The Kakuro Kraze

Kakuro puzzles are currently sweeping the world!

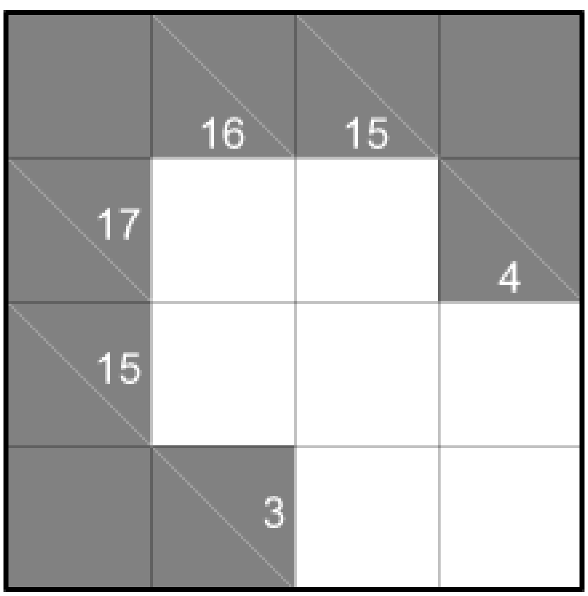
In Japan, the puzzles are well known and second in popularity only to Sudoku puzzles, which spawned a puzzle publishing craze in Europe and the United States in the spring of 2005. Various British newspapers started publishing Kakuro puzzles in late 2005, looking for a follow-up to Sudoku. The newspapers named the puzzles Kakuro, a variation of their Japanese name, Kakro. Some authors would have you believe the puzzles originated in Japan, but, like Sudoku, the puzzles actually come from the United States. Kakuro puzzles were originally published by Dell in the early 1950s under the name of Cross Sums.

But whether you call them Kakuro, Kakro, or Cross Sums, the puzzles are the same. Simple to understand, challenging to solve, and every bit as addicting as Sudoku!

All of these puzzles have only one solution, and can be solved without resorting to trial and error methods.

The Rules of Kakuro

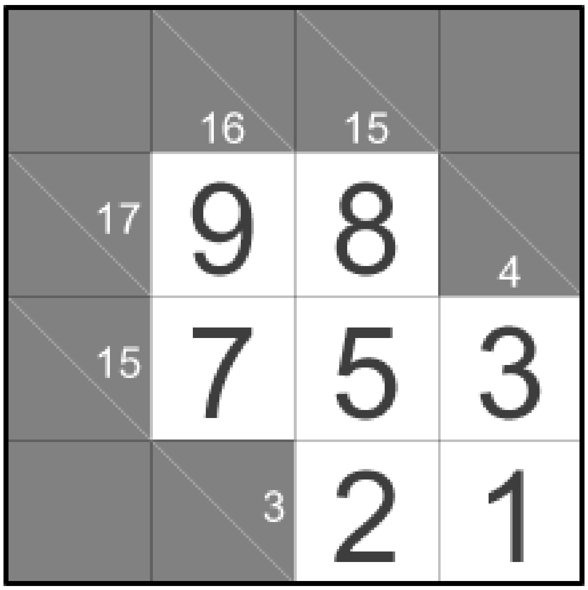
To explain kakuro, here's a simple puzzle:



As you can see, it closely resembles a crossword puzzle. Kakuro puzzles are essentially crossword puzzles with numbers instead of letters. The squares in the puzzle form *blocks* of connected squares (which would contain words in a regular crossword puzzle).

Each block has a *target,* which is simply the sum you get when you add the digits up in the block. The target serves the same function as a clue in a regular crossword. Unlike a regular crossword, the targets are shown on the puzzle diagram, rather than beneath it. The target is shown on the left side of the “across” blocks, and on the top of “down” blocks.

Here is the solved puzzle.

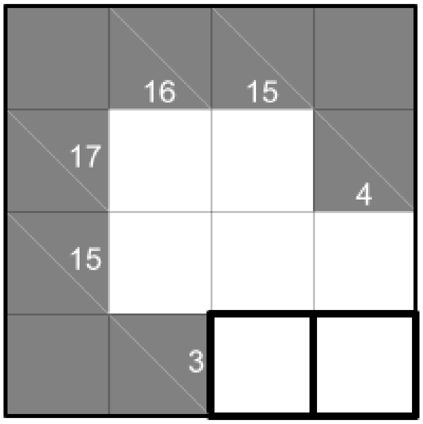


When solving a Kakuro puzzle, you take advantage of the following rules:

- Only the digits 1 through 9 appear in the puzzle. 0 is never used.
- Inside each block, the digits will never repeat. For example, in a block with two squares, and a target of 4 (what we would call a “4 in 2”) you will never see the digit 2, because in the equation 2+2=4 there are two 2s.

Solving Kakuro

There are a few basic principles you can use to solve Kakuro without trial and error. To explain them, we’ll go thru the process of solving two puzzles. We’ll start by solving small puzzle we just looked at.

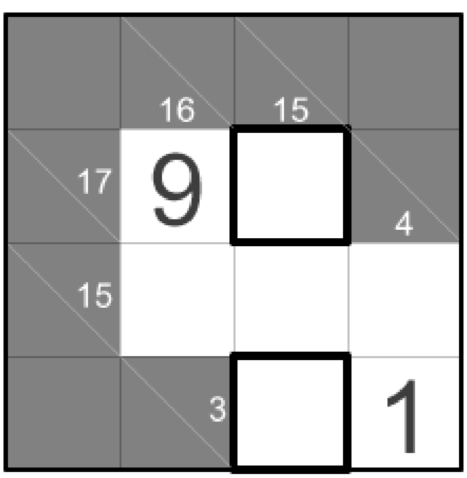


Restricted Blocks

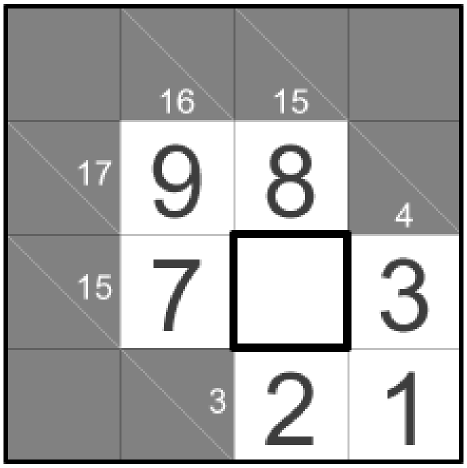
First look at the bottom two squares, which have a target of 3. There are only two possibilities for each square – 1 or 2. We don't know what order they appear, but we do know that only those two digits can be used. We call this a *restricted block*. There's another restricted block nearby. On the right side of the puzzle, the squares underneath the target 4 can only contain 1 or 3 (remember that we can't use 2, since the two would repeat in that block).

Because the two restricted blocks cross each other, the square in the bottom right corner has to be 1 - the only number both of the restricted blocks have in common. When two restricted blocks cross like this it can make a puzzle much easier to solve.

There are two more restricted blocks on the upper left side of the puzzle. Can you solve the square where they cross? The two top squares can only be 9 or 8, while the two squares on the left can only be 9 or 7. This means that the square on the top left hand corner must be 9.



Now that we've solved the corners, we can fill in the rest of the 2-square blocks, which leaves only one unsolved square in the middle, which you can easily figure out.



As you can see, restricted blocks are a very useful tool for solving Kakuro! Most solvers start by keeping a table of the restricted blocks around as a reference. Here's a table of the restricted blocks under 8.

Length of Block	Target	Possible Digits
2	3	1,2
2	4	1,3
2	16	9,7
2	17	9,8
3	6	1,2,3
3	7	1,2,4
3	23	9,8,6
3	24	9,8,7
4	10	1,2,3,4
4	11	1,2,3,5
4	29	9,8,7,5
4	30	9,8,7,6
5	15	1,2,3,4,5
5	16	1,2,3,4,6
5	34	9,8,7,6,4
5	35	9,8,7,6,5
6	21	1,2,3,4,5,6
6	22	1,2,3,4,5,7
6	38	9,8,7,6,5,3
6	39	9,8,7,6,5,4
7	28	1,2,3,4,5,6,7
7	29	1,2,3,4,5,6,8
7	41	9,8,7,6,5,4,2
7	42	9,8,7,6,5,4,3

If you examine the table carefully, you'll see that they follow a pattern. Notice that there are 4 restricted blocks for each length. This pattern changes when you get up to lengths of 8 and 9, which are special cases.

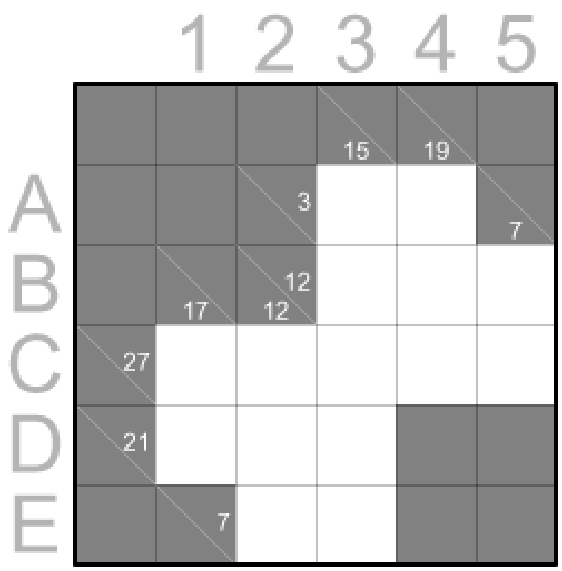
If there are 9 digits, then all the digits are used, and they will always sum to 45.

1+2+3+4+5+6+7+8+9 = 45.

All blocks with a length of 8 have a restricted set of digits - the missing digit will always be 45 minus the target. So for a block of length 8 with a target of 42, we can see that 45-42 = 3, so 3 must be the missing digit.

You can use this same principle for other long blocks as well. The missing digits will always add up to 45 minus the target. So, for example, in a block with a length of 7 and a target of 40, you know that the 2 missing digits add up to 5, and that they must be either (2,3) or (1,4). Hence the digits 5,6,7,8,9 must appear somewhere in the block.

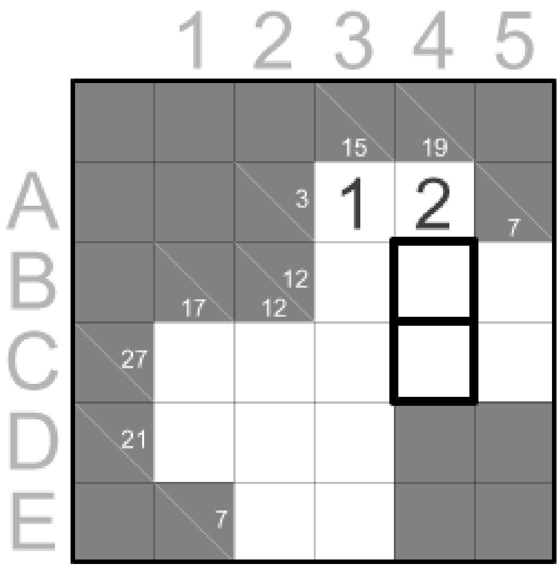
Now let's solve a larger puzzle, to see what other strategies come into play.



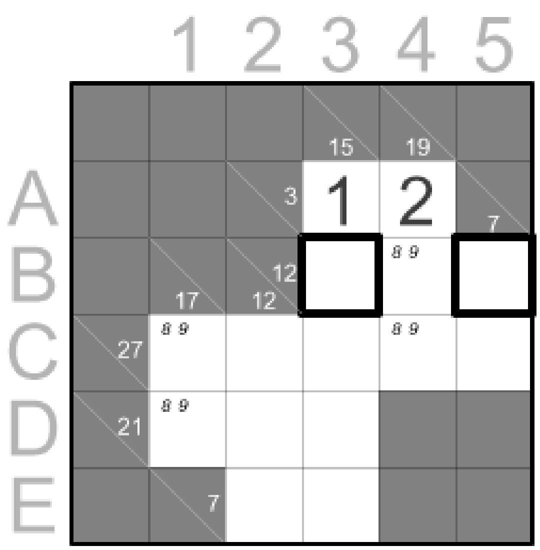
First let's look for restricted blocks. You'll notice that there are a few of them, but that none of them cross. What to do?

Eliminating the impossible

Let's start by examining square A4. Notice that because it's part of a restricted 3 in 2 block, it can only have a value of 1 or 2. Consider what would happen if it had a value of 1. If so, the remaining two squares in the 19 block would sum to 18. The only two digits that can sum to 18 are 9+9, which are not allowed. 17 is the highest sum you can have for two squares in a block. This means that we can't use a 1 in A4 – hence it must be 2, and square A3 must be the 1.



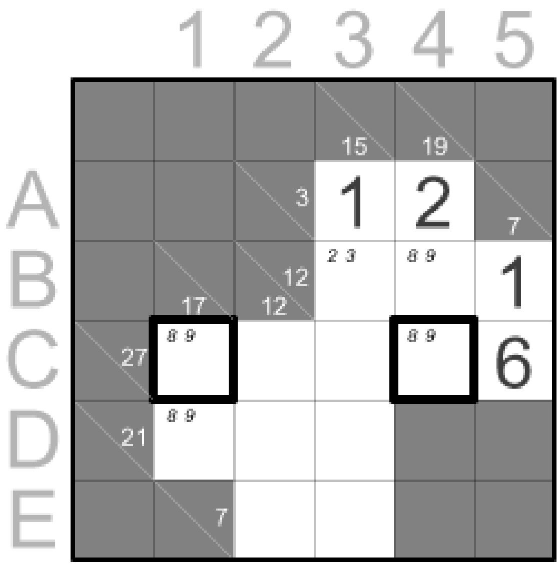
We know that the remaining two squares in the 19 block must sum to 17, so these remaining squares form a restricted 17 in 2 block, and each one must be either 9 and 8. We can write those in as possibilities. We'll also write them in as possibilities for the 17 in 2 block on the left.



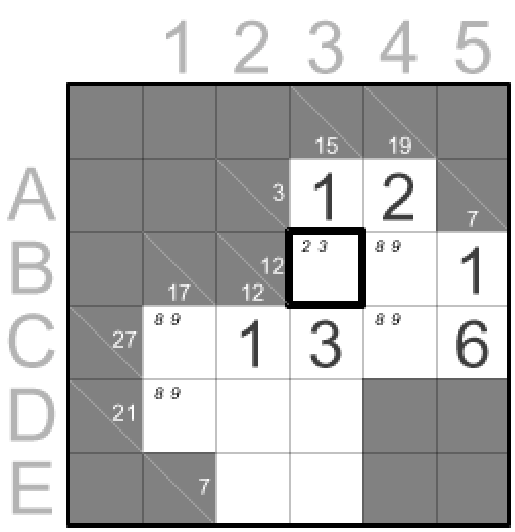
At this point it looks like we've run out of restricted blocks, or have we?

Take a look at the 12-block that goes from B3 to B5. We know that the middle square, B4, must be 8 or 9. That means the remaining squares must sum to either 12-8 or 12-9, in other words, 4 or 3, both of which are restricted! This means that one of these squares is going to be a 1, and the other square will either be a 2 or a 3. So which square is going to be 1? It can't be B3, because it is part of a column block that already contains a 1. So the 1 must go into B5, and we can put 2 and 3 as possibilities for B3. We can also finish the 7 in 2 block by putting a 6 in C5.

Naked Pairs

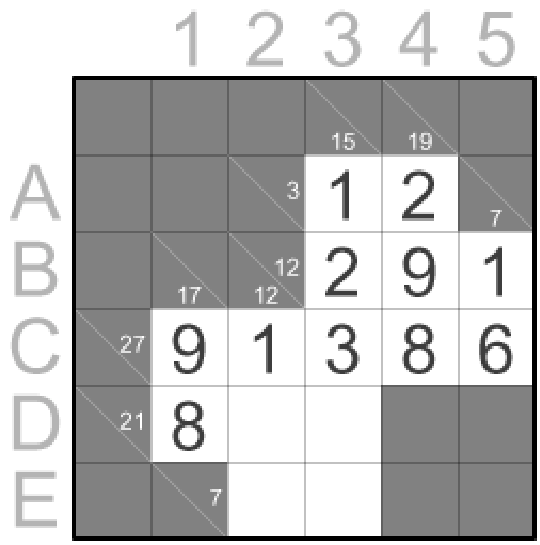


Now take a look at squares C1 and C4, which are part of a 27 in 5 block. Notice that both squares have the same two possible numbers: 8 and 9. When this happens it is called a *naked pair* (if it happens with 3 squares, it's a *naked triplet*, and so on). We know that one of these squares must be 8, and the other one must be 9. We just don't know which order. We also know that the two squares add up to 17. Since the 6 in that block is also filled in, we know that the three squares (C1,C4,C5) add up to 23, and we know that the remaining two squares must add up to 27–23 or 4. So we know that in the two remaining squares, C2 and C3, the only possible values are 1 and 3. Once again, the 1 in A3 prevents us from putting a 1 in that column, so the 1 must go in C2, and the 3 must go in C3.



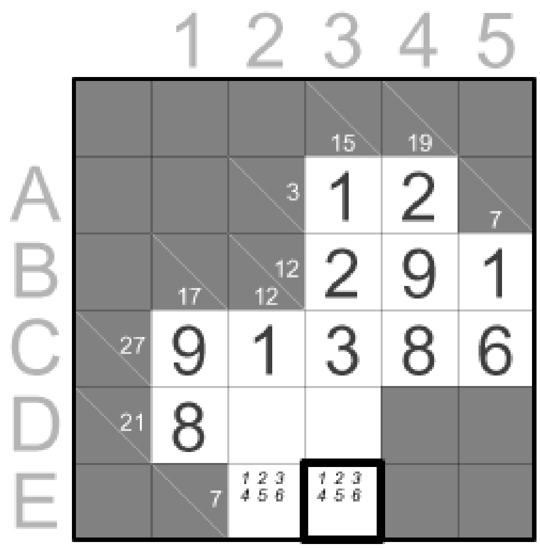
Now that we've solved C3, we've eliminated 3 from B3, which sets off a whole chain of easy solutions:

B3 must be 2
B4 must be 9
C4 must be 8
C1 must be 9
C1 must be 8

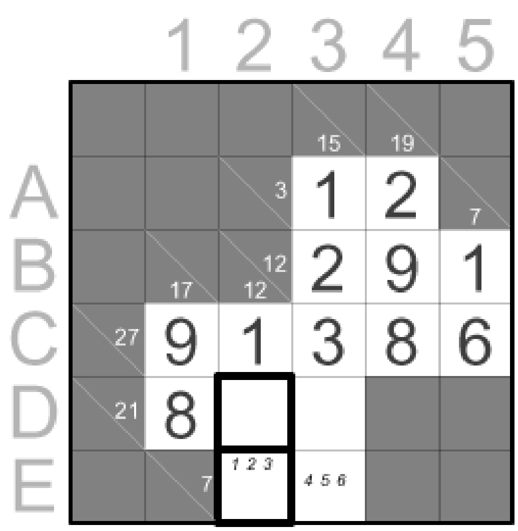


Matching Pairs

Now there are only 4 squares left, but we seem to be out of easy solutions. Let's write in the possible digits that remain. Notice that in the 7-block on the bottom, no digit can be larger than 6, so let's write those in.

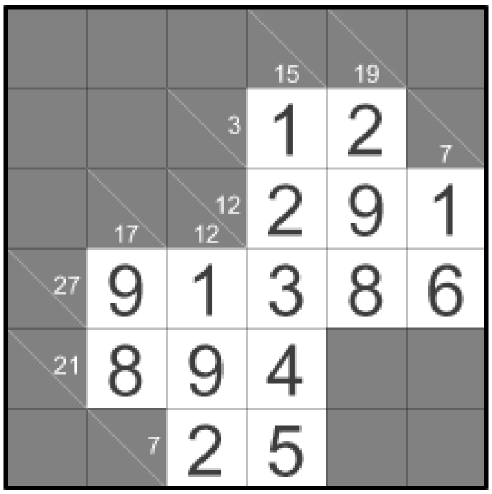


Notice that we can't actually use the 1,2 or 3 in square E3. This means that we also can't use the corresponding the 6,5 or 4 in square E2 (because they must add up to 7). This principle is called matching pairs. In this remaining part of the puzzle, we have 4 interlocking pairs, so it comes into play in a powerful way.



Now look at the E2/D2 pair. We know that these two squares must sum to 12-1 or 11. This means the possible values for D2 are 10 (which is illegal), 9, or 8 (which is a duplicate). The only legal value for D2 is 9. The remaining squares are solved like dominos falling:

E2 must be 2.
E3 must be 5.
D3 must be 4.



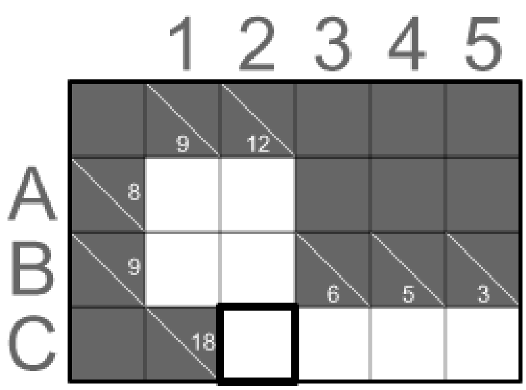
Advanced Strategies

Now we'll discuss some techniques that come into play in puzzles which contain larger blocks of 6, 7, 8 or 9 squares.

Criss-cross Arithmetic

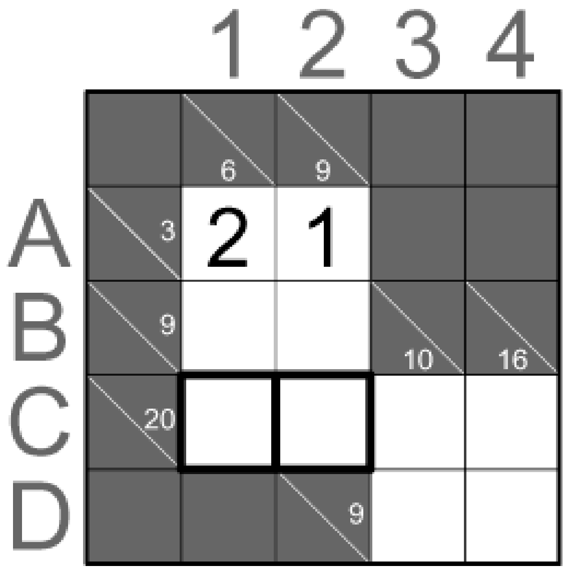
You will occasionally encounter a puzzle in which a few isolated cornersquares can be determined right away by adding or subtracting clues.

Consider the following puzzle:



You can figure out the value of C2 by adding two vertical blocks (9,12) and subtracting the two horizontal blocks (8,9). 9+12-8-9 = 4.

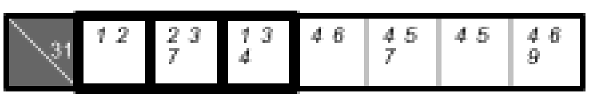
This is *criss-cross arithmetic*. Even when you can't use it to determine the value of an individual square, you can often use it to determine the sum of pairs or triplets which are part of larger blocks. If those sub-blocks have restricted sums, you can exploit them. Consider the following partially filled in puzzle fragment:



Criss-cross arithmetic tells us that squares c1+c2 must sum to a value of 6 + 9 – 3 – 9 = 3. This means those two squares must contain a 1 and 2. Since the 1 and 2 are already filled in, we know which contains which. Also, since those two squares sum to 3, we know the two remaining squares, c3 and c4 must contain either 8 or 9 (because 20 – 3 = 17).

Hidden Pairs

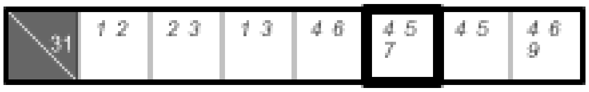
Sometimes you will encounter a longer clue in which there is a pair or triplet which contains a specific number of possibilities that aren't found in the other squares. Consider the following block, which we've already narrowed down to the following possibilities:



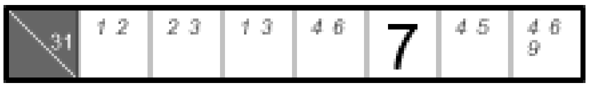
In this block the first three squares contain 1,2 or 3 as possibilities, but those digits aren't found in any of the other squares. If we knew for a certainty that 1,2,3 were required for this block, then we would also know that those 3 squares must contain the 1 2 and 3, and we could eliminate other numbers as possibilities for those first three squares, converting them into a naked triplet.

It turns out that 1,2 and 3 are indeed required! We know this because the target of the block is 31. The missing two digits must sum to 45-31, or 14, therefore neither of the missing digits can be smaller than 4. Since 1,2 and 3 are required in the block, the first three squares form a *hidden triplet*. We can eliminate all the other numbers except 1,2,3 from the first three squares.

After eliminating these, you'll notice there is only one remaining square with 7 as a possibility:



Again, we know that 7 is required in this block, because the missing two digits sum to 14. Since you can't use 7 to make a 14 in 2, 7 must appear in the block, and we now know the square in which it must appear!



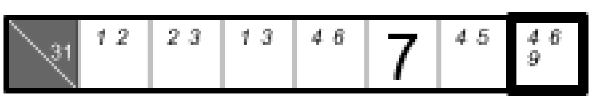
Binding Squares

For any square which is part of a block, you can narrow it down to a minimum and maximum value. This is called establishing the bounds, or *binding the square*, and it is a very useful technique on large unwieldy blocks.

To determine the maximum value of any square, figure out the smallest legal sum you can form from the remaining squares and subtract this value from the target. We know the square can't get any larger than this value, and any digits that are larger than this number can be eliminated.

Similarly, to figure out the minimum value of any square, figure out the largest legal sum you can form from the remaining squares and subtract this value from the target. We know the square can't get any smaller than this, and any digits that are smaller than this number can also be eliminated.

The best candidates to try the binding technique on first are the ones with the widest range of possible values. This is because the remaining squares will tend to have the narrowest range of possible sums. Consider the 31 in 7 that we have been working on:

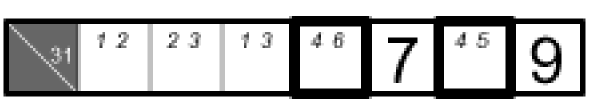


The square with the widest range of possible values is the last one.

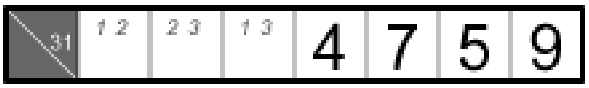
The smallest sum we can form from the remaining squares is 1+2+3+4+7+5 = 22. The largest sum we can form from the remaining squares is 1+2+3+6+7+5 = 24. So the maximum possible value for the last square is 31-22, or 9. The smallest possible value is 31-24, or 7.

Since 7 is the smallest possible value, we can eliminate 4 and 6 as possibilities for that last square, leaving only 9.

Now that we've found the 9, we can also determine the values of two more squares.



We already know that the block contains 1,2,3,7 and 9. Adding these numbers up gives us 22. This means that the two unsolved squares which don't contain 1, 2, or 3, must contain a number which sums to 31-22, or 9. The only available values that work are 4 in the first square and 5 in the second square.



Since this is just a fragment, we can't completely solve the first 3 squares, but as you can see, we've accomplished quite a lot without a whole lot to go on.

So ends our discussion of advanced techniques. Be sure to check out my website at http://www.krazykakuro.com for the latest tips & advice.

Enjoy the puzzles!

-- Jim Bumgardner