SUDOKU

|  | 8 | 9 |  |
| :---: | :---: | :---: | :---: |
|  |  | 6 |  |
|  |  | 9 |  |
|  | 678 |  |  |
|  |  |  |  |
|  |  | 1 |  |
|  | 6 | 2 |  |
| 9 |  |  |  |
|  | 14 |  |  |

## Sudoku - No more a Puzzle

## By

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Second Edition, December 2005
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You can host YOUR OWN Sudokus, and send and/or receive them at my Website: www.freesudokuhost.com

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## Introduction To Sudoku

Sujji wa dokushini ni kagiru is the original name of the game. If you think the name is complex, wait till you try the game. Sujji wa dokushini ni kagiru is of Japanese origin, now known as Sudoku (pronounced Soo Doe Koo) and meaning number game. Su stands for number and Doku for single, so say the Japanese.

It's a 9x9 number grid, with nine major squares and each major square, in turn, having nine mini squares (- we'll refer to them as Cells). All a player has to do is fill these up with numbers from 1-9 without ever repeating a single one.

A typical Sudoku puzzle would look something like this:

|  |  | 3 |  |  |  | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 |  |  | 4 |  | 5 |  | 1 |  |
| 2 |  |  | 1 |  | 7 |  |  |  |
|  |  |  | 7 |  |  |  | 9 | 3 |
|  |  |  |  | 4 |  |  |  |  |
| 7 | 1 |  |  |  | 6 |  |  |  |
|  |  |  | 5 |  | 9 |  |  | 4 |
|  | 5 |  | 2 |  | 4 |  |  | 6 |
|  |  | 2 |  |  |  | 5 |  |  |

And the solution would look somewhat like this (- the values in the originally filled squares are in black; and the values in the solved squares are in red.)

| 1 | 7 | 3 | 6 | 9 | 8 | 2 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 9 | 6 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4 | 7 | 8 | 1 | 6 | 9 | 3 |
| 6 | 3 | 8 | 9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 9 | 3 | 5 | 6 | 4 | 8 | 2 |
| 3 | 8 | 7 | 5 | 6 | 9 | 1 | 2 | 4 |
| 9 | 5 | 1 | 2 | 7 | 4 | 8 | 3 | 6 |
| 4 | 6 | 2 | 8 | 1 | 3 | 5 | 7 | 9 |

You can observe the following:

Every one of the 9 Rows has values ' 1 ' to '9' in the 9 cells. And no value repeats in any row.

Every one of the 9 Columns has values 'l' to '9' in the 9 cells. And no value repeats in any column.

Every one of the 9 Major Squares (in different colors) has values '1' to
' 9 ' in the 9 cells. And no value repeats in any Major Square.

What are the Major Squares we're refering to, above?
Each of the differently colored $3 * 3$ Squares is a Major Square.

| 1 | 7 | 3 | 6 | 9 | 8 | 2 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 9 | 6 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4 | 7 | 8 | 1 | 6 | 9 | 3 |
| 6 | 3 | 8 | 9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 9 | 3 | 5 | 6 | 4 | 8 | 2 |
| 3 | 8 | 7 | 5 | 6 | 9 | 1 | 2 | 4 |
| 9 | 5 | 1 | 2 | 7 | 4 | 8 | 3 | 6 |
| 4 | 6 | 2 | 8 | 1 | 3 | 5 | 7 | 9 |

## 2. Who Is Sudoku For?

 uduko is for everyone.Whether they know numbers or not. In fact, you can even play Sudoku without numbers; you can use any nine distinctly different symbols, say 9 alphabets in any language, 9 colors, or whatever; because you're not using the relationship between the numbers as such. So, even pre-school kids can take to Sudoku, if only they have the natural inclination.

It's a game needing no mathematical calculations. Just juggle the placement of numbers. So Sudoku is for all. That's the reason for its universal appeal. A grandfather can sit with his granddaughter in a battle of wits (- an ideal way to foster family ties, shall we say?)

## Sudoku Joke:

A newspaper publishes 3 categories of Sudoku. Instead of terming them as 'Very Easy', Moderately Difficult' and 'Very Difficult', the newspaper comes up with an interesting way to hook people of all categories on to Sudoku, including kids. And the newspaper publishes Sudoku for Kids only on Sundays and, to indicate that these could be solved by even 6 year old kids, mentions just above the Puzzle: '6+ Years'.

John buys this newspaper and reads it only on Sundays. So, he sees only the Sudoku for Kids. He struggles with it for days and months, without success.

One day, he comes excitedly to office and announces, "Hey everybody, I've done it! This Puzzle says, '6+ Years', but I solved it in 6 months flat!" udoku is much more than a game. In fact it is about living well! One lives well if one is able to field the intellectual challenges thrown one's way. This needs the presence of a sharp mind. What better way to sharpen the mind than by playing Sudoku?

University professors have now found a novel way of testing their candidates. Gone are the days of laboriously inking down answers. Solve a Sudoku and earn a university seat. For, professors know how well oiled the brain cells are by a student's Sudoko solving capacity.

Prof. Ian Robertson conducted a study on 3000 people. They were aged between 65 to 94. After regular Sudoku solving sessions, their mental age improved by 7-14 years.

Sudoku is highly recommended for children too. Mothers are now encouraging their children to play Sudoku. They know it improves their children's mental faculties.

Ever heard before? Teachers making their students play Sudokus! Well, they are merely following instructions of the government backed Teachers Magazine. Convinced you should start learning Sudoku too?

In 1783, Leonhard Euler, a Swiss father of 13, invented this game. Possibly to entertain his children. He called them as 'New kind of Magic squares'. For nearly two centuries none took much note of this game. In the 1970's, Dell Magazines published them as 'Number place'. But they failed to have much impact. Then came the Japanese who took this game home. They enjoyed it for two decades. But the outside world had still not realized the joy of Sudoku playing.

As chance would have it, a retired judge, Wayne Gould, was browsing thorough a book store in Tokyo. This was in 1977. He happened to see an unsolved number puzzle. So enticed was he that he spent six years of his life developing a software program - Pappocom. Now young and old alike download Sudokus from his website, Sudoku.com ( http://www.sudoku.com ).

Newspapers were quick to spot this frenzy. They started offering
Sudoku columns. Finally Sudokus appeared as a regular feature in the
Manhattan in 'The New York Post'. It had reached its homeland after two decades of globe trotting. tay Healthy The Sudoku Way:

1. Need a higher IQ?

## Or want to keep

Alzheimer's at bay?
Go the Sudoku way!
2. Sudoku releases chemicals that fertilize the brain cells. Acquire the mental age you had 14 years ago by solving Sudokus...

- A story in 'the Guardian' newspaper

Get Rich Quick ----- Learn the Sudoku trick.
3. Wayne Gould is a millionaire today. Pappocom churns out plenty of Sudokus for all. And Gould is its proud owner.
4. H. Bauer is all set to go the Gould way. This German is publishing a 80 puzzle Sudoku magazine. The expected sale is 100,000 .
5. Speculative bidding for Puzzler Media has touched $£ 100 \mathrm{~m}$. Bought in April 2002 for a mere $£ 36.7$ m, it turned rich featuring Sudokus.
6. Finishing six hard Sudokus in only 22 and a half minutes, Edward

Billig added $£ 5000$ to his pocket. "My head's hurting a bit now," joked the Independent Sudoku Grand Master - 2005, "I think something's broken inside it."

## Celebrities - Sudoku

7. Ever thought of naming your child Sudoku? Well, the Beckham's have. Sudoku Beckham is soon to join them, according to 'The Telegraph.'
8. Nine players in nine teams, each with a celebrity, battled Sudoku on Sky One. Carol Vorderman was the famous hostess that July 1st evening, in 2005.
9. The pop singers in 'Top of the Pop' are mixing work and play.

They are solving Sudokus while performing live. One such celebrity was Kelly Osbourne.
10. Channel 4 in August, 2005 had eight celebrities locked up in a house. And all they had to do was solve Sudokus.

## Sudoku in Print and Television

11. Five publishers together supply 666,000 homes with Sudoku magazines. Sudoku is Nikoli's registered trade mark. So others prefer calling it Number Place.
12. Sudoku books are sold in plenty. The only other books that sold more were J.K. Rowling's and Kevin Trudeau's.
13. U.S.A. TODAY'S top selling book list has six Sudoku books.
14. The top 5 slot in Switzerland's book shops has Sudoku books. So says Orell Fussli, the largest book shop owner.
15. London's Michael Mepham has produced 11 Sudoku books since May!
16. One Friday, the Guardian front page declared 'G2 - The only newspaper section with Sudoku on every page!
17. Sudoku has been taken up by more than 179 different TV and Radio channels across the U.K.

## Differing Sudoku Versions

18. Bunnydoku - Sudoku in a pocket PC.
19. Dell Number Place Challenger puzzles - the numbers in the main diagonals of the grid are to be unique.
20. Godoku - An alphabetical version of Sudoku.
21. Gnu doku - A free program for creating and solving

Sudoku puzzles.
https:/ / www.icculus.org/homepages/jcspray/GNUDoku/
22. Kokonutsu - is Sudoku $X$, the instantly recognizable $X$ factor formed by the diagonals.
23. Killer Sudoku - Also called Samunmpure. Six hours of your life are gone while you solve one.
24. Latin Squares - The original Sudoku.
25. Mobli Sudoku - Sudoku in your mobile.
26. Samurai Sudoku - Five Sudokus linked by the fifth one.
27. Su Do $12 \quad$ - A $12 \times 12$ grid for you.
28. Sudo Critters - An online version of Sudoku with pictures.
29. Sudoku 3D - Three dimensional Sudoku.
30. Wasabi - An enhanced Sudoku.

Some Ominous and Some Interesting Sudoku facts
31. Sudoku was born in America.
32. Sudoku has proved quite addictive in the lines of Tetris and chocolate .
33. Compulsory Sudoku Syndrome victims are prevented from
leaving for work without solving a Sudoku. Are you one?
34. Sudoku has been identified as a mental virus spreading to all nations.
35. Pencil sales have soared in the U.K. since Sudoku stepped in. A fact confirmed by U.K chain stores W H Smith , Borders and Woolworths.
http:/ / www.dailyrecord.co.uk/
36. Statistics claim that it is possible to create 6,670,903,752,021,072,936,960 Sudoku puzzles.
37. Sudoku is today - what Rubik's Cube was in the 1970's.
38. Chipping Sodbury, near Bristol, England - On this hill side is carved a giant Sudoku, or let's say the World's largest Sudoku.

## 6. Solving Sudoku SAP

## A simplified form of Sudoku!

[This is the most important chapter in this eBook. Read it slowly, carefully and patiently. If you don't have about 60 minutes at a stretch to read and understand at least up to 6.2 How To Solve Sudoku SAP, postpone taking it up till you can spend 60 minutes continuously, undisturbed.]

How do you eat an elephant?
A piece at a time, of course!

Sudoku is not as difficult as it may appear to be. It may appear somewhat difficult as it consists of too many squares ( 81 to be precise, constituted by $9(=3 * 3)$ major squares each consisting of $9(=3 * 3)$ minor squares), as below:

Sudoku - Regular


## Solving Sudoku SAP

The following are the 9 large major squares, shown below in different colors for clarity:


TI: Top left: Consists of 9 small squares to the Top left.
Tc: Top center: Consists of 9 small squares to the Top center.
Tr: Top right: Consists of 9 small squares to the Top right.
MI: Mid left: Consists of 9 small squares to the Mid left.
Mc: Mid center: Consists of 9 small squares to the Mid center.
Mr: Mid right: Consists of 9 small squares to the Mid right.

BI: Bottom left: Consists of 9 small squares to the Bottom left.
Bc: Bottom center: Consists of 9 small squares to the Bottom center.
Br: Bottom right: Consists of 9 small squares to the Bottom right.

What are the conditions that the game imposes?
(1) The numbers 1 to 9 MUST occur in each of the 9 rows.
(2) The numbers 1 to 9 MUST occur in each of the 9 columns.
(3) The numbers 1 to 9 MUST occur in each of the 9 mini squares (of each major square).
(4) None of the numbers 1 to 9 should repeat in any of the 9 rows.
(5) None of the numbers 1 to 9 should repeat in any of the 9 columns.
(6) None of the numbers 1 to 9 should repeat in any of the 9 mini squares within the same Major Square.

Isn't that difficult?
If you look deeply, you'll realize that conditions (4) to (6) can be just ignored, as they are redundant.

That is, if you solve for conditions (1) to (3), conditions (4) to (6) are taken care of automatically; you just need to do nothing more.

Still the puzzle is difficult, right?
Not quite!
The puzzle is fairly simple. What is slightly difficult, though, is explaining how to solve the puzzle. But we'll simplify that too. Don't worry.

First of all, we need a convention to address the Cells (the mini squares) so that, when I say - assign the value '3' to a particular square, you will know which square I am referring to.

Let's refer to each square by its (row number, column number) i.e., Cell (Row\#, Column\#).

For example, the top left square is $(1,1)$. Top right square is $(1,9)$. Bottom left square is $(9,1)$. Bottom right square is $(9,9)$. Simple enough?

So, the Sudoku Puzzle's cell addresses are:

| $(1,1)$ | $(1,2)$ | $(1,3)$ | $(1,4)$ | $(1,5)$ | $(1,6)$ | $(1,7)$ | $(1,8)$ | $(1,9)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $(2,1)$ | $(2,2)$ | $(2,3)$ | $(2,4)$ | $(2,5)$ | $(2,6)$ | $(2,7)$ | $(2,8)$ | $(2,9)$ |
| $(3,1)$ | $(3,2)$ | $(3,3)$ | $(3,4)$ | $(3,5)$ | $(3,6)$ | $(3,7)$ | $(3,8)$ | $(3,9)$ |
| $(4,1)$ | $(4,2)$ | $(4,3)$ | $(4,4)$ | $(4,5)$ | $(4,6)$ | $(4,7)$ | $(4,8)$ | $(4,9)$ |
| $(5,1)$ | $(5,2)$ | $(5,3)$ | $(5,4)$ | $(5,5)$ | $(5,6)$ | $(5,7)$ | $(5,8)$ | $(5,9)$ |
| $(6,1)$ | $(6,2)$ | $(6,3)$ | $(6,4)$ | $(6,5)$ | $(6,6)$ | $(6,7)$ | $(6,8)$ | $(6,9)$ |
| $(7,1)$ | $(7,2)$ | $(7,3)$ | $(7,4)$ | $(7,5)$ | $(7,6)$ | $(7,7)$ | $(7,8)$ | $(7,9)$ |
| $(8,1)$ | $(8,2)$ | $(8,3)$ | $(8,4)$ | $(8,5)$ | $(8,6)$ | $(8,7)$ | $(8,8)$ | $(8,9)$ |
| $(9,1)$ | $(9,2)$ | $(9,3)$ | $(9,4)$ | $(9,5)$ | $(9,6)$ | $(9,7)$ | $(9,8)$ | $(9,9)$ |

So, when we have a Sudoku puzzle as below, we can say $(1,8)$ has a ' 1 ', $(2,7)$ has a ' 6 ', $(5,2)$ has a ' 8 ', and $(8,6)$ has a '4' as
highlighted in pink.

|  |  |  |  |  |  |  | 1 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 |  |  | 5 |  |  | 6 |  |  |
| 9 |  |  | 3 |  |  |  |  |  |
|  | 5 |  |  | 1 |  |  | 3 |  |
|  | 8 |  |  | 3 |  |  | 7 |  |
|  | 6 |  |  | 2 |  |  | 9 |  |
|  |  |  |  |  | 8 |  |  | 2 |
|  |  | 3 |  |  | 4 |  |  | 1 |
| 5 | 7 |  |  |  |  |  |  |  |

You now try to solve it based on common sense. If you are a novice, you'll realize that if you try to get the rows right, you get a few columns wrong, and vice versa. And if you get both right, you get the numbers wrong in some of the major squares. Getting them all right, and in a reasonably short time, is a difficult task for a novice, but not impossible. It becomes easy when you learn how to do it!

But how do you learn?

Rocket scientists solve complex problems of sending rockets and humans into space, by first solving their simpler mathematical models. Likewise, all complex puzzles can be best solved by solving their simpler models. Sudoku can't be more complex than rocket science, can it be?

So, shall we create a simpler model of Sudoku and learn to solve it first, understanding the techniques, and using the same techniques to solve the original, more complex (looking), Sudoku?

Alright, we have created Sudoku SAP (Simple As Possible), for you, just for this purpose. This will simplify your understanding of the original Sudoku puzzle itself, and help you learn to solve Sudoku Puzzles. No matter how complex, every Sudoku puzzle will be child's play, as you'll soon see. Just have the patience to go thru the eBook once, slowly, carefully and patiently!

### 6.1 The Sudoku SAP Puzzle

Sudoku has $3 * 3(=9)$ major squares, each consisting of $3 \star 3(=9)$ minor squares, multiplying to a total of 81 squares, right? Shall we call this Sudoku of Order 3 (viz., 3 rows, 3 columns, and 3 * 3 major squares), for simplicity of our understanding?

Let's see if we could create a simple Sudoku of Order 1 with just l* major square, each consisting of ${ }^{*}{ }^{*}$ minor square. Oh, No... It multiplies up to just 1 square totally. Such a Sudoku is possible, no doubt, but wait ... how many rows and columns will be there in it? Just one row and one column. So, actually the puzzle becomes trivial, with just one minor square/cell, and just one solution viz., a 'l' in the only square! So, this won't help us understand Sudoku. There is one only one Sudoku puzzle for a Sudoku SAP (Order 1) Puzzle.

## 1

Now, let us see if a more practical Sudoku SAP puzzle of Order 2 (2 rows, 2 columns, and $2 \star 2$ major squares), exists. Hey, and Presto! Yes, such a simpler, and more practical miniature version of Sudoku, the Sudoku SAP, does exist. This will have just 16 squares, with numbers from just 1 to 4 to be filled in. It's much simpler than the 81 squares, and with numbers 1 to 9 to be filled in. Isn't it far simpler?

Let's create a Notation System to address all the minor squares, exactly as we had created earlier for the regular Sudoku.

$$
\begin{array}{ll|l|}
\hline(1,1) & (1,2) & (1,3) \\
\hline(2,4) & (2,2) & (2,3) \\
\hline(2,4) \\
\hline(3,1) & (3,2) & (3,3) \\
\hline(3,4) & (4,2) & (4,3) \\
\hline(4,4)
\end{array}
$$

Row 1 has cells $(1,1),(1,2),(1,3)$ and $(1,4)$ in that order. Similarly, Row 2, Row 3 and Row 4 have cells $(2,1)$ to $(2,4),(3,1)$ to $(3,4)$ and $(4,1)$ to $(4,4)$ respectively. We now have an address for each cell.

Let me present to you, a simple sample Sudoku SAP puzzle:

| 2 |  |  |  |
| :--- | :--- | :--- | :--- |
|  | 1 |  |  |
| 3 |  | 4 |  |
|  |  |  |  |

## Solving Sudoku SAP

Why don't you just try to solve this simple Sudoku SAP puzzle, on your own? You may be surprised to find that you can already solve it, after all! At least try as much as you can. Good Luck!

In any case, even if you have solved it yourself, learn the system below.
This method will help you solve the larger Sudoku later.

## Solving Sudoku SAP

### 6.2 How To Solve Sudoku SAP

There may appear to be many methods of solving the Sudoku puzzle, but all of them are simply variations of just one method.

And I'm going to teach you to solve it by the simplest and most effective method.

A method that never fails!
A method that helps you solve the puzzle in a definite time!
A method that will tell you if there are multiple solutions, and help you get every one of these solutions.

A method that will tell you if there is a mistake in the puzzle, and so tell you that there is no solution. And why.

Let's now see what all are the possible numbers that can occupy the empty cells. This is called construction of the Possibility Matrix.

| 2 | 3,4 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | $2,3,4$ |
| 3 | 2 | 4 | 1,2 |
| 1,4 | $2,3,4$ | $1,2,3$ | $1,2,3$ |

How did we arrive at this Possibility Matrix?
Let's see Cell $(1,2)$, that is the Cell in the Row 1 and Column 2, shown in red below:


Since Row 1 has a '2' already, Cell ( 1,2 ) can't take the value '2'. Since
Column 2 has a '1' already, Cell $(1,2)$ can't take '1' either. Since the top left major square consisting of cells $(1,1),(1,2),(2,1)$ and $(2,2)$ doesn't contain any number other than '2' and ' 1 ', no other number is precluded for Cell $(1,2)$. So, from the possible numbers $1,2,3$ and 4 , excluding ' 2 ' and 'l' for reasons above, the only possible numbers that can get into Cell $(1,2)$ are ' 3 ' and ' 4 '.

Suppose Cell $(2,1)$ had a 3 in it (which is not the case here), Cell $(1,2)$ can't take the value ' 3 ' too. And the Cell would have been forced to take only the number ' 4 '.

Likewise, Cell $(3,4)$ can't take the values ' 3 ' and ' 4 ' (since Row 3 has the values ' 3 ' and ' 4 ', and there is no number in Column 4); there are no other numbers precluded in the Bottom Right Major Squares where $(3,4)$ lies. So, the values permitted in Cell $(3,4)$ are '1 and ' 2 '.

Similarly, let's fill up the entire table.

| 2 | 3,4 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | $2,3,4$ |
| 3 | 2 | 4 | 1,2 |
| 1,4 | $2,3,4$ | $1,2,3$ | $1,2,3$ |

[There's a more detailed method for construction of the Possibility Matrix, explained in Chapter '6.6 Rigorous Method of Construction of the Possibility Matrix'. You can skip it without losing much, if you have understood so far. Or read it for a fuller understanding later. Reading this part of the chapter is optional.]

We have already solved the puzzle partly.
Now, let's continue solving the puzzle, by the process of Reduction.
That is reducing the Matrix, removing exclusions newly created, if any.

We see that cell $(2,1)$ takes the ' 4 '.
Since cell $(2,1)$ takes the value ' 4 ', we can't have any more ' 4 ' in row 2 , column 1 , and the top left major square (consisting of cells $(1,1),(1,2)$,
$(2,1)$ and $(2,2)$ ). So, delete ' 4 ' from these cells in the Possibility Matrix where the puzzle is yet to be solved (Cells $(1,2),(2,4)$ and $(4,1)$, where the '4's are shown as struck off in the figure below).

| 2 | 3,4 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | $2,3,4$ |
| 3 | 2 | 4 | 1,2 |
| 1,4 | $2,3,4$ | $1,2,3$ | $1,2,3$ |

Similarly, since cell $(3,2)$ takes the value ' 2 ', we can't have any more ' 2 ' in row 3 or column 2, or the bottom left major (consisting of cells $(3,1),(3,2),(4,1)$ and $(4,2))$. So, delete ' 2 ' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved $((3,4)$, and $(4,2))$.

Now, the puzzle reduces to the following:

| 2 | 3,4 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | $2,3,4$ |
| 3 | 2 | 4 | $1, z$ |
| 1,4 | $z, 3,4$ | $1,2,3$ | $1,2,3$ |

## Solving Sudoku SAP

Let's now see what we have, after the deletions:

| 2 | 3 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 3,4 | $1,2,3$ | $1,2,3$ |

Cells $(1,2),(3,4)$ and $(4,1)$ have got resolved as a result, and they take ' 3 ', '1' and '1' respectively.

Let's repeat the process of Reductions, deleting these values from their respective rows, columns and major squares respectively, till no more reductions are possible.

First, since cell $(1,2)$ takes the value ' 3 ', we can't have any more ' 3 ' in row 1 , or column 2 or the top left major square (consisting of cells ( 1,1 ), $(1,2),(2,1)$ and $(2,2)$ ). So, delete ' 3 ' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have:

| 2 | 3 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 3,4 | $1,2,3$ | $1,2,3$ |

## Solving Sudoku SAP

Now, since cell $(3,4)$ takes the value ' 1 ', we can't have any more ' 1 ' in row 3, or column 4 or the bottom right major square (consisting of cells $(3,3),(3,4),(4,3)$ and $(4,4))$. So, delete '1' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved.

| 2 | 3 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 3,4 | $7,2,3$ | $4,2,3$ |

Again, since cell ( 4,1 ) takes the value ' 1 ', we can't have any more ' 1 ' in row 4 , or column 1 or the bottom left major square (consisting of cells $(3,1),(3,2),(4,1)$ and $(4,2))$. So, delete ' 1 ' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have no such ' 1 ' to be deleted, though, and this is what we now have:

| 2 | 3 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 3,4 | $7,2,3$ | $\pm, 2,3$ |

Removing the deleted numbers, the puzzle reduces to the following:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2,3 | 2,3 |

We see that Cell $(1,3)$ now has a ' 1 ', Cell $(1,4)$ has '4', and $(4,2)$ has ' 4 '.

Again, let's repeat the process of deleting these values from their respective rows, columns and major squares respectively.

Since cell $(1,3)$ takes the value ' 1 ', we can't have any more ' 1 ' in row 1 , or column 3 or the top right major square (consisting of cells $(1,3),(1,4)$,
$(2,3)$ and $(2,4)$ ). So, delete ' 1 ' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have, again, no such 'l' to be deleted, and this is what we continue to have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2,3 | 2,3 |

Since cell $(1,4)$ takes the value ' 4 ', we can't have any more ' 4 ' in row 1 , or column 4 or the top right major square (consisting of cells $(1,3)$, $(1,4),(2,3)$ and $(2,4))$. So, delete ' 4 ' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have, yet again, no such '4' to be deleted, and this is what we still continue to have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2,3 | 2,3 |

Since cell $(4,2)$ takes the value ' 4 ', we can't have any more ' 4 ' in row 4, or column 2 or the bottom left major square (consisting of cells $(3,1)$, $(3,2),(4,1)$ and $(4,2))$. So, delete '4' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have, once again, no such '4' to be deleted, and this is what we again continue to have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2,3 | 2,3 |

Now, we find that we have reached an impasse, being unable to resolve between cells $(2,3),(2,4),(4,3)$ and $(4,4)$. And we need to resort to the Tie Breaker Rule.
[Note: If we reach an impasse and it becomes clear that we have reached an impasse, we don't have to repeat the previous steps where we found no further scope for reduction is possible. We would learn to skip these steps from experience, as we solve more and more puzzles.]

In a situation like this, where we have 2 or more cells with exactly the same possibility values for different cells, and if we are unable to resolve otherwise, we break the impasse using the Tie-breaker Rule.

Let's assume one of the 2 possible values for any one of the unresolved cells. Let's start with lower value for the Lower Row No., and the Lower Column no. (You start in any order, and still you will get the same results.)

Let's assume the Value '2' in $(2,3)$; so, let's delete ' 3 ' from Cell $(2,3)$. This is what we have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2,3 | 2,3 |

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Since cell $(2,3)$ takes the value ' 2 ', we can't have any more ' 2 ' in row 2 , or column 3 or the Top Right Major Square (consisting of cells $(1,3)$, $(1,4),(2,3)$ and $(2,4)$ ). So, delete ' 2 ' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | $z, 3$ |
| 3 | 2 | 4 | 1 |
| 1 | 4 | $z, 3$ | 2,3 |

Let's now remove the deleted nos. and see what we have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2 | 3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 3 | 2,3 |

Since cell $(2,4)$ takes the value ' 3 ', we can't have any more ' 3 ' in row 2 , or column 4 or the top right major square (consisting of cells $(1,3)$, $(1,4),(2,3)$ and $(2,4)$ ). So, delete ' 3 ' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2 | 3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 3 | 2,3 |

Now, removing the deleted value, we have the Final Solution, as below:

| 2 | 3 | 1 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 1 | 2 | 3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 3 | 2 |

But, hold on... is this THE Final Solution? Are you sure?

We had made an assumption along the way, didn't we? Do you remember that we deleted '3' from Cell $(2,3)$ and assumed the Value '2' in $(2,3)$ ?

What if we had deleted the Value '2' from Cell $(2,3)$ and assumed the Value '3' instead? So, let's apply the Exhaustive Tie Breaker Rules.

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Let's see what we would have had in such a case:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | $z, 3$ | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2,3 | 2,3 |

Since cell $(2,3)$ takes the value ' 3 ', we can't have any more ' 3 ' in row 2 , or column 3 or the top right major square (consisting of cells $(1,3)$, $(1,4),(2,3)$ and $(2,4))$. So, delete ' 3 ' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | $z, 3$ | 2,3 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2,3 | 2,3 |

Let's now remove the deleted nos. and see what we have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 3 | 2 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2 | 2,3 |

Since cell $(2,4)$ takes the value ' 2 ', we can't have any more ' 2 ' in row 2 , or column 4 or the top right major square (consisting of cells $(1,3)$, $(1,4),(2,3)$ and $(2,4))$. So, delete '2' from all the above cells in the Possibility Matrix where the puzzle is yet to be solved. We have:

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 3 | 2 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2 | $z, 3$ |

Now, removing the deleted value, we have a Final Solution, as below:

| 2 | 3 | 1 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 1 | 3 | 2 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2 | 3 |

Hey, this is ANOTHER Solution! Isn't that interesting?
For all practical purposes, you should be satisfied if you get one of the final solutions. In fact, most Sudoku solvers wouldn't even know that there may be more than one solution to a puzzle (when the puzzle at hand has more than one solution), if they solve it by any intuitive methods.

The long and short: A Sudoku SAP can have more than one solution; so, obviously, real Sudoku, which is more complex, can surely have more than one solution too.

Hence, when you verify answers, don't be surprised if you find that you have a right solution, but it is not the same as the one given by the Sudoku publisher. Ideally, if a puzzle has multiple solutions, the publisher should give all the possible solutions.

Most Sudoku publishers don't give puzzles with multiple solutions, if the first place.

But multiple solutions can be there, very rarely, though! We'll see about this later.

We've learnt to Solve Sudoku SAP, haven't we?
No, not quite fully. Not as yet.

Let's now see a Sudoku SAP Puzzle with another level of complexity, which we can't solve with just what we know.

|  |  |  |  |
| :--- | :--- | :--- | :--- |
| 3 | 1 |  |  |
| 1 | 3 |  |  |
|  |  |  | 3 |

Let's quickly construct the Possibility Matrix, as below:

| 2,4 | 2,4 | $1,2,3$ | $1,2,4$ |
| :---: | :---: | :---: | :---: |
| 3 | 1 | 2,4 | 2,4 |
| 1 | 3 | 2,4 | 2,4 |
| 2,4 | 2,4 | $1,2,4$ | 3 |

Now, how do we proceed? Do we have to go in for the Tie Breaker?
Not quite as yet!
Column 3 has " 2 " and " 4 " as possible values in all the 4 Cells, but 2 of these cells can take only the values ' 2 ' or ' 4 '; they can't take any other values. Either Cell $(2,3)$ must be ' 2 ' and Cell $(3,3)$ must be ' 4 ', OR Cell $(2,3)$ must be ' 4 ' and Cell $(3,3)$ must be '2'. In such a case, Cell $(1,3)$ and Cell $(4,3)$ can't take these values, ' 2 ' and ' 4 '. So, remove ' 2 ' and ' 4 ' from these Cells. We call it 'Group Reduction'.

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Now, we have:

| 2,4 | 2,4 | 1,3 | $1,2,4$ |
| :---: | :---: | :---: | :---: |
| 3 | 1 | 2,4 | 2,4 |
| 1 | 3 | 2,4 | 2,4 |
| 2,4 | 2,4 | 1 | 3 |

By the process of repeated reductions, we now get:

| 2,4 | 2,4 | 3 | 1 |
| :---: | :---: | :---: | :---: |
| 3 | 1 | 2,4 | 2,4 |
| 1 | 3 | 2,4 | 2,4 |
| 2,4 | 2,4 | 1 | 3 |

We can now apply the Tie Breaker Rule, and try to get the first solution:

| 2 | 4 | 3 | 1 |
| :---: | :---: | :---: | :---: |
| 3 | 1 | 2,4 | 2,4 |
| 1 | 3 | 2,4 | 2,4 |
| 4 | 2 | 1 | 3 |

We find that still 4 Cells remain unresolved.

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We now apply one more Tie Breaker Rule, and get, the first solution as:

| 2 | 4 | 3 | 1 |
| :--- | :--- | :--- | :--- |
| 3 | 1 | 2 | 4 |
| 1 | 3 | 4 | 2 |
| 4 | 2 | 1 | 3 |

We had assumed the value of '2' for Cell $(2,3)$. Now, let's assume the value of '4' for Cell ( 2,3 ), and get the Second Solution:

| 2 | 4 | 3 | 1 |
| :--- | :--- | :--- | :--- |
| 3 | 1 | 4 | 2 |
| 1 | 3 | 2 | 4 |
| 4 | 2 | 1 | 3 |

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Hold on! We had initially made an assumption of the value '2' for Cell $(1,1)$, when ' 4 ' was also a possible value, right? Let's now assume the value of ' 4 ' for Cell ( 1,1 ), and get the 2 more Solutions, as below:

| 4 | 2 | 3 | 1 |
| :--- | :--- | :--- | :--- |
| 3 | 1 | 2 | 4 |
| 1 | 3 | 4 | 2 |
| 2 | 4 | 1 | 3 |


| 4 | 2 | 3 | 1 |
| :--- | :--- | :--- | :--- |
| 3 | 1 | 4 | 2 |
| 1 | 3 | 2 | 4 |
| 2 | 4 | 1 | 3 |

So, this Sudoku SAP puzzle has 4 possible solutions.
How is that possible? When you do not define a Sudoku (or a Sudoku SAP) puzzle completely, it can have more than one solution. The more ill-defined the puzzle (i.e., the less fully specified the minimum necessary values for a Unique Solution), the more possible the solutions.

So, hypothetically, if you leave out a Sudoku (or a Sudoku SAP) puzzle completely unfilled (with all squares being empty), then you can find many possible solutions, but the number of such possible solutions is finite and deterministic. Isn't this an interesting observation?

Now, you are completely equipped to solve a Sudoku puzzle proper, but before you do that, let's make sure you have learnt to solve the Sudoku SAP fully. Before we proceed to try the Sudoku SAP Puzzles, let's summarize the steps.

## Summarizing:

The 4 Processes Involved in solving a Sudoku SAP (or Sudoku) Puzzle are:

1. Creating the Possibility Matrix
2. Reduction
3. Group Reduction
4. Tie Breaker

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The 7 Steps Involved in solving a Sudoku SAP (or Sudoku) Puzzle are:

1. Create the Possibility Matrix.
2. Perform Iterative (Repeated) Reductions of the Possibility Matrix.
3. Perform Group Reductions.
4. Perform Reductions \& Group Reductions alternatively.
5. Resolve impasse thru Tie Breaker and arrive at one possible Solution.
6. Repeat Tie Breakers with other possible values to arrive at all possible Solutions, if required.
7. Declare the Problem as Unsolvable If Conflicts arise which are not resolvable.

Very Simple and Simple Puzzles are solved by Stage 2.
Moderately Difficult Puzzles are solved by Stage 3.
Difficult Puzzles are solved by Stage 4.
Very Difficult Puzzles (by whatever names different composers choose to call them) are solved by Stage 5 and beyond.

### 6.3 Some Sudoku SAP Puzzles...

...to cut your teeth, before you learn to solve the original Sudoku Puzzles (of Order 3).

Why not try to solve the following Sudoku SAP Puzzles, and get your fundas right, so that you'll find it a lot easier to solve the original Sudoku Puzzles a little later? Try all of them, and you won't regret it, as each has an interesting story to tell you.


Solving Sudoku SAP


Solving Sudoku SAP


## Solving Sudoku SAP

6.4 Solutions to the Sudoku SAP Puzzle

| 2 | 3 | 1 | 4 |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 3 | 2 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2 | 3 |

2 | 1 | 2 | 4 | 3 |
| :--- | :--- | :--- | :--- |
| 3 | 4 | 1 | 2 |
| 2 | 1 | 3 | 4 |
| 4 | 3 | 2 | 1 |

3. The 2 Solutions are:

| 3 | 4 | 2 | 1 |
| :--- | :--- | :--- | :--- |
| 1 | 2 | 4 | 3 |
| 4 | 3 | 1 | 2 |
| 2 | 1 | 3 | 4 |


| 3 | 4 | 2 | 1 |
| :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 4 |
| 4 | 3 | 1 | 2 |
| 2 | 1 | 4 | 3 |

4. The 2 Solutions are:

| 4 | 1 | 3 | 2 |
| :--- | :--- | :--- | :--- |
| 2 | 3 | 1 | 4 |
| 1 | 4 | 2 | 3 |
| 3 | 2 | 4 | 1 |


| 4 | 3 | 1 | 2 |
| :--- | :--- | :--- | :--- |
| 2 | 1 | 3 | 4 |
| 1 | 4 | 2 | 3 |
| 3 | 2 | 4 | 1 |

5. The 2 Solutions are:

| 2 | 3 | 1 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 1 | 3 | 2 |
| 3 | 2 | 4 | 1 |
| 1 | 4 | 2 | 3 |


| 2 | 4 | 3 | 1 |
| :--- | :--- | :--- | :--- |
| 3 | 1 | 2 | 4 |
| 1 | 3 | 4 | 2 |
| 4 | 2 | 1 | 3 |

6. There are 4 solutions possible for this Sudoku.

| 1 | 3 | 2 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 2 | 1 | 3 |
| 2 | 4 | 3 | 1 |
| 3 | 1 | 4 | 2 |


| 3 | 1 | 2 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 2 | 3 | 1 |
| 2 | 4 | 1 | 3 |
| 1 | 3 | 4 | 2 |


| 1 | 3 | 2 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 2 | 1 | 3 |
| 2 | 4 | 3 | 1 |
| 3 | 1 | 4 | 2 |


| 3 | 1 | 2 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 2 | 3 | 1 |
| 2 | 4 | 1 | 3 |
| 3 | 1 | 4 | 2 |

7. This Puzzle has no valid Solution, as there are unsolvable Conflicts as below:

| 4 | 2,3 | 1 | 2,3 |
| :---: | :---: | :---: | :---: |
| 3 | 1 | 2 | 4 |
| 1 | 3,4 | 3 | 2,4 |
| 2,3 | 2 | 3,4 | 1 |

(This is the Step 7 we've not had a chance to see so far.)

### 6.5 Interesting Sidelights about the Sudoku SAP Puzzles and Clues.

[Avoid reading this Section till you have solved as many of the above Exercise Puzzles as possible.]

1. What's interesting about the Puzzle 1 above? This puzzle is the same as the one we have solved under 6.3 above, but with the impasse resolved. That is, in our solved puzzle, we came to a point of deadlock which we resolved thru Tie Breaker. Here, one of the Cells (where we had impasse previously) has been assigned a specific number, and so the impasse no longer exists. So, you can see that it gets solved more easily.
2. Puzzle 2 is quite simple. This is here for you to cut your teeth before going on to solve more difficult Sudoku SAP and original Sudoku Puzzles.
3. What's interesting about Puzzle 3 above? This puzzle is also the same as the one we have solved above, with the impasse in tact. How? Substitute All '1' by ' 2 ', ' 2 ' by ' 3 ', ' 3 ' by ' 4 ' and ' 4 ' by ' 1 ' in the puzzle we have solved under 6.3 above, and you have Puzzle 3. Effectively, you will get the answers by simply substituting all '1's by ' 2 's, ' 2 's by ' 3 's, ' 3 's by ' 4 's and ' 4 's by 'I's. This is just a repeat exercise to make sure you don't go wrong anywhere, and if you do, for you to be able to recover easily, by making substitutions of the above numbers from the solved example.
4. Puzzle 2 was simple, but Puzzle 4 is more complex. Despite more cells being filled with numbers than in the case of Puzzle 2, Puzzle 4 leads to Tie Breaker. This shows that for a Puzzle to be simple, it is not adequate if more cells are filled with numbers, but the right cells must be filled with their numbers.
5. Puzzle 5 is a Diagonal puzzle, and so we find that in the first step, nothing gets resolved. But the moment you start applying the Tie Breaker Rule, the puzzle gets resolved easily, with multiple answers.
6. Puzzle 6 has multiple solutions, but we need to apply the Tie Breaker Rule twice, since there are 2 Cells which can take one of two values, and there are 2 more Cells which can take two more (of other) values. First you apply the Tie Breaker for one set of 2 Cells, and for each of these two values, you again apply the Tie Breaker for the other set of 2 Cells. So, you get 4 possible solutions, overall.
7. Puzzle 7 is wrongly composed. Such a puzzle is very rarely given, if at all. And such puzzles could be given by a mistake in transcription, or deliberately to see if you are able to realize the mistake in the puzzle early.

### 6.6 Rigorous Method of Construction of the Possibility Matrix

The puzzle at hand, let's say, is:

| 2 |  |  |  |
| :--- | :--- | :--- | :--- |
|  | 1 |  |  |
| 3 |  | 4 |  |
|  |  |  |  |

Let's try to see what all values the empty cells in each row can't take.
Row 1 has a '2' in Cell (1, 1). So, all other cells in Row 1, which are empty, can't take the value '2'. Therefore, the Row Exclusions for Row 1 is just the value ' 2 '.

Similarly, let's write down the other Row Exclusions too, as below.
Likewise, Column 1 has a '2' in Cell ( 1,1 ), and a ' 3 ' in Cell ( 3,1 ). So, all other cells in Column 1 , which are empty, can't take the values ' 2 ' and '3'. Therefore, the Column Exclusions for Column 1 are the values '2' and '3'.

Similarly, let's write down the other Column Exclusions too, as below.

|  | Column 1 | Column 2 | Column 3 | Column 4 | Row Exclusions |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Row 1 | 2 |  |  |  | 2 |
| Row 2 |  | 1 |  |  | 1 |
| Row 3 | 3 |  | 4 |  | 3,4 |
| Row 4 |  |  |  |  |  |
| Column Exclusions | 2,3 | 1 | 4 |  |  |

The Top left major square has a '2' in Cell $(1,1)$ and a ' 1 ' in Cell $(2,2)$. So the other Cells in this major square, viz., Cell $(1,2)$ and Cell $(2,1)$ can't take the values 'l' or '2'. We'll represent this as

Major Square Exclusions:

| 2 | $\{1,2\}$ | $\}$ | $\}$ |
| :---: | :---: | :---: | :---: |
| $\{1,2\}$ | 1 | $\}$ | $\}$ |
| 3 | $\{3\}$ | 4 | $\{4\}$ |
| $\{3\}$ | $\{3\}$ | $\{4\}$ | $\{4\}$ |

Let's now create the Possibility Matrix for the Sudoku SAP puzzle. Fill in each square excluding the Row Exclusions, Column Exclusions and Major Square Exclusions. (Remember the possible values for the squares are only '1', '2', '3', and '4'.)

| 2 | 3,4 | 1,3 | $1,3,4$ |
| :---: | :---: | :---: | :---: |
| 4 | 1 | 2,3 | $2,3,4$ |
| 3 | 2 | 4 | 1,2 |
| 1,4 | $2,3,4$ | $1,2,3$ | $1,2,3$ |

How did we arrive at this table?

Take the case of Cell $(1,2)$, for example.
Row Exclusions for Row 1: '2'
Column Exclusions for Column '2': '1'
Major Square Exclusions for Upper Left Major Square: '1', '2'
So, excluding all these values, viz., '1' and '2', we have the Possible Values as: 3, 4

We repeat the same process for all the blank squares to arrive at the Possibility Matrix, as above.

### 6.7 How to Speed Up Your Puzzle Solving Ability?

Let's face it: Some people will always be faster than others; if you happen to solve them slower, don't worry. Try to learn to do them as fast as you can, but once you've reached your best speed, don't let the thought of your speed in comparison with someone haunt you. This is not only true of Sudoku, but of every other competitive activity in life. Do your best, and then, don't be obsessed with the thought of having to match or beat someone else in speed.

After you have learnt to solve Sudoku SAP Puzzle, learn to increase your ability to solve them quicker.

Solving Sudoku (or Sudoku SAP) is a skill. You hone it by practice. The more puzzles you solve, the faster you become.

However, there are a few shortcuts that can help you solve them faster. Here are some tips:

1. Constructing the Possibility Matrix is a mechanical process; there's no great intelligence involved. I feel that this is best got done with the help of the Computer. However, if you want to do it manually for any reason, please feel free to do so. Only, be careful; this is the one area where people go wrong most, not because they don't understand how to solve, but because we deal with a maze of numbers, and we may tend to make silly reading/ transcription mistakes.

Isn't this the same as having the Computer solve Sudokus? A kill joy?
Not really. Let's understand this one clearly.
There are mainly 2 parts to solving Sudoku our way. One:
Constructing the Possibility Matrix - this is mechanical, with almost no scope for use of one's intellect. Two: Solving the puzzle after the Possibility Matrix has been constructed - this involves using the intellect.

I do not recommend use of the Computer for the second part. But I do recommend use of the Computer for the first part.

There are the following added advantages in using the Computer to construct the Possibility Matrix:
(i) We may easily go wrong when we do it manually, but the Computer won't.
(ii) After you've constructed the Possibility Matrix a few times, you may, on your own, want to know if there's a simpler way to do it; Computer appears to be the answer.
(iii) You can solve more Sudokus in your given time, or solve them faster this way.
(iv) You won't still miss the thrill of solving your Sudokus all by
yourself. (When you solve your higher school math problems, do you miss the thrill just because you used a Calculator to do your arithmetic?)

However, I leave it to you to decide how you will solve your Sudokus.
2. Whether you're constructing a Possibility Matrix or applying the process of Reductions, you may solve the puzzles faster if you take up the most potential squares first.

The most potential squares for solution are the ones where there is the best probability of resolution. Let me explain, with the example of one of our solved exercises:

| 2 |  |  |  |
| :--- | :--- | :--- | :--- |
|  | 1 |  |  |
| 3 |  | 4 |  |
|  |  |  |  |

Now, we need to construct the Possibility Matrix. When the Computer creates a Possibility Matrix, it hardly matters in which sequence it creates. So, I had suggested that you may go from top to bottom, left to right.

However, when we do manually, since the sequence has a bearing on the speed of solution, it is a good idea to solve it in the smartest way, saving time and efforts, if we can.

In the puzzle above, we had originally solved it mechanically, top to bottom, left to right, just as a computer program might. Let's now try to do it the smarter way.

We had originally taken up Cell $(1,2)$ first for finding the possibilities, and found it can take values 3,4 .

Let's now try to see if any cell can take a unique value rather than 2 or more possible values.

How do you know if a cell can take a unique value? If 3 values are precluded, it can take a unique value. Let's inspect and see if there is any cell where 3 values are precluded.

Row 3 has 2 cells filled in, column 1 has 2 cells filled in, and top left major squares have 2 cells filled in already.

So intersection of these with rows/columns/ major squares are likely candidates for maximum exclusions.

For example, take Cell ( 2,1 ). It can take only the value ' 4 ', since Column 1 has '2' and '3' filled in already, and Row 2 has '1' filled in. So, let's first

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fill Cell $(2,1)$ with the value ' 4 '. Similarly, Cell $(3,2)$ can take only the value '2'; so, let's fill it in with '2'.

| 2 |  |  |  |
| :--- | :--- | :--- | :--- |
| 4 | 1 |  |  |
| 3 | 2 | 4 |  |
|  |  |  |  |

Cell $(4,1)$ can take only the value 'l'; let's also fill that in. Also, Cell $(1,2)$ can take only the value ' 3 ' since the other cells in the left major square have all been filled in with the other values. We'll fill in this too. Now, we have:

| 2 | 3 |  |  |
| :--- | :--- | :--- | :--- |
| 4 | 1 |  |  |
| 3 | 2 | 4 |  |
| 1 |  |  |  |

Now, Cell $(1,3)$ can take only the value ' 1 '; let's fill that in. Similarly, Cell $(4,2)$ can take only the value ' 4 '; let's also fill that in. Again, Cell $(3,4)$ can take only 'l'; let's also fill that in. Now, this is what we have:

| 2 | 3 | 1 |  |
| :--- | :--- | :--- | :--- |
| 4 | 1 |  |  |
| 3 | 2 | 4 | 1 |
| 1 | 4 |  |  |

Cell $(1,4)$ can only be '4'. Let's fill that in too.

| 2 | 3 | 1 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 1 |  |  |
| 3 | 2 | 4 | 1 |
| 1 | 4 |  |  |

We see that we're not able to fill in any more Cells likewise. Now, creating the Possibility is a lot simpler, isn't it? From experience, you'll see that this will not only make your task more interesting, but also it will be easier and take less time to solve the puzzle itself.

Smarter ways of taking up cells that can be resolved faster is a sure way to speed up solving Sudoku. And you'll be able to do this with practice.

## Solving Sudoku SAP

3. Creating a new table at every stage, or preferably, periodically will help you retract in case you have made any silly mistakes along the way. You could always go back to the last stage's results and continue from there rather than having to correct a mistake midway.

## 7. Discover the 7 Secrets...

...That Let You Quickly And Correctly Solve Any Sodoku Puzzle!

1. Solving Sudoku is Simple, if only you learn to do it right! Learn it starting with Sudoku SAP!
2. Focus initially on Accuracy than Speed. You'll gain speed with practice.
3. Possibility Matrix is better than Intuitive techniques. With it, You'll surely solve any Sudoku! Preferably use a Computer for it.
4. Apply Reduction and Group Reduction, repetitively, as much as possible. Avoid mistakes as backtracking will be difficult.
5. When you reach an impasse, apply Tie Breaker, starting with fewer option Cells.

Even if you had missed out on some reductions, Tie Breaker will take care, though you may take longer to solve.

If your solution doesn't match the given solution, try to get the matching solution too.

## Discover the 7 Secrets...

6. If you get no solution, don't despair! There may be no correct
solution. See if the puzzle is in error, and if so, prove!
7. Scan starting from the Most Potential Squares first, to solve faster!

## 8. Solving Regular Sudoku Puzzles

Let's now see a simple Conventional (or regular) Sudoku Puzzle.

|  | 1 |  |  |  |  |  | 5 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 5 | 4 | 3 | 9 | 1 |  |  |
| 6 |  |  |  | 7 |  |  |  | 9 |
| 4 |  |  | 8 | 2 | 5 |  |  | 3 |
|  |  | 8 |  |  |  | 5 |  |  |
| 5 |  |  | 6 | 9 | 7 |  |  | 2 |
| 3 |  |  |  | 1 |  |  |  | 8 |
|  |  | 7 | 9 | 6 | 8 | 4 |  |  |
|  | 9 |  |  |  |  |  | 7 |  |

- Does this have a solution or not?
- Is simple or complex?
- Would get solved with just the construction of the Possibility

Matrix?

- Would it involve making assumptions, and going thru a 'trial and error'?
- Would it involve having to resort to some of the more complex


## Solving Regular Sudoku Puzzles

techniques?

- Would it have a unique answer or multiple solutions?

No idea!

Let's apply our learning from Sudoku SAP and solve this regular and rather simple Sudoku now.

|  | 1 |  |  |  |  |  | 5 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 5 | 4 | 3 | 9 | 1 |  |  |
| 6 |  |  |  | 7 |  |  |  | 9 |
| 4 |  |  | 8 | 2 | 5 |  |  | 3 |
|  |  | 8 |  |  |  | 5 |  |  |
| 5 |  |  | 6 | 9 | 7 |  |  | 2 |
| 3 |  |  |  | 1 |  |  |  | 8 |
|  |  | 7 | 9 | 6 | 8 | 4 |  |  |
|  | 9 |  |  |  |  |  | 7 |  |

A quick scan suggests we take up Cell $(5,5)$ first: It can only take the value '4'.

## Solving Regular Sudoku Puzzles

Another scan suggests we take up Cells $(1,5),(9,5)$, and $(6,7)$.
There may be some more, but let's not waste time trying to find out all of them.

We fill them in with the only values possible in these cells.
And then construct the Possibility Matrix, as below:

| $2,7,8,9$ | 1 | $2,3,4,9$ | 2 | 8 | 6 | $2,3,6,7$, | 5 | $4,6,7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2,7,8$ | $2,7,8$ | 5 | 4 | 3 | 9 | 1 | $2,6,8$ | 6,7 |
| 6 | $2,3,4,8$ | $2,3,4$ | 5 | 7 | 1 | $2,3,8$ | $2,3,4,8$ | 9 |
| 4 | 6,7 | 6,9 | 8 | 2 | 5 | $6,7,9$ | 1 | 3 |
| $2,7,9$ | $2,6,7$ | 8 | 1 | 4 | 3 | 5 | $1,4,6,9$ | $1,4,6,7$ |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | $2,4,5,6$ | $2,4,6$ | 7 | 1 | 2,4 | $2,6,9$ | $2,6,9$ | 8 |
| 1 | 2,5 | 7 | 9 | 6 | 8 | 4 | 3 | 1,5 |
| 8 | 9 | $1,2,4,6$ | $2,3,5$ | 5 | $2,3,4$ | $2,3,6$ | 7 | $1,5,6$ |

By further reduction we get the puzzle partly solved as follows. And we resolve the value for Cell $(9,4)$ as ' 3 '.

## Solving Regular Sudoku Puzzles

| 7,9 | 1 | $3,4,9$ | 2 | 8 | 6 | 3,7, | 5 | 4,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2,7,8$ | $2,7,8$ | 5 | 4 | 3 | 9 | 1 | $2,6,8$ | 6,7 |
| 6 | $2,4,8$ | $2,3,4$ | 5 | 7 | 1 | 2,3 | $2,4,8$ | 9 |
| 4 | 6,7 | 6,9 | 8 | 2 | 5 | $6,7,9$ | 1 | 3 |
| $2,7,9$ | $2,6,7$ | 8 | 1 | 4 | 3 | 5 | 6,9 | 6,7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | $2,4,5,6$ | $2,4,6$ | 7 | 1 | 2,4 | $2,6,9$ | $2,6,9$ | 8 |
| 1 | 2,5 | 7 | 9 | 6 | 8 | 4 | 3 | 1,5 |
| 8 | 9 | $1,2,4,6$ | 3 | 5 | 2,4 | 2,6 | 7 | 1,6 |

Now, Cell $(8,9)$ reduces to ' 5 ', and $(1,9)$ takes the value '4' as the Unique values in Column 9 (let's call it C9).

| 7,9 | 1 | $3,4,9$ | 2 | 8 | 6 | 3,7, | 5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2,7 | $2,7,8$ | 5 | 4 | 3 | 9 | 1 | $2,6,8$ | 6,7 |
| 6 | $2,4,8$ | $2,3,4$ | 5 | 7 | 1 | 2,3 | $2,4,8$ | 9 |
| 4 | 6,7 | 6,9 | 8 | 2 | 5 | $6,7,9$ | 1 | 3 |
| $2,7,9$ | $2,6,7$ | 8 | 1 | 4 | 3 | 5 | 6,9 | 6,7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | $2,4,5,6$ | $2,4,6$ | 7 | 1 | 2,4 | $2,6,9$ | $2,6,9$ | 8 |
| 1 | 2,5 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | $2,4,6$ | 3 | 5 | 2,4 | 2,6 | 7 | 1,6 |

## Solving Regular Sudoku Puzzles

By reduction, we get values of the Cell $(8,2)$ as '2'. Cell $(9,9)$ takes the value 'l' being the unique value in C .

| 7,9 | 1 | 3,9 | 2 | 8 | 6 | 3,7, | 5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2,7 | 7,8 | 5 | 4 | 3 | 9 | 1 | $2,6,8$ | 6,7 |
| 6 | 4,8 | $2,3,4$ | 5 | 7 | 1 | 2,3 | 2,8 | 9 |
| 4 | 6,7 | 6,9 | 8 | 2 | 5 | $6,7,9$ | 1 | 3 |
| $2,7,9$ | 6,7 | 8 | 1 | 4 | 3 | 5 | 6,9 | 6,7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | $4,5,6$ | 4,6 | 7 | 1 | 2,4 | $2,6,9$ | $2,6,9$ | 8 |
| 1 | 2 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | 4,6 | 3 | 5 | 2,4 | 2,6 | 7 | 1 |

## Solving Regular Sudoku Puzzles

Cell $(7,2)$ takes the value ' 5 ' being the unique value in $C 2$. Similarly Cell $(5,1)$ takes the value '2' in Row 5 (let's call it R5), and Cell $(3,3)$ takes the value ' 2 ' in C 3 .

| 7,9 | 1 | 3,9 | 2 | 8 | 6 | 3,7, | 5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2,7 | 7,8 | 5 | 4 | 3 | 9 | 1 | $2,6,8$ | 6,7 |
| 6 | 4,8 | 2 | 5 | 7 | 1 | 2,3 | 2,8 | 9 |
| 4 | 6,7 | 6,9 | 8 | 2 | 5 | $6,7,9$ | 1 | 3 |
| 2 | 6,7 | 8 | 1 | 4 | 3 | 5 | 6,9 | 6,7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | 5 | 4,6 | 7 | 1 | 2,4 | $2,6,9$ | $2,6,9$ | 8 |
| 1 | 2 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | 4,6 | 3 | 5 | 2,4 | 2,6 | 7 | 1 |

## Solving Regular Sudoku Puzzles

By further reduction we get the values of Cells $(2,1)$, $(3,7)$ and $(3,8)$ as $' 7$ ', ' 3 ' and ' 8 ' respectively. Cell $(3,2)$ takes the value '4' being the unique value in C 2 .

| 7,9 | 1 | 3,9 | 2 | 8 | 6 | 3,7, | 5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 7,8 | 5 | 4 | 3 | 9 | 1 | $2,6,8$ | 6,7 |
| 6 | 4 | 2 | 5 | 7 | 1 | 3 | 8 | 9 |
| 4 | 6,7 | 6,9 | 8 | 2 | 5 | $6,7,9$ | 1 | 3 |
| 2 | 6,7 | 8 | 1 | 4 | 3 | 5 | 6,9 | 6,7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | 5 | 4,6 | 7 | 1 | 2,4 | $2,6,9$ | $2,6,9$ | 8 |
| 1 | 2 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | 4,6 | 3 | 5 | 2,4 | 2,6 | 7 | 1 |

## Solving Regular Sudoku Puzzles

Cell $(1,3)$ takes the value ' 3 ' being the unique value in C3. Similarly Cell $(2,8)$ takes the value ' 2 ' in R2. By reduction, we get the values of Cells $(1,1),(2,2),(2,9)$ and $(1,7)$ as '9', '8', '6' and '7' respectively.

| 9 | 1 | 3 | 2 | 8 | 6 | 7 | 5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 8 | 5 | 4 | 3 | 9 | 1 | 2 | 6 |
| 6 | 4 | 2 | 5 | 7 | 1 | 3 | 8 | 9 |
| 4 | 6,7 | 6,9 | 8 | 2 | 5 | $6,7,9$ | 1 | 3 |
| 2 | 6,7 | 8 | 1 | 4 | 3 | 5 | 6,9 | 6,7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | 5 | 4,6 | 7 | 1 | 2,4 | $2,6,9$ | $2,6,9$ | 8 |
| 1 | 2 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | 4,6 | 3 | 5 | 2,4 | 2,6 | 7 | 1 |

## Solving Regular Sudoku Puzzles

Cell $(4,3)$ takes the value ' 9 ' being the unique value in $C 3$, and Cell $(5,9)$ takes the value ' 7 '. Consequently, Cell $(5,8)$ takes the value '9' in R5.

| 9 | 1 | 3 | 2 | 8 | 6 | 7 | 5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 8 | 5 | 4 | 3 | 9 | 1 | 2 | 6 |
| 6 | 4 | 2 | 5 | 7 | 1 | 3 | 8 | 9 |
| 4 | 6,7 | 9 | 8 | 2 | 5 | $6,7,9$ | 1 | 3 |
| 2 | 6,7 | 8 | 1 | 4 | 3 | 5 | 9 | 7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | 5 | 4,6 | 7 | 1 | 2,4 | $2,6,9$ | 6,9 | 8 |
| 1 | 2 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | 4,6 | 3 | 5 | 2,4 | 2,6 | 7 | 1 |

## Solving Regular Sudoku Puzzles

By reduction, we get the values of Cells $(4,7),(5,2)$ and $(7,8)$ as ' 6 ', '6', and '6' respectively.

| 9 | 1 | 3 | 2 | 8 | 6 | 7 | 5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 8 | 5 | 4 | 3 | 9 | 1 | 2 | 6 |
| 6 | 4 | 2 | 5 | 7 | 1 | 3 | 8 | 9 |
| 4 | 6,7 | 9 | 8 | 2 | 5 | 6 | 1 | 3 |
| 2 | 6 | 8 | 1 | 4 | 3 | 5 | 9 | 7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | 5 | 4,6 | 7 | 1 | 2,4 | $2,6,9$ | 6 | 8 |
| 1 | 2 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | 4,6 | 3 | 5 | 2,4 | 2,6 | 7 | 1 |

## Solving Regular Sudoku Puzzles

By further reduction, we get the values of Cells $(7,3),(9,7),(4,2)$ as ‘4', '2', and '7' respectively.

| 9 | 1 | 3 | 2 | 8 | 6 | 7 | 5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 8 | 5 | 4 | 3 | 9 | 1 | 2 | 6 |
| 6 | 4 | 2 | 5 | 7 | 1 | 3 | 8 | 9 |
| 4 | 7 | 9 | 8 | 2 | 5 | 6 | 1 | 3 |
| 2 | 6 | 8 | 1 | 4 | 3 | 5 | 9 | 7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | 5 | 4 | 7 | 1 | 2,4 | 2,9 | 6 | 8 |
| 1 | 2 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | 4,6 | 3 | 5 | 2,4 | 2 | 7 | 1 |

Finally, we get the values of the Cells $(9,3),(9,6),(7,6)$ and $(7,7)$ as '6', '4', '2' and '9' respectively.

## Solving Regular Sudoku Puzzles

| 9 | 1 | 3 | 2 | 8 | 6 | 7 | 5 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7 | 8 | 5 | 4 | 3 | 9 | 1 | 2 | 6 |
| 6 | 4 | 2 | 5 | 7 | 1 | 3 | 8 | 9 |
| 4 | 7 | 9 | 8 | 2 | 5 | 6 | 1 | 3 |
| 2 | 6 | 8 | 1 | 4 | 3 | 5 | 9 | 7 |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | 5 | 4 | 7 | 1 | 2 | 9 | 6 | 8 |
| 1 | 2 | 7 | 9 | 6 | 8 | 4 | 3 | 5 |
| 8 | 9 | 6 | 3 | 5 | 4 | 2 | 7 | 1 |

Hey, we've done this one too!
Let's now see a difficult Sudoku:

|  |  | 3 |  |  |  | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 |  |  | 4 |  | 5 |  | 1 |  |
| 2 |  |  | 1 |  | 7 |  |  |  |
|  |  |  | 7 |  |  |  | 9 | 3 |
|  |  |  |  | 4 |  |  |  |  |
| 7 | 1 |  |  |  | 6 |  |  |  |
|  |  |  | 5 |  | 9 |  |  | 4 |
|  | 5 |  | 2 |  | 4 |  |  | 6 |
|  |  | 2 |  |  |  | 5 |  |  |

Looks like it may not reduce easily, and it may also not be easily possible to identify solutions for any cell. So, let's just go ahead with the Possibility Matrix, as below:

| $\begin{aligned} & 1,4,5,6, \\ & 9 \end{aligned}$ | 4,6,7,9 | 3 | 6,8,9 | 6,8,9 | 8 | 2 | $\begin{array}{\|l\|} \hline \hline 4,5,6,7, \\ 8 \\ \hline \end{array}$ | 5,7,8,9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,7,9 | 6,7,9 | 4 | 2 | 5 | 3,6,7,9 | 1 | 7,9 |
| 2 | 4,6,9 | 5 | 1 | 3 | 7 | $\begin{aligned} & 3,4,6,8, \\ & 9 \end{aligned}$ | $\begin{aligned} & 3,4,5,6, \\ & 8 \end{aligned}$ | 5,8,9 |
| 4,5,6 | 2,4,6,8 | 4,5,6,8 | 7 | 1,2,5,8 | 1,2,8 | 1,4,6,8 | 9 | 3 |
| 3,5,6,9 | $\begin{aligned} & 2,3,6,8, \\ & 9 \end{aligned}$ | 5,6,8,9 | 3,8,9 | 4 | 1,2,3,8 | 1,6,7,8 | $\begin{aligned} & 2,5,6,7, \\ & 8 \end{aligned}$ | $\begin{aligned} & 1,2,5,7, \\ & 8 \end{aligned}$ |
| 7 | 1 | 4,5,8,9 | 3,8,9 | $\begin{aligned} & 2,3,5,8, \\ & 9 \end{aligned}$ | 6 | 4,8 | 2,4,5,8 | 2,5,8 |
| 1,3,6 | 3,6,7,8 | 1,6,7,8 | 5 | $\begin{aligned} & 1,3,6,7, \\ & 8 \end{aligned}$ | 9 | 1,3,7,8 | 2 | 4 |
| 1,3,9 | 5 | 1,7,8,9 | 2 | 1,3,7,8 | 4 | $\begin{aligned} & 1,3,7,8, \\ & 9 \end{aligned}$ | 3,7,8 | 6 |
| $\begin{aligned} & 1,3,4,6, \\ & 9 \end{aligned}$ | $\begin{aligned} & 3,4,6,7, \\ & 8,9 \end{aligned}$ | 2 | 3,6,8 | $\begin{aligned} & 1,3,6,7 \\ & 8 \end{aligned}$ | 1,3,8 | 5 | 3,7,8 | 1,7,8,9 |

## Solving Regular Sudoku Puzzles

Let's now reduce it as far as possible:

| 1 | $4,6,7,9$ | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,6,7$ | $5,7,9$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, <br> 9 | $6,8,9$ | 8,9 | 4 | $1,2,3$ | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 3,9 | 5 | $1,7,8,9$ | 2 | $1,7,8$ | 4 | $1,7,8,9$ | $3,7,8$ | 6 |
| $3,4,6,9$ | $3,4,6,7$, <br> 8,9 | 2 | $3,6,8$ | $1,6,7,8$ | 1,3 | 5 | $3,7,8$ | $1,7,8,9$ |

Looking at the Major Squares, and filling in unique values, we have:

| 1 | $4,6,7,9$ | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,6,7$ | $5,7,9$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, <br> 9 | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 3,9 | 5 | $1,7,8,9$ | 2 | $1,7,8$ | 4 | $1,7,8,9$ | $3,7,8$ | 6 |
| $4,6,9$ | $4,6,7,8$, <br> 9 | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | $1,7,8,9$ |

Since Cells $(1,4)$ and $(1,5)$ can take only values ' 6 ' or ' 9 ', remove these from all other cells in the Row 1 , and reduce further:

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 3,9 | 5 | $1,7,8,9$ | 2 | $1,7,8$ | 4 | $1,7,8,9$ | $3,7,8$ | 6 |
| $4,6,9$ | $4,6,7,8$, <br> 9 | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | $1,7,8,9$ |

Cell $(8,8)$ is the only one that can take the vale ' 3 ' in Right Bottom
Square. Assign it. Now it reduces to:

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $\mathbf{¥}, 5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | $4,6,7,8$ | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | $1,7,8,9$ |

## Solving Regular Sudoku Puzzles

Since Cell $(9,9)$ is the only one that can take the value ' 9 ' in Right Bottom Square, assign it.

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $\mathbf{4 , 5 , 8}$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6$ | $2,3,6,8$, <br> 9 | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | $4,6,7,8$ | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | 9 |

Now, it further reduces to:

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | 5,8 | 1 | $4,6,8$ | 9 | 3 |
| $3,5,6$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 2 | $6,7,8$ | $5,6,7,8$ | 1 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | 6,8 | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | 9 |

Since value 'l' is unique to Cell $(9,5)$ in Row 9 , assign it, and the table reduces to:

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | 5,8 | 1 | $4,6,8$ | 9 | 3 |
| $3,5,6$ | $3,6,8,9$ | $6,8,9$ | 8,9 | 4 | 2 | $6,7,8$ | $5,6,8$ | 1 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | 1,8 | 3 | 6 |
| 4,6 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

Since value ' 7 ' is unique to Cell $(5,7)$ in Column 7 , assign it. It reduces to:

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| $4,5,6$ | $2,6,8$ | $4,6,8$ | 7 | 5,8 | 1 | $4,6,8$ | 9 | 3 |
| $3,5,6$ | $2,3,6,8$, <br> 9 | $6,8,9$ | 8,9 | 4 | 2 | 7 | $5,6,8$ | 1 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | 1,8 | 3 | 6 |
| 4,6 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

## Solving Regular Sudoku Puzzles

Since value '6' is unique to Cell (4,7) in Column 7, assign it. It reduces to:

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 4,5 | 2,8 | 4,8 | 7 | 5,8 | 1 | 6 | 9 | 3 |
| $3,5,6$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 2 | 7 | 5,8 | 1 |
| 7 | 1 | 8,9 | 3 | $5,8,9$ | 6 | 8 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | 1,8 | 3 | 6 |
| 4,6 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

Since value '2' is unique to Cell $(4,2)$ in Row 4 , assign it. Then, since value '4' is unique to Cell $(4,3)$ in Column 3, assign it. It reduces to

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4 | 7 | 8 | 1 | 6 | 9 | 3 |
| 3,6 | $3,6,8,9$ | $6,8,9$ | 8,9 | 4 | 2 | 7 | 5,8 | 1 |
| 7 | 1 | 8,9 | 3 | $5,8,9$ | 6 | 4 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | 1,8 | 3 | 6 |
| 4,6 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

Since value ' 5 ' is unique to Cell $(6,5$ ) in Column 5 , assign it; we have:

| 1 | 7 | 3 | 6 | 9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4 | 7 | 8 | 1 | 6 | 9 | 3 |
| 3,6 | $3,6,8$ | 6,8 | 9 | 4 | 2 | 7 | 5,8 | 1 |
| 7 | 1 | 8,9 | 3 | 5 | 6 | 4 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | 6 | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7 | 4 | 1,8 | 3 | 6 |
| 4,6 | 6,8 | 2 | 8 | 1 | 3 | 5 | 7 | 9 |

Since vale ' 5 ' is unique to Cell $(5,8)$ in Row 8 , assign it and we have:

| 1 | 7 | 3 | 6 | 9 | 8 | 2 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 9 | 6 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4 | 7 | 8 | 1 | 6 | 9 | 3 |
| 6 | 3 | 8 | 9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 9 | 3 | 5 | 6 | 4 | 8 | 2 |
| 3 | 8 | 7 | 5 | 6 | 9 | 1 | 2 | 4 |
| 9 | 5 | 1 | 2 | 7 | 4 | 8 | 3 | 6 |
| 4 | 6 | 2 | 8 | 1 | 3 | 5 | 7 | 9 |

## Solving Regular Sudoku Puzzles

Hey, we made it! Know what? This is supposed to be a Difficult Puzzle.

Now, we're ready to take on any Sudoku!

## Solving Regular Sudoku Puzzles

## Tips:

How do you actually solve the Sudoku in a newspaper?
I recommend that you use a sharp pencil.
Initially you could write all possible numbers in a Possibility Matrix as we have shown in all the illustrations. And cross off the numbers you want to delete. When there is only one possible number for a Cell, you may erase all the numbers in that cell and rewrite by the only possible number in a larger size, in thick/ bold/ using a pen.

If you want to retain the steps you have gone thru, you may round the only possible number preferably with a pen, or highlight it with a highlighter, so it stands out.

## Solving Regular Sudoku Puzzles

## Sudoku Exercises

Now, we need to practice solving Sudoku puzzles!
Visit http:/ /www.websudoku.com/ , select a level of difficulty, and play! There are billions of Sudokus to choose from and try! Absolutely Free! Solutions are also available for you to verify.

Here are a few Sudokus for your ready trial:
Have Fun!

Level - easy

|  |  |  | 1 | 5 |  |  | 7 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  | 6 |  |  |  | 8 | 2 |  |
| 3 |  |  | 8 | 6 |  |  | 4 |  |
| 9 |  |  | 4 |  |  | 5 | 6 | 7 |
|  |  | 4 | 7 |  | 8 | 3 |  |  |
| 7 | 3 | 2 |  |  | 6 |  |  | 4 |
|  | 4 |  |  | 8 | 1 |  |  | 9 |
|  | 1 | 7 |  |  |  | 2 |  | 8 |
|  | 5 |  |  | 3 | 7 |  |  |  |

## Solving Regular Sudoku Puzzles

Level - easy:
(2)

|  | 5 |  |  |  | 1 | 4 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 |  | 3 |  |  |  | 7 |  |  |
|  | 7 |  | 3 |  |  | 1 | 8 | 2 |
|  |  | 4 |  | 5 |  |  |  | 7 |
|  |  |  | 1 |  | 3 |  |  |  |
| 8 |  |  |  | 2 |  | 6 |  |  |
| 1 | 8 | 5 |  |  | 6 |  | 9 |  |
|  |  | 2 |  |  |  | 8 |  | 3 |
|  |  | 6 | 4 |  |  |  | 7 |  |

Level - Medium:
(1)

| 7 |  |  |  |  |  |  |  | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 2 |  |  | 1 |  |  | 3 | 4 |
|  | 6 |  | 7 |  | 9 |  | 5 |  |
|  |  |  | 6 |  | 4 |  |  |  |
| 4 |  |  |  |  |  |  |  | 7 |
|  |  |  | 2 |  | 1 |  |  |  |
|  | 5 |  | 4 |  | 8 |  | 1 |  |
| 1 | 9 |  |  | 6 |  |  | 4 | 8 |
| 8 |  |  |  |  |  |  |  | 2 |

## Level - Medium:

(2)

|  |  | 4 |  | 5 |  |  | 6 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 6 |  | 1 |  |  | 8 |  | 9 |
| 3 |  |  |  |  | 7 |  |  |  |
|  | 8 |  |  |  |  | 5 |  |  |
|  |  |  | 4 |  | 3 |  |  |  |
|  |  | 6 |  |  |  |  | 7 |  |
|  |  |  | 2 |  |  |  |  | 6 |
| 1 |  | 5 |  |  | 4 |  | 3 |  |
|  | 2 |  |  | 7 |  | 1 |  |  |

Level - Hard:
(1)

|  |  | 1 |  |  | 8 |  |  | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 3 | 2 | 1 |  |  |  |  | 4 |
|  | 7 |  | 6 |  |  |  | 3 |  |
|  | 2 | 5 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  | 8 |
|  |  |  |  |  |  | 6 | 1 |  |
|  | 9 |  |  |  | 6 |  | 4 |  |
| 3 |  |  |  |  | 4 | 9 | 7 |  |
| 1 |  |  | 3 |  |  | 8 |  |  |

## Solving Regular Sudoku Puzzles

Level - Hard:
(2)

|  | 7 |  |  | 1 |  |  | 9 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9 |  |  | 8 |  |  |  |  | 7 |
|  |  | 3 |  |  |  |  |  | 6 |
|  | 4 |  |  |  | 1 | 5 |  |  |
|  | 3 |  |  |  |  |  | 1 |  |
|  |  | 2 | 7 |  |  |  | 6 |  |
| 5 |  |  |  |  |  | 6 |  |  |
| 6 |  |  |  |  | 5 |  |  | 2 |
|  | 8 |  |  | 2 |  |  | 7 |  |

Level - Very Hard:
(1)

| 1 |  |  | 8 |  |  |  | 3 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7 |  |  |  |  |  |  | 2 |  |
|  |  |  | 5 | 6 |  |  | 7 |  |
|  |  | 8 |  |  |  | 9 |  |  |
|  |  | 5 | 2 | 1 | 7 | 4 |  |  |
|  |  | 4 |  |  |  | 7 |  |  |
|  | 3 |  |  | 8 | 9 |  |  |  |
| 2 |  |  |  |  |  |  | 8 |  |
| 8 |  |  |  | 4 |  |  | 6 |  |

## Solving Regular Sudoku Puzzles

Level - Very Hard:
(2)

| 6 |  |  |  | 1 |  | 5 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 |  | 3 |  |  |  |  |  |  |
|  |  |  |  | 6 |  |  | 2 |  |
|  | 3 |  | 1 |  | 8 |  | 9 |  |
| 1 |  |  |  | 9 |  |  |  | 4 |
|  | 5 |  | 2 |  | 3 |  | 1 |  |
|  | 7 |  |  | 3 |  |  |  |  |
|  |  |  |  |  |  | 3 |  | 6 |
|  |  | 4 |  | 5 |  |  |  | 9 |

Solutions to the Above Sudoku Puzzles:
Level - Easy:
(1)

| 4 | 2 | 8 | 1 | 5 | 9 | 6 | 7 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 9 | 6 | 3 | 7 | 4 | 8 | 2 | 5 |
| 3 | 7 | 5 | 8 | 6 | 2 | 9 | 4 | 1 |
| 9 | 8 | 1 | 4 | 2 | 3 | 5 | 6 | 7 |
| 5 | 6 | 4 | 7 | 1 | 8 | 3 | 9 | 2 |
| 7 | 3 | 2 | 5 | 9 | 6 | 1 | 8 | 4 |
| 2 | 4 | 3 | 6 | 8 | 1 | 7 | 5 | 9 |
| 6 | 1 | 7 | 9 | 4 | 5 | 2 | 3 | 8 |
| 8 | 5 | 9 | 2 | 3 | 7 | 4 | 1 | 6 |

## Solving Regular Sudoku Puzzles

Level - Easy:
(2)

| 6 | 5 | 8 | 2 | 7 | 1 | 4 | 3 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 1 | 3 | 8 | 9 | 4 | 7 | 5 | 6 |
| 4 | 7 | 9 | 3 | 6 | 5 | 1 | 8 | 2 |
| 9 | 2 | 4 | 6 | 5 | 8 | 3 | 1 | 7 |
| 5 | 6 | 7 | 1 | 4 | 3 | 9 | 2 | 8 |
| 8 | 3 | 1 | 9 | 2 | 7 | 6 | 4 | 5 |
| 1 | 8 | 5 | 7 | 3 | 6 | 2 | 9 | 4 |
| 7 | 4 | 2 | 5 | 1 | 9 | 8 | 6 | 3 |
| 3 | 9 | 6 | 4 | 8 | 2 | 5 | 7 | 1 |

Level - Medium:
(1)

| 7 | 1 | 4 | 5 | 3 | 2 | 8 | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 2 | 9 | 8 | 1 | 6 | 7 | 3 | 4 |
| 3 | 6 | 8 | 7 | 4 | 9 | 2 | 5 | 1 |
| 2 | 7 | 1 | 6 | 8 | 4 | 3 | 9 | 5 |
| 4 | 8 | 6 | 9 | 5 | 3 | 1 | 2 | 7 |
| 9 | 3 | 5 | 2 | 7 | 1 | 4 | 8 | 6 |
| 6 | 5 | 7 | 4 | 2 | 8 | 9 | 1 | 3 |
| 1 | 9 | 2 | 3 | 6 | 7 | 5 | 4 | 8 |
| 8 | 4 | 3 | 1 | 9 | 5 | 6 | 7 | 2 |

## Level - Medium:

(2)

| 2 | 1 | 4 | 9 | 5 | 8 | 3 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 6 | 7 | 1 | 3 | 2 | 8 | 4 | 9 |
| 3 | 9 | 8 | 6 | 4 | 7 | 2 | 1 | 5 |
| 4 | 8 | 1 | 7 | 9 | 6 | 5 | 2 | 3 |
| 7 | 5 | 2 | 4 | 8 | 3 | 6 | 9 | 1 |
| 9 | 3 | 6 | 5 | 2 | 1 | 4 | 7 | 8 |
| 8 | 4 | 3 | 2 | 1 | 9 | 7 | 5 | 6 |
| 1 | 7 | 5 | 8 | 6 | 4 | 9 | 3 | 2 |
| 6 | 2 | 9 | 3 | 7 | 5 | 1 | 8 | 4 |

Level - Hard:
(1)

| 5 | 4 | 1 | 7 | 3 | 8 | 2 | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 3 | 2 | 1 | 5 | 9 | 7 | 8 | 4 |
| 8 | 7 | 9 | 6 | 4 | 2 | 1 | 3 | 5 |
| 7 | 2 | 5 | 8 | 6 | 1 | 4 | 9 | 3 |
| 4 | 1 | 6 | 9 | 7 | 3 | 5 | 2 | 8 |
| 9 | 8 | 3 | 4 | 2 | 5 | 6 | 1 | 7 |
| 2 | 9 | 7 | 5 | 8 | 6 | 3 | 4 | 1 |
| 3 | 5 | 8 | 2 | 1 | 4 | 9 | 7 | 6 |
| 1 | 6 | 4 | 3 | 9 | 7 | 8 | 5 | 2 |

## Solving Regular Sudoku Puzzles

Level - Hard:
(2)

| 4 | 7 | 8 | 5 | 1 | 6 | 2 | 9 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9 | 6 | 5 | 8 | 3 | 2 | 1 | 4 | 7 |
| 2 | 1 | 3 | 4 | 9 | 7 | 8 | 5 | 6 |
| 7 | 4 | 9 | 3 | 6 | 1 | 5 | 2 | 8 |
| 8 | 3 | 6 | 2 | 5 | 9 | 7 | 1 | 4 |
| 1 | 5 | 2 | 7 | 4 | 8 | 3 | 6 | 9 |
| 5 | 2 | 4 | 9 | 7 | 3 | 6 | 8 | 1 |
| 6 | 9 | 7 | 1 | 8 | 5 | 4 | 3 | 2 |
| 3 | 8 | 1 | 6 | 2 | 4 | 9 | 7 | 5 |

Level - Very Hard:
(1)

| 1 | 5 | 9 | 8 | 7 | 2 | 6 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7 | 6 | 3 | 9 | 4 | 1 | 8 | 2 | 5 |
| 8 | 4 | 2 | 5 | 6 | 3 | 1 | 7 | 9 |
| 2 | 7 | 8 | 4 | 3 | 5 | 9 | 6 | 1 |
| 6 | 9 | 5 | 2 | 1 | 7 | 4 | 8 | 3 |
| 3 | 1 | 4 | 6 | 9 | 8 | 7 | 5 | 2 |
| 5 | 3 | 6 | 1 | 8 | 9 | 2 | 4 | 7 |
| 4 | 2 | 1 | 7 | 5 | 6 | 3 | 9 | 8 |
| 9 | 8 | 7 | 3 | 2 | 4 | 5 | 1 | 6 |

Level - Very Hard:
(2)

| 6 | 9 | 2 | 8 | 1 | 7 | 5 | 4 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 1 | 3 | 4 | 2 | 5 | 9 | 6 | 7 |
| 7 | 4 | 5 | 3 | 6 | 9 | 8 | 2 | 1 |
| 2 | 3 | 6 | 1 | 4 | 8 | 7 | 9 | 5 |
| 1 | 8 | 7 | 5 | 9 | 6 | 2 | 3 | 4 |
| 4 | 5 | 9 | 2 | 7 | 3 | 6 | 1 | 8 |
| 9 | 7 | 8 | 6 | 3 | 1 | 4 | 5 | 2 |
| 5 | 2 | 1 | 9 | 8 | 4 | 3 | 7 | 6 |
| 3 | 6 | 4 | 7 | 5 | 2 | 1 | 8 | 9 |

There are many who already solve Sudoku using a method known to them already. This method may have been self acquired intuitively, or learnt from elsewhere.

The problems with such methods are:
(i) You are not guaranteed to get the result, since you're given a number of tools, and told to apply the right one at the right times.

And no one can tell which the right method in any given situation is. So, your chances of half solving most Sudokus are very high; and half solving isn't, by any means, the same as solving. Many people have reported that they could solve no more than simple ones, at best.
(ii) The tools are hard to learn and use.
(iii) The methods used to teach them are hard too.
(iv) When multiple solutions are possible, chances are you may not know that. And consequently, you can almost never find all the solutions to any such puzzle. They presume that a given puzzle has one and only one solution. And this may well be most often true, particularly with puzzles published in reputed magazines. However, there are significant exceptions, and it is with these exceptional puzzles that the Conventional Methods may fail. A case in point is the 5,000

Pound $(\$ 8,500)$ Prize Money Sudoku published at:
http:// www.skyone.co.uk/ programme/pgefeature.aspx?pid=488fid=129
(v) When your attempts to solve a puzzle lead to a point of no correct solution, you may never know if the puzzle is in error, or you have made a mistake. Since the approaches themselves give rise to high chances of making errors, you never know whether it is the puzzle that is in error, or you have made an error along the way.

What is the main difference between these methods and my method? Most of the other methods are variations of just one method, viz., trying to find out which should take a certain value.

And my method tries to find out what value a certain should take.

What if you want to continue to solve Sudoku the way you have been used to, and use my method only when you're stuck?

No issues. My method is such that you could start using it from any situation, assuming, of course, that you have solved correctly so far.

And my method is exactly the same whether you start using it from the start or from midway; you need to make no changes. This is because, even mid way, you could treat the puzzle as the one you have after you have part solved it.

Let's see with an example. Let's take one of the difficult puzzles we've seen.

|  |  | 3 |  |  |  | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 |  |  | 4 |  | 5 |  | 1 |  |
| 2 |  |  | 1 |  | 7 |  |  |  |
|  |  |  | 7 |  |  |  | 9 | 3 |
|  |  |  |  | 4 |  |  |  |  |
| 7 | 1 |  |  |  | 6 |  |  |  |
|  |  |  | 5 |  | 9 |  |  | 4 |
|  | 5 |  | 2 |  | 4 |  |  | 6 |
|  |  | 2 |  |  |  | 5 |  |  |

Let's say you started solving it using your own method, and could get this far, but are unable to proceed; you're stuck.

| 1 |  | 3 |  |  | 8 | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 |  |  | 4 | 2 | 5 | 3 | 1 |  |
| 2 |  | 5 | 1 | 3 | 7 |  |  |  |
|  |  |  | 7 |  |  |  | 9 | 3 |
|  |  |  |  | 4 |  |  |  |  |
| 7 | 1 |  | 3 |  | 6 |  |  | 2 |
|  |  |  | 5 |  | 9 |  | 2 | 4 |
|  | 5 |  | 2 |  | 4 |  |  | 6 |
|  |  | 2 |  |  | 3 | 5 |  |  |

Just consider this as your given Sudoku, and solve it using my method. You get the following Possibility Matrix:

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 3,9 | 5 | $1,7,8,9$ | 2 | $1,7,8$ | 4 | $1,7,8,9$ | $3,7,8$ | 6 |
| $3,4,6,9$ | $3,4,6,7$, | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | $3,7,8$ | $1,7,8,9$ |

This reduces to:.

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 3,9 | 5 | $1,7,8,9$ | 2 | $1,7,8$ | 4 | $1,7,8,9$ | $3,7,8$ | 6 |
| $4,6,9$ | $4,6,7,8$, <br> 9 | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | $1,7,8,9$ |

We get the value of Cell $(8,8)$ as ' 3 ' in the Bottom Right Major Square being the unique value. Let's assign this value to it.

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 3,9 | 5 | $1,7,8,9$ | 2 | $1,7,8$ | 4 | $1,7,8,9$ | 3 | 6 |
| $4,6,9$ | $4,6,7$, | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | $1,7,8,9$ |

Removing the value ' 3 ' from Cell ( 8,1 ), we get the value of Cell $(8,1)$ as ' 9 ' by reduction.

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | $4,6,7,8$ | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | $1,7,8,9$ |

Now, we get the value of Cell $(9,9)$ as ' 9 ' being the unique value in the Bottom Right Major Square.

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7,9 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | $4,6,8,9$ | $4,6,8$ | 8,9 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | $4,6,7$, | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | 9 |

If we simplify the puzzle further we get the values for the Cells $(2,9)$ and $(3,9)$ as ' 7 ' and ' 8 ' respectively. Cell $(3,7)$ takes the value ' 9 ' being the unique value in C 7 .
Let us assign these values to their respective Cells.

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | $4,5,7$ | 5,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $6,7,9$ | $6,7,9$ | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | $4,6,9$ | 5 | 1 | 3 | 7 | 9 | $4,6,8$ | 8 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | $1,5,7,8$ |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | $4,6,7,8$ | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | 9 |

By further reduction, we get values to the Cells $(1,9)$ as ' 5 ', Cell $(3,8)$ takes the value ' 6 ', being the unique value in Top Right Major Square.

| 1 | 4,7 | 3 | 6,9 | 6,9 | 8 | 2 | 4,5 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4,6 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $1,4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $1,6,7,8$ | $5,6,7,8$ | 1,5 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | $4,6,7$, | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | 9 |

By further reduction we get the values to the Cells $(1,8),(5,9)$ and $(3,2)$ as '4', '1' and '4' respectively. Cell $(1,2)$ takes the value '7' being the unique value in R1

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| $4,5,6$ | $2,4,6,8$ | $4,6,8$ | 7 | $1,5,8$ | 1,2 | $4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $2,3,6,8$, | $6,8,9$ | 8,9 | 4 | 1,2 | $6,7,8$ | $5,7,8$ | 1 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | $4,5,8$ | 2 |
| 3,6 | $3,6,7,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | $4,6,7$, | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | 9 |

By further reduction, we get the value of Cell $(5,6)$ as ' 2 '. And, we get the value to Cell $(4,6)$ as ' 1 '.

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| $4,5,6$ | $2,6,8$ | $4,6,8$ | 7 | 5,8 | 1 | $4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $3,6,8$, | $6,8,9$ | 8,9 | 4 | 2 | $6,7,8$ | $5,7,8$ | 1 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $1,6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | $1,7,8$ | 4 | $1,7,8$ | 3 | 6 |
| 4,6 | 6,8 | 2 | 6,8 | $1,6,7,8$ | 3 | 5 | 7,8 | 9 |

Now, we get the value of Cell $(4,2)$ as '2' being the unique value in column 2. Similarly, we get the values of Cells $(9,1)$ and $(9,5)$, as '4' and '1' respectively being the unique values in rows ' 9 '. Let us assign these values to their corresponding Cells.

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5,6 | 2 | $4,6,8$ | 7 | 5,8 | 1 | $4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $3,6,8$, | $6,8,9$ | 8,9 | 4 | 2 | $6,7,8$ | $5,7,8$ | 1 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | $1,7,8$ | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | $1,7,8$ | 3 | 6 |
| 4 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7,8 | 9 |

Now, we get the value of Cell $(9,8)$ as '7' being the unique value in row '9'.

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5,6 | 2 | $4,6,8$ | 7 | 5,8 | 1 | $4,6,8$ | 9 | 3 |
| $3,5,6,9$ | $3,6,8$, | $6,8,9$ | 8,9 | 4 | 2 | $6,7,8$ | 5,8 | 1 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4,8 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | 1,8 | 3 | 6 |
| 4 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

On further simplification, we get the value of Cell $(5,7)$ as '7' being the unique value in the Mid Right Major Square.

At this stage, Cells $(7,7)$ and $(8,7)$ in column '7' can take the possible values ' 1 ' and ' 8 ' between themselves. Therefore, we can eliminate the values '1' and ' 8 ' as possible values from the other Cells in that column by applying group reduction. Consequently, we get the value of Cell $(6,7)$ as ' 4 '. Let us assign these values to their respective Cells.

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5,6 | 2 | $4,6,8$ | 7 | 5,8 | 1 | 4,6 | 9 | 3 |
| $3,5,6,9$ | $3,6,8$, | $6,8,9$ | 8,9 | 4 | 2 | 7 | 5,8 | 1 |
| 7 | 1 | $4,8,9$ | 3 | $5,8,9$ | 6 | 4 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | 1,8 | 3 | 6 |
| 4 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

By further reduction we get '6' in the Cell $(4,7)$. Consequently, we get value to the Cell $(4,1)$ as ' 5 ' by reduction.

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | $4,6,8$ | 7 | 5,8 | 1 | 6 | 9 | 3 |
| $3,6,9$ | $3,6,8$, | $6,8,9$ | 8,9 | 4 | 2 | 7 | 5,8 | 1 |
| 7 | 1 | 8,9 | 3 | $5,8,9$ | 6 | 4 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | 1,8 | 3 | 6 |
| 4 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

By further reduction we get the values to the Cell $(4,5)$ as ' 8 '. Cell $(5,8)$ takes the value ' 5 ' being the unique value in R5.

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | $4,6,8$ | 7 | 8 | 1 | 6 | 9 | 3 |
| $3,6,9$ | $3,6,8$, | $6,8,9$ | 8,9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 8,9 | 3 | $5,8,9$ | 6 | 4 | 5,8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | $6,7,8$ | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7,8 | 4 | 1,8 | 3 | 6 |
| 4 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

By further reduction we get values to the Cells $(5,4),(8,5)$ and $(6,8)$ as '9', '7' and '8' respectively.

| 1 | 7 | 3 | 6,9 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4,6 | 7 | 8 | 1 | 6 | 9 | 3 |
| $3,6,9$ | $3,6,8$, | $6,8,9$ | 9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 8,9 | 3 | 5,9 | 6 | 4 | 8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | 6,7 | 9 | 1,8 | 2 | 4 |
| 9 | 5 | $1,7,8$ | 2 | 7 | 4 | 1,8 | 3 | 6 |
| 4 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

By reduction we get the values to the Cells $(6,5),(7,5),(6,3)$ and $(1,4)$ as '5', '6', '9' and '6' respectively.

| 1 | 7 | 3 | 6 | 6,9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6,9 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4,6 | 7 | 8 | 1 | 6 | 9 | 3 |
| 3,6 | $3,6,8$ | 6,8 | 9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 9 | 3 | 5 | 6 | 4 | 8 | 2 |
| 3,6 | $3,6,8$ | $1,6,7,8$ | 5 | 6 | 9 | 1,8 | 2 | 4 |
| 9 | 5 | 1,8 | 2 | 7 | 4 | 1,8 | 3 | 6 |
| 4 | 6,8 | 2 | 6,8 | 1 | 3 | 5 | 7 | 9 |

By reduction we get the values to Cells $(7,1),(9,4),(1,5)$ and $(2,3)$ as ' 3 ', ' 8 ', '9' and '6' respectively. Cell $(7,3)$ takes the value '7' being the unique value in R 7 .

| 1 | 7 | 3 | 6 | 9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 6,9 | 6 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4,6 | 7 | 8 | 1 | 6 | 9 | 3 |
| 3,6 | $3,6,8$ | 6,8 | 9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 9 | 3 | 5 | 6 | 4 | 8 | 2 |
| 3 | 3,8 | 7 | 5 | 6 | 9 | 1,8 | 2 | 4 |
| 9 | 5 | 1,8 | 2 | 7 | 4 | 1,8 | 3 | 6 |
| 4 | 6,8 | 2 | 8 | 1 | 3 | 5 | 7 | 9 |

By reduction we get the values of the Cells $(7,2),(5,1),(2,2),(4,3)$ and $(5,3)$ as '8' , '6' , '9' , '4' and '8' respectively. Cell $(9,2)$ takes the value '6' being the unique value within that Major Square. Similarly Cell $(8,3)$ takes ' 1 ' in C3.

| 1 | 7 | 3 | 6 | 9 | 8 | 2 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 9 | 6 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4 | 7 | 8 | 1 | 6 | 9 | 3 |
| 6 | 3,8 | 8 | 9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 9 | 3 | 5 | 6 | 4 | 8 | 2 |
| 3 | 8 | 7 | 5 | 6 | 9 | 1,8 | 2 | 4 |
| 9 | 5 | 1 | 2 | 7 | 4 | 1,8 | 3 | 6 |
| 4 | 6 | 2 | 8 | 1 | 3 | 5 | 7 | 9 |

By further reduction we get the values to the Cells $(5,2),(7,7)$ and $(8,7)$ as ' 3 ', ' 1 ' and ' 8 ' respectively.

| 1 | 7 | 3 | 6 | 9 | 8 | 2 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 9 | 6 | 4 | 2 | 5 | 3 | 1 | 7 |
| 2 | 4 | 5 | 1 | 3 | 7 | 9 | 6 | 8 |
| 5 | 2 | 4 | 7 | 8 | 1 | 6 | 9 | 3 |
| 6 | 3 | 8 | 9 | 4 | 2 | 7 | 5 | 1 |
| 7 | 1 | 9 | 3 | 5 | 6 | 4 | 8 | 2 |
| 3 | 8 | 7 | 5 | 6 | 9 | 1 | 2 | 4 |
| 9 | 5 | 1 | 2 | 7 | 4 | 8 | 3 | 6 |
| 4 | 6 | 2 | 8 | 1 | 3 | 5 | 7 | 9 |

Sudoku can be solved by manual, semi-automatic or automated methods. (By automatic, I mean using the Computer as a tool.) I'll arm you with a Swiss Knife tool so that you could use whichever of the knives you like, or whichever suits you best in a given situation.

Go right ahead, and take on every possible Sudoku under the Sun! Feel free to write to us in case you need any help at any stage.

All the best!

### 10.1 The Manual Solutions

There are essentially 2 approaches to solving the Sudoku puzzle manually:

1. The Conventional (or Intuitive) Method
2. The Possibility Matrix Method (A Mathematical/ Scientific Approach).

Incidentally, it is usually the 2nd approach that is used by software that solve Sudoku (or any similar) puzzles.

Let's know something about both the approaches now.

## The Sudoku Swiss Knife

### 10.1.1 - The Conventional Method

The conventional method is a more of a 'trial and error' method, with the risk of 'hit or miss'. People do tend to learn to hit more often than miss, with experience, but it suffers from the following shortcomings:

If a puzzle has no answers, you would not be sure. This is because if you are unable to get a solution, there is always the possibility that you may have missed out on a possible answer.

If it has answer(s), you are not $100 \%$ guaranteed to find it (them).
If the puzzle has multiple solutions, you may not know that there are multiple solutions, and you may not be able to easily find out every answer, if necessary.

If the answer you get is different from the one given by the source from where you got the puzzle, you may not know if it could be because the puzzle has multiple answers.

If you are caught in a puzzle with complex multiplicity of possibilities, you may not be able to solve it practically.

You may never be able to determine a reasonable timeframe within which you could solve, as with all 'trial and error' methods of solving complex puzzles.

There is no guarantee that you will, in deed, be able to solve a given puzzle by this method.

Yet, this is the method followed by most Sudoku solvers, since many people enjoy the 'random walk' method (even if with a logic); and people are used to it for a long time, as solvers of 'Crossword Puzzles'. But there is no reason why you should solve a mathematical puzzle the same way you would solve a language related puzzle.

Typically it involves identifying the right cells for every value.
Let's skip this method in this eBook, since it is fairly cerebral and more difficult to Learn, and not necessarily the best method to learn, and in any case not the best first method to learn.

## The Sudoku Swiss Knife

### 10.1.2 - The Possibility Matrix Method (A Mathematical Approach)

As you may have guessed, the Possibility Matrix Method is the opposite of the 'trial and error' method(s), with no risk of 'hit or miss'. Though this may take a little longer to solve simpler puzzles initially, being more rigorous, people tend to learn to solve puzzles faster with experience. And this involves identifying the right values for every Cell.

And it obviously has the following advantages:
If a puzzle has no answers, you would know, surely and in a deterministic time.

If it has an answer, you are $100 \%$ guaranteed to find it.
If the puzzle has multiple solutions, you would know that there are multiple solutions, and you can find out every answer, if necessary. But you can stop at one solution, if you want.

If the answer you get is different from the one given by the source from where you got the puzzle, you know if it is because the puzzle has multiple answers.

You won't get caught in a puzzle with complex multiplicity of possibilities, as you take every probability head on.

You can work to a reasonably deterministic timeframe within which you could solve it, however complex it may be. More complex puzzles would, of course, take more time to solve.

There is a guarantee that you will, in deed, be able to solve every given puzzle by this method.

Some people enjoy a 'random walk' method than a scientific method. They usually come with 'Crossword Puzzle' solution mindset and so they can't think of any other method.

But there is every reason why you should solve a mathematical puzzle differently than you would solve a language puzzle.

We've already learnt the Possibility Matrix Sudoku Solution.

|  | 1 |  |  |  |  |  | 5 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 5 | 4 | 3 | 9 | 1 |  |  |
| 6 |  |  |  | 7 |  |  |  | 9 |
| 4 |  |  | 8 | 2 | 5 |  |  | 3 |
|  |  | 8 |  |  |  | 5 |  |  |
| 5 |  |  | 6 | 9 | 7 |  |  | 2 |
| 3 |  |  |  | 1 |  |  |  | 8 |
|  |  | 7 | 9 | 6 | 8 | 4 |  |  |
|  | 9 |  |  |  |  |  | 7 |  |

Solving the above puzzle manually, we get the following Possibility Matrix! (Verify it)

| $\left.\right\|_{9} ^{2,7,8,}$ | 1 | ${ }_{9}^{2,3,4,}$ | 2 | 8 | 6 | ${ }_{7,8}^{2,3,6,}$ | 5 | 4.6,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2,7,8 | 2,78 | 5 | 4 | 3 | 9 | 1 | 2,6,8 | 6,7 |
| 6 | $\begin{aligned} & 2,3,4,4, \\ & 88 \end{aligned}$ | 2,34 | 5 | 7 | 1 | 2,3,8 | ${ }_{8}^{2,3,4}$ | 9 |
| 4 | 6,7 | 6,9 | 8 | 2 | 5 | 6,7, | 1 | 3 |
| 2,7,9 | 2,6,7 | 8 | 1 | 4 | 3 | 5 | 6 | 1,4,6, |
| 5 | 3 | 1 | 6 | 9 | 7 | 8 | 4 | 2 |
| 3 | ${ }_{6}^{2,4,5,}$ | 2,4, | 7 | 1 | 2,4 |  | 2,6,9 | 8 |
| 1 | 2,5 | 7 | 9 | 6 | 8 | 4 | 3 | 1,5 |
| 8 | 9 | ${ }_{6}^{1,24,}$ | 2,3,5 | 5 | 2,3,4 | 2,3,6 | 7 | 6 |

## The Sudoku Swiss Knife

## 10.2 - The Software-assisted Solution

As explained already, it's best to use software rather than waste one's time on a routine task like generating the Possibility Matrix. And as our customer for the eBook on Sudoku, you will receive a free download to a copy of this software at:
http:/ / www.howtosolveeverysudokupuzzle.com/dl/pm.php (Coming Soon)

Use the username and password sent to you thru email along with our acknowledgement email, acknowledging receipt of your order for the eBook.

If you want to generate the Possibility Matrix online, you can do so at: http:/ /www.howtosolveeverysudokupuzzle.com/olacc/pm.php

## The Sudoku Swiss Knife

## 10.3 - The Software Solution

As explained already, it's best to use software to solve puzzles if you don't have the time, or to verify your results. Remember, this software has the following additional facilities than typical Sudoku Solution software:
(i) It gives you estimates of the time required to solve manually,
(ii) It tells you if there are multiple solutions,
(iii) It gives you all the multiple solutions, if you want,
(iv) It tells you if the puzzle is in error, and has no valid solutions
(v) It is updated regularly, with more features being added, and so, is available online always.
http:/ / www.howtosolveeverysudokupuzzle.com/olacc/soln.php
If you want the step-by-step computer assisted solution, here it is:
http://www.howtosolveeverysudokupuzzle.com/olacc/sbs.php (Coming Soon)

As our customer for the eBook on Sudoku, you will have unlimited access to the use of these software, for ever.

## 10.4 - More Complex Levels of Sudoku And Solving Them

We could have Sudoku of Order 4, 5 and more. However, they are more difficult to solve. I propose to come out with these in our later versions, and I will send you a Free Copy when ready.

I propose to create a Sudoku Community and I'll send you an Invite when it's ready.

This is going to be a Unique Sudoku Community with features you don't see anywhere else. Let me shut up for now, so I don't reveal too much and kill the thrill of 'suspense'.

## 12. Sudoku resources

## sudoku with the daily news.

Newspapers that supply sudokus
http:/ / www.timesonline.co.uk/ article / 0,,18209-1770888,00.html
http:/ / www.thesun.co.uk/article / 0,,2005240037,00.htmI
http:/ /www.hindu.com/thehindu/sudoku/sudokusunq.htm
http:/ / www.sundayherald.com/np/sudoku.shtml

## SUDOKUS APLENTY.

http:/ /www.pocketgear.com/software_detail.asp?id=19203\&couponcod $e=m s f t 247146$
http:/ /www.rosboroughtech.com/products.html
http:/ /sudokus.blogspot.com/
http:/ / www.baroufasoft.net/sudoku.htm
http:/ / users.pandora.be/vandenberghe.jef/sudoku
http://www.jarrahbark.com/home_sudoku_play.html
http://sudoku.cc/
http:// www.nzherald.co.nz/ category/1500944/index.cfm?c_id=1500944
http:/ /www.lovatts.com.au/sudoku/sudoku.htm

## Sudoku resources:

http://www.thevoiceofreason.com/SpeedSudoku/SpeedSudokul.htm
http:// www.playatjoes.com/
http:/ / www.playonthego.com/ids38_51/ page30/index.htm
EASY, MEDIUM, HARD--- SUDOKUS FOR ALL.
http:/ / www.tiscali.co.uk/games/play/sudoku/index.html
http://www.guardian.co.uk/sudoku
http:/ /www.krazydad.com/sudoku/
http:/ /www.goobix.com/games/sudoku/
http:/ / www.daily-sudoku.com / daily_suduko.php
http:/ / foroplus.net/sudoku/sudoku.php
http:/ /www.puzzlechoice.com/pc/Sudoku_Puzzlex.html
http:/ / freespace.virgin.net/gold.stone/sudoku/index.htm
http:/ /sudoku.com.au/default4.aspx
http://www.number-logic.com/
http:/ / www.palmsudoku.com/pages/free.php
http://goobix.com/games/sudoku/

## Sudoku resources:

## TIPS, TRICKS AND THE GAME.

http:/ / www.so-sudoku.com/play.aspx
http://www.su-doku.net/
http:/ /www.lovatts.com.au/sudoku/sudoku.htm
http:/ / www.dailymail.co.uk/pages/dmstandard/article.html?in_article_i d=3490548in_page_id=1766
http:/ / sudokuholics.blogspot.com
http:/ / www.freewebs.com/ crbaker/sudoku/index.htm
http:/ / www.cs.uky.edu/ ~raphael/ sud.htmI
http:/ /www.sudoku.com/
http:/ /www.geocities.com/binnyva/ code/javascript/sudoku/
http:/ / www.nikoli.co.jp/puzzles/1/index-e.htm

## DIFFERING SUDOKU VERSIONS.

http:// www.boxerjam.com/games/wasabi/
http://www.amuniversal.com/ups/ features/sudoku/index.htm
http:/ / sudoku.toplog.nI/
http:/ / godoku.com/

