Andrzej Malec
University of Białystok

LEGAL REASONING AND LOGIC

In this paper I discuss relations between rules of legal reasoning and formal logic. I state that some rules of legal reasoning can be arranged as formal systems. To prove my thesis I construct a formal system of the kind in question.

1. In the process of legal argumentation two kinds of rules of reasoning are used. The rules of the first kind are the well-known rules of classical logic. The rules of the second kind are usually called "the rules of legal reasoning".

2. The rules of legal reasoning can be divided into five groups:
 a) the rules of the first group (so called "rules of interpretation") are used to reconstruct the meaning of legal expressions; the famous rule *clara non sunt interpretanda* is of this kind,
 b) the rules of the second group (so called "rules of inference") are used to infer consequences from legal norms; the rules of reasoning: *per analogiam (a simili), a contrario, a fortiori (a maiori ad minus, a minori ad maius)* are of this kind,
 c) the rules of the third group (so called "rules of collision") are used to solve collisions of legal norms; the rule *lex posterior derogat legi priori* is of this kind,
 d) the rules of the fourth group are used to determine factual circumstances; the rule *in dubio pro reo (in dubio pro libertate)* is of this kind,
 e) the rules of the fifth group are the rules of procedure; the rule that a judge should consider arguments of both parties is of this kind.

3. The system of rules of legal reasoning is called "legal logic". How can we define the relation between legal logic and formal logic? Chaim Perelman opposes legal logic to formal logic in two ways. First, he maintains that legal logic is a heuristic logic, whereas formal logic is just the logic of justification. Second, he maintains that legal logic is possible only as "material logic",

"nonformal logic". This peculiarity of legal logic is connected – according to Perelman – with the fact that many rules of legal reasoning depend on subjective valuations.

4. *Ad hoc* I can agree that formal logic is not a heuristic logic. In fact, it doesn't offer us rules effective in **all** cases of legal argumentation. However, I admit that in **some** cases rules of formal logic could be effective (for example, it seems that rules of inductive logic are used in legal argumentation as a heuristic method).

5. It is reasonable to assume that legal logic is a heuristic logic. Having this assumption we can consider legal logic as a part of methodology of law: the part which deals with problems such as which legal norm should we use in legal argumentation and how should we use it? Legal logic helps us with finding the solution of a legal problem whereas formal logic (which includes both kind of rules: deductive and inductive) helps us with justification of this solution.

6. However, I can not agree that legal logic is **necessarily** nonformal. Indeed, many rules of legal logic are based on subjective valuations. Quite often this fact makes it difficult or even impossible to formalise such rules. For example, I don't know how we can formalise the rule *clara non sunt interpretanda*. However, on the other hand, many rules of legal reasoning can be formalised quite easily. For example, I formalise the rule of reasoning *a contrario* in the following way:

$$\frac{(x)\{P(x) \Rightarrow Q(x)\}}{(x)\{-P(x) \Rightarrow -Q(x)\}}$$

Moreover, sometimes it is possible not only to formalise a single rule, but also to build a formal system of rules of legal reasoning.

7. Let us consider the following rules of legal reasoning (these rules are called "rules of collision"):
- *lex posterior derogat legi priori* (later norms suppress earlier norms),
- *lex superior derogat legi inferiori* (superior norms suppress inferior norms),
- *lex specialis derogat legi generali* (particular norms suppress general norms),
- *lex superior prior derogat legi inferiori posteriori* (earlier superior norms suppress later inferior norms),

- *lex superior generalis derogat legi inferiori speciali* (superior general norms suppress inferior particular norms),
- *lex prior specialis derogat legi posteriori generali* (earlier particular norms suppress later general norms).

The first three of these rules are called "the first order rules of collision". The last three are called "the second order rules of collision". Whenever the use of first order rules leads us to a contradiction, we employ a second order rule. Respectively, a third order rule of collision would be defined and employed in the case of contradiction between second order rules.

8. Let us build a formal system for the above relations. We add some two-place predicates to the vocabulary of a system of predicate logic: $ESup(...,...)$, $ESpec(...,...)$, $EPost(...,...)$, $Sup(...,...)$, $Spec(...,...)$, $Post(...,...)$, $Der(...,...)$. The definitions: of term, of atomic formula, of formula and the definition of sentence are standard. The axioms of the system are: all sentences of the language of the system which are constructed according to the schemas of valid formulas of predicate logic and some axioms which describe the properties of $ESup(...,...)$, $ESpec(...,...)$, $EPost(...,...)$, $Sup(...,...)$, $Spec(...,...)$, $Post(...,...)$:

AXIOM 1 $(x)ESup(x,x)$,

AXIOM 2 $(x)(y)\{ESup(x,y) \Rightarrow ESup(y,x)\}$,

AXIOM 3 $(x)(y)(z)\{ESup(x,y) \mathbin{\&} ESup(y,z) \Rightarrow ESup(x,z)\}$,

AXIOM 4 $(x)Sup(x,x)$,

AXIOM 5 $(x)(y)\{Sup(x,y) \mathbin{\&} -ESup(x,y) \Rightarrow -Sup(y,x)\}$,

AXIOM 6 $(x)(y)(z)\{Sup(x,y) \mathbin{\&} Sup(y,z) \Rightarrow Sup(x,z)\}$,

AXIOM 7 $(x)(y)\{-Sup(x,y) \Rightarrow Sup(y,x)\}$,

AXIOM 8 $(x)ESpec(x,x)$,

AXIOM 9 $(x)(y)\{ESpec(x,y) \Rightarrow ESpec(y,x)\}$,

AXIOM 10 $(x)(y)(z)\{ESpec(x,y) \mathbin{\&} ESpec(y,z) \Rightarrow ESpec(x,z)\}$,

AXIOM 11 $(x)Spec(x,x)$,

AXIOM 12 $(x)(y)\{Spec(x,y) \mathbin{\&} -ESpec(x,y) \Rightarrow -Spec(y,x)\}$,

AXIOM 13 $(x)(y)(z)\{Spec(x,y) \mathbin{\&} Spec(y,z) \Rightarrow Spec(x,z)\}$,

AXIOM 14 $(x)(y)\{-Spec(x,y) \Rightarrow Spec(y,x)\}$,

AXIOM 15 $(x)EPost(x,x)$,

AXIOM 16 $(x)(y)\{EPost(x,y) \Rightarrow EPost(y,x)\}$,

AXIOM 17 $(x)(y)(z)\{EPost(x,y) \mathbin{\&} EPost(y,z) \Rightarrow EPost(x,z)\}$,

AXIOM 18 $(x)Post(x, x)$,

AXIOM 19 $(x)(y)\{Post(x, y)\ \&\ -EPost(x, y) \Rightarrow -Post(y, x)\}$,

AXIOM 20 $(x)(y)(z)\{Post(x, y)\ \&\ Post(y, z) \Rightarrow Post(x, z)\}$,

AXIOM 21 $(x)(y)\{-Post(x, y) \Rightarrow Post(y, x)\}$.

The only rules of the system are the rules of predicate logic and the following rules describing the properties of $Der(\dots, \dots)$:

RULE 1
$$\frac{\begin{array}{c} Sup(x, y) \\ -ESup(x, y) \end{array}}{Der(x, y)}$$

RULE 2
$$\frac{\begin{array}{c} ESup(x, y) \\ Spec(x, y) \\ -ESpec(x, y) \end{array}}{Der(x, y)}$$

RULE 3
$$\frac{\begin{array}{c} ESup(x, y) \\ ESpec(x, y) \\ Post(x, y) \\ -EPost(x, y) \end{array}}{Der(x, y)}$$

9. According to the axioms: AXIOM 1 – AXIOM 21, the predicates: $ESup(\dots, \dots)$, $ESpec(\dots, \dots)$, $EPost(\dots, \dots)$ denote some equivalence relations and the predicates: $Sup(\dots, \dots)$, $Spec(\dots, \dots)$, $Post(\dots, \dots)$ denote some linear order relations. According to intuition, the above relations order the set of legal norms. So, we read: $ESup(\dots, \dots)$ – "the norm … is neither superior nor inferior in relation to the norm …", $ESpec(\dots, \dots)$ – "the norm … is neither general nor particular in relation to the norm …", $EPost(\dots, \dots)$ – "the norm … is neither later nor earlier in relation to the norm …", $Sup(\dots, \dots)$ – "the norm … is not an inferior norm in relation to the norm …", $Spec(\dots, \dots)$ – "the norm … is not a general norm in relation to the norm …", $Post(\dots, \dots)$ – "the norm … is not an earlier norm in relation to the norm …".

10. The rules for $Der(\dots, \dots)$ can be called: RULE 1 – "the rule of derogation of inferior norms", RULE 2 – "the rule of derogation of general norms", RULE 3 – "the rule of derogation of earlier norms". These rules describe the order of derogation defined by the rules of legal reasoning introduced in the point 7 – so called "rules of collision". The construction of the rules:

RULE 1 – RULE 3 determines their "hierarchy": RULE 1 is the strongest rule (in the sense that one needs only two premises to use this rule) and RULE 3 is the weakest rule. So, in the above system the second order rules of collision are needless.

11. The above system is a formal system as well as a system of legal logic. So, legal logic is not necessarily nonformal.

Quod erat demonstrandum

Andrzej Malec
Department of Logic, Informatics and Philosophy of Science
University of Białystok
15–420 Bialystok, Plac Uniwersytecki 1, Poland
e-mail: malec@hum.uwb.edu.pl