The Field Axioms

(A0) (Existence of Addition) Addition is a well defined process which takes pairs of real numbers a and b and produces from then one single real number $a+b$.

(A1) (Associativity) If a, b, and c are real numbers, then

$$a+(b+c)=(a+b)+c.$$

(A2) (Additive Identity) There is a number 0 such that for all numbers a

$$a+0=a.$$

(A3) (Additive Inverse) For every real number a there is a real number $-a$ such that

$$a+(-a)=0.$$

(A4) (Commutativity) If a and b are any real numbers, then

$$a+b=b+a.$$

(M0) (Existence of Multiplication) Multiplication is a well defined process which takes pairs of real numbers a and b and produces from then one single real number ab.

(M1) (Associativity) If a, b, and c are any real numbers, then

$$a(bc)=(ab)c.$$

(M2) (Multiplicative Identity) There is a number 1 such that for all numbers a

$$a1=a.$$

(M3) (Multiplicative Inverse) For every real number $a\neq 0$, there is a real number a^{-1} such that

$$aa^{-1}=1.$$

(M4) (Commutativity) If a and b are any real numbers, then

$$ab=ba.$$

(D) (Distributive) For all real numbers a, b, and c,

$$a(b+c)=ab+ac.$$

(Z) (Non-triviality) $0\neq 1$