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Poetry Inspired by Mathematics

Sarah Glaz
Department of Mathematics
University of Connecticut
Storrs, CT 06269, USA
E-mail: glaz@math.uconn.edu

Abstract

This article explores one of the many manifestations of the mysterious link between mathematics and poetry—the phenomenon of poetry inspired by mathematics. Such poetry responds to the mathematical concerns and accomplishments of the day, be it a ground breaking definition or technique, a long standing unsolved conjecture, or a celebrated theorem. The motivation for writing the poems, their mathematical subjects, and their poetic styles, vary through history and from culture to culture. We bring a selection of poems from a variety of time periods and mathematical subjects: from a Sumerian temple hymn—where an anonymous priest counts the number of cattle in the herds of the moon god, Nanna, to contemporary poetry celebrating the proof of Fermat's Last Theorem, the still unsolved Riemann Conjecture, or the creation of fractal geometry. We also include references to additional sources of poetry inspired by mathematic, and a brief discussion on the use of such poetry in the mathematics classroom.

Counting

Writing was invented in Mesopotamia, the fertile plain between the Tigris and the Euphrates rivers, situated in the region of present-day Iraq. Fragments of kiln baked clay tablets inscribed with wedge shaped cuneiform figures unearthed at archeological digs offer tantalizing glimpses of the culture, the poetry, and the mathematical activities of the Sumerian, Akkadian, and Babylonian civilizations that succeeded each other in the region from 4000 BC until about the 2nd century BC. The mathematical and poetic gifts left to us by ancient civilizations seem to be intertwined. Below is an excerpt from a Sumerian temple hymn (circa 1800 BC) dedicated to the moon god, Nanna [4].

from: **The Herds of Nanna**
by unknown author

The lord has burnished the heavens; he has embellished the night.
Nanna has burnished the heavens; he has embellished the night.
When he comes forth from the turbulent mountains, he stands as Utu stands at noon.
When Acimbabbar comes forth from the turbulent mountains, he stands as Utu stands at noon.
……………………..
His lofty *jipar* shrines number four.
There are four cattle pens which he has established for him.
His great temple cattle pens, one *ece* in size, number four.
They play for him on the churn.

The cows are driven together in herds for him.
His various types of cow number 39600.
His fattened cows number 108000.
His young bulls number 126000.
The sparkling-eyed cows number 50400.
The white cows number 126000.
The cows for the evening meal are in four groups of five each.
Such are the various types of cow of father Nanna.

…………………………
Their herds of cattle are seven.
Their herdsmen are seven.
There are four of those who dwell among the cows.

They give praise to the lord, singing paeans as they move into the *jipar* shrines.
Nisaba has taken their grand total; Nisaba has taken their count, and she is writing it on clay.
The holy cows of Nanna, cherished by the youth Suen, be praised!

Nisaba, appearing in the penultimate line of the hymn fragment, is the grain goddess and patroness of scribal arts and mathematical calculations. It appears that the author of this hymn needed a little divine assistance with the calculation of the grand total. This poem gives credence to the theory that one of the driving forces behind the invention of both writing and numbers, and by extension—literature and mathematics, was the need to keep track of a growing quantity of riches, in particular grain and cattle.

Another ancient example of what Pablo Neruda calls "*the thirst to know how many*" (*Ode to Numbers*, by Pablo Neruda [13]), is Archimedes' *The Cattle Problem* [18]. Archimedes (287-212 BC) posed this problem in verse to the mathematicians of Alexandria in a letter he sent to Eratosthenes of Cyrene. In twenty two Greek elegiac distichs (a total of 44 lines), the poem asks for the total number of cattle—white, black, dappled, and brown bulls and cows, belonging to the Sun god, subject to several arithmetic restrictions. The restrictions may be divided into three sets. The first two sets of restrictions pose some, but not insurmountable, difficulties. The problem with these sets of restrictions was posed as a challenge; it can be solved nowadays using Linear Algebra. After describing the last set of restrictions Archimedes' poem, translated into English by Hillion & Lenstra [16, 18], says:

***from*: The Cattle Problem**
by Archimedes

friend, canst thou analyse this in thy mind,
and of these masses all the measures find,
go forth in glory! be assured all deem
thy wisdom in this discipline supreme!

Attempts to solve the problem for the last set of restrictions gave rise to the Pell Equation, $x^2 = dy^2 + 1$, where d is an integer, which is not a square; and the solutions x and y, need to be positive integers. The first mathematician to solve the *Cattle Problem* with this restriction was A. Amthor in 1880. The solution generated a number that occupied, in reduced type, twelve journal pages—the number is approximately $7.76\text{x}10^{206544}$. The Pell Equation continues to pose new "counting difficulties" to this day, as mathematicians struggle to find efficient computer-based solution methods. Interested readers may find more information and references about the Pell Equation in [18], and a number of more modern poems inspired by numbers and counting in [13].

Geometry

The British poet Samuel Taylor Coleridge (1772-1834), best known for the poem *The Rime of the Ancient Mariner*, wrote in a letter to his brother, Rev. George Coleridge, "I have often been surprised, that Mathematics, the quintessence of Truth, should have found admirers so few…." The letter included a poem that gives an account of the proof of Proposition 1, from Book I of Euclid's (325-265 BC) *Elements*. Perhaps not quite in jest, Coleridge told his brother that the poem was a sample from a more ambitious project which intends to reproduce all of Euclid's *Elements* in a series of Pindaric odes. Unfortunately, the project was not pursued any further. Proposition 1 states that given a line segment AB, one can construct, using only a ruler and compass, an equilateral triangle with AB as one of its sides. Below is an excerpt from Coleridge's poem [21]:

***from:* A Mathematical Problem**
by Samuel Taylor Coleridge

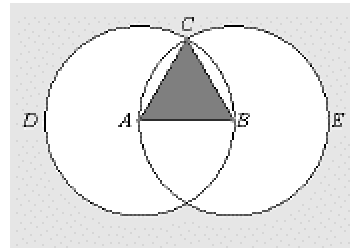

Figure 1. *Euclid: Elements, Proposition 1*

This is now--this was erst,
Proposition the first--and Problem the first.

On a given finite Line
Which must no way incline;
To describe an equi--
--lateral Tri--
--A, N, G, L, E.
Now let A. B.
Be the given line
Which must no way incline;
The great Mathematician
Makes this Requisition,
That we describe an Equi--
--lateral Tri--
--angle on it:
Aid us, Reason--aid us, Wit!

From the centre A. at the distance A. B.
Describe the circle B. C. D.
At the distance B. A. from B. the centre
The round A. C. E. to describe boldly venture.
(Third Postulate see.)
And from the point C.
In which the circles make a pother
Cutting and slashing one another,
Bid the straight lines a journeying go,
C. A., C. B. those lines will show.
To the points, which by A. B. are reckon'd,
And postulate the second
For Authority ye know.
A. B. C.
Triumphant shall be
An Equilateral Triangle,
Not Peter Pindar carp, not Zoilus can wrangle.

Coleridge was not the only poet to be moved into verse by a beautiful geometric proof. About one hundred years later Frederick Soddy (1877-1956), Nobel prize winning British chemist, rediscovered *Descartes' Circle Theorem*—originally proved by Rene Descartes (1596-1650), which involves the radii of four mutually tangent circles. In his joy he wrote the verses below [13, 19]:

***from*: The Kiss Precise**
by Frederick Soddy

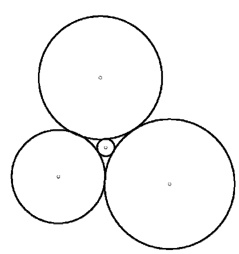

Figure 2. *Four mutually tangent circles*

For pairs of lips to kiss maybe
Involves no trigonometry.
'Tis not so when four circles kiss
Each one the other three.
To bring this off the four must be
As three in one or one in three.
If one in three, beyond a doubt
Each gets three kisses from without.
If three in one, then is that one

Thrice kissed internally.

Four circles to the kissing come.
The smaller are the benter.
The bend is just the inverse of
The distance from the center.
Though their intrigue left Euclid dumb
There's now no need for rule of thumb.
Since zero bend's a dead straight line
And concave bends have minus sign,
The sum of the squares of all four bends
Is half the square of their sum.

The last stanza of the poem, not included here, involves Soddy's proof of the analogous formula for spheres. After these verses appeared, Thorold Gosset (1869-1962) wrote *The Kiss Precise (Generalized)* (see, for example,[13]), to describe the more general case of tangency, or "kissing," of $n + 2$ hyperspheres in n dimensions. A 1980 addition to Soddy's verses is Bobo's poem, *Foursomes, Fivesomes, and Orgies* [6]. Additional poems inspired by geometry, Euclidean and otherwise, may be found in [7, 13].

Calculus

Sixteenth century Europe saw a vigorous revival of mathematical activities that culminated with the invention of Calculus in late seventeenth century—a development that marked the beginning of modern mathematics. The towering figures of the two inventors of Calculus, Isaac Newton (1643-1727) and Gottfried Leibniz (1646-1716); the *Fundamental Theorem of Calculus* they both discovered and proved—separately; and some of the history and controversy surrounding the birth of Calculus, are captured in my poem, *Calculus* [11, 13]. A fragment of this poem is given below:

***from:* Calculus**
by Sarah Glaz

I tell my students the story of Newton versus Leibniz,
the war of symbols, lasting five generations,
between The Continent and British Isles,
involving deeply hurt sensibilities,
and grievous blows to national pride;
on such weighty issues as publication priority
and working systems of logical notation:
whether the derivative must be denoted by a "prime,"
an apostrophe atop the right hand corner of a function,
evaluated by Newton's fluxions method, $\Delta y/\Delta x$;
or by a formal quotient of differentials dy/dx,
intimating future possibilities,
terminology that guides the mind.
The genius of both men lies in grasping simplicity
out of the swirl of ideas guarded by Chaos,
becoming channels, through which her light poured clarity
on the relation binding slope of tangent line
to area of planar region lying below a curve,
The Fundamental Theorem of Calculus,
basis of modern mathematics, claims nothing more.

While Leibniz—suave, debonair, philosopher and politician,
published his proof to jubilant cheers of continental followers,
the Isles seethed unnerved,

they knew of Newton's secret files,
locked in deep secret drawers—
for fear of theft and stranger paranoid delusions,
hiding an earlier version of the same result.
....
Calculus—Latin for small stones,
primitive means of calculation; evolving to abaci;
later to principles of enumeration advanced by widespread use
of the Hindu-Arabic numeral system employed to this day,
as practiced by *algebristas*—barbers and bone setters in Medieval Spain;
before Calculus came the Σ (sigma) notion—
sums of infinite yet countable series;
and culminating in addition of uncountable many dimensionless line segments—
the integral $\int$ —snake,
first to thirst for knowledge, at any price.

The wonder aroused by the sum of an infinite series that is on occasion a finite number, the ability to "add" uncountable entities, and the discovery of unexpected connections between disparate mathematical notions, inspired many other poets and mathematicians. Jacob Bernoulli (1654-1705), an important contributor to the development of Calculus, included the following verses in his posthumously published work *Ars Conjectandi* [3, 13]:

from: **Treatise on Infinite Series**
by Jacob Bernoulli

Even as the finite encloses an infinite series
    And in the unlimited limits appear,
So the soul of immensity dwells in minutia
    And in narrowest limits no limits inhere.
What joy to discern the minute in infinity!
    The vast to perceive in the small, what divinity!

Poems inspired by the mathematics and the mathematicians of that period are widely spread through literature. A small selection appears in [7, 13].

Contemporary Mathematics

It is easier to gain perspective on past mathematical accomplishments, than to characterize the mathematical landscape of the present. Nevertheless, it is possible to point out several contemporary mathematical results that had an impact on the popular and mathematical culture. Foremost among these is fractal geometry, which was created by Benoît Mandelbrot in 1970. In less than 50 years fractal geometry has become so entrenched in culture that it needs no introduction—the image of the Mandelbrot set is immediately recognizable by every literate person. One can speculate that the appeal of fractal geometry lies in the beauty of its computer generated images or its power to describe the seemingly random and chaotic order of the world. Whatever the explanation, fractal geometry stars in many poems. The verse fragment below comes from a song lyric composed by American folk-pop singer and writer Jonathan Coulton [10, 13]. Additional poems on fractal geometry may be found in [5, 7, 8, 13].

***from*: Mandelbrot Set**
by Jonathan Coulton

Pathological monsters! cried the terrified mathematician
Every one of them is a splinter in my eye
I hate the Peano Space and the Koch Curve

I fear the Cantor Ternary Set
And the Sierpinski Gasket makes me want to cry
And a million miles away a butterfly flapped its wings
On a cold November day a man named Benoît Mandelbrot was born

His disdain for pure mathematics and his unique geometrical insights
Left him well equipped to face those demons down
He saw that infinite complexity could be described by simple rules
He used his giant brain to turn the game around
And he looked below the storm and saw a vision in his head
A bulbous pointy form
He picked his pencil up and he wrote his secret down

Take a point called Z in the complex plane
Let Z1 be Z squared plus C
And Z2 is Z1 squared plus C
And Z3 is Z2 squared plus C and so on
If the series of Z's should always stay
Close to Z and never trend away
That point is in the Mandelbrot Set

Mandelbrot Set you're a Rorschach Test on fire
You're a day-glo pterodactyl
You're a heart-shaped box of springs and wire
You're one badass fucking fractal
And you're just in time to save the day
Sweeping all our fears away
You can change the world in a tiny way

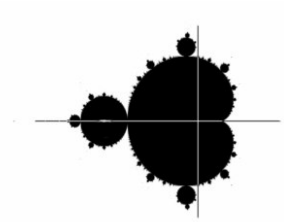

Figure 3. *The Mandelbrot set*

Several contemporary landmark mathematical results revolve around the great conjectures. In 1996 Andrew Wiles proved Fermat's Last Theorem, which states that the equations $x^n + y^n = z^n$, have no positive integer solutions for n larger than 2. The theorem was stated by Pierre de Fermat (1601-1665) on the margin of his copy of Diophantus' *Arithmetic,* accompanied by the famous words: "I have discovered a truly remarkable proof which this margin in too small to contain." The 350 years long quest for a solution and the substantial body of mathematics generated in the process made Wiles' proof the most remarkable mathematical result of the century. As expected, this event was celebrated with poetry (see, for example, [2, 20]), most of which is humorous—a genre much loved by mathematicians. Below is a small sample from [20]:

***from*: Fermat's Last Theorem Poetry Challenge, Two Limericks**

by Ted Munger

With an integer greater than 2
It's something one simply can't do.
If this margin were fat,
I'd show you all that,
But it's not, so the proof is on you!

by Jeremy Teitelbaum

We take an elliptic curve E,
Consider the points killed by 3,
This "rho" must be modular,
And by facts which are popular,
The proof of Fermat comes for free.

The Clay Mathematics Institute, founded in 1998, listed the seven most important open problems in mathematics, the *Millennium Prize Problems* [9]. One of the millennium prize problems, the Riemann Conjecture, proposed by Bernhard Riemann (1826-1866), celebrates its 150th anniversary this year. The Riemann Conjecture is a conjecture about the zeros of the Riemann zeta function. It is considered to be the most important open problem in pure mathematics, whose solution will advance our knowledge of the distribution of prime numbers. Below is Tom Apostol's Riemann Conjecture poem [1].

Where Are the Zeros of Zeta of s?
by Tom Apostol

Where are the zeros of zeta of s?
G.F.B. Riemann has made a good guess;
They're all on the critical line, saith he,
And their density's one over 2pi log t.

This statement of Riemann's has been like a trigger
And many good men, with vim and with vigor,
Have attempted to find, with mathematical rigor,
What happens to zeta as mod t gets bigger.

The efforts of Landau and Bohr and Cramer,
And Littlewood, Hardy and Titchmarsh are there,
In spite of their efforts and skill and finesse,
In locating the zeros there's been little success.

In 1914 G.H. Hardy did find,
An infinite number that lay on the line,
His theorem however won't rule out the case,
There might be a zero at some other place.

Let P be the function pi minus li,
The order of P is not known for x high,
If square root of x times log x we could show,
Then Riemann's conjecture would surely be so.

Related to this is another enigma,
Concerning the Lindelof function mu (sigma)
Which measures the growth in the critical strip,
On the number of zeros it gives us a grip.

But nobody knows how this function behaves,
Convexity tells us it can have no waves,
Lindelof said that the shape of its graph,
Is constant when sigma is more than one-half.

Oh, where are the zeros of zeta of s?
We must know exactly, we cannot just guess,
In order to strengthen the prime number theorem,
The integral's contour must not get too near 'em.

Several other poems were inspired by the millennium prize problems. In 2004, Grigory Perelman was offered the Field Medal for solving the millennium prize problem: The Poincaré Conjecture [9]. He refused to accept the award. JoAnne Growney's poem, *Perelman and Me* [15], grapples with the complex emotions aroused by this refusal. A poem by Haipeng Guo, *When a P-man Loves an NP-woman*, appearing in [13], involves the millennium prize problem: P vs. NP [9]. In this poem, the NP-woman asks the P-man to solve the conjecture as a condition for marrying him. The Navier-Stokes equations, a subject of another one of the millennium prize problems, are featured in a poetic reading and interpretation video prepared by a Kim Lasky, Peter Childs, Abdulnaser Sayma, and their students at the Engineering School of the University of Sussex, UK [17].

Concluding Remarks

Everyone's mathematical education follows the path taken by our ancestors. It starts with counting in nursery rhymes—we learn about numbers by playing with rhyming words. Behind the game lies a serious intent: to teach counting by utilizing the power of poetry to engage learners' attention and enhance retention of abstract concepts. Poetry inspired by mathematics appears in the mathematics classroom through the ages, and at all mathematical levels. The nature and frequency of its use as a tool for teaching mathematics fluctuates to reflect technological advances and changing attitudes to mathematics education. But regardless of the specific reasons for the inclusion of a poem in a class, the power of poetry to engage attention and enhance memory is always an underlying presence. In addition to enrichment of pedagogy through engagement and enhancement of retention, poetry is often used in the mathematics classroom to shape course content, to facilitate integration of material, and to ease the transition from theory to applications. Some ancient cultures, for example India during the middle-ages, imparted all mathematical knowledge in verse form. Nowadays, poetry inspired by mathematics shapes course content by focusing attention on a particular aspect of the material taught in class, and acting as a springboard to initiate class-wide or small group discussions, assignments, or projects based on the poem's content. Judicious choice of poems and careful project construction often result in additional pedagogical benefits, such as better integration of material and easier transition to its applications. A different type of poetry project, with similar aims and results, requires students to compose their own poems about mathematical techniques or concepts. The nation-wide education initiative "writing across the curriculum" generated a number of recent pedagogical experiments with poetry writing in college mathematics classes. Examples of the use of poetry projects in college mathematics classes, a survey of the efforts made by educators in this direction, and an extensive bibliography, may be found in [12, 14]. A recent project in mathematics education that evaluates the effects of presenting advanced mathematical concepts to engineering students through a media-enhanced poetic reading and interpretation may be found in [17].

The poems appearing in this article may be used in the mathematics classroom to enhance pedagogy or course content in any of the ways discussed above. They may also be used to enrich history of mathematics courses, and courses focusing on the connections between mathematics and the arts. I hope that the poems presented here will inspire mathematicians and educators to write their own mathematical poems, and to experiment with innovative uses of poetry in their classes.

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