ φανερόν. διήχθω δέ τις καὶ ἑτέρα ἡ ΓΕ, καὶ ἐπεζεύχθω ἡ ΑΕ. ἐπεὶ διάμετρός ἐστιν ἡ ΑΒ, μείζων ἐστὶν τῆς ΑΕ. καὶ αὐταῖς πρὸς ὀρθὰς ἡ ΑΓ. μείζων <ἄρα> ἐστὶν ἡ ΒΓ τῆς ΓΕ. ὁμοίως καὶ πασῶν. καὶ κατὰ τὰ αὐτὰ μείζων δειχθήσεται ἡ ΕΓ τῆς ΓΑ. ὥστε μεγίστη μὲν ἡ ΒΓ, ἐλαχίστη δὲ ἡ ΓΑ τῶν ἀπὸ τοῦ Γ σημείου πρὸς τὸν ΑΒ κύκλον προσπιπτουσῶν εὐθειῶν.

(235) τῶν αὐτῶν ὑποκειμένων, πιπτέτω ἡ κάθετος ἐκτὸς τοῦ κύκλου, καὶ ἔστω ἡ ΓΔ, καὶ ἐπὶ τὸ κέντρον τοῦ κύκλου τὸ Ε ἐπιζευχθεῖσα ἡ ΔΕ ἐκβεβλήσθω, καὶ ἐπεζεύχθωσαν αἱ ΑΓ, ΒΓ. λέγω δὴ ὅτι μεγίστη μέν ἐστιν ἡ ΒΓ, ἐλαχίστη δὲ ἡ ΑΓ πασῶν 922 τῶν ἀπὸ τοῦ Γ πρὸς τὸν ΑΒ κύκλον προσπιπτουσῶν εὐθειῶν. ὅτι μὲν οὖν μείζων ἐστὶν ἡ ΒΓ τῆς ΓΑ φανερόν. λέγω δὴ ὅτι καὶ πασῶν τῶν ἀπὸ τοῦ Γ πρὸς τὴν τοῦ ΑΒ κύκλου περιφέρειαν

‖ 1 ΑΒ Co ΑΒΓ A ‖ 4 ante ἔστω add δέον Ha ‖ 6 κύκλου — ἐντὸς τοῦ ΑΒ om A¹ add mg A² ‖ 13 τῆς Ha τῆι A ‖ 18 κύκλου ΑΒ transp Ha | ἡ add Ge (recc?) ‖ 24 ἄρα add Ha ‖ 26 μεγίστη Ha (Co) μεγίστης A ‖ 31 ὅτι δὴ transp Ha ‖ 32 ΑΒ Co ΑΒΓ A ‖ 33 δὴ] δὲ Hu let some other (line) ΓZ be dropped, and let ΔZ be joined. Then since BΔ is through the center, ΔB is greater than ΔZ (III 8).[1] And ΔΓ is at right angles to them,[2] since also to the plane. Therefore BΓ is greater than ΓZ.[3] Similarly (it is greater) than all. Thus ΓB is greatest.

(To prove) that AΓ is least. For since AΔ is less than ΔZ,[4] and ΔΓ is at right angles to them,[5] therefore AΓ is less than ΓZ.[6] Similarly (it is less) than all. Thus AΓ is least, BΓ greatest of all the straight lines that fall from Γ onto the circumference of circle AB.

(236) For the Conic Definitions.

"If (a line) from some point onto the circumference of a circle..." *(Conics 1 def. 1)* Apollonius reasonably adds also "is produced in both directions", inasmuch as he is setting out the generation of the general cone. For if the cone were isosceles, it would be superfluous to produce (the line) because the moving straight line always touches the circle's circumference, since the point would always be an equal distance from the circle's circumference. But since the cone can also be scalene, and there is, as was written above, in a scalene cone some greatest and least edge, necessarily he adds the "let it be produced" so that the least (line) by being produced always increases [with the greatest always being produced] until it becomes equal to the greatest, and touches the circle's circumference at that point.

(237) *(Prop. 168)* Let there be a curve ABΓ, and AΓ (given) in position, and let all the perpendiculars drawn from the curve to AΓ be drawn in such a way that the square of each of them equals the rectangle contained by the sections of the base cut off by each of them. I say that ABΓ is a circle's circumference, and AΓ its diameter.

For let perpendiculars ΔZ, BH, EΘ be drawn from points Δ, B, E.[1] Then the square of ΔZ equals the rectangle contained by AZ, ZΓ,[2] and the square of BH (equals) the rectangle contained by AH, HΓ,[2] and the square of EΘ (equals) the rectangle contained by AΘ, ΘΓ.[4] Now let AΓ be bisected at K,[5] and let ΔK, KB, KE be joined. Then since the rectangle contained by προσπιπτουσῶν. προσπιπτέτω γάρ τις καὶ ἑτέρα ἡ ΓΖ, καὶ ἐπε|ζεύχθω ἡ ΔΖ. ἐπεὶ οὖν διὰ τοῦ κέντρου ἐστὶν ἡ ΒΔ, |168 μείζων ἐστὶν ἡ ΔΒ τῆς ΔΖ. καὶ ἔστιν αὐταῖς ὀρθὴ ἡ ΔΓ, ἐπεὶ καὶ τῶι ἐπιπέδωι. μείζων ἄρα ἐστὶν ἡ ΒΓ τῆς ΓΖ. ὁμοίως καὶ πασῶν. μεγίστη μὲν ἄρα ἐστὶν ἡ ΓΒ. ὅτι δὲ καὶ ἡ ΑΓ ἐλαχίστη. ἐπεὶ γὰρ ἐλάσσων ἐστὶν ἡ ΑΔ τῆς ΔΖ, καὶ ἔστιν αὐταῖς ὀρθὴ ἡ ΔΓ, ἐλάσσων ἄρα ἐστὶν ἡ ΑΓ τῆς ΓΖ. ὁμοίως καὶ πασῶν. ἐλαχίστη ἄρα ἐστὶν ἡ ΑΓ, μεγίστη δὲ ἡ ΒΓ πασῶν τῶν ἀπὸ τοῦ Γ πρὸς τὴν τοῦ ΑΒ κύκλου περιφέρειαν προσπιπτουσῶν εὐθειῶν.

(236) ΕΙΣ ΤΟΥΣ ΚΩΝΙΚΟΥΣ ΟΡΟΥΣ

"ἐὰν ἀπό τινος σημείου πρὸς κύκλου περιφέρειαν..." εἰκότως ὁ Ἀπολλώνιος προστίθησιν καὶ "ἐφ' ἑκάτερα ἐκβληθῆι", ἐπειδήπερ τοῦ τυχόντος κώνου γένεσιν δηλοῖ. εἰ μὲν γὰρ ἰσοσκελὴς ὁ κῶνος, περισσὸν ἦν προσεκβάλλειν διὰ <τὸ> τὴν φερομένην εὐθεῖαν αἰεί ποτε ψαύειν τῆς τοῦ κύκλου περιφερείας, ἐπειδήπερ πάντοτε τὸ σημεῖον ἴσον ἀφέξειν ἔμελλεν τῆς τοῦ κύκλου περιφερείας. ἐπεὶ δὲ δύναται καὶ σκαληνὸς εἶναι ὁ κῶνος, ἔστιν δέ, ὡς προγέγραπται, ἐν κώνωι σκαληνῶι μεγίστη τις καὶ ἐλαχίστη πλευρά, ἀναγκαίως προστίθησιν τὸ "προσεκβεβλήσθω" ἵνα αἰεὶ προσεκβληθεῖσα ἡ ἐλαχίστη [ἀεὶ τῆς μεγίστης] αὔξηται 924 [προσεκβαλλομένης] ἕως ἴση γένηται τῆι μεγίστηι, καὶ ψαύσηι κατ' ἐκεῖνο τῆς τοῦ κύκλου περιφερείας.

(237) ἔστω γραμμὴ ἡ ΑΒΓ, καὶ θέσει ἡ ΑΓ. πᾶσαι δὲ αἱ ἀπὸ τῆς γραμμῆς ἐπὶ τὴν ΑΓ κάθετοι ἀγόμεναι οὕτως ἀγέσθωσαν, ὥστε τὸ ἀπὸ ἑκάστης αὐτῶν τετράγωνον ἴσον εἶναι τῶι περιεχομένωι ὑπὸ τῶν τῆς βάσεως τμημάτων <τῶν> ὑφ' ἑκάστης αὐτῶν τμηθέντων. λέγω ὅτι κύκλου περιφέρειά ἐστιν ἡ ΑΒΓ, διάμετρος δὲ αὐτῆς ἐστὶν ἡ ΑΓ. ἤχθωσαν γὰρ ἀπὸ σημείων τῶν Δ, Β, Ε κάθετοι αἱ ΔΖ, ΒΗ, ΕΘ. τὸ μὲν ἄρα ἀπὸ ΔΖ ἴσον ἐστὶν τῶι ὑπὸ ΑΖΓ, τὸ δὲ ἀπὸ ΒΗ τῶι ὑπὸ ΑΗΓ, τὸ δὲ ἀπὸ ΕΘ τῶι ὑπὸ ΑΘΓ. τετμήσθω δὴ δίχα ἡ ΑΓ κατὰ τὸ Κ, καὶ

‖ 6 ἐλάσσων Ha ἐλαχίστη A | ante αὐταῖς add καὶ Ha ‖ 7 ἐλάσσων Ha ἐλαχίστη A ‖ 13 καὶ ἐφ' ἑκάτερα ἐκβληθῆι] ἐφ' ἑκάτερα προσεκβληθῆι Hu ex Apollonio ‖ 16 τὸ add Hu ‖ 18 δὲ A^2 ex δ∗ ‖ 21 προσεκβεβλήσθω] προσεκβληθῆι Ha ‖ 22 ἀεὶ τῆς μεγίστης... προσεκβαλλομένης del Ha ‖ 26 ἀγόμεναι Ha ἀγομενοι A secl Hu ‖ 28 τῶν add Heiberg$_2$ | ὑφ'] ἀφ' Ha ‖ 29 αὐτῶν τμηθέντων Ha ἀπὸ τῶν τμηθέντων A ἀποτμηθέντων Heiberg$_2$ ‖ 33 δὴ] δὲ Hu

AZ, ZΓ plus the square of ZK equals the square of AK,[6] but the square of ΔZ equals the rectangle contained by AZ, ZΓ, therefore the square of ΔZ plus the square of ZK, that is the square of ΔK,[8] equals the square of AK.[7] Therefore AK equals KΔ.[9] Similarly we shall prove that each of BK, EK equals AK, or KΓ. Thus ABΓ is the circumference of the circle about center K, that is about diameter AΓ.

(238) *(Prop. 169 a – b)* (Let there be) three parallels AB, ΓΔ, EZ, and let two straight lines AHZΓ, BHEΔ be drawn across them. That as is the rectangle contained by AB, EZ to the square of ΓΔ, so is the rectangle contained by AH, HZ to the square of HΓ.

For since as AB is to ZE, that is as is the rectangle contained by AB, ZE to the square of ZE,[2] so is AH to HZ,[1] that is the rectangle contained by AH, HZ to the square of HZ,[3] therefore as is the rectangle contained by AB, ZE to the square of ZE, so is the rectangle contained by AH, HZ to the square of HZ.[4] But also as is the square of ZE to the square of ΓΔ, so is the square of ZH to the square of HΓ.[5] *Ex aequali* therefore as is the rectangle contained by AB, ZE to the square of ΓΔ, so is the rectangle contained by AH, HZ to the square of HΓ.[6]

(239) *(Prop. 170)* As is AB to BΓ, so let AΔ be to ΔΓ, and let AΓ be bisected at point E. That the rectangle contained by BE, EΔ equals the square of EΓ, and that the rectangle contained by AΔ, ΔΓ (equals) the rectangle contained by BΔ, ΔE, and that the rectangle contained by AB, BΓ (equals) the rectangle contained by EB, BΔ.

For since as AB is to BΓ, so is AΔ to ΔΓ,[1] *componendo*[2] and (taking) the halves of the leading (members)[3] and *convertendo*, as is BE to EΓ, so is ΓE to EΔ.[4] Therefore the rectangle contained by BE, EΔ equals the square of EΓ.[5] Let the square of EΔ be subtracted in common. Then the remaining rectangle contained by AΔ, ΔΓ equals the rectangle contained by BΔ, ΔE.[6] But since the rectangle contained by BE, EΔ equals the square of EΓ, let each be subtracted from the square of BE.[7] Then the remaining rectangle contained by AB, BΓ equals the rectangle contained by EB, BΔ.[8] Thus the three things result.

(240) *(Prop. 171)* Let A have to B the ratio compounded out of that which Γ has to Δ, and that which E has to Z. That also Γ has to Δ the ratio compounded out of that which A has to B, and that which Z has to E.

ἐπεζεύχθωσαν αἱ ΔΚ, ΚΒ, ΚΕ. ἐπεὶ οὖν τὸ ὑπὸ ΑΖΓ μετὰ τοῦ ἀπὸ ΖΚ ἴσον ἐστὶν τῶι ἀπὸ ΑΚ, ἀλλὰ τῶι ὑπὸ ΑΖΓ ἴσον ἐστὶν τὸ ἀπὸ ΔΖ, τὸ ἄρα ἀπὸ ΔΖ μετὰ τοῦ ἀπὸ ΖΚ, τουτέστιν τὸ ἀπὸ ΔΚ, ἴσον ἐστὶν τῶι ἀπὸ ΑΚ. ἴση ἄρα ἐστὶν ἡ ΑΚ τῆι ΚΔ. ὁμοίως δὴ δείξομεν ὅτι καὶ ἑκατέρα τῶν ΒΚ, ΕΚ ἴση ἐστὶν τῆι ΑΚ, ἢ τῆι ΚΓ. κύκλου ἄρα περιφέρειά ἐστιν ἡ ΑΒΓ τοῦ περὶ κέντρον |τὸ |168v Κ, τουτέστιν τοῦ περὶ διάμετρον τὴν ΑΓ.

(238) τρεῖς παράλληλοι αἱ ΑΒ, ΓΔ, ΕΖ, καὶ διήχθωσαν εἰς αὐτὰς δύο εὐθεῖαι αἱ ΑΗΖΓ, ΒΗΕΔ. ὅτι γίνεται ὡς τὸ ὑπὸ ΑΒ, ΕΖ πρὸς τὸ ἀπὸ ΓΔ, οὕτως τὸ ὑπὸ ΑΗΖ πρὸς τὸ ἀπὸ ΗΓ τετράγωνον. ἐπεὶ γάρ ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΖΕ, τουτέστιν ὡς τὸ ὑπὸ ΑΒ, ΖΕ πρὸς τὸ ἀπὸ ΖΕ, οὕτως ἡ ΑΗ πρὸς τὴν ΗΖ, τουτέστιν τὸ ὑπὸ ΑΗΖ πρὸς τὸ ἀπὸ ΗΖ, ὡς ἄρα τὸ ὑπὸ ΑΒ, ΖΕ πρὸς τὸ ἀπὸ ΖΕ, οὕτως τὸ ὑπὸ ΑΗΖ πρὸς τὸ ἀπὸ ΗΖ. ἀλλὰ καὶ ὡς τὸ ἀπὸ ΖΕ πρὸς τὸ ἀπὸ ΓΔ, οὕτως ἐστὶν τὸ ἀπὸ ΖΗ πρὸς τὸ ἀπὸ ΗΓ. δι' ἴσου ἄρα ἐστὶν ὡς τὸ ὑπὸ ΑΒ, ΖΕ πρὸς τὸ ἀπὸ ΓΔ 926 τετράγωνον, οὕτως τὸ ὑπὸ ΑΗΖ πρὸς τὸ ἀπὸ ΗΓ τετράγωνον.

(239) ἔστω ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, καὶ τετμήσθω ἡ ΑΓ δίχα κατὰ τὸ Ε σημεῖον. ὅτι γίνεται τὸ μὲν ὑπὸ ΒΕΔ ἴσον τῶι ἀπὸ ΕΓ, τὸ δὲ ὑπὸ ΑΔΓ τῶι ὑπὸ ΒΔΕ, τὸ δὲ ὑπὸ ΑΒΓ τῶι ὑπὸ ΕΒΔ. ἐπεὶ γάρ ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, συνθέντι καὶ τὰ ἡμίση τῶν ἡγουμένων, καὶ ἀναστρέψαντί ἐστιν ὡς ἡ ΒΕ πρὸς τὴν ΕΓ, οὕτως ἡ ΓΕ πρὸς τὴν ΕΔ. τὸ ἄρα ὑπὸ ΒΕΔ ἴσον ἐστὶν τῶι ἀπὸ ΓΕ τετραγώνωι. κοινὸν ἀφηιρήσθω τὸ ἀπὸ ΕΔ τετράγωνον. λοιπὸν ἄρα τὸ ὑπὸ ΑΔΓ ἴσον ἐστὶν τῶι ὑπὸ ΒΔΕ. ἐπεὶ δὲ τὸ ὑπὸ ΒΕΔ ἴσον ἐστὶν τῶι ἀπὸ ΕΓ, ἑκάτερον ἀφηιρήσθω ἀπὸ τοῦ ἀπὸ τῆς ΒΕ τετραγώνου. λοιπὸν ἄρα τὸ ὑπὸ ΑΒΓ ἴσον ἐστὶν τῶι ὑπὸ ΕΒΔ. γίνεται ἄρα τὰ τρία.

(240) τὸ Α πρὸς τὸ Β τὸν συνημμένον λόγον ἐχέτω ἔκ τε τοῦ ὃν ἔχει τὸ Γ πρὸς τὸ Δ, καὶ ἐξ οὗ ὃν ἔχει τὸ Ε πρὸς τὸ Ζ. ὅτι καὶ τὸ Γ πρὸς τὸ Δ τὸν συνημμένον λόγον ἔχει ἔκ τε τοῦ ὃν ἔχει τὸ Α πρὸς τὸ Β καὶ τὸ Ζ πρὸς τὸ Ε. τῶι γὰρ τοῦ Ε πρὸς

‖ 4 δὴ] δὲ Hu app ‖ 6 τοῦ Ha τῆς A ‖ 7 τοῦ Ha τῆς A ‖ 19 γίνεται Hu (B) γίνονται A ‖ 25 ἀφηιρήσθω Ha ἀφαιρείσθω A ‖ 26 ΒΕΔ Co ΒΑΔ A ‖ 27 ἀφηιρήσθω Ha ἀφαιρείσθω A ‖ 31 ἐξ οὗ ὃν] εξουσιον (sine acc.?) A[1] acc. add et σι secl A[2]

For let the (ratio) of Δ to H be made equal to that of E to Z.[1] Then since the (ratio) of A to B is compounded out of that of Γ to Δ, and that of E to Z,[2] that is of Δ to H,[3] whereas the (ratio) compounded out of that which Γ has to Δ, and that which Δ has to H is the (ratio) of Γ to H,[4] therefore as is A to B, so is Γ to H.[5] But since Γ has to Δ the ratio compounded out of that which Γ has to H, and that which H has to Δ,[6] while that of Γ to H was proved to be the same as that of A to B, and the (ratio) of H to Δ by inversion is the same as that of Z to E,[7] therefore Γ has to Δ the ratio compounded out of that which A has to B, and that which Z has to E.[8]

(241) *(Prop. 172 a– b)* Let there be two equiangular parallelograms AΓ, ΔZ, that have angle B equal to angle E. That as is the rectangle contained by AB, BΓ to the rectangle contained by ΔE, EZ, so is parallelogram AΓ to parallelogram ΔZ.

Now if angles B, E are right, it is obvious. If not, let perpendiculars AH, ΔΘ be drawn.[1] Then since angle B equals angle E,[2] and right (angle) H is (equal) to (angle) Θ, therefore triangle ABH is equiangular to triangle ΔEΘ.[3] Hence as is BA to AH, so is EΔ to ΔΘ.[4] But as BA is to AH, so is the rectangle contained by AB, BΓ to the rectangle contained by AH, BΓ,[5] while as is EΔ to ΔΘ, so is the rectangle contained by ΔE, EZ to the rectangle contained by ΔΘ, EZ.[6] Therefore *alternando* as is the rectangle contained by AB, BΓ to the rectangle contained by ΔE, EZ, so is the rectangle contained by AH, BΓ, that is parallelogram AΓ, to the rectangle contained by ΔΘ, EZ,[7] that is parallelogram ΔZ.[8]

(242) *(Prop. 173)* Let there be triangle ABΓ, and let BΓ be parallel to ΔE, and let the rectangle contained by ZA, AE be made equal to the square of ΓA. That, if ΔΓ, BZ are joined, BZ is parallel to ΔΓ.

But this is obvious. For since as is ZA to AΓ, so is ΓA to AE,[1] <while as is ΓA to AE,> so is BA to AΔ in parallels,[2] therefore as is ZA to AΓ, so is BA to AΔ.[3] Thus ΔΓ, BZ are parallel.

τὸ Ζ λόγωι ὁ αὐτὸς πεποιήσθω ὁ τοῦ Δ πρὸς τὸ Η. ἐπεὶ οὖν
<ὁ> τοῦ Α πρὸς τὸ Β συνῆπται ἔκ τε τοῦ τοῦ Γ πρὸς Δ καὶ τοῦ
τοῦ Ε πρὸς Ζ, τουτέστιν τοῦ Δ πρὸς τὸ Η, ἀλλὰ ὁ συνημμένος
ἔκ τε τοῦ ὃν ἔχει τὸ Γ πρὸς τὸ Δ καὶ ἐξ οὗ ὃν ἔχει τὸ Δ πρὸς
τὸ Η, ὁ τοῦ Γ πρὸς τὸ Η ἐστίν, <ὡς> ἄρα τὸ Α πρὸς τὸ Β, οὕτως
τὸ Γ πρὸς τὸ Η. ἐπεὶ δὲ τὸ Γ πρὸς τὸ Δ τὸν συνημμένον λόγον 928
ἔχει ἔκ τε τοῦ ὃν ἔχει τὸ Γ πρὸς τὸ Η καὶ ἐξ οὗ ὃν ἔχει τὸ Η
πρὸς τὸ Δ, ἀλλ' ὁ μὲν τοῦ Γ πρὸς τὸ Η ὁ αὐτὸς ἐδείχθη τῶι τοῦ
Α πρὸς τὸ Β, ὁ δὲ τοῦ Η πρὸς τὸ Δ ἐκ τοῦ ἀνάπαλιν ὁ αὐτός
ἐστιν τῶι τοῦ Ζ πρὸς τὸ Ε, καὶ τὸ Γ ἄρα πρὸς τὸ Δ τὸν
συνημμένον λόγον ἔχει ἔκ τε τοῦ |ὃν ἔχει τὸ Α πρὸς τὸ Β καὶ |169
ἐξ οὗ ὃν ἔχει τὸ Ζ πρὸς τὸ Ε.

(241) ἔστω δύο παραλληλόγραμμα τὰ ΑΓ, ΔΖ ἰσογώνια, ἴσην ἔχοντα τὴν Β γωνίαν τῆι Ε γωνίαι. ὅτι γίνεται ὡς τὸ ὑπὸ ΑΒΓ πρὸς τὸ ὑπὸ ΔΕΖ, οὕτως τὸ ΑΓ παραλληλόγραμμον πρὸς τὸ ΔΖ παραλληλόγραμμον. εἰ μὲν οὖν ὀρθαί εἰσιν αἱ Β, Ε γωνίαι, φανερόν. εἰ δὲ μή, ἤχθωσαν κάθετοι αἱ ΑΗ, ΔΘ. ἐπεὶ οὖν ἴση ἐστὶν ἡ μὲν Β γωνία τῆι Ε, ἡ δὲ Η ὀρθὴ τῆι Θ, ἰσογώνιον ἄρα ἐστὶν τὸ ΑΒΗ τρίγωνον τῶι ΔΕΘ τριγώνωι. ἔστιν ἄρα ὡς ἡ ΒΑ πρὸς τὴν ΑΗ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΘ. ἀλλ' ὡς μὲν ἡ ΒΑ πρὸς τὴν ΑΗ, οὕτως ἐστὶν τὸ ὑπὸ ΑΒΓ πρὸς τὸ ὑπὸ ΑΗ, ΒΓ, ὡς δὲ ἡ ΕΔ πρὸς τὴν ΔΘ, οὕτως ἐστὶν τὸ ὑπὸ ΔΕΖ πρὸς τὸ ὑπὸ ΔΘ, ΕΖ. ἔστιν ἄρα ἐναλλὰξ ὡς τὸ ὑπὸ ΑΒΓ πρὸς τὸ ὑπὸ ΔΕΖ, οὕτως τὸ ὑπὸ ΑΗ, ΒΓ, τουτέστιν τὸ ΑΓ παραλληλόγραμμον, πρὸς τὸ ὑπὸ ΔΘ, ΕΖ, τουτέστιν πρὸς τὸ ΔΖ παραλληλόγραμμον.

(242) ἔστω τρίγωνον τὸ ΑΒΓ. ἔστω δὲ παράλληλος ἡ ΒΓ τῆι ΔΕ, καὶ τῶι ἀπὸ τῆς ΓΑ ἴσον κείσθω τὸ ὑπὸ ΖΑΕ. ὅτι, ἐὰν ἐπιζευχθῶσιν αἱ ΔΓ, ΒΖ, γίνεται παράλληλος [ἐστιν] ἡ ΒΖ τῆι ΔΓ. τοῦτο δέ ἐστιν φανερόν. ἐπεὶ γάρ ἐστιν ὡς ἡ ΖΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΓΑ πρὸς τὴν ΑΕ, <ὡς δὲ ἡ ΓΑ πρὸς τὴν ΑΕ,> οὕτως ἐστὶν ἐν παραλλήλωι ἡ ΒΑ πρὸς ΑΔ, καὶ ὡς ἄρα ἡ ΖΑ πρὸς ΑΓ, οὕτως ἡ ΒΑ πρὸς ΑΔ. παράλληλοι ἄρα εἰσὶν αἱ ΔΓ, ΒΖ.

‖ 2 ὁ τοῦ Ha τὸ A | ante συνῆπται add λόγος Ha | τοῦ (Γ) Ha τῆς A | ante Δ add τὸ Hu ‖ 3 τοῦ (E) Ha τῆς A | ante Z add τὸ Ha ‖ 5 ἐστιν ante ὁ τοῦ Γ transp Ha (Hu app ad locum vix sanus) | ὡς add Ha ‖ 26 ἡ ... τῆι] τῆι ... ἡ coni Hu app ‖ 28 ἐστιν del Ha ‖ 30 ΓΑ Co ΓΔ A | ὡς – ΑΕ add Hu (Co)

(243) *(Prop. 174)* Let there be triangle ABΓ, and trapezium ΔEZH, so that angle ABΓ equals angle ΔEZ. That as is the rectangle contained by AB, BΓ to the rectangle contained by ΔH, EZ taken together and ΔE, so is (triangle) ABΓ to (trapezium) ΔEZH.

Let perpendiculars AΘ, ΔK be drawn.[1] Since angle ABΓ equals angle ΔEZ,[2] while right (angle) Θ equals right (angle) K,[3] therefore as is BA to AΘ, so is EΔ to ΔK.[4] But as is BA to AΘ, so is the rectangle contained by AB, BΓ to the rectangle contained by AΘ, BΓ,[5] while as is EΔ to ΔK, so is the rectangle contained by ΔH, EZ taken together and ΔE to the rectangle contained by ΔH, EZ taken together and ΔK.[6] And half the rectangle contained by AΘ, BΓ is triangle ABΓ,[7] while half the rectangle contained by ΔH, EZ taken together and ΔK is trapezium ΔEZH.[8] Therefore as is the rectangle contained by AB, BΓ to the rectangle contained by ΔH, EZ taken together and ΔE, so is triangle ABΓ to trapezium ΔEZH.[9]

(244) And if there is triangle ABΓ, and parallelogram ΔZ, then as is triangle ABΓ to parallelogram ΔEZH, so is the rectangle contained by AB, BΓ to twice the rectangle contained by ΔE, EZ, by the same argument. And it is obvious from these things that the rectangle contained by AB, BΓ, if parallelogram ΔZ <equals triangle ABΓ>, equals twice the rectangle contained by ΔE, EZ. In the case of the trapezium it equals *twice* the rectangle contained by ΔH, EZ taken together and ΔE. Q.E.D.

(245) *(Prop. 175)* Let there be triangle ABΓ, and with ΓA produced let some arbitrary (line) ΔE be drawn across, and let AH be drawn parallel to it, and AZ (parallel) to BΓ. That as is the square of AH to the rectangle contained by BH, HΓ, so is the rectangle contained by ΔZ, ZΘ to the square of ZA.

Let the rectangle contained by <AH, HK> be made equal to the rectangle contained by BH, HΓ,[1] <and the rectangle contained by> AZ, ZΛ (equal) <to the rectangle contained by ΔZ, ZΘ>,[3] and let BK, ΘΛ be joined. Then since angle Γ equals angle BKH,[2] and angle ΔAΛ equals angle ZΘΛ in a circle,[4] therefore angle HKB equals angle ZΘΛ.[5] But as well angle H equals angle Z.[6] Therefore as is BH to HK, so is ΛZ to ZΘ.[7] But since as is AH to HB, so is ΘE to EB,[8] while as is ΘE to EB, so is ZΘ to ZA

(243) ἔστω τρίγωνον μὲν τὸ ΑΒΓ, τραπέζιον δὲ τὸ ΔΕΖΗ, ὥστε [930] ἴσην εἶναι τὴν ὑπὸ ΑΒΓ γωνίαν τῆι ὑπὸ ΔΕΖ γωνίαι. ὅτι γίνεται ὡς τὸ ὑπὸ ΑΒΓ πρὸς τὸ ὑπὸ συναμφοτέρου τῆς ΔΗ, ΕΖ καὶ τῆς ΔΕ, οὕτως τὸ ΑΒΓ πρὸς τὸ ΔΕΖΗ. ἤχθωσαν κάθετοι αἱ ΑΘ, ΔΚ. ἐπεὶ δὲ ἴση ἐστὶν ἡ μὲν ὑπὸ ΑΒΓ γωνία τῆι ὑπὸ ΔΕΖ γωνίαι, ἡ δὲ <Θ> ὀρθὴ τῆι Κ ὀρθῆι ἴση, ἔστιν ἄρα ὡς ἡ ΒΑ πρὸς ΑΘ, οὕτως ἡ ΕΔ πρὸς ΔΚ. ἀλλ' ὡς μὲν ἡ ΒΑ πρὸς ΑΘ, οὕτως ἐστὶν τὸ ὑπὸ ΑΒΓ πρὸς τὸ ὑπὸ ΑΘ, ΒΓ· ὡς δὲ ἡ ΕΔ πρὸς τὴν ΔΚ, οὕτως ἐστὶν τὸ ὑπὸ συναμφοτέρου τῆς ΔΗ, ΕΖ καὶ τῆς ΔΕ πρὸς τὸ ὑπὸ συναμφοτέρου τῆς ΔΗ, ΕΖ καὶ τῆς ΔΚ. καὶ ἔστιν τοῦ μὲν ὑπὸ ΑΘ, ΒΓ ἥμισυ τὸ ΑΒΓ τρίγωνον, τοῦ δὲ |ὑπὸ [|169v] συναμφοτέρου τῆς ΔΗ, ΕΖ καὶ τῆς ΔΚ ἥμισυ τὸ ΔΕΖΗ τραπέζιον. ἔστιν ἄρα ὡς τὸ ὑπὸ ΑΒΓ πρὸς τὸ ὑπὸ συναμφοτέρου [καὶ] τῆς ΔΗ, ΕΖ καὶ τῆς ΔΕ, οὕτως τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΔΕΖΗ τραπέζιον.

(244) καὶ ἐὰν ἦι δὲ τρίγωνον τὸ ΑΒΓ, καὶ παραλληλόγραμμον τὸ ΔΖ, γίνεται ὡς τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΔΕΖΗ παραλληλόγραμμον, οὕτως τὸ ὑπὸ ΑΒ, ΒΓ πρὸς τὸ δὶς ὑπὸ ΔΕΖ, κατὰ τὰ αὐτά. καὶ φανερὸν ἐκ τούτων ὅτι τὸ μὲν ὑπὸ ΑΒ, ΒΓ, ἐὰν ἦι παραλληλόγραμμον τὸ ΔΖ <ἴσον τῶι ΑΒΓ τριγώνωι>, ἴσον γίνεται τῶι δὶς ὑπὸ ΔΕΖ. ἐπὶ δὲ τοῦ τραπεζίου ἴσον γίνεται τῶι δὶς ὑπὸ συναμφοτέρου τῆς ΔΗ, ΕΖ καὶ τῆς ΔΕ. ὅ(περ): —

(245) ἔστω τρίγωνον τὸ ΑΒΓ, καὶ ἐκβληθείσης τῆς ΓΑ διήχθω τις τυχοῦσα ἡ ΔΕ, καὶ αὐτῆι μὲν παράλληλος ἤχθω ἡ ΑΗ, τῆι δὲ ΒΓ ἡ ΑΖ. ὅτι γίνεται ὡς τὸ ἀπὸ ΑΗ τετράγωνον πρὸς τὸ ὑπὸ ΒΗΓ, οὕτως τὸ ὑπὸ ΔΖΘ πρὸς τὸ ἀπὸ ΖΑ τετράγωνον. κείσθω τῶι μὲν ὑπὸ ΒΗΓ ἴσον τὸ ὑπὸ <ΑΗΚ, τῶι δὲ ὑπὸ ΔΖΘ ἴσον τὸ ὑπὸ> ΑΖΛ, καὶ ἐπεζεύχθωσαν αἱ ΒΚ, ΘΛ. ἐπεὶ οὖν ἴση ἐστὶν ἡ Γ [932] γωνία τῆι ὑπὸ ΒΚΗ, ἡ δὲ ὑπὸ ΔΑΛ ἐν κύκλωι ἴση ἐστὶν τῆι ὑπὸ ΖΘΛ, καὶ ἡ ὑπὸ ΗΚΒ ἄρα ἴση ἐστὶν τῆι ὑπὸ ΖΘΛ γωνίαι. ἀλλὰ καὶ ἡ πρὸς τῶι Η γωνία ἴση ἐστὶν τῆι πρὸς τῶι Ζ. ἔστιν ἄρα ὡς ἡ ΒΗ πρὸς τὴν ΗΚ, οὕτως ἡ ΛΖ πρὸς τὴν ΖΘ. ἐπεὶ δέ ἐστιν ὡς ἡ ΑΗ πρὸς τὴν ΗΒ, οὕτως ἡ ΘΕ πρὸς τὴν ΕΒ, ὡς δὲ ἡ ΘΕ πρὸς

‖ 1 τραπέζιον Ha τραπέζειον A ‖ 2 post γωνίαι add ἡ δὲ ΔΗ τῆι ΕΖ παράλληλος Ha ‖ 5 δὲ] οὖν coni Hu app ‖ 6 Θ add Hu (Ha post ὀρθὴ) ‖ 8 ΕΔ Co ΑΔ A ‖ 11 τοῦ] τὸ Ha ‖ 12 ΔΚ Co ΑΚ A ‖ 13 καὶ (τῆς ΔΗ, ΕΖ) del Ha ‖ 16 δὲ secl Hu ‖ 18 ΑΒ, ΒΓ Ha ΑΘΒΓ A ΑΒΓ Co ‖ 19 ΑΒ, ΒΓ Ha ΑΘΒΓ A ΑΒΓ Co ‖ 20 ἴσον τῶι ΑΒΓ τριγώνωι] καὶ ἴσον τῶι ΑΒΓ τριγώνωι add Ha ‖ 22 τῶι Ha (Co) | δὶς del Co | ΔΕ ὅ(περ)] ΔΕΟ A ‖ 25 δὲ ΒΓ Ha (Co) Δ̄Ε̄Β̄Γ̄ A ‖ 27 ΔΖΘ Co ΖΘ A ‖ 28 ΑΗΚ — τὸ ὑπὸ add Co ‖ 32 Η Ha Γ A | Ζ Ha Κ A in parallels,[9] therefore as is AH to HB, so is ΘZ to ZA.[10] Hence since as AH is to HB, so is ΘZ to ZA, while as BH is to HK, so is some other (line) ΛZ to the leading (member) ZΘ,[11] *ex aequali* therefore in disturbed proportion as is AH to HK, so is ΛZ to ZA.[12] But as is AH to HK, so is the square of AH to the rectangle contained by AH, HK,[13] that is to the rectangle contained by BH, HΓ;[14] while as is ΛZ to ZA, so is the rectangle contained by ΛZ, ZA, that is the rectangle contained by ΔZ, ZΘ,[16] to the square of AZ.[15] Thus as is the square of AH to the rectangle contained by BH, HΓ, so is the rectangle contained by ΔZ, ZΘ to the square of ZA.[17]

(246) *(Prop. 175)* By means of compounded (ratio).

Since the ratio of AH to HB is that of ΘE to EB,[1] that is that of ΘZ to ZA,[2] while the ratio of AH to HΓ is the same as that of ΔE to EΓ,[3] that is that of ΔZ to ZA,[4] therefore the ratio compounded out of that which AH has to HB, and that which AH has to HΓ, which is that of the square of AH to the rectangle contained by BH, HΓ, is the same as the (ratio) compounded out of that of ΘZ to ZA and that of ΔZ to ZA,[5] which is that of the rectangle contained by ΔZ, ZΘ to the square of ZA.[6]

ΕΒ, οὕτως ἐστὶν ἐν παραλλήλωι ἡ ΖΘ πρὸς ΖΑ, ἔστιν ἄρα ὡς ἡ ΑΗ πρὸς τὴν ΗΒ, οὕτως ἡ ΘΖ πρὸς ΖΑ. ἐπεὶ οὖν ἐστιν ὡς μὲν ἡ ΑΗ πρὸς ΗΒ, οὕτως ἡ ΘΖ πρὸς ΖΑ, ὡς δὲ ἡ ΒΗ πρὸς ΗΚ, οὕτως ἄλλη τις ἡ ΛΖ πρὸς τὴν ἡγουμένην τὴν ΖΘ, δι' ἴσου ἄρα ἐν τεταραγμένηι ἀναλογίαι ὡς ἡ ΑΗ πρὸς τὴν ΗΚ, οὕτως ἡ ΛΖ πρὸς τὴν ΖΑ. ἀλλ' ὡς μὲν ἡ ΑΗ πρὸς ΗΚ, οὕτως ἐστὶν τὸ ἀπὸ ΑΗ πρὸς τὸ ὑπὸ ΑΗΚ, τουτέστιν πρὸς τὸ ὑπὸ ΒΗΓ· ὡς δὲ ἡ ΛΖ πρὸς ΖΑ, οὕτως ἐστὶν τὸ ὑπὸ ΛΖΑ, τουτέστιν τὸ ὑπὸ ΔΖΘ, πρὸς τὸ ἀπὸ ΖΑ. ἔστιν ἄρα ὡς τὸ ἀπὸ ΑΗ πρὸς τὸ ὑπὸ ΒΗΓ, οὕτως τὸ ὑπὸ ΔΖΘ πρὸς τὸ ἀπὸ ΖΑ.

(246) διὰ δὲ τοῦ συνημμένου. ἐπεὶ ὁ μὲν τῆς ΑΗ πρὸς ΗΒ λόγος ἐστὶν ὁ τῆς ΘΕ πρὸς ΕΒ, τουτέστιν ὁ τῆς ΘΖ πρὸς ΖΑ, ὁ δὲ τῆς ΑΗ πρὸς τὴν ΗΓ λόγος ὁ αὐτός ἐστιν τῶι τῆς ΔΕ πρὸς ΕΓ, τουτέστιν τῶι τῆς ΔΖ πρὸς ΖΑ, ὁ ἄρα συνημμένος ἔκ τε τοῦ ὃν ἔχει ἡ ΑΗ <πρὸς> ΗΒ καὶ τοῦ ὃν ἔχει ἡ ΑΗ πρὸς ΗΓ, ὅς ἐστιν ὁ τοῦ ἀπὸ ΑΗ πρὸς τὸ ὑπὸ ΒΗΓ, ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΘΖ πρὸς ΖΑ καὶ τοῦ τῆς ΔΖ πρὸς ΖΑ, ὅς ἐστιν ὁ τοῦ ὑπὸ ΔΖΘ πρὸς τὸ ἀπὸ ΖΑ τετράγωνον.

‖ 15 ἡ ΑΗ – ὃν ἔχει om A^1 add mg A^2 | πρὸς (ΗΒ) add Ha

(247) (Lemmas) of (Book) 2.

(Prop. 176) Given two (straight lines) AB, BΓ, and straight line ΔE, to fit a straight line equal to ΔE and parallel to it into AB and BΓ. But this is obvious. For if we draw EΓ through E and parallel to AB, and ΓA is drawn through Γ parallel to ΔE, then, because AΓΔE is a parallelogram, AΓ will be equal to ΔE, and parallel, and it has been fitted into the given straight lines AB, BΓ.

(248) *(Prop. 177)* Let there be two triangles ABΓ, ΔEZ, and as AB is to BΓ, so let ΔE be to EZ, and (let) AB (be) parallel to ΔE, and BΓ to EZ. That also AΓ is parallel to ΔZ.

Let BΓ be produced, and let it intersect ΔE and ΔZ at H and Θ. Then since as AB is to BΓ, so is ΔE to EZ,[1] and angles B and E are equal,[2] because there are two (parallels) to two (lines), therefore also (angle) Γ is equal to (angle) Z,[3] that is to (angle) Θ,[6] because EZ and HΘ are parallel.[5] For angle <E> equals (angle) H,[4] since (it) also (equals angle) B. Thus AΓ is parallel to ΔΘ.[7]

(249) *(Prop. 178)* (Let) AB (be) a straight line, and let AΓ and ΔB be equal, and let an arbitrary point E be taken between Γ and Δ. That the rectangle contained by AΔ, ΔB plus the rectangle contained by ΓE, EΔ equals the rectangle contained by AE, EB.

Let ΓΔ be bisected at Z, no matter where (Z) is with respect to point E.[1] And since the rectangle contained by AΔ, ΔB plus the square of ZΔ equals the square of ZB,[2] but the rectangle contained by ΓE, EΔ plus the square of ZE equals the square of ZΔ,[3] and the rectangle contained by AE, EB plus the square of ZE equals the square of ZB,[4] therefore the rectangle contained by AΔ, ΔB plus the rectangle contained by ΓE, EΔ and the square of ZE equals the rectangle contained by AE, EB and the square of ZE.[5] Let the square of ZE be subtracted in common. Then the remaining rectangle contained by AΔ, ΔB plus the rectangle contained by ΓE, EΔ equals the rectangle contained by AE, EB.[6]

(250) *(Prop. 179)* (Let) AB (be) a straight line, and let AΓ and ΔB be equal, and let an arbitrary point E be taken between Γ and Δ. That the rectangle contained by AE, EB equals the rectangle contained by ΓE, EΔ and the rectangle contained by ΔA, AΓ.

For let ΓΔ be bisected at Z, no matter where (Z) is with respect to point E.[1] And so all AZ equals all ZB.[2] Hence the rectangle contained by

(247) ΤΟΥ Β´

δύο δοθεισῶν τῶν ΑΒ, ΒΓ, καὶ εὐθείας τῆς ΔΕ, εἰς τὰς ΑΒ, ΒΓ ἐναρμόσαι εὐθεῖαν ἴσην τῆι ΔΕ καὶ παράλληλον αὐτῆι. τοῦτο δὲ φανερόν. ἐὰν γὰρ διὰ τοῦ Ε τῆι ΑΒ παράλληλον 934
ἀγάγωμεν τὴν ΕΓ, διὰ δὲ τοῦ Γ τῆι ΔΕ παράλληλος ἀχθῆι ἡ ΓΑ, ἔσται, διὰ τὸ παραλληλόγραμμον εἶναι τὸ ΑΓΔΕ, ἡ ΑΓ ἴση τῆι ΔΕ, καὶ παράλληλος. καὶ ἐνήρμοσται εἰς τὰς δοθείσας εὐθείας τὰς ΑΒ, ΒΓ.

(248) ἔστω δύο τρίγωνα τὰ ΑΒΓ, ΔΕΖ, καὶ ἔστω ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΕ πρὸς ΕΖ, καὶ παράλληλος ἡ μὲν ΑΒ τῆι ΔΕ, ἡ δὲ ΒΓ τῆι ΕΖ. ὅτι καὶ ἡ ΑΓ τῆι ΔΖ ἐστὶν παράλληλος. ἐκβεβλήσθω ἡ ΒΓ, καὶ συμπιπτέτω ταῖς ΔΕ, ΔΖ κατὰ τὰ Η, Θ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΕ πρὸς ΕΖ, καὶ εἰσὶν ἴσαι αἱ Β, Ε γωνίαι, διὰ τὸ εἶναι δύο παρὰ δύο, ἴση ἄρα ἐστὶν καὶ ἡ Γ τῆι Ζ, τουτέστιν τῆι Θ, διὰ τὸ παραλλήλους εἶναι τὰς ΕΖ, ΗΘ. ἴση γάρ ἐστιν ἡ <Ε> γωνία τῆι Η, ἐπεὶ καὶ τῆι Β. παράλληλος ἄρα ἐστὶν ἡ ΑΓ τῆι ΔΘ.

(249) εὐθεῖα ἡ ΑΒ, καὶ ἔστωσαν ἴσαι αἱ ΑΓ, ΔΒ, καὶ μεταξὺ τῶν Γ, Δ εἰλήφθω τυχὸν σημεῖον τὸ Ε. ὅτι τὸ ὑπὸ ΑΔΒ μετὰ τοῦ ὑπὸ ΓΕΔ ἴσον ἐστὶν τῶι ὑπὸ ΑΕΒ. τετμήσθω ἡ ΓΔ δίχα, ὅπως ἂν ἔχηι ὡς πρὸς τὸ Ε σημεῖον, κατὰ τὸ Ζ. καὶ ἐπεὶ τὸ ὑπὸ ΑΔΒ μετὰ τοῦ ἀπὸ ΖΔ ἴσον ἐστὶν τῶι ἀπὸ ΖΒ, ἀλλὰ τῶι μὲν ἀπὸ ΖΔ ἴσον ἐστὶν τὸ ὑπὸ ΓΕΔ μετὰ τοῦ ἀπὸ ΖΕ, τῶι δὲ ἀπὸ ΖΒ ἴσον ἐστὶν τὸ ὑπὸ ΑΕΒ μετὰ τοῦ ἀπὸ ΖΕ, τὸ ἄρα ὑπὸ ΑΔΒ μετὰ τοῦ ὑπὸ ΓΕΔ καὶ τοῦ ἀπὸ ΖΕ ἴσον ἐστὶν τῶι τε ὑπὸ ΑΕΒ καὶ τῶι ἀπὸ ΖΕ. κοινὸν ἀφηιρήσθω τὸ ἀπὸ ΖΕ. λοιπὸν ἄρα τὸ ὑπὸ ΑΔΒ μετὰ τοῦ ὑπὸ ΓΕΔ ἴσον ἐστὶν τῶι ὑπὸ ΑΕΒ.

(250) εὐθεῖα ἡ ΑΒ, καὶ ἔστωσαν ἴσαι αἱ ΑΓ, ΔΒ, καὶ μεταξὺ 936
τῶν Γ, Δ εἰλήφθω τυχὸν σημεῖον τὸ Ε. ὅτι τὸ ὑπὸ τῶν ΑΕΒ ἴσον ἐστὶν τῶι τε ὑπὸ τῶν ΓΕΔ καὶ τῶι ὑπὸ τῶν ΔΑΓ. τετμήσθω γὰρ ἡ ΓΔ δίχα, ὅπως ἂν ἔχηι ὡς πρὸς τὸ Ε σημεῖον, κατὰ τὸ Ζ. καὶ ὅλη ἄρα ἡ ΑΖ τῆι ΖΒ ἴση ἐστίν. τὸ μὲν ἄρα

‖ 4 Ε Co Γ Α ‖ 5 ΓΑ Co ΓΔ Α ‖ 6 ΑΓΔΕ] ΑΓΕΔ Co ‖ 16 ἴση – Β del Co | γάρ] ἄρα Α ‖ 18 ante εὐθεῖα add ἔστω Ha ‖ 21 ὅπως – σημεῖον secl Hu | ὡς] τὸ Ha ‖ 26 ἀφηιρήσθω Ha ἀφαιρείσθω Α ‖ 28 ante εὐθεῖα add ἔστω Ha ‖ 29 τῶν (ΑΕΒ) del Ha ‖ 30 τῶν (ΓΕΔ) del Ha | τῶν (ΔΑΓ) om Hu ‖ 31 ὅπως – σημεῖον secl Hu | ὡς] τὸ Ha ‖ 32 ἴση Ha ἴσον Α

AE, EB plus the square of EZ equals the square of AZ,[3] while the rectangle contained by ΔA, AΓ plus the square of ΓZ equals the square of AZ.[4] Thus the rectangle contained by AE, EB plus the square of EZ equals the rectangle contained by ΔA, AΓ plus the square of ΓZ.[5] But the square of ΓZ equals the rectangle contained by ΓE, EΔ and the square of EZ.[6] And let the square of EZ be subtracted in common. Then the remaining rectangle contained by AE, EB equals the rectangle contained by ΓE, EΔ and the rectangle contained by ΔA, AΓ.[7]

(251) *(Prop. 180)* Let there be two triangles ABΓ, ΔEZ, and let (angle) Γ equal (angle) Z, and (let) (angle) B (be) greater than (angle) E. That BΓ has to ΓA a lesser ratio than has EZ to ZΔ.

Let angle ΓBH be erected equal to angle E.[1] But (angle) Γ also equals (angle) Z.[2] Hence as BΓ is to ΓH, so is EZ to ZΔ.[3] But BΓ has to ΓA a lesser ratio than BΓ has to ΓH.[4] Therefore BΓ has to ΓA a lesser ratio than has EZ to ZΔ.[5]

(252) *(Prop. 181)* Again, let BΓ have to ΓA a greater ratio than has EZ to ZΔ, and let angle Γ be equal to (angle) Z. That again angle B is less than angle E.

For since BΓ has to ΓA a greater ratio than has EZ to ZΔ,[1] therefore if I make EZ to something as BΓ is to ΓA, it will be to something less than ZΔ.[3] Let it be to ZH,[2] and let EH be joined. And the sides around equal angles are in ratio.[4] Therefore angle B equals angle ZEH,[5] which is less than (angle) E.[6]

(253) *(Prop. 182)* Let there be similar triangles ABΓ, ΔEZ, and let AH and ΔΘ be drawn across so that as the rectangle contained by BΓ, ΓH is to the square of ΓA, so is the rectangle contained by EZ, ZΘ to the square of ZΔ. That triangle AHΓ too is similar to triangle ΔΘZ.

For since as the rectangle contained by BΓ, ΓH is to the square of ΓA, so is the rectangle contained by EZ, ZΘ to the square of ZΔ,[1] but the ratio of the rectangle contained by BΓ, ΓH to the square of ΓA is compounded out of that which BΓ has to ΓA, and that which HΓ has to ΓA,[2] while the (ratio) of the rectangle contained by EZ, ZΘ to the square of ZΔ is compounded out of that of EZ to ZΔ and that of ΘZ to ZΔ,[3] and of these the ratio of BΓ to ΓA is the same as that of EZ to ZΔ,[4] because of the similarity of the triangles, therefore the remaining ratio of HΓ to ΓA is the same as that of ΘZ to ZΔ.[5] And (they are) about equal angles.[6] Thus triangle AΓH is similar to triangle ΔZΘ.[7]

ὑπὸ ΑΕΒ μετὰ τοῦ ἀπὸ ΕΖ ἴσον ἐστὶν |τῶι ἀπὸ ΑΖ, τὸ δὲ ὑπὸ |170v ΔΑΓ μετὰ τοῦ ἀπὸ ΓΖ ἴσον ἐστὶν τῶι ἀπὸ ΑΖ· ὥστε τὸ ὑπὸ ΑΕΒ μετὰ τοῦ ἀπὸ ΕΖ ἴσον ἐστὶν τῶι ὑπὸ ΔΑΓ καὶ τῶι ἀπὸ ΓΖ. ἀλλὰ τὸ ἀπὸ ΓΖ ἴσον ἐστὶν τῶι τε ὑπὸ ΓΕΔ καὶ τῶι ἀπὸ ΕΖ. καὶ κοινὸν ἀφῃρήσθω τὸ ἀπὸ ΕΖ τετράγωνον. λοιπὸν ἄρα τὸ ὑπὸ ΑΕΒ ἴσον ἐστὶν τῶι τε ὑπὸ ΓΕΔ καὶ τῶι ὑπὸ ΔΑΓ.

(251) ἔστω δύο τρίγωνα τὰ ΑΒΓ, ΔΕΖ, καὶ ἔστω ἴση ἡ μὲν Γ τῆι Ζ, μείζων δὲ ἡ Β τῆς Ε. ὅτι ἡ ΒΓ πρὸς ΓΑ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΕΖ πρὸς ΖΔ. συνεστάτω τῆι Ε γωνίαι ἴση ἡ ὑπὸ ΓΒΗ. ἔστιν δὲ καὶ ἡ Γ τῆι Ζ ἴση. ἔστιν ἄρα ὡς ἡ ΒΓ πρὸς ΓΗ, οὕτως ἡ ΕΖ πρὸς ΖΔ. ἀλλὰ ἡ ΒΓ πρὸς τὴν ΓΑ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΒΓ πρὸς ΓΗ. καὶ ἡ ΒΓ ἄρα πρὸς ΓΑ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΕΖ πρὸς ΖΔ.

(252) ἐχέτω δὴ πάλιν ἡ ΒΓ πρὸς ΓΑ μείζονα λόγον ἤπερ ἡ ΕΖ πρὸς ΖΔ, ἴση δὲ ἔστω ἡ Γ γωνία τῆι Ζ. ὅτι πάλιν γίνεται ἐλάσσων ἡ Β γωνία τῆς Ε γωνίας. ἐπεὶ γὰρ ἡ ΒΓ πρὸς ΓΑ μείζονα λόγον ἔχει ἤπερ ἡ ΕΖ πρὸς ΖΔ, ἐὰν ἄρα ποιῶ ὡς ἡ ΒΓ πρὸς τὴν ΓΑ, οὕτως τὴν ΕΖ πρός τινα, ἔσται πρὸς ἐλάσσονα τῆς ΖΔ· ἔστω πρὸς τὴν ΖΗ, καὶ ἐπεζεύχθω ἡ ΕΗ. καὶ περὶ ἴσας γωνίας ἀνάλογόν εἰσιν αἱ πλευραί· ἴση ἄρα ἐστὶν ἡ Β γωνία τῆι ὑπὸ ΖΕΗ, ἐλάσσονι οὔσηι τῆς Ε. 938

(253) ἔστω ὅμοια τρίγωνα τὰ ΑΒΓ, ΔΕΖ, καὶ διήχθωσαν αἱ ΑΗ, ΔΘ οὕτως, ὥστε εἶναι ὡς τὸ ὑπὸ ΒΓΗ πρὸς τὸ ἀπὸ ΓΑ, οὕτως τὸ ὑπὸ ΕΖΘ πρὸς τὸ ἀπὸ ΖΔ. ὅτι γίνεται ὅμοιον καὶ τὸ ΑΗΓ τρίγωνον τῶι ΔΘΖ τριγώνωι. ἐπεὶ γάρ ἐστιν ὡς τὸ ὑπὸ ΒΓΗ πρὸς τὸ ἀπὸ ΓΑ, οὕτως τὸ ὑπὸ ΕΖΘ πρὸς τὸ ἀπὸ ΖΔ, ἀλλ᾽ ὁ μὲν τοῦ ὑπὸ ΒΓΗ πρὸς τὸ <ἀπὸ> ΓΑ λόγος συνῆπται ἔκ τε τοῦ ὃν ἔχει ἡ ΒΓ πρὸς ΓΑ καὶ τοῦ τῆς ΗΓ πρὸς ΓΑ, ὁ δὲ τοῦ ὑπὸ ΕΖΘ πρὸς τὸ ἀπὸ ΖΔ συνῆπται ἔκ τε τοῦ τῆς ΕΖ πρὸς ΖΔ καὶ τοῦ τῆς ΘΖ πρὸς ΖΔ, ὧν ὁ τῆς ΒΓ πρὸς ΓΑ λόγος ὁ αὐτός ἐστιν τῶι τῆς ΕΖ πρὸς ΖΔ διὰ τὴν ὁμοιότητα τῶν τριγώνων, λοιπὸς ἄρα ὁ τῆς ΗΓ πρὸς ΓΑ λόγος ὁ αὐτός ἐστιν τῶι τῆς ΘΖ πρὸς ΖΔ. καὶ περὶ ἴσας γωνίας. ὅμοιον ἄρα ἐστὶν τὸ ΑΓΗ τρίγωνον τῶι ΔΖΘ τριγώνωι.

‖ 1 τὸ δὲ – ΑΖ del Co ‖ 3 ΕΖ Co ΘΖ A ‖ 4 καὶ (κοινὸν) del Ha ‖ 6 καὶ τῶι] τῶι τε Ha ‖ 8 ΓΑ Co ΓΔ A ‖ 17 μείζονα Ha ἐλάσσονα A | ἡ Ha τὴν A ‖ 18 ΓΑ Co ΓΔ A | post τινα add ἄλλην Ha ‖ 19 ἐπεζεύχθω Ha ἐπιζευχθῆι A | περὶ] πρὸς Ha ‖ 21 ἐλάσσονι οὔσηι τῆς Ε Hu ἐλάσσονος οὔσης τῆς Ε A ἐλάσσονι οὔσηι τῆς ΖΕΔ Ha ‖ 27 ἀπὸ add Ha (Co) ‖ 31 λοιπὸς Ha λοιπὸν A

(254) *(Prop. 183)* Now (it is proved) by means of compound ratio as written above. Let it now be (required) to prove it not using compounded ratio.

Let the rectangle contained by ΑΓ, ΓΚ be made equal to the rectangle contained by ΒΓ, ΓΗ.[1] Then as ΒΓ is to ΓΚ, so is ΑΓ to ΓΗ.[2] Let the rectangle contained by ΔΖ, ΖΛ be made equal to the rectangle contained by ΕΖ, ΖΘ.[3] Then as ΕΖ is to ΖΛ, so is ΔΖ to ΖΘ.[4] But it was stipulated that as the rectangle contained by ΒΓ, ΓΗ, that is the rectangle contained by ΑΓ, ΓΚ, is to the square of ΑΓ, *that is as ΑΓ is to ΓΚ*,[7] so is the rectangle contained by ΕΖ, ΖΘ, that is the rectangle contained by ΔΖ, ΖΛ,[6] to the square of ΔΖ,[5] *that is ΔΖ to ΖΛ*.[7] But also as is ΒΓ to ΓΑ, so is ΕΖ to ΖΔ,[8] because of the similarity (of the triangles). And so as is ΒΓ to ΓΚ, so is ΕΖ to ΖΛ.[9] But as is ΒΓ to ΓΚ, so ΑΓ was proved to be to ΓΗ, while as ΕΖ is to ΖΛ, so ΔΖ (was proved to be) to ΖΘ. Therefore as ΑΓ is to ΓΗ, so is ΔΖ to ΖΘ.[10] And (they are) about equal angles.[11] Thus triangle ΑΓΗ is similar to triangle ΔΖΘ.[12]

Likewise also (to prove, if triangle) ΑΗΒ (is similar) to (triangle) ΔΘΕ (and the rectangle contained by ΒΓ, ΓΗ is to the square of ΓΑ as the rectangle contained by ΕΖ, ΖΘ is to the square of ΖΔ), that also (triangle) ΑΒΓ (is similar) to (triangle) ΔΕΖ.

(255) *(Prop. 184)* Let triangle ΑΒΓ be similar to triangle ΔΕΖ, and (triangle) ΑΗΒ to (triangle) ΔΘΕ. That as the rectangle contained by ΒΓ, ΓΗ is to the square of ΓΑ, so is the rectangle contained by ΕΖ, ΖΘ to the square of ΔΖ.

For since because of the similarity (of the triangles) all (angle) Α equals all (angle) Δ,[1] and angle ΒΑΗ (equals) angle ΕΔΘ,[2] therefore remainder angle ΗΑΓ equals remainder angle ΘΔΖ.[3] But also (angle) Γ (equals angle) Ζ.[4] Therefore as ΗΓ is to ΓΑ, so is ΘΖ to ΖΔ.[5] But also as ΒΓ is to ΓΑ, so was ΕΖ to ΖΔ.[6] And so the compounded (ratio) is the same as the compounded (ratio).[7] Thus as the rectangle contained by ΒΓ, ΓΗ is to the square of ΓΑ, so is the rectangle contained by ΕΖ, ΖΘ to the square of ΖΔ.[8]

(256) *(Prop. 185)* Another way, not by means of compounded (ratio).

Let the rectangle contained by ΑΓ, ΓΚ be made equal to the rectangle contained by ΒΓ, ΓΗ,[1] and the rectangle contained by ΔΖ, ΖΛ to the rectangle contained by ΕΖ, ΖΘ.[3] Again as ΒΓ is to ΓΚ, so will ΑΓ be to ΓΗ,[2] while as ΕΖ is to ΖΛ, so (will) ΔΖ (be) to ΖΘ.[4] And by the same argument as above we shall prove that as ΑΓ is to ΓΗ, so is ΔΖ to ΖΘ.[5] And so as ΒΓ is to ΓΚ, so is ΕΖ to ΖΛ.[6] But also as ΒΓ is to ΓΑ, so is ΕΖ to ΖΔ because of the similarity (of the triangles).[7] *Ex aequali* therefore as ΚΓ is to ΓΑ, that is as the rectangle contained by ΚΓ, ΓΑ, which is the rectangle contained by ΒΓ, ΓΗ, to the square of ΑΓ, so is ΛΖ to ΖΔ,[8] that is the rectangle contained by ΛΖ, ΖΔ, which is the rectangle contained by ΕΖ, ΖΘ, to the square of ΖΔ.[9] [10] Q.E.D.

(254) |διὰ μὲν οὖν τοῦ συνημμένου λόγου, ὡς προγέγραπται. |171 ἔστω δὲ νῦν ἀποδεῖξαι μὴ προσχρησάμενον τῶι συνημμένωι λόγωι. κείσθω τῶι μὲν ὑπὸ ΒΓΗ ἴσον τὸ ὑπὸ ΑΓΚ. ἔστιν ἄρα ὡς ἡ ΒΓ πρὸς τὴν ΓΚ, οὕτως ἡ ΑΓ πρὸς τὴν ΓΗ. τῶι δὲ ὑπὸ ΕΖΘ ἴσον κείσθω τὸ ὑπὸ ΔΖΛ. ἔστιν ἄρα ὡς ἡ ΕΖ πρὸς ΖΛ, οὕτως ἡ ΔΖ πρὸς ΖΘ. ὑπόκειται δὲ ὡς τὸ ὑπὸ ΒΓΗ, τουτέστιν τὸ ὑπὸ ΑΓΚ, πρὸς τὸ ἀπὸ ΑΓ, τουτέστιν ὡς ἡ ΑΓ πρὸς ΓΚ, οὕτως τὸ ὑπὸ ΕΖΘ, τουτέστιν τὸ ὑπὸ ΔΖΛ, πρὸς τὸ ἀπὸ ΔΖ, τουτέστιν ἡ ΔΖ πρὸς ΖΛ. ἀλλὰ καὶ ὡς ἡ ΒΓ πρὸς ΓΑ, οὕτως ἡ ΕΖ πρὸς ΖΔ, διὰ τὴν ὁμοιότητα. καὶ ὡς ἄρα ἡ ΒΓ πρὸς ΓΚ, οὕτως ἡ ΕΖ πρὸς ΖΛ. ἀλλ' ὡς μὲν ἡ ΒΓ πρὸς ΓΚ, οὕτως ἐδείχθη ἡ ΑΓ πρὸς ΓΗ, ὡς δὲ ἡ ΕΖ πρὸς ΖΛ, οὕτως ἡ ΔΖ πρὸς ΖΘ. καὶ ὡς ἄρα ἡ ΑΓ πρὸς ΓΗ, 940 οὕτως ἡ ΔΖ πρὸς ΖΘ. καὶ περὶ ἴσας γωνίας. ὅμοιον ἄρα ἐστὶν τὸ ΑΓΗ τρίγωνον τῶι ΔΖΘ τριγώνωι. ὁμοίως καὶ τὸ ΑΗΒ τῶι ΔΘΕ, ὅτι καὶ τὸ ΑΒΓ τῶι ΔΕΖ.

(255) ἔστω ὅμοιον τὸ μὲν ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι, τὸ δὲ ΑΗΒ τῶι ΔΘΕ. ὅτι γίνεται ὡς τὸ ὑπὸ ΒΓΗ πρὸς τὸ ἀπὸ ΓΑ, οὕτως τὸ ὑπὸ ΕΖΘ πρὸς τὸ ἀπὸ ΔΖ. ἐπεὶ γὰρ διὰ τὴν ὁμοιότητα ἴση ἐστὶν ὅλη μὲν ἡ Α ὅλῃ τῆι Δ, ἡ δὲ ὑπὸ ΒΑΗ τῆι ὑπὸ ΕΔΘ, λοιπὴ ἄρα ἡ ὑπὸ ΗΑΓ λοιπῆι τῆι ὑπὸ ΘΔΖ ἐστὶν ἴση. ἀλλὰ καὶ ἡ Γ τῆι Ζ. ἔστιν ἄρα ὡς ἡ ΗΓ πρὸς τὴν ΓΑ, οὕτως ἡ ΘΖ πρὸς ΖΔ. ἀλλὰ καὶ ὡς ἡ ΒΓ πρὸς τὴν ΓΑ, οὕτως ἦν ἡ ΕΖ πρὸς ΖΔ. καὶ ὁ συνημμένος ἄρα τῶι συνημμένωι ἐστὶν ὁ αὐτός. ἔστιν ἄρα ὡς τὸ ἀπὸ ΒΓΗ πρὸς τὸ ἀπὸ ΓΑ, οὕτως τὸ ὑπὸ ΕΖΘ πρὸς τὸ ἀπὸ ΖΔ.

(256) ἄλλως μὴ διὰ τοῦ συνημμένου. κείσθω τῶι μὲν ὑπὸ ΒΓΗ ἴσον τὸ ὑπὸ ΑΓΚ, τῶι δὲ ὑπὸ ΕΖΘ ἴσον τὸ ὑπὸ ΔΖΛ. ἔσται πάλιν ὡς μὲν ἡ ΒΓ πρὸς ΓΚ, οὕτως ἡ ΑΓ πρὸς ΓΗ, ὡς δὲ ἡ ΕΖ πρὸς ΖΛ, οὕτως ἡ ΔΖ πρὸς ΖΘ. καὶ κατὰ τὰ αὐτὰ τῶι ἐπάνω δείξομεν ὅτι ἐστὶν ὡς ἡ ΑΓ πρὸς ΓΗ, οὕτως ἡ ΔΖ πρὸς ΖΘ. καὶ ὡς ἄρα ἡ ΒΓ πρὸς ΓΚ, οὕτως ἡ ΕΖ πρὸς ΖΛ. ἀλλὰ καὶ ὡς ἡ ΒΓ πρὸς ΓΑ, οὕτως ἡ ΕΖ πρὸς ΖΔ διὰ τὴν ὁμοιότητα. δι' ἴσου ἄρα ἐστὶν ὡς ἡ ΚΓ πρὸς ΓΑ, τουτέστιν ὡς τὸ ὑπὸ ΚΓΑ, ὅ ἐστιν τὸ ὑπὸ ΒΓΗ, πρὸς τὸ ἀπὸ ΑΓ, οὕτως ἡ ΛΖ |πρὸς ΖΔ, τουτέστιν τὸ |171v ὑπὸ ΛΖΔ, ὅ ἐστιν τὸ ὑπὸ ΕΖΘ, πρὸς τὸ ἀπὸ ΖΔ, ὅπερ: —

‖ 5 τὸ Ha τῶι A ‖ 7 ΑΓ ... ΓΚ] ΚΓ ... ΓΑ Ha ‖ 8 ΔΖ ... ΖΛ] ΛΖ ... ΖΔ Ha ‖ 10 post ὁμοιότητα add τῶν τριγώνων Ha (Co) | ΖΛ Co ΖΔ A ‖ 14 ΔΖΘ Co ΑΖΘ A ‖ 15 ὅτι] ὥστε Ha ‖ 22 ante ΖΔ add τὴν Ha | ἦν del Co ‖ 23 ΖΔ Co ΖΑ A ‖ 24 ΕΖΘ Co ΕΘΖ A ‖ 29 τῶι (ἐπάνω)] τοῖς coni Hu app | ἐπάνω Ha ἐπάνωι A ‖ 30 ΔΖ Co ΕΖ A ‖ 32 ΕΖ Co ΘΖ A ‖ 33 ΓΑ Co ΓΛ A | τὸ (ὑπὸ ΒΓΗ) Ha τοῦ A ‖ 34 ΛΖ Co ΑΖ A

Likewise we shall prove, if, as is the rectangle contained by ΒΓ, ΓΗ to the square of ΑΓ, so is the rectangle contained by ΕΖ, ΖΘ to the square of ΖΔ, and triangle ΑΒΓ is similar to triangle ΔΕΖ, that also triangle ΑΒΗ is similar to triangle ΔΕΘ.

(257) *(Prop. 186)* Let there be two similar triangles ΑΒΓ, ΔΕΖ, and let perpendiculars ΑΗ, ΔΘ be drawn. That as is the rectangle contained by ΒΗ, ΗΓ to the square of ΑΗ, so is the rectangle contained by ΕΘ, ΘΖ to the square of ΘΔ. But this is obvious, because it is like the preceding ones.

(258) *(Prop. 187)* Let angle Β be equal to angle Ε, and (angle) Α less than (angle) Δ. That ΓΒ has to ΒΑ a lesser ratio than has ΖΕ to ΕΔ.

For since angle Α is less than (angle) Δ, let angle ΕΔΗ be erected equal to (angle Α).[1] Then as ΓΒ is to ΒΑ, so is ΕΗ to ΕΔ.[2] But also ΕΗ has to ΕΔ a lesser ratio than has ΖΕ to ΕΔ.[3] And so ΓΒ has to ΒΑ a lesser ratio than has ΖΕ to ΕΔ.[4]

And we shall prove all the things like that by the same procedure.

(259) *(Prop. 188)* As the rectangle contained by ΒΗ, ΗΓ is to the square of ΑΗ, so let the rectangle contained by ΕΘ, ΘΖ be to the square of ΔΘ; and let ΒΗ be equal to ΗΓ, and let ΓΗ have a lesser ratio to ΗΑ than has ΖΘ to ΘΔ. That ΖΘ is greater than ΘΕ.

For since the square of ΓΗ has a lesser ratio to the square of ΗΑ than has the square of ΖΘ to the square of ΘΔ,[1] but the <square of> ΓΗ is equal to the rectangle contained by ΒΗ, ΗΓ,[2] therefore the rectangle contained by ΒΗ, ΗΓ has to the square of ΑΗ a lesser ratio than has the square of ΖΘ to the square of ΘΔ.[3] But as the rectangle contained by ΒΗ, ΗΓ is to the square of ΑΗ, so was the rectangle contained by ΕΘ, ΘΖ stipulated to be to the square of ΘΔ.[4] Therefore the rectangle contained by ΕΘ, ΘΖ has to the square of ΘΔ a lesser ratio than has the square of ΖΘ to the square of ΘΔ.[5] Hence the square of ΖΘ is greater than the rectangle contained by ΕΘ, ΘΖ.[6] Thus ΖΘ is greater than ΘΕ.[7]

ὁμοίως δὴ δείξομεν καὶ ἐὰν ἧι ὡς τὸ ὑπὸ ΒΓΗ πρὸς τὸ ἀπὸ 942
ΑΓ, οὕτως τὸ ὑπὸ ΕΖΘ πρὸς τὸ ἀπὸ ΖΔ, καὶ ὅμοιον τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι, ὅτι καὶ τὸ ΑΒΗ τρίγωνον τῶι ΔΕΘ τριγώνωι ὅμοιον.

(257) ἔστω δύο ὅμοια τρίγωνα τὰ ΑΒΓ, ΔΕΖ, καὶ κάθετοι ἤχθωσαν αἱ ΑΗ, ΔΘ. ὅτι ἐστὶν ὡς τὸ ὑπὸ ΒΗΓ πρὸς τὸ ἀπὸ ΑΗ, οὕτως τὸ ὑπὸ ΕΘΖ πρὸς τὸ ἀπὸ ΘΔ. τοῦτο δὲ φανερόν, ὅτι ὅμοιον γίνεται τοῖς πρὸ αὐτοῦ.

(258) ἔστω ἴση ἡ μὲν Β γωνία τῆι Ε, ἐλάσσων δὲ ἡ Α τῆς Δ. ὅτι ἡ ΓΒ πρὸς ΒΑ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΖΕ πρὸς ΕΔ. ἐπεὶ γὰρ ἐλάσσων ἡ Α γωνία τῆς Δ, συνεστάτω αὐτῆι ἴση ἡ ὑπὸ ΕΔΗ. ἔστιν ἄρα ὡς ἡ ΓΒ πρὸς ΒΑ, οὕτως ἡ ΕΗ πρὸς ΕΔ. ἀλλὰ καὶ ἡ ΕΗ πρὸς ΕΔ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΖΕ πρὸς ΕΔ. καὶ ἡ ΓΒ ἄρα πρὸς τὴν ΒΑ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΖΕ πρὸς τὴν ΕΔ. καὶ πάντα δὲ τὰ τοιαῦτα τῆι αὐτῆι ἀγωγῆι δείξομεν.

(259) ἔστω ὡς τὸ ὑπὸ ΒΗΓ πρὸς τὸ ἀπὸ ΑΗ, οὕτως τὸ ὑπὸ ΕΘΖ πρὸς τὸ ἀπὸ ΔΘ, καὶ ἡ μὲν ΒΗ τῆι ΗΓ ἔστω ἴση, ἡ δὲ ΓΗ πρὸς ΗΑ ἐλάσσονα λόγον ἐχέτω ἤπερ ἡ ΖΘ πρὸς ΘΔ. ὅτι μείζων ἐστὶν ἡ ΖΘ τῆς ΘΕ. ἐπεὶ γὰρ τὸ ἀπὸ ΓΗ πρὸς τὸ ἀπὸ ΗΑ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ἀπὸ ΖΘ πρὸς τὸ ἀπὸ ΘΔ, ἀλλὰ τὸ <ἀπὸ> ΓΗ ἴσον ἐστὶν τῶι ὑπὸ ΒΗΓ, τὸ ἄρα ὑπὸ ΒΗΓ πρὸς τὸ ἀπὸ ΑΗ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ἀπὸ ΖΘ πρὸς τὸ ἀπὸ ΘΔ. ἀλλ' ὡς τὸ ὑπὸ ΒΗΓ πρὸς τὸ ἀπὸ ΑΗ, οὕτως ὑπόκειται τὸ ὑπὸ ΕΘΖ πρὸς τὸ ἀπὸ ΘΔ. καὶ τὸ ὑπὸ ΕΘΖ ἄρα πρὸς τὸ ἀπὸ ΘΔ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ἀπὸ ΖΘ πρὸς τὸ ἀπὸ ΘΔ. μεῖζον ἄρα ἐστὶν τὸ ἀπὸ ΖΘ τοῦ
ὑπὸ ΕΘΖ. ὥστε μείζων ἐστὶν ἡ ΖΘ τῆς ΘΕ. 944

‖ 1 καὶ del Ha ‖ 12 ἀλλὰ καὶ] ἀλλ' ἐπεὶ coni Hu app ‖ 14 ΖΕ Co ΖΘ A ‖ 15 τὰ bis A corr Ha ‖ 20 ἀπὸ (ΓΗ) add Ha (Co) ‖ 23 ὑπόκειται Ha ὑπέκειτο A | ΕΘΖ Co ΕΖΘ A ‖ 24 ΕΘΖ Co ΕΖΘ A ‖ 25 μεῖζον Ha μείζων A ‖ 26 ὑπὸ ΕΘΖ Co ἀπὸ ΘΗ A

(260) (Lemmas) of (Book) 3.

(Prop. 189) (Let there be) figure ABΓΔEZH. Let BH equal HΓ. That EZ is parallel to BΓ.

Let ΘK be drawn through A parallel to BΓ,[1] and let BZ and ΓE be produced to points K and Θ. Then since BH equals HΓ,[2] therefore also ΘA equals AK.[3] Hence as is BΓ to ΘA, that is, as BE is to EA,[5] so is BΓ to KA,[4] that is ΓZ to ZA.[6] Thus EZ is parallel to BΓ.[7]

(261) *(Prop. 190)* Let there be two triangles ABΓ, ΔEZ, that have angles A and Δ equal. Let the rectangle contained by BA, AΓ equal the rectangle contained by EΔ, ΔZ. That also triangle equals triangle.

Let perpendiculars BH, EΘ be drawn.[1] Then as HB is to BA, so is EΘ to EΔ.[2] And so as is the rectangle contained by BH, AΓ to the rectangle contained by BA, AΓ, so is the rectangle contained by EΘ, ΔZ to the rectangle contained by EΔ, ΔZ,[3] *Alternando*, as the rectangle contained by BH, AΓ is to the rectangle contained by EΘ, ΔZ, so is the rectangle contained by BA, AΓ to the rectangle contained by EΔ, ΔZ.[4] But the rectangle contained by BA, AΓ equals the rectangle contained by EΔ, ΔZ.[5] Therefore the rectangle contained by BH, AΓ equals the rectangle contained by EΘ, ΔZ.[6] But half the rectangle contained by BH, AΓ is triangle ABΓ,[7] and half the rectangle contained by EΘ, ΔZ is triangle ΔEZ.[8] Thus triangle ABΓ equals triangle ΔEZ.[9]

Obviously also the parallelograms that are twice them are equal.

(262) *(Prop. 191)* (Let there be) triangle ABΓ, and ΔE parallel to BΓ. That as the square of BA is to the square of AΔ, so is triangle ABΓ to triangle AΔE.

For since triangle ABΓ is similar to triangle AΔE,[1] therefore triangle ABΓ has to triangle AΔE twofold the ratio that BA has to AΔ.[2] But also the square of BA has to the square of AΔ twofold the ratio that BA has to AΔ.[3] Thus as the square of BA is to the square of AΔ, so is triangle ABΓ to triangle AΔE.[4]

(263) *(Prop. 192)* (Let) AB and ΓΔ (be) equal, and E an arbitrary point. That the rectangle contained by ΓE, EB exceeds the rectangle contained by ΓA, AB by the rectangle contained by ΔE, EA.

Let BΓ be bisected by Z.[1] Then Z is also the bisection of AΔ.[3] And since the rectangle contained by ΓE, EB plus the square of BZ equals the square of EZ,[2] but also the rectangle contained by ΔE, EA plus the square

(260) ΤΟΤ Γ´

καταγραφὴ ἡ ΑΒΓΔΕΖΗ. ἔστω δὲ ἴση ἡ ΒΗ τῆι ΗΓ. ὅτι
παράλληλός ἐστιν ἡ ΕΖ τῆι ΒΓ. ἤχθω |διὰ τοῦ Α τῆι ΒΓ |172
παράλληλος ἡ ΘΚ, καὶ ἐκβεβλήσθωσαν αἱ ΒΖ, ΓΕ ἐπὶ τὰ Κ, Θ
σημεῖα. ἐπεὶ οὖν ἴση ἐστὶν ἡ ΒΗ τῆι ΗΓ, ἴση ἄρα ἐστὶν καὶ ἡ
ΘΑ τῆι ΑΚ. ἔστιν ἄρα ὡς ἡ ΒΓ πρὸς τὴν ΘΑ, τουτέστιν ὡς ἡ ΒΕ
πρὸς τὴν ΕΑ, οὕτως ἡ ΒΓ πρὸς τὴν ΚΑ, τουτέστιν ἡ ΓΖ πρὸς ΖΑ.
παράλληλος ἄρα ἐστὶν ἡ ΕΖ τῆι ΒΓ.

(261) ἔστω δύο τρίγωνα τὰ ΑΒΓ, ΔΕΖ ἴσας ἔχοντα τὰς Α, Δ γωνίας. ἴσον δὲ ἔστω τὸ ὑπὸ ΒΑΓ τῶι ὑπὸ ΕΔΖ. ὅτι καὶ τὸ τρίγωνον τῶι τριγώνωι ἐστὶν ἴσον. ἤχθωσαν κάθετοι αἱ ΒΗ, ΕΘ. ἔστιν ἄρα ὡς ἡ ΗΒ πρὸς τὴν ΒΑ, οὕτως ἡ ΕΘ πρὸς τὴν ΕΔ. καὶ ὡς ἄρα τὸ ὑπὸ ΒΗ, ΑΓ πρὸς τὸ ὑπὸ ΒΑ, ΑΓ, οὕτως τὸ ὑπὸ ΕΘ, ΔΖ πρὸς τὸ ὑπὸ ΕΔΖ. ἐναλλὰξ ὡς τὸ ὑπὸ ΒΗ, ΑΓ πρὸς τὸ ὑπὸ ΕΘ, ΔΖ, οὕτως τὸ ὑπὸ ΒΑΓ πρὸς τὸ ὑπὸ ΕΔΖ. ἴσον δέ ἐστιν τὸ ὑπὸ ΒΑΓ τῶι ὑπὸ ΕΔΖ. ἴσον ἄρα ἐστὶν καὶ τὸ ὑπὸ ΒΗ, ΑΓ τῶι ὑπὸ ΕΘ, ΔΖ. ἀλλὰ τοῦ μὲν ὑπὸ ΒΗ, ΑΓ ἥμισύ ἐστιν τὸ ΑΒΓ τρίγωνον, τοῦ δὲ ὑπὸ ΕΘ, ΔΖ ἥμισύ ἐστιν τὸ ΔΕΖ τρίγωνον. καὶ τὸ ΑΒΓ ἄρα τρίγωνον τῶι ΔΕΖ τριγώνωι ἴσον ἐστίν. φανερὸν δὴ ὅτι καὶ τὰ διπλᾶ αὐτῶν παραλληλόγραμμα ἴσα ἐστίν.

(262) τρίγωνον τὸ ΑΒΓ, καὶ παράλληλος ἡ ΔΕ τῆι ΒΓ. ὅτι 946
ἐστὶν ὡς τὸ ἀπὸ ΒΑ πρὸς τὸ ἀπὸ ΑΔ, οὕτως τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΑΔΕ τρίγωνον. ἐπεὶ γὰρ ὅμοιόν ἐστιν τὸ ΑΒΓ τρίγωνον τῶι ΑΔΕ τριγώνωι, τὸ ἄρα ΑΒΓ τρίγωνον πρὸς τὸ ΑΔΕ τρίγωνον διπλασίονα λόγον ἔχει ἤπερ ἡ ΒΑ πρὸς ΑΔ. ἀλλὰ καὶ τὸ ἀπὸ ΒΑ πρὸς τὸ ἀπὸ ΑΔ διπλασίονα <λόγον> ἔχει ἤπερ ἡ ΒΑ πρὸς τὴν ΑΔ. ἔστιν ἄρα ὡς τὸ ἀπὸ ΒΑ πρὸς τὸ ἀπὸ ΑΔ, οὕτως τὸ [ἀπὸ] ΑΒΓ <τρίγωνον> πρὸς τὸ ΑΔΕ τρίγωνον.

(263) ἴσαι αἱ ΑΒ, ΓΔ, καὶ τυχὸν σημεῖον τὸ Ε. ὅτι τὸ ὑπὸ ΓΕΒ τοῦ ὑπὸ ΓΑΒ ὑπερέχει τῶι ὑπὸ ΔΕΑ. τετμήσθω ἡ ΒΓ δίχα τῶι Ζ. τὸ Ζ ἄρα διχοτομία ἐστὶν καὶ τῆς ΑΔ. καὶ ἐπεὶ τὸ ὑπὸ ΓΕΒ μετὰ τοῦ ἀπὸ ΒΖ ἴσον ἐστὶν τῶι ἀπὸ ΕΖ, ἀλλὰ καὶ τὸ ὑπὸ

‖ 2 ἔστω – ΒΗ bis A corr Co ‖ 4 ἐπὶ Ha ἐπεὶ A ‖ 7 ΕΑ Co ΓΑ A ‖ 13 τὸ ὑπὸ ΒΗ, ΑΓ – οὕτως bis A corr Co ‖ 14 ἐναλλὰξ – ΕΔΖ del Ha ‖ 19 δὴ] δὲ Ha ‖ 21 τῆι Ha τῆς A ‖ 22 ΒΑ] *ΒΑ A | ΑΔ Co ΑΒ A ‖ 25 (ΑΔΕ) τρίγωνον del Ha ‖ 26 διπλασίονα λόγον Ha διπλάσιον A ‖ 28 ἀπὸ (ΑΒΓ) del Ha (Co) | (ΑΒΓ) τρίγωνον add Ha (Co) of AZ equals the square of EZ,[4] and the square of AZ equals the rectangle contained by ΓA, AB plus the square of BZ,[5] let the square of BZ be removed in common. Then the remaining rectangle contained by ΓE, EB equals the rectangle contained by ΓA, AB plus the rectangle contained by ΔE, EA.[6] Thus the rectangle contained by ΓE, EB exceeds the rectangle contained by BA, AΓ by the rectangle contained by ΔE, EA.[7] Q.E.D.

(264) *(Prop. 193)* But if the point (E) is between points A and B, the rectangle contained by ΓE, EB will be less than the rectangle contained by ΓA, AB by the same area. The proof of this is by the same argument. *(Prop. 194)* But if the point is between B and Γ, the rectangle contained by ΓE, EB will be less than the rectangle contained by AE, EΔ by the rectangle contained by AB, BΔ, by the same procedure.

(265) *(Prop. 195)* (Let) AB equal BΓ, and (let there be) two points Δ, E. That four times the square of AB equals twice the rectangle contained by AΔ, ΔΓ plus twice the rectangle contained by AE, EΓ and twice the squares of BΔ and BE.

But this is obvious. For twice the square of AB, because of the bisections, equals twice the rectangle contained by AΔ, ΔΓ plus twice the square of ΔB, while twice the square of AB equals twice the rectangle contained by AE, EΓ plus twice the square of EB.

(266) *(Prop. 196 a– d)* (Let) AB equal ΓΔ, and (let there be) point E. That the squares of AE and EΔ equal the squares of BE and EΓ plus twice the rectangle contained by AΓ, ΓΔ.

Let BΓ be bisected at Z.[1] Then since twice the square of <ΔZ> equals twice the rectangle contained by AΓ, ΓΔ plus twice the square of ΓZ,[2] with twice the square of EZ added in common, twice the rectangle contained by AΓ, ΓΔ plus twice the squares of EZ and ZΓ equals twice the squares of ΔZ and ZE.[3] But the squares of AE and EΔ equal <twice> the squares of ΔZ and ZE,[4] while the squares of BE and EΓ equal <twice> the squares of ΓZ and ZE.[5] Thus the squares of AE and EΔ equal the squares of BE and EΓ plus twice the rectangle contained by AΓ, ΓΔ.[6]

ΔΕΑ μετὰ τοῦ ἀπὸ ΑΖ ἴσον ἐστὶν τῶι ἀπὸ ΕΖ, καὶ ἔστιν τὸ ἀπὸ ΑΖ ἴσον τῶι |ὑπὸ ΓΑΒ μετὰ τοῦ ἀπὸ ΒΖ, κοινὸν ἐκκεκρούσθω τὸ |172v ἀπὸ ΒΖ. λοιπὸν ἄρα τὸ ὑπὸ ΓΕΒ ἴσον ἐστὶν τῶι τε ὑπὸ ΓΑΒ καὶ τῶι ὑπὸ ΔΕΑ. ὥστε τὸ ὑπὸ ΓΕΒ τοῦ ὑπὸ ΒΑΓ ὑπερέχει τῶι ὑπὸ ΔΕΑ. ὅπερ: — (264) ἐὰν δὲ τὸ σημεῖον ἧι μεταξὺ τῶν Α, Β σημείων, τὸ ὑπὸ ΓΕΒ τοῦ ὑπὸ ΓΑΒ ἔλασσον ἔσται τῶι αὐτῶι χωρίωι, οὗπέρ ἐστιν κατὰ τὰ αὐτὰ ἡ ἀπόδειξις. ἐὰν δὲ τὸ σημεῖον ἧι μεταξὺ τῶν Β, Γ, τὸ ὑπὸ ΓΕΒ τοῦ ὑπὸ ΑΕΔ ἔλασσον ἔσται τῶι ὑπὸ ΑΒΔ, τῆι αὐτῆι ἀγωγῆι.

(265) ἴση ἡ ΑΒ τῆι ΒΓ, καὶ δύο σημεῖα τὰ Δ, Ε. ὅτι τὸ τετράκις ἀπὸ τῆς ΑΒ τετράγωνον ἴσον ἐστὶν τῶι δὶς ὑπὸ ΑΔΓ μετὰ τοῦ δὶς ⌜ὑπὸ ΑΕΓ καὶ δὶς ἀπὸ τῶν ΒΔ, ΒΕ τετραγώνων. τοῦτο δὲ⌝ φανερόν. τὸ μὲν γὰρ ⌜δὶς ἀπὸ ΑΒ διὰ τῶν⌝ 948 διχοτομιῶν ἴσον ἐστὶν τῶι τε δὶς ὑπὸ ΑΔΓ καὶ τῶι ⌜δὶς ἀπὸ ΔΒ, τὸ⌝ <δὲ> δὶς ἀπὸ ΑΒ ἴσον ἐστὶν τῶι τε δὶς ὑπὸ ΑΕΓ καὶ τῶι ⌜δὶς ἀπὸ ΕΒ⌝ τετραγώνωι.

(266) ἴση ἡ ΑΒ τῆι ΓΔ, καὶ σημεῖον ⌜τὸ Ε⌝. ὅτι τὰ ἀπὸ τῶν ΑΕ, ΕΔ τετράγωνα ἴσα τοῖς ἀπὸ τῶν ΒΕ, ΕΓ τετραγώνοις καὶ τῶι δὶς ὑπὸ τῶν ΑΓΔ. τετμήσθω <δίχα> ἡ ΒΓ κατὰ τὸ Ζ. ἐπεὶ οὖν τὸ δὶς ἀπὸ τῆς <ΔΖ> ἴσον ἐστὶν τῶι τε δὶς ὑπὸ ΑΓΔ καὶ δὶς ἀπὸ ΓΖ, [ἀλλὰ] κοινοῦ προστεθέντος τοῦ δὶς ἀπὸ ΕΖ, ἴσον ἐστὶν τό τε δὶς ὑπὸ ΑΓΔ καὶ τὰ δὶς ἀπὸ τῶν ΕΖ, ΖΓ τοῖς δὶς ἀπὸ τῶν ⌜Δ⌝Ζ, ΖΕ τετραγώνοις. ἀλλὰ τοῖς μὲν <δὶς> ἀπὸ τῶν ΔΖ, ΖΕ ἴσα ἐστὶν τὰ ἀπὸ τῶν ΑΕ, Ε⌜Δ τετράγωνα, τοῖς⌝ δὲ <δὶς> ἀπὸ τῶν ΓΖ, ΖΕ ἴσα ἐστὶν τὰ ἀπὸ τῶν ΒΕ, ΕΓ τετράγωνα. τὰ ἄρα ἀπὸ τῶν ΑΕ, ΕΔ τετράγωνα ἴσα ἐστὶν τοῖς τε ⌜ἀπὸ τῶν ΒΕ, Ε⌝Γ τετραγώνοις καὶ τῶι δὶς ὑπὸ τῶν ΑΓΔ.

‖ 1 καὶ ἔστιν] ἔστιν ἄρα καὶ coni Hu app ‖ 2 ΓΑΒ Co ΑΓΒ A ‖ 3 ἴσον — ΓΕΒ bis A corr Co ‖ 5 ante σημεῖον add E Co ‖ 6 τοῦ (ὑπὸ ΓΑΒ) Ha τὸ A | ἔλασσον Ha ἐλάσσων A ‖ 7 post χωρίωι add τῶι ὑπὸ ΔΕΑ Ha (Co) | οὗπέρ Ha ὅπερ A | ante σημεῖον add E Co ‖ 8 ΓΕΒ Co ΓΕΔ A ‖ 11 τετράκις Ha (Co) δεκάκις A ‖ 12 ὑπὸ ΑΕΓ — τοῦτο rescripta manu recentiore A | ΑΕΓ Co ΚΑΓ A (manu rec.) | ΒΕ Co ΔΕ A ‖ 13 δὶς — τῶν rescripta manu rec. A | διὰ τῶν Ha (Co) δὶς ἀπὸ A (manu rec.) ‖ 14 δὶς ἀπὸ ΔΒ τὸ] δὶς ἀπὸ ΑΒ τῶι rescripta manu rec. A (ΔΒ Co, τὸ Ha, fortasse olim A) ‖ 15 δὲ add Ha fortasse evanidum in A ‖ 16 δὶς ἀπὸ ΕΒ rescripta manu rec. A ‖ 19 δίχα add Ha (Co) | ΒΓ Co ΒΕ A ‖ 20 ΔΖ add Co ‖ 21 ἀλλὰ del Ha | post κοινοῦ add ἄρα Hu app ‖ 22 ΕΖ, ΖΓ] ΕΖΓ A ΓΖ, ΖΕ Ha ‖ 23 δὶς add Ha (Co) ‖ 24 (Ε)Δ — τοῖς rescripta manu rec. A | δὲ Ha ΔΕ A ‖ 25 δὶς add Ha (Co)

(267) *(Prop. 197)* Let the rectangle contained by BA, AΓ plus the square of ΓΔ equal the square of ΔA.[1] That ΓΔ equals ΔB.

For let the square of ΓΔ be subtracted in common. <Then the remaining rectangle contained by BA, AΓ equals the difference of the squares of AΔ and ΔΓ,[2] that is the rectangles contained by ΔA, AΓ and AΓ, ΓΔ.[3] But since the rectangle contained by BA, AΓ equals the rectangle contained by ΔA, AΓ plus the rectangle contained by BΔ, AΓ,[4] let the rectangle contained by ΔA, AΓ be subtracted in common.> Then the remaining rectangle contained by AΓ, ΔB equals the rectangle contained by ΔΓ, ΓA.[5] Thus ΔΓ equals ΔB.[6] Q.E.D.

(268) *(Prop. 198)* Let the rectangle contained by AΓ, ΓB plus the square of ΓΔ equal the square of ΔB. That AΔ equals ΔB.

Let ΔE be made equal to ΓΔ.[1] Then the rectangle contained by ΓB, BE plus the square of ΔE, that is the square of ΓΔ,[3] equals the square of ΔB,[2] that is the rectangle contained by BΓ, ΓA plus the square of ΓΔ.[4] Hence the rectangle contained by ΓB, BE equals the rectangle contained by BΓ, ΓA.[5] Therefore AΓ equals EB.[6] But also ΓΔ equals ΓE.[7] Thus all AΔ equals all ΔB.[8]

(269) *(Prop. 199)* Again, let the rectangle contained by BA, AΓ plus the square of ΔB equal the square of AΔ. That ΓΔ equals ΔB.

Let AE be made equal to ΔB.[1] Then since the rectangle contained by BA, AΓ plus the square of ΔB, that is the square of EA,[3] equals the square of AΔ,[2] let the rectangle contained by ΔA, AΓ be subtracted in common. Then the remaining rectangle contained by BΔ, AΓ, that is the rectangle contained by EA, AΓ,[5] plus the square of EA, which is the rectangle contained by ΓE, EA,[6] equals the rectangle contained by AΔ, ΔΓ.[4] Thus EA, that is BΔ, equals ΔΓ (see commentary).[7] [8]

(270) *(Prop. 200)* (Let there be) a line AB, on which are three points Γ, Δ, E, so that BE equals EΓ, and the rectangle contained by AE, EΔ (equals) the square of EΓ. That as BA is to AΓ, so is BΔ to ΔΓ.

For since the rectangle contained by AE, EΔ equals the square of EΓ,[1] in ratio[2] and *convertendo*[3] and (taking) twice the leading (members)[4] and *separando*, therefore, as is BA to AΓ, so is BΔ to ΔΓ.[5]

(271) *(Prop. 201)* Again, let the rectangle contained by BΓ, ΓΔ equal the square of ΓE, and (let) AΓ equal ΓE. That the rectangle contained by <AB, BE equals the rectangle contained by> ΓB, BΔ.

For since the rectangle contained by BΓ, ΓΔ equals the square of ΓE,[1] in ratio BΓ is to ΓE, that is to ΓA,[3] as ΓE, that is AΓ, is to ΓΔ.[2] And sum to sum,[4] and *convertendo*[5] and area to area, therefore, the

(267) ἔστω τὸ ὑπὸ ΒΑΓ μετὰ τοῦ ἀπὸ ΓΔ ἴσον τῶι ἀπὸ ΔΑ. ὅτι ἴση ἐστὶν ἡ ΓΔ τῆι ΔΒ. κοινὸν γὰρ ἀφηιρήσθω τὸ ἀπὸ ΓΔ. <λοιπὸν ἄρα τὸ ὑπὸ ΒΑΓ ἴσον ἐστὶν τῆι τῶν ἀπὸ ΑΔ, ΔΓ ὑπεροχῆι, τουτέστιν τοῖς ὑπὸ τῶν ΔΑΓ, ΑΓΔ. ἐπεὶ δὲ τὸ ὑπὸ ΒΑΓ ἴσον ἐστὶν τῶι ὑπὸ ΔΑΓ καὶ τῶι ὑπὸ ΒΔ, ΑΓ, κοινὸν ἀφηιρήσθω τὸ ὑπὸ ΔΑΓ.> λοιπὸν ἄρα τὸ ὑπὸ ΑΓ, ΔΒ ἴσον ἐστὶν τῶι ὑπὸ ΔΓΑ. ἴση ἄρα ἐστὶν ἡ ΔΓ τῆι ΔΒ. ὅ(περ): —

(268) ἔστω τὸ ὑπὸ ΑΓΒ μετὰ τοῦ ἀπὸ ΓΔ ἴσον τῶι ἀπὸ ΔΒ
τετραγώνωι. ὅτι ἴση ἐστὶν ἡ ΑΔ τῆι ΔΒ. κείσθω τῆι ΓΔ ἴση ἡ 950
ΔΕ. τὸ ἄρα ὑπὸ ΓΒΕ μετὰ τοῦ ἀπὸ ΔΕ, τουτέστιν τοῦ ἀπὸ ΓΔ,
ἴσον <ἐστὶν> τῶι |ἀπὸ ΔΒ, τουτέστιν τῶι ὑπὸ ΒΓΑ μετὰ τοῦ |173
ἀπὸ ΓΔ. ὥστε τὸ ὑπὸ ΓΒΕ ἴσον ἐστὶν τῶι ὑπὸ ΒΓΑ. ἴση ἄρα
ἐστὶν ἡ ΑΓ τῆι ΕΒ. ἀλλὰ καὶ ἡ ΓΔ τῆι ΓΕ. ὅλη ἄρα ἡ ΑΔ ὅληι
τῆι ΔΒ ἴση ἐστίν.

(269) ἔστω πάλιν τὸ ὑπὸ ΒΑΓ μετὰ τοῦ ἀπὸ ΔΒ ἴσον τῶι ἀπὸ ΑΔ. ὅτι ἴση ἐστὶν ἡ ΓΔ τῆι ΔΒ. κείσθω τῆι ΔΒ ἴση ἡ ΑΕ. ἐπεὶ οὖν τὸ ὑπὸ ΒΑΓ μετὰ τοῦ ἀπὸ ΔΒ [ἴσον ἐστίν], τουτέστιν τοῦ ἀπὸ ΕΑ, ἴσον ἐστὶν τῶι ἀπὸ ΑΔ τετραγώνωι, κοινὸν ἀφηιρήσθω τὸ ὑπὸ ΔΑΓ. λοιπὸν ἄρα τὸ ὑπὸ ΒΔ, ΑΓ, τουτέστιν τὸ ὑπὸ ΕΑΓ, μετὰ τοῦ ἀπὸ ΕΑ, ὅ ἐστιν τὸ ὑπὸ ΓΕΑ, ἴσον ἐστὶν τῶι ὑπὸ ΑΔΓ. ἴση ἄρα ἐστὶν ἡ ΕΑ, τουτέστιν ἡ ΒΔ, τῆι ΔΓ.

(270) εὐθεῖα ἡ ΑΒ, ἐφ' ἧς γ̄ σημεῖα τὰ Γ, Δ, Ε, οὕτως ὥστε ἴσην μὲν εἶναι τὴν ΒΕ τῆι ⌜ΕΓ⌝, τὸ δὲ ὑπὸ ΑΕΔ τῶι ἀπὸ ΕΓ. ὅτι γίνεται ὡς ἡ ΒΑ πρὸς ΑΓ, οὕτως ἡ ⌜ΒΔ⌝ πρὸς ΔΓ. ἐπεὶ γὰρ τὸ ὑπὸ ΑΕΔ ἴσον ἐστὶν τῶι ἀπὸ ΕΓ, ἀνάλογον καὶ ἀναστρέψαντι καὶ δὶς τὰ ἡγούμενα καὶ διελόντι, ἔστιν ἄρα ὡς ἡ ΒΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΒΔ πρὸς ΔΓ.

(271) ἔστω πάλιν τὸ ὑπὸ ΒΓΔ ἴσον τῶι ἀπὸ ΓΕ, ἴση δὲ ἡ ΑΓ τῆι ΓΕ. ὅτι τὸ ὑπὸ <ΑΒΕ ἴσον ἐστὶν τῶι ὑπὸ> ΓΒΔ. ἐπεὶ γὰρ τὸ ὑπὸ ΒΓΔ ἴσον ἐστὶν τῶι ἀπὸ ΓΕ, ἀνάλογόν ἐστιν ἡ ΒΓ πρὸς ΓΕ, τουτέστιν πρὸς τὴν ΓΑ, οὕτως ἡ ΓΕ, τουτέστιν ἡ ΑΓ, πρὸς τὴν ΓΔ. καὶ ὅλη πρὸς ὅλην, καὶ ἀναστρέψαντι καὶ χωρίον

‖ 2 ἀφηιρήσθω Ha ἀφαιρείσθω A | τὸ Ha ‖ 3 λοιπὸν — τὸ ὑπὸ ΔΑΓ add Hu (eadem fere Ha) ‖ 8 ΑΓΒ Co ΑΒΓ A ‖ 9 ΔΒ Co ΑΒ A ‖ 10 τοῦ Ha (Co) τὸ A ‖ 11 ΒΓΑ] ΕΑΓ A ΑΓΒ Co ‖ 12 ΒΓΑ] ΒΓΔ A ΑΓΒ Co ‖ 17 ἴσον ἐστίν del Co | τοῦ (ἀπὸ ΕΑ) Ha (Co) τὸ A ‖ 18 ἀφηιρήσθω Ha ἀφαιρείσθω A ‖ 23 ὑπὸ ΑΕΔ Ha (Co) ἀπὸ ΔΕ A ‖ 29 ΑΒΕ — ὑπὸ (ΓΒΔ) add Ha (Co) | ΓΒΔ Co ΕΒΔ A rectangle contained by AB, BE equals the rectangle contained by ΓB, BΔ.[6]

And it is obvious that the rectangle contained by AΔ, ΔE equals the rectangle contained by BΔ, ΔΓ too. For if the square of ΓΔ is subtracted in common from the equation of the square of ΓE to the rectangle contained by BΓ, ΓΔ, (the equation of the rectangle contained by AΔ, ΔE to the rectangle contained by BΔ, ΔΓ) results.

(272) *(Prop. 202)* Let three (straight lines) AEΔ, BEΓ, ZEH be drawn across two parallels AB, ΓΔ, and through the same point E. That as the rectangle contained by AE, EB is to the rectangle contained by AZ, ZB, so is the rectangle contained by ΓE, EΔ to the rectangle contained by ΓH, HΔ.

It is obvious by means of compound (ratio). For as AE is to EΔ, so is AZ to HΔ, while as BE is to EΓ, so is ZB to HΓ, and the areas are composed out of these. Thus (the theorem) holds true.

It is also possible (to prove it) as follows, not using compound (ratio). For since as AE is to EB, so is EΔ to EΓ,[1] therefore as the rectangle contained by AE, EB is to the square of EB, so is the rectangle contained by ΔE, EΓ to the square of EΓ.[2] But also as the square of BE is to the square of BZ, so is the square of EΓ to the square of ΓH.[3] *Ex aequali* therefore as the rectangle contained by AE, EB is to the square of ZB, so is the rectangle contained by ΓE, EΔ to the square of ΓH.[4] But also as is the square of ZB to the rectangle contained by BZ, ZA, so is the square of ΓH to the rectangle contained by ΓH, HΔ.[5] *Ex aequali,* therefore, as the rectangle contained by AE, EB is to the rectangle contained by AZ, ZB, so is the rectangle contained by ΓE, EΔ to the rectangle contained by ΓH, HΔ.[6]

χωρίωι, τὸ ἄρα ὑπὸ ΑΒΕ ἴσον ἐστὶν τῶι ὑπὸ ΓΒΔ. φανερὸν δὲ ὅτι καὶ τὸ ὑπὸ ΑΔΕ ἴσον ἐστὶ τῶι ὑπὸ ΒΔΓ. ἐὰν γὰρ ἀφαιρεθῆι τὸ ἀπὸ ΓΔ κοινὸν ἀπὸ τῆς τοῦ ἀπὸ ΓΕ πρὸς τὸ ὑπὸ ΒΓΔ ἰσότητος, γίνεται.

(272) εἰς δύο παραλλήλους τὰς ΑΒ, ΓΔ διά τε τοῦ αὐτοῦ σημείου τοῦ Ε τρεῖς διήχθωσαν αἱ ΑΕΔ, ΒΕΓ, ΖΕΗ. ὅτι ἐστὶν ὡς τὸ ὑπὸ ΑΕΒ πρὸς τὸ ὑπὸ ΑΖΒ, οὕτως τὸ ὑπὸ ΓΕΔ πρὸς τὸ ὑπὸ ΓΗΔ. διὰ τοῦ συνημμένου φανερόν. ὡς μὲν γὰρ ἡ ΑΕ πρὸς τὴν ΕΔ, οὕτως ἡ ΑΖ πρὸς τὴν ΗΔ, ὡς δὲ ἡ ΒΕ πρὸς τὴν ΕΓ, οὕτως ἡ ΖΒ πρὸς τὴν ΗΓ, καὶ σύγκειται ἐκ τούτων τὰ χωρία. μένει ἄρα.

ἔστιν δὲ καὶ οὕτως, μὴ προσχρησάμενον τῶι συνημμένωι. ἐπεὶ γάρ ἐστιν ὡς ἡ ΑΕ πρὸς τὴν ΕΒ, οὕτως ἡ ΕΔ πρὸς τὴν ΕΓ, καὶ ὡς ἄρα τὸ ὑπὸ ΑΕΒ πρὸς τὸ ἀπὸ ΕΒ, οὕτως τὸ ὑπὸ ΔΕΓ πρὸς τὸ ἀπὸ ΕΓ. ἀλλὰ καὶ ὡς τὸ ἀπὸ ΒΕ πρὸς τὸ ἀπὸ ΒΖ, οὕτως τὸ ἀπὸ ΕΓ πρὸς τὸ ἀπὸ ΓΗ. δι' ἴσου ἄρα ἐστὶν ὡς τὸ |ὑπὸ ΑΕΒ πρὸς τὸ ἀπὸ ΖΒ, οὕτως τὸ ὑπὸ ΓΕΔ πρὸς τὸ ἀπὸ ΓΗ. ἀλλὰ καὶ ὡς |173v τὸ ἀπὸ ΖΒ πρὸς τὸ ὑπὸ ΒΖΑ, οὕτως τὸ ἀπὸ ΓΗ πρὸς τὸ ὑπὸ ΓΗΔ. δι' ἴσου ἄρα ἐστὶν ὡς τὸ ὑπὸ ΑΕΒ πρὸς τὸ ὑπὸ ΑΖΒ, οὕτως τὸ ὑπὸ ΓΕΔ πρὸς τὸ ὑπὸ ΓΗΔ.

‖ 1 ΑΒΕ Co ΑΕΒ A ‖ 2 ΒΔΓ Co (k) ΒΑΓ A ‖ 3 ΓΔ Co ΑΔ A ‖ 6 αἱ Ha ἡ A ‖ 10 μένει ἄρα Heiberg$_2$ μενῒ ἄρα A γίνεται ἄρα Hu ἀνάλογον ἄρα ἐστί Ha ‖ 11 post οὕτως add ἀπόδειξαι Ha (Co) ‖ 17 ΓΗΔ Co ΓΗΑ A

(273) (Lemmas) of (Book) 5.

(Prop. 203) (Let there be) triangle ABΓ, and let perpendicular AΔ be drawn. I say that if the rectangle contained by BΔ, ΔΓ equals the square of AΔ, then angle A is right; if greater, obtuse; if less, acute.

First let it be equal.[1] Then (BΔ, AΔ, ΔΓ are) in ratio and about equal angles. Thus angle A equals the angle at Δ.[2] Hence the angle at A is right.[3]

But let it be greater,[4] and let the square of ΔE be made equal to it,[5] and let BE and EΓ be joined. Then angle BEΓ will be right.[6] And angle A is greater than it.[7] Thus angle A is obtuse.[8]

But again let it be less,[9] and let the square of ΔZ be made equal to it,[10] and let BZ and ZΓ be joined. Then angle BZΓ will be right,[11] and the angle at A less than it.[12] Thus angle A is acute.[13]

(274) *(Prop. 204)* Two straight lines AB, BΓ being (given) in position, and point Δ given, to draw through Δ a hyperbola about asymptotes AB, BΓ.

Let it be accomplished. Then its center is B. Let ΔB be joined and produced. Then it is (the hyperbola's) diameter. Let BE be made equal to ΔB. Then it is given. Hence E is given, and it is an end of the diameter. Let perpendicular ΔZ be drawn onto BΓ from Δ. Then Z is given. And let ZΓ be made equal to BZ. Then Γ too is given. And let ΓΔ be joined and produced to A. Then (ΓΔ) is (given) in position. But AB too (is given) in position. Thus A is given. But also Γ is given. Therefore AΓ is given in magnitude. And AΔ will be equal to ΔΓ, because BZ equals ZΓ. Let ΔH be the *latus rectum* of the 'figure' on EΔ. Then each of AΔ, ΔΓ is in square one quarter the rectangle contained by EΔ, ΔH (Conics II, 3). But (they are also one quarter in square) of the square of AΓ. Hence the rectangle contained by EΔ, ΔH equals the square of AΓ. But the square of AΓ is given. Hence also the rectangle contained by EΔ, ΔH is given. And EΔ is given. Therefore HΔ too is given. And thus H is given. Then since with two straight lines EΔ, ΔH given in position in a plane and situated at right angles to each other, and with angle AΔB given, there is a hyperbola whose diameter is EΔ, vertex Δ, and the ordinates drawn at the given angle AΔB

(273) ΤΟΥ Ε΄

τρίγωνον τὸ ΑΒΓ, καὶ κάθετος ἤχθω ἡ ΑΔ. λέγω ὅτι, εἰ μὲν ἴσον ἐστὶν τὸ ὑπὸ ΒΔΓ τῶι ἀπὸ ΑΔ τετραγώνωι, γίνεται ὀρθὴ ἡ Α γωνία, εἰ δὲ μεῖζον, ἀμβλεῖα, εἰ δὲ ἔλασσον, ὀξεῖα. ἔστω πρότερον ἴσον. ἀνάλογον ἄρα καὶ περὶ ἴσας γωνίας. ἴση ἄρα ἐστὶν ἡ Α γωνία τῆι πρὸς τῶι Δ. ὥστε ὀρθή ἐστιν ἡ πρὸς τῶι 954 Α γωνία. ἀλλὰ ἔστω μεῖζον, καὶ αὐτῶι ἴσον κείσθω τὸ ἀπὸ ΔΕ, καὶ ἐπεζεύχθωσαν αἱ ΒΕ, ΕΓ. ἔσται ἄρα ὀρθὴ ἡ ὑπὸ ΒΕΓ γωνία. καὶ αὐτῆς μείζων ἡ Α γωνία. ἀμβλεῖα ἄρα ἐστὶν ἡ Α γωνία. ἀλλὰ ἔστω πάλιν ἔλασσον, καὶ αὐτῶι ἴσον κείσθω τὸ ἀπὸ ΔΖ, καὶ ἐπεζεύχθωσαν αἱ ΒΖ, ΖΓ. ἔσται δὴ ὀρθὴ ἡ ὑπὸ ΒΖΓ γωνία, καὶ αὐτῆς ἐλάσσων ἡ πρὸς τῶι Α γωνία. ὀξεῖα ἄρα ἐστὶν ἡ Α γωνία.

(274) θέσει οὐσῶν δύο εὐθειῶν τῶν ΑΒ, ⌜ΒΓ⌝, καὶ σημείου δοθέντος τοῦ Δ, γράψαι διὰ τοῦ Δ ὑπερβολὴν περὶ ἀσυμπτώτους τὰς ΑΒ, ΒΓ. γεγονέτω. κέντρον ἄρα αὐτῆς ἐστιν τὸ Β. ἐπεζεύχθω οὖν ἡ ΔΒ, καὶ ἐκβεβλήσθω. διάμετρος ἄρα ἐστίν. κείσθω τῆι ΔΒ ἴση ἡ ΒΕ. δοθεῖσα ἄρα ἐστίν. ὥστε δοθέν ἐστιν τὸ Ε καὶ πέρας τῆς διαμέτρου. ἤχθω ἀπὸ τοῦ Δ ἐπὶ τὴν ΒΓ κάθετος ἡ ΔΖ. δοθὲν ἄρα ἐστὶν τὸ Ζ. καὶ κείσθω τῆι ΒΖ ἴση ἡ ΖΓ. δοθὲν ἄρα ἐστὶν καὶ τὸ Γ. καὶ 956 ἐπιζευχθεῖσα ἡ ΓΔ ἐκβεβλήσθω ἐπὶ τὸ Α. θέσει ἄρα ἐστίν. θέσει δὲ καὶ ἡ ΑΒ. δοθὲν ἄρα ἐστὶν τὸ Α. ἔστιν δὲ καὶ τὸ Γ δοθέν. δέδοται ἄρα ἡ ΑΓ τῶι μεγέθει. καὶ ἔσται ἴση ἡ ΑΔ τῆι ΔΓ, διὰ τὸ καὶ τὴν ΒΖ τῆι ΖΓ ἴσην εἶναι. ἔστω δὴ ὀρθία τοῦ πρὸς τῆι ΕΔ εἴδους ἡ ΔΗ. ἑκατέρα ἄρα τῶν ΑΔ, ΔΓ δυνάμει ἐστὶν <δ΄> τοῦ ὑπὸ ΕΔΗ. ἀλλὰ καὶ τοῦ ἀπὸ ΑΓ. ἴσον ἄρα ἐστὶν τὸ ὑπὸ ΕΔΗ τῶι ἀπὸ ΑΓ τετραγώνωι. δοθὲν δὲ τὸ ἀπὸ ΑΓ τετράγωνον. δοθὲν ἄρα καὶ τὸ ὑπὸ ΕΔΗ. καὶ ἔστιν δοθεῖσα ἡ ΕΔ. δοθεῖσα ἄρα καὶ ἡ ΗΔ. ὥστε δοθὲν τὸ Η. ἐπεὶ οὖν θέσει δεδομένων δύο εὐθει⌊ῶ⌋ν <ἐν> ἐπιπέδωι τῶν ΕΔ, ΔΗ ὀρθῶν |174 ἀλλήλαις κειμένων, καὶ [ἀπὸ] δοθείσης τῆς ὑπὸ ΑΔΒ γωνίας γίνεται ὑπερβολὴ ἧς διάμετρος μὲν ἡ ΕΔ, κορυφὴ δὲ τὸ Δ, αἱ δὲ καταγόμεναι κατάγονται ἐν τῆι δοθείσηι γωνίαι, τῆι ὑπὸ

‖ 5 ἴσον Ha (Co) ἴση A ‖ 7 μεῖζον Ha μείζων A | ἴσον Ha ἴση A ‖ 10 ἔλασσον Ha ἐλάσσων A ‖ 12 ἐλάσσων Ha ἔλασσον A | τῶι Ge (S) τὸ A ‖ 14 post οὐσῶν add πρὸς ὀρθὰς Ha | post ΒΓ add πρὸς ὀρθὰς ἀλλήλαις Hu app ‖ 18 δοθεῖσα Ha δοθὲν A ‖ 22 ἐπιζευχθεῖσα Ha (Co) ἐπεζεύχθω A ‖ 27 δ΄ add Co | ἀλλὰ — ΕΔΗ om A^1 add mg A^2 alia manu ‖ 30 ὥστε Ha ἔστω A καὶ ἔστι Co ‖ 31 εὐθειῶν ἐν ἐπιπέδωι Co εὐθειαι ἐπιπέδων A ‖ 32 δοθείσης] δοθέντος A, post quod add τοῦ Δ Co | τῆς ὑπὸ] ὑπὸ τῆς transp Ha | γωνίας Ha ΓΕ̄ A ‖ 34 κατάγονται del Hu app are equal in square to the (rectangles) applied to ΔH that have the breadth that they cut off of the continuation of the diameter on the side of Δ, and that exceed it by a figure similar to the rectangle contained by EΔ, ΔH, therefore the section is (given) in position (cf. Conics I, 53).

(275) *(Prop. 204)* The synthesis of the problem will be made as follows. Let the two straight lines (given) in position be AB, BΓ, and the given (point) Δ, and let ΔB be joined and produced to E, and let BE be made equal to it, and let perpendicular ΔZ be drawn, and let ZΓ be made equal to BZ, and let ΓΔ be joined and produced to A, and let ΔH be erected on ΔE, and let the rectangle contained by EΔ, ΔH be made equal to the square of AΓ, and let there be drawn, as we said in the analysis, a hyperbola about diameter ΔE. I say that it solves the problem.

For since BZ equals ZΓ,[1] therefore AΔ too equals ΔΓ.[2] Hence each of AΔ, ΔΓ in square is one quarter the square of AΓ,[3] that is the rectangle contained by EΔ, ΔH,[4] that is the 'figure' on diameter EΔ. But if this is so, then it has been proved in the second (book) that AB, BΓ are the hyperbola's asymptotes (Conics II 1).

(276) *(Prop. 205)* (Let) straight line AB (be given) in position. Let Γ (be) given. Let BΓ be drawn across. Let BΔ be made given. Let ΔE be erected at right angles. That E touches a section of a cone (given) in position, a hyperbola, passing through Γ.

Let perpendicular ΓZ be drawn. Then Z is given. <ZA equals BΔ. Then A is given.> Let AH be erected at right angles. Then AH is (given) in position. Let it intersect BΓ produced at H. And with BA, AH given in position and point Γ given, <let> a hyperbola <be drawn> about asymptotes HA, AB. Then it will pass through E too, because BΓ equals EH, since also <all> (BE equals) all (ΓH). And it is possible (to draw) according to the foregoing (lemma).

ΑΔΒ, δυνάμεναι τὰ παρὰ τὴν ΔΗ παρακείμενα, πλάτη ἔχοντα <ἃ> αὐταὶ ἀφαιροῦσιν ἀπὸ τῆς ἐπ᾽ εὐθείας τῆι διαμέτρωι πρὸς τῶι Δ, ὑπερβάλλοντα εἴδει ὁμοίωι τῶι ὑπὸ ΕΔΗ, θέσει ἄρα ἐστὶν ἡ τομή.

(275) συντεθήσεται δὴ τὸ πρόβλημα οὕτως. ἔστωσαν αἱ τῆι θέσει δύο εὐθεῖαι <αἱ> ΑΒ, ΒΓ, τὸ δὲ δοθὲν τὸ Δ, καὶ 958 ἐπιζευχθεῖσα ἡ ΔΒ ἐκβεβλήσθω ἐπὶ τὸ Ε, καὶ αὐτῆι ἴση κείσθω ἡ ΒΕ, καὶ ἤχθω κάθετος ἡ ΔΖ, καὶ τῆι ΒΖ ἴση κείσθω ἡ ΖΓ, καὶ ἐπιζευχθεῖσα ἡ ΓΔ ἐκβεβλήσθω ἐπὶ τὸ Α, καὶ τῆι ΔΕ προσανήχθω ἡ ΔΗ, καὶ τῶι ἀπὸ ΑΓ ἴσον κείσθω τὸ ὑπὸ ΕΔΗ, καὶ γεγράφθω, [καὶ] ὡς ἐν τῆι ἀναλύσει ἐλέγομεν, περὶ διάμετρον ΔΕ ὑπερβολή. λέγω ὅτι ποιεῖ τὸ πρόβλημα. ἐπεὶ γὰρ ἴση ἐστὶν ἡ ΒΖ τῆι ΖΓ, ἴση ἄρα ἐστὶν καὶ ἡ ΑΔ τῆι ΔΓ. ἑκάτερον ἄρα τῶν [ἀπὸ] ΑΔ, ΔΓ δυνάμει δ´ ἐστιν τοῦ ἀπὸ τῆς ΑΓ τετραγώνου, τουτέστιν τοῦ ὑπὸ ΕΔΗ, τουτέστιν τοῦ πρὸς τῆι ΕΔ διαμέτρωι εἴδους. ἐὰν δὲ ⌜ἧι τοῦτο⌝, δέδεικται ἐν τῶι δευτέρωι ὅτι ἀσύμπτωτοί εἰσιν ⌜αἱ ΑΒ, ΒΓ⌝ τῆς ὑπερβολῆς.

(276) θέσει εὐθεῖα ἡ ΑΒ. δοθὲν τὸ Γ. διήχθω ἡ ΒΓ. κείσθω δοθεῖσα ἡ ΒΔ. ὀρθὴ ἀνήχθω ἡ ΔΕ. ὅτι τὸ Ε ἅπτεται θέσει κώνου τομῆς ὑπερβολῆς ἐρχομένης διὰ τοῦ Γ. ἤχθω κάθετος ἡ ΓΖ. δοθὲν ἄρα ἐστὶν τὸ Ζ. <τῆι ΒΔ ἴση ἡ ΖΑ. δοθὲν ἄρα ἐστὶν τὸ Α.> ἀνήχθω ὀρθὴ ἡ ΑΗ. θέσει ἄρα ἐστὶν ἡ ΑΗ. συμπιπτέτω τῆι ΒΓ ἐκβληθείσηι κατὰ τὸ Η. καὶ θέσει δοθεισῶν τῶν ΒΑ, ΑΗ, καὶ σημείου δοθέντος τοῦ Γ, <γεγράφθω> ὑπερβολὴ περὶ ἀσυμπτώτους τὰς ΗΑ, ΑΒ. ἐλεύσεται ἄρα καὶ διὰ τοῦ Ε, διὰ τὸ ἴσην εἶναι τὴν ΒΓ τῆι ΕΗ, ἐπεὶ καὶ <ὅλη> ὅληι. καὶ ἔσται διὰ τὸ προγεγραμμένον.

‖ 1 δυνάμεναι] δύνανται coni. Hu app | ΔΗ Co ΔΑ A ‖ 2 ἃ add Ha (Co) | τῆι διαμέτρωι Hu τῆς διαμέτρου A ‖ 5 δὴ] δὲ A ‖ 6 αἱ (ΑΒ) add Ha ‖ 7 ἐπιζευχθεῖσα Hu (Co) ἐπεζεύχθω A ‖ 9 ἐπιζευχθεῖσα Hu (Co) ἐπεζεύχθω A ‖ 11 καὶ (ὡς) del Ha | ἐλέγομεν Hu λέγομεν A ‖ 13 ἑκάτερον Hu ἑκατερα A ‖ 14 δυνάμει del Hu ‖ 15 τουτέστιν] καὶ ἔστι Ha | τοῦ (ὑπὸ ΕΔΗ) Hu τῶν A ‖ 16 διαμέτρωι εἴδους Co διαμέτρου εἴδει A ‖ 18 ante δοθὲν add καὶ Ha | δοθὲν Ha δοθεῖσα A | ante κεῖσθω add καὶ Ha ‖ 19 post ὀρθὴ add δὲ Ha | θέσει κώνου τομῆς secl Hu (Ha) ‖ 21 post ΓΖ add καὶ τῆι ΒΔ ἴση κείσθω ἡ ΖΑ Co | Ζ] Δ Α Α Co ‖ 23 συμπιπτέτω] συμπίπτουσα A | συμπιπτέτω — Η secl Hu | ἐκβληθείσηι Hu ἐκβεβλήσθω A ἥτις ἐκβεβλήσθω Ha ‖ 25 τὰς Hu ἡ A, om Ha ‖ 27 ὅλη ὅληι] ὅλη ἡ ΒΕ τῆι ΗΓ Ha (Co)

The synthesis of it will be made as follows. Let the straight line given in position be AB, the given (point) Γ, the (line) drawn across BΓ, the given (line) Θ, and, with perpendicular ΓZ drawn, let ZA be made equal to (Θ), and let AH be erected at right angles and let it intersect BΓ at H, and about asymptotes HA, AB and through given Γ let a hyperbola be drawn. I say that it solves the problem, that is that, if perpendicular EΔ is drawn, BΔ is equal to Θ.

But this is obvious because of the asymptotes. <For> EH equals ΓB (Conics II 8), so that AΔ too equals ZB. Hence all AZ, that is Θ, equals BΔ.

(277) *(Prop. 206)* As BA is to AΓ, so let the square of BΔ be to the square of ΔΓ.[1] That the mean proportional of BA and AΓ is AΔ.

Let ΔE be made equal to ΓΔ.[2] *Separando*, then, as BΓ is to ΓA, that is as the rectangle contained by ΓB, BE is to the rectangle contained by AΓ, EB,[4] so is the rectangle contained by ΓB, BE to the square of EΔ.[3] Therefore the rectangle contained by AΓ, EB equals the square of ΔE,[5] that is the rectangle contained by ΓΔ, ΔE.[6] In ratio[7] and *componendo*, as BΔ is to ΔE, that is to ΔΓ,[9] so is ΔA to AΓ.[8] Therefore sum to sum, as BA is to AΔ, so is AΔ to AΓ.[10] Thus AΔ is mean proportional of BA and AΓ.

(278) *(Prop. 207)* Let the rectangle contained by AB, BΓ equal twice the square of AΓ.[1] That AΓ equals ΓB.

Let AΔ be made equal to AΓ.[2] Then the rectangle contained by ΓΔ, ΔA will be equal to the rectangle contained by AB, BΓ,[3] and (they are applied) to the same (line AΓ). Thus ΔA, that is AΓ, equals ΓB.[4] [5]

(279) *(Prop. 208 a)* About the same asymptotes AB, BΓ let hyperbolas HE, ΔZ be drawn. I say that they do not meet each other.

For if possible, let them intersect at Δ, and from Δ let straight line AΔZEΓ be drawn across the sections. Because of section ΔZ, AΔ will equal ZΓ, and because of section ΔE, AΔ (will) equal) EΓ (Conics II 8), so that ΓZ equals ΓE, which is impossible. Thus the sections do not meet each other.

συντεθήσεται δὴ οὕτως. ἔστω ἡ μὲν τῆι θέσει δεδομένη 960 εὐθεῖα ἡ ΑΒ, τὸ δὲ δοθὲν τὸ Γ, ἡ δὲ διηγμένη ἡ ΒΓ, ἡ δὲ δοθεῖσα ἡ Θ, καὶ αὐτῆι ἴση ἔστω, καθέτου ἀχθείσης τῆς ΓΖ, ἡ ΖΑ, καὶ ὀρθὴ ἀναχθεὶς ἡ ΑΗ. συμπιπτέτω τῆι ΒΓ κατὰ τὸ Η, καὶ περὶ ἀσυμπτώτους τὰς ΗΑ, ΑΒ διὰ δοθέντος [ἔντος] τοῦ Γ γεγράφθω ὑπερβολή. λέγω ὅτι ποιεῖ τὸ πρόβλημα, τουτέστιν ὅτι, ἂν κάθετος ἀχθῆι ἡ ΕΔ, ἴση γίνεται ἡ ΒΔ τῆι Θ. τοῦτο δὲ φανερὸν διὰ τὰς ἀσυμπτώτους. ἴση <γὰρ> ἡ ΕΗ τῆι ΓΒ. ὥστε καὶ ἡ ΑΔ τῆι ΖΒ. καὶ ὅλη ἄρα |ἡ ΑΖ, τουτέστιν ἡ Θ, ἴση ἐστὶν |174v τῆι ΒΔ.

(277) ἔστω ὡς ἡ ΒΑ πρὸς τὴν ΑΓ, οὕτως τὸ ἀπὸ ΒΔ πρὸς τὸ ἀπὸ ΔΓ. ὅτι τῶν ΒΑ, ΑΓ μέση ἀνάλογόν ἐστιν ἡ ΑΔ. κείσθω τῆι ΓΔ ἴση ἡ ΔΕ. κατὰ διαίρεσιν ἄρα γίνεται ὡς ἡ ΒΓ πρὸς τὴν ΓΑ, τουτέστιν ὡς τὸ ὑπὸ ΓΒΕ πρὸς τὸ ὑπὸ ΑΓ, ΕΒ, οὕτως τὸ ὑπὸ ΓΒΕ πρὸς τὸ ἀπὸ ΕΔ. ἴσον ἄρα ἐστὶν τὸ ὑπὸ ΑΓ, ΕΒ τῶι ἀπὸ ΔΕ, τουτέστιν τῶι ὑπὸ ΓΔΕ. ἀνάλογον καὶ συνθέντι ἐστὶν ὡς ἡ ΒΔ πρὸς τὴν ΔΕ, τουτέστιν πρὸς τὴν ΔΓ, οὕτως ἡ ΔΑ πρὸς ΑΓ. ὅλη ἄρα πρὸς ὅλην ἐστὶν ὡς ἡ ΒΑ πρὸς τὴν ΑΔ, οὕτως ἡ ΑΔ πρὸς τὴν ΑΓ. ὥστε τῶν ΒΑ, ΑΓ μέση ἀνάλογόν ἐστιν ἡ ΑΔ.

(278) ἔστω τὸ ὑπὸ ΑΒΓ ἴσον τῶι δὶς ἀπὸ ΑΓ. ὅτι ἴση ἐστὶν ἡ Α⌜Γ τῆι ΓΒ. κείσθω⌝ τῆι ΑΓ ἴση ἡ ΑΔ. ⌜ἔσται ἄρα τὸ ὑπὸ⌝ ΓΔΑ ἴσον τῶι ὑπὸ ΑΒΓ, καὶ παρὰ τὴν αὐτήν. ἴση ἄρα ⌜ἐστὶν ἡ ΔΑ, τουτέστιν⌝ ἡ ΑΓ, τῆι ΓΒ.

(279) περὶ τὰς αὐτὰς ἀσυμπτώτους τὰς ΑΒ, ΒΓ ὑπερβολαὶ 962 γεγράφθωσαν αἱ ΗΕ, ΔΖ. λέγω ὅτι οὐ συμβάλλουσιν ἀλλήλαις. εἰ γὰρ δυνατόν, συμπιπτέτωσαν κατὰ τὸ Δ, καὶ ἀπὸ τοῦ Δ διήχθω εἰς τομὰς εὐθεῖα ἡ ΑΔΖΕΓ. ἔσται δὴ διὰ μὲν τῆς ΔΖ τομῆς ἴση ἡ ΑΔ τῆι ΖΓ, διὰ δὲ τῆς ΔΕ τομῆς ἴση ἡ ΑΔ τῆι ΕΓ, ὥστε ἡ ΓΖ τῆι ΓΕ ἴση ἐστίν, ὅπερ ἀδύνατον. οὐκ ἄρα συμβάλλουσιν αἱ τομαὶ ἀλλήλαις.

‖ 1 δὴ] δὲ Ge (S) ‖ 2 ἡ δὲ διηγμένη Ha ἡ δὲ διάμετρος A καὶ διήχθω Co ‖ 4 ἀναχθεὶς] ἀνήχθω A | ante συμπιπτέτω add καὶ Hu | ΒΓ Ha ΒΗ A | post ΒΓ add ἐκβληθείσηι Ha ‖ 5 ἔντος del Ha ‖ 6 ὑπερβολὴ Ha ὑπερβολῆι A ‖ 7 post ὅτι add οἷα (scil. οἵα) Ha ‖ 8 γὰρ add Ha ‖ 14 ΓΑ Co ΓΔ A | ΑΓ, ΕΒ Co ΑΓΕ A ‖ 19 ΑΓ (ὥστε) Co ΔΓ A | ἀνάλογον Ha ἀνάλογος A ‖ 25 ΗΕ, ΔΖ] ΔΕ, ΔΖ A ΔΖ, ΗΕ Hu ‖ 26 post συμπιπτέτωσαν add ἀλλήλαις Ha ‖ 27 διήχθω... εὐθεῖα ἡ ΑΔΖΕΓ Co διήχθωσαν... εὐθεῖαι αἱ $\overline{\text{ΗΑΔΖΕΓ}}$ A

(Prop. 208 b) *I say that as they grow indefinitely they draw closer to each other and approach to a lesser distance. For let some other (line) ΘK be drawn, and let there be the diameter, and let its end be M. Then as is the rectangle contained by MΛ, ΛN to the square of ΛΞ, so will the *latus transversum* be to the *latus rectum* (Conics I, 12). But as the rectangle contained by MO, OΠ is to the square of OP, so is the *latus transversum* to the *latus rectum* (Conics I, 12). Hence as the rectangle contained by MΛ, ΛN is to the square of ΛΞ, so is the rectangle contained by MO, OΠ to the square of OP. *Alternando*, <as the rectangle contained by MΛ, ΛN is to the rectangle contained by MO, OΠ, so is the square of ΛΞ to the square of OP.> But the rectangle contained by MΛ, ΛN is greater than the rectangle contained by MO, OΠ. Therefore ΞZ is greater than PΣ. And because of the sections the rectangle contained by ZΔ, ΔΞ equals the rectangle contained by ΣP, PΘ. Hence ΞΔ is less than ΘP. Thus they always approach to a lesser distance.*

But (the theorem) is also to hand. For if each of them draws closer to the asymptotes (Conics II 14), obviously (they approach) each other too.

(280) *(Prop. 209)* As AB is to BΓ, so let ΔE be to EZ, and as BA is to AH, so let EΔ be to ΔΘ. That as the solid that has as base the square of AΓ, as height AB, is to the solid that has as base the square of ΔZ, as height ΔE, so is the cube of AH plus that which has the ratio to the cube of HB that the square of AΓ has to the square of ΓB, to the cube of ΔΘ plus that which has the ratio to the cube of ΘE that the square of ΔZ has to the square of ZE.

For since as ΓA is to AB, so is ZΔ to ΔE,[1] therefore as the square of ΓA is to the square of AB, so is the square of ZΔ to the square of ΔE.[2] But as the square of ΓA is to the square of AB, with common height AB, so is the solid with the square of AΓ as base, height AB, to the cube of AB;[3] and as the square of ZΔ is to the square of ΔE, with common height ΔE, so is the solid with the square of ΔZ as base, ΔE as height, to the cube of ΔE.[4] Hence these things also by inversion and *alternando.*[5] But also as the cube of AB is to the cube of ΔE, so is the cube of AH to the cube of ΔΘ,[6] and the cube of HB to the cube of ΘE.[7] <But as the cube of HB is to the cube of ΘE,> so is that which has the ratio to the cube of HB that the square of AΓ has to the square of ΓB, to that which has the ratio to the λέγω δὴ ὅτι καὶ εἰς ἄπειρον αὐξόμεναι ἔγγιον προσάγουσιν ἑαυταῖς καὶ <εἰς> ἔλαττον ἀφικνοῦνται διάστημα. ἤχθω γάρ τις καὶ ἑτέρα ἡ ΘΚ, καὶ ἔστω ἡ διάμετρος ἧς πέρας ἔστω τὸ Μ. ἔσται ἄρα ὡς μὲν τὸ ὑπὸ ΜΛΝ πρὸς τὸ ἀπὸ ΛΞ, οὕτως ἡ πλαγία πρὸς τὴν ὀρθίαν. ὡς δὲ τὸ ὑπὸ ΜΟΠ πρὸς τὸ ἀπὸ ΟΡ, οὕτως ἡ πλαγία πρὸς τὴν ὀρθίαν. ὥστε ἐστὶν <ὡς> τὸ ὑπὸ ΜΛΝ πρὸς τὸ ἀπὸ ΛΞ, οὕτως τὸ ὑπὸ ΜΟΠ πρὸς τὸ ἀπὸ ΟΡ. ἐναλλὰξ ἐστιν <ὡς τὸ ὑπὸ ΜΛΝ πρὸς τὸ ὑπὸ ΜΟΠ, οὕτως τὸ ἀπὸ ΛΞ πρὸς τὸ ἀπὸ ΟΡ.> μεῖζον δέ ἐστιν τὸ ὑπὸ ΜΛΝ τοῦ ὑπὸ ΜΟΠ. μείζων ἄρα ἐστὶν ἡ ΞΖ τῆς ΡΣ. καὶ ἔστιν διὰ τὰς τομὰς ἴσον τὸ ὑπὸ ΖΔΞ τῶι ὑπὸ ΣΡΘ. ἐλάσσων ἄρα ἐστὶν ἡ ΞΔ τῆς ΘΡ. ὥστε αἰεὶ εἰς ἔλαττον ἀφικνοῦνται διάστημα.

ἀλλὰ καὶ παράκειται. εἰ γὰρ ἑκατέρα αὐτῶν ταῖς 964
ἀσυμπτώτοις ἔγγιον προσάγει, δηλονότι καὶ ἑαυταῖς.

(280) |ἔστω ὡς μὲν ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΕ πρὸς τὴν ΕΖ,
ὡς δὲ ἡ ΒΑ πρὸς ΑΗ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΘ. ὅτι γίνεται ὡς |175
τὸ στερεὸν τὸ βάσιν μὲν ἔχον τὸ ἀπὸ ΑΓ τετράγωνον, ὕψος δὲ τὴν ΑΒ, πρὸς τὸ στερεὸν τὸ βάσιν μὲν ἔχον τὸ ἀπὸ ΔΖ τετράγωνον, ὕψος δὲ τὴν ΔΕ, οὕτως ὁ [τε] ἀπὸ τῆς ΑΗ κύβος μετὰ τοῦ λόγον ἔχοντος πρὸς τὸν ἀπὸ τῆς ΗΒ κύβον ὃν τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΓΒ, πρὸς τὸν ἀπὸ τῆς ΔΘ κύβον μετὰ τοῦ λόγον ἔχοντος πρὸς τὸν ἀπὸ τῆς ΘΕ κύβον ὃν τὸ ἀπὸ ΔΖ πρὸς τὸ ἀπὸ ΖΕ. ἐπεὶ γάρ ἐστιν ὡς ἡ ΓΑ πρὸς τὴν ΑΒ, οὕτως ἡ ΖΔ πρὸς τὴν ΔΕ, καὶ ὡς ἄρα τὸ ἀπὸ ΓΑ πρὸς τὸ ἀπὸ ΑΒ, οὕτως τὸ ἀπὸ ΖΔ πρὸς τὸ ἀπὸ ΔΕ. ἀλλ' ὡς μὲν τὸ ἀπὸ ΓΑ πρὸς τὸ ἀπὸ ΑΒ, κοινὸν ὕψος ἡ ΑΒ, οὕτως τὸ στερεὸν τὸ βάσιν μὲν ἔχον τὸ ἀπὸ ΑΓ τετράγωνον, ὕψος δὲ τὴν ΑΒ, πρὸς τὸν ἀπὸ τῆς ΑΒ κύβον· ὡς δὲ τὸ ἀπὸ ΖΔ πρὸς τὸ ἀπὸ ΔΕ, κοινὸν ὕψος ἡ ΔΕ, οὕτως τὸ στερεὸν τὸ βάσιν μὲν ἔχον τὸ ἀπὸ ΔΖ τετράγωνον, ὕψος δὲ τὴν ΔΕ, πρὸς τὸν ἀπὸ τῆς ΔΕ κύβον. καὶ ταῦτα ἄρα ἀνάπαλιν καὶ ἐναλλάξ ἐστιν. ἔστιν δὲ καὶ ὡς ὁ ἀπὸ τῆς ΑΒ κύβος πρὸς τὸν ἀπὸ τῆς ΔΕ κύβον, οὕτως ὅ τε ἀπὸ τῆς ΑΗ κύβος πρὸς τὸν ἀπὸ τῆς ΔΘ κύβον, καὶ ὁ ἀπὸ τῆς ΗΒ κύβος πρὸς τὸν ἀπὸ τῆς ΘΕ κύβον. <ἀλλ' ὡς ὁ ἀπὸ τῆς ΗΒ κύβος πρὸς τὸν ἀπὸ τῆς ΘΕ κύβον>, οὕτως τὸ λόγον ἔχον πρὸς τὸν ἀπὸ τῆς ΗΒ κύβον <ὃν> τὸ ἀπὸ

‖ 1 ἔγγιον Ha ἔγγειον A ‖ 2 εἰς add Hu (Co) αἰεὶ εἰς Hu ‖ 3 post διάμετρος add ΜΝ Ha, lacunam indicavit Hu ‖ 4 ἔστω del Ha | post Μ add ἔστω ἡ τῆς ΔΠΖ διάμετρος ἡ ΠΗ Ha, lacunam indicavit Hu ‖ 5 ΜΟΠ] ΗΟΠ Ha ‖ 6 ὀρθίαν Ha ὀρθήν A | ὡς add Ha ‖ 7 ΜΟΠ] ΗΟΠ Ha ‖ 8 ἐναλλάξ ἐστιν] καὶ ἐναλλάξ Ha ‖ 9 μεῖζον Ha μείζων A | ΜΛΝ Co ΛΜΝ A | ΜΟΠ] ΗΟΠ Ha ‖ 10 ΡΣ] ΘΣ Ha ‖ 11 ΖΔΞ ... ΣΡΘ] ΖΞΔ ... ΣΘΡ Ha, post quae add ἕκαστον γὰρ τῶι ἀπὸ ΠΓ ἴσον ‖ 13 ἀλλὰ – ἑαυταῖς secl Hu | παράκειται] παράκεινται Ha ‖ 14 ἔγγιον Ha ἔγγειον A ‖ 16 ante ΑΗ add τὴν Ge (recc?) ‖ 19 τε del Ha ‖ 20 ὃν Ha (Co) ὅτι A ‖ 24 ΑΒ Co ΓΒ A | ΖΔ Co ΖΔΘ A ‖ 25 ΔΕ Co ΔΘ A | κοινὸν Ha κύβου A ‖ 29 ΔΖ Hu (Co) ΑΖ A ‖ 30 ἀνάπαλιν] ἀνάλογον A ‖ 31 κύβος Ha καὶ A ‖ 34 ἀλλ' – κύβον add Co ‖ 35 τὸ (λόγον) Ha τὸν A | ὃν add Ha (Co) cube of ΘE that the square of ΔZ has to the square of ZE.[8] Therefore as one of the leading (members) is to one of the following (members), so are all to all. Thus as is the solid that has the square of AΓ as base, AB as height, to the solid that has the square of ΔZ as base, ΔE as height, so is the cube of AH plus that which has the ratio to the cube of HB that the square of AΓ has to the square of ΓB, to the cube of ΔΘ plus that which has the ratio to the cube of ΘE that the square of ΔZ has to the square of ZE.[9]

(281) *(Prop. 210)* Let A plus B equal Γ plus Δ. That the amount by which A exceeds Γ is the amount by which Δ exceeds B.

For let the amount by which A exceeds Γ be E.[1] Then A equals Γ and E.[2] Let B be added in common. Then A and B equal Γ and E and B.[3] But A and B are stipulated to be equal to Γ and Δ.[4] Therefore Γ and Δ equal Γ and E and B.[5] Let Γ be subtracted in common. Then the remainder, Δ, equals B and E,[6] so that Δ exceeds B by E.[7] Thus the amount by which A exceeds <Γ> is the amount by which Δ exceeds B.[8]

Similarly we shall prove that if the amount by which A exceeds Γ is the amount by which Δ exceeds B, then A and B equal Γ and Δ.

(282) *(Prop. 211)* Let there be two magnitudes AB, BΓ. That if BA exceeds AΓ by ΓB, then that which has a ratio to AB exceeds that which has the same ratio to AΓ by that which has the same ratio to ΓB.

For let that which has a certain ratio to AB be ΔE,[1] and ΔZ that which has the same ratio to AΓ.[2] Then the remainder, EZ, has to BΓ the same ratio.[3] And EZ is the difference by which ΔE exceeds ΔZ,[4] that is (by which) that which has a ratio to AB (exceeds) that which has the same ratio to AΓ.

(283) *(Prop. 212)* Let A exceed Γ by a lesser amount than Δ (exceeds) B. That A and B are less than Γ and Δ.

For let E be the amount by which A exceeds Γ.[1] Then A and B equal Γ and E and B.[2] But since A exceeds Γ by a lesser amount than Δ (exceeds) B,[3] and A exceeds Γ by E, therefore E is less than the difference

ΑΓ πρὸς τὸ ἀπὸ ΓΒ, πρὸς τὸ λόγον ἔχον πρὸς τὸν ἀπὸ τῆς ΘΕ κύβον ὃν τὸ ἀπὸ ΔΖ πρὸς τὸ ἀπὸ ΖΕ. καὶ ὡς ἄρα ἓν τῶν ἡγουμένων πρὸς ἓν τῶν ἑπομένων, <οὕτως> ἅπαντα πρὸς ἅπαντα. ἔστιν ἄρα ὡς τὸ στερεὸν <τὸ> βάσιν μὲν ἔχον τὸ 966 ἀπὸ τῆς ΑΓ τετράγωνον, ὕψος δὲ τὴν ΑΒ, πρὸς τὸ στερεὸν τὸ βάσιν μὲν ἔχον τὸ ἀπὸ τῆς ΔΖ τετράγωνον, ὕψος δὲ τὴν ΔΕ, οὕτως ὁ ἀπὸ τῆς ΑΗ κύβος μετὰ τοῦ λόγον ἔχοντος πρὸς τὸν ἀπὸ τῆς ΗΒ κύβον <ὃν> τὸ ἀπὸ ΑΓ πρὸς τὸ ἀπὸ ΓΒ, πρὸς τὸν ἀπὸ τῆς ΔΘ κύβον καὶ τὸ λόγον ἔχον πρὸς τὸν ἀπὸ τῆς ΘΕ κύβον ὃν τὸ ἀπὸ τῆς ΔΖ πρὸς τὸ ἀπὸ τῆς ΖΕ.

(281) ἔστω τὸ Α μετὰ τοῦ Β ἴσον τῶι Γ μετὰ τοῦ Δ. ὅτι ὧι ὑπερέχει τὸ Α τοῦ Γ, τούτωι ὑπερέχει καὶ τὸ Δ τοῦ Β. ἔστω γὰρ ὧι ὑπερέχει τὸ Α τοῦ Γ τὸ Ε [τοῦ Α]. τὸ Α ἄρα ἴσον ἐστὶν τοῖς Γ, Ε. κοινὸν προσκείσθω τὸ Β. τὰ Α, Β ἄρα ἴσα ἐστὶν τοῖς Γ, Ε, Β. |ἀλλὰ τὰ Α, Β τοῖς Γ, Δ ἴσα ὑπόκειται. καὶ τὰ Γ, Δ ἄρα τοῖς Γ, Ε, Β ἴσα. κοινὸν ἀφηιρήσθω τὸ Γ. λοιπὸν ἄρα τὸ |175v Δ ἴσον τοῖς Β, Ε, ὥστε τὸ Δ τοῦ Β ὑπερέχει τῶι Ε. ὧι ἄρα ὑπερέχει τὸ Α <τοῦ Γ>, τούτωι ὑπερέχει καὶ τὸ Δ τοῦ Β. ὁμοίως δὴ δείξομεν ὅτι, ἐὰν ὧι ὑπερέχηι τὸ Α τοῦ Γ, τούτωι ὑπερέχει καὶ τὸ Δ τοῦ Β, ὅτι τὰ Α, Β ἴσα ἐστὶν τοῖς Γ, Δ.

(282) ἔστω δύο μεγέθη τὰ ΑΒ, ΒΓ. ὅτι εἰ ὑπερέχει τὸ ΒΑ τοῦ ΑΓ τῶι ΓΒ, ὑπερέχει καὶ τὸ λόγον ἔχον πρὸς τὸ [ἀπὸ] ΑΒ τοῦ λόγον ἔχοντος πρὸς τὸ ΑΓ τὸν αὐτὸν τῶι λόγον ἔχοντι πρὸς τὸ [ἀπὸ] ΓΒ τὸν αὐτόν. ἔστω γὰρ τὸ μὲν πρὸς τὸ ΑΒ λόγον τινὰ ἔχον τὸ ΔΕ, τὸ δὲ πρὸς τὸ ΑΓ τὸν αὐτὸν λόγον ἔχον τὸ ΔΖ. λοιπὸν ἄρα τὸ ΕΖ πρὸς τὸ ΒΓ λόγον ἔχει τὸν αὐτόν. καὶ ἔστιν τὸ ΕΖ ἡ ὑπεροχὴ ἧι ὑπερέχει τὸ ΔΕ τοῦ ΔΖ, τουτέστιν τὸ λόγον ἔχον πρὸς τὸ ΑΒ τοῦ λόγον ἔχοντος πρὸς τὸ ΑΓ τὸν αὐτόν.

(283) τὸ Α τοῦ Γ ἐλάσσονι ὑπερεχέτω ἤπερ τὸ Δ τοῦ Β. ὅτι τὰ Α, Β ἐλάσσονά ἐστιν τῶν Γ, Δ. ἔστω γὰρ ὧι ὑπερέχει τὸ Α 968 τοῦ Γ τὸ Ε. τὰ Α, Β ἄρα ἴσα ἐστὶν τοῖς Γ, Ε, Β. ἐπεὶ δὲ τὸ Α τοῦ Γ ἐλάσσονι ὑπερέχει ἤπερ τὸ Δ τοῦ Β, τὸ δὲ Α τοῦ Γ

‖ 2 ὃν τὸ Ha (Co) οντα A ‖ 3 οὕτως add Ha ‖ 4 τὸ (βάσιν) add Ge (S) ‖ 8 ὃν add Ha (Co) ‖ 9 καὶ τὸ... ἔχον] μετὰ τοῦ... ἔχοντος Ha (Co) ‖ 13 τοῦ Α del Co ‖ 16 ἀφηιρήσθω Ha ἀφαιρείσθω A ‖ 17 (Β) Ε] Γ A^1 corr A^2 | ὧι Ha (Co) ὡς A ‖ 18 τοῦ Γ add Ha (Co) ‖ 19 ὅτι del Hu | ὑπερέχηι Hu ὑπερέχει A ‖ 20 ἴσα Ha ἴσον A ‖ 21 εἰ] ὧι A ἐὰν Ha | εἰ – τωι ΓΒ secl Hu ‖ 22 τῶι ΓΒ] τούτωι A del Ha | καὶ del Hu | ἀπὸ del Co ‖ 23 λόγον (ἔχοντος) Ha λόγου A | τὸ (ΑΓ) Ha τὸν A | λόγον (ἔχοντι) Ha λόγωι A ‖ 24 ἀπὸ del Co ‖ 25 τὸ (ΔΖ) Co τῶι A ‖ 27 ΕΖ ἡ Hu app $\bar{\mathrm{E}}\bar{\mathrm{Z}}\bar{\mathrm{H}}$ A, Η del Co ‖ 32 Α (τοῦ Γ) Co ἀπὸ A ‖ 33 ἐλάσσονι Ha ἐλάσσον· A of Δ and B.[4] Hence E and B are less than Δ.[5] Let Γ be added in common. Then Γ and E and B are less than Γ and Δ.[6] But Γ and E and B were proved to equal A and B. Thus A and B are less than Γ and Δ.[7]

The converse similarly, and the (lemmas) for the ellipse similarly.

ὑπερέχει τῶι Ε, τὸ Ε ἄρα ἔλασσόν ἐστιν τῆς τῶν Δ, Β ὑπεροχῆς. ὥστε τὰ Ε, Β ἐλάσσονά ἐστιν τοῦ Δ. κοινὸν προσκείσθω τὸ Γ. τὰ Γ, Ε, Β ἄρα ἐλάσσονά ἐστιν τῶν Γ, Δ. ἀλλὰ τὰ Γ, Ε, Β ἴσα ἐδείχθη τοῖς Α, Β. τὰ Α, Β ἄρα ἐλάσσονά ἐστιν τῶν Γ, Δ. ὁμοίως καὶ τὸ ἀναστρόφιον, καὶ τὰ ἐπὶ τῆς ἐλλείψεως ὁμοίως.

‖ 3 τῶν (Γ, Δ) Hu (Co) τοῖς A ‖ 5 τῶν (Γ, Δ) Hu (Co) τοῖς A | ἐπὶ del Ha

(284) (Lemmas) of (Book) 6.

1. *(Prop. 213)* Let there be two obtuse-angled triangles ABΓ, ΔEZ, that have angles Γ, Z obtuse, and angles A and Δ acute and equal. Let ΓH and ZΘ be drawn at right angles to BΓ and EZ. As the rectangle contained by BA, AH is to the square of AΓ, so let the rectangle contained by EΔ, ΔΘ be to the square of ΔZ. That triangle ABΓ is similar to triangle ΔEZ.

For let semicircles be drawn on HB and EΘ. They will pass through Γ and Z. Let them pass, and let them be HΓB and EZΘ. Now either AΓ and ΔZ are (both) tangent to the semicircles or (both are) not. Then if they are (both) tangent *(Prop. 213 a)*, obviously triangles ABΓ and ΔEZ are similar. For if I take the centers M and N, and join MΓ and NZ, then angles MΓA and NZΔ will be right.[1] And angles A and Δ are equal.[2] Therefore angle AMΓ (equals) angle ΔNZ.[3] And the halves too (are equal). Therefore angle B equals angle E (III 20).[4] But also (angle) A (equals angle) Δ. Therefore the triangles are similar.[5]

Now, however *(Prop. 213 b – c)*, let them not be tangent, but let them cut the semicircles at some points K, Λ, and let perpendiculars MΞ, NO be drawn. Then KΞ equals ΞΓ,[6] and ΛO (equals) OZ.[7] But (triangle) AMΞ is similar to triangle ΔNO.[8] Therefore as ΞA is to AM, so is OΔ to ΔN.[9] But since as the rectangle contained by BA, AH is to the square of AΓ, so is the rectangle contained by EΔ, ΔΘ to the square of ΔZ,[10] therefore as the rectangle contained by KA, AΓ is to the square of AΓ, that is as KA is to AΓ, so is the rectangle contained by ΛΔ, ΔZ to the square of ΔZ,[11] that is ΛΔ to ΔZ.[12] Hence also ΞA is to AΓ as OΔ is to ΔZ.[13] But also as ΞA is to AM, so is OΔ to ΔN,[14] because of the similarity of the triangles. *Ex aequali* therefore as ΓA is to AM, so is ZΔ to ΔN.[15] And (the sides) about equal angles A, Δ are in ratio.[16] Therefore angle AMΓ equals angle ΔNZ.[17] And the halves (are equal). Therefore angle B too equals angle E.[18] But also by hypothesis (angle) A (equals angle) Δ. Thus triangle ABΓ is similar to triangle ΔEZ.[19]

(285) *(Prop. 213)* The converse of it is apparent, namely with (triangle) ABΓ similar to (triangle) ΔEZ, and angles BΓH and EZΘ right, to prove that as the rectangle contained by BA, AH is to the square of AΓ, so is the rectangle contained by EΔ, ΔΘ to the square of ΔZ. For because of the similarity of the triangles, as BA is to AΓ, so is EΔ to ΔZ, while as HA is to AΓ, so is ΘΔ to ΔZ. And the compounded (ratio is therefore equal to the compounded ratio).

(284) ΤΟΥ Ϛ´

<α´.> ἔστω δύο τρίγωνα ἀμβλυγώνια τὰ ΑΒΓ, ΔΕΖ, ἀμβλείας ἔχοντα τὰς Γ, Ζ γωνίας, καὶ ἴσας τὰς Α, Δ ὀξείας· ὀρθαὶ ταῖς ΒΓ, ΕΖ ἤχθωσαν αἱ ΓΗ, ΖΘ. ἔστω δὲ ὡς τὸ ὑπὸ τῶν ΒΑΗ πρὸς τὸ ἀπὸ τῆς ΑΓ τετράγωνον, οὕτως τὸ ὑπὸ τῶν ΕΔΘ πρὸς τὸ ἀπὸ τῆς ΔΖ. ὅτι ὅμοιόν ἐστιν τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι. γεγράφθω γὰρ ἐπὶ τῶν ΗΒ, ΕΘ ἡμικύκλια· ἐλεύσεται δὴ καὶ διὰ τῶν Γ, Ζ. ἐρχέσθω, καὶ ἔστω τὰ ΗΓΒ, ΕΖΘ. ἤτοι |δὴ |176
ἐφάπτονται αἱ ΑΓ, ΔΖ τῶν ἡμικυκλίων ἢ οὔ. εἰ μὲν οὖν ἐφάπτονται, φανερὸν ὅτι γίνεται ὅμοια τὰ ΑΒΓ, ΔΕΖ τρίγωνα. ἐὰν γὰρ λάβω τὰ κέντρα τὰ Μ, Ν, καὶ ἐπιζεύξω τὰς ΜΓ, ΝΖ, ἔσονται ὀρθαὶ αἱ ὑπὸ ΜΓΑ, ΝΖΔ γωνίαι. καὶ εἰσὶν αἱ Α, Δ γωνίαι ἴσαι. καὶ ἡ ὑπὸ ΑΜΓ ἄρα τῆι ὑπὸ ΔΝΖ γωνίαι. καὶ τὰ ἡμίση. καὶ ἡ Β ἄρα γωνία τῆι Ε ἐστὶν ἴση. ἀλλὰ καὶ ἡ Α τῆι 970
Δ. ὅμοια ἄρα ἐστὶν τὰ τρίγωνα.

ἀλλὰ δὴ μὴ ἐφαπτέσθωσαν, ἀλλὰ τεμνέτωσαν τὰ ἡμικύκλια κατά τινα σημεῖα τὰ Κ, Λ, καὶ ἤχθωσαν κάθετοι αἱ ΜΞ, ΝΟ. ἴση ἄρα ἐστὶν ἡ μὲν ΚΞ τῆι ΞΓ, ἡ δὲ ΛΟ τῆι ΟΖ. ὅμοιον δὲ τὸ ΑΜΞ τῶι ΔΝΟ τριγώνωι. ἔστιν ἄρα ὡς ἡ ΞΑ πρὸς ΑΜ, οὕτως ἡ ΟΔ πρὸς ΔΝ. ἐπεὶ δέ ἐστιν ὡς τὸ ὑπὸ ΒΑΗ πρὸς τὸ ἀπὸ ΑΓ, οὕτως τὸ ὑπὸ ΕΔΘ πρὸς τὸ ἀπὸ ΔΖ, καὶ ὡς ἄρα τὸ ὑπὸ ΚΑΓ πρὸς τὸ ἀπὸ ΑΓ, τουτέστιν ὡς ἡ ΚΑ πρὸς ΑΓ, οὕτως τὸ ὑπὸ ΛΔΖ πρὸς τὸ ἀπὸ ΔΖ, τουτέστιν ἡ ΛΔ πρὸς ΔΖ. ὥστε καὶ ὡς ἡ ΞΑ πρὸς ΑΓ, οὕτως ἡ ΟΔ πρὸς ΔΖ. ἀλλὰ καὶ ὡς ἡ ΞΑ πρὸς ΑΜ, οὕτως ἐστὶν ἡ ΟΔ πρὸς ΔΝ, διὰ τὴν ὁμοιότητα τῶν τριγώνων. δι' ἴσου ἄρα ἐστὶν ὡς ἡ ΓΑ πρὸς ΑΜ, οὕτως ἡ ΖΔ πρὸς ΔΝ. καὶ περὶ ἴσας γωνίας τὰς Α, Δ ἀνάλογόν εἰσιν. ἴση ἄρα ἐστὶν ἡ ὑπὸ τῶν ΑΜΓ τῆι ὑπὸ τῶν ΔΝΖ γωνίαι. καὶ τὰ ἡμίση. καὶ ἡ Β ἄρα γωνία ἴση ἐστὶν τῆι Ε. ἀλλὰ καὶ ἡ Α τῆι Δ καθ' ὑπόθεσιν. ὅμοιον ἄρα ἐστὶν τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι.

(285) συμφανὲς δὲ τὸ ἀντίστροφον αὐτῶι, τὸ ὄντος ὁμοίου τοῦ ΑΒΓ τῶι ΔΕΖ, καὶ ὀρθῶν τῶν ὑπὸ ΒΓΗ, ΕΖΘ, δεῖξαι ὅτι γίνεται ὡς τὸ ὑπὸ ΒΑΗ πρὸς τὸ ἀπὸ ΑΓ, οὕτως τὸ ὑπὸ ΕΔΘ πρὸς τὸ ἀπὸ ΔΖ. ἔστιν γὰρ διὰ τὴν ὁμοιότητα τῶν τριγώνων ὡς μὲν ἡ ΒΑ πρὸς ΑΓ, οὕτως ἡ ΕΔ πρὸς ΔΖ, ὡς δὲ ἡ ΗΑ πρὸς ΑΓ, οὕτως ἡ ΘΔ πρὸς ΔΖ. καὶ ὁ συνημμένος. 972

‖ 2 α´ add Hu (BS) ‖ 3 ἔχοντα τὰς Ha ἔχον τὰς A ‖ 8 ἐρχέσθω – ΕΖΘ secl Hu | ΕΖΘ Co ΒΕΖ A ‖ 9 ἢ οὔ] ἤγου A¹ corr A² ‖ 16 τεμνέτωσαν Co τεμνέτω A ‖ 23 ΛΔ Co ΛΑ A ‖ 25 διὰ – τριγώνων secl Hu ‖ 26 περὶ] παρὰ A ‖ 28 γωνίαι Ha γωνιῶν A ‖ 31 τὸ] τοῦ A om Hu | τὸ – ΔΕΖ] τοῦ ΑΒΓ ὄντος ὁμοίου τῶι ΔΕΖ Ha ‖ 33 ΕΔΘ Co ΕΛΘ A ‖ 35 ΗΑ Co ΚΑ A ‖ 36 ΘΔ Co ΛΔ A

(286) 2. *(Prop. 214)* Let there be two similar segments greater than a semicircle, namely the (segments) on AB, ΓΔ, and let perpendiculars EZH, ΘΚΛ be drawn. And as EH is to HZ, so let ΘΛ be to ΛΚ. It is required to prove that arc BZ is similar to arc ΔK.

Let the centers M, N be taken, and let perpendiculars MΞ, MO, NΠ, NP be drawn,[1] and let MB, NΔ be joined. Then angle OMB equals angle PNΔ;[2] for the (angles) in the segments are equal, and the halves. And (angles) O and P are right.[3] Therefore also angle MBO equals angle NΔP.[4] Let ZΣ, KT be drawn parallel to AB, ΓΔ,[5] and let MZ, NK be joined. Then also angle MΣZ equals angle NTK.[6] But since as EH is to HZ, so is ΘΛ to ΛK,[7] and therefore as ΞH is to HZ, so is ΠΛ to ΛK,[8] and so also as HΞ is to ΞZ, that is MB to MΣ,[10], that is ZM to MΣ,[11] so is ΛΠ to KΠ,[9] that is ΔN to NT,[12] <that is KN to NT>,[13] while angles MΣZ and NTK are equal, and angles MZΣ and NKT acute,[14] therefore angle ΣMZ equals angle TNK.[15] Thus arc BZ is similar to arc ΔK.[16]

(287) *(Prop. 215)* Let there be two right-angled (triangles) ABΓ, ΔEZ, that have angles Γ and Z right, and let AH and ΔΘ be drawn across at equal angles BAH and EΔΘ. And as is the rectangle contained by BΓ, ΓH to the square of AΓ, so let the rectangle contained by EZ, ZΘ be to the square of ZΔ. That triangle ABΓ is similar <to triangle ΔEZ>.

For let segments of circles BHA, EΘΔ be drawn about triangles ABH and ΔEΘ. Hence they are similar.[1] Now either AΓ and ΔZ are tangent to the segments, or not. First let them be tangent *(Prop. 215 a)*. Then the rectangle contained by BΓ, ΓH equals the square of AΓ,[2] that is, if I draw AK at right angles to AH,[3] (the rectangle contained by BΓ, ΓH equals) the rectangle contained by HΓ, ΓK,[4] while the rectangle contained by EZ, ZΘ (equals) the square of ΔZ,[5] that is, if I draw ΔΛ at right angles to ΔΘ,[6] (the rectangle contained by EZ, ZΘ equals) the rectangle contained by ΘZ, ZΛ.[7] Hence BΓ equals ΓK, and EZ equals ZΛ.[8] And AΓ and ΔZ are at right angles (to BΓ and EZ).[9] Therefore angle BAK is twice angle BAΓ, and angle EΔΛ (twice) angle EΔZ.[10] And angles BAK and EΔΛ are equal;[13] for angle BAH equals angle EΔΘ,[11] and right angle HAK (equals) right angle ΘΔΛ.[12] Therefore angles BAΓ and EΔZ are equal.[14] But also

(286) β´. ἔστω δύο ὅμοια τμήματα μείζονα ἡμικυκλίου τὰ ἐπὶ τῶν ΑΒ, ΓΔ, καὶ ἤχθωσαν κάθετοι αἱ ΕΖΗ, ΘΚΛ. ἔστω δὲ ὡς ἡ ΕΗ πρὸς ΗΖ, οὕτως ἡ ΘΛ πρὸς ΛΚ. δεικτέον ὅτι ὁμοία ἐστὶν ἡ ΒΖ περιφέρεια τῆι ΔΚ περιφερείαι. εἰλήφθω τὰ κέντρα τὰ Μ, Ν, καὶ κάθετοι ἤχθωσαν αἱ ΜΞ, ΜΟ, ΝΠ, ΝΡ, |καὶ ἐπεζεύχθωσαν αἱ ΜΒ, ΝΔ. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΟΜΒ γωνία τῆι ὑπὸ ΡΝΔ γωνίαι. |176v ἴσαι γάρ εἰσιν αἱ ἐν τοῖς τμήμασιν, καὶ τὰ ἡμίση. καὶ εἰσὶν ὀρθαὶ αἱ Ο, Ρ. ἴση ἄρα ἐστὶν καὶ ἡ ὑπὸ ΜΒΟ γωνία τῆι ὑπὸ ΝΔΡ γωνίαι. ἤχθωσαν ταῖς ΑΒ, ΓΔ παράλληλοι αἱ ΖΣ, ΚΤ, καὶ ἐπεζεύχθωσαν αἱ ΜΖ, ΝΚ. ἴση ἄρα ἐστὶν καὶ ἡ ὑπὸ ΜΣΖ γωνία τῆι ὑπὸ ΝΤΚ γωνίαι. ἐπεὶ δέ ἐστιν ὡς ἡ ΕΗ πρὸς ΗΖ, οὕτως ἡ ΘΛ πρὸς ΛΚ, καὶ ὡς ἄρα ἡ ΞΗ πρὸς ΗΖ, οὕτως ἐστὶν ἡ ΠΛ πρὸς ΛΚ, ὥστε καὶ ὡς ἡ ΗΞ πρὸς ΞΖ, τουτέστιν ἡ ΜΒ πρὸς ΜΣ, τουτέστιν ὡς ἡ ΖΜ πρὸς ΜΣ, οὕτως ἡ ΛΠ πρὸς ΚΠ, τουτέστιν ἡ ΔΝ πρὸς ΝΤ, <τουτέστιν ἡ ΚΝ πρὸς ΝΤ>, καὶ εἰσὶν αἱ μὲν ὑπὸ ΜΣΖ, ΝΤΚ ἴσαι, αἱ δὲ ὑπὸ ΜΖΣ, ΝΚΤ ὀξεῖαι, ἴση ἄρα ἐστὶν ἡ ὑπὸ ΣΜΖ γωνία τῆι ὑπὸ ΤΝΚ. ὁμοία ἄρα ἐστὶν ἡ ΒΖ περιφέρεια τῆι ΔΚ περιφερείαι.

(287) ἔστω δύο ὀρθογώνια τὰ ΑΒΓ, ΔΕΖ, ὀρθὰς ἔχοντα τὰς Γ, Ζ γωνίας, καὶ διήχθωσαν αἱ ΑΗ, ΔΘ ἐν ἴσαις γωνίαις ταῖς ὑπὸ ΒΑΗ, ΕΔΘ. ἔστω τε ὡς τὸ ὑπὸ τῶν ΒΓΗ πρὸς τὸ ἀπὸ τῆς ΑΓ, 974 οὕτως τὸ ὑπὸ τῶν ΕΖΘ πρὸς τὸ ἀπὸ ΖΔ. ὅτι ὅμοιόν ἐστιν τὸ ΑΒΓ τρίγωνον <τῶι ΔΕΖ τριγώνωι>. γεγράφθω γὰρ περὶ τὰ ΑΒΗ, ΔΕΘ τρίγωνα τμήματα κύκλων τὰ ΒΑΗ, ΕΔΘ. ὅμοια ἄρα ἐστίν. ἤτοι δὴ ἐφάπτονται αἱ ΑΓ, ΔΖ τῶν τμημάτων ἢ οὔ. ἐφαπτέσθωσαν πρότερον. ἴσον ἄρα ἐστὶν τὸ μὲν ὑπὸ ΒΓΗ τῶι ἀπὸ ΑΓ, τουτέστιν, ἐὰν πρὸς ὀρθὰς ἀγάγω τῆι ΑΗ τὴν ΑΚ, τῶι ὑπὸ τῶν ΗΓΚ, τὸ δὲ ὑπὸ τῶν ΕΖΘ τῶι ἀπὸ ΔΖ, τουτέστιν, ἐὰν ὀρθὴν ἀγάγω τὴν ΔΛ τῆι ΔΘ, τῶι ὑπὸ ΘΖΛ. ὥστε ἴση ἐστὶν ἡ μὲν ΒΓ τῆι ΓΚ, ἡ δὲ ΕΖ τῆι ΖΛ. καὶ ὀρθαὶ αἱ ΑΓ, ΔΖ. διπλῆ ἄρα ἐστὶν ἡ μὲν ὑπὸ ΒΑΚ γωνία τῆς ὑπὸ ΒΑΓ γωνίας, ἡ δὲ ὑπὸ ΕΔΛ γωνία τῆς ὑπὸ ΕΔΖ. καὶ εἰσὶν ἴσαι αἱ ὑπὸ ΒΑΚ, ΕΔΛ. ἴση γάρ ἐστιν ἡ μὲν ὑπὸ ΒΑΗ τῆι ὑπὸ ΕΔΘ, ὀρθὴ δὲ ἡ ὑπὸ ΗΑΚ ὀρθῆι

‖ 4 πεφέρεια A^1 ρι add supr A^2 ‖ 7 αἱ Hu ταῖς A | καὶ τὰ ἡμίση] κατὰ μίαν A ὥστε καὶ ἡμίσειαι Hu ‖ 12 ΘΛ Co ΕΛ A ‖ 13 ὡς del Ha | post ΜΒ add ἤτοι ΖΜ Ha ‖ 14 τουτέστιν ὡς ἡ ΖΜ πρὸς ΜΣ del Ha | ὡς del Hu ‖ 15 post ΔΝ add ἤτοι ΚΝ Ha | τουτέστιν ἡ ΚΝ πρὸς ΝΤ add Co ‖ 16 ΝΤΚ] ΝΤ και A^1 -αι secl A^2 ‖ 19 ὀρθογώνια] τρίγωνα Ge (S) ‖ 21 τε ὡς Ha τέως A ‖ 22 τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι Co τῶι ΑΒΓ τριγωνον A^1 -ώνωι A^2 ‖ 24 ΔΕΘ Ha ΕΔΘ A | τρίγωνα Ha τρίγωνον A | ΒΑΗ, ΕΔΘ Co ΒΑΗ, ΒΔΘ A ΒΗΑ ΔΘΕ Ha | ὅμοια ἄρα ἐστίν secl Hu ‖ 25 δὴ add Co ‖ 26 ἐφαπτέσθωσαν Ha (Co) ἐφαπτέσθω A ‖ 27 τῶι Co τὸ A ‖ 29 ΔΛ Co ΔΑ A | ΘΖΛ Co ΖΛ A ‖ 30 ΕΖ Co ΗΖ A | ὀρθαὶ Ha ὀρθὴ A πρὸς ὀρθὰς coni. Hu app | ΑΓ, ΔΖ] Γ, Ζ Ha ‖ 32 γωνία — εἰσὶν in ras. A | ΒΑΚ Co ΑΒΚ A right (angles) Γ, Z (are equal).[15] Thus triangle ABΓ is similar to triangle ΔEZ.[16] Q.E.D.

Now, however *(Prop. 215 b – c)*, let AΓ and ΔZ not be tangent, but let them cut (the segments) at points K and Λ. <Then as is the rectangle contained by KΓ, ΓA to the square of ΓA, that is> as KΓ is to ΓA, so is the rectangle contained by ΔZ, ZΛ to the square of ΔZ,[17] that is ΛZ to ZΔ.[18] And segments BAH and EΔΘ are similar and greater (than a semicircle).[19] Therefore arc AH is similar to arc ΔΘ (lemma 7.221).[20] Hence angle B is equal to angle E.[21] Therefore triangle ABΓ is similar to triangle ΔEZ.[22]

(288) *(Prop. 215 d)* The same thing in another way. Let there be two triangles that have angles Γ, Z right, and let AH and ΔΘ be drawn across at equal angles BAH and EΔΘ. And as the rectangle contained by BΓ, ΓH is to the square of AΓ, so let the rectangle contained by EZ, ZΘ be to the square of ΔZ. That triangle ABΓ is similar to triangle ΔEZ.

Let AK and ΔΛ be drawn at right angles to AH and ΔΘ.[1] Then the square of AΓ equals the rectangle contained by HΓ, ΓK, while the square of ΔZ (equals) the rectangle contained by ΘZ, ZΛ.[2] Thus as the rectangle contained by BΓ, ΓH is to the rectangle contained by HΓ, ΓK, that is as BΓ is to ΓK, so is the rectangle contained by EZ, ZΘ to the rectangle contained by ΘZ, ZΛ,[3] that is EZ to ZΛ.[4] Let ΓM and ZN be drawn parallel to AK and ΔΛ.[5] Hence as BM is to MA, so is EN to NΔ.[6] And (the angles) at points Γ, Z are right,[7] and the (angles) at points M, N equal;[9] for so are angles BAK and EΔΛ. By the foregoing (lemma) triangle ABΓ is similar to triangle ΔEZ.[10]

(289) *(Prop. 216)* Let there be two triangles that have the angles at points B and E right, and let BH and EΘ be drawn across at equal angles AHB and ΔΘE. And as the rectangle contained by AH, HΓ is to the square of HB, so let the rectangle contained by ΔΘ, ΘZ be to the square of ΘE. It is required to prove that triangle ABΓ is similar to triangle ΔEZ.

Let circles be circumscribed, and let their centers K, Λ be taken. Now it is obvious that they are on the same side of H, Θ (as each other). For if possible, let K be between points Γ, H, and Λ between Δ, Θ,[1] and let BH, EΘ be produced to points M, N. And from K let perpendicular KΞ be drawn upon MB.[2] Then it will fall between H and B,[3] and angle AHB is obtuse.[4] And it equals angle ΔΘE.[5] Hence angle ΔΘE too is obtuse.[6]

τῆι ὑπὸ ΘΔΛ. αἱ ἄρα ὑπὸ ΒΑΓ, ΕΔΖ ἴσαι εἰσίν. ἀλλὰ καὶ ὀρθαὶ αἱ Γ, Ζ. ὅμοιον ἄρα ἐστὶν τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι. ὅπερ: —

ἀλλὰ δὴ μὴ ἐφαπτέσθωσαν αἱ ΑΓ, ΔΖ, ἀλλὰ τεμνέτωσαν κατὰ τὰ Κ, Λ σημεῖα. ἔστιν <οὖν ὡς τὸ ὑπὸ τῶν ΜΓΑ πρὸς τὸ ἀπὸ ΑΓ, τουτέστιν> ὡς ἡ ΚΓ πρὸς ΓΑ, οὕτως τὸ ὑπὸ τῶν ΔΖΛ πρὸς τὸ
ἀπὸ ΔΖ, τουτέστιν ἡ ΛΖ πρὸς ΖΔ. καὶ ἔστιν ὅμοια μείζονα 976
τμήματα <τὰ> ΒΑΗ, ΕΔΘ. ὁμοία ἄρα ἐστὶν ἡ ΑΗ περιφέρεια τῆι
ΔΘ περιφερείαι. |ὥστε ἴση ἐστὶν ἡ Β γωνία τῆι Ε. ὅμοιον ἄρα |177
ἐστὶν τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι.

(288) ἄλλως τὸ αὐτό. ἔστω δύο τρίγωνα ὀρθὰς ἔχοντα τὰς Γ, Ζ γωνίας, καὶ διήχθωσαν αἱ ΑΗ, ΔΘ ἐν ἴσαις γωνίαις ταῖς ὑπὸ ΒΑΗ, ΕΔΘ. ἔστω τε ὡς τὸ ὑπὸ ΒΓΗ πρὸς τὸ ἀπὸ ΑΓ, οὕτως τὸ ὑπὸ ΕΖΘ πρὸς τὸ ἀπὸ ΔΖ. ὅτι ὅμοιον τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι. ἤχθωσαν ταῖς ΑΗ, ΔΘ ὀρθαὶ αἱ ΑΚ, ΔΛ. ἴσον ἄρα τὸ μὲν ἀπὸ ΑΓ τῶι ὑπὸ ΗΓΚ, τὸ δὲ ἀπὸ ΔΖ τῶι ὑπὸ ΘΖΛ. ἔστιν οὖν ὡς τὸ ὑπὸ ΒΓΗ πρὸς τὸ ὑπὸ ΗΓΚ, τουτέστιν ὡς ἡ ΒΓ πρὸς τὴν ΓΚ, οὕτως τὸ ὑπὸ ΕΖΘ πρὸς τὸ ὑπὸ ΘΖΛ, τουτέστιν ἡ ΕΖ πρὸς ΖΛ. ἤχθωσαν ταῖς ΑΚ, ΔΛ παράλληλοι αἱ ΓΜ, ΖΝ. καὶ ὡς ἄρα ἡ ΒΜ πρὸς ΜΑ, οὕτως ἡ ΕΝ πρὸς ΝΔ. καὶ εἰσὶν ὀρθαὶ μὲν αἱ πρὸς τοῖς Γ, Ζ σημείοις, ἴσαι δὲ αἱ πρὸς τοῖς Μ, Ν· καὶ γὰρ αἱ ὑπὸ ΒΑΚ, ΕΔΛ. διὰ δὴ τὸ προγεγραμμένον ὅμοιόν ἐστι τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι.

(289) ἔστω δύο τρίγωνα ὀρθὰς ἔχοντα τὰς πρὸς τοῖς Β, Ε σημείοις γωνίας, καὶ διήχθωσαν αἱ ΒΗ, ΕΘ ἐν ἴσαις γωνίαις ταῖς ὑπὸ ΑΗΒ, ΔΘΕ. ἔστω τε ὡς τὸ ὑπὸ τῶν ΑΗΓ πρὸς τὸ ἀπὸ ΗΒ, οὕτως τὸ ὑπὸ τῶν ΔΘΖ πρὸς τὸ ἀπὸ ΘΕ. δεικτέον ὅτι ὅμοιόν
ἐστιν τὸ ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι. περιγεγράφθωσαν 978
κύκλοι, καὶ εἰλήφθω αὐτῶν τὰ κέντρα τὰ Κ, Λ. φανερὸν δὴ ὅτι ἐπὶ τὰ αὐτὰ τῶν Η, Θ σημείων εἰσίν. εἰ γὰρ δυνατόν, ἔστω τὸ μὲν Κ μεταξὺ τῶν Γ, Η σημείων, τὸ δὲ Λ μεταξὺ τῶν Δ, Θ, καὶ ἐκβεβλήσθωσαν αἱ ΒΗ, ΕΘ ἐπὶ τὰ Μ, Ν σημεῖα. καὶ ἀπὸ τοῦ Κ ἐπὶ τὴν ΜΒ κάθετος ἤχθω ἡ ΚΞ. πεσεῖται ἄρα μεταξὺ τῶν Η, Β, ἀμβλεῖά τε γίνεται ἡ ὑπὸ ΑΗΒ γωνία. καὶ ἔστιν ἴση τῆι ὑπὸ

‖ 1 αἱ ἄρα Ha καὶ αἱ A | post ΕΔΖ add ἄρα Hu ‖ 5 Κ, Λ] Μ, Ν Ha | οὖν — τουτέστιν add Ha (Co) ‖ 6 ΚΓ] ΜΓ Ha | ΔΖΛ] ΔΖΝ Ha ‖ 7 ΛΖ] ΝΖ Ha | ὅμοια Co ὅμοιον A ‖ 8 τὰ add Ha ‖ 9 Ε Ha Θ A ‖ 13 τε ὡς Ha τέως A ‖ 18 ΕΖ Co ΘΖ A ‖ 21 καὶ γὰρ αἱ Co καὶ τῶν αἱ A γωνίαι ταῖς Ha ἐπεὶ καὶ αἱ item Co ‖ 26 τε ὡς Ha τέως A ‖ 28 περιγεγράφθωσαν κύκλοι Ha (Co) περιγεγράφθω κύκλος A ‖ 30 εἰσίν Ha εἶναι A ‖ 33 πεσεῖται] πιπτέτω Ha | τῶν (Η, Β) Ha τὴν A

Therefore angle ΔΘN is acute.[7] Hence the perpendicular drawn from Λ upon EN falls between Θ, N.[9] Let it fall, and let it be ΛO.[8] Then NO equals OE.[10] Thus NO is greater than ΘE.[11] Hence NΘ is much greater than ΘE.[12] And the rectangle contained by NΘ, ΘE, that is the rectangle contained by ΔΘ, ΘZ,[14] is greater than the square of EΘ.[13] And as the rectangle contained by ΔΘ, ΘZ is to the square of ΘE, so is the rectangle contained by AH, HΓ to the square of HB;[15] which is absurd. For it is also less, since MH is less than HB, and the rectangle contained by MH, HB than the square of HB. Thus if center K is between H and Γ, Λ will not be between Δ, Θ.

(290) *(Prop. 216)* So let (Λ) be between Θ, Z, and in the same way let perpendicular ΛO be drawn.[15]

Then since as the rectangle contained by AH, HΓ, that is the rectangle contained by MH, HB,[17] is to the square of HB, that is as MH is to HB, so is the rectangle contained by ΔΘ, ΘZ, that is the rectangle contained by NΘ, ΘE,[18] to the square of ΘE,[16] <that is NΘ to ΘE>.[19] And BM and NE have been bisected by Ξ, O.[20] Therefore as BΞ is to ΞH, so is EO to OΘ.[21] But also as HΞ is to ΞK, so is ΘO to OΛ;[24] for (angles) Ξ, O are right,[22] and the angles at points H, Θ are equal.[23] *Ex aequali*, therefore, as BΞ is to ΞK, so is EO to OΛ.[25] And they are about equal angles.[26] Therefore angle BKΞ equals angle EΛO.[27] But also angle ΞKH equals angle OΛΘ.[28] Hence all angle BKH equals all angle EΛΘ.[29] And the halves (are equal). Hence angle AΓB equals angle ΔZE.[30] And angles B, E are right.[31] Thus triangle ABΓ is similar to triangle ΔEZ.[32]

(291) *(Prop. 216)* The converse of this too is obvious, namely if triangle ABΓ is similar to triangle ΔEZ, and (triangle) HBΓ to (triangle) ΘEZ, that as the rectangle contained by AH, HΓ is to the square of HB, so is the rectangle contained by ΔΘ, ΘZ to the square of ΘE, because of the similarity of the triangles.

ΔΘΕ. ἀμβλεῖα ἄρα |ἐστὶν καὶ ἡ ὑπὸ ΔΘΕ γωνία. ὀξεῖα ἄρα |177v ἐστὶν ἡ ὑπὸ ΔΘΝ. ὥστε ἡ ἀπὸ τοῦ Λ ἐπὶ τὴν ΕΝ κάθετος ἀγομένη πίπτει μεταξὺ τῶν Θ, Ν. πιπτέτω, καὶ ἔστω ἡ ΛΟ. ἴση ἄρα ἐστὶν ἡ ΝΟ τῆι ΟΕ. ὥστε μείζων ἐστὶν ἡ ΝΟ τῆς ΘΕ. πολλῶι ἄρα ἡ ΝΘ τῆς ΘΕ ἐστὶν μείζων. καὶ τὸ ὑπὸ ΝΘΕ, τουτέστιν τὸ ὑπὸ ΔΘΖ, μεῖζόν ἐστιν τοῦ ἀπὸ ΕΘ τετραγώνου. καὶ ἔστιν ὡς τὸ ὑπὸ ΔΘΖ πρὸς τὸ ἀπὸ ΘΕ, οὕτως τὸ ὑπὸ ΑΗΓ πρὸς τὸ ἀπὸ ΗΒ· ὅπερ ἐστὶν ἄτοπον· ἔστιν γὰρ καὶ ἔλασσον, ἐπειδήπερ ἐλάσσων ἐστὶν ἡ ΜΗ τῆς ΗΒ, καὶ τὸ ὑπὸ ΜΗΒ τοῦ ἀπὸ ΗΒ. οὐκ ἄρα τοῦ Κ κέντρου ὄντος μεταξὺ τῶν Η, Γ, τὸ Λ ἔσται μεταξὺ τῶν Δ, Θ.

(290) ἔστω οὖν μεταξὺ τῶν Θ, Ζ, καὶ κατὰ τὰ αὐτὰ ἤχθω ἡ ΛΟ κάθετος. ἐπεὶ οὖν ἐστιν ὡς τὸ ὑπὸ ΑΗΓ, τουτέστιν τὸ ὑπὸ 980 ΜΗΒ, πρὸς τὸ ἀπὸ ΗΒ, τουτέστιν ὡς ἡ ΜΗ πρὸς ΗΒ, οὕτως τὸ ὑπὸ ΔΘΖ, τουτέστιν τὸ ὑπὸ ΝΘΕ, πρὸς τὸ ἀπὸ ΘΕ, <τουτέστιν ἡ ΝΘ πρὸς ΘΕ.> καὶ τέτμηνται αἱ ΒΜ, ΝΕ δίχα τοῖς Ξ, Ο. ἔστιν ἄρα ὡς ἡ ΒΞ πρὸς ΞΗ, οὕτως ἡ ΕΟ πρὸς ΟΘ. ἀλλὰ καὶ ὡς ἡ ΗΞ πρὸς ΞΚ, οὕτως ἡ ΘΟ πρὸς τὴν ΟΛ. ὀρθαὶ μὲν γὰρ αἱ Ξ, Ο, ἴσαι δὲ αἱ πρὸς τοῖς Η, Θ σημείοις γωνίαι. δι' ἴσου ἄρα ἐστὶν ὡς ἡ ΒΞ πρὸς ΞΚ, οὕτως ἡ ΕΟ πρὸς ΟΛ. καὶ περὶ ἴσας γωνίας. ἴση ἄρα ἐστὶν ἡ ὑπὸ τῶν ΒΚΞ γωνία τῆι ὑπὸ τῶν ΕΛΟ γωνίαι. ἔστιν δὲ καὶ ἡ ὑπὸ ΞΚΗ γωνία τῆι ὑπὸ ΟΛΘ ἴση. ὅλη ἄρα ἡ ὑπὸ ΒΚΗ ὅληι τῆι ὑπὸ ΕΛΘ ἐστὶν ἴση. καὶ τὰ ἡμίση. καὶ ἡ ὑπὸ τῶν ΑΓΒ ἄρα γωνία ἴση ἐστὶν τῆι ὑπὸ τῶν ΔΖΕ. καὶ εἰσὶν ὀρθαὶ αἱ Β, Ε γωνίαι. ὅμοιον ἄρα ἐστὶν τὸ ΑΒΓ <τρίγωνον> τῶι ΔΕΖ τριγώνωι.

(291) φανερὸν δὲ καὶ <τὸ> τούτωι ἀναστρόφιον, τὸ ἐὰν ἦι ὅμοιον τὸ μὲν ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι, τὸ δὲ ΗΒΓ τῶι ΘΕΖ, ὅτι γίνεται ὡς τὸ ὑπὸ ΑΗΓ πρὸς τὸ ἀπὸ ΗΒ, οὕτως τὸ ὑπὸ ΔΘΖ πρὸς τὸ ἀπὸ ΘΕ, διὰ τὴν ὁμοιότητα τῶν τριγώνων.

‖ 3 ΛΟ Co ΛΘ A ‖ 4 ΝΟ (τῆι ΟΕ) Co ΝΘ A | ΟΕ ex *Ε A | ΝΟ (τῆς ΘΕ) Co ΝΘ A ‖ 5 πολλῶι – ΘΕ bis A corr Co ‖ 6 τὸ ὑπὸ (ΔΘΖ) Hu τοῦ A | ΔΘΖ Co ΔΕΖ A ‖ 8 καὶ del Ha ‖ 11 Θ Co Ε A ‖ 15 τουτέστιν – πρὸς ΘΕ add Co ‖ 16 τέτμηνται] τέμνονται Ha ‖ 23 ΕΛΘ Co ΕΛΟ A | ἡμίση Ge (S) ἡμίσεια A ‖ 25 τρίγωνον add Ge (recc?) ‖ 27 τὸ add Hu | ἀναστρόφιον Hu ἀναστρέφον A αντίστροφον Ha | τὸ del Hu ‖ 30 διὰ – τριγώνων secl Hu

(292) *(Prop. 217 a – b)* Let there be two triangles ABΓ, ΔEZ, that have angles A, Δ equal, but not right, and let perpendiculars AH, ΔΘ be drawn, and as the rectangle contained by BH, HΓ is to the square of AH, so let the rectangle contained by EΘ, ΘZ be to the square of ΔΘ, and let BH, EΘ be greater parts of straight lines BΓ, EZ. I say that triangle ABH is similar to (triangle) ΔEΘ, and the rest (triangle HAΓ) to the rest (triangle ΘΔZ).

Let circles be circumscribed, and let AH, ΔΘ be produced to points K, Λ, and let the centers M, N of the circles be taken, and from them let perpendiculars MΞ, MO, NΠ, NP be drawn upon AK, BΓ, ΔΛ, EZ.[1] Now by the same argument as in the foregoing (lemmas), as KH is to HA, so is ΛΘ to ΘΔ.[2] Hence also as AΞ is to ΞH, so is ΔΠ to ΠΘ.[3] Let AM, ΔN be joined. But as AΞ is to ΞH, so is AM to MΣ,[4] while as ΔΠ is to ΠΘ, so is ΔN to NT.[5] And so as AM is to MΣ, so is ΔN to NT.[6] Let BM, EN be joined. Then since segment BAΓ is similar to segment EΔZ,[7] therefore the remaining segment BKΓ is similar to the remaining segment EΛZ.[8] Hence the angles in them are equal, and also their halves are equal. Thus angles BMO, ENP are equal,[9] in the first pair of cases (Prop. 217 a). In the second (Prop. 217 b) it is manifest from what is already there that angle BMO equals angle ENP; for the angles in segments BAΓ, EΔZ too (are equal). Thus as BM is to MO, that is as AM is to MO, so is EN to NP,[10] that is ΔN to NP.[11] But also as AM is to MΣ, so is ΔN to NT.[12] *Ex aequali*, therefore, as MO is to MΣ, so is PN to NT.[13] And angles O, P are right,[14] and each of (angles) Σ, T acute.[15] Therefore angle OMΣ equals angle PNT.[16] But angle BMO too equals angle ENP.[17] Thus angle BMΣ too equals angle ENT,[18] and so also angle Γ equals angle Z.[19] Thus all (the triangles) are similar to all.[20]

(293) *(Prop. 217 c)* It is also possible, having the proof of one case, either that of the obtuse or of the acute, already written, to supply the remaining one, as follows. For let it be supposed that (the case) of the angles being equal and obtuse has been proved first, in the way written above. And let it be required to prove, having two equal acute angles BAΓ,

(292) ἔστω δύο τρίγωνα τὰ ΑΒΓ, ΔΕΖ, ἴσας ἔχοντα τὰς Α, Δ γωνίας, μὴ ὀρθὰς δέ, καὶ κάθετοι ἤχθωσαν αἱ ΑΗ, ΔΘ, ἔστω τε <ὡς> τὸ ὑπὸ τῶν ΒΗΓ πρὸς τὸ ἀπὸ τῆς ΑΗ, οὕτως τὸ ὑπὸ τῶν ΕΘΖ πρὸς τὸ ἀπὸ τῆς ΔΘ, καὶ ἔστω τῶν ΒΓ, ΕΖ εὐθειῶν μείζονα τμήματα τὰ ΒΗ, ΕΘ. λέγω ὅτι ὅμοιόν ἐστιν τὸ μὲν ΑΒΗ τρίγωνον τῶι ΔΕΘ, τὸ δὲ λοιπὸν τῶι λοιπῶι. περιγεγράφθωσαν 982 κύκλοι, καὶ ἐκβεβλήσθωσαν αἱ ΑΗ, ΔΘ ἐπὶ τὰ Κ, Λ σημεῖα, καὶ εἰλήφθω τὰ κέντρα τῶν κύκλων τὰ Μ, Ν, καὶ ἀπὸ αὐτῶν |ἐπὶ τὰς |178 ΑΚ, ΒΓ, ΔΛ, ΕΖ κάθετοι αἱ ΜΞ, ΜΟ, ΝΠ, ΝΡ. ἔστιν δὴ κατὰ τὰ αὐτὰ τοῖς προγεγραμμένοις ὡς ἡ ΚΗ πρὸς ΗΑ, οὕτως ἡ ΛΘ πρὸς ΘΔ. ὥστε καὶ ὡς ἡ ΑΞ πρὸς ΞΗ, οὕτως ἡ ΔΠ πρὸς ΠΘ. ἐπεζεύχθωσαν αἱ ΑΜ, ΔΝ. ἀλλ' ὡς μὲν ἡ ΑΞ πρὸς ΞΗ, οὕτως ἡ ΑΜ πρὸς ΜΣ. ὡς δὲ ἡ ΔΠ πρὸς ΠΘ, οὕτως ἡ ΔΝ πρὸς ΝΤ. καὶ ὡς ἄρα ἡ ΑΜ πρὸς ΜΣ, οὕτως ἡ ΔΝ πρὸς ΝΤ. ἐπεζεύχθωσαν δὴ αἱ ΒΜ, ΕΝ. ἐπεὶ οὖν ὅμοιόν ἐστι τὸ ΒΑΓ τμῆμα τῶι ΕΔΖ τμήματι, καὶ λοιπὸν ἄρα τὸ ΒΚΓ τμῆμα λοιπῶι τῶι ΕΛΖ τμήματι ὅμοιόν ἐστιν. αἱ ἄρα ἐν αὐτοῖς γωνίαι ἴσαι εἰσίν, καὶ εἰσὶν αὐτῶν καὶ τὰ ἡμίση ἴσα. αἱ ὑπὸ τῶν ΒΜΟ, ΕΝΡ ἄρα γωνίαι ἴσαι εἰσίν, ἐπὶ τῆς πρώτης δυάδος τῶν πτώσεων· ἐπὶ δὲ τῆς δευτέρας ἐκ παρακειμένου δῆλον ἐστὶν ὡς ἴση ἡ ὑπὸ τῶν ΒΜΟ γωνία τῆι ὑπὸ τῶν ΕΝΡ· καὶ γὰρ αἱ ἐν τοῖς ΒΑΓ, ΕΔΖ τμήμασιν γωνίαι. γίνεται οὖν ὡς ἡ ΒΜ πρὸς ΜΟ, τουτέστιν ὡς ἡ ΑΜ πρὸς ΜΟ, οὕτως ἡ ΕΝ πρὸς ΝΡ, τουτέστιν ἡ ΔΝ πρὸς ΝΡ. ἔστιν δὲ καὶ ὡς ἡ ΑΜ πρὸς ΜΣ, οὕτως ἡ ΔΝ πρὸς ΝΤ. δι' ἴσου ἄρα ἐστὶν ὡς ἡ ΜΟ πρὸς ΜΣ, οὕτως ἡ ΡΝ πρὸς ΝΤ. καὶ εἰσὶν ὀρθαὶ μὲν αἱ Ο, Ρ γωνίαι, ὀξεῖα δὲ ἑκατέρα τῶν Σ, Τ. ἴση ἄρα ἐστὶν ἡ ὑπὸ τῶν ΟΜΣ γωνία τῆι ὑπὸ τῶν ΡΝΤ γωνίαι. ἀλλὰ καὶ ἡ ὑπὸ τῶν ΒΜΟ τῆι ὑπὸ ΕΝΡ ἐστὶν ἴση. καὶ ἡ ὑπὸ τῶν ΒΜΣ ἄρα τῆι ὑπὸ τῶν ΕΝΤ ἐστὶν ἴση. ὥστε καὶ ἡ Γ γωνία τῆι Ζ ἐστὶν ἴση. ὅμοια ἄρα ἐστὶν πάντα πᾶσιν.

(293) δυνατὸν δὲ καὶ τῆς μιᾶς πτώσεως, ἢ τῶν ἀμβλειῶν ἢ 984 ὀξειῶν, προγεγραμμένης τῆς δείξεως, τὸ λοιπὸν ἀποδοῦναι οὕτως. ὑποκείσθω γὰρ ἀποδεδεῖχθαι, οὐσῶν ἴσων ἀμβλειῶν τῶν γωνιῶν τὸ πρότερον, κατὰ τὸν προγεγραμμένον τρόπον. καὶ ἔστω, δυεῖν ὀξειῶν οὐσῶν ἴσων τῶν ὑπὸ ΒΑΓ, ΕΔΖ, δεῖξαι

‖ 2 ὀρθὰς δέ Co ὀρθή τε A | κάθετοι Ha (Co) κάθετος A | ἔστω τε Co ὥστε A ὡς addidi ‖ 7 ἐπὶ ex ἐπεὶ A ‖ 9 post ΕΖ add ἤχθωσαν Ha | ἔστιν Ha εἰσὶν A | δὴ] δὲ Ha ‖ 18 καὶ τὰ ἡμίση ἴσα] κατὰ μίαν ἴσαι A καὶ ἡμίσειαι ἴσαι Hu ‖ 19 ἐπὶ — τμήμασιν γωνίαι secl Hu ‖ 20 δῆλον] δηλονότι A | ἐστὶν ὡς ἴση] ἴση ἐστὶν Ha ἐστὶν ἴση Hu | ΒΜΟ Co ΒΜΘ A ‖ 21 τοῖς] ἴσοις Ha ‖ 22 (ΒΜ πρὸς) ΜΟ Co ΜΘ A | (ΑΜ πρὸς) ΜΟ Co ΜΘ A ‖ 24 ΜΟ Co ΜΘ A ‖ 27 ΒΜΟ... ΕΝΡ Co ΒΟΜ... ΕΡΝ A ‖ 31 δυνατὸν Hu δύναται A | πτώσεως Hu γωνίας A | ἢ τῶν — ὀξειῶν secl Hu | τῶν (ὀξειῶν) add Co ‖ 32 ὀξειῶν Co ὀξεῖα A ‖ 33 ἀποδεδεῖχθαι Hu ἀποδεδειχέναι A ‖ 35 δυεῖν ex δυιν A | ὑπὸ del Ha | ΒΑΓ Co ΑΒΓ A

ΕΔΖ, that the triangles are similar.

And again let the circles be circumscribed, and with ΑΗ, ΔΘ produced to Κ, Λ, let ΒΚ, ΚΓ, ΕΛ, ΛΖ be joined. Then obtuse angles ΒΚΓ, ΕΛΖ too are equal.[1] And since as is the rectangle contained by ΒΗ, ΗΓ, that is the rectangle contained by ΑΗ, ΗΚ,[3] to the square of ΑΗ, that is ΚΗ to ΗΑ, so is the rectangle contained by ΕΘ, ΘΖ, that is the rectangle contained by ΔΘ, ΘΛ,[4] to the square of ΔΘ,[2] that is ΛΘ to ΘΔ.[5] And so as is the square of ΑΗ to the square of ΗΚ, so is the square of ΔΘ to the square of ΘΛ.[6] But also as is the rectangle contained by ΒΗ, ΗΓ to the square of ΑΗ, so is the rectangle contained by ΕΘ, ΘΖ to the square of ΔΘ.[7] *Ex aequali* therefore, as is the rectangle contained by ΒΗ, ΗΓ to the square of ΗΚ, so is the rectangle contained by ΕΘ, ΘΖ to the square of ΘΛ.[8] And angles ΒΚΓ, ΕΛΖ are equal and obtuse; and ΚΗ, ΛΘ are perpendiculars.[9] Because of the foregoing (lemma), triangle ΒΚΗ is similar to triangle ΕΛΘ, and (triangle) ΓΚΗ to (triangle) ΖΛΘ.[10] Thus also triangle ΑΒΗ is similar to triangle ΔΕΘ, and (triangle) ΑΗΓ to (triangle) ΔΘΖ (see commentary).[11] Hence also all triangle ΑΒΓ is similar to all triangle ΔΕΖ.[12]

(294) *(Prop. 218)* With ΑΒ, ΑΓ given in position, to draw ΔΕ parallel to a (line given) in position, making ΔΕ given (in magnitude).

Let it have been accomplished, and let ΑΖ be drawn through Α and parallel to ΔΕ. Then it is parallel to a (line given) in position. And Α is given. Therefore ΑΖ is (given) in position. Let ΕΖ be drawn through Ε parallel to ΑΒ. Then ΑΖ is equal to ΔΕ. But ΔΕ is given. Therefore ΑΖ too is given. But (it is given) also in position. And Α is given. Therefore Ζ too is given. Now ΖΕ has been drawn through a given point Ζ parallel to a (line given) in position. Therefore ΔΕ is (given) in position.

The synthesis of the problem will be made as follows. Let the two straight lines given in position be ΑΒ, ΑΓ, and let the (line) given in magnitude be Η, and let (the line) parallel to which (lines) are drawn be ΑΖ, and let ΑΖ be made equal to Η.[1] And let ΖΕ be drawn through Ζ <parallel> to ΑΒ,[2] and ΕΔ through Ε parallel to ΑΖ.[3] I say that ΔΕ solves the problem.

For since ΔΕ equals ΑΖ,[4] but ΑΖ equals Η, that is the given (line), therefore ΔΕ too equals the given, Η.[5] Hence ΔΕ solves the problem. And obviously it alone (solves the problem); for the (line) nearer Α is always less than the farther (line).

ὅτι ὅμοια τὰ τρίγωνα. καὶ πάλιν περιγεγράφθωσαν οἱ κύκλοι, καὶ ἐκβεβλημένων τῶν ΑΗ, ΔΘ ἐπὶ τὰ Κ, Λ, ἐπεζεύχθωσαν αἱ ΒΚ, ΚΓ, ΕΛ, ΛΖ. ἴσαι ἄρα εἰσὶν καὶ αἱ ὑπὸ ΒΚΓ, ΕΛΖ γωνίαι ἀμβλεῖαι. καὶ ἐπεί ἐστιν ὡς τὸ ὑπὸ ΒΗΓ, του|τέστιν τὸ ὑπὸ |178v ΑΗΚ, πρὸς τὸ ἀπὸ ΑΗ, τουτέστιν ἡ ΚΗ πρὸς ΗΑ, οὕτως τὸ ὑπὸ ΕΘΖ, τουτέστιν τὸ ὑπὸ ΔΘΛ, πρὸς τὸ ἀπὸ ΔΘ, τουτέστιν ἡ ΛΘ πρὸς ΘΔ. καὶ ὡς ἄρα τὸ ἀπὸ ΑΗ πρὸς τὸ ἀπὸ ΗΚ, οὕτως τὸ ἀπὸ ΔΘ πρὸς τὸ ἀπὸ ΘΛ. ἔστιν δὲ καὶ ὡς τὸ ὑπὸ ΒΗΓ πρὸς τὸ ἀπὸ ΑΗ, οὕτως τὸ ὑπὸ ΕΘΖ πρὸς τὸ ἀπὸ ΔΘ. δι' ἴσου ἄρα ἐστὶν ὡς τὸ ὑπὸ ΒΗΓ πρὸς τὸ ἀπὸ ΗΚ, οὕτως τὸ ὑπὸ ΕΘΖ πρὸς τὸ ἀπὸ ΘΛ. καὶ εἰσὶν ἴσαι ἀμβλεῖαι αἱ ὑπὸ τῶν ΒΚΓ, ΕΛΖ γωνίαι. καὶ 986 κάθετοι αἱ ΚΗ, ΛΘ. διὰ δὴ τὸ προγεγραμμένον, ὅμοιόν ἐστι τὸ <μὲν> ΒΚΗ τρίγωνον τῶι ΕΛΘ τριγώνωι, τὸ δὲ ΓΚΗ τῶι ΖΛΘ. ὥστε καὶ τὸ μὲν ΑΒΗ τρίγωνον τῶι ΔΕΘ τριγώνωι ἐστὶν ὅμοιον, τὸ δὲ ΑΗΓ τῶι ΔΘΖ. ὥστε καὶ ὅλον τὸ ΑΒΓ ὅλωι τῶι ΔΕΖ ἐστὶν ὅμοιον.

(294) θέσει δεδομένων τῶν ΑΒ, ΑΓ, ἀγαγεῖν παρὰ θέσει τὴν ΔΕ καὶ ποιεῖν δοθεῖσαν τὴν ΔΕ. γεγονέτω, καὶ διὰ τοῦ Α τῆι ΔΕ παράλληλος ἤχθω ἡ ΑΖ. παρὰ θέσει ἄρα ἐστίν. καὶ ἔστιν δοθὲν τὸ Α. θέσει ἄρα ἐστὶν ἡ ΑΖ. διὰ δὲ τοῦ Ε τῆι ΑΒ παράλληλος ἤχθω ἡ ΕΖ. ἴση ἄρα ἐστὶν ἡ ΑΖ τῆι ΔΕ. δοθεῖσα δέ ἐστιν ἡ ΔΕ. δοθεῖσα ἄρα ἐστὶν καὶ ἡ ΑΖ. ἀλλὰ καὶ θέσει. καὶ δοθέν ἐστιν τὸ Α. δοθὲν ἄρα ἐστὶν καὶ τὸ Ζ. διὰ δὴ δεδομένου τοῦ Ζ παρὰ θέσει τῆι ΑΒ ἦκται ἡ ΖΕ. θέσει ἄρα ἐστὶν ἡ ΖΕ. θέσει δὲ καὶ ἡ ΑΓ. δοθὲν ἄρα ἐστὶν τὸ Ε. καὶ διὰ αὐτοῦ παρὰ θέσει ἦκται ἡ ΔΕ. θέσει ἄρα ἐστὶν ἡ ΔΕ.

συντεθήσεται δὴ τὸ πρόβλημα οὕτως. ἔστωσαν αἱ μὲν τῆι θέσει δεδομέναι δύο εὐθεῖαι αἱ ΑΒ, ΑΓ, ἡ δὲ δοθεῖσα τῶι μεγέθει ἔστω ἡ Η, παρ' ἣν δὲ ἄγονται ἔστω ἡ ΑΖ, καὶ τῆι Η ἴση κείσθω ἡ ΑΖ. καὶ διὰ μὲν τοῦ Ζ τῆι ΑΒ <παράλληλος> ἤχθω ἡ ΖΕ, διὰ δὲ τοῦ Ε τῆι ΑΖ παράλληλος ἤχθω ἡ ΕΔ. λέγω ὅτι ἡ ΔΕ ποιεῖ τὸ πρόβλημα. ἐπεὶ γὰρ ἴση ἐστὶν ἡ ΔΕ τῆι ΑΖ, ἀλλὰ ἡ ΑΖ τῆι Η ἐστὶν ἴση, τουτέστιν τῆι δοθείσηι, καὶ ἡ ΔΕ ἄρα ἴση ἐστὶν τῆι Η, τῆι δοθείσηι. ἡ ΔΕ ἄρα ποιεῖ τὸ πρόβλημα. καὶ φανερὸν ὅτι μόνη. αἰεὶ γὰρ ἡ ἔγγιον τοῦ Α τῆς ἀπώτερόν ἐστιν ἐλάσσων.

‖ 3 ΕΛΖ Co ΕΔΖ A ‖ 5 ΗΑ Ha ΚΑ A ‖ 8 ΘΛ Co ΘΑ A ‖ 12 κάθετοι Ha (Co) κάθετος A ‖ 13 μὲν add Hu ‖ 15 ΔΘΖ Ha ΔΖΘ A ‖ 21 ΑΖ Co ΑΗ A | ante δοθεῖσα add καὶ Ha ‖ 22 δέ Co ἄρα A del Ha | ΑΖ ex A* A ‖ 25 ΑΓ Co ΗΑΓ A ‖ 29 ἄγονται] ἄγεται Hu ἄγεσθαι δεῖ Ha ‖ 30 παράλληλος add Ha (Co) ‖ 31 ΔΕ Co ΑΕ A ‖ 33 Η Co Ν A ‖ 35 μόνη Ha μόνηι A | ἔγγιον Ha ἔγγειον A

(295) *(Prop. 219)* Let there be two planes BΔ, BZ standing on the same straight line BΓ, at right angles to the same plane, namely the plane of reference. I say that straight lines AB, BE, BΓ are in one plane.

For let HB be drawn from B and at right angles to BΓ in the plane of reference. Then HB will be at right angles also to plane EΓ. Hence it is also at right angles to BE. By the same argument (it is at right angles) to AB as well. But it is also (at right angles) to BΓ. Now straight line BH has been set up at right angles to three straight lines AB, BE, BΓ, from the point of contact B. Hence by the Element, straight lines AB, BE, BΓ are in one plane (XI 5).

(296) *(Prop. 220 a)* Let there be two triangles ABΓ, ΔEZ, that have angles A, Δ right, and let AH, ΔΘ be drawn across at equal angles AHB, ΔΘE. And as BH is to HΓ, so let EΘ be to ΘZ. That triangle ABΓ is similar to triangle ΔEZ, and (triangle) AHΓ to (triangle) ΔΘZ, and furthermore triangle ABH to triangle ΔEΘ.

Let AH be produced, and let ΓH be made to HK as ΔΘ is to ΘE,[1] and let BK, KΓ be joined. Then angle ΔEΘ equals angle ΓKH.[2] But since as BH is to HΓ, so is EΘ to ΘZ,[3] while as ΓH is to HK, so is ΔΘ to ΘE, *ex aequali* therefore in disturbed proportion, so BH is to HK, so is ΔΘ to ΘZ.[4] And (they are) about equal angles.[5] Therefore angle BKH equals angle Z.[6] But it was proved that angle ΓKH equals (angle) E;[7] and (angles) E, Z equal a right (angle).[8] Therefore angle BKΓ is right.[9] But by hypothesis also angle BAΓ is right.[10] Therefore points A, B, Γ, K are on a circle.[11] Hence angle AKΓ, that is angle ΔEΘ,[13] equals angle ABΓ.[12] But also angle AHB by hypothesis equals angle ΔΘE.[14] Therefore triangle ABH is similar to triangle ΔEΘ.[15] By the same argument also triangle AHΓ is similar to (triangle) ΔΘZ.

(295) ἔστω δύο ἐπίπεδα τὰ ΒΔ, ΒΖ ἐπὶ τῆς αὐτῆς εὐθείας 988
τῆς ΒΓ ἐφεστῶτα, τῶι αὐτῶι ἐπιπέδωι τῶι ὑποκειμένωι ὀρθά.
λέγω |ὅτι ἐν ἑνὶ ἐπιπέδωι εἰσὶν αἱ ΑΒ, ΒΕ, ΒΓ εὐθεῖαι. ἤχθω |179
γὰρ ἀπὸ τοῦ Β τῆι ΒΓ ἐν τῶι ὑποκειμένωι ἐπιπέδωι ὀρθὴ ἡ ΗΒ.
καὶ τῶι ΕΓ ἄρα ἐπιπέδωι ἔσται ὀρθὴ ἡ ΗΒ· ὥστε καὶ τῆι ΒΕ
ἐστὶν ὀρθή. κατὰ τὰ αὐτὰ καὶ τῆι ΑΒ. ἔστι δὲ καὶ τῆι ΒΓ.
εὐθεῖα δὴ ἡ ΒΗ τρισὶν εὐθείαις ταῖς ΑΒ, ΒΕ, ΒΓ ὀρθὴ ἐπὶ τῆς
ἁφῆς τῆς Β ἐφέστηκεν. διὰ ἄρα τὸ στοιχεῖον ἐν ἑνὶ εἰσὶν
ἐπιπέδωι αἱ ΑΒ, ΒΕ, ΒΓ εὐθεῖαι.

(296) ἔστω δύο τρίγωνα τὰ ΑΒΓ, ΔΕΖ, ὀρθὰς ἔχοντα τὰς Α, Δ
γωνίας, καὶ διήχθωσαν αἱ ΑΗ, ΔΘ ἐν ἴσαις γωνίαις ταῖς ὑπὸ
ΑΗΒ, ΔΘΕ. ἔστω δὲ ὡς ἡ ΒΗ πρὸς τὴν ΗΓ, οὕτως ἡ ΕΘ πρὸς τὴν
ΘΖ. ὅτι ὅμοιόν ἐστιν τὸ μὲν ΑΒΓ τρίγωνον τῶι ΔΕΖ τριγώνωι,
τὸ δὲ ΑΗΓ τῶι ΔΘΖ, καὶ ἔτι τὸ ΑΒΗ τρίγωνον τῶι ΔΕΘ τριγώνωι.
ἐκβεβλήσθω ἡ ΑΗ, καὶ πεποιήσθω ὡς ἡ ΔΘ πρὸς ΘΕ, οὕτως ἡ ΓΗ
πρὸς ΗΚ, καὶ ἐπεζεύχθωσαν αἱ ΒΚ, ΚΓ. ἴση ἄρα ἐστὶν ἡ ὑπὸ ΔΕΘ
γωνία τῆι ὑπὸ ΓΚΗ γωνίαι. ἐπεὶ δέ ἐστιν ὡς μὲν ἡ ΒΗ πρὸς
ΗΓ, οὕτως ἡ ΕΘ πρὸς ΘΖ, ὡς δὲ ἡ ΓΗ πρὸς ΗΚ, οὕτως ἡ ΔΘ πρὸς
ΘΕ, δι' ἴσου ἄρα ἐστὶν ἐν τεταραγμένηι ἀναλογίαι ὡς ἡ ΒΗ
πρὸς ΗΚ, οὕτως ἡ ΔΘ πρὸς ΘΖ. καὶ περὶ ἴσας γωνίας. ἴση ἄρα
ἐστὶν ἡ ὑπὸ τῶν ΒΚΗ γωνία τῆι Ζ γωνίαι. ἐδείχθη δὲ καὶ ἡ
ὑπὸ ΓΚΗ γωνία ἴση τῆι Ε. καὶ εἰσὶν αἱ Ε, Ζ ὀρθῆι ἴσαι. ἡ ἄρα 990
ὑπὸ ΒΚΓ γωνία ἐστὶν ὀρθή. ἀλλὰ καθ' ὑπόθεσιν καὶ ἡ ὑπὸ ΒΑΓ
γωνία ὀρθή. ἐν κύκλωι ἄρα ἐστὶν τὰ Α, Β, Γ, Κ σημεῖα. ἴση
ἄρα ἐστὶν καὶ ἡ ὑπὸ ΑΚΓ, τουτέστιν ἡ ὑπὸ ΔΕΘ, τῆι ὑπὸ ΑΒΓ.
ἀλλὰ καὶ ἡ ὑπὸ ΑΗΒ γωνία καθ' ὑπόθεσιν ἴση ἐστὶν τῆι ὑπὸ
ΔΘΕ γωνίαι. ὅμοιον ἄρα ἐστὶν τὸ ΑΒΗ τρίγωνον τῶι ΔΕΘ
τριγώνωι. κατὰ τὰ αὐτὰ καὶ τὸ ΑΗΓ τρίγωνον τῶι ΔΘΖ ἐστὶν
ὅμοιον.

‖ 1 ΒΔ, ΒΖ] ΑΒΓ, ΕΒΖ Ha ΑΒΓ, ΒΖ coni. Hu app ‖ 2 ἐφεστῶτα τῶι Ha ἐφεστάτω A | ὀρθά Ha ὀρθῶι A ‖ 4 ἐν] καὶ A ‖ 5 ΕΓ] ΕΒΖ Ha ΒΖ coni. Hu app ‖ 7 εὐθεῖα] εὐθεῖαι A | δὴ] ΔΗ A del Hu | post ΒΗ add ὀρθή. ἡ ΒΗ ἄρα Ha ‖ 8 τὸ στοιχεῖον] τὸ δέκατον πρῶτον στοιχεῖον Ha τὰ στοιχεῖα Ge τὸ ιά στοιχείων Hu ‖ 13 ΑΒΓ... ΔΕΖ] ΑΒΗ... ΔΕΘ Ha ‖ 14 καὶ ἔτι — τριγώνωι] καὶ ὅλον ὅλωι Ha del Hu ‖ 19 (ἐν) τῆι add Ha ‖ 22 ὀρθῆι Co ὀρθαὶ A ‖ 25 ΔΕΘ Co ΔΖΘ A ΔΕΖ Ha ‖ 27 ΔΘΕ Co ΔΕΘ A

(297) *(Prop. 220 b)* In another, better way.

Let ΒΓ, ΕΖ be bisected by points Κ, Λ,[1] and let ΑΚ, ΔΛ be joined. Then since as ΒΗ is to ΗΓ, so is ΕΘ to ΘΖ,[2] *componendo*[3] and (taking) the halves of the leading (members),[4] and *convertendo*, as ΓΚ, that is as ΑΚ, is to ΚΗ, so is ΛΖ, that is ΔΛ,[6] to ΛΘ.[5] And the angles at points Η, Θ are equal,[7] and angles ΚΑΗ, ΛΔΘ both at once acute.[8] Therefore angle ΑΚΗ equals angle ΔΛΘ;[9] and the halves too (are equal). Therefore angle Β too equals (angle) Ε.[10] But also angle Η equals (angle) Θ.[11] Therefore triangle ΑΒΗ is similar to triangle ΔΕΘ.[12] By the same argument also triangle ΑΗΓ is similar to triangle ΔΘΖ.

(297) ἄλλως ἄμεινον. τετμήσθωσαν δίχα τοῖς Κ, Λ σημείοις αἱ ΒΓ, ΕΖ, καὶ ἐπεζεύχθωσαν αἱ ΑΚ, ΔΛ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΒΗ πρὸς ΗΓ, οὕτως ἡ ΕΘ πρὸς ΘΖ, συνθέντι καὶ τὰ ἡμίση τῶν ἡγουμένων καὶ ἀναστρέψαντι γίνεται ὡς ἡ ΓΚ, τουτέστιν ὡς ἡ ΑΚ, πρὸς ΚΗ, οὕτως ἡ ΛΖ, τουτέστιν ἡ ΔΛ, πρὸς ΛΘ. καὶ εἰσὶν ἴσαι μὲν αἱ πρὸς τοῖς Η, Θ σημείοις γωνίαι, αἱ δὲ ὑπὸ ΚΑΗ, ΛΔΘ ἑκα|τέρα ἅμα ὀξεῖα. ἴση ἄρα ἐστὶν καὶ ἡ ὑπὸ ΑΚΗ γωνία |179v τῆι ὑπὸ ΔΛΘ γωνίαι. καὶ τὰ ἡμίση. καὶ ἡ Β ἄρα γωνία ἴση ἐστὶν τῆι Ε. ἀλλὰ καὶ ἡ Η γωνία τῆι Θ ἴση ἐστίν. ὅμοιον ἄρα ἐστὶν τὸ ΑΒΗ τρίγωνον τῶι ΔΕΘ τριγώνωι. κατὰ τὰ αὐτὰ καὶ τὸ ΑΗΓ τρίγωνον τῶι ΔΘΖ τριγώνωι ἐστὶν ὅμοιον.

‖ 3 συνθέντι Ha (Co) συντεθήσεται A ‖ 4 ΓΚ Co ΗΓΚ A | ὡς (ἡ ΑΚ) del Ha ‖ 7 ΛΔΘ Co ΔΛΘ A ‖ 11 ΑΗΓ Co ΑΚ A | ΔΘΖ Co ΔΛΖ A

(298) (Lemmas) of (Books) 7, 8.

1. *(Prop. 221)* *(Let) ΑΓ (be) a right-angled parallelogram, and let ΕΖΑ be drawn across. That the rectangle contained by ΕΑ, ΑΖ equals the rectangle contained by ΖΒ, ΒΓ plus the rectangle contained by ΕΔ, ΔΓ.

For since the square of ΕΖ equals the squares of ΕΓ, ΓΖ,[1] and out of these the squares of ΕΑ, ΑΖ equal the squares of ΕΔ, ΔΑ, that is the squares of ΕΔ, ΓΒ, plus the squares of ΑΒ, ΒΖ,[2] that is the squares of ΓΔ, ΒΖ,[3] therefore the remaining twice the rectangle contained by ΕΑ, ΑΖ equals twice the rectangle contained by ΕΔ, ΔΓ plus twice the rectangle contained by ΖΒ, ΒΓ.[4] Hence once the rectangle contained by ΕΑ, ΑΖ equals the rectangle contained by ΕΔ, ΔΓ plus the rectangle contained by ΖΒ, ΒΓ.[5] *

(299) 2. *(Prop. 222)* (Let) ΑΓ be a right-angled parallelogram, and let ΕΑΖ be drawn across. That the rectangle contained by ΕΔ, ΔΓ plus the rectangle contained by ΓΒ, ΒΖ equals the rectangle contained by ΕΑ, ΑΖ.

For since the square of ΕΖ equals the squares of ΕΓ, ΓΖ,[1] and the squares of ΕΑ, ΑΖ equal the squares of ΕΔ, ΔΓ, ΓΒ, ΒΖ,[2] therefore twice the rectangle contained by ΕΑ, ΑΖ equals twice the rectangle contained by ΕΔ, ΔΓ plus twice the rectangle contained by ΖΒ, ΒΓ.[3] Thus also once (the rectangle contained by ΕΑ, ΑΖ) equal once (the rectangles contained by ΕΔ, ΔΓ and ΖΒ, ΒΓ).[4]

(300) 3. *(Prop. 223)* Let ΑΒ be greater than ΓΔ, and the rectangle contained by ΑΕ, ΕΒ equal to the rectangle contained by ΓΖ, ΖΔ. And let ΑΕ, ΓΖ be the greater parts. That ΑΕ is greater than ΓΖ.

Let the wholes ΑΒ, ΓΔ be bisected by points Η, Θ.[1] Then ΗΒ is greater than ΔΘ.[2] Hence also the square of ΗΒ is greater than the square of ΔΘ.[3] But also the rectangle contained by ΑΕ, ΕΒ equals the rectangle contained by ΓΖ, ΖΔ.[4] Therefore the square of ΗΕ is greater than the square of ΘΖ.[5] Hence ΗΕ is greater than ΘΖ.[6] But also ΑΗ is greater than ΓΘ.[7] Thus the whole ΑΕ is greater than the whole ΓΖ.[8]

(298) ΤΟΥ Ζ΄, Η΄

<α΄.> παραλληλόγραμμον ὀρθογώνιον τὸ ΑΓ, καὶ διήχθω ἡ ΕΖΑ. ὅτι τὸ ὑπὸ ΕΑΖ ἴσον ἐστὶν τῶι τε ὑπὸ ΖΒΓ καὶ τῶι ὑπὸ ΕΔΓ. ἐπεὶ γὰρ τὸ ἀπὸ τῆς ΕΖ ἴσον ἐστὶν τοῖς ἀπὸ τῶν ΕΓ, ΓΖ, 992
ὧν τὰ ἀπὸ τῶν ΕΑ, ΑΖ τετράγωνα ἴσα ἐστὶν τοῖς ἀπὸ τῶν ΕΔ, ΔΑ, τουτέστιν τοῖς ἀπὸ τῶν ΕΔ, ΓΒ, καὶ τοῖς ἀπὸ τῶν ΑΒ, ΒΖ, τουτέστιν τοῖς ἀπὸ τῶν ΓΔ, ΒΖ τετραγώνοις, λοιπὸν ἄρα τὸ δὶς ὑπὸ τῶν ΕΑΖ ἴσον ἐστὶν τῶι τε δὶς ὑπὸ τῶν ΕΔ, ΔΓ καὶ τῶι δὶς ὑπὸ τῶν ΖΒ, ΒΓ. καὶ τὸ ἅπαξ ἄρα ὑπὸ τῶν ΕΑΖ ἴσον ἐστὶν τῶι τε ὑπὸ ΕΔΓ καὶ τῶι ὑπὸ ΖΒΓ. ὅ(περ): —

(299) <β΄.> παραλληλόγραμμον ὀρθογώνιον τὸ ΑΓ, καὶ διήχθω ἡ ΕΑΖ. ὅτι τὸ ὑπὸ τῶν ΕΔ, ΔΓ μετὰ τοῦ ὑπὸ ΓΒΖ ἴσον ἐστὶν τῶι ὑπὸ ΕΑΖ. ἐπεὶ γὰρ τὸ ἀπὸ τῆς ΕΖ ἴσον ἐστὶν τοῖς ἀπὸ τῶν ΕΓ, ΓΖ, ἔστιν δὲ καὶ τὰ ἀπὸ τῶν ΕΑ, ΑΖ τετράγωνα ἴσα τοῖς ἀπὸ τῶν ΕΔ, ΔΓ, ΓΒ, ΒΖ, καὶ τὸ δὶς ὑπὸ τῶν ΕΑΖ ἄρα ἴσον ἐστὶν τῶι δὶς ὑπὸ τῶν ΕΔΓ μετὰ τοῦ δὶς ὑπὸ τῶν ΖΒΓ. ὥστε καὶ τὸ [ἀπὸ] ἅπαξ τοῖς ἅπαξ.

(300) <γ΄.> ἔστω μείζων ἡ ΑΒ τῆς ΓΔ, καὶ ἴσον τὸ ὑπὸ ΑΕΒ [γωνία] τῶι ὑπὸ ΓΖΔ. καὶ ἔστω μείζω τμήματα τὰ ΑΕ, ΓΖ. ὅτι μείζων ἐστὶν ἡ ΑΕ τῆς ΓΖ. τετμήσθωσαν αἱ ὅλαι αἱ ΑΒ, ΓΔ δίχα τοῖς Η, Θ σημείοις. μείζων ἄρα ἐστὶν ἡ ΗΒ τῆς ΔΘ. ὥστε καὶ τὸ ἀπὸ ΗΒ μεῖζον τοῦ ἀπὸ ΔΘ τετραγώνου. ἔστιν δὲ καὶ 994
τὸ ὑπὸ ΑΕΒ ἴσον τῶι ὑπὸ ΓΖΔ. καὶ τὸ ἀπὸ ΗΕ ἄρα μεῖζόν ἐστιν τοῦ ἀπὸ ΘΖ. μείζων ἄρα ἐστὶν ἡ ΗΕ τῆς ΘΖ. ἔστι δὲ καὶ ἡ ΑΗ μείζων τῆς ΓΘ. ὅλη ἄρα ἡ ΑΕ ὅλης τῆς ΓΖ μείζων ἐστίν.

‖ 2 α΄ add Hu (BS) ‖ 3 ΕΖΑ Co ΕΖ A | ΖΒΓ Co ΖΓΒ A ‖ 4 ΕΔΓ Co ΕΓΔ A ‖ 5 ὧν] καὶ Co ‖ 6 ΓΒ Co Γ A ‖ 7 post τετραγώνοις add ἀλλὰ τὸ μὲν ἀπὸ ΕΖ μετὰ τοῦ δὶς ὑπὸ ΕΑΖ ἴσον ἐστὶ τοῖς ἀπὸ ΕΑ, ΑΖ, τὰ δ' ἀπὸ ΓΕ, ΓΖ μετὰ τοῦ δὶς ὑπὸ ΕΔΖ καὶ τοῦ δὶς ὑπὸ ΖΒΓ ἴσα ἐστὶ τοῖς τε ἀπὸ ΕΔ, ΒΓ καὶ τοῖς ἀπὸ τῶν ΓΔ, ΒΖ τετραγώνοις Ha (Co) ‖ 9 ΖΒ] *ΖΒ A ‖ 11 β΄ add Hu (BS) ‖ 13 ΕΑΖ Co ΕΔΖ A ‖ 14 τετράγωνα Ha τετραγώνων A ‖ 15 ΔΓ Co ΔΖ A ‖ 16 ΖΒΓ Ha (Co) ΖΓ A ‖ 17 ἀπὸ del Ha | τοῖς Hu app τῶι A ‖ 18 γ΄ add Hu (BS) | τὸ Co ἡ τη A ‖ 19 γωνία del Co | τῶι Co τῆι A | μείζω Ge (S) μείζων A | τὰ Ge (BS) αἱ A ‖ 20 αἱ (ὅλαι) del Ha ‖ 21 ΔΘ Co ΔΕ A ‖ 22 τὸ Co τὰ A | μεῖζον Co μείζων A | καὶ] καθ' ὑπόθεσιν coni. Hu app ‖ 24 μείζων Ha μεῖζον A | ΘΖ Co ΘΔ A

(301) 4. *(Prop. 224)* (Let) the rectangle contained by AE, EB equal the rectangle contained by ΓZ, ZΔ, with AB, ΓΔ equal. That the greater parts AE, ΓZ are <equal>. What comes after: For let AB, ΓΔ be bisected by H, Θ....

(302) 5. *(Prop. 225)* Let AB be greater than ΓΔ, and BE less than ΔZ, with AB being greater than BE, and ΓΔ than ΔZ. That the difference of AB, BE is greater than the difference of ΓΔ, ΔZ.

For since <AB> is greater <than ΓΔ,[1] therefore the difference of AB, BE is greater> than the difference of ΓΔ, EB.[2] But the (difference) of ΓΔ, EB is greater than the difference of ΓΔ, ΔZ;[4] for EB is less than ΔZ.[3] Therefore the difference of AB, BE is much greater than the difference of ΓΔ, ΔZ.[5]

(303) 6. *(Prop. 226)* Let AB equal BΓ, and ΔE (equal) EZ. That the rectangle contained by AΓ, ΔZ is four times the rectangle contained by AB, ΔE.

For since ΓA is twice AB,[1] with common height ΔE, therefore the rectangle contained by ΓA, ΔE is twice the rectangle contained by AB, ΔE.[2] Again, since ΔZ is twice ΔE,[3] with common height AΓ, therefore the rectangle contained by AΓ, ΔZ is twice the rectangle contained by AΓ, ΔE.[4] But the rectangle contained by AΓ, ΔE <is twice the rectangle contained by AB, ΔE. Thus the rectangle contained by AΓ, ΔZ is four times> the rectangle contained by AB, ΔE.[5]

(304) 7. *(Prop. 227)* As AB is to BΓ, so let ΔE be to EZ, and as AB is to BH, so let ΔE be to EΘ. That as the rectangle contained by AB, BH is to the rectangle contained by AH, HΓ, so is the rectangle contained by ΔE, EΘ to the rectangle contained by ΔΘ, ΘZ.

For since as AB is to BH, so is ΔE to EΘ,[1] *convertendo*, as BA is to AH, so is EΔ to ΔΘ.[2] Hence also as the square of BA is to the square of AH, so is the square of ΔE to the square of ΔΘ.[3] But also as the square of AB is to the rectangle contained by AB, BH, so is the square of ΔE to the rectangle contained by ΔE, EΘ.[4] Therefore as the square of AH is to the rectangle contained by AB, BH, so is the square of ΔΘ to the rectangle contained by ΔE, EΘ.[5] But since it was stipulated that as AB is to BΓ, so is ΔE to EZ,[6] by inversion[7] and *componendo*, therefore, as ΓA is to AB, so is ZΔ to ΔE.[8] But also as BA is to AH, so is EΔ to ΔΘ.[9] *Ex aequali*

(301) <δʹ.> |ἴσον τὸ ὑπὸ ΑΕΒ τῶι ὑπὸ ΓΖΔ, ἴσων οὐσῶν τῶν ΑΒ, ΓΔ. ὅτι τὰ μείζονα τμήματα τὰ ΑΕ, ΓΖ <ἴσα> ἐστιν. τὸ δ᾽ ἐφεξῆς· τετμήσθωσαν γὰρ αἱ ΑΒ, ΓΔ δίχα τοῖς Η, Θ: — |180

(302) <εʹ.> ἔστω μὲν μείζων ἡ ΑΒ τῆς ΓΔ, ἐλάσσων δὲ ἡ ΒΕ τῆς ΔΖ, οὔσης μείζονος τῆς μὲν ΑΒ τῆς ΒΕ, τῆς δὲ ΓΔ τῆς ΔΖ. ὅτι ἡ τῶν ΑΒ, ΒΕ ὑπεροχὴ μείζων ἐστὶν τῆς τῶν ΓΔ, ΔΖ ὑπεροχῆς. ἐπεὶ γὰρ μείζων ἐστὶν <ἡ ΑΒ τῆς ΓΔ, καὶ ἡ τῶν ΑΒ, ΒΕ ὑπεροχὴ ἄρα μείζων ἐστὶν> τῆς τῶν ΓΔ, ΕΒ ὑπεροχῆς. ἀλλὰ ἡ τῶν ΓΔ, ΕΒ μείζων ἐστὶν τῆς τῶν ΓΔ, ΔΖ ὑπεροχῆς· ἐλάσσων γάρ ἐστιν ἡ ΕΒ τῆς ΔΖ. ὥστε ἡ τῶν ΑΒ, ΒΕ ὑπεροχὴ πολλῶι μείζων ἐστὶν τῆς τῶν ΓΔ, ΔΖ ὑπεροχῆς.

(303) <ϛʹ.> ἔστω ἴση ἡ μὲν ΑΒ τῆι ΒΓ, ἡ <δὲ> ΔΕ τῆι ΕΖ. ὅτι τὸ ὑπὸ ΑΓ, ΔΖ τετραπλάσιόν ἐστιν τοῦ ὑπὸ ΑΒ, ΔΕ. ἐπεὶ γὰρ διπλῆ ἐστιν ἡ ΓΑ τῆς ΑΒ, κοινὸν ὕψος ἡ ΔΕ, τὸ ἄρα ὑπὸ ΓΑ, ΔΕ διπλάσιόν ἐστιν τοῦ ὑπὸ ΑΒ, ΔΕ. πάλιν ἐπεὶ διπλῆ ἐστιν ἡ ΔΖ τῆς ΔΕ, κοινὸν ὕψος ἡ ΑΓ, τὸ ἄρα ὑπὸ ΑΓ, ΔΖ διπλάσιόν ἐστιν τοῦ ὑπὸ ΑΓ, ΔΕ. ἀλλὰ τὸ ὑπὸ ΑΓ, ΔΕ <τοῦ ὑπὸ ΑΒ, ΔΕ διπλάσιόν ἐστιν. τὸ ἄρα ὑπὸ ΑΓ, ΔΖ τετραπλάσιόν ἐστιν> τοῦ ὑπὸ ΑΒ, ΔΕ.

(304) <ζʹ.> ἔστω ὡς μὲν ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΕ πρὸς τὴν ΕΖ, ὡς δὲ ἡ ΑΒ πρὸς τὴν ΒΗ οὕτως ἡ ΔΕ πρὸς τὴν ΕΘ. ὅτι γίνεται ὡς τὸ ὑπὸ τῶν ΑΒΗ πρὸς τὸ ὑπὸ τῶν ΑΗΓ, οὕτως τὸ ὑπὸ τῶν ΔΕΘ πρὸς τὸ ὑπὸ τῶν ΔΘΖ. ἐπεὶ γάρ ἐστιν ὡς ἡ ΑΒ πρὸς 996 τὴν ΒΗ, οὕτως ἡ ΔΕ πρὸς τὴν ΕΘ, ἀναστρέψαντί ἐστιν ὡς ἡ ΒΑ πρὸς τὴν ΑΗ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΘ. ὥστε καὶ ὡς τὸ ἀπὸ ΒΑ πρὸς τὸ ἀπὸ ΑΗ, οὕτως τὸ ἀπὸ ΔΕ πρὸς τὸ ἀπὸ ΔΘ. ἀλλὰ καὶ ὡς τὸ ἀπὸ ΑΒ πρὸς τὸ ὑπὸ ΑΒΗ, οὕτως τὸ ἀπὸ ΔΕ πρὸς τὸ ὑπὸ ΔΕΘ. καὶ ὡς ἄρα τὸ ἀπὸ ΑΗ πρὸς τὸ ὑπὸ ΑΒΗ, οὕτως τὸ ἀπὸ ΔΘ πρὸς τὸ ὑπὸ ΔΕΘ. ἐπεὶ δὲ ὑπόκειται ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΕ πρὸς τὴν ΕΖ, ἀνάπαλιν καὶ συνθέντι, ὡς ἄρα ἡ ΓΑ πρὸς τὴν

‖ 1 δʹ add Hu (BS) ‖ 2 ἴσα add Co | τὸ δ᾽ ἐφεξῆς Hu τὸ δεφῆς A ‖ 3 τετμήσθωσαν Co τμήματα A | αἱ Ha (Co) τῶι A | post Θ add καὶ τὰ ἐφεξῆς Ha ‖ 4 εʹ add Hu (BS) ‖ 7 ἡ ΑΒ — ἐστὶν add Hu ἡ ΑΒ τῆς ΓΔ, μείζων ἄρα ἡ τῶν ΑΒ, ΒΕ ὑπεροχὴ Co ‖ 12 (303-307) passim his propositionibus K pro E posuit Ha | ϛʹ add Hu (BS) | δὲ add Ha ‖ 16 (τῆς) ΔΕ Co ΖΕ A ‖ 17 ΔΕ (τοῦ) Co ΔΖ A | τοῦ — τετραπλάσιόν ἐστιν add Hu (Co) ‖ 19 ΑΒ Co ΑΓ A ‖ 20 ζʹ add Hu (BS) ‖ 22 ΑΒΗ Co ΑΗΒ A | ΑΗΓ Co ΑΓΗ A ‖ 23 ΔΘΖ Co ΔΕΖ A ‖ 26 ΔΘ Co ΕΘ A ‖ 28 ΔΘ Co ΛΘ A ‖ 29 ΔΕΘ Co ΔΕΖ A ‖ 30 καὶ συνθέντι] συνθέντι καὶ A transp Ha therefore, as ΓA is to AH, so is ZΔ to ΔΘ.[10] And so as ΓH is to HA, so is ZΘ to ΘΔ.[11] And as the rectangle contained by (ΓH, HA) is to the square of (AH), so is the rectangle contained by (ZΘ, ΘΔ) to the square of (ΘΔ).[12] But also as is the square of AH to the rectangle contained by AB, BH, so is the square of ΔΘ to the rectangle contained by ΔE, EΘ.[13] Thus as the rectangle contained by AB, BH is to the rectangle contained by AH, HΓ, so is the rectangle contained by ΔE, EΘ to the rectangle contained by ΔΘ, ΘZ.[14]

(305) 8. *(Prop. 228)* Let the squares of AB, BΓ <taken together> be given, and the difference of the squares of AB, BΓ be given. That each of AB, BΓ is given. For let BΔ be made equal to ΓB.[1] Then the squares of ΓA, AΔ (taken together) is given.[2] But also twice the rectangle contained by ΓA, AΔ is given,[5] since also the rectangle contained by ΓA, AΔ is given,[4] for it is the difference of the squares of AB, BΓ.[3] Hence also the square of ΓA, AΔ taken together is given.[6] And so ΓA, AΔ taken together are given.[7] And half of this is BA;[8] so that BA is given.[9] Thus BΓ too is given.[10]

(306) 9. *(Prop. 229)* Let AB be <equal> to BΓ, and ΔE to EZ, and furthermore as ΓB is to BH, so let ZE be to EΘ. That as the rectangle contained by AH, HB is to the rectangle contained by BΓ, ΓH, so is the rectangle contained by ΔΘ, ΘE to the rectangle contained by EZ, ZΘ.

For since as ΓB is to BA, so is ZE to EΔ,[1] but also as ΓB is to BH, so is ZE to EΘ,[2] therefore also as the square of AH is to the rectangle contained by AH, HB, so will the square of ΔΘ be to the rectangle contained by ΔΘ, ΘE.[3] But also as the square of AH is to the square of BΓ, so is the square of ΔΘ to the square of EZ,[4] while as the square of BΓ is to the rectangle contained by BΓ, ΓH, so is the square of EZ to the rectangle contained by EZ, ZΘ.[5] Therefore *ex aequali* as is the rectangle contained by AH, HB to the rectangle contained by BΓ, ΓH, so is the rectangle contained by ΔΘ, ΘE to the rectangle contained by EZ, ZΘ.[6]

(307) 10. *(Prop. 230)* Let AB be equal to BΓ, and BΔ less than <BE. That the rectangle contained by AΔ, ΔB> has <to the rectangle contained by> BΓ, ΓΔ a lesser ratio than has the rectangle contained by ΓE, EB to the rectangle contained by BA, AE.

For since AB equals BΓ,[1] while BΔ is less than BE,[2] therefore ΓΔ is greater than AE.[3] Hence also ΓE is greater than AΔ.[4] Therefore the

ΑΒ, οὕτως ἡ ΖΔ πρὸς ΔΕ. ἔστιν δὲ καὶ ὡς ἡ ΒΑ πρὸς ΑΗ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΘ. δι' ἴσου ἄρα ἐστὶν ὡς ἡ ΓΑ πρὸς ΑΗ, οὕτως ἡ ΖΔ πρὸς ΔΘ. καὶ ὡς ἄρα ἡ ΓΗ πρὸς ΗΑ, οὕτως ἡ ΖΘ πρὸς ΘΔ. καὶ ὡς τὸ ὑπὸ πρὸς τὸ ἀπό, τὸ ὑπὸ πρὸς τὸ ἀπό. ἀλλὰ καὶ ὡς τὸ ἀπὸ ΑΗ πρὸς τὸ ὑπὸ ΑΒΗ, οὕτως τὸ ἀπὸ ΔΘ πρὸς τὸ ὑπὸ ΔΕΘ. καὶ ὡς ἄρα τὸ ὑπὸ ΑΒΗ πρὸς τὸ ὑπὸ ΑΗΓ, οὕτως τὸ ὑπὸ ΔΕΘ πρὸς τὸ ὑπὸ ΔΘΖ.

(305) <η´.> ἔστω δοθέντα <συναμφότερα> τὰ ἀπὸ τῶν |ΑΒ, |180v ΒΓ, καὶ δοθεῖσα ἡ τῶν ἀπὸ ΑΒ, ΒΓ ὑπεροχή. ὅτι δοθεῖσά ἐστιν ἑκατέρα τῶν ΑΒ, ΒΓ. κείσθω γὰρ τῆι ΓΒ ἴση ἡ ΒΔ. δοθέντα ἄρα ἐστὶν καὶ τὰ ἀπὸ τῶν ΓΑ, ΑΔ· ἀλλὰ καὶ τὸ δὶς ὑπὸ τῶν ΓΑΔ δοθέν ἐστιν, ἐπεὶ καὶ τὸ ὑπὸ ΓΑΔ δοθέν ἐστιν· ὑπεροχὴ γάρ ἐστιν τῶν ἀπὸ ΑΒ, ΒΓ τετραγώνων. δοθὲν ἄρα ἐστὶν καὶ τὸ <ἀπὸ> συναμφοτέρου τῆς ΓΑ, ΑΔ. ὥστε δοθεῖσά ἐστι συναμφότερος ἡ ΓΑ, ΑΔ. καὶ ἔστιν αὐτῆς ἡμίσεια ἡ ΒΑ. 998 δοθεῖσα ἄρα ἐστὶν ἡ ΒΑ. ὥστε καὶ ἡ ΒΓ δοθεῖσά ἐστιν.

(306) θ´. ἔστω <ἴση> ἡ μὲν ΑΒ τῆι ΒΓ, ἡ δὲ ΔΕ τῆι ΕΖ, ἔτι δὲ ἔστω ὡς ἡ ΓΒ πρὸς ΒΗ, οὕτως ἡ ΖΕ πρὸς ΕΘ. ὅτι γίνεται ὡς τὸ ὑπὸ ΑΗΒ πρὸς τὸ ὑπὸ ΒΓΗ, οὕτως τὸ ὑπὸ ΔΘΕ πρὸς τὸ ὑπὸ ΕΖΘ. ἐπεὶ γάρ ἐστιν ὡς ἡ ΓΒ πρὸς ΒΑ, οὕτως ἡ ΖΕ πρὸς ΕΔ, ἀλλὰ καὶ ὡς ἡ ΓΒ πρὸς ΒΗ, οὕτως ἡ ΖΕ πρὸς ΕΘ, ἔσται ἄρα καὶ ὡς τὸ ἀπὸ ΑΗ πρὸς τὸ ὑπὸ ΑΗΒ, οὕτως τὸ ἀπὸ ΔΘ πρὸς τὸ ὑπὸ ΔΘΕ. ἀλλὰ καὶ ὡς μὲν τὸ ἀπὸ ΑΗ πρὸς τὸ ἀπὸ ΒΓ, οὕτως τὸ ἀπὸ ΔΘ πρὸς τὸ ἀπὸ ΕΖ, ὡς δὲ τὸ ἀπὸ ΒΓ πρὸς τὸ ὑπὸ ΒΓΗ, οὕτως τὸ ἀπὸ ΕΖ πρὸς τὸ ὑπὸ ΕΖΘ. ἔσται ἄρα δι' ἴσου ὡς τὸ ὑπὸ ΑΗΒ πρὸς τὸ ὑπὸ ΒΓΗ, οὕτως τὸ ὑπὸ ΔΘΕ πρὸς τὸ ὑπὸ ΕΖΘ.

(307) ι´. ἔστω ἴση ἡ μὲν ΑΒ τῆι ΒΓ, ἐλάσσων δὲ ἡ ΒΔ τῆς <ΒΕ. ὅτι τὸ ὑπὸ τῶν ΑΔΒ πρὸς τὸ ὑπὸ> τῶν ΒΓΔ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ὑπὸ τῶν ΓΕΒ πρὸς τὸ ὑπὸ τῶν ΒΑΕ. ἐπεὶ γὰρ ἴση μέν ἐστιν ἡ ΑΒ τῆι ΒΓ, ἐλάσσων δὲ ἡ ΒΔ τῆς ΒΕ, ἡ ΓΔ

‖ 2 ΕΔ Co ΕΑ A | ΑΗ Co ΔΗ A ‖ 4 post πρὸς τὸ ἀπὸ add οὕτως Co | τὸ ὑπὸ Co τοῦ A ‖ 5 ΔΘ Co ΔΕ A ‖ 7 ὑπὸ (ΔΘΖ) Ha (Co) ἀπὸ A | ΔΘΖ Co ΔΕΖ A ‖ 8 η´ add Hu (BS) | συναμφότερα add Ha (Co) ‖ 9 δοθεῖσα Ha δοθέντα A ‖ 10 δοθέντα] δοθὲν ὅτι A (ὅτι del Co) ‖ 11 τὰ ἀπὸ] τὸ ὑπὸ Co | ΓΑ, ΑΔ] ΓΑΔ A | ἀλλὰ – δοθέν ἐστιν post τετραγώνων transp. Ha ‖ 14 ἀπὸ συναμφοτέρου Ha συναμφότερον A | δοθεῖσά Ha δοθέν A ‖ 15 συναμφότερος Hu συναμφοτέρου A | ἡ (ΒΑ) Co δύο αἱ A ‖ 17 θ´ mg A | ἴση add Co (k) | ΒΓ] ΓΔ A, ἐστιν τῇ ΒΓ mg A (ἐστιν: compendium) ‖ 18 ΖΕ... ΕΘ Co ΘΕ... ΕΖ A ‖ 19 ΑΗΒ ex ΑΗΘ A ‖ 22 ὑπὸ (ΔΘΕ) Ha (Co) ἀπὸ A ‖ 27 ι´ mg A ‖ 28 ΒΕ – πρὸς τὸ ὑπὸ add Co rectangle contained by ΑΔ, ΔΒ is less than the rectangle contained by ΓΕ, ΕΒ;[5] while the rectangle contained by ΒΓ, ΓΔ is greater than the rectangle contained by ΒΑ, ΑΕ.[6] Thus the rectangle contained by ΑΔ, ΔΒ has to the rectangle contained by ΒΓ, ΓΔ a ratio less than has the rectangle contained by ΓΕ, ΕΒ to the rectangle contained by ΒΑ, ΑΕ.[7]

(308) 11. *(Prop. 231)* But now let it be required to prove the converse of the foregoing (lemmas), namely with ΑΒ equal to ΒΓ, and ΔΕ to ΕΖ, and furthermore as the rectangle contained by ΑΗ, ΗΒ to the rectangle contained by ΒΓ, ΓΗ, so the rectangle contained by ΔΘ, ΘΕ to the rectangle contained by ΕΖ, ΖΘ, to prove that as ΓΒ is to ΒΗ, so is ΖΕ to ΕΘ.

Let the rectangle contained by ΓΗ, ΑΚ be made equal to the rectangle contained by ΑΗ, ΗΒ, and the rectangle contained by ΖΘ, ΔΛ equal to the rectangle contained by ΔΘ, ΘΕ.[1] Then as is the rectangle contained by ΑΚ, ΓΗ to the rectangle contained by ΒΓ, ΓΗ, that is ΑΚ to ΒΓ, so is the rectangle contained by ΔΛ, ΖΘ to the rectangle contained by ΕΖ, ΖΘ,[2] that is ΔΛ to ΕΖ.[3] But also as ΓΒ is to ΒΑ, so is ΖΕ to ΕΔ.[4] Therefore ΑΒ, ΒΓ, ΓΚ are in similar positions to ΔΕ, ΕΖ, ΖΛ, and in the same ratio, that is, as ΚΓ is to ΓΒ, so is ΛΖ to ΖΕ.[5] But since the rectangle contained by ΑΗ, ΗΒ equals the rectangle contained by ΑΚ, ΓΗ,[6] let each be subtracted from the rectangle contained by ΑΚ, ΗΒ.[7] Then the remaining rectangle contained by ΒΗ, ΗΚ equals the rectangle contained by ΑΚ, ΒΓ.[8] Hence as is the rectangle contained by ΑΚ, ΒΓ to the square of ΒΚ, so is the rectangle contained by ΒΗ, ΗΚ to the square of ΒΚ.[9] For the same reasons also as the rectangle contained by ΔΛ, ΕΖ is to the square of ΕΛ, so is the rectangle contained by ΕΘ, ΘΛ to the square of ΕΛ.[10] And as the rectangle contained by ΑΚ, ΒΓ is to the square of ΒΚ, so is the rectangle contained by ΔΛ, ΕΖ to the square of ΕΛ,[11] because the similarly positioned segments are in ratio. Hence as the rectangle contained by ΒΗ, ΗΚ is to the square of ΒΚ, so is the rectangle contained by ΕΘ, ΘΛ to the square of ΛΕ.[12] And ΒΗ, ΕΘ are the same segments. Therefore as ΚΒ is to ΒΗ, so is ΛΕ to ΕΘ.[13] And thus as ΗΒ is to ΒΓ, so is ΘΕ to ΕΖ.[14]

ἄρα μείζων ἐστὶν τῆς ΑΕ· ὥστε καὶ ἡ ΓΕ μείζων ἐστὶν τῆς ΑΔ. ἔλασσον ἄρα ἐστὶν τὸ ὑπὸ ΑΔΒ τοῦ ὑπὸ ΓΕΒ, μείζων δὲ τὸ ὑπὸ τῶν ΒΓΔ τοῦ ὑπὸ ΒΑΕ. τὸ ἄρα ὑπὸ ΑΔΒ πρὸς τὸ ὑπὸ ΒΓΔ ἐλάσσονα λόγον ἔχει ἤπερ τὸ ὑπὸ ΓΕΒ πρὸς τὸ ὑπὸ ΒΑΕ.

(308) ια´. ἔστω δὲ νῦν τὸ τοῖς προηγουμένοις ἀναστρόφιον
δεῖξαι, οὔσης ἴσης τῆς μὲν ΑΒ τῆι ΒΓ, τῆς δὲ ΔΕ τῆι ΕΖ, καὶ 1000
ἔτι ὡς τὸ ὑπὸ ΑΗΒ πρὸς τὸ ὑπὸ ΒΓΗ οὕτως τὸ ὑπὸ ΔΘΕ πρὸς τὸ
ὑπὸ ΕΖΘ, δεῖξαι ὅτι γίνεται ὡς ἡ ΓΒ πρὸς ΒΗ, οὕτως ἡ ΖΕ πρὸς
ΕΘ. κείσθω τῶι μὲν ὑπὸ ΑΗΒ ἴσον τὸ ὑπὸ ΓΗ, ΑΚ, τῶι δὲ ὑπὸ
ΔΘΕ ἴσον [ἐστὶν] τὸ ὑπὸ ΖΘ, ΔΛ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΑΚ, ΓΗ
πρὸς τὸ ὑπὸ ΒΓΗ, τουτέστιν ἡ ΑΚ πρὸς ΒΓ, οὕτως τὸ ὑπὸ ΔΛ, ΖΘ
πρὸς τὸ ὑπὸ ΕΖΘ, τουτέστιν ἡ ΔΛ πρὸς ΕΖ. ἀλλὰ καὶ ὡς ἡ ΓΒ
πρὸς ΒΑ, οὕτως ἐστὶν ἡ ΖΕ πρὸς ΕΔ. αἱ ΑΒ, ΒΓ, ΓΚ ἄρα ταῖς ΔΕ,
ΕΖ, ΖΛ ὁμοταγεῖς εἰσιν ἐν τῶι αὐτῶι λόγωι, τουτέστιν |ὡς ἡ |181
ΚΓ πρὸς ΓΒ, οὕτως ἡ ΛΖ πρὸς ΖΕ. ἐπεὶ δὲ τὸ ὑπὸ τῶν ΑΗΒ ἴσον
ἐστὶν τῶι ὑπὸ τῶν ΑΚ, ΓΗ, ἀμφότερον ἀφῃρήσθω ἀπὸ τοῦ ὑπὸ
τῶν ΑΚ, ΗΒ. λοιπὸν ἄρα τὸ ὑπὸ τῶν ΒΗ, ΗΚ ἴσον ἐστὶν τῶι ὑπὸ
τῶν ΑΚ, ΒΓ. ἔστιν ἄρα ὡς τὸ ὑπὸ τῶν ΑΚ, ΒΓ πρὸς τὸ ἀπὸ τῆς
ΒΚ, οὕτως τὸ ὑπὸ τῶν ΒΗΚ πρὸς τὸ ἀπὸ τῆς ΒΚ. διὰ ταὐτὰ δὴ
καὶ ὡς τὸ ὑπὸ τῶν ΔΛ, ΕΖ πρὸς τὸ ἀπὸ τῆς ΕΛ, οὕτως ἐστὶν τὸ
ὑπὸ τῶν ΕΘΛ πρὸς τὸ ἀπὸ τῆς ΕΛ. καὶ ἔστιν ὡς τὸ ὑπὸ τῶν ΑΚ,
ΒΓ πρὸς τὸ ἀπὸ τῆς ΒΚ, οὕτως τὸ ὑπὸ τῶν ΔΛ, ΕΖ πρὸς τὸ ἀπὸ
τῆς ΕΛ, διὰ τὴν ἀναλογίαν τῶν ὁμοταγῶν τμημάτων. καὶ ὡς
ἄρα τὸ ὑπὸ ΒΗΚ πρὸς τὸ ἀπὸ ΒΚ, οὕτως τὸ ὑπὸ ΕΘΛ πρὸς τὸ ἀπὸ
ΛΕ. καὶ ἔστιν τὰ αὐτὰ τμήματα τὰ ΒΗ, ΕΘ. ἔστιν ἄρα ὡς ἡ ΚΒ
πρὸς ΒΗ, οὕτως ἡ ΛΕ πρὸς ΕΘ. καὶ ὡς ἄρα ἡ ΗΒ πρὸς ΒΓ, οὕτως 1002
ἐστὶν ἡ ΘΕ πρὸς ΕΖ.

‖ 2 ἔλασσον Ha ἐλάσσων A ‖ 3 ΑΔΒ Co ΔΑΒ A ‖ 5 ια´ mg A | ἀναστρόφιον Hu ἀναστρεφειν A ἀντίστροφον Ha ‖ 6 καὶ ἔτι] ἔστω Hu ‖ 7 ΑΗΒ πρὸς τὸ ὑπὸ bis A corr Co ‖ 10 ἐστὶν del Hu ἔστω Ha | ΖΘ, ΔΛ Co ΖΘΔ A | ΓΗ Co ΓΖ A ‖ 11 ΔΛ (ΖΘ) Co ΔΑ A ‖ 12 ΔΛ (πρὸς) Co ΔΑ A ‖ 13 ΑΒ, ΒΓ, ΓΚ] ΑΚ, ΒΓ, ΒΚ Hu | ΔΕ, ΕΖ, ΖΛ] ΔΛ, ΕΖ, ΕΛ Hu ‖ 14 τουτέστιν – ΖΕ secl Hu τουτέστιν ὡς ἡ ΒΓ πρὸς ΓΚ, οὕτως ἡ ΕΖ πρὸς ΖΛ. καὶ ὡς ἄρα ἡ ΒΓ πρὸς τὴν ΒΚ, οὕτως ἡ ΖΕ πρὸς τὴν ΕΛ Ha ‖ 15 ἴσον Ha ἴση A ‖ 16 ἀμφότερον Co ἀμφοτέρων A ‖ 17 ΑΚ, ΗΒ Co ΑΚΒ A | ΒΗ, ΗΚ] ΒΑ, ΗΚ A ΗΚ, ΗΒ Ha ‖ 20 ΔΛ Co ΑΛ A ‖ 23 ὁμοταγῶν Hu ὁμοιοτάτων A ‖ 24 ΕΘΛ Co ΕΘΑ A ‖ 25 ΛΕ] ΔΕ A ΕΛ Co | ΚΒ... ΒΗ... ΛΕ... ΕΘ] ΗΒ... ΒΚ... ΘΕ... ΕΛ Co ‖ 26 ΒΗ Ha ΒΑ A | καὶ ὡς – ΕΖ] ἀλλ' ἐδείχθη ὡς ἡ ΒΓ πρὸς τὴν ΒΚ, οὕτως ἡ ΖΕ πρὸς τὴν ΔΛ. δι' ἴσου ἄρα ὡς ἡ ΒΓ πρὸς ΒΗ, οὕτως ἐστὶν ἡ ΖΕ πρὸς ΕΘ Ha | ΗΒ... ΒΓ... ΘΕ... ΕΖ] ΓΒ... ΒΗ... ΖΕ... ΕΘ Hu

(309) 12. *(Prop. 232)* Let AB be <equal> to BΓ, and ΔE to EZ, and furthermore let BΓ have to ΓH a greater ratio than has EZ to ZΘ. That in the first case AH has to BΓ a greater ratio than has ΔΘ to EZ, in the second a lesser (ratio).

For since BΓ has to ΓH a greater ratio than has <EZ to ZΘ,[1] in the first case ΓB has to BH a lesser ratio than has> ZE to EΘ, but in the second, a greater (ratio).[2] And so also AB has to BH, in the first case, a lesser ratio than has ΔE to EΘ, but in the second, a greater (ratio).[3] Hence HA has to AB, in the first case, a greater ratio than ΘΔ to ΔE, but in the second, a lesser (ratio).[4] And as AB is to BΓ, so is ΔE to EZ.[5] *Ex aequali* therefore, in the first case AH has to BΓ a greater ratio than ΔΘ to EZ, but in the second, a lesser (ratio).[6]

(310) 13. *(Prop. 233)* *Again let AB be equal to BΓ, and ΔE to EZ, and furthermore let AH have to HB a *lesser* ratio than has ΔΘ to ΘE. That also BΓ has to ΓH a greater ratio than has EZ to ZΘ.

For since *convertendo*[1] and *separando*[2] HB has to BA, that is to BΓ, a *greater* ratio than has ΘE to EΔ,[3] that is to EZ,[4] *convertendo*[5] and *separando*, BΓ has to ΓH a greater ratio than EZ to ZΘ.[6] *

(311) 14. *(Prop. 234)* (Let) AB (be) equal to BΓ, and ΔE to EZ, and furthermore let AH have to HB a greater ratio than has ΔΘ to ΘE. That BH has to HΓ a lesser ratio than has EΘ to ΘZ.

For since *separando* AB, that is BΓ, has to BH a greater ratio than ΔE, that is EZ,[2] has to EΘ,[1] *convertendo*[3] and *separando* BH has to HΓ a lesser ratio than EΘ to ΘZ.[4]

(309) ιβʹ. ἔστω <ἴση> ἡ μὲν ΑΒ τῇ ΒΓ, ἡ δὲ ΔΕ τῇ ΕΖ, ἔτι δὲ ἡ ΒΓ πρὸς ΓΗ μείζονα λόγον ἐχέτω ἤπερ ἡ ΕΖ πρὸς τὴν ΖΘ. ὅτι ἐπὶ μὲν τῆς αʹ πτώσεως καὶ ἡ ΑΗ πρὸς τὴν ΒΓ μείζονα λόγον ἔχει ἤπερ ἡ ΔΘ πρὸς τὴν ΕΖ, ἐπὶ δὲ τῆς δευτέρας ἐλάσσω. ἐπεὶ γὰρ ἡ ΒΓ πρὸς ΓΗ μείζονα λόγον ἔχει ἤπερ <ἡ ΕΖ πρὸς ΖΘ, ἐπὶ μὲν τῆς πρώτης πτώσεως ἡ ΓΒ πρὸς ΒΗ ἐλάσσονα λόγον ἔχει ἤπερ> ἡ ΖΕ πρὸς ΕΘ, ἐπὶ δὲ τῆς βʹ μείζω. ὥστε καὶ ἡ ΑΒ πρὸς τὴν ΒΗ, ἐπὶ μὲν τῆς πρώτης πτώσεως ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΔΕ πρὸς ΕΘ, ἐπὶ δὲ τῆς δευτέρας μείζω. καὶ ἡ ΗΑ ἄρα πρὸς τὴν ΑΒ, ἐπὶ μὲν τῆς πρώτης πτώσεως, μείζονα λόγον ἔχει ἤπερ ἡ ΘΔ πρὸς ΔΕ, ἐπὶ δὲ τῆς δευτέρας ἐλάσσω. καὶ ἔστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΕ πρὸς ΕΖ. δι' ἴσου ἄρα [ἐστὶν] ἐπὶ μὲν τῆς πρώτης πτώσεως ἡ ΑΗ πρὸς τὴν ΒΓ μείζονα λόγον ἔχει ἤπερ ἡ ΔΘ πρὸς τὴν ΕΖ, ἐπὶ δὲ τῆς δευτέρας ἐλάσσω.

(310) ιγʹ. ἔστω πάλιν ἴση ἡ μὲν ΑΒ τῇ ΒΓ, ἡ δὲ ΔΕ τῇ ΕΖ, ἔτι δὲ ἡ ΑΗ πρὸς τὴν ΗΒ ἐλάσσονα λόγον ἐχέτω ἤπερ ἡ ΔΘ πρὸς τὴν ΘΕ. ὅτι καὶ ἡ ΒΓ πρὸς τὴν ΓΗ μείζονα λόγον ἔχει ἤπερ ἡ ΕΖ πρὸς τὴν ΖΘ. ἐπεὶ γὰρ κατὰ ἀναστροφὴν καὶ διαίρεσιν ἡ ΗΒ πρὸς τὴν ΒΑ, τουτέστιν τὴν ΒΓ, μείζονα λόγον ἔχει ἤπερ ἡ ΘΕ πρὸς τὴν ΕΔ, τουτέστιν πρὸς τὴν ΕΖ, ἀναστρέψαντι καὶ 1004 διελόντι ἡ ΒΓ πρὸς τὴν ΓΗ μείζονα λόγον ἔχει ἤπερ ἡ ΕΖ πρὸς τὴν ΖΘ.

(311) <ιδʹ.> |ἴση ἡ μὲν ΑΒ τῇ ΒΓ, ἡ δὲ ΔΕ τῇ ΕΖ, καὶ ἔτι ἡ |181v ΑΗ πρὸς τὴν ΗΒ μείζονα λόγον ἐχέτω ἤπερ ἡ ΔΘ πρὸς τὴν ΘΕ. ὅτι ἡ ΒΗ πρὸς τὴν ΗΓ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΕΘ πρὸς τὴν ΘΖ. ἐπεὶ γὰρ κατὰ διαίρεσιν ἡ ΑΒ, τουτέστιν ἡ ΒΓ, πρὸς τὴν ΒΗ μείζονα λόγον ἔχει ἤπερ ἡ ΔΕ, τουτέστιν ἡ ΕΖ, πρὸς τὴν ΕΘ, ἀναστρέψαντι <καὶ> κατὰ διαίρεσιν ἡ ΒΗ πρὸς τὴν ΗΓ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΕΘ πρὸς τὴν ΘΖ.

‖ 1 ιβʹ mg A | ἴση add Co ‖ 3 αʹ] ΕΑ A πρώτης Co (k) | ΒΓ Co ΗΓ A ‖ 5 ἐλάσσω Ge (S) ἐλάσσων A | ἡ ΕΖ — ἤπερ add Co ‖ 8 μείζω Ge (S) μείζων A ‖ 10 μείζω Ge (S) μείζων A | ΗΑ Co ΗΛ A ‖ 12 ἐλάσσω Ge (S) ἐλάσσων A ‖ 13 ἐστὶν del Co ‖ 15 ἐλάσσω Ge (S) ἐλάσσων A ‖ 16 ιγʹ mg A ‖ 17 ἐλάσσονα] μείζονα Hu | ΔΘ Co ΔΕ A ‖ 20 μείζονα] ἐλάσσονα Hu ‖ 24 ιδʹ add Hu (BS) | ἔτι Ge (recc?) ἔστιν A ‖ 26 ἐλάσσονα Co μείζονα A | ΕΘ Ha ΕΒ A ‖ 29 καὶ add Ha (Co)

(312) For the (Loci) on Surfaces

(Prop. 235) If there is (given) a straight line AB, and ΓΔ parallel to (a line given) in position, and the ratio of the rectangle contained by AΔ, ΔB to the square of ΔΓ is (given), Γ touches a conic line. Then if AB is deprived of (being given) in position, and A and B are deprived of being given, but are on straight lines (given) in position AE, EB, Γ elevated is on a surface (given) in position. But this was proved.

(313) *(Prop. 235 bis)* If straight line AB is (given) in position, and Γ given in the same plane, and ΔΓ is drawn across, and ΔE drawn parallel to (a line given) in position, and the ratio of ΓΔ to ΔE (given), Δ touches a conic section (given) in position. Now it is required to prove which curve. It will be proved as follows, after this locus (7.314-317) has first been written.

(314) Given two (points) A, B, and ΓΔ at right angles, let the ratio of the square of AΔ to the squares of ΓΔ, ΔB (together be given). I say that Γ touches a section of a cone, whether the ratio is equal to equal or greater to less or less to greater.

(315) *(Prop. 236 a)* For first let the ratio be equal to equal.

And since the square of AΔ equals the squares of ΓΔ, ΔB,[1] let ΔE be made equal to BΔ.[2] Then the rectangle contained by BA, AE equals the square of ΔΓ.[3] Let AB be bisected by Z.[4] Then Z is given.[5] And AE is twice ZΔ.[6] Hence the rectangle contained by BA, AE is twice the rectangle contained by AB, ZΔ.[7] And twice AB is given.[8] Therefore the rectangle contained by a given (line) and ΔZ equals the square of ΔΓ.[9] Thus Γ touches a parabola (given) in position and passing through Z.[10]

(Prop. 236 b) The synthesis of the locus will be made as follows. Let the given (points) be A, B, and let the ratio be equal to equal, and let AB be bisected by Z. Let P be twice AB, and with ZB being a straight line (given) in position terminated at Z, and with P given in magnitude, let parabola HZ

(312) ΕΙΣ ΤΟΥΣ ΠΡΟΣ ΕΠΙΦΑΝΕΙΑΙ

ἐὰν <*ἦι*> *εὐθεῖα ἡ* ΑΒ, *καὶ παρὰ θέσει ἡ* ΓΔ, *καὶ ἦι λόγος τοῦ ὑπὸ* ΑΔΒ *πρὸς τὸ ἀπὸ* ΔΓ, *τὸ* Γ *ἅπτεται κωνικῆς γραμμῆς. ἐὰν οὖν ἡ μὲν* ΑΒ *στερηθῆι τῆς θέσεως καὶ τὰ* Α, Β *στερηθῆι τοῦ δοθέντος εἶναι, γένηται δὲ πρὸς θέσει εὐθείαις ταῖς* ΑΕ, ΕΒ, *τὸ* Γ *μετεωρισθὲν γίνεται πρὸς θέσει ἐπιφανείαι. τοῦτο δὲ ἐδείχθη.*

(313) *ἐὰν ἦι θέσει εὐθεῖα ἡ* ΑΒ, *καὶ δοθὲν τὸ* Γ *ἐν τῶι αὐτῶι ἐπιπέδωι, καὶ διαχθῆι ἡ* ΔΓ, *καὶ παρὰ θέσει ἀχθῆι ἡ* ΔΕ, 1006 *λόγος δὲ ἦι τῆς* ΓΔ *πρὸς* ΔΕ, *τὸ* Δ *ἅπτεται θέσει κωνικῆς τομῆς. δεικτέον δὴ ἥτις γραμμή. δειχθήσεται δὲ οὕτως, προγραφέντος τόπου τοῦδε.*

(314) *δύο δοθέντων τῶν* Α, Β, *καὶ ὀρθῆς τῆς* ΓΔ, *λόγος ἔστω τοῦ ἀπὸ* ΑΔ *πρὸς τὰ ἀπὸ* ΓΔ, ΔΒ. *λέγω ὅτι τὸ* Γ *ἅπτεται κώνου τομῆς, ἐάν τε ἦι ὁ λόγος ἴσος πρὸς ἴσον ἢ μείζων πρὸς ἐλάσσονα ἢ ἐλάσσων πρὸς μείζονα.*

(315) *ἔστω γὰρ πρότερον ὁ λόγος ἴσος πρὸς ἴσον. καὶ ἐπεὶ ἴσον ἐστὶν τὸ ἀπὸ* ΑΔ *τοῖς ἀπὸ* ΓΔ, ΔΒ, *κείσθω τῆι* ΒΔ *ἴση ἡ* ΔΕ. *ἴσον ἄρα ἐστὶ τὸ ὑπὸ* ΒΑΕ *τῶι ἀπὸ* ΔΓ. *τετμήσθω δίχα ἡ* ΑΒ *τῶι* Ζ. *δοθὲν ἄρα τὸ* Ζ. *καὶ ἔστιν διπλῆ ἡ* ΑΕ *τῆς* ΖΔ. *ὥστε τὸ ὑπὸ* ΒΑΕ *τὸ δίς ἐστιν ὑπὸ τῶν* ΑΒ, ΖΔ. *καὶ ἔστιν ἡ διπλῆ τῆς* ΑΒ *δοθεῖσα. τὸ ἄρα ὑπὸ δοθείσης καὶ τῆς* ΔΖ *ἴσον ἐστὶν τῶι ἀπὸ τῆς* ΔΓ. *τὸ* Γ *ἄρα ἅπτεται θέσει παραβολῆς ἐρχομένης διὰ τοῦ* Ζ.

συντεθήσεται δὴ ὁ τόπος οὕτως. ἔστω τὰ δοθέντα Α, Β, *ὁ δὲ λόγος ἔστω ἴσος πρὸς ἴσον, καὶ τετμήσθω ἡ* ΑΒ |*δίχα τῶι* Ζ. |182
τῆς δὲ ΑΒ *διπλῆ ἔστω ἡ* Ρ, *καὶ θέσει οὔσης εὐθείας τῆς* ΖΒ

‖ 1 *τοὺς* Ge *τὰς* A | *ἐπιφανείαι* Hu *ἐπιφάνειαν* A ‖ 2 *ἦι* add Hu ‖ 4 *τὰ* Α, Β] *ἑκάτερον τῶν* Α, Β coni. Hu app ‖ 5 *εὐθείαις* Tannery *εὐθεῖα* A ‖ 6 *ἐπιφανείαι*] *ἐπιφανείας* A ‖ 9 *παρὰ θέσει*] *πρὸς ὀρθὰς* Hu ‖ 10 *ἦι* Ge (Co) *ἦν* A | Δ Co Ε A ‖ 11 *δεικτέον – γραμμή*] *δείκνυται δὲ ὅτι γραμμῆς* A *δεικτέον* pro *δείκνυται* coni. Hu app | post *γραμμή* add *μέρος ποιεῖ τὸν τόπον* Ge ‖ 12 *τοῦδε* Ge (recc?) *τοῦ* $\bar{\Delta}\bar{E}$ A ‖ 14 ΓΔ, ΔΒ Co ΓΔΒ A ‖ 18 ΑΔ *τοῖς ἀπὸ* bis A corr Co | ΓΔ, ΔΒ Co ΓΔΒ A ‖ 20 *ἔστιν* Hu (Co) *ἔσται* A ‖ 22 *δοθείσης*] compendium *δοθ* (*θ* supr) A | ΔΖ] ΒΓ A ΖΔ Co ‖ 23 *παραβολῆς ἐρχομένης* Co *παραβολὴ ἐρχομένη* A be drawn about axis ZB so that if a point such as Γ is taken on it, and perpendicular ΓΔ is drawn, the rectangle contained by P, ZΔ equals the square of ΔΓ. And let BH be drawn at right angles. I say that part ΓH of the parabola <solves the locus>.

For let perpendicular ΓΔ be drawn,[1] and let ΔE be made equal to BΔ.[2] Then since AB is twice BZ,[3] and EB (twice) BΔ,[4] therefore AE too is twice ZΔ.[5] Hence the rectangle contained by BA, AE equals twice the rectangle contained by AB, ZΔ,[6] that is the square of ΔΓ.[7] Let the square of EΔ be added in common, which equals the square of ΔB.[8] Therefore the sum, the square of AΔ, equals the squares of ΓΔ, ΔB.[9] Thus curve ZΓH solves the locus.

(316) *(Prop. 237 a– b)* Again let the two given points be A, B, and (let) ΔΓ (be) at right angles, and let *the ratio of the square of AΔ to the squares of BΔ, ΔΓ* be, in the first case, less to greater, in the second greater to less. I say that Γ touches a section of a cone, in the first case an ellipse, in the second a hyperbola.

For since *the ratio of the square of AΔ to the squares of BΔ, ΔΓ* (is given),[1] let the (ratio) of the square of BΔ to the square of ΔE be the same as it.[2] Now in the first case BΔ is less than ΔE, in the second BΔ is greater than ΔE.[3] Then let ΔZ be made equal to EΔ.[4] Since the ratio of the square of AΔ to the squares of ΓΔ, ΔB (is given), and the (ratio) of the square of EΔ to the square of ΔB is the same as it, therefore the remainder, the ratio of the rectangle contained by ZA, AE to the square of ΔΓ is given.[5] But since the ratio of EΔ to ΔB,[6] and of ZΔ to ΔB,[7] and (so that) of ZB to BΔ (is given),[8] let the (ratio) of AB to BH be the same as it.[9] Hence the sum, the ratio of AZ to ΔH, is given.[10] Again, since the ratio of EΔ to ΔB is given,[11] therefore the ratio of EB to BΔ too is given.[12] Let the (ratio) of AΘ to BΘ be the same as it.[13] Then the ratio of AB to BΘ too is given.[14] Hence Θ is given.[15] And the remainder, the ratio of AE to ΘΔ, is given.[16] Therefore also the ratio of the rectangle contained by ZA, πεπερασμένης κατὰ τὸ Ζ, τῆς δὲ Ρ δεδομένης τῶι μεγέθει, γεγράφθω περὶ ἄξονα τὸν ΖΒ παραβολὴ ἡ ΗΖ ὥστε οἷον ἐὰν ἐπ' αὐτῆς σημεῖον ληφθῆι ὡς τὸ Γ, κάθετος δὲ ἀχθῆι ἡ ΓΔ, ἴσον εἶναι τὸ ὑπὸ Ρ, ΖΔ τῶι ἀπὸ ΔΓ· καὶ ἤχθω ὀρθὴ ἡ ΒΗ· λέγω ὅτι 1008 τὸ ΓΗ μέρος τῆς παραβολῆς [ἐστιν] <ποιεῖ τὸν τόπον>. ἤχθω γὰρ κάθετος ἡ ΓΔ, καὶ τῆι ΒΔ ἴση κείσθω ἡ ΔΕ. ἐπεὶ οὖν διπλῆ ἐστιν ἡ μὲν ΑΒ τῆς ΒΖ, ἡ δὲ ΕΒ τῆς ΒΔ, διπλῆ ἄρα καὶ ἡ ΑΕ τῆς ΖΔ. τὸ ἄρα ὑπὸ ΒΑΕ ἴσον ἐστὶν τῶι δὶς ὑπὸ τῶν ΑΒ, ΖΔ, τουτέστιν τῶι <ἀπὸ> ΔΓ. κοινὸν προσκείσθω τὸ ἀπὸ ΕΔ ἴσον ὂν τῶι ἀπὸ ΔΒ. ὅλον ἄρα τὸ ἀπὸ ΑΔ ἴσον ἐστὶν τοῖς ἀπὸ ΓΔ, ΔΒ. ἡ ΖΓΗ ἄρα γραμμὴ ποιεῖ τὸν τόπον.

(316) ἔστω δὴ πάλιν τὰ δύο δοθέντα σημεῖα τὰ Α, Β, καὶ [ἐφάπτεται] ἡ ΔΓ πρὸς ὀρθάς, λόγος δὲ ἔστω τοῦ ἀπὸ ΑΔ πρὸς τὰ ἀπὸ ΒΔ, ΔΓ, ἐπὶ μὲν τῆς πρώτης πτώσεως ἐλάσσων πρὸς μείζονα, ἐπὶ δὲ τῆς δευτέρας μείζων πρὸς ἐλάσσονα. λέγω ὅτι τὸ Γ ἅπτεται κώνου τομῆς, ἐπὶ μὲν τῆς πρώτης πτώσεως ἐλλείψεως, ἐπὶ δὲ τῆς δευτέρας ὑπερβολῆς. ἐπεὶ γὰρ λόγος ἐστὶν τοῦ ἀπὸ ΑΔ πρὸς τὰ ἀπὸ ΒΔ, ΔΓ, ὁ αὐτὸς αὐτῶι γεγονέτω ὁ τοῦ ἀπὸ ΒΔ πρὸς τὸ ἀπὸ ΔΕ. ἐπὶ μὲν οὖν τῆς πρώτης πτώσεως, ἐλάσσων ἐστὶν ἡ ΒΔ τῆς ΔΕ, ἐπὶ δὲ τῆς δευτέρας μείζων ἐστὶν ἡ ΒΔ τῆς ΔΕ. κείσθω οὖν τῆι ΕΔ ἴση ἡ ΔΖ. ἐπεὶ λόγος ἐστὶν τοῦ ἀπὸ ΑΔ πρὸς τὰ ἀπὸ ΓΔ, ΔΒ, καὶ ἔστιν αὐτῶι ὁ 1010 αὐτὸς ὁ τοῦ ἀπὸ ΕΔ πρὸς τὸ ἀπὸ ΔΒ, καὶ λοιπὸς ἄρα τοῦ ὑπὸ ΖΑΕ πρὸς τὸ ἀπὸ ΔΓ λόγος ἐστὶν δοθείς. ἐπεὶ δὲ λόγος ἐστὶν τῆς ΕΔ πρὸς ΔΒ, καὶ τῆς ΖΔ πρὸς ΔΒ, καὶ τῆς ΖΒ πρὸς ΒΔ, ὁ αὐτὸς αὐτῶι γεγονέτω ὁ τῆς ΑΒ πρὸς ΒΗ. καὶ ὅλης ἄρα τῆς ΑΖ πρὸς ΔΗ λόγος ἐστὶν δοθείς. πάλιν ἐπεὶ λόγος ἐστὶν τῆς ΕΔ πρὸς ΔΒ δοθείς, καὶ τῆς ΕΒ ἄρα πρὸς ΒΔ λόγος ἐστὶν δοθείς. ὁ αὐτὸς αὐτῶι γεγονέτω ὁ τῆς ΑΘ πρὸς ΒΘ. λόγος ἄρα καὶ τῆς ΑΒ πρὸς ΒΘ ἐστὶν δοθείς. δοθὲν ἄρα τὸ Θ. καὶ λοιπὸς τῆς ΑΕ

‖ 9 ἀπὸ (ΔΓ) add Ge (Co) ‖ 11 ΖΓΗ Co ΖΓ η A ‖ 13 ἐφάπτεται — ὀρθάς] ἐφάπτεται ἡ ΔΓ καὶ ὀρθὴ A κατήχθω ὀρθὴ ἡ ΔΓ Co ‖ 14 ἐλάσσων... μείζονα... μείζων... ἐλάσσονα] μείζων... ἐλάσσονα... ἐλάσσων... μείζονα Co ‖ 18 ΒΔ, ΔΓ Co ΒΔΓ A ‖ 19 ΒΔ... ΔΕ] ΕΔ... ΔΒ Co ‖ 20 ἐλάσσων... ΒΔ... ΔΕ] μείζων... ΕΔ... ΔΒ Co | ΒΔ Co ΒΑ A ‖ 21 μείζων... ΒΔ... ΔΕ] ἐλάσσων... ΕΔ... ΔΒ Co | οὖν] ὅτι A del Co ‖ 22 ΓΔ, ΔΒ Co ΓΔΒ A ‖ 23 λοιπὸς Co λοιπὸν A | τοῦ Co τὸ A ‖ 24 δοθείς Co (k) δοθέντα A ‖ 25 καὶ τῆς ΖΔ πρὸς ΔΒ del Co ‖ 27 δοθεις compendium A ‖ 28 ΔΒ δοθείς Co (restituens lacunam in k) ΔΘ δοθέντα A | καὶ τῆς ΕΒ — δοθείς del Co | δοθείς Co δοθέντα A ‖ 29 ΑΘ Co ΑΒ A ‖ 30 δοθείς Ge (S) δοθὲν A | δοθὲν ἄρα τὸ Θ del Hu | δοθὲν (ἄρα) compendium A | λοιπὸς Ge (S) λοιπὴ A | post λοιπὸς add ἄρα Hu

AE to the rectangle contained by ΘΔ, ΔH is given.[17] But the ratio of the rectangle contained by ZA, AE to the square of ΓΔ is given.[18] Therefore the ratio of the rectangle contained by HΔ, ΔΘ to the square of ΔΓ too is given.[19] And Θ, H are two given (points).[20] Hence in the first case Γ touches an ellipse, in the second a hyperbola. *Greater to less, less to greater.*

(317) *(Prop. 237 c – d)* The synthesis of the locus will be made as follows. Let the two given points be A, B, the given ratio that of <the square> of PT to <the square> of TΣ, in the first case *less to greater*, in the second *greater to less*. And let TT be made equal to PT, and let AB be made to BH as TΣ is to ΣT. And let AΘ be made to ΘB as is PT to TΣ. And let there be drawn about axis ΘH, in the first case an ellipse, in the second a hyperbola, so that if a point such as Γ is taken on it, and perpendicular ΓΔ is drawn, the ratio of the rectangle contained by ΘΔ, ΔH to the square of ΔΓ is compounded out of that which TΣ has to ΣT and that which TΣ has to ΣP and that which the given ratio has, which is that of the square of PT to the square of TΣ. Let BK be drawn at right angles. I say that ΘK solves the assignment.

For let perpendicular ΓΔ be drawn,[1] and let ZB be made to BΔ as AB is to BH,[2] and EΔ to ΔB as AΘ is to ΘB.[3] Hence the ratio of ΔH to AZ is the same as that of HB to BA,[4] that is that of TΣ to ΣT.[5] Whereas the ratio of ΘΔ to AE is the same as that of TΣ to ΣP,[6] for this was proved in the analysis. Hence the ratio of the rectangle contained by ΘΔ, ΔH to the rectangle contained by ZA, AE is compounded out of that which TΣ has to ΣT and TΣ to ΣP.[7] But since the rectangle contained by ΘΔ, ΔH has to the square of ΔΓ the ratio compounded out of that which TΣ has to ΣT and TΣ to ΣP and the given ratio, that of the square of PT to the square of TΣ[8] [less to greater], <while the (ratio)> of the rectangle contained by πρὸς ΘΔ λόγος ἐστὶν δοθείς· καὶ τοῦ ὑπὸ ΖΑΕ ἄρα πρὸς τὸ ὑπὸ ΘΔΗ λόγος ἐστὶ δοθείς· τοῦ δὲ ὑπὸ ΖΑΕ πρὸς τὸ ἀπὸ ΓΔ λόγος ἐστὶν δοθείς· καὶ τοῦ ὑπὸ ΗΔΘ <ἄρα> πρὸς τὸ ἀπὸ ΔΓ λόγος ἐστὶν δοθείς. καὶ ἔστιν δύο δοθέντα τὰ Θ, Η. ἐπὶ μὲν |ἄρα τῆς πρώτης πτώσεως τὸ Γ ἅπτεται ἐλλείψεως, ἐπὶ δὲ τῆς δευτέρας ὑπερβολῆς· μείζων πρὸς ἐλάσσονα, ἐλάσσων πρὸς μείζονα.

(317) συντεθήσεται δὴ ὁ τόπος οὕτως. ἔστω τὰ μὲν δύο δοθέντα σημεῖα τὰ Α, Β, ὁ δὲ δοθεὶς λόγος ὁ τοῦ <ἀπὸ> ΡΤ πρὸς <τὸν ἀπὸ> ΤΣ, ἐπὶ μὲν τῆς πρώτης πτώσεως, ἐλάσσων πρὸς μείζονα, ἐπὶ δὲ τῆς δευτέρας μείζων πρὸς ἐλάσσονα. καὶ τῆι ΡΤ ἴση κείσθω ἡ ΤΥ, καὶ πεποιήσθω ὡς ἡ ΤΣ πρὸς τὴν ΣΥ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΗ. πεποιήσθω δὲ καὶ ὡς ἡ ΡΤ πρὸς τὴν ΤΣ, οὕτως ἡ ΑΘ πρὸς τὴν ΘΒ. καὶ γεγράφθω περὶ ἄξονα τὸν ΘΗ, ἐπὶ μὲν τῆς πρώτης πτώσεως ἔλλειψις, ἐπὶ δὲ τῆς δευτέρας ὑπερβολή, ὥστε οἷον ἐὰν ἐπ' αὐτῆς ληφθῆι σημεῖον ὡς τὸ Γ, καὶ κάθετος ἀχθῆι ἡ ΓΔ, λόγον εἶναι τοῦ ὑπὸ τῶν ΘΔΗ πρὸς τὸ ἀπὸ ΔΓ τὸν συνημμένον ἔκ τε τοῦ ὃν ἔχει ἡ ΤΣ πρὸς ΣΥ καὶ ἐξ οὗ ὃν ἔχει ἡ ΤΣ πρὸς ΣΡ καὶ ἐξ οὗ ὃν ἔχει ὁ δοθεὶς λόγος ὅς ἐστιν ὁ τοῦ ἀπὸ ΡΤ πρὸς τὸ ἀπὸ ΤΣ. κατήχθω ὀρθὴ ἡ ΒΚ. λέγω ὅτι ἡ ΘΚ ποιεῖ τὸ ἐπίταγμα. ἤχθω γὰρ κάθετος ἡ ΓΔ, 1012
καὶ πεποιήσθω ὡς μὲν ἡ ΑΒ πρὸς τὴν ΒΗ, οὕτως ἡ ΖΒ πρὸς τὴν ΒΔ, ὡς δὲ ἡ ΑΘ πρὸς τὴν ΘΒ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΒ. ὥστε ἔσται ὁ μὲν τῆς ΔΗ πρὸς τὴν ΑΖ λόγος ὁ αὐτὸς τῶι τῆς ΗΒ πρὸς τὴν ΒΑ, τουτέστιν τῶι τῆς ΤΣ πρὸς ΣΥ. ὁ δὲ τῆς ΘΔ πρὸς ΑΕ λόγος ὁ αὐτός ἐστιν τῶι τῆς ΤΣ πρὸς ΣΡ· τοῦτο γὰρ ἐν τῆι ἀναλύσει ἀπεδείχθη· ὥστε τοῦ ὑπὸ ΘΔΗ πρὸς τὸ ὑπὸ ΖΑΕ λόγος συνῆπται ἐξ οὗ ὃν ἔχει ἡ ΤΣ πρὸς ΣΥ καὶ ἡ ΤΣ πρὸς ΣΡ. ἀλλ' ἐπεὶ τὸ ὑπὸ ΘΔΗ πρὸς τὸ ἀπὸ ΔΓ τὸν συνημμένον ἔχει λόγον ἐξ οὗ ὃν ἔχει ἡ ΤΣ πρὸς ΣΥ καὶ ἡ ΤΣ πρὸς ΣΡ καὶ ἐκ τοῦ δοθέντος

‖ 1 ΘΔ Co ΕΔ Α | δοθείς compendium A | τοῦ (ὑπὸ ΖΑΕ) Ge (recc?) τὸ A ‖ 2 δοθείς compendium A | τοῦ (δὲ) Ge (BS) τὸ A ‖ 3 δοθείς compendium A | ἄρα add Hu ‖ 4 δοθείς compendium A ‖ 6 μείζων – μείζονα del Co ‖ 8 δὴ] δὲ A ‖ 9 τοῦ ἀπὸ – ἀπὸ Co τῆς ΡΤ πρὸς ΤΣ A ‖ 10 ἐλάσσων… μείζονα… μείζων… ἐλάσσονα] μείζων… ἐλάσσονα… ἐλάσσων… μείζονα Co ‖ 20 ΡΤ Co ΡΣ A ‖ 21 ΘΚ Co ΒΚ A ‖ 26 ἐστιν secl Hu | τοῦτο Hu app τὸ αὐτὸ A ‖ 29 ἐπεὶ Ge (BS) ἐπὶ A | ΔΓ Co ΑΓ A ‖ 30 post ΣΡ add καὶ ἐξ οὗ ὃν ἔχει ὁ δοθεὶς λόγος Co | ἐκ τοῦ δοθέντος λόγου τοῦ] ἔστιν ὁ δοθεὶς λόγος ὁ A

ΘΔ, ΔΗ to the square of ΔΓ is compounded out of that which the rectangle contained by ΘΔ, ΔΗ has to the rectangle contained by ΖΑ, ΑΕ and the rectangle contained by ΖΑ, ΑΕ to the square of ΔΓ,[9] and the ratio of the rectangle contained by ΘΔ, ΔΗ to the rectangle contained by ΖΑ, ΑΕ is the same as that compounded out of that which ΤΣ has to ΣΥ and ΤΣ to ΣΡ,[10] therefore the remaining ratio of the rectangle contained by ΕΑ, ΑΖ to the square of ΔΓ is the same as that of the square of ΡΥ to the square of ΤΣ,[11] that is that of the square of ΕΔ to the square of ΔΒ.[12] And all to all, therefore, as is the square of ΑΔ to the squares of ΒΔ, ΔΓ, so is the square of ΡΥ to the square of ΤΣ,[13] that is the given ratio. Thus part ΘΚ of the section solves the locus.

(318) *(Prop. 238 a)* These things being so, we go back to the original (problem). Let line ΑΒ be (given) in position, and Γ given in the same plane, and let ΔΓ be drawn across, ΔΕ a perpendicular, and let the ratio of ΓΔ to ΔΕ (be given). I say that Δ touches a section of a cone, and if the ratio is equal to equal a parabola, if less to greater an ellipse, if greater to less a hyperbola.

For first let the ratio be equal to equal, that is first let ΓΔ equal ΔΕ. To prove that Δ touches a parabola.

Let perpendicular ΓΖ be drawn[1] – hence it is (given) in position[2] – and ΔΗ parallel to ΑΒ.[3] And since the square of ΕΔ equals the square of ΔΓ,[4] and ΕΔ equals ΖΗ,[5] and the square of ΔΓ equals the squares of ΔΗ and ΗΓ,[6] therefore the square of ΖΗ equals the squares of ΔΗ, ΗΓ.[7] And ΖΓ is (given) in position,[8] and Ζ, Γ are two given (points).[9] Thus Δ touches a parabola;[10] for this was proved above.

(Prop. 238 b) The synthesis will be made as follows. Let the (line given) in position be ΑΒ, the given (point) Γ, and let perpendicular ΓΖ be drawn, and with ΓΖ (given) in position and two given (points) Ζ, Γ, let parabola ΔΘ be found so that if a point such as Δ is taken (on it), and perpendicular ΔΗ is drawn, the square of ΖΗ equals the squares of ΔΗ, ΗΓ. I say that curve ΔΘ <solves> the locus, that is, whatever (line) ΓΔ is

λόγου τοῦ τοῦ ἀπὸ ΡΤ πρὸς τὸ ἀπὸ ΤΣ [ἐλάσσων πρὸς μείζονα], <ὁ δὲ> τοῦ ὑπὸ ΘΔΗ πρὸς τὸ ἀπὸ ΔΓ συνῆπται ἐξ οὗ ὃν ἔχει τὸ ὑπὸ ΘΔΗ πρὸς τὸ ὑπὸ ΖΑΕ καὶ τὸ ὑπὸ ΖΑΕ πρὸς τὸ ἀπὸ ΔΓ, καὶ ἔστιν ὁ τοῦ ὑπὸ τῶν ΘΔΗ πρὸς τὸ ὑπὸ ΖΑΕ λόγος ὁ αὐτὸς τῶι συνημμένωι ἐξ οὗ ὃν ἔχει ἡ ΤΣ πρὸς ΣΤ καὶ ἡ ΤΣ πρὸς ΣΡ, λοιπὸς ἄρα τοῦ ὑπὸ ΕΑΖ πρὸς τὸ ἀπὸ ΔΓ λόγος ὁ αὐτός ἐστιν τῶι τοῦ ἀπὸ ΡΤ πρὸς τὸ ἀπὸ ΤΣ, τουτέστιν τῶι τοῦ ἀπὸ ΕΔ πρὸς τὸ ἀπὸ ΔΒ. καὶ πάντα πρὸς πάντα, ὡς ἄρα τὸ ἀπὸ ΑΔ πρὸς τὰ ἀπὸ ΒΔ, ΔΓ, οὕτως ἐστὶν τὸ ἀπὸ ΡΤ πρὸς τὸ |ἀπὸ ΤΣ, |183
τουτέστιν ὁ δοθεὶς λόγος. ὥστε τὸ ΘΚ μέρος τῆς τομῆς ποιεῖ τὸν τόπον.

(318) τούτων οὕτως ἐχόντων, ἐλευσόμεθα ἐπὶ τὸ ἐξ ἀρχῆς. ἔστω θέσει εὐθεῖα ἡ ΑΒ, καὶ δοθὲν τὸ Γ ἐν τῶι αὐτῶι ἐπιπέδωι. καὶ διήχθω ἡ ΔΓ, κάθετος ἡ ΔΕ, λόγος δὲ ἔστω τῆς ΓΔ πρὸς ΔΕ. λέγω ὅτι τὸ Δ ἅπτεται κώνου τομῆς, καὶ ἐὰν μὲν ὁ λόγος ἦι ἴσου πρὸς ἴσον, παραβολῆς, ἐὰν δὲ ἐλάσσονος πρὸς 1014
μείζονα, ἐλλείψεως, ἐὰν δὲ μείζονος πρὸς ἐλάσσονα, ὑπερβολῆς. ἔστω γὰρ πρότερον ὁ λόγος ἴσου πρὸς ἴσον, τουτέστιν ἔστω πρότερον ἴση ἡ ΓΔ τῆι ΔΕ. δεῖξαι ὅτι τὸ Δ ἅπτεται παραβολῆς. ἤχθω κάθετος ἡ ΓΖ — θέσει ἄρα ἐστί — τῆι δὲ ΑΒ παράλληλος ἡ ΔΗ. καὶ ἐπεὶ τὸ ἀπὸ ΕΔ ἴσον τῶι ἀπὸ ΔΓ, ἴση δὲ ἡ μὲν ΕΔ τῆι ΖΗ, τὸ δὲ ἀπὸ ΔΓ ἴσον τοῖς ἀπὸ ΔΗ, ΗΓ, τὸ ἄρα ἀπὸ ΖΗ ἴσον ἐστὶν τοῖς ἀπὸ ΔΗ, ΗΓ. καὶ ἔστιν θέσει ἡ ΖΓ. καὶ δύο δοθέντα τὰ Ζ, Γ. τὸ Δ ἄρα ἅπτεται παραβολῆς. τοῦτο γὰρ προδέδεικται.

συντεθήσεται δὴ οὕτως. ἔστω ἡ τῆι θέσει ἡ ΑΒ, τὸ δὲ δοθὲν τὸ Γ, καὶ ἤχθω κάθετος ἡ ΓΖ, καὶ θέσει οὔσης τῆς ΓΖ καὶ δύο δοθέντων τῶν Ζ, Γ, εὑρήσθω παραβολὴ ἡ ΔΘ, ὥστε οἷον ἐὰν ληφθῆι σημεῖον ὡς τὸ Δ, ἀχθῆι δὲ κάθετος ἡ ΔΗ, ἴσον εἶναι τὸ ἀπὸ ΖΗ τοῖς ἀπὸ ΔΗ, ΗΓ. λέγω ὅτι ἡ ΔΘ γραμμὴ

‖ 1 ἐλάσσων πρὸς μείζονα del Co | post μείζονα add καὶ Hu ‖ 2 τοῦ] τὸ A ‖ 6 λοιπὸς Co λοιπὸν A | ΕΑΖ Co ΘΔΗ A ‖ 8 ΔΒ Co ΑΒ A ‖ 9 ΒΔ, ΔΓ (vel ΒΔΓ)] ΒΗ A ΓΔ, ΔΒ Co ‖ 16 ἴσου] ἴσος Ge (recc?) | παραβολῆς Hu παραβολή A | ἐλάσσονος] ἐλάσσων Ge (BS) ἐλασσον A ‖ 17 μείζονα Ge (recc?) | ἐλλείψεως Hu ἐλλείπει A | μείζονος] μείζων A ‖ 18 ὑπερβολῆς Hu ὑπερβολή A | γὰρ Hu τῶν A ‖ 21 δὲ (ΑΒ) Ge (recc?) ΔΕ A ‖ 23 ΔΗ, ΗΓ Co ΔΗΓ A ‖ 30 εἶναι] ἐστὶν A | ΔΗ, ΗΓ Co ΔΗΓ A drawn across, and perpendicular ΔE (drawn), ΓΔ equals ΔE.

Let perpendicular ΔH be drawn.[1] Then because of the parabola the square of ZH equals the squares of ΔH, HΓ.[2] And EΔ equals ZH,[3] and the square of ΔΓ equals the squares of ΔH, HΓ.[4] Therefore the square of ΔΓ equals the square of ΔE.[5] Thus ΓΔ equals ΔE.[6] Thus curve ΔΘ solves the locus.

The Seventh (Book) of the Collection of Pappus of Alexandria, which contains the arrangement and the content and the lemmas of the Domain of Analysis.

<ποιεῖ> τὸν τόπον, τουτέστιν οἵα τις ἂν διαχθῆι ὡς ἡ ΓΔ, καὶ κάθετος ἡ ΔΕ, ἴση ἐστὶν ἡ ΓΔ τῆι ΔΕ. ἤχθω κάθετος ἡ ΔΗ. διὰ ἄρα τῆς παραβολῆς ἴσον ἐστὶν τὸ ἀπὸ ΖΗ τοῖς ἀπὸ ΔΗ, ΗΓ. καὶ ἔστιν τῆι μὲν ΖΗ ἴση ἡ ΕΔ, τοῖς δὲ ἀπὸ ΔΗ, ΗΓ ἴσον τὸ ἀπὸ ΔΓ. τὸ ἄρα ἀπὸ ΔΓ ἴσον ἐστὶν τῶι ἀπὸ ΔΕ. ἴση ἄρα ἐστὶν ἡ ΓΔ τῆι ΔΕ. ἡ ἄρα ΔΘ γραμμὴ [τομὴ] ποιεῖ τὸν τόπον.

ΠΑΠΠΟΥ ΑΛΕΞΑΝΔΡΕΩΣ ΣΥΝΑΓΩΓΗΣ Ζ´ Ο ΠΕΡΙΕΧΕΙ ΤΗΝ ΤΑΞΙΝ ΚΑΙ ΤΗΝ ΠΕΡΙΟΧΗΝ ΚΑΙ ΤΑ ΛΗΜΜΑΤΑ ΤΟΥ ΑΝΑΛΥΟΜΕΝΟΥ ΤΟΠΟΥ

‖ 1 ποιεῖ add Ge (Co) | ἂν Hu ἐὰν A ‖ 3 ἴσον Hu ἴση A | τὸ (ἀπὸ) Hu παράλληλος A | ΔΗ, ΗΓ Co ΔΗΓ A ‖ 4 ΔΗ, ΗΓ Co ΔΗΓ A ‖ 6 τομὴ del Hu

(319) Lemma of the (Domain) of Analysis

(Prop. 239) Let there be right triangle ABΓ, that has angle ABΓ right, and let AZ be to ZB, and BH to HΓ, as AB is to BΓ; and let AEH, ΓEZ, BEΔ be joined. That BΔ is a perpendicular upon AΓ.

Since as AB is to BΓ, so is AZ to ZB, and BH to HΓ,[1] therefore as AZ is to BZ, so is BH to HΓ.[2] *Componendo*[3] and *alternando*, as AB is to BΓ, so is ZB to HΓ.[4] But as AB is to BΓ, so is BH to HΓ.[5] Therefore as ZB is to HΓ, so is BH to HΓ.[6] Hence ZB equals BH.[7] Therefore with ZH joined, angle BZΘ equals angle BHΘ.[8] And straight line ZΘ is greater than ΘH.[14] For if we draw HIK through H parallel to AΓ,[9] angle BΘH, which equals the opposite angles ΘHI and ΘIH,[10] is greater than angle HΘI,[11] that is acute angle ZΘB.[12] Hence also the remaining angle HBΘ is less than angle ZBΘ.[13] (Let) ZH (be) bisected by Λ.[15] Then the circle drawn with center Λ, radius one of ΛZ, ΛB, ΛH, will pass through Δ too, and quadrilateral ΔZBH will be in a circle;[16] for this (will be proved) next. Angle BΔZ equals angle BΔH.[17] And each is half a right angle[19] (III 21) – for each of angles BHZ, BZH is half a right angle[18] – and (so) angle ZΔH is right.[20] Then I say that angle AΔB is right.

For if not, then it is either greater or less than a right angle. First let it be greater than a right angle, and let angle BΔM be right,[21] with HΓ and MΔ produced and intersecting at N. Then since right triangle MBΔ is similar to right triangle MBN,[22] and each of angles BΔZ, ZΔM is half a right angle,[23] therefore as MZ is to ZB, so is MΔ to ΔB.[24] But as MΔ is to ΔB, so is BΔ to ΔN,[25] that is BH to HN;[27] for angle BΔN too is bisected by ΔH.[26] Hence as MZ is to ZB, so is BH to HN.[28] Again, since as AZ is to ZB, so BH was stipulated to be to HΓ,[29] therefore MZ has to ZB a lesser ratio than has BH to HN;[30] which is impossible. For it was proved that as MZ is to ZB, so is BH to HN. Thus angle BΔA is not greater than a right angle. Similarly we shall prove that angle AΔB is not less than a right angle, by drawing ΞΔO through Δ and at right angles to ΔB. For again as ΞZ is to ZB, so will BH be to HO. And AZ will be shown to have a much

(319) |ΛΗΜΜΑ ΤΟΥ ΑΝΑΛΥΟΜΕΝΟΥ |183v 1016

ἔστω τρίγωνον ὀρθογώνιον τὸ ΑΒΓ, ὀρθὴν ἔχον τὴν ὑπὸ ΑΒΓ γωνίαν, καὶ ἔστω ὡς ἡ ΑΒ πρὸς ΒΓ, οὕτως ἡ ΑΖ πρὸς τὴν ΖΒ, καὶ ἡ ΒΗ πρὸς ΗΓ, καὶ ἐπεζεύχθωσαν αἱ ΑΕΗ, ΓΕΖ, ΒΕΔ. ὅτι ἡ ΒΔ κάθετός ἐστιν ἐπὶ τὴν ΑΓ. ἐπεὶ ὡς ἡ ΑΒ πρὸς ΒΓ, ἡ ΑΖ πρὸς ΖΒ, καὶ ἡ ΒΗ πρὸς ΗΓ, ὡς ἄρα ἡ ΑΖ πρὸς ΒΖ, ἡ ΒΗ πρὸς ΗΓ. συνθέντι καὶ ἐναλλὰξ ὡς ἡ ΑΒ πρὸς ΒΓ, ἡ ΖΒ πρὸς ΗΓ. ἀλλ' ὡς ἡ ΑΒ πρὸς ΒΓ, ἡ ΒΗ πρὸς ΗΓ. ὡς ἄρα ἡ ΖΒ πρὸς ΗΓ, ἡ ΒΗ πρὸς ΗΓ. ἴση ἄρα ἡ ΖΒ τῆι ΒΗ. ὥστε ἐπιζευχθείσης τῆς ΖΗ καὶ γωνία ἡ ὑπὸ ΒΖΘ τῆι ὑπὸ ΒΗΘ ἐστὶν ἴση. καὶ μείζων ἡ ΖΘ εὐθεῖα τῆς ΘΗ. ἐὰν γὰρ διὰ τοῦ Η τῆι ΑΓ παράλληλον ἀγάγωμεν τὴν ΗΙΚ, ἡ ὑπὸ ΒΘΗ γωνία ταῖς ἀπεναντίον ὑπὸ ΘΗΙ, ΘΙΗ ἴση οὖσα μείζων ἐστὶν τῆς ὑπὸ ΗΘΙ, τουτέστιν τῆς ὑπὸ ΖΘΒ ὀξείας. ὥστε καὶ λοιπὴν τὴν ὑπὸ ΗΒΘ ἐλάσσονα γίνεσθαι τῆς ὑπὸ ΖΒΘ. δίχα ἡ ΖΗ τῶι Λ. ὁ ἄρα κέντρωι τῶι Λ, διαστήματι δὲ ἑνὶ τῶν ΛΖ, ΛΒ, ΛΗ γραφόμενος κύκλος ἥξει καὶ διὰ τοῦ Δ. καὶ ἔσται <ἐν> κύκλωι τὸ ΔΖΒΗ τετράπλευρον. τοῦτο γὰρ ἑξῆς. ἴση ἐστὶν ἡ ὑπὸ ΒΔΖ γωνία τῆι ὑπὸ ΒΔΗ. καὶ ἔστιν ἑκατέρα ἡμίσεια ὀρθῆς — καὶ γὰρ ἑκατέρα τῶν ὑπὸ ΒΗΖ, ΒΖΗ ἡμίσειά ἐστιν ὀρθῆς — 1018 καὶ ὀρθὴ ἡ ὑπὸ ΖΔΗ. λέγω οὖν ὅτι ἡ ὑπὸ ΑΔΒ ὀρθή ἐστιν. εἰ γὰρ μή, ἤτοι μείζων ἐστὶν ἢ ἐλάσσων ὀρθῆς. ἔστω πρότερον μείζων ὀρθῆς, καὶ ἔστω ὀρθὴ ἡ ὑπὸ ΒΔΜ, τῶν ΗΓ, ΜΔ ἐκβληθεισῶν καὶ συμπιπτουσῶν κατὰ τὸ Ν. ἐπεὶ οὖν τὸ ΜΒΔ τρίγωνον ὀρθογώνιον ὅμοιόν ἐστιν τῶι ΜΒΝ τριγώνωι ὀρθογωνίωι, καὶ ἔστιν ἡμίσεια ὀρθῆς ἑκατέρα τῶν ὑπὸ ΒΔΖ, ΖΔΜ, ὡς ἄρα ἡ ΜΖ πρὸς ΖΒ, ἡ ΜΔ πρὸς ΔΒ. ἀλλ' ὡς ἡ ΜΔ πρὸς ΔΒ, ἡ ΒΔ πρὸς ΔΝ, τουτέστιν ἡ ΒΗ πρὸς ΗΝ. δίχα γὰρ τέτμηται καὶ ἡ ὑπὸ ΒΔΝ γωνία τῆι ΔΗ. ὡς ἄρα ἡ ΜΖ πρὸς ΖΒ, ἡ ΒΗ πρὸς ΗΝ. πάλιν ἐπεὶ ὡς ἡ ΑΖ πρὸς ΖΒ, ἡ ΒΗ πρὸς ΗΓ ὑπόκειται, ἡ ΜΖ ἄρα πρὸς ΖΒ ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΒΗ πρὸς ΗΝ. ὅπερ ἀδύνατον. ἐδείχθη γὰρ ὡς ἡ ΜΖ πρὸς ΖΒ, ἡ ΒΗ πρὸς ΗΝ. οὐκ ἄρα μείζων ἐστὶν ὀρθῆς ἡ ὑπὸ ΒΔΑ γωνία. ὁμοίως δὴ δείξομεν ὅτι οὐδὲ ἐλάσσων ἐστὶν ὀρθῆς ἡ ὑπὸ ΑΔΒ, διὰ τοῦ Δ τῆι ΔΒ πρὸς ὀρθὰς ἀγαγόντες τὴν ΞΔΟ. ἔσται γὰρ πάλιν ὡς ἡ ΞΖ πρὸς ΖΒ, ἡ ΒΗ πρὸς ΗΟ. καὶ <δειχθήσεται ἡ> ΑΖ πρὸς ΖΒ πολλῶι

‖ 1 (319-321) om Co ‖ 9 ὥστε — finem capitis secl Hu ‖ 13 ΖΘΒ] ΖΒΘ A ‖ 16 ἐν add Ge (S) ‖ 19 ἡμίσειά Ge (BS) ἡμι in fine versus A ‖ 24 τριγώνωι Ge (BS) τριγώνων A ‖ 27 ΒΔ Hu ΜΔ A ‖ 28 ΔΗ Ge (S) ΒΗ A ‖ 34 τὴν (ΞΔΟ) Ge (S) τῶν A | ΞΔΟ Hu ΔΞΟ A ‖ 35 ΖΒ Hu ΖΘ A | δειχθήσεται add Hu ἡ addidi | (Α)Ζ in ras. A | ΖΒ in ras. A lesser ratio to ZB than BH to HΓ; which is impossible. For it was stipulated that as AZ is to ZB, so is BH to HΓ.

(320) *(Prop. 240)* As AB is to BΓ, so let AZ be to ZB, and BH to HΓ. That ZB equals BH.

For since as AZ is to ZB, so is BH to HΓ,[1] *componendo*[2] and *alternando*, as AB is to BΓ, that is as BH is to HΓ,[4] so is ZB to HΓ.[3] Thus ZB equals BH.

(321) *(Prop. 253)* (Let there be) right triangle ABΓ, and (angle) B right, and let AZ be to ZB and BH to HΓ as AB is to BΓ. And let ΓEZ, AEH, BEΔ be joined. That BΔ is a perpendicular upon AΓ.

Let it be so.[1] Then triangles ABΔ, BΔΓ are similar to the whole (triangle) ABΓ and to each other.[2] Hence as AB is to BΓ, that is as AZ is to ZB,[4] so is AΔ to ΔB.[3] Thus angle AΔB is bisected by ZΔ.[5] Therefore angle ZΔB is half a right angle.[6] For the same reasons also angle BΔΓ is bisected by ΔH.[7] Thus angle BΔH is half a right angle.[8] Hence angle ZΔH is right.[9] But also angle ZBH is right.[10] Therefore quadrilateral BZΔH is in a circle.[11] And angle ZΔB equals angle BΔH.[12] Thus also ZB equals BH.[13] But it is (equal), because of the (lemma) proved above.

ἐλάσσονα λόγον ἔχουσα ἤπερ ἡ ΒΗ πρὸς ΗΓ. ὅπερ ἀδύνατον. ὑπόκειται γὰρ ὡς ἡ ΑΖ πρὸς ΖΒ, ἡ ΒΗ πρὸς ΗΓ.

(320) |ἔστω ὡς ἡ ΑΒ πρὸς ΒΓ, ἡ ΑΖ πρὸς ΖΒ, καὶ ἡ ΒΗ πρὸς ΗΓ. |184
ὅτι ἴση ἐστὶν ἡ ΖΒ τῆι ΒΗ. ἐπεί ἐστιν ὡς ἡ ΑΖ πρὸς ΖΒ, ἡ ΒΗ πρὸς ΗΓ, συνθέντι καὶ ἐναλλὰξ ὡς ἡ ΑΒ πρὸς ΒΓ, τουτέστιν ὡς ἡ ΒΗ πρὸς ΗΓ, ἡ ΖΒ πρὸς ΗΓ. ἴση ἄρα ἡ ΖΒ τῆι ΒΗ.

(321) τρίγωνον ὀρθογώνιον τὸ ΑΒΓ, ὀρθὴ ἡ Β, καὶ ἔστω ὡς ἡ ΑΒ πρὸς ΒΓ, ἡ ΑΖ πρὸς ΖΒ καὶ ἡ ΒΗ πρὸς ΗΓ. καὶ ἐπεζεύχθωσαν αἱ ΓΕΖ, ΑΕΗ, ΒΕΔ. ὅτι ἡ ΒΔ κάθετός ἐστιν ἐπὶ τὴν ΑΓ. γεγονέτω. ὅμοια ἄρα τὰ ΑΒΔ, ΒΔΓ τρίγωνα τῶι ὅλωι ΑΒΓ καὶ
ἀλλήλοις. ὡς ἄρα ἡ ΑΒ πρὸς ΒΓ, τουτέστιν ἡ ΑΖ πρὸς ΖΒ, οὕτως 1020
ἡ ΑΔ πρὸς ΔΒ. ἡ ἄρα ὑπὸ ΑΔΒ γωνία δίχα τέτμηται ὑπὸ τῆς ΖΔ. ἡμίσεια ἄρα ὀρθῆς ἐστιν <ἡ> ὑπὸ ΖΔΒ. διὰ ταῦτα δὴ καὶ ἡ ὑπὸ ΒΔΓ δίχα τέτμηται ὑπὸ τῆς ΔΗ. ἡμίσεια ἄρα ὀρθῆς ἡ ὑπὸ ΒΔΗ. ὀρθὴ ἄρα ἡ ὑπὸ ΖΔΗ. ὀρθὴ δὲ καὶ ἡ ὑπὸ ΖΒΗ. ἐν κύκλωι ἄρα τὸ ΒΖΔΗ τετράπλευρον. καὶ ἔστιν ἡ ὑπὸ ΖΔΒ τῆι ὑπὸ ΒΔΗ ἴση. ἴση ἄρα καὶ ἡ ΖΒ τῆι ΒΗ. ἔστιν δὲ διὰ τὸ προδειχθέν.

‖ 10 τῶι ὅλωι] ὅλωι τε τῶι coni. Hu app ‖ 13 ἡ add Ge (BS) ‖ 16 ΒΖΔΗ Hu ΒΔΖΗ A ‖ 17 ἔστιν – προδειχθέν secl Hu

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