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# MODELING THEORY IN SCIENCE EDUCATION

Ibrahim A. Halloun

Springer

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# Modeling Theory in Science Education

*by*

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To

Alexia Catherina and Gabriella Christiana

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# PREFACE

Modeling theory is originally a theory *of* science, a theory about scientific theory and practice that emerged lately in the philosophy of science. It draws on the practice of prominent figures in the history of science, as well as on observation of modern day scientists at work (Bronowsky, 1953; Bunge, 1973; Giere, 1988; Harré, 1970; Hesse, 1970; Hestenes, 1992; Leatherdale, 1974; Nersessian, 1995; Wartofsky, 1968). The theory basically asserts that models are at the core of any scientific theory and that model construction and deployment are fundamental, if not the most fundamental, processes in scientific inquiry. This book is the culmination of over twenty years of work to deploy modeling theory *in* physics then science education with the prospect to turn it eventually into a theory *of* science education.

The last half-century has witnessed numerous calls and movements to reform the state of science education. A plethora of research articles has constantly shown that under conventional instruction of lecture and demonstration, students of all levels fail to develop a meaningful understanding of scientific theory and practice. Reformists have virtually all agreed that in order to change the state of things, a science student must become actively engaged in scientific inquiry, just like an apprentice does in any art or trade. With science perhaps as the most counter-intuitive trade of them all, science educators are being called upon to take special advantage of cognitive science, and particularly of the two-way stream that has been growing recently between cognition and philosophy of science. (Duschl, 1988; Duschl & Hamilton, 1992; Gentner & Stevens, 1983; Giere, 1992, 1994; Lakoff, 1987; Redish, 1994; Reif & Larkin, 1991). This call resonates well with our work on modeling theory.

As presented in this book, modeling theory in science education is grounded in a number of tenets about the nature of scientific knowledge and inquiry, as well as about learning processes in which students ought to become engaged in order to develop a meaningful understanding of science. The scientific perspective is offered in Chapter 1 of the book. It emphasizes the central role of models in putting together scientific theory and of modeling in conducting various forms of scientific inquiry. Related cognitive aspects are presented in part in the same chapter and are further developed from a pedagogical perspective in Chapter 3. The emphasis in the former chapter is on the need for students to develop experiential knowledge about physical realities, knowledge that comes about mainly as the result of interplay between people's own ideas about the physical world and particular patterns in this world.

Special attention is paid in our work to course content. This interest is driven by the conviction that knowledge organization is crucial for effective and efficient thought and inquiry. It is also implied by the fact that we are catering to science education standards and curricula that continue to be content-driven, which is justifiable as long as the drive for process is also there. Our focus with regard to content is primarily on models and modeling schemata in the manner discussed in Chapter 2. A scientific model is, for us, a conceptual system mapped, within the context of a specific theory, onto a specific pattern in the real world so as to reliably represent the pattern in question and serve specific functions in its regard. A model may serve an exploratory function (pattern description, explanation, post-diction and/or prediction), and/or an inventive function (control or change of existing physical systems to produce the pattern, and/or pattern reification into new physical systems and phenomena). Model constitution and function are laid out explicitly in Chapter 2 and extrapolated to the case of concepts in accordance with specific modeling schemata. A modeling schema serves students as an organizational tool for structuring models or related conceptions in a meaningful and productive way. It also provides teachers with reliable means for planning instruction and for the assessment of student learning and teaching practice.

A person's ideas about the physical world are spread across what we call a paradigmatic profile. Such a profile consists of a mix of paradigms, one of which is dominated by naïve realism. As discussed in Chapter 3, modeling theory in science education is set to help students through a paradigmatic evolution with reasonable expectations. Individual students are anticipated not to achieve a radical paradigm shift in the direction of scientific realism. Instead, they are moved to curtail naïve realism in their paradigmatic profiles and build up scientific realism to realistic levels. Meaningful paradigmatic evolution takes off from a threshold that is attainable by any student willing to invest the necessary effort. The threshold is defined in a given science course by the set of basic models in the scientific theory that is the object of the course. A basic model is a model that provides an affordable and efficient framework for students to develop fundamental tenets and conceptions (concepts, laws, and various theoretical statements) of the respective theory, as well as essential tools and skills of scientific inquiry.

A particular modeling program presented in Chapter 4 is designed to help students to achieve the target paradigmatic evolution. The program concentrates on the common denominator among all scientific disciplines: model-laden theory and inquiry. Implicit in the program is the recognition that students at the college and pre-college levels cannot be brought to develop scientific theory and inquiry with uncompromising rigor. The compromise is however significantly reduced through didactic transposition of the content of scientific theory, a transposition that revolves around the set of basic models in the theory. Appropriate activities are designed for students to develop these and other models from different rational and empirical perspectives, along with the fundamental tools and skills that are necessary for various forms of scientific inquiry. Activities are associated with particular norms and guidelines for a variety of assessment and evaluation processes that allow students to reflect on their own ideas and regulate them in an insightful manner.

The program is implemented in structured learning cycles described in Chapter 5. A learning cycle is, for us, a five-phase modeling cycle. The five phases are exploration, model adduction, model formulation, model deployment, and paradigmatic synthesis. Each cycle is devoted to the development of a specific model along with particular modeling processes that can best be developed in the context of the model in question. A cycle takes off with subsidiary models, i.e., counterpart models that have limited viability by comparison to the target model that students develop, intuitively sometimes, by correspondence to familiar situations. A cycle proceeds through student-centered investigative activities that allow groups of students progressively to refine their subsidiary models until they take the form of the target model. The entire process is teacher-mediated so as to bring to the surface various student ideas, especially those that are at odds with science, and to help students to mutually ascertain their ideas and regulate them in the light of empirical evidence and in conformity with scientific theory and practice.

Modeling instruction as presented in this book has been systematically corroborated, mostly within the context of secondary school and university physics courses, and primarily in U.S.A. and Lebanon (Halloun, 1984, 1994, 1996, 1998a, 2001b, 2004; Halloun & Hestenes, 1987; Wells, Hestenes & Swackhamer, 1995). Discussion is often illustrated with examples from Newtonian mechanics, examples that are within the scope of virtually any science teacher and chosen so as to keep an affordable storyline across various chapters. Modeling theory as presented in this book is now being deployed into other scientific fields and educational levels. Early results are consistent with what we have been able to achieve in the context of physics. They show that the theory in question actually fosters the paradigmatic evolution we are calling for, and that it brings about an equitable learning experience that narrows significantly the traditional gap between students at opposite ends of the competence spectrum, i.e., those students that enter a science course with high competence and those that do so with low competence.

This book is the culmination of over twenty years of work. It presents aspects of modeling theory that have repeatedly demonstrated their value when deployed in physics education, and lately in science education. The book is intended primarily for researchers and graduate students in science education. It can serve as well as a major reference work for in-service and pre-service science teachers who want to go into their classroom not to promote canned texts but to foster the sort of meaningful understanding of science called for in this book. Interested educators are invited to contribute to our drive to turn modeling theory into a theory of science education. This still requires hard work at the level of theoretical foundations and structure of the prospective theory, as well as further systematic deployment and corroboration in a variety of scientific disciplines and educational levels. Meanwhile, one cannot but acknowledge that, given the intricacy of our endeavor and especially the seemingly endless list of hard to control cognitive and affective factors involved in any learning process, we may never get to a theory of science education that is as rigorous and as viable as a scientific theory. Nevertheless, this author is determined to bring modeling theory in science education to as high an efficacy level as any educational theory can possibly achieve.

Numerous people have contributed in one way or another to the appearance of this book. I am above all indebted to my family for putting up with my long days of isolation while writing the book. I am grateful to Professor Bill Cobern for his trust, and to Kluwer's staff for their kind cooperation in bringing this work to press. Special acknowledgments are due to the modeling research team headed by Professor David Hestenes at Arizona State University and to many other colleagues around the world with whom I keep exchanging ideas about modeling theory in science and science education. I am especially grateful to the numerous teachers and professors who have been diligently implementing modeling instruction in their classes and providing me with valuable feedback, and to their students and mine who have endured with us the hardship of bringing this work to its current state. In a sense, colleagues and students have all been part of this work. Their contributions are acknowledged throughout this book with collective attribution of work and points of view. Still, because ideas might have come about without consultation with any or some of these people, no one but myself should be held responsible for the way modeling theory is presented in this book.

*Chapter 1*

# FUNDAMENTAL TENETS OF MODELING THEORY

Modeling theory is promoted in this book as a pedagogical theory for science education. It is thereby concerned with cognitive processes and curriculum aspects leading students at different educational levels to the formation of particular knowledge and skills commonly associated with scientific theory and practice. As such, we acknowledge in our proposed theory that, in content and respective skills, scientific knowledge is distinguished in specific respects from other forms of knowledge, just as we acknowledge that there are common factors underlying the formation of knowledge of any type in humans' minds. In the same way, we acknowledge that various scientific disciplines have many features in common, just as we recognize that they may be distinguished from one another in some aspects. This is at least a practical position that stands as long as there are demarcation lines among these disciplines that are commonly recognized within the broad scientific community, as well as within the educational community, and irrespective of how artificial or how blurred these lines may sometimes seem to be. Nevertheless, we stand firmly in our theory for the position that various scientific disciplines share by and large enough common features to bear the common label of "science", and to be set apart all together from other forms of human endeavors. These features constitute the main concern of modeling theory, both as a theory of science and as a theory of science education.

Science is primarily concerned with the development of human knowledge (subject matters and processes) that helps us to understand the real world as objectively as possible and interact with this world as constructively as possible. Science education is primarily concerned with helping people to develop ways of knowing and learning that are as closely aligned as possible with scientific judgment and inquiry. Various science educators, teachers included, thus need to have a basic understanding and appreciation of the intricacies that govern the relationship between what we know and the things we know about in the real world, both as ordinary people and as scientists. Such knowledge, that is in part the object of this chapter, is indispensable for educators to guide science students in efficacious learning paths.

The nature of human knowledge about the real world has long been debated among philosophers, and most recently among cognitive researchers. Viewpoints have ranged between two extreme positions, mostly distinguished by their ontological and their epistemological premises. At one end of the spectrum lays *positivism*, a philosophical school that finds its roots in the works of Aristotle (384-322 BC), and various forms of which were held by August Comte (1798-1857), Claude Bernard (1813-1878), Ernst Mach (1838-1916), Bertrand Russell (1872-1970), and Rudolf Carnap (1891-1970). The main ontological premise of most positivists is that no physical object exists unless it can be humanly perceived. The epistemological consequence is that the physical world is knowable, and that it is the way it is perceived by our senses. Our knowledge of this world is thus conceived to constitute a photographic replica of whatever may be directly exposed to our senses. At the opposite end of the spectrum lays *nominalism*, a philosophical school that is commonly associated with the works of Henri Poincaré (1854-1912) and that finds its roots in the less radical works of Emmanuel Kant (1724-1804), Friedrich Hegel (1770-1831), and Friedrich Schelling (1775-1854). The main premise of nominalists is that the reality of physical things and events in the universe is completely independent of any human perception or conception, and that it is humanly unknowable. We thus can develop knowledge *about* but not *of* the physical world, nominalists argue, knowledge that consists of pure fabrications of our brains and that does not correspond in any form to this world.

In the middle of the spectrum are many *realism* schools that hold, to various degrees, that the real world is independent of human perception but that it is knowable in specific respects and to certain extents. As presented in this chapter, modeling theory is based on a number of tenets regarding the real world and our knowledge about this world, tenets that draw on certain aspects of *scientific realism* as advocated primarily by Mario Bunge (1967), Ronald Giere (1988), Rom Harré (1961), and George Lakoff (1987). The tenets also draw on certain foundations of non-realist schools that have valuable implications to science education, primarily those underlying the work of Thomas Kuhn (1970).

Major tenets and aspects of modeling theory pertaining to human knowledge in general are discussed in the first four sections of this chapter. Those pertaining specifically to scientific knowledge are discussed in the following four sections. Pedagogical consequences are discussed throughout this book, but primarily in Chapter 3. Discussion is limited, in this and following chapters, to those tenets and related cognitive and philosophical aspects that bear directly on the pedagogical concerns of this book, in line with Gruender's (2001) principle of demarcation:

> *If the application or resolution of an issue in the history or the philosophy of science has no implications, however general, for current work in the science of a field or for its teaching, then it is not one which scientists or science teachers have a professional duty to trouble themselves about.*

## 1.1 PHYSICAL REALITIES AND HUMAN COGNITION

> *In the absence of human intervention, physical systems exist, interact, and evolve, producing certain phenomena in the universe, all independently of human existence and activity. Humans can eventually come to realize the existence of such systems and phenomena, and develop about them ideas of variable degrees of viability.*

We hold in modeling theory a clear distinction between two worlds, the physical universe (or the real world) and human mind (or the mental world). The physical universe, i.e., the real world about which science is concerned, consists of physical systems (i.e., material systems, including biological ones) that interact and evolve in ways that give rise to specific phenomena. As discussed in § 2.1, a *physical system* is an entity in the real world that may consist of a single physical object or of many physical objects that interact with one another in specific ways. A *phenomenon* is an event, a *change* in spacetime, or a series of events that could result from the interaction among the constituents of a particular system and/or among different systems. An atom, the human body, the solar system are examples of physical systems. Electromagnetic radiation, human reproduction, planets' movement around a sun, are examples of physical phenomena.

Physical systems and phenomena, hereafter referred to as *physical realities*, are the object of natural sciences (e.g., physical and biological), as well as of technology and engineering. Physical realities are distinguished from *social realities* (e.g., a particular community of people and the activities of its members) that are the object of sociology and some branches of philosophy. They are especially distinguished from *intellectual* or *mental realities* that consist of cognitive structures and processes that are developed as a result of individual or collective human enterprises, and that are the object of cognition, psychology and some other branches of philosophy.

As long as humans do not intervene, whether consciously or unconsciously, with any aspect of a physical reality, the state and evolution of the reality in question remain independent of human existence and activity. This independence does not hold when humans intervene in the process, for making certain measurements, or for exploiting the reality one way or another. This is the case of technology where humans invent new systems or processes to make use of existing realities in specific respects, and/or to control or modify the state of such realities. This is also the case of ecological changes caused by human activities.

The existence and evolution of a physical reality is especially independent of whether or not humans could come to realize its existence. Yet, and as we shall see later, if a physical reality exists, humans could eventually realize its existence and develop particular ideas about it. They can do this: (a) *empirically*, i.e., through immediate perception or with the help of appropriate instruments, should they be available, or (b) *rationally*, through inference from established knowledge and related empirical data. For example, long before scientists were able to "detect" quarks in their laboratories, they inferred their existence from established knowledge about more complex atomic structure and phenomena. The same is true for distant galaxies that no one has ever "seen", not with the naked eye or with any available instrument.

The distinction we maintain between physical and mental realities does not necessarily imply total ontological independence of one another, especially not of mental realities from physical realities. We shall come back later to this point. The relative independence of the real world from the mental world in the manner postulated above should especially not be misinterpreted to imply the existence of an objective reality, a reality that our mind can eventually come to mirror in its "true" state. As our discussions throughout this book will hopefully make it clear, truth is for us a relative and partial predicate that humans can gradually develop through successive approximations (Bunge, 1973, p. 169).

The mental world of a given person includes *structures* and *processes* of two cognitive levels. In the first level are *implicit* structures and processes that are constructed involuntarily, and even unconsciously, in the person's mind, and that cannot be subject to conscious scrutiny by the same person or to direct scrutiny by others. In the second level are *explicit* structures and processes that: (a) are developed and evaluated voluntarily and consciously by the person through pure thought (intrinsic intellectual experience) and/or through an experience with physical and/or social realities, and that (b) can be communicated to other people and shared with them. The explicit part of the mental world is thereafter referred to as the *conceptual* world of a person. Modeling theory in science education is only concerned with student conceptual world, mainly in relation to physical realities and by contrast to science.

*Conceptual structures* include *conceptions*, i.e., concepts, theoretical statements (axioms, laws, theorems, definitions), models, theories, as well as conceptual *tools* used in the development and employment of various conceptions (e.g., language, pictures, mathematics, and related semantics and syntax). *Conceptual processes* include all conscious mental procedures, and associated norms and rules that a person follows in the construction and deployment of conceptual structures. Through practice, conceptual processes evolve gradually in their autonomy until they develop into *skills*. These are processes that are driven by internal needs and controlled by spontaneous habits, and that can be actuated autonomously outside typical situations within the context of which they were originally developed. The merits of a person's conceptions, tools and skills with regard to specific physical realities depend mostly on the extent to which they correspond to such realities and serve specific functions in their respect. Their merits primarily depend on whether they constitute knowledge or beliefs about such realities.

*Knowledge* consists of conceptual structures and processes that have been *corroborated* in specific respects. Corroboration consists of some sort of evidence, the most reliable of which being empirical or real world evidence that meets specific norms. Reliable evidence is an objective datum, or set of data, that is independent of personal idiosyncrasies and acceptable by a group of people according to well-defined criteria, that is open to scrutiny, and that stands firmly enough certain tests of refutability. These and other conditions for data to constitute reliable evidence from a scientific perspective are discussed in details in § 2.7. Not all evidence accepted by a given person or a given group of people is necessarily reliable; and thus, what might constitute knowledge for one person or group of people may not be considered as such by other individuals or groups. For example, when an event follows in some respects an astrological prediction, astrologers and their followers consider this to be a reliable evidence in their favor, whereas scientists and other people who do not believe in astrology consider the prediction to be a lucky guess, and the subsequent fact to be a mere coincidence or, at best, some event that can be statistically inferred. Similarly, the apparent motion of the sun still constitutes for many people reliable evidence for the sun's translation around the Earth rather than for the Earth's rotation around itself.

*Beliefs* are ideas that one holds about certain realities, individually or in common with others, without due corroboration. For example, when you hear somebody talk about a certain subject matter, you "know" that this person is in the process of speaking, and you can either "know" or "believe" that s/he is telling the truth or not. To know it one way or the other, you must have experienced what the person is talking about and/or possess some tangible data about the topic, like a photograph or a reliable record of some sort. In the absence of such empirical evidence, one can make an inferential judgment based on prior knowledge of the person and/or body language, and end up believing, or not, what the person is saying.

In this sense, we can speak of scientific knowledge (corroborated) but of religious beliefs (uncorroborated). In the same way, we can distinguish between student knowledge *of* a physical reality (i.e., that it exists) and *about* it (i.e., of its properties) on the one hand, and student belief in what science says about such a reality, on the other. Student knowledge would be based on some direct experience with the reality in question and/or on learning science meaningfully in the manner described in this book. In contrast, student belief would be based on authoritative instruction and following memorization by rote of scientific texts.

Once a system or a phenomenon becomes a physical reality, it makes it possible, but not necessary, for humans to know of it and about it*. Once this book has been printed, it became a physical reality that any person could know of, by seeing it on the shelf of a bookstore or by learning of its existence in a reference or through the media. One can further know what the book consists of and what it is about, by directly examining and reading it, as you are doing now, or indirectly, from a reliable third party. Seeing the book and reading it allows one to develop *experiential knowledge* about this work. Learning about it from another source may result in *traded knowledge*. The book also allows one to know of the existence of the author of this book, and perhaps to make some valid judgment about him from reading the book. This is *inferred knowledge*. Some beliefs (uncorroborated ideas) about the author and the book topics could also be generated in the process. As a theory of science, modeling theory is concerned with human knowledge, and especially scientists' knowledge. As a pedagogical theory, it is concerned with helping students turn, preferably in experiential forms, all sorts of knowledge and beliefs about physical realities into knowledge that is reliable by scientific standards.

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* Unless otherwise specified in the rest of this book, knowledge or belief "about" something refers to knowledge "of and about" it or belief "in and about it".

## 1.2 EXPERIENTIAL KNOWLEDGE

> *A person's new knowledge about a given physical reality exposed directly or indirectly to the person's senses results from interaction between the person and the reality. New knowledge thus depends on: (a) the existing knowledge of the person, (b) the actual (ontological) state of the reality that the person is interacting with and of its environment, (c) the condition of the person's senses and state of mind, and (d) the state of employed instruments, if any.*

The decision to read this book was triggered by your interest and other control factors in your mind that depend, in part, on your current knowledge about the topics discussed in the book. Once the book is in your hands, similar control factors will make you decide whether to read or skip particular paragraphs. While you are reading the book, you are interpreting words and sentences of your selection in terms of your current knowledge of the English language (and of other tools) and discussed topics. Without such knowledge, you would be able neither to make meaningful interpretation of what you are reading nor to determine whether or not you are offered something new to learn about. The reading process also depends on the affective state of your mind, the presence of any distracters around you, as well as on the quality of your eyes and of any seeing aids you might be using. The entire experience further depends on the book whose existence made your original decision possible, and whose layout has some influence on the ease with which you would be reading it and perhaps on what you might decide to read or to skip.

Hence, the reading experience in question is of you and of the book. The selection of what this experience involves and the process it follows depend on the state of both your mind and the book, as well as on the state of the mutual environment. The same holds for any human experience, especially when it results in knowledge development or learning. According to Johnson-Laird (1983, p.402), "our view of the world, is causally dependent both on the way the world is and on the way we are", and, according to Lakoff and Johnson (1980, p. 163), properties we attribute to physical objects "are not properties of objects *in themselves* but are, rather, interactional properties, based on the human perceptual apparatus, human conceptions of function, etc.".

Similarly, Bunge argues that empirical experience "is not a self-subsistent object but a certain *transaction* between two or more concrete systems, at least one of which is the experient organism. Experience is always of *somebody and* of *something*" (Bunge, 1967, p. 162, italics added), and the resulting knowledge "is attained jointly by experience (in particular experiment) and by reason (in particular theorizing)" (Bunge, 1973, p. 170). The transaction involves inputs from both knower and known, and the resulting knowledge reflects not only the reality of the known but also that of the knower.

The notion of experiential knowledge as resulting from a transaction, i.e., of an interaction that depends on the state of both the knower's mind and the surrounding environment including the object of study, is also at the core of Dewey's philosophy of education. Some, like Wong, Pugh, *et al.* (2001), have pushed Dewey's notion to the point of assuming that following such transaction, "both the person and world are necessarily transformed". Our interpretation is however that the world is transformed by the person as *conceived* in the mind, or even as *perceived* with senses, and not necessarily as it really exists. It is true that, sometimes, the person's environment could be physically "transformed", like when the intention is to modify the object of study or when some measurement done on the object affects the object itself. However, we do not admit that every learning activity entails a physical transformation of the "world", unless we take the person as part of it, so that when the person's mind is transformed *in* the world to which it belongs, so does the latter. Observing an object from a distance without any instrument that might affect the state of the object may help you to learn something about the object without affecting the object. As a result, your mind becomes transformed, but not the object. Similarly, information as transcribed in the selection you are now reading is not transformed because of your reading or because of some notes you might be jotting on the side. Such a physical transformation could only take place in a new edition of the book, should you kindly relay your notes to this author. Still, and because of all the influences mentioned above, and because of the limitations of our perceptual and conceptual systems, the transaction between knower and known results in knowledge (or belief) that does not mirror the perceived world. The outcome of the transaction is an *emergence* from both knower and known, i.e., a product that may share some properties of both but that also holds properties of its own that are not necessarily shared by either, and especially not the known object. For example, we attribute colors to physical objects while color is not an intrinsic property of real things but a consequence of the interaction of our visual sensory system with light in the real world.

In general, a person's experiential knowledge about a number of physical realities consists, from our point of view, of *conceptions* that could *correspond*, within certain limits, to specific structural or behavioral details in those realities. Some of these details may be common to all realities in question, while others may be particular to individual realities. From an ontological perspective, the correspondence might be: (a) *analogical*, like the picture of a familiar person we might have in our mind or like a circle drawn on a piece of paper to represent a round object, or (b) *analytical*, like the name of a person or like a point representing the person in a kinematical diagram. From an epistemological point of view, the correspondence might be subjective or objective. *Subjective* knowledge is often tainted with emergent details that are relatively detached from the real world and that may be entirely dependent on the idiosyncrasies of an individual's mental state. Such details are not necessarily reproducible or subject to similar interpretations by different people. In these and other respects, subjective knowledge is unreliable by many concerned people standards. In contrast, *objective* knowledge is characterized with details, including emergent details, that are kept in close and explicit correspondence to the real world, and detached in the best possible ways from particular human interests and mental states. When objective, experiential knowledge is shared by a group of people who can supply, by various standards, reliable evidence to their shared conceptions.

Scientific knowledge is in this respect the most objective form of experiential knowledge about physical realities. A scientific conception always corresponds to a set of physical realities in some analogical and/or analytical way, and with such a degree of precision that we can say that it reliably *represents* what it corresponds to in the real world (§ 2.7). The object of modeling theory in science education is to help students to develop norms and rules that allow them develop experiential knowledge that may be characterized as objective and reliable by scientific standards.

Our position in this matter is opposed to those who argue, with Latour and Woolgar (1979, p. 129), that scientific knowledge consists of mere artifacts "constituted through the artful creativity of scientists", with no necessary correspondence of whatever form to existing physical realities. Giere (1988, p. 59) points out that this nominalist position regarding experiential knowledge in science "comes from the fact that [people who hold it] typically argue that there is no fundamental difference between the social sciences and the natural sciences". Giere (*ibid*) then rightfully argues that the "general idea of 'social construction'… can be accepted for many aspects of *social* reality. But this, by itself, provides no evidence that *natural* reality is similarly constructed". If experiential knowledge, and especially scientific knowledge, consisted of mere conceptual inventions, and if physical realities were unknowable, "there would be no point in investigating things" in the first place (Bunge, 1973, p. 171), and, we add, there would be no point either in distinguishing science from other enterprises and thus in having separate curricula for science education.

## 1.3 TRADED KNOWLEDGE

> *The real world may be humanly knowable indirectly through knowledge trade, i.e., through interaction with other people and/or with public knowledge. Traded knowledge may contribute to experiential knowledge, and is sometimes indispensable for human knowledge to develop.*

Experiential knowledge about physical realities, i.e., knowledge developed through direct transaction with those realities, is perhaps the most meaningful form of knowledge. However, it is humanly impossible that all the knowledge of a person be experiential, both from a practical point of view and from a cognitive perspective. No human being can possibly know all s/he wants to learn about particular physical realities through direct transaction with those realities. Even when such a transaction is possible, knowledge development may also be affected by some social realities. There are times when knowledge may not even be developed without interaction with other people and/or with some public knowledge, i.e. knowledge of an individual person or of a community of people available through various media forms. Indeed, some knowledge, like new words and their meanings, can only be developed through such interaction. We call *traded knowledge* about a physical reality all forms of knowledge that a person develops about the reality not through direct transaction with it but following discourse with other people an/or exposure to public knowledge regarding the reality in question.

Human knowledge about physical realities is actually a mix of experiential and traded knowledge. Most of the knowledge our students develop about the real world in conventional science courses of lecture and demonstration is purely traded knowledge, and in some places all this knowledge is. Our position in modeling theory is to put more emphasis on experiential knowledge, especially at the pre-college level, and to promote student transaction with physical realities and empirical data, be it individually or in-group work. We hereby do not underestimate the importance of social factors in knowledge development, and we acknowledge unequivocally the role of public knowledge in the process, and especially the role of science textbooks. However, we admit neither that all human knowledge, including scientific knowledge, is purely traded, nor that social interaction involving other people, especially classroom peers, is always necessary or sufficient for developing meaningful knowledge.

Science education is concerned with helping students to develop knowledge about physical realities that is in line with scientific knowledge. To this end, science teachers must especially account in their courses, and on almost equal footings, for the established knowledge included in these courses (scientific subject-matter and related processes, along with underlying canons, norms and rules), and for the four dimensions involved in the development of such knowledge and listed in the experiential knowledge tenet (§ 1.2). The educational transaction facilitated by a science teacher thus involves primarily three major entities or sets of entities: (a) individual learners and their knowledge and beliefs about the world and science, (b) physical realities addressed in the course, and (c) related scientific knowledge. More specifically, knowledge (and beliefs) we are referring to, whether personal or scientific, consists of specific paradigms, and the transaction we are promoting in this book is to result in a paradigmatic evolution whereby students align their personal paradigms with those of science to certain reasonable levels.

## 1.4 PARADIGMS

A paradigm is, for us, a *conceptual* system that governs *explicitly* a person's *conscious* experience in a given situation, somewhat in the manner described by Kuhn (§ 1.5). The experience, though conscious, may also be affected implicitly by some mental structures and processes that are beyond the scope of this book. It may entail a single activity (thought or behavior, voluntary or involuntary) or a number of activities of one sort or another. It results in some form of *learning*, i.e., in the transformation of the involved paradigm and/or in the creation of a new one.

A paradigm, from our point of view, governs a person's conscious experience in the following respects:

1. It (the paradigm) determines the conditions that trigger every voluntary activity in the experience.
2. It sets forth standards, rules and guidelines for choosing and processing all that is necessary for the reification and continuous evaluation of the activity. This includes selection and analysis of empirical data when the experience is with physical realities.
3. It provides necessary conceptions, conceptual tools and methodology for conducting the activity, and for refining the paradigm subsequently.
4. It supplies appropriate mnemonics for consciously retrieving necessary means and method from memory.

Every human experience is thus paradigm-laden. Even blind perception (without aim) is. For, according to Kuhn (1970, p.113), "something like a paradigm is prerequisite to perception itself. What a man sees depends both upon what he looks at and also upon what his previous visual-conceptual experience has taught him to see". The paradigmatic dependence is not only about the interpretation of what one "looks at" in a perceptual experience. As mentioned in the second point above, it is foremost about sorting out primary from secondary details in a perceived reality. *Primary* details are salient details on which one decides to concentrate, and to retrieve from appropriate data for subsequent paradigm-laden analysis and interpretation. *Secondary* details are insignificant details that one decides to ignore or not to look at in the first place.

A given person possesses a number of paradigms of different natures, each tailored to a specific type of experience. These include, among others: *natural* paradigms for studying physical systems and phenomena in the universe, *technical* paradigms for conducting manual tasks with appropriate equipment, *social* paradigms for interaction with other people, and *metaphysical* paradigms (religion included, if any) for establishing beliefs about some ultimate "truths" within oneself and/or out in the cosmos, and for conducting oneself accordingly.

An individual's constellation of paradigms makes up her/his *worldview*, or world picture, somewhat in the sense advanced by Holton (1993). Holton defines a person's world view (or Weltbild) as "a generally robust, map-like constellation of the individual's underlying beliefs of how the world as a whole operates", beliefs that guide, to some degree, all opinions and actions of the person. Holton (1993, pp. 157-163) outlines his notion of worldview as the "constellation of underlying beliefs" in a concise list of 28 features. These features are virtually all attributable to our notion of worldview as the "constellation of paradigms".

Paradigms of different nature are not necessarily independent. Social paradigms are normally affected by metaphysical paradigms, technical paradigms by natural paradigms, and vice versa. Mutual dependence though does not necessarily imply coherence and consistency. As Holton (1993) argues, a person's worldview is "not necessarily internally coherent or noncontradictory". This can be reflected by a lack of coherence within the same paradigm or by a lack of consistency among different paradigms. Furthermore, a particular paradigm is "not necessarily stable over time" (Holton, 1993), and it may not be equally developed in the minds of different people. Various paradigms of a given person's worldview are not necessarily equally developed in the mind of this person. Among various paradigms possessed by a given person, those associated with the person's line of work are normally best developed. Among paradigms of the same nature held by different people, those held by concerned professionals are normally better developed than others'. That is why, for example, natural paradigms of scientists, i.e., scientific paradigms (§ 1.5), are better developed than those of lay people.

No two people can ever share exactly the same paradigm, whatever the nature of the paradigm or the profession that the two people might be having in common, and this, because of biological and cultural differences in people's history. For paradigms of a particular nature, differences are significantly more pronounced within the lay community than within a professional community guided by such paradigms. For instance, members of a given religious order (priests, nuns, pastors) share very similar religious beliefs and practice that make up the proclaimed metaphysical paradigm of their order, and more so do members of a given scientific community with respect to the natural paradigms associated with their fields of expertise. In fact, a scientific paradigm may be delimited in a specific field in such a way that we can practically ignore paradigmatic differences among scientists working in this field, and say that all those scientists share virtually the same paradigm. These scientists make up "a uniquely competent professional group [that should be recognized] as the exclusive arbiter of professional achievement… The group's members, as individuals or by virtue of their shared training and experience, must be seen as the sole possessors of the rules of the game or of some equivalent basis for unequivocal judgments" (Kuhn, 1970, p. 168).

## 1.5 SCIENTIFIC PARADIGMS: A MODELING PERSPECTIVE

According to Kuhn, a scientific "paradigm is what the members of a scientific community share, *and*, conversely, a scientific community consists of men who share a paradigm." (Kuhn, 1970, p. 176). However, and as Giere (1988, pp. 34ff) points out, Kuhn was so much involved in discussing the development or the evolution of scientific paradigms – and more specifically of scientific practice – in his book, that he neglected to specify paradigms with a clear structure. Kuhn recognized this fact indirectly in the epilogue of his book (1970), and in his reply (in Lakatos and Musgrave, 1970, p. 231-278) to Masterman (*ibid*, p. 59-89) who identified at least 21 different senses of the word paradigm as used by Kuhn. In an attempt to circumvent the problem, Kuhn defined a scientific paradigm as a conceptual system consisting of what he calls a "disciplinary matrix" associated with "symbolic generalizations", "beliefs in particular models", and a particular system of "values" (Kuhn, 1970, p. 182ff).

Our position regarding paradigms, and especially scientific paradigms, converges in part with Kuhn's position. We do not fully subscribe to Kuhn's work (1970), or any other work in the philosophy of science for that matter, and we acknowledge the merits of some of the criticism that this work has been subjected to (e.g., Lakatos & Musgrave, 1970). However, we believe that Kuhn's account of the development of scientific paradigms provides significant insights not only into those paradigms, but also into the natural paradigms of science students. In this respect, the cognitive implications of Kuhn's work bear for us a special value that will hopefully become evident in subsequent chapters, and especially in Chapter 3.

Scientific paradigms are natural paradigms. They are concerned only with physical systems and phenomena. Each scientific paradigm has a well-defined and exclusive scope. It can provide, in particular ways and with certain limits of *viability* (§ 2.7), particular answers to specific questions about physical realities; questions that are of interest to a particular community of scientists. Conceptual building blocks of a scientific paradigm are constructed, corroborated and deployed in the real world following generic tenets, principles and rules so as to provide nothing but reliable knowledge about this world.

We thus define a *scientific paradigm* as a natural paradigm shared by the members of a particular scientific community, of well-defined scope in the real world, and consisting of:

1. Ontological tenets about physical realities.
2. A scientific theory, or a set of theories about such realities, along with epistemological: (a) tenets that underline the nature of various conceptions that make up any scientific theory, and that establish the correspondence of theory and conceptions to the real world, and (b) principles and rules for conceptual structure and categorization, and for theory organization.
3. Specific methodology (including standards, tools, rules, guidelines, processes) for: (a) theory construction, corroboration and deployment (to borrow Hestenes' (1987) and Giere's (1988) terminology for various forms of theory implementation), and (b) continuous evaluation and refinement of all related conceptual structures and processes.
4. Axiological tenets some of which set the "value" of scientific theory and others govern scientist practice from an ethical point of view.

Among these four components of a scientific paradigm, only theory is formulated explicitly by the concerned community. In line with the position of many philosophers of science or mathematics (Casti, 1989; Giere, 1988; Harré, 1970; Hesse 1970; Wartofsky, 1968), and some science educators (Hestenes, 1987, 1992; Johsua & Dupin, 1999), a *scientific theory* is, for us, a conceptual system consisting of: (a) *a set of models* or families of models, and (b) *a set of particular rules and theoretical statements* that govern model construction and deployment and that relate models to one another and to specific patterns in the real world, and this in accordance with various tenets of the respective paradigm (§ 1.7 and § 2.6). These tenets, and, to a lesser extent, other paradigmatic components are often implicit in scientists' practice and literature. Philosophers of science have long been preoccupied in making them explicit, and cognitive scientists and science educators have lately joined them in this endeavor.

The scope of a scientific paradigm is set in accordance with the preoccupations of the scientific community with which it is associated. More specifically, it is function of the theory or set of theories that the designated community works on (§ 1.7). Each of the paradigmatic sets of tenets mentioned above is made up of two subsets, a subset of generic tenets and a subset of specific tenets. Generic subsets are common to practically all scientific communities, while specific subsets and any methodological differences that might distinguish one community from another are mainly due to the nature of respective theory. We may thus distinguish one or more paradigms within a given discipline (e.g., physics, chemistry, biology), depending on whether we group together all theories of the discipline or a limited number of those theories. For practical reasons, especially from a pedagogical point of view, and until the day we end up with a unified theory of science, we prefer to group together in a given paradigm a limited number of theories that correspond closely to one another and to the real world. This is how for example, in physics, we may group together, in what we call the *classical mechanistic paradigm*, Newtonian theory of translation, Euler theory of rotation, kinetic theory, thermodynamics, and classical electrodynamics.

Modeling theory in science education is concerned with helping students, especially those at the college and high school levels, develop natural paradigms that are *in line* with scientific paradigms (or that are *commensurable* with the latter, as we shall see in § 3.6). We do not pretend that modeling theory can help targeted populations to develop fully-fledged "scientific" paradigms by the time they graduate from high school or even college. This is an involved process that takes long years of actual scientific practice and that formal education alone can never accomplish under any educational theory, at least not by the time students graduate from college (Chapter 3).

Any scientific paradigm is distinguished from its natural counterparts held by ordinary people, students included, in virtually every aspect of the four dimensions distinguished above. Major aspects that set scientific paradigms apart from their counterparts are discussed in the following three sections. Each section is devoted to a specific philosophical dimension. These are respectively ontology (§ 1.6), epistemology (§ 1.7), and methodology (§ 1.8). Axiological issues are deferred to § 2.7. The following sections highlight our stand on scientific paradigms from a modeling perspective, and set what we believe is at stake in the educational enterprise, mainly with respect to helping students to reconsider their own paradigms and evolve into the realm of science (Chapter 3).

## 1.6 PATTERNS

*Physical realities that are of particular interest to scientists exhibit universal patterns.*

The "final desideratum" of scientific research, according to Bunge (1967, p. 190), "is the disclosure of patterns". Bunge is, of course, referring here to what we call exploratory research, and this is one of two types of scientific research, the other being inventive research. *Exploratory* research is about *describing*, *explaining*, and/or *predicting* patterns. A pattern may be reflected in the structure or behavior of a number of physical systems spread throughout space and time under certain similar conditions. Every scientific theory is originally conceived to explore certain patterns in the real world. *Inventive* research is about using the corroborated theory for pattern *reification*. This may be done by *controlling* or *modifying* existing physical realities so that they produce a specific pattern that the theory is concerned with, or by *devising* new physical realities to produce such a pattern.

Patterns treated in a given scientific theory are never restricted to the physical realities where those patterns were originally disclosed; otherwise scientific theory would lose its predictive power. Under similar conditions, a given pattern may be reproduced anywhere and at any time in the universe. The scope of any scientific theory thus extends to all physical realities in the universe that could possibly exhibit the patterns that the theory describes and explains. Some of the realities in question may not be already known by humans; scientists may eventually discover them or even predict their existence long before they are discovered, thanks to the already established patterns.

For example, in 1869, Mendeleev inferred a specific pattern in the chemical properties of about sixty elements that were known in his time, and proposed the first periodic table of the elements. Based on this pattern, he was able to predict the existence of many elements that were not then known, and he allocated specific cells in his periodic table for those elements. He was convinced that these elements would eventually be discovered, and he gave each element the name of an adjacent element that was then known with an "eka" prefix. For example, he allocated next to aluminum a cell for what he called eka-aluminum, and next to silicium a cell for what he called eka-silicium. Eka-aluminum and eka-silicium were actually discovered in 1875 and 1886 respectively, and were given the respective names gallium and germanium. The stories of the six quarks and of many astronomical objects that were long predicted before they were actually discovered testify to the importance of patterns in science.

The dominance of patterns in the universe does not exclude the existence of irregularities (or anomalies), and it does not preclude scientists' interest in such irregularities. On the contrary, irregularities are captivating to scientists. They incite them to go deeper in their investigations, and, as a result, some apparent irregularities may turn out to be disguised instances of known patterns, while others will not. The latter often lead to new discoveries, and more specifically to new patterns. The search for patterns is now getting to the heart of every scientific discipline, even those disciplines, like ecology, that are primarily interested with irregularities and weak trends, and for which the search for patterns and universal laws has always been "a touchy subject" (Harte, 2002).

Scientific theory, though, is about patterns. As Harré (1970, p. 35) argues, scientific "theories are seen as solutions to a peculiar style of problem: namely, 'Why is it that the patterns of phenomena are the way they are?' A theory answers this question by supplying an account of the constitution and behavior of those things whose interactions with each other are responsible for the manifested patterns of behavior [and constitution]". Helping students to develop systematic ways for identifying, exploring and reifying patterns in the real world must thus be at the core of science education. Such ways, as we shall see next, come about by following systematic model construction and deployment.

## 1.7 MODEL–CENTERED EPISTEMOLOGY

> *Models are at the center of a middle-out structure of scientific theory. A scientific model is mapped onto a particular pattern in the real world so as to reliably represent the pattern in question and serve specific functions in its regard.*

Categorization is one of the most important processes, if not the most important one, in human cognition. Construction and organization of categories have thus been a focal point in cognitive research. Many cognitive scientists have shown that, in accordance with the *theory of prototypes and basic-level categories* of Eleanor Rosch, "categories are not merely organized in a hierarchy from the most general to the most specific, but are also organized so that the categories that are cognitively basic are 'in the middle' of a general-to-specific hierarchy… Categories are not organized just in terms of simple taxonomic hierarchies. Instead, categories 'in the middle' of a hierarchy are the most *basic*, relative to a variety of psychological criteria" (Lakoff, 1987, pp. 13 and 56). For example, "dog" is "in the middle" of a hierarchy between "animal" and "retriever", just as "chair" is between "furniture" and "rocker" (Figure 1.1). Categories *in the middle* are *basic* in the sense that: (a) they ensure best a cohesive structure of human knowledge of any type, and that (b) they constitute the most accessible, efficient and reliable building blocks in knowledge construction and deployment.

The *middle-out* hierarchy extends, for us, from physical systems in the real world to conceptual systems in the paradigmatic world as indicated in Figure 1.1. Theories constitute the "content" of a scientific paradigm (§ 1.5), and models are ‘in the middle’ of conceptual hierarchy, between theory and concept. The model-centered, middle-out structure of scientific theory ensures theory coherence and consistency from an epistemological perspective, and it facilitates the development of scientific knowledge from a cognitive perspective.

A scientific model is to theory and concept what an atom is to matter and to elementary particles. Each elementary particle is essential in the structure of matter but its importance cannot be conceived independently of its interaction with other particles inside an atom. It is the atom and not elementary particles that give us a coherent and meaningful picture of matter, and it is the atom that displays best the role of each elementary particle in matter structure. Now, Bohr’s model of the atom is essential for understanding hydrogen-like atoms, and is often referred to as a “model” in physical science textbooks. However, other scientific models are seldom referred to or even presented as such, which would give students the false impression that Bohr’s model is about the only scientific “model”. Furthermore, various concepts and laws are often presented episodically, one after another in a given chapter, without relating them to one another in the context of appropriate models, whether

*Categories Hierarchy*
(according to Eleanor Rosch & George Lakoff)

| | | |
|---|---|---|
| SUPERORDINATE | Animal | Furniture |
| BASIC LEVEL | Dog | Chair |
| SUBORDINATE | Retriever | Rocker |

*Real World Structural Hierarchy:*

| | | |
|---|---|---|
| SUPERORDINATE | Matter | Galaxy |
| BASIC LEVEL | Atom | Solar System |
| SUBORDINATE | Elementary particle | Planet |

*Conceptual Hierarchy:*

| | |
|---|---|
| SUPERORDINATE | Theory |
| BASIC LEVEL | Model |
| SUBORDINATE | Concept |

*Figure 1.1.* Middle-out hierarchies.

implicitly or explicitly. Students are thus deprived of the opportunity of developing a coherent, model-based, picture of scientific theory, and end up with a piecemeal, fragmented picture of the world. To get a feel of this picture, imagine what your knowledge about physical realities would look like should you have learned at school that matter consists of elementary particles and no mention was ever made to you about the atom.

"When viewing the content of a science, Giere (1988) argues, we find the models occupying center stage… Theoretical [i.e., conceptual] models are the means by which scientists represent the world – both to themselves and for others. They are used to represent the diverse systems found in the real world (p. 79, 80). Our models shape the way we think and talk (p. 111)". What a scientific model represents, for us, is specifically a particular pattern in the real world that the model was originally conceived to disclose. As Harré puts it (1970, p. 35), the "chief means by which this is done [i.e., pattern disclosure] is by the making or imagining of models… The rational construction of models [is] to proceed under the canons of a theory of models" which is the epistemological theory of all scientific theories. In fact, Harré continues, scientific "theory can fruitfully be looked upon as the imaginative construction of models, according to well-chosen principles".

There is no unique definition of the word "model" in the literature, and there is no consensus on the use of the term even among advocates of modeling theory, be it philosophers of science or science educators (Fig. 1.2). Most think of a conceptual model as a complex theoretical structure while some bring it down to the level of a diagram or a mathematical equation. Harré (1970, p. 37) rightfully warns people who "still talk of equations as models of motions and processes" that at "that rate every vehicle for thought would become a model, and a valuable and interesting distinction would be lost… It's well to remember the old saying, if our eyes were made of green glass then *nothing* would be green". All modelers however agree that a model is always *of* some things and *for* a specific purpose. It has a well-defined scope. The scope is delimited in terms of the set of physical realities it is a model of, as well as in terms of the model function, i.e., questions it allows us to ask about those realities and the nature of the answers it is expected to furnish.

*Models are for the most part caricatures of reality, but if they are good, then, like good caricatures, they portray, though perhaps in distorted manner, some of the features of the real world... The main role of models is not so much to explain or to predict – though ultimately these are the main functions of science – as to polarize thinking and to pose sharp questions.*

Mark Kac, 1969 (in Pollak, 1994)

*Men do tend to employ familiar systems of relations as models in terms of which initially strange domains of experience are intellectually assimilated.*

Nagel, 1979

*A mental model is a knowledge structure that incorporates both declarative knowledge (e.g., device models) and procedural knowledge (e.g., procedures for determining distributions of voltages within a circuit), and a control structure that determines how the procedural and declarative knowledge are used in solving problems (e.g., mentally simulating the behavior of a circuit).*

White & Frederiksen, 1990

*A model is a surrogate object, a mental and/or conceptual representation of a real thing.*

Andaloro, Donzelli, & Sperandeo-Mineo, 1991

*A theoretical model of an object or phenomenon is a set of rules or laws that accurately represents that object or phenomenon in the mind of an observer.*

Swetz & Harzler, NCTM, 1991

*The term mental model refers to knowledge structures utilized in the solving of problems. Mental models are causal and thus may be functionally defined in the sense that they allow a problem solver to engage in description, explanation, and prediction. Mental models may also be defined in a structural sense as consisting of objects, states that those objects exist in, and processes that are responsible for those objects' changing states.*

Hafner & Stewart, 1995

*A scientific model is a set of ideas that describe a natural process. A scientific model (constructed of objects and the processes in which they participate) so conceived can be mentally "run", given certain constraints, to explain or predict natural phenomena.*

Passmore & Stewart, 2002

*A model is a representation, usually visual but sometimes mathematical, used to aid in the description or understanding of a scientific phenomenon, theory, empirical law, physical entity, organism, or part of an organism.*

NSTA, 1995

*A model represents a physical structure or process by having surrogate objects with relations and/or functions that are in correspondence with it.*

Nersessian, 1995

*Models are tentative schemes or structures that correspond to real objects, events, or classes of events, and that have explanatory power. Models help scientists and engineers understand how things work.*

NRC, 1996

*A model is a representation of structure in a physical system and/or its properties.*

Hestenes, 1997

*Models are mappings of functional correspondences between the structures of different domains of our knowledge... Pattern recognition also is a form of modelling.*

Glas, 2002

*Figure 1.2.* Sample model definitions.

A *scientific model* is, for us, a conceptual system mapped, within the context of a specific theory, onto a specific *pattern* in the structure and/or behavior of a set of physical systems so as to reliably represent the pattern in question and serve specific functions in its regard. These functions may be *exploratory* (pattern description, explanation, and prediction or post-diction), or *inventive* (pattern reification in existing physical realities or in newly devised realities). Mapping is done so that the model captures the essence of the pattern, and this by concentrating on specific but not all details in the physical realities exhibiting the pattern, particularly on *primary* details that are salient to the model function.

A scientific model can be defined and situated in a specific scientific theory following a four-dimensional *schema*. Two of the four dimensions, composition and structure, set the ontology and function of the model, and the other two, domain and organization, set its scope, all in terms of the scientific theory it belongs to, and by correspondence to physical realities exhibiting the modeled pattern.

The *domain* of a scientific model includes all physical realities exhibiting the pattern in question. Model *composition* consists of conceptions representing physical constituents and respective properties that are salient to the pattern. Model *structure* spells out relevant relationships among the pattern's salient features, especially in the form of laws that set the *distinctive* descriptive and/or explanatory *function* of the model. Model *organization* establishes the relationship of this particular model to other models in the corresponding scientific theory. The four-dimensional model schema is discussed in detail in Chapter 2.

A scientific theory consists primarily of a set of models, and its function in the real world is determined by those models, chiefly by correspondence to the set of patterns that they represent in this world. A theory's coherence is ensured by the inner structure of its individual models and by the mutual relationships among those models. Lower-level conceptions (concepts, laws and other theoretical statements) gain their theoretical significance through model composition and structure. In the latter respect, Giere (1988, p. 82) further argues that there is "no direct relationship between sets of statements [lower-level conceptions] and the real world. The relationship is indirect through the intermediary of a theoretical [i.e., conceptual] model".

Some cognitive scientists, linguists and other researchers have argued that model-based epistemology is not restricted to scientific paradigms, but that it extends to all sorts of human knowledge, and even to that of some animals (Johnson-Laird, 1983, p. 405 ff.). Bower and Morrow (1990) argue that "we build mental models that represent significant aspects of our physical and social world, and we manipulate elements of those models when we think, plan, and try to explain events of that world". Meanwhile, Johnson-Laird, Hestenes and others express a more radical position. According to Johnson-Laird (1983, p. 402), "*all* our knowledge of the world depends on our ability to construct models of it", and according to Hestenes (1995) "we come to know real objects (their properties and processes) *only* by constructing models to represent them in the mind" [italics added]. A more moderate position is expressed by Lakoff (1987) who argues that we "use cognitive models in trying to understand the world. In particular, we use them in theorizing about the world, in the construction of scientific theories as well as in theories of the sort we all make up" (p. 118). "The main thesis" of Lakoff's *experiential realism* "is that we organize our knowledge by means of structures called *idealized cognitive models,* or ICMs" (*ibid*, p. 68).

In an analysis of categorization data, Lakoff (1987) shows, and Giere (1994) supports, that human categorization is based on ICMs and not on similarity between individual features. ICMs not only govern the middle-out hierarchy among categories, but they also imply similar graded structures within individual categories. In the latter respect, Giere (1994) argues that models of any scientific theory can be graded with some basic models in the middle. *Basic models* are most fundamental to develop the elementary building blocks of all models in a given scientific theory and corresponding rules of model construction and deployment. They thus need to be given special attention in science education. We shall come back to this point often in our discussion.

## 1.8 MODELING METHODOLOGY

*Primary details of physical realities are not necessarily exposed directly to our senses. Disclosure and study of relevant patterns in the real world require some model-based* *idealization of physical realities that is often beyond the reach of ordinary people.*

Scientific conceptions are distinguished from lay conceptions of ordinary people, not only because of epistemological differences between scientists and ordinary people, but more importantly, because of methodological differences between the two groups when it comes to investigating physical realities. In everyday life, people develop and apply experiential knowledge about all sorts of realities mostly following tacit rules of thumb. These rules are concealed in people's unconscious to a point that it is often hard, if not impossible, to subject them to scrutiny. In contrast, scientific research is done according to systematic rules that are either spelled out explicitly in scientific literature or can be disclosed through meticulous scrutiny of scientists' practice.

The difference between scientific and lay methodology has long been, and still is, at the core of debates among philosophers of science. Some philosophers read in scientists' practice, just like in that of ordinary people, a wide diversity of research methods, while others have spoken of a unique scientific methodology that is common to all scientists irrespective of their discipline or their field of specialty. Some have argued that scientific methodology is predominantly inductive while others have argued that it is predominantly deductive or hypothetico-deductive. Some have spoken for a variant of either approach, while others recognized the merits of both induction and deduction in science. Some in the last camp have also identified processes, and especially "model generation processes", that are "neither inductive nor deductive" (Clement, 1989, 1993).

Scientific exploration starts by asking a particular question about specific physical realities within the framework of an appropriate paradigm. The paradigm then helps us to formulate an appropriate hypothesis, i.e., conjecture a tentative answer to the question. The paradigm also guides our observation of the realities of interest in two respects. First, the paradigm helps us to sort out primary from secondary details, and determine, subsequently, what data are salient for assessing the hypothesis we made. Next, the paradigm helps us interpret selected data, analyze them, and decide whether they corroborate or refute the hypothesis (Bunge, 1967, pp. 162-169, 177-184; Kuhn, 1970, pp. 111, 120-124).

We will come back to hypothesis testing later in this section. An important aspect of scientific exploration is that primary details and data may not be exposed directly to our senses. About twenty-four centuries ago, Democritus (c.460-c.370 B.C.) pointed out that "nothing do we know from having seen it; for the truth is hidden in the deep" (Miller, 1985, p. 32). Unfortunately, this point was fully appreciated only about twenty centuries later when Galileo (1564-1642) warned us that ordinary lay experience that relies heavily on sense perception is often deceiving, because reliable knowledge of the world resides in primary data that are not exposed directly to our senses. This position is nowadays at the foundations of modern science. As Bunge (1967, p. 169) argues, "patterns are sought and found beyond appearance, in a reality that is supposed to be there, that must be hypothesized since it cannot be directly perceived". Science, according to Bunge, is indeed interested in "the finding and making of nonordinary" realities. These are "iceberg-like [realities]: they are mostly submerged under the surface of immediate experience". They "are not within the reach of the layman" because they "are not purely empirical" and they require "the invention of theories going beyond the systematization of experiential items and requiring consequently ingenious [conceptual tools and] test procedures". In science, Bunge adds, "theory and experience are interpenetrating rather than separate, and theory alone can lead us beyond appearances, to the core of reality" (Bunge, 1967, pp. 155-158).

Bunge's "hypothesized realities" are, from our point of view, *idealized conceptual realities* (somewhat in the sense of Lakoff's ICMs), the most effective and efficient of which are *scientific models*. Such realities may or may not be conceived by *re*construction of a set of physical realities. In the former event, the conceptual reconstruction is partial. It is done within the framework of an appropriate paradigm in order to display the best specific primary details in the corresponding physical realities and optimize their exploitation. In the latter event, i.e., when our idealized conceptual realities do not consist of conceptually *re*constructed physical realities that are known to us and are exposed to our senses in one form or another, these conceptual realities may be constructed following conjectures about the existence of some physical realities that are as yet unknown. This was for example the case when Gell-Mann first hypothesized the existence of quarks by pure rational *inference* from some mathematical manipulations, or when Bohr proposed his atomic model by analogy to the planetary model. This was also the case with Darwin, who proposed his evolution theory following a rational inference from Malthus' theory on populations' evolution as a function of natural resources, and by analogy to what was then known about natural selection among plants competing for survival in certain territories. Construction of idealized conceptual realities about unknown physical realities is indeed, as Harré argues (1970, p.40), "the creative process of science, by which potential advances are initiated, while" idealization of known physical realities "has, generally speaking a more heuristic value".

Leonardo Da Vinci (1452-1519) was perhaps the most impressive figure among those who started the campaign against the Baconian inductive approach. Da Vinci argued that this approach does not allow us to disclose primary details and relationships in the real world. Instead, he argued, and showed through practice, that to this end, we need to begin exploratory research not with data collection but with the construction of *idealized models*, including mathematical models, and then follow with mapping those models onto physical realities. Galileo (1564-1642) picked up later on Da Vinci's approach and developed it in a way that laid the early foundations of a modeling theory of science.

Modeling processes can yet be traced to the early days of scientific enterprise. In their discussion of "seven ideas that shook the universe" (from Copernican astronomy to quantum theory), Spielberg and Anderson (1995, p. 302-304) recognize that the use of models made it possible for major break-throughs to take place in the history of physics (and thus science), especially because models make it "possible to synthesize (in our minds)" major aspects of physical realities "that we might otherwise not have guessed".

Reviews of landmark works in the history of modern science, like those of Newton (Hestenes, 1992, 1997), Maxwell (Nersessian, 1995) or Darwin (Harré, 1970, 1978), and observation of scientists presently at work (Clement, 1989; Gentner & Gentner, 1983; Giere, 1988) reveal that modeling is a major form of scientific reasoning – if not "the" major form – whereby scientists generate, test and reify creative and viable ideas about physical realities through the successive refinement of generic models. A particular model is constructed, deployed and continuously evaluated within the framework of the theory it belongs to, and by correspondence to physical realities exhibiting the pattern that the model represents in the real world.

We admit that various scientific groups may have their methodological particularities. However, we maintain that they do share generic practices with one another, as well as with other creative groups, like artists. Modeling processes are the most important generic processes that scientists share and follow more systematically than any other group, though implicitly or even unconsciously at times. All modeling advocates agree, to various degrees, with Johnson-Laird (1983, p. 417-418) that we do not only use models to "make sense" of the world around us and to coherently and efficiently structure our knowledge, but we also "impose" them on ourselves as "regulative principles of behavior". However, and like in the case of "model", there is no consensus yet as to how we do so and what "model*ing*" entails in the first place. Some modelers, like Johsua and Dupin (1999, p. 17) talk of a single modeling process, while others talk of a variety of modeling processes and make a distinction, say, between model construction and model deployment (Hestenes, 1987), or of a variety of modeling "activities" considered as "variations of a single modeling process" (Hestenes, 1995). Yet they all agree that some form of modeling is always involved in any scientific activity.

Scientific knowledge is the result of transactions between the empirical world of physical realities and the rational world of scientists along the lines discussed in § 1.2. It is especially the result of continuous *empirical-rational dialectics* between physical patterns and scientific models within the framework of appropriate paradigms. Such dialectics always start with the construction of a tentative model followed by the collection of appropriate empirical data that will be analyzed to test the validity of the model and subsequently make the appropriate judgment as to the acceptance, refinement or rejection of the model. In short, scientific methodology is primarily about making, testing and using conceptual models of patterns in physical realities, with the use of various conceptual tools, and following well-defined principles and rules of engagement.

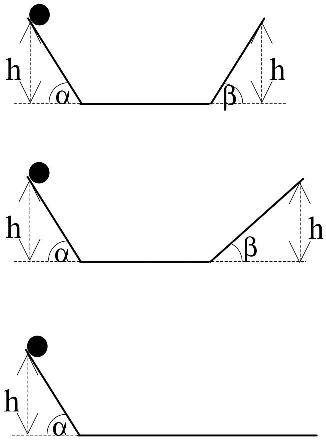
Pattern description and explanation are prime goals of the scientific enterprise. Pattern description may be carried out through *observation* of physical realities exhibiting the pattern. A *descriptive* model (§ 2.5) may be constructed to this end, that may be directly mapped onto observable data and duly corroborated. However, possible causes that explain the pattern, or the absence of any cause, cannot be determined directly through observation. One needs only to remember that explanatory concepts like force, field, and energy are not observable. Pattern explanation can only be carried out through *explanatory* models (§ 2.5) *inferred* from descriptive models the way Newton explained the motion of physical objects (Hestenes, 1992, 1995) and the way Darwin explained the evolution of species (Harré, 1970, 1978).

Model construction is often accompanied by the construction of new lower-level conceptions (concepts, specific laws). In fact, we maintain that all sorts of scientific conceptions are developed in the process of, or for the purpose of, modeling physical realities. A concept or a law is always conceived within the context of a specific model, or set of models, in order to contribute to model formulation and subsequently to theory construction and deployment. Theory construction and validation in exploratory research is, for us, primarily a process of *model induction* and corroboration. Theory deployment is a process of *model adduction* and analysis in problem solving in the traditional sense, and a process of *model deduction* in theory reification and inventive research.

Let us go back to hypothesis making and testing, which is an integral part of any scientific research, whether exploratory or inventive. A hypothesis is a conjecture, a tentative statement about a specific relationship within or among physical realities. It is more specifically, as Giere (1988, p. 80, italics added) puts it, "a statement asserting some sort of *relationship between a model and a designated real system* (or class of real systems). A theoretical hypothesis, then, is true or false according to whether the asserted relationship holds or not". The relationship, Giere continues (*ibid,* p. 81), is "*similarity* between models and real systems [in some] relevant *respects* and *degrees*". Testing a hypothesis thus consists of assessing the model-system relationship, and not the actual relationship between the elements of concerned physical realities. Otherwise, rejecting a hypothesis would be like rejecting the physical realities in question. When the outcome of hypothesis testing is positive, the relationship between model and realities is sustained and the model is corroborated (or reinforced, if it already exists). When the outcome is negative, one of the following scenarios could take place: (a) the relationship between model and realities is reconsidered while the model is preserved, (b) the relationship is sought with an alternative model without losing the original model, the issues addressed then turning out to be outside the scope of the model, (c) the model is refined (and perhaps falsified) and the relationship reevaluated.

Modeling does not always have to proceed in the empirical world. It may proceed exclusively in the rational world of scientists where most of the creative inventive research actually takes place. Hypothesis making, for example, does not have to pertain directly to empirical data (in the Baconian sense), and hypothesis testing does not always have to start in the empirical world, though it has to get there ultimately. When Galileo postulated and corroborated his version of the principle of inertia, he was not thinking directly about physical realities, but more in terms of a particle model that he contrived for a thought experiment depicted in Figure 1.3. A particle model consists of an idealized, dimensionless object of no internal structure. The particle represents all objects whose translation is not affected by their own shape and dimensions. The situation involved in Galileo's thought experiment is an altogether idealized situation. All resistive forces of the real world, like friction and air resistance, have been removed so that when the particle is on a horizontal track, it will be subject only to two forces that cancel each other out. These are the object's weight and a normal force exerted by the track. The same sort

A particle glides on a frictionless track having the shape shown in the accompanying figure. The left ramp of the track has a fixed slope of angle α, while the right ramp can be tilted to any slope angle β. Because of energy conservation, when released from a point located at a height h on the left side, the particle reaches the same height h on the right side, irrespective of the value of the angle β. The smaller β is, the longer the distance traveled by the particle on the right ramp to reach the same height h. When β is zero, the particle travels an infinite distance to reach height h. In other words, once it hits the bottom of the left ramp, the particle will be subject to no net force, and it will keep gliding indefinitely at constant speed, and in a straight line*, on the now horizontal part of the track.



* Galileo had actually thought of a curved path around the earth.

*Figure 1.3*. Galileo's thought experiment about the principle of inertia.

of particle model idealization was involved in the development of the Newtonian theory of mechanics. This theory, as we shall illustrate in Chapter 2, is entirely about Galilean particle models, each model describing and explaining a specific pattern in the translation of physical objects (e.g., free particle in uniform motion, forced particle in uniformly accelerated motion, bound particle in circular motion or in harmonic oscillations).

Modeling requires a number of conceptual tools for knowledge organization, depiction and representation, processing and communication. Not all tools used by scientists are as explicitly formulated as mathematics, either in scientists' minds, or in science textbooks. In fact, the most important tools advocated in our modeling theory are entirely tacit in scientists' minds and texts. These are modeling schemata. As we shall discuss in the next chapter, a *modeling schema* is an organizational tool that helps us to "define" explicitly specific conceptions, concepts or models, and "situate" them appropriately in the corresponding theory. With these schemata are associated explicit rules, especially modeling rules, for using conceptions in both the rational and the empirical worlds. These rules, as well as those associated with other tools, are the object of Chapter 4.

Mathematics offers scientists the most efficient tools of expression and rational operations. The practical utility of mathematical symbols, equations, diagrams, graphs, etc., along with associated semantics and syntax, is best realized in the construction and deployment of scientific models. In fact, and as we shall see in Chapters 4 and 5, the utility of a scientific model, and especially one of physical sciences, is primarily determined by the degree to which it can be *transformed* into a *mathematical model*. At this point, and as Harré (1978) puts it, the umbilical cord between the scientific model and the real world can be cut, and the model can be entirely processed rationally, in dissociation from the empirical world. The return to this world will only be needed to interpret and justify the outcomes. Successful modeling in the rational world is in fact, at some level, an indicator of mastery in science. Theoretical scientists often construct new scientific conceptions, models included, based entirely on theoretical premises. This is in sort what Galileo did in his thought experiment (Fig. 1.3) whereby construction and initial validation of his free particle model were first done exclusively in his rational world. Empirical corroboration followed later, actually after his death.

## *Chapter 2*

# MODELING SCHEMATA

The merits of modeling theory are being more and more recognized among scientists and philosophers of science, and more notably among science educators. All major reform movements that are nowadays in effect in science and mathematics education subscribe to some aspects of modeling theory, without necessarily recognizing it explicitly. Virtually all admit that *models* and *modeling* processes are as much important in science and mathematics education as in the original disciplines (AAAS, 1990, 1993; AMATYC, 1995; NCTM, 1989, 1991; NRC, 1996; NSTA, 1995). Special sessions are being devoted to modeling in annual meetings of prominent science education organizations (ESERA, August 2003; NARST, March 2003). Models are deemed vital for instilling conceptual order in the apparent chaos of the real world as perceived through our senses. More importantly, models are considered crucial pedagogical tools for the meaningful and efficient learning of science. Unless students are “introduced to the game that professional scientists play called ‘creating and shooting down models’”, many reformists argue, we “do not let them in on the game of ‘being’ a scientist” (Pollak, 1994), and we end up driving them instead into a state that “is likely to contribute to [scientific] illiteracy” (Erduran, 2001).

In contrast to the consensus that philosophers and educators are getting close to regarding the importance of models, both in science and science education, there is little agreement yet as to what scientific model and modeling are all about. In this chapter we introduce a particular set of modeling tools that we call *modeling* *schemata* and that we consider most generic and most important for model construction and deployment. The tools are deployed to lay out, as explicitly and as comprehensively as possible, the *content* of scientific models and their conceptual building blocks. Use of these and other tools in model deployment is reserved for subsequent chapters.

Our emphasis in this and subsequent chapters is on aspects that are immediately related to science education. Philosophical aspects will hereafter be referred to only within the limits of their pertinence to education. Whenever necessary, reference is briefly made to student states regarding the issues under discussion, with just enough details to point out the pedagogical importance of these issues. Student states about these and other issues is discussed in more detail in the next chapter.

## 2.1 SYSTEMS

Scientific paradigms are about physical realities, i.e., systems and phenomena of the real world. They are natural paradigms especially concerned with exploratory and inventive research about patterns manifested by such realities, research conducted via conceptual models that are supposed to represent the patterns under study. A model is mapped on a specific pattern following explicit rules laid out by the scientific theory that the model belongs to (§ 2.3 and § 2.6). The pattern may be about the structure or the behavior of a number of physical systems, spread out throughout space and time in the universe.

A physical system may consist of a single physical body (*simple* system) or of a number of interacting physical bodies (*composite* system). The constituent(s) of a system may interact with physical bodies outside the system. The latter make up the *environment* of the system. In order to distinguish constituent bodies from those in the environment, we shall refer to the former as *objects* and to the latter as *agents*. The boundaries of a system, and thus objects that need to be considered as part of the system, are normally chosen by convenience, mainly in terms of the pattern under study, and of some theoretical controls. The same goes for the scope of the environment, and thus for the choice of agents of interest.

The system-environment demarcation line can most conveniently be set in terms of the nature and significance of interaction among various bodies of interest. Two bodies are normally included as *objects* in a system whenever it is necessary that both of the actions that they mutually exert on one another be accounted for. When only the action of a body on an object needs to be accounted for, but not the reciprocal action of the object on that body, the body in question will be incorporated into the environment of the system, and thus considered as an agent (Fig. 2.1). Interactions that are accounted for, and subsequently selected objects and agents, are those considered to be significant within the degrees of approximation and precision set for the study of the pattern in question.

For example, our solar system consists of our sun, along with nine planets and their moons. Depending on the nature of the study we are interested in with this system, and especially the significance of interaction with other celestial bodies, the corresponding environment may extend from a limited number of neighboring stars to the entire Milky Way galaxy, or even beyond. For the same reasons, we may

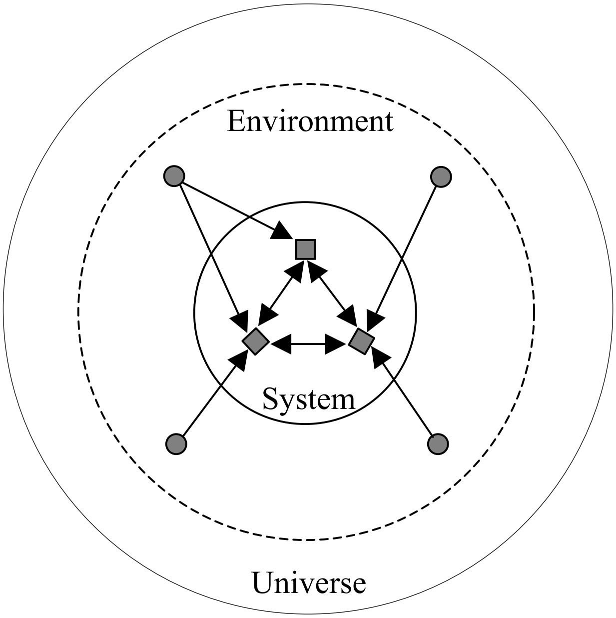


*Figure 2.1*: System and environment.

Arrows indicate that mutual interactions are considered between the objects (squares) of a system, while only the action of agents (dark circles) on objects is accounted for.

either: (a) widen the system boundaries so as to include our solar system in a bigger composite system like the entire galaxy, or (b) narrow those boundaries down, and break up the composite solar system into simple systems, each consisting of a single planet.

Physical systems may be natural or artificial. *Natural systems* are those developed in the universe without human intervention (or the intervention of any other intelligent form of life). *Artificial systems* are human-made systems. An artificial system is called a *physical model* when it is built to represent specific aspects (a pattern) that are common to a number of other physical systems (e.g., mock-ups, miniature cars, dolls). A physical model does not necessarily represent all aspects of the systems it stands for. It usually stands as a *partial representation* of those systems. As such, we distinguish a physical model from a prototype. A *prototype* is, for us, a system chosen among many included in the same category so as to represent this particular category. It stands, at least for the people who choose it, as a comprehensive representative of the members of its category. It is usually the one that people are most familiar with and that first comes to mind when one thinks of the category to which it belongs. For example, the robin is for many people the prototype-bird. It is the kind of bird that first comes to their mind, and in terms of which they think, every time they become engaged in a discussion about birds. Similarly, and because they are the celestial bodies that we are most familiar with, our sun, our terrestrial globe and our moon are the respective prototypes of the sun, planet and moon categories for virtually all humans living on Earth.

Systems are not necessarily physical. There are systems of different natures (social, political, industrial, etc.). Natural science is concerned with *conceptual systems* (§ 1.1) that correspond to physical realities, and so is modeling theory. Paradigms, and then theories, are the most complex conceptual systems in science, and conceptual models are the most fundamental (§ 2.6). Like a physical model, a *conceptual model* is an artificial system. It is however, made up of conceptual, and not physical components (§ 2.4 and § 2.5), and it always constitutes a partial representation of the physical realities it represents (§ 2.3). Scientists rely on both physical and conceptual models in their endeavors. However, conceptual models always precede physical models ontologically. A physical model is always the reification of some conceptual model; the conceptual model is conceived ahead as the blueprint of the physical one. Because of that, and especially because of their higher viability in studying natural systems and phenomena, conceptual models are more important than physical models (and prototypes) in science. Modeling theory is mainly concerned with conceptual models, and, in this book, we are more specifically interested with teasing out from scientists' products and conduct generic tools and systematic rules for laying out model ontology and for model construction and deployment. The most generic of these tools are modeling schemata, to which we next turn our attention in this chapter. Rules of interest are discussed throughout the rest of this book, and especially in the last two chapters.

## 2.2 MODELING SCHEMATA

Philosophers of science have long argued that scientists and lay people are distinguished from one another not only by the nature and amount of knowledge that their natural paradigms comprise, but primarily by the way the knowledge is organized in these paradigms. Scientific paradigms are better organized, which makes them more productive and especially more efficient than their common-sense counterparts. Two key features stand out more than others, we believe, in the organization of scientific paradigms. One is the *model-centered, middle-out* structure of scientific theory discussed in the previous chapter. The other is the existence of generic tools for the construction of all sorts of conceptions. The most important of these tools are templates that scientists use, often implicitly and even unconsciously, for the layout of any conception and its integration with the rest of their knowledge. We call these templates *schemata*, and we hereby concentrate on modeling schemata, i.e., schemata used for the construction of models and their conceptual building blocks.

For reasons that will become obvious during our discussion, we will concentrate on two modeling schemata. One is the model schema (Halloun, 1996, 2001a); the other is the concept schema (Halloun, 1998a, 2001a). The *model schema* is a four-dimensional template that can be used for the construction and deployment of any scientific model. Two dimensions, *composition* (§ 2.4) and *structure* (§ 2.5) set the ontology and function of any model. One dimension, *domain* (§ 2.3), sets its empirical scope in the real world, and another, *organization* (§ 2.6), relates it to other models and conceptions in a given scientific theory. Similarly, the *concept schema* is another four-dimensional template concerned with any scientific concept (§ 2.8). The model schema is especially helpful at the secondary school and college levels. It may have limited utility at lower levels, especially in the early schooling years, where the concept schema is most suitable.

A *modeling schema* is an organizational template used to ensure that any conception, and especially a model, be built comprehensively without missing any primary feature, and integrated coherently in a given theory, and this in the most efficient, compact and coherent way possible. It also offers, directly or indirectly, well-defined rules for evaluating and employing the corresponding scientific conception. In a sense, modeling schemata are, along with other tools, to meaningful learning of science what semantics and syntax are to mastering any language. A modeling schema sets the rules of correspondence of a conception to the real world just like semantics do with vocabulary. It also sets the guidelines for putting the conception together and relating it to other conceptions just like syntax in grammar.

At an early age, babies pick up their native language, or any other language for that matter, and after a few years of practice, they start using it for fluent communication with others without developing explicitly the corresponding grammar. However, they remain unable to master this language and make creative literal composition until they develop grammar through formal education. Learning a new language at a later age is not as straightforward, and neither is learning science. Learning science without modeling schemata can be (and actually has always been for most students) as treacherous as attempting to learn a new language at adulthood without learning its grammar explicitly, say, from reading texts in this language or listening to native people speaking it.

Some educators have duly pointed out that: (a) science education has so far blurred out the distinction between “procedures that are useful for encoding [information] and those that are useful for retrieval”, and that (b) encoding procedures are not necessarily convenient for retrieval (Duschl, Hamilton & Grandy, 1990). Modeling schemata help in solving this problem. When covered properly, some dimensions (composition and structure in model schema) help students to develop appropriate encoding procedures for any particular conception, while others (domain/scope and organization) help them to develop appropriate mnemonics for deploying the conception.

Modeling schemata are as helpful for science teachers as they are for students. They are used for planning and teaching lessons and for assessing student learning and teaching practice. Under modeling instruction, the content of a teaching unit is usually organized around a specific model (§ 4.2). Planning and teaching a lesson following modeling schemata ensures that students develop the model in question (or any necessary conception) without missing any salient feature. The same schemata can subsequently be used to develop an appropriate assessment taxonomy that covers all salient features and that help in logging the evolution of every student (§ 4.6).

Before we turn to a detailed discussion of our modeling schemata, we need to call the reader's attention to two points concerning these schemata. One has to do with the common use of the word schema, the other, with the targeted users of modeling schemata.

Our use of the word schema is different from its common use in cognition. There, it means "a cognitive unit for representing an individual's theoretical knowledge… a unit of knowledge with thematic content". It is a mental system that consists of a set of structures and processes related to a particular "theme" (Duschl, Hamilton & Grandy, 1990). It resembles, in many aspects, what diSessa (1993) calls phenomenological primitives, or p-prims, and which he defines as "small knowledge structures, typically involving configurations of only a few parts, that act largely by being recognized in a physical system or in the system's behavior or hypothesized behavior". They "occupy midlevels" between "data-driven sensory elements" and "the world of ideas, or named concepts and categories". "P-prims, it seems", diSessa argues, "lie systematically at the interface between experience and formalizable physics, both in the genetic sense (providing an important knowledge base for learning physics), and later, for interpreting the real world in terms of the formal theory and vice versa". To each physical reality or theme corresponds a particular p-prim so that "the set of physical p-prims is, in fact, rather large and loosely coupled" (diSessa, 1993).

In contrast, a modeling schema is, for us on the one hand, a generic tool for explicitly organizing and deploying a particular class of conceptions (i.e., models or concepts, in this chapter). It is a sort of conceptual template with no specific content, but with a set of cells (dimensions) and associated guidelines for content development. It lies at the interface between not a single physical reality (or particular theme) and the corresponding idea, but among many physical realities exhibiting a certain pattern and the corresponding model (or related conception). On the other hand, and unlike cognitive schemata and p-prims mentioned above, the number of modeling schemata is very limited, and the most important of them are the two discussed in this chapter.

Teachers and not students are the main target of our discussion in this chapter. The way they are presented, modeling schemata serve as comprehensive templates or check-lists for planning, conducting and evaluating instruction, and for putting more structure and coherence in the presentation of various models, laws and concepts in any scientific theory. Students need to systematically construct their conceptions following these schemata, but they need not, at least at first, to do so by going linearly and explicitly through each of the four dimensions of a given schema. In fact, a schema and its dimensions should not even be presented as such to students, at least not freshmen. As instruction progresses, teachers may encourage students to develop, for each schema, some sort of a flowchart or check-list for comprehensive model or concept development (§ 4.4).

## 2.3 MODEL DOMAIN

A scientific model is mapped in specific respects on a particular pattern in the real world. The pattern can be in the structure and/or behavior of a multitude of physical systems. The *domain* of the model is confined to this particular pattern in the real world, and the model is said to *correspond* to all physical realities manifesting the pattern. Every physical system that the model corresponds to is called a model *referent*, and the entire set of systems exhibiting the pattern constitutes the model *reference class*. The model can also be said to *represent* its referents in the *specific respects* that are particular to the pattern and are accounted for in the model.

Like the pattern it represents, the domain of a model is universal. Model referents may exist anywhere and anytime in the universe. However, no model can represent any of its referents in an exhaustive manner. A scientific model represents primary aspects of its referents that are salient to the corresponding pattern. However, it can never represent all possible primary aspects of any referent, as it can never answer all questions about the structure or behavior of such a system. As such, the model representation of its reference class is always *partial*, and is governed by appropriate correspondence rules.

*Correspondence rules* spell out the conditions for a physical system to be the referent of a model, i.e., to be partially represented by the model. Some of these rules are common to all models belonging to the same scientific theory, or to families of models belonging to various theories in the same paradigm. Others are particular to the model in question. Common rules stem from the generic laws of the theory (and/or paradigm) and the corresponding limits of precision. Particular rules stem from the specific pattern represented by the model.

For example, many theories in the classical mechanistic paradigm (§ 1.5) are about *particle models*. A common correspondence rule for all these models is that their referents must behave in a way that is not affected by their geometric properties of shape and dimensions, or such that we can ignore any such effect within the limits of precision set by the corresponding theory. To highlight this fact, referents in question are each represented in the corresponding model by a particle, a fictitious point-like, dimensionless object (§ 2.4). This is the case of translation models in Newtonian theory of mechanics and of ideal gas models in kinetic theory. Every particle model obeys this and other general rules, as well as particular rules, in the manner illustrated in Figure 2.2. In order to be a referent of a given model, a system must obey all correspondence rules of the model, both general (like the ones marked P and N in this figure) and particular. If any one of these rules is broken, the model in question can no longer represent the system.

The reference class of any scientific model is not exclusive. The same physical system may manifest more than one pattern, and would thus be the referent of various models that may or may not belong to the same scientific paradigm. For example, in its continuous movement, the Earth exhibits two motion patterns that are the object of the classical mechanistic paradigm in physics. These patterns result in two other patterns; two sidereal cycles that govern life on our planet and are the concern of a different paradigm in earth or biological sciences. One motion pattern is in the elliptical translation of our globe around the sun; it is represented by a particle model governed by Newtonian theory of mechanics. The other motion pattern is in the Earth's rotation around itself; it is represented by a rigid body model governed by Euler theory. The first motion results in various seasonal cycles around the globe. The second motion results in day and night cycles. In these respects, Earth is the referent of four different models, two models of physical sciences and two of earth or biological sciences.

The same physical pattern may be represented with different scientific models. However, no two models can represent the pattern in exactly the same way, though they may both have the same reference class. Alternatively, and unless one pattern can be seen as a

*A physical system, microscopic or macroscopic, may be represented with the Newtonian uniformly accelerated particle model if:*

- The system is in translation in a specific reference system. (N)
- The reference system is inertial. (P)
- Objects in the system may be moving relative to one another, but their relative motion does not affect the translation of the entire system. (N)
- Objects in the system may be disintegrating, but the mass of the entire system remains constant. (N)
- The translation is not affected by the system's geometric properties of shape and dimensions. (N)
- Intrinsic properties (e.g., mass and dimensions) of the system are not significantly affected by its speed within the adopted limits of precision. The speed of the system remains extremely small by comparison to the speed of light. (P)
- The translation of the system is linear or parabolic.
- The system is subject to a net constant force from its environment. Its acceleration is constant.

*Figure 2.2*: Correspondence rules for a uniformly accelerated particle model in Newtonian theory.

This and other Newtonian models are discussed in more detail later in this chapter. Rules marked with (P) are common to many theories in the mechanistic paradigm. Rules marked with (N) are common to all models in Newtonian theory. Unmarked rules are particular to the model in question.

special case of another more complex pattern, no model can represent two patterns in exactly the same respects, even when the two patterns are exhibited by the same physical realities. Outside the two noted cases, and unlike its referents, the domain of a scientific model is both restrictive and exclusive.

A model can represent two patterns only when one pattern can be seen as a special case of a more complex pattern. Alternatively, the same pattern can be represented by two different models within the same theory only when one model, that originally represents a more complex pattern, can be totally reduced to the model that represents the pattern in question. For example, the same physical objects may undergo different types of translation in inertial reference systems*, and be represented by various particle models in Newtonian theory (e.g., linear uniform motion, uniformly accelerated motion, or simple harmonic motion). No two Newtonian models represent the same translation pattern in exactly the same ways. However, linear uniform translation (zero acceleration) can be seen as a special case of accelerated translation, and the uniformly accelerated particle model (constant acceleration under constant force) can be reduced to the free particle model (constant velocity under no net force) so as to represent linear uniform motion. All components of the latter model can be drawn from the former model by setting force and acceleration to zero (§ 2.5).

Two scientific models may represent the same pattern without having one model completely reducible into the other. However, the two models may not then belong to the same scientific theory, and they cannot allow us look at the pattern from the same perspective.

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* A *reference system* consists of a reference frame and a clock. It is represented mathematically with an appropriate *coordinate system*. A *reference frame* is a physical system relative to which various spatial components of a physical reality can be conveniently defined and measured. Inertial reference systems are commonly used in the classical mechanistic paradigm whereby time is assumed not to vary from one system to another. Consequently, corresponding coordinate systems are strictly spatial; they represent only the reference frame but not the clock. Cartesian systems like the one shown later in Figure 2.3b are typically used in this respect.

A pattern, and especially the composition and structure of its referents, is always defined in an appropriate reference system; composition and structure of the corresponding model are specified in a convenient coordinate system. The value of some properties and the form in which they are represented and related may vary from one reference system (or coordinate system) to another.

Alternatively, every time we need to look at the same pattern from different practical or theoretical perspectives, or when we need to represent the same pattern with different levels of precision, we have to resort to different models belonging to different scientific theories within or without the same paradigm. An instance of different practical perspectives is in the Earth's dual movement accompanied by two different sidereal cycles. In such an instance, and depending on the interest of study, one may resort to a paradigm of either physical or biological sciences. When interested in the motion of our globe from a mechanical perspective, and for theoretical convenience purposes, one may resort to either Newtonian models or to Lagrangian or Hamiltonian models within the confinement of classical physics. If the interest were in significantly high precision, one would shift the paradigm to relativity. The same paradigm shift would also be required if we were examining the same pattern in non-inertial reference systems.

The domain of a model or of any scientific conception for that matter, let alone the notion of a model, is seldom discussed explicitly in traditional science instruction. Consequently, students are often confused as to what a model represents in the real world and under what conditions. The confusion is especially manifested in problem solving where students often resort to wrong models either because they are unable to tease out the correct pattern(s) manifested in a problem situation, and/or because they are unable to match patterns with appropriate models. For instance, many students wrongly believe that the reference classes of particle models of Newtonian theory of mechanics are restricted to macroscopic terrestrial objects, and that they exclude microscopic or astronomical objects of any sort. Students who miss the universality of Newtonian models are unable to realize that these models can be deployed in the resolution of problems pertaining to the atomic world, e.g., in situations involving charged particles moving under Coulomb's interaction. Alternatively, and because of the confusion they have among various models, especially with regard to domain restriction and exclusivity, many students deploy wrong models in problem solving. For example, in classical mechanics students often deploy by mistake the uniformly accelerated particle model for solving problems pertaining to translation patterns of variable acceleration, like simple harmonic motion. Student failure results either from missing the correspondence rules of the deployed model (Fig. 2.2), or from erroneous problem-solving routines. In the latter case, students concentrate on secondary features in a given situation, thus missing the actual pattern(s) involved, and/or they follow purely mathematical routines that ignore any explicit correspondence of a model to the real world.

## 2.4 MODEL COMPOSITION

A scientific model is a conceptual system. Like any system, it is composed of particular entities with specific characteristics. While a physical system is composed of material entities of given physical properties, a scientific model is composed of conceptions. More specifically, a model composition consists of *concepts* that represent *primary* features of the modeled pattern. These are features that are common to all systems in the model reference class and are responsible for producing the pattern. They include certain bodies (objects and agents) and/or fields, in the make-up of model referents, and certain structural and behavioral properties shared by such entities and systems they belong to. Salient entities (bodies or fields) and primary properties are represented in the model with appropriate entity-concepts and property-concepts, respectively.

An entity-concept, or *depictor*, represents physical *bodies*, objects or agents, in the constitution of model referents, bodies that contribute significantly to the existence of the pattern. Fields are also represented by appropriate depictors, but, for convenience purposes, we shall hereafter concentrate our discussion on physical bodies. An entity-concept is often a geometric concept that depicts primary morphologic aspects of represented bodies. Such a depictor is a particle, a dimensionless point, in Newtonian models of translation, in Bohr's model of the atom, and in the ideal gas model. It is a three-dimensional figure in Euler's models of rotation, in Ohmic models of resistive circuits and in some models of biological systems in living organisms. Like a system, a model is said to be *simple* when it consists of a single depictor (e.g., Newtonian particle models), and *composite* when it consists of many depictors (Bohr atomic model).

A property-concept, or *descriptor*, represents a common primary *property* in the structure and/or behavior of systems exhibiting the pattern. Descriptors are of two broad categories, object descriptors and interaction descriptors. All descriptors are quantitative descriptors in physical sciences, and they often bear the same names as the properties they represent.

An *object descriptor* represents a characteristic property of a physical body, and can be either an intrinsic descriptor or a state descriptor. An *intrinsic descriptor*, often referred to as a *parameter* in mathematical language, represents a characteristic property that is usually unaffected by the behavior of the corresponding body (or system) or by its interaction with other bodies. Mass, charge, and dimensions are examples of intrinsic descriptors in classical theory. A *state descriptor*, often referred to as a *variable* in mathematical language, represents a property that characterizes the behavior of a physical body and that may vary under interaction with other bodies. Velocity, momentum and electric current are examples of state descriptors.

An *interaction descriptor* represents mutual actions that two or more physical bodies exert on one another. Force, field, and energy are examples of interaction descriptors in science. As noted in § 2.1, the demarcation line between a system and its environment can be conveniently chosen so as not to account for the action *of* objects inside the system *on* agents in its environment. Interaction descriptors would then be needed to represent mutual actions of objects inside the system (*internal* interactions), but only the action of agents on objects (*external* interactions). The situation is especially simplified in the case of simple models. A simple model consists of a single depictor. Since no object can interact with itself, simple models include no internal interaction descriptors.

Should there be an ideal referent, a physical system that consists only of those bodies that are necessary and sufficient to produce the modeled pattern and that possesses just the required properties, there would be a one-on-one mapping between that referent and the corresponding model. In the case of such a fictitious referent, and only in this case as we shall discuss in § 2.5, model-referent *isomorphism* would be comprehensive. Model depictors would then represent all physical objects and agents pertaining to the ideal referent, and they would be divided along the same demarcation line that typically sets the boundaries between a system and its environment (Fig. 2.1). Model descriptors would represent all physical properties of the ideal referent. Like in the case of a model, each concept has a set of correspondence rules that establish what the concept actually represents in the real world and under what conditions (§ 2.8).

Various referents of the same model may have different physical composition. However, the composition of the model is always limited to those depictors and descriptors that would be mapped on the ideal referent. Thus, when the model is deployed in a physical situation exhibiting the modeled pattern, one needs to concentrate only on those primary features of the situation that have a match in the model composition and represent them accordingly. Any other feature would be secondary and should be ignored. Otherwise, the model would fail to serve its purpose.

Take for example the situation of Figure 2.3. A loaded trolley is pushed by a mailman so as to undergo a linear, uniformly accelerated translation (constant acceleration **a**). For convenience purposes, consider the *simple* system consisting of the trolley and its load taken together as a single *object*. The system environment consists of three *agents*, ignoring air resistance. These are the mailman, the Earth, and the ground on which the trolley is moving. When interested in the motion of the trolley, and deploying to this end the Newtonian uniformly accelerated particle model (Fig. 2.2), a *depictor* (a particle) will be needed to represent only this object but not the corresponding agents (Fig. 2.3a). As discussed in § 2.1, agents concern us only in terms of their actions on the object (Fig. 2.3c). In situations like that of Figure 2.3a, the particle model in question requires, in an appropriate coordinate system (§ 2.5): (a) only one *object descriptor*, mass, (b) a number of kinematical concepts like position, displacement, velocity, acceleration, linear momentum and kinetic energy as *state descriptors* (Fig. 2.3b), and (c) at least one dynamical concept, force, as an *interaction descriptor* (Fig. 2.3c). Not all listed descriptors may be needed to treat the situation at hand. However, no descriptor outside this list may be involved. Similarly, no depictor but the particle may be used to represent any object in the situation as long as a Newtonian particle model is concerned. Other depictors, like a box or any other geometric figure, would distract users, to say the least, from the idea that object descriptors of shape and dimensions are not accounted for in the composition of Newtonian particle models.

Educational researchers have often complained that a major deficiency in student problem-solving skills resides in their inability to isolate the appropriate systems in a given situation, and to correctly

*Figure 2.3a*: When in translation with acceleration **a**, the trolley can be represented by an appropriate Newtonian particle model. Depending on the convenience of the situation, the particle (depictor) may represent a system consisting of the trolley and the load, or of either object taken separately. Agents are represented by no depictors.

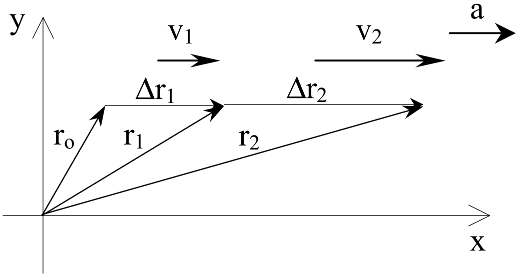


*Figure 2.3b*: Kinematical diagram representing, in a Cartesian coordinate system, some state descriptors of a Newtonian accelerated particle model. These are position ($\mathbf{r}_i$), displacement ($\Delta\mathbf{r}_i$), velocity ($\mathbf{v}_i$), and acceleration ($\mathbf{a}$) at particular instants $t_i$. Such state descriptors may well represent kinematical properties of the above trolley.

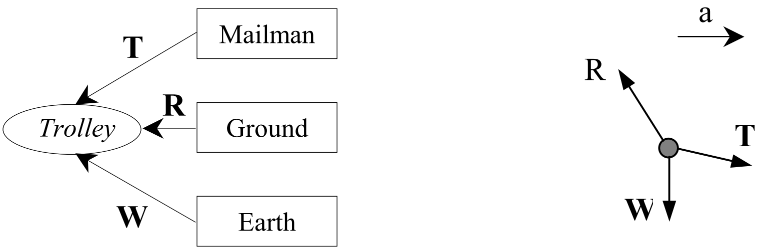


*Figure 2.3c*: Interaction diagram (left) showing the simple system (trolley) of Figure 2.3a and the three agents (mailman, ground and Earth) interacting with it. Labeled arrows indicate that the action of each agent on the system is represented with a force to which is attributed the label shown. The interaction diagram is translated into a force diagram on the right where each arrow represents the direction and magnitude (to a scale not shown) of the force bearing the same label. All force arrows have their origins on a particle depicting the trolley to remind us that where an agent applies its force on the object is here a secondary matter, and to subsequently facilitate mathematical operations with force vectors (§ 2.8).

*Figure 2.3*: Composition of a Newtonian accelerated particle model deployed in the situation of a loaded trolley in translation.

identify primary physical entities and properties (Chi, Feltovich and Glaser, 1981). Our view is that students are actually incapable of matching the situation at hand with the correct pattern(s) and of subsequently *adducing* the appropriate model(s) for deployment. Even when they are capable of categorizing a problem in a manner that is acceptable to some science educators like Chi et al., i.e., by attributing the correct "principles" to the problem, students often concentrate on the wrong physical bodies and properties, and/or on the wrong depictors and descriptors, or they miss primary features in the composition of the system(s) involved and the corresponding model(s). The latter failure is due, at least in part, to the fact that students had not developed explicitly in the first place a systematic way for developing and deploying model composition. As a result of all the above, students will continue to follow wrong routines in conceptual development and in problem solving, routines that make them concentrate on physical and conceptual features that are not necessarily what science considers primary features in both the physical and conceptual worlds.

## 2.5 MODEL STRUCTURE

Model composition sets conceptual building blocks that are necessary for model construction. Unrelated to one another, the building blocks constitute nothing but a heap of stone in the sense referred to by Poincaré (1902, p. 158) when he noted that science is built up of facts the way a house is built of stones; but an accumulation of facts, he continued, is no more a science than a heap of stone is a house. Model structure is the schematic dimension that converts the heap into a coherent conceptual system of well-defined function. It does so by relating various model components with appropriate laws (or principles) and other theoretical statements (axioms, theorems, definitions). Some of these laws and statements are generic; they are provided by the scientific theory containing the model or by the corresponding paradigm, and they are common to all models in the theory. Others are particular to the model at hand and reflect the particularities of the modeled pattern.

"The essential characteristic of a model is its functional role" (Johnson-Laird, 1983, p. 403). It is the characteristic that distinguishes most a particular model from other models representing or not the same pattern, and that determines the viability of the model in the real world. The *function* of a model refers to the questions it can answer about the corresponding pattern. Pattern description and explanation are the two major functions of scientific models. All other functions (prediction or postdiction, control or modification of existing realities, invention of new realities) follow from these two. Depending on its structure, a model may assume either or both descriptive and explanatory functions.

Model structure can be defined along four subdimensions, or *facets*, each dealing with a specific aspect of model referents in relation to pattern formation. These are the topology facet, the state facet, the interaction facet, and the cause-effect or causal facet. Each facet is distinguished conceptually by the nature of the descriptors involved and the ways they are related in space and time. Various relationships (laws and other theoretical statements) are expressed in an appropriate reference system relative to which the pattern is conveniently identified (e.g., Earth in Figure 2.3a represented by the Cartesian system of Figure 2.3b).

Within the *topology facet* of a model structure, the relative positions of various depictors are set in an appropriate coordinate system so as to conveniently represent the *spatial configuration* of primary objects and agents that are responsible for pattern production in a given reference system. The topology of object depictors is relevant only in composite models but not in simple models. A simple model consists of a single object depictor, and thus it has no internal topology, and no internal interaction. That is why it is often convenient to define the boundaries of a system in a way that it may be represented by a simple model (Fig. 2.4a). When the composition of a model is broken along the lines of Figure 2.1, agents' topology will matter only with respect to quantifying agents' action on objects they interact with (i.e., with respect to expressing interaction laws discussed below). Hence, only the position of an agent relative to an object will be considered, and agents' positions relative to one another will be ignored. In some cases, agent topology may matter for some but not other forms of interaction. For example, when Newtonian particle models are deployed in the macroscopic world, the position of an agent relative to an object matters only for long-range interactions, but not for contact interactions.

*Figure 2.4a*: Topology of a Newtonian particle model deployed in the situation of Figure 2.3a. No agent is represented since the force exerted by any agent, Earth included in this case, is assumed constant and independent of the position of the agent relative to the object (trolley).

| | | |
|---|---|---|
| *Acceleration*: | $\mathbf{a}$ = constant | |
| *Velocity at a time t*: | $\mathbf{v} = \mathbf{v}_o + \mathbf{a}t$, | *$v_o$ being the velocity at the time of origin (0s)* |
| *Position at a time t*: | $\mathbf{r} = \mathbf{r}_o + \mathbf{v}_o t + \frac{1}{2}\mathbf{a}t^2$ | *$r_o$ being the position at the time of origin* |
| *Mean velocity rule*: | $\mathbf{v} + \mathbf{v}_o = 2\Delta\mathbf{r}/t$ | *originally formulated by Hentisberus (c. 1340)* |

*Figure 2.4b*: State laws of a uniformly accelerated particle model expressed algebraically. These laws may well *describe* the state of the trolley in Figure 2.3a.

$$\text{Newton: } \vec{F} = G\frac{m_{agent}m_{object}}{r^2}\hat{r} \qquad \text{Coulomb: } \vec{F} = k\frac{q_{agent}q_{object}}{r^2}\hat{r}$$

$$\text{Hooke: } \vec{F} = -\,k_{agent}\Delta r_{object}\ \hat{r}$$

*Figure 2.4c*: Interaction laws of universal gravitation (Newton), electrostatic interaction (Coulomb) and elastic binding (Hooke). $\hat{r}$ denotes unit position vector.

| | | |
|---|---|---|
| *Newton's $1^{st}$ law*: | If $\mathbf{F} = 0$, then | $\mathbf{p} = m\mathbf{v}$ = constant |
| | or | $\mathbf{v}$ = constant, *if mass m is constant* |
| *Newton's $2^{nd}$ law*: | $\mathbf{F} = d\mathbf{p}/dt$ or | $\mathbf{F} = m\mathbf{a}$, *if mass m is constant* |
| *Work–Energy principle*: | $W_F = \int \mathbf{F}.d\mathbf{r} = \Delta(\frac{1}{2}mv^2)$ | |

*Figure 2.4d*: Causal laws of any Newtonian particle model expressed algebraically (**F** represents a net force). These laws may well *explain* the state of the trolley.

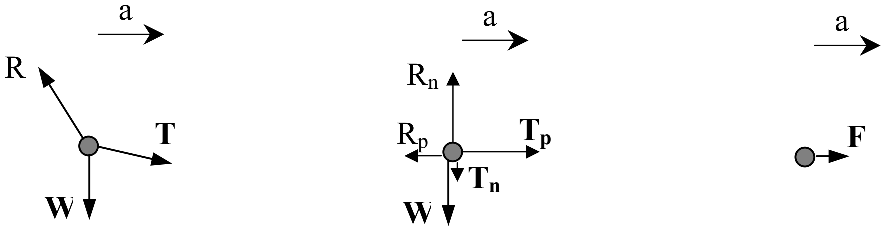


*Figure 2.4e*: Force diagrams corresponding to the Newtonian particle model deployed in the situation of the trolley of Figure 2.3a, and developed according to Newton's fourth law or law of composition. The diagram on the left is the same as in Figure 2.3c. The middle diagram shows the same forces with **R** and **T** broken into two components each, one component in the horizontal direction of motion, the other in the normal direction. The diagram on the right shows the resultant **F** of all three forces (**F** = **T** + **R** + **W**). Acceleration, **a**, is shown to remind us of the nature and direction of translation, and to help us check that **F** and **a** be in the same direction according to Newton's second law (**F** = m**a**).

*Figure 2.4*: Structure of a Newtonian accelerated particle model deployed in the situation of Figure 2.3a.

The *state facet* is concerned with pattern *description*. Appropriate *state laws* express *how* the model referents are *structured* and/or how they *behave* in a specific reference system so as to bring about the pattern in question. More specifically, a state law expresses how a given state descriptor pertaining to a specific physical object (or physical system) varies or not in time. Figure 2.4b shows certain state laws pertaining to the Newtonian uniformly accelerated particle model. These are laws of *kinematics* often referred to in physics textbooks as "equations of motion". They answer the question "*how* does a kinematical concept, like position, velocity or acceleration, vary over time?" They *describe* how a particle-like object moves in an inertial reference system under a net constant force.

The state facet is the structure facet that most distinguishes one model from another, whether or not the two models belong to the same scientific theory, and whether or not they represent the same pattern. When representing the same pattern, the difference could be in that different models are concerned with different state descriptors, or that they may be concerned with the same descriptors from different theoretical perspectives. In the latter case, state laws belonging to different models lead to the same result when the models belong to the same paradigm, but to different (but not contradictory) results when belonging to different paradigms. The first scenario (same results) takes place when, for example, the descriptors of Figure 2.4b are related to one another in Hamiltonian models instead of Newtonian models. The second scenario takes place when the same descriptors are interrelated in relativistic models (results differ in degree of precision).

Within the *interaction facet* of model structure, interaction descriptors are expressed primarily with interaction laws. An *interaction law* expresses a particular interaction descriptor in terms of object descriptors pertaining to the interacting bodies. Newton's law of universal gravitation, Coulomb's law of electrostatic interaction and Hooke's law of elastic binding are typical interaction laws in the classical mechanistic paradigm (Figure 2.4c). The first two laws express force (F), an interaction descriptor exchanged between an agent and an object, in terms of an intrinsic descriptor of both object and agent (mass, m, in Newton's law and charge, q, in Coulomb's law) and a common state descriptor (their relative position, r). In contrast, Hooke's law expresses force in terms of one agent's intrinsic descriptor (stiffness or force constant, k) and one object's state descriptor (displacement from equilibrium position, Δr). Interaction laws are common to all models within a given paradigm, and they may even be shared by many paradigms. The latter is the case, for example, of Newton's law of universal gravitation and Coulomb's law of electrostatic interaction.

The *causal facet* is concerned with pattern *explanation*. Appropriate *causal laws* assert *why* model referents have the common structure they have, and/or why they behave the way they do in a specific reference system. They specify the conditions for model referents to be in a given state as described by the topology and state facets. They determine the causes of such a state, if any, causes often identified with specific interactions. Alternatively, a causal law associates a specific effect on model referents with a specific cause (whence the commonly used label of cause-effect). The cause-effect relationship is commonly expressed in a spatio-temporal function between an interaction descriptor (cause) and some state descriptors (effect). The law then states how the latter descriptors evolve over time because of the interaction in question (or in the absence of any interaction).

Newton's second law of dynamics and the work-energy principle are causal laws that specify what happens to an object under certain interactions. Newton's second law specifies how the momentum (or velocity if mass is constant) of an object varies under a net force exerted on this object by one or many agents. The work-energy principle specifies how the kinetic energy (state descriptor) of the object varies under the work (interaction descriptor) of the agents in question. The two laws answer the question "*why* does a kinematical concept like velocity, momentum, or kinetic energy, vary in time?", or why does it become conserved (Figure 2.4d). They do so by attributing specific *causes* (force and work respectively), if any, to the kinematical state in effect.

Causal laws are generic laws. Some, like the work-energy principle, are provided by a paradigm and apply (in various forms) to all theories in the paradigm. Others, like Newton's laws of dynamics, are provided by a particular theory in the paradigm and apply to all models in the theory. Causal laws provided by a particular theory set modeling rules within the context of the theory in question (Hestenes, 1992). In the case of Newtonian theory, these laws indicate that in order to model a given pattern or to determine the appropriate model(s) for deployment in a given physical situation, one needs to start by identifying the forces involved and then applying Newton's second law (and/or the work-energy principle). The type of translation can subsequently be determined, and the appropriate model composition and structure completed to fulfill the objectives at hand.

It has often been said that the most important function of scientific theory is to explain the "change of state" of a number of physical realities. Among all causal laws in any scientific theory, there is always one law that is perhaps the most fundamental law in this respect. It is the law that specifies what is to be considered as a "change of state" according to the theory. Every theory is concerned with a particular state of physical realities. The law in question specifies a particular descriptor to characterize this state and to subsequently determine what sort of change needs to be explained in the structure or behavior of the realities in question. For example, Newtonian theory is concerned with the state of translation at low speeds in inertial reference systems. The law we are referring to is, in this theory, the first law of dynamics or the law of inertia. Aside from our concern, this law sets the main correspondence rule for the free particle model (no net force) and the condition for a reference system to be inertial (a free particle remains as such, i.e., it maintains a constant velocity in any inertial reference system).

Newton's first law of dynamics states that, contrary to what most students and other ordinary people think, the state of a physical object in translation is characterized by the concept of velocity (or linear momentum if mass is not constant) and not by the concept of position. The law further states that a change of velocity (or momentum) and not a change of position constitutes a change of state, and thus that the former not the latter change needs to be explained by the concept of force. In other words, Newton's law of inertia states that a force needs to be exerted on an object to change its velocity (or momentum) but not its position. No force is needed and no cause is to be sought as long as the change of position takes place with constant velocity, i.e., as long as the object moves in uniform linear motion (or as long as it stays at rest in certain inertial reference frames).

The structure of a model does not always consist of all four facets, and its function regarding the corresponding pattern is then determined by the nature of the included facets. As noted at the beginning of this section, description and explanation are the two major functions of a model. These two functions depend specifically on the state facet and the causal facet, respectively. A model is said to assume only a descriptive function, and is then called a descriptive model, when its structure consists of only the state facet (and perhaps the topology facet). A *descriptive model* is thus a model concerned exclusively with *pattern description*. It answers "what" and "how" questions about the structure and/or behavior of model referents. A model is said to assume only an explanatory function, and is then called an explanatory model, when its structure includes only the causal facet (along with the interaction facet). An *explanatory model* is a model concerned exclusively with *pattern explanation*. It answers "why" questions about the structure and/or behavior of model referents. The model is called *comprehensive* when it assumes both functions and includes all four structure facets described above. A comprehensive model emerges from putting together a descriptive model and an explanatory model dealing with the same pattern under the same theoretical and empirical conditions. In Newtonian theory, particle models of kinematics are descriptive models, whereas particle models of dynamics are explanatory models. A comprehensive model of classical mechanics emerges from putting together a kinematics model with the corresponding dynamics model. No comprehensive model is comprehensive in the sense that it can answer any question about its referents. It can answer only those questions about pattern description and explanation that are dealt with in the various facets of model structure.

Three issues are worth mentioning at this point regarding model structure. The first issue pertains to generic laws. The second issue deals with the distinguishing features of the interaction facet and the causal facet. The third and most important issue is concerned with the type of isomorphism between a model and its referents.

Generic laws in a given scientific theory include interaction and causal laws as well as other laws that apply across the board in a model structure. Some of these laws are superordinate or dominant laws in the sense that they apply to any scientific paradigm, and they govern the derivation of generic laws pertaining to individual theories.

This is the case of the energy conservation principle whose ramification, say for conservative interaction in Newtonian theory, leads to the principle of mechanical energy conservation. Other generic laws are theory specific; they seep throughout many structure facets of every model in the theory. This is the case of Newton's fourth law or law of force composition that governs the composition of many forces into a single net force (resultant) or the decomposition of a given force into specific components (Fig. 2.4e). This law can be used within the interaction facet as well as within the causal facet in the structure of all particle models of Newtonian theory. The law of force composition is a subordinate of a dominant (or superordinate) principle, the superposition principle. Another subordinate of this principle is the kinematical superposition principle used in the state facet for composing or decomposing kinematical descriptors like velocity. The two subordinates share common aspects set by the dominant principle like the independence of two forces or two velocities that are being composed.

The second issue pertains to the distinction that should be unequivocally maintained between the interaction facet and the causal facet of a model structure. The interaction facet is concerned with choosing convenient descriptors, like force, to represent and quantify interaction between physical bodies that contribute to pattern formation. Interaction laws spell out the characteristics, like magnitude and direction, of the chosen interaction descriptors. The causal facet is concerned with identifying the effect of interaction on the state of physical realities involved in the pattern. Causal laws may do so by specifying the effect of, say, the force exerted by a given agent on the velocity of an object that the agent acts on. From a functional point of view, an interaction law relates an interaction descriptor to intrinsic descriptors pertaining to both agent and object, whereas a causal law relates the interaction descriptor to state descriptors pertaining only to the object. Figures 2.4c and 2.4d show algebraic expressions of respectively three interaction laws and three causal laws. F denotes in Figure 2.4c the force exerted by agent on object or the opposite force exerted by object on agent, whereas it denotes in Figure 2.4d only the force exerted by agent on object. On the right-hand side of each equation are object descriptors of both object and agent in Figure 2.4c, whereas there are descriptors of only object and no agent in Figure 2.4d.

The third issue has to do with the ontological relationship between a model and its referents, mainly at the level of composition and structure. A model is mapped in specific respects onto the physical pattern it represents. The *scope* of the model (domain and function), as well as its viability (§ 2.7), depends on the sort of mapping involved, i.e., on the level of isomorphism between the model and its referents. It is primarily determined by what Hempel (1965) calls *nomic isomorphism* between laws in model structure and actual relationships among physical properties of the referents.

Hempel (1965, p. 436) argues, in line with Lord Kelvin, for the importance of analogical mechanical models in understanding physical realities. "The relevant similarity or 'analogy' between a model of the kind here considered [mechanical model], Hempel argues, and the modeled type of a phenomenon consists in a *nomic isomorphism*, i.e., a *syntactic isomorphism between two corresponding sets of laws*". One set of laws belongs to model structure, and the other set, according to Hempel, is inherent into physical realities that the mechanical model is "analogous" to. Hempel's position on nomic isomorphism somewhat echoes the position that Klein held about half a century earlier on mathematical models. "Models [in mathematics, according to Klein] are mapping of functional correspondences between the structures of different domains of our knowledge. Like geographical maps, their aim is not to produce a 'resembling picture' of the original; they just represent its *structural* features in such a manner (often highly abstract and symbolic) as to be *functional* in the exploration of the target domain on which they are brought to bear" (Glas, 2002; italics added).

Hempel's isomorphism pertaining to analogical, mechanical models can actually be extended to all scientific models. Accordingly, nomic isomorphism between a conceptual model and the physical pattern it is modeled on entails selective mapping, at the level of both composition and structure, between the model and its referents. At the level of composition, model *depictors* correspond to salient physical bodies, and model *descriptors* correspond to primary physical properties. At the level of structure, each *relationship* expressed among certain model descriptors (laws and other theoretical statements) corresponds to a specific relationship among pattern physical properties. Appropriate *semantics* (correspondence rules) establish the model-pattern correspondence in all respects, so that the isomorphism between the two can be expressed explicitly.

Hempel's nomic isomorphism at the level of laws remains the most important for setting the function of a model. However, and unlike Hempel, we maintain that "natural laws" or laws inherent to physical realities in the universe may not be directly exposed to our scrutiny, and, thus, that an ontological mapping between model and pattern is not straightforward in this respect. Nomic isomorphism between model and pattern, hence model corroboration (§ 2.7), can be established either: (a) indirectly, through the model's viability in providing acceptable answers about the corresponding physical pattern, and especially in providing good predictions about the physical realities exhibiting the pattern, or/and (b) directly, and *à posteriori*, through model reification, say, in the invention of new physical realities, mainly technological ones, that exhibit the pattern in the manner depicted by the model.

Nomic isomorphism between scientific model and physical pattern allows us to say, as we mentioned in § 1.2, that the model *represents* all physical realities in those respects exhibited by the pattern and described and/or explained by the model to a certain degree of precision. The representation is established according to well-defined *semantics* that include model correspondence rules and particular semantics associated with individual conceptions and establishing what a conception represents in the real world, e.g., what physical bodies a depictor refers to and what physical property a descriptor represents (§ 2.8). Semantics help us to *interpret* the model in the real world, as well as empirical and rational outcomes obtained in model deployment. Objectivity of scientific knowledge is ensured, at least in part, by the ontological nature of model-pattern isomorphism, and by associated semantics. Scientists strive to make the isomorphism as independent as possible from all human influence, and this by concentrating on primary details that are inherent to physical realities and independent of human perception. Furthermore, they associate with the model appropriate semantics that would allow all concerned scientists to interpret the model virtually the same way whether in establishing its correspondence to the real world or in deploying it in this world.

A model is thus for us more than a tool of a well-defined function. It represents specific aspects of reality in the sense that, as a whole or in parts, it has a “factual content” (Bunge, 1973, p. 38). In this respect, our position goes well beyond instrumentalism for which a model is a mere “tool for reality organization, i.e., a tool for ordering experiences rather than a *description* of reality” (Casti, 1989, p. 458). Yet our notion of representation does not go as far as positivism for whom scientific conceptions are objective mirrors of reality. Model-pattern mapping does not necessarily involve a one-to-one correspondence, in the classical sense, between conceptual components of the model and the constituents of its referents. For example, a wave function in quantum mechanics does not correspond to the real world in the same way that a Newtonian particle corresponds to a physical object in translation.

Three levels of isomorphism can be distinguished between model and referents. At the lowest level is the isomorphism restricted to a *morphological* mapping between the two, i.e., to an analogy at the level of physical form and structure. At this level are some physical models like miniature cars and airplanes, wooden or plastic globes representing the Earth, and plastic models of the human body or of molecular structures that we commonly find in our classrooms. At the next and more meaningful level is the nomic isomorphism dealing exclusively with *behavioral* analogy between model and physical realities. This is the case of Newtonian particle models, the ideal gas model and neuroscience models of synaptic signal transmission across neurons. At the highest level is the nomic isomorphism, revealing both morphological and behavioral analogy between model and referents, and thus making the model *representation* of physical realities most meaningful, but never comprehensive. This is the case of Eulerian models of classical mechanics in physics, molecular models of crystals in chemistry, and neuronal models dealing with detailed anatomic structure of the nervous system in biology. In addition to motion, a Eulerian model accounts for the shape and dimensions of a rotating physical object, and the object is accordingly represented by a convenient geometric figure in the model, a figure whose shape is as close as possible to that of the object. This is not the case in Newtonian models where all objects in translation are represented by a particle, an imaginary shapeless and dimensionless object. The same contrast holds between crystal models and the ideal gas model, and between neuronal models in biology and those in neuroscience.

We have mentioned at the end of the previous section that students' failure to tease out primary from secondary features in a given physical situation is due primarily to their inability to adduce the appropriate model to the situation and subsequently to their failure to decide on the appropriate model composition. Deficiency in model adduction is due to missing correspondence rules and, especially, the function of the model as determined by its structure. Model structure is thus more critical than model composition in model deployment. It is in fact at a higher order, both from ontogenic and ontological perspectives. From an ontological perspective, model structure is more involved than model composition. Model structure includes various theoretical statements, especially laws that express functional relationships among various descriptors. Model composition is limited to the identification and definition of appropriate depictors and descriptors. From an ontogenic perspective, model structure, and more precisely model function, determines model composition in science and not the other way around. A model is always built to serve a well-defined function in science, and all low level conceptions (depictors and descriptors) are developed as conceptual building blocks to achieve this end.

Looking at things deeper from an ontogenic perspective, we note that humans tend to concentrate for quite some time during their development more on isolated objects in physical realities than on primary structure or phenomenon, and more on morphological isomorphism than on behavioral isomorphism between the conceptual realm and the physical world. Objects are conceived first, then their properties followed by mutual relationships among corresponding descriptors (Bachelard, 1934, p. 34). Objects are placed before phenomena (or structure), and morphology before behavior; it is then assumed that one needs to concentrate on objects before phenomena because objects are considered to produce phenomena (Bachelard, 1940, p. 34). The pattern idea is then not central in natural (naïve or common sense) human thinking, and neither are models in the sense presented in this book. Conceptual models of most lay people either lack structure and have composition as the dimension of prime concern, or they concentrate more on morphological than on behavioral aspects. For most lay people, model structure follows from, and does not instigate, model composition. Moreover, model composition consists then of depictors and descriptors that do not necessarily correspond to what science considers primary features in physical realities. As a consequence, pre-scientific models are doomed to have critical flaws. A good proportion of high school and college students hover about this phase of development, whence the failure in question.

## 2.6 MODEL ORGANIZATION

The fourth dimension of a model schema pertains to organization, i.e., to the status of a given model in the corresponding scientific theory. It pertains particularly to the relation of the model to other models in the theory, and to what the model can offer to expand the scope of the theory. Models occupy, for us, the basic level in a middle-out conceptual hierarchy (Fig. 1.1) that places theory at the top of the hierarchy (superordinate level), models in the middle, and concepts at the bottom (Fig. 2.5). Theories provide, through their models, the conceptual *content* of a scientific paradigm, with concepts as the most elementary building blocks, and laws of various order as regulators of the construction process.

Every theory includes a characteristic set of generic laws that "function more like recipes for constructing models than like general statements" (Giere, 1994). These laws are complemented by more global laws that are applicable to every model in any theory belonging to the paradigm containing the theory of concern. For example, all particle models of the classical theory of mechanics are built following rules implied by Newton's four laws of dynamics (Hestenes, 1992). The "first" is the law of inertia, the "second", the dynamical law of cause-and-effect, the "third", the law of mutual interaction, and the "fourth", the law of force composition.

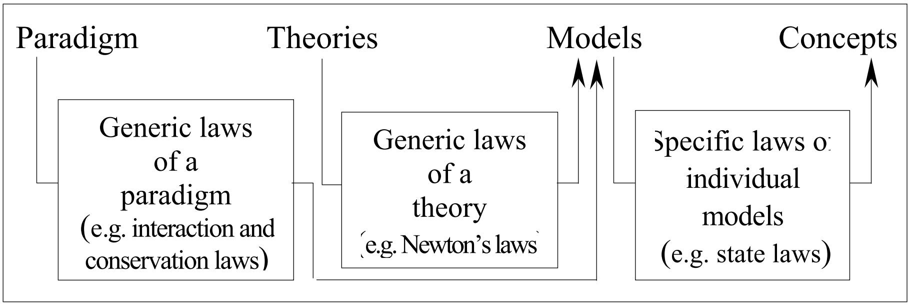


*Figure 2.5*: Conceptual organization in a scientific paradigm.

Construction of Newtonian models can further benefit from higher order laws like conservation laws that are applicable to all theories in the classical mechanistic paradigm (Fig. 2.5).

Models are most fundamental in the middle-out hierarchy of Figures 1.1 and 2.5, and their role in a theory goes beyond the physical patterns they are modeled after. In exploratory research, models offer the conceptual lenses through which we see the real world, we describe, explain and predict the constitution and behavior of its systems. Once a model is developed, scientists deploy it every time they investigate any of its referents so that they set the kind of questions that they can ask about these referents, ascertain how they need to go about answering those questions, and assess subsequent answers. In inventive research, a model offers the rational context for constructing and corroborating lower-level conceptions (concepts, state laws and other theoretical statements); it sets the guidelines for controlling model referents; it serves as a blueprint for altering existing physical realities or inventing new realities in order to produce the pattern that the model represents. Model functions are discussed at various points throughout this book; so we concentrate in this section on the relation of a given model to other models belonging to the same theory.

Every scientific theory includes a set of models all governed by the same generic laws offered by the theory. These models may share the same referents with one another as well as with models in other theories, but each model represents in particular ways a given pattern produced under particular conditions. Models in the same theory may also have similar components (depictors and descriptors), and they may share some, but not all, structural aspects. Different models are, however, distinguished by their scope (respective patterns and functions). The distinction is mostly apparent at the level of the state facet of model structure.

For example, the classical mechanistic paradigm includes, among others, Newton’s theory and Euler’s theory (§1.2). Newton’s theory consists of a set of models, each representing a particular translational motion of physical objects (e.g., uniform linear motion like that of an object in free space, or curvilinear motion like the elliptical translation of the Earth around the sun). Euler’s theory consists of another set of models, each representing a particular rotational motion of physical objects (e.g., uniform rotational motion like that of the Earth around itself). Models in both theories are governed by generic interaction and conservation laws that apply, in different forms, to all theories in the paradigm (Fig. 2.5). Meanwhile, various forms of motion represented in translational models are further *explained* by Newton's laws of dynamics, while those represented in rotational models are explained by Euler's laws. Among these laws are respectively Newton's second law, sometimes referred to as the fundamental principle of translational dynamics, and commonly expressed in the form: $\mathbf{F} = m\mathbf{a}$, and the corresponding Eulerian fundamental principle of rotational dynamics commonly expressed in the form: $\tau = I\alpha$ ($\tau$ being the net torque exerted on an object, I the object's moment of inertia, and $\alpha$ its angular acceleration). The distinct motion represented in a given model is *described* by particular *state* laws that establish exclusive relationships among certain kinematical concepts (e.g., the so-called equations of motion $r(t)$ or $\theta(t)$).

According to their scope, and to the level of their conceptual complexity, the set of models included in a scientific theory may also be distributed in a middle-out hierarchy (consisting of models only) that places the family of what we call basic models in the middle of the hierarchy. At the superordinate level of the hierarchy is the family of emergent models, and at the subordinate level is a repertoire of subsidiary models. The family of *basic models* includes models of the theory that cover virtually all fundamental conceptions (depictors, descriptors, and various theoretical statements), rules and tools that are necessary to build and deploy any model in the theory. Basic models are comprehensive models (descriptive and explanatory). They are generic enough to allow, when two or more of them are put together, the construction of superordinate, more complex models, i.e., emergent models. An *emergent model* is a model whose composition and, especially, structure can be obtained from putting together the composition and structure of two or more other models (mostly basic models). The new model will have a scope that is entirely different from the scope of either model that served in its construction. A *subsidiary model* is a simplified version of a basic model and serves as a major stepping-stone in the comprehensive development of the basic model in question. The scope of a subsidiary model is considerably narrower than that of the corresponding basic model. Its reference class is limited to everyday life physical realities that one (especially a lay person) is most familiar with. It may have a strictly descriptive or a strictly explanatory function, or it may assume both functions.

For example, the family of basic particle models in Newtonian theory consists of the five models shown in Figure 2.6. Examples of subsidiary models include a free particle at rest in a given inertial reference system, or a uniformly accelerated particle restricted to linear translation (like in the case of free fall), or a bound particle constrained to linear harmonic oscillation (like in the case of a block suspended from a spring). An example of emergent models in the same theory is the bound particle in accelerated circular motion (Figure 2.7). This model can be composed of two models: the bound particle in uniform circular motion and the uniformly accelerated particle.

***Free particle***

Physical objects subject to no net force ($\Sigma \boldsymbol{F}_i = 0$), and thus maintaining constant velocity in any inertial reference system ($\boldsymbol{a} = 0$, $\boldsymbol{v} =$ constant).

***Uniformly accelerated particle***

Physical objects in linear or parabolic translation with constant acceleration ($\boldsymbol{a} =$ constant) under a net constant force ($\Sigma \boldsymbol{F}_i =$ constant).

***Bound particle in harmonic oscillation***

Physical objects undergoing periodic back and forth translation (sinusoidal $\boldsymbol{a}$ function) under a net force that is proportional to their displacement from a center of force ($\Sigma \boldsymbol{F}_i \propto \Delta \boldsymbol{r}$). This model is often called a simple harmonic oscillator.

***Bound particle in uniform circular motion***

Physical objects in uniform circular translation ($a = v^2/r$) under a net centripetal force ($\Sigma \boldsymbol{F}_i \propto \boldsymbol{r}/r^2$) of constant magnitude.

***Particle under impulsive interaction***

Physical objects whose linear momentum changes significantly, and almost instantaneously, like in the case of collision, under a variable net force ($\Sigma \boldsymbol{F}_i = \boldsymbol{f}(t)$) exerted for a very short period.

*Figure 2.6*: Basic particle models in Newtonian theory of classical mechanics, with an outline of the translational pattern that each model represents in inertial reference systems.

Science education may rely on subsidiary models to introduce models and the model schema in the manner discussed in the following chapters, and especially in Chapter 5. However, basic models are more important for meaningful understanding of various modeling rules and tools, and especially modeling schemata. In this respect, we take the position of Lakoff (1987, p. 269-271) that "our experience is preconceptually structured at [the basic level]… 'basic' does not mean 'primitive; that is, basic-level concepts are not atomic building blocks without internal structure. The basic level is an intermediate level; it is neither the highest nor the lowest level of conceptual organization". A family of basic models may be identified in every scientific theory, and students may subsequently develop the theory around these models in a middle-out approach.

Before we close this section, let us briefly point out some common mix-ups between models and other conceptions. There is sometimes confusion in the literature, as well as in many students' minds, between terms like theory and hypothesis, theory and law, model and prototype, model and concept, model and law, and especially between model and a mathematical representation of a law. Each of these terms corresponds, for us, to a distinct conceptual entity in the structure of a scientific paradigm. Except for hypothesis, the various terms that are the object of confusion are discussed at various lengths throughout this chapter. The confusion between theory and hypothesis stems perhaps from everyday life vocabulary where the word theory often refers to an uncorroborated viewpoint. This is in a sense what the word hypothesis refers to in science. A hypothesis is a tentative statement, a speculation that needs to be put to the test, about particular aspects of a model or a set of models. Once corroborated, the hypothesis often takes the form of a law, a generic law that applies to an entire paradigm (e.g., conservation laws), or to an entire theory

| CATEGORY | MODEL TYPE | EXAMPLES |
|---|---|---|
| SUPERORDINATE | Emergent model | Bound particle accelerating on circular paths |
| BASIC LEVEL | Basic model | Uniformly accelerated particle |
| SUBORDINATE | Subsidiary model | Particle in free fall |

*Figure 2.7*: Middle-out hierarchy of models in a scientific theory.

(e.g., Newton's laws of dynamics), or a specific law that applies to a particular model (e.g., state laws like the so-called equations of motion in particle models). In contrast, a scientific theory is a conceptual system that has already been corroborated and that consists of: (a) a set of scientific models, and (b) a set of rules and theoretical statements for model construction and deployment (§ 1.5). The theory allows us to reliably describe and explain a set of physical patterns, each represented by an appropriate model, and, consequently, to predict and control those patterns, and to reproduce them in altered or invented physical realities.

Perhaps the most serious problem is the confusion that some make between scientific models and mathematical representations of individual laws in model structure. Many people, science teachers and professors included, often consider that a mathematical representation of a particular law or even of a particular descriptor, like either equation or diagram of Figures 2.3 and 2.4, constitutes a suitable "model" for the situation at hand. As can be seen in this chapter, a model is for us a far more complex conceptual structure. Watering things down to this simplistic level makes not only scientific models loose their significance, but more seriously, it depletes scientific theory and paradigm to the point of making them no different from the naïve theory and paradigm of lay people (§ 3.2).

It is true that for the purpose of optimizing efficiency and objectivity, one likes to convert a scientific model into a purely mathematical model, and thus operate with the latter model rationally and independently of the real world. Scientists and philosophers of science may disagree on many things, including the extent to which various scientific disciplines should be formalized mathematically and the implications of such formalism. However, they all agree that mathematics constitutes the universal language of science, and that "the description of physical reality provided by mathematical [science] is far more capable of disclosing the actual character of this reality than descriptions framed in ordinary language" (Kafatos & Nadeau, 1990, p. 4).

Whether in construction or deployment, scientific models are ultimately converted into mathematical models, i.e. models whose composition and structure are expressed in the form of mathematical variables and relationships. Scientific models are then concisely expressed, and thanks to the relatively culture-free semantics and syntax of mathematics, scientists all around the globe can interpret each other's models with a high level of objectivity and precision. Model deployment is also optimized with mathematics. Expressed mathematically, a scientific model can be analyzed rationally, independently of the empirical world, and it can subsequently be expanded to new horizons. However, the mathematical model into which a scientific model is converted would not simply consist of a single mathematical representation. Instead, it would be a complex model consisting of a variety of representations including, but not limited to, equations and diagrams similar to those of Figures 2.3 and 2.4, in order to cover all required structural facets. Even when one needs to concentrate on a single theoretical statement, say a particular law, a single mathematical representation cannot provide a comprehensive depiction of all possible aspects of the law. To this end, one needs to put together various sorts of mathematical representations (equations, diagrams, graphs) of the law in question. On the other hand, model deployment cannot stop at the point where rational analysis of the mathematical model ends. One always needs to go back to the actual scientific model in order to establish the correspondence to the real world of whatever outcomes the mathematical model may lead to, and interpret these outcomes empirically.

## 2.7 MODEL VIABILITY

The merits of a natural paradigm are by and large determined by its *viability* in the real world, i.e.: (a) its *validity* for asking specific questions about certain physical realities, (b) the *reliability* and the degree of precision with which it allows us answer those questions, (c) the *efficiency* with which questions can be asked and answers can be obtained, and (d) the extent of *coverage* in the real world, i.e., the number of physical realities the paradigm corresponds to and their distribution in space and time. From a scientific perspective, the answer to a given question comes only from a particular model in an appropriate theory included in the paradigm. The viability of a scientific paradigm (or theory) thus depends on the way models that make up its theories are constructed, corroborated and deployed in the real world.

Model viability is for us not a question of whether a model is "true" or "false". It is rather a question of: (a) how well (in what respects, and to what extent) the model represents a specific pattern in the real world, *and* (b) how suitable (or "instrumental") it is for answering specific questions about physical realities exhibiting the pattern in question. In these respects, viability depends above all on the way the model is corroborated, i.e. on the nature of sought evidence and the manner in which it is sought and established, and on the consequences of such corroboration. Corroboration of scientific models entails a continuous search for evidence that is both empirically reliable and rationally conforming to accepted theoretical canons. It may subsequently entail refinement of the model so as to boost model stability and reduce inherent falsifiability.

The following discussion is limited to viability aspects that we deem most important for our science teachers and their students to realize, but that both groups commonly miss; teachers following professional training, and students under conventional science instruction (Gilbert, 1991; Grosslight, Unger, Jay & Smith, 1991; Halloun & Hestenes, 1998; Smit & Finegold, 1995). There are of course many other aspects to which scientific models owe their viability (Bunge, 1967; Holton, 1993; Lakoff, 1987). These aspects are well beyond the scope of this book.

### *2.7.1 Empirical corroboration:*

A conceptual model, or any other conception, must be corroborated empirically in order to be inducted with no reluctance in the hall of science and to preserve its status there. Empirical evidence for the viability of a model is established according to objective, theory-laden norms and criteria, and it continues to be sought every time the model is deployed in the real world. Continuous empirical corroboration is a major feature that distinguishes scientific paradigms from all other sorts of paradigms, mathematical ones included.

Many a mathematician like Klein "construed the whole of mathematics as a collection of interconnected models, and its method as consisting essentially of 'imaginative' modeling and model-based reasoning, all the way 'up' to the axioms and all the way 'down' to its remotest social and cultural implications… Surely mathematics [though] is not an empirical science, nor is it aimed merely or even mainly at application in scientific *explanations* of nature" (Glas, 2002). Once "the axioms and the rules [of mathematics] are fully formulated, everything else is built up from them, without recourse to the outside world" (Mac Lane, 1988). "For all its faults, as Matthews argues (2000, p. 349), the scientific tradition has promoted rationality, critical thinking and objectivity. It instills a concern for evidence, and for judging ideas not by personal or social interest, but by how the world is."

Empirical corroboration is primarily established with respect to the function of the model, i.e., by seeking direct or indirect *evidence* to nomic isomorphism between model and represented pattern. As already mentioned in § 2.5, indirect evidence is established when model predictions and postdictions (for past phenomena) are matched with actual instances of the pattern that the model is supposed to represent. Direct evidence is established through model reification, either by controlling existing physical realities or by inventing new ones. By way of control, pattern conditions stated in the model correspondence rules are imposed on existing physical realities. By means of invention, new physical realities are created (or simulated) bearing primary features represented in model composition and structure. Corroboration will then be established if realities in either case produce the modeled pattern to an acceptable degree of precision.

For data to count as evidence in science, it has to meet a number of criteria. According to Bunge (1967, pp. 178-181), such criteria "must be agreed upon in advance of observation and on the basis of [scientific] theory". For "every evidence [in science] is relative to some hypothesis by virtue of a body of theoretical knowledge: no evidence is absolute and no evidence is prior to every theory". Acceptable evidence, Bunge continues, requires "an objective referent [whereby] personal experiences are irrelevant…, [as well as] a public control, and a minimum of interpretation in terms of accepted theories". Bernard (1865, p. 77), quoting a French poet stresses that, unlike art that reflects the personality of the artist, science should reflect a community consensus: "l'art, c'est *moi*; la science, c'est *nous*". "The tolerance of impersonality is at the very heart of conventional science" according to Holton (1993, p. 168) who quoted Max Planck as saying in this respect that "he was above all motivated by a search for…knowledge valid not only for all people but even, if they existed, for extraterrestrial beings".

Even when all criteria are met, according to Bunge (*ibid*, 163-183), "in order to be scientific, [evidence] must not be irrefutable but testable". In this direction, scientists normally have their data "*critically scrutinized* in search for errors of observation… Secondly, the data are universalized or *standardized* [i.e., *refined*, in order to optimize] the precision and relevance of universal information… Thirdly… the raw data are subject to *reduction*… The emphasis is on relevancy: excess of detail, if irrelevant, can be as encumbering in science as it is in everyday life". Data that count, Bunge continues, "are not given but must be produced [in the empirical word], very often by hard work", and they "must be *reproducible* by other workers in similar conditions", notwithstanding the fact that there are and will always be some "nonrepetitive facts" in the universe. Appropriate instruments are often needed to produce required evidence, instruments that, according to Lakoff (1987, p. 298, 300), "all extend basic-level perception… [according to well-defined] standards of consistency, reliability, and (to a lesser extent) rational explanation [of why these instruments work]… It is the technological extension of basic-level perception and manipulation that makes us confident that science provides us with real knowledge… Well, it's as real as our knowledge ever gets – real enough for all but the most seasoned skeptics."

In sum, for evidence to be reliable and thus to count as empirical corroboration of a scientific model, it has to meet a number of criteria. Among these criteria:

- Norms must be set *à priori*, within the framework of an appropriate *theory*, for what counts, and what not, as reliable evidence or counterevidence.
- These norms must be independent of the idiosyncrasies and expectations of individual researchers, and accepted by a concerned community of scientists.
- Data must be gathered following the methodology set by the corresponding paradigm (choice and use of instruments included), and interpreted in terms of the corresponding theory.
- Evidence must be optimized and reported in a way that makes it testable and open to scrutiny.

- In addition to routine checks of data for flawlessness, plausibility, and consistency with theory (next point), it must be checked that agreement between model and empirical data is not an accident due to concomitant factors.
- Evidence must be reproducible, whenever possible.

### *2.7.2 Rational coherence and conformity:*

Empirical corroboration is necessary but not sufficient for a model to be inducted into the halls of science. Scientists may reject a model that agrees with empirical data when the model does not blend with accepted theory or paradigm. For example, both Ptolemy's planetary model, with its geocentric epicycles, and Copernicus model, with its heliocentric elliptical orbits, fit the same planetary data. The Copernican model kept being rejected up until the sixteenth century despite the fact that it is simpler than the Ptolemaic model, and that it obeys better Occam's Razor principle that characterized the reductionism school of the Middle Ages. The Ptolemaic model prevailed until then mainly because it was consistent with the geocentric philosophy that dominated all sorts of paradigms (and worldviews) in the pre-Galilean era, and that put Earth and its human inhabitants at the center of the universe. Only the rejection of such philosophy paved the way for the Copernican model to replace its predecessor.

In addition to empirical evidence, a model must then pass rational tests of theoretical conformity and internal coherence. *Theoretical consistency* or *conformity* tests are concerned with the extent to which the model actually belongs to the theory it is supposed to belong to. These tests are made at the level of all four schematic dimensions to ensure that the model obeys all generic canons (tenets, axioms, laws, norms and rules) of the theory and the corresponding paradigm. C*oherence* tests are especially concerned with model structure. They are meant to assess the syntax of various laws and other theoretical statements included in all four facets of model structure, as well as the harmony among various relationships within and between facets, and especially between the causal facet and the state facet.

The rational underpinnings of a scientific model, or even of an entire scientific theory, may be well developed before the model becomes corroborated empirically, and it may even be inducted into the halls of science before then. This was the case for example with the entire theory of general relativity. Einstein was so sure of his theory that when he was informed of the detection of light ray's deviation near the sun in accordance with his calculations, his reaction was that, had the evidence turned out differently, he "would have to be sorry for dear God. The theory is correct" (quote cited in *American Journal of Physics*, 1994, *62* (1), p. 14).

### *2.7.3 Falsifiability:*

Scientific paradigms are not dogmatic. Their canons and theories are always put to the test indirectly through their models, and they may subsequently be modified, at least in some respects. No matter how valid and reliable a scientific model might be, we can never claim that any answer it provides is absolute, exact, or final. Falsifiability is an inherent feature of scientific models (and thus of theory and paradigm). In fact, science owes its objectivity, at least in part, to the *testability* of scientific models, testability that is not limited to the empirical and rational evidence discussed in the previous two points. Testability norms and criteria extend to those of model refutation, i.e., norms and criteria that set not only reliable evidence but also possible counter-evidence that could make the model lose its viability in the defined scope. This is perhaps what distinguishes science most from all other forms of human knowledge and belief systems, from religion to mathematics (Bernard, 1865; Bunge, 1973; Popper, 1959, 1963).

There are practical reasons that make scientific models, theory and paradigm, falsifiable. Some are related to the inherent nature of scientific models. Others are related to the nature of judgments involved in scientific activities. The idealization involved in pattern disclosure and model construction is reductionist. A scientific model's representation of its referents is always partial. It is constructed by *reduction* and *parsimony* in order to simplify and optimize the way it describes and/or explains the corresponding pattern. It reduces the pattern to specific primary features, and it eliminates all secondary details that conceal primary features, or that infiltrate them with noise. It is parsimonious in the sense that the isomorphism with preserved primary details is idealized to a minimum that is sufficient to describe and/or explain the pattern within the acceptable limits of validity, reliability and precision. Consequently, there will always be something missing in a scientific model, and no matter how secondary or negligible this might appear to be, it will always prevent us from claiming that the model provides us with an exact picture of the pattern it represents.

Scientific paradigms are value-laden like any other human paradigm. Scientists strive to keep all causes of partiality under control, as mentioned in § 2.7.1, they work hard to tame their personal interests so as not to bias their search for the "reality" of things or their exploitation of this reality. However, as individuals or as a community, scientists are not completely immune to psychological, social and political interference, and their activities are always driven by some value judgments that reflect the common interests of their community. Thus, they can never completely eliminate human impact in their activities, not even in exploratory research where they may be, knowingly or not, tampering with reality as they interact with it.

### *2.7.4 Stability and plasticity:*

Scientific paradigms enjoy a relative stability despite their falsifiability, and perhaps because of it, because of scientists' awareness of the limitations of their endeavors, and because of the special care they take to come as close as possible to an objective reality and to keep their work open to scrutiny. Once a model is reliably corroborated, and after it is validated over and over again through long decades of successful deployment, it becomes extremely hard, yet not impossible, to question its validity and reliability within its scope. This relative stability feature is what distinguishes modern scientific paradigms from their predecessors in the history of science.

For instance, for the last three centuries, particle models of Newtonian mechanics have been allowing us to describe, explain and predict, within certain limits of precision, low speed translation of physical objects with such reliability that one can hardly question their viability in these respects. The same can be said about the ideal gas model in chemistry, and the cell model of the structure of organisms in biology. These models and the corresponding theories enjoy the kind of stability that may be described by a quote attributed, in the late nineteenth century, to Fitzgerald: "if a yardstick does not give readings that fit the predictions of theory, then the length of the yardstick should be changed" (Whittaker, 1957). Other modern theories may not yet be as stable, but as Bunge (1973, p. 31) put it, should they ever fail, this would not refute them; it would “only weaken our interest” in them.

The stability of modern science is ensured, according to Lakoff (1987, p. 265), by “reasonable standards of objectivity and correctness” that the scientific community has set for its theories, and that represent “the best we can do” when “no God’s eye view standard is possible”. As such, modern scientific paradigms enjoy the kind of stability that characterizes Kuhn’s (1970) “normal science”, at least at the level of the “hard core” that Lakatos attribute to those paradigms or to what he calls “research programmes” in science (Lakatos & Musgrave, 1970, p. 132).

Model stability does not preclude plasticity or flexibility to accommodate unexpected findings. A scientific theory “should be capable to ‘learn’ from the new experience it was not able to predict… Good theories, like good cars, are not those that cannot collide but rather those which can stand some repair” (Bunge, 1967, p. 353, 354). A scientific model is originally conceived by correspondence to specific physical realities. However, the scope of the model is not necessarily limited to those realities. There may always be other realities that can be discovered eventually and that exhibit the same pattern represented in the model or that would help us to realize new potentials of the model. “One way science advances, Giere (1988, p. 107) argues, is by discovering new aspects of the world, that is, new respects in which our models might resemble the world. Science also advances by discovering some respects in which similarities between model and world are not as commonly thought”. As a consequence, the model may undergo some refinements to accommodate new findings, especially at the level of nomic isomorphism, and improve its efficiency. Furthermore, model deployment may eventually lead to construction of new conceptions that would enhance the precision of the model, or contribute to the construction, corroboration and/or deployment of other models within the framework of the same paradigm. This is a natural consequence of continuous paradigm refinement.

For example, Newton originally developed his classical theory of mechanics to *explain* the motion of planets and other celestial objects. Newtonian scholars found out later that the theory can actually explain the translational motion of terrestrial objects as well. They have further expanded the theory conceptually so as to additionally *describe* the motion of such objects. Nowadays, Newtonian particle models are used to describe and explain, under certain conditions, the translation of objects of all size, from the microscopic atomic world to the macroscopic and astronomic worlds. Similarly, Darwin developed Malthus theory and took it to new evolutionary horizons (§ 1.9), and Mendel's work on peas' crossbreeding evolved, about a century later, into a comprehensive genetics theory.

Some fields in modern science, and especially in physics, have reached such a stability level that some, like Kafatos and Nadeau (1990, p. 17) have come to the conviction that "most of the basic features of the universe appear to have been disclosed, and continued extensions and refinements in quantum physics are not likely to fundamentally alter present assumptions about the actual character of physical reality".

All in all, models of modern science have become so viable that they have brought scientific theory and paradigm close to the state advocated by a number of realists who believe, like Matthews (2000, pp. 329, 330), "that science aims to tell us about reality, not about our experiences; that its knowledge claims are evaluated by reference to the world, not by reference to personal, social, or national utility or viability [i.e., conformity]; that scientific methodology is normative, and consequently distinctions can be made between good and bad science; that science is objective in the sense of being different from personal, inner experience; that science tries to identify and minimize the impact of noncognitive interests (political, religious, gender, class) in its development; that decision-making in science has a central cognitive element and is not reducible to mere sociological considerations; and so on."

## 2.8 CONCEPT SCHEMA

Concepts are the most elementary components in the middle-out conceptual hierarchy that includes theory at the top and models at the basic, middle level. Concepts of different types constitute the ingredients for formulating various theoretical statements, and like these conceptions they gain their full significance only after entering in the composition of models and contributing to model structure. As in the case of models, concepts are defined in modeling theory and situated in scientific theory according to a particular schema, a concept schema. A concept schema serves, in modeling theory, as a template for the construction and deployment of all concepts of a given type that are needed in the edification of a scientific theory, and this within the framework of the theory's basic models.

Within the epistemological confinements of this book, we have recourse to three types of concepts: object-concepts or *depictors*, property-concepts or *descriptors*, and operation-concepts or *operators* (mostly, of logico-mathematical nature). The categorization could of course have been extended, following different criteria, to include other types of concepts. Should we have been interested explicitly, say, in the axiology of scientific theory, we would have included a fourth type of concept, value-concepts, that pertain to value-judgment about model and theory viability.

Descriptors are the most important concepts because they are the ones mostly used in model structure. State laws, interaction laws, and causal laws express relationships among descriptors of object and/or agent, often in mathematical symbolism that involves particular operators. Operators, along with their semantics and syntax, are the object of mathematics. That is why we concentrate our attention in this section on the concept schema as implemented exclusively in the development of descriptors among all concept types.

In line with the model schema, the concept schema is a four-dimensional schema. The four dimensions, scope, expression, organization and quantification, are concisely discussed in this section and illustrated with the concept of force. Discussion is limited to particular aspects that distinguish concept schema from model schema and those that allow linking the two schemata together without encumbering the reader with redundant or unnecessary details. The concept of force is chosen for illustration because it relates to Newtonian particle models that have been used the most in our discussion of model schema, and because it is the most fundamental interaction descriptor, and the most difficult descriptor to learn in introductory physics courses (Halloun, 1986; Halloun & Hestenes, 1985; Halloun & Rabah, 2004).

### 2.8.1 Scope:

A descriptor represents, to a certain degree and within certain limits, a particular physical property shared by many real world systems or phenomena. It has a *domain* confined to the represented property and a particular *function* depending on the nature of the property. Domain and function constitute concept *scope*. Physical realities bearing the property in question can exist anywhere and at any time in the universe. Like in the case of a model, each of these realities is called a concept *referent*, and the set of all referents are said to constitute the concept *reference class*. As discussed in § 2.4, a descriptor can be either an *object descriptor* or an *interaction descriptor*, and an object descriptor can be either an *intrinsic descriptor* or a *state descriptor*. Object descriptors, and especially state descriptors, have a *descriptive* function, whereas interaction descriptors have an *explanatory* function.

Different descriptors may share the same reference class, and they may even represent the same physical property (which is not so frequent within the same scientific theory), but they may never represent this property in exactly all the same respects. For example, force and field are two interaction descriptors. They may represent interaction between the same physical objects. However, force is measured in terms of a common intrinsic property of both agent and descriptor, whereas field is measured only in terms of an agent's intrinsic property. For example, expressions of gravitational and electrostatic forces between two bodies involve mass or charge of both bodies (Fig. 2.4c), whereas expressions of the respective fields (commonly denoted by **g** and **E**) involve only mass or charge of the agent:

$$\vec{g} = G\frac{m_{agent}}{r^2}\hat{r} \text{ and } \vec{E} = k\frac{q_{agent}}{r^2}\hat{r}.$$

Unlike a model, a descriptor may never accumulate two functions. It can never be an object descriptor and an interaction descriptor at the same time, just like it can never be a state descriptor and an intrinsic descriptor at the same time. Similarly, a descriptor may not swap functions, especially not descriptive and explanatory functions. Only interaction descriptors can serve an explanatory function, and only state descriptors can serve a descriptive function. When such subtleties are not made explicit in science instruction, students will inevitably end up confusing concepts of a different nature with one another. Research shows that in fact students often use state and interaction descriptors interchangeably. For example, physics students often speak of velocity, acceleration or kinetic energy as "causes" of motion, thus mixing up these state descriptors with interaction

1. Force is an interaction descriptor.
2. Its reference class includes physical bodies that may belong to the microscopic world of atoms or to the macroscopic world (as extended to the astronomical level).
3. No object can interact with itself, and intrinsic forces (like impetus) do not exist.
4. Interaction takes place between at least two physical bodies. Force may then represent the action of either body on the other.
5. When two or more physical bodies interact, we call *object* a body under study, and *agents* all other bodies interacting with it.
   Respective depictors are attributed the same names.
6. Object and agent exert simultaneous and reciprocal actions on one another. The respective interaction is then represented by a pair of equal and opposite forces (Newton's third law). For convenience purposes, we concentrate exclusively on the force exerted by agent on object.
7. Interaction between physical bodies is not directly perceptible. It may be manifested by a particular change of state, in a specific reference system, of either body involved (i.e., change of linear momentum or of velocity in inertial reference systems).
8. No change of state takes place in the absence of any (net) interaction. A change of state reflects the presence of unbalanced interactions, and may be *explained* with the force descriptor.
9. Four types of interaction, and thus four types of force, are distinguished in the universe: gravitational, electromagnetic, weak and strong. The first two take place in the microscopic world as well as in the macroscopic world, whereas the last two take place only in the microscopic world, at the nuclear level. Except for gravitational interaction, a specific type of boson mediates interaction between object and agent.
10. All forms of interaction take place at a distance in the microscopic world. For convenience purposes, and to a good level of approximation, we may assume in the macroscopic world, and by virtue of Newtonian theory, that two bodies may "touch" one another so that no distance separates them. A different force taxonomy can then be established in this world including "contact" forces of different types, each associated with particular types of agents (Halloun, 1998a, 2001a). Bosons are then ignored.

*Figure 2.8*: Correspondence rules pertaining to the concept of force within the framework of Newtonian theory of classical mechanics.

descriptors like force or potential energy, and they attribute to them sometimes an intrinsic nature like that of impetus, which has long been proven to be wrong (*op. cit.*).

These functional subtleties, like other aspects related to concept domain, are explicitly expressed in appropriate *correspondence rules*. These rules delimit the scope of a concept within the framework of a particular scientific theory (or a particular paradigm, if the concept is generic enough to be used throughout the paradigm, like in the case of force). They subsequently prevent us from confusing two concepts with one another. Correspondence rules associated with the Newtonian concept of force are given in Figure 2.8 for illustration.

### *2.8.2 Expression:*

Objectivity is a major viability aspect of scientific conceptions. This aspect is guaranteed in part by expressing each conception in a unique way associated with particular semantics that establish what the expression actually delineates in the real world or the rational world of scientific theory and paradigm. A mix of verbal, symbolic, iconic, and especially mathematical forms of expression is commonly used to communicate any scientific conception. The mix is necessary to come as close as possible to a comprehensive expression of the conception, descriptors included, since no single form can actually do so alone.

*Verbal expression* of a given descriptor (or depictor) consists of a particular name, i.e., a particular word used to denote *exclusively* the concept in question. No two scientific concepts can bear the same name, and no two names can be given to the same concept. There are neither homonyms nor synonyms in science. A concept name serves as an exclusive *mnemonic key* for retrieving the particular concept from memory, and actuating whatever is necessary of its schematic structure.

Unlike colloquial vocabulary, scientific vocabulary is precise and exclusive. Student confusion among different concepts, whether of the same nature or of different natures as mentioned above, is due in part to the indiscriminate use of colloquial and scientific terminology. That is how, for example, students mistakenly use homonyms in science just because this is common practice in everyday life. That is also how they bring into their science courses the confusion they typically make in everyday life between descriptors like “mass” and “weight”, “force” and “power”, or between depictors like “virus” and “bacteria”. They may even extend the mix-ups to new concepts they learn in science courses when the exclusivity of concept names is not enforced explicitly during instruction.

*Symbolic expression* of a given descriptor (or depictor) consists of alphanumeric labels typical of the symbols associated with mathematical variables (e.g., $m$ or $m_i$ for the mass of a body $i$). Unlike verbal expressions, symbolic expressions are usually not exclusive. Except for symbols associated with chemical elements, atomic particles and the like, a symbol does not designate a particular concept (whether a depictor or a descriptor) in science, but a particular instance of the concept in a particular situation (e.g., a particular value of a given descriptor). Hence different symbols may be used to denote different instances of the same concept in different situations, and the same symbol may be used to denote instances of two different concepts (preferably not in the same situation). Some symbols are uniformly used in various textbooks to denote instances of particular concepts (e.g., $\boldsymbol{F}$ for force, $\boldsymbol{a}$ for acceleration). This is misleading for some students. They end up believing that such a symbol can only be attributed to the concept in question, and that no other symbols can be used to designate the same concept.

*Iconic* or *pictorial expressions* consist of geometric figures (icons). They may serve as depictors, as well as means to designate some descriptors in relatively tangible ways. An icon may or may not resemble what it represents in the real world. In the first case, the icon is said to offer an *analogical representation* of what it stands for (e.g., a sphere depicting our planet). In the second case, it is said to offer an *analytical representation* (e.g., a particle depicting physical bodies in translation). Not all descriptors may be designated with iconic expressions. For example, no icons are commonly associated with scalars like the intrinsic descriptors of mass and charge. In contrast, vectorial descriptors, like force and acceleration, can be expressed iconically with arrows drawn to scale in appropriate coordinate systems, and bearing distinctive symbols (Fig. 2.4e).

*Mathematical expressions* include all sorts of tables, equations (chemical formulas included), graphs and diagrams used to denote particular *relationships* among certain descriptors. A particular mathematical mode of expression reflects only certain but not all aspects of a given relationship, and no two modes may express exactly all the same aspects. For example, the equation $\boldsymbol{F} = m\boldsymbol{a}$ expresses Newton's second law. The equation can be deployed only to find out the magnitude of the force $\boldsymbol{F}$ needed to move an object of mass $m$ with an acceleration $\boldsymbol{a}$ of particular value. The equation does not show the direction of either force or acceleration; a vectorial diagram like that of Figure 2.4e is needed to this end. Both modes (equation and diagram) express snapshots of the relationship between the descriptors involved; when deployed in instances of variable acceleration, each mode can allow us to obtain only one particular value of either descriptor at a particular point of spacetime. A graph $F(a)$ can show magnitudes of either descriptor at various points of spacetime; yet the graph does not show, say, the direction of either descriptor or the time at which those values are taken. A variety of mathematical modes are then needed to express all possible aspects of Newton's second law, or of any other relationship in which one might be interested.

Particular s*emantic rules* are associated with each mode of expression to help interpreting it in both the real and rational worlds. Such rules are perhaps more crucial for mathematical expressions than for other forms of expression, given the complexities and subtleties involved. Semantic rules spell out the potentials and limitations of what each expression represents, and this along the lines of what we just did with Newton's second law. Semantic rules are coupled with *syntactic rules* that set, in the manner discussed next, the way in which each form of expression can be put together and used in the development of various conceptions.

### *2.8.3 Organization:*

A descriptor gains its significance only after being related to other descriptors within the contexts of models, and especially basic models. Concept organization sets rules and guidelines for establishing such relationships, beginning with the categorization of the concept at hand. Such a categorization is necessary to determine the scope of the concept, and especially the function it can play in model structure.

Different descriptor categories can be distinguished, based on the nature of adopted criteria. We have already classified descriptors from an epistemological perspective into object and interaction descriptors of descriptive and explanatory function, respectively. We shall maintain this classification throughout this book since it best suits our needs. From an ontological perspective, a descriptor may either be a prime descriptor or a derived one. A *prime descriptor* is one that cannot be defined explicitly in terms of other descriptors. It can only be defined implicitly through a set of axioms. This is the case, for example, with the concept of force in the classical mechanistic paradigm. Force is defined implicitly through Newton's four laws of dynamics (whence the reference to these laws as Newtonian axioms of force). A *derived concept* is one that can be expressed explicitly in terms of other concepts of the same epistemological nature through so-called "definitions". For example, in Newtonian theory, acceleration is a kinematical (state) descriptor derived from velocity, another kinematical descriptor, and work is a dynamical (interaction) descriptor derived from the dynamical descriptor of force. Laws or axioms, but not definitions, can relate one descriptor to another of different nature (e.g., force to acceleration in Newton's second law). Along the same ontological lines, depictors can be classified into *elementary depictors* when of no internal structure (like particles of Newtonian models of translation), and *composed depictors* otherwise. Once the nature of a concept is determined, appropriate *syntactic rules* can be set, within the framework of an appropriate scientific theory, to relate it to other concepts (Fig. 2.9), and this along with appropriate *semantic rules* for interpreting established relationships.

Concept categorization is especially important from a pedagogical perspective. It is necessary for teachers to understand beforehand the cognitive demands of learning a specific concept. In this respect, and given their axiomatic nature, prime descriptors are usually harder to learn than derived descriptors, and given their simple structure, elementary depictors are usually easier to learn than composed depictors. As for descriptive and explanatory descriptors, the former are usually easier to conceive and deploy than the latter. For explanatory descriptors represent properties that are not directly exposed to our senses, and they subsequently involve more complex rational-empirical dialectics. Consequently, teachers should expect to deploy more effort and spend more time with prime and explanatory descriptors, as well as with composed depictors, than with their counterparts. This is also more so the case of laws by comparison to definitions, and of causal laws by comparison to state laws. For, on the one hand, definitions and state laws involve concepts of the same nature, whereas causal laws involve concepts of different natures. On the other hand, definitions express local relationships between descriptors (they allow finding one descriptor in terms of another at a particular instance of spacetime), whereas state laws express the evolution in spacetime of one descriptor in terms of others.

1. Force is a prime, interaction descriptor. No explicit definition can be formulated for this concept. It is defined axiomatically through all Newton's laws of dynamics.
2. The pair of forces exchanged between an object and an agent depends on the type of interaction between them, and is governed by an appropriate interaction law.

   Interaction laws express the concept of force in terms of intrinsic descriptors of object and/or agent.
3. Causal laws determine the effect of interaction on the state of a given object.

   The effect of the force $F$ exerted by an agent on the state of an object of constant mass m can be assessed with the concept of acceleration $a$ within the context of Newtonian theory of mechanics.

   Newton's second law is a causal law that relates cause ($F$) and effect ($a$): $\boldsymbol{F} = \mathrm{m}\boldsymbol{a}$. The equality does not imply an identity in this expression. We cannot say that F "is" ma. The expression says that "if $\boldsymbol{F}$ then $\boldsymbol{a}$" of value $\boldsymbol{F}/\mathrm{m}$.
4. Force is a vectorial concept. Any law or other theoretical statement about it involves other vectorial concepts.
5. The concept of force may serve to define derived dynamical concepts like work.
6. $F$ and $a$ take different forms in different particle models (Fig. 2.6). Thus, conceptualization and interpretation of the two descriptors and corresponding logico-mathematical operations vary from one model to another (Kuhn, 1970, p. 188; Lakoff, 1987, p. 305, 6).
7. Unlike state descriptors, force, like any interaction or intrinsic descriptor, stays the same in all inertial reference systems.
8. The expression of any theoretical statement involving the concept of force does not change from one inertial reference system to another.

   Subsequently, and despite possible variation of state descriptors, the nature of any Newtonian particle model does not vary from one inertial system to another.

*Figure 2.9*: Syntactic rules pertaining to the Newtonian concept of force.

### *2.8.4 Quantification:*

> A century ago, Lord Kelvin (1891) argued:
> "when you can measure what you are speaking about, and express it in *numbers*, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thought, advanced to the stage of science".

A descriptor cannot be scientific unless it is measurable according to well-defined laws and rules. Measurement of a descriptor consists of its *comparison* to a certain *standard.* The result of the comparison is a *value* reported on an appropriate *scale.* The scale can be nominal, ordinal, interval-type or ratio-type. A *nominal scale* is one that allows only a comparison of identity or difference with the chosen standard (e.g., gender). In an *ordinal scale*, the value of a descriptor can further be said to be either inferior or superior to the chosen standard, and various values of the descriptor may subsequently be ranked in a certain order. The difference between two values remains however unquantifiable (e.g., pain). When such a difference can be calculated, the scale will be said *interval*-type (e.g., temperature), and when we can further determine a proportionality coefficient between the two values, the scale will be called *ratio*-type (e.g., force). A descriptor is usually said to be quantifiable in the latter two cases. Biological sciences include descriptors of all four types, whereas physical sciences are concerned only with interval and, especially, ratio type descriptors. As such, all descriptors of physical sciences are *quantifiable*, which makes them relatively more objective and more precise than descriptors of other disciplines.

Descriptor quantification is concerned with setting appropriate laws and rules for measuring interval and ratio types of descriptors within the context of an appropriate scientific paradigm or theory. *Quantification laws* set the nature of the descriptor, specify measurement standard and scale, and determine logico-mathematical operations that the descriptor can undergo, all along with underlying assumptions. *Quantification rules* state how one can go about determining empirical values of the descriptor, each value specified with at least a number and a unit, within well-defined limits of precision and approximation. Some of the laws and rules are generic, and apply to many descriptors across various theories (or a particular theory) in a given paradigm, while others may be particular to the descriptor at hand. Figure 2.10 shows quantification laws and rules associated with the concept of force within the context of Newtonian theory of mechanics.

Quantification laws and rules are seldom presented as explicitly as in Figure 2.10 in conventional science instruction. As a consequence, students encounter many insurmountable difficulties that are well documented in the literature. Among these difficulties: (a) confusion between interval and ratio type descriptors, especially with regard to permissible logico-mathematical operations, e.g., calculating by error the sum or ratio of two temperatures; (b) confusion between scalar and vectorial concepts; (c) confusion between instantaneous and average values of a variable descriptor, mainly because of the mathematical confusion between derivatives and change ratio; (d) attribution of wrong units to a given descriptor; (e) accepting values that are out of empirical range, e.g., a gravitational acceleration greater than 10 $m/s^2$ near the surface of Earth.

Modeling involves model construction and deployment, as well as continuous model evaluation throughout both processes. The emphasis in this chapter has been on the use of modeling schemata more in model construction than in model deployment, and especially in setting the content of models in a given scientific theory. Within each dimension, and especially at the level of composition and structure, model construction requires specific tools like those of Figures 2.3 and 2.4, as well as others discussed later in Chapter 4. Model deployment requires the same tools as well as mnemonics for deciding on the appropriate model(s) for a particular situation, and retrieving from memory whatever is necessary from a model composition and structure to deal with the situation. Modeling schemata can be extrapolated in the manner discussed in Chapters 4 and 5 in order to help in answering these concerns.

1. Force is a *vectorial* concept (as opposed to *scalar,* like the concepts of mass or temperature). Characteristics (magnitude and direction) of a force exerted by a given agent on a given object depend on interaction parameters (intrinsic properties and relative position) determined by the nature of the interaction between the two (Fig. 2.8).
2. A single force is needed to represent a specific action of an agent on an object.
3. The force exerted by a given agent on an object depends only on this particular agent and on no other agents with which the object may be interacting.
4. The force exerted by an agent on an object remains the same as long as the interaction parameters are unchanged. As long as this is the case, the force does not change with the state of the object.
5. The force exerted by an object on a given agent is opposed only to the one exerted by the same agent on the object (Newton's third law).
6. The particular force exerted by an agent on an object affects only the state of motion of this particular object in accordance with Newton's second law. It does not affect the kinematical state of the agent or that of a different object.
7. The effect of a force on an object is instantaneous and lasts as long as the force is exerted on this particular object. It stops only when the force is no longer exerted on the object.
8. Force and acceleration as expressed in Newton's second law are assessed at the same time.
9. When an object in translation interacts with many agents, the net interaction is equivalent to one with a single agent that exerts a force on the object equal to the vectorial sum of all forces exerted by the original agents.
   This is Newton's fourth law or law of force composition. It assumes that various forces are independent of one another, and it requires that all forces being added are exerted on the same object at the same time.
   Alternatively, a single force may be decomposed into a number of components that may be treated as independent forces.
10. The force exerted by an agent on an object cannot be directly measured empirically. It can be measured indirectly through its effect on that object (by applying Newton's second law). Two forces are then said to be equal if they produce the same effect on the same object (This assumes that after each measurement, the object can be brought back exactly to the same initial conditions).
11. In SI, Newton is the unit of force. In Newtonian theory, a force of one Newton is required to impart to an object of mass 1 kg an acceleration of 1 m/s$^2$.
12. Force is an *extensive* descriptor (as opposed to *intensive*, like temperature), i.e., a single force of magnitude zero indicates no net interaction.
13. Force is an *additive* descriptor (as opposed to *non-additive*, like temperature); two or more forces can be added vectorially following the law of composition.
14. Force is a *ratio-type* descriptor (as opposed to *interval-type,* like temperature); two forces exerted on the same object can be compared by a ratio.
15. The characteristics of a force are invariant under Galilean transformations, i.e., when changing inertial reference systems.

*Figure 2.10a*: Quantification laws.

1. In order to determine the characteristics of a given force, refer to the corresponding interaction law corresponding to the specific object-agent pair (L1, L2, L3).

   A detailed force catalog is given elsewhere (Halloun, 1998a, 2001a).
2. The pair of forces mutually exerted by an object and an agent have the same magnitude and opposite directions (L5).
3. The net force exerted on an object resulting from its interaction with many agents can be determined from a force diagram whereby each force is represented by an arrow to scale as shown in Fig. 2.4e (L9). Rules of vector addition need then to be followed (L13), while ensuring that the net force is parallel and proportional to the acceleration of the object at the instant the net force is evaluated (L6, L7, L8).

   The net force must be zero when the object is at rest or when moving with constant velocity in an inertial reference system (L12).
4. The outcomes of force composition or decomposition do not change from one inertial reference system to another, as long as the interaction parameters remain unchanged (L4, L15).
5. The dimension of a force is given symbolically by (L6 and L11):

   $$[Force] = \frac{[Mass][Length]}{[Time]^2}$$
6. Conversion from one unit system to another is governed by the above dimension rule. Accordingly, conversion from SI to cgs or the old British unit system follows:

   1 dyne = $10^{-5}$ N (cgs) and 1 lb = 4.448 N (Brit.).
7. Any force probe must be graduated in accordance with L10 and L14. A dynamometer is such a probe governed by Hooke's law.
8. No measurement in the empirical word is perfect because of many constraints and limitations, some inherent to measurement tools and governing theory, and others related to human nature.

*Figure 2.10b*: Quantification rules.
Corresponding law statements in Fig. 2.10a are referred to as Li (i=1…15).

*Figure 2.10*: Quantification laws and rules pertaining to the Newtonian concept of force.

## *Chapter 3*

# PARADIGMATIC EVOLUTION

"Professor Mazur, how should I answer these questions? According to what you taught us, or by the way I *think* about these things?" (Mazur, 1997a, p.4).

This question sums up the state of conventional science instruction of lecture and demonstration. It was asked by a college student taking the Force Concept Inventory (FCI), a test assessing conceptual understanding of mechanics in the context of everyday life situations, the description and explanation of which require no quantitative problem-solving heuristics (Halloun, Hake, Mosca & Hestenes, 1995; Halloun & Hestenes, 1985b). The question reflects at least three drawbacks about student natural paradigms and the way they evolve under conventional science instruction.

First, student conceptions about physical realities consist more of mixed beliefs and knowledge of vague correspondence to the real world than of viable knowledge about physical realities. Mazur's student asked whether she should answer questions the way she "thinks" about physical realities and not the way she "knows" them. The term may have merely slipped, or it may just be a common way of saying things. Yet, it is not as simple as it sounds. It reflects the actual state of most, if not all, ordinary people's natural paradigms, our students included. People seldom pause in their everyday life to assess empirically, or even rationally, their "thoughts" or ideas about natural systems and phenomena. Without due corroboration, these ideas remain mere beliefs; and with ordinary common sense corroboration, they seldom have the chance to be transformed into reliable/scientific knowledge about physical realities.

Second, conventional science education does little, if any, to change the situation. Students are seldom given the opportunity in science courses to reflect on their own ideas, so as to regulate them, whether internally, by assessing the mutual consistency and coherence of these ideas, or externally, by comparison to empirical data and scientific paradigms.

Third, and more importantly, what the quoted student got out of physics lectures appears to her, like many of her fellow students around the world, as a system of conceptual structures and processes that does not relate to her own natural paradigm. It appears as if scientist and student natural paradigms were about two independent worlds or worldviews, or even as if scientific paradigms were about a foreign culture that is only remotely relevant to the student's own life. As a consequence, the student thinks by mistake that the teacher-delivered conceptual system and her own natural paradigm can cohabit freely in her mind. Due to the incompatibility or even the conflict between the two conceptual systems, cohabitation does not last for long, and the newly delivered system fades away with time.

In 1910, Dewey argued that "the future of civilization depends upon the widening spread and deepening hold of the *scientific habit of mind*…, [the kind of habit] that to some extent the natural common sense of mankind has been interfered with to its detriment…; the problem of problems in our education is therefore to discover how to mature and make effective this scientific habit" (Archambault, 1974, pp. 190, 191, italics added). About a century later, and despite numerous similar calls worldwide, Dewey's "creed of life" is not yet fulfilled. The reason is partly because, as Dewey argued, "science has been so frequently presented just as so much ready-made knowledge, so much subject-matter of fact and law, rather than as the effective method of inquiry into any subject-matter… a method of thinking, an attitude of mind, after the pattern of which mental habits are to be *transformed*" (*ibid*, pp. 183, 187, italics added).

The transformation Dewey is calling for is, from our point of view, a comprehensive transformation in student natural paradigms, a *paradigmatic evolution* from the realm of naïve realism or common sense to the realm of science. This entails an evolution of all aspects of student paradigms, aspects that extend from underlying canons to various conceptions, tools and processes, and that encompass various cognitive factors that affect learning.

## 3.1 PARADIGMATIC PROFILE

Educational research on students' inquiry and conceptions about the real world reveals that their natural paradigms have many components that differ significantly from those of scientific paradigms, and that they are by far not as systematically, reliably or coherently articulated as their scientific counterparts. This should come of no surprise to us. Students are not afforded in their everyday life, or even at school, the sort of physical environment or the kind of social interaction that scientists are afforded in their observatories, research facilities and professional organizations. In this respect, scientists and students live in different worlds, and the two groups are driven by two different cultures (Cobern, 1995) with different goals, commitments, concerns and requirements (Reif & Larkin, 1991). Lakoff & Johnson (1980, p. 146) argue that "conceptual systems of various cultures partly depend on the physical environments they have developed in", and that "the social reality defined by a culture affects its conception of physical reality". Thus, and according to the epistemic tenets discussed in Chapter 1, the mismatch between student and scientist natural paradigms is to be expected in virtually all paradigmatic respects.

To each scientific paradigm corresponds a variety of student natural paradigms, irrespective of the demarcation lines we might draw between various scientific paradigms. A student paradigm often consists of a mix of components some of which may be somewhat compatible with modern scientific paradigms, others at odds with the latter and often reminiscent of paradigms that dominated the pre-Galilean era of science, and that relied heavily on common sense perceptual experience (Cobern, 1993; Halloun & Hestenes 1985b; Helm & Novak, 1983; Novak, 1987, 1993).

An alternative look at student natural paradigms is offered by Bachelard (1940) who suggests that nobody holds a single natural paradigm, but that every human being, individual students and scientists included, holds a mix of natural paradigms, some in agreement with science others at odds with it. This, as Lakoff (1987, p. 121, 122) points out, should come as no surprise, since "most of us do not have a single coherent understanding of how the physical world works". Depending on personal convenience and past experience with a given conception, one may deploy the conception in the framework of one paradigm (e.g., a positivist one) in a particular situation, then in the framework of another paradigm (e.g., classical mechanistic) in a different situation.

According to Bachelard (1940), no conception held by an individual, whether an ordinary person or a scientist, falls from a single philosophy about the real world and the conceptual realm. Every conception is distributed throughout what Bachelard calls an *epistemological profile*. The profile suggested by Bachelard has five dimensions drawn from five different philosophical schools that dominate all sorts of paradigms, and that are sometimes in contradiction with one another. These schools extend from what we call *subjective concretism,* a school based solely on perceptual experience that Bachelard calls *naïve realism* (a label that we reserve for subsequent use in a more generic sense), to *dialectical idealism* that is totally independent of such an experience (Figure 3.1). The profile has the following major characteristics, mostly inferred from the work of Bachelard:

1. A conception may be exposed to other people in a particular philosophy, but, in one's own mind, it can never be grounded in a single philosophy. A spectrum of different philosophies (epistemological profile) is required to envisage the conception from different practical and rational perspectives.
2. Any conception, be it a simple concept or a complex model, be it held by an ordinary person or by a scientist, is distributed throughout various dimensions (philosophies) of a specific epistemological profile. The conception is evoked within the framework of a particular philosophical dimension (SR, PE, CR, RR or DI in Fig. 3.1), the choice of which depends on the person's experience with the empirical context where the conception is being evoked. Personal experience is the major determinant of the scope of a given conception within each of the five dimensions.
3. The epistemological profile is not necessarily complete for all the conceptions a person holds. It is rare that a conception be distributed throughout all five dimensions of the spectrum shown in Figure 3.1. In fact, there are conceptions relating to particular scientific disciplines where rationalism is barely apparent, and others where concretism is completely absent.

The epistemological profile of any concept (e.g., mass) has five dimensions, or levels according to Bachelard (1940):

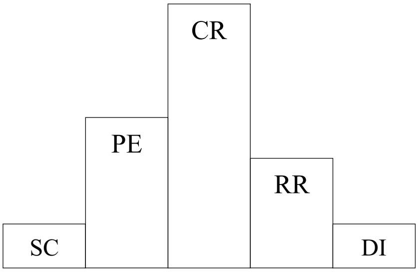
1. *Subjective Concretism* (SC), called *naïve realism* by Bachelard. Interaction with the real world concentrates more on objects than on phenomena. Properties of an object are roughly assessed by looking at the object. As far as mass is concerned, the bigger an object, the heavier it is (mass proportional to volume, irrespective of density). This level is dominated by *animism.*

2. *Positivist Empiricism* (PE), or *clear and positivist empiricism* according to Bachelard. Concepts become more objective and precise. Quantitative assessment is made with the use of appropriate instruments, like the scale for mass. Yet instrument development and use are not theory laden. Theory follows from exploratory, Baconian research.

3. *Classical Rationalism* (CR) of Newton, Lagrange, Poisson and Hamilton. Conceptual systems are built primarily on the basis of inner theoretical (mathematical) coherence. Empirical, experimental corroboration comes later. CR fulfils all functions of a Kantian *à priori*. The merits of a concept are determined more by its *predictive* power with regard to natural *phenomena* than by its immediate correspondence to physical objects. Mass is now rationally conceived as the ratio of two concepts, force and acceleration. The relationship among the three concepts can be fully analyzed with the rational laws of arithmetic.

4. *Relativistic Rationalism* (RR), or *complete rationalism* according to Bachelard. The *closed* rationalism of Newton and Kant is now *open* so that there are no more absolute concepts, not even of space, time and mass. Basic concepts in CR remain as such in RR, but they are no longer as simple as before. Mass is still a basic concept, but it is now a complex function of speed. This level is completely dominated by *noumena* in search for their phenomena.

5. *Dialectical Idealism* (DI), or the *open and discursive (sur)rationalism* of Dirac. Modern science tends to put reality aside, to open rational parentheses within reality often in a paradoxical manner. Concepts like negative mass that could not be conceived in any of the previous four philosophies are now conceivable, even when corresponding empirical corroboration seems far fetched. Scientists do not know reality until they realize it, until they turn the wheel around and become masters of the eternal new beginning of things. This is where *anagogic dreams* take place.



Bachelard's own epistemological profile of the concept of mass (Bachelard, 1940)

*Figure 3.1.* Bachelard's epistemological profile.

4. Profile dimensions form a continuous and orderly spectrum whereby no dimension can totally overtake or annihilate another.
5*. The epistemological profile of a conception is context-dependent. The spectrum of dimensions (philosophies) it includes and the relative importance of each dimension depend on individual's personal experience. However, the nature of each dimension is context-independent; it is determined by pre-Galilean "science" for the first two dimensions, and by modern science for the last three.
6. The relative importance (size) of each of the five dimensions varies from one person to another for the same conception, and from one conception to another for the same person.
7. The epistemological profile of any conception is not static. It evolves continuously following clashes with obstacles and contradictions.
8. Evolution of a conception takes place progressively across all five dimensions, and in the order shown in Figure 3.1 from SC to DI. In the process, dimensions on the right of SC become, to various degrees, increasingly more important.
9. The ontogenetic order of the three fundamental philosophies (concretism-empiricism-rationalism) is genetic. [The orderly ontogeny matches, to different degrees, developmental stages suggested by Piaget and Perry].
10. Evolution into the scientific realm requires one to transcend the perceptual and empirical world so that rationalism becomes significantly more important. It requires, especially at the fourth level of relativistic rationalism, an "internal openness" toward individual conceptions rather than an external openness toward the empirical world or other conceptions. The relative importance of each rationalist dimension depends on expertise in specific scientific arenas.

Bachelard's profile pertains to a single conception, and it concentrates on epistemological aspects. We prefer to extend it to more practical horizons with respect to individual conceptions, as well as with respect to the array of all natural paradigms that a person might possess. In the latter respect, what a person considers to be personal "knowledge" about the real world consists of conceptual

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* This feature is actually suggested by Mortimer (1995) and not by Bachelard.

structures and processes, and underlying tenets that may be distributed across natural paradigms of different trends. These paradigms make up the person's *paradigmatic profile*. With respect to individual conceptions (especially concepts or models), we prefer to speak, in line with Mortimer (1995), not of an epistemological profile but of a *conceptual profile* of the particular conception. A conceptual profile encompasses methodological as well as epistemological aspects of the conception. Depending on personal experience, the conception may be confined to a single paradigm, or it may have different alternatives distributed across different paradigms along the lines of Figure 3.1. Every person possesses a single paradigmatic profile about physical realities, but a multitude of conceptual profiles each associated with a particular conception. A paradigmatic profile covers all paradigmatic aspects distinguished in § 1.5, but each aspect is at different levels of maturity and complexity across various paradigms depending on the individual's personal experience. A conceptual profile covers all aspects outlined in the corresponding modeling schema, as well as underlying canons. Both types of profiles can be composed in accordance with Bachelard's taxonomy and broken along the lines of Figure 3.1. However, for practical purposes, we prefer to adopt a modified scheme discussed below.

A scientist's paradigmatic profile is dominated by relatively viable dimensions (CR, RR, DI in Fig. 3.1). The viability of each dimension (i.e., paradigm) is well-established by a concerned scientific community within well-defined scopes and limits of approximation, and various dimensions complement one another in specific respects. A physicist may use a Newtonian model (CR-type in Fig. 3.1) to study a typical translation represented in Fig. 2.6, and then shift to a relativistic model (RR) to study the same translation or a similar one, should s/he desire to significantly improve the precision of the outcomes. In contrast, paradigmatic profiles of ordinary people, science students included, do not have their scopes and limits of viability well delineated, and various paradigms often overlap in conflicting ways. A student might have recourse to a particular model with one particular instance of a given pattern (say, a positivist, PE model in Fig. 3.1), and then a contradictory model (say, a Newtonian, CR one) with another instance of the same pattern considered under the same rational and empirical conditions as before.

As Nagel (1979, p. 6) puts it, “the occurrence of conflicts between judgments is one of the stimuli to the development of science”; yet, in the realm of common sense, ordinary people entertain with ease and without reluctance “incompatible and inconsistent beliefs”, as well as “conflicting judgments” about all sorts of realities. This, according to Nagel, is “the result of an almost exclusive preoccupation with the immediate consequences and qualities of observed events [in our everyday life]. Much that passes as common-sense knowledge certainly is about the effects familiar things have upon matters that men happen to value; the relating of events to one another, independent of their incidence upon specific human concerns, is not systematically noticed and explored”.

Demarcation lines among various dimensions of a conceptual profile, and especially of a paradigmatic profile, are subsequently not as easy to delineate for ordinary people as for scientists. This is especially true between SC and PE which, together, form the foundations of what we call *naïve realism*, as well as among the three scientific dimensions CR, RR, and DI, on the right-hand side of Figure 3.1, and especially between RR and DI. It is thus more convenient for us, and even more realistic, to combine SC and PE together in one dimension, hereafter referred to as naïve realism (NR). It is more so with RR and DI, which we include in a single dimension, hereafter referred to as *modern scientific realism* (MR) and pertaining to all non-classical theories of so-called modern science (mainly with quantum and relativistic foundations). We prefer the label “realism” to “rationalism” for all scientific dimensions, including Bachelard’s CR which hereafter refers to *classical scientific realism* and pertains to all classical theories of science. For all scientific conceptions, and especially models, *correspond* to physical *realities*, and *represent* these realities in the sense discussed in the previous chapter, even though they are human conceptual *re*construction of such realities. Bachelard’s dimensions are thereby reduced to three dimensions, naïve realism, NR, classical scientific realism, CR and modern scientific realism, MR, with the realism of NR distinctively different from the *scientific realism* of CR and NR as we shall see in the next section.

The profile of an ordinary person, whether conceptual or paradigmatic, seldom includes an MR dimension, and when it does, it virtually never includes DI. Still, no scientific dimension, including

CR, can be as important for an ordinary person as it is for a scientist. Hence, while the profile of a scientist is dominated, to variable degrees, by the two broad scientific dimensions CR and MR, the profile of an ordinary person often consists of an unbalanced and incoherent mix of NR and CR (Figure 3.2). We shall hereafter refer to the latter mix: (a) as *common sense (CS) profile*, when there is some balance between NR and CR, and (b) as *naïve profile,* when it is dominated by NR. Similarly, a non-scientific natural paradigm will hereafter be referred to as *naïve paradigm* when dominated by NR, and as *common sense paradigm* when underlined more by CR than NR. A person who holds a naïve paradigmatic profile will be called *naïve realist* (§ 3.2).

What corresponds to a given science course in the dimensions of a student paradigmatic profile (mostly NR and CR) varies in content and size from one course to another, depending on the nature of the physical realities involved, and on student familiarity with these realities. The corresponding naïve realism dimension often consists of two parts. The first NR part corresponds to situations where the expressed naïve ideas may be locally coherent in the sense that they may allow apparently consistent inferences in closely related domains;

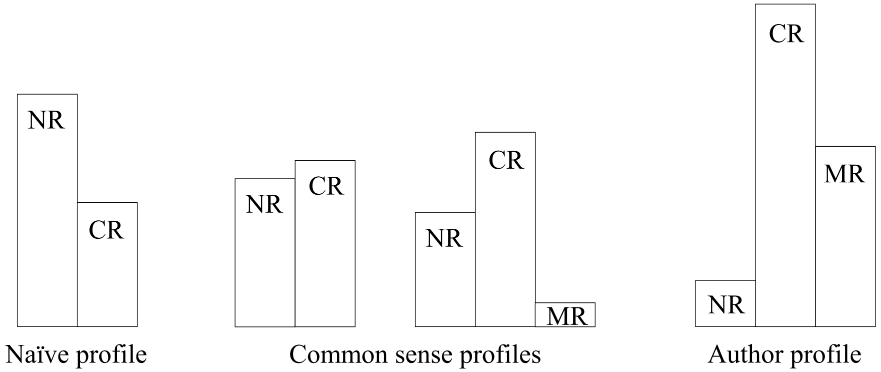


*Figure 3.2*. Paradigmatic profiles.

Bars are not to scale in the above bar charts, and bars' relative heights reflect an ordinal and not a proportional order of magnitude.

My own natural paradigmatic profile is currently dominated more by classical scientific realism (CR) than by modern scientific realism (MR) because my professional experience has so far been concerned more with CR than with MR. The naïve realism (NR) dimension is mostly about physical realities that are the object of scientific fields outside my domain of expertise, and which I casually contemplate.

these ideas may be considered as *viable* (with trepidation) when confined to these domains (Reif & Larkin, 1991). Some of the viable ideas might still be at the level of uncorroborated beliefs, while others could have already been corroborated in some respects, though insufficiently, in the student personal experience. The second NR part corresponds to situations where naïve realism could not apply under any circumstances, and where CR could be more appropriate from a scientist perspective. Like the first part, this one includes uncorroborated beliefs, as well as other ideas that appear to be duly corroborated in the student mind but whose claimed evidence is actually unreliable, or not conforming to accepted theory from a scientific perspective. The NR dimension is thus incoherent, and it often leads to inconsistent inferences and contradictions.

In contrast, the CR dimension of a student paradigmatic profile consists of ideas that are all relatively viable. The size of this dimension by correspondence to a given science course is evidently smaller for students than for scientists concerned with the content of the course, and the more remote the course is from everyday life, the smaller the dimension in question. Moreover, viability is not ascertained here as rigorously as discussed in § 2.7 and other parts of Chapter 2. A component of a student profile is considered "viable" only to the extent that it is closer to a scientific viewpoint rather than to a naïve perspective.

## 3.2 NAÏVE REALISM

Our research has shown that virtually every high school or college student has a natural paradigmatic profile that is a mix of naïve realism (NR) and classical realism (CR), and that virtually no student shows a profile that consists exclusively of one dimension or the other. It has also shown that students who consistently express some naïve philosophical tenets make up less than a quarter of the population in question (Halloun, 2001b; Halloun & Hestenes, 1998). Yet, when it comes to actual exploration of physical realities and problem solving in science courses, an overwhelming majority of students demonstrate naïve conceptions and procedures on numerous occasions (Driver & Erickson, 1983; Garnet, Tobin & Swingler, 1985; Halloun, 1986; Halloun & Hestenes, 1985b, 1995; Novak, 1987, 1993). Naïve paradigmatic profiles, i.e. profiles dominated more by NR than by CR, seem by-and-large to be more frequent than common sense paradigmatic profiles among high school and college students. At any rate, and according to the experiential knowledge tenet of § 1.2, naïve paradigms are there to affect student learning of science to various degrees. For this reason, they deserve our special attention.

Relative to a given science course, naïve paradigms vary from one student to another, whether in terms of the nature of viable ideas (viable in the sense discussed above), or in terms of the relative size of viable and non viable parts of the paradigm. There is no unique naïve paradigm that stands in contrast to a given scientific paradigm but to a multitude of such paradigms. Furthermore, no student demonstrates a paradigm that is as coherently structured or as systematically deployed as a scientific paradigm (§ 1.5). Nevertheless, various naïve paradigms predominantly share some common features that are next presented in this section. Common sense paradigmatic profiles share variably a number of those naïve characteristics, while they include more scientifically accepted features (mostly of classical nature) than their naïve counterparts.

A good proportion of naïve realists hold, in many respects, a positivist perspective on physical realities, and believe mistakenly that modern science does the same (Aikenhead, 1987; Cobern, 1993; Edmondson & Novak, 1993; Halloun, 2001b; Halloun & Hestenes, 1998; Songer & Linn, 1991). They believe, like Mach, that scientists do not admit the existence of any physical reality unless they can perceive it directly with their bare senses or with some instruments ("esse est percipi"). They also maintain that, contrary to the experiential knowledge tenet (§ 1.2), one should, and can, observe physical realities without any influence of prior knowledge in order to guarantee the objectivity of constructed knowledge. In this respect, they believe that scientists collect and analyze empirical data in an inductive Baconian approach, without any *a priori* hypotheses or any *a priori* judgment regarding primary and secondary details they need to concentrate on.

The ontological tenet about patterns discussed in § 1.6 takes different meanings and/or implications in naïve paradigms, and is sometimes missing altogether (Chi, Feltovich & Glaser, 1981; Halloun & Hestenes, 1998; Reif & Larkin, 1991). A few naïve realists hold that physical objects exist in the universe independently of one another, and that all properties of any given object are independent of other objects in the universe. The structure and behavior of physical realities are thus governed by local and not universal laws. Other naïve realists acknowledge the pertinence of patterns in the real world, but they seek them more in *objects* than phenomena (SC in Fig. 3.1), and thus more on the basis of *secondary-level morphological* and not behavioral isomorphism. At this level, people look for apparent features to establish the analogy between various physical realities, features often ascertained as secondary by scientists and ignored in their models. Furthermore, the analogy is established by comparison to a physical *prototype* and not by correspondence to a conceptual model like scientists (Giere, 1988; Lakoff, 1987). This is best reflected in problem solving.

Chi et al. (1981) have shown that when solving physics problems, students often invoke a problem that they have solved before and that involves *objects* (not phenomena) that have the same *shape* as the ones involved in a new problem. Subsequently, they apply the solution of the familiar problem to the new one, without checking for nomic isomorphism between the two problems. The set of objects in a familiar problem constitutes for those students a physical prototype with which they may associate a conceptual model that bears some secondary-level morphological isomorphism with the prototype. Once faced with a new problem, students check first not their repertoire of models but their repertoire of prototypes for one that best matches the situation at hand, and they then deploy the corresponding model for solving the problem. Sometimes, a prototype may ultimately serve students as a good instance for the development of an appropriate scientific model. However, and because classification criteria often vary between scientists and naïve realists, a mismatch often results between what scientists consider as an appropriate model for a given situation and what naïve realists may consider as an appropriate prototype for the situation.

At any rate, naïve realists believe that primary details of physical realities are exposed directly to our senses and that human knowledge, including scientific knowledge, mirrors the apparent world. They often ascertain that experiential knowledge about physical realities must be comprehensive and not selective, and that it must reflect all apparent details without exception. Consequently, they agglomerate conceptions meant to portray all available empirical data in an objectivist sense. These conceptions consist of prepositional statements and depictions that do not necessarily constitute a model in the sense discussed in this book, and they do not necessarily serve the same function attributed to a model (Gilbert, 1991; Grosslight, Unger & Lay, 1991; Moreira & Greca, 1995; Smit & Finegold, 1995). Nevertheless, and in order to systematize our discussion, we will loosely refer to any conceptual agglomeration shared by some naïve realists in parallel with a given scientific model as a naïve "model". Conceptions are also loosely agglomerated, though to a lesser extent, within common-sense paradigmatic profiles, the classical (CR) dimension included. We will also loosely refer to these agglomerations as common-sense (CS) "models".

*Naïve models* are incompatible with their scientific counterparts both internally, i.e., with respect to model composition and structure, and externally, i.e., with respect to model correspondence to the real world and its function there. Naïve models have much narrower scopes (domain and function) and are far less viable than scientific models. In contrast, *common sense models* demonstrate some compatibility with scientific models, internally and externally, but they are not as thorough, as coherent, or as systematic as the latter. They are obviously less viable than their scientific counterparts.

The domain of naïve models is localized in space and time, and their viability is restricted accordingly. For naïve realists, a model that applies to microscopic realities cannot apply to macroscopic realities, or one that applies to terrestrial objects cannot apply to other objects in the universe. Furthermore, for some of these students, the universe evolves in such a way that the laws governing the structure and behavior of physical realities change in time. Sometimes, and especially in physical sciences, there are students who hold an anti-realist position, believing that scientific models are fictitious mathematical models with no correspondence to the real world. This belief is unfortunately reinforced in conventional instruction that presents scientific conceptions with excessive mathematical formalism, and that concentrates more on specific mathematical routines for problem solving than on generic rules and processes of scientific theory construction and deployment.

The problem with the scope of naïve models is especially important at the functional level, and this mostly because of their lack of internal coherence. A naïve model typically consists of a loose bundle of conceptions that are mutually confused. For example, in Newtonian mechanics, students – including some of those who are not naïve realists – often confuse velocity with acceleration, and these two concepts with force and energy. Subsequently, they confuse between *state laws* that *describe* the motion of an object (commonly expressed in the so-called equations of motion), and *causal laws* that *explain* the motion (Newton's four laws of dynamics). A descriptive model is thus not discriminated from an explanatory model, and, in the realm of naïve realism, either model type is deemed appropriate to assume both descriptive and explanatory functions.

Naïve methodology lacks well-defined rules of engagement with physical realities. As mentioned above, naïve realists are often Baconian explorers. Yet, and perhaps because of their extreme positivist position regarding objectivity, they do not develop their assumed knowledge following systematic heuristics. They follow instead rules of thumb, and trial-and-error techniques, in order to identify all sorts of variables in a given situation, and relate them in ways dissociated from any reference to specific theories and corresponding models. The relationship is generally sought in a theoretical statement (mostly in the form of a formula or a mathematical equation), or in a set of statements, that apparently involves all identified variables, irrespective of the validity of the statement(s) to actually describe or explain the situation at hand.

Naïve realists fail to realize the limitations of their models mainly because they do not attempt to systematically evaluate these models, neither internally for coherence and consistency, nor externally by correspondence to the real world or by comparison to scientific models. When it comes to empirical and rational corroboration, naïve realists do not proceed in the manner discussed in § 2.7.1 and § 2.7.2. They are often content with *à posteriori* "verification" that a particular model can fulfil, from an idiosyncratic perspective, particular purposes in particular situations. The situation does not get any better in conventional classroom settings. Learning science takes place passively and it relies far more on traded knowledge than on experiential knowledge. Naïve students generally accept at face value what an authority (teacher or textbook) dictates to them, and they often end up with a set of beliefs about science rather than with reliable knowledge about physical realities.

The mismatch between student and scientist natural paradigms takes in many respects the form of a “clash of cultures”. For many students, and especially naïve realists, science looks like a foreign culture that is being forced upon them and that can only be met with resistance (Cobern, 1995). At best, students resign themselves to the authority of teacher and textbook and learn things by rote (mostly in the form of traded knowledge) only to satisfy curriculum requirements. They often end up with a sort of *cognitive dissonance* between what they learn in science courses and the way they interact with the physical realities of everyday life, a way that remains mostly driven by naïve realism. What naïve realists learn under conventional science instruction consists of a loose collection of theoretical statements and routines for problem solving, which these students memorize episodically, i.e., one after another, without a coherent big picture. The collection may be extensive, but it is so shaky that it can be described the way Mach described advanced science students in 1886:

> “I know of nothing more terrible than the poor creatures who have learned too much [passively, we might add]. Instead of that sound powerful judgment which would probably have grown up if they had learned nothing, their thoughts creep timidly and hypnotically after words, principles, and formulae, constantly by the same paths. What they have acquired is a spider’s web of thoughts too weak to furnish some supports, but complicated enough to produce confusion”. (Mach, 1986, p. 369).

## 3.3 PARADIGMATIC PROFILE EVOLUTION

Dissonance between scientific paradigms and their naïve or common sense counterparts extends from the generic tenets discussed in Chapter 1 to knowledge form and content. The dissonance is often so deep that no educational theory or schooling system can ever close the gap, especially between scientific and naïve realism. We do not claim that modeling theory can do so either. In fact, no formal education should even consider a radical paradigmatic evolution whereby secondary school or even college students transform their common sense or naïve paradigms entirely into scientific paradigms. A more reasonable credo is to transform naïve and common sense paradigmatic profiles, and not paradigms, into more viable profiles whereby the naïve dimensions (NR in Figure 3.2) would be significantly reduced in favor of the scientific dimensions (mainly CR in this figure). Eliminating naïve realism (NR) altogether from any person's paradigmatic profile would be a far-fetched target for at least three reasons.

First, naïve paradigms are intuitive and grounded in some successful personal experience. Unless one desires to become a scientist, it will be practically impossible to be convinced of the need to give up what apparently comes naturally and works conveniently. People usually get by in their everyday life by relying on their intuition. Intuitive conceptions, tools and processes seem to be reliable enough to answer routine questions about, and interact successfully with, common physical realities. The sun "rises" every morning and "sets" every evening in a manner that could not affect ordinary people's lives differently if they were to think that this is only an apparent motion of the sun. Practically speaking, the heavier an object is, the harder one needs to push or pull in order to move the object on a rough surface; so what difference would it make for an ordinary person to think that it is friction and not weight that one needs to overcome? Harmful bacteria and viruses could cause illness alike, and should be treated by medications that only physicians know of and can legally prescribe; so why would a layperson bother learning about differences between these two sorts of organisms? All in all, typical daily experience with physical realities does not usually present people with situations that cannot be *conveniently* solved within the confinements of naïve realism. As such, an ordinary person is seldom faced with the kind of crisis Kuhn (1970) speaks about in order to consider giving up naïve paradigms in favor of scientific alternatives.

Unless people become accustomed to asking questions and seeking answers differently, beginning at an early age, it will be practically impossible to convince them to give up their naïve paradigms altogether by the time they get to college, or even to secondary school. By then, we can open students' eyes to asking questions about daily situations and seeking answers in ways that are *more reliable and more efficient* than what they are used to, and only hope that they would be motivated enough to reconsider their own paradigms in specific respects that touch their personal lives. Otherwise, scientific paradigms will remain part of an irrelevant foreign culture.

One would then ask why should people even bother to reconsider their convenient paradigmatic profiles, and then why should they bother taking science courses in the first place. The answer is that science is nowadays as much a personal necessity as it is a universal one. Science touches all aspects of our lives, at least indirectly through all sorts of equipment we use daily and through decisions related to science and technology that we need to contribute to, or at least abide by, at the socio-political level. Some scientific conceptions and processes are necessary to use technology safely and efficiently, as well as to conduct one's life with an understanding of science-related regulations and with a commitment to continuously improve such regulations. At the broader level, the world needs science for human progress. Scientists are not born as such, and without formal exposure to science, no one would wake up one day and decide to become a scientist just out of the blue. It is thus a moral obligation, a humane one, for our educational systems to give students the chance to assess whether they could ultimately make valuable contributions to human progress, and thus opt for a profession in or related to science. In this respect, even the most liberal educational systems should make it mandatory for students to complete some basic science courses before they graduate from secondary school. When I used to teach introductory college physics courses for non science majors, no semester would pass by without having at least one student coming to me by the end of the semester and telling me that should s/he have taken physics before, s/he would have opted for a scientific major instead. Many of those students actually changed their majors in this direction.

Second, scientific paradigms are not required in their integrity for ordinary people to take advantage of science in their life (not even classical realism, CR, in Figures 3.1 and 3.2), not even if they choose science as a way of life (but not necessarily as a profession). Dewey was absolutely right in ascertaining that our education needs to "mature and make effective" the scientific habits of mind in our everyday life. Yet he was also realistic in calling not to turn every person into a scientist, but to "transform" an ordinary person's habits of mind "after the pattern" of science. This transformation calls for a *relative alignment* of some fundamental intuitive habits, beliefs and knowledge with generic tenets of science (§ 1.6 – § 1.8), and not for a complete transformation of a person's conceptual world into scientific paradigms.

Third, scientific paradigms are so much involved, and in some respects so much counterintuitive, especially those of modern scientific realism (MR), that no one scientific paradigm can be fully developed by the end of schooling years. In fact, it takes prospective scientists long years of post-graduation practice to overcome naïve and common sense realisms to a significant degree but not entirely (Fig. 3.2), and develop respective scientific paradigms comprehensively.

These reasons and others make it impossible, from our point of view, to completely skew a person's paradigmatic profile in one direction or another. As suggested by Bachelard (1940) and Mortimer (1995), educators should instead concentrate on: (a) making students realize the limitations of the naïve part of their profile, and thus (b) the necessity to build up the viable counterpart in the scientific direction. Modeling theory calls for an evolution of students' paradigmatic profiles along these lines, an evolution that significantly reduces the naïve realism dimension and that builds up to realistic levels the classical and modern scientific realism dimensions (Fig. 3.2). These levels, as we shall argue next, correspond to basic models in any scientific theory that is the object of instruction.

## 3.4 PARADIGMATIC THRESHOLD

The gap between naïve realism and scientific realism may be so deep, and the amount of knowledge in any scientific field of which ordinary people are usually unaware may be so extensive that, in the words of Viau (1994), we "must learn to compromise [in quantity, not quality]… We need a model of education in which performance is not central, in which information is not central, and in which thought, and even wisdom, are." This model, according to Viau, is one that concentrates, from "the first few years of schooling", on the construction and use of models on which all sorts of reasoning draw, from "comparison and classification" to "metaphoric and allegorical thinking". "Students need to learn and practice the use of models… to think critically about them", so that they can: process information about the real world, make sense of it, and "bring coherence to the chaos of data stream"; organize information and experiences; bring knowledge to life and give it relevance, utility, and purpose; "see knowledge as something that empowers them – to do, to produce, to create". It "behoves us to ensure that our future citizens have a wide repertoire of possible models to choose from and a critical approach to any model that is chosen, no matter how attractive it may seem" (Viau, 1994).

All major reform movements in science and mathematics education have lately emphasized the importance of *models* for coherent knowledge organization and of *modeling* for the development of scientific methodology (AAAS, 1990, 1993; AMATYC, 1995; NCTM, 1989, 1991; NRC, 1996; NSTA, 1995):

- Models are considered as "fundamental and comprehensive… unifying" structures (NRC, 1996, p. 115) that "transcend disciplinary boundaries and prove fruitful" in all scientific, technological and mathematical enterprises (AAAS, 1990, p. 165).
- Models have proven to facilitate "efficient entry" into the realm of science and mathematics and, subsequently, meaningful learning of such disciplines (Casti, 1989; Clement, 1989, 1993; Erduran, 2001; Giere, 1992; Gilbert, 1991; Glas, 2002; Halloun, 1984, 1998a, 2000; Hestenes, 1992, 1997; Justi & Gilbert, 2002; Moreira & Greca, 1995; Nersessian, 1995; Passmore and Stewart, 2002; Shore et al., 1992; Smit & Finegold 1995; Steen, 1990).
- Models can become "pedagogical tools" that provide learners with suitable contexts for expressing, exploring and refining their own knowledge (Bullock, 1979; Doerr, 1996; Gee, 1978; Hafner & Stewart, 1995; Halloun, 1998b, 2000; Redish, 1994; White, 1993).
- Modeling is pivotal in problem solving (NCTM, 1991) as well as in all sorts of educational activities (Clement, 1989; Halloun, 1984, 1994, 1996; Halloun & Hestenes, 1987; Hestenes, 1992; Nersessian, 1995).

Modeling theory, the way we see it, accounts for all the above in its drive for paradigmatic profile evolution in the sense discussed. In this direction, we undertake a program of reasonable expectations. The program aims at helping every secondary school or college science students to reach *at least* what we call a *paradigmatic threshold* by the time s/he achieves a well-designed course of science.

The threshold corresponds in a given science course to the development and successful deployment of *basic models* in the scientific theory that is the object of the course. We advocate this threshold because our experience suggests that, in any scientific theory, *basic models constitute the minimum requirement for meaningful and equitable learning experience*.

As discussed in § 2.6, basic models are usually the simplest models of the theory, both from scientific and cognitive perspectives. They offer the context required to develop meaningfully all lower-level conceptions of the theory (concepts, laws and other theoretical statements) and required tools. They relate these conceptions to the real world and to one another in coherent structures. Most importantly, basic models foster efficient knowledge growth and development of scientific "habits of mind" called for by Dewey and other reformists, along with all sorts of reasoning and inquiry skills associated with scientific thinking, from induction and deduction to the use of analogies and metaphors.

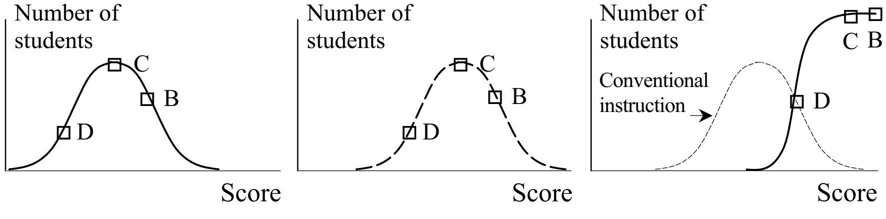
The paradigmatic threshold is what we consider a *minimum* competence level required to enable students to cross the kind of barrier discussed by Margolis (1993). Margolis (1993, p. 41) argued that a Kuhnian paradigm shift took place in the history of science every time scientists were able to cross a particular barrier that "is *unique* in the sense that a person competent to make the discovery who somehow gets beyond that barrier is likely to go on to make the discovery, and not so otherwise". The barrier in question may be one of two sorts. The first is a "habit of mind that is both highly robust and critical for the emergence of a new idea" (*ibid*, p. 32). Kepler crossed such a barrier when he dispensed with "the long-standing commitment of astronomy to the principle of uniform circular motion" (*ibid*, p. 33). The other sort of barrier is one that comes "as a consequence of the absence of a facilitating habit of mind" (*ibid*, p. 36). In a given science course, such a barrier can be associated with the generic laws of the theory that is the object of the course (Fig. 2.5). For example, in classical mechanics, the barrier may be primarily associated with student beliefs that: (a) position (rather than velocity) is the descriptor that characterizes the mechanical state of an object, and a force would thus be required every time the object changed its position, and that (b) two bodies of different mass or size exchange forces of different magnitudes. To cross such a barrier, students need to develop meaningful understanding of Newton's four laws of dynamics, which may only occur after they develop the basic models of Figure 2.6. Once they cross the barrier, students will be ready to autonomously undertake meaningful transformation of their paradigmatic profiles so that these profiles become significantly dominated by scientific realism rather than by naïve realism.

The paradigmatic evolution we aim at is *realistic* in the sense that its threshold is attainable by any student willing to make the necessary effort (Halloun, 1984, 1996, 2001a; Halloun & Hestenes, 1987). Under our program, science education becomes an *efficient* and *equitable* enterprise. It satisfies the "less is more" philosophy advocated long ago by Philip Morrison, by eliminating noise and redundancies in any course while maintaining reasonable coverage breadth. It thus resolves the long-standing issue of topics coverage under such a philosophy, the issue that "cognitive need for systematic 'in-depth' coverage of a few science topics is [seen to be] at odds with the conventional 'in-breadth' coverage of many science topics" (Eylon and Linn, 1988).

Because of individual differences in their initial paradigmatic profiles as well as in affective factors that control the learning process, we cannot expect to bring all students in any science course to the same achievement level by the end of the course. However, they must all reach at least the threshold in question so that the Deweyan transformation of mental habits becomes possible. Once students reach this level, they are able to develop more complex models and corresponding paradigmatic requirements, more effectively than before, and with less teacher assistance. Student evolution into the realm of science does not only become possible then, but efficiently manageable, and realizable with an exponential course.

The lack of efficiency, and especially of equity, has long been a downside of science education. The ranking of students in a science course is virtually the same before and after conventional instruction. Figure 3.3 shows a typical normalized distribution of student competence levels at the beginning (Fig. 3.3.a) and at the end (Fig. 3.3.b) of such a course as measured by parallel pretests and posttests assessing basic student understanding of course-related materials. The figure shows that pretest and posttest scores usually correlate with one another to a high level of statistical significance. A student D who begins the course with a low competence level usually finishes it by conserving her/his poor standing with respect to the rest of the class. The same is true for an average competence student (C) or a high competence student (B). Conventional instruction thus shifts the entire class by about the same amount roughly equal to the change in the class average, while preserving the relative competence of individual students (i.e., the class standard deviation remains practically unchanged). By the end of a conventional course, only high competence students (B and above) reach or cross the paradigmatic threshold that we are promoting. That is roughly at the level of student B in Figure 3.3.b. Rarely an average student and virtually never a low competence student can do so. Equitable instruction should turn the situation around into what Figure 3.3.c roughly shows and what modeling instruction actually does (Halloun, 1984, 1996, 1998b, 2004; Halloun & Hestenes, 1987).

Our drive for equity and our conviction that science is learnable by anyone willing to invest the necessary effort do not imply that all students in a given science course can be expected to reach exactly the same level by the end of the course. Many cognitive and affective factors control the learning process (§ 3.7), and no educational theory can ever claim to provide sufficient control for all these factors. There will always be differences among students in these respects, after instruction as well as before. Despite these differences, though, we maintain that our paradigmatic threshold can be attainable by all willing students. Our index of equity is set at this threshold, while we acknowledge that there could always be some non-controllable factors that would leave one or two students a little behind (Fig. 3.3.c). We also acknowledge that there will always be some students who can



(a) Before instruction (b) After conventional instruction (c) After equitable instruction

*Figure 3.3.* Performance in science courses following two different types of instruction.

easily align their own natural paradigms with the corresponding scientific paradigm, and who can exceed by far the threshold in question. Our drive for equity is not at the detriment of these advanced students. They are not only afforded to reach higher levels of achievement under modeling instruction than under conventional instruction, but also to exceed their peers to whatever attainable levels (*op. cit.*).

## 3.5 FROM MIXED BELIEFS ABOUT SCIENCE TO RELIABLE KNOWLEDGE ABOUT PHYSICAL REALITIES

The paradigmatic threshold we are advocating in a given science course is about both content and processes. At the epistemological level of knowledge content and structure, it is about coherent organization of scientific knowledge around basic models of the theory of concern. At the level of methodology, it is about the development of necessary tools and processes for scientific inquiry (both exploratory and inventive), mainly as conducted in the construction and deployment of these models. In contrast, conventional science instruction results in piecemeal epistemology and reproductive methodology. At the content level, concept definitions, laws and other theoretical statements are stated *episodically*, i.e., one isolated conception after another with little coherent structure, if any, which in no respect comes close to our modeling schemata. At the process level, conventional instruction concentrates on limited aspects of model deployment, and it virtually ignores model construction. Various theoretical statements are *delivered* or *transmitted* to students for memorization. Students are not afforded the chance to actually construct these statements, whether empirically or rationally, not even in the laboratory. Conventional laboratory experiments are done separately from the corresponding "course", and they are often meant to *verify* not to construct theoretical statements delivered in the course. As for model deployment, it is limited to routine applications of isolated theoretical statements, not of models *per se*, and this only in the resolution of specific paper-and-pencil exercises. The exercises are frequently about situations that appear to be fictitious and unrelated to the real world, and their resolution is usually done following some cookbook recipes.

In conventional instruction, students are normally conditioned to verbally *re*produce theoretical statements and heuristics in situations typical of, if not identical to, the ones discussed in class. The whole learning experience is primarily about inscription of traded knowledge (§ 1.3) in student minds, mostly in the short-term memory, and seldom about formation of experiential knowledge (§ 1.2). In the process, the student "learns about the real world from an intellectual distance, by reading about it", and is being filled "with information *about* the world, information that, in true written-word fashion, is removed from its context, at least to some extent, and represented rather than experienced directly" (Viau, 1994). Furthermore, students are seldom afforded the chance to go through a reflective experience whereby they could relate what they were told and shown in class to their own paradigms, and subsequently regulate, *à la* Dewey or *à la* Piaget, whatever "conflict" that might emerge in the process. As a consequence, students usually memorize by rote various theoretical statements and related problem-solving routines with the only interest of passing course exams rather than learning something that could be personally relevant and meaningful. Their naïve realism remains entrenched in their minds, and they keep resorting to it in their everyday life and not to what they learned in class, thus ending up with the state of cognitive dissonance referred to in § 3.2.

Science remains to secondary school and college students, especially non-science majors, an irrelevant foreign culture. Whatever novelties they learn under conventional instruction consist more of *beliefs* about this culture than of reliable knowledge about physical realities. For, and as many an educator has constantly pointed out, students "accept theories on the authority of teacher and text, not because of evidence" Kuhn (1970, p. 80). Our research has shown that this is actually the case with virtually all naïve students and some others. It has also shown that students bring along to their science courses mixed ideas about physical realities (vague, confused ideas, often inclined toward naïve realism), and that students seldom explicitly compare these ideas to what the authority dictates so that they get them regulated and transformed into scientific knowledge. These students believe that, as presented in conventional courses, science is mostly unrelated to physical realities they encounter in their personal lives, and that it is good only for those who would like to become scientists and live in a different mental world (Halloun, 2001b; Halloun & Hestenes, 1998). Most naïve students, as well as others, remain then below the promoted paradigmatic threshold (Fig. 3.3.b), with their original paradigmatic profiles virtually unfettered.

A major target in modeling theory is to reverse the situation and help students to develop not beliefs about science but reliable knowledge about physical realities, knowledge that is personally relevant. This entails developing all sorts of conceptual structures and processes that are necessary for student paradigmatic evolution more in the form of experiential than traded knowledge. To begin with, the objective is first to help individual students to distinguish in their own conceptual world between viable and naïve ideas. It is then to help them: (a) consolidate viable ideas and reinforce them empirically and rationally, and (b) regulate naïve ones, so that all conceptual structures and processes be transformed into reliable knowledge. The *transformation* of existing ideas is by itself insufficient to realize our goal. The targeted paradigmatic evolution also calls for the simultaneous *formation* of novel knowledge, i.e. of some scientific knowledge that has no match in student preexistent conceptual world. A major *learning tenet* in modeling theory is then:

> *Paradigmatic evolution involves* ***transformation*** *of existing constituents of a person's initial paradigmatic profile, as well as* ***formation*** *of new paradigmatic constituents. Transformation extends from the refinement to the rejection and replacement of existing conceptual structures and processes.*

Paradigmatic evolution then proceeds in a variety of directions depending on the state of a student's paradigmatic profile. From a scientific perspective, a particular component of a student profile, be it a tenet, a conception, a tool or a process, can be in one of six forms. These are:

1. *Naïve belief*, i.e., an uncorroborated constituent of a naïve paradigm that is, in many respects, at odds with science. For example, naïve realists "believe" that scientists accept the existence of a physical object only after they detect it directly with their bare senses or with some instruments. Naïve beliefs can be regulated and transformed into reliable knowledge based belief, for example, on some counter-evidence offered directly in the empirical world and/or through scientific documentations.

2. *Naïve knowledge*, i.e., knowledge at odds with science and developed based on some unreliable evidence or on some misinterpretation of empirical observations, like the idea that an object falls to earth because the air pushes it down. Naïve knowledge is a bit more involved than naïve beliefs and cannot be treated as easily. Unlike beliefs, naïve knowledge is developed through experience with physical realities, which makes it more deeply rooted in people's minds and thus harder to treat.
3. *Viable belief*, i.e., an uncorroborated idea that is, to a large extent, in agreement with scientific knowledge (mainly of CR type in Figs. 3.1 and 3.2). Such a belief needs to be backed with empirical evidence in order to be transformed into reliable knowledge.
4. *Viable knowledge* that is, to a large extent, in agreement with scientific knowledge (mainly CR), and backed by some form of reliable evidence.

The reader is reminded that, when viable, a student belief or piece of knowledge does not necessarily meet the viability criteria discussed in Chapter 2 as rigorously as scientific knowledge. When it comes to student paradigms, viability is relative and delineated by contrast to naïveté. Naïve beliefs and knowledge, often referred to in the literature as "misconceptions", need to be "replaced" by other ideas. In contrast, viable ideas may need only to be "refined", modified in some respects and reinforced with more empirical evidence.

5. *Missing, derivable knowledge*. This is scientific knowledge that has no match in students' minds, but that can be entirely constructed from preexisting viable ideas. This is for example the case of constructing a new concept from preexisting concepts, like the concept of acceleration in physics from the concepts of velocity and time.
6. *Missing, prime knowledge*. This is scientific knowledge that has no match in students' minds, and cannot be constructed only from preexisting viable ideas. This is, for example, the case of learning quantum mechanics. While missing derivable knowledge can be developed through processes similar to those involved in the transformation of viable ideas, missing prime knowledge has higher paradigmatic requirements.

The three broad categories, naïve ideas (belief or knowledge), viable ideas and missing knowledge, differ significantly from one another and require different approaches in education. The same goes for the two subcategories within each. The transformation of naïve ideas into scientific knowledge, or more generally of a naïve paradigm into a scientific paradigm, follows what Kuhn (1970) calls a "scientific revolution" whereby an existing paradigm "in crisis" is *replaced* by a new one (but not necessarily along exactly the same lines discussed by Kuhn). The transformation of viable ideas and development of missing, derivable knowledge require the sort of activities involved in Kuhnian "normal science". Development of missing prime knowledge has similar requirements to the development of a new scientific paradigm that has no rivals in science.

According to Kuhn (1970): " 'normal science' means research firmly based upon one or more past scientific achievements, achievements that some particular scientific community acknowledges for a time as supplying the foundation for its further practice [while being] sufficiently open-ended to leave all sorts of problems for the redefined group of parishioners to resolve" (p.10). Three "classes of problems – determination of significant facts [i.e., those "revealing the nature of things"], matching of facts with theory [i.e., those that "can be compared directly with predictions from the paradigm theory"], and [empirical] articulation of theory – exhaust… the literature of normal science, both empirical and theoretical" (*ibid,* pp. 25-34). The "results gained in normal research are significant because they add to the scope and precision with which the paradigm can be applied" (*ibid,* p. 36).

In contrast, when "an anomaly comes to seem more than just another puzzle of normal science" [i.e., a game of "paradigm-nature fit"] and "to evoke crisis", a scientific revolution characterized by the "transition to a new paradigm" becomes inevitable (Kuhn, 1970, pp. 82, 90). A scientific revolution is characterized by a "community's *rejection* of one-time honored *scientific theory* in favor of another *incompatible* with it." (e.g., revolutions associated with Copernicus, Newton, Lavoisier and Einstein). It produces a "shift in the problems available for scientific scrutiny and in the *standards* by which the profession determined what should count as an *admissible problem* or as a *legitimate problem-solution*" (*ibid*, p.6, italics added). It requires "a change in the rules of the game" (*ibid,* p. 40), a "reconstruction of the field from new fundamentals, a reconstruction that changes some of the field's most elementary theoretical generalizations as well as many of its paradigm methods and applications." (*ibid,* p. 85).

On another account, "a new theory does not have to conflict with any of its predecessors. It might deal exclusively with phenomena not previously known, as the quantum theory deals (but, significantly, not exclusively) with subatomic phenomena unknown before the twentieth century. Or again, the new theory might be simply a higher-level theory than those known before, one that linked together a whole group of lower level theories without substantially changing any… In the evolution of science new knowledge would replace ignorance rather than replace knowledge of another and incompatible sort." (Kuhn, 1970, p. 95).

The content of any science course is by and large missing (mostly as prime knowledge) in students' initial knowledge state, and most of what students think they know about the course actually consists of mixed beliefs, naïve or viable, about science and to a lesser extent about physical realities. The relative amount of knowledge and the proportion of naïve knowledge to viable knowledge vary from course to course, and from one student to another in a given course.

At first sight, one may be driven to conclude from all the above that the nature, the level and the amount of various learning activities required for student paradigmatic evolution are so diverse that no one teacher could ever handle them alone in a science classroom. This would be true if we were to opt for some sort of personalized instruction, or if we were to divide a science class into homogeneous groups of students, with students in each group sharing the same ideas. These two approaches, as research has long shown, are neither practical nor viable. Educational research has actually shown that the underlying canons of naïve realism are more or less homogeneous across student populations, and that naïve conceptions and processes pertaining to a given science course can be classified in a limited number of categories (Halloun & Hestenes, 1985b, 1998). This makes it feasible to find a teaching approach that can be implemented by a single teacher and that allows every student to succeed in all the knowledge transformation and formation that are necessary for reaching at least the paradigmatic threshold that we are promoting in any science course.

Modeling theory suggests that such an approach would be determined primarily: (a) by the content of basic models and corresponding epistemological and methodological requirements, and (b) by the initial state of students' paradigmatic profiles. The theory then tells us that the paradigmatic profile evolution called for requires that students get engaged in *rational-empirical dialectics* that help them to assess their own paradigmatic profiles and regulate them in an insightful manner (§ 3.6). It also tells us that for such dialectics to be effective and efficient, science teachers need to control, to the extent that is possible, effects that have a direct impact on the learning experience (§ 3.7), but, most importantly, that students need to be situated in a *structured learning environment* that keeps student dialectics in line with scientific discourse and inquiry (§ 3.8). These premises are discussed in the following three sections. Their practical implications for modeling instruction are discussed in the next two chapters.

## 3.6 INSIGHTFUL REGULATION

Science educators have been drawing recently on what they consider to be points of convergence between cognitive science and philosophy of science. One major point of convergence emerges from similarities between processes involved in the evolution of human knowledge and those involved in the evolution of scientific theory. What many cognitive scientists call *weak restructuring* is seen to be in line with Kuhn's normal science, and what the former call *radical restructuring* is seen to converge with Kuhn's revolutionary science. "Weak restructuring results in the accumulation of new facts and the formation of new relations between existing concepts. In contrast, radical restructuring consists of a change in core concepts, structure and phenomena to be explained" (Duschl, Hamilton & Grandy, 1990).

Many historians and philosophers of science have long argued that major developments have taken place in science during crises that resulted in the kind of paradigmatic shifts discussed by Kuhn. Similarly, many educators have argued, along with Holton (1993, p. 162), that the "increase in awareness of internal contradictions in a world picture, brought about by external stress, can provide the *opportunity for the most effective educating intervention* to take place". In this regard, and following the footsteps of Dewey and Piaget, science teachers are being called upon to confront their students with rational or empirical situations that stir up some "conflict" or "cognitive disequilibrium" in students' minds, so that they are incited to reconsider and regulate those ideas appropriately (Siegler, 1978).

A major condition for cognitive disequilibrium to be brought about meaningfully is that students need to admit beforehand the falsifiability of their own ideas, just like scientists do. According to Wartofsky (1968, pp. 66, 67), "Common sense itself may be said not to be aware of 'incompatible beliefs' or 'inconsistencies' within its own structure, precisely because it is so largely tacit. Such incompatibility becomes apparent only upon critical reflection. Such reflection therefore requires some notion of systematicity, of how the various proposals of common sense bear on each other, of how one concept relates to another, or of how one judgment entails another. What further needs to be made explicit are such systematic criteria as consistency and noncontradiction… The most common difference between science and common sense lies in the explicitness and the refutability of the scientific proposition and in the aim of science to be consciously and deliberately critical as a matter of course."

Kuhn argues that revolutionary changes throughout the history of science are characterized by the total rejection of some existing paradigm. This may be especially the case of students with naïve paradigms. According to Kuhn (1970, p. 77), the "decision to reject one paradigm is always simultaneously the decision to accept another, and the judgment leading to that decision involves the comparison of both paradigms with nature *and* with each other". The latter comparison is indeed required of all science students, irrespective of the level of transformation and/or formation processes that they need to go through. Depending on the initial state of a student paradigmatic profile, paradigmatic profile evolution may involve anything from a fine-tuning of some original paradigmatic components, and/or formation of new components, to a radical change of naïve components. Either way, the process obeys the following tenet about *regulatory dialectics*, a tenet that is in fact a corollary of the epistemic tenet discussed in § 1.2:

> *Meaningful paradigmatic evolution results from insightful regulatory dialectics. Some of these dialectics pertain to intrinsic negotiations among various components of one's*

> *own paradigmatic profile. Others pertain to extrinsic negotiations with empirical data and with other people's paradigms, especially scientific paradigms.*

Figure 3.4 depicts the three "negotiation" or assessment modes in which science students need to get engaged accordingly. A student paradigmatic profile needs to be assessed in all three directions (coherence, correspondence and commensurability) across all its dimensions (Fig. 3.2), but this is especially needed for the dimension or paradigm dominated by naïve realism. That is why we concentrate on naïve paradigms in the rest of this section. One of the three modes is an intrinsic *rational* negotiation, an assessment of internal *coherence* of a given naïve (or even common sense) paradigm. The other two are extrinsic negotiations. One involves an *empirical* assessment of *correspondence* of a student paradigm to physical realities. Another involves a *rational* assessment of *commensurability* between the student paradigm and the corresponding scientific paradigm. Depending on whether assessed paradigmatic components are originally viable, naïve or missing, a negotiation of any type may result respectively in the reinforcement, modification or replacement of existing paradigmatic components, and/or the construction of new ones. The outcome, in other words, consists of the possible transformation of viable or naïve components and/or the formation of missing ones.

Before we discuss the three types of assessment depicted in Figure 3.4, let us point out the importance that all negotiations they involve be *insightful* as stated in the tenet above. To paraphrase the definition of Feuerstein, Rand, Hoffman and Miller (1980, pp. 278, 279), insight is here viewed as an internally generated feedback

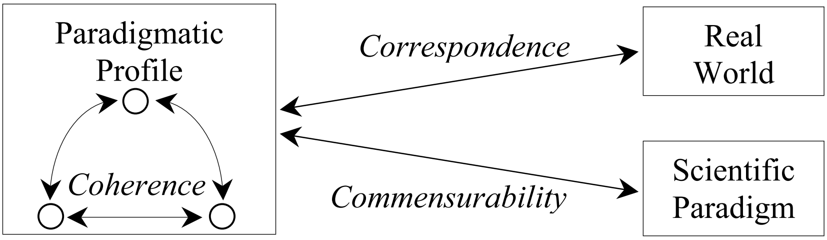


*Figure 3.4*. Rational-empirical dialectics for the evolution from the realm of naïve realism or common sense to the realm of science.

mechanism whereby a learner, through rational and empirical tests, can realize the limitations of her/his own conceptual structures or processes, the sources of error when committed, and the advantages of scientific alternatives. We shall come back to this point later in this section. However, let us point out here that the sort of assessment and insight that we are promoting constitutes a major deviation from ways of assessment in everyday life. Because of that, and because people are normally comfortable with their naïve paradigms (§ 3.3), students may meet our approach with some resistance. "In everyday life, errors are usually well enough prevented and corrected by informal means, e.g., by noticing unsatisfactory results or by heeding comments from other people. Many errors are regarded as mere slips because they do not usually cause much trouble. Thus they are remedied if necessary, but usually not examined at greater length" (Reif & Larkin, 1991). Students' resistance can be tamed by helping them to shift in evaluation norms and processes gradually and not abruptly in the more serious and systematic direction that we are promoting.

*Coherence assessment* involves rational dialectics among various components of a paradigmatic profile, especially when of common sense nature, in the absence of any external input, be it empirical or rational, in order to resolve any mutual incongruence. It is a purely reflective process in one's own rational world that involves mainly the comparison of various theoretical statements with one another within a given dimension of the paradigmatic profile (e.g., naïve realism dimension) and with parallel or related statements that might exist in another dimension of the profile. This sort of assessment is enhanced, especially in the physical sciences, with the use of appropriate mathematical models whereby the relationships in question can be conveniently represented and manipulated.

Coherence assessment is usually productive when one is negotiating among personal viable ideas, or between naïve ideas and viable counterparts in one's own paradigmatic profile. This sort of assessment goes often in vain with naïve realists when they negotiate exclusively within the naïve dimension of the profile. In such an event, i.e., when one's own profile is strictly a naïve profile, negotiations with the outside world become necessary to allow an evolution into the realm of science.

*Correspondence assessment* involves empirical dialectics between components of a given paradigm and corresponding physical realities.

It is meant in principle to corroborate paradigmatic components, i.e. to reliably establish their validity for exploring what they are supposed to refer to in the real world. In the case of a particular model, correspondence assessment is meant to assess whether the model is valid for *describing* or *explaining* a given pattern in the structure or behavior of certain physical realities. This external form of negotiation always starts by asking specific questions about particular realities assumed to manifest the pattern that the model is supposed to represent. Appropriate descriptive or explanatory predictions (hypothetical answers) are next made using the model in question within the framework of the corresponding theory and paradigm. The predictions are then compared to what actually takes place in the real world, and appropriate judgments are made as to the viability of the assessed model and required paradigmatic changes, if any. Similar measures are taken when assessing lower level conceptions (especially concepts and laws) within the framework of a particular model.

When a student possesses a viable conception that has no major flaws from a scientific perspective, correspondence assessment begins by establishing the viability of the conception within its own reference class. To this end students are presented with a variety of real world situations, some of which fall within the scope of the conception and others not. A physical reality is said to be within the scope of the conception if specific aspects of it can actually be described or explained by this conception to an acceptable degree of precision. In such a corroboration exercise, students need first to determine whether the conception actually corresponds to a given physical reality, and, if so, use the conception to predict specific aspects in the structure or behavior of this reality. Students then compare their prediction to the actual state of things, refine the conception (in minor details) if needed, and decide to hold it for subsequent corroboration exercises discussed later in this section.

When a conception is of limited viability, i.e., when it presents some flaws but remains significantly closer to scientific realism than to naïve realism, correspondence assessment serves to pinpoint the limits of viability of the conception and to properly *modify* it so as to widen its scope and/or enhance its viability. This is for instance the case when a student mistakenly believes, based on everyday observations of objects falling in air, that objects of the same mass and different shapes are subject to gravitational interactions of different magnitudes. The student needs then to realize that, say, a flat sheet of paper falls "slower" than an identical sheet crumpled into a lump not because of different gravitational pulls but because of different drag forces exerted by the air. In such cases, students need to be presented with empirical situations where the conception allows good predictions and confronted with others where it does not. As Smith, diSessa and Roschelle (1993) argue, students with conceptions of limited viability need to be "encouraged to consider the limits of their conceptions" without denying the validity of these conceptions, and to become engaged in activities that allow them to use what they "already know in more general and powerful ways" and learn "where and why pieces of knowledge that are conceptually correct may only work in more restricted contexts".

When a student possesses a naïve idea, correspondence assessment serves to *replace* the conception in question with its scientific counterpart. This is the case when a student believes for example that terrestrial objects fall only because air pushes them down from above (Halloun & Hestenes, 1985a). Students holding this belief are *wrong* in at least two counts. First, they actually do attribute the gravitational pull to air and not to earth (wrong agent). Second, they consider air to be conducive of motion instead of being a resistive agent as it actually is in this situation (wrong direction of air drag). In such cases, students need to be confronted with empirical situations where the naïve idea leads to inconsistent and contradictory predictions so that they realize the need to consider an alternative one, and become involved in a process of *conceptual change* similar to the one described by Posner, Strike, Hewson and Gertzog (1982).

In the case of naïve realism, correspondence assessment should result in a radical shift, often a paradigm shift similar to the one involved in scientific revolutions (Kuhn, 1970). It entails a complete "change in the rules of the game" and reconstruction of the naïve paradigm "from new fundamentals", and thus a change in the nature of questions naïve realists need to ask about physical realities and of the way they seek their answers. For example, in the case of moving objects that are the object of Newtonian mechanics, naïve realists are accustomed to asking the question: "what *maintains* an object in motion?", be it linear or not, uniform or accelerated, and to seek answers more from within the moving object than from its environment. That is how most of these students come to think that an intrinsic power, an impetus, maintains all sorts of motion, including linear uniform motion that actually requires no cause to be maintained (Halloun, 1986; Halloun & Hestenes, 1985a). The question naïve realists need to ask is: “what *changes* the velocity of an object?”, whether in direction or magnitude, and the answer they need to seek is only in external causes coming from the environment of the object.

Naïve realists may not easily undertake this radical shift. Students who have lived comfortably with their naïve paradigms cannot be expected to give up easily on them. In fact, naïve realists often cling fervently and blindly to their ideas. When faced with empirical evidence that flatly contradicts a particular idea, they often tend not to reconsider their own idea, but to reject the evidence as forged or hiding some primary aspects so as to render it unrelated to the questionable idea (Halloun & Hestenes, 1985b).

*Commensurability assessment* involves rational dialectics between one’s own paradigmatic profile, especially its naïve components, and a corresponding scientific paradigm. It aims at assessing whether naïve paradigmatic components are “commensurable” with their scientific counterparts, i.e. whether the two sorts are “compatible in a measurable way”. For objective and reliable comparison from a scientific perspective requires that it be conducted quantitatively with specific degrees of precision. This is perhaps what distinguishes science most from other human and social endeavors. We thus ought to proceed the same way in order to reliably compare a naïve conception to a scientific one (whence our use of the term “commensurability” that implies some form of explicit measurement rather than the term “compatibility” that does not necessarily carry the same connotation).

Commensurability assessment is mainly needed to treat naïve ideas that are at odds with science and that cannot be straightened out by coherence and correspondence assessment. This form of assessment can also serve to further reinforce or consolidate viable ideas that are commensurable with science. When everything else fails with naïve realists, a teacher will be left with the only choice of proposing a scientific alternative to student ideas. The teacher would propose the scientific alternative as an option to ponder in the light of some empirical and/or rational evidence, but not as a scientific fact that students have to take for granted. Students would be asked to compare the proposed alternative to their own ideas, especially in making predictions about specific physical realities, so that they would realize the advantage of replacing their naive ideas with the scientific alternative.

An obstacle similar to the one discussed above with correspondence assessment arises here. Sometimes, naïve realists do not realize at first that a naïve idea that they hold and the presented scientific alternative are related. The two may appear to them as corresponding to two different sorts of reality or even to two different worlds, one being the fictitious realm of scientists and the other the real world of everyday life. The teacher needs thus to bring naïve idea and scientific alternative to a state of conscious conflict so that students work at resolving the conflict explicitly in favor of the scientific alternative. If the conflict between the two remains unconscious, it does not get resolved, and, in a Piagetian sense, the scientific idea ends up being completely annihilated by its naïve counterpart, and the latter will dominate in students' minds. This is often the case in conventional instruction whereby presented conceptions and problem solving routines are memorized by rote without comparison to one's own ideas. As a consequence, and like Mazur's student referred to in the introduction of this chapter, naïve realists are driven into a form of cognitive dissonance whereby they continue to deal with everyday life situations "the way [they] *think* about these things", and to answer conventional exam questions using what is dictated to them by teacher or textbook. Once the course is completed, the passively assimilated traded knowledge gets gradually eradicated from students' minds to give way for the naïve paradigm to thrive again.

We have so far concentrated our discussion on the transformation of existing ideas into reliable knowledge about physical realities. Formation of missing knowledge can also be constructed along the lines of Figure 3.4, while remembering that we are now dealing with the construction of "new knowledge [that] would replace ignorance rather than replace knowledge of another and incompatible sort." (Kuhn, 1970). A variant of correspondence assessment is often the best place to start. Students need to be presented with a number of physical realities whereby they can detect a *new pattern* that cannot be predicted, described or explained from available knowledge. They would then be guided to construct needed conceptions, and perhaps develop new skills as well.

Knowledge *formation* may proceed by *anchoring*, i.e., by taking advantage of existing viable ideas when it is about missing derivable knowledge, or it may start from entirely new grounds when about missing prime knowledge. In the first case, one of two anchoring scenarios can take place, branching-out or bridging. *Branching-out* refers to rational manipulation of existing knowledge in order to develop new knowledge corresponding to the same physical realities. This is the case of deriving one concept from another, or combining two known models to constitute a new model that represents a new pattern exhibited by the same referents as those of the original models. *Bridging* refers to the extension of existing knowledge to new physical realities (Brainin, 1985) but especially to new patterns. The extension may be done by *analogy* or through *metaphor*. For example, students who are familiar with the solar system may construct Bohr's model of hydrogen-like atoms by analogy to this system. Interaction among atomic particles may also be realized by analogy to interaction among magnets, or, say, through a game metaphor. I vaguely remember a movie where a science teacher was able to bring his class to a meaningful understanding of the atomic model using a football game as a metaphor.

Branching-out and analogical bridging are not appropriate in the case of missing prime knowledge, especially when knowledge formation has to take place on entirely new grounds, and "may … happen independently of previous conceptions" (Mortimer, 1995). The new grounds may be either empirical or rational. However, if feasible, empirical grounds are usually more convenient to start with.

The suitability, the course and the prospects of each of the three modes of assessment depend primarily on the initial knowledge state of concerned students in relation to the target knowledge that is the subject of instruction (Table 3.1). Students need normally not to be engaged in all three modes of negotiations, and one mode may be more suitable for a specific group of students than others. Coherence assessment is more suitable for viable knowledge than naïve knowledge, and definitely not suitable for missing knowledge. Once viable knowledge is in place, whether by transformation or formation, all students need to be encouraged to go through an exercise of coherence assessment in order to put their knowledge together in an insightful and productive way. Commensurability assessment is mainly a last resort for naïve realists whose ideas could not be transformed otherwise. Correspondence assessment is required for all students because it offers them the chance to develop experiential knowledge in the context of real world situations. This motivates them to go for meaningful rather than rote learning, in at least two respects. First, people feel more at ease learning things in empirical than in

*Table 3.1*

Regulatory processes that students need to be engaged in depending on the state of their initial knowledge as compared to the target scientific knowledge

| *State of target knowledge in student minds* | *Naïve* | *Viable* | *Missing* | |
|---|---|---|---|---|
| | | | *derivable* | *prime* |
| Anticipated evolution (cognitive sciences) | Radical restructuring | Weak restructuring | Weak restructuring | Novel structuring |
| Parallel development in the history of science (Kuhn) | Scientific revolution | Normal science | Normal science | Scope expansion: New paradigm to complement existing ones |
| Anticipated paradigmatic profile evolution | Comprehensive transformation:<br>Curtail naïve paradigm and build up scientific alternative | Local transformation:<br>Paradigm refinement and articulation | Formation from existing grounds:<br>Paradigm expansion | Formation from new grounds:<br>Paradigm/scope expansion |
| Pedagogical strategy from a Deweyan or Piagetian perspective | Cognitive conflict leading to replacement of existing ideas | Cognitive disequilibrium leading to modification and/or expansion of existing knowledge | Cognitive disequilibrium leading to knowledge expansion | Cognitive disequilibrium leading to knowledge expansion |
| Learning processes | Conceptual change | Conceptual change and/or anchoring | Anchoring | Metaphoric bridging |
| Primary assessment dialectics | Coherence + Correspondence (with focus on counter-evidence) + commensurability | Coherence + Correspondence | Correspondence | Correspondence |

rational contexts. Second, and when explored physical realities are conveniently drawn from everyday life situations, this gives students the chance to see in science a way to enhance their own personal life. They would thus be motivated to learn science more for personal benefit than for fulfilling some awkward curriculum requirements.

Regulatory dialectics fostered in modeling theory require that all knowledge transformation or formation be *insightful* as pointed out at the beginning of this section. Students and teachers should not be merely satisfied when a scientific model (or any other conception) is constructed and deployed appropriately. Insight entails that students be consciously aware of what makes scientific realism superior to naïve realism from all perspectives. A number of conditions need to be satisfied for various forms of dialectics to be insightful. These conditions have originally been proposed by Feuerstein et al. (1980, p. 279) in such a clear and generic way that we cite them verbatim:

> "Insight implies an awareness not only of the functions that must be used in order to produce a given mental act and solve a problem, but also of the specific needs generated by situations that elicit the successful use of such mental operations and cognitive functions… lack of insight is one of the primary reasons for the… very limited learning through trial and error.
>
> Insight is produced by the teacher in discussions that deal with the following:
>
> 1. An analysis of the various functions involved in the proper completion of a task.
> 2. An investigation of the types of errors produced and the specific reasons for their appearance.
> 3. A development of an awareness of the changes or modifications occurring in the cognitive processes following exposure to the learning experience.
> 4. A search for and formulation of the most efficient, as well as the most economic, strategies for successful mastery of the task.
> 5. Creation of an awareness of the role played by the cognitive functions, strategies, planning behavior, and insight dealt with in the [specific task] in a variety of other life situations [so as to *bridge* over into an ever-widening areas of application and concern].

Perry's (1970) *Scheme of Intellectual and Ethical Development* and, especially, King and Kitchener's (1994) *Reflective Judgment Model* (RJM) show that the nature of judgments a person makes about the viability of one's own ideas and other people's ideas significantly depends on the person's own epistemic assumptions. These assumptions also determine the way the person would go about transforming or forming his/her own ideas. In a longitudinal study involving thousands of people over about a fifteen-year period, King and Kitchener (1994) found that people's judgments in ill-structured situations evolve following a seven-stage RJM model. Various stages are characterized with "remarkably consistent interrelationships between individuals' assumptions about the nature of knowledge and how they justify beliefs in the face of uncertainty" (*ibid.*, p. 24). King and Kitchener's results "are quite consistent across domains as divergent as science and history" (*ibid.*, p. 25), and show that most students hover about the third RJM stage throughout their high school years, and barely exceed the fourth stage of the model by the time they finish their undergraduate studies.

Characteristics of the two stages of concern are given in Table 3.2. They clearly show how futile regulatory dialectics would be at the level of coherence and correspondence assessment (Fig. 3.4) should students proceed following their own views about knowledge viability. These views need to be explicitly treated for the dialectics in question to be insightful and thus productive. To this end, students need to be engaged in reflective judgments of the sort called for by King and Kitchener (1994, p. 8, italics added) and "based on the *evaluation* and *integration* of existing *data* and *theory* into a solution about the problem at hand, a solution that can be *rationally* defended as most plausible and reasonable, taking into account the sets of *conditions* under which the problem is being solved."

The problem shown by King and Kitchener with regard to knowledge transformation has worse repercussions when it comes to knowledge formation. In the latter event, the promoted paradigmatic evolution cannot entirely proceed from students' own paradigms. For the evolution is not limited then to resolving "rational inconsistencies" in the manner promoted by some advocates of *conceptual change*, and the missing prime knowledge that it may be about often has requirements that are outside the scope of students' own paradigms, a fact that many *constructivists* refuse to admit. The target paradigmatic evolution, and especially in the case of naïve realists, is meant to bring students about to work "in a different world", just like scientists do after a paradigm shift (Kuhn, 1970, p. 135).

Many advocates of both schools have sometimes misled educators to believe that missing prime knowledge can be treated on the same cognitive footing as naïve and viable ideas. Dreyfus, Jungwirth and

Eliovitch (1990) have demonstrated how far from the truth this can be. The authors have shown for instance that when “input knowledge... has no counterpart in the experience of the student”, guiding students through a process of conceptual change can be a futile endeavor, even when students first become dissatisfied with existing conceptions, and when the new conceptions are intelligible, plausible and fruitful, as recommended by Posner et al. (1982). Wong et al. (2001) showed that, under many instances of instruction that follows Posner tenets of conceptual change, “students neither changed their conceptions as anticipated nor sought to reduce logical inconsistencies as expected”, and this even when the proposed conceptions had counterparts in

*Table 3.2*

Relationship* between high school and college students’ assumptions about knowledge viability and the way they justify their beliefs about the world

| RJM* stage | *View of knowledge* | *Concept of justification* |
|---|---|---|
| *Stage 3*<br>(High school students) | Knowledge is assumed to be absolutely certain or temporarily uncertain. In areas of temporary uncertainty, only personal beliefs can be known until absolute knowledge is obtained. In areas of absolute certainty, knowledge is obtained from authorities. | In areas in which certain answers exist, beliefs are justified by reference to authorities’s views. In areas in which answers do not exist, beliefs are defended as personal opinion since the link between evidence and beliefs is unclear. |
| | “*When there is evidence that people can give to convince everybody one way or another, then it will be knowledge, until then, it’s just a guess* .” | |
| *Stage 4*<br>(University graduates) | Knowledge is uncertain and knowledge claims are idiosyncratic to the individual since situational variables (such as incorrect reporting of data, data lost over time, or disparities in access to information) dictate that knowing always involves an element of ambiguity. | Beliefs are justified by giving reasons and using evidence, but the arguments and choice of evidence are idiosyncratic (for example choosing evidence that fits an established belief). |
| | “*I’d be more inclined to believe evolution if they had proof. It’s just like the pyramids: I don’t think we’ll ever know. Who are you going to ask? No one was there.*” | |

* According to the Reflective Judgment Model (RJM) of King and Kitchener (1994, pp. 14, 15).

students' minds. Mortimer (1995) argues that comparison of a student's conception to a scientific alternative does not necessarily imply the "suppression" of the student's conception, "neither does it raise or lower" the intelligibility, the plausibility or the fruitfulness of this conception. It can "only show in what domain [the conception] can be considered as plausible and fruitful". Another implication comes about from Mortimer' work, and it is that teachers can only hope that in any conceptual profile possessed by individual students some but not all of its naïve components become modified. Thus, one can never expect an integral paradigm shift, from naïve to scientific realism, but only a partial, though significant, change of the paradigmatic or conceptual profile in the positive direction.

## 3.7 AFFECTIVE CONTROLS

Paradigmatic evolution in any science course does not depend only on natural paradigms, whether student or scientist paradigm. Learning science, like any other discipline, is also affected by many psychological and cognitive factors, the most important of which are affective controls. Among these controls are interest, motivation, locus of control, as well as attitudes toward science and science education. Affective controls are by and large the object of cognitive psychology, and as such, they are beyond the scope of this book. However, they do present some aspects that are perhaps more related to science education than to other fields, and in this respect they deserve at least the limited attention we give them in this section.

Science educators have always been concerned about getting students of all levels, and especially high school and college students, interested in taking science courses, and in getting them subsequently motivated to learn science meaningfully. "*It is easier to learn something that is motivated than something that is arbitrary*. It is also easier to *remember* and *use* motivated knowledge than arbitrary knowledge" (Lakoff, 1987, p. 346). In liberal educational systems, a majority of high school students take advantage of the choice of not taking science courses. The rate of drop outs and the achievement level of those who do take science are alarming. The situation is no better in other educational systems where all high school students are mandated to take science courses. Educational research constantly indicates that a good proportion of these students prefer not to take science if they have the choice, and that their conceptual understanding of science is no better than their peers under liberal systems.

The problem resides, at least in part, in the way science is presented in conventional courses, and in the locus of control in the general educational enterprise. Our research has shown that the overwhelming majority of non-interested, non-motivated, and authority-driven students have poor achievements in science courses of all levels (Halloun, 2001b; Halloun & Hestenes, 1998). This is more specifically the case with students who: (a) consider that science is irrelevant to everyday life, (b) take science more to fulfil course requirements than to develop useful knowledge, (c) believe that it takes more talent than personal effort to learn science, (d) value a little, if any, positive learning attitudes like openness, critical mind, independence, perseverance, curiosity, and creativity, and who subsequently (e) rely heavily on the authority of teacher and textbook, and assimilate blindly what either authority dictates without reflecting back on their own ideas. The interest of these students is: (a) to pass course exams by memorizing just what it takes and only for how long it takes, and (b) to escape reprimand or to please other people, be it parents, teachers or administrators, more than themselves. For these *passive learners*, learning science is a frustrating experience that they wish they could do without. Their attitudes toward science and science education remain unchanged following conventional instruction, and, if any, the change is in the negative direction.

Our research has also shown that naïve realists are most likely to be passive learners in the way just described, and significantly more so than any other group of students (Halloun, 2001b; Halloun & Hestenes, 1998). Many researchers had actually argued that student beliefs about the nature of scientific knowledge are closely related to their learning styles (Edmonson & Novak, 1993; Hammer, 1994; Reif & Larkin, 1991; Songer & Linn, 1991; Tobias, 1990). Special attention needs thus to be paid to students' affects in relation to their learning styles in order to foster the paradigmatic evolution we are arguing for.

In this direction, we first need to invert the locus of control inwards so that each passive learner sees a *personal need* to go through such paradigmatic evolution. This has at least two requirements. First, passive learners need to realize that their naïve paradigms are not as viable as they think. Second, they need to recognize that science is not a foreign culture but a *way of life* that is worth considering. To this end, we concentrate in modeling theory on scientific tools and processes significantly more than is commonly the case, and we do this, as often as we can, in the context of everyday life situations. Subsidiary models that students are familiar with are major stepping stones in the last respect. Students then become intrinsically motivated to develop scientific conceptual processes and to get them driven by personal habits that are in line with scientific habits of mind (Chapter 4).

No affective factor can yet be fully controlled by any teacher, or any other person for that matter. That is why students cannot all evolve in the same way and to the same level in any given course, and why some students may always fall behind the paradigmatic threshold we are advocating. Our goal in modeling theory is to prevent any student willing to make the effort from falling behind this threshold, and to allow all willing students to exceed it with least variation, i.e. with a significantly reduced standard deviation. Our experience suggests that this goal is achievable in a conveniently structured learning environment.

## 3.8 STRUCTURED EVOLUTION

John Dewey (1897) characterized education as "a process of living not a preparation for future living" (Archambault, 1974, p.430), a process that should treat students as stakeholders who become actively engaged in the educational enterprise not as mere consumers of scientific products. Yet, and as Eger (1993 pp. 20, 323) rightfully argues, a "*sender-channel-receiver* model of information theory" governs conventional instruction. In this model, the research scientist is portrayed as a "producer-sender" of a scientific commodity, the teacher as a "retailer" or, like the textbook, as a "channel" of information transmission, and the student is considered as a "receiver" or "consumer of science's cognitive products". In this process, both teacher and student "see themselves as *outsiders* to science 'itself', identified with the production process". The situation needs to be turned around at the level of both teachers and students. Teachers need to refrain from delivering canned lectures, and conducting rigidly prescribed demonstrations and experiments. They need to assume instead an active role in adapting course content to students' cognitive level and needs and in designing appropriate learning activities. Students need to see science education not as an imposed requirement and not as an enterprise that depends entirely on the authority of teacher and textbook. They rather need to see in science a relevant way of life and to become subsequently engaged in a meaningful evolution from the realm of naïve realism (or common sense, in general) to the realm of science.

Students of any level cannot be expected to embark on such an involved enterprise on their own. They are generally not motivated enough to do so in the first place, and when they are, they need to be placed in an educational environment that is conducive to such an evolution. In an analysis of research published in the last two decades, Taconis, Fergusson-Hessler and Broekkamp (2001) found that teaching approaches that improve significantly student problem solving in science courses share the following three characteristics: (a) the deliberate intention "to enhance the quality of [students'] knowledge base", with a special "attention for schema construction", (b) "the availability of external guidelines and criteria…, i.e. objective [guidelines and criteria] provided by experimental set up or the teacher", and (c) "the presence of immediate feedback". The authors also found that "letting students work in small groups does not improve problem-solving education unless the group work is combined" with all three features just mentioned. Our modeling approach (Halloun, 1996; Halloun & Hestenes, 1987) came on top of the list of best practices identified by the authors. Taconis et al. (2001) had also noted that "a considerable part of the teaching experiments over the past 10 years has been devoted to aspects of learning tasks that are not effective, such as group work without immediate feedback or external guidelines and criteria", and that "treatments focused on the knowledge base have been given comparatively little interest". Many educators have come lately to recognize the shortcomings of modern educational trends that emphasize student-centered environments that are relatively free from all sorts of structuring. Even some constructivists have come lately to realize the need "to structure the environment in ways that would lead to a deeper understanding of science" and to recognize teachers' pivotal role in the process by admitting that "until each student respects the teacher and is willing to construct that person as his/her teacher, there is little point in proceeding with a curriculum that provides students with autonomy and opportunities to learn through inquiry" (Seiler, Tobin & Sokolic, 2001).

The educational environment called for in modeling theory is a structured environment, constantly monitored by the teacher, so as to keep student paradigmatic evolution in line with scientific theory and practice. This environment is, however, flexible enough to allow individual students reflect on their own paradigmatic profiles and regulate them properly. The learning processes involved are parallel to the three forms of paradigmatic evolution distinguished by Kuhn (§ 3.5). However, and because of major differences between scientists' and students' paradigms, students cannot follow scientists' footsteps all the way through in any one of the three directions. As we shall discuss in the next two chapters, learning a scientific theory requires some cognitive transformation of the content of scientific theory, a transformation that follows a middle-out approach centered on basic models. It also requires that students reflect continuously on their own natural paradigms while developing particular models along with necessary tools and skills, and this through well-designed modeling cycles whereby the teacher mediates the learning process in different forms.

## *Chapter 4*

# MODELING PROGRAM

Modeling theory calls for a paradigmatic profile evolution whereby individual students significantly curtail their naïve realism in favor of scientific realism. To this end, a given science course should be geared to help all students to cross a paradigmatic threshold established by the set of basic models in the scientific theory that is the object of the course. Efficacy of science instruction is thus determined by the extent to which respective programs bring students to such an achievement.

In science, modeling theory concentrates on the common denominator among all scientific disciplines, this being the model-centered content of scientific theory and the modeling processes of scientific inquiry. Similarly, in education, modeling theory concentrates on generic curricular aspects that apply uniformly across various science courses at all educational levels, and especially at the high school and college levels. As such, the theory advocates that a generic program of instruction be implemented in structured learning environments. The program revolves around model-centered course content and the tools and skills of modeling inquiry, and takes into account student paradigmatic profiles. The learning environment is structured so that students become empowered with all that is necessary for them to insightfully regulate their own paradigmatic profiles in the direction of scientific realism. Students proceed in this direction through structured learning cycles, each cycle being devoted to the development of a particular scientific model.

The methodology of modeling instruction throughout a learning cycle is the object of the next chapter. In this chapter, we concentrate on the modeling program, i.e., on the way the content of a science course needs to be structured for modeling theory to meet its ends. This includes particular restructuring of any scientific theory around basic models, and the design of appropriate activities that help students to develop most of the theory and required skills in the form of experiential knowledge. It also includes specification of necessary tools and rules for conducting such activities and evaluating their products in an insightful manner.

## 4.1 DIDACTIC TRANSPOSITION

Science instruction within the framework of modeling theory, or modeling instruction for short, is concerned with helping students transcend the realm of naïve realism or common sense and evolve into the realm of science. The transcendence level is not set so as to completely abolish naïve realism from students' paradigmatic profiles (§ 3.3). It is rather kept within realistic margins that are manageable by any student willing to invest the necessary effort. The level is set at a paradigmatic threshold that is characteristic of the set of basic models in a given scientific theory (§ 3.4). The threshold does not account for everything scientists know about the theory in question, not even about its basic models. Furthermore, it is not expected that students attain the threshold following scientists' footsteps for every step of the way. Aside from the reasons discussed in § 3.3, there are other reasons closely related to the educational process that make it practically impossible for students to learn science in exactly the same way as scientists do science, or to develop their paradigmatic profiles so that they be completely commensurable with scientific paradigms.

Firstly, the starting point is not the same for scientists taking up a new research project and students beginning a corresponding science course. Our students enter a science course, especially an introductory one, with naïve or common sense paradigmatic profiles while scientists start their work with profiles dominated by scientific paradigms; and we already know how far apart naïve and scientific paradigms are from one another. Students' paradigms are often reminiscent in some respects of those held by Middle-Ages scientists. Yet, they are no match, even remotely, to the paradigm of a Copernicus, a Galileo, or a Lavoisier, so that they could overcome on their own the hurdles of their naïve ideas, and consider alternative paths without external guidance.

Secondly, when in need of help, a scientist can rely on peers who have a comparable background and with whom s/he can objectively communicate following well-established rules of engagement. This is not the case with our students. Though they need to be encouraged to work in groups (§ 4.4), students cannot rely solely on one another. There are always times, especially in the case of missing knowledge, when even students with the best viable ideas would fall short of knowing in which direction to head.

Thirdly, scientists have access through their observatories and research facilities to physical realities, and to empirical data about such realities, that are not normally available to science students at the college and pre-college levels. At these levels, students and their teachers have to live with whatever limited resources they have at their disposal. Experiential knowledge that can be developed in a normal science course, as well as correspondence assessment (§ 3.6), is often constrained to rely heavily on empirical data about physical realities rather than on direct interaction with explored realities. Moreover, the data may be available in some forms but not others. These include tables, diagrams, pictures, computer or laboratory simulations, documentary films, etc. At times, the needed data may not be available in any form. Students are then left with the only option of developing traded knowledge.

Fourth, practical constraints are imposed in the classroom, the most restrictive of which is the obligation to complete a fixed curriculum in a fixed timetable. Teachers and students are not afforded the flexibility of scientists' agendas. A science course may include other materials (models, if the course is done correctly) beside basic models. A delicate balance needs then to be maintained between fulfilling the credo of modeling theory and completing the course as designed often by people other than the teacher in charge of the course.

Fifth, science teachers are not normally actual scientists – and they need not be at the pre-college level –, and they may not be reasonably aware of what the scientific enterprise entails. In this respect, and at least in high school, a clear distinction needs to be maintained between the science produced by scientists and the science that is at teachers' disposal.

The list could go on and on but the point remains the same: students cannot – and should not – learn science in the way scientists do science, and they cannot end up with a product that is commensurate with scientific paradigm in every respect. In order for the target paradigmatic evolution to be achieved, science instruction needs to account for the constraints imposed by the initial state of students' paradigmatic profiles and by school environment, as well as for the general cognitive demands of the target evolution. Students are then mandated to re-construct scientific theory that has undergone some sort of *cognitive transformation*, or what the French call "*transposition didactique*" or "*didactic transposition*" (Johsua & Dupin, 1999, p. 193-247, and references therein). In the process, a theory is not as well articulated as originally conceived by scientists, especially when the theory is first disintegrated in order to allow students to develop lower-level conceptions (concepts, laws) and associated tools and processes before they go back to develop the theory and its models in their integrity. Explicit pedagogical norms and guidelines thus need to be specified so that teachers can prevent students from losing the rigor of scientific theory in the process and from wandering into futile paths, and so that learners achieve the target paradigmatic evolution in the most efficient way possible. These norms and guidelines emerge from natural and cognitive sciences. They are about course content, including scientific tools and processes, as well as about appropriate learning styles. They are about *what* things students need to learn and *how* they should go about learning them.

In the last two decades significant efforts have been deployed to enhance science education at both levels. At the level of course content, innovations go from: (a) the development of supplementary tools for structuring existing materials like concept maps (Novak, 1990), Vee diagrams (Novak, Gowin & Johansen, 1983), and semantic networks (Fisher, 1990; Goldberg, Bendall and Bach, 1991), to (b) fundamental restructuring of course content on new foundations as in the case of Reif's hierarchical organization (Eylon & Reif, 1984; Reif & Allen, 1992; Reif & Heller, 1982). At the level of learning styles, the spectrum extends from getting students engaged in: (a) autonomous activities that are virtually free from all constraints (e.g., radical constructivism) to (b) structured activities that follow specific guidelines and sequences as in the case of Karplus' learning cycle

(Karplus, 1977). Teachers' involvement is marginal at the former end of the spectrum, and it takes many forms elsewhere. These go from: (a) inducing a cognitive disequilibrium for subsequent self-regulation *à la* Dewey or Piaget, sometimes through Socratic dialogues (Hake, 1992; Raman, 1980), to (b) modeling experts' behavior through cognitive apprenticeship (Heller, Foster & Heller, 1997; Shore et al., 1992). Throughout the spectrum, students are often engaged in teamwork. Groups are sometimes homogeneous (students of similar competence level), but often heterogeneous. In the latter case, each group is either put together and controlled by the teacher following specific criteria, or freely organized by students. As Hake (2001) puts it, all these instructional methods, and especially "non-traditional interactive-engagement methods appear to be much more effective than traditional methods. [However], there is need for more research to develop better strategies for the enhancement of student learning… History … suggests that the present educational reform effort may, like its predecessors, have little lasting impact."

The limited success of all new approaches is due, at least in part, to the fact that most concentrate on one aspect of the educational enterprise and ignore others. More specifically, they either concentrate on content aspects and ignore learning styles, or *vice versa*. In our modeling theory we account for both sides of the coin, as we shall see in this chapter and the following one. To this end, we benefit from the practices of others whose success is not limited to specific domains or specific student populations, and we add our own touch with particular aspects that have been duly corroborated while modeling instruction has been put to practice (Halloun, 1984, 1994, 1996, 1998a, 2000, 2001a, 2004; Halloun & Hestenes, 1987; Wells, Hestenes & Swackhammer, 1995).

## 4.2 MODEL-BASED CONTENT

The content of a science course is primarily determined by the scientific theory that is the object of the course, and the didactic transposition of the theory is implied by the paradigmatic aspects discussed in the previous chapters. In conventional instruction, a scientific theory is disintegrated so much that it loses its structural and functional power. Concepts, laws and other theoretical statements are covered episodically so that students can hardly distinguish theory structure and function (Fig. 4.1). As a consequence, students end up with loose bundles of theoretical statements, mostly in the form of algebraic equations and formulas, whose scope is limited to solving specific paper-and-pencil exam problems.

Modeling theory sets a minimum level of structural coherence and practical function for any course so as to significantly reduce the loss in scientific rigor. This level is the one associated with paradigmatic threshold. Accordingly, a theory that is the object of a science course is developed and deployed in a *middle-out* approach centered on the *basic models* of the theory. Basic models represent simple patterns in the real world, and as such they are within reach of any interested student. They constitute not only the core content of a science course but also some sort of *pedagogical tools* that help students develop the corresponding theory most meaningfully, along with its epistemological and methodological requirements. From an epistemological perspective, basic models provide students with the necessary rational and empirical contexts for coherent development of all generic laws and lower-level conceptions of the theory, as well as with generic building blocks for the construction of more complex models in the theory. From a methodological perspective (§ 4.3), basic models offer students the chance to develop systematically all conceptual tools and rules that are necessary for theory construction and deployment.

***Fundamentals of Physics***
**(Halliday, Resnick & Walker, 1997):**

1. Measurement
2. Motion along a straight line
3. Vectors
4. Motion in two and three dimensions
5. Force and motion - I
6. Force and motion - II
7. Kinetic energy and work
8. Potential energy and conservation of energy

***Physics***
**(Hecht, 1994):**

1. An introduction to physics
2. Kinematics: Speed and velocity
3. Kinematics: Acceleration
4. Newton's three laws: Momentum
5. Dynamics: Force and acceleration
6. Equilibrium: Statics
7. Gravity, according to Newton
8. Rotational motion
9. Energy

*Figure 4.1*: Partial table of contents of two conventional physics textbooks covering Newtonian theory of classical mechanics.

Notice the explicit accent, especially in Hecht (1994), on individual concepts or laws in virtually every chapter.

1. Introduction to modeling translational motion:
   Reference systems, translation and rotation, Galilean particle models, position, time and duration, distance, displacement, trajectory
2. Free particle: Kinematical model
3. Uniformly accelerated particle: Kinematical model
4. Free particle and uniformly accelerated particle: Dynamical models with Newton's laws
5. Free particle and uniformly accelerated particle: Dynamical models with conservation laws
6. Bound particle in harmonic oscillation (Comprehensive, mechanical model)
7. Bound particle in uniform circular motion (Comprehensive, mechanical model)
8. Particle under impulsive interaction (Comprehensive, mechanical model)
9. Newtonian basic models: A synthesis (Paradigmatic perspective)

*Figure 4.2*: Partial table of contents of a course manual that covers the materials of Figure 4.1 in modeling instruction, at the high school or college levels.

In modeling instruction, the content of a science course is divided into units dealing not with individual concepts or laws as in the case of Figure 4.1, but with specific models, starting with basic models (Fig. 4.2). Furthermore, a modeling unit consists not of a chapter detailing finished scientific products, but of a set of instructions guiding students through specific modeling phases and leading to the development of a particular model (Fig. 4.3). The *development* process begins with the *construction* of the model by correspondence to a limited set of referents, and it continues through the *deployment*

1. Solicitation or construction of subsidiary models (through observation of physical realities exhibiting the modeled pattern, home experiments, and/or case studies).
2. Exploration of subsidiary models (establishing the scope of the new model), and proposal of a candidate model (hypotheses formulation, design of appropriate classroom activities).
3. Teacher intervention. It begins while exploring subsidiary model and continues in different forms throughout subsequent activities.
4. Formulation of the new model following the model schema.
5. Deployment of the new model.
6. Synthesis and integration of the new model in the corresponding theory.

*Figure 4.3*: Sequence of activities in a modeling unit.

of the model thus constructed in novel empirical and rational situations so that students get a chance to develop as comprehensively as possible every *schematic* dimension of the model (i.e., any of the four dimensions that make up the model schema). Whenever possible, students begin a modeling unit by examining *subsidiary models* that correspond to the target model, and that students might be familiar with from everyday life and/or from other courses. Otherwise, students would be confronted, in class, with simple physical situations that would allow them begin the unit with the construction of such models. Students are then guided in the manner discussed below to progressively transform subsidiary models into the corresponding model that is the object of the unit under study.

The family of basic models in every scientific theory includes a limited subset of models that are the simplest in the family and that initiate students to the most fundamental concepts and generic laws, tools and rules needed for the construction of any model in the theory. The subset consists of the free particle model and the uniformly accelerated particle model in the Newtonian theory of mechanics (Figures 2.6 and 4.2). Didactic transposition comes best into play, and with the least damage possible to scientific rigor, with this subset of models hereafter referred to as *elementary basic models* or *elementary models* for short. The conceptual disintegration process that didactic transposition involves can be contained from the onset, and most effectively, within the context of these models. Virtually every modeling element (conception, tool or rule) is conceived not for its own sake but for the purpose of constructing an elementary model, and not independently of but in relation to other modeling elements, and this while constructing and/or deploying the model in question. As such the conception begins to gain its significance from the very moment of its inception.

A new modeling element is thus invoked only on a need basis, mainly to contribute to the composition or structure of an elementary basic model under development. Notice for example that the concept of velocity, the concept that defines the “state” of an object in Newtonian theory, is not introduced in Unit 1 in Figure 4.2. It is reserved for Unit 2 where it gains its significance as part of the free particle model. A new conception is often constructed *progressively* within the context of a particular model, especially when it is as involved as a law. For example, Newton’s second law is gradually developed in the context of the uniformly accelerated particle model (Unit 4 in Fig. 4.2) so that students: (a) overcome the paradigmatic barrier discussed in § 3.4 consisting in the mistaken belief that a force is required for an object to change its position, and (b) meaningfully develop semantic and syntactic aspects of the functional relationship that the law expresses. In contrast, in conventional textbooks this law usually constitutes part a chapter devoted to the introduction of a number of theoretical statements, the merits of which are left to be discovered in subsequent chapters (Fig. 4.1). In such textbooks, Newton's second law, like any other theoretical statement, is first stated verbally and mathematically in its formal form ($\mathbf{F} = m\mathbf{a}$ or $\mathbf{F} = d\mathbf{p}/dt$, with $\mathbf{p} = m\mathbf{v}$), and then the functional relationship expressed in the law statement is somewhat detailed. Conventional laboratory experiments that students conduct to "verify" or come up with the law in question are normally constrained to data collection and analysis that lead directly to the formal law statement and end there. In modeling instruction, students progressively develop an understanding of the functional relationship expressed in the law before they come up with the corresponding formal statement. As outlined in Figure 4.4 and discussed in the next chapter, students begin exploring physical situations pertaining to the free particle model and others pertaining to the uniformly accelerated particle model so as to come up with a *nominal* expression of the law. Subsequently, students further explore the same or other situations in order to gradually develop the nominal

1. *Nominal expression*: An object needs to interact with some agent(s) to change its velocity (in direction or magnitude), and not its position, in a given reference system. In the absence of any interaction, the object maintains a constant velocity in any inertial reference system.
2. *Ordinal expression*: When an object interacts with an agent that exerts a given force on the object, the velocity (or linear momentum) of the object changes in the direction of the force. The bigger the change of the object velocity in a given time (acceleration) for a particular mass of the object, or the bigger the mass of the object for a particular change in its velocity in a given time, the bigger the required force.
3. *Proportional expression*: Under the condition above, the required force is proportional to the object acceleration and mass.
4. *Formal expression*: Under the same condition, the required force vector $\mathbf{F}$ is equal to the product of the object acceleration vector $\mathbf{a}$ and mass $m$ ($\mathbf{F} = m\mathbf{a}$).

*Figure 4.4*: Successive forms in which students progressively develop Newton's second law of dynamics.

expression into an *ordinal* form, and then a *proportional* form, before they get to the formal statement of the law (**F** = m**a**).

The *progressive* approach is a major feature of modeling instruction. Students gradually develop a given scientific theory as required in a given course, starting with elementary basic models. Each of these models and of the respective building blocks is constructed along the lines of Figure 4.4. When students possess alternative conceptions of limited viability, modeling activities begin with these conceptions and proceed to gradually refine them until they become commensurate with scientific theory. In this respect, the construction of a given model may begin, as noted above, with a subsidiary model that students might possess. In the absence of a familiar model, students begin the process with the construction of a new subsidiary model. Various schematic dimensions of the subsidiary model are then gradually refined until the model acquires the desired form. This is how, for example, construction of the uniformly accelerated particle model may begin with the free-fall subsidiary model. At some educational levels, and especially at the elementary level, successive refinements of the subsidiary model or of any student conception of limited viability may follow the approach prescribed by Barbara White in her ThinkerTools. White (1993) developed a software whereby, among others, students develop the concept of constant force through a hierarchy of simulation activities, beginning with an activity that simulates the force with identical pulses imparted to a dot on a computer screen.

Development of basic models, and especially elementary models, follows a *spiral* approach whereby empirical and rational complexity increases progressively within and across modeling units, i.e. within a given model and from one model to the next (Fig. 4.5). Models in the same theory may share, partially or entirely, the same reference class. Common referents are gradually added from one model to the next. The first elementary model is normally constructed by reference to a limited number of referents. New referents are gradually added in subsequent models, and old models are then revisited to specify the conditions under which they may represent those referents. In order to emphasize the universality of scientific theory, and especially the *multidisciplinary* value of modeling processes and products, explicit reference is continuously made in modeling instruction to physical realities that are typically the object of different courses pertaining to the same discipline or to different disciplines in conventional instruction. For example, Newtonian particle models are constructed and/or deployed in situations involving living organisms that are typically the object of biology courses, as well as in situations involving atomic particles that are typically the object of electrostatics or chemistry courses.

The function of elementary basic models is also built up gradually. Students develop the descriptive model corresponding to each of these models in a separate unit (Units 2 and 3 in Figure 4.2). They then develop corresponding explanatory models in following units (Units 4 and 5 in this figure). Once all elementary models have been constructed, first as descriptive models and then as explanatory models, each of the subsequent modeling units can be devoted for the comprehensive construction of a new model, i.e., a model with both descriptive and explanatory functions (Units 6, 7 and 8 in Figure 4.2). Students can proceed with the development of subsequent models faster than they did with elementary models, and they gradually gain more insight into various modeling elements and processes as they proceed from one model to the next. When they are done developing the entire family of basic models, students proceed to a synthesis of all completed units before they move on to emergent and more complex

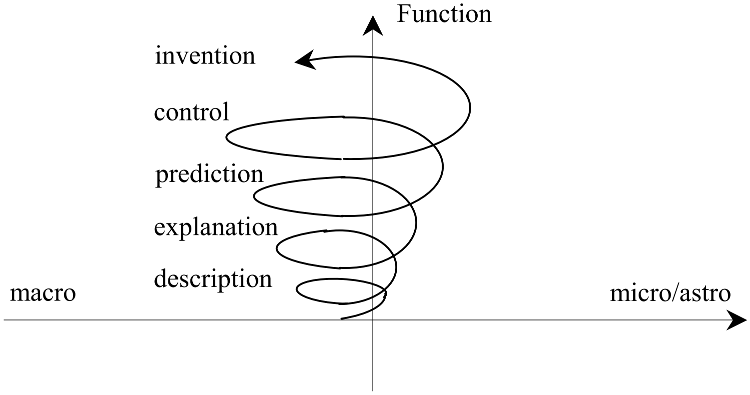


*Figure 4.5*: Spiral development of models, especially elementary basic models.

The scope of a model is gradually extended: its reference class, from the macroscopic world, if applicable, to the microscopic and astronomical worlds, and its function, from pattern description and explanation to invention of new physical realities.

models (Unit 9 in Figure 4.2). In this synthesis, students recap major lessons learned from constructing and deploying basic models, especially with regard to generic aspects pertaining to the theory under study and the paradigm to which it belongs.

In modeling instruction, students are afforded the chance to develop *experiential knowledge* about physical realities, especially when it comes to basic models and their conceptual building blocks. This not only helps students to transform their conceptual and paradigmatic profiles meaningfully, it foremost gets them motivated to do so. Two major conditions need to be satisfied so that students become motivated to construct a new model (or any new conception for that matter): personal relevance and necessity. In order to meet the first condition, modeling units begin with activities pertaining to everyday life. These may consist of observation of familiar physical realities and collection of related empirical data, and/or of home experiments with such realities (§ 1 in Figure 4.3). Such observations or experiments may be supplemented with *case studies* documented in various media forms, and pertaining to current events or to historical development of scientific theory. Case studies are especially important when observations and experiments are not possible or feasible like in the case of microscopic or astronomical realities. The second condition, i.e., necessity, is met by inducing students into a state of cognitive disequilibrium. This is achieved by directing modeling activities so that students encounter, at the onset, obstacles that they cannot overcome with available knowledge. Students would then realize the need to construct a new model (or conception) with well-defined scope.

Any new conception, from concept to model, any new tool or rule is introduced on a need basis in modeling instruction. Research suggests that discussing, say, vectors from a pure mathematical perspective long before they are used for representing physics concepts may impede understanding of both vectors and physics concepts (Ahlgren & Wheeler, 2002). This is why vectors, major concepts of mechanics and Newtonian laws do not figure in separate modeling units the way they do in conventional textbooks (Figure 4.2). Vectors and their properties are discussed when representing vectorial descriptors like position, velocity, acceleration and force, and not independently of such descriptors, only after students realize that a scalar quantification of such descriptors is inappropriate.

Acceleration is introduced in the context of the uniformly accelerated particle model (Unit 3 in Figure 4.2) as a descriptor needed to quantify the change in velocity of a moving object, and only after students realize that the concept of velocity is not convenient to study the kinematics of an accelerating object. Newton's laws of dynamics are introduced when needed, in the development of elementary basic models (Unit 4 in Figure 4.2).

Students cannot always be expected to develop a new model entirely on their own, especially when it comes to conceptions in model composition (new concepts) or in model structure (new laws) that are totally *lacking* in student knowledge (§ 3.5). The teacher then needs to lend a hand, preferably in line with a written text available in a modeling unit (§ 3 in Figure 4.3). Intervention may come in the form of guidelines for designing or conducting specific activities, or in the form of empirical data when no observation or experiment can be conducted, so that students develop required conceptions in the form of experiential knowledge. Teacher intervention may also come in the form of lecture so that students develop those conceptions in the form of *traded knowledge*. Lecturing is inevitable when no facilities are available for data generation or analysis, and thus for developing experiential knowledge, or when required conceptions impose cognitive demands that are beyond students' potentials. Teacher intervention (or mediation as we actually call it for reasons discussed in the next chapter) comes into play when students start putting their subsidiary models to the test. The form and level of this intervention vary depending on the state of student paradigmatic profiles and the kind of modeling activity in which they are involved (details in the next chapter).

There is currently no available textbook that lends itself directly to modeling instruction. Some textbooks come closer to Figure 4.2 than others in arranging their content, but no chapter in any textbook is ever devoted, in whole or in part, for explicit model formulation. Virtually all textbook's chapters concentrate exclusively, and only partially, on sections 3 and 5 of Figure 4.3. That is why, and until an appropriate textbook becomes available, teachers who follow modeling instruction with conventional textbooks supplement these textbooks with modules that they design in order to align their instruction with Figures 4.2 and 4.3. At least three supplementary modules are usually needed for every modeling unit. One module, handed out before the beginning of class discussion on a particular model, prescribes pre-class activities and/or case studies that lead to the emergence of subsidiary models. A second module, handed out after students attempt model construction in class, formulates the model under study following the model schema of Chapter 2. A third module, handed out along with the second module or shortly afterwards, provides extra activities designed to coherently articulate the four schematic dimensions of the model and develop systematic schemes for model deployment.

## 4.3 MODEL DEPLOYMENT ACTIVITIES

Model construction proceeds from the start, and all the way through, as a series of inquiry activities (problem solving included) in both the empirical world of physical realities and/or related data, and the rational world of scientific theory and paradigm. As discussed in the next chapter, model construction begins with a particular form of *inductive* inquiry that delimits the essence of a specific pattern in the real world and sets mapping conditions and details between pattern and representing model. Other forms of inquiry can be undertaken at certain stages of the process, especially model *adduction* and *deduction*, in order to articulate the conditions of nomic isomorphism between model and referents and thus the function of the model. The latter forms of inquiry are associated with model deployment that is then conceived as an integral part of model development. In modeling instruction this goes contrary to conventional wisdom whereby "problems encountered by a student in laboratories or in science texts… are thought to supply only practice in the application of what the student already knows. He cannot, it is said [in conventional instruction], solve problems at all unless he has first learned the theory and some rules for applying it. Scientific knowledge is embedded in theory and rules; problems are supplied to gain facility in their application… this localization of the cognitive content of science is wrong". Unless the student learns theory from the start by doing problems, by inquiry, "the laws and theories he has previously learned would have little empirical content" (Kuhn, 1970, pp. 187, 188). Yet a model cannot acquire its full meaning unless it is deployed, after construction, in a rich set of activities that allow students consolidate their conceptions, tools and skills, and delimit the scope of the model.

Model deployment activities are not limited to conventional end-of-chapter paper-and-pencil problems. They include, like in the case of model construction, observations in the real world, empirical experiments, thought experiments (*à la* Galilée, Fig. 1.3), field projects, case studies, all chosen with a special attention to interdisciplinarity and designed to provide, every now and then, the opportunity for team work. Most importantly, deployment activities are not limited to the "application" of conceptual models in solving empirical problems. They involve a variety of dialectics within each of two worlds, as well as between the two, the empirical world of physical realities and related data, and the rational world of scientific theory. In other words, model deployment activities are not confined to exercises of exploratory inquiry as in conventional instruction, which has exercises limited to the application of specific theoretical statements to certain physical or fictitious realities. Instead, activities are diversified so as to help individual students to develop a balanced diversity of skills pertaining to *both exploratory* research (through model adduction) and *inventive* research (through model-based deduction), while they meaningfully realize, and take advantage of, the potentials of every model in a given scientific theory. As such, modeling instruction maintains a balance between four categories of model deployment activities. Each category involves a particular type of dialectics within the rational world or the empirical world, or between the two worlds. The four categories are: application, analogy, reification, and extrapolation.

*Application* activities are exploratory activities that involve the deployment of conceptual models for describing, explaining or predicting particular aspects in the structure or behavior of certain physical realities. In an application activity, students are confronted with an empirical situation, directly in the real world, computer simulated, or on paper, and asked to solve a certain problem regarding it. The problem solution requires that a convenient conceptual model (or set of models) be adduced to the situation so as to come up with empirical answers to certain questions. *Model adduction* requires primarily mapping of given empirical data onto appropriate facets of model structure. It thus involves what we refer to as <E → R> dialectics whereby students begin the adduction process by teasing out primary details in the empirical (E) world in order to match the situation at hand with a familiar pattern(s), and subsequently with the appropriate model(s) chosen from students' repertoires of conceptual models included in their rational (R) world.

A major function of application activities is to help students to learn how to deploy model correspondence rules in the empirical world, and subsequently delimit the scope of the model. Activities are diversified so that students develop a repertoire of referents for each model, rich enough to allow students to realize the conditions and potentials of nomic isomorphism between model and referents, and learn how to choose exactly what of a model composition and structure they need to deploy, and how to deploy them, in order to solve any problem about model referents.

The most critical step in an application problem is the choice of the appropriate model(s) from the start, just after physical systems and phenomena have been delineated in the situation at hand (Fig. 4.6). When a science course is about more than one scientific theory, which is seldom the case in conventional instruction, the choice would first be a paradigmatic choice. The problem solution would begin with the

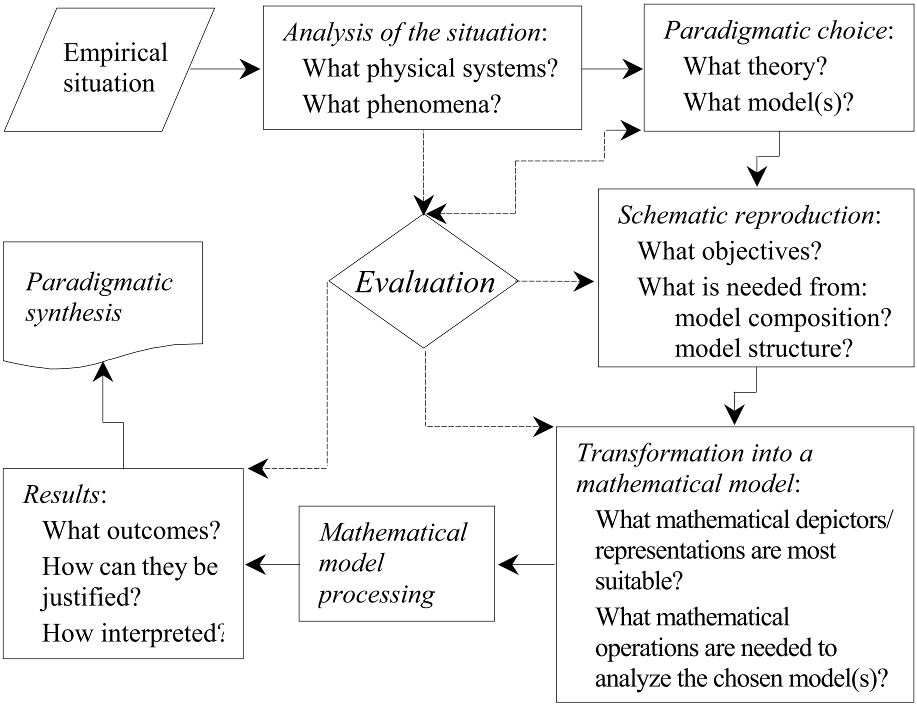


*Figure 4.6*: Model deployment scheme in an application activity.

choice of an appropriate theory within the context of a specific scientific paradigm (e.g., the choice of Newton theory, Euler theory, or Hamilton-Jacobi theory for classical mechanics situations), followed by the choice of appropriate model(s). Then, once the problem objectives are determined, one can choose what part of a selected model is required for solving the problem. Usually, and especially in physical sciences, the process is accompanied by the transformation of a chosen scientific model into a mathematical model that optimizes the efficiency with which a problem can be solved and the outcomes analyzed. The entire process is constantly evaluated, especially in terms of nomic isomorphism between chosen models and delineated physical realities, and it ends with a paradigmatic synthesis whereby major lessons learned from the deployment activity are integrated into the paradigmatic profile of each student. The process of model deployment in an application activity as outlined in Figure 4.6 will be discussed later in this chapter and in the following one.

Naïve realists are used to reasoning backwards in end-of-chapter application problems. The main message in the model deployment strategy of Figure 4.6 is that one should start solving an application problem by analyzing problem givens and asking what models are appropriate for the given situation, before even knowing what the "unknowns" in the problem might be. The answer to any question would immediately follow from model structure, provided that one has already constructed such a structure meaningfully. By contrast, naïve realists look first for the questions asked in the problem in order to determine the "unknowns". Then, they move back to the problem givens, not to analyze the situation as recommended above, but to tease out "givens" so that they can determine, not the appropriate model, but candidate formulas or equations that relate givens to unknowns, irrespective of whether or not these formulas are appropriate to the situation. All that matters for naïve realists is to find a set of formulas, any set of formulas, that helps them to find numerical answers for asked questions in terms of *all* givens in the problem statement. Any problem with superfluous data or with missing information will get them lost. Unfortunately, conventional end-of-chapter and exam problems are designed so that they can often be solved by plug-and-chuck, which makes it hard to sway naïve realists away from their practice. These students get so hooked on manipulating formulas by trial-and-error that they completely block out other alternatives. Any problem that cannot be solved with formula manipulation is sure to bring them immediately to a deadlock.

A physics professor wanted once to test the waters in this regard with his senior physics students. He used to follow conventional lecturing in his upper level classical mechanics course, and concentrate on conventional application problems in his homework and midterms. At the time, his students were about the best that can be, as assessed both in his lecture course and the corresponding laboratory course. He gave a final exam problem consisting of a three-column table including empirical data about three unidentified variables. The question was to check whether the data correspond to any particular phenomenon that students might be familiar with. The first column displayed position data of a simple harmonic oscillator; the second column displayed respective instants, and the third a constant value that could be attributed to mass or to any secondary variable. Among the twenty or so students in the class, and to the stupefied dismay of both "course" and lab professors, only one student tried to plot and analyze a graph of the data. The rest of the students where divided between two camps. In the first camp was a minority of students who tried, to no avail, to find by trial-and-error a set of equations that would fit all given data. In the second camp were students who gave up on the problem altogether after scribbling a few insignificant notes. Nevertheless, the majority of the class was able to solve other exam problems that were far more complex application problems!

*Analogy* activities are exploratory activities about matching with one another different empirical situations exhibiting the same pattern represented by a particular model. These activities primarily involve comparison of empirical data between two or more physical situations, thus what we refer to as <E → E> dialectics within the empirical world, aimed at determining common and different physical features. Dialectics are conducted by indirect adduction of conceptual models, since any comparison between referents of any given model has to be made by reference to model composition and structure. Analogy activities are conducted to help students identify primary features of physical situations that are responsible for pattern production in the real world, and subsequently determine analogy criteria between various referents of the model representing the pattern. These criteria are primarily set in terms of nomic isomorphism between a model and its referents.

Whenever possible, an application activity is immediately followed by a related analogy activity. In the new exercise, students are confronted with a new empirical situation and asked to assess the analogy between the new situation and the one just treated in the application activity. Alternatively, students are better asked to describe a familiar situation, or invent a new one, that is analogous to the old situation (the analogy exercise is then turned into an activity of inventive rather exploratory research). Students are then guided through the first four cells of Figure 4.6 in order to determine the extent of analogy between the two situations and thus determine whether or not they exhibit the same pattern. Special care is paid in the process of teasing out primary features from secondary features, and determining what makes a particular feature a primary or a secondary one. In order to further reinforce analogical reasoning skills, students are confronted with, or asked to invent, no only analogous situations but counter-examples as well, so that they would realize the limits of analogy criteria they came up with and especially the limits of the corresponding model scope (domain and function).

Naïve realists are used to assess the analogy between two empirical situations, not systematically and explicitly by reference to conceptual models, but following rules of thumb that often distract them from primary features, and that concentrate more on objects than on phenomena. Sometimes, when solving an application problem, naïve realists proceed by looking for an analogous problem in their repertoires of familiar problems. The analogy is sought by reference to a physical prototype and not to a conceptual model, and it often concentrates rather on secondary features and not primary ones (§3.2). When appropriately conducted, model deployment in analogy activities offers students the chance to develop reliable criteria for assessing the analogy between empirical situations, and thus a reliable approach for identifying patterns in the real world.

*Reification* activities involve the use of a conceptual model as a master plan for the control of an existing physical reality so that it produces the pattern represented by the model, or as a blueprint for inventing a new referent of the model. Reification activities are thus activities of inventive research involving primarily *model-based deduction*. As in technology, they involve what we refer to as <R → E> dialectics that go in a direction opposite to application activities, from the rational world to the empirical world in order to modify this world in certain respects, and submit it to the conditions of the model. Depending on the feasibility of the situation, students may propose a way to control an existing physical reality or design a new physical reality on paper; they may have either reality computer simulated, or they may construct it in the laboratory or the field. The modified or invented reality must satisfy the conditions of being a referent of the model in question, and primarily those of nomic isomorphism.

Analogy activities may also involve the invention of new referents as mentioned above. However, they remain in this respect less involved than reification activities. Students have then at their disposal a physical reality to emulate, which is not the case of reification activities, at least not explicitly. The starting point in the latter case is an abstract conception, a model, and not a concrete thing. Preliminary reification activities are sometimes designed so that students may reason implicitly by analogy to a familiar referent. However, the main objective of reification activities is to help students gradually to disengage from their dependence on familiar referents, and be creative (productive rather than reproductive) in the empirical world, i.e., be capable of inventing new referents exclusively by reference to the model.

Reification activities are the least afforded in conventional instruction, if ever. Students miss then the chance to reach the top level of Figure 4.5 (control and invention), and hence the chance to become engaged in a critical aspect of inventive research, scientific creativity. Most importantly, they miss the chance to articulate semantics associated with various components of conceptual models. Such articulation takes place progressively beginning with partial reification of a particular model. It may begin by providing students with a partial mathematical model, i.e. a set of mathematical depictions and representations pertaining to certain facets in the structure of the scientific model of concern (e.g., Fig. 2.4e). Students would then be asked to reify those facets in the manner described above. The reification process would follow Figure 4.6 backwards, from the mathematical model cell to the empirical situation cell.

At the highest level of the taxonomy of model deployment activities are *extrapolation* activities. These are activities of inventive research that require higher levels of deduction than reification activities, and that are thus characteristic of *mastery* in scientific thinking. Extrapolation activities get students engaged in <R → R> dialectics that take place entirely in the rational world. They enable students to ultimately cut the umbilical cord between a scientific model and its referents, i.e., to reason about the model in the abstract without any reference to the empirical world. As in the case of model reification, model extrapolation involves creative processes that lead, however, to the invention of new conceptions, not of new physical realities. Included in this category of model deployment are thought experiments like those conducted by Galileo to conceive the particle models of classical mechanics (Fig. 1.3). At another level, model extrapolation entails predicting the existence of new physical realities before they are discovered and the conception of new prospects for a given model and respective pattern. This was the case with Mendeleïev and his periodic table of the elements and with Gell-Mann and his quarks.

At the high school and college level, extrapolation activities are mainly concerned with the conception of new elements in the composition or structure of a particular model, or of an emergent model out of familiar ones. As such, they constitute an integral part of model construction. For example, in the classical mechanistic paradigm, students can extrapolate Newtonian particle models of translation for the construction of Eulerian rigid body models of rotation. To this end, students would be assigned to transform linear descriptors and state laws (equations of motion) into angular counterparts, and to emulate Newton's laws of translational dynamics in the formulation of Euler's laws of rotational dynamics. Students can also be assigned to formulate the model of a bound particle in uniformly accelerated circular motion by emergence from the two basic particle models, the uniformly accelerated particle model and the model of a particle in uniform circular motion (Fig. 2.6). As such, extrapolation activities relieve teachers from lecturing and help students construct conceptions of different levels meaningfully.

The four categories of model deployment activities are outlined in Table 4.1. In modeling instruction, activities of any category are diversified so as to allow learners to explicitly develop all the schematic aspects of a model (i.e., aspects delineated in the model schema), realize model viability in all conceivable and affordable contexts, and, in the process, reflect explicitly on their own paradigmatic profiles and regulate them in the direction of scientific realism. The prime objective of model deployment is thus not to inculcate a newly constructed model in the memory of students through repeated drilling exercises. It is not to assess student knowledge about the model in the traditional sense either. Drilling and assessment take different meanings in modeling instruction. Modeling theory recognizes the need for drilling to consolidate newly developed knowledge. However, drilling does not take place with similar tasks as is often the case in conventional instruction, which has been shown not to have an impact on student knowledge state (Wollman, 1984). Instead it is conducted with tasks of different empirical and rational contexts so that students become familiar with a diversity of referents

*Table 4.1*
Taxonomy of model deployment activities

| Category | Dialectics | Inquiry | Objectives |
|---|---|---|---|
| Application | Empirical → Rational | Exploratory | Develop a rich repertoire of model referents<br>Delimit model scope and set conditions of nomic isomorphism<br>Develop rules of model adduction for pattern description, explanation and prediction or post-diction |
| Analogy | Empirical → Empirical | Exploratory<br>Inventive | Develop rules for pattern identification<br>Develop criteria for establishing analogy between model referents<br>Apply these criteria for designing new referents |
| Reification | Rational → Empirical | Inventive | Develop rules of model-based deduction for pattern reification through:<br>Control or modification of existing physical realities<br>Invention of new physical realities |
| Extrapolation | Rational → Rational | Inventive | Articulate deduction rules to:<br>Refine a model<br>Develop new concepts or laws<br>Construct a new model<br>Predict the existence of unfamiliar referents or primary details of the modeled pattern |

for every model and develop sufficient experience in deploying the model in a variety of situations. Familiarity and experience have proven to be key elements of experts' success in problem solving. In this regard, Singh (2002) has shown that college physics professors struggle in solving an unfamiliar problem even when the "inherent difficulty of the problem posed in [the] study is comparable to problems the professors can solve without much difficulty. This study suggests that the perceived complexity of a problem not only depends on its inherent complexity but also on the experience, familiarity, and intuition we have built about a certain class of problems". Modeling theory also recognizes the need for teachers to conduct external assessment of student knowledge. However, such an assessment is not conceived as an end by itself, but as a means of diagnosing students' conceptual and paradigmatic profiles, and prescribing necessary activities for steering student paradigmatic evolution in the right direction. Special care is paid in the process for promoting student self-evaluation and self-regulation (§ 4.5 and § 4.6).

All in all, and although it follows model construction chronologically (Fig. 4.3), model deployment does *not* strictly follow *from* model construction and it does not subserve the latter. The two modeling processes complement one another with respect to helping students develop a scientific model as comprehensively as possible, and gradually evolve into the realm of science. Model construction is not a one time shot, especially not when it follows the model schema. Schematic aspects cannot all be realized in a single round following the first four steps in the sequence of Figure 4.3. For instance, the domain of any model is so vast that the respective correspondence rules and conditions of nomic isomorphism with its referents cannot be fully inferred through the sequence in question. Such inference requires that students deploy the model in a rich array of situations within each of the four deployment categories distinguished above. Furthermore, model deployment offers learners a more flexible and effective platform than model construction to articulate various modeling tools and rules, including but not limited to those that govern negotiations within and between the rational and empirical worlds.

## 4.4 MODELING TOOLS

In modeling instruction, we assume that unless armed with the "tools of the trade" and the rules that govern the use of such tools, a science student, like any apprentice, will not be enabled to come out with meaningful scientific products, not even a simple concept. The tools in question are primarily modeling tools of a conceptual nature employed at different levels of model construction and deployment. They also include mnemonics for integrating every new conception in memory and efficiently retrieving it when necessary.

Conventional science instruction relies on a limited set of modeling tools, mainly select iconic (pictorial and diagrammatic) depictions, and mathematical formulas and representations. These tools are often employed blindly in passive lectures, without letting students realize the semantic and syntactic rules that govern their use, or helping them to develop corresponding mnemonics. Some other tools that are made possible by modern technology, like educational software and computer-based laboratories and simulations, are nowadays being integrated in some classrooms, but often under the same philosophy that governs the use of iconic and mathematical tools.

A wider and balanced diversity of such tools constitutes only a part of what learning science under modeling theory entails. There are other indispensable tools, perhaps more indispensable than the former, that are commonly neglected in conventional instruction. Among these are *organizational* tools that help students to coherently organize various conceptions, and subsequently deploy them in the most effective and efficient ways possible. In this regard, research has continuously shown that the "critical aspect of experts' working memory is not the amount of information stored *per se* but rather how the information is stored and indexed in long-term memory" (Ericsson & Charness, 1994).

According to modeling theory, effectiveness and efficiency of scientific paradigms are optimized with the middle-out, model-centered structure of scientific theory. *Modeling schemata* are in this respect the most indispensable generic tools for theory construction and organization. As presented in Chapter 2, model and concept schemata are geared more to teachers than to students. As such, they provide teachers with reliable means for *planning instruction*, as well as for *assessing student learning and teaching practice*. Students need to be guided to "transpose" these schemata so that they become readily available for them in model construction and deployment. Didactic transposition of either schema can take place by the time students finish building elementary basic models in a given course. One way to do it then is to help students to reflect back on the way they have built the models in question, and extract some sort of a check-list that includes major issues that they systematically addressed in the process of building these models. The list would take the form of Figure 4.7, and it will subsequently be implemented in constructing the rest of the models in the course, and refined if necessary.

A similar approach is followed for helping students to put together generic model *deployment schemes* in line with the scheme shown in Figure 4.6. The schemes emphasize the central role of models in all sorts of inquiry, including traditional problem solving that falls mostly under the application category of model deployment. In this respect, we agree with Giere (1988, p. 177) that student failure to solve such problems should often be regarded "not as evidence of deficiencies in reasoning ability, but simply as indicators of ignorance of the most appropriate models for the situation". Students are thus induced in modeling instruction to realize that the solution to any problem can be efficiently attained by identifying (or adducing) at first the appropriate model(s) for the situation. To this end students develop a scheme similar to the one shown in Figure 4.8. Once the model(s) is identified in an application problem, the answer to any question follows directly from model structure (provided that one has already developed such a structure following the model schema).

With the more general tools mentioned above are associated particular *procedural tools*. These tools are needed for building up particular schematic aspects of a model (or concept) in the process of model construction, or for carrying out particular routines of model deployment. With each tool is associated a number of semantic and syntactic rules that students should become well versed in. *Semantic rules* establish the correspondence between the tool being used, on the one hand, and the empirical world and other representational tools on the other. They set the norms for interpreting various elements of the tool or whatever product that the tool may be bring about when used, both in the empirical and rational worlds. *Syntactic rules* spell

***Domain***:

- What physical systems does the model refer to in the real world?
- What pattern do these systems share in their structure and/or their behavior?
- In what sort of reference systems?
- Under what physical conditions?
- Under what limits of approximation and precision?

***Composition***:

- Of what *object depictors* does the model consist? (e.g., particles, solids of specific geometry)
- What agents in the respective environment interact with these objects?
- What coordinate system is most convenient for depicting the physical realities under study?
- What *intrinsic descriptors* characterize each object? (e.g., mass, charge)
- What *state descriptors* characterize each object? (e.g., position, momentum and other kinematical concepts)
- What descriptors characterize object-object and/or object-agent interactions? (e.g., force, field and other dynamical concepts)
- What symbolic, pictorial, diagrammatic, graphical depictions can most conveniently be used to represent all objects and descriptors above?

***Structure***:

- What function does the model serve? (descriptive and/or explanatory; e.g., a kinematical or a dynamical model)
- Of what does its *topology* consist? (e.g., none for particle models in Figure 2.6, discrete topology of many-particle models)
- What *interaction laws* quantify best the interaction of each object with other objects and agents? (e.g., Newton's law of universal gravitation, Hooke's law, Coulomb's law)
- What *state laws* best describe the behavior of each object? (e.g., so-called kinematical equations of motion, like r(t))
- What *causal laws* best explain the behavior of each object? (e.g., Newton's second law)
- What symbolic, pictorial, diagrammatic, graphical representations can be used to depict all the above conveniently?

***Organization***:

- What are the limitations of the model?
- What features does it share with other models in the theory to which it belongs?
- How does it differ from other models?
- What other models complement it in the theory?
- Can it be merged with other models to form a new model that answers questions that cannot be answered with either model separately? If so, how?

*Figure 4.7*: A student formulation of a model schema.

Until they get used to the formal jargon, students can use, instead of terms in italics, colloquial terms or theory-specific terms like the respective ones listed in parentheses.

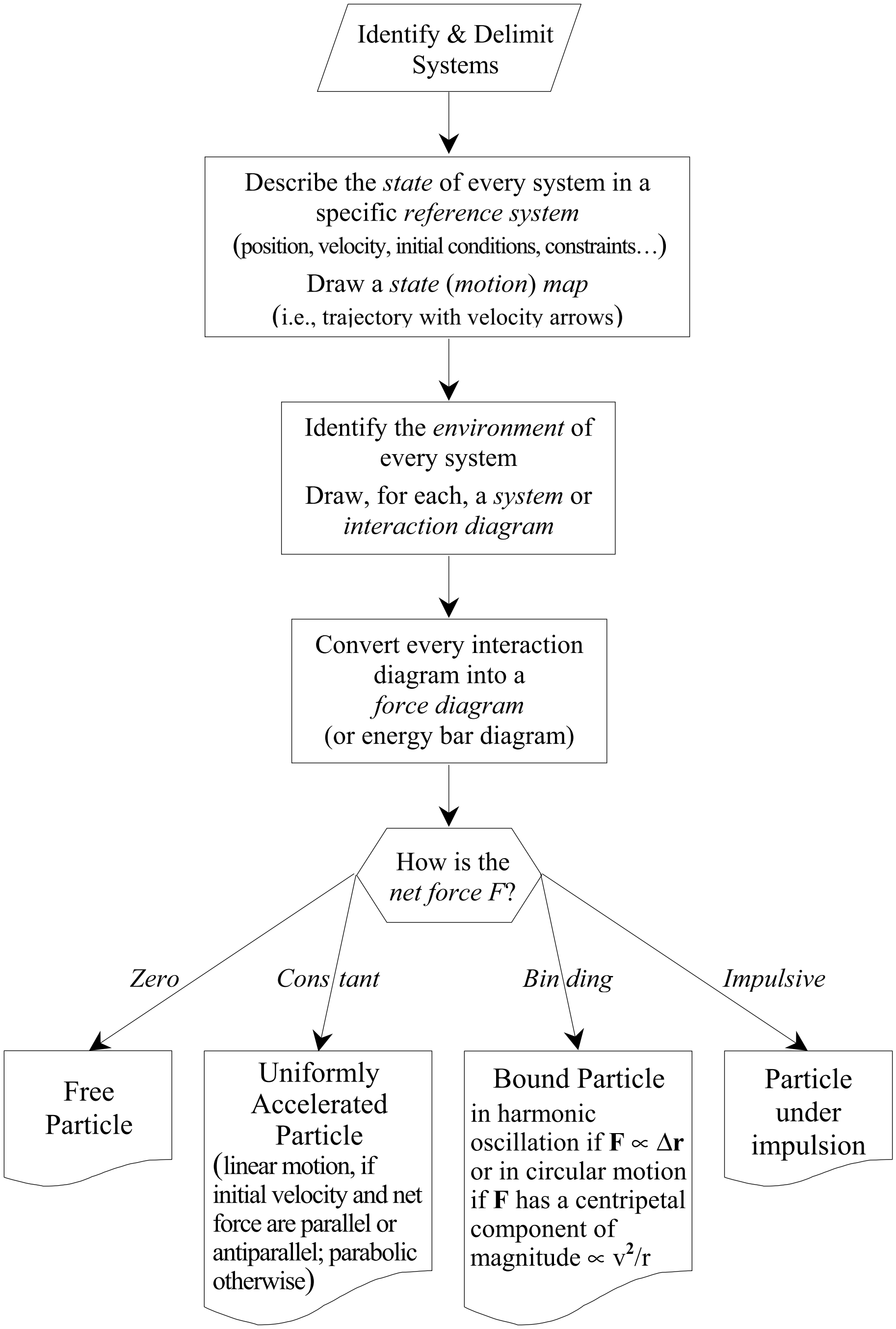


*Figure 4.8*: A scheme for identifying appropriate basic Newtonian particle models in application problems of classical mechanics.

the conditions and guidelines for relating various elements of the tool to one another and to those of other tools, and for manipulating the tool in specific empirical and rational contexts.

For example, the first step in both model construction (Fig. 4.7) and model application (Figures 4.6 & 4.8) is to delineate the systems of concern, along with their respective environments. An interaction diagram (or system diagram) similar to the one shown in Figure 2.3c comes in as a very handy tool in this respect. Semantic rules associated with such a diagram indicate that only primary bodies that significantly affect the structure or behavior of a system under study should be represented in the diagram, and they stipulate, among others, that objects inside the system and agents outside should be represented in a discriminating way. To the latter end, an object inside the system is depicted by an ellipse in Fig. 2.3c, and an agent in the environment of the system is depicted by a rectangle. One-directional arrows pointing in the direction of the object indicate that only actions of agents on the object need to be accounted for; object actions on its agents are ignored. Each action is represented by a particular force depicted with an appropriate arrow on the right side of Figure 2.3c. The set of arrows constitute a so-called force diagram, a representation tool needed in model composition and structure (Fig. 4.7) as well as in the schematic model reproduction phase of model deployment (Fig. 4.6). The latter tool leaves out any depiction of object or agent as physical bodies, and concentrates on the actions *of* agents *on* an object. Semantic rules associated with the force diagram set the correspondence between the two diagrams in Figure 2.3c, as well as between the force diagram and the real world in accordance with the expression dimension of the concept schema discussed in § 2.8. Some of the syntactic rules associated with the force diagram are spelled out in Figure 2.9. Other rules set how various arrows should be drawn so as to display, in the best way possible, force features in relation to the model being constructed or deployed. One of these rules is expressed in the caption of Figure 2.3c. It states that, in the case of Newtonian particle models, all arrows need to originate from the same point to highlight the fact that an object's translation is not affected by its geometric properties of shape and dimension, and to facilitate subsequent vectorial operations with the represented forces.

A tool like the aforementioned interaction or system diagram may look trivial to some. It is seldom used, if ever, in conventional instruction; yet it has proven to be indispensable for students in at least two respects in modeling instruction. First, it helps students to think systematically of identifying systems and their environments in any model construction or deployment activity, and especially to isolate primary objects and agents that are salient to the structure or phenomenon (pattern) under study. Second, and most importantly, it allows students to identify all instances of the chosen interaction descriptor that are truly necessary for setting the interaction facet of model structure. In the case of Figure 2.3, this corresponds to identifying all forces of concern, without missing any relevant force or adding a superfluous one, two common mistakes that students often commit in mechanics courses. In modeling instruction, students become convinced of using such tools after they realize in practice how important they are in allowing them to avert pitfalls, like the two just mentioned.

Modeling instruction does not trivialize any tool unless it proves unhelpful to students when put to the test under a variety of contexts. Students are encouraged to consider whatever tool used by scientists, irrespective of the discipline into which the tool was originally developed, or of the time at which its development took place. Some long forgotten tools may sometimes be more efficient than ones that are currently adopted and even revered. For example, it has long been a practice in conventional physics instruction to decompose, into independent Cartesian components, vectorial descriptors or expressions of state laws that involve such descriptors. This is typically the case with projectile motion studied within the context of the Newtonian uniformly accelerated particle model. As can be seen in Figure 4.9, the structure of such a model can be constructed and deployed more efficiently in projectile problems or the like by doing without traditional graphical representation and vectorial decomposition, and resorting instead to the long forgotten coordinate free theorem developed by Hentisberus more than six centuries ago. It is unfortunate that long time traditional practice in one direction or another may acquire an inertia that is hard to break even within a community like that of physics educators. A form of Hentisberus' theorem was brought to the attention of this community in *The Physics Teacher* more than three decades ago (Winans, 1971); yet it has not so far been adopted in any physics textbook with which this author is familiar.

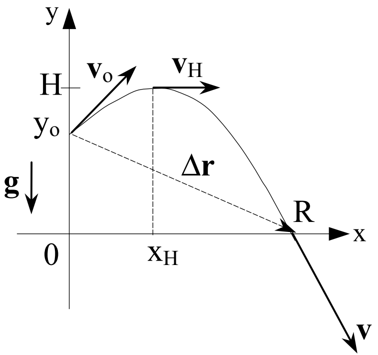
***Traditional approach***:

The duration of flight of a projectile fired, from altitude $y_o$, with initial velocity $\mathbf{v_o}$ at an angle $\theta_o$ with the horizontal is usually assessed via the equation of motion pertaining to the vertical component of the trajectory:

$$y = y_o + v_{oy}t + \tfrac{1}{2}at^2$$

with: $a = -g$, $y = 0$ and $v_{oy} = v_o\sin\theta_o$

$$t_R = \frac{v_{oy} + \sqrt{v_{oy}^2 + 2gy_o}}{g}$$



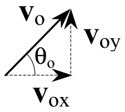
The maximal height H and range R are obtained respectively via the equations:

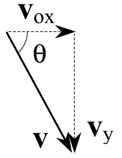
$$v_{Hy}^2 - v_{oy}^2 = -2g\Delta y$$

$$x = v_{ox}t_R$$

where: $v_{Hy} = 0$; $\Delta_y = H - y_o$; $v_{ox} = v_o\cos\theta_o$

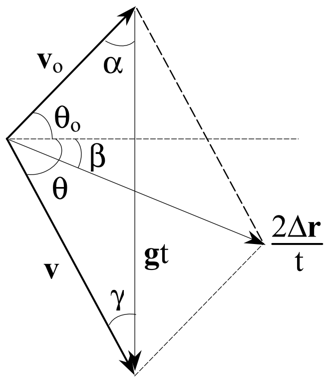
$$H = y_o + \frac{v_o^2}{2g}\sin^2\theta_o \quad \text{and} \quad R = v_o\cos\theta_o\, t_R$$





***Hentisberus Theorem:***

The same results could have been obtained with Hentisberus theorem (c. 1340) according to which one needs to represent in a single diagram the sum of the projectile final and initial velocities, **v** and $\mathbf{v}_o$ respectively, ($\mathbf{v} + \mathbf{v_o} = 2\Delta\mathbf{r}/t$, expression known as the mean speed rule) and the difference between them ($\mathbf{v} - \mathbf{v_o} = \mathbf{g}t$). All one needs to do then is to apply the law of sines in, say, the top and left triangles of the adjacent diagram:



$$\frac{u}{\sin(\alpha+\gamma)} = \frac{v}{\sin(\theta_o+\beta)} = \frac{v_o}{\sin(\theta - \beta)}$$

$$\frac{gt}{\sin(\theta_o+\beta)} = \frac{v}{\sin\alpha} = \frac{v_o}{\sin\gamma} \quad \text{with } u = \frac{2\Delta r}{t}$$

*Figure 4.9*: Projectile motion studied following two different ways within the context of Newtonian theory of classical mechanics.

No tool is used in modeling instruction under the assumption that students know how to use it, even when the tool is supposed to be fully developed in other courses. Mathematics is for science the main source of representation and especially operation tools. Students often learn these tools in mathematics without getting the chance to realize their utility in science. Furthermore, conventional mathematics courses are so overwhelmed by symbolic and abstract reasoning, and dissociated from the empirical world, that students seldom realize for what mathematics is good. Take, for example, the case of derivatives. Science teachers, and especially physics teachers of all levels, are often faced with students who encounter particular difficulties in interpreting the derivative of a scientific descriptor with respect to another descriptor, or even mixing up the derivative with the average rate of change of one descriptor relative to another. Nevertheless, the same students may be able to correctly complete any mathematical operation involving derivatives or ratios of any sort. For instance, students of introductory physics courses may be able to solve correctly any mathematical equation involving the expression of instantaneous velocity as the derivative of position with respect to time, or even of acceleration as the derivative of velocity with respect to time. However, many students who succeed to do so are unable to realize exactly what it means that instantaneous velocity is the derivative of position, or that acceleration is the derivative of velocity. The same students often fail to relate instantaneous and average values of either descriptor and to discriminate between the two sorts of value. From a formal perspective, many students think in error that the expression of a derivative is about the ratio between two variables just like that of an average rate of change (e.g., that instantaneous velocity like that of average velocity is the ratio of displacement, i.e., position change, to the time interval during which the change takes place). As a consequence, these students resort, by mistake, to the expression of average velocity to evaluate instantaneous velocity when this descriptor is not constant. From an empirical perspective, when the students in question are told that, say in free fall, an object has a given velocity at a given time and then asked to tell what would be its velocity one second later, a few of them can answer the question without going through the manipulation of some mathematical equation(s) of motion. Yet, all it takes to answer the question is to increase the given velocity by an amount equal in magnitude to the acceleration of motion.

In modeling instruction, the problem in question is treated empirically while introducing the concept of instantaneous velocity and then of instantaneous acceleration in the descriptive uniformly accelerated particle model. Through an appropriate experiment or observation(s), students collect empirical data related to the kinematical model in question. Students analyze the data such as to first realize empirically the difference between instantaneous and average velocity, and then infer the corresponding mathematical expressions and the associated semantic and syntactic rules, rules that they generalize progressively to other derivatives and average rates of change. Semantic rules help students explicitly to realize what each expression means in the empirical world and the conditions under which it can be used. Syntactic rules help them to manipulate the expression correctly, especially in relation to other expressions involving the same or other descriptors.

The same care is taken with all sorts of mathematical representations and operators in modeling instruction. Associated rules are developed in a variety of contexts within the framework of basic models. In general, by the time elementary models are covered in a given course, students would be capable of meaningful manipulation of a good proportion of mathematical tools required for the construction and deployment of all models in the course. Additional mathematical tools are usually developed at a faster pace with subsequent basic models. By the time all basic models are covered, little time, if any, is usually needed for developing the necessary tools for the course.

Model deployment requires virtually the same tools as model construction. Moreover, and like for the deployment of any other conception, model deployment requires additional tools, specifically, say, for invoking or *adducing* the appropriate model(s) for a given situation, and then *recalling* what is needed of a model structure to achieve the deployment activity at hand. These tools are of a *mnemonic* nature. Students develop them following the deployment of a model in a rich repertoire of situations. The scheme of Figure 4.8 is a typical scheme for model adduction. As for recalling the necessary part of a model structure, students are encouraged to infer appropriate mnemonics from a qualitative analysis of the semantics and syntax of every law involved.

Modeling instruction can be enhanced with the tools of modern technology when appropriately designed. None of the currently available tools, from MBLs and CBLs to various simulation and instructional software, is however underlined by modeling theory in the way presented in this book. Teachers thus have to invest particular effort in order to adapt any of these tools to modeling instruction. Some of these tools can be adapted so as to be helpful in specific modeling respects, and especially to improve the efficiency of learning and instruction. More specifically, they can enhance the logistics of some model construction and deployment activities, and cut the cost and/or time needed to conduct such activities. However, and because of different underlying philosophies, and especially because commercialized tools seldom make any of the semantic and syntactic rules discussed above transparent to users, these tools cannot foster paradigmatic evolution in accordance with the schemas and schemes we are promoting or to the level for which we are pushing. As such, and until appropriate modeling software and related tools are developed, the use of modern technology in modeling instruction may increase the breadth, but not so much the depth, of covered content in science courses. Thus, and as our experience actually shows in the current state of things, schools that cannot afford such technology do not necessarily suffer a significant handicap.

## 4.5 REFLECTIVE INQUIRY

The paradigmatic evolution targeted in modeling theory requires that students: (a) pick up special skills of a rational nature so that they can efficiently and meaningfully develop model-centered scientific theory and necessary tools, and (b) subsequently develop the habit of systematically deploying these tools and skills within and outside the context of the course they are taking. The most important skills are perhaps those of reflective inquiry. In modeling instruction, inquiry, whether exploratory or inventive, is focused primarily on patterns in the empirical world, and, in this regard, students develop the habit of thinking of any inquiry as being one of either model construction or model deployment. Students develop various inquiry skills through structured learning cycles that are the object of the next chapter. In the process, students are guided to constantly reflect on their own conceptual and paradigmatic profiles and regulate them so as to develop the habit of self-evaluation and self-regulation in an insightful manner.

In conventional instruction, students are wrongly assumed to be ready to assimilate everything dictated to them by teacher or textbook, and to be capable to decipher and coherently organize on their own any information presented to them, in whatever form the information may be presented. Our research has shown that this is far from being the case, especially with *passive* learners who accept at face value anything dictated to them by an authority (Halloun, 2001b; Halloun & Hestenes, 1998). These students are often unable to put their fingers on relevant aspects and decipher presented information in a science course. They blindly memorize lecture notes and textbook statements and problem-solving routines that have been emphasized in teacher lectures. They seldom attempt to piece conceptions together in a meaningful way, or to identify and develop generic tools and systematic schemes for constructing or deploying conceptions. When a teacher offers a piece of information that is at odds with what they think they know, or a problem solution that differs from one that they have produced on their own, they seldom compare their position to that of their teacher in a self-regulation process. Instead, and just like Mazur's student quoted in the introduction of the previous chapter, they evolve into a state of cognitive dissonance whereby they adopt what the teacher dictates only for use in subsequent exams, and they keep what they originally had in mind for use elsewhere.

The situation is countered in modeling instruction by engaging students in well-structured *learning cycles* that treat students as stakeholders in the learning process and that gets them actively engaged in every learning activity that takes place inside or outside the classroom. As described in Chapter 5, each cycle has five stages (exploration, model adduction, model formulation, model deployment, and paradigmatic synthesis) and is devoted to the development of a specific model, along with the necessary conceptions, tools and skills. All along a cycle, individual students ascertain their ideas in light of empirical and rational evidence at their disposal, and regulate them appropriately. By contrast to the target scientific knowledge, students' initial ideas can fall in one of three categories, missing, viable, or naïve (Table 3.1). Accordingly, students are engaged in appropriate activities of reflective inquiry leading, at an intermediate stage, to: (a) the construction of missing knowledge by inference from a natural pattern, (b) the preservation of existing viable ideas that are free of any flaw, (c) the modification of viable ideas that have inherent flaws

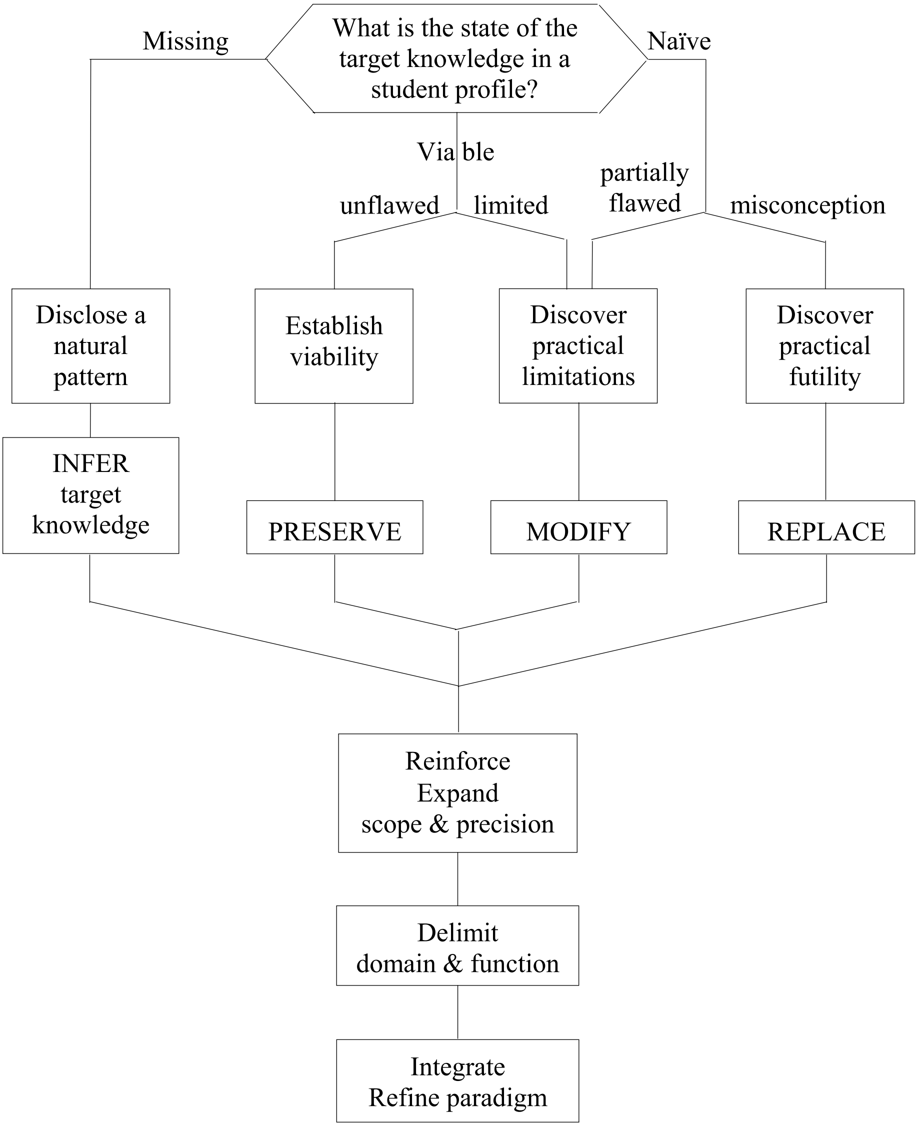


*Figure 4.10.* Self-regulation processes required for paradigmatic evolution.

after realizing their limitations, or (d) the replacement of naïve ideas that are entirely at odds with their scientific counterparts after realizing the futility of these ideas (Fig. 4.10).

The type and level of inquiry and underlying cognitive processes with which students become engaged depends mostly on the state of student initial ideas by comparison to the target scientific knowledge (Table 3.1). When the target knowledge is missing (leftmost branch in Figure 4.10), students go through a process of knowledge *formation*. To this end, and when the target knowledge is a new model, students are presented with a number of empirical situations displaying the same pattern represented by the model, and guided so as to disclose the pattern in question and construct the target model following the model schema. Branching-out and bridging analogies *à la* Clement (1993) are resorted to in the process. All along, students are guided to go through correspondence and coherence assessment to ensure the internal and external viability of the constructed knowledge (§ 3.6).

When the target knowledge has some counterparts in student initial knowledge, one of two situations may take place (middle two branches in Figure 4.10). In the first instance, students already possess the target knowledge in a satisfactory state. Through limited exercises of coherence and correspondence assessment, students establish the viability of what they already know and *preserve* their knowledge virtually the way it already exists in their mind. In the second instance, students' knowledge would be incommensurate with the target knowledge in some or all respects. They would then become engaged in a process of knowledge *transformation* through exercises involving one or more of the three types of assessment discussed in § 3.6 (coherence, correspondence, and commensurability assessment).

In the latter instance, and when student knowledge is not entirely at odds with the target scientific knowledge, knowledge transformation involves the modification or refinement of existing knowledge, mostly through coherence and correspondence assessment. This is the case of viable knowledge of limited scope or of partially flawed naïve knowledge. Students are guided then to modify their knowledge after they realize the limitations or flaws of this knowledge, especially while predicting particular aspects of physical realities that are the object of the target knowledge (Dykstra, Boyle & Monarch, 1992; Hashweh, 1986; Minstrell, 1982, 1989, 1991; Smith, Blakeslee, & Anderson, 1993). A case of viable knowledge with limited scope is manifested when a student believes that Bohr's model of an atom is entirely analogous to the solar system and that it applies to all sorts of atoms. In such a case, the student is guided to realize first to what extent the analogy is valid between the model and the system in question. Then, Bohr's model is gradually modified to embrace all features of the standard model. A case of partially flawed naive knowledge presents itself when a student believes that, under any conditions, the heavier an object is, the faster it accelerates when falling near the surface of the Earth. Such a student is guided to realize through appropriate observations, and/or through rational inference, that, for the same volume, "heavier" objects fall faster in air or through any other fluid, not because of their increased mass but because of reduced air resistance. Subsequently, the student is aware of the fact that the intrinsic properties of a falling object, mass, shape and size included, have no effect on free fall in vacuum.

Knowledge transformation takes a different direction when student knowledge falls under the misconceptions category (rightmost branch of Figure 4.10). A *misconception* is, for us, a naïve conception that is entirely at odds with its scientific counterpart, and that is futile in all practical respects. This is the case, for example, when a student believes that objects fall near the surface of the earth because "air pushes down from above". As already discussed in § 3.6, this is a misconception in at least two respects. First, people who believe this attribute the gravitational field not to the Earth but to air, and/or they mistakenly believe that a gravitational field cannot be manifested in the absence of a medium (as in the case of sound propagation). Second, air resists fall, whether in the form of buoyancy according to Archimedes principle or in the form of drag, and it does not provoke it or expedite it.

Knowledge thus formed, preserved, or transformed is subsequently *reinforced* in the exploration of new physical realities so as, like in the case of Kuhn's normal science, to *expand* its scope and enhance its precision in the real world and realize where its viability ends in this world. Required activities should be contextually rich, and involve both instances and counter-instances of the now viable knowledge, so that, say in the case of a conception, students can: (a) specify what sort of physical realities the conception corresponds to and under what conditions (domain), (b) indicate what questions can it answer and to what degree of precision (function), (c) distinguish primary features from secondary features in physical realities by correspondence to the conception in question, and (d) specify criteria that determine in any subsequent exercise what constitutes reliable evidence for the conception and what does not. Various corroboration, reinforcement and delimitation exercises help students to develop a rich repertoire of familiar situations that they can refer to in subsequent exercises of any nature. Finally, and as noted in Figure 4.10, the reliable knowledge just developed is integrated in the corresponding paradigm, and the paradigm is refined accordingly.

Various forms of knowledge transformation and formation are often negotiated with peers, whether in teamwork or through class discussions. In modeling instruction, we favor a “blend of whole class and small group activity” which Doerr (1996) shows “to focus the students on the questions and bring them back to the essential problem while fostering and encouraging diversity and student autonomy”. Many cognitive scientists and philosophers have long argued with Lakoff and Johnson (1980, p. 230) that “understanding emerges from interaction, from constant negotiation with the environment and other people”, and, with Wartofsky (1968, p. 240), that to “explain something is to have come to an understanding of it in such a way that one can bring another to understand it”. In order to reach such a constructive level, and to avoid dead ends, students cannot be left to interact with one another at their free will and outside a clearly defined educational framework (Hake, 2001; Shore et al., 1992). Peer interaction is encouraged in modeling instruction following explicit guidelines set and enforced by the instructor(s), and to the extent that it helps students reflect on their own ideas and articulate them in meaningful and efficient ways (§ 5.7). Such interaction may take place at the level of an entire class, or within and among the groups of students.

A learning cycle is usually most effective and most efficient when students are engaged in teamwork to develop experiential knowledge. At various points of a cycle, specific matters may be brought up for discussion by the entire class. Similar large-scale peer interaction takes place, and more frequently then, when practical constraints make it impossible for students to develop experiential knowledge. The class resorts then to traded knowledge that may be developed through short lectures and/or demonstrations offered by the instructor in the manner discussed in § 5.7. In either case, discourse is managed mostly following Mazur's norms of "Peer Instruction" (Crouch & Mazur, 2001; Mazur, 1997a & 1997b). The class then becomes "surprisingly animated", and "a minute or so of discussion [may be] sufficient to dramatically improve the level of understanding of the class… But best of all, testing shows that this teaching style [whereby the teacher focuses peer interaction on a specific issue, and manages it according to a preset agenda] engenders a better understanding of the fundamental concepts and discourages a number of bad study habits such as rote memorization and an excessive focus on problem solving" (Mazur, 1997b, p. 983).

Case studies from the history of science come in very handy when no experiential knowledge can be developed in the classroom, especially when it comes to modifying or replacing student knowledge (Fig. 4.10). Research has long shown that student ideas about physical realities are often reminiscent of paradigms that dominated science up to the twentieth century, and especially in the pre-Galilean era. History of science thus offers teachers insights into their students' problems, as well as guidelines for overcoming these problems. A naïve idea can be brought to the surface for evaluation when students are exposed to a historical case underlined by such an idea. Peer interaction is first steered so as to let students analyze the merits of the case in its own historical context, and identify the paradigmatic norms that then guided scientific enterprise. Students are then guided to subject the case to coherence and correspondence assessment (Fig. 3.4) in order to discover the limitations or the futility of the underlying paradigmatic tenet(s) and/or conception(s) (Fig. 4.10). When these two forms of assessment do not lead to regulation of the naïve idea in question, students proceed to explicitly compare the presented case to a modern science alternative offered by the teacher (commensurability assessment in Fig. 3.4) and discover the advantages of the latter.

Galilean thought experiments (Fig. 1.1) offer another suitable venue when students cannot interact with the empirical world, or when such an interaction falls short of developing meaningful knowledge. As Kuhn (1970, p. 88) puts it, "analytical thought experimentation that bulks so large in the writings of Galileo, Einstein, Bohr, and others are perfectly calculated to expose the old paradigm to existing knowledge in ways that isolate the root of crisis with a clarity unattainable in the laboratory". The merits of thought experiments, as well as of historic case studies, are not however restricted to instances where student knowledge is partially or totally incommensurate with scientific knowledge. These two venues are well suited for the development of missing knowledge, as well as for the reinforcement of traded or experiential knowledge, whether formed or transformed, whether in large scale peer interaction or in team work.

- Students are organized into heterogeneous groups of three. Based on appropriate pretests administered at the beginning of a course (e.g., FCI), student competence is ranked high, medium or low. Every competence level is represented in each group.
- Groups are rotated at least twice during a given semester. Student competence levels are reassessed before each rotation.
- Icebreaker, short activities are administered every time new groups are formed so that members of the same group become comfortable with one another. These activities may be more of a social than a scientific nature (introducing members to one another, sharing personal stories or anecdotes, solving a riddle together, etc.).
- Teamwork is collaborative. No particular role is assigned to any group member. Group presentations, class discussion, and assessment exercises are conducted so as to ensure that members of a group behave as equal stakeholders.
- Teamwork is resorted to in various learning activities conducted inside and outside the classroom.
- Interaction takes place between groups at the beginning and end of each activity or phase of a learning cycle, and within groups all the way through.
- The instructor steers teamwork according to a pre-established agenda. S/he sets and enforces regulations and guidelines for interaction within and between groups. S/he sets the objectives of each learning activity and guides it in the manner discussed in the next chapter. Nevertheless, each group has the flexibility to change direction at opportune times in order to efficiently fulfil the goals of a given activity.
- The instructor maintains a log for each group based on modeling schemata and required tools and skills so as to keep track of the evolution of the group and assign appropriate activities to bring all groups to virtually the same level at specific points during the semester (preferably at the end of each learning cycle).
- Regular assessment is conducted for individuals and groups of students.
- The instructor provides, directly or preferably by soliciting student ideas, immediate feedback to keep all sorts of interaction and negotiations in line with modeling theory.

*Figure 4.11.* Some norms and guidelines for team work in modeling instruction.

Modeling instruction is most fulfilling through teamwork conducted inside and outside the classroom in well-defined conditions (Fig. 4.11). For good management of team work, it is preferred that a science class does not exceed ten groups (i.e., thirty students) when only one instructor is in charge of the course, especially when students are constantly engaged in the development of experiential knowledge. Nevertheless, teamwork may be conducted in large lecture halls, but then the focus will be more on short time interactions *à la* Mazur, and far more within groups than between groups. In all cases, the prime concern of teamwork is to foster reflective inquiry through peer interaction. In general, students are more comfortable sharing and discussing ideas with peers than with instructors. With appropriate guidelines and discourse management offered by the instructor, peer negotiations optimize instruction outcomes by allowing individual students to assume control of their own paradigmatic evolution while reducing instructors' involvement.

## 4.6 ASSESSMENT AND EVALUATION

Student self-evaluation and self-regulation are main pillars of reflective inquiry. Individual students constantly reflect on their own knowledge (paradigmatic or conceptual profiles) while engaged in any sort of activity. The self-evaluation process takes a more structured and systematic direction during formal assessment (tests, homework, and other assignments) conducted by the teacher. In modeling instruction, formal assessment, or assessment for short, is not an end in itself, and its main objective is not to ascertain the knowledge state of individual students for grading or ranking purposes. The main objective is to provide individual students with reliable means for evaluating their own knowledge, so that they can reflect on their conceptual or paradigmatic profiles and regulate them appropriately (Halloun, 2004). The locus of control throughout the learning process, and especially through assessment, is thus turned inwards. Individual students take control of their own paradigmatic evolution. They are mainly driven to this end not by the outside authority of the teacher but by intrinsic motivation to transcend their naïve or common sense profiles after detecting their flaws and/or limitations and realizing the value of evolving into the realm of science.

Assessment in modeling instruction differs in many respects from assessment in conventional instruction in order to fulfil the objective above and promote an equitable and meaningful learning experience. From a practical perspective, the two modes of assessment are perhaps distinguished most in the following respects: (a) assessment taxonomy, (b) nature of assessment means, (c) interpretation of outcomes, (d) data management, and (e) feedback.

Assessment in modeling instruction is a *normative* process whereby student knowledge state is measured by comparison to clearly defined norms and standards that are set primarily in terms of: (a) schematic dimensions of the concerned conceptions (i.e., dimensions of the respective modeling schemata, and especially the model schema), at the level of content, and (b) modeling tools, rules and schemes, at the level of inquiry processes. At certain points during the course of instruction, particular student *trends* are also evaluated. Under the label of trends we include learning styles and certain metacognitive aspects like student attitudes toward, and views about, science and science education that are nowadays considered as integral parts of scientific literacy in local and national science education standards, and that are suspected to have an impact on learning outcomes (Halloun, 2001b). Two major steps are taken at the beginning of a course for the normative process to result in reliable (criterial) indicators of student knowledge state (Halloun, 2004):

1. Establish a detailed *taxonomy* of conceptions, processes and trends that would make up the profile that students are anticipated to develop following the completion of a course or of a given part of the course.
2. Set *criteria* that establish whether individual students have actually developed each element of the anticipated profile, and to what extent they have done so.

In parallel, teachers are encouraged to adopt similar taxonomy and criteria for their own practice in the classroom in order to subsequently assess whether they have actually done everything that is necessary for students to develop the anticipated profile. Teachers then assume the role of action-researchers who insure that our promoted paradigmatic profile evolution is being achieved in the most meaningful and equitable ways possible. Assessment of student knowledge is thus for us an integral part of a more comprehensive, and continuous, normative evaluation process that ascertains altogether the merits of modeling theory in science education (Halloun, 2004).

In order for the evaluation process to focus on realistic aspects and result in meaningful outcomes, taxonomy and criteria are *graded* between two crucial levels or *thresholds*:

1. *Basic threshold*. This is the most fundamental level. It corresponds to the *minimum standards* of meaningful understanding that *any* student should meet, irrespective of the initial competence level and interests of the student. This threshold corresponds to *elementary* basic models in a given scientific theory.
2. *Mastery* or *critical threshold*. This is the highest threshold that students need to cross in order to master all fundamental conceptions and processes in a given course. This threshold corresponds to the entire set of *basic* models in the theory of concern.

The efficacy of a given science course, from our point of view, is primarily determined by the extent to which it allows individual students to cross the basic threshold, irrespective of whether or not the course is being taught in the framework of modeling theory. Students' failure to reach the basic threshold is, in general, an indication of deficits in instruction more than anything else. In such an event, teachers ought to significantly reconsider their own practice. The more students are capable of crossing the basic threshold and getting close to the critical threshold, the better the efficacy of instruction. In an ideal and truly equitable situation, all students willing to invest the necessary efforts should actually be capable of reaching the critical threshold. In traditional classroom settings, this threshold is however reserved for the minority of students who enter a science course with high competence. This inequity case is significantly resolved in modeling instruction (Halloun, 1984, 2004).

A diversified array of assessment means is required for normative evaluation with graded taxonomy and criteria to bring about reliable indicators of the state of student paradigmatic profiles. Conventional instruction of lecture and demonstration relies heavily on paper-and-pencil tests and homework for assessing student understanding (or rather recall capacity) of mostly traded knowledge covered in science courses. Such tests and homework are usually about solving problems falling almost exclusively in the application category of deployment activities (Table 4.1). Teachers choose problems not necessarily to cover a specific taxonomy of conceptions or processes, but mostly to cover what they arbitrarily think are the most important scientific theoretical statements and/or problem situations discussed during instruction. Students can often solve any administered problem by reproducing a specific problem-solving routine that they have learned by rote from teacher or textbook, and they seldom evaluate their solutions following systematic norms and criteria. Development of supposedly experiential knowledge is often restricted to traditional laboratory experiments in conventional instruction. Assessment of such knowledge is usually limited to laboratory reports. These reports have a rigid structure imposed by the teacher, and they are mostly about pre-designed experiments conducted following cookbook recipes, not to construct particular models or other conceptions, but to verify traded knowledge discussed separately in a corresponding course.

Modeling instruction relies on similar assessment tools, but only as part of a wider battery of assessment instruments pertaining, in balanced ways, to experiential and traded knowledge, and to individual and group activities (Halloun, 2004). Assessment means include: (a) oral and written reports on observation of physical realities displaying specific patterns, on particular student-designed experiments, on field trips, on case studies from the history of science or daily life, as well as (b) the design or construction of simple physical realities for model reification. Paper-and-pencil tests and homework and other assessment means just mentioned are all based on well-defined taxonomies laid out in the manner described above. However, the focus of assessment in a modeling course is more on the big picture, i.e., on the conceptual organization and merits of scientific paradigm and theory (Fig. 2.5) and especially on models (Fig. 4.12). For, to “learn something one may have to learn how to perform a certain task in a certain way, in accordance with certain rules or canons of performance… (at the lower end of the scale)”, but most importantly one has to come “to understand the framework within which [performance is approved, and] the approval is understood in terms of its reasons (at the upper end of the scale)” (Wartofsky, 1968, pp. 242, 243). That framework is offered by scientific theory and paradigm. Moreover, assessment is as much about model construction as model deployment (Fig. 4.12). The latter cover the four categories discussed in § 4.3. With insightful self-regulation as a main objective of assessment, students are constantly urged to approach any assessment task from different perspectives and in different ways, and to evaluate their work from rational and empirical perspectives (details in § 5.5 and § 5.6).

Interpreting student performance on a given assessment exercise is a delicate issue in any educational enterprise. Assessment in conventional instruction is underlined by the assumption of transparency, i.e., the assumption that performance on paper-and-pencil instruments (mostly problem-solving tests in science courses) reflects what a student actually knows about the subject of assessment. Subsequently, student scores on such tests are being interpreted as sufficient and reliable indicators of student understanding of covered materials. Educational research has long shown that this is far from being the case. Most students who are capable of doing well on traditional paper-and-pencil tests do so because they are capable of recalling specific statements or problem solutions that they have already memorized by rote. The same students are by and large incapable of answering qualitative questions about everyday life situations pertaining to the same conceptions that they manage to manipulate correctly in conventional tests (Halloun and Hestenes,

1. Subsidiary model analysis and construction of the target model.
2. Application of the model requiring partial reproduction of its schematic dimensions in familiar situations, first qualitatively then quantitatively.
3. Application and then analogy deployment of the model in describing, explaining, and predicting physical realities in unfamiliar situations (qualitatively and quantitatively).
4. Reification of the model in the control of existing physical realities, and in the design and construction of new referents.
5. Ascertaining assumptions underlying the viability of the model within its own scope.
6. Establishing norms and criteria for model corroboration and falsification.
7. Refinement of the model as a consequence of all the above.
8. Comparison of the model to other models in the same theory.
9. Extrapolation of the model in constructing new conceptions.

*Figure 4.12.* Partial taxonomy for assessing student knowledge of a model. Notice that assessment pertains to model construction (1 and 7) as well as to model deployment, and that the level of assessment gradually increases from reproduction or recall (2) to creativity (4 and 9), and from conceptual building blocks (2) to underlying tenets and viability conditions (5 and 6).

1985a; Mazur, 1997a; Tobias, 1990). Lack of reliability of conventional instruments finds its roots in many aspects extending from deficient taxonomies and validity problems to student inability to express themselves adequately in conventional settings. That is why, in the latter respect, modeling instruction diversifies assessment tools so that a student can always find at least one suitable way to communicate what s/he actually knows about the subject matter. In any case, student performance on a given assessment instance is never considered as a sufficient indicator of the student's knowledge. Such an indicator can come, and only to a certain degree, from a diversity of tasks, contexts, and assessment tools.

Outcomes of student assessment are considered as much an indicator of instruction viability as of student knowledge state. As mentioned above, teachers in modeling instruction are constantly in a state of action-research so that, like their students, they continuously evaluate their teaching practice in order to regulate it in the direction of facilitating equitable and meaningful learning. To this end, and in the light of student performance, teachers reconsider their practice, if needed, especially with regard to the choice of appropriate learning activities and the way activities need to be conducted. A helpful tool in this regard is a log of student evolution.

A *log of student evolution* constitutes an efficient way for managing assessment outcomes and making appropriate inferences. The log is meant to allow teachers to trace the performance of every student throughout a given learning cycle, and student evolution across consecutive cycles. Different teachers may adopt different schemes for such a log. A log that has proven to be useful and efficient for many teachers is one consisting of a number of grids or spreadsheets where the teacher can check whether every student has satisfactorily accomplished individual elements of at least two taxonomies, one conceptual and one procedural. The conceptual taxonomy pertains to the four schematic dimension of a model (or those of a concept). The other taxonomy pertains to tools and skills required for accomplishing tasks of Table 4.1 and Figure 4.12. The process evaluation grid may be complemented with subsidiary grids pertaining to schemes like those of Figures 4.6 and 4.8. In the case of a deficiency in a given cell of the log, and depending on the situation, the teacher may assign follow-up, remedial activities to individual students or groups of students falling behind the rest of the class, or to the entire class.

Immediate feedback and follow-up activities are crucial in modeling instruction. Feedback is usually solicited in many forms from individuals or groups of students. Teacher intervention is kept for a last resort. Teacher feedback normally comes in the form of guidelines for students to conduct on their own coherence, correspondence and then commensurability assessment. The teacher refrains as much as possible from pointing out directly what is wrong in student work and providing scientific alternatives as correct answers or solutions. When all other attempts fail to lead students to self-regulation, the teacher presents the scientific position as an alternative for students to compare to their own positions, without telling them that this is the correct position or even a position better than theirs. Students are first given the chance to discover this fact by putting the new alternative to rational and empirical tests. If this does not work, the teacher may then intervene to show how the scientific alternative is a better choice (§ 5.7).

Immediate feedback is meant primarily to enhance self-evaluation and self-regulation while in process. The two reflective processes may be further facilitated with appropriate follow-ups after class. A follow-up activity may be about the same task being assessed or it may involve a new task. In the former case, the work of a group (or student) is returned to this particular group for reconsideration in light of negotiations that took place in the classroom. The work may instead be returned to a different group for assessment along the same guidelines mentioned above for peer negotiations. Alternatively, and when reassessment of the same task cannot lead to the desired outcome, a new task may be assigned to complement the prior task and make it possible to achieve the goal of instruction.

Efficiency of feedback through peer discussion is enhanced by asking each group (and sometimes each student) to expose their work (designs, experiment reports, problem solutions, etc.) in a way that is accessible to all peers. One efficient way to do so is through the use of whiteboard in the manner discussed by Wells, Hestenes, and Swachammer (1995). Whiteboards are displayed all around the classroom so that every student can have a clear view of them all. Each group (or student) is asked to concisely present its work while justifying various elements. Once all groups, or a representative sample, are finished with their presentations, members of a given group are asked to discuss their work with other groups, mainly those who express opposite viewpoints. Each group is first asked to criticize opposing views and then to put themselves in the shoes of others so as to find the merits of these views. Mutual understanding between people who do not share common knowledge is possible, according to Lakoff and Johnson (1980, p. 231), "through the negotiation of meaning. To negotiate meaning with someone, you have to become aware of and respect both the differences in your backgrounds and when these differences are important." Such negotiations help students to develop critical thinking, and especially systematic and objective assessment criteria. These criteria are meant not to lead to an absolute judgment about the various alternatives being assessed, so that one alternative is adopted as being right while others are dropped as being wrong. They are rather meant to lead to a relative judgment whereby the merits and limitations of different alternatives are weighed in order to adopt the most viable alternative in the given circumstances.

Feedback through peer negotiations is more delicate than through direct teacher intervention. In both instances, but especially in the former case, the teacher has to pay special attention to keeping student discourse as scientific as possible with respect to both expression modes and argumentation content. In this direction, the teacher needs not only to help students ascertain the face validity of their expressions and the merits of their arguments. S/he especially needs to ensure that students actually mean what they say, and that what they say (or write in an assessment exercise) reflects what they actually know or do not know. For instance, a student may sometimes refer to a concept that s/he has in mind with a term that is different from its scientific label (e.g., a student speaking of "force" and actually meaning "energy", "power", "impetus", or even a kinematical concept). Teacher and peers may then be misled twice: when the student uses the appropriate label to refer to an inappropriate conception s/he has in mind, and when the student uses an inappropriate label to actually refer to the appropriate conception. The problem may be countered by asking students, even in formal assessment exercises, to express their ideas in more than one form, i.e., verbally, pictorially and mathematically, and to assess the mutual coherence of the various expressions. The issue of multiple expression shares in this respect the same concerns and prospects we expressed with respect to multiple representation in § 4.4.

Many important aspects of normative assessment and evaluation have not been the object of our discussion. They are beyond the scope of this book. Some of these aspects are treated in a related publication (Halloun, 2004), and others are commonly treated in textbooks concerned with performance assessment, course and curriculum evaluation, or psychometrics. We have limited our discussion to those aspects that are of particular concern to modeling instruction and that somewhat distinguish this form of instruction from conventional instruction and some other forms of instruction. The same is true for other matters discussed in this chapter. These distinguishing aspects are further developed in the next chapter, especially from a practical perspective that bears directly on model construction and deployment through a particular learning cycle.

## *Chapter 5*

# LEARNING CYCLES

Modern educational trends virtually all call for “student-centered” education. They all converge on recognizing the importance of student’s initial knowledge state in any learning process and the need for individual students to submit their own ideas for self-evaluation and self-regulation. However, these trends diverge significantly in their expectations about student ideas and about the nature and course of the self-evaluation and self-regulation processes. They especially disagree on the appropriate directions that the learning process should take in current school settings. Some trends advocate for students to rely entirely and exclusively on their initial ideas in any learning activity, while others recognize, as we do, that these ideas may, or may not, serve as a stepping-stone in the right direction. Some call for letting students on their free will in the entire learning process, from laying down objectives and agenda to setting course and even outcomes, while others recognize, like we do, that such a track is practically inefficient if not futile. Some, unlike many others including ourselves, have even gone as far as rejecting the notion of “teaching” or “instruction” altogether as being antagonist to “learning”, and concentrating on “learning” in a way that denies teachers any active role in the process.

We do recognize that given the diversity in student initial paradigmatic profiles and various factors that control the learning process, there is no single approach that is equally effective in bringing all students to the paradigmatic evolution called for in modeling theory. Yet we also recognize that it is not feasible for any single teacher to allow different students or groups of students to follow learning paths that are significantly different from one another, at least not in the current or even prospective state of things in our schools, colleges and universities. The alternative, as our experience and that of numerous teachers in the past two decades shows, is in mediated learning. It is in the sort of student-centered instruction that does not let students wander on their own in the learning process, but that infuses instead some structure in this process. The alternative, as we see it, is more specifically in learning cycles that engage students in model construction and deployment in accordance with the norms of scientific inquiry, while they are still afforded the opportunity of meaningful and efficient self-evaluation and self-regulation.

## 5.1 MODELING CYCLES

The idea of a learning cycle as a structured, mediated form of learning was first proposed by Karplus (1977), primarily for teaching *concepts* of elementary school science within the framework of Piaget's theory of intellectual development. Karplus' "learning cycle consists of three instructional phases that combine experience with social transmission and encourage self-regulation… These three phases are *exploration*, *concept introduction*, and *concept application*". In the first phase, students are invited to *explore* an unfamiliar empirical situation in ways that "raise questions or complexities that they cannot resolve with their accustomed patterns of reasoning… As a result, mental disequilibrium will occur and the students will be ready for self-regulation". A new concept or principle is *introduced* in the second phase to resolve the problem at hand, and then *applied* in the third phase where "familiarization takes place as students apply the new concept and/or reasoning pattern to additional situations". Social transmission (i.e., teacher lecture for transfer of traded knowledge) is reduced in the first stage. It reaches its peak in the second phase where teachers reclaim their conventional role of lecture and demonstration, and it winds down in the third phase where "physical experience with materials and social interactions with teacher and peers play a role" (Karplus, 1977).

Karplus' learning cycle was conceived by many science educators not only as a method of teaching, but also as "a curriculum organization principle" (Renner, Abraham and Birnie, 1985). The cycle was implemented in a variety of ways throughout the years with relative success in various scientific disciplines and at all educational levels. Some variants of the cycle were specifically designed for certain forms of modeling by Clement (1989), Hestenes (1987, 1992, 1997) and White (1993).

Following the analysis of a number of experts' schemes for solving physics problems, Clement (1989) proposed a "model construction cycle" consisting of three phases: hypothesis conjecture, evaluation, and modification or rejection. The cycle begins with the *conjecture* of a hypothesis about some observed phenomenon, often by analogy to familiar situations. The hypothesis undergoes a series of empirical and rational tests in the second or *evaluation* phase. As a consequence, the hypothesis may be *modified* or *rejected* altogether in the third phase. Unlike Karplus' cycle, Clement's cycle is not linear. One may go endlessly back and forth between the three phases of the cycle so that hypotheses "undergo a *series of successive refinements*". Clement called on science educators to design instructional activities so as to turn his cycle into a learning cycle for students to develop scientific models and skills of scientific inquiry.

White (1993) designed a curriculum to enable sixth graders to "develop a conceptual model that embodies the principles underlying Newtonian mechanics, and to apply their model in making predictions, solving problems, and generating explanations". The curriculum, called ThinkerTools, includes "computer microworlds," which simulate real-world situations underlined with causal reasoning about force and motion, and making use of iconic representations. The curriculum activities are embedded in a four-phase instructional cycle (motivation, model evolution, formalization, and transfer). In the *motivation phase*, students are asked to make predictions about simple real-world situations. In the *model evolution phase*, students work in groups on a series of activities of increasing complexity so that they discover the causal principles and concepts embedded in a given computer microworld. In the *formalization phase*, students formalize what they have learned so far "into a law that describes the behavior of the microworld". To facilitate the process in the early stages of the curriculum, and until they can go through on their own, students are "presented with examples of 'good' and 'bad' laws before they have to construct their own". In the *transfer phase*, students first apply the formalized law to the predictive questions asked in the first phase of the cycle, and then apply the law to new real-world situations. Throughout an instructional cycle, students proceed through a *gradual* process of *conceptual change* by "building on correct intuitions…, refining simplified conceptual models into more sophisticated models and generalizing the evolving conceptual model to a wide range of contexts" (White, 1993).

Wells, Hestenes and Swackhamer (1995) proposed a "modeling cycle" for high school and college physics students that "can be regarded as a refinement of the *learning cycle* developed by physicist Robert Karplus... The modeling cycle has two stages, involving the two general classes of modeling activities: *Model development and model deployment… Stage I* is designed to lead students systematically through the four main *phases of model development*: *description*, *formulation*, *ramification* and *validation… Stage II* is devoted to *deployment* of the model developed in Stage I to a variety of new physical situations in a variety of different ways. This helps to free the students' understanding of the model from the specific context in which it was developed". Hestenes (1987, 1992) had already laid out in detail the four phases of model development, specifically in the context of problem solving. In the *description phase*, individual systems and phenomena involved in a given empirical situation are isolated, and the corresponding primary properties are identified and mathematically represented. In the *formulation phase*, properties thus identified are related to one another with appropriate laws, and an abstract, mathematical model is formulated. The model is analyzed in the *ramification phase* so as to come up with a solution to the problem at hand, interpret the solution and consider its implications. The model and ensuing solution are assessed in the *validation stage*. "Throughout the modeling cycle the teacher has a definite agenda and specific objectives for every class activity, including concepts and terminology to be introduced, conclusions to be reached, issues to be raised, and misconceptions to be addressed. Though the teacher sets the goals of instruction and controls the agenda, this is done unobtrusively. The teacher assumes the roles of activity facilitator, Socratic inquisitor, and arbiter (more the role of a physics coach than a traditional teacher). To the students, the skilled teacher is *transparent*, appearing primarily as a facilitator of student goals and agendas". Teacher involvement decreases progressively from one modeling cycle to another as students gradually "become more independent in formulating and executing tasks and more articulate in presenting and defending their points of view." (Wells, Hestenes, and Swackhamer, 1995).

Modeling instruction, as we see it, mediates learning in successive learning cycles. A science course is completed through a limited number of cycles, each cycle being devoted to the development of a particular model and of appropriate modeling tools and skills for which the model of concern offers the most convenient context. A modeling learning cycle is conducted according to the canons of modeling theory laid out in the previous chapters. It draws in some respects on the four learning/modeling cycles described above, as well as on other successful practices in science education, so as to help in fulfilling the credo of modeling theory in the most effective and efficient way possible. As presented in this chapter, a modeling learning cycle has the following major characteristics.

- *Structured five-phase cycle.* A modeling learning cycle is devoted to the development of a specific model. A cycle consists of five consecutive phases: exploration, model adduction, model formulation, model deployment, and paradigmatic synthesis (Fig. 5.1). The five phases are discussed in the following five sections. One cannot proceed to a given phase, or to given stage within a phase, before going through the preceding ones. However, one can go back at any time to a prior phase to reconsider problematic issues. A cycle ends with a synthesis that delimits the scope of the newly developed model and paves the way for the development of a new model in a new cycle.
- *Transparent, realistic objectives*. Each learning cycle has a well-defined objective: development of a specific conceptual model by correspondence to a particular pattern in the real world and in conformity with a particular scientific theory. Students set the scope of the desired model in the first phase of a cycle and then proceed to develop the model accordingly. The broad objective of the series of cycles in a given science course is to help all students to cross the critical paradigmatic threshold defined by the set of basic models in the course. As discussed in § 4.6, the threshold is within reach of any student willing to make the necessary effort, and as such it is an indicator of equity as well as of effectiveness of instruction.
- *Middle-out, progressive cognition.* Generic canons of a scientific theory and various conceptions, tools and skills associated with

the theory are all developed within the context of, and for the purpose of developing, models of the theory (primarily basic models). A learning cycle starts with subsidiary models and it evolves so as to progressively encompass all schematic aspects of the target model and broaden its scope. Like the model, any new conception required for model development is invoked by necessity, and it is then gradually formalized, beginning with a nominal form of the conception. Accordingly, the complexity of the model gradually increases in the process and so do cognitive requirements.

- *Didactic transposition*. Given the limitations of student paradigmatic profiles and of available resources in the classroom, students cannot develop a model, or any other conception, tool or skill, in its full scientific rigor or thoroughly following the canons of scientific inquiry. In modeling learning cycles, didactic

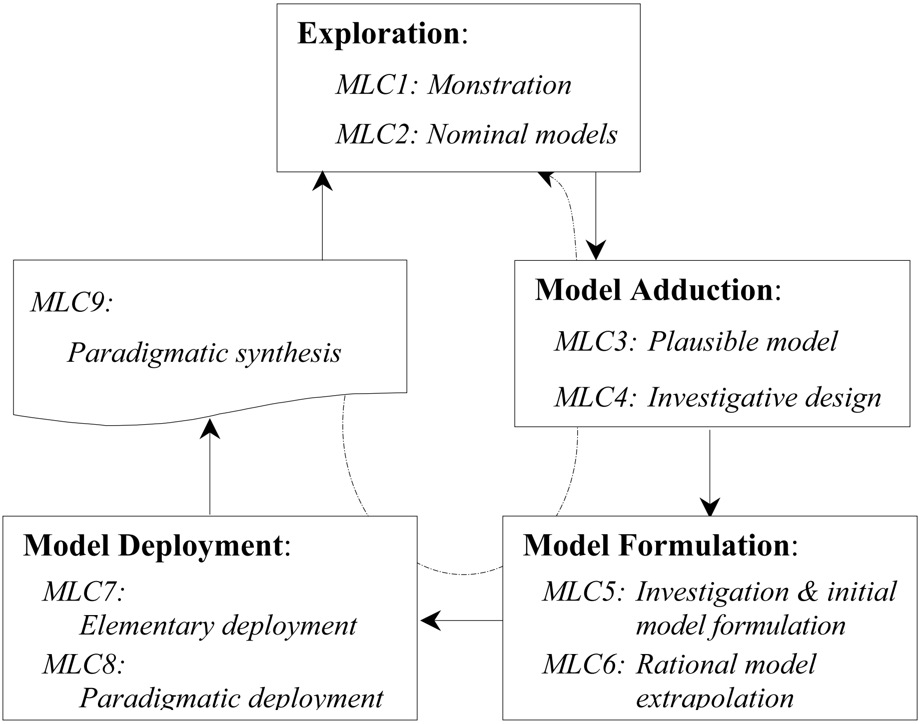


*Figure 5.1*: Modeling learning cycle.

The dashed, curved arrow indicates that one may go back to any preceding phase as a result of rational and empirical evaluation that takes place continuously throughout the cycle.

transposition follows, instead of a piecemeal, episodic approach, model-centered epistemology and modeling methodology for the development of mostly experiential knowledge so as to significantly preserve the rigor of student inquiry and its products.

- *Product-process balance.* A learning cycle is equally concerned with the schematic dimensions of a particular model (product) and with underlying paradigmatic canons and related tools, schemes and rules. Various model construction and deployment activities are chosen so as to maintain a balance between exploratory aspects (description, explanation, prediction of a pattern) and inventive aspects (pattern reification) of modeling inquiry.
- *Transferability of modeling kits.* A modeling kit (tools, schemes and rules) that students develop within the context of a given model in a given learning cycle is mostly generic and transportable within and outside the discipline of instruction. By the time they reach the critical paradigmatic threshold, students muster sufficient competence and autonomy to deploy modeling kits that they have so far put together in the development of more complex models, and to transfer those kits to other courses.
- *Resolution of the breadth-depth paradox.* A limited number of learning cycles is needed to cover models around which a science course can be structured. Unnecessary redundancies are eliminated so that coverage at the level of both content and process is at least as broad in modeling instruction as in conventional instruction, and that it gets deeper under the modeling approach than under the conventional one.
- *Insightful, reflective dialectics.* To the extent that is possible, students are driven, in every stage of the learning cycle, into a state of cognitive disequilibrium (or conflict) so that they realize the necessity for paradigmatic evolution through self-regulation. Whenever necessary, the process begins by breaking major paradigmatic barriers. Students then, and subsequently, reflect on their own ideas, and proceed through rational-empirical dialectics pertaining to either mode or all three modes of assessment (coherence, correspondence and commensurability). Self-evaluation and self-regulation proceed in an insightful way that helps students to realize, whenever such is the case, the limits of viability or the futility of their own ideas and the advantage of

transforming their paradigmatic profiles in the direction of scientific realism.

- *Conditional anchoring*. Paradigmatic profile evolution involves transformation of some ideas already existing in student paradigmatic profiles, as well as formation of entirely new knowledge. Anchoring onto existing ideas, whether for knowledge transformation or formation, can take place as long as existing ideas have at least some limited viability. Otherwise, knowledge development may take place from entirely new grounds and without explicit anchoring onto existing ideas.
- *Locus of control*. Students are stakeholders in the learning process. They assume responsibility of their own paradigmatic evolution, as individuals or as groups. They negotiate with the teacher and among themselves the agenda pertaining to each phase of the learning cycle, as well as the norms and criteria that govern the successful completion of any activity. Any learning activity, including formal assessment exercises, provides means for student self-evaluation and self-regulation.
- *Teacher-mediated learning and timely feedback*. Teachers maintain a pivotal role in the learning process, primarily a mediation role that takes the form of moderation, arbitration, or scaffolding (§ 5.7). Students are not expected to complete any learning cycle entirely on their own. Feedback is often needed, among others, to: (a) answer specific questions asked by students, (b) enhance student discourse and negotiation of ideas, (c) bring off-course students back on track, (d) reinforce and expand good student ideas. Teachers provide feedback in a timely manner, either by soliciting student ideas or, when this fails, by guiding their students to ask appropriate questions and answer them properly. When all other attempts fail, teachers may ask the proper question and even provide the proper answer, but only in the form of an alternative that students need to assess in order to be convinced of its viability on their own.

A modeling learning cycle (hereafter referred to as MLC, or learning cycle, or modeling cycle, for short) as discussed in this chapter represents, according to our experience in teaching science and especially physics at the secondary school and college level, the most effective learning cycle for the comprehensive development of a particular model. The cycle may be easily adapted, at any educational level, for the construction of individual concepts or laws within the context of appropriate models or theory. Depending on the integral complexity and the projected scope of the target model, a cycle may require five to ten 50-minute periods. Should any practical constraints prevent a teacher from proceeding through any cycle in the manner described below, some of the stages, particularly MLC3, MLC4 and MLC5, may be curtailed but never circumvented. Curtailment may be such as to develop in these stages some but not all laws (and other theoretical statements) in the structure of a model, and then develop the other laws by rational extrapolation in subsequent stages, especially in MLC6, in the manner discussed in § 5.4.

## 5.2 EXPLORATION

The exploration phase of a learning cycle proceeds in two stages, monstration and nominal models proposition (MLC1 and MLC2 respectively in Fig. 5.1), so as to induce students to construct a new model of well-defined scope and then roughly consider possible candidates in this direction. A cycle begins by bringing students to a state of cognitive disequilibrium in MLC1 whereby they realize (details below): (a) the inadequacy of existing viable knowledge (mostly models constructed in previous cycles) for describing, explaining and/or predicting in some respects a presented pattern that is outside the scope of prior knowledge, and thus (b) the necessity to construct a new model in order to come up with the correct inferences about the newly presented pattern. Construction of the target model begins in the second stage of the cycle (MLC2) with rough subsidiary models. In subsequent phases, students are brought closer and closer to the target scientific model through *progressive refinement or approximations*. For, according to Bunge (1973, p. 169), things are knowable only through such a process “rather than exhaustively and at one stroke”.

Conventional science instruction follows by and large a deductive approach whereby a new theoretical statement like a concept definition or law statement is first enunciated and then “explained” by the teacher. The statement is first inferred, often by deduction from old conceptions, and then it is applied in a series of exercises. In contrast, many modern educational trends advocate instead an inductive approach whereby students are first engaged in a series of empirical observations and/or experiments so that they become exposed to comparable data in a diversity of empirical contexts and then induce the appropriate conceptions. However, some educators have argued that while the conventional, deductive approach cannot bring about meaningful learning experience for science and mathematics students, the alternative, inductive approach as advocated remains inefficient in this regard. The same educators have shown that an *intermediate* approach is significantly more effective, which starts with partial and incomplete models, whether subsidiary, analogue, or iconic, and which proceeds so as to allow students to progressively refine these models to a satisfactory state (Clement, 1989; Lochhead, 1985; White, 1993; White & Fredericksen, 1990). Our modeling cycles follow the latter approach.

### *MLC1. Monstration:*

The first stage of a learning cycle overlaps with the last stage of the cycle that students have just completed. As we shall see in § 5.6, the synthesis of a just completed cycle (MLC9 in Fig. 5.1) involves delimitation of the viability of a newly developed model, a process that includes identifying instances situated outside the scope (domain and function) of the model. Some of these instances are chosen so as to make the case for the model to be developed in the subsequent cycle. Students are first exposed to such instances in the classroom by "monstration", a term originally coined by Joshua and Dupin (1989, 1999) to denote a constrained form of conventional teacher demonstration that is intended to help students to come up with a problem proposition leading to the construction of a new abstract model.

Depending on the convenience of the situation, *monstration* exercises in our learning cycle may come in the form of empirical demonstrations, video clips, empirical case studies, computer simulations, and/or Galilean thought experiments, all about a *new pattern* that is the object of the target model. To the extent that is possible, and for motivational purposes, monstration exercises are chosen so as to pertain to everyday life situations on the one hand, and to invoke, on the other, a state of cognitive disequilibrium. In the latter respect, monstration has, for us, two complementary objectives. First, it aims at helping students to realize that their existing knowledge, and especially knowledge consisting of viable models, is inadequate for studying a newly presented pattern. Second, it is intended to roughly pinpoint some characteristic features of this pattern and delimit, to a first approximation, the scope of the target model. For students to be able to fulfil these objectives unambiguously, monstration should satisfy at least the following four conditions (based, in part, on the work of Joshua & Dupin, 1989, pp.20, 21):

1. Monstration should make the problem transparent to students. To this end, and whenever possible, students should be presented with situations that are familiar in some respects. Otherwise, monstration runs the risk of preventing students from identifying novelties that constitute the core of the problem at hand.
2. Monstration, however, should not be restricted to familiar features so that it would not run the same risk of preventing students from identifying the nature of the pattern and the problem in question (i.e., the need for a new model of particular scope).
3. Monstration is not an end in itself; it aims at bringing students to a state of cognitive disequilibrium and evoking in their minds the need for a new model to be mapped on the presented pattern. To this end, monstration should bear on primary features of the new pattern in the simplest ways possible.
4. Monstration does not demonstrate all there is about the pattern or the model under study. It mostly calls students' attention to the existence of the new pattern and thus to the need for constructing the representational model.

The nature of monstration exercises and the extent to which they need to be diversified depend on students' paradigmatic profiles, and primarily on the repertoire of basic scientific models they have already developed in a given course. When this repertoire is rich enough, a single exercise may be sufficient to fulfill monstration objectives. Otherwise, and especially at the beginning of a course when students would not yet have formed an adequate paradigmatic framework, monstration exercises need to be multiplied just the way scientists multiply exercises of *exploratory experimentation* with subsidiary, rudimentary models when they develop an entirely new theory.

Ribe & Steinle (2002) classify scientists' experimental practice into two categories, one governed by the "standard theory-oriented experimentation view", the other by "exploratory* experimentation".

According to the standard theory-oriented experimentation view, "it makes sense to perform an isolated experiment, and in particular an *experimentum crucis*, designed to judge between competing hypotheses" [à la Galileo or Newton]. This approach "is of little help" when "studying complex systems that consist of numerous interacting elements". Scientists then "often start with a multitude of empirical findings whose interconnections and underlying principles are unclear. They must use experiments not so much to demonstrate propositions as to develop the concepts needed to make sense of multiplicity". "Exploratory experimentation typically comes to the fore in situations in which no well-formed conceptual framework for the phenomena being investigated is yet available; instead, experiments and concepts codevelop, reinforcing or weakening each other in concert".

A typical monstration exercise has two parts. The first part deals with a familiar situation that falls within the scope of previously constructed models. The second part deals with the same situation altered in a way that it can no longer be explored with any of these models. The first part is intended to satisfy the first monstration condition mentioned above by helping students to associate themselves with familiar model referents and realize that the target model is within their reach. The second part is meant to satisfy the second monstration condition by showing students that, given some alterations in the state of the referents at hand, their existing models will fall short of adequately describing, explaining, and/or predicting the altered state.

Monstration exercises have limited requirements at this stage, in line with the third condition mentioned above. The second part of a monstration exercise is designed in such a way as to direct students' attention to the possible domain (pattern and possible reference class) and composition of the target model, and to help them to stipulate the desired function of the model (description and/or explanation of a particular *state*, a particular structure or behavior, of certain physical systems) without going into any detail pertaining to any facet in model structure at this point. In the current state of things, all models that need to be constructed in a given science course belong to the same theory or to related theories in such a way that various models may

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* Note that «exploratory» is used here in a narrower sense than our use of the term in "exploratory" inquiry, research or investigation.

have the same reference class and the same or parallel descriptors may be used in the composition of various models. For example, introductory high school or college mechanics courses deal with Newtonian particle models of translation (Fig. 2.6) and/or Eulerian models of rotation. Composition of all Newtonian models is made up of the same descriptors (e.g., kinematical concepts of linear velocity and acceleration, and interaction concept of force). The same goes for Eulerian models whose descriptors can easily be derived from those of Newtonian models (e.g., kinematical concepts of angular velocity and acceleration, and interaction concept of torque). As such, the reference class of a model that is the target of a new cycle may be virtually the same as that of preceding models, and the difference in model domain would be restricted to referents' behavior and not constitution. Furthermore, the composition of the target model may virtually consist of the same descriptors as prior models, and the difference in model composition would be restricted to the way these descriptors change.

Such a state of affairs makes it easy to satisfy all monstration conditions, and especially the first two. Take, for example in introductory physics courses, the case of monstrating the Newtonian model of the bound particle in uniform circular motion. This model is, in modeling instruction, the object of the third or fourth learning cycle in a series of five cycles devoted to the development of basic models of Newtonian theory of mechanics (Fig. 2.6). At least two basic particle models are thus developed before the bound particle model in question; these are the free particle model and the uniformly accelerated particle model. A monstration exercise pertaining to the target bound particle model consists of showing students an object in three-phase motion. In the first phase, the object is driven in linear uniform or uniformly accelerated translation on a horizontal surface. In the second phase of the motion, an additional constant force is applied at an angle to the object so that it changes direction and undergoes a parabolic, uniformly accelerated motion. In the third phase, constant force(s) previously applied are suddenly replaced with a central force that puts the object in uniform circular motion. Students are asked to observe and comment on the three phases of the motion. The first two phases of the motion fall within the scope of previously developed Newtonian models. Students should thus be able to answer any question about them, be it qualitative or quantitative, of a descriptive, explanatory or predictive nature. The third phase falls outside the scope of these familiar models, and students are not expected to answer precisely just any question about it. This phase presents the sort of novelties required in the second monstration condition.

The teacher starts a monstration exercise by showing an appropriate system or phenomenon for a few minutes like the three-phase motion just described. S/he then follows up with simple questions asking students to *compare* various parts of the system or phases of the phenomenon, and possibly to make some reasonable predictions about each part or phase. Questions are constrained at this point so as only to allow students to realize empirical and rational novelties in the situation while making use of familiar depictors and descriptors and representation modes. Typical questions follow, some phrased in the context of our example about the three-phase motion:

- What is the demonstrated situation about?
- What systems and phenomena are involved in the situation?
- What is the state of every system? Is it conserved throughout the demonstration, or does it change? If so how?
- What concepts are necessary to describe and/or explain the state of each system? In what reference system?
- How can these concepts be represented? In what coordinate system?

In the particular case of the three-phase motion:

- What is the trajectory of the moving object? How does its position vary throughout its motion?
- How does the velocity of the object vary (in each phase of the motion)?
- What could possibly maintain the motion of the object (in each phase)?
- How does the motion evolve beyond the last shown instance: (a) should the same conditions be maintained, and (b) should these conditions change one way or another?

Students are not anticipated at this stage to answer quantitatively and precisely all questions, especially questions like those pertaining to the third phase of our example. It is enough at this stage that students realize that, say, in the third phase, and unlike the preceding two phases, the velocity vector changes with a nonconstant acceleration, and that a variable force is required to maintain the motion. Ultimately, the discussion could possibly be pushed to the extent that students realize that, in the third phase, both acceleration and force are oriented towards a fixed point, but not any further.

Monstration of a new cycle normally takes place towards the end of the period devoted to the last stage of a preceding cycle. Depending on the complexity of the monstration exercise, students may be asked to answer respective questions in the classroom immediately following teacher demonstration, or they may be given some time for preparation and then come back the following period with their answers. Student answers are exposed and negotiated at this point under teacher *moderation* in the manner discussed towards the end of this section.

Once students become aware, at least in part, that they are up to a new pattern, they are driven into a state of cognitive disequilibrium (à la Dewey or Piaget) whereby they realize that their existing knowledge is insufficient to deal with the pattern in question, especially the part of this knowledge consisting of previously developed scientific models. To this end, monstration exercises are further developed and/or supplemented with other exercises whereby students are asked certain quantitative questions about the pattern (mostly of a predictive nature), questions that could not possibly be answered by deploying the structure of any of the old models. Students are thereby intentionally driven by the teacher to a state of *impasse* that would convince them of the *need* to construct the target model.

Students may sometimes not be easily convinced of the inadequacy of prior knowledge. In such an event, the teacher would ask them to use this knowledge for making certain predictions about the new pattern and compare their predictions to actual empirical data. For example, many students of introductory physics courses have the tendency to deploy equations of motion of the uniformly accelerated particle model (Fig. 2.4b) to any translational motion. These students would be asked to deploy the equations in question in the monstration exercise given above about the three-phase motion so that they be convinced that these equations do not allow them make correct predictions in the third phase of motion.

Student state of cognitive disequilibrium is first brought about in the classroom with respect to limited aspects of the new pattern and then further amplified to encompass other aspects through home assignments. For example, the Newtonian uniformly accelerated particle model of Figure 2.6 is first introduced for linear motion (e.g., free fall) and then developed for parabolic motion (e.g., projectile motion). Home assignments may consist of empirical experiments or observations that are within student means, and/or they may bear on empirical data pertaining to monstration exercises of the previous stage. They are intended to fulfil three main objectives. The first is to expose students to the same pattern in different empirical contexts so that they could infer as many features as possible that distinguish the new pattern from other familiar patterns. The second objective is to provide students with the opportunity to: (a) consciously and practically realize how short old scientific models fall from allowing correct inferences about the new pattern, while (b) reinforcing the idea that there are some generic laws in every scientific theory (like Newton's laws of dynamics) that apply uniformly to all models in the theory. The third goal is to prepare students for coming MLC stages, and especially the next one, by asking them to attempt to formulate the problem as explicitly as possible (presence of a new pattern that requires a new model of well-defined scope), and to encourage every student (or group of students) to conjecture a crude candidate model along with plausible ways for evaluating this model and completing its composition and structure.

A multitude of subsidiary models may be conjectured at this point, one by each student or group of students. A conjectured model is typically a subsidiary model expected to bear on some but not all aspects of the composition and structure of the target model. In general, a subsidiary model proposed in MLC1 can be extrapolated from previously constructed models and/or mapped on a particular referent of the target model, a real world referent that a student is familiar with or exposed to in monstration exercises. The subsidiary model in question is thus inferred not by induction, but by deduction from a previous model(s) (as in the third phase of the example above) and/or by one-on-one mapping that does not quite reveal nomic isomorphism between model and referents at this stage. It may be an entirely new model constructed as a result of exploration before a learning cycle is truly conducted in class (as home assignment), or an old model that the student had formerly developed and simply evoked through the assignment.

### *MLC2. Nominal models proposition:*

The part of monstration exercises conducted at first in class normally takes no more than a quarter of a 50-minute class period. It is followed with a home assignment as discussed above so that students come back to class the following period ready to begin constructing the new model. At the beginning of this period, students expose, individually or in groups, what they came up with in their assignment with regard to two main issues. The first is a problem statement, and the second is a set of particular hypotheses about the candidate model, hypotheses that students can conjure up within the context of their subsidiary models. The issue of hypotheses evaluation is deferred until the next stage when students are expected to come to a consensus on the general outlines of a single candidate model.

The second MLC stage begins then with fulfilling the main objective of monstration, i.e., the agreement among students on a clear problem statement about the scope of the target model, its possible domain and function; the latter being only defined qualitatively at this point (e.g., in the case of our example, to describe and/or explain physical objects in circular uniform translation, a function that cannot be fulfilled with previously constructed models). Students' propositions are next solicited and negotiated regarding the following issues:

1. *Theory*: What scientific theory is most appropriate for model construction.
2. *Composition*: What concepts of the theory are needed to construct the target model. These would typically be concepts already used in the construction of previous models.
3. *Generic structure*: What generic laws of the theory already used in the construction of previous models (mostly causal and interaction laws) can be used in the construction of this particular model.
4. *Particular structure*: What particular laws (mostly state laws) need to be formulated (in subsequent stages) in order to complete the structure of the model.

The set of required hypotheses are mostly about the fourth issue, i.e. about state (and topology) laws that would distinguish the target model from other models in the theory. Students first indicate possible primary constituents and properties of the new pattern, and represent each with the appropriate concept. They then sort descriptors out into what is commonly referred to as dependent and independent variables, and take a first guess as to what (independent) descriptor affects another (dependent). As such, the formulated hypotheses are called *nominal hypotheses* (Fig. 4.4), and a subsidiary model thus proposed is called a nominal model. If possible, students may further enhance their guess by specifying in what direction a presumed independent descriptor would affect the corresponding dependent descriptor. In other words, students would ascertain whether the dependent descriptor would increase or decrease when the independent descriptor varies one way or another. This advanced guess brings about what we call *ordinal hypotheses*.

A *nominal model* is a generalization of a subsidiary model. A subsidiary model is originally mapped on a particular instance of a pattern, a particular system or phenomenon that a student is familiar with, whereas the emerging nominal model is about the pattern itself, i.e., about all physical realities exhibiting the pattern. Both subsidiary and nominal models are entirely constructed by students, and the nature of the corresponding conceptual profile may be anywhere in the spectrum extending from naïve to scientific (Fig. 3.2). The structure of the subsidiary model may be well developed in student minds. However, students are intentionally guided to generalize the particular structure of this model in a nominal form at this stage in order to ensure that peer negotiations and self-regulation in the coming stage be headed in the right direction, and reduce the chances of coming back to refine model composition in subsequent stages.

Student naïve models are often encumbered in their composition by secondary features that are irrelevant to the represented pattern from a scientific perspective. For example, student naïve counterparts of Newtonian particle models of mechanics include by mistake shape and dimensions of moving objects as primary properties. Naïve models miss their function and fail to reliably depict what they are supposed to represent by accounting for such secondary features. When students are asked to formulate nominal hypotheses about these features and justify them by reference to their subsidiary models, they often realize their flaws and appropriately reconsider their position. This helps in cutting down on the time and effort required to set students on the correct path in the coming phase, which is the most crucial phase in actual model construction. In order to enhance self-regulation in this direction, enough nominal models are solicited to cover all possible naïve alternatives in student paradigms.

Students negotiate these models among themselves under teacher *moderation*. As a *moderator*, the teacher brings students together to discuss their own models among themselves and practically refine them on their own. S/he could intervene when students fail to do so, but with the only purpose of clearing the way of student negotiations from any noise. This may involve clarification of some student views to the rest of the class, reminding students of conceptions they ask about, passive supply of some empirical data, historical cases or any information that may help students to brainstorm and bring their naïve ideas to the surface or get out of any possible gridlock in their negotiations. By the end of this stage, which takes about half a class period, students eliminate all models *they* duly consider nonplausible so that they are left with no more than three candidate models for consideration in the coming stage.

## 5.3 MODEL ADDUCTION

The exploration phase is intended to direct students' attention to a new pattern that necessitates the construction of a new model. During that phase, students have almost a free-hand exploring different possible ideas about the pattern in question and plausible models to represent it. The more subsidiary or nominal models students come up with that are incommensurate with scientific theory, the more chance they get to eliminate early on non-viable models, and thus to efficiently evolve into the realm of science. Because teacher intervention was so far restricted to moderation, non-viable models may, however, not be completely eliminated in the first MLC phase. As mentioned above, students may be left by the end of the exploration phase with more than one candidate model (three at the most) with varying degrees of viability and naïveté. These models are carefully scrutinized in the second MLC phase under teacher arbitration, a more involved role than moderation, so that only one model is finally considered for evaluation and systematic formulation.

The adduction phase is thus intended to focus students' attention on one plausible model that appears to be reliably mapped on the new pattern investigated in the cycle. The model structure is hypothetically formulated as comprehensively as possible in the first stage of the phase (MLC3). Appropriate empirical experiments and/or observations are subsequently designed to assess the model (MLC4) so that it is ready for refinement in the following phase. By the end of this phase, students resolve major incommensurabilities between their own models and the target scientific model while enhancing their methodology of inquiry. As a consequence, they significantly reduce the naïve dimension and build up the scientific dimension of the corresponding conceptual profile.

The teacher whose role was restricted to moderation of student brainstorming and negotiation in the previous phase now assumes a more active role, an *arbitration* role. As an *arbitrator*, in MLC3 the teacher intentionally steers student interaction in the direction of a single candidate model that is proposed in accordance with the model schema and that has a relatively high degree of viability and low degree of naïveté by comparison to its subsidiary and nominal predecessors. If necessary, s/he also ensures that appropriate new conceptions be constructed to this end. In MLC4, the teacher makes certain that students come up with a sound investigative design to assess the tentative model, along with necessary norms and criteria for model acceptance and refutation.

### *MLC3. Plausible model proposition:*

Subsidiary models and nominal models that are the object of the preceding phase are entirely student constructed. The scope of those models may be limited or incongruent with the scope of the target scientific model, and their composition and structure may be less developed than this model. They may even be cluttered with secondary details and naïve conceptions (concepts and laws). The current stage is devoted to help students to refine their nominal models and come to a consensus on a single plausible model. This is a hypothetical model that students conjecture explicitly according to the model schema. It may still include some residual secondary/naïve elements that students could not entirely resolve, residues shared by all or some groups of students. These residues will be sifted out in the next phase.

The plausible model conjectured by the end of this stage may or may not entirely evolve from students' grounds, depending on the state of students' knowledge about the target model. The negative case is especially true when students' ideas are not viable. In order to preserve the scientific rigor of didactic transposition and complete the learning cycle efficiently, the starting point of MLC3 (or of any subsequent learning activity for that matter) is then at an intermediate level between where students stand and the target model. This point is determined by the teacher depending on the state of students' conceptual profiles pertaining to the target model. The teacher may intervene by putting at students' disposal a set of hypotheses and asking them to consider the merits of those hypotheses. S/he may even put at their disposal an incomplete model and then guide the cognitive transformation through successive refinements in the manner described, say, by White (1993) in her ThinkerTools, and leading to a plausible model with the least secondary and naïve elements possible.

Progressive construction of the plausible model is carried out *vis à vis* model scope, composition and especially particular model structure (mostly state facet). Hypotheses of the nominal models kept by the end of MLC3 are sifted and further refined under explicit teacher arbitration so as to gradually improve the viability of the remaining hypotheses, as well as their objectivity and precision. In the latter respect, strictly nominal hypotheses are gradually converted into ordinal hypotheses, and then, if possible, into ratio-type hypotheses. A nominal hypothesis, we recall, is one that only states whether or not a given descriptor (presumed independent) affects another (dependent) descriptor. An ordinal hypothesis specifies, in a first approximation, whether the dependent descriptor increases or decreases when the independent descriptor varies one way or another. At a higher precision level, relationships between descriptors are conjectured in *proportional hypotheses* (Fig. 4.4). In such a hypothesis, a mathematical expression of proportionality is expressed between a dependent descriptor and presumed independent descriptors, without necessarily specifying the order of magnitude of the descriptors' change. In the case of our example, a proportional hypothesis about a state law of the circular motion may take the algebraic form:

$$a \propto \frac{v^n}{r^m}$$

where *a* denotes the acceleration of the moving object, *v*, its velocity, and *r*, the radius of its circular orbit, and where *n* and *m* are unspecified power orders. When the precision of conjecture is further enhanced such that proportionality is converted into equality, and the power orders are specified, the hypothesis becomes a higher-order *ratio-type*. Conversion from one hypothesis form to another takes place gradually, and only to the extent that student knowledge allows it, in order not to impose cognitive demands that are beyond their reach on students and scare them away from actively participating in the learning process. Most importantly, the gradual conversion process allows students to carefully reflect on their own knowledge and regulate it so as to meaningfully understand what a relationship between descriptors is about through corresponding semantics and syntax, especially when this relationship is of a functional nature and conjectured as a law.

The model so far proposed does not necessarily consist entirely of uncorroborated hypotheses, and it does not have to come about so as to cover primary aspects of the pattern under study all at once. When students have already developed some models that belong to the same scientific theory as the target model, they may readily take advantage of generic aspects of previously constructed models in the formulation of the new model. This is especially the case with generic laws that make up the interaction and causal facets of all models belonging to the same theory. Newton's laws of dynamics, and especially the second law, are examples of such generic laws. Students may, for example, readily include Newton's second law in its formal form ($\mathbf{F}=d\mathbf{p}/dt$ or $\mathbf{F}=m\mathbf{a}$) within the structure of any particle model that is being built following the uniformly accelerated particle model (Fig. 2.6). On the other hand, students may formulate a given schematic dimension of a plausible model, and especially its structure dimension, progressively within MLC3. For example, the uniformly accelerated particle model just mentioned represents both linear and parabolic translation of objects subject to a net constant force. When developing this model, students may first propose a plausible model representing linear translation (e.g., by reference to free fall), and then they may gradually extrapolate things to various cases of parabolic motion.

As the plausible model is being formulated, students continuously evaluate it empirically and rationally. Empirical assessment is performed against data available from referent(s) of student subsidiary models, as well as against data presented in monstration exercises and collected in home assignments. Rational assessment is about mutual coherence and conformity. *Mutual coherence* assessment is conducted to ensure that various hypotheses conjectured in the plausible model are consistent with one another. *Conformity* assessment is conducted to ascertain whether those hypotheses are consistent with the scientific theory that the plausible model is supposed to belong to, and especially with generic laws of the theory.

Proposition of the plausible model is achieved by the end of the same class period that follows monstration and that began with the proposition of nominal models. In other words, MLC2 and MLC3 are achieved together in a single period. Should the teacher judge that students are still capable of further refining the plausible model beyond what has already been achieved in class, students would be assigned to do so as a homework. In any event, students are asked by the end of MLC3 to prepare at home an investigative design that will be conducted in class the following period in order to assess the plausible model and refine it in a way that leads to the target model in almost its final form. This may be the design of an appropriate experiment with equipment that students are familiar with by the end of MLC3. It may bear as well, or instead, should laboratory equipment be not available, on field observations made by students as part of the homework, or on empirical data provided by the teacher should such observations not be feasible or possible.

### *MLC4. Investigative design:*

Modeling learning cycles are supposed to allow students to develop experiential knowledge about models of concern in the most meaningful ways possible. This sort of knowledge evolves progressively throughout a cycle beginning, in a rough form, with the monstration stage (MLC1). The evolution becomes especially meaningful in the current stage (MLC4) and the following one, and it continues at different levels until the cycle ends (and even beyond). In the current stage, students expose and negotiate investigative designs prepared at home to inquire in the empirical world about the viability of the model that they presumed most plausible in the previous stage. Depending on equipment availability and procedural feasibility, the design can pertain to a classroom or field experiment, to observations in the real world, or to empirical data provided by the teacher or any other reliable source about the pattern under study.

Classroom experiments constitute the most efficient and effective ways for students to develop experiential knowledge under teacher supervision. Such experiments are designed and then conducted when appropriate equipment is available at school; appropriate in the sense that it can be handled by students and that it allows didactic transposition with an acceptable level of scientific rigor. Experimental designs in our modeling cycles are distinguished from their conventional counterparts in at least two major respects. First, MLC experiments are designed to assess the viability of a conjectured model and not to: (a) "verify" a scientifically established model already transferred to students in the form of traded knowledge, or (b) to induce a new model from experimental data. Second, a variety of designs originally proposed by students are put forth for consideration instead of a unique design imposed in a prescriptive way by an authority, be it teacher or laboratory manual.

Students propose their experimental designs in the context of the plausible model, to come up with sufficiently reliable data that help them to assess, and subsequently complete, the formulation of this model. Designed experiments constitute part of a more comprehensive scheme for developing the target model in the form of experiential knowledge. In addition to classroom experiments, the scheme includes other investigative actions and deployment activities, some assigned by the teacher and others taken up by students on their own initiative. These activities are conducted throughout the rest of the cycle in order to inquire about the model under construction from different perspectives and ensure that it reliably fulfils its function within the assigned domain.

A typical experimental design includes a list of required apparatus chosen from what students already know is available at school, blueprints for setting up the apparatus, and plans for conducting and analyzing a set of experimental activities, each designed to assess a particular hypothesis in the plausible model. The design also includes a set of practical guidelines and rational constraints derived from the scientific theory to which the model is supposed to belong. The latter set includes among others: (a) general guiding principles implied by the generic laws of the theory (and corresponding paradigm), (b) acceptable approximation limits and precision intervals to be respected in data collection and analysis, (c) convenient modes for logging and handling data, (d) appropriate mathematical representations and operations, and, most importantly, (e) norms and criteria that specify when experimental outcomes constitute a corroboration for the plausible model and when not, and, if deemed necessary, (f) what avenues might be considered to refine, modify or replace the model in question.

When appropriate experiments cannot be conducted in the classroom, e.g., when the required apparatus is not available at school or otherwise is beyond students' reach, students are directed to seek other alternatives for data collection. Alternatives include field experiments or observations about everyday life systems bearing on the pattern under study. Students, however, do not always have to come up with their own data in order to evaluate the plausible model. There are instances in science courses where appropriate experiments may not be affordable, and observation of real world systems exhibiting the pattern in question may not be possible. In such an event, and depending on the convenience of the situation, students may be asked to acquire appropriate data from reliable sources (e.g., scientific laboratories or publications), or the teacher may provide such data. Students are then asked to propose ways for analyzing data and assessing correspondence to data of the plausible model. In any event, any investigative design must fulfil the conditions of experimental designs discussed above.

Student proposals prepared at home about the investigative design are negotiated in class under teacher arbitration. Student negotiations culminate with a single design to be implemented in the next stage. The time it takes to reach a consensus on such a design does not exceed a quarter of a two-period session devoted to planning (MLC4) and conducting the actual investigation in the classroom (MLC5). It is the teacher's responsibility to ensure: (a) that the final design can be executed with the available apparatus, (b) that the design is in line with scientific methodology, (c) that it is not doomed to bring students to a dead end, and (d) that it is flexible enough so that students can change direction at any time, if necessary. It is also the teacher's responsibility to ensure: (a) that all hypotheses in the plausible model are conveniently accounted for, and (b) that, if appropriately conducted, student investigation should result in sifting away all secondary residues left in the plausible model, thus clearing the way to comprehensively formulate the scientific model in the following phase.

## 5.4 MODEL FORMULATION

Efficacy and efficiency of a learning cycle depend mostly on the prior two phases, and especially on the model adduction phase. The more of their own ideas they expose in these two phases and assess in class by comparison to one another, and the more transparent and focused the investigative design gets in their minds, the better the chances are for students to bring the self-regulation process to a meaningful conclusion at this point and progress in the direction of a truly scientific model. Gradual formulation of this model takes place for the most part in the two stages of the third MLC phase, and is achieved in the last phase following model deployment.

In the first formulation stage (MLC5), students carry out the investigation they designed in the previous stage, and refine the plausible model of MLC3 in light of the investigation outcomes. The model is rationally analyzed and extrapolated in the following stage (MLC6) so as to come close to a comprehensive model formulation. The formulation is not exhaustive at this stage because the model has not yet been sufficiently deployed. Model deployment takes place in the fourth MLC phase. New insights will subsequently be gained into various aspects of the model, and the model will be brought then to a maturity level that is high enough to conclude the learning cycle.

The teacher assumes consecutively two different roles in this phase of the learning cycle that may take up to three class periods. At the beginning of MLC5, and all through the actual investigation process (that takes up to one and a half periods), the teacher retracts from the arbitration role to supervise the process from a distance as a moderator. Once the investigation is completed and students have prepared their reports, the teacher becomes again more involved as an arbitrator of students' interaction to ensure that the model is properly formulated in MLC6. Sometimes, the teacher may even find it necessary to step up her/his intervention to take the form of *scaffolding*. As such, the teacher may instruct students to follow explicitly a specific path and/or provide them with necessary concepts and tools that are totally missing from their paradigmatic profiles and that are indispensable to complete model composition and structure. Scaffolding may take the form of a lecture when practical constraints prevent students from coming up with any of these concepts and tools on their own (§ 5.7).

### *MLC5. Investigation and initial model formulation:*

Once all students agree on a common plausible model in MLC3 (with perhaps minor distinguishing features among groups) and on a common investigative design in MLC4, they proceed to conduct the investigation in collaborative groups. Groups carry out the designed investigation independently of one another; yet groups may intermittently confer with one another to share helpful ideas. The teacher may be called upon at any time for helpful hints. A group's investigation culminates with the preparation of a report on the group's work, in class if time allows it, or otherwise after class.

A group report is drafted on a whiteboard (or a shared computer platform) with an outline of major outcomes pertaining to various model dimensions and especially model structure. Appropriate depictions, and especially mathematical representations (mainly graphs) are used to this effect. Reports of various groups are exposed and discussed under teacher arbitration in the following class session (for about one period), in the manner discussed in § 4.6. Special care should be taken to settle points of disagreement among groups that were left unresolved in the plausible model, and to filter out any residual secondary/naïve elements in light of the investigation outcomes.

By the end of this stage, students achieve a preliminary formulation of the target model, a viable but incomplete formulation. The model thus formulated is a refined, formalized form of the plausible model conjectured in MLC3. Hypotheses in the latter are corroborated, modified or replaced, and the originally conjectured relationships are more precisely expressed (e.g., ordinal or proportional hypotheses are turned into ratio-type law statements). Seldom do new major elements that have not been thought about before in one form or another emerge in the newly refined model. There is virtually a one-on-one mapping between this model and its predecessor. It is thus common that some primary features of the target model may still be missing. Aside from issues that may be brought about only after model deployment, and because of reasons discussed in § 4.1, student investigations cannot possibly cover all model aspects at this stage. Various constraints may make it impossible for students to empirically corroborate or even induce certain model aspects. Such aspects may then only be inferred by rational analysis and extrapolation, and perhaps only through teacher scaffolding. These aspects would be the object of the next stage.

### *MLC6. Rational model extrapolation:*

The model formulated by now is the best model students can come up with on their own, given all practical constraints, and with teacher intervention being so far limited to moderation or arbitration. It is a viable model, but not necessarily a complete one. In the latter event, various schematic dimensions of the model (domain, composition, structure and organization) may be at different completeness or limitation levels. Ordinarily, the structure dimension is the one that needs attention the most. Missing elements can sometimes still be inferred from available data with more direct teacher guidance. However, they often need to be rationally deduced through appropriate extrapolation activities (§ 4.3), guided or even carried out by the teacher.

Rational extrapolation can take place within the model so far constructed, and/or from previously constructed models. For instance, by the time they have achieved MLC5, students would have already refined, in the form of scientific laws, all hypotheses conjectured in MLC3. When these laws are insufficient to complete the model structure, students can be guided to induce missing laws from available data if possible. If not, they can be guided to formulate these laws by rational extrapolation of laws they have already formulated in MLC5, and/or laws formulated in previous learning cycles, be it generic laws or laws that are particular to some old models. Students however can be assigned such extrapolation exercises only after they have developed elementary basic models (§ 4.2). By then, students would have come up with fundamental concepts and generic laws of the theory that the model under construction belongs to, along with associated tools and skills. Only by then, would they have sufficient backbone to carry out required extrapolation activities with some teacher guidance. The more basic models students develop afterwards, the less teacher intervention and student effort will be needed to extrapolate old models in the construction of new models of any sort. Until students develop elementary models, teacher intervention remains highly critical and can sometimes take the form of conventional lecture. This is especially true at the beginning of a science course when fundamental conceptions are entirely missing from student paradigms and when practical constraints make it impossible for students to infer such conceptions from their existing ideas or from empirical investigations.

By the end of the model formulation phase, student conceptual profiles about the target model must evolve in such a way as to significantly reduce naïve aspects and increase scientific aspects along all schematic dimensions of the model under construction. More specifically, the evolution in question should proceed in the direction of achieving at least the following points regarding the respective dimensions. Depending on the complexity of the target model and the initial state of students' conceptual profiles, some of these points may not actually be fully achieved until the end of the cycle.

*Model domain*:

- The model is about a particular pattern in the real world, and not about structure or behavior that is particular to a given physical system or to a narrow set of physical systems.
- Correspondence rules between model and referents are clearly stated.
- The model reference class is no longer restricted in space or time to specific real world realities (systems or phenomena) that students may be closely familiar with and on which they may have originally mapped their subsidiary models. The reference class is instead universal and open to embrace more elements in the future.
- The reference class no longer includes physical realities that cannot actually be represented by the model.
- The reference class no longer includes fictitious systems or phenomena (purely mathematical, imaginary or metaphysical). It consists only of physical realities that actually belong to the real world.

*Model composition*:

- Appropriate depictors are used to represent physical objects that are primary to pattern existence and model function, and all such primary objects are duly depicted.
- No depictors are included to represent physical objects that are secondary to model scope.
- Appropriate descriptors are used to represent primary properties of the pattern, and all such properties are duly represented.
- No descriptors are included to represent physical properties that are of secondary nature.
- All depictors and descriptors are appropriately classified (e.g., elementary vs. composed depictors; prime vs. derived, object vs. interaction descriptors) and defined following the concept schema.
- Appropriate representations are associated with each depictor and descriptor, along with corresponding semantics.
- Operations allowed with each descriptor are identified, along with appropriate syntax.

*Model structure*:

- Every descriptor in model composition is used in at least one facet of model structure. An omitted descriptor indicates that either it is a secondary descriptor that should not have been included in the model composition in the first place, or that something is still missing in the model structure.
- No descriptor figures in model structure that has not been included in model composition.
- Every required facet of model structure is adequately formulated, and various facets adequately related.
- The model function is well established and conditions of nomic isomorphism between model and referents are explicitly stated.
- Descriptive function of the model is clearly distinguished from its explanatory function.
- Generic laws provided by corresponding theory and paradigm are clearly distinguished from laws (mostly state laws) that are particular to the constructed model.
- The functional relationship expressed in every law is meaningfully understood and corresponding semantics and syntax clearly established.

- Any incommensurability with student ideas has been resolved in favor of the corresponding scientific law or other theoretical statement.

*Model organization*:

- The scope, and especially function, of the new model is compared to, and clearly distinguished from, the scope of any other model in the family of models that the new model belongs to.
- Reasons that necessitated the construction of new concepts, if any, are clearly established.
- Particular laws in model structure that characterize its function are distinguished from particular laws of other models.
- Fundamental concepts and generic laws that are common to other models in the respective family of models are identified with the recognition that generic laws keep this model with others in the same family and provide common guiding principles for model construction and deployment.
- No subsidiary model remains in student profiles that has not been converted into the more generic model that is the one formulated in this cycle.
- Reasons that necessitated changes in students' conceptual profiles are recapitulated in a way that elucidates the advantages of scientific over naïve model formulation.
- Norms and criteria are clearly established for what would constitute in the future reliable evidence or counterevidence to the model and the theory it belongs to.

## 5.5 MODEL DEPLOYMENT

A model gains its full significance only after deployment in the real world for describing, explaining, predicting and controlling the structure and/or behavior of a variety of existing physical realities, for inventing new conceptual or physical realities, and for subsequently bringing to new horizons the theory and paradigm which the model belongs to. New insights are gained as the model is deployed in different contexts and envisaged from different perspectives. It gradually gains in scope (domain and function), and it becomes better and better situated in the corresponding theory. This is as much the case with scientists that it is with students. From an ontological point of view, a model gains its scientific identity only after it becomes sufficiently corroborated in the real world, especially after it constantly allows scientists to make reliable predictions about the pattern it is supposed to represent, and to invent or discover new physical realities.

From a pedagogical point of view, construction of a new model is not fully achieved by the end of the formulation phase. While being deployed in new situations, and provided that students are engaged in appropriate deployment activities, the model becomes to students increasingly more meaningful, and it ends up being reinforced with a broader reference class. Meanwhile, the conceptual profile of the model continues to evolve in every student's mind, hopefully in favor of the scientific perspective, and so does the student paradigmatic profile. The evolution reaches a satisfactory state when students become capable of severing the umbilical cord between the empirical world of model referents and the rational world of the model itself, and of conducting rational extrapolation exercises without any reference to the empirical world. The MLC deployment phase can then come to an end and students can proceed to the phase of paradigmatic synthesis that marks, from a practical educational perspective, the end of a learning cycle, but not quite the end of model development. Students may keep gaining insights into a given model as they proceed to construct new models, and in this respect, the construction of a model may be looked upon as an open process.

Model deployment is thus not an end in itself, and it does not follow conventional problem solving philosophy. It is conceived in MLC neither as a drilling avenue nor as an assessment end point. Model deployment, as we argued in § 4.3, subserves model construction, and not the other way around, while it helps students to develop various inquiry skills. It does not even mark the end of the learning process about a particular model. Moreover, model deployment is not envisaged in the narrow sense of conventional problem solving. Hestenes (1997) describes the state of all conventional science courses as he criticizes conventional physics courses by saying that these "courses lay heavy emphasis on problem solving. This has the undesirable consequence of directing student attention to *problems and their solutions as units of scientific knowledge.* Modeling theory tells us that *these are the wrong units; the correct units are models*. Problem solving is important, but it should be subservient to modeling… It [modeling theory] tells us that most physics *problems are solved by constructing or selecting a model from which the answer to the problem is extracted by model-based inference.* In a profound sense the model provides the solution to the problem". The picture becomes even gloomier when we look closer at problem solving strategies promoted in conventional science instruction. For instance, most high school and college physics textbooks promote the kind of strategy outlined in Figure 5.2, a strategy that unfortunately often works out for solving conventional homework and exam problems. Such a strategy has the drawback of allowing students to lose sight of the actual model-centered structure of scientific theory and model-based inference in scientific inquiry. It even encourages students to hang on tight to their strategies of solving problems backwards by starting from problem questions and looking back in problem "givens" for information that helps them to determine a convenient "relationship between the given quantities and the one(s) to be found" (Hecht, 1994, p. 19).

Model deployment activities are chosen so as to allow students to complete and reinforce all four schematic dimensions of a model (especially points mentioned at the end of § 5.4 that are still pending), and develop efficacious schemes for model-based inference in various empirical and rational contexts (e.g., Figs. 4.6 and 4.8). Activities are conducted in ways that allow students to develop tools and systematic rules that are necessary for model adduction in exploratory research

Suggestions on Problem Solving

1. Problems are usually posed in nonscientific prose… (a) Decide on what is happening as far as the physics is concerned and lift that information from the inconsequential background… (b) Translate the statement into the language of physics… (c) Symbolically represent the variables… Once the problem has been so restated, write down what was *given*... and what you must *find*… That completes the crucial translation phase of the solution.
2. Determine a relationship between the given quantities and the one(s) to be found. It is the business of physics to produce such relatioships in the form of laws and definitions…
3. As a rule, it is always a good idea to draw a diagram. The process helps to organize your thoughts.
4. Wherever possible, check your calculation…
5. Show all your work when making a calculation…
6. [A hint on unit conversion]

*Figure 5.2*: Outline of a problem solving strategy (Hecht, 1994, p. 19) typically recommended in conventional college physics textbooks.

and for model-based deduction in inventive research (Table 4.1). In the process, students are brought to realize the limitations and even the futility of rules of thumb or context-specific routines that are commonly prescribed to determine convenient “relationship(s)” between givens and unknowns. Modeling schemes and required tools and associated rules are progressively developed in two deployment stages in our learning cycle. In the first stage (MLC7), students deploy in each activity only parts of the model formulated in MLC6. Deployment situations become progressively more complicated until they reach the level of paradigmatic situations (MLC8), situations that require deployment of the model structure in virtually its integrity. In addition to conventional paper-and-pencil exercises and problems that fall mostly within the application category, modeling activities in both deployment stages are chosen to cover all four categories distinguished in § 4.3 (application, analogy, reification, and extrapolation). Activities are also diversified in context to include field projects, thought experiments, historical and contemporary case studies, especially of an interdisciplinary nature, and other contexts related to modern-day life.

### *MLC7. Elementary deployment:*

Model deployment begins with simple activities in the empirical and/or rational world. A given activity may then pertain to one particular aspect (e.g., a particular facet in model structure) or to a limited number of schematic aspects of the new model considered separately or in relation to other models. It may also pertain to the development of a certain general tool or skill of scientific inquiry. In some respects, and especially with regard to their cognitive requirements, these elementary deployment activities are similar to end-of-chapter exercises found in conventional textbooks.

Special care is devoted in this deployment stage to the following issues:

- Recognition of patterns both in the empirical world of physical realities and the rational realm of scientific conceptions.
- Understanding of the fundamental assumptions underlying the viability of any conception (concept, law) used in model composition and structure, especially those related to the limits of approximation, precision and estimation.

- Development of semantic and syntactic rules associated with every conception.
- Classification of conceptions and the clear distinction between concepts or laws that students normally confuse with one another (e.g., instantaneous vs. average descriptors, scalar vs. vectorial descriptors, state law vs. interaction law or causal law, interaction law vs. causal law).
- Understanding the nature of pseudo-descriptors, i.e., fictitious descriptors that correspond to no physical properties (e.g., pseudo-forces like so-called centrifugal forces).
- Development of scientific discourse, and especially the translation of everyday, colloquial language into scientific language and the recognition of respective constraints (e.g., when two moving objects “meet”, they occupy the “same position at the same instant in a given reference system” without necessarily having, at that instant, the same speed or other descriptors of the same magnitude).
- Use of efficient representation and operation modes, especially those not commonly used in conventional instruction (e.g., Hentisberus Theorem in Fig. 4.9).
- Understanding of semantics and syntax associated with a given mode of representation, and recognition of the corresponding limits.
- Coordination of various modes of representation of a given conception.
- Model universality especially at the level of various laws in model structure.
- Invariance of generic laws (like Newton’s laws of dynamics) vs. variation of state laws from one model to another.
- Invariance of generic laws (like Newton’s laws of dynamics) vs. variation of state laws for the same model under certain transformations (change of reference system).
- Articulation of every facet in model structure, first separately and then in relation to one another, and subsequently of conditions of nomic isomorphism between model and represented pattern.
- Regulation, along Figure 4.10, of particular student conceptions that are not entirely commensurate with their scientific counterparts.

### *MLC8. Paradigmatic deployment:*

Educational research about problem solving in the last two decades has constantly shown that novices (mostly naïve students) and experts (mostly university professors) approach science problems (especially in physics) from opposite directions, and that the two groups focus on different features when analyzing a problem situation. Novices are shown to start analyzing a problem situation "*backward* from the goal to the information given in the problem. In contrast, more experienced subjects proceed by *forward* reasoning. As they [experts] read the description of the problem situation, an integrated representation is generated and updated, so when they finally encounter the question in the problem text, they simply retrieve a solution plan from memory" (Ericsson & Charness, 1994). As for analysis of problem givens, novices are shown to concentrate on "surface features" that are viewed by experts as secondary and irrelevant, whereas experts are portrayed to concentrate on "underlying principles" (Chi, Feltovich & Glaser, 1981).

Our position, like that of other modeling advocates (Giere, 1994; Hestenes, 1997; Nersessian, 1995), is that the prevalent position on solving application problems in science education literature may be well taken with regard to the schemes of novices but not of experts. Experts analyze problem givens primarily to *adduce appropriate model(s)* and not principles to the situation at hand, and they do this *progressively* along the lines of Figure 4.6. They first isolate various systems and phenomena in the situation, along with their primary features, and they roughly depict them so that they can envision the pattern(s) involved and match it (them) with the appropriate model(s). This critical step is often carried out implicitly, and is not as transparent in experts' schemes as other steps. Experts subsequently develop their depictions in the context of chosen models so as to decide on conceptions in model composition and structure needed to represent the primary features of the problem givens ("underlying principles" included). They do this at first irrespective of problem goals. After they identify the goals, they refine their depictions and their choice of conceptions so as to build and then process the mathematical model for solving the problem.

For example, when faced with a mechanics problem, experts first roughly depict the situation at hand in order to decide on the appropriate theory (classical or relativistic, Newtonian or Eulerian), and then the appropriate model(s). In the case of, say, translational motion with relatively small velocities by comparison to the speed of light, the choice will be for Newtonian particle models (chosen following the scheme of Fig. 4.8). Experts then depict each body with a single particle (geometric point) with which they associate one descriptor representing only one intrinsic property (mass) and a number of descriptors representing some state (kinematical) properties, if necessary. Experts' focus on such properties and not others could not have been possible outside the context of the chosen models, and neither were their depictions and related assumptions (e.g., the limits of approximation and range of acceptable values for each descriptor). The same is true for interaction among various bodies and the choice of the convenient interaction descriptor(s), and subsequently for the choice of appropriate state, interaction and causal laws. The choice of appropriate descriptors, theoretical statements (laws included), and depictions (e.g., vector diagrams, motion maps, graphs, algebraic equations) becomes more focused once the problem goals are identified.

The process of model adduction and subsequent analysis of model

1. The situation involves new systems and /or phenomena with which students are not familiar.
2. The situation requires the adduction of one model or more, and the deployment of the most part possible of the structure of any model.
3. Appropriate models or conceptions are not explicitly suggested in the statement of the situation.
4. The situation is presented with superfluous data.
5. The situation as presented misses primary data, estimations and/or underlying assumptions.
6. The situation is interdisciplinary; it involves aspects of scientific disciplines (or branches of the same discipline) other than the one that is the object of the course.
7. Analysis of the situation requires the coordination of different representation modes (without an excess of mathematical calculations).
8. Outcomes are extrapolated in other situations or in the same situation extended to new conditions.
9. The same situation is modified so as to require the adduction of different familiar models.
10. The situation is modified so as to ultimately require the construction of an entirely new model.

*Figure 5.3*: Certain characteristics of a paradigmatic situation.

composition and structure in the manner just described is so involved that students cannot develop it through elementary model deployment exercises (MLC7). In order to develop required schemes (e.g., Fig. 4.6), tools and rules, students need to become engaged in the study of what we call paradigmatic situations. A *paradigmatic situation* is one that requires the comprehensive use of the composition and structure of at least one model, especially in a way: (a) that requires explicit negotiations between the empirical and rational worlds, (b) that fulfils a number of the objectives outlined toward the end of § 5.4, and (c) that allows students to develop generic features and skills associated with the theory and paradigm to which the model belongs (Fig. 5.3). The context of paradigmatic situations is varied so as to extend from investigative activities to conventional paper-and-pencil problems.

Paradigmatic situations provide students with the opportunity to complete model construction and then reinforce and consolidate their knowledge about various schematic aspects of the particular model under construction. They are especially important for helping students to give up their "backward", trial-and-error problem solving tactics in favor of "forward", systematic model-based inference. They consolidate conceptions and skills developed in prior stages, and foster development of new ones, all in a way to articulate the paradigmatic evolution called for in modeling theory.

Deployment activities, and especially those of paradigmatic deployment, are conducted under teacher arbitration following the same guidelines as the previous two stages. Students are encouraged more in this stage than ever before to rely on themselves and collaborate with the members of their groups whenever possible, and carry out every deployment activity while reflecting on their own knowledge and regulating it in the most insightful way possible. To this end, they conduct every deployment activity in the manner they conducted the investigation of MLC4 and MLC5, individually or in groups, during class hours whenever that is possible. They expose their work afterwards and discuss it in class the same way they did in MLC5. When peer negotiations head to a dead-end after all possible arbitration, and only then, the teacher may intervene to resolve the issue one way or another by scaffolding.

*Table 5.1*

Model nature and features as evolving progressively through a modeling learning cycle

| MLC stage | Nature of model(s) | Characteristics of the model(s) | Teacher mediation |
|---|---|---|---|
| 1. Monstration | Subsidiary (localized) | Crude models inferred, from previous personal experience or following monstration, (a) not by induction from a pattern but through mapping on a particular referent a student is familiar with or exposed to in class, and/or (b) by emergence from familiar model(s). | Moderation |
| 2. Nominal propositions | Subsidiary (generalized) | Nominal subsidiary model(s) (up to three) inferred from all subsidiary models proposed above by individual students and generalized to the pattern of interest. As much secondary/naïve details are sifted out of student models as possible. | Moderation |
| 3. Plausible model | Hypothetical, schematic | A single tentative model proposed following the model schema. The model may still incorporate secondary/naïve residues. Model structure, especially state facet, consists of hypotheses, preferably of proportional nature. | Arbitration |
| 5. Initial formulation | Formal, corroborated empirically | Viable, but perhaps incomplete, model formulated following due empirical hypotheses testing. Secondary/naïve residues are filtered out. Various structure facets are scientifically formulated. | Arbitration |
| 6. Rational extrapolation | Formal, expanded rationally | Almost complete scientific model. Missing structure is deduced from theory, and from the same and previous models. | Arbitration<br>Scaffolding if necessary |
| 7&8. Model deployment | Formal, articulated through model adduction and deduction | Complete scientific model. New empirical and rational insights are gained about the model. Modeling tenets and inquiry schemes, tools and rules are further developed. | Moderation<br>Arbitration<br>Scaffolding as last resort |
| 9. Paradigmatic synthesis | Formal, well-integrated in the respective theory/paradigm | Consolidation of conceptions and skills developed throughout the cycle. Recapitulation of the paradigmatic evolution gradually achieved therein. | Arbitration |

## 5.6 MODEL EVALUATION AND PARADIGMATIC SYNTHESIS

A learning cycle, and thus a modeling unit of instruction (often equivalent to a chapter, in conventional instruction) is not brought to closure with the deployment activities. New insights are gained about the model under construction in the deployment phase. Students need then to consolidate their experience in this phase with what they achieved in previous phases. The last stage of the learning cycle is devoted to this end and to subsequent recapitulation of the paradigmatic evolution gradually achieved by students in the cycle (Table 5.1). Consolidation and recapitulation are conducted as critically as any other process undertaken during the cycle. Every point is systematically evaluated in this stage as in any other stage. If determined to be viable, students proceed to the following point. Otherwise, students go back to a previous stage where the source of the problem might lie so that they can reconsider things and refine them appropriately. The process continues under teacher arbitration until students complete the synthesis of the current cycle and set the stage for the following cycle. Such an evaluation is in fact not limited to this stage, but it is carried out throughout an entire cycle, as implied in the dashed, counterclockwise arrow of Figure 5.1. It is conducted in the manner discussed next, before we turn our attention back to the paradigmatic synthesis of MLC9.

### *Model evaluation:*

A model is continuously evaluated, from the early moments of its inception in MLC2 and all the way through its deployment in MLC8. Model evaluation goes hand-in-hand with students' self-evaluation of their own conceptual profiles pertaining to the model under construction, and along the same lines of Figure 3.4. By the end of every stage, and especially by the end of MLC6 in the formulation phase and MLC8 in the deployment phase, students evaluate the model as framed by then, and this rationally in relation to the scientific theory it belongs to, and empirically by correspondence to the real world.

*Rational evaluation* bears on two complementary aspects, internal coherence and external consistency or conformity. *Internal coherence* is meant to ascertain whether or not various depictors and descriptors in model composition belong together in this dimension, and whether various laws of model structure, especially those particular to the model in topology and state facets, are consistent with one another. It is also intended to assess the semantics of theoretical statements, and especially of particular laws that distinguish the model under construction from other models, and whether descriptors are thereby duly related to one another. *External consistency* or *conformity* is about the relationship of the model, and especially of its particular structure, to the generic laws of the theory and paradigm that the model belongs to, and the extent to which the model structure accounts for such laws. It is about the function of the model within the context of the respective scientific theory, and thus about whether or not the model actually belongs to this theory. Rational evaluation of model internal coherence is concerned with intrinsic aspects of model composition and structure, and it goes in parallel with coherence assessment of student profiles discussed in § 3.6. Similarly, evaluation of conformity of model to theory goes in parallel with commensurability assessment of student profiles (§ 3.6).

*Empirical evaluation* is concerned with the mapping between model and pattern it is supposed to represent. It is meant to ascertain whether or not each presumed referent is actually so, and to assess the model rules of correspondence. It is also intended to ascertain the extent to which model composition represents all primary aspects of presumed referents, the conditions of nomic isomorphism between model structure and pattern, and the extent to which the model fulfils its function with regard to its reference class. Empirical evaluation is also about all assumptions underlying model viability in the real world (e.g., limits of approximation and precision with which it fulfils its function in this world). This aspect of model evaluation goes in parallel with correspondence assessment of student profiles (§ 3.6).

Students develop the necessary tools and skills of model evaluation most systematically in the paradigmatic deployment stage (MLC8). As indicated in Figure 4.6 and discussed in § 4.3, evaluation is an integral part of any deployment activity. Evaluation pertains then, as at any other point in a learning cycle, to the particular modeling act carried out at this point as well as to students' conceptual profiles about the model(s) of concern and to their overall paradigmatic profiles. The merits of a model, as well as respective modeling procedures and outcomes, are assessed with respect to the situation under investigation. Meanwhile, students reflect on their own profiles and have them self-regulated, within the context of the situation at hand and, when appropriate, within the context of follow-up activities that the teacher might assign specifically to sustain the evaluation and regulation processes.

The solution of a problem of any sort consists, from a modeling perspective, of a deployed model (or a deployed set of models). The model may be an existing one or one that needs to be newly constructed in order to attend to the needs of the problem at hand. Evaluation of a problem solution is thus as much about deployed model(s) and related conceptions, tools and skills, as it is about problem solving procedures and outcomes. The evaluation takes place throughout an entire model deployment process (Fig. 4.6), and it bears on every step of the process. Figure 5.4 shows various evaluation measures that need to be attended to in solving an application problem. Similar aspects are assessed in analogy, reification and exploitation problems (Halloun, 2001a).

The foremost evaluation of a problem solution is about the *viability* of the chosen theory and model(s). It is ascertained by

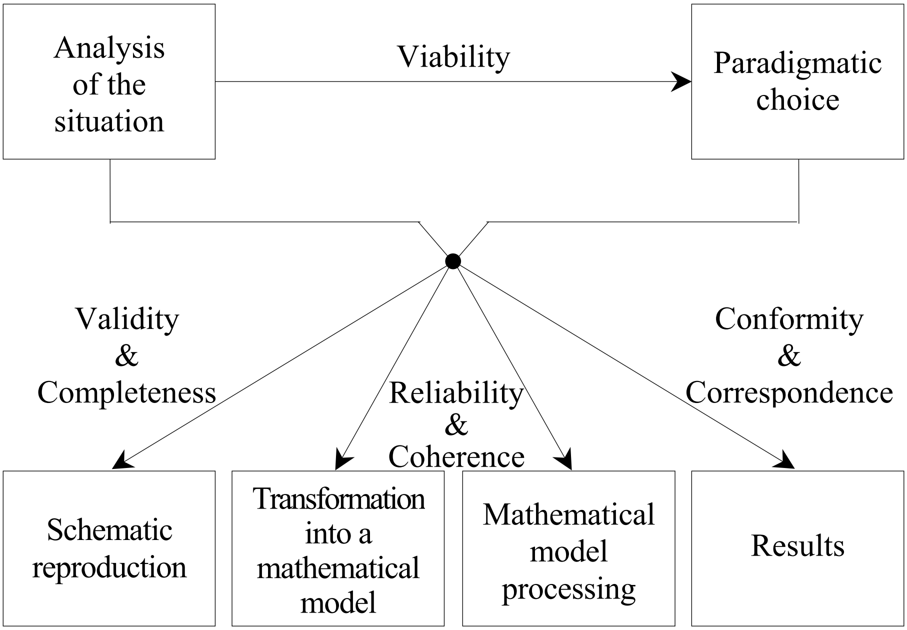

*Figure 5.4*: Evaluation measures pertaining to the solution of an application problem (Fig. 4.6).

assessing whether each adduced model is appropriate for representing systems and/or phenomena involved in the situation under study, and for answering related questions. Viability assessment is first done in terms of the correspondence rules of the chosen model(s). It continues throughout the problem solving process as every step is evaluated in terms of both the empirical grounds of the situation being investigated and the rational premises of the paradigm and theory whose models are being deployed in the situation (Fig. 5.4). Once a model is adduced (produced or reproduced, in part or in its integrity) to solve a problem, deployed elements of model composition and structure are assessed as to their *validity* to represent primary features of the situation. The deployed part of the model is afterwards assessed for its *completeness* in representing all primary features without exception, before the deployed elements are converted into a mathematical model (Fig. 4.6). As this model is subsequently processed, two measures of evaluation are undertaken. One pertains to the *reliability* of each component of the mathematical model to express what it is supposed to express and/or to process related information for obtaining specific outcomes (answers to questions), all to acceptable limits of objectivity and precision. The other measure is about the internal *coherence* of various components of the mathematical model (e.g., assessment is made as to whether different elements of a particular diagram are consistent with one another, and whether the diagram is consistent with a corresponding graphical or algebraic representation).

Once the model is processed and results are obtained, outcomes are ascertained with respect to their conformity to scientific theory and their correspondence to empirical evidence, in a sort of *summative evaluation*. In this respect, it is first assessed whether all the questions of the problem have been answered without exception, and whether all assumptions and constraints have been accounted for. *Conformity* of outcomes to scientific theory is ascertained in at least two different ways. First, each outcome is checked to see whether it follows conceptually from the respective facet(s) of model structure (e.g., the algebraic equation expressing an estimated descriptor is checked against expressions of laws it was inferred from, and/or in terms of descriptors' dimensions on both sides of the equation). Second, an attempt is made to answer the same question in different ways; it is then checked that the answer does not change from one way to another. *Correspondence* evaluation is intended to ascertain that results fall within acceptable empirical intervals, in terms of limits set by the chosen theory and by comparison to values obtained in prior deployment activities. This evaluation also extends, whenever possible, to the extrapolation of outcomes in the same situation under modified conditions and/or in other situations related to the one under study and falling within the scope of deployed models. The extrapolation is meant to reinforce students' knowledge about these models as well as to delimit their scopes and determine the boundaries outside of which a particular model loses its viability.

Evaluation of a deployed model in the manner outlined above is accompanied with an evaluation of students' conceptual profiles about the particular model. The latter evaluation is conducted explicitly following Figure 3.4 in order to help students to detect and resolve any incommensurability with the scientific model, so as to significantly reduce the corresponding naïve dimension in their conceptual profiles and build up the respective scientific dimension (Fig. 3.2). The particular deployment activity that is being carried out may sometimes not offer sufficient grounds for students' profiles to evolve as desired. The teacher may then help the process by framing unresolved matters in different contexts, e.g., by asking simple questions about related real world systems and phenomena, say, in the manner recommended by Mazur (1997a). Students are then guided in the manner discussed in § 4.6 to uncover the limitations or non-viability of their own ideas and regulate things appropriately.

### *MLC9. Paradigmatic synthesis:*

Students are engaged in self-evaluation and self-regulation throughout a learning cycle in order to bring their conceptual and paradigmatic profiles to evolve in the most meaningful and stable way possible (Fig. 4.10). This is achieved with appropriate reinforcement activities including deployment exercises and, especially, with periodic checks of major lessons learned by the end of each stage. Individual students keep a log of what they learn throughout a cycle and periodically recapitulate major lessons so as to consolidate the learning experience. Recapitulation involves model content, modeling methodology and various underlying tenets and assumptions, all considered by comparison to students' initial profiles. As in the case of evaluation, recapitulation is most comprehensive when carried out by the end of the model formulation phase (MLC6) and, especially, by the end of the deployment phase (MLC8).

Recapitulation may be informally, and even intermittently carried out at the end of each stage, but not at the end of MLC6, and especially not at the end of MLC8. It is conducted then rigorously and systematically to take the form of a *paradigmatic synthesis*, a synthesis that consolidates the desired paradigmatic evolution. Individual students would have already been directed to keep personal logs of major lessons learned throughout various MLC stages. A log consists of two parts. In the first part, a student records an outline of what has been agreed upon in class (as a scientific perspective) regarding: (a) the four schematic dimensions of the model under construction, (b) the four schematic dimensions of any new concept constructed in the process, and especially semantics and syntax of every new conception involved in model composition and structure, (c) approved methodology, including various tools, rules and schemes of modeling (including problem solving) and of reflective inquiry, (d) generic, theory and paradigm, tenets underlying all former points. The second part of the log consists of major student ideas regarding each of the four points logged in the first part from a scientific perspective, ideas that emerged in class discussions and that are incommensurate with science (naïve or of limited viability). Each of these ideas is recorded along with: (a) reasons offered in class to show its deficiencies from rational and empirical perspectives, and (b) the way(s) incommensurability has been resolved. By the end of MLC6 and/or MLC8, students are asked to compare notes within their own groups and come up with a group log to be exposed and discussed in class like any other group activity. Group logs are then fused under teacher arbitration into a common paradigmatic synthesis. In order to ensure that no salient point is missed, and that the synthesis is written in a scientific language, it is preferred that the teacher puts it together and hands it out to students, but only after group logs are discussed in class and a consensus has been reached about most if not all elements of the synthesis. An alternative would be to assign synthesis composition to a particular group, and then have it reviewed and put in its final form by the teacher. Group discussion leading to the paradigmatic synthesis may take up to an entire class period by the end of MLC8. The discussion ends with a monstration exercise (MLC1 of the next cycle) that shows the limitations of the newly constructed model and sets the stage for the construction of a new model.

Paradigmatic synthesis has many advantages over end-of-chapter summaries presented in conventional textbooks or by teachers. From a procedural perspective, the synthesis is not conveyed in one shot by an authority, be it teacher or textbook, for rote memorization. Students are instead provided ample opportunities to put it together progressively throughout a cycle and then to recapitulate it in the last stage of a learning cycle, in an insightful, self-regulatory process. All along, they reflect on their own conceptual and paradigmatic profiles under a variety of settings: as they compose their individual logs on their own, as they discuss logs in groups without teacher intervention and among groups under teacher arbitration thereafter, and finally, as they write down the final synthesis, or as they read it in the form presented by the teacher and compare it to what they had achieved on their own. From a content perspective, and unlike end-of-chapter summaries, the paradigmatic synthesis is not a rundown of theoretical statements and formulas. It is instead a systematic recapitulation, following modeling schemata, of a model and its building blocks, as well as of tools, processes and underlying tenets involved in model construction and deployment. It also involves a review of all student ideas that emerged throughout the cycle and that are incommensurate with science, a critical review that ingrains in students' minds the shortcomings of such ideas and the advantage of scientific alternatives, which would help in stabilizing their paradigmatic evolution.

## 5.7 TEACHER-MEDIATED LEARNING

Modeling learning cycles are student-centered. They are flexible enough to allow individual students to reflect on their own ideas and regulate them in the light of the rational and empirical evidence at their disposal. Yet, cycles are structured in the manner discussed in this chapter so as to ensure that student paradigmatic evolution toward the realm of science takes place in the most effective and efficient way possible. It is the teacher's responsibility to ensure that students proceed systematically from one phase to the next and that all sorts of dialectics and negotiations be controlled so that self-regulation takes place in a meaningful and insightful manner. Modeling instruction is thus student-centered in the sense that it actively engages individual students in the learning process, but it does not leave them out entirely of their own free will. It has a specific agenda to fulfil, meaningful and insightful paradigmatic evolution within the confinements of a given curriculum, an agenda that cannot be fulfilled without teacher mediation.

Advocates of modern educational trends have often pushed the slogan that, for meaningful learning of science, teacher role needs to change from “a sage on the stage to a guide on the side”. We do sympathize with the idea that a science teacher should not behave like “a sage on the stage” who does nothing but deliver sermons about science, and expects the message to sink deep into students’ minds. However, we do not subscribe to the second part of the slogan in a manner that gives teachers a marginal role in the classroom, or that portrays them as mere catalysts of classroom activities.

There is no meaningful learning without teaching, at least not for the overwhelming majority of students who cannot be self-educated. “We have centuries of evidence to show that natural thinking is neither rational nor scientific. Scientific thinking has to be cultivated and nurtured. It is the result of education… Without teachers there are neither scientists nor scientifically literate citizens” (Matthews, 2000, pp. 332, 349). Research has shown that students “do not have the maturity to work either independently or in small groups” without guidance, and that it is practically unreasonable to expect that “students could be scientific inquirers in the classroom and generate meaning more or less independently of the teacher”. Even when “students are encouraged to generate their own questions, the teacher must always be prepared to intervene in order to keep the content in line with the expectations of curricular experts” (Deboer, 2002). Research has even shown that “gifted” or “genius” experts who out-perform their peers in arts and science owe their achievement to the fact that they benefit from “sustained and specialized intervention from skilled teachers and parents” and especially “master teachers who either themselves had reached that level or had previously trained other individuals to that level” (Ericsson & Charness, 1994). Teacher intervention is advocated not in the form of conventional lecture and demonstration, but in the perspective of “mediated learning experience” whereby the teacher “mediates, transforms, reorders, organizes, groups, and frames” appropriate learning activities (Feuerstein & Jensen, 1980).

“In seeking to understand new ideas”, Swann (1950) argued more than half a century ago, “the student must, in a sense, travel the same path as the originator of the ideas. To do this, however, he does not have to be a Newton or an Einstein, for he has beside him his teacher to steer him away from unfruitful paths and illuminate the beauties of the true path as he develops eyes to see it”. Our modeling learning cycle is teacher-mediated in this sense. Depending on the circumstance, mediation may take the form of moderation, arbitration or scaffolding, and it always involves teacher feedback (§ 4.6) so as to prevent students from going astray and to keep their reflective inquiry aligned as closely as possible with scientific inquiry.

Moderation and arbitration are appropriate when students have their own ideas about topics of instruction. Scaffolding is most appropriate when students lack any knowledge about such topics, but it may also be resorted to in order to enhance moderation and arbitration. For mediation to be efficiently carried out, the teacher needs then to be equipped with a battery of diagnostic instruments that would help identify and categorize student preinstructional knowledge state, and decide subsequently for the appropriate mediation strategy (Halloun, 2004).

As a *moderator*, the teacher solicits ideas about a particular topic, and then guides students to compare ideas and resolve possible incompatibilities to the extent that they can do it on their own. The teacher does not intervene directly in the process to resolve the matter in favor of one idea or another. As noted in § 5. 2, s/he can only passively supply some rational or historical details, or some empirical data that may help students brainstorm, clarify to one another specific ideas of their own, or bypass a stalemate that they may get to. The teacher gets more involved in the mediation process as an *arbitrator*. This role is especially important when students have conceptions or follow rules of engagement that are incommensurate with science (naïve or of limited viability). The teacher would then bring concerned students first to a conscious state of cognitive disequilibrium, and direct them next to negotiate things with their colleagues so as to get them resolved in favor of a particular position that is viable from a scientific perspective. The teacher does so first by invoking among students a sort of Socratic dialogue (Hake, 1987, 1992). When this fails to bring things to a satisfactory closure in due time, the teacher shifts to scaffolding and offers the scientific position as an alternative that students are invited to contemplate.

*Scaffolding* is, for us, the type of mediation whereby the teacher gets most involved in directing the learning experience in the scientific direction. This sort of mediation is resorted to when arbitration fails to bring about students' self-regulation, but especially when students' knowledge about the topic of instruction is totally lacking. In the latter event, the teacher intervenes by confronting students with empirical situations or data from which they are guided to infer the appropriate conception(s) (Fig. 4.10), and/or by helping students rationally infer such conceptions from prior knowledge by anchoring (Table 3.1). The teacher may provide students with appropriate tools in the process. When students fail to construct the target conception or conduct a particular modeling process, the teacher induces them to do so in a more direct way by presenting them with the scientific conception or process. The scientific position is however not imposed in an authoritative way, but is offered only as an alternative that students are asked to consider and ascertain on their own in order to be convinced of its viability. The teacher does the same when arbitration fails to meet its ends. Students would subsequently be asked to deploy the scientific conception or process in a sequence of modeling activities where the teacher can gradually retreat from direct intervention, somewhat in the manner promoted in *cognitive apprenticeship* and similar modes of instruction (Heller, Foster & Heller, 1997; Shore et al., 1992; Roychoudhury & Roth, 1996). Scaffolding is especially needed in the first few cycles of a science course, cycles devoted to the construction and deployment of elementary basic models (§ 4.2). The teacher progressively moves away from this mediation form through subsequent cycles as students become more and more autonomous in model construction and deployment.

Mediation is the principal role of a teacher in modeling instruction. It is a role that preserves in all its forms a central role for student engagement and active participation in decision making. Nevertheless, not all aspects of modeling instruction can be dealt with that way. There are matters that are entirely reserved for the teacher to decide upon, matters with respect to which students are not prepared to have a say. This is the case for example of setting the objectives of a science course and the criteria for meeting the objectives (primarily in terms of the paradigmatic thresholds we are promoting). It is the sole responsibility of the teacher to set such matters and to decide on appropriate learning activities and the mediation process that best suits each activity. In parallel, there are matters about which students may have a say, but where the final word is always reserved to the teacher. This is the case for example with the agendas of learning activities and with assessment and evaluation criteria.

The modeling program of instruction and learning cycles require of a science teacher particular proficiencies that go beyond what is normally required in any other form of instruction. The modeling program is unique in many respects. It requires that the content of a science course be explicitly restructured around models, that each model and any conception it requires be constructed according to modeling schemata, that necessary schemes and tools be developed along with explicit rules of engagement, that particular activities be designed for students to conceive all aspects of exploratory and inventive inquiry, and that assessment and evaluation be conducted according to particular norms and guidelines. Modeling learning cycles have many things in common with modern educational trends, yet they are also unique in some respects. Some distinctive aspects pertain to the progressive approach in model construction and deployment. Others pertain to management of student discourse, particularly in relation to the rational-empirical dialectics set in the program and related evaluation norms and guidelines. The modeling program thus requires of a teacher particular understanding of the scientific paradigms governing a course of instruction, as well as of the epistemology and methodology of science as envisaged in its historical context. Modeling cycles further require that the teacher embrace a cognitive perspective on scientific and naïve paradigms pertaining to the course in question. They also entail mastery of various mediation skills and tools, especially those required to guide students through insightful processes of self-evaluation and self-regulation.

All in all, teaching science in the framework of modeling theory is an involved process that requires particular teacher training, support systems, and classroom environments. It requires of science educators a pedagogic paradigmatic evolution that may look radical in several respects, but that is attainable, as our experience suggests, by any educator willing to invest the necessary time and effort. In his acceptance speech for the 1989 Oersted Medal presented to him by the American Association of Physics Teachers, A. P. French (1989) noted:

> *"When it comes to curriculum, anyone who studies such matters must, I think, be struck by two things. One is the enormous amount of dedicated effort that has gone into the design of new curricula. The other is the way in which the results of such efforts tend to disappear from the scene."*

Our experience during the past two decades has systematically shown that dedicated efforts within the framework of modeling theory pay off in bringing about the paradigmatic evolution promoted in this book. Learning outcomes in a given course are significantly more meaningful, and especially more equitable, under modeling instruction than under some other forms of instruction, and they are sustainable within and beyond the course discipline (Halloun, 2000, 2004; Taconis et al., 2001; Wells et al., 1995). Our results should thus not "disappear from the scene", especially if all dedicated educators who are implementing, or considering to implement, one variant or another of modeling theory consolidate their efforts. This book will hopefully serve this end.

# REFERENCES

Ahlgren, A. & Wheeler, S. (2002). Mapping the steps toward basic understanding of scientific inquiry. *Science & Education, 11* (3), 217-230.

Aikenhead, G.S. (1987). High school graduates beliefs about Science-technology-society. III. Characteristics and limitations of scientific knowledge. *Science Education 71* (4), 459-487.

American Association for the Advencement of Science. (1993). *Benchmarks for Science Literacy.* Project 2061. New York: Oxford University Press.

American Association for the Advencement of Science. (1990). *Science for All Americans. Project 2061*. New York: Oxford University Press.

American Mathematical Association of Two-Year Colleges. (1995). *Crossroads in Mathematics. Standards for Introductory College Mathematics Before Calculus*. Memphis, TN: AMATYC.

Andaloro, G., Donzeli, V., & Sperandeo-Mineo, R.M. (1991). Modelling in physics teaching: the role of computer simulation. *International Journal of Science Education, 13* (3), 243-254.

Archambault, R.D. (Ed.). (1964). *John Dewey on Education: Selected Writings*. Chicago, IL: The University of Chicago Press.

Bachelard, G. (1940). *La Philosophie du Non*. Paris: Quadrige (4th edition, 1994) / Presses Universitaires de France.

Bachelard, G. (1934). *Le Nouvel Esprit Scientifique*. Paris: Quadrige (6th edition, 1999) / Presses Universitaires de France.

Bernard, C. (1865). *Introduction à l'Étude de la Médecine Expérimentale*. Paris: Flammarion.

Bower, G.H., & Morrow, D.G. (1990). Mental Models in Narrative Comprehension. *Science, 247*, 44-48.

Brainin, S. H. (1985). Mediating learning: Pedagogic issues in the improvement of cognitive functioning. In: E. W. Gordon (Ed.), *Review of Research in Education,* 121-155. Washington, DC: American Educational Research Association.

Bronowski, J. (1953). *The Common Sense of Science.* Cambridge, MA: Harvard University Press.

Bullock, B. (1979). The use of models to teach elementary physics. *Physics Education, 14,* 312-317.

Bunge, M. (1973). *Method, Model and Matter.* Boston, MA: D. Reidel.

Bunge, M. (1967). *Scientific Research I & II.* New York, NY: Springer-Verlag.

Casti, J. L. (1989). *Alternate Realities. Mathematical Models of Nature and Man*. New York, NY: Wiley-Interscience.

Chi, M.T.H., Feltovich, P.J., & Glaser, R. (1981). Categorization and Representation of Physics Problems by Experts and Novices. *Cognitive Science, 5,* 121-152.

Clement, J. (1993). Using bridging analogies and anchoring intuitions to deal with students' preconceptions in physics. *Journal of Research in Science Teaching, 30* (10), 1241-1257.

Clement, J. (1989). Learning via Model Construction and Criticism. In: G. Glover, R. Ronning & C. Reynolds (Eds.), *Handbook of Creativity, Assessment, Theory and Research.* New York, NY: Plenum.

Cobern, W. W. (1995). Science education as an exercise in foreign affairs. *Science & Education, 4* (3), 287-302.

Cobern, W.W. (1993). College students' conceptualizations of nature: An interpretive world view analysis. *Journal of Research in Science Teaching, 30* (8), 935-951.

Crouch, C. H. & Mazur, E. (2001). Peer Instruction: Ten years of experience and results. *American Journal of Physics, 69* (9), 970-977.

Deboer, G. E. (2002). Student-centered teaching in a Standards-based world: Finding a sensible balance. *Science & Education, 11* (4), 405-417.

diSessa, A.A. (1993). Toward an epistemology of physics. *Cognition & Instruction, 10* (2&3), 105-225.

Doerr, H. M. (1996). Integrating the study of trigonometry, vectors, and force through modeling. *School Science and Mathematics, 96* (8), 407-418.

Dreyfus, A., Jungwirth, E., & Eliovitch, R. (1990). Applying the "cognitive conflict" strategy for conceptual change – Some implications, difficulties and problems. *Science Education, 74* (5*),* 555-569.

Driver, R., Erickson, G. (1983). Theories-in-action: some theoretical and empirical issues in the study of students' conceptual frameworks in science. *Studies in Science Education, 10*, 37 – 60.

Duschl, R. A. (1988). Abandoning the scientistic legacy of science education. *Science Education, 72*, 51-62.

Duschl, R.A. & Hamilton, R. (Eds.). (1992). *Philosophy of Science, Cognitive Psychology, and Educational Theory and Practice*. Albany, NY: SUNY Press.

Duschl, R. A., Hamilton, R. & Grandy, R. E. (1990). Psychology and epistemology: Match or mismatch when applied to science education? *International Journal of Science Education, 12* (3), 230-243.

Dykstra, D.I., Boyle, C.F. & Monarch, I.A. (1992). Studying conceptual change in learning physics. *Science Education 76* (6), 615-652.

Edmondson, K. M. and Novak, J. D. (1993). The interplay of scientific epistemological views, learning strategies, and attitudes of college students. *Journal of Research in Science Teaching, 30* (6), 547-559.

Eger, M. (1993). Hermeneutics as an approach to science: Parts I & II. *Science & Education, 2* (1 & 4), 1-29 & 303-328.

Erduran, S. (2001). Philosophy of chemistry: An emerging field with implications for chemistry education. *Science & Education, 10* (6), 581-593.

Ericsson, K. A. & Charness, N. (1994). Expert performance. Its structure and acquisition. *American Psychologist, 49* (8), 725-747.

Eylon, B. S. & Linn, M. C. (1988). Learning and instruction: An examination of four research perspectives in science education. *Review of Educational Research, 58* (3), 251-301.

Eylon, B.S., & Reif. (1984). Effects of Knowledge Organization on Task Performance. *Cognition and Instruction, 1* (1), 5-44.

Feuerstein, R. & Jensen, M. R. (1980). Instrumental enrichment: Theoretical basis, goals, and instruments. *The Educational Forum*, 401-423.

Feuerstein, R., Rand, Y., Hoffman, M. B., & Miller, R. (1980). *Instrumental Enrichment. An Intervention Program for Cognitive Modifiability.* Baltimore, MD: University Park Press.

Fisher, K.M. (1990). Semantic networking: the new kid on the block. *Journal of Research in Science Teaching, 27* (10), 1001-1018.

French, A. P. (1989). Learning from the past; looking to the future. *American Journal of Physics, 57* (7), 587-592.

Garnett, P.J., Tobin, K. & Swingler, D.G. (1985). Reasoning abilities of secondary school students aged 13-16 and implications for the teaching of science. *European Journal of Science E*ducation*, 7*, 387-397.

Gee, B. (1978). Models as a pedagogical tool: can we learn from Maxwell? *Physics Education, 13,* 287-291.

Gentner, D., & Stevens A. L. (Eds.). (1983). *Mental Models.* Hillsdale, NJ: Lawrence Erlbaum.

Giere, R.N. (1994). The cognitive structure of scientific theories. *Philosophy of Science, 61,* 276-296.

Giere, R.N., Editor (1992) *Cognitive Models of Science.* Minnesota Studies in the Philosophy of Science, Vol. XV. Minneapolis, MN: University of Minnesota Press.

Giere, R.N. (1988). *Explaining Science: A Cognitive Approach.* Chicago, IL: University of Chicago Press.

Gilbert, S. W. (1991). Model building and a definition of science. *Journal of Research in Science Teaching, 28* (1), 73-79.

Glas, E. (2002). Klein's model of mathematical creativity. *Science & Education, 11* (1), 95-104.

Goldberg, F., Bendall, S., & Bach, N. (1991). Development of computer-video-based learning activities and assessment of student undertanding in the domain of geometrical optics. Paper presented at the *Annual Meeting of the American Educational Research Association.*

Grosslight, L., Unger, C., Jay, E., and Smith, C. (1991). Understanding models and their use in science: Conceptions of middle and high school students and experts. *Journal of Research in Science Teaching, 28* (9), 799-822.

Gruender, D. (2001). A new principle of demarcation: A modest proposal for science and science education. *Science & Education, 10* (1), 85-95.

Hafner, R., & Stewart, J. (1995). Revising Explanatory Models to Accommodate Anomalous Genetic Phenomena: Problem Solving in the "Context of Discovery". *Science Education, 79* (2), 111-146.

Hake, R. R. (2001). Lessons from the physics-education reform effort. Available at: http://*www.physics.indiana.edu/~hake.*

Hake, R. R. (1992). Socratic pedagogy in the introductory physics laboratory. *The Physics Teacher, 30,* 546-552.

Hake, R. R. (1987). Promoting student crossover to the Newtonian world. *American Journal of Physics, 55* (10), 878-884.

Halliday, D., Resnick, R., & Walker, J. (1997). *Fundamentals of Physics*. 5th Edition. New York: John Wiley & Sons.

Halloun, I. (2004). Normative evaluation of mathematics, science and technology curricula: The case of the modeling curriculum. *Proceedings of the 12th annual meeting of the South African Association for Research in Mathematics, Science and Technology Education*. Cape Town, South Africa: SAARMSTE.

Halloun, I. (2001a). *Apprentissage par Modélisation : La Physique Intelligible.* Beirut, Lebanon: Phoenix series / Librairie du Liban Publishers.

Halloun, I. (2001b). *Student Views about Science: A comparative Survey.* (2001). Beirut, Lebanon: Phoenix series / Educational Research Center, Lebanese University.

Halloun, I. (2000). Model-laden inquiry: A prescription for effective physics instruction. (2000). *THEMES, 1* (4), 339-355.

Halloun, I. (1998a). Schematic concepts for schematic models of the real world. *Science Education, 82* (2), 239-263.

Halloun, I. (1998b). Interactive model-based education: An alternative to outcomes-based education in physics. *South African Journal of Science, 94,* 313-318.

Halloun, I. (1996a). Schematic modeling for meaningful learning of physics. *Journal of Research in Science Teaching, 33* (9), 1019-1041.

Halloun, I. (1994). Teaching model construction for solving physics problems. *Recherches Pédagogiques, 19,* 5-17.

Halloun, I. (1986). Le réalisme naif et l'apprentissage de la physique. *Recherches Pédagogiques, 17*, 23-47.

Halloun, I. (1984). *The Use of Models in Teaching Newtonian Mechanics*. Unpublished PhD dissertation. Tempe, AZ: Arizona State University.

Halloun, I., Hake, R., Mosca, E., & Hestenes, D. (1995). *Revised Force Concept Inventory*. Available at: http://asu.edu/modeling.

Halloun, I., & Hestenes, D. (1998). Interpreting VASS dimensions and profiles. *Science & Education, 7* (6), 553-577.

Halloun, I., & Hestenes, D. (1995). Interpreting the Force Concept Inventory. *The Physics Teacher, 33* (8), 502-506.

Halloun, I., & Hestenes, D. (1987). Modeling Instruction in Mechanics. *American Journal of Physics*, *55* (5), 455-462.

Halloun, I., & Hestenes, D. (1985a). The initial knowledge state of college physics Students. *American Journal of Physics, 53* (11), 1043-1055.

Halloun, I., & Hestenes, D. (1985b). Common sense concepts about motion. *American Journal of Physics, 53* (11), 1056-1065.

Halloun, I. & Rabah, H. (2004). Interaction and force according to secondary school students. *In preparation*.

Hammer, D. (1994). Epistemological beliefs in introductory physics. *Cognition and Instruction, 12* (2), 151-183.

Harré, R. (1978). Models in science. *Physics Education*, *13* (5), 275-278.

Harré, R. (1970). *The Principles of Scientific Thinking.* Chicago, IL : The University of Chicago Press.

Harré, R. (1961). *Theories and Things.* London, UK : Sheed & Ward.

Harte, J. (2002). Toward a synthesis of the Newtonian and Darwinian worldviews. *Physics Today, 55* (10), 29-34.

Hashweh, M. (1986). Towards an explanation of conceptual change. *European Journal of Science Education, 8*, 129 – 289.

Hecht, E. (1994). *Physics.* Pacific Grove, CA: Brooks/Cole.

Heller, J.I., & Reif, F. (1984). Prescribing Effective Human Problem Solving Processes: Problem Description in Physics. *Cognition and Instruction, 1* (2), 177-216.

Heller, P., Foster, T., & Heller, K. (1997). Cooperative group problem solving laboratories for introductory classes. In: E. F. Redish & J. S. Rigden (Eds), *The Changing Role of Physics Departments in Modern Universities. Proceedings of ICUPE*, 913-933. College Park, MD: American Institute of Physics.

Helm, H. & Novak, J. (Eds.). (1983). Proceedings of the International Seminar: Misconceptions in Science and Mathematics. Ithaca, NY: Cornell University.

Hempel, C. G. (1965). *Aspects of Scientific Explanation.* New York: The Free Press, Macmillan.

Hesse, M.B. (1970). *Models and Analogies in Science.* South Bend, IN: University of Notre Dame Press.

Hestenes, D. (1997). Modeling methodology for physics teachers. In: E. F. Redish & J. S. Rigden (Eds), *The Changing Role of Physics Departments in Modern Universities. Proceedings of ICUPE*, 935-957. College Park, MD: American Institute of Physics.

Hestenes, D. (1995). Modeling software for learning and doing physics. In: C. Bernardini, C. Tarsitani & M. Vincentini (Eds.), *Thinking Physics for Teaching,* 25-66. New York: Plenum.

Hestenes, D. (1992). Modeling games in the Newtonian World. *American Journal of Physics*, *60* (8), 732-748.

Hestenes, D. (1987). Toward a Modeling Theory of Physics Instruction. *American Journal of Physics*, *55* (5), 440-454.

Holton, G. (1993). *Science and Anti-Science*. Cambridge, MA: Harvard University Press.

Johnson-Laird, P.N. (1983). *Mental Models.* Cambridge, MA: Cambridge University Press.

Joshua, S. & Dupin, J.J. (1999). *Introduction à la Didactique des Sciences et des Mathématiques*. 2nd Ed. Paris : Presses Universitaires de France.

Joshua, S. & Dupin, J.J. (1989). *Représentations et modélisations : Le débat scientifique dans la classe et l'apprentissage de la physique*. Berne : Peter Lang.

Justi, R. & Gilbert J. K. (2002). Models and Modelling in Chemical Education. In: J. K. Gilbert, O. de Jong, R. Justi, D. F. Treagust & J. H. van Driel (Eds), *Chemical Education: Towards Research-based Practice*, 47-68. Dordrecht: Kluwer Academic Publishers.

Kafatos, M. & Nadeau, R. (1990). *The Conscious Universe. Part and Whole in Modern Physical Theory*. New York: Springer-Verlag.

Karplus, R. (1977). Science Teaching and the Development of Reasoning. *Journal of Research in Science Teaching, 14* (2), 169-175.

Kelvin, W.T., Lord. (1891). *Popular Lectures and Addresses*. London: Macmillan.

King, P. M. & Kitchener, K. S. (1994). *Developing Reflective Judgment. Understanding and Promoting Intellectual Growth and Critical Thinking in Adolescents and Adults*. San Francisco, CA: Jossey-Bass.

Kuhn, T. (1970). *The Structure of Scientific Revolutions.* 2nd Ed. Chicago, IL: University of Chicago Press.

Lakatos, I., Musgrave, A, (Eds.). (1970). *Criticism and the Growth of Knowledge.* Cambridge: Cambridge University Press.

Lakoff, G. (1987). *Women, Fire, and Dangerous Things*. *What Categories reveal about the Mind*. Chicago, IL: The University of Chicago Press.

Lakoff, G. & Johnson, M. (1980). *Metaphors we live by.* Chicago, IL: The University of Chicago Press.

Latour, B. & Woolgar, S. (1979). *Laboratory Life.* Beverly Hills, CA: Sage.

Leatherdale, W.H. (1974). *The Role of Analogy, Model and Metaphor in Science.* New York, NY: American Elsevier Pub.

Lockhead, J. (1985). New horizons in educational development. In: E. W. Gordon (Ed.), *Review of Research in Education,* 3-9. Washington, DC: American Educational Research Association.

Mac Lane, S. (1988). Mathematical Models: A Sketch for the Philosophy of Mathematics. *The American Mathematical Monthly*, *95*, 462-471.

Mach, E. (1986). *Popular Scientific Lectures.* Illinois: Open Court.

Margolis, H. (1993). *Paradigms & Barriers. How Habits of Mind Govern Scientific Beliefs*. Chicago, IL: The University of Chicago Press.

Matthews, M. R. (2000). *Time for Science Education. How Teaching the History and Philosophy of Pendulum Motion Can Contribute to Science Literacy*. New York: Kluwer Academic/Plenum Publishers.

Mazur, E. (1997a). *Peer Instruction: A User's Manual.* Englewood Cliffs, NJ: Prentice Hall.

Mazur, E. (1997b). Peer instruction: Getting students to think in class. In: E. F. Redish & J. S. Rigden (Eds), *The Changing Role of Physics Departments in Modern Universities. Proceedings of ICUPE*, 981-988. College Park, MD: American Institute of Physics.

Miller, D. (1985). *Popper Selections.* Princeton, N.J.: Princeton University Press.

Minstrell, J. (1991). Facets of students' knowledge and relevant instruction. In: R. Duit, F. Goldberg & H. Niedderer (Eds.), *Research in physics learning: Theoretical issues and empirical studies*, 110-128. Kiel: IPN.

Minstrell, J. (1989). Teaching science for understanding. In L.B. Resnick & L.E. Klopfer (eds.), *Toward the Thinking Curriculum: Current Cognitive Research*. 129-149.

Minstrell, J. (1982). Explaining the 'at rest' condition of an object. *The Physics Teacher, 20*, 10-14.

Moreira, M. A. & Greca, H. (1995, April). Kinds of mental representations – models, propositions, and images – used by students and physicists regarding the concept of field. Paper presented at the *Annual meeting of the National Association for Research in Science Teaching*, San Francisco, CA.

Mortimer, E. F. (1995). Conceptual change or conceptual profile change? *Science & Education, 4* (3), 267-285.

Nagel, E. (1979). *The Structure of Science.* Indiananpolis, IN: Hackett.

National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM.

National Research Council. (1996). *National Science Education Standards*. Washington, DC: National Academy Press.

National Science Teachers Association. (1995). *Scope, Sequence, and Coordination of Secondary School Science. Volume 3. A High School Framework for National Science Education Standards*. Washington, DC: NSTA.

Nersessian, N.J. (1995). Should physicists preach what they practice? Constructive modeling in doing and learning physics. *Science & Education, 4* (3), 203-226.

Novak, J. (Ed.). (1993). *Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics*. Ithaca, NY: Cornell University (distributed electronically).

Novak, J.D. (1990). Concept mapping: a useful tool for science education. *Journal of Research in Science Teaching, 27* (10), 937-949.

Novak J. (Ed.). (1987). *Proceedings of the Second International Seminar: Misconceptions and Educational Strategies in Science and Mathematics*, Vol. I, II, III. Ithaca, NY: Cornell University.

Novak, J.D., Gowin, D.B. & Johansen, G.T. (1983). The use of concept mapping and knowledge vee mapping with Junior High School science students. *Science Education, 67*, 625-645.

Passmore, C. & Stewart, J. (2002). A modeling approach to teaching evolutionary biology in high schools. *Journal of Research in Science Teaching, 39* (3), 185-204.

Perry, W.G. (1970). *Forms of Intellectual and Ethical Development in the College Years: A Scheme*. New York: Holt, Rinehart, and Winston.

Poincaré, H. (1902). *La science et l'hypothèse.* Paris: Flammarion.

Pollak, V.L. (1994). Science Education II: Scientific Literacy and the Karplus Taxonomy. *Journal of Science Education and Technology*, 3 (2), 89-97.

Popper, K. R. (1963). *Conjectures and Refutations*. New York: Basic Books.

Popper, K. R. (1959). *The Logic of Scientific Discovery*. London: Hutchinson.

Posner, G.J., Strike, K.A., Hewson, P.W. & Gertzog, W.A. (1982). Accommodation of a scientific conception: Toward a theory of conceptual change. *Science Education*, 66 (2), 211-227.

Raghavan K., & Glaser, R. (1995). Model-Based Analysis and Reasoning in Science: The MARS Curriculum. *Science Education*, 79 (1), 37-61.

Raghavan, K., Sartoris, M.L., & Glaser, R. (1998).Why does it go up? The impact of the MARS curriculum as revealed through changes in student explanations of a helium balloon. *Journal of Research in Science Teaching, 35* (5), 547-567.

Raman, V. V. (1980). Teaching Aristotelian physics through dialogue. *The Physics Teacher, 18* (8), 580-583.

Redish, E. (1994). Implications of Cognitive Studies for Teaching Physics. *American Journal of Physics*, *62* (9), 796-803.

Reif, F., & Heller, J.I. (1982). Knowledge structure and problem solving in physics. *Educational Psychologist, 17* (2), 102-127.

Reif, F. & Larkin, J.H. (1991). Cognition in scientific and everyday domains: Comparison and learning implications. *Journal of Research in Science Teaching, 28* (9), 733-760.

Renner, J.W., Abraham, M.R., & Birnie, H.H. (1985). The importance of the *Form* of Student Acquisition of Data in Physics Learning Cycles. *Journal of Research in Science Teaching, 22* (4), 303-325.

Ribe, N., & Steinle, F. (2002). Exploratory experimentation: Goethe, Land, and color theory. *Physics Today, 55* (7), 43-49.

Roychoudhury, A. & Roth, W. M. (1996). Interactions in an open-inquiry physics laboratory. *International Journal of Science Education, 18* (4), 423-445.

Seiler, G., Tobin, K., & Sokolic, J. (2001). Design, technology, and science: Sites for learning, resistance, and social reproduction in urban schools. *Journal of Research in Science Teaching, 38* (7), 746-767.

Shore, L. S., Erickson, M. J., Garik, P., Hickman, P., Stanley, E., Taylor, E. F., & Trunfio, P. A. (1992). Learning fractals by "doing science": Applying cognitive apprenticeship strategies to curriculum design and instruction. *Interactive Learning Environments, 2* (3 & 4), 205-226.

Siegler, R. S. (1978). The origins of scientific reasoning. In: R. Glaser (Ed.), *Advances in Instructional Psychology*, 109-149. Hillsdale, N.J.: Lawrence Erlbaum Associates.

Singh, C. (2002). When physical intuition fails. *American Journal of Physics, 70* (11), 1103-1109.

Smit, J. J. A. & Finegold, M. (1995). Models in physics: perceptions held by final-year prospective physical science teachers studying at South African universities. *International Journal of Science Education, 17* (5), 621-634.

Smith, E.L., Blakeslee, T.D., & Anderson, C.W. (1993). Teaching Strategies Associated with Conceptual Change Learning in Science. *Journal of Research in Science Teaching, 30* (2), 111-126.

Smith, J.P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *Journal of Learning Sciences, 3(2)*, 115-163.

Smith, K.A. & Starfield, A. M. (1993). Building models to solve problems. In: J. H Clarke & A. W. Biddle (Eds.). *Teaching critical thinking: Reports from across the curriculum,* 254-263. Englewood Cliffs, NJ: Prentice-Hall.

Songer, N. B. and Linn, M. C. (1991). How do students' views of science influence knowledge integration? *Journal of Research in Science Teaching, 28* (9), 761-784.

Spielberg, N. & Anderson, B. D. (1995). *Seven Ideas that shook the Universe.* 2nd edition. New York: Wiley.

Steen, L.A. Editor (1990). *On the Shoulders of Giants. New Approaches to Numeracy.* Washington, DC: National Academy Press.

Swann, W.F.G. (1950). The Teaching of Physics. *American Journal of Physics*, *19* (2), 182-187.

Swetz, F., & Hartzler, J.S. (Eds.). (1991). *Mathematical Modeling in the Secondary School Curriculum.* Reston, VA: National Coucil of Teachers of Mathematics.

Taconis, R., Ferguson-Hessler, M.G.M., & Broekkamp, H. (2001). Teaching science problem solving: An overview of experimental work. *Journal of Research in Science Teaching, 38* (4), 442-468.

Tobias, S. (1990). *They're Not Dumb, They're Different. Stalking the Second Tier*. Tucson, AZ: Research Corporation.

Viau, E. A. (1994). The mind as a channel: A paradigm for the information age. *Educational Technology Review, Autumn/Winter 1994,* 5-10.

Wartofsky, M.W. (1968). *Conceptual Foundations of Scientific Thought.* New York, NY: MacMillan.

Wells, M., Hestenes, D., & Swackhamer, G. (1995). A Modeling Method for High School Physics Instruction. *American Journal of Physics, 64* (7), 606-619.

White, B.Y. (1993). ThinkerTools: Causal Models, Conceptual Change, and Science Education. *Cognition and Instruction, 10* (1), 1-100.

White, B.Y., & Frederiksen, J.R. (1990). Causal Model Progressions as a Foundation for Intelligent Learning Environments. *Artificial Intelligence, 42*, 99-157.

Whittaker, E., Sir. (1948). G. F. Fitzgerald. In: *Lives in Science.* A Scientific American book, 75-83. New York: Simon & Schuster, 1957.

Winans, J.G. (1971). Trajectories without trigonometry. *The Physics Teacher*, *9*, 341-343.

Wollman, W. (1984). Models and procedures: Teaching for transfer of pendulum knowledge. *Journal of Research in Science Teaching, 21* (4), 399-415.

Wong, D., Pugh, K., & the Dewey Ideas Group at Michigan State University. (2001). Learning Science: A Deweyan Perspective. *Journal of Research in Science Teaching*, *38* (3), 317-336.

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