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CHAOS:
A NEW MATHEMATICAL PARADIGM

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"You believe in a God who plays dice, and I in complete law and order."

Albert Einstein in a letter to Max Born

Introduction

Mathematics is traditionally done in obscurity and mathematicians go about their business with the realization that hardly any non-specialist is willing to invest the time to grasp the significance of the important ideas and theorems of mathematics. Recently, however, this situation changed when the new mathematical theory referred to as chaos[1] burst on the popular scene in an unprecedented fashion.[2] Articles on chaos routinely appeared in the popular press. James Gleick's book, *Chaos*, became a New York Times best seller. Non-specialists have jumped on the bandwagon with almost a religious fervor.

> The most passionate advocates of the new science go so far as to say that twentieth-century science will be remembered for just three things: relativity, quantum mechanics, and chaos. . . . Of the three, the revolution in chaos applies to the universe we see and touch, to objects at human scale. Everyday experience and real pictures of the world have become legitimate targets for inquiry. (Gleick 5-6)

Because of all the hype and exaggerated claims that have accompanied this phenomenon, it is important to have some idea of what the mathematical theory of chaos is. The aim of this essay is to briefly explain what chaos is and to outline how this theory can inform and modify our mathematical world view. I hope this information will help take some of the mystery out of what chaos theory is and will aid readers in applying the theory to their own disciplines.[3]

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1 The mathematical term *chaos* was coined by James Yorke in 1975.

2 Certainly Gödel's Theorem has had a popular impact but probably not as much as chaos.

3 For an informative treatment of some possible applications of chaos theory outside of mathematics the reader is directed to the essay in this same series by Lynden Rogers.

Two Mathematical Cautions

All disciplines use jargon and technical words; for example, biologists use Latin names and chemists use symbolic names such as NaCl. In contrast, mathematicians use everyday words but give them very different meanings.[4] Mathematicians have a completely different meaning for the word *chaos* and the reader should guard against thinking in terms of the way it is commonly used. For example, mathematically, the word chaos is not synonymous with disorder, clutter, pandemonium, or confusion. The mathematical meaning of the word chaos is outlined in a later section entitled "What Is Chaos?" In this essay the term *chaos* will always be used in its technical, mathematical sense.

It is important to keep in mind that the *mathematical world* is a very abstract and precise concept. Many times it is not at all clear how a mathematical idea or theorem informs our *real world* and the tendency is to overreach in the application of mathematics when there is no real connection. An example of this "mathamorphic" mode of thought is to say that since mathematicians believe in the fourth dimension (which they do), therefore God must live in the fourth dimension. The point is not whether God lives in the fourth dimension but whether the abstract mathematical concept has anything directly to say about where God lives. However, if a new paradigm is discovered in one area it can direct one's mind to the discovery of a paradigm in another area that uses similar modes of thought. The mathematical theory of chaos seems to be an important new paradigm of this type.

The Old Paradigm[5]

In the rise of classical science and mathematics, several "axioms" that still affect our thinking

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4 For example, to a mathematician a *ring* is (roughly) a set of numbers that can be added, subtracted, and multiplied.

5 Once again Lynden Rogers treats this specific topic in more detail.

today became part of the prevailing paradigm. Five of these are: 1) determinism—complete current knowledge yields complete predictability; 2) linearity—the standard linear mathematical models suffice; 3) reductionism—the belief that a complex system can be analyzed in terms of its constituent parts;[6] 4) complexity—complex problems must have complex solutions;[7] 5) randomness—seemingly random phenomena do not have natural patterns.

In many ways classical science can be viewed in terms of a search for certainty. De Jong describes this view as follows:

> The Cartesian-Newtonian paradigm contends that the physical world is made up of basic entities with distinct properties distinguishing one element from another. Isolating and reducing the physical world to its most basic entities, its separate parts, provides us with completely knowable, predictable, and therefore controllable physical universe. . . . The Cartesian-Newtonian paradigm contends that the physical universe is governed by immutable laws and therefore is determined and predictable, like an enormous machine. In principle, knowledge of the world could be complete in all its details. (De Jong 100-01)

Certainly during the twentieth century this search for certainty has been under siege (e.g., by relativity and quantum mechanics in physics and Gödel's Theorem in mathematics).[8] Chaos can be viewed as the next "nail in the coffin" in the search for certainty.

It is ironic that the Cartesian-Newtonian paradigm was motivated by "the Judeo-Christian conviction that God is a rational being and thus created a rationally knowable world to be one of the inspirations for the emergence of modern science" (Beck 154). Alfred North Whitehead said it this way:

> The greatest contribution of medievalism to the formation of the scientific movement . . . [was] the inexpungable belief that every detailed occurrence can be correlated with its antecedents in a perfectly definite manner. . . . How has this conviction been so vividly implanted in the European mind? . . . It must come from the medieval insistence on the rationality of God. (Whitehead 12)

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6 E.g., Elementary particles in physics, atoms in chemistry, or genes in biology.

7 Statistics is certainly a method to discern patterns in seemingly random data. However, statistics came fairly late into classical science and mathematics and addresses randomness and patterns in a much different manner than chaos.

8 Hofstadter's book *Gödel, Escher, Bach* addresses these issues.

What Is Chaos?

Chaos (in a mathematical sense) is very difficult to define[9] and is hard to handle theoretically; however, it is often easy to “recognize it when you see it.” For our purposes all we need is the idea of a dynamical system and the concept of *sensitive dependence on initial conditions.*

To a scientist a “system” is a collection of objects that are interrelated.[10] Examples of systems are: the population of rabbits and foxes in Yosemite Valley; the solar system; the Landers earthquake. A mathematical system is one in which the salient features can be quantified mathematically in terms of variables. A particular set of values for the system variables is called a “state” of the system (e.g., the number of rabbits and foxes on July 1 would be a state of the system). A dynamical system is one that changes in time and it is usually defined by an “initial” state and rules for changing a given state into some future (or past) state. The rules for changing states are usually given by a set of equations. Historically, dynamical systems were defined in terms of differential equations but difference equations work just as well and are much easier to deal with.

An example of a dynamical system, due to Robert May, for the population of some species of imaginary fish is given by the single rule—the Logistic Equation: $x_{t+1} = r\,(x_t - x_t^2)$. One can think of x as the fraction of the maximum allowable population of fish and x_t as the population fraction at time t. Here time can only be discrete quantities[11] such as 0, 1, 2; hence, this is a difference equation dynamical system.[12] An initial value, x_0, is chosen; it is substituted into the rule and x_1 is computed. The process is repeated and values for x_2, x_3, x_4, . . . are computed.[13] The rule or function is repeated or iterated to compute the population at any future time.

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9 There is no accepted mathematical definition of chaos and some argue, on philosophical grounds, that there cannot be any (Davis 6).

10 Here I am closely following Stephen Kellert’s ideas outlined in his book, *In the Wake of Chaos*.

11 Think of a time unit as a year or a season, etc.

12 A differential equation dynamical system would allow for continuous time.

13 The rule or function is repeated or iterated to compute the population at any future time. For this reason, a dynamical system is often called an *iterated function system.*

The symbol r is called a parameter and can be viewed as some type of growth or fitness factor for the fish. For a particular population of fish the value of the parameter, r, remains constant but different populations of fish could have different r values. In contrast the variable x_t changes over time for a particular population of fish. One goal of chaos theory is to study how the value of r affects the behavior or dynamics of the population over time. The Logistic Equation is an example of a nonlinear equation due to the x_t^2 term. Chaos can only be found in nonlinear systems and this is essentially the simplest such system.[14]

> It could be argued that a study of very simple nonlinear difference equations . . . should be part of high school or elementary college mathematics courses. They would enrich the intuition of students who are currently nurtured on a diet of almost exclusively linear problems. (May and Oster 573)

Following May's and Oster's advice and studying the Logistic Equation gives insight into the nature of chaos. Readers are encouraged to play with the Logistic Equation[15] and discover for themselves how complicated this simple dynamical system is. I will outline some of the features.

For example, if the parameter r is 2.1, then the population soon settles down to a final, stable population of approximately 0.52 no matter what the starting population is (as long as it is strictly between 0 and 1).[16] If the parameter drops below 1 the population becomes extinct. At first, when the parameter rises past 2.1, the final population also rises but soon settles down to a fixed value. Everything seems to be progressing in a predictable fashion until the parameter passes 3. Suddenly the final population does not settle down to a single value, rather it oscillates between two values every other year. This correlates to a "boom–bust" 2-cycle of the fish in alternating years. As the parameter is increased the 2-year cycle continues but the high and low values move further apart.

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14 While a single (nonlinear) difference equation can exhibit chaos, a system of differential equations must have at least three equations before it can exhibit chaos.

15 On a hand-held calculator it is easier to study Mandelbrot's equation: $x_{t+1} = x_t^2 + c$. The dynamics of this equation and the Logistic Equation are exactly the same. Explore values of c from +0.25 to –2.

16 This means that the population (quickly) heads toward 52% of the maximum allowable population.

Increasing the parameter still more, all of a sudden[17] the population moves out of a 2-cycle into a 4-cycle.[18]

These splits from a single value into a 2-cycle and then from a 2-cycle into a 4-cycle are called bifurcations. As the parameter is further increased these bifurcations come faster and faster and one finds 4, 8, 16, . . . -cycles. At a certain point all periodicity seems to disappear and the population appears to fluctuate in a very complicated and seemingly random fashion that is called chaotic. However, as the parameter increases through the chaotic region, the population will again become regular and settle down in some odd period, such as a 5-cycle.[19] Period-doubling bifurcations into 10, 20, 40, . . .-cycles now occur much like before but they come at a much faster rate and then when the parameter is raised just a little more the system returns to chaos. When the parameter becomes 4 the behavior of the population dynamics appears completely random.[20]

Sensitive Dependence on Initial Conditions

There are several features of chaos that are nicely illustrated by the Logistic Equation. For certain values of the parameter the population settles down to periodic behavior. For other values the population never approaches a periodic population. The values of the parameter that do not yield periodic populations are called the chaotic region and we say that the system exhibits chaos (in the mathematical sense) in this region.

The chaotic region of the Logistic Equation is separated into subregions by nonchaotic regions called periodic windows. If one looks closely inside one of these periodic windows there are period doubling bifurcations that lead back into a smaller chaotic subregion. This smaller chaotic subregion

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17 As r passes $1+\sqrt{6}$ or about 3.4495.

18 When r is 3.5 the population settles down to: 0.39, 0.83, 0.49, 0.87 repeated every four years.

19 This is known as a period-5 window. A period-5 window appears at a parameter value of approximately 3.74 and a period-3 window at approximately 3.83. It is not just chance that the period-3 window comes after the period-5 window. A wonderful theorem by Sharkovskii states that this must be the case.

20 If the parameter is larger than 4 the population rapidly goes to minus infinity and hence the system has no physical meaning.

has still smaller windows of periodicity. When any portion of this subregion is magnified, the small region turns out to behave similarly to the whole region. This property of subregions within regions happens at every possible magnification. The property of similar detail at every level of scale is a hallmark of all chaotic systems. A chaotic system cannot be studied by breaking it down into smaller pieces since the pieces will be as complicated in detail as the complete system. For this reason chaos is inherently antireductionist and thus a holistic theory.

For a parameter in the chaotic region of the Logistic Equation, the population fluctuates in an unpredictable manner. However, if the population is plotted for many generations patterns emerge. Population points will cluster together in some places and might completely miss other places.[21] These patterns in the long range dynamics of a chaotic system are another hallmark of chaotic systems.

For a parameter in the periodic region of the Logistic Equation, the population settles down to the same value independent of the starting value. In the chaotic region the situation is very different. Two different starting values can yield completely different results, even if the populations are run for an infinite length of time. This phenomenon happens with starting values that are arbitrarily close together. For example we could begin with two starting values that only differ in the millionth decimal place and run them through the dynamical systems for many generations. Quickly the two runs will drift apart and follow completely different population paths. This is what is meant by *sensitive dependence on initial conditions* and is the most important feature shared by all chaotic dynamical systems.

An interesting historical note is that the first dynamical system in which chaos was observed and clearly recognized was probably Edward Lorenz's model of the weather.[22] Lorenz realized that his system was chaotic and expressed its sensitivity to initial conditions by his famous "butterfly effect."[23] A single butterfly flying by today would alter the initial state of the weather a little bit.

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21 These patterns of long range chaotic dynamics are called *strange attractors* and are examples of geometric objects known as fractals.

22 Lorenz's dynamical system is a differential equation model of three equations and hence is the simplest possible.

23 Lorenz gave a talk at the 1979 annual meeting of the American Association for the Advancement of Science titled "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?"

Over a period of time the weather could turn out to be drastically different than it would have been if the butterfly had not flown by.

To understand the concept of sensitive dependence on initial conditions it is instructive to look at a system that does not exhibit it. An example is: $x_{t+1} = x_t + c$. If two different starting populations are used the two populations might drift apart, as far as you like, but over a long period of time. There are two key points when comparing this system with a chaotic system such as the Logistic Equation. When close starting values are used each system's population will diverge; however, the rate of divergence is much different and faster for the chaotic system. Also for a non-chaotic system the dynamics tends to be qualitatively similar for nearby starting values. This is not the case for a chaotic system.

Chaos and a Linear Versus Nonlinear World View

An over-simplified description of the Cartesian-Newtonian method is to first reduce a physical system to a set of equations and then solve the equations. The belief is that then one has complete predictability. Ian Stewart calls this a "clockwork world" view.

> The revolution in scientific thought that culminated in Newton led to a vision of the universe as some gigantic mechanism, functioning "like clockwork", a phrase that we still use – however inappropriate it is in an age of digital watches – to represent the ultimate in reliability and mechanical perfection. In such a vision, a machine is above all predictable. Under identical conditions it will do identical things. An engineer who knows the specifications of the machine, and its state at any one moment, can in principle work out exactly what it will do for all time. (Stewart 9)

Certainly this has been a very productive approach for science for the last three hundred years.[24]

[24] For example, it has been exceedingly effective in astronomical predictions such as solar eclipses, the return of comets, and sunset times.

In a clockwork universe it appears as if there is no place for chaos and its sensitive dependence on initial conditions. Why has this Newtonian "clockwork" approach been so effective for so long and why has it taken so long for chaos to be discovered? The answer is, of course, complex but a large part of the answer is that until the mid-twentieth century mathematicians and physicists concentrated almost exclusively on linear dynamical systems and, as we have seen earlier, chaos can only occur in a nonlinear system.

> The solvable systems are the ones shown in textbooks. They behave. Confronted with a nonlinear system, scientists would have to substitute linear approximations or find some other uncertain backdoor approach. Textbooks showed students only the rare nonlinear systems that would give way to such techniques. They did not display sensitive dependence on initial conditions. Nonlinear systems with real chaos were rarely taught and rarely learned. When people stumbled across such things—and people did—all their training argued for dismissing them as aberrations. Only a few were able to remember that the solvable, orderly, linear systems were the aberrations. Only a few, that is, understood how nonlinear nature is in its soul. Enrico Fermi once exclaimed, "It does not say in the Bible that all laws of nature are expressible linearly!" The mathematician Stanislaw Ulam remarked that to call the study of chaos "nonlinear science" was like calling zoology "the study of nonelephant animals." (Gleick 68)

The question, "What kind of equations can we solve?" was answered pragmatically: "Only linear equations." This answer became a pillar of the Cartesian-Newtonian paradigm and permeated the world view of the scientific community.

> Classical mathematics concentrated on linear equations for a sound pragmatic reason: it couldn't solve anything else. In comparison to the unruly hooligan antics of a typical differential equation, linear ones are a bunch of choirboys. . . . So docile are linear equations that the classical mathematicians were willing to compromise physics to get them. . . .
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> So ingrained became the linear habit that by the 1940s and 1950s many scientists and engineers knew little else. 'God would not be so unkind,' said a prominent engineer, 'as to make the equations of nature nonlinear.' Once more the Deity was carrying the can for humanity's obtuseness. The engineer meant he didn't know how to solve nonlinear equations, but wasn't honest enough to admit it.
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> Linearity is a trap. The behaviour of linear equations – like that of choirboys – is far from typical. But if you decide that only linear equations are worth thinking about, self-censorship sets in. Your textbooks fill with triumphs of linear analysis, its failures buried so deep that the graves go unmarked and the existence of the graves goes unremarked. (Stewart 83)

What had started out by Newton as a belief in a "clockwork world" had been replaced by the middle of the twentieth century by a belief in a "linear world." "Linear worlds" do not allow for chaos; thus, sensitive dependence on initial conditions was not even an issue.

The "linear world" view came under attack during the last half of the twentieth century. It became apparent that what Lorenz had noticed in his weather model was not an odd exception but rather it was a common occurrence. His butterfly effect acquired a technical name: sensitive dependence on initial conditions. Stewart points out that "sensitive dependence on initial conditions was not an altogether new notion" by quoting the old rhyme from the past (Stewart 23):

> For the want of a nail, the shoe was lost;
> For the want of a shoe, the horse was lost;
> For the want of a horse, the rider was lost;
> For the want of a rider, the battle was lost;
> For the want of a battle, the kingdom was lost!

Chaos and Determinism

The Cartesian-Newtonian plan has always been mathematically ambitious—even for linear systems the equations are usually very hard or beyond our abilities to solve. The dream of actually achieving complete predictability for any complicated system has always been more of an ideal rather than a practical reality. We now understand from the theory of chaos and its sensitive dependence on initial conditions that total predictability has always been a "false god." Nonlinear systems have an inherent limitation since, in the chaotic region, absolute accuracy is necessary for useful predictions. "This does not mean that all chaotic systems require impossible accuracy for all useful prediction tasks; some systems may manifest a mixture of predictable and unpredictable behaviors. But when we speak of chaotic phenomena as being impossible to predict, I would maintain that this is a good way to spell out that limitation" (Kellert 35).

At this point it is fair to ask why we are so preoccupied by determinism. The answer lies in what we accept as ultimate explanations. For the last three hundred years the "holy grail" of science has been prediction. A litmus test of good scientific research is repeatability. In the Cartesian-

Newtonian paradigm to say that we can explain an event usually means that we can predict the event. "If the only alternatives to determinism are final causes (e.g., divine intervention) and hazard (e.g., accident or chance), then determinism is attractive as an *a priori* truth or a methodological imperative of scientific inquiry" (Earman 23). Chaos has given another option to this dichotomy.

Stephen Kellert's *In the Wake of Chaos* is an outstanding book that discusses in detail what we mean by "predictable," the layers of meaning of the word "determinism" and the impact of the theory of chaos. I will just touch on one aspect of Kellert's argument and hope that the reader will be stimulated to read Kellert's book, on which the rest of this section heavily relies.

Kellert points out that the term "determinism" can be viewed in at least four ways and lists them in the following "rough order, from the simpler and less restrictive to the more robust and full-blown" (Kellert 50).

1. Differential or Difference Equations from a sufficient model of the system.
2. Unique evolution, i.e., the evolution of the system is uniquely fixed once we specify the state of the system at any one moment.
3. Value determinateness. Do all properties of the system have well-defined real values?
4. Total predictability ("whatever that means" (Davis 6)).

Each of the four shadings of the meanings of "determinism" listed above includes the ones listed before it.

The fourth meaning of total predictability is the goal of the Cartesian-Newtonian paradigm. Karl Popper calls this meaning "scientific determinism," namely,

> the doctrine that the state of any closed physical system can be predicted, even from within the system, with any specified degree of precision, by deducing the prediction from theories, in conjunction with initial conditions whose required degree of precision can always be calculated (in accordance with the principle of accountability) if the prediction task is given. (Popper 36)

Nonlinear models and chaos have made determinism unreachable if we employ the criteria of total predictability.

"Value determinateness, the third level of determinism, means that physical quantities have exact values" (Kellert 60). Some systems could have parameter values that are spread out or somehow indistinct. Some authors have argued that a system without the precision of value determinateness could not qualify for being a deterministic system.[25]

Unique evolution[26] says that the complete description of a system at a certain time completely fixes the future (and past) of the system. This means that each set of initial conditions dictates a unique trajectory or history of the system. Chaos brings a real tension to this meaning of determinism. While a system can exhibit unique evolution,[27] sensitive dependence on initial conditions "means that we will never be able to tell *which* unique trajectory a system is following, but that does not mean such trajectories do not exist" (Kellert 63).

Chaos has forced us to reconsider three of the four meanings Kellert has given to determinism. At this point all we have left is the first meaning: Differential or Difference equation models. The obvious question is what do we have left and what type of information can we get from such models?

> The activity of building and using these models [differential or difference dynamic systems] has three important characteristics . . . the behaviour of the system is not studied by reducing it to its parts; the results are not presented in the form of deductive proofs; and the systems are not treated as if instantaneous descriptions are complete. (Kellert 85)

These three characteristics point to important ways in which chaos is changing the operating paradigm of mathematics; consequently a new paradigm for how we view our world becomes possible.

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25 Chaos reveals little about value determinateness.

26 Sometimes called "Laplacian determinism."

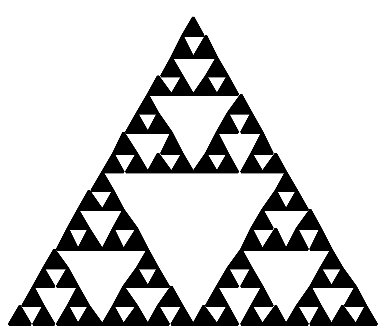
27 The Logistic Equation is such a system.

Chaos Versus Reductionism, Complexity, and Randomness

Chaos is an intrinsically a holistic, nonreductionistic paradigm. A nonlinear system is not the sum of its parts. In fact, as we have seen, in the chaotic region a nonlinear system has similar detail at every level of magnification. The Cartesian-Newtonian paradigm focused on quantitative solutions. Chaos focuses on qualitative aspects such as when does a system have periodic behavior and, when the behavior is not periodic, what is the general shape of its attractor. Determining the general shape of the attractor in phase space becomes a central problem. The geometry rather than the algebra of the dynamical system becomes the focus.

The Cartesian-Newtonian paradigm assumes that complex outcomes must have complex explanations. This is not true of many chaotic systems. The Logistic Equation is an example of a very complex outcome with a very simple explanation—a simple quadratic equation.

A delightful example of a system that clearly demonstrates the nonreductionistic nature of chaos, illustrates that a complex outcome can have a very simple explanation, and which shows a pattern in a seemingly random process is the Chaos Game. The game is started by drawing a triangle and choosing a random point inside the triangle. The game proceeds by choosing, at random, one of the three vertices of the triangle and moving half the distance from the current point to that vertex. Another vertex is randomly chosen, the point is moved half the distance to that vertex and so on. If the successive points are marked in the triangle the pattern illustrated below emerges (5,000–10,000 points should suffice to get a good picture). The pattern (called the



Sierpinski Gasket) is independent of the starting point or of the random sequence of vertices chosen. The Chaos Game is a picture of a chaotic system. This system clearly shows a pattern in a seeming random process. If an infinite number of points were plotted the pattern would look *exactly the same* at every level of magnification and thus a reductionist approach to studying the figure would be useless.[28]

Conclusions

It is certainly fair to ask questions such as: Is chaos theory a fad? Will it go the direction of mathematical catastrophe theory and become more of a belief/religion than a valid theory? Has chaos been overly hyped? Will chaos revolutionize science? For questions of this type only time will tell; however, I am firmly convinced that mathematically the theory of chaos is valid, important, here to stay, and will significantly change the way we do mathematics. Chaos theory helps us to focus on important features that have been neglected. "Far better to consider chaos theory as a search for *order*, a concept broader than law. . . models, not laws, form the heart of a science" (Kellert 117). Chaos does not eliminate prediction; rather it addresses the issue of what kind of prediction[29] is possible and aids in predicting the limits of our predictions.

One view is that the Cartesian-Newtonian paradigm was developed in order to answer questions about what appeared to be a disordered world (chaotic in a non-mathematical sense). The result of this was the "clockwork" world paradigm. The theory of mathematical chaos has brought us full circle. "There is a theory that history moves in cycles. But, like a spiral staircase, when the course of human events comes full circle it does so on a new level" (Stewart 1). I believe that chaos theory has moved us up to a new level.

Kellert proposes that chaos theory gives us a "dynamic understanding."

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28 The property of similar detail at every level of scale is the reason fractals can be used to describe chaotic systems. A fractal (loosely speaking) is a geometric object that looks similar at every level of magnification. Fractals have the curious property that they are not one, two or three dimensional, rather they have fractional dimensions. The dimension of the Sierpinski Gasket is approximately 1.585.

29 E.g., qualitative or quantitative.

> First, it calls to mind the connection with dynamical systems theory, the qualitative study of the behavior of simple mathematical systems. Second, it connotes change and process, tying together the various uses of the word "how," . . . Chaos theory lets us understand how patterns and unpredictability arise by showing us how certain geometric mechanisms bring them forth. (114)
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> Some may contend that this search for patterns actually strives to discover new laws governing qualitative features of systems. (112)

It is important to understand that the patterns referred to are at the heart of the qualitative not quantitative aspects of chaos; furthermore these patterns have little to do with the type of patterns that statistical techniques address.

Chaos theory is intrinsically holistic and antireductionist. In my judgment this is one of the most important aspects of chaos from a Christian point of view. It changes the way we view and analyze our world.

> . . . you know the right equations but they're just not helpful. You add up all the microscopic pieces and you find that you cannot extend them to the long term. They're not what's important in the problem. It completely changes what it means to know something. (Gleick 175)

When Albert Einstein wrote to Max Born, "You believe in a God who plays dice, and I in complete law and order," it was in the context of quantum mechanics not chaos (after all, the theory of chaos did not exist at that time). At the time Einstein wrote this, the Cartesian-Newtonian paradigm was under siege. The goal of complete predictability was being replaced by the theory that nature has a fundamentally unpredictable or even random aspect. Einstein was hoping for a more satisfactory explanation than the hypothesis of a God who acts in a random manner. I agree with Stewart that "The question is not so much *whether* God plays dice, but *how* God plays dice" (2). Currently my answer is that God does not play dice in a traditional random sense but, as Stewart says, "He's playing a much deeper game that we have yet to fathom" (293). It is my belief that chaos theory is one component of this deeper game of "dice" that God is playing.

As a mathematician the new paradigm of chaos theory has been important to my gaining a deeper understanding of mathematical phenomena. I now study mathematics from a more holistic, nonreductionist point of view. I view complexity and randomness in a completely different light.

Above all I have come to realize that I really do not understand what mathematical determinism is and that it is a far deeper concept than I had realized. I do not know how these ideas will ultimately transfer to other areas of my non-mathematical life but I do know that the mathematical paradigm of chaos has opened my eyes to exciting possibilities about the nature of God.

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