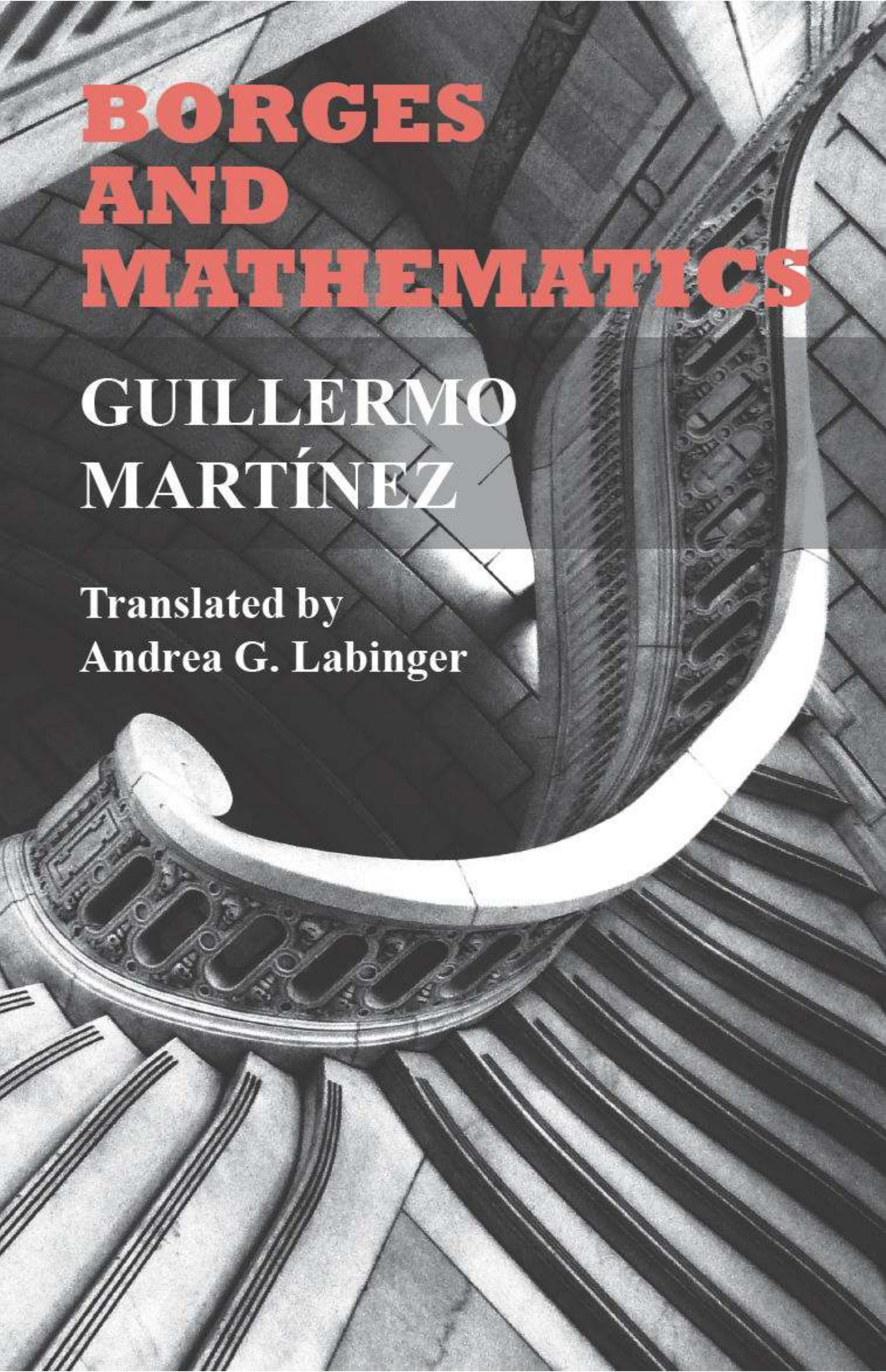
BORGES AND MATHEMATICS

GUILLERMO MARTÍNEZ

Translated by
Andrea G. Labinger



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LECTURES AT MALBA

Borges y la matemática
Clases del MALBA

GUILLERMO MARTÍNEZ

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My very special thanks to the MacDowell Foundation, to my anonymous benefactors, and to Mr. and Mrs. Putnam for two stays in that paradise for artists known as Colonia MacDowell, where a portion of this book was written.

PREFACE

I owe the impulse and initial idea for this book to a trip I made to the United States in 2001, and more specifically to an invitation by Professor Alicia Borinsky during that visit to lecture at Boston University on the connection between Borges and mathematics.

Until then, and ever since adolescence, I had read Borges with the same intensity and hypnotic sense of wonder his writings provoke in any aspiring writer, but only in successive, partial attempts, in cautious doses, with the hesitant attraction of an iron filing toward a powerful magnet. Perhaps for that reason, and despite the fact that while reading Borges I myself was "an arduous disciple of Pythagoras," I hadn't paid too much attention to the obvious and apparent references to mathematical concepts in his texts. Interspersed with so many other markers of erudition and knowledge, these can easily be passed over at first glance.

And yet, as soon as I began to review his complete works in a methodical, "mathematical" way, Borges' lifelong, enduring fascination and curiosity about that discipline emerged in all their lucidity: his pride in those aspects he had mastered, the little lessons he presented in some of his essays, the reviews, and readings of mathematical books. Suffice it to say that I was able to find almost 200 citations with mathematical allusions throughout his oeuvre, and a bibliography of 45 mathematical works consulted or cited.[1] Beyond this rather impressive body of references, most interesting and revealing for me was the discovery of clear traces of some mathematical ideas behind several of his fictions, and an awareness of the subtle way in which mathematical concepts were transmuted and imbued with new life within a context of literary intentions. To study Borges' appropriation of these ideas and to analyze them *within* each fictional work without separating them from those literary intentions, with the facets and layers of meaning they add, is the fundamental purpose of this book. I believe that to disregard the profusion of mathematical footprints would be to ignore some of the objectives and deliberate meanings that Borges attempted to convey, and thus to miss one of the dimensions of his work. On the other hand, to isolate or overemphasize this aspect, to examine the references with a high-powered magnifying glass or an arsenal of hyper-sophisticated mathematical tools would be an error of excess. Therefore I have tried to maintain a careful equilibrium in my approach: for me the game of interpretation is, above all, a balancing act.

In Borges the essayist there is also the unmistakable sign of a way of thinking that is akin to logical argumentation and mathematics, both in his choice and execution of style and in various statements of his artistic credo. This is something that this book also proposes to explore.

On too many occasions, when I was still a professional mathematician and I was publishing my first novels, I heard the incredulous question: "But how is it possible . . . when literature and mathematics are such opposite worlds?" It was of little use to invoke the names Lewis Carroll, Raymond Queneau, Stanisław Lem, Robert Musil, and Nicanor Parra. The suspicions hardly abated, as if I were referring to exotic anomalies. The articles on mathematics and literature in the second half of this book suggest examples of ties between the two worlds, the two cultures, with the objective of allaying that skepticism and showing, as Borges himself has said, that “Imagination and mathematics are not contradictory; they complement one another like lock and key.”

I would like to thank Ariel de la Fuente for his interest, diligence, and encouragement in publishing this book with Purdue University Press, and also my editor, Charles Watkinson, for his patient efforts in acquiring the translation rights for the citations. Very special thanks to Andrea Labinger for her outstanding, meticulous work on the translation. Finally, I am grateful to the Latino Cultural Center at Purdue University and its Director, Maricela Alvarado. Their generous support helped make the book available in English.

Buenos Aires, May 2012

Note

1. This list (in Spanish) can be found at www.guillermomartinezweb.blogspot.com.

Editor’s note

MALBA stands for *Museo de Arte Latinoamericano de Buenos Aires*.

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FIRST LECTURE

February 19, 2003[1]

Angle, Slope, and Interpretation
Thomas Mann and the Twelve-Tone Scale
The Game of Interpretation as a Balancing Act

Whenever one chooses an angle or a theme, the phenomenon to be studied is often distorted, something physicists know well. It also happens whenever one tries to approach an author from a particular angle: one finds oneself mired in the quicksand of interpretation. In this regard, it's good to keep in mind that the game of interpretation is a balancing act that allows for errors of omission or of commission. If we approach a Borges text, let's say, from a purely mathematical, very specialized standpoint, we may end up above the text. Here, "above" really means outside: we might skew the text to say things it really doesn't say and never intended to say. An error of erudition. On the other hand, if we completely ignore the mathematical elements in Borges' work, we might find ourselves below the text. Therefore, I'm going to attempt an exercise in equilibrium. I realize that among my readers there might be people who know a great deal about mathematics, but I'm going to address those who know only how to count to ten. This is my personal challenge. Everything I'm about to say should be understandable to those who can only count to ten.

There is a second, even more delicate question that Thomas Mann referred to when he was compelled to add a note at the end of *Doktor Faustus* in recognition of Arnold Schönberg's intellectual authorship of the twelve-tone theory of musical composition. Mann did this reluctantly because he believed that this particular musical theory had been transmuted into something different, molded "into an ideal context for a fictional character" (his fictitious composer, Adrian Leverkühn). Similarly, the mathematical elements that appear in Borges' work are also molded and transmuted into "something different"—literature. We will try to recognize these elements without removing them from that context of literary intentionality.

For example, when Borges begins his essay "Avatars of the Tortoise" by saying: "There is a concept which corrupts and upsets all others. I refer not to Evil, whose limited realm is that of ethics; I refer to the infinite" (202), the connection he establishes between the infinite and Evil, the playful but accurate pride of place that he assigns it among other iniquities, immediately removes the infinite from the serene world of mathematics and casts all the tidy, formulaic, almost technical discussions that follow in a slightly menacing light. When he goes on to say that the "numerous hydra" is a foreshadowing or symbol of geometric progressions, he repeats the game of projecting monstrosity and "convenient horror" onto a precise mathematical concept.

How Much Mathematics Did Borges Know? Proceeding with Caution in His Library Truth in Mathematics and Literature

How much mathematics did Borges know? In the same essay, he says: "Five or seven years of metaphysical, theological and mathematical apprenticeship would allow me (perhaps) to plan decorously [a history of the infinite]" ("Avatars of the Tortoise" 202). The sentence is ambiguous enough to make it hard to determine if he really devoted that many years to studying mathematics or if it was just a future plan, although it remains clear that Borges was familiar, at the very least, with the subjects contained in the book *Mathematics and the Imagination*, for which he wrote a prologue.[2] Those topics are a good sample of what can be learned in an introductory university course in algebra and analysis. The book deals with logical paradoxes, the question of the various types of infinities, some basic problems in topology and probability theory. In the prologue to this book, Borges recalls in passing that, according to Bertrand Russell, vast mathematics might not be anything more than a vast tautology, and from this observation it is clear that he was aware—at least at the time—of what was a crucial and hotly debated topic in the foundations of mathematics: what is *true* versus what is *demonstrable*.

In their usual task of closely examining the universe of forms and numbers, mathematicians find recurrent connections and patterns, certain relationships that are always verified. They are accustomed to believing that these relationships, if true, are true for some reason, that they are organized according to an external Platonic scheme that must be deciphered. When they find that profound—and generally hidden—explanation they express it in what is called a demonstration or proof.

Thus, in mathematics, as in art, there are two moments: one that we might call the moment of illumination or inspiration, a solitary and even "elitist" moment when the mathematician glimpses, in an elusive, Platonic world, a result that he considers to be true; and a second, let's say democratic, moment when he has to convince the community of his peers of this truth. This is exactly analogous to the way the artist first sees fragments of a vision and later must execute that vision in writing, painting, or whatever form it happens to take. In that sense, the creative processes are similar. What's the difference? In mathematics there are formal protocols governing the way mathematicians systematically demonstrate the truth to their peers, step by step, proceeding from certain principles and "rules of the game" on which all mathematicians agree. On the other hand, the demonstration of an aesthetic fact is not so clear-cut. An aesthetic "fact" is always subject to the criteria of authority, fashion, cultural journals, personal opinion, and ultimately—that often capricious element—taste.

For centuries mathematicians believed that in their world, these two concepts—what is true and what is demonstrable—were essentially the same. They thought that if something was true, the reason for that truth could always be explained through the logical steps of a proof. However, in other professions—the law, for example—it has always been understood that truth and demonstrability are not the same thing. Let's suppose a crime has been committed in a sealed-off room (or, in more modern terms, a sealed-off country), with only two possible suspects. Both of them know the whole truth about the crime: *I did it* or *I didn't do it.* There is a single truth and they know it, but justice has to arrive at this truth through other, indirect, means: fingerprints, cigarette butts, connections, alibis. Often the judicial system doesn't succeed in proving the guilt of one party or the innocence of the other. Something similar happens in archeology: there are only provisional truths. The ultimate truth remains out of reach, in the never-ending bone pile of the demonstrable.

Thus, in other fields, truth doesn't necessarily coincide with demonstrability. Bertrand Russell may have been the strongest supporter of the idea that in mathematics the two terms might be made to coincide, and that mathematics is nothing more than "a vast tautology." In a sense that was also Hilbert's program, a great effort on the part of mathematicians to guarantee that everything that can be proven true, through any method whatsoever, can also be demonstrated a posteriori by following a formal protocol, an algorithm that can corroborate the truth in a mechanical way and that can be modeled on a computer, without the use of intelligence. That method would have essentially reduced mathematics to whatever could be proven by a computer.

It was finally demonstrated—by Kurt Gödel's dramatic incompleteness theorem in the 1930s—that things just don't work that way, that mathematics is more like criminology in that regard: there are certain assertions that are true but nonetheless remain beyond the reach of formal theories. Or rather, formal theories can neither affirm nor deny those assertions; they cannot prove their innocence or their guilt. What I would like to point out is that Borges had already foreseen the germ of this discussion (although it doesn't seem that he was aware of its resolution).

Mathematical Elements in Borges' Work

There are very diverse mathematical elements throughout Borges' work.[3] The obvious, natural way to approach this topic would be to track all those mathematical footprints in his texts. That has been done, and done well, in the book *Borges y la ciencia*, which contains essays on Borges and mathematics, Borges and scientific investigation, Borges and memory, Borges and physics. I've occasionally joked that my favorite is "Borges and Biology." After a few false starts, the author somewhat apologetically writes that he's read Borges' complete works and is obliged to say that there is no connection between Borges and biology. None! The man had discovered, to his horror, one topic in this world—biology—that Borges never touched.

And yet mathematical elements abound. In fact, a careful review of all his works reveals over 180 mathematical references. I'm going to take advantage of my situation as a writer in order to do something a little different: I'm going to try to connect mathematical elements and stylistic procedures in Borges. I will attempt to find a connection that is stylistic, rather than thematic. Here are some of the texts where mathematical concepts are most evident: the short stories "The Disk," "The Book of Sand," "The Library of Babel," "The Lottery in Babylon," "On Exactitude in Science," "Examination of the Work of Herbert Quain," "*Argumentum Ornithologicum*," the essays "The Perpetual Race of Achilles and the Tortoise" together with "Avatars of the Tortoise," "The Analytical Language of John Wilkins," "The Doctrine of Cycles," "Pascal," in addition to "Pascal's Sphere," and others. Among these texts there are even little mathematical lessons. Even so, despite this ample selection, I believe that there are just three main recurrent themes. These three themes come together in the short story "The Aleph." Let's begin with that one.

Cantor's Infinity

I'm going to mention the three themes in reverse order from that in which they appear. The first element is infinity or, more accurately, infinities. Toward the end of the story, Borges says:

> There are two observations that I wish to add: one, with regard to the nature of the Aleph; the other, with respect to its name. Let me begin with the latter: "aleph," as well all know, is the name of the first letter of the alphabet of the sacred language. Its application to the disk of my tale would not appear to be accidental. In the Kabbalah, that letter signifies the En Soph, the pure and unlimited godhead; it has also been said that its shape is that of a man pointing to the sky and the earth, to indicate that the lower world is the map and mirror of the higher. For the *Mengenlehre,* the aleph is the symbol of the transfinite numbers, in which the whole is not greater than any of its parts. ("The Aleph" 285)

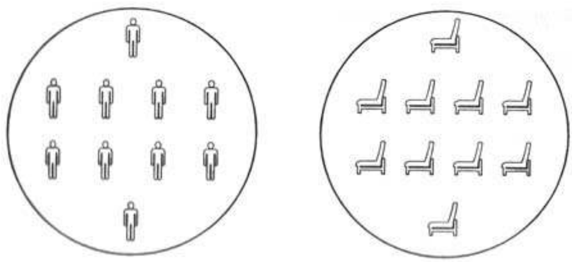
The *Mengenlehre* is a German term for the theory of quantities. The Aleph symbol, which mathematicians write in simplified form, looks like this:

$$\aleph$$

One arm pointing toward heaven and the other pointing toward earth. The symbol of transfinite numbers, in which, as Borges says, *the whole is not necessarily greater than each of the parts.* This is one of the mathematical concepts that really fascinated Borges. It represents a departure from the Aristotelian postulate that says the whole must be greater that any of the parts. I would like to offer a brief explanation of how this idea of infinity arises in mathematics.

Until 1870, when Georg Cantor began his work with set theory (theory of aggregates), mathematicians used a different symbol for infinity, the number 8 lying on its side: ∞. They believed that there was really only one unique infinity. They didn't consider the possibility that there might be various kinds of infinities. How did Cantor arrive at his idea of infinity, the one that raises this first paradox?

In order to understand this, we have to remember what it means to count. We can think of the process of counting in two ways: let's suppose that in a first set we have ten people—our numerical limit—and in a second set we have ten chairs.



Fine, you might say, I know that there are as many people as chairs, because here I count ten people and there I count ten chairs, or rather, I assign to the first set a quantity that I know—ten—and also to the second set a quantity I know—ten. And as 10 = 10, I conclude that the two sets have the same number of elements. However, let's suppose I'm playing cards with a three-year-old boy. The boy, like us today, cannot count past ten, but he knows that if he gives me the first card and keeps the second, then gives me the third and keeps the fourth, and so on, when he finishes dealing out the deck, even though he's unable to tell me the *number* of cards he has in his hands (because he only knows how to count to ten), he can still say something; he still possesses one element of certainty, which is that *both he and I have the same number of cards.* This much he knows, although he doesn't know *how many* there are.

In the example of the chairs, we might also have concluded that there is the same quantity of people as of chairs by making each person sit in one chair, thereby establishing a perfect correspondence in which there is no chair left without a person and no person without a chair. Similarly, when watching a military parade, it's not possible to tell at first glance how many riders or horses there are, but it's still possible to determine something: that there are as many riders as horses.

This is trivial, I know, but sometimes from trivialities great ideas are born. Here comes the mathematician's sleight of hand. Notice what Cantor does: it's essentially something very simple, but extraordinary. What he discovered is a concept that in the finite context is the equivalent of "having the same quantity of elements." He says: "In the finite context, sets A and B have the same quantity of elements if and only if I can establish a perfect one-to-one correspondence between them." This affirmation is very simple to prove. But what happens when we take the leap to the infinite? One of the two equivalent concepts—"quantity of elements"—ceases to have meaning. What does "quantity of elements" mean when it's impossible to finish counting? That part is no longer of any use to us, but we can still use the second part. The second part survives; we can still establish perfect one-to-one correspondences for infinite sets just as we did between people and chairs.

But then strange things start happening. Because there is an obvious way to establish a perfect one-to-one correspondence between all natural numbers (the numbers we use for counting—1, 2, 3, etc.) and even numbers. To 1 we assign the number 2; to 2 we assign 4; to 3 we assign 6, and so on. And here we are compelled by Cantor's definition to say that there are "as many" natural numbers as even numbers. However, the even numbers make up "half" of the natural numbers in the sense that we obtain natural numbers by joining even numbers to odd ones. Then, there is effectively one part, the even numbers, which is as great as the whole. *There is a part that is equivalent, in this sense, to the whole.* This is the sort of paradox that fascinated Borges: in the mathematical infinite, the whole is not necessarily greater than each of the parts. There are certain parts that are as great as the whole. There are parts that are equivalent to the whole.

Recursive Objects

It is possible to isolate this curious property of infinity and apply it to other objects or other situations in which a part of the object contains key information to the whole. We'll call them recursive objects.

Thus, Borges' Aleph, the small sphere that holds all the images in the universe, would be a fictional recursive object. When Borges says that the application of the name "Aleph" to this sphere is no coincidence, immediately calling our attention to its connection with that property of infinities, he is inserting his idea into a setting that makes it seem feasible, as he himself illustrates in his essay, "Narrative Art and Magic." He inserts it into a setting of similar ideas that makes it plausible: just as in infinity a part can equal the whole, one might conceive a part of the universe that stores the universe in its entirety.

Borges plays with other recursive objects in his work. For example, in the expanding maps in the essay "On Exactitude in Science," where the map of a single province occupies an entire city, and "whose tattered ruins are inhabited by animals and beggars" ("On Exactitude in Science" 325). Similarly, from a biological point of view, a human being is a recursive object. A single human cell is enough to generate a clone. Mosaics are also clearly recursive objects: the design of the first pieces recurs throughout the whole.

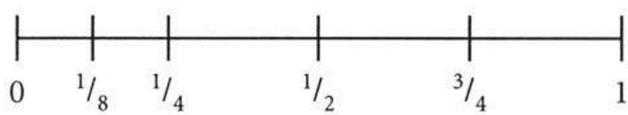
Now let's think about objects that possess the *opposite* property. What would be some examples of anti-recursive objects, those in which no part replaces the whole, but in which each part is essential? An immediate example is finite sets. Jigsaw puzzles, as well. In a reasonable jigsaw puzzle, repeated designs should be avoided in order to make things more challenging. Another example, from an existential point of view, would be the human being. There's a very intimidating phrase by Hegel (not Sartre) that goes: "Man is no more than the series of his acts." It doesn't matter how unimpeachable a man's conduct has been every day of all the years of his life: there is always time to commit one final act that will contradict, ruin, and destroy whatever it has been up until that moment. Or the converse, to take Thomas Mann's approach in *The Holy Sinner,* based on the *Life of Saint Gregory*: it makes no difference how incestuous or sinful a man may have been during his lifetime; he can always repent and become Pope.

Infinity and "The Book of Sand"

What I've said up to now about infinity would be enough to clarify this brief fragment. I'm going to expand a little more in order to explain something that is related to "The Library of Babel" and "The Book of Sand." We've just seen that there are "as many" natural numbers as even ones. What happens if we consider fractions? Fractions are very important in Borges' reasoning. Why? Let's recall that fractions (etymologically "broken numbers"), also called rational numbers, are obtained by dividing integers by one another. More precisely, we can think of them as pairs of integers: an integer as numerator and an integer (other than zero) as denominator.

$$3/5, -2/7, 4/-9, -1/-3 \ldots$$

What property do these numbers have in common, the property that Borges uses in his tales? *For any two fractions there is always another one in between.* Between 0 and 1 is ½; between 0 and ½ is ¼; between 0 and ¼ is 1/8; etc. In particular, any rational number can always be divided in half.



Thus, when I want to go from 0 to the first fraction, I can never find that first number in the usual order because there is always another one in between. This is exactly the same notion that Borges borrows in "The Book of Sand." You'll recall that there is a point in the story when the character named Borges is challenged to open to the first page of the Book of Sand, and he comments:

> He told me his book was called the Book of Sand because neither sand nor this book has a beginning or an end. He suggested I try to find the first page. I took the cover in my left hand and

> opened the book, my thumb and forefinger almost touching. It was impossible: several pages always lay between the cover and my hand. It was as though they grew from the very book. ("The Book of Sand" 481)

The Book of Sand's front cover would be zero; the back cover would be one; and the pages would then correlate with the fractions between zero and one. Among these fractions it would be impossible to find the first number after zero or the last number before one. There would always be numbers in between. It would be tempting to imagine the infinity of fractional numbers as tighter, denser, or richer than that of natural numbers. However, the second surprise that infinities offer us is that this isn't the case; as strange as it may seem, there are "as many" rational numbers as there are natural numbers. How can this be demonstrated?

As all fractions are pairs of integers, with a numerator and a denominator, all (positive) fractions are represented in the following table:

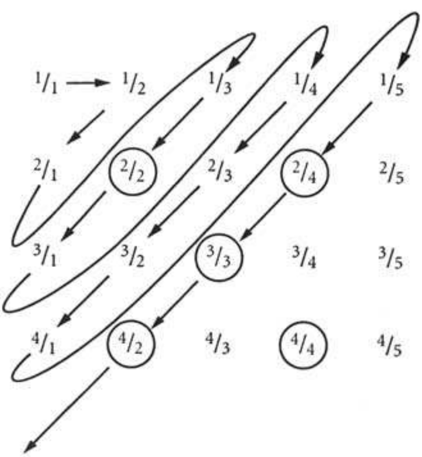
1/1	1/2	1/3	1/4	1/5 . . .
2/1	2/2	2/3	2/4	2/5 . . .
3/1	3/2	3/3	3/4	3/5 . . .
4/1	4/2	4/3	4/4	4/5 . . .

In the first row I've included all the fractions with the numerator 1; in the second row all those with the numerator 2; in the third row all those with the numerator 3; and so forth. Obviously, by organizing them in this way some will inevitably be repeated: for example, 3/3 is the same as 2/2 or 1/1. In other words, some fractions will appear several times, but that doesn't matter. If I can do more, I can do less. If I can count with repetitions, I can also count without them. What interests me is that all the positive fractional numbers appear at some point in the table. I still have to contend with the negative ones. But if I know how to count the positive fractions, it's easy to count the negative ones. Mathematicians will have to forgive my lack of specificity here.

What I want to make you see, what I want to convince you of, is that this infinite table I've devised, with its infinite rows and columns, contains all the positive fractions.

In order to demonstrate that there are "as many" fractions as natural numbers, it would be sufficient to assign a natural number to each element of this table so that as we go along we may be sure that no element remains unnumbered. How can I do this? For example, it won't work to begin by trying to cover the first row, because I'd never get to the second one. The path must alternate elements of the various rows in order to be sure to cover the entire table. Cantor discovered a way to assign numbers to fractions; this is known as "Cantor's diagonal argument."

Here's how it works:



To the fraction 1/1 we assign number 1.
To the fraction 1/2 we assign number 2.
To the fraction 2/1 we assign number 3.
To the fraction 1/3 we assign number 4.
We skip the fraction 2/2 because we've already counted it (1/1 = 2/2).
To the fraction 3/1 we assign number 5.
To the fraction 1/4 we assign number 6, etc.

The path advances along increasingly longer diagonals, sweeping through all the rows and columns. As I progress, we can be sure that we're assigning a natural number to all the fractions, and we simply pass over any repeated fractions that we've already assigned numbers to, like 3/3 or 2/4. What does this demonstrate? That although the infinite of fractions may appear to be "richer," there are "as many" fractions as there are natural numbers. Further, with this numbering system it's possible to organize fractions in consecutive order, an order that differs, of course, from the one they have in the straight line. This new order allows us to understand how the pages can be numbered in the Book of Sand. This is something that Borges might not have known about. The numbering of pages that Borges finds mysterious in the story, and to which he ascribes an equally mysterious explanation, is really no mystery at all. There is no contradiction between the fact that between any two pages of the Book of Sand there is always another one inserted and the fact that each page can have a number. The same skillful bookbinder who could sew together the infinite pages of the Book of Sand could also number them perfectly, just as we are doing here.

Infinity and the "Library of Babel"

Mathematicians—and Borges, as well—like to repeat ideas and squeeze them for all they're worth. Now that I've described Cantor's diagonal argument, I can't resist applying it one more time to another recurrent Borgesian theme, that of languages, as is presented, for example, in "The Library of Babel" or in the article titled "The Total Library." Let's think for a moment about the underlying idea in "The Library of Babel," a total library whose books are not necessarily intelligible. This library's fundamental law is: "[I]t suffices that a book be possible for it to exist" (57). Borges establishes an alphabet consisting of twenty-five symbols, but in order to allow ourselves even more latitude, we will consider books written in every possible language. We will create a single list, a universal alphabet that combines all the symbols of all existing alphabets. We'll begin with the twenty-five orthographic symbols that Borges mentions (thus assuring that all the books in the Library of Babel will also be included on our shelves). To these we'll incorporate the twenty-seven symbols of the Spanish alphabet. Then we'll add the five accented vowels as new symbols. We can continue, for instance, with Cyrillic symbols, later including the German ö and the various other symbols found in every language. Thus the basic alphabet will continue to grow. In order to give ourselves some leeway for future developments, we might assume that the symbols of our alphabet are natural numbers; in so doing, we will always have room available for adding new alphabets, new symbols (like @), or symbols from extraterrestrial languages that we might happen upon at some point. The numbers 1-25 would correspond to the orthographic symbols of the Library of Babel; the number 26 would be A; 27 would represent B; 526 might well be a Chinese ideograph; and so forth.

You will recall that in "The Library of Babel," Borges specifies the number of pages that each book may have: 410. This might make us wonder what sort of infinity would result if we included all the books that could be written in our universal alphabet containing *any* number of pages and words of *any length.*

Following Cantor's diagonal argument, it can be demonstrated that this set of books *is also enumerable.* The idea, of course, is to put all the one-page books in the first row, all the two-page books in the second row, the three-page books in the third, and so forth, and then to number them according to Cantor's system. Since all the books in the Library of Babel are also included on our shelves, we can conclude that the set of books in the Library of Babel is enumerable. Why is this so essential to understanding Borges' story?

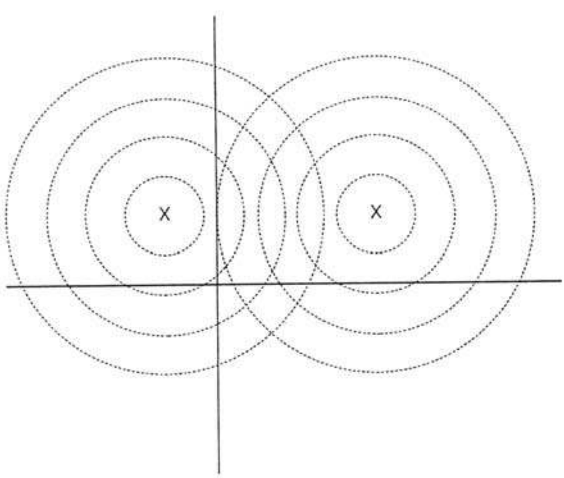
In a footnote at the end of his story, Borges writes that a friend of his had observed that the entire structure of the Library of Babel was superfluous or excessive (Borges uses the word "useless") because, in fact, all the books in the Library would fit into a single volume. This unique book would consist of an infinite number of infinitely thin pages, a "silky *vademecum*" in which "each page would unfold into other analogous ones . . ." (58). The threading together of all those books, one behind the other, into that single volume would be none other than Cantor's diagonal argument.

This concluding footnote was the germ of an idea that would later become “The Book of Sand,” a transition that evokes a very mathematical way of thinking. The first example, “The Library of Babel,” is laborious and dense. Of course it contains other riches and meanings as well—I don’t mean to suggest that it’s reduced to this point alone. But at the conclusion, Borges comes up with a simpler idea: that all books can be combined into a single volume of an infinite number of pages, or as he says, a book such that each page would be divisible. This is an anticipation of the story “The Book of Sand.” I want to emphasize Borges’ way of reflecting on his own texts in order to extract an essential idea that he will then repeat or duplicate elsewhere. It’s the first example of a general procedure, an operation that reminds us of the way mathematicians work. We will study this subject in more detail later on.

The Sphere Whose Center is Everywhere and Circumference Nowhere

Let us now examine the second mathematical element in “The Aleph.” It appears just as Borges is about to describe the Aleph and asks: “How can one transmit to others the infinite Aleph, which my timorous memory can scarcely contain?” (“The Aleph” 282). I will add something else about the Aleph symbol. It can be seen as the figure of a man with one arm touching earth and the other pointing toward heaven, which seems particularly appropriate because in a certain sense counting is a human effort to reach the infinite. That is to say, a human being cannot, in his finite life—in his “*vidita*,” as Bioy Casares would say—effectively count all numbers, but he does have a way of generating them, of conceiving them and arriving at numbers as large as necessary. Ever since the discovery of decimal writing, of ten digits, he can attain numbers as large as he wishes. As limited as he is by his earthly boundaries, he can still extend his arm toward heaven. That is the purpose and challenge of counting. Borges writes something comparable: “How can one transmit to others the infinite Aleph, which my timorous memory can scarcely contain?” In a similar situation, mystics have employed a wealth of emblems: to signify the deity, a Persian mystic speaks of a bird that somehow is all birds; Alain de Lille speaks of a sphere whose center is everywhere and circumference nowhere" (282). A little later he says: "And besides, the central problem—the enumeration, even partial enumeration, of infinity—is irresolvable" (282). In effect, Borges is attempting to describe the Aleph, which is infinite. And he cannot achieve it in writing, because writing is sequential; language is "successive." That is the problem we've just been considering. Instead he must present a sketch, a sample, a list of sufficiently convincing images. The result is the famous enumeration of images that follows and to which we'll refer later.

But in fact the second idea I'd like to examine now is that sphere, which also appears in "Pascal's Sphere" (translated by Anthony Kerrigan as "The Fearful Sphere of Pascal"). A sphere *whose center is everywhere and circumference nowhere.* Here Borges points out: "It is not for nothing that I call to mind these inconceivable analogies" ("The Aleph" 282). It's a very precise analogy that adds plausibility to the little sphere he wishes to describe. In order to understand this geometric idea, which in principle seems like a word game, let us first think of a plane; instead of spheres, let us think of circles. The idea goes like this: all points on the plane are reachable by increasingly large circles, the location of whose center doesn't really matter; the center might be anywhere.



We can place the center at any point (it doesn't matter where) and describe increasingly large circles. As we enlarge the radii, those circles begin to occupy the entire surface of the plane. In his essay "Pascal's Sphere," in an attempt to clarify this image a little, Borges writes: "Calogero and Mondolfo reasoned that Parmenides intuited an infinite, or *infinitely expanding* sphere, and that the words just transcribed possess a dynamic meaning" (189). In other words, we can replace the image of the plane with a circle that grows and grows, because all points on the plane are covered by the circle. The circumference of this infinitely expanding circle will eventually be lost in infinity. We cannot fix a circumference. This, I believe, is the idea to which Borges is referring. By making the leap to infinity, we can conclude that the entire plane is a circle whose center is any point and whose circumference is nowhere.

The same sort of design holds true if we think of tridimensional space: that is, a globe-like sphere that expands infinitely until it comes to occupy all points. The universe can therefore be understood as an infinitely expanding sphere. This is, incidentally, contemporary physics' notion of the universe: a little sphere of infinitesimal size and infinitely concentrated mass that at a given point, during the big bang, expanded in all directions. Why is this "inconceivable analogy" so interesting? Because the Aleph is a little sphere. If we can understand the entire universe as a giant sphere, the idea that all the images in the universe might be reproduced in the little sphere at the foot of the staircase becomes much more plausible. Through simple contraction the whole universe can be condensed into that little sphere. Of course, this is only one of the ways in which Borges uses the analogy, the mathematician's-eye-view we've chosen to adopt here. But, as stated earlier, mathematics slips into Borges' work within a context of philosophical and literary references: the idea of the universe as a sphere is connected to a long mystical, religious, and kabbalistic tradition. These other connotations are explained in more detail in "Pascal's Sphere."

Russell's Paradox

The third idea is one I would call "the paradox of magnification." (Technically, it's known in logic as "self-reference," but the expression "self-reference" means something different in literature, and I don't want to confuse the definitions). This paradox appears when Borges decides to give us a partial enumeration of the images contained in the Aleph. But it also occurs in other narratives, whenever Borges invents worlds that are so vast and all-encompassing that they end up by including Borges himself—or his readers—within their boundaries. In "The Aleph," this can be seen in the following passage: "[I] . . . saw the circulation of my dark blood, saw the coils and springs of love and the alterations of death, saw the Aleph from everywhere at once, saw the earth in the Aleph, and the Aleph once more in the earth and the earth in the Aleph, saw my face and my viscera, saw your face, and I felt dizzy, and I wept . . ." ("The Aleph" 283-84).

The postulation of very vast objects, of magnification, gives rise to some strange paradoxes, and Borges must have been perfectly aware of the most famous of these, attributable to Bertrand Russell, which rattled set theory and caused one of the most serious cracks in the foundations of mathematics. According to Russell's paradox, the existence of a set containing all sets cannot be postulated; that is, an Aleph of sets cannot be postulated. Here is a quick explanation of this idea: let us observe that the most common sets we can think of are not elements of themselves. For example, the set of all natural numbers is not itself a natural number. The set of all trees is not a tree. But now let's think for a moment about the set of all concepts. The set of all concepts *is* in itself a concept. Then, strange as it may seem, there is the possibility that a set could be an element of itself. As a second example, if I propose a set of all sets, such a thing would have to be an element of itself simply because it is a set.

Indisputably, there are some sets that are elements of themselves and others that are not. Let us now consider the set of all sets that are not elements of themselves.

X = {A such that A is a set and A is not an element of A}

X will therefore contain the set of natural numbers, the set of all trees, the set of people reading this book, and so forth. But now we can ask ourselves: is X an element of X? The answer must be yes or no. In order for X to be an element of itself, it would have to meet the above definition. In other words, if X belongs to X, then X cannot belong to X. But this is absurd. Is it possible, then, that X is not an element of itself? If X is not an element of itself, it satisfies the definition inside the brackets and therefore it has to belong to X. That is to say, if X is not an element of X, then X is an element of X. Also absurd. Here we have a set that is in no-man's-land, a set that both *is and is not* an element of itself.

This is the paradox Russell discovered when he was young. He wrote a letter to Gottlob Frege, one of the most renowned logicians of the time, who was about to publish the last volume of his great treatise on the foundations of mathematics, based on set theory. Frege acknowledged Russell's letter at the end of his book with the following pathetic words: "Hardly anything more unfortunate can befall a scientific writer than to have one of the foundations of his edifice shaken after the work is finished. This is the position I was placed in by a letter of Mr. Bertrand Russell, just when the printing of this volume was nearing its completion" (Appendix to "The Foundation of Arithmetic"). With these few lines Russell not only demolished ten or fifteen years of Frege's work, but also provoked one of the most significant crises in the foundations of mathematics.

To popularize this paradox, Russell came up with the notion of a village barber who only shaves those men who don't shave themselves. In theory the existence of a man with such an honest profession seems reasonable: a village barber, one might say, is precisely the sort of man that shaves all men who don't shave themselves. But then, should the barber shave himself or shouldn't he? If he does shave himself, he ceases to belong to the category of men whom he may shave. Therefore, he cannot shave himself. But, on the other hand, if he doesn't shave himself, that puts him in the category of men who don't shave themselves, and therefore he must shave himself. The barber is trapped in logical limbo: his beard keeps growing, and he can neither shave nor not shave himself!

There's another variation attributed to Russell that Borges alludes to in "The Library of Babel." At the beginning of that story, the librarian is searching for the catalog of all catalogs. I'll leave it to you to think about the formulation of the paradox in terms of catalogs. Because, what are catalogs, after all? They're books whose text consists of the titles of other books. There are catalogs that include themselves among their titles and others that don't. In this way one can arrive at the same kind of paradox.

Why Does Borges Interest Mathematicians?

The three elements that we have just examined appear again and again in Borges' work, shaped into literary form in various ways. In the essay "El cartesianismo como retórica, o ¿por qué Borges interesa a los científicos?" (Cartesianism as Rhetoric, or Why Borges Interests Scientists), which appears in the anthology *Borges y la ciencia* (*Borges and Science*), Lucila Pagliai wonders why Borges' texts are so pleasing to physicists, mathematicians, and other scientists. She concludes that there is a fundamentally essayistic matrix in Borges, especially in his mature work. And, of course, her entire text attempts to document this. It's an incisive essay, and I believe it touches on the truth. Borges is a writer who proceeds from an idea—"In the beginning was the Idea"—and who conceives his tales as embodiments or avatars of abstract notions. There are also bits of logical arguments in many of his tales. This sort of essayistic matrix to which she refers is undoubtedly one of the elements that reveal a certain similarity to scientific thought.

In a brief article I wrote on the same subject, "Borges y tres paradojas matemáticas" (Borges and Three Mathematical Paradoxes), I point out the stylistic elements that have an affinity with mathematical aesthetics. Here is the main thesis:[4]

> I've stated earlier that there are multiple mathematical allusions in Borges' work. This is true, but even if there weren't any, even in those texts having nothing to do with mathematics, there is something, a stylistic element in his writing, that is particularly pleasing to the mathematical aesthetic. I believe that the key to this element is inadvertently expressed in the following extraordinary passage from "A History of Eternity": "I do not wish to bid farewell to Platonism (which seems icily remote) without making the following observation, in the hope that others may pursue and justify it: *The general can be more intense than the concrete.* There is no lack of examples to illustrate this. During the boyhood summers I spent in the north of the province of Buenos Aires, I was intrigued by the rounded plain and the men who drank *mate* in the kitchen, but great indeed was my delight when I learned that the circular space was the 'pampa' and those men 'gauchos' . . . The general . . . takes priority over individual features. . . ." ("A History of Eternity" 129)
>
> When Borges writes, he typically accumulates examples, analogies, related stories, and variations of whatever he has decided to narrate. In this way, the main fiction he develops is at once specific and general, and his texts sound as if the specific example carries within it and permanently alludes to a universal form. Mathematics proceeds in the same way. When mathematicians study an example, a particular case, they examine it in the hope of discovering in it a more widespread, general feature, which they might abstract into a theorem. Borges, mathematicians like to believe, writes exactly as they would write if they were put to the test: with a proud Platonism, as if there existed a heaven made up of perfect fictions and a precise notion of truth for literature.

This summarizes what I think about the articulation of mathematical thought in Borges' style. For now it is not much more than what mathematicians call a *claim*, something that is affirmed in advance but which must be proven at some point. In the next chapter I will attempt to justify this statement and will explore some of Borges' non-mathematical texts in this light.

Notes

1. In editing these lectures for publication, the colloquial tone has been maintained.

2. See Appendices A and B.

3. The reader is referred to Appendix A.

4. Another excellent essay from *Borges y la ciencia,* "Indicios," by Humberto Alarga, called my attention to the excerpt from "A History of Eternity" that I cite in this passage.

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2 | BORGES AND MATHEMATICS
SECOND LECTURE

February 26, 2003

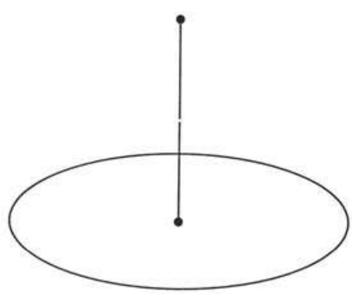
I'd like to begin with a brief recapitulation of what we've seen in the first chapter; then I will bring in additional evidence to support what we've already said. I want to call your attention to the book *Borges: Textos recobrados*, part of an effort to collect all his writings. It contains some truly remarkable essays, and Borges the polemicist is revealed. At the very beginning we spoke of Borges' mathematical education. In *Textos recobrados*, there is a fairly technical article called "La cuarta dimensión," which allows us to appreciate the fact that Borges was capable of reading mathematical texts, especially geometry, in depth. It is somewhat related to the topics we left pending at the end of the last chapter, the question of the generic and the concrete, the formation of concepts, Platonism, and so forth. At one point he states:

> [T]he surface, the point, and the line are geometric ideals, but so too is volume, and so too might be the hypervolume of four dimensions. In the material world there is no absolutely equilateral triangle, but we can intuit it. There may not be a single hypercone,

> either, but someday we will intuit that, as well. . . . ("La cuarta dimensión" 97)[1]

He goes on to say: "Hinton promises us as much in his book, *A New Era of Thought.*" Then he adds, referring to that book, "I have bought it; I've begun reading it; I have lent it to others" (97). This last statement confirms what I've been trying to say: the number of mathematics books in a library is no indication of the mathematical education of the library's owner because mathematics books are easy to start and hard to finish. Let's say that by the middle, they are easily lent to others.

Borges continues: "One irrefutable fact remains: to reject the fourth dimension is to limit the world; to affirm it is to enrich it. Through the third dimension, height, a point imprisoned within a circle can escape without touching the circumference" (97).



In effect, the point "escapes" upward. Borges then adds—in what might be seen as the germ of a potential story and illustrative of what we've already discussed (the transition from an abstract problem to a work of fiction, the path to the literary embodiment of a mathematical idea): "Through the unimaginable fourth dimension, a man held captive in a prison cell could escape without passing through the ceiling, the floor, or the walls" (97).

In the previous chapter we also discussed infinity. We showed that parts of infinities may be equivalent to the whole, and by abstracting this property we defined what we've called recursive objects. We have already provided some examples. Here are some others. A periodic number is a recursive object: it suffices to know the periodicity in order to know the entire number. A number like π is an anti-recursive object because it's impossible to anticipate what the rest of the number will be, no matter how much of it is already known. Another example of an anti-recursive object is the list of numbers corresponding to where the ball lands on successive spins of a roulette wheel in a casino. There's even a definition of chance based on this idea.[2] These are some mathematical examples.

It has been pointed out to me that languages can be seen as recursive objects. And so they are: the hieroglyph-covered stone from the city of Rosetta sufficed to reconstruct the ancient Egyptian language. Also, any room containing a mirror becomes a recursive object. Or a painting like Velázquez' *Las Meninas*, or Magritte's *La Condition Humaine,* in which a portion of the painting consists of the canvas where the entire work is reflected. Conversely, another anti-recursive object would be a short story, one that's taut enough to preclude the elimination of any of its parts.

As I mentioned earlier, up until 1870 mathematicians thought that there was a single infinity, which they designated with the symbol ∞, until Cantor demonstrated that there is a first infinity, that of natural numbers, to which he assigned the symbol ℵ. We have proved that this first infinity of natural numbers is the same kind as that of fractions as well as that of the infinity of all imaginable books. But I didn't make any further comment about that. In fact, the infinity of natural numbers is the flimsiest one possible. There is a whole chain of infinities, each richer than the last, beyond that one. Real numbers have a vaster and different sort of infinity. These ever-growing infinities can be constructed by adding all the parts of the last infinite set, resulting each time in a richer, more populous infinity. There is an entire tower of infinities, an interminable hierarchy of different infinities.

We've also observed that the totality of fractions between zero and one comprises the Book of Sand. Its front cover is zero, its back cover one, and all the pages lie in between. We said that there is no contradiction between the fact that the book could not be opened to its first page and the fact that all the pages were numbered. Using

Cantor's diagonal argument, we showed that if a sufficiently skilled bookbinder can sew together all the pages of the Book of Sand, he can also enumerate them.

Next we spoke of Alain de Lille's sphere, whose center is everywhere and circumference nowhere. I've been told that Borges technically should have described this sphere as having its "center everywhere and its *surface* nowhere," since the concept of circumference when dealing with a circle becomes a surface when applied to a sphere. In my opinion, that expression loses some of its immediate evocative impact, and I'd like to remind you here of what we've said about distilling mathematics into literary vessels. Borges, I believe, introduces the example of the circle here to illustrate his point, thus producing the "imprecision." But we can also conceive of the circumference of a sphere as an equator that encircles and delimits the sphere in the finite situation.

Then we proceeded to the third paradox: the barber who shaves all those men who don't shave themselves. I indicated that there is a variant of the paradox with the catalogs in a library. In effect, there are some catalogs that should mention themselves in their listing of titles. For example, if the catalog of all books in Spanish is written in Spanish, it should include itself. We might imagine the catalog of all books that do not mention themselves. And through that line of reasoning we would arrive at the same absurd conclusion: such a hypothetical book could neither mention nor neglect to mention itself. In other words, the catalog of all possible catalogs does not exist. Borges was well acquainted with this version of the paradox because he slips it into "The Library of Babel": "I have wandered in search of a book, perhaps the catalog of catalogs . . ." (53).

In fact, the sphere with its center everywhere and its circumference nowhere reappears in "The Library of Babel." But here Borges has decided to replace the sphere with hexagons. I believe that he makes the rooms hexagonal because the hexagon is a polygon whose form sufficiently suggests the idea of circumference. It would be very awkward and would clash with concrete reality as we know it to conceive of rounded shelves when books are rectangular. Borges considers this possibility for a moment, attributing it to a mystical vision: "The mystics claim that their ecstasy reveals to them a circular chamber containing a great circular book, whose spine is continuous and which follows the complete circle of the walls obscure. . . . This cyclical book is God" (52). And so he finds a geometrical figure that approximates circularity—the hexagon—and states, with this minor variation: "The Library is a sphere whose exact center is any one of its hexagons and whose circumference is inaccessible" (52).

The Generic Versus the Concrete
"The God's Script" and "Funes the Memorious"
The Strategy of the Universal

Now we finally arrive at a discussion of the generic versus the concrete, which is the first stylistic element I'm interested in examining. Let's take a look at how this idea is manifested in other, non-mathematical Borges stories. One of these is "The God's Script." In this story, as you may recall, a priest is trapped in a cave; once a day he can see the spots of a leopard. His god had written a sacred word somewhere in the universe, and he conjectures that the word might be encoded among the leopard's moving spots:

> I shall not recite the hardships of my toil. More than once I cried out to the vault that it was impossible to decipher that text. *Gradually the concrete enigma I labored at disturbed me less than the generic enigma of a sentence written by a god.* What type of sentence (I asked myself) will an absolute mind construct? (171; italics mine)

Here we see once more the articulation between a concrete situation and an abstract problem.

We've discussed the way in which Borges likes to bolster his tales with related examples. This is a recurrent process, even in "The Aleph." At one point he observes that the Aleph on Calle Garay might be a false one, and he lists other possible avatars of it, including a stone column in a mosque that contains the sounds of the entire universe. Something similar happens in "Funes the Memorious": "Ireneo began by enumerating, in Latin and in Spanish, the cases of prodigious memory recorded in the *Naturalis historia:* Cyrus, king of the Persians, who could call every soldier in his armies by name; Mithridates Eupator, who administered the law in the twenty-two languages of his empire, Simonides, inventor of the science of mnemonics . . ." (63). This, I should emphasize, is not just a "mathematical" procedure that amasses examples in order to clarify what is essential or what is general, but also a strategy that Ricardo Piglia describes quite well in his essay, "¿Existe la novela argentina?":

> What happens when one writes in a marginal language? Gombrowicz reflects on this question in his *Diary*, using Argentine culture as a laboratory ground for his hypothesis. On this point Borges and Gombrowicz present similar views. Consider, for example, one of the fundamental texts of Borges' poetics: *The Argentine Writer and Tradition.* What do we mean by Argentine tradition? Borges poses this question as his point of departure, and the essay is a manifesto that accompanies the fictional construction of "The Aleph," his tale about Argentine "national" writing. How is it possible to become universal in that outpost of the world? How can one shrug off nationalism without ceasing to be "Argentine" (or "Polish")? Is it necessary to be "Polish" (or "Argentine"), or must one resign oneself to being a "European in exile" (like Gombrowicz in Buenos Aires)? (46)

Let's say that the references Borges liberally sprinkles throughout his work are not arbitrary. They are always important examples drawn from some universal tradition, selected as part of his strategy of inserting his writings in a universal context. Somehow Borges' eternal complex is always present: although he may write about the working-class outskirts of Buenos Aires and the "compadritos" or street toughs that inhabit them, he is still concerned with demonstrating, sometimes ironically (for example, by calling Ireneo Funes "a vernacular and rustic Zarathustra"), that his "South American destiny" is a legitimate avatar of any universality. This cosmopolitanism always plays a role in his choice of examples.

The Generic and the Concrete in the Formation of Concepts

The subject of the abstract and the concrete were of special theoretical interest to Borges; he also chose it as the theme of some of his short stories.

The following description occurs in the story "Funes the Memorious":

> A circle drawn on a blackboard, a right triangle, a lozenge—all these are forms we can fully and intuitively grasp; Ireneo could do the same with the stormy mane of a stallion, with a herd of cattle on a hill, with the changing fire and its innumerable ashes, with the many faces of a dead man throughout a long wake. I don't know how many stars he could see in the sky. (64)

Oliver Sacks cites the same passage in his essay "The Twins" (from his extraordinary book *The Man Who Mistook His Wife for a Hat*) when he reflects on intelligence and memory. That essay, and in fact the entire book, introduce an unexpected angle, neurophysiology, into this philosophical discussion. Borges tells us that for Funes, abstract and concrete are one and the same. The concrete never quite consolidates, resolves, or distills into abstraction. Everything occupies the same plane. That's why he can envision a numbering system with twenty thousand symbols. Borges describes this project as follows:

> Locke, in the seventeenth century, postulated (and rejected) an impossible language in which each individual thing, each stone, each bird and each branch, would have its own name; Funes once projected an analogous language, but discarded it because it seemed too general to him, too ambiguous. In fact, Funes remembered not only every leaf of every tree of every wood, but also every one of the times he had perceived or imagined it. . . .
>
> The two projects I have indicated (an infinite vocabulary for the natural series of numbers, a useless mental catalogue of all the images of his memory) are senseless, but they betray a certain stammering grandeur. They permit us to glimpse or infer the nature of Funes' vertiginous world. He was, let us not forget, almost inca-

> pable of ideas of a general, Platonic sort. Not only was it difficult for him to comprehend that the generic symbol *dog* embraces so many unlike individuals of diverse size and form; it bothered him that the dog at three-fourteen (seen from the side) should have the same name as the dog at three-fifteen (seen from the front). His own face in the mirror, his own hands, surprised him every time he saw them. ("Funes the Memorious" 65)

And finally he says:

> With no effort, he had learned English, French, Portuguese and Latin. I suspect, however, that he was not very capable of thought. To think is to forget differences, generalize, make abstractions. In the teeming world of Funes, there were only details, almost immediate in their presence. ("Funes the Memorious" 66)

This idea, the notion that "to think is to forget differences, generalize, make abstractions," may be linked to a text that was found among Nietzsche's posthumous papers about the development of logic in the human brain. Nietzsche argues that logic arises, in essence, as the triumph of bestiality or instinct, the part that reacts quickly and equalizes things that are inherently different. In primitive times the man that survived was the one who understood that the wolf about to attack him at three-fourteen (seen from the front) was more or less the same as the wolf that was about to attack him from three-fifteen (seen from the side). And maybe a prehistoric Funes would have died in his attempt to establish the subtle differences. What I'm trying to say is that there might be a dialectical principle at work in the formal process of equalizing, a principle that is present in the origins of logic. Formal identity and logic might come from their exact opposite.

The Generic and the Concrete in Writing

Concerning the question of the generic and the concrete, there are also interesting stylistic consequences, which have been assembled in an issue of the cultural supplement of *Clarín* dedicated to Carlos Mastronardi, an Argentine writer and a close friend of Borges (15

February 2003). At one point Mastronardi says of Borges: "He feels and suffers like few others over that dramatic paradox so characteristic of writers: a generic or vague language to express a detailed, differentiated, singular reality."

The topic of the generic versus the concrete is among the most crucial for any writer. It is an everyday concern, a difficult question of balance: how much detail to use in describing a character; how much shape to give it; how much free rein to allow the reader's imagination for filling in the gaps. Borges had his own ideas about this. We might, for example, contrast Borges with Juan José Saer in this regard, or we might compare Borges with Nabokov's obsession with the precious details. Borges, I would say, preferred to set down few details, letting his readers complete the figures by themselves.

There is an article—or rather a critical review of a novel by Norah Lange (*45 días y treinta marineros*)—in which Borges states: "The central problem of the novel is causality. If circumstantial details are missing, everything seems unreal; if there are a great many of them (as in Bove's novels or in Mark Twain's *Huckleberry Finn*), we become suspicious of that documented truth and its compelling evidence. Here is the solution: to invent minutiae that are so believable as to make them seem inevitable or so dramatic that the reader will prefer to accept, rather than dispute, them."[3]

More than once Borges declared that he preferred to set his stories in relatively distant eras so that the details would be hard to verify and the reader could accept them more or less on blind faith. This procedure has the same purpose: the suspension of disbelief. In another one of his notes in the same article, Mastronardi remarks:

> In narrative, as Borges observes, it is not a good idea to provide all the psychological facts. Highly detailed accounts, in fact, work against the impression of reality that we're aiming for. According to Borges, the more sensible approach is to identify with the characters' inner being, in order to portray them later through certain significant signs or crucial brushstrokes He understands that judicious omissions make them appear more vivid and concrete in the eyes of the reader. (*Clarín,* 15 February 2003)

How Abstraction Works

Now I'd like to offer a first example of the assertion we left dangling in the last chapter about how Borges approaches fiction. We said that he tracks the themes of his short stories through universal literature, accumulating examples, comparing them in order to abstract a general pattern, and finally he adds his own fiction, like just one more version. We'll do this by examining an essay called "Laberintos" (Labyrinths), also from *Textos recobrados*.

I'm going to excerpt a few parts. The first one goes like this:

> The concept of the labyrinth—that of a house whose brazen purpose is to confound its guests and drive them to despair—is far stranger than the edifice itself or the law behind those incoherent palaces. The name, however, comes from an ancient Greek term meaning "tunnels of the mines," which would seem to indicate that labyrinths existed prior to the idea of the labyrinth. Daedalus, in short, had simply replicated an effect already produced by chance. As for the rest, a timid dose of alcohol—or of distraction—suffices to convert any building equipped with stairs and corridors into a labyrinth. . . . Thomas Ingram's recent book . . . is perhaps the first monograph devoted to this subject. [In an appendix] he tries to pinpoint "the immutable and true principles that the architect-gardener must observe in all labyrinths." Those principles can be reduced to just one: economy. If the space is vast, the design must be simple; if it is restricted, detours are less intolerable. ("Laberintos" 158)

And he adds, quoting Ingram:

> With two square miles of terrain and two hundred forking paths, curves, and right angles, the worst bungler is capable of creating a good labyrinth. The ideal is the psychological labyrinth, based, let us say, on the widening divergence of two paths that the explorer or the victim imagines as parallel. ("Laberintos" 158-59)

Notice how Borges shapes, stretches, and spins out the idea of the labyrinth. He starts off with its most rudimentary definition, its etymology, but he immediately observes that the concept of the labyrinth does not necessarily depend on the edifice, of the architecture itself, but rather sometimes on the person's psychological state. Then he adds an aesthetic requirement: a labyrinth cannot be a jumble of forking paths. This condition is analogous to the exploration of mathematical ideas. Mathematicians don't accept just any arbitrary proof; they aren't satisfied with whatever proof works. They always bear certain aesthetic considerations in mind. A good mathematical solution is not just any solution; it must possess a certain beauty. It must conform to certain criteria of scale, of economy of tools, and so on. There is a saying in mathematics: you can't kill a mosquito with a bazooka. This is the same idea as the square miles of terrain with two hundred forking paths.

Now, on a new level of abstraction, Borges says: "The ideal labyrinth would be a straight, unobstructed line, one hundred paces long, where disorientation would be produced by some psychological factor" ("Laberintos" 158-59). As we can see, his intention is to achieve maximum simplicity without losing the essence of the labyrinth concept: disorientation. "We will never find it on this earth," he says, "but the more closely our design approximates that classical archetype and the less it resembles an arbitrary jumble of broken lines, the better. A labyrinth should be a sophism, not a muddle" (159). We will encounter this idea again, connected to the paradox of Achilles and the tortoise, at the end of the story "Death and the Compass."

Following these observations, the article reviews some of the most famous labyrinths, including that of Crete. Finally, Borges says: "From the first appendix of the work, we have copied a brief Arabic legend translated into English by Sir Richard Burton. Its title is "The Two Kings and the Two Labyrinths" ("Laberintos" 160).

Here we see a perfect confirmation of Lucila Pagliai's thesis of the essayistic matrix in Borges' work. In this essay Borges outlines the main ideas, extracts a generalization that interests him, and as if it were a prolongation of the essay, derives one of his own short stories from it. Because the story of "The Two Kings and the Two Labyrinths" is, in fact, one of Borges' stories. Here it is:

> It is said by men worthy of belief (though Allah's knowledge is greater) that in the first days there was a king of the isles of Babylonia who called together his architects and his priests and bade them build him a labyrinth so confusèd and so subtle that the most prudent men would not venture to enter it, and those who did would lose their way. Most unseemly was the edifice that resulted, for it is the prerogative of God, not man, to strike confusion and inspire wonder. In time there came to the court a king of the Arabs, and the king of Babylonia (to mock the simplicity of his guest) bade him enter the labyrinth, where the king of the Arabs wandered, humiliated and confused, until the coming of the evening, when he implored God's aid and found the door. His lips offered no complaint, though he said to the king of Babylonia that in his land he had another labyrinth, and Allah willing, he would see that someday the king of Babylonia made its acquaintance. Then he returned to Arabia with his captains and his wardens and he wreaked such havoc upon the kingdoms of Babylonia, and with such great blessing by fortune, that he brought low its castles, crushed its people, and took the king of Babylonia himself captive. He tied him atop a swift-footed camel and led him into the desert. Three days they rode, and then he said to him, "O king of time and substance and cipher of the century! In Babylonia didst thou attempt to make me lose my way in a labyrinth of brass with many stairways, doors, and walls; now the Powerful One has seen fit to allow me to show thee mine, which has no stairways to climb, nor doors to force, nor wearying galleries to wander through, nor walls to impede thy passage." Then he untied the bonds of the king of Babylonia and abandoned him in the middle of the desert, where he died of hunger and thirst. Glory to Him who does not die. ("The Two Kings" 263-64)

What we've just witnessed is a typical mathematical operation: the absolute abstraction of the labyrinth concept and the demonstration that a labyrinth can also be a desert. This process of abstraction is one of the recurrent mathematical procedures we see in Borges' work.

Here is a second example, taken from another article in the same book, titled "The Dialogues of Ascetic and King." It's exactly the same sort of process:

> A king is a plenitude, an ascetic is nothing or wants to be nothing, and so people enjoy imagining a dialogue between these two archetypes. Here are a few examples, from Eastern and Western sources. . . . (382)

Borges begins to list various examples, like that of Diogenes:

> The sixth book has another version, from sources unknown, whose protagonists are Alexander and Diogenes the Cynic. The former had arrived in Corinth to lead the war against the Persians, and everyone had come out to see and welcome him. Diogenes refused to leave his house, and there Alexander found him one morning, taking the sun. "Ask me for anything you'd like," said Alexander, and Diogenes, lying on the ground, asked him to move a little, so as not to block the light. (382)

Then he comments on a novel called *Preguntas de Milinda* (Milinda's Questions), in which the king ultimately becomes an ascetic, assuming the ascetic's habit. Let me share this brief paragraph with you:

> Dressing himself as an ascetic, the King becomes indistinguishable from one, and he brings to mind another king of the Sanskrit epic who left his palace to beg alms in the streets and who said these dizzying words: "From now on I have no kingdom or my kingdom is limitless; from now on my body does not belong to me or the whole earth belongs to me." (383-84)

Here I find a conceptual thread, a trail that leads to "The God's Script." You may recall the ending of that short story, the resolution of the priest when the name of god is revealed to him, when he finally manages to read the sacred word and decides not to pronounce the phrase that would set him free, choosing instead to remain lying in the cave because he possesses everything and possesses nothing, and at that moment he finds it is all the same:

> Whoever has seen the universe, whoever has beheld the fiery designs of the universe, cannot think in terms of one man, of that man's trivial fortunes or misfortunes, though he be that very man. That man *has been he* and now matters no more to him. What is the life of that other to him, the nation of that other to him, if he, now, is no one. This is why I do not pronounce the formula, why, lying here in the darkness, I let the days obliterate me. (173)

The essay "The Dialogues of Ascetic and King" continues, offering similar examples and variants of the same idea:

> In the stories I have mentioned, the ascetic and the king symbolize nothing and plentitude, zero and infinity. More extreme symbols of that contrast would be a god and a dead man, and their fusion would be more economical: a god that dies. Adonis wounded by the boar of the moon goddess, Osiris thrown by Set into the waters of the Nile, Tammuz carried off to the land from which he cannot return, are all famous examples of this fusion. No less poignant is this, which tells of the modest end of a god. (385)

And at this point, again he inserts his own story, this time about the death of Odin. Borges closes the essay with an observation that neatly sums up his almost scientific interest in abstraction:

> Apart from their greater or lesser virtues, these texts, scattered in time and space, suggest the possibility of a morphology (to use Goethe's word) or science of the fundamental forms of literature. I have occasionally speculated in these pages that all metaphors are variants of a small number of archetypes; perhaps this proposition is also applicable to fables. (385)

Logical Structuring in Borges' Short Stories

Up to this point we have examined a first mathematical-type operation, which we've called generalization or abstraction. The second, which I'd like to refer to now, is one that I would call the logical structuring of the tales. Let's start by giving Borges the floor in order to demonstrate how his theory coincides with his practice, something that doesn't always have to be true. Borges was enormously interested in questions of structure; he was convinced that narratives, even genres, were governed by laws. This is the point I'd like to address. For example, in another one of the essays in his *Textos recobrados,* called "Leyes de la narración policial" ("Rules for Crime Fiction"), he attempts to abstract the fundamental laws of any detective tale. I won't quote all of it, but it states: "The commandments for writing crime fiction might be the following"—and he outlines a list:

> A) *A prudent limit to the number of characters.* The heedless violation of this rule can be blamed for the confusion and tedium of all detective films.
>
> B) *Statement of all terms of the problem.* If my memory (or lack thereof) serves, an assortment of violations of this second rule is Conan Doyle's preferred defect. Sometimes it's a matter of wispy ash particles swept up behind the reader's back by the privileged Holmes and traceable only to a certain Burmese cigar dispensed in only one shop to a single client, etc.
>
> C) *Strict economy of explanations.*
>
> D) *Precedence of "how" over "who."*
>
> E) *The modesty of death.* Homer told of how a sword sliced off Hypsenor's hand, and how the bloody hand fell to earth, and how blood-colored death and harsh destiny took possession of his eyes, but that sort of funereal pageantry has no place in the crime narrative, whose aloof muses are hygiene, fallacy, and order.
>
> F) *Necessity and wonder of the solution.* (36)

This last requirement is very similar to what mathematicians demand, namely that the theorem inevitably be derived from the premises, and yet there should be a certain surprise effect (the "punch line," as the unexpected conclusion of a theorem is sometimes called). In other words, the result or thesis should not be totally foreseeable from the initial data, but rather it should astonish, disconcert, and reveal something innovative, original, and different from whatever had been suspected until then.

There is another Borges essay that may be even more precise regarding the mechanisms of creativity. I believe it expresses his thoughts on the subject very clearly. This essay is called "La génesis de *El cuervo* de Poe" (The Genesis of Poe's "The Raven"). Borges recalls that in April 1846, *Graham Magazine* in Philadelphia published a two-column article by its correspondent, Mr. Poe, titled "The Philosophy of Composition." In that article Edgar Allan Poe endeavored to explain the genesis of his glorious poem, "The Raven":

> He begins by announcing the phonetic motifs that suggested the melancholy refrain *never more.* He then expresses his need to justify the periodic use of that word in a plausible way. How best to reconcile that monotony, that "eternal return," with the exercise of reason? An irrational being, capable of articulating the precious adverb, was the obvious solution. The first candidate was a parrot, but immediately the raven, gloomier and more genteel, took its place. The raven's plumage then suggested the addition of a marble bust, to contrast with that blackness. It was a bust of Pallas Minerva, because of the Greek euphony of the name and also to complement the narrator's books and scholarly mind. And so on with the rest of the poem. . . . I will not transcribe the elegant reconstruction Poe attempted; I will simply recall a few links [. . .] It is pointless to add that his long retrospective process was met with disbelief, if not scorn or outrage, by the critics. To go from being the interlocutor of the Muses, the poet-scribe of a dark god, to a mere spinner of explanations. Lucidity instead of inspiration, comprehensible intelligence rather than genius. What a disappointment for Hugo's contemporaries, and even for those of Bréton and Dalí! There were some who refused to take Poe's declarations seriously. . . . Others, overly credulous, feared that Poe had desecrated the essential mystery of poetic creation and rejected the entire article. As one might guess, I do not share those opinions. . . . I—naïvely, perhaps—believe Poe's explanations. Discounting any possible display of showmanship on his part, I think that the mental process he adduces corresponds more

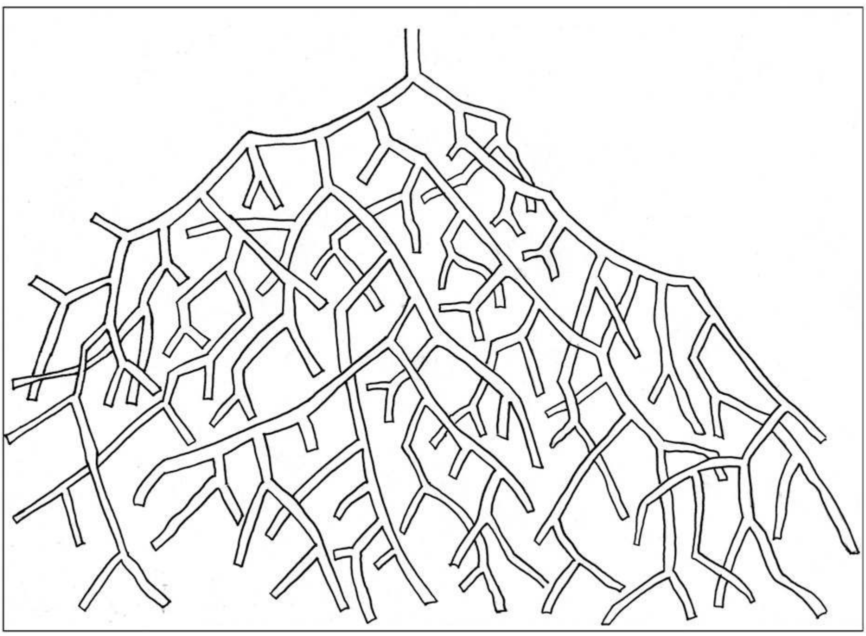
> or less to the actual creative process. I'm sure this is how intelligence works: through changes of mind, obstacles, elimination. The complexity of the operations he describes doesn't bother me; I suspect that the real approach must have been even more complex and much more chaotic and hesitant. . . . All this does not mean to suggest that the *arcana* of poetic creation—of *that* poetic creation—were revealed by Poe. In the links that the writer explores, the conclusion he draws from each premise is logical, of course, but not the only one necessary. (120)

Here we have a key point: in this brief speech Borges may have gone as far as possible (disappointingly not all that far, though) in trying to say something about the creative process in general. And again, in this discussion of "divine, winged" intuition versus the prosaic, tortoise pace of logic, I would like to contradict a myth about mathematics: the process Borges describes is exactly the same as what happens in mathematical creation. Let's consider the mathematician who has to prove a theorem for the first time, not someone who follows the demonstration of an already-proved theorem line by line, a process that would be akin to what a reader does with an already-completed work. Our mathematician sets out to prove a result without even knowing if such a proof really exists. He gropes his way through an unknown world, proving and making mistakes, refining his hypothesis, starting all over again and trying another approach. He too has infinite possibilities within his grasp and with every step he takes. And so each attempt will be logical, but by no means the only one possible. It is like the moves of a chess player. Each of the chess player's moves conforms to the logic of the game in order to entrap his rival, but none is predetermined. This is the critical step in artistic and mathematical elaboration, and in any imaginative task. I don't believe there is anything unique to literary creation as far as the duality of imagination/intuition versus logic/reason is concerned. Now let's return to Borges:

> In the steps we have explored, the conclusion the writer draws from each premise is, of course, logical, but not the only one necessary. For example, from the need for an irrational being ca-

> pable of articulating an adverb, Poe comes up with a raven, after having rejected a parrot. He might just as easily have produced a lunatic, a solution that would have changed the poem utterly. I raise this objection out of a thousand possible others. Each link is valid, but between one link and another there remains a trace of shadow or of unbridled, unconstrained inspiration. ("La génesis de *El cuervo* de Poe" 121-22)

Exactly the same thing happens in mathematics: between one link and another there must be human intelligence and inventiveness to decide that this, and not the other, is the proper path to follow.



The labyrinth of possibilities in a mathematical reasoning. In the links that the mathematician *explores the conclusion he draws from each premise is logical, of course, but not the only one necessary.*

Borges adds: "I will say it differently: Poe articulates various steps in the poetic process, but between each one and the next remains the infinitesimal step of invention" (122).

Now, using these ideas of Borges' as a base, I would like to refer to an article I wrote in which I compare the short story with a logical system, slightly modifying an idea that Ricardo Piglia expresses quite eloquently in an article called " Tesis sobre el cuento" (Thesis on the Short Story) in *Crítica y ficción*. The germ of that idea, in fact, should also be attributed to Borges, as Leopoldo Brizuela has pointed out to me. In effect, Borges writes in his prologue to María Esther Vázquez's *Los nombres de la muerte*: "Since our contemporary reader is also a critic, a man who knows—and can anticipate—literary artifice, the short story must contain two plots: a false one, vaguely hinted at, and the other, authentic one, which will remain secret until the end" (*Obras completas* 234).

This is the same idea that Piglia later elaborates, that every short story is the interplay of two stories: one that is told on the surface and another that is subterranean, secret, one that the writer gradually unearths during the course of the story and reveals completely only at the very end.

In my slight variation, "The Short Story as Logical System,"[4] I note that it seems a bit excessive, when analyzing concrete examples of short stories, to insist that there really be two arguments. Often there isn't even *one* story to be found in some contemporary tales. I propose replacing this rather demanding requirement with a slightly laxer, more general scheme by thinking about two different sorts of logic. I observe that, in general, short stories start out in the realm of common sense, the initial logic of some form of "normalcy," and that there is another, hidden, logic that only the narrator knows about at the beginning, one that concerns what he or she wants to disclose at the end. The narrator's magic consists of successfully transmuting that initial logic, little by little, into the second, fictional logic. So, for example, an element that is introduced as random or haphazard in the first logic can turn out to be absolutely necessary for the second logic.

"Death and the Compass"

I now propose that we follow one of Borges' short stories, "Death and the Compass," paying close attention to that transmutation of logical systems. Of course, this is not something unique or peculiar to Borges' stories. It's related to the structure of the traditional tale, but Borges was especially conscious of these levels. In general his stories are conceived and structured in this way. Let's take a look at the first paragraph:

> Of the many problems which exercised the reckless discernment of Lönnrot, none was so strange—so rigorously strange, shall we say—as the periodic series of bloody events which culminated at the villa of Triste-le-Roy, amid the ceaseless aroma of the eucalypti. It is true that Erik Lönnrot failed to prevent the last murder, but that he foresaw it is indisputable. Neither did he guess the identity of Yarmolinsky's luckless assassin, but he did succeed in divining the secret morphology behind the fiendish series as well as the participation of Red Scharlach, whose other nickname is Scharlach the Dandy. (76)

One observation here: notice that Borges writes "the periodic series of bloody events" because in this tale he wants to adhere to what he himself has said about the detective genre; that is, he's trying to lay all his cards on the table. And so he uses what might appear to be a euphemism, "bloody events," in order to avoid the word "crimes." For those who aren't familiar with the story, suffice it to say that not all of them are crimes. If the omniscient narrator were to say "crimes" here, it would lead the reader to form a mistaken idea, and the two logics must not be contradictory, but rather they must overlap.

Now let's go to the second paragraph: "The first murder occurred in the Hôtel du Nord—that tall prism which dominates the estuary whose waters are the color of the desert" (76).

In principle, what we register as an important fact is that the first crime took place in a hotel. Here, in the details, we see the theme of the contingent and the necessary. In the initial logic of the story, the

Hôtel du Nord, as described, is just a random hotel, the name of a hotel. But the important detail is one that at first seems arbitrary or haphazard, the word "Nord," because it will represent the northern cardinal point. In other words, the name of the hotel, which at first we skim over without paying any particular attention, will later become important in the narration. The same thing occurs in the following passage: "To that tower . . . there came on the third day of December the delegate from Podolsk to the Third Talmudic Congress, Doctor Marcelo Yarmolinsky. . . ." (76).

We read the "third of December" as if it were any ordinary day. The third, the fifth—what difference does it make? Dates and numbers don't mean that much to the reader, especially if that reader is a mathematician! All numbers are the same. We suppose that the author chose the date fairly casually. But later the fact that it was the third *will* become important.

Observe how in these first two paragraphs Borges has already mentioned all the crucial elements of the story: the investigator, the criminal, the name of the first victim, and so on. He has positioned his pieces as in a chess opening. Here again we can see his intention of "stating all the terms of the problem."

And then we have the first crime. Yarmolinsky, a scholar of Judaic sects, turns up dead in his hotel room. Soon after, a meeting takes place between Treviranus, the "official" detective, the detective of the prosaic order of reality, and Lönnrot, the Borgesian detective, the detective of the fictional order. Borges continues:

> "No need to look for a three-legged cat here," Treviranus was saying as he brandished an imperious cigar. "We all know that the Tetrarch of Galilee owns the finest sapphires in the world. Someone, intending to steal them, must have broken in here by mistake. Yarmolinsky got up; the robber had to kill him. How does it sound to you?"
>
> "Possible, but not interesting,' Lönnrot answered. "You'll reply that reality hasn't the least obligation to be interesting. And I'll answer you that reality may avoid that obligation but that hy-

> potheses may not. In the hypothesis that you propose, chance intervenes copiously. Here we have a dead rabbi. I would prefer a purely rabbinical explanation, not the imaginary mischances of an imaginary robber." (77)

This dialogue is very important. Treviranus' explanation conforms to the chaos and fortuitousness of reality. The crime has an accidental aspect. What Lönnrot objects to is the aesthetic imbalance, the fact that it isn't "literary." He would prefer a hypothesis that would make sense of that chaos. Underlying is the conflict between reality and fiction. I say this because Borges imagines a solution in which both elements appear. Or rather, both the detective of "reality" and the "fictional" one are partially right. Borges' resolution is very interesting, if not altogether innovative, it must be admitted. There is a novel by Agatha Christie, a writer that many people publicly deride but nonetheless continue to read clandestinely, that contains a very similar idea. We'll get back to this later.

Treviranus replies:

> "I'm not interested in rabbinical explanations. I am interested in capturing the man who stabbed this unknown person."
>
> "Not so unknown," corrected Lönnrot. (77)

He proceeds to comment on the Yarmolinsky writings that were found there, a whole series of works on kabbalah, the Hasidic sect, books on Judaism, and so on. Once again, this is an apparently arbitrary element: there might or might not have been books in the room. But as narrator, what does Borges need? He needs to give his readers a mini-lesson in the ABC's of kabbalah in order to facilitate the subsequent development of the tale. The books that he finds have a double function here. How does Borges manage to provide this lesson without falling into the trap of didacticism? The solution is to imagine that his detective is also ignorant of these subjects. Then, as his detective goes off to read about kabbalah and the history of these Jewish sects, the reader, too, acquires the information necessary to proceed. Clearly there is a technical device at work here. But, again, a large part of a writer's mastery consists of converting a technical device into something necessary, integrating it naturally into the tale. In the essay I mentioned earlier, "The Short Story as Logical System," I compare the writer to an illusionist who uses one hand to perform the trick and the other to conceal it. And then I say that the true artist among writers should ideally be a magician like René Lavand, who, as you may know, has only one hand.

Lönnrot, then, as we've said, devotes himself to studying the books he has found and informs us of the essential principles of kabbalah. Let us recall that lying beside the dead man was a paper containing the phrase: "The first letter of the Name has been uttered."

In the lesson we are given we learn that one of the books speaks of "the virtues and terrors of the Tetragrammaton, which is the unutterable name of God"; another, of the "thesis that God has a secret name, in which is epitomized (as in the crystal sphere which the Persians ascribe to Alexander of Macedonia) by his ninth attribute, eternity—that is to say, the immediate knowledge of all things that will be, which are, and which have been in the universe" (78). This same idea, that the name of god, a certain combination of letters, might be a door leading to absolute knowledge, reappears in "The God's Script."

As the story advances, there is another digression that also has special meaning. An article appears in a popular newspaper about the murder. Borges inserts this odd paragraph:

> One of those enterprising shopkeepers who have discovered that any given man is resigned to buying a given book published a popular edition of the *History of the Hasidic Sect.* (78)

What is the meaning of this deviation from the narrative that is unfolding in the foreground? In principle it can be read as one of the many possible consequences of the murder. But in fact this "digression" is inserted in order to solve a technical problem of credibility that will arise later. The problem is that the man behind the series of murders, that man who devises this series as a trap for ensnaring Lönnrot, is Scharlach. And Scharlach is a criminal from the outskirts of Buenos Aires. This character creates several problems for Borges. I believe that in order to suggest a certain refinement of character, he assigns him the nickname “Scharlach the Dandy.” But in any case, how is it possible for a lowlife from a rough neighborhood to suddenly become so well-acquainted with the Hasidic sect? That’s why a popular edition had to be published. This apparently “loose end” is tied up at the conclusion. Ricardo Piglia offers a similar explanation in his “Tesis sobre el cuento” (76).

What I want you to notice is how Borges assembles the second logical structure of the tale. Looking backward from the conclusion, we can see how many of the details can be explained differently. But this second structure is present from the beginning, lying in wait, concealed within the logical sequential order of the plot.

With the second crime the regularity of elements in the series emerges. “The second crime occurred on the evening of the third of January.” The number three reappears, and by now we know that it’s no coincidence. The second victim, a thug by the name of Azevedo, has features “masked in blood”: “a deep knife wound had split his breast. On the wall, across from the yellow and red diamonds, were some words written in chalk” (“Death and the Compass” 79). The words, of course, were: “The second letter of the Name has been uttered.”

Thus, with the second crime, the detail of the rhombuses appears, a detail that seems circumstantial with regard to the number three, but one that will become essential with regard to the number four, the true number of the series. The rhombuses foretell the ultimate solution. Then Borges writes: “The third crime occurred on the night of the third of February. A little before one o’clock, the telephone in Inspector Treviranus’ office rang. . . .” (79).

Once again the number three makes its appearance. The police receive a phone call from a certain Ginzberg or Ginsburg, “prepared to communicate, for reasonable remuneration, the events surrounding the two sacrifices of Azevedo and Yarmolinsky” (79). The word “sacrifice” slips in here as one of the possible variants of the word “death.” However, as will become evident toward the end of the story, the word "sacrifice" is essential to the narrative. There is then a third death (although later we discover that it's a sham). The "victim" is a man who walks between two masked harlequins:

> Twice he stumbled; twice he was caught and held by the harlequins. Moving off toward the inner harbor which enclosed a rectangular body of water, the three got into the cab and disappeared. From the footboard of the cab, the last of the harlequins scrawled an obscene figure and a sentence on one of the slates of the pier shed. (80)

The sentence was: "The last of the letters of the Name has been uttered." The last. It would appear from this message that the series of crimes ends here: three crimes, the third day. Treviranus, the detective of reality, isn't so sure: "'What if all this business tonight were just a mock rehearsal?'" he asks (81).

Borges, as we can see, plays fair till the end: the story *is* a sham, and the detective of reality discovers this.

But the reader, already trapped in the fictional logic, knows that something else is going to happen. In effect, the second logic, the fictional one, has already contaminated the tale. And what does the reader foresee? As in any classic detective story, the reader anticipates that Lönnrot will be the one to provide the definitive explanation and that the "realistic" detective will be more inept. Borges plays with that relationship of superiority, slowly constructed in thousands of examples of crime fiction. Here Lönnrot lets slip the first detail that allows the reader to reconstruct the whole story—the detail about the start of the Hebrew day—"'The Hebrew day begins at sundown and lasts until the following sundown'" (81).

The above observation lends a different meaning to the theme of the number three in the dates of the murders: three becomes four if the crimes occur close to nightfall.

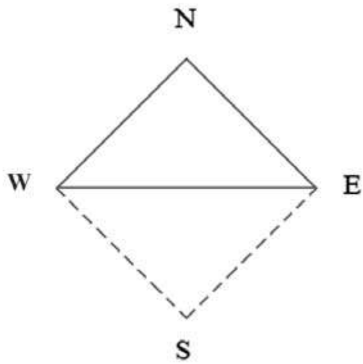
> The inspector attempted an irony.
>
> "Is that fact the most valuable one you've come across tonight?"
>
> "No. Even more valuable was a word that Ginzberg used." (81)

That word is "sacrifice." What happens next? As the plot unravels, Treviranus receives a letter with the first solution, the "false" one of the series: "The letter prophesied that on the third of March, there would not be a fourth murder, since the paint shop in the west, the tavern on the rue de Toulon and the Hôtel du Nord were 'the perfect vertices of a mystic equilateral triangle'" (82). Thus, the first "solution" to the puzzle is the equilateral triangle.

> Erick Lönnrot studied them. The three locations were in fact equidistant. Symmetry in time (the third of December, the third of January, the third of February); symmetry in space as well . . . Suddenly, he felt as if he were on the point of solving the mystery. He smiled, pronounced the word Tetragrammaton (of recent acquisition) and phoned the inspector. He said:
>
> "Thank you for the equilateral triangle you sent me last night. It has enabled me to solve the problem. This Friday the criminals will be in jail, we may rest assured."
>
> "Then they're not planning a fourth murder?"
>
> "Precisely because they *are* planning a fourth murder we can rest assured." (82)

Of course the real solution is the one that has to do with the Hebrew name for God, JHVH (or YHVH), which has four letters. And in fact the figure to be completed will indicate the place where Scharlach will ambush Lönnrot. That is, what Lönnrot does is to complete a rhombus by extending the triangle with a fourth point, without knowing that the murderous Scharlach waits for him there.

The puzzle is a trap, a labyrinth (as Borges calls it). North, east, and west are the three points in the city that he uses to calculate, with the help of a compass and some calipers, the fourth point in the south where his own death awaits him. Because Scharlach has a score to settle with Lönnrot. This is something the readers don't know until the end; it's part of what Scharlach reveals in the final explanation.



The triangle turns into a rhombus, pointing out the place of a fourth murder.

Let's look at this monologue, which takes place when he finds himself in Triste-le-Roy, confronting Lönnrot:

> "On those nights I swore by the God who sees with two faces and by all the gods of fever and of the mirrors to weave a labyrinth around the man who had imprisoned my brother. I have woven it and it is firm: the ingredients are a dead heresiologist, a compass, an eighteenth-century sect, a Greek word, a dagger, the diamonds of a paint shop.
>
> "The first term of the sequence was given to me by chance. I had planned with a few colleagues—among them Daniel Azevedo—the robbery of the Tetrarch's sapphires. Azevedo betrayed us: he got drunk with the money that we had advanced him and he undertook the job a day early. He got lost in the vastness of the hotel; around two in the morning he stumbled into Yarmolinsky's room. The latter, harassed by insomnia, had started to write. He was working on some notes, apparently, for an article on the Name of God; he had already written the words: *The first letter of the Name has been uttered.* Azevedo warned him to be silent; Yarmolinsky reached out his hand for the bell which would awaken the hotel's forces; Azevedo countered with a single stab in the chest. It was almost a reflex action; half a century of violence had taught him that the easiest and surest thing is to kill. . . ." (85)

The first crime is framed within the confines of reality: it's an accident, just as Treviranus had foreseen. Here is where the slippage begins, the transition to fictional logic:

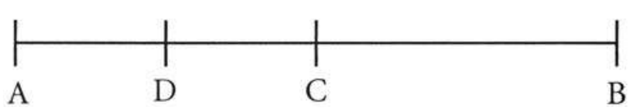
> "Ten days later I learned through the *Yidische Zaitung* that you were seeking in Yarmolinsky's writings the key to his death. I read the *History of the Hasidic Sect*; I learned that the reverent fear of uttering the Name of God had given rise to the doctrine that that Name is all powerful and recondite. I discovered that some Hasidim, in search of that secret Name, had gone so far as to perform human sacrifices. . . . I knew that you would make the conjecture that the Hasidim had sacrificed the rabbi; I set myself the task of justifying that conjecture." (85)

That is to say, a stroke of chance, Yarmonlinsky's unpremeditated murder, unexpectedly provides Scharlach with the possibility of luring Lönnrot into a trap. From that moment on, building on that first death, which chance had brought his way, Scharlach constructs his series, keeping in mind *what the detective wants to discover.* This is the interesting twist to the tale that I referred to earlier and that also appears in one of Agatha Christie's earliest novels, *Murder on the Links.* In this novel, Christie wages a small battle against Conan Doyle by confronting her psychological detective, Hercule Poirot, with a French counterpart, Giraud, who imitates Sherlock Holmes' methods. She invents a detective who works like Holmes, sniffing around, crouching down on all fours to examine cigarette butts and footprints on the lawn, things like that. She ridicules Sherlock Holmes, let us say. And truly the clever touch in this novel is that the criminal leaves little clues all around precisely so that this sort of detective can find them. The criminal conforms to the detective's modus operandi. The criminal cracks the theory, and the two planes merge. Exactly the same thing happens here. That's why I maintain that in this tale the two planes—the real and the fictional—coexist. The criminal introduces into reality those elements that are theoretically appealing to the detective's methods. He converts what is fictional and theoretically "interesting" to Lönnrot into real crimes.

Here I will repeat something that caused quite a stir last year when I gave a series of lectures on this topic: I find the final dialogue unconvincing. It goes like this:

> "In your labyrinth there are three lines too many," he said at last. "I know of one Greek labyrinth which is a single straight line. Along that line so many philosophers have lost themselves that a mere detective might well do so, too. Scharlach, when in some other incarnation you hunt me, pretend to commit (or do commit) a crime at A, then a second crime at B, eight kilometers from A, then a third crime at C, four kilometers from A and B, half-way between the two. Wait for me afterwards at D, two kilometers from A and C, again halfway between both. Kill me at D, as you are now going to kill me at Triste-le-Roy." (86-87)

This variation, this double ending, does not convince me, either from a literary or a mathematical point of view. Literarily, it seems to me that some of the final drama is lost with this overly sophisticated explanation. For me, this theoretical refinement doesn't suit the ambience or the rhythm of the action. But above all, I think that in this case what Proust called the three-adjective rule is not borne out. Apparently, at one time in Paris it became fashionable to toss out three adjectives as a sign of admiration, but of course that requires a certain gradation: the third adjective must surpass the other two. I think that the linear geometrical trap Borges proposes through Lönnrot as a "simpler" alternative isn't as clear or as neat as the graphic image of the diamond represented by the four cardinal points. Let me explain why. Here again is the sketch that corresponds to Lönnrot's explanation, which we've just seen; it's the same sketch that Borges drew in the margin of his original manuscript.



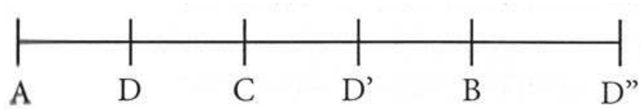
Remember that the series has to form a trap leading the detective inevitably to the fourth point. Lönnrot continues:

> "[P]retend to commit (or do commit) a crime at A, then a second crime at B, eight kilometers from A, then a third crime at C, four kilometers from A and B, half-way between the two." (87)

In other words, following an imaginary straight line, our detective goes first to point A, then proceeds to B, and finally retreats to C. That is the trajectory following the order in which the crimes have been committed. Lönnrot then says:

> "Wait for me afterward at D, two kilometers from A and C, again halfway between both. Kill me at D, as you are now going to kill me at Triste-le-Roy." (87)

Thus, D would be the fourth, imaginary point; the trajectory would be A, B, C, D. Of course this has to do with one of Borges' favorite themes, the paradox of Achilles and the tortoise. That's why he mentions a "Greek labyrinth which is a single, straight line." It's an important, time-honored idea, but it doesn't work here. According to the three initial markers—points A, B, and C, why would Lönnrot have to go to D and not D' or D", for example?



What I'm trying to say is that point D, as Lönnrot describes it, is not unambiguously or logically determined by the three previous points. Or rather, what is there to favor this point over others? Nothing. Underlying this idea is a more profound one, studied by Ludwig Wittgenstein and connected with the many possible continuations of logical series. It's important to be aware that, in general, there are no unique solutions. Because Borges has the paradox of Achilles and the tortoise in mind, he feels that point D as the fourth point on this line is as obvious, as inevitable, and as fatal as the southern point suggested by the three other cardinal points. But D is not so clear: I walk eight kilometers to reach B; then I go backward four kilometers to arrive at C. It might be a movement defined as: advance eight/

retreat four/advance eight/retreat four; or it might be: advance eight/ retreat four/advance two/retreat one; and so on—or any other possibility you can think of. There are many equally "logical" continuations. And so it seems to me that this coda detracts from the precision of the ending, which was really quite good enough. Lönnrot arrives at the fourth point, explains the meaning of the series to himself, and is killed.

I would have liked to analyze one more story in this same way, returning to "The Aleph" and looking at it from the point of view of its "construction," but my remarks can be found in an article that appeared in *Clarín* on the centenary of Borges' birth. It's called "Un regreso a *El Aleph,*" and it can also be read on my website (www.guillermomartinezweb.blogspot.com), where all my articles appear. It was also published in the MALBA literary review, www.elhilodeariadna.com.ar. Now let's have some questions.

Questions

Q1: Regarding the series of points, it may be true that what Borges proposes isn't the only possible solution, but it *does* seem like the most immediate one, the one that corresponds to 1, ½, ¼ . . .

GM: Well, maybe it seems like the most immediate one to you because he suggests it.

Q1: It's the one we'd most naturally think of before looking for another one.

GM: What I was trying to explain is that it depends on how we read the points. Let's think of a real situation where a person turns up dead at point A. The only thing we can really be sure of is that a dead person has turned up at that point. Then a second dead person turns up at point B, and then another at point C. That's what we know.

Q1: Point B is 1; C is ½. Then to go to point D' would be to go from 1 to ½ to ¾, which doesn't seem so attractive.

GM: But it depends on how you "read" the series. Series, as you know, can differ greatly from one another. A series can be, as Lenin would say, one step forward, two steps backward. Why not? One step forward, a half-step backward, one step forward, a half-step backward. In principle there's no single, privileged continuation.

Q1: No, no. I agree, but it's more of a stretch.

GM: I don't know. The idea that I go from A to B and then I start to return to A without ever recovering the forward movement again doesn't seem so evident to me. I advance, I retreat, and then I keep retreating—that's not so evident to me either. Obviously, everything becomes evident once you explain it enough. What I mean is that there's no clear uniqueness. In the first series, the one with the cardinal points, the whole structure of the story unambiguously determines the fourth point. The uniqueness rests on the shape of the rhombus, the cardinal points, and so on. Otherwise the southern point wouldn't be such an obvious solution.

Q1: Okay, fine. But I was referring to the continuation that a reader . . .

GM: It can also seem evident to a reader of Borges, I agree. But a mathematical reader . . .

Q1: . . . likes things to be more complicated.

GM: No. A reader of Borges might possibly also have the paradox of Achilles and the tortoise in mind. Then he would immediately read that into it. Borges clearly wasn't thinking of other options; he didn't think of a different possibility.

Q2: A very long, very intellectualized, and complicated conclusion. Very different from other deaths in other Borges stories, that's for sure. But the fact that here the victim is the detective himself—couldn't it fit with the ending? I found the conclusion long and "speechy," but I thought it was logical because the detective is killed. The victim is precisely the one who's searching for the killer.

GM: I agree completely. It's fine that the detective dies, that the last victim is the detective. What I'm saying is that I, as reader, would have preferred omitting the second explanation. It seems like it leads to a mathematical discussion with a thug from the 'hood. Even the language Scharlach uses is strange; he almost sounds like Borges, although Borges is aware of Scharlach's educational deficit because he provides that initial didactic exposition in a popular edition of the history of the sect exclusively for him. On the one hand, he realizes that Scharlach is a gangster from the slums. And yet, when the moment comes to have him speak, in my opinion Scharlach is infected with an overly intellectual tone.

Q2: Sure, but the intellectual discussion reveals that Scharlach isn't just any old thug. The two faces of Janus, everything he describes previously about the garden, etc. That conclusion has a certain logic considering all that comes before in the story.

GM: Of course. I'm always going to be in the minority on this point—I'm totally aware of that. Borges has an essay on the classics where he tries to define what a classic is. He says that a classic is that book or author that people or nations have decided to read with a priori devotion and mysterious loyalty. *A priori devotion and mysterious loyalty.* I think that Borges has achieved precisely that: people read him with a religious zeal that often prevents the possibility of thinking he might have left some loose ends or that certain allusions were just private jokes. People read Borges like kabbalists read the Bible, believing that all the connections are there and if we don't see them it's because we haven't thought it through sufficiently, or we don't have enough faith, that nothing is superfluous, nothing is lacking, that everything can be interpreted, and everything has a reason for being there. I don't think that's the case, but I do think that there is something prodigious about Borges. That's what I would like to conclude with: that his work succeeds in creating that illusion. Let's say that if literature were a recursive object, Borges would aspire to being the part that equals the whole. And in effect, many people believe that by reading Borges, they are reading all of literature.

There are even those who announce with pride, a pride they think demonstrates their intellectual refinement: "I *only* read Borges," as if they've tried the most exquisite dish and can no longer be nourished by anything else. But, after we chuckle a little at those people, we have to recognize that Borges has achieved what Piglia calls the microcosm of literature. He displays extraordinary feats of synthesis. And he achieves them, I believe, in the way I was trying to explain: he provides essential, critical examples, and we have the feeling that his stories generate all possible variants, or that they are a synthesis of all possible variants. That is Borges' immense literary achievement. But even so, I still think that in the diagram point D is not so clear!

Notes

1. All translations from *Textos recobrados* are by Andrea G. Labinger unless otherwise indicated.

2. See "The Music of Chance" later in this book.

3. Prologue to *Los nombres de la muerte* by María Esther Vázquez, in "Prólogos, con un prólogo de prólogos," *Borges: Obras Completas,* Ed. Sudamericana, 2011, 234.

4. Also published in this volume.

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3 | THE GOLEM AND ARTIFICIAL INTELLIGENCE[1]

Although it is not yet clear if something that might properly be called "artificial intelligence" really exists (beyond certain possible, convincing simulations), through the miracle of theorizing specialists now speak of an "ancient era" and a "modern era" in this quest. In the "ancient era," investigators tried to model intelligence as an algorithm distinct from the physical, a gigantic program designed for an ideal computer. In the "modern era," efforts are being made to "embody" intelligence within an organic-spatial context through robots, the latter-day golem.

Now I'd like to remind you of some verses from Borges' poem about the Rabbi of Prague and his creature, using this reading to make certain observations about the distinction between these two "eras." In one of the first stanzas of "The Golem," Borges says:

> So, composed of consonants and vowels,
> there must exist one awe-inspiring word
> that God inheres in—that, when spoken, holds
> Almightiness in syllables unslurred. ("The Golem" 81)

This is a subject that Borges also deals with in the story "The God's Script." In that tale a priest is trapped in a well, together with a jaguar. Once a day, whenever the trapdoor on top is opened to feed him, the priest can see the jaguar's spots and at last discovers that the design made by those spots contains the encoded message of a sentence written by the god, a fourteen-word phrase that implicates the entire universe. To pronounce those words would give the priest the *summum* of power; it would, in fact, turn him into the god. This is a variation on a kabbalist belief that Borges has repeated several times, the idea that syntactical manipulation, the mere combination and pronunciation of certain symbols, can generate life. Not only is it the process used by the Rabbi of Prague, but it is also found in some pre-biblical creation myths, and it corresponds perfectly with what has been called the "ancient era" of disembodied artificial intelligence, because a program is, after all, nothing more than a bit of language, a fistful of commands and words.

Following is another verse that reads:

> To it the rabbi would explain the universe—
> "This is my foot, this yours, this is a clog"—
> year in, year out, until the spiteful thing
> rewarded him by sweeping the synagogue.

We can compare the traditional, ominous image of a golem that keeps growing disproportionately to Borges' ironic, condescending vision in this poem. Borges' golem, closer to its roots, is an amorphous thing that never quite manages to attain its potential and to which its creator resigns himself: "until the spiteful thing/rewarded him by sweeping the synagogue." It should be noted that the original Spanish reads "*perverso*" ("spiteful" in Alan Trueblood's translation), a word that here signifies "thwarted in its nature," without any connotation of evil. I don't know if robotics has yet managed to sweep the synagogue properly; that is something we would have to verify. But the verse I'd like to emphasize is: "This is my foot, this yours." This lesson, the sense of possession of one's own body—perhaps the most basic of all—has to do with self-awareness, one of those implicit senses of which we are not conscious. We have five senses that we recognize and other, more hidden ones, which allow us to function as an integrated whole and that, when affected by cerebral damage (as in the cases explored by Oliver Sacks in *The Man Who Mistook His Wife for a Hat*), can be lost or dislocated. It's possible to feel that one of our limbs no longer belongs to us. There are cases of patients that fall out of bed trying to remove one of their own feet, which they believe has been placed there, separately, like someone's idea of a practical joke. These senses "behind the five senses" should also be taken into account, I think, when discussing intelligence as a physical embodiment.

Borges' irony returns, more pronounced, in a later stanza:

> Perhaps the sacred name had been misspelled
> or in its uttering been jumbled or too weak.
> The potent sorcery never took effect:
> man's apprentice never learned to speak.

This rather dismissive view of "sorcerers' apprentices"—whether those sorcerers be rabbis, alchemists, or scientists—is quite commonplace in literature. Here, in a way, we see the clash of the two cultures: the humanistic versus the scientific. In literature (with the specific exception of science fiction), scientific efforts are usually doomed to failure. The prototypical example, of course, is *Frankenstein*, in which the monster turns against its creator. If the portrayal of the golem seems somewhat ominous, Shelley's creature, used symbolically in the title of a conference on robotics, comes across as even more unfriendly. Yet they aren't all that distant: Mary Shelley's *Frankenstein* is subtitled *The Modern Prometheus*. And rightly so: the golem is also connected to the Promethean notion of endowing man with all the divine attributes. Further, the Prometheus myth apparently shares a common origin with the story of Adam and the creation of man from clay.

Now I'd like discuss the limits or possible limits of artificial intelligence, as reflected in the last stanza of Borges' poem. We will see a mechanism that Borges has perfected and repeated, one that is especially significant in this context. The rabbi reflects on his creation, his slightly dimwitted child. He says:

> In his hour of anguish and uncertain light,
> upon his Golem his eyes would come to rest.
> Who is to say what God must have been feeling,
> Looking down and seeing His rabbi so distressed? (193-97)

This is a very frequent Borgesian technique: I'd call it "the backward step." He does the same thing, for example, in the short story "The Circular Ruins." At the last moment the man that steps into the fire is spared from burning because he, too, is the dream of another, higher creator. That backward step of reason, I believe, is one of the fundamental attributes of the human being. It is what lies behind Kurt Gödel's theorem. In effect, even before Alan Turing, Gödel was the first to realize the intrinsic limitations of all formal systems (which, on careful consideration, is a problem of the limitations of language). Once we establish the syntactical and logical rules of the game in a formal system, once we find a way to model an algorithm and can perceive it as a separate object of study, we are somehow able to take that "step backward" and formulate a question that lies beyond the reach of that system. This is the idea that Roger Penrose later takes up in the book *The Emperor's New Mind.* He observes that Gödel's theorem allows us to show a true proposition, one that we *know* is true, but whose truth lies beyond the reach of the computer's ability to verify it. This illustrates the gap that exists between the truth and the *demonstrable*—or verifiable—aspect of the truth. I think that it is the very same mechanism we find in these two verses. Borges achieves it, as poetry does, through the ancient magic of "affinities" (or "sympathies"), through plausible analogy. In other words, he presents us with a rabbi who tries fruitlessly to educate his creature and then takes a step backward, and suddenly we become the creatures of a higher creator who also strives . . . without, at least until now, very impressive results.

Note

1. Excerpt of a presentation given at a multidisciplinary conference, "Golem Project," in collaboration with the Czech Republic, Museum of Fine Arts, October 2003.

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4 | THE SHORT STORY AS LOGICAL SYSTEM[1]

There are certain elements in the structure of the short story—brevity and rigor, for example—that too easily tempt us into formulating rules for the genre and dreaming up possible classifications and commandments. These efforts usually turn out either too vague and general to be of interest or else, regardless of how many carefully thought-out axioms are presented and precautions taken, they fail to consider some perfectly legitimate example of a short story that mocks the laws. And just as in that old-fashioned book *One Hundred Ways to Say NO to a Sexual Proposition,* the hundredth answer is YES; in every list of Ten Commandments, the tenth seems condemned to be (as Argentine writer Abelardo Castillo has suggested): Don't take the previous nine too seriously.

This deficiency in all attempts to formalize the genre may produce a sigh of relief and the swift and misleading conclusion that there really are no guidelines to consider when tackling the task of writing a short story. And yet—as anyone who has seriously attempted it knows—it doesn't take too long for us to realize that the rules we thought we'd thrown out the door come flying back through the window. They're slippery, intangible rules that can be recognized in specific examples but which don't lend themselves to generalization and cannot be readily articulated. I'll mention two that strike me as especially profound. The first is one that Borges suggests (by opposition) in a paragraph where he tries to contrast the short story with the novel. Borges skips the most obvious and superficial difference—length—and observes that what characterizes the novel, above all, is the evolution of the characters. In short stories the preeminent aspect is the plot; the characters are important only as agents of the plot, and therefore they lose a certain degree of liberty.

The second rule is one Ricardo Piglia declares in his "Tesis sobre el cuento," in an article that appeared in *Clarín* a few years ago.[2] There he maintains that every short story is the interplay of two tales, one told on the surface and another, subterranean and secret, that the writer gradually unfolds throughout the course of the story and reveals in its entirety only at the end. This idea coincides with my own most frequent image of the short story writer: an illusionist who diverts the public's attention with one hand while he executes his magical act with the other. An added benefit to this approach is that it allows us to see the short story not as a finished product, ready to be dismantled by critics, but rather as a living process, from its inception on.

A slight variation of this idea allows us to think of the short story as a logical system. The word "logical," inserted into an artistic context, shouldn't necessarily be alarming. Logic—not to be confused with the rigid syllogisms we learned in high school or the binary fragment used by mathematics—has proved to be a very malleable substance. From the historic moment in the early 1800s when the young student Nikolai Lobachevsky denied Euclid's fifth postulate in the belief that it would lead to an absurd conclusion, and when a new, perfectly strange but perfectly consistent geometric world emerged instead, a silent revolution has erupted in human thinking. Since then, various disciplines and branches of thought have developed their own logic. Thus, the field of law formalizes and attempts to introduce automatic procedures into its criteria for evidence and validity; mathematics begins to reason with polyvalent logics; psychiatry attempts to formulate models for the logic of schizophrenia; and washing machines incorporate fuzzy logic.

What is, in fact, a logical system? It's a set of initial assumptions and a series of deductive rules—which can be thought of as rules of the game—that allow us to proceed "legitimately" from the initial assumptions to new assertions. The variety and diversity of types of logic basically depend on which rules of deduction are chosen. In intuitionist logic, for example, demonstrations *per reductio ad absurdum* are not allowed, and in trivalent logic it is possible to affirm and deny the same proposition simultaneously without causing too much of a scandal.

On close inspection, short stories also operate and proceed according to this design. In effect, every short story—just like a horror film—begins with the illusion of normalcy, in the realm, let us say, of common sense. But from the very beginning, by definition, this state of affairs is furtively threatened, in a tacit pact between author and reader, by the expectation that "something is going to happen." The first bits of information, which might appear to be casual, are accepted within this context of normalcy. That is, at the beginning of the story, fictional logic coincides with (or rather, conceals itself beneath) the usual logic of common sense.

In our scheme, the initial assumptions are these first bits of information that are laid out like chess pieces on a board at the beginning of a game. But of course these initial data, which might seem more or less interchangeable or random to the reader, are not arbitrary for the author. What is contingent in the initial logic is necessary in the logic of fiction: the author needs it somehow for a second order that, momentarily, only he knows. This second order is ruled by a different logic, and the whole magic act—the short story writer's sleight-of-hand—consists of the transmutation of the original logic of "normalcy" into this second, fictional logic, which gradually takes over and from which the ending is deduced (if things work out well) as inevitable rather than startling. In this way, Piglia's idea of two tales can be replaced by the less restrictive, and therefore more general, notion of two possible logical orders, or more precisely, a single logic that splits in two in the course of the narrative.

Up to now I have spoken of the writer as a more or less astute manipulator of logical systems, but the writer is also—sometimes—an artist. Returning to the image of the illusionist, not long ago I saw a television show featuring an old, one-handed Argentine magician doing card tricks in Las Vegas. He was sitting at a table with his single, bare hand lying on the card table, completely surrounded by people watching his routine from all angles. The demonstration was simple. He dealt six cards onto the table face up, one at a time, alternating colors: red, black, red, black, red, black. He picked them up in the same order, and when he dealt them again, the colors had grouped together: red, red, red; black, black, black. "It can't be done more slowly," he then said. "Or maybe . . . just maybe it can be done more slowly." Then he dealt the cards again, unhurriedly: red, black, red, black, red, black. And again he smiled to himself and repeated the phrase: "It can't be done more slowly . . . or maybe, just maybe, it can be done more slowly." This would be the artist among writers, an illusionist with just one hand who can always say, with all eyes upon him: Maybe, just maybe, it can be done more slowly.

Note

1. Published in *V de Vian,* no. 20 (Feb. 1998) and in *Vox* (1998).

2. In *Crítica y ficción* (Fausto 1993).

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5 A MARGIN TOO NARROW[1]

A man leans over a book at night. He is a high-ranking official in the court system in seventeenth-century France who filters petitions to the king and can send the accused to the bonfires of the Inquisition. His name is Pierre Fermat. Due to the gravity of his role and so as to avoid bribes or favoritism, he is not permitted a social life, but this proscription, far from being disturbing, allows him to devote himself to a secret passion for numbers. He spends his nights making notations in the margins of his copy of Diophantus' *Arithmetica.*

On one of the pages appears Pythagoras' age-old equation for right triangles, which establishes that the square of the hypotenuse is equal to the sum of the squares of the other two sides. In this volume there also appears the method for finding right triangles whose three sides are all integers. The triad 3, 4, 5 is just the first in an infinite series of such integral solutions, zealously guarded by the Pythagorean brotherhood. Fermat wonders if these integral solutions could still be found if the exponent 2 in the equation were replaced by a larger number. Around 1667, on another such arduous night, he jots down his negative conclusion in the page margin: "It is impossible, if *n* is greater than two, to find an integral solution to the equation $z^n = x^n + y^n$."

Thereupon he adds a comment that will change the history of mathematics: "I have found a truly remarkable proof of this fact, but this margin is too narrow to contain it."

Fermat died thirty days later, and his son, who foresaw the importance of those nighttime labors, published the *Arithmetica* with all the notations. Mathematicians at the time were in possession of many propositions and conjectures, but they rarely found the hints to prove them. Throughout his life, Fermat—"that braggart," "that damned Frenchman"—preferred to keep the proofs to himself. Instead, he enjoyed writing letters to English mathematicians, defying them to rework the proofs. Even so, using the elementary techniques of that time, one by one Fermat's affirmations were proved to be true. But Pythagoras' generalized equation, like a final challenge, resisted all efforts, and no one was able to reconstruct the "truly remarkable" proof announced by Fermat. Leonhard Euler, the greatest mathematical genius of the century, barely managed to prove the case $n = 3$ and desperately begged Fermat's son to search for another clue among his father's papers.

From generation to generation, employing ever more painstaking efforts and sophisticated techniques, more particular cases were proved, but the demonstration of the general case seemed only to grow more distant with each new attempt. In order to understand this, one must remember that mathematical reasoning differs from that of other scientists. Ian Stewart tells a familiar story about an engineer, a physicist, and a mathematician who, on entering Scotland by train, see a black sheep in the middle of a field. "How strange," the engineer observes, "in Scotland sheep are black." "No," retorts the physicist, "in Scotland *some* sheep are black." "No, no," the mathematician patiently corrects, "in Scotland there is at least one field that has at least one sheep *whose only visible side from the train* is black."

Mathematicians are indeed cautious in their affirmations, and any given number of particular cases to support a conjecture is not enough to establish a general proof. To complicate matters even more, the particular cases that were resolved showed how enormously complex any global proof would have to be, a fact that planted a seed of doubt as to whether Fermat really had found a "remarkable," relatively concise proof. In 1847, in the middle of a battle between Augustin-Louis Cauchy and Gabriel Lamé, both of whom thought they had arrived at a solution, a seminal work by Ernst Kummer revealed that Fermat's last theorem was hopelessly beyond the reach of all known lines of attack, and that in any case an essentially new idea, transcending traditional algebra, would be required. Thus, at the beginning of the twentieth century, serious mathematicians had given it up as a lost cause: no one was prepared to devote his career to a problem that had always seemed difficult and that Kummer had again left in the dark. Three hundred years after it was first announced, Fermat's theorem had become an inaccessible myth, the paradigm of what mathematicians consider to be an "intractable" problem. And yet, the most exciting part of the story was yet to come.

With the adroitness of a novelist, Simon Singh—PhD in physics at Imperial College and scientific advisor to the BBC program *Horizon*—has written a fascinating book on one of the greatest achievements in contemporary thought, comparable perhaps only to Einstein's theory of relativity. *Fermat's Enigma* (alternate title: *Fermat's Last Theorem*) is not, as one might fear, a book on mathematics. With compassionate regard for the general reader, yet without sacrificing intellectual rigor, Singh manages to convey the sleepless nights and the maze of passions behind each and every formula. Along the way he touches upon some of the most vivid portraits in the history of mathematics, from the dramatic end of the Pythagorean School to the political and romantic trap that leads Évariste Galois to a fatal duel against France's best sharpshooter; from the male disguise worn by Sophie de Germain in order to gain admission to university to a spy novel by Alan Turing, who breaks the code of the Nazi Enigma machine and dies after the war, persecuted for his homosexuality and poisoned by an apple. Equally excellent is the chapter on the amazement and philosophical crisis produced by Russell's paradox and Gödel's incompleteness theorem.

The Fortunate Suicide

In the main line of the story, at the beginning of the twentieth century there is an unexpected bit of comic relief that brings new life to the problem. Paul Wolfskehl, the son of a family of German industrialists with a huge fortune of his own, was also a devotee of mathematics and one of many who had tried their luck with the theorem. At some point in his youth, he became smitten with a very beautiful woman who rejected him. The broken-hearted young Wolfskehl decided to commit suicide by shooting himself in the head at the stroke of midnight. But, after making all the preparations, since he still had some time left to wait, he reopened his book of mathematics containing Kummer's great calculus, which had stymied all efforts of classical algebra and which seemed to the prospective suicide like an appropriate bit of reading for such a solemn occasion. It seemed to him that he might have found a small defect in one of the implications; he thought Kummer might have made an error, which would reawaken the hope of an elementary proof. He stayed up till dawn making feverish calculations. Kummer, of course, hadn't made a mistake, but by then the designated hour for Wolfskehl's suicide had passed, and he discovered that, unexpectedly, his desire to go on living had returned. He tore up his previous evening's farewell notes and rewrote his will that same day. On his death, his family discovered that he had bequeathed a large portion of his fortune to whoever could publish the first complete demonstration of Fermat's theorem. The prize, which at that time was the equivalent of more than two million dollars, had a one hundred year time limit that would expire in September 2007. Strangely enough, the sum would be awarded only to the person who could prove the theorem true: anyone providing a counter-example wouldn't collect a single pfennig.

The contest, despite the publicity in all mathematical journals and the enormous amount of the prize, didn't generate too much interest among professional mathematicians, who were familiar with the true nature of the equation behind its innocent-looking façade. But it did immediately attract thousands of optimistic amateurs, unwary students, and all sorts of adventurers. Some submitted the first part of a proof, with promises of the second if they could collect a portion of the prize in advance. One person offered a percentage of any future reward in exchange for assistance in finishing the proof and even threatened to send his notes off to a Soviet department of mathematics if no one was willing to collaborate with him. Professor Edmund Landau, one of those who received the avalanche of flawed proofs, discovered that replying to the letters took up all his time and decided to print a terse card: "Dear . . . Thank you very much for your manuscript. The first error can be found on page . . . This invalidates your proof." One of his colleagues chose to return the manuscripts with a note in the margin: "I have a truly remarkable refutation of your proof, but this margin is too narrow to contain it."

Even so, the Wolfskehl competition kept the aura surrounding this enigma alive, and in all books of mathematical puzzles and dilemmas, Fermat's theorem occupies first place. It was because of one of these books, Eric Bell's *The Last Theorem,* that a ten-year-old boy read about the enigma for the first time and quietly developed an obsession to resolve it.

An Arduous Disciple of Pythagoras

Around 1975, that boy, whose name was Andrew Wiles, received his degree from Cambridge and began his postgraduate studies. Although he hadn't given up his childhood obsession, he too now understood the risk involved in devoting himself to a problem that had been shunted aside by mathematics, almost like a historical curiosity, one that might consume his entire career without giving him anything to show for it in return.

His supervisor, John Coates, convinced him to concentrate on a closely related field instead, the so-called *elliptical curves.* Suffice it to say that Fermat's equation can be thought of as a particular case of an elliptical curve. And so Wiles, after earning his doctorate, became another "serious" mathematician, a professor at Princeton, following the usual routine: delivering lectures, supervising students, and regularly publishing scholarly papers. Meanwhile, in postwar Japan, a parallel story was developing. Two young mathematicians, trying to recover the spirit of investigation, noticed that certain intensely studied mathematical objects of that time, known as *modular forms,* gave rise to elliptical curves. These Japanese mathematicians formulated what eventually became known as the Taniyama-Shimura conjecture, which says that *all* modular forms can be associated with an elliptical curve. If this conjecture proved to be true, it would open up the possibility of transferring, through parallelism, results from the modular world to the elliptical world, and vice-versa. This was the kind of essentially innovative approach that nineteenth-century mathematics could not have brought forth: the insight that there are profound connections between diverse areas that have historically been developed as separate entities, using totally different techniques, so that if the proper precautions are taken, the results in one area can be translated and exported into the other.

The Beginning of the End

One afternoon in 1986, while having tea with some colleagues, Wiles learned some news that would change his life. A specialist named Ken Ribet, through this sort of parallelism, had proved that if the Taniyama-Shimura conjecture was correct, Fermat's theorem could also be proved as a corollary. That is to say, whoever could prove the Taniyama-Shimura conjecture would at the same time prove Fermat's last theorem. For Ribet, this fact meant only that he had reached a dead end; his result simply showed that the Japanese conjecture was as difficult (or more difficult) to prove as the most difficult of theorems. But Wiles realized that his moment had come. Instead of devoting himself to a direct proof of Fermat's theorem, he could now concentrate on a problem that was much more appreciated in the academic world. Even if he failed to attain his ultimate goal, all the partial results he might obtain would be publishable. He immediately abandoned all but his most unavoidable duties concerning the supervision of his students. He disappeared from the lecture circuit and locked himself up at home for seven years without telling anyone of his plan, the monumental task of reviewing, one by one, all the methods and historical attempts to prove the theorem. He re-emerged in June 1993, at a Theory of Numbers conference in Cambridge, his native city. All his colleagues suspected that he was about to present important results, especially when he was allotted three lecture slots—an unusually high number. In the first two of these, Wiles did not show his entire hand. Even so, e-mails began circulating furiously all around the world, trying to determine how far he would get in his last lecture. Among those in attendance was Shimura, but not Taniyama; he had committed suicide a few years earlier, without ever seeing the importance his conjecture would attain. In his farewell note, he calmly explained that he could not see a future for himself. An unusual number of curious attendees gathered for the final lecture, attracted by the rumor that something major was about to happen. That day a bookmaker received the same strange bet five times—that a certain, very old theorem would be proved that afternoon—but his intuition led him not to accept the wager. The press wasn't called in, but some mathematicians brought their cameras, just in case. In a charged atmosphere, Wiles developed his demonstration of the Taniyama-Shimura conjecture, a proof that he had created in the utmost secrecy, finally writing on the blackboard the statement of Fermat's theorem, which (as everyone knew) was now automatically proved. "I think I'll stop here," he said. After three hundred and fifty years, Fermat's last enigma had been defeated. *But had it really?*

Another Turn of the Screw

The news made the headlines in all the papers. Wiles' photo at the blackboard traveled around the world. The *New York Times* proclaimed: "At Last! Shout of 'Eureka!' in Age-Old Math Mystery."[2] Meanwhile, Wiles presented the manuscript of his proof, a two-hundred page document, to experts for examination. It was clearly not the same proof that Fermat believed he had found. However, it did represent an astonishing synthesis of mathematical thought over the course of three centuries, an amalgam of old and new ideas, of revived, strengthened techniques together with unpublished inventions: the confirmation that, in mathematics as in literature, all profound work establishes a much more intricate and complex relationship with tradition than the obvious fidelity-betrayal dichotomy.

Even so, during the revision process, as in a horror film, the monster rose up for the last time and threatened to destroy its would-be conqueror. This second, lesser-known ending to the story was an embarrassing secret in the mathematical community for over a year. The description of this tension-filled period is one of the best parts of Singh's book. Suffice it to say that Wiles was finally able to claim the Wolfskehl prize—which, after the devaluation of the German mark during the war, had been reduced to fifty thousand dollars.

Obviously, it wasn't the money that guided Wiles in his thirty-year quest. And it wasn't any subsequent idea of "utility," either. Fermat's theorem, like much of mathematics, has little to do with anything that might be considered "practical" or "useful." What is the real motivation, then, for this fraternity that has never stopped being somewhat clandestine? Perhaps the certainty that its work is the only kind that can endure throughout all time: the confidence that, when the pyramids once more crumble into dust in the desert and men have disappeared, Pythagoras' theorem, and all theorems, will still be true. As G. H. Hardy says in the epigraph chosen by Singh: "'Immortality' may be a silly word, but probably a mathematician has the best chance of whatever it may mean" (6).

Notes

1. Published under the title “La fórmula de la inmortalidad,” *Clarín,* 1 Aug. 1999. Review of *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem,* Simon Singh, New York: Walker and Company, 1997.

2. June 4, 1993.

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6 | EUCLID, OR THE AESTHETICS OF MATHEMATICAL REASONING

At the end of the 1930s, a diminutive man with a fragile demeanor and a broad forehead arrived at the Universidad Nacional del Litoral, persecuted by Mussolini. He was Beppo Levi, among the most important mathematicians of the twentieth century. He had been hired as a researcher at one of the first specialized institutes in Argentina, but, due to a typical Argentine paradox, there was a sudden, devastating intervention, and Levi ended up teaching ordinary classes in mathematical analysis to first-year students. It was also in the city of Rosario in Argentina where his *Leyendo a Euclides* (Reading Euclid) was first published. Nearly fifty years later, a group of his academic disciples issued a new edition of this detective-like incursion into Socratic thought.

In order to understand the importance of this book, it's important to keep in mind that Euclid's geometrical axioms not only were—and still are, to a great extent—the paradigm of the way mathematical reasoning operates, but that they also forged a profound and almost imperative aesthetic for that reasoning, with multiple philosophical implications that endure to this day. That aesthetic is the delicate balance between simplicity and scope, between the minimum number of assumptions and the maximum number of consequences that can be derived from those assumptions.

In effect, the beauty and seductiveness of the Euclidean model lies in that fact that, by using basic concepts such as point, straight line, circle, and only five axioms to connect those concepts in a fairly obvious way, proceeding from theorem to theorem, all of classical geometry can be derived: that is, the sum total of geometry known to humanity until not very long ago, a geometry that Kant believed to be the only one possible. It is the geometry that corresponds to the way in which we see the world, and the one that mapmakers, architects, and surveyors employ for all their daily needs.

This age-old influence of the axiomatic approach to philosophy can be found in Spinoza's *Ethics*, whose subtitle is *Demonstrated in Geometric Order*, as well as in Descartes' search for a truth "beyond all reasonable doubt," one that might serve as first principle and foundation on which to build an impregnable system of thought through purely logical steps. But perhaps the best-known story about Euclidean geometry is the one having to do with the fifth postulate: Given a straight line and a point outside of it, there can be only one straight line parallel to the given line that passes through that point.

Of the five axioms, this last one was the least obvious, even to Euclid himself, and he tries to use it in his proofs only when strictly necessary. For two thousand years it was thought that it might be possible to prove this fifth axiom by using the four previous ones, like just one more theorem. To find that elusive proof became geometricians' primary unresolved problem. At last, in 1926, a Russian student named Nikolai Lobachevsky discovered that it was completely possible to develop a new geometry in which the first four axioms were valid, *but not the fifth.* Later, Hungarian mathematician János Bolyai proved something even more curious: that the new geometry, strange as it might have seemed intuitively, was as legitimate and solid as Euclidean geometry in the sense that if it happened to lead to a logical contradiction, the "fault" of that contradiction could not be attributed to the negation of the fifth postulate, but rather to the four previous ones, which are shared with classical geometry.

German mathematician Carl Friedrich Gauss, who had arrived at the same conclusions on his own, was one of the first to observe that the existence of a non-Euclidean geometry threatened the Kantian idea of an a priori notion of space. This was one of the harshest blows to Kant's philosophy, later compounded by experiments in the geometry of visual perception, also not wholly Euclidean, by German physicist Hermann von Helmholtz.

Hilbert's Program and Incompleteness

Euclid's spirit was revived with special vigor in early 1900 with Hilbert's program for laying the foundations of mathematics. Certain logical paradoxes, as pointed out by Russell in set theory, had caused the venerable edifice of mathematics to creak for the first time, revealing the need to look for principles and corroborative methods that would allow the careful review of each result. The idea behind Hilbert's program was that all mathematics should be endowed with a set of well-determined axioms, like Euclid's five postulates, so that any result mathematicians might declare to be true—through any method whatsoever—could be verified and reproduced by means of those axioms, through a purely mechanical process, in a finite succession of steps. In short, Hilbert was trying to identify the concept of *true* with the concept of *demonstrable.*

In real life we're well aware that these two concepts are not necessarily equivalent. Let's go back to the example we discussed in Chapter One: a crime with just two suspects. Both of them know the truth of their guilt or innocence: *I did it* or *I didn't do it.* Nevertheless, the court system must produce material evidence through other means in order to reach a conclusion, and all too often there are insufficient data for attaining that truth. Furthermore, it's also possible that neither the guilt of one *nor the innocence of the other* can be proved.

In 1930, Kurt Gödel showed—in a dramatic, unexpected coup—that precisely the same thing happens in mathematics. His famous incompleteness theorem toppled Hilbert's program by revealing that even within the limited realm of basic arithmetic—natural numbers, addition, and multiplication—it is impossible to produce a finite number of postulates, in Euclidean fashion, that would allow us to obtain all true assertions in theorem form. That is, arithmetic, unlike classical geometry, cannot be reduced to axiomatic treatment.

Gödel's theorem, too readily adopted as a fetish of postmodernism and Lacanian psychology, should be seen as a result that points out the limitations of formal axiomatic methods, and in general, as a result concerning the limitations of language. From a mathematical point of view, it says that there is more complexity in the world of mathematical objects than can be accounted for by finitary methods of proof. It also says that intelligence and human discernment are irreplaceable: it is impossible to design a computer that can obtain as outputs all true assertions of natural numbers. The special human factor is the ability to interpret and assign meaning.

At the same time, Gödel's result challenges the simplicity-scope aesthetic for the first time, an aesthetic that had been deeply entrenched in mathematical thought ever since Euclid: arithmetic, and many other fragments of mathematics, cannot be rendered axiomatic without losing some of their scope along the way.

Beppo Levi's Book

In an earlier and perhaps lesser-known investigation, French mathematician Henri Poincaré returned to Euclid in order to expose the hidden premises behind the five axioms: for example, the tacit admission that figures cannot be distorted by rotations or transpositions. In a world of fluids, Euclidean geometry would be meaningless. This mathematical way of paying attention to what is unsaid and of questioning what every era converts into an unconscious, automatic truth prefigures what would later become Foucault's archeological techniques in the social sciences.

Leyendo a Euclides belongs more to this second tendency and can be considered a revision made beneath the powerful magnifying glass of the ages in order to understand the *corpus* of geometrical knowledge and the way of reasoning during Euclid's era. In his prologue, Levi states that all his efforts in writing the book would be wasted if he failed to capture the attention of non-mathematical readers. Those readers will have the unique opportunity to relearn geometry at the hands of a truly renowned mathematician (one theorem of analysis bearing his name has already become a classic), while at the same time—as Mario Bunge says at the conclusion—holding an intelligent conversation with the dead, without the intervention of mediums or ouija boards.

What is there after all, one might wonder, behind this aesthetic that spans centuries, behind this desire to capture all the consequences of a system in just a few properties? Perhaps axioms express human boundaries. Human beings have always struggled with their finitude, and in mathematics they sometimes cleverly manage to defeat it: no one can count all numbers, but we know how to write any given one of them, and we can do so using just ten symbols. No one can write the infinite theorems of geometry, but Euclid taught us that with enough patience we might derive any one of them from only five axioms. In other situations, though, no amount of cleverness is sufficient. We humans are limited creatures, but we launch children whose steps we cannot follow, gods that succeed us eternally, and objects whose complexity escapes our grasp.

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7 | SOLUTIONS AND DISILLUSIONMENT[1]

In mathematics there is an elitist moment that corresponds to the correct intuition of the solution to a problem and is reserved for the enlightened few, and a second, genuinely democratic moment when that solution is revealed to one and all through a proof. On closer inspection, a mathematical proof is a succession of small, logical steps, connected to one another so that anyone may examine the links as thoroughly as possible. Ideally, each one of the steps should be so simple that any person possessing even the most basic acquaintance with symbols could check it almost automatically, verifying each connection in a "local" way, just as a computer traces innocent little lines in minuscule squares on a screen without knowing that they will ultimately form a portrait of the Mona Lisa.

This combination of imagination and freedom to conjecture solutions, and of transparency and rigor in proofs, might well be the key to the depths that mathematical thought has reached, as compared to the relatively horizontal accumulation of knowledge found in other disciplines. Nevertheless, the complexity of certain problems and the use of computers can dramatically change the concept of "solution" and the nature of proofs.

One of the most important problems in algebra—how to classify certain mathematical objects known as finite groups—required a Herculean effort involving a team of dozens of mathematicians. It's very likely that only the director was able to perceive the outlines of the larger picture in the puzzle being assembled: no single mathematician trying to convince himself could have reproduced all the details in a human lifetime. For many years Russian mathematicians in the former Soviet Union would put an asterisk of warning on their work whenever they found themselves obliged to use this theorem. They considered it to be more an act of faith on the part of their Western colleagues than an admissible piece of mathematical reasoning. Similarly, it is interesting to note the shiver of anxiety produced throughout the mathematical world when Andrew Wiles announced his solution to Fermat's last conjecture, an open wound for over three centuries. His original proof contained an error that only three or four specialists could detect; they were the same three of four specialists that certify that the error has now been corrected. I don't mean to suggest that there's any doubt that the theorem has been proved at last. But the proof covers one hundred pages that refer to one hundred algebra books and three centuries of the history of mathematics. This naturally alters the democratic character of the proof. If Fermat could come back to life, he would surely protest. He believed he had a brief, basic, admirable argument—a good, old-fashioned proof.

Things can get worse when computers come into play. One of the most famous problems in mathematics is that of the four colors: given a map of any arbitrary countries, what is the minimum number of colors necessary to paint the map so that neighboring countries have different shades? It was known that five colors were sufficient and three were not enough. For many years, people tried to prove that the minimum number was four. Finally, a "demonstration" was produced: it's a book of programs that, once run, exhaust thousands of ramifications of a classification that is as detailed as it is discouraging. No mathematician would be willing to accept something like that as a demonstration strictly from the standpoint of aesthetics or mathematical need. It wins, but it doesn't convince, just like Deep Blue, the computer that was able to defeat Garry Kasparov at chess, but which didn't really play the same game. Without a doubt, an acute *aesthetic* problem emerges into focus here.

I've read that in the United States they're offering a million dollars to anyone that can solve any of the seven pending mathematical problems. Perhaps they should add that the solution must be verifiable in human time. Deep Thought, the supercomputer imagined by Douglas Adams in *The Hitchhiker's Guide to the Galaxy*, completes its calculations and prints the final answer, "42," in a future so distant that no one can remember the question.

Note

1. Published in "Radar," *Página/12*, 20 Jan. 2002.

8 | THE PYTHAGOREAN TWINS

In May 2003 I had the opportunity to review Oliver Sacks' *The Man Who Mistook His Wife for a Hat* for the Argentine newspaper *La Nación.* Among this extraordinary collection of clinical tales, one of the most astonishing for any mathematician is "The Twins," which reveals an unexpected source of "biological," or more precisely, "neurophysiological" evidence for the formulation of a critical, still-unanswered question in the history of mathematics about prime numbers.

Sacks relates that "The Twins . . . had been variously diagnosed as autistic, psychotic, or profoundly retarded" (195). In 1966, when Sacks began observing them, most of the reports concluded, as is often the case with "idiot savants," that there was nothing special about them "except for their remarkable 'documentary' memories of the tiniest visual details of their own existence, and their use of an unconscious, calendrical algorithm that enabled them to say at once on what day of the week a date far in the past or future would fall" (195). This ability, incidentally, earned them some television appearances.

"The reality," says Sacks, "is far stranger, far more complex . . . than any of these studies suggests" (196). Following is part of his description, written from a naturalist's perspective:

> The twins say, 'Give us a date—any time in the last or next forty thousand years'. You give them a date, and, almost instantly, they tell you what day of the week it would be. . . . One may observe, though this is not usually mentioned in the reports, that their eyes move and fix in a peculiar way as they do this—as if they were unrolling, or scrutinising, an inner landscape, a mental calendar. They have the look, of 'seeing', of intense visualisation, although it has been concluded that what is involved is pure calculation. . . .
>
> Their memory for digits is remarkable—and possibly unlimited. They will repeat a number of three digits, of thirty digits, of three hundred digits, with equal ease. This too has been attributed to a 'method'.
>
> But when one comes to test their ability to calculate—the typical forte of arithmetical prodigies and 'mental calculators'—they do astonishingly badly, as badly as their IQs of sixty might lead one to think. *They cannot do simple addition or subtraction with any accuracy, and cannot even comprehend what multiplication or division means.*

Sacks again emphasizes the extent of the twins' memory:

> [If] you ask them how they can hold so much in their minds—a three-hundred-figure digit, or the trillion events of four decades—they say, very simply, 'We see it'. And 'seeing'—'visualising'—of extraordinary intensity, limitless range, and perfect fidelity, seems to be the key to this. It seems a native physiological capacity of their minds, in a way which has some analogies to that by which A. R. Luria's famous patient, described in *The Mind of a Mnemonist,* 'saw'. . . . But there is no doubt, in my mind at least, that there is available to the twins a prodigious panorama, a sort of landscape or physiognomy, of all they have ever heard, or seen, or thought or done, and that in the blink of an eye, externally obvious as a brief rolling and fixation of the eyes, they are able (with

> the 'mind's eye') to retrieve and 'see' nearly anything that lies in this vast landscape.
>
> Such powers of memory are most uncommon, but they are hardly unique. We know little or nothing about why the twins or anyone else have them. Is there then anything in the twins that is of deeper interest. . . ? (198-99)

At this point Sacks describes his first contact with the twins' "natural" powers.

> A box of matches on their table fell, and discharged its contents on the floor. '111', they both cried simultaneously, and then, in a murmur, John said '37'. Michael repeated this, John said it a third time and stopped. I counted the matches it took me some time—and there were 111.
>
> 'How could you count the matches so quickly?' I asked. 'We didn't count', they said. 'We *saw* the 111. . . .'
>
> 'And why did you murmur "37", and repeat it three times?' I asked the twins. They said in unison, '37, 37, 37, 111'.
>
> . . . That they should *see* 111—'111-ness'—in a flash was extraordinary, but perhaps no more extraordinary than Oakley's 'G sharp'—a sort of 'absolute pitch', so to speak, for numbers. But they had then gone on to 'factor' the number 111—without having any method, without even 'knowing' (in the ordinary way) what factors meant. . . .
>
> 'How did you work that out?' I said, rather hotly. They indicated, as best they could . . . that they did not 'work it out', but just 'saw' it, in a flash . . . or that it 'came apart' of its own accord, into these three equal parts, by a sort of spontaneous, numerical 'fission'. They seemed surprised at my surprise—as if *I* were somehow blind. . . . Is it possible, I said to myself, that they can somehow 'see' the properties, not in a conceptual, abstract way, but as *qualities,* felt, sensuous, in some immediate, concrete way? . . . If they could see '111-ness' at a glance (if they could see an entire 'con-

> stellation' of numbers), might they not also 'see', at a glance—see, recognise, relate and compare, in an entirely sensual and non-intellectual way—enormously complex formations and constellations of numbers? . . . I thought of Borges's 'Funes': 'We, at one glance, can perceive three glasses on a table: Funes, all the leaves and tendrils and fruit that make up a grape vine . . . A circle drawn on a blackboard, a right triangle, a lozenge—all these are forms we can fully and intuitively grasp; Ireneo could do the same with the stormy mane of a stallion, with a herd of cattle on a hill . . . I don't know how many stars he could see in the sky'.
>
> Could the twins . . . perhaps see in their minds a numerical 'vine', with all the number-leaves, number-tendrils, number-fruit, that made it up? (199-201)

Sacks describes as follows a second revealing encounter that he accidentally witnessed:

> [T]hey were seated in a corner together, with a mysterious, secret smile on their faces, a smile I had never seen before, enjoying the strange pleasure and peace they now seemed to have. I crept up quietly, so as not to disturb them. They seemed to be locked in a singular, purely numerical converse. John would say a number—a six-figure number. Michael would catch the number, nod, smile and seem to savour it. Then he, in turn, would say another six-figure number, and now it was John who received, and appreciated it richly. They looked, at first, like two connoisseurs wine-tasting, sharing rare tastes, rare appreciations. . . .
>
> It was perhaps a sort of game, but it had a gravity and an intensity, a sort of serene and meditative and almost holy intensity. . . . I contented myself with noting down the numbers they uttered—the numbers that manifestly gave light, and which they 'contemplated', savoured, shared, in communion. . . .

When he returned home, Sacks realized that those numbers were, in fact, prime numbers, and at their next meeting, he brought along a book with a list of large prime numbers:

> I again found them closeted in their numerical communion, but this time, without saying anything, I quietly joined them. They were taken aback at first, but when I made no interruption, they resumed their 'game' of six-figure primes. After a few minutes I decided to join in, and ventured a number, an eight-figure prime. . . . There was a long pause—the longest I had ever known them to make, it must have lasted a half-minute or more—and then suddenly, simultaneously, they both broke into smiles.
>
> They had, after some unimaginable internal process of testing, suddenly seen my own eight-digit number as a prime. . . . *There is no simple method, for primes of this order—and yet the twins were doing it*. (201-04)

Finally, Sacks concludes:

> I believe the twins, who have an extraordinary 'feeling' for numbers . . . actually feel them, in themselves, as 'forms', as 'tones', like the multitudinous forms that compose nature itself. They are not calculators, and their numeracy is 'iconic'. They summon up, they dwell among, strange scenes of numbers; they wander freely in great landscapes of numbers. . . . The twins, though morons, hear the world symphony . . . but hear it entirely in the form of numbers. . . .
>
> [They] . . . have not just a strange 'faculty'—but a sensibility, a harmonic sensibility, perhaps allied to that of music. One might speak of it . . . as a 'Pythagorean' sensibility—and what is odd is not its existence, but that it is apparently so rare. . . . Mathematics has always been called the 'queen of sciences', and mathematicians have always felt . . . the world as organised, mysteriously, by the power of number.
>
> The twins live exclusively in a thought-world of numbers. . . . And yet numbers for them, I believe, are not 'just' numbers, but significances, signifiers whose 'significand' is the world.
>
> They do not approach numbers lightly, as most calculators do . . . They are, rather, serene contemplators of number—and approach

> numbers with a sense of reverence and awe. Numbers for them are holy, fraught with significance. This is their way . . . of apprehending the First Composer. (206-08)

Then, as a postscript, he adds mathematician Israel Rosenfield's reaction on reading his manuscript:

> "Their ability to determine the days of the week within an eighty-thousand-year period suggests a rather simple algorithm. One divides the total number of days between 'now' and 'then' by seven. If there is no remainder, then that date falls on the same day as 'now'; if the remainder is one, then that date is one day later; and so on. Notice that modular arithmetic is cyclic: it consists of repetitive patterns. Perhaps the twins were visualising these patterns, either in the form of easily constructed charts, or some kind of 'landscape' like the spiral of integers shown on page 30 of [Ian] Stewart's book [*Concepts of Modern Mathematics*].
>
> "This leaves unanswered why the twins communicate in primes. But calendar arithmetic requires the prime of seven. And if one is thinking of modular arithmetic in general, modular division will produce neat cyclic patterns *only* if one uses prime numbers. Since the prime number seven helps the twins to retrieve dates, and consequently the events of particular days in their lives, other primes, they may have found, produced similar patterns to those that are so important for their acts of recollection. . . . In fact, only the prime patterns could be 'visualised'. . . . In short, modular arithmetic may help them to retrieve their past, and consequently the patterns created in using in these calculations (which only occur with primes) may take on a particular significance for the twins." (210-11)

The foregoing all comes from Sacks' book. The possibility that prime numbers might be "seen" directly, like landscapes or particularly pleasing geometrical forms, and the mention of the spiral of integers in Ian's Stewart's book reminded me of a classic biology text that I consulted recently, D'Arcy Wentworth Thompson's *On Growth and Form*, which resurrects the Pythagorean (and even before that, the Egyptian) idea of "gnomons" to explain the spiral growth pattern of snail shells, horns, and so on. Thompson recalls the notion of the "gnomon" with some numerical and geometrical examples:

> Thus if we add to a square an L-shaped portion, shaped like a carpenter's square, the resulting figure is still a square and the portion which we have so added, with this singular result, is called in Greek a 'gnomon' . . . Euclid extends the term to include the cases of any parallelogram . . . and Hero of Alexandria specifically defines a gnomon . . . as any figure which, being added to any figure whatsoever, leaves the resultant figure similar to the original. Included in this important definition is the case of numbers, considered geometrically . . . which can be translated into *form*, by means of rows of dots or other signs . . . or in the pattern of a tiled floor; all according to 'the mystical way of Pythagoras, and the secret magick [*sic*] of numbers.' (181-82)

Thompson goes on to say:

> There are other gnomonic figures more curious still. For example, if we make a rectangle such that the two sides are in the ratio of 1:√2**,** it is obvious that, on doubling it, we obtain a similar figure; for 1:√2::√2:2; and each half of the figure, accordingly, is now a gnomon to the other. . . . For another elegant example, let us start with a rectangle whose sides are in the proportion of the 'divine' or 'golden section', that is to say as 1:½ (√5-1), or, approximately, as 1:0.618. . . . The gnomon to this rectangle is the square erected on its longer side, and so on successively. (182)

Thompson uses the concept of gnomon in his description of the chambered nautilus and other related organic forms in the explanation of his law of growth:

> [I]t is characteristic of the growth of the horn, of the shell, and of all other organic forms in which an equiangular spiral can be recognised, that *each successive increment of growth is similar, and similarly magnified, and similarly situated to its predecessor, and is in consequence a gnomon to the entire pre-existing structure*

> We see that the successive chambers of a spiral *Nautilus* . . . each new increment of the operculum of a gastropod, each additional increment of an elephant's tusk . . . has its leading characteristic at once described and its form so far explained by the simple statement that it constitutes a *gnomon* to the whole previously existing structure. (184-86)

So much for Thompson.

Just as the Pythagoreans geometrically conceived of "triangular" and "rectangular" numbers, the most immediate question is what kind of "especially pleasing" visual form might be associated with prime numbers. But it's also possible that in the twins' process of recognizing a number as prime, there might be a "gnomonic" principle at work, having to do with the way in which numerical concepts are naturally, "biologically" registered (or inscribed) in the brain. The gnomons (with regard to sums) of the first prime numbers, for example, are listed in Wacław Sierpiński's *Elementary Theory of Numbers* (115) as the table of differences between successive prime numbers. This first "biological" hypothesis, in which prime numbers are somehow pre-inscribed in the brain's right hemisphere and can be "read" visually, appears to be compatible with the explanations Sacks provides of the different specializations of the hemispheres. The twins, despite having serious defects in the logical and algorithmic functions corresponding to the left hemisphere, nonetheless might still be able to access those visual forms of memory corresponding to the right hemisphere.

Perhaps the most interesting (and maddening) aspect of this biological phenomenon is that there appears to be no way to ask them about it because they cannot provide "reasons," nor do they understand what division and multiplication are. But then, is our only recourse simply to observe them as if they were incomprehensible, natural prodigies? What is the "intelligible" key—if indeed it exists—the aesthetic pattern that remains invisible to us, but evident to them, to the "visual" recognition of prime numbers?

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9 | THE MUSIC OF CHANCE (INTERVIEW WITH GREGORY CHAITIN)[1]

Gregory Chaitin is an extraordinary mathematician. He spent half his youth in Manhattan and the other half in Buenos Aires. In 1957, when the Russians succeeded for the first time in placing a satellite in space, the North Americans, alarmed, created a series of advanced courses for students who were interested in science. So it was that at age twelve, and despite the fact that his father is a playwright, Chaitin began to study quantum physics and the theory of relativity at Columbia University. At fifteen he discovered a variation of Gödel's theorem that allowed him to define the idea of chance in computational terms. In his recent book, *The Limits of Mathematics,* he shows that there are fragments of arithmetic that are impenetrable to thought, and that God plays dice, not only with physics, but also with mathematical reasoning. In this interview at the Café Tortoni in Buenos Aires, he quickly recovers his Spanish and discusses the future of scientific thought, artificial intelligence, the new generation of computers, and the Machiavellian machine that defeated Garry Kasparov.

What brought your parents to Argentina?

My parents were actually born here. They were the children of immigrants from Eastern Europe who decided to go to the United States after the Second World War. When they returned to Buenos Aires in 1966, I became involved in a variety of things: I joined the IBM labs and also taught courses at the Facultad de Ciencias Exactas, the only time in my life that I've taught "normal" courses, with a final exam, etc. The atmosphere was electric; there were some very capable people there. It's a great pleasure to teach when students are interested.

What were your first research interests?

From a very early age, I was interested in the theory of relativity, quantum physics, and cosmology. But in order to understand physics, you have to learn some mathematics, and I stuck with mathematics from then on. I tried to understand what I considered to be the most profound problem: the question of the limits of mathematical reasoning, Gödel's theorem. It was something very mysterious to me; I suspected that it was as profound a subject as the theory of relativity. At age fifteen I formed the key idea that governs all my research. That is, for thirty-five years I've been devoted to a single idea: to define a measure of the complexity of information.

Can you explain it in simple terms?

The key idea in all my efforts is to measure the minimum number of words required for defining something, but this is an ambiguous quantity; it varies from one language to another. So the next step was to formulate a precise mathematical notion in an artificial language. And to accomplish that I used computers.

Was your initial objective to find another proof for Gödel's theorem?

No. My original intention was to define the idea of chance through this new notion of complexity. That is, to come up with a "computational" definition of chance. A number is random if the information about its digits cannot be compressed using a smaller program. For example, the number formed by a million nines is a very large number, but its description is very short. It's what we call compressible information: the digits of this number behave regularly; they can be grasped by that program. On the other hand, if the most concise description of a number is to give all its digits, this means that the number has no regularity, no pattern; there's no way a clever gambler can develop a winning strategy by betting on its digits. One of the paradoxes resulting from this definition is that the vast majority of numbers are random, but there is no way to give a mathematical demonstration to prove that a particular number is random! Here we have a mathematical fact with a very high probability of being true, and even so, we can never be absolutely sure. This is the fundamental paradox of my focus on the limits of mathematics.

Looking for Gödel

Did you already know this when you tried to speak to Gödel?

Yes, that was the novelty of it, the new approach that I had. As you can imagine, Gödel was my hero, and I wanted to get his reaction to this idea. So I phoned him.

Was he at Princeton at that time?

Yes. And the only person he spoke to was Einstein. I was very young, half the age I am now, and I had no references. I called him on the phone and said, "Look, I have this new approach, and I'd very much like to discuss it with you." Amazingly, he didn't hang up on me. Instead he replied, "All right, send me some of your work with this information, then call me again and we'll see if I'll give you an interview." I sent him my work, and when I called him back, he granted me the interview! It was a glorious moment for me: I was a visiting researcher at the Watson Laboratory, and I started looking at a map to try to figure out the best way to get to Princeton by train. I was in my office, about to leave, when the phone rang and a voice (a horrid voice), introducing itself as Gödel's secretary, said that it had started to snow in Princeton, and as Gödel was in fragile health, he preferred to postpone the interview. It was already springtime; normally it wouldn't have been snowing. But so it was, and my appointment was canceled. I had to return to Argentina the following weekend, and I suspected I'd never have another opportunity. That proved to be true, because Gödel died soon after.

Einstein was horrified by a notion of intrinsic chance behind physical phenomena.

Chance is a fundamental, but very controversial, notion in twentieth-century physics. Why did Einstein say that God doesn't play dice with the universe? Because in subatomic physics we lose the possibility of determining the future in an unambiguous way.

Fundamental laws are statistics. And Einstein was frightened by something like that; he was trained in the classical, Newtonian school.

He believed in hidden variables.

Exactly. He thought there had to be hidden variables and that when they were discovered, the element of chance would disappear and the behavior of particles could be predicted exactly. However, physicists today believe that chance is structural. I've followed the whole Einstein-Bohr polemic on quantum physics. Einstein was one of the founders of quantum physics, but he didn't believe in chance: he rejected it, which nearly reduced Bohr to tears, because he considered Einstein his hero, his master. However, Bohr was convinced that chance plays a fundamental role. I was studying Gödel's results and some mathematical problems that have remained unsolved to this day. And I began thinking: Couldn't it be that chance itself, the lack of structure or laws that prevails in basic physics, might also be found in pure mathematics? Everything I've done, really, could be said to derive from these ideas from physics. And physicists feel more comfortable with my results than mathematicians do.

It's because you proved something that is very alien to mathematical intuition and practice: that there are results in arithmetic that are true, not for any particular reason, but rather through pure chance.

Yes, in particular I was able to define a number with a very curious property: it's perfectly defined as a mathematical object, but its digits cannot be determined. Each one of these digits has to be a number between 0 and 9, but it's impossible to know which one. Mathematical custom dictates that if something is true, it's true for a reason. The mathematician's task is to figure out that reason and turn it into a proof. But it turns out that the digits of this number are so delicately balanced that they're impenetrable to any kind of reasoning. Mathematicians loathe this: anything that escapes reason is terrible; it's dangerous. It frightens mathematicians.

Questions for God

This number that you've defined has been called the "number of wisdom."

It turns out that this number codifies a great deal of information, compressed in an extreme way. If we knew the first hundred digits, we'd know many, many things. I'd be able to resolve a bunch of mathematical hypotheses. Let's put it this way: If a mathematician could ask God one hundred questions, the most effective way to utilize those questions would be to ask him for the first hundred digits of this number. Some people are interested in this number in a mystical way. It excites their imagination. The fact that this number escapes reason makes them imbue it with mystical powers. But I'm not a mystic; I'm a rational man who wants to follow a tradition that comes to us from ancient Greece. And yet there's something paradoxical about it, because by reasoning like a mathematician, I reach the limits of comprehension. From a philosophical point of view, I'm in a very uncomfortable position. I love mathematics, but I see that there are limits to what mathematical thought can achieve. And this is sometimes hard to tolerate: it plants seeds of doubt about what I've done for my entire life. Because if mathematics is nothing more than a game we invent, then I've wasted my life. There's a personal paradox that emerges when working beyond limits. From a psychological point of view it's pretty . . . delicate.

In any case, within the context of contemporary mathematical practice, there are probably few results that are subject to chance.

Yes, my results have no impact on everyday mathematics. But in certain fields they're conceptually important and should be considered. Some mathematicians are even developing an innovative, almost empirical way of doing mathematics, working as physicists would, by adding hypotheses for which there is plenty of evidence, but no absolute certainty. This is due to the possibility of experimenting on a large scale with computers.

Truth and Real Life

What were you thinking during those ten years when you already had formed the idea of your notion of complexity but hadn't managed to find the exact formulation?

What happens is that we mathematicians are sort of artists, I think. Pure mathematics is really an art, and I have an aesthetic sense. How do you know if a definition is correct? A concept is good if the resulting theorems are beautiful and natural. You have to get the concepts to combine and work harmoniously together. When I began working on my theory, I tried out a first definition that made the work easier, but I felt I had lost something with regard to other definitions that I had already considered and which caused me technical difficulties. During my first visit to the Watson Laboratory in the United States, I concentrated only on that. And then I realized that it was possible to get everything to fall into place as if it were preordained. In mathematics there's a certain amount of freedom to change the rules of the game if the game isn't going well. Now 99 percent of my theory is working better, but there's a small percentage that was hopelessly lost.

In the epigraph to your book you say, "He thought he possessed the Truth." How did you feel when you managed to prove the first important theorem?

On the one hand, in normal life we know that the truth doesn't exist. Everything is very complicated. We have to look at things from many perspectives. In mathematics we used to think that we could all agree, that mathematics was different from normal life in that sense. But Gödel and Turing's theorems, and my own result, show that it's impossible to possess the whole truth. What *is* certain is that in the course of doing research there is a moment of ecstasy, of euphoria. Because research is really arduous: most of the time you're struggling and everything seems ugly; nothing goes right; ideas contradict one another; and you feel like you're wasting time on it. But then there comes a moment when you see the light and you realize what the correct insight is.

Can you describe such a moment?

Once I was climbing a mountain in northern New York State. I was hiking in the rain with a group of friends. We were slogging through mud all the way up. But when we reached the summit, the mountaintop pierced the cloud layer, the sun shone brilliantly, and you could see the white surface of the clouds and, in the distance, other peaks emerging. It's the same feeling of euphoria you get when, after many years of struggling with your own ignorance, you suddenly understand how to look at something. Everything becomes more beautiful, and you have the feeling you can see farther than before. It's a glorious moment, but you pay a great price for it, which is your obsession with the problem, like a constant wound or a pebble in your shoe. I wouldn't recommend that sort of life to anyone. Einstein had a close friend, Michele Besso, with whom he discussed many details of the theory of relativity. But Besso himself never accomplished anything important in science. His wife once asked Einstein why, if in fact her husband was so gifted. "Because he's a good person!" Einstein replied. And I think it's true: You have to be a fanatic, and that ruins your life and the lives of those who are close to you.

What is your relationship with real life? Do you read newspapers, for example?

Well, when I was young I liked to go backpacking, rowing on the Tigre River, and running after pretty girls in Buenos Aires, and I would laugh at those eccentric images people have of mathematicians. But with the passage of time, God has taken his revenge: I'm shocked to look in the mirror and discover that I've become that stereotypical mathematician that I used to think of as a joke! But the truth is that in order to work on these topics I've really isolated myself from the world. I live in a house in the country, half an hour by car from the nearest café. Now that I'm back in Buenos Aires, I realize how much I miss it. This is wonderful: the people in the streets, the cafés. I live near New York City, which isn't as beautiful as Buenos Aires but is a great city anyway, and I hardly ever go there. I'd rather go hiking in the hills, the countryside—well, that's the sort of life I lead these days.

In Vienna you visited places where Gödel had been. What was he like as a young man?

From photos we have an image of Gödel as an extremely skinny, very serious man who wasn't interested in the real world. But when he was young, he spent all his time in Viennese nightclubs. That's where he met his wife, who was a dancer.

That sort of nightlife was normal for the children of wealthy families, like Gödel. What wasn't normal was that he also liked mathematics! A friend told me that one day in Princeton he saw Gödel coming down the street toward him, and he thought about stopping him to introduce himself and shake his hand. But at that moment, a young, very pretty student passed by on the opposite sidewalk. She was scantily dressed because it was summer. It seems that, just as my friend was about to extend his hand, Gödel focused his attention on that girl, and he didn't dare interrupt him. This proves that Gödel was no mathematical saint, and that's just fine. After all, we're flesh-and-blood men, aren't we?

Supercomputers

What is that idea behind the new generation of supercomputers that are being envisioned?

Well, they offer the very interesting technological possibility of taking advantage of subatomic phenomena: quantum parallelism. It so happens that a subatomic physical system simultaneously fulfills all possible scenarios. As if we were to say: my plane arrived six hours late, but at the same time, I arrived on schedule, and at the same time the plane exploded in midflight. The final result in quantum physics, what is measured, is a kind of average sum covering all the possibilities; all paths must be taken into account, all intersections and interferences. Once it was thought that this was paradoxical, but now there is a new generation of young people that grew up thinking in this way. They've passed over the hump and somehow find it natural. Instead of fighting against these concepts, they think of ways to take advantage of this subatomic madness: how to take this crazy behavior to its limits and bring it to the surface, and how to turn this parallelism into a computer capable of doing millions of parallel computations simultaneously. Just one of these processors would replace a million computers running at the same time. What I find especially interesting is this idea of forcing the subatomic world to reveal itself, and to reveal itself as quantum to the extreme. Something like thinking: if that's the way the world is, let's exaggerate it!

The New Golem

Where do you stand in the polemic about the possibility of creating artificial intelligence?

I'm glad you asked. I think that artificial intelligence is already being achieved, only we don't yet realize it. It used to be generally thought that artificial intelligence had to resemble human intelligence. There's not much development in that direction: it's very, very hard to speak, to understand a natural language, to recognize faces, to walk. All those things that are simple for humans turn out to be complex for computers. But computers are very good at tasks that we find difficult: symbolic calculations, for example. There's a program by Stephen Wolfram, called Mathematics, that I'd say really possesses artificial intelligence. It's not human intelligence, but it can help me a great deal in my research.

Also in chess: my laboratory worked on the supercomputer that defeated Kasparov, but, again, it wasn't done in human style, but rather through brute force, with a large-scale engineering project. They didn't simulate the way a chess player thinks. Instead, hundreds of very fast, interconnected machines, known as massively parallel computers, were used.

I was actually referring to Roger Penrose's central argument against the possibility of artificial intelligence: the impossibility of the computer to be self-reflective.

Penrose's book is quite interesting. He did very important work on black holes, and later he was Stephen Hawking's dissertation advisor. But I must say that I totally disagree with the thesis of his book. My personal opinion is that the problem with artificial intelligence is not a mathematical or theoretical problem, but rather one of engineering. I know that this position seems a little strange for a mathematician. However, I think of the human being as a work of engineering, very well adapted for getting along in this world. It often happens that a theory demonstrates that something cannot be achieved in practice. But engineers manage to find a reasonably good solution, or a good approximation thereof, in the majority of cases. I believe human intelligence is something like that. We've already come part of the way, although we don't realize it yet. Within fifty years we'll be very close to having true artificial intelligence, and later people will wonder why it was ever considered so hard to achieve. It's not going to be the result of a mathematical theorem, but rather the product of the work of many engineers, bit by bit, growing. . . . It's a little like what's happening in biology. Biologists say that God is a cobbler.

A cobbler?

Exactly. Human beings weren't designed like works of art. A new patch was added every time there was an emergency. That's how we are. A little outlandish, but we function. I think artificial intelligence will turn out to be something like that.

Like Dolly the sheep?

Yes, like a succession of grafts, a Frankenstein's monster that will gradually become more sophisticated, until one day we'll realize that the monster is already pretty intelligent. So now you see: my point of view here isn't that of a mathematician, but rather that of an engineer.

A New Renaissance

Do you think that the conclusions of your work will inspire some sort of pessimism with regard to science or to reasoning in general?

Some of the things I've said might seem a little pessimistic. I've even been interviewed for a book called *The End of Science.* The man who wrote this book thought that my results supported his thesis that science is coming to an end. But in the interview I emphatically stated that I don't agree at all. I prefer another book, *The New Renaissance* by Douglas Robertson. His thesis is that we're living in a new stage of society and science, due to the incorporation of computers at all levels. According to him, what separates man from animal, in principle, is language. Civilization begins with reading and writing, which allows us to know and remember more things. Then comes the European Renaissance, with the invention of print and the democratization of knowledge (before that, books were objects of luxury, reserved only for bishops and kings). And now we're about to enter the next level, in which the computer will make its real impact felt. The personal computer, the Internet, and the World Wide Web were all necessary. There's still a copyright problem with the Web, but once this is solved, we'll have within our reach, on our screen, the sum total of all global and historic knowledge. The Web will be an immense library, the universal human library. The important thing, according to Robertson, is the quantity of information within the reach of every member of society. With each one of the historical steps (language, writing, printing, the Internet), society grows and disseminates information more effectively. Robertson says that the computer will, besides, cause a conceptual revolution in the way we do science and mathematics. The concept of solution has changed, and the methods are gradually changing as well. Very complex systems can now be studied. Analytical problems have become elementary problems.

And yet, with this new focus something is lost: the idea of elegance, of concision, of mathematical beauty. Ideas that are derived from a human aesthetic . . .

It's true, and the beauty of mathematical reasoning is what I love most. When I was young, I used to say that the beauty of certain proofs could be compared to that of a beautiful woman. Obviously it's not the same thing, but in a certain sense they produce the same powerful emotion. But mathematics is constantly evolving, and I'm afraid the problems that allow for a lovely, concise solution are now like child's play. Of course, this is just my personal opinion, which is very controversial. But since we're at Café Tortoni, I feel very Argentine again and able to talk about everything.

Note

1. Published in "Radar," *Página/12,* 7 June 1998. For more information on Gregory Chaitin, see http://www.cs.auckland.ac.nz/CDMTCS/chaitin.

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10 | LITERATURE AND RATIONALITY[1]

A particularly extremist thesis of our modern age, yet one that is widely accepted and repeated like a bromide of the times, proclaims all philosophical systems ineffective, all great syntheses of thought impossible, and reason's ambition to account for reality unfeasible. It's not hard to imagine why this thesis is so popular: there are too many philosophers; philosophy books are long; thinking is exhausting and causes headaches. And then, of course, in order to read Schopenhauer, we need to go back to Hume and Kant; in order to read Sartre, we must return to Heidegger; and we can't get to Marx without first stopping at Hegel, Ricardo, and Feuerbach. In order to understand Wittgenstein, we need to know logic; to read Vico, history; to tackle Saint Augustine, theology. How tempting it is, then, to succumb to a good argument that none of this is necessary, that all those guys were wrong, and that we can guiltlessly forget about those three or four thousand books.

Instead of a good argument, there is a misleading sleight of hand: the critics' point of departure is that human reason is limited (which, of course, is as certain and as original as saying, for example, that men are mortal, or that no matter how fast you flap your arms, you'll never leave the ground), and from there the entire history of thought comes undone by confusing limitation with impotence.

But limitation, as an exhausted Casanova would protest, has nothing to do with impotence. The error—always the same—lies in considering the domain of the rational in an unfairly narrow way, like a completed, fixed set of logical operations, a sort of definitive syllogistic table: in short, in confusing reason with the package used especially by mathematicians and scientists. But not even in those domains is reason a finished, rigid thing. Thus Lobachevsky, for instance, in refuting Euclid's fifth postulate, not only expanded geometry, but also mathematical reasoning itself, and in contemporary physics, to provide an adequate model for the subatomic world is equivalent to finding a sufficiently elastic logic to explain it.

What is invariably is left out of the picture is that rationality, like any other human faculty, has been developing in human beings over time, in permanent conflict and division, and sometimes even in paradoxical alliances, with irrationality. Nietzsche's page on the formation of logic in the human mind as the result of the brutal suppression of nuance, of primitive simplifications and instinctive equating, necessary for survival but fatally "illogical," allows a momentary glimpse of the unsuspected drama behind the *modus ponens*, or traces of barbarity in the remainder theorem. Rationality, then, is a process that proceeds through contradictions, successive approximations, vague limits, and precarious, always provisional theories, in the no-man's-land of reality.

Looking at things for a moment in this light, considering reason as a living, changing faculty, it makes sense to ask ourselves if it might not be possible to reconstruct understanding on the basis of a new, expanded, subtler, and more potent form of rationality, one that escapes Kant and Gödel, one in which philosophical reason, as it has been recognized until now, would be a "limited" and particular case. My novel *Regarding Roderer* hinges on this question, which at heart is the equivalent of wondering about the possibility or impossibility of reestablishing a Promethean vision in these times of Faustian pacts.

Narrative and the Fin de Siècle

This positioning in the face of rationality does not lack consequences in contemporary narrative. Unlike religions, which impassively resist God's silence, thought—far more skittish—flees toward irrationality or discouragement at the first crack in its edifices. The legitimate criticism of nineteenth-century Positivism seems to have brought about, as a strange corollary, the return of witches in the twentieth. Just as from the stagnation of psychoanalysis, self-help manuals and Bach's floral remedies have sprung forth, so too has literature quickly leaped from vast, totalizing efforts to the restrained recipe book of postmodernism.

One automatic response to the lack of confidence in great syntheses is to take refuge in minimalism. This literature of minimalist intentions can be seen in a certain way as the continuation of Hemingway's work, with the difference that it generally doesn't distinguish between the tip of the iceberg and the ice cube in a gin and tonic. Beyond minimalism there are other, much more extensive and recurrent elements that make up an authentic rhetoric of "contemporaneity" and could practically constitute a how-to manual for the modern novel. (It's the old paradox of time: although no one likes to admit it, there is also at this point a classical, traditional way of writing "modern" literature.)

The new rhetoric's point of departure is the skeptical, though not very original, opinion that in literature essentially "everything has been said." From this standpoint—as Thomas Mann understood more than fifty years ago—creativity is condemned to two dead ends: parody and repetition. Today repetition bears the more prestigious label of "intertextuality." Parody tends to be that of genre, with constant appeals to the reader not to be a dolt and to appreciate the author's conspiratorial winks and architectural talents.

There also exists a cliché for character portrayal: the hero must be a skeptic, or better yet, a downright cynic. Nothing unnerves him: he kills with disaffection, shoots heroin with ennui, makes love with a single hand. He's the typical, hardboiled-ironic-nocturnal-marginal—though not necessarily bad—boy of North American noir literature dredged out again and again with the excuse that it's a touch of parody. But if we look carefully at these elements of cynicism, parody, intertexuality, literature about literature, and self-referentiality—what do they have in common? An overriding fear of being caught off guard, the desire to never again be vulnerable. A non-believer can't be accused of being naïve. Someone who stands for nothing can't be contradicted. In the same way, it's impossible to parody a parody, and intertextuality can't be sorted out or remixed. Our own fin-de-siècle, with a "once bitten, twice shy" attitude, seeks refuge in the terminal stages of skepticism. Isn't it touching to be reminded paternalistically by these authors every three pages that what we're reading is "just fiction"? They want to save us (for our own good, no doubt) even from that minimal bit of temporary credulity known as reading. But skepticism as a position is as sterile as it is unassailable, and in the realm of literature—as is plain to see—it quickly leads to dead ends.

At this point, the natural question is if there is another option. It's true, of course, that in literature a great deal has already—and definitively—been said, and for that reason the other option cannot be a state of innocence. Any alternative must depart from the recognition that literature is also a form of knowledge—literary knowledge—and this obliges us to keep in mind a long history of permanent invention, variety, and exhaustion of resources, effects, theories, rhetoric, and genres. But why suppose that this history has come to an end? What is necessary, therefore, is to distinguish among the tide of literary works what has, in effect, "been said" from what still remains to be said. Or, to express this as a program: to write against everything that has been written.

Naturally, "to write against everything that has been written" becomes increasingly difficult over time, not only because of the expanded inventory, the extent of what has been touched, but also because literature's self-awareness has grown keener, so that formal mechanisms and successive rhetorics are quickly worn out. Thus, every new piece of work in our time has to contend with a second demand for originality on a formal level: it must establish its own rhetoric.

This growing difficulty in the field of writing also presents, like a tempting escape route, the seduction of giving in to the idea that "everything has been said." Strangely, through two different routes—one "external" and social, linked to our era and its disillusionments, and the other "interior," related to the intimate history of writing—we arrive at the same crossroads where skepticism and originality intersect.

It is possible that all convictions may also be a kind of ingenuousness, but after all, convictions and a sprinkle of ingenuousness are the ingredients of all the works of humankind.

Skepticism, in times of collapse, can easily pass for intelligence. But the real question of intelligence is how to create again.

Note

1. Published in *La Nación,* 13 Feb. 1994.

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11 | WHO'S AFRAID OF THE BIG BAD ONE?[1]

It's well known that there is only one more effective way to kill conversation in a waiting room than to open a book, and that is to open a book of mathematics. The mere mention of the word "mathematics" induces chills and terror and can reduce the most confident adult to the tremors of division of fractions and other numerological nightmares of childhood. And despite the fact that mathematical thought has left its fingerprints all over the so-called humanities, from the Pythagoreans to the Vienna Circle, from Pascal's theological wager to ethics according to Spinoza's geometrical order, from Descartes' first principles to Gödel's theorem, and despite the fact that mathematics has, throughout history, proven to be an incredibly changeable and Protean science, it's all been in vain. The immense majority of people still confuse it with those rather tedious bits that used to be (and still are, perhaps) dished out in secondary schools.

The Number Devil, by German essayist and poet Hans Enzensberger, bears the subtitle *A Mathematical Adventure,* and, in the Spanish translation, is dedicated to "*todos aquellos que les temen a* *las matemáticas*" ("all those who are afraid of mathematics"). And of course it would be a universal bestseller if it weren't for one small detail. Those who really are afraid of mathematics will never, ever open a book with that word appearing in the title, because they anticipate—and rightly so—what awaits them: that beneath the insidious promise of simplicity, of the rudimentary, lurks a treacherous attempt to teach them some tremendously difficult things. They're simply following the impeccable logic of that child of Simone de Beauvoir who refused to learn the letter "a" because he knew that next would come "b" and "c" and "z" and all of French grammar and literature.

The book's protagonist, Robert, is an eleven-year-old boy who is not very fond of numbers either. His teacher torments him with the rule of three, and at night he has recurrent, monotonous nightmares. In one of these dreams the devil Teploxtal appears to him, sent from the hell-heaven of mathematicians into order to initiate him into that accursed science.

For twelve nights—twelve elementary math lessons—the devil manages to dispel Robert's skepticism and gradually awaken his enthusiasm. So successful are his attempts that Robert keeps pondering mathematical dilemmas throughout his waking hours and even gives up playing soccer with his friends (remember, Robert is a German child who doesn't know that soccer is the only important thing in this world). Toward the end of the book, Robert is rewarded for his efforts with an invitation to dine with a group of immortal mathematicians: Gauss, Klein, Russell, and Fibonacci. During this dinner, they award him the Order of Pythagoras, a magical sorcerer's apprentice medallion, which he takes back to earth with him, allowing him to solve his annoying math teacher's annoying problems with panache.

The Number Devil has several important virtues. The first of these is that Enzensberger, who is not a mathematician and who has written on such diverse subjects as political ecology and the sinking of the *Titanic*, manages to cover the twelve topics quite neatly (along with good counsel), even though they contain some not-at-all trivial subtleties. (There is a serious error in the Spanish edition in the statement of Goldbach's conjecture.) The second point in the book's favor is the selection of these topics, all of them curious, attractive, and successful in bringing the ancient magic of mathematics to the forefront: in particular, the Romans' unfortunate ignorance of the concept of zero; the impact of Fibonacci's succession on the growth of trees and the proliferation of rabbits; triangular numbers; Klein's bottle—whose inside is impossible to distinguish from its outside—the diversity of infinities; and Sierpiński's magic triangle. It also provides a glimpse of what mathematicians really engage in: the solution of open (unsolved) problems, the demonstration of conjectures, and the eternal machinery of formulating more questions, which is the heart of all science. The explanations are very clear and do not require any previous knowledge other than remembering that 1 + 1 = 2 (this is unequivocally the case). The author knows how to stop in time, without trying to include all possible proofs: as in an illusionist's session, what is shown is more important than what is proven. On the other hand, the deliberate, jarring distortion of certain mathematical terms to make them seem more . . . familiar? . . . is questionable. For example, he calls prime numbers "prima donna numbers," square roots "rutabagas," and irrational numbers "unreasonable numbers." Is mathematics more palatable with rutabagas than with roots? Do unreasonable numbers sound less intimidating than irrational numbers?

The weakest part of the book, paradoxically, is the literary aspect. The plot comes across as very flimsy, and the story, in its naïveté and gracelessness, doesn't achieve the same level as the mathematical content. It seems to be designed for much younger children and isn't much more than a precarious wrapping, an excuse for a series of lessons. Thus, what with a little more imagination might have become a work in the manner of *Alice in Wonderland,* with mathematics dramatically integrated into the story, becomes simply a well-chosen, well-explained collection of lessons for beginners. Designed especially for children, it also will appeal to anyone that wants to give mathematics a second chance, with the bonus of pleasant little illustrations by Rotraut Susanne Berner.

Note

1. Originally published in "Radar," *Página/12*, 1998.

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12 | A SMALL, SMALL GOD[1]

How many possible choices did God have in constructing the universe? This question, posed by Einstein, which in other eras might have been of concern to philosophers or theologians, through a paradox of postmodernism is about to be answered by modern physics. The point of departure for this journey to the end of night is a crucial astronomical observation made in 1929: no matter where a telescope is pointed, distant galaxies move away from us. Or, to express it more dramatically: the universe is expanding.

It took physicists several decades to process this news theoretically; the belief in an essentially stationary cosmos was so firm that Einstein himself—in the only error of his career—had introduced, deus ex machina*,* the notion of a "cosmological constant" to keep the universe in balance. And yet it moves. A movement that has profound consequences for ideas about God.

In effect, an immediate deduction says that if the galaxies are drifting apart, in previous eras they must have been closer together. By extending the calculations backward, it was conjectured that at some point all matter in the universe must have been concentrated as in a sinkhole, in a single, infinitesimal point. From there to the big bang theory is just one step. Roger Penrose and his doctoral student Stephen Hawking took that step in 1970 when they demonstrated—under the assumption that the general theory of relativity still prevailed in the sinkhole—that at Moment Zero the universe must have effectively consisted of a point with no dimension and infinite density, what mathematicians call a singularity. Specifically, they also proved that if there had been any events prior to that initial instant, they would not have affected in any way what happens in the present; they would have no observable consequences. Thus, time does not continue indefinitely backward, as Kant believed, but rather, as Saint Augustine had suspected, it is a property inseparable from the universe and has its origins in the big bang.

The theological implication of this first conjecture is somewhat uncomfortable. In a stationary universe there is no physical need for a beginning, and one can imagine that God freely chose the moment of Creation. On the other hand, in an expanding universe, the beginning of time can no longer be chosen arbitrarily. One could still imagine that God created the universe at the moment of the big bang, but it would make no sense to suppose that it had been created before that, and this imposes a precise limit on a Creator.

Even so, the Church enthusiastically approved this first formulation. After all, there was still a little room remaining at the beginning of time to accommodate the fiat of a Creator. But, above all, the fact that the origin of the universe was a singularity left physicists defenseless to continue investigating Moment Zero, simply because when dealing with singularities, all general rules fail. Thus, Genesis remained protected by an aura of mystery that was very convenient for ecclesiastical purposes.

However, they neglected an essential detail: in physics, all theories are provisional, and each new theory is supported only until a new observation or experiment reveals some inconsistency, forcing physicists to correct their formulas or radically change their point of view about some paradigm. The Catholic Church had already committed the error of tying the sacred texts to Ptolemy's cosmological interpretation, fixing a motionless Earth at the center of the universe. That error, which lasted more than four hundred years, earned Galileo his prison sentence.

This time the bad news came more quickly. At a cosmology conference organized at the Vatican by the Jesuits, to which leading experts had been invited, participants had an audience with the Pope. Hawking comments ironically about this meeting in his *A Brief History of Time:*

> He told us that it was all right to study the evolution of the universe after the big bang, but we should not inquire into the big bang itself because that was the moment of Creation and therefore the work of God. I was glad then that he did not know the subject of the talk I had just given at the conference—the possibility that space-time was finite but had no boundary, which means that it had no beginning, no moment of Creation. I had no desire to share the fate of Galileo . . . ! (116)

What had just happened was that Hawking himself had revised his theory and—in a new version—had managed to eliminate the initial singularity. The brand-new formulas, which he presented to cardinals and bishops, leave God without any role in Creation.

In order to understand this change, one must remember that today there exist two partial theories that describe the universe: the general theory of relativity, which explains the laws of gravity and the large-scale structure of the cosmos, and quantum mechanics, which is concerned with the subatomic world, the infinitesimally small. It is recognized that both these theories cannot be correct simultaneously. Indeed, physicists today direct their greatest efforts toward the formulation of a single, unified theory that might amalgamate the results of both worlds. The main difficulty to surmount is that of the subatomic world, where Heisenberg's uncertainty principle rules. This establishes limits to the possibilities of observation and prediction and points to an irreducible element of chance in the subatomic world. This theory elicited from Einstein, who never resigned himself to accepting it, his famous expression of displeasure: "God does not play dice with the universe."

The general theory of relativity, on the other hand, does not take the uncertainty principle into account. The coexistence of these two contradictory theories is possible because the phenomena at work are of different scales. But the hypothesis that the universe at some point was infinitely small makes it clear that, at those minimal dimensions, the quantic effects must be considered. They can no longer be ignored: general relativity, which was the hypothesis Penrose and Hawking used in their first big bang theory, must be replaced—combined with the uncertainty principle—by a new quantum theory of gravity.

Once the quantic effects are considered, the singularity can be eliminated, and a new picture of the universe becomes possible: space-time, in Hawking's most recent conjecture, is finite in extension but has no limits. It can be thought of as a smooth, closed surface, like the surface of the Earth, on which one can walk indefinitely without falling off a cliff. Neither are there any singularities before which the laws of science fail, or boundaries that oblige one to resort to God, or a new law to establish surrounding conditions. But if the universe is really self-contained, if it has no limits or boundaries, it would also have no beginning or end. It would simply be. Thus, there is no place for a Creator.

Therefore, if Hawking's new conjecture is confirmed, the answer to Einstein's question about how many choices God had in conceiving the universe would be *none.* And like that astronomer whose king asked him where he placed God in his system of spheres, he might answer, with a Mephistophelian smile, "Sire, I had no need of that hypothesis."

Note

1. Published under the title "Laws of the Universe," *Clarin*, 16 Aug. 1998.

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13 | GOD'S SINKHOLE[1]

I remembered this little story recently when I heard Stephen Hawking predict in an interview that soon, perhaps in the first decade of the millennium,[2] physics will arrive at a unified theory of the laws of the universe, with a mathematical explanation of the first moment of Creation.

I recalled, as the reporter asked Hawking the inevitable question about what role would be left for God to play, Professor Katz's cosmology classes at the Facultad de Ciencias Exactas and the terror he instilled in his students. Katz had studied at Oxford with Roger Penrose, Hawking's dissertation director, and during his brief return visit to Argentina he taught cosmology as the capstone course for the physics degree. He soon became famous for the swiftness with which he filled blackboards, the force with which he snapped the chalk as he wrote, and the superhuman difficulty of his assignments. He had requested a graduate student in mathematics to be his teaching assistant, and Pablo Marín, who was a friend of mine at the time, accepted the challenge. At the university bar, Pablo enjoyed regaling me with stories of Katz's sarcastic barbs and the students' desperation at his formulas. In particular, he told me about a somewhat older female student who had already failed the course twice and tailed him like a shadow to all his office hours in order to consult him, with obsessive persistence, about every exercise, one after another.

The term went by and final exam period arrived.

Pablo had scheduled a final review right before the exam. That day, as we ate lunch together at the bar, he was called back to the department office for a phone call. He came back distraught: his old girlfriend, who was in Buenos Aires for a brief visit, wanted to see him again. He asked me to go to the classroom in fifteen minutes to notify his students that there would be no class that day, and then he took off, loping toward the bus stop. I ordered another coffee, waited fifteen minutes, and headed for the classroom. There was only one girl standing next to the platform, swaying nervously and clutching a black portfolio: the student Pablo had told me about. As I approached her I noticed that the arm clutching the portfolio was twitching, her fist tightly squeezed shut as if she were concealing something, and that her chin trembled involuntarily. She looked as though her teeth were about to start chattering. I told her that Pablo had canceled the class. She stood there for a moment, defeated and speechless, and then she looked at me imploringly, like a last resort.

"Maybe you can help me," she said. "You're a mathematician too, right?" and she clumsily opened the portfolio before I could say a word. The assignment bore a strange title: *God's Sinkhole.* It might have been another example of Katz's sarcasm, or maybe it was just a waggish term physicists used to refer to the singularity of the initial instant. Beneath that title were the most impenetrable equations I had ever seen in my entire academic career. The first one occupied three lines, in which I could barely recognize two or three symbols. I realized that an hour wouldn't be enough for me even to decipher the notations. I looked up again, and before I could say anything she understood that her last hope had vanished. I saw that she was trembling and that her fist, which had been hanging by her side, was squeezing convulsively. For a moment I stood there, frozen: from that fist, from the juncture between her fingers, a thread of blood trickled silently to the floor without her appearing to notice. I reached out my hand in order to steady her wrist, and before she could move it away, with my other hand I pried her fingers apart. What that physics student had concealed and squeezed until they dug into the palm of her hand were the metal points of a crucifix.

Notes

1. Published under the title "Una cuestión de tiempo" in "Viva," *Clarín,* 2000.

2. Obviously, Hawking's prophecy has not yet been fulfilled.

APPENDIX A
MATHEMATICAL THEMES IN BORGES' WORK

A review of the most recent edition of Borges' complete works (*Borges: Obras Completas,* Sudamericana, 2011) allowed me to discover a surprising number of mathematical citations, somewhat over one hundred eighty. The complete list that I have compiled can be found (although not in English translation) at www.guillermomartinezweb.blogspot.com. Mathematical references are also included in verses of Borges' poetry, as well as in fictional tales, reviews, and even in mini-lessons embedded in his essays.

Despite the quantity of citations, a few recurrent themes connected with philosophical traditions or logical paradoxes clearly emerge, appearing with slight variations in different contexts. For readers interested in mathematics, we offer the following list of those major themes (Titles for as yet untranslated material appear in Spanish):

The concept of infinity (from a philosophical point of view, the generation of the multiple from the singular): "Acerca de Unamuno, poeta"; *El idioma de los argentinos*; "The Perpetual Race of Achilles and the Tortoise"; "Avatars of the Tortoise"; "The Aleph"; "Pascal"; "El pudor de la historia"; "When Fiction Lives in Fiction"; Arthur Waley's *Three Ways of Thought in Ancient China* (London: Allen and Unwin, 1939)

Infinite regression (in particular a study of infinite regression traced through Aristotle, Zeno of Elea, Lewis Carroll, and Saint Augustine): "Avatars of the Tortoise"; "Coleridge's Flower"; "Time and J. W. Dunne"

Fractions tending to zero: "A Defense of Basilides the False"; "The Marked Dyer, Hakim of Merv"

Ascending progression: "Two Notes"

Transfinite numbers and Cantor's infinite sets: "Ramón Gómez de la Serna: *La sagrada cripta de Pombo*"; "Acotaciones"; "There Are More Things"; "Alguien sueña"; "The Total Library"; "The Perpetual Race of Achilles and the Tortoise"; "A History of Eternity"; "The Doctrine of Cycles"; "The Aleph"; "Nihon"; "El tiempo"

Infinitely sub-divisible segments: "The Lottery in Babylon"; "The Perpetual Race of Achilles and the Tortoise"; "The Library of Babel"; "The Book of Sand"

Russell's Paradox: (in the "Catalog of Catalogs" version): "The Library of Babel"

Pascal's Sphere: "The Library of Babel"; "The Aleph"; "Pascal's Sphere"

Royce's Map: "Another Poem of Gifts"; "When Fiction Lives in Fiction"

Line, plane, volume: "The Book of Sand"; "Descartes"; Review of Edward Kasner and James Newman's *Mathematics and the Imagination*

The fourth dimension: "La cuarta dimensión"; "Emanuel Swedenborg, Mystical Works"; "Ibn-Hakam al-Bokhari, Murdered in His Labyrinth"; "A New Refutation of Time"

The Euclidean Circle: "The Disk"; "Epílogo"

Numbering systems: "Tlön, Uqbar, Orbis Tertius"; "A Survey of the Works of Herbert Quain"; "Funes the Memorious"; "John Wilkins' Analytical Language"; "Doctor Brodie's Report"; Review of *Men of Mathematics* by E. T. Bell; "Homenaje a Xul Solar"; "The Total Library"; "Duodecimal Arithmetic, Longmans"

Artificial languages: "*Delphos or the Future of International Language*, de E. S. Pankhurst"; "The Total Library"

Huxley's monkeys: "The Total Library"

Self-reference: "The Library of Babel"; "The Aleph"; "Partial Magic in the *Quixote*"; "Nathaniel Hawthorne"; "Metaphors of *The Thousand and One Nights*"; "When Fiction Lives in Fiction"

The Laplace Formula: "Gilbert Waterhouse: *A Short History of German Literature* (London: Methuen, 1934)"; "Notas"; "M. Davidson: *The Free Will Controversy* (Watts, London, 1934)"; "The Creation and P. H. Gosse"; "Pragmatismo"; Prologue to *Canto* *a mí mismo* (León Felipe's translation of Walt Whitman's *Song of Myself*, Editorial Losada, Buenos Aires, 1941) ;"Observación final"

Computers and Llull's Cycle: "Ars Magna"

Occam's Razor: "A Defense of Basilides the False"; "On Literary Description"

APPENDIX B
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APPENDIX C
SOURCES OF ENGLISH TRANSLATIONS AND EXCERPTS

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