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Combustion Chambers for Jet Propulsion Engines

BY

V. S. ZUYEV

AND

L. S. SKUBACHEVSKII

TRANSLATED FROM THE RUSSIAN
BY
WILLIAM E. JONES
United Kingdom Atomic Energy Authority

TRANSLATION EDITED BY
B. P. MULLINS
Deputy Chief Scientific Officer
Ministry of Aviation, Farnborough, Hampshire

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TRANSLATION EDITOR'S PREFACE

IN AIRCRAFT gas turbine systems the combustion chamber usually occupies a small volume between the compressor and turbine. Similarly, in books on gas turbine engines, if any treatment of the combustion chamber is given at all this generally comprises a small chapter sandwiched between extensive discussions of the aerodynamic theory of compressor and turbine design.

Hence the present 244-page monograph, devoted entirely to the design of combustion chambers for turbo-jet engines (including reheat systems) and ramjet engines and the underlying combustion processes, is particularly welcome. Now that this has been translated into English it should be of considerable value to British and American students of aeronautical engineering generally and combustion engineers in particular.

At the suggestion of the Publishers and the Editor-in-Chief of the International Series of Monographs on Aeronautics and Astronautics, Propulsion Division, a short additional bibliography of related books and papers for further study, selected from British and American works, has been included in this book. Several of the illustrations of, or relating to, Western-type combustion chambers included in Part Two will be found described in greater detail in several of the earlier references in this additional bibliography.

B. P. MULLINS

Farnborough, Hants

PREFACE

THE present book is a training manual for the combustion chamber course held in the Moscow Aeronautical Institute.

In principle, the manual assumes a synopsis of lectures delivered by V. S. Zuyev to students of the engine-construction faculty during the 1954–1955 Academic year.

The work does not pretend to embrace all the problems related to the operating process, characteristics and construction of combustion chambers, but is aimed at giving only a general presentation of the basic elements of the process of operation, characteristics and design of combustion chambers.

Such essential problems as heterogeneous combustion under the conditions of the combustion chamber of a jet propulsion engine and the operating processes in the combustion chamber are described, on the whole, only from the qualitative point of view in the absence at the present time of sufficiently reasonable concrete methods for their evaluation.

Anyone interested in more profound information on the theory of combustion and the processes in combustion chambers should turn to specialist works, in particular the works of L. N. Khitrin, G. F. Knorr and other authors.

In the first section of the manual, an account is given of the elements of chemical kinetics and the theory of combustion of a homogeneous mixture in gas streams. The second section is devoted to the thermodynamics of the combustion chamber, aerodynamic and thermal losses, construction of the combustion chamber and a description of the operating process. The problem concerning the effect of losses in combustion chambers on the characteristics of jet propulsion engines is discussed in the work.

In view of the fact that the design of fuel injectors and the process of mixture formation are discussed in other courses, this problem has not been practically discussed in the present manual.

In the compilation of Chapter VII, "Combustion Chambers for Gas Turbine Engines", Lecturer of the Moscow Institute of Aviation G. M. Gorbunov participated, and has written the section on Aerodynamic Rating.

The authors are extremely grateful to Tutor G. M. Gorbunov and Engineer P. I. Shevchenko for their advice and the valuable comments offered by them in criticizing the manuscript.

INTRODUCTION

THE combustion chamber is the unit of a jet propulsion engine which mainly determines the efficiency and reliability of operation of the engine. The processes taking place in combustion chambers are complex and until recently have been relatively little studied.

The construction of the combustion chamber of a jet propulsion engine is an extremely difficult task. Solution of this problem requires an understanding of the physical and chemical processes taking place in a combustion chamber.

Considerable experience has been accumulated in technology on the construction of boiler furnaces operating on liquid fuel. Such furnaces are employed in the engines of railway and sea transport and also in fixed power units.

However, the combustion chambers of aero engines are much smaller in diameter and length than boiler furnaces and other corresponding parameters differ, for example, the higher gas velocity, the greater quantity of heat given off per unit volume, etc.

The dimensions used in furnace technology can be gauged according to the magnitude of the specific load or throughput and the volumetric heat liberation of the furnace.

By throughput (specific load) W is understood the quantity of heat liberated in a combustion chamber over 1 hr and relative to the operating cross-sectional diameter of the combustion chamber.

It is obvious that the magnitude of the specific load for a given supply of heat determines the cross-sectional dimensions of the furnace, or, as we shall call it in future, the combustion chamber.

The second characteristic value is the volumetric heat liberation Q_V, which is equal to the quantity of heat liberated in a combustion chamber volume of 1 m^3 in 1 hr. This value characterizes the volume of the combustion chamber.

If the quantity of heat Q liberated is expressed in kcal/hr, the volume V_K of the combustion chamber in m^3 and the characteristic area of the chamber F_K in m^2, then the dimensions W and Q_V are determined from the obvious relationships:

$$W = \frac{Q}{F_K} \text{ kcal/m}^2 \text{ hr.}$$

and

$$Q_V = \frac{Q}{V_K} \text{ kcal/m}^3 \text{ hr}\,.$$

The values W and Q_V can be easily determined, knowing the rate of supply of fuel G_f, the combustion efficiency ξ_z, the calorific value of the fuel H_u and, correspondingly, the cross-sectional area of the combustion chamber F_K or its volume V_K.

Assuming that the rate of fuel supply G_f is measured in kg/sec, we obtain:

$$W = \frac{G_f\, H_u\, \xi_z\, 3600}{F_K} \text{ kcal/m}^2 \text{ hr}\ ;$$

$$Q_V = \frac{G_f\, H_u\, \xi_z\, 3600}{V_K} \text{ kcal/m}^3 \text{ hr}\,.$$

For a comparative estimation of the dimensions of the combustion chamber of different engines, it is convenient to use the "reduced" heat liberation per unit volume combustion intensity and, correspondingly, the reduced loading

$$Q_p = \frac{Q_V}{p_2}\,,$$

and

$$W_p = \frac{W}{p_2}\,,$$

where p_2 is the pressure in the chamber in atm. It is obvious that

$$Q_p = \frac{G_f\, H_u\, \xi_z\, 3600}{V_K\, p_2} \text{ kcal/m}^3 \text{ hr atm};$$

$$W_p = \frac{G_f\, H_u\, \xi_z\, 3600}{F_K\, p_2} \text{ kcal/m}^2 \text{ hr atm}\,.$$

Some appropriate values for the specific loading, combustion intensity, mean flow velocity, relative to the maximum diametrical cross-section, the excess air coefficient and other values characterizing modern combustion chambers are presented in Table 1.

The figures given in the table show that the combustion chambers of aircraft engines have a considerably smaller volume than the combustion chambers of other power units.

The high mean flow velocity of the air in a combustion chamber (up to 50 m/sec in the chambers of gas turbine engines and 100 m/sec and more in the combustion chambers of pulse-jet engines and reheat combustion chambers compared with a velocity of a few metres per second in furnaces) and the small residence time of the fuel in a combustion chamber in view of its small length, causes considerable difficulty in establishing the combustion process.

The layouts and designs of combustion chambers for jet propulsion engines differ substantially in the majority of cases from the layouts and designs of the usual boiler furnaces for liquid fuel.

Until recently, in view of the complexity of the physical and chemical processes taking place in combustion chambers, it has not been possible to formulate an adequate theory of the operating processes by means of which, for given conditions, the most favourable layout of a combustion chamber could be chosen and the dimensions of its components determined. Designing and completing a combustion chamber has had to be based, until recently, upon available experimental data.

TABLE 1

Purpose of combustion chamber	$W \cdot 10^{-6}$ kcal/m² hr	$W_p \cdot 10^{-6}$ kcal/m² hr atm	$Q_V \cdot 10^{-6}$ kcal/m³ hr	$Q_p \cdot 10^{-6}$ kcal/m³ hr atm	w_K* m/sec.	α
Fuel oil furnace for fixed boiler	2·5–4·0	2·5–4·0	0·2–0·6	0·2–0·6	0·5–1·0	1·2–1·4
Fuel oil furnace for locomotive	2·5–4·0	2·5–4·0	1·5–2·5	1·5–2·5	1·0–2·0	1·15–1·25
Petroleum marine furnace	—	—	2·6	$\sim$2·6	—	—
Combustion chamber of gas turbine engine	10	2–10	$>$150	up to 50	25–50	3·5–4·0
Combustion chamber of pulse-jet engine and reheat chamber of gas turbine engine	500	250–500	up to 250	125–250	up to 100 and greater	$\geq$1·2

* w_K is the velocity in a representative cross-secti onof the combustion chamber.

Despite the fact that the state of scientific knowledge concerning the *modus operandi* in a combustion chamber is at the present time far from complete, theory is still of considerable help to designers and experimentalists. To the designer, theory gives the possibility of arriving at a reasonable choice of layout for a combustion chamber and to resolve, rationally, its design; to the experimentalist, working on the finished combustion chamber, theory gives the possibility of presenting clearly the reasons for difficulties encountered and of finding proper methods for their elimination.

It should be noted that great attention is paid to exploitation of the theoretical problems associated with the operating process in a combustion chamber. The results of scientific research work, carried out in the Scientific-Research Institute and in the Design office give, with each year, still more essential data for the exploitation of a strictly scientific method for calculating and designing combustion chambers.

The development of the science of combustion, the operating processes in a combustion chamber, is continually associated with the works of Russian scientists. Thus, for example, the most profound theoretical and experimental exploitation of the kinetics of chemical reactions is attributed to Academician N. N. Semenov, who obtained the Nobel Prize in 1957 for his research into the field of combustion. To N. N. Semenov is credited the profound exploitation of the theory of chain reactions which are of great theoretical and practical value.

Work on heterogeneous combustion, conducted by Fellow-Readers of the Academy of Sciences U.S.S.R. A. S. Predvoditel'ev, L. N. Khitrin and their co-workers, is of great national economic importance.

The scientific works of the Soviet scientists K. I. Shchelkin, Ya. B. Zel'dovich, D. A. Frank-Kamenetskii, and also a number of foreign scientists, have laid the foundation to the basic theory of combustion in gas streams.

These works are directly utilized in the designing of combustion chambers.

Extremely valuable theoretical and experimental work on the study of the operating process of combustion chambers are conducted in the Institutes of the Academy of Sciences, U.S.S.R., Moscow Academy of Research, and in other Russian and foreign educational and scientific research institutes.

The wide front of the work carried out, and moreover the results obtained, give everywhere reason to believe that the task of studying the process of combustion in the combustion chambers of jet propulsion engines will be successfully solved in the very near future.

REQUIREMENTS OF COMBUSTION CHAMBERS AND THEIR OPERATING CONDITIONS

A combustion chamber functions within the normal system of a jet propulsion engine. This superimposes upon the design performance of the chamber and upon the organization of the process of combustion in it, a succession of specific conditions.

In order to meet these conditions, the following requirements are put forward for the combustion chambers of jet propulsion engines:

(1) Combustion chambers should ensure a high combustion efficiency at the principal operating conditions of the engine. This requirement is

dictated by the desire to make possible a more efficient engine. It is obvious that the efficiency of an engine (specific fuel consumption) for a given heat output (given temperature on exit from the combustion chamber) is directly proportional to the combustion efficiency.

The maximum value attained by the combustion efficiency is $\xi_z = 0{\cdot}97 - 0{\cdot}98$ for the combustion chamber of a gas turbine engine, and $\xi_z = 0{\cdot}9 - 0{\cdot}95$ for the combustion chamber of a pulse-jet engine.

(2) The combustion chamber of jet propulsion engines should have minimum dimensions.

The greater the dimensions of the combustion chamber and the greater the girth of the engine, the greater are the difficulties and disadvantages which arise in installing it on an aircraft.

Modern aircraft are characterized by high flight speeds, as a result of which the drag of the various components of the aircraft attain extremely high values. If an engine with a given thrust has large dimensions then it can be shown that a very considerable fraction of the thrust will be expended in overcoming the resistance of the engine installation alone, and the thrust, on the whole, will be inadequate for propelling the aircraft.

It is estimated that the thrust of an engine of the RD-45 or RD-45F type at a flight velocity corresponding to $M = 1{\cdot}2\text{–}1{\cdot}3$ will be equal to the drag of the nacelle in which the engine is housed.

With the object of decreasing the drag of an aircraft it is extremely desirable that the dimensions of the nacelle shall be a minimum. Consequently, one strives to make the primary combustion chambers and reheat chambers of jet propulsion engines of such dimensions that they do not exceed the dimensions of the compressor or the turbine.

The combustion chamber of a pulse-jet engine directly defines the diametrical dimensions of the engine and sometimes also of the fuselage of the aircraft; consequently its dimensions should also be the smallest possible.

The longitudinal dimensions of primary combustion chambers for gas turbine engines with the usual engine layout, when the combustion chambers are arranged directly on the compressor, determine the length of the shaft, the distance between the bearings and influence the power arrangement of the engine. The shorter the combustion chamber of a gas turbine engine, the simpler are the engine components and the less is its weight. The length of a reheat chamber and the combustion chamber of a pulse-jet engine is usually of no significance; however, a decrease in length is always advantageous with the object of decreasing the weight and improving the engine components generally.

In general it is always necessary to bear in mind that small linear dimensions stipulate a small volume and a corresponding small weight.

(3) Combustion chambers should have minimum aerodynamic pressure loss, since the presence of aerodynamic pressure loss leads to a decrease in the total pressure of the working substance.

As is well known, aerodynamic pressure losses are usually rated by the pressure loss coefficient ξ, which defines the loss of total pressure as a proportion of the dynamic pressure in any specific section of the combustion chamber.

The expression for determining the relative drop in total pressure has the form:

$$\frac{p_2^* - p_3^*}{p_2^*} = \xi \frac{\varrho\, w_2^2}{2\, p_2^*} .$$

It is convenient to use such a method for evaluating the "aerodynamic" efficiency of a combustion chamber, since the coefficient ξ is practically invariable within a large range of values of the mean flow velocity w_2 (in view of the sufficiently large Reynolds numbers for which the flow is maintained the same).

As a representative cross-section, the maximum diametrical cross-section of the combustion chamber $F_{k\,\mathrm{max}}$ may be assumed, in terms of which is also defined the mean flow velocity w_2 (M_{av} or λ_2).

The pressure loss coefficient ξ for modern combustion chambers has the following values: for the combustion chambers of gas turbine engines, $\xi \approx 8$–12: for reheat chambers and chambers of pulse-jet engines $\xi \leq 3$.

In addition to this method considered for evaluating pressure losses via the coefficient ξ, the losses can also be evaluated by means of the coefficient of pressure recovery $\sigma = \frac{p_3^*}{p_2^*}$. The following relationship exists between the pressure loss coefficient and σ:

$$\sigma = 1 - \frac{k}{2} \xi\, M_2^2 .$$

The use of σ as a value rating losses in the combustion chambers is convenient in that σ is determined by the ratio of the two values p_3^* and p_2^* which are directly determinable by experiment. However, σ for a given combustion chamber essentially depends on M_2, whereas ξ is a more stable characteristic, depending only slightly on M_2. Consequently, ξ more correctly rates the combustion chamber from the point of view of aerodynamic pressure losses.

Decrease in the pressure loss of combustion chambers of gas turbine engines, especially associated with reheat at high velocities, is of great importance for increasing the efficiency of an engine.

(4) Combustion chambers should start up rapidly and reliably under any flight conditions, including at altitude — for low temperatures and

pressures of the air entering the chamber. This requirement is obvious and requires no special explanations.

(5) The process of combustion in combustion chambers should be stable for all flight conditions. The chamber should not suffer flame-out on transition from condition to condition.

The excess air coefficient for primary combustion chambers in gas turbine engines with uncooled turbines amounts to approximately $\alpha \approx 3{\cdot}5$–$6{\cdot}5$ at maximum and cruising conditions; for the combustion chambers of pulse-jet engines $\alpha \approx 1{\cdot}2$–$3{\cdot}0$ and for reheat chambers $\alpha \approx 2{\cdot}0$.

For the primary combustion chambers of gas turbine engines a very weak mixture ($\alpha > 40$–50) is possible over a short interval of time, occurring usually at the instant of sudden removal of a zone of gas, for example, as a result of a manoeuvre. In these cases, the combustion chamber should not flame-out.

The data presented show only series of values typical of the types of engines mentioned and are not intended to be precise.

(6) A combustion chamber, just as any other unit of an engine, should have a guaranteed life during which the (combustion) chamber should not require any servicing whatsoever.

Dependent upon the type and purpose of the aircraft, this life nowadays may be reduced from several hundreds of hours to a few minutes. The required life of a combustion chamber is influenced by its construction, the maximum temperature attained and by other characteristics of the combustion chamber.

(7) On exit from the combustion chamber of a gas turbine engine, a specific temperature distribution should be maintained across the duct.

This requirement is dictated by the conditions of durability of the turbine blades.

At the periphery, where the turbine blades have the least stresses, it is possible to attain a somewhat higher temperature relative to the average value of the temperature. At the root of the blade, tensile and bending stresses have considerably higher values. At this point it is desirable to have a somewhat lower temperature in order that the durability will be increased.

The requirement mentioned is extremely difficult to satisfy on the finished engine, since the temperature field at the exit depends upon a whole series of factors which cannot always be allowed for in the designing.

It is possible to point out further series of secondary requirements which combustion chambers should satisfy.

For the combustion chambers of pulse-jet engines and reheat chambers of turbo-jet engines, uniformity is highly desirable of temperature, total pressures and static pressures along the exit section. Non-uniformity of total pressures, static pressures and especially of temperature may significantly reduce the momentum of the emergent jet and hence the thrust of the engine.

The difficulty of fulfilling the requirements enumerated above is explained to a considerable extent by the fact that measures taken to meet one of these requirements can interfere with the others. Thus, for example, the manufacture of a combustion chamber with a good combustion efficiency at high gas flow velocities and for a small value of the pressure loss coefficient ξ is extremely difficult, first and foremost because a reduction in ξ for approximately the same conditions promotes a reduction in the efficiency of combustion and narrows the limits of stable operation of the combustion chamber with respect to the excess air coefficient and reheat.

CHAPTER I

ELEMENTS OF THE KINETICS OF CHEMICAL REACTIONS

CHEMICAL kinetics is the science of study of the mechanism and velocity of chemical reactions.

In the present book we shall confine ourselves to a brief account of certain fundamental ideas and aspects of chemical kinetics, for the further explanation of which a number of examples are presented.

A knowledge of the principles of chemical kinetics is essential for all persons occupied with problems of combustion and the operation of combustion chambers. Without a knowledge of chemical kinetics it is impossible to represent the conditions necessary for the rapid development of a chemical reaction. The basic principles of chemical kinetics are extensively used in practice for designing new combustion systems and also for the improvement of the existing ones.

1. IDEAS CONCERNING THE VELOCITY OF A CHEMICAL REACTION

By the velocity of a chemical reaction is understood the change of concentration of one of the reacting substances (initial reactants or final reaction products) in unit time.

The concentration of any substance determines its quantity in unit volume and can be expressed in various units, for example: kg/m^3, mol/m^3, number of moles/cm^3.

Since gases have low densities, it is difficult to determine accurately the weight of a gas. It is considerably more convenient to determine the concentration of a substance in a gas by the number of moles, since the number of moles is proportional to the volume occupied by the gas. In this case, change of weight is measured by the changes of volumes and pressures which are much more easy to determine. Consequently, in technology, concentration is usually determined by the number of moles in unit volume. We shall denote molar concentrations by the letter C, and molecular concentrations by the letter n.

The following relationship exists between molar and molecular concentrations:

$$n = \frac{N\,C}{10^6}, \tag{I.1}$$

where N is the number of molecules in one mole (the number 10^6 in the denominator is fixed by the dimensions of the volume to which the molar and molecular concentrations refer. In the given case this number indicates that 1 m^3 consists of 10^6 cm^3).

The number of molecules in one gram-mole $N_A = 6{\cdot}06 \times 10^{23}$ molecules/mole. If C is the molar concentration expressed in kilogram-moles, then $N = 1000\,N_A$. It is obvious that any dimension can be chosen to express concentrations. The principal criterion for the soundness of choice of the dimensions is the convenience of their usage.

Let us define the concept of the velocity of a chemical reaction. As already indicated above, by the velocity of a chemical reaction is understood the change of concentration of the initial reactants or final products in unit time. As for any process, in indicating the velocity of a chemical reaction, a distinction should be made between the average and actual velocity of a chemical reaction.

By the average velocity of a chemical reaction is understood the change in concentration of any of the reacting substances over a certain interval of time, i.e.,

$$\left.\begin{aligned} w_C &= \pm\frac{\Delta C}{\Delta\tau} \\ \text{or} \qquad & \\ w_n &= \pm\frac{\Delta n}{\Delta\tau} \end{aligned}\right\} \tag{I.2}$$

(the sign "—" is taken in order to obtain a positive value of velocity by determining w with respect to concentration of the initial reactants which decrease with time).

By decreasing the time interval $\Delta\tau \to 0$ we approach a limit which determines the velocity of the chemical reaction at a specific instant of time:

$$\left.\begin{aligned} w_C &= \pm\frac{\mathrm{d}C}{\mathrm{d}\tau} \\ \text{or} \qquad & \\ w_n &= \pm\frac{\mathrm{d}n}{\mathrm{d}\tau}. \end{aligned}\right\} \tag{I.3}$$

It is obvious that the values for the velocity of a chemical reaction, determined with respect to different concentrations (molar, molecular, etc.), are different.

However, there exists a well-defined relationship between them, resulting from their dimensions. For example, if C is in kg.mol/m^3 and n is in molecules/cm^3, then

$$w_n = \frac{N}{10^6}\,w_C,$$

where N is the number of molecules in 1 gram-mole.

The velocity of a chemical reaction can be determined by the change of concentration of any substance participating in the reaction.

Let us explain the relationship between the velocities of chemical reactions calculated with respect to the concentrations of the various substances taking part in the reaction.

We write the stoichimetric equation for the chemical reaction in the following general form:

$$aA + bB + \cdots \rightarrow cC + dD + \cdots,$$

where A, B etc., indicate the substances participating in the reaction;
a, b etc., are the number of reacting molecules of substances A, B etc.

The concentrations of substances A and B in the course of the reaction do not vary in the same manner, since $a \neq b$.

Consequently, in the general case, the velocities

$$w_A = -\frac{\mathrm{d}n_A}{\mathrm{d}\tau} \quad \text{and} \quad w_B = -\frac{\mathrm{d}n_B}{\mathrm{d}\tau} \tag{I.4}$$

are not equal to one another, $w_A \neq w_B$.

However, there is obviously a well-defined relationship between w_A and w_B

$$\frac{w_A}{w_B} = \frac{a}{b}. \tag{I.5}$$

Consequently, it does not matter, with respect to which substance's change of concentration the velocity of a chemical reaction is determined, since they are inter-related by equation (I.5). It is only necessary to know with respect to the change of concentration of which substance the velocity is determined.

By way of example, let us consider the reaction between hydrogen and oxygen

$$2\,H_2 + O_2 = 2\,H_2O\,.$$

It can be seen from the equation that the velocity of the chemical reaction based on the hydrogen concentration is twice as great as the velocity based on the oxygen concentration.

Therefore it follows that, in the reaction, for each two reacting molecules of hydrogen only one molecule of oxygen is involved.

2. MECHANISM OF CHEMICAL REACTIONS

Let us write the chemical reaction between two substances A and B:

$$A + B \rightarrow C + D\,.$$

From the point of view of modern ideas, the reaction can proceed only as a result of collision of molecules of the reacting substances amongst themselves.

However, not all colliding molecules are involved in the reaction. Collisions of "active" molecules only are involved in a chemical reaction, by which is understood molecules possessing an initial energy greater than or equal to a certain energy, which we denote by the letter E.

If the energy of the colliding molecules is less than the value E, then reaction between them does not occur.

The necessity for the reacting molecules present to possess a certain energy level E is explained by the fact that in order that chemical reaction should take place it is necessary at the beginning to break down, or weaken to a suitable extent, the intra-molecular bonds. The magnitude of E also determines the minimum energy at which rupture of primary molecular bonds is possible, essential for the course of the reaction.

The magnitude of the energy of the intra-molecular bonds can be extremely large, for example for hydrogen it amounts to $\sim$ 103 kcal/mole. or 51,500 kcal/kg, whereas the calorific value of gasoline amounts to only 10,000 - 11,000 kcal/kg.

It is well-known from molecular physics that the distribution of energy between molecules is extremely non-uniform. Thus for example, data are presented in Table 2 concerning the non-uniformity of distribution of energy of mechanical motion of molecules in terms of the distribution of the velocities of molecules of oxygen.

TABLE 2

Velocity of O_2 molecule in m/sec for $t = 0°C$	41–82	205–246	410–450	615–655	820–860	1230–1270
Percentage of molecules possessing the given velocity	0·22	4·39	8·3	5·36	1·65	0·03

Other forms of energy of the molecules, in particular the energy of vibrational and rotational motion, are also non-uniformly distributed between the molecules.

Thus, some molecules are energy-rich — "active", others are relatively energy-poor.

With a large concentration of "active" molecules, a reaction proceeds rapidly and the process accelerates. For a small number of active molecules chemical reaction barely proceeds.

If it is assumed that the energy necessary for the occurrence of a chemical process is determined by the kinetic energy of the colliding molecules, then it is possible to derive simply a formula for determining the velocity of a chemical reaction.

3. DETERMINATION OF THE VELOCITY OF A CHEMICAL REACTION

The velocity of a chemical reaction is easily determined by the number of colliding "active" molecules, i.e., in our concept, those molecules which possess the necessary kinetic energy, adequate for rupture of the intramolecular bonds.

Let us imagine two gases A and B which react with each other according to the scheme

$$A + B \rightarrow C + D\,.$$

From the kinetic theory of matter it is well known that the number of molecules of one gas colliding with the molecules of the other gas for a time $\Delta\tau$ in unit volume is equal to

$$Z_{A1} = \sqrt{(2)}\, n_B \pi d^2 w \Delta\tau\,, \tag{I.6}$$

where n_B is the concentration of substance B;

d is the mean "effective" diameter of the molecules

$$d = \frac{d_A + d_B}{2}\,;$$

w is the arithmetic mean velocity, equal to

$$w = \sqrt{\frac{8RT}{\pi}\left(\frac{1}{M_A} + \frac{1}{M_B}\right)}\,, \tag{I.7}$$

where M_A and M_B are the molecular weights of the reacting substances;

R is the gas constant ($R = 8{\cdot}313 \times 10^7$ g cm^2/sec^2 g mol deg).

Having used the expression for w (I.7) and the value of the gas constant R, we shall carry out a transformation of equation (I.6). Denoting constant values by "const." we obtain the relationship for determining the number of collisions of one molecule of substance A with molecules of substance B as a function of temperature and concentration of the other substance

$$Z_{A1} = 4\, d^2 \sqrt{\pi R\left(\frac{1}{M_A} + \frac{1}{M_B}\right)} \sqrt{T}\, n_B \Delta\tau = \text{const.}\, n_B \sqrt{T}\, \Delta\tau\,. \tag{I.8}$$

The total number of collisions of all molecules of substance A with molecules of substance B over a time $\Delta\tau$ will be

$$z_A = \text{const.} \sqrt{T}\, n_A\, n_B \Delta\tau\,. \tag{I.9}$$

In Table 3 the values are shown for the number of molecular collisions for various gases at a pressure of 760 mm Hg and temperature 0 °C.

However, we are interested only in those collisions of molecules which lead to a reaction, i.e., those for which the energy of the molecules is equal to or greater than a certain value E, which is taken to denote the energy of activation.

TABLE 3

Gas	H_2	O_2	CO	CO_2	H_2O
d cm	$2{\cdot}77\times10^{-8}$	$2{\cdot}71\times10^{-8}$	$2{\cdot}9\times10^{-8}$	$2{\cdot}9\times10^{-8}$	$2{\cdot}27\times10^{-8}$
w m/sec	1619	425	453	357	566
z 1/sec.cm^3	$10{\cdot}3\times10^{9}$	$4{\cdot}5\times10^{9}$	$5{\cdot}2\times10^{9}$	$6{\cdot}05\times10^{9}$	$8{\cdot}5\times10^{9}$

The kinetic theory shows that the number of collisions with energies, equal to or greater than E is equal to the total number of collisions multiplied by the exponent $e^{-E/RT}$, where E is the energy of activation in kcal/kg. mol;

R is the gas constant in kcal/kg.mol.deg;

T is the temperature of the reaction in deg absolute.

Consequently, the number of active collisions will be equal to

$$z_{A_{\text{act}}} = z_A \Delta^{E/RT} = \text{const} \cdot n_A n_B \sqrt{(T)}\, e^{-E/RT} \Delta\tau . \qquad (I.10)$$

It is obvious that $z_{A_{\text{act}}}$ defines the number of reacting molecules of the gas A over time $\Delta\tau$ for maintaining the reaction conditions unchanged.

Resulting from the determination of the value $z_{A_{\text{act}}}$ we can write:

$$z_{A_{\text{act}}} = -\Delta n_A . \qquad (I.11)$$

Now, from formulae (I.10), (I.11) and (I.4) it is easy to write down the expression for the velocity of the reaction with respect to the change of concentration of substance A:

$$w = \text{const.} \sqrt{T}\, e^{-E/RT} n_A n_B , \qquad (I.12).$$

It can be seen from this formula that the velocity of a specified chemical reaction depends upon the temperature and concentration

$$w = f(T, n) .$$

All discussions have been conducted for a bimolecular reaction when the molecules of the two substances are colliding according to the scheme

$$A + B \rightarrow C + D .$$

In practice, reactions of other types are also encountered, for example:

(1) mono-molecular reactions, for which disintegration of molecules takes place, having energies $\geq E$.

In this case the equation for velocity has the form

$$w = \text{const.} \sqrt{T}\, e^{-E/RT} n_A ; \qquad (I.13)$$

(2) tri-molecular reactions

(a) $A + B + C \rightarrow D + E + F + \cdots$, for which

$$w_A = \text{const.} \sqrt{T}\, e^{-E/RT} n_A n_B n_C\,; \tag{I.14}$$

(b) $2A + B \rightarrow D + E + \cdots$,

for which the formula for the velocity of the chemical reaction has the form

$$w_A = 2\,\text{const.} \sqrt{T}\, e^{-E/RT} n_A^2\, n_B\,. \tag{I.15}$$

The presence in the latter expression of the index "2" and n_A^2 is explained by the fact that the reaction takes place between two molecules of substance A and one molecule of substance B.

It is obvious that the velocity of the reaction, determined with respect to substance B, will be equal to

$$w_B = \frac{1}{2} w_A\,. \tag{I.16}$$

The probability of trimolecular reactions is small, since the simultaneous interaction of three molecules participating in the reaction is necessary. Consequently, the velocity of such a reaction is insignificant.

A bimolecular reaction of the type $A + B \rightarrow C + D + \cdots$ is of prime importance in practice.

As can be seen from equation (I.12), the velocity of a reaction depends on the nature of the reacting substance (E), the temperature (T) and the concentrations.

It can be concluded from the form of equation (I.12) that if the value of E is "large", then the velocity of the reaction will be very strongly dependent on the temperature and, vice versa; if $E \rightarrow 0$ then $e^{-E/RT} \rightarrow 1$ and the velocity of the chemical reaction will be determined essentially by the concentrations of the reacting substances. The velocity of such a reaction should be very large since the number of active collisions sharply increases (see Table 3).

In order to determine the quantitative effect of temperature and concentration on the velocity of a chemical reaction, let us consider the following two examples.

EXAMPLE 1. Let us determine the velocity and time of chemical conversion of 1 per cent of the reacting substance for the reaction

$$H_2 + I_2 = 2\,HI$$

as a function of the initial temperature. We shall take as starting data:

(1) $n_{H_2} = n_{I_2} = 1{\cdot}35 \times 10^{19}$ mol/cm³ (for $p = 760$ mm Hg and $t = 0$ °C)

(2) Energy of activation 39,000 kcal/kg mol.

(3) Gas constant $R = 1{\cdot}985$ kcal/kg mol deg.

The constant entering into the equation for determining the velocity is equal to $5{\cdot}48 \times 10^{-12}$.

Let us substitute these values in the velocity equation:

$$w_{H_2} = -\frac{dn_{H_2}}{d\tau} = \text{const.}\sqrt{T}\, e^{-E/RT}\, n_{H_2} n_{I_2} \text{ mol/cm}^3 \text{sec.}$$

and let us carry out the calculation for the temperatures $T_1 = 273$ °K; $T_2 = 400$ °K; $T_3 = 600$ °K; $T_4 = 800$ °K.

As a result of the calculation we obtain the data presented in Table 4.

TABLE 4.

T °K	w_{H_2}
273	$1{\cdot}64 \times 10^{-3}$
400	$2{\cdot}00 \times 10^{7}$
600	$2{\cdot}44 \times 10^{14}$
800	$2{\cdot}82 \times 10^{17.5}$

These results show that with increase of temperature the velocity of the chemical reaction increases sharply. For $T = 273$ °K the velocity is very small and the reaction barely proceeds.

At a temperature of the reacting substances equal to 600 °K, the velocity of the chemical reaction exceeds the value for the velocity at $T = 273$ °K by a factor of more than 10^7, and for $T = 800$ °K by a factor of more than 10^{20}. Increase of temperature from 600° to 800 °K leads to an increase of w by a factor of 3000.

Let us determine the time τ for which 1 per cent of the initial substances react.

It is obvious that

$$\tau = \frac{\text{number of molecules} \times 0.01}{\text{velocity of reaction}} \text{ sec}.$$

For a temperature of $T = 273$ °K,

$$\tau = \frac{1{\cdot}35 \times 10^{19} \times 0{\cdot}01}{1{\cdot}64 \times 10^{-3}} \approx 10^{20} \text{ sec} \approx 3 \times 10^{12} \text{ years}.$$

In the case of $T = 800$ °K, a sharp reduction in the time of the reaction occurs on account of the increase in velocity of the chemical reaction:

$$\tau = \frac{1{\cdot}35 \times 10^{19} \times 0{\cdot}01}{2{\cdot}82 \times 10^{17.5}} \approx 0{\cdot}15 \text{ sec.}$$

The time of the reaction, as can be seen from the example considered, depends very strongly on the temperature.

EXAMPLE 2. Let us determine the velocity of the reaction when the reaction proceeds in a medium vitiated by the burning reaction products.

We shall assume that half the mixture consists of the burning reaction products HI, the temperature of which is equal to 2000 °K; for this, the concentrations of the reactants are reduced by half, and will be equal to

$$n_{H_2} = n_{I_2} = 0.675 \times 10^{19} \text{ mol/cm}^3 .$$

Before determining the velocity of the chemical reaction it is necessary to know the temperature of the mixture T_{mix}.

We shall use the following relationship, presented as a thermal balance

$$G_1 c_{p_1} T_1 + G_2 C_{p_2} T_2 = (G_1 + G_2) c_{p_{\text{mix}}} T_{\text{mix}} .$$

For our case $G_1 = G_2$.

In order to simplify the calculation we shall assume that the specific heats of the initial substances are all equal:

$$c_{p_1} = c_{p_2} = c_{p_{\text{mix}}} .$$

Hence, assuming that the initial temperature of the mixture was equal to 400 °K

$$T_{\text{mix}} = \frac{T_1 + T_2}{2} = \frac{400 + 2000}{2} = 1200 \text{ °K}.$$

The velocity of the chemical reaction in the case of vitiation will be equal to

$$w_{1200} = 5{\cdot}48 \times 10^{-12} \sqrt{1200} \cdot e^{-\frac{39{,}000}{2 \times 1200}} \times 0{\cdot}675 \times 10^{19} \times 0{\cdot}675 \times 10^{19}$$
$$\approx 8{\cdot}5 \times 10^{20} \text{ mol/cm}^3 \text{ sec} .$$

By comparing the values of the velocities of the chemical reactions it can be seen that as a result of vitiation of the mixture by burning reaction products, the velocity has been increased approximately by a factor of 10^{13}.

Thus, the temperature exerts a considerably greater influence on the velocity of a chemical reaction than does the concentration, in that the greater it is, the greater is the value of E.

If it is desired to accelerate the course of a chemical reaction, vitiation of the fuel mixture should not be feared if this "vitiation" is accompanied by increase of temperature the excess of which compensates for the reduction of concentration of the reactants. We shall become further acquainted with the practical application of this result and with its further definition.

The effect of temperature may not be uniform for different reactions. It is clear that temperature influences the velocity of a chemical reaction the more, the greater is the energy of activation and, conversely, the lower the energy of activation, the less is the influence of temperature.

For small values of energy of activation, the change of concentration of the substance exerts a more powerful influence on the velocity of a chemical reaction than does change of temperature.

The velocity of a chemical reaction depends very strongly on the magnitude of the energy of activation. Thus for example, if we take $E = 25{,}000$ kcal/kg.mol, $T = 300$ °K and $R = 1.985$ kcal/kg.mol.deg, we obtain

$$e^{-\frac{E}{RT}} \approx 2{\cdot}72^{-\frac{25{,}000}{1{\cdot}985\times 300}} \approx 10^{-18{\cdot}3} \approx 5\times 10^{-19}\,.$$

Thus, for approximately the same conditions, the velocity of the chemical reaction for $E = 25{,}000$ kcal/kg.mol is approximately 10^{19} times less than the velocity of the reaction for which $E = 0$.

Reactions with a small energy of activation do exist. Such reactions are all reactions in which molecules participate with weak molecular bonds, in particular, reactions between atoms; for example

$$\mathrm{H} + \mathrm{H} = \mathrm{H_2} \qquad \text{or} \qquad \mathrm{O} + \mathrm{O} = \mathrm{O_2}\,.$$

Atomic reactions have a small energy of activation and consequently practically every collision of atoms leads to reaction.

It is precisely as a result of this that the fact is explained that gases in the atomic state are not encountered in nature, for example oxygen, hydrogen etc., since the atoms formed soon combine into molecules.

4. ENERGY OF ACTIVATION

One of the principal problems of chemical kinetics is the determination of the energy of activation for different reactions. The determination of the magnitude of the energy of activation in the general case is extremely complex. Nowadays, the values of the energy of activation are known for only relatively small numbers of reactions. The energy of activation can be determined approximately by an empirical formula. In particular, for bimolecular reactions of the type $B + C \rightarrow D + E + \cdots$, the energy of activation can be calculated according to the following formula:

$$E = \frac{1}{4}(E_B + E_C)\,, \qquad \text{(I.17)}$$

where E_B and E_C are the energies which it is necessary to expend on rupturing the intra-molecular bonds for the molecule of substance B and for the molecule of substance C.

The values for the energy of activation, determined experimentally and calculated according to the empirical formula (I.17) for certain reactions, are presented in Table 5.

Examples of the magnitude of the energy of rupture of the intra-molecular bonds for certain simple and complex substances are presented in Table 6.

The first two reactions given in Table 5 have been sufficiently well studied, since they may take place fairly slowly.

TABLE 5.†

Reaction		$H_2 + I_2 = 2\,HI$	$H_2 + Br_2 = 2\,HBr$	$H_2 + ICl = HI + HCl$	Gasoline or kerosene with oxygen
E kcal/kg.mol	Calculated	40×10^3	43×10^3	42×10^3	—
	By experiment	39×10^3	43×10^3	34×10^3	30×10^3

TABLE 6.*

Substance	H_2	O_2	I_2	$>C-H$	$-\overset{\vert}{\underset{\vert}{C}}-\overset{\vert}{\underset{\vert}{C}}-$	$\overset{\vert}{\underset{\vert}{C}}=\overset{\vert}{\underset{\vert}{C}}$
E kcal/kg.mol	103×10^3	116×10^3	$35{\cdot}4 \times 10^3$	$85{\cdot}6 \times 10^3$	$62{\cdot}8 \times 10^3$	101×10^3

5. CHAIN REACTIONS

If we consider the reaction

$$H_2 + I_2 = 2\,HI \tag{1}$$

then the ultimate quantity of the end products of the reaction between H_2 and I_2 is formed as a result of collision of their molecules.

However, side by side with the principal reaction determining the formation of the end-product HI, intermediate reactions also take place which in their turn may also lead to formation of the end product.

For example, the following reaction always takes place

$$I_2 + M \rightleftarrows 2\,I + M$$

(M is any molecule, collision with which of the molecule of I_2 leads to dissociation of the latter).

The presence of free atomic I may lead to the following reactions, as a result of which the formation of the end product takes place

$$I + H_2 = HI + H \tag{2}$$

$$H + I_2 = HI + I \tag{3}$$

$$\text{Summing:} \quad I + H_2 + I_2 = 2\,HI + I\,.$$

These reactions, although also taking place, have little influence on the formation of the end product, since the velocity of reaction (1) is substantially greater than the velocity of (2) and (3), and consequently the end product is formed mainly on account of reaction (1).

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† Data taken from the work of A. B. Nalbandyan and V. V. Voyevodskii, *Mechanism of Oxidation and Combustion of Hydrogen*, published by Academy of Sciences, U.S.S.R., 1949.

We shall arbitrarily call reactions, the velocity and nature of the occurrence of which are determined by the form of the stoichiometric equation, "simple" reactions.

Let us consider a second possible case, when the formation of the end products is principally determined by the periodically-recurrent intermediate reactions.

As a result of this, direct collision of active molecules of the initial substances may also lead to the formation of end products, but in relatively small quantity. The intermediate reactions provide the main fraction of the end products. Let us take the reaction

$$H_2 + Cl_2 = 2\,HCl\,. \tag{1'}$$

Experiment shows that the nature of the reaction is different in its radical form from the previous case ($I_2 + H_2 = 2\,HI$).

Intermediate reactions also take place here associated with the formation of atomic chlorine the initial quantity of which is determined by the equilibrium state of the system

$$Cl_2 + M \rightleftarrows 2\,Cl + M\,.$$

The presence of atomic chlorine leads to the reactions

$$Cl + H_2 = HCl + H \tag{2'}$$

$$H + Cl_2 = HCl + Cl \tag{3'}$$

$$\text{Summing:}\quad Cl + H_2 + Cl_2 = 2\,HCl + Cl\,.$$

The atom of chlorine remaining from these reactions enters into a new series of reactions in accordance with the equations described.

In this case we have the reverse picture for the formation of the end products. Since the energy of activation of reaction (1′) is considerably greater than the energy of activation of reaction (2′), which in its turn is greater than the energy of activation of reaction (3′),

$$E_1 > E_2 \qquad \text{and} \qquad E_2 > E_3\,,$$

then the formation of the end products proceeds mainly on account of the intermediate reactions (2′) and (3′). Since in the system of reactions (2′) and (3′) the velocities of the chemical reactions are different, then the rate of formation of the end-product (HCl) is determined by the velocity of reaction (2′) being the slower ($E_2 > E_3$).

Thus,

$$w_{HCl} = 2\,\text{const.}\sqrt{T}\,e^{-E/RT}\,n_{Cl}\,n_{H_2}\,. \tag{I.18}$$

The factor "2" enters into the formula determining the rate of formation of HCl, since in the reactions (2′) and (3′) considered, we see that two molecules of HCl are obtained per one atom of Cl.

The concentration of chlorine atoms can be determined from the equilibrium conditions, i.e., the equations for the velocities of the reaction for the formation of the chlorine atoms, and for the reverse reaction of their recombination:

$$Cl_2 + M \rightleftarrows Cl + Cl + M .$$

Denoting the velocity constants of the forward and reverse reactions by K_1 and K_1', we can write down this condition in the form

$$K_1 n_{Cl_2} = K_1' n^2{}_{Cl} ,$$

whence

$$n_{Cl} = \sqrt{\frac{K_1}{K_1}} \sqrt{n_{Cl_2}} . \tag{I.19}$$

Consideration of the reactions described leads to the conclusions that

(1) the reaction $Cl_2 + M \rightleftarrows 2\,Cl + M$ leads to the formation of a relatively large quantity of free chlorine;

(2) the velocity of reaction (1′) is approximately 100,000 times less than the velocity of reaction (2′), which in its turn is substantially less than the velocity of reaction (3′) which occurs practically instantaneously, and therefore reaction (2′) also mainly determines the formation of the end product.

The atom of Cl obtained as a result of these reactions becomes the source of new reactions, which are also repeated further.

However, one should pay attention to the fact that the quantity of atomic chlorine is not decreased.

Thus, the reaction

$$Cl_2 + H_2 = 2\,HCl$$

takes place according to the following scheme:

$$\begin{array}{c} \text{Element} \\ \text{of} \\ \text{chain} \end{array} \left\{ \begin{array}{l} Cl + H_2 = HCl + H \\ \downarrow \\ H + Cl_2 = HCl + Cl \\ \hline \qquad\quad 2\,HCl + Cl \end{array} \right.$$

$$\begin{array}{c} \text{Element} \\ \text{of} \\ \text{chain} \end{array} \left\{ \begin{array}{l} \downarrow \\ Cl + H_2 = HCl + H \\ \downarrow \\ H + Cl_2 = HCl + Cl \\ \hline \qquad\quad 2\,HCl + Cl \end{array} \right.$$

$$\begin{array}{c} \text{Element} \\ \text{of} \\ \text{chain} \end{array} \left\{ \begin{array}{l} \downarrow \\ Cl + H_2 = HCl + H \\ H + Cl_2 = HCl + Cl \\ \hline \qquad\quad 2\,HCl + Cl \end{array} \right.$$

etc.

The active source in the sequence of reactions produced is the atom of chlorine.

Reactions for which the rate of formation of the end products is mainly determined by the velocity of the intermediate, periodically-recurring reactions, are called chain reactions.

A system of periodically-recurring reactions is called an element of chain. Chain reactions are of two types: with non-branching and with branching chains.

By chain reactions with non-branching chains are implied such reactions for which, for each element of chain, the number of active centres in the initial and final reactions is maintained unchanged.

The reaction broken down above between chlorine and hydrogen is an example of a chain reaction with non-branching chains.

Under chain reactions with branched chains we shall consider such chain reactions in which, for each resulting element of chain, an increase in the number of active centres will take place relative to their initial quantity.

The reaction for the oxidation of hydrogen, $2\,H_2 + O_2 = 2\,H_2O$ can serve as an example of such a reaction.

If the mechanism of this reaction should correspond to the scheme for the course of a simple reaction, then for the formation of H_2O it should be necessary to have the simultaneous collision of three energy-rich molecules, the probability of which is extremely small. Consequently, the rate of formation of H_2O should be insignificant.

In practice this reaction, under certain conditions, takes place very rapidly.

The reason for such rapid progress of the reaction consists in the fact that it involves chain reactions with branched chains.

Let us consider one of the possible schemes for this reaction.

In the mixture of molecules of oxygen and hydrogen, a certain quantity of atomic hydrogen is always present, produced as a result of the reaction

$$H_2 + M = 2\,H + M\,.$$

It should be noted that this reaction proceeds slowly and consequently hydrogen atoms may only be present in small quantities.

The presence of atomic hydrogen may occur by the following reaction:

$$H + O_2 = OH + O\,. \tag{I}$$

The atom of hydrogen so-formed takes part in the reaction

$$O + H_2 = OH + H\,. \tag{II}$$

The two OH radicals produced in reactions (I) and (II), colliding with molecules of H_2, react with them forming as a result H_2O and a free atom of hydrogen:

$$OH + H_2 = H_2O + H; \tag{III}$$

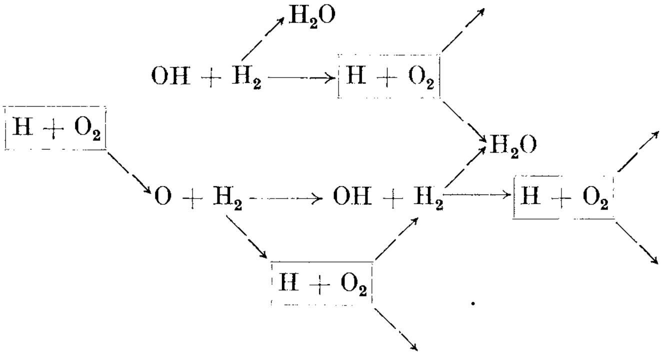
$$OH + H_2 = H_2O + H\,. \tag{III'}$$

From the sum total of reactions (I), (II), (III) and (III′) we have

$$H + O_2 + 3\,H_2 = 2\,H_2O + 3\,H\,.$$

It can be seen from the system of reactions (I – III) that the presence initially of a single free atom of hydrogen has led, as a result of the system of reactions shown above, to the appearance of three atoms of hydrogen, which in turn are sources for similar reactions.

The reaction can be represented schematically in the following form:



The number of free hydrogen atoms as a result of the occurrence of these systems of reactions, increases very rapidly.

The rate of formation of the end product H_2O is determined by the slowest reaction in the system. Reaction (I) proves to be this reaction, having the greatest energy of activation.

$E_I = 18{,}000$ kcal/kg.mole compared with $E_{II} = 6000$ kcal/kg.mole, and $E_{III} = 10{,}000$ kcal/kg.mole. Consequently, the rate of formation of H_2O is determined by the following formula

$$w_{H_2O} = 2\ \text{const.}\sqrt{T}\cdot e^{-E_I/RT}\, n_H\, n_{O_2} \qquad (I.20).$$

The factor "2" is inserted in view of the fact that as a result of the system of reactions (I–III), two molecules of H_2O are produced per one atom of H absorbed in reaction (I).

From expression (I.20) it follows that determination of the velocity of a reaction according to its scheme is only possible for a known concentration of atomic hydrogen in the mixture.

In order to determine the concentration of atomic hydrogen, it is not possible to apply equilibrium conditions, as was done previously for calculating the concentration of atomic chlorine, since the formation of active centres is determined mainly by the system of reactions (I–III).

It should be emphasized that the value of our scheme deduced above, which taken by itself is only approximate†, consists in the fact that it gives the possibility of representing correctly the basic qualitative characteristic of the course of a chain reaction with branched chains.

The special features of the course of such reactions are determined by the increase in concentration of active centres according to the measure of the branching reaction, which leads to its auto-acceleration. In order to represent more fully the nature of the course of chain reactions with branched chains, we shall consider the initial stage of development of such reactions.

6. DEVELOPMENT OF A CHAIN REACTION

It is obviously always important to know how a reaction branches and what conditions determine its development in the initial phase.

As an example we shall choose the development of the chain reaction for $2\,H_2 + O_2$ given above, with branched chains. We shall assume that the reaction commences under the following conditions:

(1) the quantity of free hydrogen atoms is determined mainly by reactions (I), (II), (III), where the very slow reaction (I) is of primary importance. Consequently, the reaction $H + H = H_2$ can be neglected, since at the beginning of the reaction the number of hydrogen atoms is relatively small and consequently, the probability of their collision is also small.

(2) the concentrations of the initial substances n_{O_2} and n_{H_2}, at the very commencement of the reaction are unchanged and are equal to the original concentrations, i.e.,

$$n_{H_2} \approx (n_{H_2})_0 \, .$$

(3) the reaction proceeds initially with small change of temperature, which permits the assumption to be made that $T = \text{const.}$, as for an isothermal reaction.

Having made these assumptions, it is easy to determine the change of concentration of atomic hydrogen on account of branching of the reaction and consequently also to determine the velocity of the primary reaction.

It was shown above that the rate of formation of H_2O is determined by equation (I.20), which can be written in the following form:

$$w_{H_2O} = K\, n_H\, n_{O_2} \, , \tag{I.21}$$

where K is the velocity constant; $K = 2\ \text{const.}\ \sqrt{T} \cdot e^{-E/RT}$.

From the system of reactions (I), (III) and (III′) we can see that the rate of formation of active centres w_H is equal to the rate of formation of the end product H_2O, i.e.,

$$w_{H_2O} = w_H \, ,$$

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† To the choice of the reaction $2\,H_2 + O_2$, much literature is devoted see for example, A. B. Nalbandyan, V. V. Voyevodskii *Mechanism of Oxidation and Combustion of Hydrogen*, published by Academy of Sciences USSR, 1949.

since as a result of the reaction, two molecules of H_2O and two new atoms of hydrogen were formed. The equation for the velocity in a differential form is:

$$\frac{dn_{H_2O}}{d\tau} = \frac{dn_H}{d\tau}. \tag{I.22}$$

Hence,

$$\frac{d(n_H)}{d\tau} = K\,(n_{O_2})_0\,n_H\,.$$

Integrating within the limits from $(n_H)_0$ to n_H and from $\tau = 0$ to τ, we have

$$\ln\frac{n_H}{(n_H)_0} = K(n_{O_2})_0\,\tau$$

or

$$n_H = (n_H)_0\,e^{K(n_{O_2})_0\tau}\,.$$

Hence we obtain the velocity of the reaction as a function of time:

$$w_{H_2O} = K(n_{O_2})_0\,(n_H)_0\,e^{K(n_{O_2})\tau}\,, \tag{I.23}$$

where the factor $(n_H)_0\,e^{K(n_{O_2})_0\tau}$ represents the instantaneous concentration of atomic hydrogen.

The function $w_{H_2O} = f(\tau)$ corresponds to the nature of the change of the factor $e^{K(n_{O_2})_0\tau}$ with time.

Let us consider a numerical example. For the initial conditions we shall take $T = 1000$ °K, $p = 2$ atm,

$$(n_{H_2})_0 = (n_{O_2})_0 = 7\times10^{18}\ \text{mol/cm}^3\,;$$

$(n_H)_0 = 1\ \text{cm}^{-3}$ (there is only one atom of hydrogen in the volume among the whole multitude of molecules).

The velocity constant $K = 2\times10^{-14}$.

From the pressure, temperature and ratio between the molecules of the final products it is possible to calculate the concentration of any of the final substances.

Let us substitute in the formula for velocity (I.23) the value $(n_{O_2})_0$, and we obtain the numerical expression

$$w_{H_2O} = 1.4\times10^5\times1\times10^{1.3\times10^5\times\tau}\,.$$

Let us take an interval of time equal to 0.0001 sec. For this value of τ the velocity is equal to

$$w_{H_2O} = 1.4\times10^5\times10^{13} = 1.4\times10^{18}\ \text{mol/cm}^3\ \text{sec}\,.$$

If we take as the next value $\tau = 0.001$ sec,

$$w_{H_2O} = 1.4\times10^5\times10^{130} = 1.4\times10^{135}\ \text{mol/cm}^3\ \text{sec}\,.$$

Such a value for the velocity is not achieved in reality, since the final assumptions made concerning the extent of branching of the reaction are incorrect. In fact, it is impossible even to neglect the "termination" reaction $H + H = H_2$, which, according to the extent of increase in the number of active centres, will have a great influence on the velocity of the reaction.

However, the examples considered indicate the immensely rapid development of the chain reaction with branched chains with respect to time. Even under isothermal conditions of flow the velocity increases sharply.

Figure 1 shows the nature of the change of velocity of a chain reaction with time at constant temperature.

For comparison, Fig. 2 shows the change of reaction velocity with time for the simple bimolecular reaction discussed earlier; the velocity of the reaction ($w = K\, n_1\, n_2$) falls with time since the concentration of the substances n_1 and n_2 decreases.

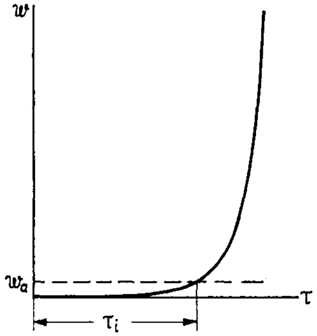

FIG. 1. Change of velocity of a chain reaction with time.

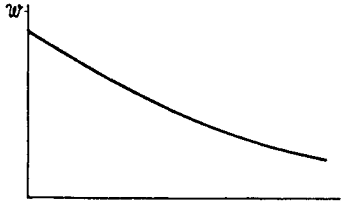

FIG. 2. Change of velocity of a bimolecular reaction with time.

Mathematically, the velocity of the reaction in this case is expressed by the following formula:

$$w_{H_2} = K\,\alpha\,(n_{H_2})_0^2 \left(\frac{1}{1 + K\,\alpha\,\tau}\right)^2, \qquad \text{(I.24)}$$

where α is a coefficient of proportionality, indicating in which proportion the end products are found at the commencement of the reaction $n_{H_2} = \alpha\, n_{I_2}$.

For a chain reaction with non-branched chains (Fig. 3) two cases are possible:

(1) if the number of active centres has already attained a maximum value and they exist in an equilibrium condition, then according to the extent of burn-out, the velocity of the reaction is reduced. In this case, the nature of the curve is the same as that for a simple reaction (curve 1 in Fig. 3).

(2) if at the commencement of the reaction the number of active centres does not attain a maximum, then the velocity of the reaction is small at first and increases according to the extent of increase of the quantity of active centres (curve 2, shown dashed in Fig. 3).

For a chain reaction with branched chains the curve of change of velocity with time (see Fig. 1) is distinguished by its radically different shape from the shape of the curves considered in the previous cases. It has a characteristically small increase of velocity initially, and then an abrupt increase of velocity over an extremely small interval of time.

For a given temperature the reaction does not develop at once, but there is a certain period of time during which the velocity of the reaction is extremely small.

The presence of these two portions is typical of a chain reaction with branched chains. The first portion, in which the reaction develops slowly, is characterized by the accumulation in the system of active centres to the extent necessary for subsequent vigorous development of the reaction.

The time elapsing from the instant of mixture formation up to the instant when the velocity of the reactions acquires a measurable value w_{τ_i} is called the induction period and is denoted by τ_i.

The induction period is not a physical constant and depends on many factors, in particular on the initial temperature, the material of the container etc. For example, an increase in temperature of the mixture leads to a decrease in the induction period.

In speaking of chain reactions it should be borne in mind that side by side with the formation of active centres, reverse reactions also take place and physical processes which lead to the destruction of active centres without the formation of the end products.

Such destruction of active centres is called breaking (or termination) of the chain.

Breaking of the chain may originate via a chemical route — as a result of recombination of active centres (for example, by the reaction $H + H = H_2$).

This is unimportant for the commencement of the reaction, but according to the extent of increase of concentration of hydrogen atoms, the velocity of this reaction increases and a time is reached when the concentration n_H ceases to increase. This naturally has an effect on the velocity of the chain reaction, the variation of which is shown in Fig. 4 versus time.

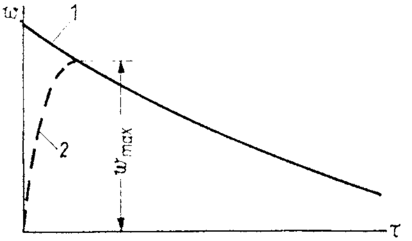

FIG. 3. Change of velocity of a chain reaction with non-branched chains with time.

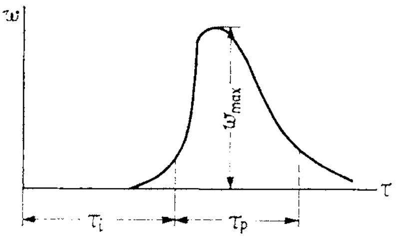

FIG. 4. Change of velocity of a chain reaction with time.

In addition, breaking of the chain can occur as a result of absorption of active centres by the surface of the vessel in which the reaction is taking place.

As a result of this the condition of the surface and the container material have a substantial influence on the absorption process.

It is wellknown that the reaction between hydrogen and oxygen ($2\,H_2 + O_2 = 2\,H_2O$) is relatively difficult to bring about in a vessel in which there is broken glass, which absorbs the atomic hydrogen, etc.

CHAPTER II

THE NATURE OF THE PROGRESS OF CHEMICAL REACTIONS. THE CONCEPT OF AUTO-IGNITION

WE SHALL consider as the basic relationships characterizing the progress of chemical reactions the functions $w = f(\tau)$ and $n = f(\tau)$, n is the instantaneous concentration of one of the reactants, which express the variation of the velocity of a chemical reaction and the concentrations of the reactants with time.

Dependent upon the nature of the reactants and the conditions under which the chemical reactions take place (for example, temperature, pressure, physical state of the reactants, shape and material of the reaction vessel etc.), it is possible to distinguish in practice "slow" reactions, for which the velocity w is small and the formation of the end products in significant quantities requires relatively large intervals of time, and reactions taking place within short intervals of time $\Delta\tau$, during which the bulk of the reagents reacts. The velocity w of these "fast" reactions increases rapidly, attaining extremely high values and only falls after an interval $\Delta\tau$ because of the reduction in the concentration of the original substances. The principal characteristic of a "fast" reaction is its marked auto-acceleration.

The reaction between one and the same reactants may be "fast" or "slow", depending upon the conditions.

The concept of "fast" and "slow" reactions is somewhat arbitrary and is introduced in a given case in order to emphasize the main difference in the possible nature of the course of a reaction. Thus for example, the form of the functions $w = f(\tau)$ and $n = f(\tau)$ for a "fast" or "explosive"† reaction is quite distinct from the form of the same functions for a slowly-developing reaction.

The typical nature of the change of $w = f(\tau)$ and $n = f(\tau)$ for explosive reactions is depicted in Fig. 5.

Three characteristic regions can be distinguished in the general case. The first region (1) is a slowly developing reaction in which the velocity

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† See for example, Ya. B. Zel'dovich and V. V. Voyevodskii *Thermal Explosion and Flame Propagation in Gases*, published by MMI, 1947.

The term "explosive" reaction should not be confused with the idea of "explosion", which will be discussed below.

is very small (unobservable). The quantity of the reaction products is also very insignificant. The second region (2) is characterized by a sharp increase in the velocity of the chemical reaction. In this region the bulk of the reagents react. The third region (3) is that in which the velocity decreases and tends to zero as $\tau \to \infty$. In this region reaction takes place between the "residual" reactants, the concentration of which is relatively small.

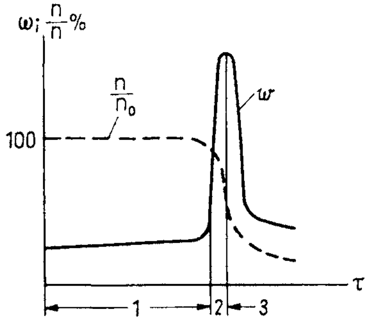

FIG. 5. The variations of reaction velocity and reactant concentration as functions of time.

1 — Region of slow increase of reaction relocity;
2 — Vigorous development of reaction;
3 — Fall in reaction velocity.

This characteristic of explosive reactions may be attributed to various causes: thus, if the reaction is "stationary" the deciding factor is the accumulation of heat in the system. The reaction acquires an explosive nature because of the increase of temperature within the volume in which the reaction takes place. For chain reactions with branched chains such a nature is dependent upon the accumulation of active centres. A chain reaction may also have an explosive nature although it is isothermal (there is a correspondingly appropriate elimination of heat); increase in temperature does, however, intensify its progress.

The time intervals $\Delta\tau_1$ and $\Delta\tau_2$ are of practical interest, essentially determining the time of the reaction and depending very markedly on the type and conditions of the course of the reaction.

In the technology of heat engines we are interested only in "fast" reactions, since only a very small interval of time is assigned to the process of combustion.

Suffice to say that the resident time of particles in the combustion chamber of a gas turbine engine may be less than 0·005 – 0·006 sec. In this time all the preliminary processes (for example, the process of atomization) and in particular the process of combustion should be completed (by completion of combustion is meant that not less than 90–97 per cent of the initial components should have reacted). Only reactions taking place with a sudden auto-acceleration under the influence of external effects correspond to this requirement, i.e., "fast" or explosive reactions.

The sudden increase in the velocity of a reaction under the influence of the physical and chemical processes arising in the combustion of a mixture is usually called auto-ignition.

Two types of ignition are distinguished in the theory of combustion; namely, chain ignition — which is dependent upon the accumulation of active centres, and thermal ignition — which is determined by the accumulation of heat (increase of temperature) from the combustible mixture.

We shall analyse briefly the nature of the course of chemical reactions in relation to the effect of external conditions.

1. THERMAL IGNITION OR THERMAL EXPLOSION

Brief mention was made earlier of thermal ignition, occurring as a result of the accumulation of heat in a system. We shall now discuss the problem in more detail.

(1) The time of elapse of the reaction under adiabatic conditions.
(2) The conditions necessary for development of the reaction.
(3) The concept of auto-ignition temperature as a result of a thermal explosion.

In practice it is extremely important to known what conditions it is necessary to create in order that the combustible mixture should burn within the limits of the combustion chamber.

For this, in particular, it is obviously necessary to calculate the time over which the combustible mixture burns and to know upon what this time depends.

Let us imagine a reheat chamber or the combustion chamber of a ram-jet engine, for which the fuel supply system ensures uniformity of combustion of the mixture throughout the combustion chamber, and that the initial temperature is sufficiently high to ensure ignition of the mixture without the aid of a special ignition source.

In considering the process taking place in the combustion chamber of such an arrangement, it can be assumed that combustion takes place over the entire volume of the chamber and without exchange of heat with the surrounding medium (i.e., "adiabatic" in the sense of absence of heat exchange with an external medium).

In order to ensure the necessary completeness of combustion in the chamber, it is necessary in the first instance to know the burning time of the fuel. Being able to determine $\tau = f(n)$ and knowing the rate of flow of the gas w, it is obvious that the length L_{ξ_z} can be calculated (Fig. 6), required for the combustion of any relative quantity of the initial combustible mixture.

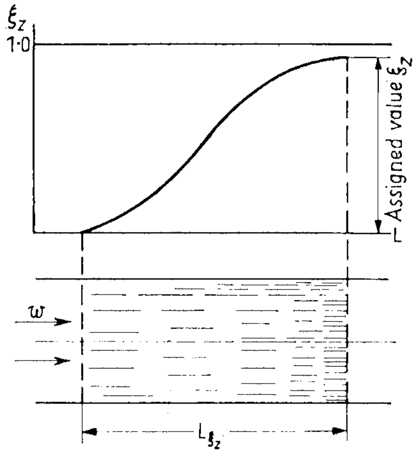

FIG. 6. Determination of the combustion chamber length for a given completeness of combustion.

The burning time of the fuel is obviously determined by the relationship $n = f(\tau)$, which can be determined by making known assumptions concerning the nature of combustion. Thus, for example, for the simplest case of the course of bimolecular chemical reactions under adiabatic conditions (i.e., without heat exchange with an external medium), this problem can be resolved.

For such reactions, the relationships $w = f(\tau)$ and $n = f(\tau)$, which, we established earlier, are the initial steps for determining the extent of combustion with time.

We find from the equation for the reaction velocity,

$$w = \text{const.} \sqrt{T}\, e^{-E/RT}\, n_1\, n_2 , \qquad \text{(II.1)}$$

or

$$\frac{dn_1}{d\tau} = \text{const.} \sqrt{T}\, e^{-E/RT}\, n_1\, n_2 ,$$

where n_1 is the fuel concentration;

n_2 is the concentration of oxidant.

Let us find T and n_2 as a function of n_1, i.e., $T = f(n_1)$ and $n_2 = f(n_1)$.

As a result of the reaction, an increase in temperature occurs from T_0 to an instantaneous value T. Let us derive the equation for the heat balance for the reaction between molecules

$$Q(n_0 - n) = c_v (T - T_0)\,, \tag{II.2}$$

where Q is the quantity of heat liberated by the reaction of one molecule; $(n_0 - n)$ is the quantity of the reacting molecules in unit volume; c_v is the specific heat relative to unit volume of gas.

For convenience we shall transform equation (II.2), substituting the value for Q as determined by the maximum heat liberated $Q \cdot n_0$ for complete combustion.

$$Q\, n_0 = c_v (T_{\max} - T_0)\,; \tag{II.3}$$

whence

$$Q = c_v \frac{T_{\max} - T_0}{n_0}\,. \tag{II.4}$$

After substitution in the equation for the heat balance (II.2) we have

$$\frac{n_0 - n}{n_0} c_v (T_{\max} - T_0) = c_v (T - T_0)\,. \tag{II.5}$$

By carrying out a series of simple transformations and taking $c_v = \text{const.}$, we obtain the required relationship between concentration and temperature

$$n = n_0 - n_0 \frac{T - T_0}{T_{\max} - T_0}\,;$$

$$n = n_0 \left(1 - \frac{T - T_0}{T_{\max} - T_0}\right) = n_0 \frac{T_{\max} - T}{T_{\max} - T_0}\,, \tag{II.6}$$

or

$$\frac{n}{n_0} = \frac{T_{\max} - T}{T_{\max} - T_0}\,. \tag{II.7}$$

Formula (II.7) is depicted graphically in Fig. 7.

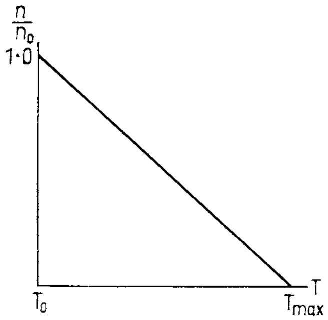

FIG. 7. Change of concentration of components as a function of temperature.

By using the expression thus obtained, the temperature can be determined as a function of the ratio of the concentrations n/n_0 and $T_{\max}$:

$$T = T_{\max} - \frac{n}{n_0}(T_{\max} - T_0)\,;$$

$$T = T_{\max}\left[1 - \frac{n}{n_0}\frac{\Delta T_{\max}}{T_{\max}}\right] = f(n)\,, \tag{II.8}$$

where $\Delta T_{\max} = T_{\max} - T_0$ is the maximum increment of temperature due to combustion.

The concentrations n_1 and n_2 enter into the velocity equation. According to the circumstances of the problem, the relationship between concentration of combustible and oxidant is always given by

$$n_1 = a\, n_2\,,$$

where a is a coefficient of proportionality. In the case of a stoichiometric mixture this coefficient, for bimolecular reactions — which in our case also determines the rate of formation of the end product — is constant and equal to unity. Further, when $\alpha \neq 1$, the coefficient a also is not equal to unity and is changed according to the extent of combustion of the mixture.

The relationship between a and α is expressed by the following formula:

$$a = \frac{n_1}{(\alpha - 1)\, n_{10} + n_1} = \frac{1}{(\alpha - 1)\dfrac{n_{10}}{n_1} + 1}\,, \tag{II.9}$$

where n_1 is the instantaneous concentration;
n_{10} is the initial concentration.

Expression (II.9) is easily obtained from the following obvious relationship

$$n_{10} - n_1 = n_{20} - n_2\,, \tag{II.10}$$

where n_{20} and n_2 are respectively the initial and instantaneous concentrations of the second reactant — to be exact — the oxidant.

From (II.10) we obtain immediately

$$n_2 = n_{10}\left(\frac{n_{20}}{n_{10}} - 1\right) + n_1\,.$$

The ratio n_{20}/n_{10} represents the ratio of the initial molecular concentrations of oxidant and combustible, i.e., the excess oxidant coefficient α. Thus, the expression for a can be written in the form of (II.9).

Substituting in place of n_1 and T their values in the function for n in the formula for the reaction velocity (II.1), we have

$$w = \text{const.}\, \sqrt{T}\, e^{-E/RT}\, n^2\, a\,,$$

or finally

$$w = \text{const.}\ \sqrt{f(n)}\ \mathrm{e}^{-E/Rf(n)}\, n^2 a\,. \tag{II.11}$$

Having the relationship (II.11), we can determine the change of concentration with time by a graphical integration method:

$$-\frac{\mathrm{d}n}{\mathrm{d}\tau} = \text{const.}\ \sqrt{f(n)}\ \mathrm{e}^{-E/Rf(n)}\, n^2 a = \text{const.}\, n_0^2\ \sqrt{f(n)}\ \mathrm{e}^{-E/Rf(n)} \left(\frac{n}{n_0}\right)^2 a\,.$$

The right-hand portion of the equation is always known for any extent of combustion. Let us solve the equation by a numerical method by assigning a number of values for n/n_0 and calculating $\mathrm{d}\tau/\mathrm{d}n$. For the calculation of these data, we construct the graph of $-\mathrm{d}\tau/\mathrm{d}n = f(n/n_0)$ (Fig. 8).

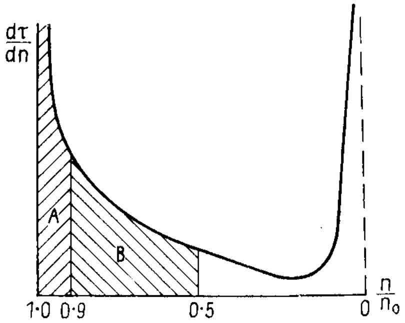

FIG. 8. Determination of time of combustion of mixture.

A — Time of combustion of 10 per cent mixture;

B — Time of combustion of 40 per cent mixture.

The area enclosed below the curve in Fig. 8 gives the combustion time of the mixture.

Thus we can solve the problem at last, and we obtain

$$n = f(\tau)\,.$$

For greater clarity we combine the graphical representation of the relationship obtained above into a single graph (see Fig. 9).

It can be seen from the curves that the higher T_0, the more rapidly does the curve for w begin to increase. Further, by analysing the relationship for change of concentration it is possible to determine the quantity of the substance which has reacted with a low velocity. Moreover, the relationships show that the process of oxidation is very slow.

Data are presented in Tables 7 and 8, characterizing the development of of a bimolecular reaction with time, taking into account all conditions.

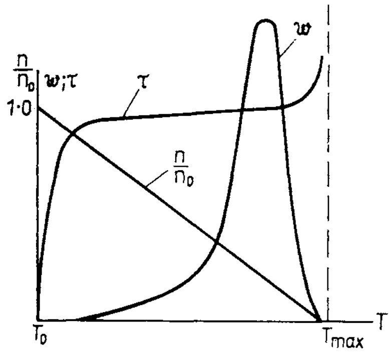

FIG. 9. Variation of velocity, reaction time and concentration of components as functions of temperature.

In the computation of the tables, vitiation by the hot combustion products has been taken into account, as a consequence of which preheating of the unburnt mixture occurs; in Table 7 the initial temperature of the mixture was taken as 300 °C.

TABLE 7.

Quantity of reacting substance, per cent	0·56	1·3	5·6	5·6	56
Temperature increase	300–310	310–333	800–900	900–1000	1000–2000
Time τ sec	$7{\cdot}5 \times 10^{10}$	10^{10}	$3{\cdot}6 \times 10^{-3}$	6×10^{-4}	$3{\cdot}7 \times 10^{-4}$

TABLE 8.

Quantity of reacting substance, per cent		50	90	97
Time in sec	$T_0 =$ 800 °K	$4{\cdot}40 \times 10^{-3}$	$4{\cdot}51 \times 10^{-3}$	—
	$T_0 =$ 1000 °K	$2{\cdot}9 \times 10^{-4}$	$3{\cdot}1 \times 10^{-4}$	$3{\cdot}6 \times 10^{-4}$

Mechanism of Thermal Ignition

For simple reactions the nature of the progress of the reaction depends in the first instance on the thermal regime set up in the space in which the reaction takes place. Together with the accumulation of heat in the system, heat exchange with the external medium will also take place on account of the chemical reaction.

It is well known that the quantity of heat removed from or supplied to the volume under consideration through its surface depends on the magnitude of the external surface area, the temperature difference between the inside and outside of the volume and the heat transfer coefficient.

The quantity of heat removed is

$$Q_{\text{loss}} = F_v \alpha_v (T - T_0), \tag{II.12}$$

where T is the temperature of the reacting mixture;
T_0 is the temperature of the surrounding medium;
α_v is the heat transfer coefficient;
F_v is the surface area of the container.

The effect of heat loss on the development of a chemical reaction can be established graphically by considering the progress of a reaction in three containers of similar volume, but with different surface areas; for example a sphere, a cube and a parallelepiped; the value of α_v for all three cases we shall take as identical.

It is well-known that, for a given volume, the sphere will have the minimum surface area and the parallelepiped the maximum surface area.

It is obvious from formula (II.12) that the curve of heat loss, as a function of reaction temperature, has a straight-line form (Fig. 10). The slope for direct heat removal will be determined by the value of the numerical factor $\alpha_v F_v$ and will be greater, the greater the heat transfer coefficient and the cooling surface of the container.

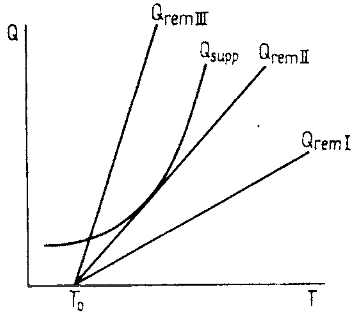

FIG. 10. Variation with temperature of the quantity of heat lost from systems with different amounts of surface cooling.

Moreover, the heat removal curve can be displaced with constant slope to left or right, depending upon the value of T_0 (Fig. 11).

Supply of heat to the reacting substances is brought about because of chemical reaction.

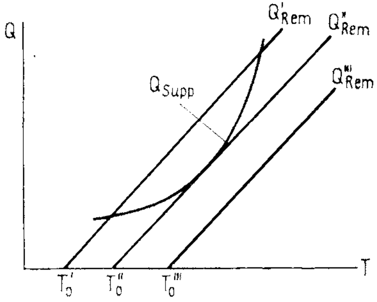

FIG. 11. Effect of magnitude of initial temperature T_0 on the position of the heat removal curve.

If q is the heat released by the combustion of one molecule, then

$$Q_{\text{supp}} = qw = q \text{ const. } \sqrt{T}\, e^{-E/RT}\, n_1 n_2 \,. \tag{II.13}$$

Uniting constant values we obtain:

$$Q_{\text{supp}} = B\, e^{-E/RT} \sqrt{T} \,. \tag{II.14}$$

The dependence of the heat supply on the reaction temperature is of exponential form, since the dominant term is the factor $e^{-E/RT}$ (see Fig. 10).

Let us now consider both processes together (Fig. 12). Let the supply of heat be changed in accordance with the curve Q_{supp}, and the loss of heat with respect to the straight line Q_{loss}. The curve of Q_{supp} can intersect the direct heat loss curve twice at points 1 and 2.

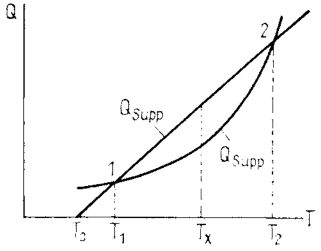

FIG. 12. Determination of ignition temperature with respect to the equilibrium condition for the supply and loss of heat.

At a temperature T_0 the temperature of the gas is equal to the temperature of the walls, and there is no heat loss. Heat is released in the reaction taking place in the container and the temperature increases. At a certain temperature T_1 the rate of generation and loss of heat become equal and the reaction cannot develop further.

Development of the reaction at a temperature T_x ($T_x > T_1$) is also impossible, since more heat is removed through the walls of the container than is liberated by the chemical reaction, i.e.,

$$Q_{\text{supp}} < Q_{\text{loss}} .$$

In this case the reaction will be displaced into the region of lower temperature until it attains the value of the temperature corresponding to the intersection of the curves for Q_{supp} and Q_{loss}, i.e. the equilibrium state (point 1).

Let us increase the temperature of the mixture to T_2, i.e. to the point 2 of intersection of the heat-release curve with the line Q_{loss}. As a result of this we also reach the equilibrium state. If, however, there occurs even a trifling increase in temperature, then Q_{supp} will be greater than Q_{loss} and the reaction commences to develop very vigorously. The temperature corresponding to equality of supply and loss of heat at point 2 we shall call the auto-ignition temperature. It should be noted that the auto-ignition temperature is not a constant characterizing a particular combustible mixture, since it depends not only on Q_{supp} but also on Q_{loss}, which is entirely determined by purely external conditions. For example, the quantity of heat lost from a container of different shape at a given temperature is not identical, and consequently the auto-ignition temperatures likewise do not coincide.

For a supply of heat in excess of that lost, accumulation of heat occurs in the system which leads to an increase in temperature and a vigorous development of the reaction or, as it is frequently called, to a thermal explosion.

2. CHAIN REACTIONS

It has been shown previously that the velocity of a chain reaction depends on the concentration of active centres and the temperature of the mixture.

When the concentration of active centres is relatively large and the temperature of the mixture is sufficient for the slowest intermediate reaction to occur intensively, and if as a result of this there is no large removal of heat from the system in which the reaction occurs, then the quantity of active centres will be increased and the velocity of the reaction, already considerable, will increase abruptly.

It is easy to arrive at such a conclusion by considering the data presented in Chapter I.

However, the case in which the state of the combustible mixture corresponds to the condition for development of a chain reaction, is a particular case. As a rule, this stage of a reaction arises owing to the effect of external conditions on the reacting mixture, the reaction velocity of which is initially quite trivial and practically imperceptible.

In order to understand the process of development of a chain reaction with branched chains, we shall consider it in the initial state when the concentration of active centres is very trivial.

In this case, clearly, the reaction velocity will depend only upon the concentration of active centres and on the conditions which give rise to their increase.

As was shown above, besides the reaction leading to the formation of active centres, reactions and processes take place (for example, the process of absorption of active centres) which lead to "non-productivity" (without the formation of end products) by destruction of active centres — to breaking of the chain.

The reaction between hydrogen and oxygen, discussed above, may be cited as an example.

As a result of the system of intermediate reactions I–IV the number of atoms of hydrogen (active centres) increases. However, not all the atoms of hydrogen, formed as a result of reactions I–IV, lead to the formation of the end product (H_2O); as a result of the presence of chain-breaking reactions (absorption of atoms, the reaction $H + H = H_2$), hydrogen atoms are "non-productively" destroyed and do not lead to the formation of H_2O.

In order to determine the conditions for the development of a chain reaction with branched chains we shall write down the equation for the rate of formation of active centres.

We shall denote the velocity of direct formation of active centres (for example, $H_2 + M = 2\,H + M$) by the symbol w_1, the velocity of the chain-branching reaction by w_2, and the velocity of the chain-breaking reaction by w_3.

These reactions determine the formation of active centres. The first and second reactions promote an increase, but the third reaction, on the contrary, promotes a decrease of active centres. We can now write the expression for the velocity of formation of active centres via the velocities w_1, w_2, and w_3.

$$\frac{dn}{d\tau} = w_1 + w_2 - w_3\,. \tag{II.15}$$

It should be recalled that the velocity w_1 is extremely small in magnitude, since the first reaction has a large energy of activation.

The velocities w_2 and w_3 are proportional to the concentration of active centres and they can be written down in the following form

$$w_2 = f\,n; \tag{II.16}$$

$$w_3 = q\,n, \tag{II.17}$$

where n is the concentration of active centres;
f and q are coefficients, depending on the temperature, energy of activation and other factors.

The velocity w_2 is determined by the slowest reaction from the system of chain reactions.

In the example given above, the reaction $H + O_2 = OH + O$ is the slowest, for which

$$w_H = 2 \text{ const} \sqrt{T}\, e^{-E/RT}\, n_{O_2}\, n_H . \tag{II.18}$$

In the given case

$$f = 2 \text{ const} \sqrt{T}\, e^{-E/RT}\, n_{O_2} .$$

The rate of formation of active centres can now be written down in the following form:

$$\frac{dn}{d\tau} = w_1 + f\,n - q\,n \tag{II.19}$$

or

$$\frac{dn}{d\tau} = w_1 + \varphi\,n\,, \tag{II.20}$$

where $\varphi = f - q$.

The velocity w_1, in view of its smallness, does not as a rule exert a significant influence on the development of the chain reaction. Branching reactions and chain-breaking reactions exert the main influence on the formation of active centres.

External conditions (temperature and pressure) exert a considerable influence on the rate of branching of the chain as well as on the rate of chain breaking. However, the extent of their influence is different. Thus, the velocity of the chain-breaking reaction is, in practice, very small, depending on the temperature, since the energy of activation in the reaction for destruction of active centres is extremely small (such reactions proceed without rupture of intramolecular bonds). At the same time, the velocity of the chain-branching reaction depends, on the contrary, very strongly on the temperature because it has a sufficiently large value for the energy of activation.

In our example, the reaction determining the velocity of the chain branching reaction has $E = 18{,}000$ kcal/mole. Consequently, at low temperatures this reaction will take place very slowly and the difference $f - q < 0$.

It is obvious that in this case the reaction will not be branched. This fact is explained by the extremely slow development of chain reactions at low temperatures†. The reaction between hydrogen and oxygen may be cited as an example. Hydrogen and oxygen may actually exist together for as long a time as is desired.

The velocity of the chain-branching reaction w_2 increases according to the extent of increase of temperature (the coefficient f will increase), but the velocity of the chain-breaking reaction is practically unchanged. Consequently, the difference $f - q$ changes sign and becomes greater than zero ($f - q > 0$), which leads, over a certain time interval, to an increase of concentration of active centres and to a vigorous development of the reaction because of the rapid increase of active centres in the system.

Reactions from the "invisible", unobservable region pass over into the "visible" region and auto-ignition takes place, as it is usually called, or in other words — a chain explosion.

The temperature T_i of the mixture, corresponding to the condition $f - q = 0$, we shall call the temperature of chain auto-ignition. For the mixture of hydrogen and oxygen under consideration the temperature of auto-ignition can, under certain conditions, be equal to $T_i = 550$ °C. This auto-ignition temperature is not a physical constant characterizing a specific combustible mixture, but has an even more arbitrary nature than the auto-ignition temperature resulting from the thermal mechanism of the development of a reaction.

For the slightest increase in temperature of the mixture above the temperature T_i, a process of accumulation of active centres is initiated in the system. The reaction velocity as a result of this remains small as long as the quantity of active centres is insufficient for rapid development of the reaction, when the course of the reaction now becomes observable.

Such an accumulation of active centres occurs over a definite interval, which is called the induction period.

It is important, in practice, that the induction period should be substantially less than the residence time of the fuel in the combustion chamber, which may amount to less than 0.005–0.006 sec.

Academician N. N. Semenov, in developing the theory of chain reactions, has given a method for determining the induction period and the velocity of a chain reaction.

This method makes it possible to explain the principal features of the course of chain reactions and in a number of the simplest cases to carry out certain quantitative calculations.

† The values of these "low" temperatures will be different for different reactions.

It should be noted that the discussion is confined only to the commencement of the process, and consequently the change of concentration of the initial products cannot be taken into consideration.

Let us determine the concentration of active centres, for which we separate the variables in equation (II.20)

$$\frac{dn}{w_1 + \varphi n} = d\tau .$$

By integrating we obtain

$$\ln \frac{w_1 + \varphi n}{w_1 + \varphi n_0} = \varphi \tau ,$$

or

$$\frac{w_1 + \varphi n}{w_1 + \varphi n_0} = e^{\varphi \tau} ,$$

where n is the instantaneous concentration of active centres;
n_0 is the initial concentration of active centres.

In the case when the initial concentration of active centres is very small, $n_0 \rightarrow 0$, the equation takes the following form:

$$n = \frac{w_1}{\varphi} (e^{\varphi \tau} - 1) . \qquad \text{(II.21)}$$

The equation obtained is primarily for determining the concentration of active centres with time.

It can be seen from the formula that increase in τ gives a sharp increase in n, since time enters into the relationship in the form of a power. Knowing the concentration of active centres, the change in velocity can be easily determined for the primary chain reaction (in our example, the reaction for the formation of H_2O) as a function of time

$$w = afn , \qquad \text{(II.22)}$$

where a is a numerical coefficient, denoting how many molecules of the end product are formed as a result of the entry of one active centre into the reaction.

For our example

$$w_H = 2\,\text{const.}\, \sqrt{T} \cdot e^{-E/RT}\, n_{O_2} n_H = f\, n_H ,$$

where
$$f = 2\,\text{const.}\, \sqrt{T}\, e^{-E/RT}\, n_{O_2} ;$$

$$w_{H_2O} = 2\,\text{const.}\, \sqrt{T}\, e^{-E/RT}\, n_{O_2} n_H .$$

Comparing expressions, we see that $a = 1$, i.e., the rate of formation of the main product and the rate of formation of active centres are determined by one and the same reaction.

Eliminating n from expressions (II.21) and (II.22), we obtain

$$w = af \frac{w_1}{\varphi} (e^{\varphi \tau} - 1) . \qquad \text{(II.23)}$$

Hence, a number of conclusions can be drawn relative to the nature of the progress of the reaction. In future we shall assume that the accumulation of active centres is determined only by the branching and breaking reactions, since the velocity w_1 is usually small; thus

$$w = \varphi n .$$

First case. We shall assume $\varphi > 0$. In this case there will always be an accumulation of active centres and the reaction will be branched.

By constructing the function (II.23) we obtain the graphical presentation for $w = f(\tau)$, the form of which is shown in Fig. 13.

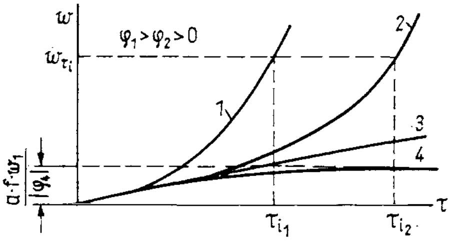

FIG. 13. Various cases for the progress of a chain reaction.

It has already been shown that for a chain reaction the presence of a time interval τ_i is characteristic, during which the reaction, superficially does not develop at all. Over this period the reaction velocity increases up to a certain minimum observed value w_{τ_i}, which under normal conditions lies on the steep portion of the curve of the function $w = f(\tau)$ (curves 1 and 2 in Fig. 13).

Thus for $\varphi > 0$ the chain reaction has a "concealed" portion, the main processes of which (influencing the development of the reaction) are processes connected not with the formation of the end products but with the formation of active centres in a quantity giving rise to a vigorous development of the reaction.

Whilst simple reactions acquire an explosive nature only under the influence of the accumulation of heat in the system (increase of temperature of the mixture), chain reactions with branched chains, on the other hand, have an explosive nature owing to the processes of accumulation of active centres.

The difference between these reactions is shown up particularly clearly by considering their progress under isothermal conditions (the temperature of the reaction mixture T = const.). Figure 14 shows the graph of the progress of such a reaction for T = const.

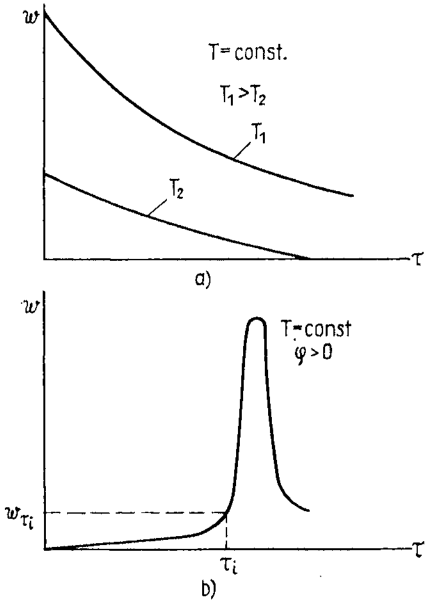

FIG. 14. Nature of change of reaction velocity under isothermal conditions (a) — For a simple reaction; (b) — For achain reaction with branched chains.

It can be seen from Fig. 14 that the velocity of the chain reaction increases up to a certain maximum value and then rapidly decreases. This is explained, just as for simple reactions, by the decrease in concentration of the reactants. Moreover, the chain-breaking reactions, which develop according to the increase in the quantity of active centres, exert an influence on the nature of the relationship $w = f(\tau)$.

Under conditions which ensure the accumulation of heat in the system, the progress development of both types of reaction will be more intensive: the induction period will be contracted, the development of the chain reaction will be more abrupt. The normal thermal reaction, as was mentioned earlier, also acquires an explosive character.

Second case, when $f = q$. In this case the rate of formation of active centres and the chain-breaking velocity are equal. The coefficient φ is equal to zero. By solving equation (II.23) an indeterminate expression is obtained, which is revealed by analysis to be a function of $e^{\varphi\tau}$ in the form

$$e^{\varphi\tau} = 1 + \frac{\varphi\tau}{1} + \frac{(\varphi\tau)^2}{2!} + \cdots . \tag{II.24}$$

After substituting in equation (II.23) and carrying out a number of simple transformations, we obtain a relationship which characterizes the change of velocity with time, when $f = q$

$$w \approx a\,f\,w_1\,\tau \tag{II.25}$$

(we shall neglect terms in the expansion, commencing with the third, in view of their smallness).

This function is presented graphically in Fig. 13 (curve 3). It is characteristic for this case that the reaction velocity increases according to a linear law (the reaction loses its explosive nature), until combustion of the mixture is affected. Of course, what has been said is true for $T =$ const. By increasing T, the character of the reaction is changed because of, for example, change in f.

Third case. This case differs from the previous ones in that, here, the chain-breaking velocity is greater than the rate of their formation, i.e., $\varphi < 0$. It follows immediately from formula (II.23) that $e^{\varphi\tau} - 1$ tends in the course of time to -1. The value for the velocity, as a result of this, will not be sharply increased, but will tend towards the limit

$$w = \frac{a\,f\,w_1}{|\varphi|}.$$

The nature of this relationship can be seen in Fig. 13 (curve 4).

In this case the chain reaction also does not lead to an explosion.

3. DETERMINATION OF THE INDUCTION PERIOD

It is clear from what has been said earlier how important it is to know the induction period, particularly in those cases when the hot mixture is in the combustion chamber for a limited time interval. In addition it is also essential to know the parameters which affect the induction period.

By analysing the change of velocity of a reaction with time (see Fig. 14b) we see that within the period of time the change of velocity is very insignificant and it is only possible to observe it for the value w_{τ_i}. The quantity w_{τ_i} in practice depends on the apparatus which is used for the experiment, but τ_i depends slightly on the value of w_{τ_i}, since w_{τ_i} lies on the steep portion of the function $w = f(\tau)$.

We shall arrange to call the interval of time, over which the velocity of the chemical reaction attains a perceptible value, the induction period.

From the expression for the reaction velocity

$$w_{\tau_i} = \frac{a\,f\,w_1}{\varphi}\left[e^{\varphi\tau_i} - 1\right] \tag{II.26}$$

it is possible to determine the induction period, which is equal to

$$\tau_i = \frac{1}{\varphi} \ln\left(\frac{\varphi\, w_{\tau_i}}{a\, f\, w_1} + 1\right). \tag{II.27}$$

In the majority of cases the unity can be neglected; then

$$\tau_i \approx \frac{1}{\varphi} \ln \frac{\varphi}{a\, f} \frac{w_{\tau_i}}{w_1}. \tag{II.28}$$

Since the ln changes slowly, the quantity φ exerts the dominant influence, i.e.,

$$\tau_i \sim \frac{1}{\varphi}.$$

Hence the conclusion can be drawn that the induction period is approximately inversely proportional to the difference between the velocity coefficients of the branching and breaking chain reactions.

The conclusion as stated has been verified in practice and is used in technology for influencing the progress of chemical reactions.

CHAPTER III

FLAME PROPAGATION IN A GAS STREAM

1. CONCEPT OF THE FLAME FRONT AND VELOCITY OF FLAME PROPAGATION

By observing a stationary combustion process in a gas stream, independent of the mode of origination (simple burner, combustion chamber of a gas turbine engine, etc.), a certain zone can always be distinguished in which the chemical transformations are mainly observed.

The zone of chemical transformation is of paramount importance for combustion chambers, since its position and dimensions, on the whole, make it possible to visualize the intensity of the combustion process and to choose rational dimensions for the "primary zone" of a combustion chamber, etc.

The position of the combustion zone mainly determines the operating lifetime of the chamber. High temperatures are usually developed in the combustion zone and its close proximity to the walls of the chamber (flame tubes) or the girth of its stabilizing devices can lead to destruction of the corresponding units of the combustion chamber.

It is obvious that the distribution of the combustion zone is dependent on the aerodynamic layout of the chamber, and the organization of the preparatory processes; for example, the process of fuel preparation (vaporization) and its mixing with the oxidant can vary.

The position of the combustion zone can be determined experimentally by various methods; for example, by measuring the temperature of the gases, by chemical analysis of a selected gas sample and, finally, by photography and visual observation of the flame.

Depending on the method used, results can be obtained which differ somewhat from one another: thus, chemical methods give the most accurate location of the zone of chemical transformation; distribution of temperature in the gas stream also makes it possible to infer the position of the combustion zone, although in this case it is necessary to take into account the phenomenon of heat transfer; for example, the region of temperature increase is not necessarily the region of chemical reaction but may be determined primarily by heat exchange phenomena. Visual observation and photography of the flame give certain outer limits for the flame, determined with respect to its brightness. However, the location of these limits will also depend on the sensitivity and selectivity of the photographic plate.

Figure 15 shows a picture which was obtained by the combustion of a combustible mixture in a turbulent flow with a centrally located ignition source, by the method of temperature measurement (lines *oa* and *ob*) and by means of photography (lines *oc*).

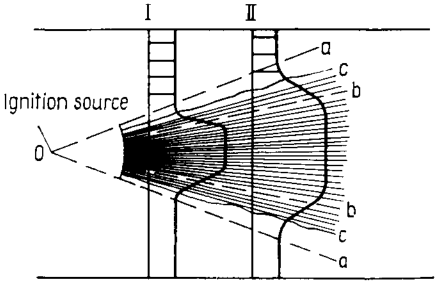

FIG. 15. Determination of flame boundaries.

It is obvious that it is not possible to establish accurately the position of the zone of chemical transformation by either one of these methods. Actually, the lines *oa* and *ob* give the range of temperature increase, which, in a given zone, is determined by thermal conductivity and not by chemical transformations.

A photograph of the flame gives essentially an "instantaneous" (if the exposure is made very small) or a somewhat average (with respect to time) position of the flame, which in itself still depends on the characteristics of the photographic material and does not give sufficiently accurate data for judging the position of the combustion zone.

However, both methods still make it possible to judge approximately the position of the combustion zone in space. Thus, for example, for homogeneous mixtures, knowing the dependence of the reaction velocity on the temperature, it is possible to conclude reasonably that the combustion zone lies between the rays *oa* and *ob* in the high temperature region, i.e., closer to the line *ob*.

Visual or photographic observations usually make it possible to establish the outer "visible" boundaries of the flame (lines *oc*) which lie between the lines *oa* and *ob*.

Obviously it is always possible to represent the surface which characterizes the position of the combustion zone in space; such a surface, found within the boundaries of the combustion zone will be called in future the flame front.

The representation of the flame front has been found to be extremely useful, since it has made it possible to evolve a reasonably consistent set of views regarding the velocity of flame propagation, to establish the conditions for the existence of a flame in a gas stream, and in certain cases to make an estimation of the dimensions of the flame "core" in combustion chambers.

The theory of flame propagation based on the concept of the flame front, for the simplest layout of combustion chambers, makes it possible to calculate the length of the hot section of a combustion chamber.

The combustion zone is frequently of such extent that it is difficult to determine the position of the arbitrary surface of the flame front.

It should be noted that the correct choice of the position of the flame front is of paramount importance for the experimental determination of the velocity of flame propagation as a function of the various parameters affecting it.

In the case of an incorrect choice of position for the flame front, the relationship obtained will bear a special character.

Usually, under the conditions existing in combustion chambers, the combustion zone has a considerable width, which, in the absence of limiting positions makes choice of the position of the flame front somewhat arbitrary.

It is precisely by this circumstance that the diverse experimental values for the velocity of flame propagation obtained by various authors are chiefly explained.

It follows by posing the same question, that the most accurate results can be obtained at the time when the combustion zone has a small width, since in this case the substitution of the particular volume occupied by the combustion zone by a surface — the flame front — is most justified.

Determination of many positions of the combustion zone indicate that the outer limits of the zone do not remain invariable. Figure 16 shows the position of the outer limits of the combustion zone at various instants of time.

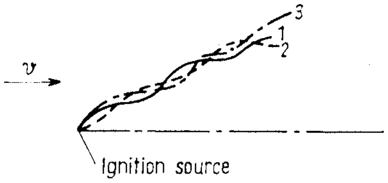

FIG. 16. Position of the outer limits of the flame at various instants of time.

However, if the external conditions are changed, relative to the zone, as a result of combustion, then it is always possible to measure its average position over long intervals of time.

In future, in talking of the position or width of the combustion zone, we shall take its mean position and average width over a long time interval.

Without pausing on the nature of this special problem, we shall pass on to a discussion of the results which can be obtained, emerging from the concept of the flame front.

2. VELOCITY OF FLAME PROPAGATION. CONDITIONS FOR THE EXISTENCE OF A FLAME FRONT

It has already been mentioned that for a stationary process of combustion of a homogeneous mixture, the position of the combustion zone is completely definable.

By substituting the combustion zone by the surface of the flame front we obtain a geometrical diagram of combustion, depicted in Fig. 17.

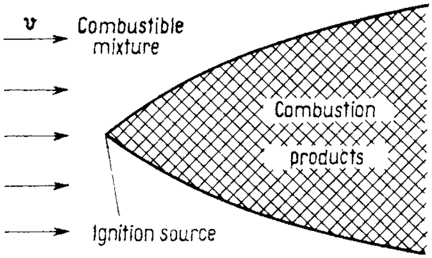

FIG. 17. Schematic diagram of the flame core.

Let us isolate a small element *ab* of the flame front, which, in view of its smallness can be considered as a straight line, and let us consider the conditions for its existence; for this, we resolve the velocity of the incident gas stream into two components: v_n in a direction normal to the surface of the flame front and v_k in a direction formed by *ab* with the separated section of the flame front (Fig. 18).

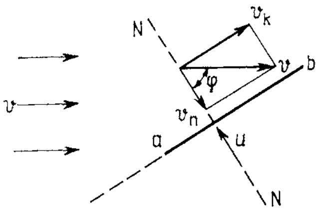

FIG. 18. Component velocities of the gas stream in an elementary section of the flame front.

The velocity v_n tends to displace the front in the direction $k - k$; the velocity v_k tends to displace the front along its surface in the direction $a - b$.

It is obvious that if the combustion conditions are unchanged, then the mean position of the front with respect to time should also be unchanged. Consequently, the velocities of the incident flame front v_n and v_k, tending to blow-away the flame front, should be opposed by the corresponding velocities of propagation of the same process of combustion.

Let us consider the velocity component v_n. Obviously, v_n is tending as if to close-up the flame core, increasing the magnitude of the angle φ. For $\varphi = 90°$ the surface of the jet will be directed parallel to the velocity of the stream v, and the velocity component v_n will be equal to zero. However the flame itself propagates towards the stream with a certain velocity and consequently the angle φ will always be less than its threshold value, but the component $v_n > 0$.

It is clear that a stationary flame core is established at such an angle φ when the velocity of flame propagation in a direction normal to the surface of the flame front ab, and the velocity v_n are identical. Usually, the velocity of flame propagation in the direction normal to the flame front is called the normal velocity of flame propagation or simply, the velocity of flame propagation.†

Denoting the velocity of flame propagation by u, we obtain the first condition for the existence of a stationary flame core

$$|u| = |v_n| \qquad \text{or} \qquad |u| = |v \cos \varphi| . \tag{III.1}$$

The expression obtained, being extremely important in the theory of flame propagation, was established in 1890 by the Russian physicist Mikhel'son as a result of studying combustion in laminar flow, and it is frequently called Mikhel'son's Law, or the Cosine Law.

The practical importance and application of the relationship obtained will be mentioned below.

Let us consider with what the tangential component velocity v_k is in equilibrium. Under the influence of v_k, particles on the surface of the flame front are transported in a direction from point a to point b, as shown in Fig. 18. It is obvious that the flame front can exist only when at point a, at the site of the particle which is removed, another similar particle arrives from the preceding zones of the front.

By considering the zone of the flame front adjacent to the base of the flame core, it can easily be seen that burning particles from the preceding zones of the flame front will not reach here in place of the burning particles being carried away.

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† In the literature, normal velocity usually implies normal velocity of flame propagation in laminar flow.

Hence it is clear that the flame front cannot exist without a constantly operating source of ignition located near the base of the flame core.

Actually, in the reverse case (if there is no ignition source) the particles removed from near the base of the flame core will assume a velocity v_k, and the flame core will be carried along the gas stream.

It is often possible to observe how the jet of flame breaks away from the origin and is blown off, as shown in Fig. 19.

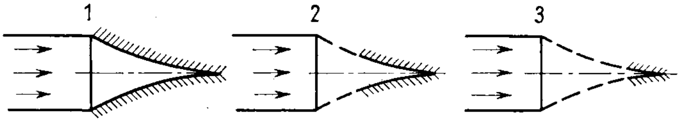

FIG 19. Flame blow-off from the base of the flame core.

1 — Flame core initiated at base of flame cone. *2* — Flame core has lifted from the base of flame cone and is blown off by the stream. *3* — State of flame core before its total dissipation.

Flame blow-off always occurs when the source of ignition either ceases to be operative or its power proves to be inadequate.

It should be noted that the reliable, effective and constantly operating ignition source in our discussion has been determined by the presence of the "blow-off" velocity v_k directed tangentially to the flame front.

It is obvious that if $v_k = 0$ (in the plane normal to the stream front) then a constantly operating ignition source is not guaranteed, since there is no transfer of particles along the front (Fig. 20). In this case the velocity of flame propagation is equal to the velocity of the moving stream.

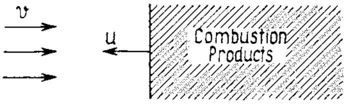

FIG. 20. Diagrammatic representation of an element of a plane flame front.

In practice, the planes of the combustion front are very seldom observed in view of their extreme instability and also because the velocity of the gas stream is always non-uniform along the chamber and the condition $v = u$ for the whole stream is never achieved.

Consequently, one is obliged to deal technologically with flames extended along the gas stream, the stability of which is achieved by the presence of powerful ignition sources.

Thus there are two necessary and sufficient conditions for the existence of a stationary flame in a gas stream.

(1) the presence of a constantly operating ignition source with sufficient intensity to achieve this purpose (under given conditions);

(2) maintenance of parity between the velocity of flame propagation u, and the component velocity v_k, normal to the flame front, i.e.,

$$|u| = |v_n| \qquad \text{or} \qquad |u| = |v \cos \varphi| .$$

It follows immediately from the second condition that if the stream velocity v is less than the velocity of flame propagation u, then the existence of a stationary flame core is impossible† since the condition $u = v$ $(v < u)$ cannot be maintained.

For $v > u$, the equation $u = v_n$ is automatically maintained as a consequence of change of the angle φ, and the second condition for the existence of a stationary flame cone surface determines only the shape of the flame jet.

To conclude this section it should be noted that the velocity of flame propagation is of great practical importance.

Actually, as we shall see later, the velocity of flame propagation u for specified conditions (mixture composition, degree of turbulence of the gas flow, etc.) has a quite definite value and can be determined by an experimental method.

Hence, by using Mikhel'son's Law it is possible to solve, for example, such a problem as the calculation of the length of the flame, important in practice. It is also possible to make a number of general remarks concerning the shape of the flame front resulting from a change in the velocity v, composition of the combustible mixture, etc.

Thus, for example, with increase of velocity of the mixture and for conditions in which u is changed only insignificantly (which occurs in many cases of practical interest), it follows immediately from the relationship

$$u = v \cos \varphi$$

that $\cos \varphi$ is reduced, the angle φ is increased, the flame front becomes sloping and the flame front is extended.

On the contrary, by reducing v the value of $\cos \varphi$ should be increased, the angle φ decreased, and the flame front should become steeper. Change of shape of the flame front for different flow velocities of the combustible mixture is shown in Fig. 21.

With change of composition of the mixture or change of the characteristics of the flow turbulence, u is changed.

In this case, the shape of the flame front is also changed (Fig. 22) and the angle of inclination of the flame front to the direction of motion of the moving gas stream is determined, just as in the first case, from the relationship $u = v \cos \varphi$.

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† It is obvious that in this case combustion is propagated against the gas flow — the flame "jumps".

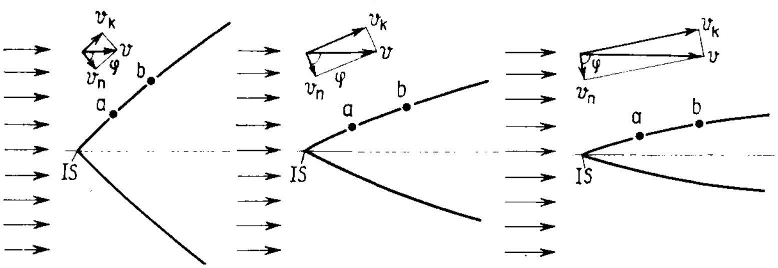

FIG. 21. Change of shape of jet as a result of increasing the flow velocity. IS — Ignition source; *ab* — Element of flame front.

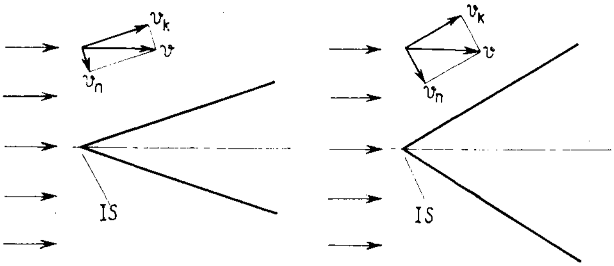

FIG. 22. Change of shape of flame core as a function of velocity of combustion

3. DETERMINATION OF THE LENGTH OF THE FLAME

This problem, as already mentioned, can be solved if the velocity of flame propagation is known (and, of course, the parameters of the "cold" gas stream).

The practical importance of this problem is defined almost entirely by the fact that the length of the flame determines, to a considerable extent, the length of the combustion chamber.

The determination of the flame length is of particular importance for turbo-jet combustion chambers where the principal dimensions of the chamber are determined by the size of the flame core.

Passing on now to the solution of the problem posed, we would point out that the comments presented below concerning the method of calculation are aimed at acquainting the student only with results arising from "geometrical" representations of the flame front.

It is obvious that the ignition source for the mixture can be disposed around the periphery as well as in the centre of the gas stream.

Let us consider this case. Figure 23 shows the position of the flame front when the ignition source is arranged on the periphery of the gas stream.

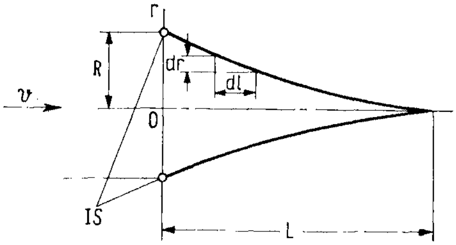

FIG. 23. Determination of the flame length for an ignition source located on the periphery of the gas stream.

We lay off the coordinate axes such that the zero of the abscissa is in the plane of the ignition source and that of the ordinate axis along the axis of the jet.

It is clear that for any elementary section of the front the following relationship is true: $-\frac{dl}{dr} = \tan\varphi$, resulting directly from Mikhel'son's Law, where φ is the angle between the direction of the flow velocity and the normal to the flame front.

If we substitute $\tan\varphi$ by $\cos\varphi$, then

$$-\frac{dl}{dr} = \sqrt{\frac{1}{\cos^2\varphi} - 1}\,. \qquad \text{(III.2)}$$

As is well-known, $\cos\varphi = u\,v$. If we make the substitution in the previous expression, we obtain as a result of this

$$-\frac{dr}{dl} = \sqrt{\left(\frac{v}{u}\right)^2 - 1}\,,$$

whence

$$dl = -\sqrt{\left\{\left(\frac{v}{u}\right)^2 - 1\right\}}\,dr\,. \qquad \text{(III.3)}$$

In order to determine the flame length it is necessary to integrate the left-hand side of the equation within the limits from zero to L, and the right-hand side within the limits R to zero.

However, it is essential to know the velocity distribution along the cross-section, i.e., $v = f(r)$.

The final expression is written in the following form:

$$L = \int_R^0 \sqrt{\left\{\left[\frac{f(r)}{u}\right]^2 - 1\right\}}\,dr\,. \qquad \text{(III.4)}$$

Integration of this expression usually presents some difficulty, since the velocity distribution with respect to the flame length is, in many cases, non-uniform. For a few of the simplest cases it can be assumed that $v = \text{const}$.

If we integrate the right-hand portion of the equation within the limits R to r,

$$L_r = \sqrt{\left(\frac{v}{u}\right)^2 - 1} \cdot (R - r). \qquad \text{(III.5)}$$

For $r = 0$ the expression has the following form:

$$L_{\max} = \sqrt{\left(\frac{v}{u}\right)^2 - 1} \cdot R. \qquad \text{(III.6)}$$

In practice, more complex cases are encountered. If we conceive, for example, the case when the velocity throughout varies according to the law of laminar flow,

$$v = f(r) = v_0\left[1 - \left(\frac{r}{R}\right)^2\right], \qquad \text{(III.7)}$$

where v_0 is the velocity on the axis of flow.

Substituting in equation (III.3) we obtain

$$\mathrm{d}l = -\sqrt{\left\{\frac{v_0\left[1 - \left(\frac{r}{R}\right)^2\right]}{u}\right\}^2 - 1} \cdot \mathrm{d}r. \qquad \text{(III.8)}$$

If the region close to the walls is not considered† and we put

$$\left\{\frac{v_0\left[1 - \left(\frac{r}{R}\right)^2\right]}{u}\right\}^2 \gg 1,$$

then

$$\mathrm{d}l = -\frac{v_0}{u}\left[1 - \left(\frac{r}{R}\right)^2\right]\mathrm{d}r. \qquad \text{(III.9)}$$

Integrating the equation thus obtained within the previous limits, we obtain, with a certain amount of over-estimation, the maximum possible length of the jet $L_{\max}$,

$$L_{\max} = \int_r^R \frac{v_0}{u}\left[1 - \left(\frac{r}{R}\right)^2\right]\mathrm{d}r = \frac{v_0}{u}\left[(R - r) - \frac{R}{3}\left(1 - \frac{r^3}{R^3}\right)\right], \qquad \text{(III.10)}$$

or,

$$L_{\max} = \frac{2}{3} R \frac{v_0}{u}. \qquad \text{(III.11)}$$

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† The formation of flame near the walls for a peripheral source of ignition is determined only by the source.

Let us consider now a second case, when the ignition source is located on the axis of the gas stream (Fig. 24). We construct the coordinate axes as for the previous case.

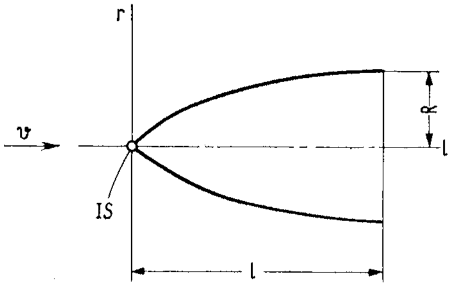

FIG. 24. Determination of the flame length for a point source of ignition.

The resulting equation differs from the previous one only in sign, viz:

$$\frac{\mathrm{d}l}{\mathrm{d}r} = \tan\varphi = \sqrt{\frac{1}{\cos^2\varphi} - 1}\,. \qquad \text{(III.12)}$$

On carrying out the transformation, we obtain

$$\mathrm{d}l = \sqrt{\left(\frac{u}{v}\right)^2 - 1}\cdot\mathrm{d}r\,. \qquad \text{(III.13)}$$

As in the first case, it is necessary, therefore, to know the variation of velocity along the flame front. If combustion takes place at constant pressure then the required function $v = f(r, l)$ can be determined approximately, for turbulent flow just as for laminar flow. However, a pressure drop is observed in a cylindrical combustion chamber, dependent upon the supply of heat to the gas stream, which affects the velocity distribution with respect to l and r. Without dwelling on this problem we note that in certain cases (for example, for the simplest cylindrical chambers) the form of the function $v = f(r, l)$ can be determined approximately.

Sometimes an average value is taken for the velocity

$$v_{\text{av}} = \frac{v_1 + v_2}{2}\,,$$

where v_1 is the velocity prior to the flame;

v_2 is the velocity at the end of the flame.

It should be noted that the specified assumption is quite rough, and is introduced with the object of obtaining a very approximate estimation of the possible length of the flame. In this case

$$L = \int_0^R \sqrt{\left(\frac{v_{\text{av}}}{u}\right)^2 - 1} \cdot \mathrm{d}r = \sqrt{\left(\frac{v_{\text{av}}}{u}\right)^2 - 1} \cdot R. \qquad \text{(III.14)}$$

The complexity of determining L is also made worse by the fact that u does not remain constant throughout the length of the flame core.

CHAPTER IV

DETERMINATION OF THE VELOCITY OF FLAME PROPAGATION

ACCORDING to modern views, the velocity of flame propagation, u, depends on the composition of the combustible mixture, its pressure and temperature and also the parameters of flow turbulence — the intensity of turbulence $\varepsilon = v'/v$, and the scale of turbulence l.

The complexity of the theoretical determination of u leads to the fact that the theoretical relationships determining it are often established by proportional relationships, the form of which is presented below.

Let us analyse some typical cases.

1. VELOCITY OF FLAME PROPAGATION IN LAMINAR FLOW

It is well known that laminar flow is characterized by the fact that transport processes only take place in it because of the motion of molecules. This fact determines the nature of the distribution of velocity of motion of the gas particles in the gas stream and the presence of a clearly defined stream flow, for which transfer by momenta and masses between adjacent layers of gas is quite insignificant.

Laminar motion covers the region of low velocities (Re up to 2000–3000) and consequently laminar flow does not, as a rule, occur in combustion chambers.

However, the velocity of flame propagation in laminar flow, being a physico-chemical constant which determines the "flammability" of a given mixture, also characterizes the velocity of flame propagation in turbulent flow. Consequently, its determination is of great value for practical problems encountered in the study of the combustion process in the combustion chamber of gas turbine engines. Usually, the velocity of flame propagation in laminar flow is called the normal flame speed (velocity).

For simplicity of reasoning we shall consider a plane flame front. Figure 25 depicts the change in temperature along the flame front. Assuming a strong dependency of combustion rate on temperature, it can be assumed that the process of combustion will take place mainly at a temperature close to the maximum T_z, and that increase in the temperature of the unburned mixture will occur only because of heat transfer. As a result of this condition, the process is controlled by the attainment of equilibrium between the heat released by combustion

$$Q_1 = w \varrho c (T_z - T_0) = u_n c \varrho (T_z - T_0), \tag{IV.1}$$

and the heat given up to the fresh combustible mixture by means of heat transfer

$$Q_2 = -\lambda \frac{dT}{dn}, \tag{IV.2}$$

where w is the flow velocity*;
ϱ is the density of the combustible mixture;
c is the specific heat of the combustible mixture;
T_z is the combustion temperature;
u_n is the normal flame speed;
λ is the heat transfer coefficient.

We shall call the distance between points A and B, determined by the intersection of the tangent at the point of maximum temperature gradient with the isotherms T_z = const. and T_0 = const., the width of the flame front. It is usual to call the proportion which is determined by the temperature rise due to heat transfer, the physical width of the front l_{phys} (Fig. 25). The portion in which chemical transformations are initiated and mainly completed (for example, 90 per cent) we shall call the chemical width of the front, l_{chem}. For simplicity we shall assume that the temperature is changed along the axis proportionally to x, i.e., we shall substitute the true picture by an approximation (see Fig. 26).

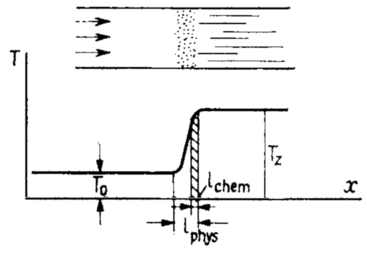

FIG. 25. Variation of temperature in the flame front.

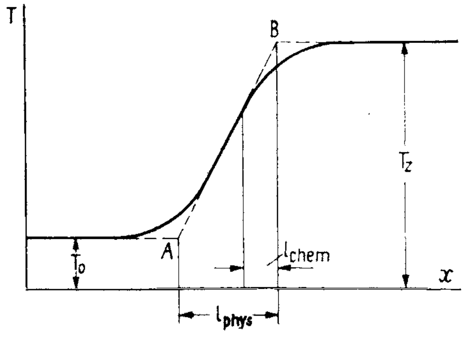

FIG. 26. Determination of width of flame front.

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* In the case given, we assume that the flame front is located in the plane perpendicular to the direction of the flow velocity, i.e., $w = w_{\text{norm}}$.

Then,

$$\frac{dT}{dx} \sim \frac{T_0 - T_z}{l_{\text{phys}}} . \tag{IV.3}$$

The heat balance equation is now written in the following form:

$$\lambda \frac{T_z - T_0}{l_{\text{phys}}} = u_n \varrho c (T_z - T_0) , \tag{IV.4}$$

whence

$$l_{\text{phys}} = \frac{\lambda}{u_n \varrho c} = \frac{a}{u_n} . \tag{IV.5}$$

It follows from this equation that the normal velocity of combustion is directly proportional to the thermal diffusivity of the mixture, a.

In addition to the velocity of flame propagation, it is interesting to know the chemical width of the flame front.

The chemical width of the flame front in our case can be defined as the product of u_n and the time of chemical reaction, i.e.,

$$l_{\text{chem}} = u_n \tau_{\text{chem}} , \tag{IV.6}$$

where τ_{chem} is the time during which the major portion of the combustible mixture reacts (for example, 90 per cent).

The physical width of the flame front, l_{phys} is always greater than l_{chem}. Consequently it can be assumed that

$$l_{\text{chem}} = \Phi \, l_{\text{phys}} , \tag{IV.7}$$

where Φ is a coefficient, less than unity, depending on the chemical reaction.

Using relationships (IV.5), (IV.6) and (IV.7), we obtain the expression for the normal velocity of combustion

$$u_n = \sqrt{\frac{a}{\tau_{\text{chem}}}} \sqrt{\Phi} . \tag{IV.8}$$

The determination of Φ is complex and falls outside the scope of this monograph, consequently for the ultimate theoretical result we shall assume the relationship

$$u_n \sim \sqrt{\frac{a}{\tau_{\text{chem}}}} \sim \sqrt{a e^{-E/RT_z}} . \tag{IV.9}$$

The values of u_n are usually determined by an experimental method.

The normal velocity of combustion is determined by the composition of the mixture and by such parameters of the state of the combustible mixture as temperature and pressure.

Let us consider the effect of these factors in more detail.

2. VARIATION OF THE NORMAL VELOCITY OF COMBUSTION WITH MIXTURE PARAMETERS

It has already been mentioned that u_n depends on the components of the combustible mixture, their ratio, pressure and temperature of the mixture. Moreover, it is well known that combustion does not take place for all ratios between the components: there are defined combustion limits for every combustible mixture, outside of which combustion is impossible.

Figure 27 shows the variation of the quantity u_n for mixtures of different fuels with air. The graphical relationships obtained by an experimental method indicate that the maximum values of u_n for the different fuels do not agree in magnitude but are attained for a different percentage content of fuel in air.

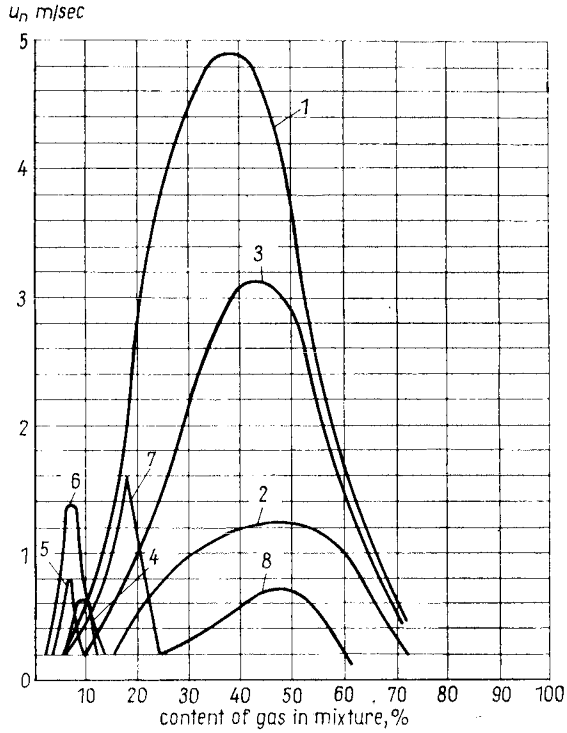

FIG. 27. Variation of normal velocity of combustion as a function of the content of the gas in air.

1 — Hydrogen; *2* — Carbon monoxide; *3* — Mixture of hydrogen and carbon monoxide; *4* — Methane; *5* — Ethane; *6* — Ethylene; *7* — Illuminating gas; *8* — Producer gas.

Homogeneous fuel — air mixtures are usually characterized by the excess air coefficient α; this coefficient represents the ratio of the actual quantity of air present in the mixture to the theoretical quantity necessary for complete combustion of the fuel.

The factor α is expressed by the following formula:

$$\alpha = \frac{G_A}{G_F L_0},$$

where G_A is the quantity of air in the mixture;
G_F is the quantity of fuel;
L_0 is the theoretical quantity of air necessary for combustion of 1 kg of fuel.

It should be noted that the maximum u_n for different fuels is attained for somewhat rich mixtures.

The variation $u_n = f(\alpha)$ is shown in Fig. 28.

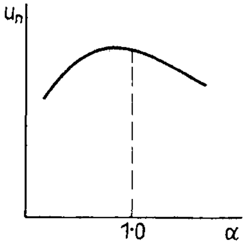

Fig. 28. Variation of normal velocity of combustion u_n as a function of the excess air coefficient α.

Table 9 shows values for the normal velocity of combustion and the limits of flammability for various fuel–air mixtures.

Table 9. Normal velocity of flame propagation and limits of flammability for mixtures of fuels and air.

Name of Fuel	Formula	Value of u_n, m/sec			Limits of flammability in terms of composition of the mixture	
		for $\alpha = 1$	for $\alpha = \alpha_{opt}$ $(u_n = u_{n_{max}})$	α_{opt}	Lower	Upper
Hydrogen	H_2	1·6	2·67	0·58	10·1	0·1405
Carbon monoxide	CO	0·3	0·41	0·556	2·94	0·146
Methane	CH_4	0·28	0·37	0·9	5·0	0·54
Acetylene	C_2H_2	1·0	1·35	0·751	3·28	0·021
Ethylene	C_2H_4	1·5	0·63	0·924	2·48	0·175

For determining the characteristics of a mixture (upper and lower limits of flammability and also the concentration for which $u_{n_{\max}}$ is attained) composed of several fuels, the well-known rule of averaging should be used

$$N = \frac{p_1 + p_2 + p_3 + \cdots + p_n}{\frac{p_1}{N_1} + \frac{p_2}{N_2} + \frac{p_3}{N_3} + \cdots + \frac{p_n}{N_n}},$$

where N is the required characteristic;

$p_1, p_2, \ldots, p_n$ are the percentage compositions of the various components in the complex mixture under consideration;

$N_1, N_2, \ldots, N_n$ are the same characteristics for simple pure mixtures.

In addition to the composition of the mixture, the initial temperature and pressure of the mixture exert a considerable influence on the magnitude of the normal velocity.

Increase of temperature leads to increase of u_n (Fig. 29) and also extends the limits of flammability of the mixture. Conversely, increase of pressure for air mixtures leads to reduction of the normal velocity of combustion (Fig. 30).

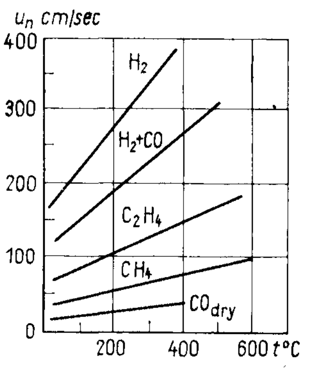

FIG. 29. Effect of temperature of mixture on normal velocity of combustion.

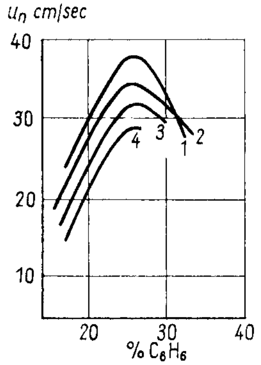

FIG. 30. Dependence of normal velocity of combustion of benzene at various pressures.

1 — $p = 1$ atm; *2* — $p = 1.5$ atm; *3* — $p = 2$ atm; *4* — $p = 3$ atm.

Increase of u_n as a result of increase of temperature of the mixture can be explained, from the point of view of the thermal theory of combustion, by the fact that with increase of T_0, T_z also increases, and consequently also u_n, which is arbitrarily proportional to $e^{-E/2RT}$.

3. EXPERIMENTAL METHODS OF MEASURING THE NORMAL VELOCITY OF FLAME PROPAGATION

Nowadays, there are a number of methods for determining the normal velocity of combustion. They may be divided into two groups: dynamic and static.

Burner Method. Amongst the dynamic methods, Mikhel'son's method, or as it is sometimes called the burner method, is worthy of the most attention. This method consists in feeding a fuel mixture into a straight, vertically mounted tube and burning it at the free end of the tube.

Two coloured zones can be easily distinguished in the burner flame (Fig. 31): an inner zone *1*, conical in shape, bright blue in colour, depending on the composition of the mixture, and an outer zone *2*, the so-called outer cone, which has a yellow colour, sometimes tinged with violet.

Combustion takes place at the surface of the inner cone. Since the oxygen present in the mixture is sometimes inadequate, final combustion of the fuel takes place in the outer cone of the flame, by combining with the oxygen of the surrounding air.

Analysis of the relationships connecting the shape and size of the inner cone with the flow velocity of the fresh combustible mixture and the normal velocity of combustion was first carried out by V. A. Mikhel'son.

Let us find the mathematical relationships for determining the normal velocity of combustion. For this we shall consider a geometrical representation of the inner cone (Fig. 32).

FIG. 31. Diagrammatic representation of burner flame.

1 — Inner cone; *2* — Outer cone.

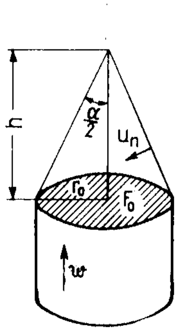

FIG. 32. Determination of the normal velocity of combustion.

We shall denote the area of the orifice of the burner by F_0, and the mean flow velocity of fresh combustible mixture by w. In order that the flame should not flash back into the burner, the condition $u_n \leq w$ should be fulfilled.

On the other hand, the entire quantity of gas, equal to $\varrho F_0 w$, should be burned at the surface of the cone F. In addition to this, we shall make the assumption that the normal velocity is constant over the whole surface of the cone and that the density of the mixture does not change.

From these considerations the following equation can be written down:

$$F_0 w = F u_n \,. \tag{IV.10}$$

Hence,

$$u_n = w \frac{F_0}{F} \,. \tag{IV.11}$$

We note further that the surface of combustion is a cone with an angle α at the apex. The area of the exit cross-section of the burner, as is well known, is equal to $F_0 = \pi r_0^2$, where r_0 is the radius of the exit cross-section of the burner.

The lateral area of the cone can be expressed in terms of the area of the base

$$F = \frac{\pi r_0^2}{\sin \frac{\alpha}{2}} \,. \tag{IV.12}$$

Substituting the value of F in equation (IV.11) we obtain the following formula:

$$u_n = w \sin \frac{\alpha}{2} \,. \tag{IV.13}$$

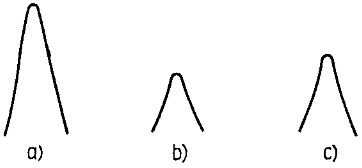

FIG. 33. Shape of burner flame.
(a) — With a deficiency of air; (b) — Mixture, corresponding to the maixmum value of the velocity of combustion; (3) — With excess of air.

Hence it follows that if the average velocity of the mixture and the height of the cone, h, (which determines the angle of the cone, since the radius of the burner is known) are measured, it is also possible to calculate the normal velocity of combustion.

The assumption that the surface of combustion is the surface of the cone is, strictly speaking, incorrect. Actually, the surface of combustion is distorted near the base of the cone, due to removal of heat through the wall of the burner and is also distorted at the apex of the cone because of the presence of preheating of the fresh mixture and an increase in the normal velocity of combustion.

Distortion of the surface near the base of the cone and at the apex are shown in Fig. 33.

In order to measure directly the area of the surface of the flame, it is necessary to photograph it. For determining the height of the cone the reading should be taken from the base of the flame. The error in determining the normal velocity of combustion for such a measurement of the surface of the cone is considerably reduced.

The Soap Bubble Method. Amongst the static methods used for determining the normal velocity of combustion, we shall consider the soap bubble method. This method consists in blowing out a soap bubble in which is located a combustible mixture of definite composition. The mixture in the centre of the bubble is then burned and the change of position of the flame surface is focussed on a moving photographic film. The main feature of the soap bubble method consists in that the gas, on combustion, is able to freely expand, and consequently the process takes place at constant pressure.

Over an interval of time Δt after combustion of the mixture, the flame is propagated to a distance $\mu u \Delta t$. Over the same interval of time the film, on which the position of the flame front is recorded, passes through a track equal to $v_f \Delta t$, where v_f is the linear velocity of the film.

The angle α can be determined directly from the photograph, the sine of which is equal to $\frac{\mu u}{v_f}$:

$$\sin \alpha = \frac{\mu u}{v_f}, \tag{IV.14}$$

where μ is the scale of the photograph.

From this equation we find that

$$u = \frac{v_f}{\mu} \sin \alpha . \tag{IV.15}$$

The value of u as measured by this method represents the flame velocity relative to the centre of the sphere, i.e., the velocity relative to the combustion products u_v ($u = u_v$).

In addition to change of the sphere of flame during combustion, the radius of the bubble is also registered on the film at the initial time instant (see Fig. 34). From the law of conservation of mass it follows that the velocity u_v is related to the normal velocity by the relationship

$$u_n = u_v \frac{\varrho_v}{\varrho_0} . \tag{IV.16}$$

In its turn $\frac{\varrho_v}{\varrho_0}$ is equal to the inverse ratio of the volumes, and consequently also to the inverse ratio of the cubes of the radii of the spheres:

$$\frac{\varrho_v}{\varrho_0} = \frac{v_0}{v_v} = \left(\frac{r_0}{r_v}\right)^3. \tag{IV.17}$$

Thus,

$$u_n = u_v\left(\frac{r_0}{r_v}\right)^3. \tag{IV.18}$$

A diagram of the apparatus for blowing a soap bubble is shown in Fig. 36.

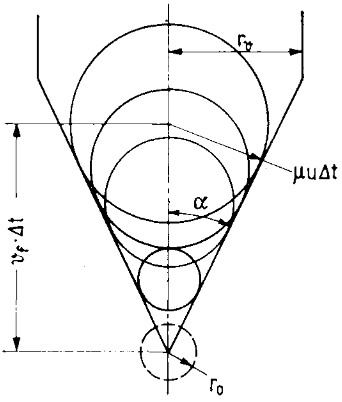

FIG. 34. Determination of normal velocity by the soap bubble method.

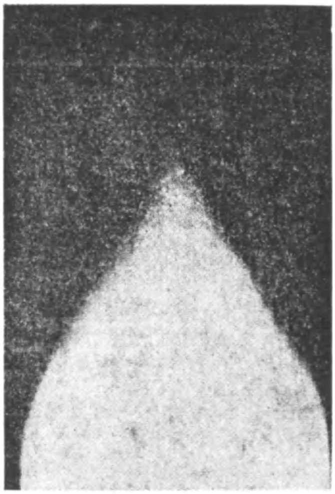

FIG. 35. Photorecording of propagation of a flame in a soap bubble, on a moving film for a mixture of 28·6 per cent C_2H_2 + 71·4 per cent O_2.

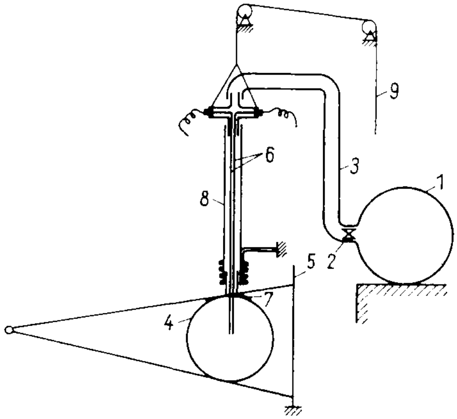

FIG. 36. Diagram of apparatus for blowing the soap bubble.
1 — Globe; *2* — Stopcock; *3* — Pipeline; *4* — Soap bubble; *5* — Screen; *6* — Ignition device; *7* — Funnel; *8* — Rubber tube; *9* — Cord for raising the conductors of the ignition device.

The mixture of gaseous fuel in a fixed proportion is accommodated prior to the experiment in a rubber gas-holder — globe *1*, fed by the stopcock *2*. Prior to the experiment, the pipe line for supplying the combustible is, as a rule, carefully flushed out. A soap bubble is then blown of definite size and an ignition device *6* is installed into the centre of it. The dimensions of the soap bubble, as well as the position of the ignition device, are monitored by their shadows on the illuminated screen *5*.

At the instant of blowing the bubble, the conductors are raised up above the funnel *7* by stretching the rubber tube *8* and by means of the cord *9* passing over a pulley.

Propagation of the flame is recorded on a moving photographic film through a horizontal slit. A photograph is shown in Fig. 35 as an illustration, which shows the propagation of the flame in the soap bubble as a result of igniting a combustible mixture of acetylene and air.

In view of the fact that the film moves according to the development of the combustion process, the image is obtained in the form of a triangle. This picture is shown diagrammatically in Fig. 34.

A whole series of methods exists for determining the normal flame velocity. For example, the method of determining the normal flame velocity by means of the change of pressure with time is based on the use of the momentum theorem and involves photographing the motion of the flame front in a tube, etc.

In addition, methods are available for determining the normal flame velocity theoretically for which it is essential to know the kinetics of the process.

4. VELOCITY OF FLAME PROPAGATION IN TURBULENT FLOW

Turbulent flow is distinguished from laminar flow in that exchanges of momentum, heat etc. within the gas take place mainly not as a result of molecular transference, but as a result of random motion of the ultimate components the molecules. The intensity of exchange as a result of this increases substantially. The profile of the mean velocities for turbulent flow is considerably more uniform than for laminar flow (Fig. 37).

The true velocity at any point in turbulent flow does not remain constant, but varies with time and fluctuates about a certain mean value.

This change of velocity is called the fluctuating velocity. In order to determine the magnitude of the fluctuation, the mean square value of the deviation from the mean flow velocity over a certain interval of time is taken as a parameter. The presence of a fluctuating velocity is a characteristic sign of turbulent flow. In order to evaluate it, a quantity ε is introduced, called the turbulence factor. The turbulence factor is defined by the relationship

$$\varepsilon = \frac{v'}{\text{av}}$$

where v' is the root mean square fluctuating velocity component;
v_{av} is the mean flow velocity.

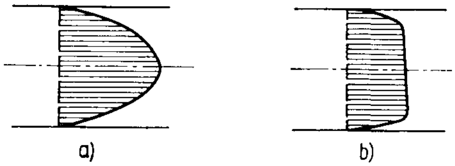

FIG. 37. Velocity distribution along a gas stream.
(a) — For laminar flow conditions; (b) — For turbulent flow.

Moreover, the quantities characterizing turbulent flow are the scale of turbulence and the fluctuation frequency. By the scale of turbulence is usually understood the linear dimensions of the average molecule or the displacement proportional to it†, denoted by the symbol l.

The structure of the gas stream exerts a substantial influence on the velocity of flame propagation.

Let us consider the possible cases of combustion in turbulent flow.

First case ($v' \ll u_n$, $l < l_{\text{chem}}$)

The turbulence of the gas stream is extremely small, the scale of turbulence is small in comparison with the chemical width of the flame front and the fluctuating component is less than the combustion velocity. As a result of the turbulent fluctuations from the flame front, there will be certain volumes, the linear dimensions of which are in the neighbourhood of the magnitude of l. Since the magnitude of l is extremely small in comparison with l_{chem}, then no substantial distortion in the surface of the flame front is caused.

The flame front, instead of remaining smooth, becomes undulatory, which leads to an increase of its surface area. This is illustrated in Fig. 38, where the flame front in the case under consideration is depicted diagrammatically. The qualitative aspect of the combustion process as a result of this is unchanged, but the processes connected with heat transfer proceed more intensely because of the turbulent fluctuations.

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† A given mean path which the average molecule traverses between two successive fluctuations.

K. I. Shchelkin made the assumption that the transference of heat in this case is dependent upon molecular motion, as well as on turbulent fluctuations, so that heat conduction in the gas stream is determined by the sum $a_m + a_t$.

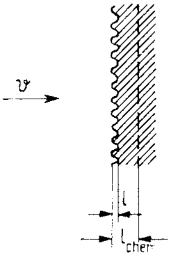

FIG. 38. Schematic diagram of the flame front in turbulent flow. $(v' < u_n;\ l < l_{\text{chem}})$.

The expression for the velocity of combustion (IV.8) is modified in the following manner:

$$u_{T_{\mathrm{I}}} = \sqrt{\frac{a}{\tau_{\text{chem}}}}\sqrt{\Phi} = \sqrt{\frac{a_m + a_t}{\tau_{\text{chem}}}}\sqrt{\Phi}\,, \qquad \text{(IV.19)}$$

where a_m is the coefficient of thermal conductivity, dependent upon molecular transfer;

a_t is the coefficient of thermal conductivity dependent upon turbulent fluctuations.

Without considering the value of the function Φ, for the reason indicated above, we shall have as the ultimate result

$$u_{T_{\mathrm{I}}} \sim \sqrt{\frac{a_m}{\tau_{\text{chem}}}}\sqrt{1 + \frac{a_t}{a_m}}\,. \qquad \text{(IV.20)}$$

In the case of laminar flow, the normal velocity of combustion was

$$u_n \sim \sqrt{\frac{a_m}{\tau_{\text{chem}}}}\,.$$

Making the substitution in equation (IV.20), we obtain the relationship which characterizes the change of velocity of flame propagation

$$u_{T_{\mathrm{I}}} \sim u_n\sqrt{1 + \frac{a_t}{a_m}}\,. \qquad \text{(IV.21)}$$

From the expression obtained, it follows that the combustion velocity has been increased because of the flow turbulence. As a result of this the width of the flame front also increases somewhat, as can be seen from the following relationship:

$$l_{\text{front}} \sim u_{T_{\text{I}}} \tau_{\text{chem}} \tag{IV.22}$$

or,

$$l_{\text{front}} \sim u_n \sqrt{1 + \frac{a_t}{a_m}} \cdot \tau_{\text{chem}} \cdot \tag{IV.23}$$

Second Case ($v' < u_n$, $l > l_{\text{chem}}$)

Let the root mean square velocity be less than the velocity of flame propagation ($v' < u_n$). Because of turbulent fluctuations, certain quantities of the fresh mixture may be "projected" through the flame front into the combustion products and conversely, quantities of combustion products are "projected" into the stream of pure combustible mixture. The mutual "projection" of the moles is shown schematically in Fig. 39.

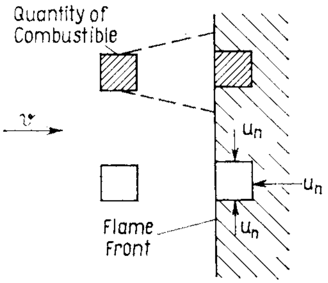

FIG. 39. Schematic diagram of the flame front in turbulent flow.

In contrast to the previous case, the flame front becomes more undulatory and its profile is reduced with time (Fig. 40).

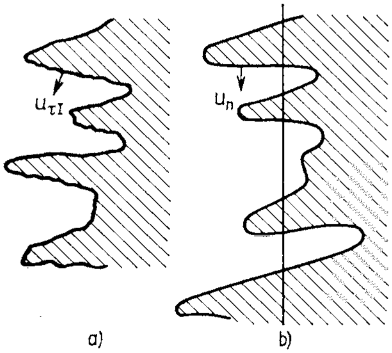

FIG. 40. Instantaneous position of the flame front.

In order to determine the velocity of flame propagation, we shall make a number of assumptions.

Firstly, since the process of stationary combustion is being considered, then a certain average position of the flame front can be represented, which also determines the velocity of flame propagation. For this, the flame front is maintained stationary by the velocity of the inflowing stream, equal to v. It is obvious that equilibrium should exist between the velocity of combustion and the velocity of the incoming stream of fresh mixture, $u_{T_{\mathrm{II}}} = v$.

It is obvious that the condition $u_{T_{\mathrm{II}}} = v$ supposes that we shall determine the velocity of flame propagation relative to the same surface F_{front} (Fig.41), representing an arbitrarily flat front, normal to the direction of the flow velocity v.

Secondly, we shall assume that the flame front consists of individual cones (see Fig. 41), the surface area of which is equal to some average surface area of the flame front over a certain interval of time

$$F_{\mathrm{con}} = \frac{\int\limits_0^\tau F_{\mathrm{front}}\,\mathrm{d}\tau}{\tau}. \tag{IV.24}$$

Let us determine $u_{T_{\mathrm{II}}}$ for this idealized case.

The flame front can exist only in the case when all the fuel mixture fed to it is consumed, i.e. the equation

$$v\,F = u_n\,F_{\mathrm{front}} \tag{IV.25}$$

should be satisfied.

Hence, taking into account the assumption made that $u_{T_{\mathrm{II}}} = v$, we obtain

$$u_{T_{\mathrm{II}}} = u_n \frac{F_{\mathrm{front}}}{F}. \tag{IV.26}$$

It should be noted that this condition is independent of the limits imposed on the shape of the flame front.

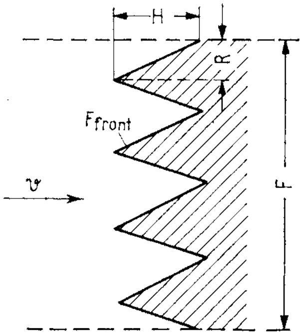

FIG. 41. Determination of the surface area of the flame front.

Thus, in the given case the problem of determining u_{T} mainly reduces to finding the lateral face of the surface of the cone, and it is not difficult to express it in the following form:

$$F_{\mathrm{front}} = F_b = \pi R \sqrt{R^2 + H^2} = \pi R^2 \sqrt{1 + \left(\frac{H}{R}\right)^2}\,, \tag{IV.27}$$

where H is the height of the cone;
R is the radius of the base.

It follows from formula (IV.26) that in order to find the value of the velocity of combustion it is necessary to determine the ratio of the lateral face of the cone to the area of its base, which, as follows from formula (IV.27) is determined by the ratio H/R.

If the proportionality is taken of the path displacement l, to the linear dimensions of the intermixed molecules, i.e. the scale of turbulence, then one may assume that $R \sim l$. Let us determine the magnitude of H. The time of intermixing of a volume of mixture is $\tau \sim l/u_n$, since it burns upwards from the surface with a velocity u_n.

The magnitude of H is approximately equal to the product of τ and v', i.e.,

$$H \approx \frac{l}{u_n} v'\,.$$

The assumption made above makes it possible to determine the ratio H/R which enters into formula (IV.27). After a number of substitutions and simple transformations we obtain

$$\left(\frac{H}{R}\right)^2 \approx B \frac{\left(l \frac{v'}{u_n}\right)^2}{l^2} \approx B \left(\frac{v'}{u_n}\right)^2. \tag{IV.28}$$

Consequently, the velocity of flame propagation for the second case of turbulent combustion may be expressed as

$$u_{T_{\mathrm{II}}} \approx u_n \sqrt{1 + B \left(\frac{v'}{u_n}\right)^2}\,. \tag{IV.29}$$

Carrying out the substitution of v' by $\varepsilon\, v$, we obtain the expression

$$u_{T_{\mathrm{II}}} \approx u_n \sqrt{1 + B\,\varepsilon^2 \left(\frac{v}{u_n}\right)^2}\,. \tag{IV.30}$$

Accurate calculations have been carried out which show that the value of the coefficient B can be taken as constant, and is close to unity.

Determination of the velocity of combustion in terms of another shapes for the flame front, either with rectangular (*a*) or with elliptical irregularities (*b*) (Fig. 42) gives similar results.

A more accurate solution leads to a somewhat modified formula

$$u_{T_{\text{II}}} \approx u_n \sqrt{1 + B\left(\frac{v'}{u_n}\right)^2 + f\left(\frac{v'}{u_n}\right)}, \tag{IV.31}$$

where B is a factor of the order of unity;

$f\left(\frac{v'}{u_n}\right)$ is a function always less than unity

(for the convergence $v' \to 0$, the given function always converges to zero).

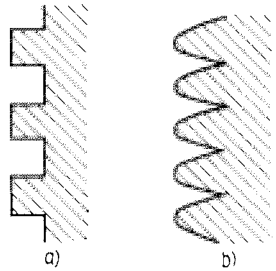

FIG. 42. Shapes of flame front.
(a) — With rectangular irregularities; (b) — With elliptical irregularities.

This relationship does not permit a determination to be made of the quantitative values of the parameters characterizing the course of the process, consequently the sign of the equation should be substituted by the proportional sign.

Thus,

$$u_{T_{\text{II}}} \sim u_n \sqrt{1 + B\left(\frac{v'}{u_n}\right)^2}, \tag{IV.32}$$

or

$$u_{T_{\text{II}}} \sim u_n \sqrt{1 + B\,\varepsilon^2\left(\frac{v}{u_n}\right)^2}. \tag{IV.33}$$

It should be noted that if there is fine-scale turbulence near the surface of the flame front (Fig. 40a), then in place of the velocity u_n (Fig. 40b) it is more accurate to substitute the velocity $u_{T_{\text{I}}}$, as a result of which the formula has the following form:

$$u_{T_{\text{II}}} \approx u_n \sqrt{1 + \frac{a_t}{a_m}} \sqrt{1 + B\left(\frac{v'}{u_n}\right)^2}. \tag{IV.34}$$

Third Case ($v' > u_n$, $l \gg l_{\text{chem}}$)

If the root mean square fluctuating velocity of turbulent flow is larger than the normal velocity of combustion ($v' > u_n$) and the path displacement in its turn is greater than the chemical width of the flame front ($l > l_{\text{chem}}$), then the case occurs when the flame front ceases to be continuous.

Ejection of burning "fragments" into an unburnt mixture does not always lead to its ignition, but on the other hand ejection of quantities of fresh mixture into the combustion products does not lead to the formation of gaps.

Figure 43 shows a diagram of the phenomenon. We shall assume that in the right, cross-hatched region there are chiefly combustion products, and in the left side — chiefly unburnt mixture.

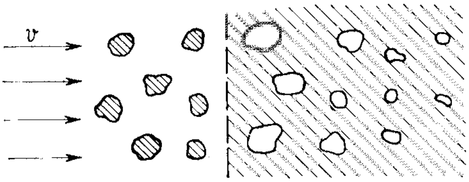

FIG. 43. Schematic diagram of the flame front in turbulent flow, for $v' > u_n$ and $l > l_{\text{chem}}$.

The volumes of combustible mixture penetrating into the combustion products burn at the surface with a velocity u_n or $u_{T_{\text{I}}}$.

In this case, the quantity of burning mixture is determined by the surface of the burning volumes of combustible mixture and by the velocity u_n (or $u_{T_{\text{I}}}$). The quantity of burning mixture for a stationary process is equal to the quantity of mixture fed in, i.e.,

$$v\,F = u_n\,F_{\text{front}}\,.$$

Since the normal velocity of flame propagation for a stationary process $u_{T_{\text{III}}} = v$, then consequently

$$u_{T_{\text{III}}} = u_n \frac{F_{\text{front}}}{F}\,. \tag{IV.35}$$

The problem reduces, as in the previous case, to determining the ratio F_{front}/F.

In order to determine this relationship we shall introduce a physical model of the course of the process under consideration.

Let us suppose that, on the basis of the theory of turbulent flow, quantities of unburnt gas having linear dimensions $\sim l$, are periodically projected into the combustion products with a relative velocity v' and that they burn at the surface with a velocity u_n (or $u_{T_{\text{I}}}$).

In order to determine F_{front} we shall assume that the volumes projected in are moving in a straight line with a velocity v', since the change of nature of the motion does not affect the final results.

Figure 44 shows the position of the volumes at a certain instant of time. In order to determine the surface area of these volumes, it is essential to know their quantity and dimensions. We shall consider the determination of these values.

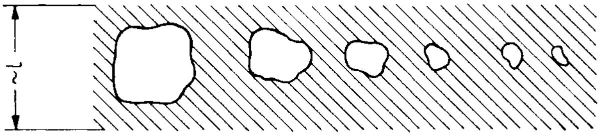

FIG. 44. Position of the projected volumes of unburnt mixture at some instant of time.

(a) Determination of the quantity of volumes of fresh mixture projected into the combustion products.

It is natural to suppose, of course, that the number of volumes of fresh mixture projected into the combustion products as a result of turbulent fluctuations during the time of combustion of one volume is proportional to the number of turbulent fluctuations for the same period of time. This follows immediately from the fact that it is precisely the fluctuations which give rise to the projected volumes of unburnt mixture.

It is obvious that the number of fluctuations during the time of combustion of a single projected volume will be equal to the ratio of the time during which the volume burns, to the time interval between fluctuations, $\tau_{\text{comb}}/\tau_{\text{fluct}}$. These values are determined by the following obvious relationship.

The time of combustions is $\tau_{\text{comb}} \approx l/2\,u_n$, and the inverval between fluctuations is $\tau_{\text{fluct}} = l/v'$. Thus, the number of fluctuations during the time of combustion will be proportional to the ratio

$$\frac{1}{2}\frac{l}{u_n} : \frac{l}{v'} \approx \frac{1}{2}\frac{v'}{u_n}. \qquad \text{(IV.36)}$$

It was shown above that this ratio should be proportional to the number of projected volumes during the time τ_{comb}.

Thus, the number of unburned volumes existing in the combustion products at any instant of time will be determined by this ratio.

If the result of these considerations is depicted diagrammatically, then in any given instant of time we shall have in the combustion products n volumes of unburnt mixture (Fig. 45), where $n \approx v'/2\,u_n$, if it be assumed that each fluctuation leads to projection of one volume of unburnt mixture.

(b) Determination of the surface area of the volumes of fresh mixture projected into the combustion products.

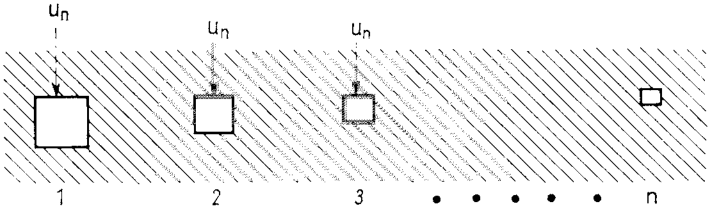

FIG. 45. Determination of the number of projected volumes.

The surface areas of the volumes are equal to the following values respectively:

$$\begin{array}{lll}
\text{1st volume} & -\left[l-2\dfrac{l}{v'}u_n\right]^2=l^2\left(1-\dfrac{1}{n}\right)^2; & \\
\text{2nd volume} & -\left[l-4\dfrac{l}{v'}u_n\right]^2=l^2\left(1-\dfrac{2}{n}\right)^2; & \\
\text{3rd volume} & -\left[l-6\dfrac{l}{v'}u_n\right]^2=l^2\left(1-\dfrac{3}{n}\right)^2; & \\
 & \dots\dots\dots\dots\dots\dots\dots\dots & \text{(IV.37)} \\
(n-1)\text{-th volume} & l^2\left(1-\dfrac{n-1}{n}\right)^2; & \\
n\text{-th volume} & \dots\dots\; l^2\left(1-\dfrac{n}{n}\right)^2=0\,. &
\end{array}$$

By summing these terms we obtain a value proportional to the surface area:

$$F_{\text{front}}\approx\frac{l^2\left[(n-1)^2+(n-2)^2+\cdots+1\right]}{n^2}$$

$$=\frac{l^2}{n^2}\left[1+2^2+3^2+\cdots+(n-1)^2\right]\,. \qquad \text{(IV.38)}$$

As is well-known, the sum of such a series is

$$S=\frac{(n-1)\,n\,(2n+1)}{6}\,. \qquad \text{(IV.39)}$$

If we multiply the terms in the numerator and we put n^2 before the brackets, then

$$S=\frac{n^2}{6}\left(2n-1-\frac{1}{n}\right). \qquad \text{(IV.40)}$$

Substituting expression (IV.40) in (IV.38) we have

$$F_{\text{front}}\approx\frac{l^2}{n^2}\frac{n^2}{6}\left(2n-1-\frac{1}{n}\right)=\frac{l^2}{6}\left(2n-1-\frac{1}{n}\right). \qquad \text{(IV.41)}$$

In place of n we substitute its value in the result, and we obtain the final relationship

$$F_{\text{front}} \approx \frac{l^2}{6}\left(\frac{v'}{u_n} - 1 - \frac{2u_n}{v'}\right). \tag{IV.42}$$

Having determined the number of projected volumes and their surface area, we shall revert to consideration of the expression for the velocity of flame propagation.

We shall substitute in formula (IV.35) the value so-obtained in place of F_{front}. For this it is necessary to remember that the quantity F_{front} only is proportional to the square of the linear dimension.

$$u_{T_{\text{III}}} \sim u_n \frac{\frac{l^2}{6}\left(\frac{v'}{u_n} - 1 - \frac{2u_n}{v'}\right)}{l^2} = u_n \frac{\left(\frac{v'}{u_n} - 1 - 2\frac{u_n}{v'}\right)}{6} \tag{IV.43}$$

or

$$u_{T_{\text{III}}} \sim u_n\left(\frac{v'}{u_n} - 1 - 2\frac{u_n}{v'}\right). \tag{IV.44}$$

In the case when $v' \gg u_n$,

$$u_{T_{\text{III}}} \sim v' . \tag{IV.45}$$

A more accurate analysis also leads to this result.

Expression (IV.44) shows that the velocity of flame propagation, as a result of the turbulence developed for $v' \gg u_n$ depends only on the fluctuation velocity. The composition of the mixture a, the variety of the fuel and of the oxidant, the temperature and other parameters of the mixture have no influence on the velocity of flame propagation.

It is impossible to understand literally the result obtained. One should only understand it in the sense of the predominant influence on the velocity of flame propagation of the fluctuating velocity relative to the other factors mentioned above (amongst them u_n), unconditionally affecting u_T. Actually, many phenomena which reduce the time required for combustion of a mole, were neglected in the conclusion, for example, the phenomena of heat transfer, mixing of unburnt mixture with the combustion products and the origination of the "volume" reaction, etc.

These circumstances are confirmed directly by experiment in a number of combustion chambers of aero-jet engines with large flow velocities.

By considering the possible cases of flame propagation, a table can be compiled (see Table 10).

It should be noted that the result obtained in the 3rd case also follows directly from the formula obtained for the 2nd case for $v' \gg u_n$.

In conclusion it is necessary to mention that the results obtained are factual for combustion of a homogeneous combustible mixture, i.e. a homogeneous fuel–air mixture.

TABLE 10

Flow	Defining equation	Expression for velocity of flame propagation	Shape of flame front
Laminar	$v' = 0$	$u_n \sim \sqrt{\frac{a_m}{\tau_{\text{chem}}}}$	Smooth
Turbulent (1st case)	$v' < u_n$ $l < l_{\text{chem}}$	$u_{T_{\text{I}}} \sim u_n \sqrt{1 + \frac{a_t}{a_m}}$	Undulating
Turbulent (2nd case)	$v' < u_n$ $l > l_{\text{chem}}$	$u_{T_{\text{II}}} \sim u_n \sqrt{1 + B\left(\frac{v'}{u_n}\right)^2}$	Strongly undulating
Turbulent (3rd case)	$v' > u_n$	$u_{T_{\text{III}}} \sim v'$	Flame front should revert to continuous

Increase of the velocity of flame propagation as a result of the presence of turbulence is explained by increase of the surface area on which combustion takes place. The greater the turbulence, the greater is the increase of the combustion surface.

The normal velocity of flame propagation is determined not with respect to the normal to the surface on which combustion actually proceeds, but with respect to the normal to some arbitrary front, which, on the whole, determined the position of the entire combustion zone in space.

An example of the determination of the direction of the velocity is shown in Fig. 46.

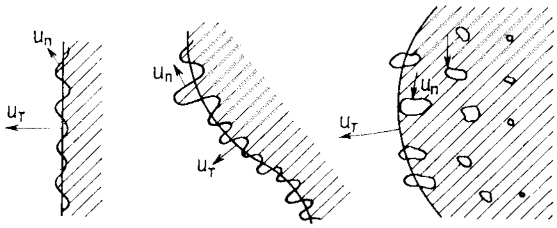

FIG. 46. Determination of the direction of the velocity of flame propagation for different positions of the arbitrary flame front.

The experimental determinations of u_T agree quite well with empirical relationships of the form

$$\frac{u_T}{u_n} = 1 + A\left(\frac{v'}{u_n}\right)^n, \tag{IV.46}$$

where n lies within the limits 0.5–1.0, and the coefficient A is approximately equal to 5.

5. THE CONCEPT OF DIFFUSION AND THE KINETICS OF COMBUSTION

The process of combustion in a gas stream with a separate combustible and oxidant feed is characterized by the simultaneous development of the processes of mixture formation and oxidation of the fuel. Consequently, by investigating the processes of combustion in identical gas streams, it is necessary to take into account not only the kinetics of the oxidation process but also the diffusion processes of mixing of the fuel with air with respect to time.

Depending upon a number of conditions, the development of the controlling process with time is either kinetic or diffusion.

If the overall velocity of the process (or the overall time during which the process takes place) is determined by the velocity of mixture formation, i.e., when the velocity of the chemical reactions is many times greater than the velocity of mixture formation, then the process of combustion is represented by the case of so-called diffusion combustion.

Alternatively, if the velocity of mixture formation is greater than the velocity of combustion, then the latter limits the entire process altogether The time of the process in this case is determined by the kinetics.

In the combustion chambers of aero-jet engines, extremely complex combustion processes take place which are limited not only by kinetic factors but also by the preparation of the fuel–air mixture, and consequently the possibility should be taken into account that the combustion process is diffusion-controlled as well as kinetic-controlled. We shall examine this case in more detail.

The total time of combustion is composed of two parts: the time required for mixing of the fuel with air (we shall denote this by τ_d) and the time of the chemical oxidation reactions (we shall denote this time by τ_k). If the partial superimposition of the processes of mixture formation and combustion is ignored, then the overall time of combustion can be written as the sum:

$$\tau_z = \tau_d + \tau_k \, .$$

Mixture formation of the fuel with air, in its turn, can be divided into a number of processes such as: turbulent mixing and molecular diffusion:

$$\tau_d = \frac{1}{\dfrac{1}{\tau_{dm}} + \dfrac{1}{\tau_{dt}}} \, .$$

If the time of mixture formation or, in other words, the physical state of the process proves to be considerably greater than the time necessary for the chemical reaction to occur, i.e. $\tau_d \gg \tau_k$, then in practice

$$\tau_z \approx \tau_d.$$

This implies that the time of the combustion process is determined on the whole only by diffusion processes. Combustion of a gaseous fuel can serve as an example of such a process in which the fuel is introduced into the combustion chamber separately from the oxidant. The temperature in the combustion zone of the chamber can become very high, as a result of which, chemical reactions proceed very rapidly (practically instantaneously). In this case, the time of combustion is determined entirely by the time of mixture formation.

When the time of the physical portion of the process is much less than the time necessary for the course of the chemical reactions, i.e., $\tau_d \ll \tau_k$, then $\tau_z \approx \tau_k$, the process proceeds in the kinetic region. An example of such a process is the case when a previously-prepared combustible mixture (a mixture of a gaseous fuel with a gaseous oxidant) is fed into a combustion chamber.

For the process to proceed in the kinetic region, the rate of combustion is determined by the laws of chemical kinetics, i.e., by the properties of a given combustible mixture, the temperature and pressure in the chamber, the concentration of reactants in the combustion zone, and depends slightly on diffusion and aerodynamic factors such as: flow velocity, shape and dimensions of stablizing devices.

On the other hand, in the diffusion region the time of the process is determined, on the whole, by aerodynamic factors and is independent of kinetic factors.

In addition to these extreme cases of the course of a process, combustion in an intermediate region is possible, when the time of mixture formation is commensurate with the time of the chemical reactions. This case is the most complex, since the time of combustion in this region also depends simultaneously on physical and on chemical factors.

The modern theory of diffusion-kinetics of combustion gives the most clear representation concerning the nature of the phenomenon for heterogeneous processes†.

Let us consider briefly the process of combustion on a solid surface.

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† By heterogeneous processes are understood processes which take place between substances existing in different states, for example, solids or liquids or gases etc. Processes in which substances participate in one and the same state, in contrast to the cases mentioned above, are called homogeneous. The process of oxidation of a gaseous fuel in air serves as an example of a homogeneous process.

At the commencement of the combustion process, the concentration of the oxidant is relatively large in the low temperature zone. The reaction velocity will be determined by kinetic factors and will follow kinetic relationships (curve 1 in Fig. 47).

Further, the concentration of the oxidant near the walls decreases according to the extent of burn-out, which begins to limit the velocity of the reaction. The possibilities for the kinetic phase are exhausted and the velocity of the process with further increase of temperature commences to depart abruptly from the kinetic curve. The further course of the combustion is entirely dependent on the rate of feed of the oxidant to the combustion zone, i.e., on diffusion processes.

The supply of oxidant is characterized by the gas volume coefficient β. The greater β, the higher is the velocity of the process (see Fig. 47). The value of the coefficient β exerts a substantial influence on the transition to the diffusion phase of combustion.

This transition takes place the more rapidly the less is the gas volume coefficient β, which can be clearly seen in Fig. 47.

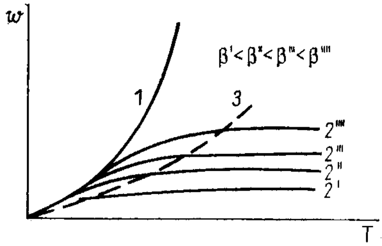

FIG. 47. Variation of reaction velocity of combustion as a function of temperature for a heterogeneous process.

The process of combustion takes place in an intermediate region when the velocities of both phases are approximately commensurate. In Fig. 47 the intermediate region for the course of the process of heterogeneous combustion is confined to curve 1 of the kinetic region and the broken curve 3 passing through the transition point into the purely diffusion region.

As mentioned earlier, the velocity of the process in the intermediate region depends simultaneously on chemical as well as physical factors.

The case considered for heterogeneous combustion of a hydrocarbon surface is the simplest. Cases encountered in actual practice present a complexity of intricate processes taking place as a result of combustion.

CHAPTER V

THERMODYNAMICS OF THE COMBUSTION CHAMBERS OF JET PROPULSION ENGINES

1. PROCESS OF HEAT SUPPLY

ALL types of aero-jet engines are, by their very nature, "jet" machines. The heat in these machines is transferred to the moving air and the velocity of motion may be very high, for example in ram-jet engines and in reheat chambers of turbo-jet engines. Owing to the fact that the transverse cross-sections of the combustion chamber of an aero-jet engine vary but little, as a rule, the process of heat supply is accompanied by a considerable increase in flow velocity with a corresponding pressure drop (the static pressure may be reduced by more than a factor of two).

The chief characteristic of heat supply in a flow system consists in a drop in the total pressure of the moving gas which leads to a reduction of its work capability and reduces the overall efficiency of the cycle and it also reduces the useful output of the engine.

It is extremely difficult to determine the parameters of a moving, heated gas in the general case, with a varying cross-section of the chamber and an arbitrary law of heat supply. Usually, as a result of this, extremely complex expressions are obtained which can be solved only by a method of successive approximation. Consequently, we are limited to the solution of two cases, typical for aero-jet engines:

(1) the process of heat supply takes place according to $p =$ const. (conical diffuser the transverse cross-section of which varies in accordance with the law of heat supply);

(2) the process of heat supply takes place in a cylindrical chamber of $F =$ const.

The first case is of a certain theoretical interest: it may be encountered in practice, for example, as a result of relatively slight heating. The second case is very general and is characteristic of the combustion chambers of ram-jet engines, reheat chambers and in certain cases also the main chambers of turbo-jet engines.

The main problem to the student for the process of heat supply to a moving gas is the determination of the gas parameters after heating up (p_2, T_2, M_2)* as a function of the heat supplied by the known initial state of flow (p_1, M_1, T_1).

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* The suffixes "1" and "2" refer: "1"— to the state of the gas prior to supply of heat; "2" — to the state of the gas after supply of heat.

At constant static pressure, the velocity of motion of the gas remains unchanged, i.e.,

$$w_2 = w_1 \, .$$

The thermodynamic temperature of the gas, as a result of heating up, increases in direct proportion to the quantity of heat supplied.

Actually, if

$$Q = C_p \left(T_2^* - T_1^*\right), \tag{V.1}$$

where

$$T_2^* = T_2 + A \frac{w_2^2}{2\, g\, c_p}$$

and

$$T_1^* = T_1 + A \frac{w_1^2}{2\, g\, c_p} \, ,$$

then for $w_2 = w_1$

$$Q = c_p \left(T_2 - T_1\right) . \tag{V.1a}$$

The total pressure of the gas is related to the static pressure by the relationship

$$p_2^* = p_2 \left(1 + \frac{k-1}{2} M_2^2\right)^{\frac{k}{k-1}} . \tag{V.2}$$

The quantity M_2 at the end of heating up can be expressed via M_1:

$$M_2^2 = \frac{w_2^2}{g k R\, T_2} = \frac{w_1^2}{g\, k\, R\, T_1 \dfrac{T_2}{T_1}} = M_1^2 \frac{1}{\dfrac{T_2}{T_1}} .$$

It is more convenient to use the stagnation temperature in this relationship, in such a form that

$$T = \frac{T^*}{1 + \dfrac{k-1}{2} M^2} .$$

After substitution and transformation we obtain

$$M_2^2 = \frac{M_1^2 \left(\dfrac{T_1^*}{T_2^*}\right)}{1 + \dfrac{k-1}{2} M_1^2 \left(1 - \dfrac{T_1^*}{T_2^*}\right)} . \tag{V.3}$$

Substituting (V.3) in expression (V.2) and assuming that

$$p_1 = \frac{p_1^*}{\left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}}} = p_2$$

after simple transformations we shall finally have

$$p_2^* = p_1^* \frac{1}{\left[1 + \frac{k-1}{2} M_1^2 \left(1 - \frac{T_1^*}{T_2^*}\right)\right]^{\frac{k}{k-1}}} . \tag{V.4}$$

If there is no heating up, then $\frac{T_1^*}{T_2^*} = 1$ and $p_2^* = p_1^*$; the same result is obtained for $M_1 = 0$; for large heating up, when the ratio $\frac{T_1^*}{T_2^*}$ is negligibly small, p_2^* will have the minimum value

$$p_{2\,\mathrm{min}}^* = \frac{p_1^*}{\left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}}} = p_1 ,$$

i.e., $p_{2\,\mathrm{min}}^*$ will be equal to the static pressure at the commencement of heating up.

Expressions (V.1), (V.1a), (V.3) and (V.4) are fundamental for determining the gas parameters (T, T^*, p^*, M) after heating up.

Change of cross-section, essential for realizing the process in question with $p =$ const., is determined by the increase of temperature or the quantity of heat supplied. Actually, under steady conditions for the supply for any cross-section x of the combustion chamber, we have

$$\frac{F_x}{F_1} \frac{w_x}{w_1} \frac{\gamma_x}{\gamma_1} = 1 .$$

Assuming that $w_x = w_1$ and $p =$ const, we shall have

$$\frac{F_x}{F_1} \frac{T_1}{T_x} = 1 .$$

Expressing the temperature T_x in terms of the initial temperature T_1 and the quantity of heat supplied Q by (V.1a), and T_1 in its turn determining the stagnation temperature T_1^* and the quantity M_1, we have

$$\frac{F_x}{F_1} = \frac{Q_x}{c_p T_1^*} \left(1 + \frac{k-1}{2} M_1^2\right) + 1 . \tag{V.5}$$

The term $\frac{Q}{c_p T_1^*}$, representing the ratio of the heat imparted to the gas or the total increase of enthalpy, to the initial enthalpy of the gas, we shall call the relative preheating τ_{rel}. The value τ_{rel} can also be similarly expressed as the ratio of the increase of the total temperatures to the total initial temperature of the gas stream T_1^*:

$$\tau_{\text{rel}} = \frac{Q_x^*}{c_p\, T_1^*} = \frac{\Delta i^*}{i_1^*} = \frac{\Delta T_{\text{comb}}^*}{T_1^*}\,.$$

The process of heat supply is shown in Fig. 48a by pv- and is-diagrams. The equations for the area $ab2^*2 = ac1^*1$ and for the segments $1 - 1^* = 2^* - 2$ follow immediately from the condition $w = \text{const.}$, which takes place for $p = \text{const.}$

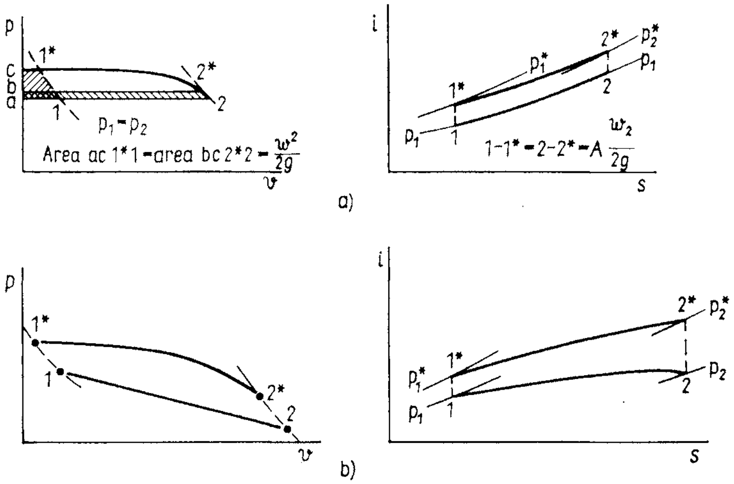

FIG. 48. Process of heat supply in pv- and is-coordinates. (a) — For $p = \text{const}$; (b) — For $F = \text{const}$.

The Heat Supply Process for $F = \text{const.}$

In a chamber of constant cross-section, gas, as a result of preheating, is dissipated, its velocity is increased and the static pressure falls.

For the basic parameters of the gas stream, relative to the change of which we shall determine the change of all other parameters, we shall take $\frac{\Delta p}{p}$ — the relative variation of static pressure. The choice of this parameter as a fundamental value characterizing the process is not accidental. The static pressure can be measured relatively simply; the drop in static pressure associated with preheating of the gas frequently determines the essential experimental conditions. Thus, the choice of the static pressure as a fundamental quantity characterizing the process is completely justified in practice.

In order to determine the gas parameters after preheating we shall use:

(1) The equation of mass supply in the form

$$\frac{w}{v} = \text{const}. \tag{V.6}$$

where the constant $= w_1 \gamma_1 = \frac{G}{f} = G_f$, the supply by weight per unit area.

(2) The equation of momentum

$$v \, \mathrm{d}p = \frac{\mathrm{d}w^2}{2g}. \tag{V.7}$$

(3) The equation of state

$$p v = R T. \tag{V.8}$$

(4) The equation of the First Law of Thermodynamics

$$\mathrm{d}Q = c_p \, \mathrm{d}T - A \, v \, \mathrm{d}p. \tag{V.9}$$

We shall determine the drop in static pressure as a function of the initial parameters of the gas flow and of the heat supplied. In order to solve the problem it is necessary to express T and v in the equation of the First Law of Thermodynamics in terms of p.

Differentiating the equation of state, we can write

$$R \, \mathrm{d}T = p \, \mathrm{d}v + v \, \mathrm{d}p,$$

whence

$$\mathrm{d}T = \frac{p \, \mathrm{d}v + v \, \mathrm{d}p}{R}.$$

Substituting the expression obtained in (V.9) in place of $\mathrm{d}T$ we obtain

$$\mathrm{d}Q = \frac{c_p}{R} p \, \mathrm{d}v + \frac{c_v}{R} v \, \mathrm{d}p. \tag{V.10}$$

In accordance with expression (V.6) it is possible to write $w^2 = (v G_f)^2$. Substituting the value $(G_f v)^2$ in place of w^2 in (V.7) we obtain

$$- v \, \mathrm{d}p = \mathrm{d}\frac{(G_f v)^2}{2g},$$

or

$$- \mathrm{d}p = \frac{G_f^2}{g} \mathrm{d}v;$$

whence

$$\mathrm{d}v = - \frac{g}{G_f^2} \mathrm{d}p \qquad \text{and} \qquad v - v_1 = \frac{g}{(G_f)^2} (p_1 - p).\dagger$$

---

† It is obvious from the expression obtained that in pv-coordinates (p is the static pressure) the process of preheating for $f = \text{const.}$ is represented by a straight line.

Substituting in (V.10) for $\mathrm{d}v$ and v, we have

$$\mathrm{d}Q = \frac{c_v}{R}\left(\frac{g}{G_f^2} p_1 + v_1\right)\mathrm{d}p - \frac{(c_p + c_v)}{R}\frac{g}{G_f^2} p\,\mathrm{d}p\,. \qquad \text{(V.11)}$$

Denoting the coefficients for $\mathrm{d}p$ and $p\,\mathrm{d}p$ by A and B respectively, integrating (V.11) within the limits from zero to Q, and from p_1 to p, we obtain

$$Q = A\,(p - p_1) - B\frac{p^2 - p_1^2}{2}\,.$$

Denoting $p_1 - p$ by Δp, the following quadratic equation can be written:

$$\frac{B}{2}\Delta p^2 - (B\,p_1 - A)\,\Delta p + Q = 0\,,$$

the solution of which relative to Δp gives

$$\Delta p = \left(p_1 - \frac{A}{B}\right) - \sqrt{\left(p_1 - \frac{A}{B}\right)^2 - \frac{2}{B}Q}\,. \qquad \text{(V.12)}$$

We express the coefficients A and B in a form convenient for solving the problem. Assuming k and c_v constant, we transform the expression

$$\frac{c_v}{R}\left(\frac{g}{G_f^2} p_1 + v_1\right)$$

and we obtain

$$A = c_v\frac{(1 + kM_1^2)}{k\,M_1^2}\frac{T_1}{p_1}\,.$$

Correspondingly, for B we have

$$B = c_v\frac{k + 1}{k\,M_1^2}\frac{T_1}{p_1^2}\,.$$

If we take k and c_v variable, then the expressions for A and B will be

$$A = \frac{c_{v_{\text{av}}}}{R_{\text{av}}}\frac{(1 + k_1 M_1^2)}{k_1 M_1^2}\frac{T_1 R_1}{p_1}\,;$$

$$B = \frac{c_{v_{\text{av}}}(k_{\text{av}} + 1)}{R_{\text{av}}}\frac{1}{k_1 M_1^2}\frac{T_1 R_1}{p_1^2} \approx c_{v_{\text{av}}}(k_{\text{av}} + 1)\frac{1}{k_1 M_1^2}\frac{T_1}{p_1^2}\,.$$

Substituting the expressions obtained for A and B in (V.12) and carrying out a few elementary transformations, we obtain final expressions for the values of the relative static pressure drop $\overline{\Delta p} = \dfrac{\Delta p}{p_1}$.

(a) k and c_v constant

$$\overline{\Delta p} = \frac{k}{k+1}(1-M_1^2) - \sqrt{\left[\frac{k}{k+1}(1-M_1^2)\right]^2 - 2\frac{k^2}{k+1}M_1^2\left(1+\frac{k-1}{2}M_1^2\right)\frac{Q}{c_p\,T_1^*}}\,. \qquad \text{(V.13)}$$

(b) k and c_v variable

$$\overline{\Delta p} = \frac{k_{\text{av}}}{k_{\text{av}}+1}\left(1-\frac{k_1}{k_{\text{av}}}M_1^2\right) - \sqrt{\left[\frac{k_{\text{av}}}{k_{\text{av}}+1}\left(1-\frac{k_1}{k_{\text{av}}}M_1^2\right)\right]^2 - 2\frac{k_1^2}{k_1-1}\left(\frac{k_{\text{av}}-1}{k_{\text{av}}+1}\right)M_1^2\left(1+\frac{k_1-1}{2}M_1^2\right)\frac{Q}{c_{p1}\,T_1^*}}\,. \qquad \text{(V.14)}$$

It is obvious that the static pressure p_2 after preheating is determined from the relationship

$$\frac{p_2}{p_1} = 1 - \overline{\Delta p}\,. \qquad \text{(V.15)}$$

It is clear that for an increase of preheating, $\overline{\Delta p}$ is also increased, but p_2 is correspondingly decreased. The static pressure drop $\overline{\Delta p}$ attains its maximum value when the square-root expression becomes equal to zero.

In this case

$$\overline{\Delta p}_{\max} = \frac{k}{k+1}(1-M_1^2) \qquad \text{(V.15a)}$$

for $k_1 = k_2 = k$;

$$\overline{\Delta p}_{\max} = \frac{k_{\text{av}}}{k_{\text{av}}+1}\left(1-\frac{k_1}{k_{\text{av}}}M_1^2\right) \qquad \text{(V.15b)}$$

or $k_1 \neq k_2$.

Let us find the velocity and temperature characteristics of the flow after preheating: w_2 and M_2, T_2 and T_2^*.

From the equation of momentum for the case $F = \text{const.}$, we have

$$p_1 - p_2 = \varrho_1 w_1 (w_2 - w_1)\,,$$

when the relationship immediately follows

$$\frac{w_2}{w_1} = \frac{\Delta p}{\varrho_1 w_1^2} + 1$$

or finally,

$$\frac{w_2}{w_1} = \frac{\overline{\Delta p}}{k_1 M_1^2} + 1\,. \qquad \text{(V.16)}$$

Knowing w_2 it is easy to determine T_2 also. Actually, from the equation of mass supply we have

$$\frac{p_2}{R_2 T_2} w_2 = \frac{p_1}{R_1 T_1} w_1 ,$$

whence, assuming $R_2 \approx R_1$,

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \frac{w_2}{w_1} ,$$

or finally,

$$\frac{T_2}{T_1} = (1 - \overline{\Delta p}) \left(\frac{\overline{\Delta p}}{k_1 M_1^2} + 1 \right) . \tag{V.17}$$

The velocity w_2 and the temperature T_2 determine the quantity M_2:

$$M_2^2 = \frac{w_2^2}{g k_2 R_2 T_2} = \frac{w_1^2}{g k R T_1} \left(\frac{w_2}{w_1} \right)^2 \frac{T_1}{T_2} \cdot \frac{k_1}{k_2} ;$$

$$M_2^2 = M_1^2 \frac{\dfrac{\overline{\Delta p}}{k_1 M_1^2} + 1}{1 - \overline{\Delta p}} \frac{k_1}{k_2} , \tag{V.18}$$

if we take $k_1 = k_2 = k$, then

$$M_2^2 = M_1^2 \frac{\dfrac{\overline{\Delta p}}{k M_1^2} + 1}{1 - \overline{\Delta p}} . \tag{V.19}$$

Finally, let us find the total pressure and the total temperature in the gas stream after preheating.

From the relationship

$$p^* = p \left(1 + \frac{k - 1}{2} M^2 \right)^{\frac{k}{k-1}}$$

we can write

$$\frac{p_2^*}{p_1^*} = \frac{p_2}{p_1} \frac{\left(1 + \dfrac{k_2 - 1}{2} M_2^2 \right)^{\frac{k_2}{k_2 - 1}}}{\left(1 + \dfrac{k_1 - 1}{2} M_1^2 \right)^{\frac{k_1}{k_1 - 1}}} .$$

Assuming $k_1 = k_2 = k$ and denoting $\frac{p_2^*}{p_1^*}$ by σ_T, we obtain

$$\frac{p_2^*}{p_1^*} = \sigma_T = (1 - \overline{\Delta p}) \frac{\left[1 + \dfrac{k - 1}{2} M_1^2 \dfrac{(\overline{\Delta p}/k M_1^2) + 1}{1 - \overline{\Delta p}} \right]^{\frac{k}{k-1}}}{\left[1 + \dfrac{k - 1}{2} M_1^2 \right]^{\frac{k}{k-1}}} \cdot \tag{V.20}$$

For $k_1 \neq k_2$

$$\frac{p_2^*}{p_1^*} = \sigma_T = (1 - \overline{\Delta p}) \cdot \frac{\left[1 + \frac{k_2 - 1}{2} \frac{k_1}{k_2} M_1^2 \frac{(\overline{\Delta p}/k_1 M_1^2) + 1}{1 - \overline{\Delta p}}\right]^{\frac{k_2}{k_2 - 1}}}{\left[1 + \frac{k_1 - 1}{2} M_1^2\right]^{\frac{k_1}{k_1 - 1}}} . \qquad (V.21)$$

It follows from physical considerations and directly from expressions (V.20) and (V.21) that the maximum reduction in total pressure $\sigma_T = \sigma_{T_{\min}}$ for a given value of M_1 is obtained for $\overline{\Delta p} = \overline{\Delta p}_{\max}$ i.e., maximum preheating is obtained by equating to zero the square-root quantities in equations (V.13) or (V.14) and examining the conditions which this represents.

The value $\sigma_{T_{\min}}$ is easy to determine by substituting in expressions (V.20) and (V.21) the value of $\overline{\Delta p}_{\max}$ taken from expression (V.15a) and (V.15b). Carrying out the substitution and after simple transformations we obtain

$$\sigma_{T_{\min}} = \frac{1 + k M_1^2}{k + 1} \left(\frac{\frac{k + 1}{2}}{1 + \frac{k - 1}{2} M_1^2} \right)^{\frac{k}{k-1}} \qquad (V.22)$$

for $k_1 = k_2 = k$
and

$$\sigma_{T_{\min}} = \frac{1 + k_1 M_1^2}{k_{\mathrm{av}} + 1} \left(\frac{1 + \frac{k_2 - 1}{2} \frac{k_{\mathrm{av}}}{k_2}}{1 + \frac{k_1 - 1}{2} M_1^2} \right)^{\frac{k_2}{k_2 - 1}} \qquad (V.22a)$$

for $k_1 \neq k_2$.

The total temperature is determined by the relationship $\Delta i^* = Q$, whence

$$\frac{T_2^*}{T_1^*} = \frac{Q}{c_{p_{\mathrm{av}}} T_1^*} + 1 , \qquad (V.23)$$

where $c_{p_{\mathrm{av}}}$ is the average value of the specific heat within the temperature interval $T_2^* - T_1^*$.

Figure 48b depicts the process of heat supply for $F =$ const. and $w > 0$. The course $i^* = f(s)$ with greater slope is characteristic of the process. For identical values of $\Delta i^* = i_2^* - i_1^*$ in the cases "a" and "b", the increase of entropy Δs_b in case "b" is greater than Δs_a. The enthalpy of the moving gas and, correspondingly, the thermodynamic temperature may be reduced, regardless of the heat supplied. The entire process proceeds in the region of lower static pressures and larger M numbers than the process $p_1 = p_2 =$ const. It can be said that each subsequent "portion" of heat is imparted to the gas stream at a higher velocity (with a large M number) and correspondingly at a lower static pressure than the previous one.

Concept of Relative Boundary Heating

It follows from expressions (V.13) and (V.14) that the relative drop of static pressure $\overline{\Delta p}$ cannot be arbitrarily large. There is a certain finite value $Q_{\max}$ which for specified initial characteristics of the gas stream is critical. It is impossible to impart to the gas stream a larger quantity of heat than $Q_{\max}$ without reducing the quantity M_1 or increasing T_1^* for a given M_1.

The maximum values of the relative preheating $\left(\frac{Q}{c_p T_1^*}\right)_{\max}$ and correspondingly $\overline{\Delta p}_{\max}$ are found from the conditions represented by equating to zero the square-root terms in equations (V.13) and (V.14). Thus, for $k_1 = k_2 = k$ the maximum relative preheating will be

$$\left(\frac{Q}{c_p T_1^*}\right)_{\max} = \frac{(1 - M_1^2)^2}{2\,(k + 1)\, M_1^2 \left(1 + \frac{k-1}{2} M_1^2\right)}; \qquad \text{(V.24)}$$

for $k_1 \neq k_2$

$$\left(\frac{Q}{c_p T_1^*}\right)_{\max} = \frac{\left(1 - \frac{k_1}{k_{\text{av}}} M_1^2\right)^2}{2\left(\frac{k_1}{k_{\text{av}}}\right)^2 \left(\frac{k_{\text{av}}^2 - 1}{k_1 - 1}\right) M_1^2 \left(1 + \frac{k-1}{2} M_1^2\right)}. \qquad \text{(V.24a)}$$

The maximum static pressure drop $\overline{\Delta p}_{\max}$ is determined by expressions (V.15a) and (V.15b):

(1) $k_1 = k_2$

$$\overline{\Delta p}_{\max} = \frac{k}{k+1}(1 - M_1^2); \qquad \text{(V.15a)}$$

(2) $k_1 \neq k_2$

$$\overline{\Delta p}_{\max} = \frac{k_{\text{av}}}{k_{\text{av}} + 1}\left(1 - \frac{k_1}{k_{\text{av}}} M_1^2\right). \qquad \text{(V.15b)}$$

It is obvious from the expressions obtained that for $M_1 = 1$, $Q = 0$ and $\overline{\Delta p} = 0$ (for $Q = 0$, $k_{\text{av}} = k_1$). Thus, the gas stream can move in a cylindrical tube with the velocity of sound ($M_1 = 1$) only in the absence of heat supply.

The physical results obtained can be explained in the following manner: by supplying heat to the moving gas for $F =$ const., the density of the gas falls proportionally to the temperature.

The fall in density of the gas for $F = \text{const.}$ should be accompanied by an increase in velocity or by a reduction in mass supply. In order that the initial parameters and flow of gas should remain unchanged it is necessary to ensure a corresponding drop in static pressure along the length of the chamber which is also a direct cause of the increase in velocity.

By further increasing the temperature and reducing the static pressure, the specific volume of the gas is increased and may become somewhat large, making it impossible to pass through the chamber, with $F = \text{const.}$, a specified supply of gas having an initial pressure p_1. This moment approaches when sonic velocity or a velocity close to sonic ($k = \text{const.}$) is attained in a duct of constant cross-section. The nearness of the flow velocity to sonic velocity is characterized by an extremely marked drop in static pressure.

It is not difficult to be convinced of the fact that the maximum relative preheating, not affecting the initial gas parameters, is limited by the attainment of sonic velocity by the gas stream. Actually, by substituting the values for $\overline{\Delta p}_{\max}$ from (V.15a) in the expression for $M_{2\max}$ we shall have

$$M^2_{2\max} = M_1^2 \frac{\left[\frac{k}{k+1}(1 - M_1^2) \Big/ k M_1^2 + 1\right]}{1 - \frac{k}{k+1}(1 - M_1^2)} = 1\,.$$

In the case of $k_1 \neq k_2$, the maximum preheating does not correspond precisely to the velocity of sound. Substitution of $\overline{\Delta p}_{\max}$ from (V.15b) in (V.18) gives

$$M^2_{2\max} = M_1^2 \frac{\left[\frac{k_{\text{av}}}{k_{\text{av}}+1}\left(1 - \frac{k_1}{k_{\text{av}}} M_1^2\right) \Big/ k_1 M_1^2\right] + 1}{1 - \frac{k_{\text{av}}}{k_{\text{av}}+1}\left(1 - \frac{k_1}{k_{\text{av}}} M_1^2\right)} \frac{k_1}{k_2}\,,$$

whence

$$M^2_{2\max} = \frac{k_{\text{av}}}{k_2} < 1\,.$$

Thus, strictly speaking, under ideal conditions (since for preheating $k \neq \text{const.}$ always applies) it is impossible to attain a sonic velocity in a cylindrical tube, but for normal limits of variation of k this velocity will be close to sonic.

Further preheating, having no effect on the gas parameters, is possible only in a divergent duct for which the flow velocity can, of course, exceed the velocity of sound.

In Fig. 49, numerical values are shown for the maximum relative preheating $\left(\frac{Q}{c_p T_1^*}\right)_{\max}$ as a function of the initial Mach number M_1 for the flow, determined by expression (V.24) for $k = 1{\cdot}4$ and $c_p = 0{\cdot}24$.

It can be seen from Fig. 49 that the maximum possible preheating for small values of M_1 is very large; thus for $M_1 = 0{\cdot}1$, which corresponds, in the primary combustion chambers of aero-jet engines to velocities of $w_1 = 40$–50 m/sec and in reheat chambers to approximately 60–65 m/sec, $\left(\frac{Q}{c_p\, T_1^*}\right)_{\max} > 20$.

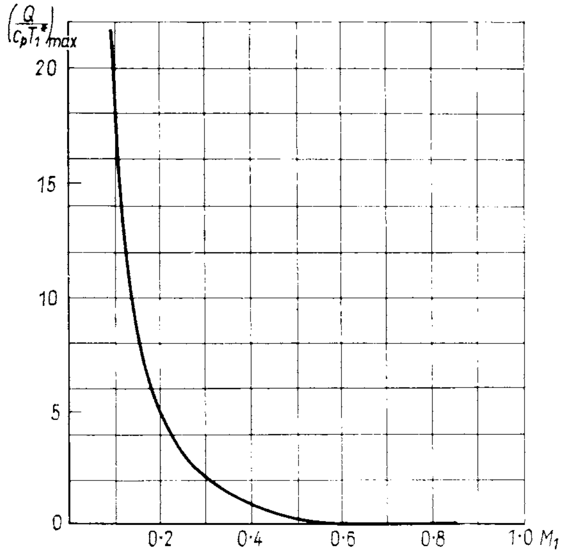

FIG. 49. Variation of maximum preheating $\tau_{\max} = \left(\frac{Q}{c_p\, T_1^*}\right)$ for heat supply in a cylindrical duct as a function of velocity at the commencement of preheating.

If it be assumed that the temperature prior to combustion is equal to 500 °K, then the fuel-air mixture is able to ensure relative preheating, in the best case, approximately equal to 5·5. It is clear from Fig. 49 that for $T_1 \approx 500$ °K and $M_1 < 0.18$ we cannot obtain surface heating for the combustion of hydrocarbon fuels because $\left(\frac{Q}{c_p\, T_1^*}\right)_{\max} > 5{\cdot}5$. Conversely, for $M_1 > 0{\cdot}18$ in a duct of constant cross-section we cannot achieve the whole of the preheating which should be obtained by combustion of the fuel, since $\left(\frac{Q}{c_p\, T_1^*}\right)_{\max} < 5{\cdot}5$.

Thus, there are always two regions, in one of which surface heating is not possible in practice because of insufficiency of the heat released by combustion, and in the second region it is not possible to impart to the gas all the heat capable of being released as a result of chemical reaction, without having an effect on the initial parameters of the gas stream.

Variation of Static Pressure

Flow with preheating in a duct with $F = \text{const.}$ is accompanied by dispersion of the gas and a drop in static pressure. Consequently, it is necessary to compare the pressure drop $\overline{\Delta p}$ resulting from preheating with that for $\overline{\Delta p}_{\text{disp}}$ which can be ensured by external conditions, for example, by an experimental rig. It is obvious that it is necessary to fulfil the condition

$$\overline{\Delta p} \leq \overline{\Delta p}_{\text{disp}} .$$

Figure 50 depicts the relationship

$$\left(\frac{p_2}{p_1}\right)_{\min} = 1 - \overline{\Delta p}_{\max} = f(M_1) ,$$

calculated according to expression (V.15). Particular attention is attracted by the substantial change of static pressure within the range of values $M_1 < 0{\cdot}3$ of practical interest, where the ratio $\left(\frac{p_2}{p_1}\right)_{\min} < 0{\cdot}5$.

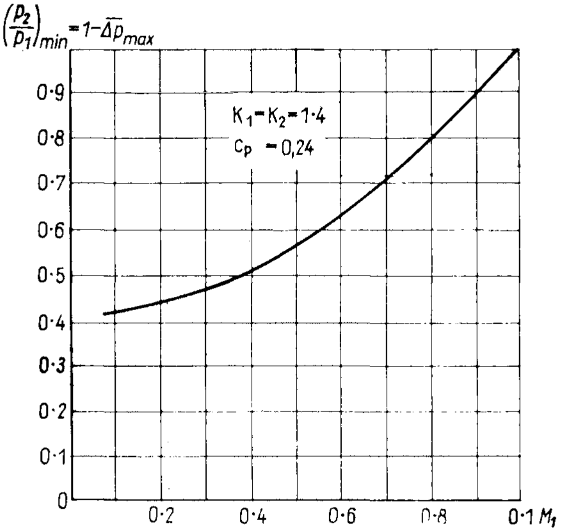

FIG. 50. Variation of static pressure as a function of flow velocity at the commencement of preheating.

Consequently, the pressure before preheating should be at least twice as great as at the end of the heat supply. Only by satisfying this condition can the maximum supply of heat be achieved. Thus, if the exhaust from the duct in which the gas temperature is increased passes directly into the atmosphere without any device at the end of the duct (having in mind a diffuser or ejector) and the pressure p_1 is less than 2 atm ($M_1 < 0.3$), then it is impossible to realize critical preheating. Attempts to do this in practice lead to the fact that commencing with a certain value for the fuel supply, the flow is reorganized: either the velocity decreases (quantity M_1) if the rig assembly permits this, or an unstable pulsation regime is created, often dangerous to the operation of the unit. It is obvious that in both cases, the initial conditions will be disturbed.

The remarks made refer not only to critical preheating but also have a general significance. As a result of any preheating of the moving gas, it is necessary to keep track of the fact that the assembly on the whole should ensure the necessary pressure drop for given preheating.

The total pressure in the gas flow for given values of M_1 and $\frac{Q}{c_p T_1^*}$ should be satisfied by the inequality

$$p_1^* \geq \frac{p_2}{1 - \overline{\Delta p}} \left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}} \tag{V.25a}$$

or, in a dimensionless form

$$\frac{p_1^*}{p_2} \geq \frac{\left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}}}{(1 - \overline{\Delta p})}, \tag{V.25b}$$

where $\overline{\Delta p}$ is the relative static pressure drop corresponding to given preheating $\left(\frac{Q}{c_p T_1^*}\right)$ and M_1, and is determined by formulae (V.13) and (V.14).

p_2 is the static pressure of the medium into which discharge takes place (the pressure on entry into the turbine; if discharge takes place into the atmosphere, then p_2 is the atmospheric pressure; if an ejector is installed at the end of the duct, then p_2 is the pressure in the ejector chamber).

Formulae (V.25a) and (V.25b) are mathematical expressions for the conditions necessary for achieving a preheating process in a duct of constant cross-section.

This should always be taken into account in practical operation and no attempt should be made to burn more fuel in combustion chambers with

$F =$ const., reheated to M_1, at a pressure differing only slightly from the pressure of the medium into which discharge takes place.

Nature of Variation of Thermodynamic Temperature

From expression (V.17), it can be seen that in order to determine the thermodynamic temperature, the temperature

$$\frac{T_2}{T_1} = (1 - \overline{\Delta p})\left(\frac{\overline{\Delta p}}{k_1 M_1^2} + 1\right)$$

is related to the static pressure drop in two ways. It is obvious that for $k_1 M_1^2 < 1$ and small $\overline{\Delta p}$, the second factor of the expression has a predominant effect, i.e., with increase of $\overline{\Delta p}$ the value of T_2 is also increased. According to the extent of increase of $\overline{\Delta p}$, the first factor should exert an even greater influence, i.e., with increase of $\overline{\Delta p}$ the thermodynamic temperature (not the stagnation temperature) can even fall, despite the continued supply of heat. If the value of $\overline{\Delta p}$, at which the increase in T_2 is initiated, is less than $\overline{\Delta p}_{\max}$, then this will imply that a drop in temperature can take place in an ideal process.

Let us determine the value of $\overline{\Delta p}_T$ for which T_2 attains its maximum. Taking the derivative of $\frac{T_2}{T_1}$ with respect to $\overline{\Delta p}$, we obtain

$$\frac{\partial \frac{T_2}{T_1}}{\partial \overline{\Delta p}} = \left(1 - \overline{\Delta p}\right)\frac{1}{k_1 M_1^2} - \left(\frac{\overline{\Delta p}}{k_1 M_1^2} + 1\right);$$

equating the value of the derivative to zero we have

$$\overline{\Delta p}_T = \frac{1}{2}(1 - k_1 M_1^2)\,.$$

Assuming that $k_1 = k_2 =$ const, we compare the expression obtained for $\overline{\Delta p}_T$ with the expression for $\overline{\Delta p}_{\max}$

$$\overline{\Delta p}_{\max} = \frac{k}{k+1}(1 - M_1^2)\,. \tag{V.15a}$$

It can be seen from the comparison that at all times $\overline{\Delta p}_T < \overline{\Delta p}_{\max}$. Actually, the value of $\frac{k}{k+1}\,(1 + M_1^2)$ is greater than $\frac{1}{2}\,(1 - k\,M_1^2)$, since $\frac{k}{k+1}$ is always greater than $\frac{1}{2}$.

Thus, for preheating, corresponding to $\overline{\Delta p} > \frac{1}{2} \times (1 - k_1 M_1^2)$, a reduction of the thermodynamic temperature will be observed.

It is also interesting to note that for $M_1^2 \geq \frac{1}{k}$, increase of the thermodynamic temperature of the gas flow does not, in general, take place. Despite the supply of heat, the temperature of the gas flow in combustion chambers of constant cross-section falls. Figure 51 depicts the relative change of thermodynamic temperature for the two values of M_1 ($M_1 = 0{\cdot}2$ and $M_1 \approx 0{\cdot}845$), which corresponds to $M_1^2 = 1/k$ (k is taken equal to 1·4).

As was to be expected, the relationship $T_2/T_1 = f(\overline{\Delta p}) = \psi(Q/c_p\, T_1^*)$, is a maximum if $M_1^2 < 1/k$, for preheating corresponding to $\overline{\Delta p_T}$. This maximum lies close to $\left(\frac{Q}{c_p\, T_1^*}\right)_{\max}$.

For $M_1^2 > \frac{1}{k}$ the temperature is reduced as a result of any preheating. For comparison, Fig. 51 shows the variation of total temperature which, naturally, increases the whole time with increased preheating.

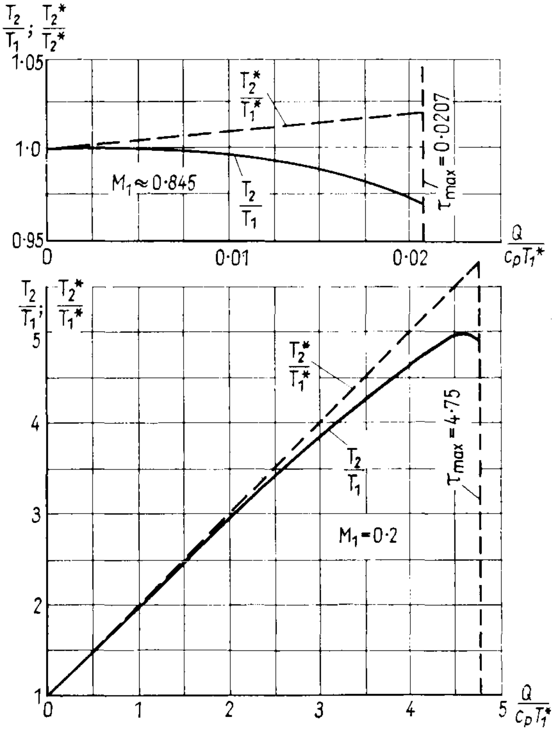

FIG. 51. Variation of thermodynamic and total (stagnation) temperature as a result of preheating (F = const., k = 1·4).

The possibility of decreasing the thermodynamic temperature as a result of preheating is a special feature of the process of heat supply to the gas for a duct with constant cross-section. This special physical characteristic can be explained in the following manner: the thermodynamic temperature is determined by the simultaneous action of two opposite-acting causes related to the supply of heat. The heat supplied, acting directly on the gas, always tends to increase the temperature of the gas.

However, the supply of heat requires at the same time an increase in the fall of pressure ensuring increase of velocity, i.e., a corresponding decrease of static pressure, and as a result of this the gas expands and this temperature tends to be decreased†. In the region of small values of M_1 the first factor is of chief importance, since the reduction in pressure and the expansion of the gas associated with it alone are insignificant and consequently have little effect on the temperature. Because of acceleration of the gas (increase of Mach number of the gas stream), every calorie supplied to the gas calls for an even greater and greater drop in static pressure. The specific volume of the gas as a result of this increases still more. This also leads to the fact that the effect of expansion of the gas on its temperature is still further increased. At first this is manifested in a slow increase of temperature with the supply of heat, and then for $Ap\,\mathrm{d}v > \mathrm{d}Q$ this leads to a reduction of T_2.

Figure 52 shows the variation of $\overline{\Delta p}$ and p_2/p_1 for the first example (see Fig. 51) (it can be seen that in the region close to maximum preheating, $(Q/c_pT_1^*)_{\max}$ falls sharply, the static pressure $\overline{\Delta p}$ increased sharply, p_2/p_1 rapidly decreases). In general form this is seen clearly from consideration of the expression for the increment $\overline{\Delta p}$. Actually, on differentiating (V.13) we obtain

$$\mathrm{d}(\overline{\Delta p}) = \frac{b}{2}\,\frac{\mathrm{d}\left(\dfrac{Q}{c_p T_1^*}\right)}{\sqrt{a^2 - b\left(\dfrac{Q}{c_p T_1^*}\right)}},$$

where

$$a = \frac{k}{k+1}(1 - M_1^2);$$

$$b = 2\frac{k^2}{k+1}M_1^2\left(1 + \frac{k-1}{2}M_1^2\right).$$

It is obvious that for small preheating, such that $b\dfrac{Q}{c_pT_1^*} \ll a^2$, $\mathrm{d}(\Delta p)$ is directly proportional to $\mathrm{d}\left(\dfrac{Q}{c_pT_1^*}\right)$; with further increase of preheating

---

† This can be seen immediately from the expression for the First Law of Thermodynamics $\mathrm{d}Q = c_v\,\mathrm{d}T + Ap\,\mathrm{d}v$, where $\mathrm{d}T = \dfrac{\mathrm{d}Q}{c_v} - \dfrac{A}{c_v}p\,\mathrm{d}v$ increases or descreases, depending upon the form of the inequality $\mathrm{d}Q > Ap\,\mathrm{d}v$ or $\mathrm{d}Q < pA\,\mathrm{d}v$.

and its approach to critical, the difference $a^2 - b\left(\frac{Q}{c_p T_1^*}\right)$ tends to zero and $d(\overline{\Delta p})$ increases indefinitely relative to $d\left(\frac{Q}{c_p T_1^*}\right)$, i.e., the static pressure falls sharply.

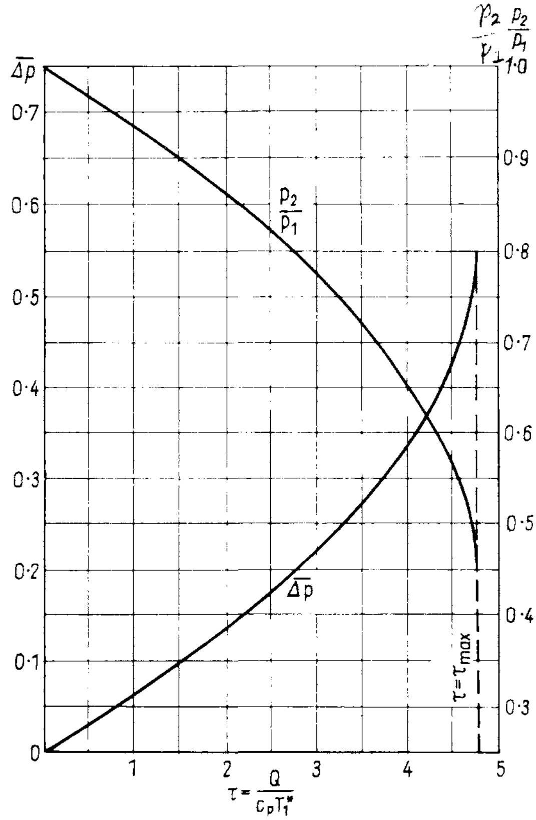

FIG. 52. Variation of static pressure as a result of preheating (F = const; $M = 0{\cdot}2$; $k = 1{\cdot}4$).

Hence, in passing, the phenomenon of "choking" becomes more understandable because of the sharp increase of volume in the region of $M \approx 1$. It should be noted that under the influence of heat supply, the phenomenon of "choking" can occur in ducts of the most diverse shape (divergent, cylindrical, tapered). This phenomenon is particularly sharply marked for a flow of preheated gas in a tapered duct, where, for specified intake conditions the values of τ_{max} are reduced, and vice versa: for specified τ_{max} the intake conditions should be altered.

Variation of Total Pressure

The reduction in the total pressure of the gas stream as a result of preheating is extremely important in practice. Thus, the total pressure of the gas determines, for given preheating, its work capability. The value of the coefficient $\sigma = p_2^*/p_1^*$ depends on M_1 and on $Q/c_pT_1^*$. For a given M_1 the value of σ varies from 1 (for $Q/c_pT_1^* = 0$) to $\sigma_{\min}$ [for $(Q/pT_1^*)_{\max}$]. The values of $\sigma_{\min}$ are also increased with increase of M_1, attaining 1 for $M_1 \approx 1{\cdot}0$. The latter result is explained by the fact that together with an increase of M_1, the maximum preheating $\tau_{\max} = (Q/c_pT_1^*)_{\max}$ is decreased, and consequently the values of $\sigma_{\min}$ with respect to M_1, correspond to even less and less preheating. Thus, if $M_1 \approx 1{\cdot}0$, then also $\tau_{\max} = (Q/c_p\, T_1^*) = 0$ and of course, as a result of this, preheating for $\sigma_{\min} = 1{\cdot}0$, i.e., there are no total pressure losses.

For a constant value of preheating, σ is less the larger is the quantity M_1. The nature of the variation of $\sigma_{\min}$ with respect to M_1 for different values of τ is shown in Fig. 53.

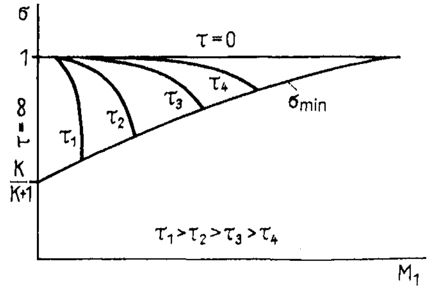

FIG. 53. Nature of variation of the loss coefficient $\sigma_{\min}$ as a function of flow velocity at the beginning of a duct, for different values of preheating.

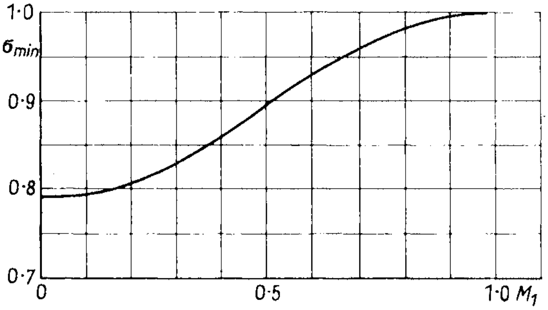

FIG. 54. Variation of the coefficient $\sigma_{\min}$ as a function of flow velocity at the commencement of preheating.

In Fig. 54, numerical values are shown for $\sigma_{\min} = f(M_1)$. The calculations show that the total pressure losses for values of $M_1 < 0{\cdot}3$ of practical interest can attain very substantial magnitudes — up to 20 per cent of the initial value ($p_2^* \approx 0{\cdot}8\, p_1^*$).

2. THE THERMODYNAMIC CYCLE AND THE EFFICIENCY OF A TURBO-JET ENGINE

When speaking of the thermodynamic cycle of a turbo-jet engine for continuous operation, the cycle $p = \text{const}$ is often represented as a cycle comprising two isobars and two adiabatics. However, such a representation of the cycle is much over-simplified.

It was shown above that the condition for supply of heat for $p = \text{const}$* in ideal conditions is not observed as a rule. In order to assess the influence of the "thermodynamics" of the combustion chamber on the functioning of an engine, it is necessary to consider the effect of the nature of the course of the heat supply process on the efficiency of a turbo-jet engine cycle. With this as objective, we shall consider briefly certain thermodynamic cycles, in which the supply of heat is derived as a result of maintenance of the conditions: (1) $p = p^* = \text{const}$ (normal cycle $p = \text{const.}$), (2) $p = \text{const.}$, and (3) $f = \text{const.}$ The processes of compression and expansion are assumed to be, in all cases, adiabatic.

Figure 55 shows the thermodynamic cycles of a turbo-jet engine for identical preheating:

(1) cycle with constant total pressure in the direction of heat supply $p_1^* = p_2^*$. For this, the static pressure is also $p_1 = p_2$;

(2) cycle with constant static pressure $p_1 = p_2$ and $M > 0$;

(3) cycle with falling static pressure $p_1 > p_2$ and $F = \text{const.}$

The corresponding suffices are adopted for the parameters of the cycles.

It was shown above that the characteristic special feature of thermodynamic cycles of turbo-jet engines consists in a reduction of the total pressure in the direction of the heat supply, with the exception of the cycle for a turbo-jet engine with infinitely large dimensions for which $p^* = p = \text{const}$ in the direction of the heat supply.

It is well known that the useful work of a turbo-jet engine as a thermal machine is determined by the difference in the kinetic energies of the combustion products discharging from the exit nozzle and the air entering the engine

$$L = (M + m)\frac{w_3^2}{2} - M\frac{w_0^2}{2},$$

where M and m are respectively the mass of air passing through the engine, and the mass of the fuel. If it is not assumed, as is usually the case in

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* Cycles for turbo-jet engines with reheat chambers and cycles for other possible engine layouts are not considered here.

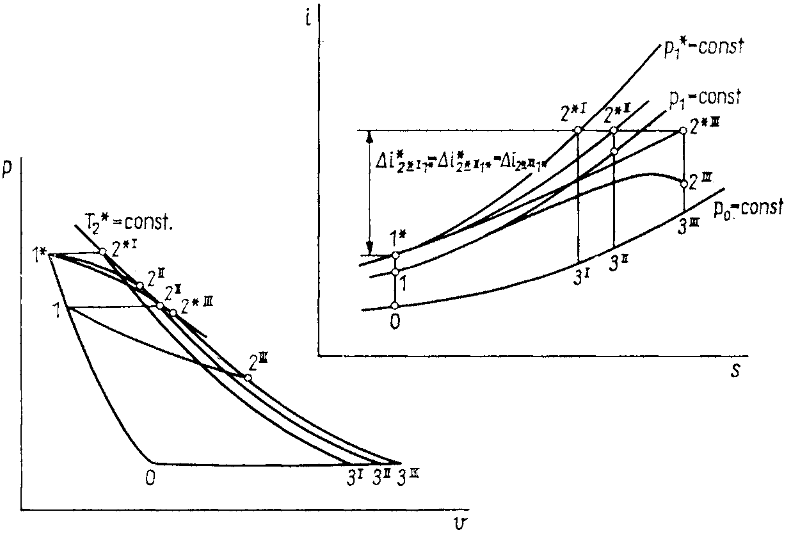

FIG. 55. Thermodynamic cycles of turbo-jet engines in *pv*- and *is*-coordinates.

thermodynamics, that the mass of the working substance increases as a result of the supply of heat, then

$$L = M \frac{w_3^2 - w_0^2}{2},$$

whence the useful work per 1 kg of air is

$$l = \frac{w_3^2 - w_0^2}{2g}.$$

It is obvious that the thermal efficiency of the engine η_t in all cases is determined by the expression

$$\eta_t = \frac{Al}{q} = \frac{A(w_3^2 - w_0^2)}{2gq},$$

where q is the heat supplied to 1 kg of air, equal to

$$q = c_p(T_2^* - T_1^*).$$

Assuming that $\frac{Aw_3^2}{2g} = i_2^* - i_3^*$ and $\frac{Aw_0^2}{2g} = i_1^* - i_0$, we obtain the expression for η_t via the change of heat content (enthalpy) or temperature in the form

$$\eta_t = \frac{(i_2^* - i_3^*) - (i_1^* - i_0)}{i_2^* - i_1^*} = 1 - \frac{i_3^* - i_0}{i_2^* - i_1^*} = 1 - \frac{T_3 - T_0}{T_3^* - T_0^*}.\ *$$

---

* It should be borne in mind that $i_1^* = i_0^*$.

The expression for η_t can be transformed for a number of actual cases, presenting it in a form more suitable for calculation.

(1) The cycle for $p^* = p = \text{const.}$ $(w_1 = w_2 = 0)$ in the direction of preheating.

As already mentioned, this cycle is possible for an aero-jet engine only when the flow velocity of the gas (air), as a result of preheating, is infinitely small.

Consideration of such a cycle is useful if only because of the fact that its parameters will be used in calculating the parameters of more ideal cycles in order to simplify mathematical expressions; in addition, the efficiency of the cycle determines, as it were, the thermodynamic limits of efficiency of aero-jet engine cycles with a continuous supply of heat, which is of definite significance.

The *pv*- and *is*-diagram of such a cycle is shown in Fig. 56.

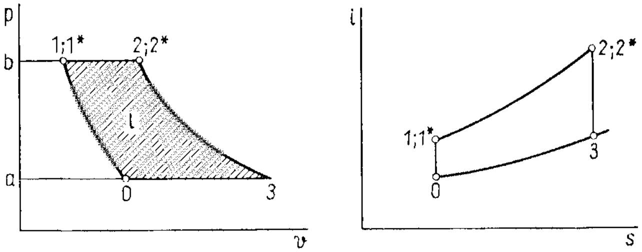

FIG. 56. Diagram of the thermodynamic cycle of an engine $p = \text{const.}$

The useful work of the cycle, as is well known, is represented in thermodynamic coordinates by the difference in areas l = area $ab2^*\,3$ — area abl^*0 (*pv*-diagram), or the difference in the segments $Al = 2^*\,3 - 1^*\,0 = \Delta i_{23} - \Delta i_{10}$ (*is* - diagram).

It is well known from thermodynamics that the areas $b\;2^*\,3$ and $b\;1^*\,0$ are equal to the following expressions:

$$\text{area } a\,b\,2^*\,3 = \frac{c_p\,T_2^*}{A}\left[1 - \left(\frac{p_3}{p_2^*}\right)^{\frac{k-1}{k}}\right]; \qquad \text{(V.26a)}$$

$$\text{area } a\,b\,1^*\,0 = \frac{c_p\,T_0^*}{A}\left[1 - \left(\frac{p_0}{p_1^*}\right)^{\frac{k-1}{k}}\right]. \qquad \text{(V.26b)}$$

Thus

$$\eta_t = \frac{A\,l}{q} = \frac{c_p\,T_2^*\left[1 - \left(\frac{p_3}{p_2^*}\right)^{\frac{k-1}{k}}\right] - c_p\,T_1^*\left[1 - \left(\frac{p_0}{p_1^*}\right)^{\frac{k-1}{k}}\right]}{q}$$

where $q = c_p\,(T_2^* - T_1^*)$ is the heat supplied.

Substituting the value obtained for q in the expression for η_t we obtain

$$\eta_t = 1 - \frac{T_2^* \left(\frac{p_3}{p_2^*}\right)^{\frac{k-1}{k}} - T_1^* \left(\frac{p_0}{p_1^*}\right)^{\frac{k-1}{k}}}{(T_2^* - T_1^*)} ;$$

since by the presence of total expansion $p_3 = p_0$, then

$$\eta_t = 1 - \frac{T_2^* \left(\frac{p_0}{p_1^*}\right)^{\frac{k-1}{k}} - T_1^* \left(\frac{p_0}{p_1^*}\right)^{\frac{k-1}{k}}}{T_2^* - T_1^*} . \qquad (V.27)$$

It should be noted that formula (V.27) is the most general expression for the efficiency of a cycle, comprising two adiabatics, for any law of heat supply whatsoever, since the work of expansion is always expressed by relationship (V.26a) and the work of compression by relationship (V.26b), just as q in all cases is also determined by the relationship given above.

In our case $p_1^* = p_2^*$ and consequently the efficiency of the cycle η_{tp^*} is equal to

$$\eta_{tp^*} = 1 - \left(\frac{p_0}{p_1^*}\right)^{\frac{k-1}{k}} = 1 - \frac{1}{\pi^{*\frac{k-1}{k}}} . \qquad (V.28)$$

The expression obtained represents a formula well known from thermodynamics for the efficiency of a cycle with $p =$ const., where π^* is the degree of pressure increase (total and static), equal also in the given case to the degree of expansion relative to the static and total pressures.

(2) The cycle for $p =$ const. and $w_2 > 0$ in the direction of preheating.

Such a cycle is possible when the cross-sectional area of the duct (chamber) where the heat supply occurs increases according to the extent of the supply of heat by the relationship

$$\frac{F}{F_1} = \frac{Q}{c_p T_1^*}\left(1 + \frac{k-1}{2} M_1^2\right) + 1 . \qquad (V.29)$$

This cycle too is little realized, since it usually requires an undesirable increase in the transverse dimensions of the combustion chamber; however its consequence permits certain general rules connected with the supply of heat to a moving gas to be established relatively simply. The *pv*- and *is*-diagrams for such a cycle are given in Fig. 57.

Just as in the previous case, the useful work is represented in the *pv*-diagram by the difference in areas:

$$l = \text{area } ab\, 2^*\, 3 - \text{area } ac\, 1^*\, 0 .$$

Thus, the work is represented graphically by the difference in the shaded areas in Fig. 57a. Since the velocities w_1 and w_2 are equal by hypothesis, then consequently they should also be equivalent to the areas $g\,c\,1^*\,1$ and $gb\,2^*\,2$, equal to $w_1^2/2\,g = w_2^2/2\,g$. Thus $l =$ (area ag 2 3 + area $gb2^*2$) − (area ag 1 0 + area gc 1* 1) = area 0 1 2 3 .

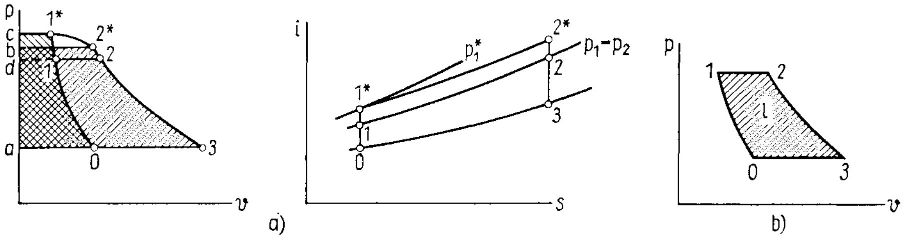

FIG. 57. Diagrams of thermodynamic cycles, taking into account the supply of heat to the moving gas.

Consequently, the useful work for the cycle p = const. is represented by the area 0 1 2 3 (see Fig. 57b), bounded above by the static pressure line 1 2. It should be possible to draw exactly the same conclusion directly from the is-diagram, in which the work will be the difference

$$Al = (i_2^* - i_3) - (i_1^* - i_0) \quad \text{or} \quad Al = (i_2 - i_3) - (i_1 - i_0)\,,$$

since the segments $1\,1^* = A\,w_1^2/2\,g$ and $2\,2^* = A\,w_2^2/2\,g$ are identical.

Let us determine the thermal efficiency of the cycle η_{tp}, for which we shall transpose the quantities entering into formula (V.26) in the following manner:

$$\text{area } ab\,2^*\,3 = \frac{c_p\,T_2}{A}\left[1 - \left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}}\right] + \frac{w_2^2}{2\,g}\,;$$

$$\text{area } ac\,1^*\,0 = \frac{c_p\,T_1}{A}\left[1 - \left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}}\right] + \frac{w_1^2}{2\,g}\,.$$

Assuming that $p_3 = p_0$ and, in addition, for our case that $p_2 = p_1$; $w_1 = w_2$ and $T_2^* - T_1^* = T_2 - T_1$, we obtain an expression for the thermal efficiency:

$$\eta_{tp} = \frac{c_p\,T_2\left[1 - \left(\frac{p_0}{p_2}\right)^{\frac{k-1}{k}}\right] - c_p\,T_1\left[1 - \left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}}\right]}{c_p\,(T_2 - T_1)} = 1 - \left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}}. \qquad \text{(V.30)}$$

As we can see, the expression for η_{tp} does not differ in form from formula (V.28). It can be seen from expression (V.30) that η_{tp} is entirely independent of the total pressure, determining identically the static pressure at the end of the compression. The efficiency of the cycle is determined entirely by the degree of pressure increase and, correspondingly, by the degree of expansion relative to the static pressure. It follows directly from expression (V.30) that if heat is transferred to the free stream ($p_1 = p_0$), then the efficiency of such an engine should be equal to zero. This also is clear, since the transfer of heat to the free stream does not lead to change of kinetic energy of the latter (the gas cannot increase its velocity, thus $p = p_0 =$ const.), and consequently also the work capability of the gas is also unchanged.

Expression (V.30) shows that the less the static pressure at which heat is transferred, the less efficiently is it utilized — the useful work of the engine is reduced. We shall consider this conclusion as the basis for the section under discussion.

It is obvious that η_{tp} for a given value of p_1^* will depend on the velocity (M_1) at the beginning of the duct (chamber) into which the heat is fed. Let us find an expression for the efficiency of the cycle, taking M_1 into account.

It is evident that

$$p_1 = \frac{p_1^*}{\left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}}};$$

substituting the value of p_1 in (V.30) we shall have

$$\eta_{tp} = 1 - \left(\frac{p_0}{p_1^*}\right)^{\frac{k-1}{k}} \left(1 + \frac{k-1}{2} M_1^2\right),$$

or finally,

$$\eta_{tp} = 1 - \frac{1}{\pi^{*\frac{k-1}{k}}} \left(1 + \frac{k-1}{2} M_1^2\right). \tag{V.31}$$

The expression for η_{tp} is easily associated with η_{tp^*}. Actually, from equation (V.28) it follows that

$$\frac{1}{\pi^{*\frac{k-1}{k}}} = 1 - \eta_{tp^*},$$

whence we obtain

$$\eta_{tp} = 1 - (1 - \eta_{tp^*}) \left(1 + \frac{k-1}{2} M_1^2\right). \tag{V.31a}$$

A considerable reduction in the efficiency of the cycle can occur, for example, for a ram-jet engine, particularly for small values of M_0.

Thus, for $M_0 = 0{\cdot}8$ und $M_1 \approx 0{\cdot}2$,

$$\eta_{tp} = 0{\cdot}94\, \eta_{tp^*},$$

i.e., the efficiency of the cycle is reduced approximately by 6 per cent relative to the case $M_1 = 0$. Finally, such a case can be presented when the efficiency of the cycle with $p = \text{const.}$ and $w_1 > 0$ will be little different from the efficiency of a "throttled" cycle; for this, obviously, the value of M_1 should be small, and it will be smaller, the smaller is π^*.

(3) Cycle with supply of heat for $F = \text{const.}$

This thermodynamic cycle corresponds most closely to ideal conditions and consequently is of prime importance. The *pv*- and *is*-diagrams for the cycle are given in Fig. 58a.

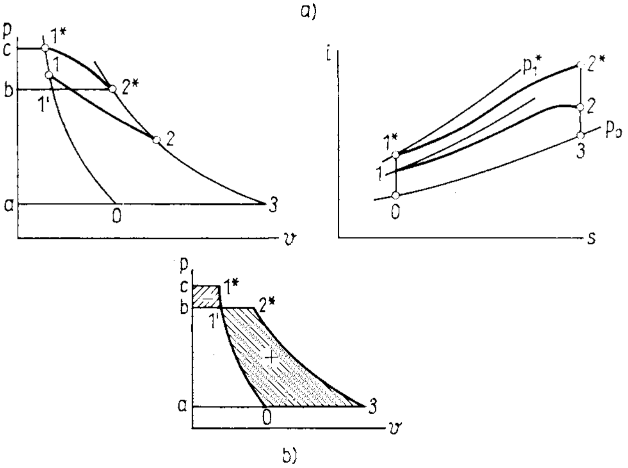

FIG. 58. Thermodynamic cycles with supply of heat for $F = \text{const.}$

The useful work of the cycle l, is depicted as usual the *pv*-diagram by the difference in areas:

$$l = \text{area } a\, b\, 2^*\, 3 - \text{area } a\, c\, 1^*\, 0\,.$$

Thus, the useful work in the *pv*-diagram will be, graphically,

$$l = \text{area } 0\, 1'\, 2^*\, 3 - \text{area } b\, c\, 1^*\, 1'\,.$$

as shown in Fig. 58b.

In the *is*-diagram, the work l is determined, as always, by the difference in enthalpy in the direction of expansion and compression

$$Al = (i_2^* - i_2) - (i_1^* - i_0)\,.$$

Let us determine the thermal efficiency.

From equations (V.26a) and (V.26b) we have

$$\eta_{tf} = \frac{c_p\, T_2^* \left[1 - \left(\dfrac{p_0}{p_2^*}\right)^{\frac{k-1}{k}}\right] - c_p\, T_1^* \left[1 - \left(\dfrac{p_0}{p_1^*}\right)^{\frac{k-1}{k}}\right]}{c_p\,(T_2^* - T_1^*)}\,.$$

By transforming the expression obtained we have

$$\eta_{tf} = 1 - \frac{c_p \left(\frac{p_0}{p_1^*}\right)^{\frac{k-1}{k}} \left[\left(\frac{p_1^*}{p_2^*}\right)^{\frac{k-1}{k}} T_2^* - T_1^*\right]}{c_p \left(T_2^* - T_1^*\right)} . \qquad (V.32)$$

Expression (V.32) derived without any limits, just like expression (V.27) has an overall significance for all cycles with adiabatic compression and expansion. The quantity $\frac{p_0}{p_1^*}$ is obviously none other than $\frac{1}{\pi^*}$, where π^* is the degree of pressure increase

$$\pi^* = \frac{p_1^*}{p_0} .$$

Hence,

$$\eta_{tf} = 1 - \frac{1}{\pi^{*\frac{k-1}{k}}} \frac{\left(\frac{p_1^*}{p_2^*}\right)^{\frac{k-1}{k}} T_2^* - T_1^*}{T_2^* - T_1^*} ;$$

if there is no loss of total pressure in the direction of heat supply, then it is obvious that

$$\eta_{tf} = \eta_{tp^*} = 1 - \frac{1}{\pi^{*\frac{k-1}{k}}} .$$

Denoting the ratio p_2^*/p_1^* by σ, where σ is a coefficient determining the reduction in total pressure as a result of preheating*, we obtain

$$\eta_{tf} = 1 - \frac{1}{\pi^{*\frac{k-1}{k}}} \frac{T_1^* \left[\frac{T_2^*}{T_1^*} \frac{1}{\sigma^{\frac{k-1}{k}}} - 1\right]}{T_3^* - T_2^*}$$

$$= 1 - \frac{1}{\pi^{*\frac{k-1}{k}}} \frac{\left(\frac{T_2^* - T_1^*}{T_1^*} + 1\right) \frac{1}{\sigma^{\frac{k-1}{k}}} - 1}{\frac{T_2^* - T_1^*}{T_1^*}} .$$

The ratio $(T_2^* - T_1^*)/T_1^*$ is none other than the relative preheating

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* $\sigma = f\left(M_1; \frac{Q}{c_p T_1^*}\right)$ is found from expressions (V.20) or (V.21).

$Q/c_p\, T_1^* = \tau$; thus, we can finally write

$$\eta_{tf} = 1 - \frac{1}{\pi^{*\,\frac{k-1}{k}}} \, \frac{(\tau + 1)\dfrac{1}{\sigma^{\frac{k-1}{k}}} - 1}{\tau}, \tag{V.33a}$$

or via η_{tp^*}

$$\eta_{tf} = 1 - \frac{(1 - \eta_{tp^*})\left[(\tau + 1)\dfrac{1}{\sigma^{\frac{k-1}{k}}} - 1\right]}{\tau}. \tag{V.33b}$$

It is easy to be convinced by direct substitution that η_{tf} transforms into η_{tp} or η_{tp^*}, if, in place of σ the corresponding value is substituted. It can be seen from expression (V.33) that η_{tf} in contrast to η_{tp} and η_{tp^*} depends on the extent of the preheating. The greater the preheating, the less is p_2^* for approximately the same conditions, which also gives rise to a reduction in η_{tf}.

The minimum value of η_{tf} occurs for maximum preheating $\left(\dfrac{Q}{c_p\, T_1^*}\right)_{\max}$ and correspondingly for $\sigma_{\min}$.

Figures 59 and 60 show the absolute and relative variation of η_{tf} with M_1 for different values of π^* and the equivalent quantities M_0 for a given π^*. It is obvious that π^* is the aggregate degree of total pressure increase equal to the product $\pi^* = \pi_k^*\, \pi_d^*$, where π_d^* is the increase of pressure due to complete throttling of the inflowing gas stream, and π_k^* is the increase of total pressure in the compressor. The values of M given in Figs. 59 and 60 correspond to the case $\pi_k^* = 1{\cdot}0$, i.e. a ram-jet engine with maximum preheating (see table to Fig. 59). As should also be expected, the reduction in the efficiency of the cycle is greater, the less is the extent of pressure increase π^*.

It can be seen from Figs. 59 and 60 that the maximum preheating gives rise to an extremely marked deviation of efficiency of the cycle from η_t. If the calculated values of η_{tf} and η_{tp^*} are compared, it can be seen that for small degrees of pressure increase η_{tf} can be many times less than η_{tp^*}.

The efficiency of the cycle η_{tf} can usually become equal to zero long before $M_1 \approx 1{\cdot}0$. Thus, for the example discussed for $\pi^* \approx 1{\cdot}52$, this occurs at $M_1 \approx 0{\cdot}66$.

Finally, for small values of M_1 ($M_1 < 0{\cdot}15 - 0{\cdot}17$) preheting at the boundaries is practically eliminated (dashed portion in Figs. 59 and 60), and therefore the likely values for η_{tf} will be higher than indicated in Figs. 59 and 60. However, it should be borne in mind that in the case of combustion chambers reheated by high velocity gas (ram-jet chambers, reheat chambers and in certain cases also the primary chambers of turbo-jet engines) we shall at all times be in the region where losses resulting from heat supply can exert a substantial influence on the efficiency of the cycle.

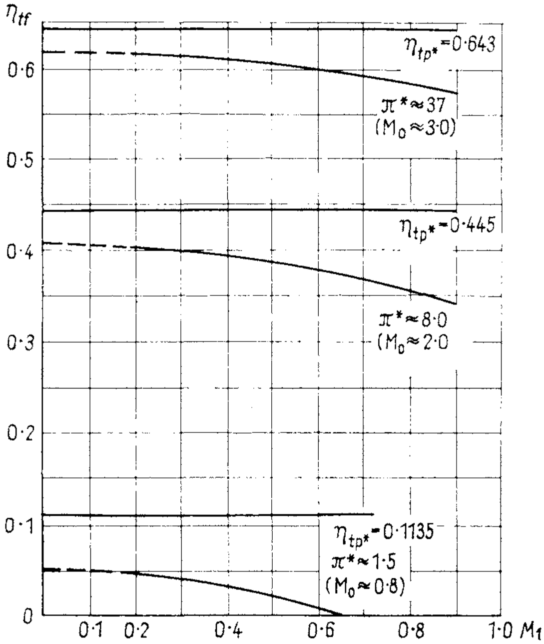

FIG. 59. Variation of efficiency of a cycle, in which supply of heat takes place in duct with F = const., as a function of the quantity M_1.

M_1	0	0·1	0·2	0·3	0·4	0·6	0·8	1·0
$\left(\frac{Q}{c_pT}\right)_{\max}$	∞	20·4	4·76	1·885	0·89	0·22	0·0374	0

We have discussed briefly the thermodynamics of aero-jet engines for certain simple cases. The choice of a much more general case undoubtedly presents practical and theoretical interest — the supply of heat to a moving gas in a duct with arbitrarily-varying cross-section. In practice it is extremely important to be able to assess the thermodynamic losses of efficiency of the cycle, the reduction of the total pressure for slightly divergent and particularly for convergent ducts.

However, in view of its complexity, this problem falls outside the scope of the present work. We shall confine ourselves here only to general observation that for convergent ducts all the phenomena accompanying heat supply are manifested in a more definite form (for divergent ducts it is the reverse). Thus for example, the phenomenon of "choking" will set in at lower preheating values, the total pressure losses for a given degree of preheating will be greater and the efficiency less than in the case when $F =$ const. is maintained.

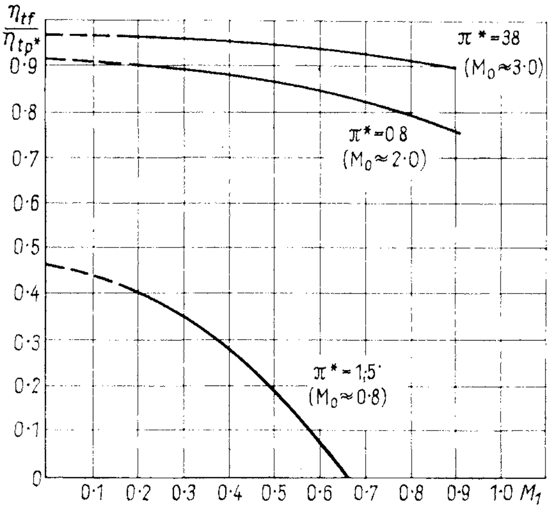

FIG. 60. Variation of the ratio η_{tf}/η_{tp^*} as a function of the quantity M_1.

In ending this section a few general conclusions will be drawn.

(1) In all cases when heat is supplied to a moving gas, a reduction will take place of the total pressure and a corresponding reduction of the work capability of the gas.

(2) Reduction of the total pressure and the corresponding loss of work capability of the gas will be greater, the greater is the heat supplied (bearing in mind that $Q \leq Q_{\max}$) and the Mach number M_1 of the gas stream.

(3) The efficiency of thermal utilization is determined only by the value of the static pressure p in the gas stream; the higher p (relative to p_0), the better will the heat be utilized; if $p = p_0$ (free flow), then the efficiency of thermal utilization will be equal to zero.

(4) For identical initial conditions and preheating, η_{tf} is always less than η_{tp^*} and η_{tp}, since if $F =$ const., then as a result of the drop in static pressure, the efficiency of each "succeeding" portion of heat worsens whilst for $p =$ const. it remains unchanged.

(5) For known conditions (e.g., $f =$ const. or an insufficiently divergent or convergent duct) it is impossible to supply as great a quantity of heat

to the moving gas as is desired, without affecting the initial parameters of the gas stream. There is a quite definite value for the relative preheating $\left(\frac{Q}{c_p T_1^*}\right)_{\max}$ for which "choking" of the chamber occurs.

(6) The magnitude of $\left(\frac{Q}{c_p T_1^*}\right)_{\max}$ depends on the shape of the duct and in the general case is decreased with increase of M_1 for $F = \text{const}$;

$$\left(\frac{Q}{c_p T_1^*}\right)_{\max} = 0, \text{ if } M_1 = 1{\cdot}0.$$

(7) It is essential for the static pressure drop in the combustion chamber, distributed within the engine, to be greater than or equal to the static pressure drop caused by the supply of a given quantity of heat, $\Delta p \geq p_1 - p_2$.

3. CALCULATION OF THE THEORETICAL COMBUSTION TEMPERATURE

Excess Air Coefficient

The determination of the temperature of combustion of fuel in industrial furnaces and in the combustion chambers of jet propulsion engines can be carried out by means of thermochemical calculations.

In order to perform these thermochemical calculations it is sufficient to know the chemical composition of the fuel, the excess air coefficient and the pressure in the combustion chamber.

In modern jet propulsion engines hydrocarbon fuels (kerosene and gasoline) are used. Chemical analysis shows that aviation kerosene consists approximately of 86 per cent carbon and 14 per cent hydrogen, by weight.

The quantity of air required for the combustion of 1 kg of fuel can easily be determined from the chemical composition.

The chemical reaction for the oxidation of carbon is written thus:

$$C + O_2 = CO_2.$$

The ratio of the weights of the substances participating in the reaction will be

$$12 + 32 = 44,$$

or,

$$1 + \frac{8}{3} = \frac{11}{3}.$$

Hence it follows that for the complete combustion of 1 kg of carbon, 2·67 kg $\left(\frac{8}{3}\text{ kg}\right)$ of oxygen are required, and as a result of combustion 3·67 kg of carbon dioxide are obtained.

The chemical reaction for the oxidation of hydrogen is

$$H_2 + O = H_2O\,.$$

Hence, the ratio of the weights will be

$$2 + 16 = 18, \quad \text{or} \quad 1 + 8 = 9\,,$$

i.e., 8 kg of oxygen are required for 1 kg of hydrogen, and as a result of combustion of 1 kg of hydrogen, 9 kg of H_2O are formed.

The minimum quantity of air necessary for the combustion of 1 kg of hydrocarbon fuel is determined by weight proportion of carbon and hydrogen. Assuming that 1 kg of air contains 0·232 kg of oxygen, we obtain:

$$L_0 = \left(\frac{8}{3}\,C + 8\,H\right)\frac{1}{0{\cdot}232}\,. \tag{V.34}$$

If oxygen is contained in the fuel, then the corresponding term with the sign "O" enters into the bracket of the formula.

For kerosene, with the composition given above, $L_0 = 14{\cdot}7$ kg/kg fuel, i.e., for the complete combustion of 1 kg of fuel, 14·7 kg of air are required.

The excess air coefficient is determined by the following relationship:

$$\alpha = \frac{G_A}{G_F L_0}\,, \tag{V.35}$$

and expresses the ratio of the actual quantity of air to the theoretical required.

Temperature of the Combustion Products

The temperature of the combustion products can be determined by compiling the equation of thermal balance.

The quantity of heat actually supplied by 1 kg of working mixture is characterized by the calorific value of the fuel H_u, the coefficient α, and the combustion coefficient ξ_z. For fuels used in jet-propulsion engines the lower calorific value of the fuel can be taken,† equal to $H_u \approx 10{,}250$ kcal/kg.

The heat released by combustion of the fuel is equal to

$$Q_{\text{rel}} = G_f\, H_u\, \xi_z\,, \tag{V.36}$$

where G_f is the quantity of fuel burned in kg/sec;

H_u is the lower calorific value of the fuel in kcal/kg;

ξ_z is the combustion efficiency.

On the other hand, the quantity of heat released is equal to the difference in enthalpy of the products of combustion and of the oxidant and the fuel admitted into the combustion chamber.

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† By the lower calorific value of a fuel is understood the total heat released by the combustion of a fuel, without taking into account the heat consumed by the formation of water vapour (steam) in the combustion products.

Since the quantity of fuel admitted into the combustion chamber of a jet-propulsion engine is small in comparison with the quantity of oxidant, then the enthalpy of the fuel can be neglected in a number of cases.

The equation for the thermal balance is written down in the following form:

$$G_f H_u \xi_z = I_3 - I_2 = G_{\text{comb}}\, cp_{\text{comb}}\, t_3 - G_a\, cp_{\text{air}}\, t_2\,, \tag{V.37}$$

where G_{comb} is the quantity of combustion products in kg/sec;
cp_{comb} is the mean specific heat of the combustion products at constant pressure in kcal/kg °C;
G_a is the quantity of air admitted into the chamber, in kg/sec;
t_3 is the temperature of the combustion products in °C;
cp_{air} is the specific heat of the air at constant pressure in kcal/kg °C;
t_2 is the temperature of the air on entry into the combustion chamber, in °C.

We can express the temperature of the combustion products from the equation of thermal balance

$$t_3 = \frac{G_f H_u \xi_z - G_a\, cp_{\text{air}}\, t_2}{G_{\text{comb}}\, cp_{\text{comb}}}. \tag{V.38}$$

For convenience of calculation we shall carry out a number of subsidiary transformations.

In place of G_f we substitute its values

$$G_f = \frac{G_a}{L_0} \tag{V.39}$$

and then from the numerator we take G_a outside a bracket.

It is not difficult to satisfy oneself that

$$G_{\text{comb}} = G_a\left(1 + \frac{1}{\alpha L_0}\right).$$

Then finally we shall have:

$$t_3 = \frac{H_u \xi_z + L_0\, cp_{\text{air}}\, t_2}{(1 + \alpha L_0)\, cp_{\text{comb}}}. \tag{V.38a}$$

It should be noted that the specific heat of the combustion products cp_{comb} is a function of the temperature and of the excess air coefficient α.

For calculating the temperature of the combustion products the value of cp_{comb} is taken from specific heat tables, assigning an approximate value for t_3. If the difference between the values of t_3 obtained by calculation and those given for a specified cp_{comb} attains a relatively large magnitude, then the calculation is repeated until the difference is found to be small.

For low values of the excess air coefficient ($\alpha = 1 - 2$) and relatively high temperatures of the air on entry into the combustion chamber, the combustion products may attain a value exceeding 2000 °C.

For temperatures of the combustion products above 2000 °C, marked variations of chemical composition, decomposition temperature and pressure of the combustion products are observed as a result of thermal dissociation.

By thermal dissociation is implied the phenomenon of decomposition of the molecules of the combustion products into more simple molecules, atoms and groups. Decomposition of the molecules takes place because at elevated temperatures the velocities of motion of the molecules are considerably increased.

In a number of cases the energy of the moving particles becomes sufficient to overcome the intra-molecular forces and the molecules dissociate.

The dissociation of the molecules of CO_2 and H_2 can serve as an example:

$$CO_2 \rightarrow CO + O;$$
$$H_2 \rightarrow H + H.$$

Simultaneously with dissociation of the molecules of the combustion products, the reverse process also takes place — the formation of molecules of the original products, called association or recombination;

$$CO + O \rightarrow CO_2;$$
$$H + H \rightarrow H_2.$$

With increase in the number of dissociating molecules, the number of collisions of dissociation products is increased, which leads to an increase in the rate of recombination.

As a result, dynamic equlibrium is established in the combustion products between the original substances and the products of their dissociation.

The ratio of the number of dissociated molecules to the number of molecules of the same substance, without taking into account dissociation, is called the degree of dissociation.

The degree of dissociation of the combustion products depends on the temperature and pressure. An increase of the temperature above 2000 °C leads to a significant increase in the degree of dissociation. Pressure exerts the reverse effect on the process of dissociation, i.e., an increase in the degree of dissociation is observed for a reduction in pressure of the combustion products.

It is well-known that molecular dissociation takes place with the absorption of heat. Thus, as a result of the process of dissociation the overall enthalpy is decreased which must be allowed for in the calculation of the temperature and the specific heat of the combustion products.

Since in the combustion chambers of gas turbine engines the discharge temperatures of the gases do not usually exceed 1000 °C, no significant reduction of enthalpy will be observed. However, for calculating local values of the temperature of the combustion products in the combustion zone of the chambers of gas turbine engines, and also at the exit from the combustion chambers of ram-jet engines and reheat chambers where the value of the temperature may be higher than 2000 °C, the phenomenon of dissociation cannot be neglected.

A brief account is given below of the method of calculation of enthalpy in the presence of dissociation of the combustion products, given in a work by A. P. Vanichev †.

In order to calculate the enthalpy in the presence of dissociation, the pressure of the gases, the composition of the combustion products (or the ultimate analysis of the fuel) and the excess air coefficient α must be known.

(1) We determine the composition of the combustion products, cooled to a temperature t_1, at which the determination of the calorific value of the fuel was carried out.

For this we assume that for $\alpha > 1$, products of total oxidation (CO_2 and H_2O) and excess oxygen, O_2, are present in the combustion products.

The weight fractions of the gases comprising the combustion products are determined by the following formulae:

$$G_{CO_2} = \frac{\frac{11}{3}\frac{C}{\alpha L_0}}{1+\frac{1}{\alpha L_0}} = \frac{\frac{11}{3}C}{\alpha L_0 + 1}; \tag{V.40}$$

$$G_{H_2O} = \frac{9\frac{H}{\alpha L_0}}{1+\frac{1}{\alpha L_0}} = \frac{9\,H}{\alpha L_0 + 1}; \tag{V.41}$$

$$G_{O_2} = \frac{0{\cdot}232 - 3{\cdot}435\frac{1}{\alpha L_0}}{1+\frac{1}{\alpha L_0}} = \frac{0{\cdot}232\,\alpha L_0 - 3{\cdot}435}{\alpha L_0 + 1}; \tag{V.42}$$

$$G_{N_2} = \frac{0{\cdot}768}{1+\frac{1}{\alpha L_0}} = \frac{0{\cdot}768\,\alpha L_0}{\alpha L_0 + 1}; \tag{V.43}$$

(2) We determine the enthalpy of the cold combustion products

$$I_1 = \sum G_i \frac{I_i}{\mu_i}, \tag{V.44}$$

where G_i is the fraction by weight of gas entering into the composition of the combustion products;

I_i is the enthalpy of the gas at temperature T_1 in kcal/mole;

μ_i is the molecular weight of the gas.

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† A. P. Vanichev, *Thermodynamic Calculation of Combustion and Flow in the High Temperature Region*, BNT, 1947.

(3) We denote the probable value of the combustion temperature by T_3 and we find the composition of the combustion products by means of the dissociation constant.

The composition of the gaseous mixture is determined primarily by the following reactions:

$$CO_2 \rightleftarrows CO + \frac{1}{2} O_2 ;$$

$$H_2O \rightleftarrows H_2 + \frac{1}{2} O_2 ;$$

$$H_2O \rightleftarrows OH + \frac{1}{2} H_2 ;$$

$$H_2 \rightleftarrows H + H ;$$

$$O_2 \rightleftarrows O + O ;$$

$$\frac{1}{2} N_2 + \frac{1}{2} O_2 \rightleftarrows NO .$$

As a result of these reactions, 10 gases can enter into the composition of the combustion products:

CO_2, H_2O, N_2, H_2, CO, O_2, OH, H, O, NO.

The partial pressure of these gases will be

p_{CO_2}, p_{H_2O}, p_{N_2}, p_{H_2}, p_{CO}, p_{O_2}, p_{OH}, p_{H}, p_{O}, p_{NO} .

In order to find the values of the stated partial pressures, 10 equations are necessary.

The expressions for the dissociation constants give six equations, well-known from chemical thermodynamics

$$\frac{p_{CO}\sqrt{p_{O_2}}}{p_{CO_2}} = K_{p_1} ; \tag{1}$$

$$\frac{p_{H_2}\sqrt{p_{O_2}}}{p_{H_2O}} = K_{p_2} ; \tag{2}$$

$$\frac{p_{OH}\sqrt{p_{H_2}}}{p_{H_2O}} = K_{p_3} ; \tag{3}$$

$$\frac{p_{H}^2}{p_{H_2}} = K_{p_4} ; \tag{4}$$

$$\frac{p_{O}^2}{p_{O_2}} = K_{p_5} ; \tag{5}$$

$$\frac{p_{NO}}{\sqrt{p_{N_2}}\sqrt{p_{O_2}}} = K_{p_6} . \tag{6}$$

We shall obtain the next three equations, comprising the equations of material balance, which express the equations for the ratios by weight of any two elements entering into the composition of the combustible mixture prior to combustion, and after combustion.

Thus, the ratio of the elements prior to combustion is calculated according to the ultimate analysis of the fuel, and the ratio of the elements after combustion according to the corresponding partial pressures.

We compile the equations of material balance for the ratios by weight of the following elements: $\frac{C}{O}, \frac{H}{C}, \frac{N}{C}$.

$$\frac{C}{\alpha L_0\, 0{\cdot}232} = \frac{12\,(p_{CO_2} + p_{CO})}{16\,(2\,p_{CO_2} + 2\,p_{O_2} + p_{CO} + p_{H_2O} + p_{OH} + p_O + p_{NO})}\,; \tag{7}$$

$$\frac{H}{C} = \frac{2\,p_{H_2O} + 2\,p_{H_2} + p_{OH} + p_H}{12\,(p_{CO_2} + p_{CO})}\,; \tag{8}$$

$$\frac{0{\cdot}768\,\alpha L_0}{C} = \frac{14\,(2\,p_{N_2} + p_{NO})}{12\,(p_{CO2} + p_{CO})}\,. \tag{9}$$

The last equation expresses the equating of the sum of the partial pressures of all the gases and the pressure in the combustion chamber:

$$\sum p_i = p_3\,. \tag{10}$$

The system of equations thus obtained enables the composition of the combustion products to be determined for a specified temperature T_3.

In order to solve the system, use is made of the method of trial and error, since it is not solvable in its explicit form. Usually three partial pressures are assigned for O_2 to check the accuracy of choice with respect to equation (10).

Knowing the composition of the combustion products we can calculate the enthalpy of the combustion products for a specified temperature T_3 with respect to the enthalpy of individual gases:

$$I = \frac{1}{\mu_{mix}} \sum I_i \frac{p_i}{p_3}\ \text{kcal/kg}\,,$$

where p_i is the "found" partial pressure of each gas;
I_i is its enthalpy in kcal/mole;

$\mu_{mix} = \sum \frac{p_i}{p_3}\mu_i$ is the apparent molecular weight of the mixture of gases.

Usually the calculation of the enthalpy of the combustion products is carried out for several values of the temperature T_3 and the variation of $I = f(T_3)$ is plotted graphically (Fig. 61).

The enthalpy of the combustion products can be determined through the calorific value of the fuel by the relationship

$$I_3 = I_1 + H_u ,$$

where I_1 is the enthalpy of the cold combustion products (at $t = 20$ °C);
H_u is the calorific value of the combustible mixture.

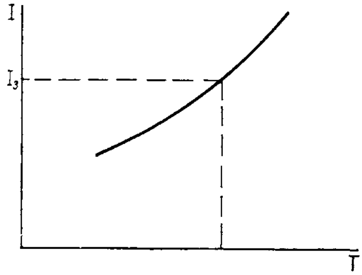

FIG. 61. Determination of the temperature of the combustion products.

According to the value found for the enthalpy I_3, and making use of the computed relationship $I = f(T)$ (see Fig. 61), we find the true value of the temperature of the combustion products.

CHAPTER VI

LOSSES IN COMBUSTION CHAMBERS

THE mechanical and thermal processes taking place in the combustion chamber, just as in any other unit of a jet engine (machines in general), are accompanied by losses which, for a given quantity of heat supplied, lead overall to a reduction in the work-potential of the gas. These losses can be divided into the following two groups:

(1) Thermal losses, which originate as a result of incomplete combustion of the fuel and the transfer of part of the heat released in the combustion chamber through the walls to the surrounding medium.

(2) Gas-dynamic losses, dependent upon the presence in the combustion chamber of aerodynamic resistances, and also upon the fact that the heat in the combustion chamber of gas turbine engines is supplied by a gas moving at a relatively high velocity.

1. THERMAL LOSSES

(a) *Incomplete Combustion*

The losses of heat due to incomplete combustion are assessed by the combustion efficiency ξ_z, represented by the relationship

$$\xi_z = \frac{Q_1}{Q_2}, \tag{VI.1}$$

where Q_1 is the heat actually released in the combustion chamber by the combustion in a certain time τ, for example in 1 sec;

Q_2 is the heat supplied by the fuel;

$$Q_2 = H_u G_f,$$

where G_f is the supply of fuel over the time τ. If $\tau = 1$ sec, then G_f is the supply per sec in kg/sec;

H_u is the lower calorific value of the fuel in kcal/kg.

The combustion efficiency is a very important characteristic of a combustion chamber.

In the modern combustion chambers of gas turbine engines, the maximum value of combustion efficiency ξ_z amounts to around 0·97–0·98; for reheat chambers and the chambers of ram-jets, ξ_z has a somewhat lower value and amounts to 0·9–0·95. The combustion efficiency for a given chamber design is a function of the chamber inlet pressure P_2, the excess air coefficient α, the temperature of the incoming air T_2 and the mean flow velocity:

$$\xi_z = f(p_2, \alpha, T_2)\,.$$

The nature of the variation of ξ_z relative to p_2 and α is shown in Figs. 62 and 63. For the relationship $\xi_z = f(p_2)$, for a constant value of α and T_2 (see Fig. 62), there is a very slight variation of ξ_z for large values of p_2 ($p_2 > 1$) and a very marked reduction for "small" values of p_2 ($p_2 \leqslant 1$ atm) is characteristic.

The variation of ξ_z with respect to α (see Fig. 63) is usually characterized by the presence of a maximum value of ξ_z; the curve of $\xi_z = f(\alpha)$ in the region of maximum may be flat or extremely steep, depending on the layout of the combustion chamber.

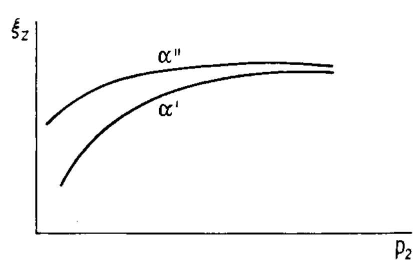

FIG. 62. Variation of the coefficient of combustion efficiency ξ_z as a function of chamber inlet air pressure.

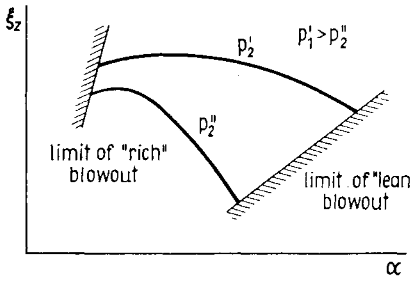

FIG. 63. Variation of the combustion efficiency ξ_z as a function of the excess air coefficient.

The magnitude of the excess air coefficient α exerts a particularly large influence on the variation of the combustion efficiency for small values of p_2. Thus for example, in the combustion chamber of a gas turbine engine, the variation of α from 3 to 8 can, for $p_2 \leqslant 1$, reduce ξ_z by more than 0·2. In general it should be borne in mind that for small pressures the combustion process becomes extremely susceptible to variation of each of the parameters affecting it.

It would be expected that the function $\xi_z = f(p_2, \alpha)$ in a quantitive relationship will differ particularly sharply for chambers for different purposes (combustion chambers of gas turbine engines, reheat chambers, chambers of ram-jet engines).

Until recently it has not been possible to establish the form of the function $\xi_z = f(p_2, \alpha)$. Empirical relationships, obtained as a result of innumerable experiments, indicate that the connection between ξ_z, p_2 and α can be approximately established by an equation of the following form:

$$\frac{\xi_z}{\xi_{z0}} = \varphi_1(p_2) + \varphi_2(\alpha, p_2), \tag{VI.2}$$

where ξ_z and ξ_{z0} are the instantaneous and maximum value of the combustion efficiency, respectively;

$\varphi_1(p_2)$ is a function determining the effect of pressure for the most favourable value of the excess air coefficient;

$\varphi_2(\alpha, p_2)$ is a function determining the effect of the mixture composition, taking into account the pressure variation.

A very rough estimate of the nature of the variation of ξ_z for the combustion chamber of a gas turbine engine as a function of p_2 ($p_2 < 1$ atm) and α can be derived by an empirical formula (VI.3), comprising the corresponding formula (VI.2) but which is appropriate only for one type of combustion chamber:

$$\frac{\xi_z}{\xi_{z0}} = 1{\cdot}025 - \left[\frac{1}{1{\cdot}3\, e^{3{\cdot}5 p_2}} + (\bar{\alpha}^2 - 1)(0{\cdot}055 - 0{\cdot}04\, p_2^2)\right], \tag{VI.3}$$

where p_2 is the pressure in the combustion chamber in atm;

$\bar{\alpha}$ is the ratio of the instantaneous excess air coefficient to its most favourable value.

In the given case, the minimum value of α is taken as the most favourable value.

The formula can be made use of for $p_2 < 1$ atm. For $p_2 > 1$ atm, the fixed value $p_2 = 1$ atm should be substituted in the formula, which expresses arbitrarily the independence of the combustion efficiency on the pressure for $p_2 > 1$ atm.

In Fig. 64 is depicted the dependence of combustion efficiency on p_2, calculated according to formula (VI.3). It can be seen from the graph, that for small values of p_2 an extremely marked reduction in the combustion efficiency can occur to $\xi_z = 0{\cdot}6$ and below; the effect of the excess air coefficient (mixture composition) as a result of this is likewise increased.

As mentioned earlier, besides p_2 and α, the temperature of the air T_2 on entry into the combustion chamber affects the combustion efficiency. By reducing the temperature, ξ_z is, as a rule, reduced.

Figure 65 shows the nature of the variation of ξ_z as a function of air temperature for the combustion chamber of a gas turbine engine of rather obsolete design. It should be borne in mind that the value of the air temperature in the combustion chambers of a number of aircraft engines under the principal operating conditions is usually quite large (starting conditions and idling conditions are not considered here), and there appears to be no factor substantially affecting the completeness of combustion. Consequently, its effect on ξ_z can, to a first approximation, be neglected.

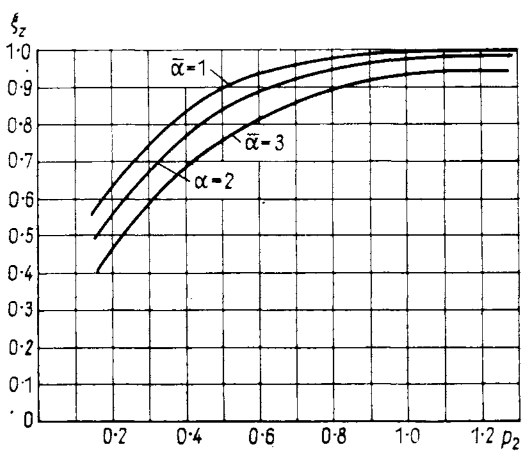

FIG. 64. Variation of combustion efficiency calculated according to the empirical formula (VI.3) as a function of chamber pressure inlet.

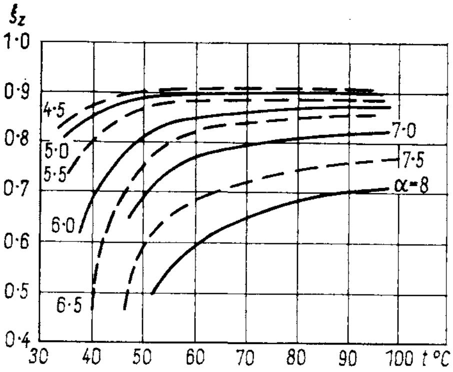

FIG. 65. Effect of temperature of air entering the combustion chamber on the value of the combustion efficiency for various values of α.

The maximum value of ξ_z and the nature of its variation with respect to p_2, α and T_2 depends, to a large extent, on the organization of the combustion process in the chamber, i.e., on the structural design of the chamber, the nature of the fuel atomization and mixing with air, intensity and position of the ignition source, the nature of entry of the air into the combustion zone, etc. By changing the structural layout of the combustion chamber it is possible to affect substantially all its characteristics, changing them in the required direction. This problem will be considered below in more detail in the description of the structural layouts of combustion chambers.

In designing an engine — calculation of its characteristics — losses due to incompleteness of combustion are always taken into account. For this, use is made of experimental data.

Usually, in assessing and designing an engine it is necessary to produce a thrust controlled by operating conditions for given values of the operating temperature. Consequently, incompleteness of combustion will only affect the specific fuel consumption, which varies inversely as the combustion efficiency. For example, if $\xi_z = 0{\cdot}95$, then this implies that C_{sp} will be greater by 5 per cent in comparison with C_{sp} for $\xi_z = 1$.

However, it should also be noted that there is a second possible case, when low combustion efficiency makes it impossible to produce the required temperature prior to the turbine even by increasing the fuel supply. As a result of this, the operation of the combustion chamber exerts an influence on a given engine not only with respect to specific consumption but also with respect to thrust (power, rev/min, etc.).

(b) *Heat Losses through the Wall*

The heat losses through the wall of the combustion chamber of a gas turbine engine to the surrounding medium are usually small. They are not separately taken into account for rating an engine, but they are included in the value of ξ_z. The combustion chambers of ram-jet and reheat chambers constitute special exceptions, operating on an excess air factor close to unity, i.e., with a high temperature of the combustion products. However, even in this case, these losses can be eliminated by a rational choice of design.

The maximum values for heat loss, other conditions being the same, are attained when the surface of the chamber is cooled only by the external air flow which does not participate in the combustion process and does not mix with the combustion products (Fig. 66).

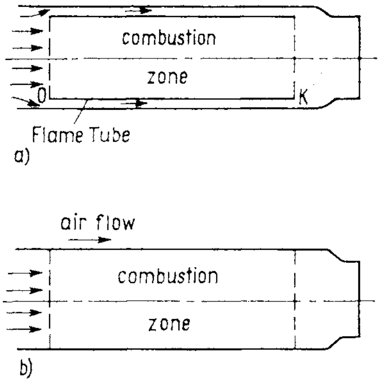

FIG. 66. Diagram of a combustion chamber with different methods of cooling the walls.

This is explained by the fact that the quantity of heat transferred through the wall to the surrounding medium is proportional to the difference in temperature of the gas flowing inside the combustion chamber and that of the external cooling air. The difference in temperature between the gas flowing through the duct *OK* and the surrounding medium in the chamber of design (a) will be less than in the chamber of design (b), in which there is no internal jacket and the temperature near the outer surface is high.

Consequently, the quantity of heat transmitted through the walls of the chamber to the surrounding medium will be greater in the chamber (b) than in chamber (a). It should be borne in mind that the installation of a cooling system decreases the heat losses but at the same time leads to an increase in the aerodynamic resistance of the combustion chamber (pressure loss).

The majority of combustion chambers for gas-turbine and ram-jet engines have "internal" (see layout (a)) cooling†. In reheat chambers, heat insulating chamber walls are used. However, in this case one has in mind not only the reduction of losses by transfer of heat to the external medium, but also the protection of the airframe against the effect of high temperatures.

It is obvious that heat losses through the walls affect the characteristics of the engine to the same extent as do losses from incompleteness of combustion.

2. GAS-DYNAMIC LOSSES

In order to improve the combustion process in the chambers of jet propulsion engines, various devices are installed which have considerable aerodynamic resistances. Thus, in the chambers of gas-turbine engines, the air usually enters the combustion zone and is mixed by injectors and orifices located in the walls of the flame tube; the ducts, along which the air and the combustion products flow, may have diffusors, recesses, etc. In reheat chambers and in ram-jet engines there are special devices (stabilizers) of badly streamlined shape, fuel manifolds, pre-combustion chambers, etc.

As a result of the flow of air along ducts with complex configurations, streamlined injectors, stabilizers, fuel manifolds, pre-combustion chambers, and also as a result of swirl and mixing of various gas streams possessing different velocities and temperatures, the dynamic pressure of the gases at the end of the combustion chamber p_3^* becomes less than the dynamic pressure p_2^* at the entry of the chamber.

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† It should be noted that for the combustion chambers of gas-turbine engines the presence of internal cooling is frequently employed not to reduce loss of heat through the wall but to distribute rationally the air along the flame-tubes, and to ensure by this the maximum efficiency of the combustion process.

In certain cases the dynamic pressure losses in the combustion chambers of jet propulsion engines may amount to 10 per cent because of aerodynamic resistance.

Dynamic pressure losses give rise to a reduction in the work capability of the gas l†.

It is obvious, that the greater is p^*, the higher is the working efficiency of the gas; for aircraft engines this means that for every kilogram of air more work is transferred at the airscrew turbine (turbo-prop engine), it increases the kinetic energy of the jet issuing from the nozzle (turbo-jet and ram-jet engines) and consequently increases the specific thrust R_{sp}. The efficiency of the engine (C_{sp}) is also improved by this.

In order to assess the characteristics of an engine, it is convenient to assess the dynamic pressure losses, wherever they may occur, by the dynamic pressure loss coefficient $\sigma^* = p_3^*/p_2^*$.

The dynamic pressure loss coefficient σ^*_{dyn} for a given chamber is a variable quantity, depending upon the velocity of the air passing through the combustion chamber. Consequently, it is inconvenient to use σ^*_{dyn} as a quantity which characterizes the aerodynamic losses.

A far more stable quantity for any given combustion chamber is the so-called aerodynamic drag ξ, it being in essence the criterion of Euler's equation $\varepsilon_u = \dfrac{\Delta p}{\varrho w^2}$. In view of the fact that the mean velocity w_2 is relatively small, ξ can be linked with the change of dynamic pressures:

$$\xi = \frac{p_2^* - p_3^*}{\dfrac{\varrho_2 w_2^2}{2}}, \tag{VI.5}$$

where p_2^* and p_3^* are the dynamic pressures at the entry to and exit from the combustion chamber, in kg/m²; $\varrho_2 w_2^2/2$ is the dynamic pressure in any characteristic section of the combustion chamber.

The convenience of application of the coefficient ξ consists in the fact that it is only relatively slightly changed, for any specified combustion chamber, as a result of a considerable change of dynamic pressure.

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† It is well-known that the adiabatic work l is a measure of the efficiency of the gas, i.e. the work that can be accomplished by 1 kg of gas as a result of reduction of the dynamic pressure p^* to the pressure of the surrounding medium p_0:

$$l = \frac{k}{k-1} RT^* \left[1 - \left(\frac{p_0}{p^*}\right)^{\frac{k-1}{k}}\right] \text{kg/kg}, \tag{VI.4}$$

where T^* is the total temperature of the gas in deg. K;
p_0 is the atmospheric pressure (surrounding medium);
p^* is the dynamic pressure of the gas.

It can be assumed that, to a first approximation, under the main operating conditions of the chamber ξ is constant.

The small effect of dynamic pressure on the value of ξ is explained by the fact that for large Reynolds numbers the flow in the chamber becomes self-determined and consequently also the pressure losses remain proportional to the dynamic pressure, just like all other aerodynamic factors.

Usually, either the entry or the maximum cross-section of the combustion chamber is taken as the characteristic section.

The aerodynamic coefficient of a combustion chamber is determined by an experimental method, usually by cold flow tests. It should be noted here that depending upon the design of the combustion chamber, the combustion process can be affected to a greater or lesser extent by its "aerodynamics", as a result of which the nature of the flow differs in general from the flow for a "cold" test. In this case, ξ will depend on the extent of preheating τ. If data concerning the relationship $\xi = f(\tau)$ are lacking, then ξ can be used for "cold" experiments, assuming that the effect of τ on ξ is insignificant, which is permissible in the majority of cases.

It is obvious that σ^*_{dyn} can be obtained easily from equation (VI.5). Actually,

$$\xi = \frac{p_2^*\left[1 - \dfrac{p_3^*}{p_2^*}\right]}{\dfrac{\varrho_2 w_2^2}{2}} = \frac{1 - \sigma^*_{\text{dyn}}}{\dfrac{\varrho_2 w_2^2}{2\, p_2^*}}$$

whence

$$\sigma^*_{\text{dyn}} = 1 - \xi \frac{\varrho_2 w_2^2}{2} \frac{1}{p_2^*}.$$

Substituting $\dfrac{\varrho w^2}{2}$ by

$$\frac{p}{gRT} \frac{w^2}{2} = \frac{k}{2} M^2 p$$

and p^* by

$$p\left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}} \approx p\left(1 + \frac{k}{2} M^2\right),$$

we obtain

$$\sigma^*_{\text{dyn}} = 1 - \xi \frac{k}{2} M_2^2 \frac{1}{\left(1 + \dfrac{k}{2} M_2^2\right)}.$$

For the determination of ξ^*_{dyn} with an accuracy sufficient for practical usage, the quantity $\frac{k}{2} M_2^2$ can be neglected†, since it is small in comparison with unity; then

$$\sigma^*_{\text{dyn}} = 1 - \frac{k}{2} M_2^2 . \qquad \text{(VI.6)}$$

Expression (VI.6) is essential for the determination of

$$\sigma^*_{\text{dyn}} = f(\xi, M_2^2).$$

In designing combustion chambers, one should always aim at reducing ξ. However, for this it should be borne in mind that the smaller ξ, the more difficult it is to arrange an efficient combustion process in a small volume. Actually, the process of combustion, roughly speaking, is more intense the greater is the blocking of the chamber by special equipment promoting flame stabilization and high completeness of combustion or, in other words, the combustion will be more intense, the larger is the value of ξ.

In the combustion chambers of jet propulsion engines, the dimensionless mean velocities relative to the centre of the chamber and to the coefficient ξ can have approximately the following values:

(a) Combustion chambers of gas-turbine engines . . . $M \approx 0{\cdot}07 - 0{\cdot}14$; $\xi = 8 - 12$;

(b) Combustion chambers of ram-jet engines . . . . $M \approx 0{\cdot}09 - 0{\cdot}15$ (sub-sonic ram-jet engines)
$M \approx 0{\cdot}15 - 0{\cdot}20$ (super-sonic ram-jet engines)

(c) Reheat chambers of turbo-jet engines . . . . . . $M \approx 0{\cdot}175$–$0{\cdot}30$.

$\xi = 1 - 4$; (for b and c)

It has already been mentioned that such a difference in the values of ξ is explained by the difference in velocities of motion of the gas through the combustion chambers of gas-turbine engines, ram-jet engines and reheat

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† Actually, for the combustion chambers of ram-jet engines, usually $M_2 < 0{\cdot}25$ and $\xi < 4$; calculating σ^*_{dyn} according to the expression for the limiting values of M and ξ, we obtain respectively:

(1) $\sigma^*_{\text{dyn}} = 1 - 4 \times 0{\cdot}7\,(0{\cdot}25)^2 \dfrac{1}{1 + 0{\cdot}7\,(0{\cdot}25)^2} = 1 - 0{\cdot}168 = 0{\cdot}832$;

(2) $\sigma^*_{\text{dyn}} = 1 - 4 \times 0{\cdot}7\,(0{\cdot}25)^2 = 1 - 0{\cdot}175 = 0{\cdot}825$.

The relative error in per cent amounts to $\dfrac{0{\cdot}832 - 0{\cdot}825}{0{\cdot}832} \cdot 100 \approx 0{\cdot}84$, i.e., less than one per cent. It is obvious that with decrease of M, the accuracy of the expression increases.

chambers. Actually, the dynamic pressure losses for $M = 0{\cdot}2$ and $\xi = 10$ will attain 28 per cent ($\sigma^*_{\text{dyn}} = 1 - 0{\cdot}7 \times 10 \times 0{\cdot}2^2 = 0{\cdot}72$), whereas for $\xi = 2$, the dynamic pressure losses amount to ~5–6 per cent in all ($\sigma^*_{\text{dyn}} = 1 - 0{\cdot}7 \times 0{\cdot}2^2 = 0{\cdot}946$). By decreasing M, the large value of ξ does not lead to large dynamic pressure losses. In fact, for $M = 0{\cdot}07$ and $\xi = 10$, the dynamic pressure losses amount in all to around 4 per cent ($\sigma^*_{\text{dyn}} = 1 - 0{\cdot}7 \times 10 \times 0{\cdot}07^2 = 0{\cdot}965$).

It follows immediately from formula (VI.6) that for small values of M, relatively large values of ξ can be permitted without the fear of producing considerable dynamic pressure losses.

Figure 67 depicts the relationship $\sigma^*_{\text{dyn}} = f(M)$ for $\xi = 3{\cdot}0$ and $\xi = 10$, illustrating the case under discussion.

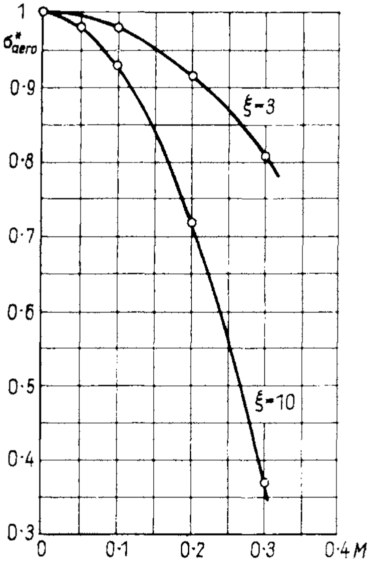

FIG. 67. Variation of dynamic pressure loss as a function of Mach number at the intake.

Figure 68 shows the effect of σ^*_{dyn} on the characteristics of ram-jet engines and turbo-jet engines for different flight velocities and values of the coefficients of pressure recovery in the engine diffuser σ_d and in the nozzle σ_n, efficiency etc., typical for these engines.

It is important here to note two cases.

(1) σ^* exerts a very strong influence on the characteristics of ram-jet engines for $M_0 < 1{\cdot}0$ (a variation of σ^*_{dyn} by 0·01 corresponds to ~ 2 per cent or more variation of R_{sp} or C_{sp}).

(2) A considerably smaller influence of σ^*_{dyn} on the characteristics of ram-jet engines is observed for $M_0 > 1{\cdot}0$ [variation of σ^*_{dyn} by 0·01 corresponds to not more than 0·2–0·4 per cent variation of $R_{sp}(C_{sp})$; for turbo-jet engines the variation of specific thrust correspondingly will be equal to

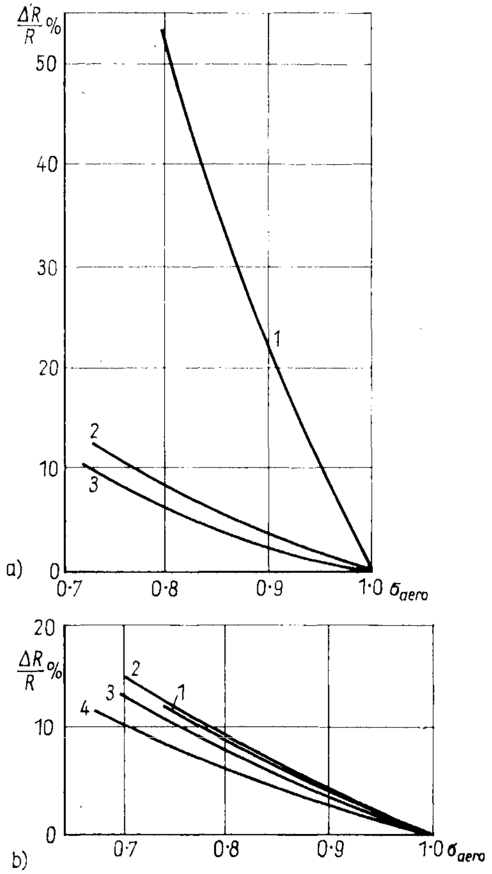

FIG. 68. Effect of aerodynamic losses on the characteristics of jet propulsion engines.

(a) — Ram-jet engine.

1 — $M_0 = 0{\cdot}9$; $T_3^* = 2000$ °K;
2 — $M_0 = 3{\cdot}0$; $T_3^* = 1500$ °K;
3 — $M_0 = 3{\cdot}0$; $T_3^* = 2000$ °K.

(b) — Turbojet engine.

1 — $M_0 = 0{\cdot}9$; $\pi^*_{K_0} = 6{\cdot}0$; $T_3^* = 1200$ °K;
2 — The same with a reheat chamber $\Delta T_F = 0°$;
3 — The same with a reheat chamber $T^*_{4F} = 2000$ °K;
4 — $M_0 = 2{\cdot}0$; $\pi^*_{K_0} = 6{\cdot}0$; $T_3^* = 1200$ °K; $T^*_{4F} = 2000$ °K.

$\sim$ 0·35–0·45 per cent depending on the operating conditions and the flight velocity]†.

Hence it follows that it is necessary to aim at obtaining a possibly greater value of σ^*_{dyn} for ram-jet engines, calculated on a flight at $M_0 < 1{\cdot}0$.

If it be assumed that $\Delta R_{sp}/R_{sp}$ should not be greater than 5 per cent, then for our case $\sigma^*_{\text{dyn}} \leq 0{\cdot}97$.

Such a value for σ^*_{dyn} can be obtained, for example, for $M_2 = 0{\cdot}06$ and $\xi = 10$, or for $M_2 = 0{\cdot}12$ and $\xi = 3{\cdot}0$. The first value does not fit since it leads to inadmissible transverse dimensions for the engine.

Hence it becomes apparent why the combustion chamber of a sub-sonic ram-jet engine should have (and has) a relatively small value of ξ.

If we take a ram-jet engine, calculated for flight at supersonic velocities, then it can be said that for "small" Mach number M_2 †† in the chamber ($M_2 \leq 0{\cdot}1$), large values of ξ should not be feared. Thus for example, for $M_0 = 3{\cdot}0$, the variation of ξ from 3·0 to 10 for $M_2 = 0{\cdot}08$ decreases R_{sp} by less than 5 per cent.

Assuming that for large values of ξ completeness of combustion is more easily attained, the possibility should be considered in this case ($M_2 < 0{\cdot}1$) of using combustion chambers with large values of ξ in supersonic ram-jet engines.

If the Mach number M_2 is large in the combustion chamber ($M_2 \leq 0{\cdot}15$–$0{\cdot}20$) and $\Delta R_{sp}/R_{sp}$ must not exceed 2–3 per cent, then it is possible to attain this only for small values of ξ. Assuming that $\xi = 3{\cdot}0$ and $M_2 = 0{\cdot}2$, we obtain $\Delta R_{sp}/R_{sp} = 1{\cdot}5$–$2{\cdot}5$ per cent; further, if $\xi = 10$, then $\Delta R_{sp}/R_{sp} = 10$–$13$ per cent. Taking $M_2 = 0{\cdot}15$, we shall have correspondingly $\Delta R_{sp}/R_{sp} \approx 1$ per cent for $\xi = 3$ and $\Delta R_{sp}/R_{sp} = 5$–6 per cent for $\xi = 10$.

Hence, the conclusion can be drawn that the use of combustion chambers with large values of ξ ($\xi \geq 10$) is possible in principle for supersonic ram-jet engines with a low velocity in the combustion chamber ($M_2 \leq 0{\cdot}1$).

An increase of ξ for ram-jet engines is advantageous only in the case which leads to a corresponding increase of ξ_z or to an extension of the range of stable operation etc., i.e. in those cases when it is required to improve any operating characteristics of the combustion chamber.

In order to make comparison possible, we shall make a few remarks concerning the value ξ for combustion chambers of gas-turbine engines. In a gas-turbine engine it is essential that complete combustion be ensured in short chambers ($l = 500$–600 mm), since the chamber determines the distance between the turbine and the compressor, which cannot be made larger

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† The figures quoted for the variation of R_{sp} for variations of $\Delta\sigma_{\text{dyn}} = 0{\cdot}01$ refer mainly to the range $\sigma^* \geq 0{\cdot}8$.

†† Greater transverse dimensions of the engine are permitted.

(otherwise a considerable increase in weight and complexity of design of the chamber is inevitable). This is possible only in the case when the combustion chamber will have a large ξ and a relatively small air stream velocity (the latter requirement is brought about by attempting to ensure the necessary residence time of the fuel in the chamber for its complete combustion).

Owing to the presence of the compressor, which compresses the air, relatively low velocities ($M_2 < 0{\cdot}1$) are produced in the combustion chamber of a gas-turbine engine without increasing the transverse dimensions of the engine which are defined by the turbine or by the compressor.

It can be seen from the examples given that the combination of large values of ξ ($\xi \geq 10$) and low velocities ($M_2 < 0{\cdot}1$) is permissible since this does not lead to a significant worsening of the engine characteristics. Thus for example, if $\xi = 10$ and $M_2 = 0{\cdot}08$, then $\Delta R_{sp}/R_{sp}$ amounts in all to 1–2 per cent.

Figure 69 depicts a diagram showing values of ξ which do not lead to a significant deterioration of engine characteristics and which are typical for the combustion chambers of jet-propulsion engines. The cross-hatched region between the rectangles denotes deviations of the values of ξ possible in isolated cases for gas-turbine and ram-jet engines.

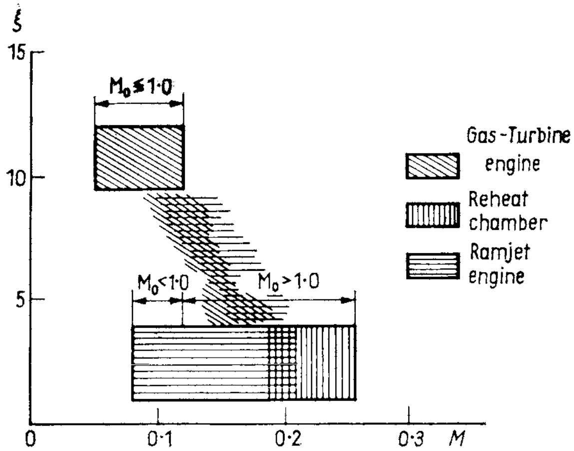

FIG. 69. Values of the aerodynamic drag coefficient ξ for combustion chambers of jet propulsion engines.

3. THERMODYNAMIC LOSSES

It was shown above that the supply of heat to a moving gas is accompanied by a reduction in the dynamic pressure and consequently also a reduction in its efficiency. Thermodynamic losses can be assessed by various methods.

Because of the ease of calculation of the engine characteristics and the simplicity of its experimental determination, thermodynamic losses in a combustion chamber will, in future, be defined by the dynamic pressure coefficient

$$\sigma_T = \frac{p_3^*}{p_{2\,T}^*},$$

where p_3^* is the dynamic pressure at the outlet of the combustion chamber (after preheating);

p_{2T}^* is the dynamic pressure of the flow prior to supply of heat.

Let us choose three cases:

(1) A conical chamber in which the static pressure remains constant ($p_3 = p_2$);

(2) A cylindrical chamber ($F_{\text{throat}} = \text{const.}$);

(3) A chamber with arbitrary variation of cross-section.

In all cases we shall assume that, for known initial data determining σ_T, the conditions at the intake (prior to preheating), for example, main flow rate, pressure, temperature etc., and also the preheating, are given either by the excess air coefficient or by the increase of temperature ΔT^*, or by the temperature after preheating T_3^*, etc. Thus, the problem consists of the fact that σ_T is determined with respect to the known flow parameters at the intake and with respect to preheating.

The well-known relationship

$$\sigma_T = \frac{p_3^*}{p_{2\,T}^*} = \frac{p_3\left(1 + \frac{k_3 - 1}{2} M_3^2\right)^{\frac{k_3}{k_3 - 1}}}{p_{2T}\left(1 + \frac{k_2 - 1}{2} M_{2\,T}^2\right)^{\frac{k_2}{k_2 - 1}}}; \qquad \text{(IV.7)}$$

will always serve as the initial formula for σ_T.

The solution of the problem will reduce to obtaining the relationships

$$\frac{p_{2\,T}}{p_3} = f(M_{2T}, \tau)$$

and

$$M_3 = \varphi(M_{2T}, \tau)\ ,$$

where τ is the relative preheating.

Conical Chamber with $p = const.$

Figure 70 shows a diagram of a combustion chamber and the nature of the variation of the gas parameters along the chamber. The nature of the variation of the parameters follows from the thermodynamic characteristics of the process, concerning which mention was made above.

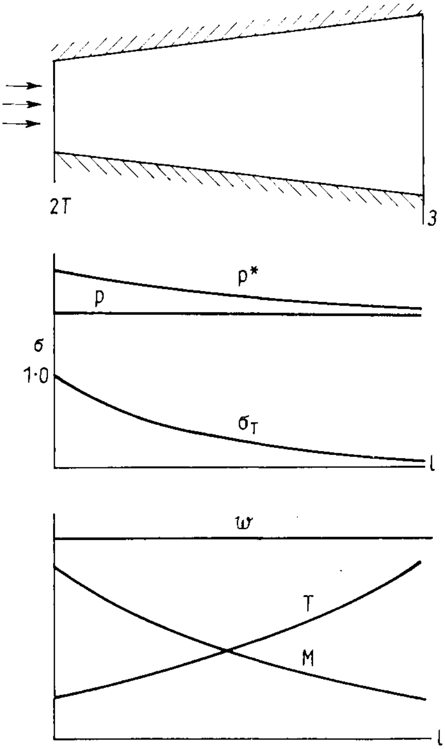

FIG. 70. Nature of variation of gas parameters along a conical chamber with the supply of heat, for p = const.

In the given case, expression (VI.7) takes the following form:

$$\sigma_T = \frac{\left(1 + \frac{k_3 - 1}{2} M_3^2\right)^{\frac{k_3}{k_3 - 1}}\dagger}{\left(1 + \frac{k_{2T} - 1}{2} M_{2\,T}^2\right)^{\frac{k_{2T}}{k_{2T} - 1}}}. \tag{VI.8}$$

It should be noted that expression (VI.8) can be simplified for many cases of practical interest.

The compressibility of the gas after preheating can be neglected with a high degree of accuracy. Then,

$$\sigma_T = \frac{1 + \frac{k_3}{2} M_3^2}{\left(1 + \frac{k_{2T} - 1}{2} M_{2T}^2\right)^{\frac{k_{2T}}{k_{2T} - 1}}}. \tag{VI.9}$$

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† Static pressures are maintained, since in our case $p_{2\,T} = p_3$.

In order to determine the Mach number M_3 relative to M_2, and the preheating (T_3^*/T_{2T}^*) it should be borne in mind that in a conical chamber as a result of maintaining the condition $p_{2T} = p_3$, as was mentioned earlier, equality of velocities occurs, $w_{2T} = w_3$. Assuming that the gas constants of the fuel-air mixture and of the combustion products are equal, we shall have

$$M_3^2 k_3 T_3 = M_{2T}^2 k_{2T} T_{2T}$$

or

$$\frac{M_3^2}{\left(1 + \frac{k_3 - 1}{2} M_3^2\right)} = \frac{k_{2T}}{k_3} \frac{T_{2T}^*}{T_3^*} \frac{M_{2T}^2}{\left(1 + \frac{k_{2T} - 1}{2} M_{2T}^2\right)}. \tag{VI.10}$$

By carrying out elementary transformations, we obtain

$$M_3^2 = \frac{\frac{k_{2T}}{k_3}\left(\frac{T_{2T}^*}{T_3^*}\right) \frac{M_{2T}^2}{\left(1 + \frac{k_{2T} - 1}{2} M_{2T}^2\right)}}{1 - \frac{k_3 - 1}{2} \frac{k_{2T}}{k_3}\left(\frac{T_{2T}^*}{T_3^*}\right) \frac{M_{2T}}{\left(1 + \frac{k_{2T} - 1}{2} M_{2T}^2\right)}}. \tag{VI.10a}$$

In view of the fact that M_3 for a conical chamber with $p_{2T} = p_3$ has, as a rule, an extremely small value (for example $M_3 < 0{\cdot}1-0{\cdot}5$), then it can be assumed with a high degree of accuracy that

$$M_3^2 \approx \frac{k_{2T}}{k_3} \frac{T_{2T}^*}{T_3^*} \frac{M_{2T}^2}{\left(1 + \frac{k_{2T} - 1}{2} M_{2T}^2\right)}.\dagger \tag{VI.11}$$

It should be noted that better results for determining σ_T can frequently be obtained by completely neglecting compressibility (for example if $M_{2T} \leq 0{\cdot}5$); then in place of (VI.8) we can write

$$\sigma_T \approx \frac{1 + \frac{k_3}{2} M_3^2}{1 + \frac{k_{2T}}{2} M_{2T}^2}; \tag{VI.12a}$$

as a result of this, the values obtained for σ_T are somewhat excessive.

For approximate estimates of σ_T it is possible to simplify the expression for σ_T even further, particularly in those cases when the value of $\frac{k_3}{2} M_3^2$

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† Expression (VI.11) follows immediately from the formula (VI.10) for $\left(1 + \frac{k_3 - 1}{2} M_3^2\right) \approx 1$.

can be (for example for $M_3 \leq 0{\cdot}12 - 0{\cdot}125$) small in comparison with unity. In this case it can generally be neglected. If, as a result of this, one allows for the fact that the denominator of expression (VI.12a) has a somewhat lower value than $\left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$, then the limit of the value of M_3, for which the value of $\frac{k_3}{2} M_3$ can be neglected for determining σ_T is somewhat extended. Thus, for $M_3 \leq 0{\cdot}15 - 0{\cdot}16$, the magnitude of the error in determining σ_T will not exceed 1 per cent.

Thus, the expression for σ_T reduces to the following form:

$$\sigma_T \approx \frac{1}{1 + \frac{k_{2T}}{2} M_{2T}^2}. \tag{VI.12b}$$

It should be noted that conical combustion chambers, in the case of aircraft application, should have high intake velocities (otherwise they will have intolerable exhaust dimensions).

Therefore, in determining σ_T expressions (VI.12a) and (VI.12b) should be used with discretion.

For comparison of the results obtained by calculating σ_T according to the various expressions, we shall determine σ_T in a conical combustion chamber for $p = \text{const}$ and for the following initial data:

$T_{2T}^* = 350$ °K; $T_3^* = 1400$ °K; $M_{2T} = 0{\cdot}5$; fuel of kerosene type.

(1) *Exact solution.*

We determine M_3^2 according to expression (VI.10); for this we find the values of k_{2T} and k_3 from the tables; thus, for air† with $T_{2T}^* \approx 350$ °K, $k_{2T} \approx 1{\cdot}4$ and for the combustion products with $\Delta T = 1050°$ and $T_3^* \approx$ $\approx 1400°$ K, $k_3 = 1{\cdot}295$,

$$M_3^2 = \frac{\frac{k_{2T}}{k_3}\left(\frac{T_{2T}^*}{T_3^*}\right) M_{2T}^2 \Big/ \left(1 + \frac{k_{2T}-1}{2} M_{2T}^2\right)}{1 - \frac{k_3 - 1}{2} \frac{k_{2T}}{k_3}\left(\frac{T_{2T}^*}{T_3^*}\right) M_{2T} \Big/ \left(1 + \frac{k_{2T}-1}{2} M_{2T}^2\right)}$$

$$= \frac{\frac{1{\cdot}4}{1{\cdot}295} \frac{350}{1400} 0{\cdot}5^2 \Big/ \left(1 + \frac{0{\cdot}4}{2} 0{\cdot}5^2\right)}{1 - \frac{1{\cdot}295 - 1}{2} \frac{1.4}{1{\cdot}295} \frac{350}{1400} 0{\cdot}5^2 \Big/ \left(1 + \frac{1{\cdot}4 - 1}{2} 0{\cdot}5^2\right)}$$

$$= \frac{0{\cdot}0643}{0{\cdot}936} = 0{\cdot}0688 .$$

---

† We shall neglect the change of k_{2T} associated with the presence of fuel vapour.

According to expression (VI.8)

$$\sigma_T = \frac{\left(1+\frac{k_3-1}{2}M_3^2\right)^{\frac{k_3}{k_3-1}}}{\left(1+\frac{k_{2T}-1}{2}M_{2T}^2\right)^{\frac{k_{2T}}{k_{2T}-1}}} = \frac{(1+0\cdot148\times0\cdot0688)^{1\cdot4}}{(1+0\cdot2\times0\cdot5^2)^{3\cdot5}} = \frac{1\cdot045}{1\cdot186} = 0\cdot88\,.$$

(2) *Approximate solution according to formula* (VI.9).

$$\sigma_T = \frac{1+\frac{k_3}{2}M_3^2}{\left(1+\frac{k_2-1}{2}M_{2T}^2\right)^{\frac{k_{2T}}{k_{2T}-1}}} = \frac{1+\frac{1\cdot295}{2}0\cdot0688}{\left(1+\frac{1\cdot4-1}{2}0\cdot5^2\right)^{3\cdot5}} = 0\cdot88\,.$$

As we can see, the solution according to formula (VI.8) and according to the approximate expression (VI.9) are in agreement.

(3) *Approximate solution according to formulae* (VI.11) *and* (VI.12a).

It follows from the foregoing that the determination of M_3^2 according to expression (VI.11) gives $M_3^2 = 0\cdot0643$ and

$$\sigma_T = \frac{1+\frac{k_3}{2}M_3^2}{1+\frac{k_{2T}}{2}M_{2T}^2} = \frac{1+0\cdot648\times0\cdot0643}{1+0\cdot70\times0\cdot5^2} \approx 0\cdot89\,.$$

The result obtained is considered to be good.

The determination of σ_T according to expression (VI.12b) gives

$$\sigma_T = \frac{1}{1+\frac{k_{2T}}{2}M_{2T}^2} = \frac{1}{1+0\cdot70\times0\cdot5^2} = 0\cdot85\,.$$

Thus, the use of expression (VI.12b), as would be expected, gives poor accuracy in the determination of σ_T for large values of M_{2T}.

In conclusion we shall make the following remarks. All the calculated expressions are written on the assumption that $k_3 \neq k_{2T}$; however, if it is borne in mind that the determination is only a numerical value for σ_T, then no large error should be made if it is assumed that $k =$ const. (for example $k = 1\cdot4$).

In order to verify the justification for such an assumption, expressions (VI.11) and (VI.12a) were used in determining σ_T, and it was assumed in them that $k_3 = k_{2T} = 1\cdot4$ for the conditions of the previous example.

We have

$$M_3^2 = \left(\frac{T^*_{2T}}{T^*_3}\right)\frac{M^2_{2T}}{1+\dfrac{k_{2T}-1}{2}M^2_{2T}} = \frac{350\times 0{\cdot}5^2}{1400\,(1+0{\cdot}2\times 0{\cdot}5^2)} = 0{\cdot}0595\,;$$

$$\sigma_T = \frac{1+\dfrac{k}{2}M_3^2}{1+\dfrac{k}{2}M_2^2} = \frac{1+0{\cdot}7\times 0{\cdot}0595}{1+0{\cdot}7\times 0{\cdot}5^2} = 0{\cdot}89\,,$$

which is in complete agreement with the result obtained previously.

It should be noted that by assuming $k_3 = k_{2T}$ for determining M_3^2 we introduce a considerable error ($M_3^2 = 0{\cdot}0595$ in place of $M_3^2 = 0{\cdot}0643$).

Cylindrical Combustion Chamber $F_{\text{throat}} = const.$

In Fig. 71 are depicted diagrams of a combustion chamber and the nature of the variation of the parameters along its length. Just as in the previous case, the nature of the variation of the gas parameters has already been explained.

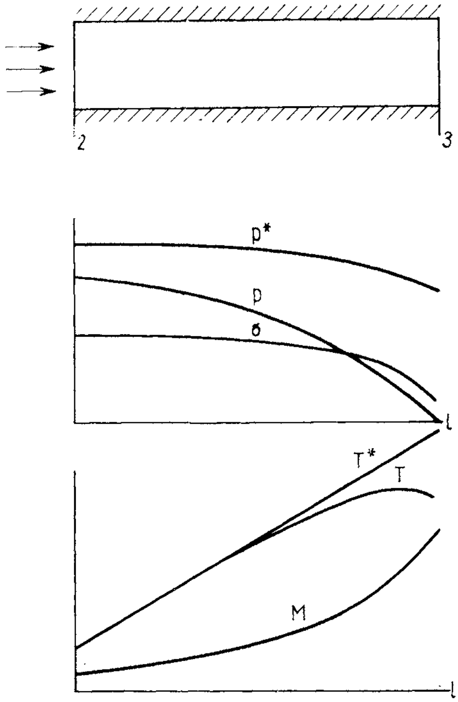

FIG. 71. Nature of variation of gas parameters along a cylindrical combustion chamber.

The relationships $\dfrac{p_{2T}}{p_3} = f(M_{2T}, \tau)$ and $M_3 = \varphi(M_{2T}, \tau)$, essential for determining σ_T according to equation (VI.7) are, as is well known, expressed by the formulae:

$$\frac{p_{2T} - p_3}{p_{2T}} = \overline{\Delta p} = \frac{k_{av}}{k_{av}+1}\left(1 - \frac{k_2}{k_{av}} M_{2T}^2\right) -$$

$$-\sqrt{\left[\left(\frac{k_{av}}{k_{av}+1}\right)\left(1 - \frac{k_2}{k_{av}} M_{2T}^2\right)\right]^2 - 2\frac{k_2}{k_2-1}\left(\frac{k_{av}-1}{k_{av}+1}\right) M_{2T}^2 \left(1 + \frac{k_2-1}{2} M_{2T}^2\right)\frac{Q}{c_{p_2} T_2^*}}$$

and

$$M_3^2 = M_{2T}^2 \frac{(\overline{\Delta p}/k_2 M_{2T}^2) + 1}{(1 - \overline{\Delta p})} \frac{k_2}{k_3}.$$

Equation (VI.7), after the appropriate substitutions, takes the following form:

$$\sigma_T = (1 - \overline{\Delta p}) = \frac{\left[\left(1 + \frac{k_3 - 1}{2}\frac{k_2}{k_3} M_{2T}^2\right)\left(\frac{(\overline{\Delta p}/k_2 M_{2T}^2) + 1}{(1 - \overline{\Delta p})}\right)\right]^{\frac{k_3}{k_3-1}}}{\left(1 + \frac{k_2 - 1}{2} M_{2T}^2\right)^{\frac{k_2}{k_2-1}}}. \quad \text{(VI.13)}$$

For convenience in calculating σ_T, graphs (Figs. 72 and 73) are constructed for a cylindrical chamber, calculated by taking into account the variation of the adiabatic index k for an initial temperature of $T_{2T} = 600$ °K.

In view of the fact that the variation of T_{2T} and k do not exert a substantial effect on σ_T, then the graphs, as given, can be used with sufficient practical accuracy and for a value of T_{2T} differing from 600 °K.

Figure 72 shows the variation of static pressure along the combustion chamber as a function of M_{2T} and the relative preheating $Q/c_p\, T_2^*$. Figure 73 shows the variation of σ_T as a function of the same parameters.

In order to explain the method of using the graph, let us solve an example.

EXAMPLE. To determine σ_T for a combustion chamber, if $F_{\text{throat}} = \text{const}$; $M_{2T} = 0{\cdot}20$; $T_{2T} = 500$ °K; $T_3^* = 2000$ °K; Fuel — kerosene.

In order to solve the problem we find the value of the parameter $Q/C_p\, T_2^*$.

The quantity of heat supplied can be calculated from the expression

$$Q = C_{p_3} T_3^* - C_{p_{2T}} T_{2T}^*,$$

where C_{p_3} and $C_{p_{2T}}$ are respectively the mean specific heats of the combustion products and of the air within the temperature range from zero to T_3^* and from zero to T_{2T}^*. The mean specific heats can be determined by calculation (according to the appropriate formulae) or from tables by means of the known initial (T_{2T}^*) and final (T_3^*) temperatures.

In our case $C_{p_3} \approx 0{\cdot}288$ and $C_{p_2} \approx 0{\cdot}240$. Thus,

$$Q = 0{\cdot}288 \times 2000 - 0{\cdot}240 \times 500 = 456 \text{ kcal/kg}.$$

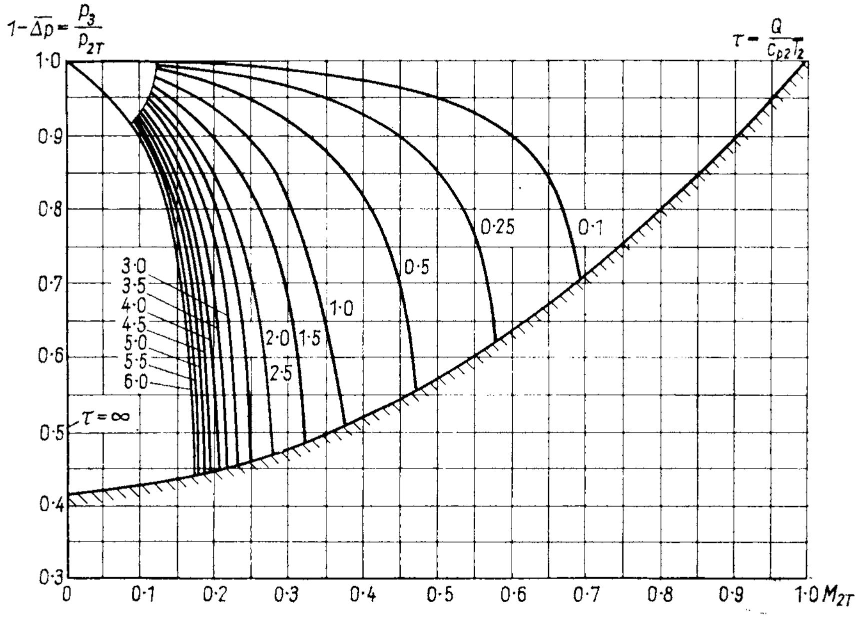

FIG. 72. Variation of static pressure along a cylindrical combustion chamber as a function of flow velocity prior to supply of heat.

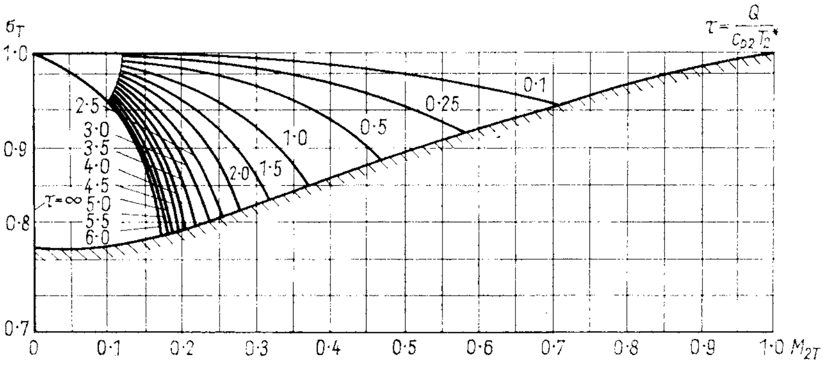

FIG. 73. Variation of dynamic pressure loss coefficient σ_T as a function of Mach number M_{2T}.

Estimating $C_{p_{2T}} \approx 0{\cdot}245$, we obtain $\frac{Q}{C_{p_2} T^*_{2T}} = \frac{456}{0{\cdot}245 \times 300} = 3{\cdot}7$.

We find from Fig. 73 that to the Mach number $M_{2T} = 0{\cdot}20$ and $\frac{Q}{C_{p_2} T^*_{2T}} = 3{\cdot}7$ corresponds $\sigma_T = 0{\cdot}85$.

If we had not taken into account the variation of specific heat, then we should have obtained

$$\frac{Q}{C_p\,T^*_{2T}} = \frac{C_p\,\varDelta T^*}{C_p\,T^*_{2T}} = \frac{2000-500}{500} = 3{\cdot}00\,,$$

whence $\sigma_T = 0{\cdot}89$, i.e., a considerable overestimate (by $\sim$5 per cent).

It is obvious that if for a given M_{2T} and τ it is not possible to find the corresponding value of σ_T in Fig. 73, then this signifies that the preheating τ is impossible for this value of M_{2T}. For example, it is impossible for $M_{2T} = 0{\cdot}25$ to have τ greater than $\approx 2{\cdot}5$.

Combustion Chamber with Variable Cross-section

This case is the most usual, but at the same time it is also the most complex. One is unsuccessful in solving the problem of calculating the parameters of the preheated gas and the dynamic pressure losses in a final form, because the system of equations obtained is not integrable, in quadratic form.

However, an approximate solution is possible. One of the possible methods of approximate solution is based on the substitution of the conical chamber by a "stepped" one (Fig. 74).

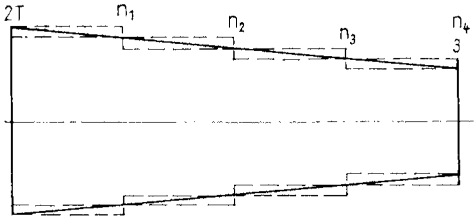

FIG. 74. Determination of the parameters in a combustion chamber of variable cross-section.

It is obvious that knowing the law of supply of heat along (or through a section of) the chamber all the flow parameters can be determined at each cylindrical section. In order to make it possible to judge the accuracy of the solution, it should be considered as a circumscribed as well as an inscribed stepped chamber. It is obvious that the flow parameters in the conical chamber will lie intermediate between the flow parameters obtained for the circumscribed and inscribed version of the stepped chamber. In order to determine σ_T with a linear law of heat supply longitudinally it is sufficient to divide up the chamber into 6–8 cylindrical sections.

It should be noted that in a convergent chamber the limiting preheating is reduced (in comparison with a cylindrical chamber) and the chamber, for given flow parameters (M_{2T}, T_{2T}), at the intake, cannot absorb that quantity of heat which it is necessary to impart to the gas in order to ensure the required engine characteristics.

It is obvious that in this case the quantity of heat Q_n, which should be supplied at the n-th section of the combustion chamber will be greater than the limiting preheating $Q_{n_{\max}}$ or, in other words, the relationship

$$\frac{Q_n}{c_{p_n} T^*_{2n}} > \left(\frac{Q_{n_{\max}}}{c_{p_n} T^*_{2n}}\right) \approx \frac{(1 - M_n^2)^2 \left(\dfrac{k+1}{k-1}\right)}{2\, M_n^2 \left(1 + \dfrac{k-1}{2} M_n^2\right)},$$

will hold, where Q_n is the heat supplied at the n-th section;

c_{p_n}, T^*_n and M_n are the specific heat, total temperature and Mach number at the beginning of the n-th section.

In order to illustrate this assumption, Fig. 75 depicts the relationship $Q_{\max} = f(n)$ for a conically convergent chamber with the area ratios $F_3/F_2 = 2{\cdot}0$; $T^*_{2\,T} = 500$ °K; $M_{2T} = 0{\cdot}2$. The law of heat supply was taken to be linear, except for the last section in which Q_n was determined from limiting conditions (maximum possible preheating).

It can be seen from Fig. 75 that the maximum quantity of heat which it is possible to absorb in a convergent chamber amounts to $\sim$150 kcal instead of 600 kcal for a cylindrical chamber with an area equal to F_2.

From the disposition of the curves in Fig. 75 it follows also that in order to determine the maximum preheating (with the required degree of accuracy) it is necessary to divide the cylindrical chamber into a considerable number of elements (in the given case – 8).

Attention should be paid to the fact that the calculation with respect to the inscribed cylindrical elements gives a considerably better approximation. This fact can be utilized for calculating the losses in convergent combustion chambers by carrying out the calculation only for the inscribed stepped chamber for $n = 6$–8 bearing in mind that the error in determining $Q_{\max}$ will amount to $\sim$5–6 per cent and that the error in determining σ_T is approximately 1 per cent.

To conclude the present section, we shall carry out a number of calculations of the losses in a stepped chamber.

The dynamic pressure loss coefficient in a stepped chamber is defined as the product

$$\sigma_T = \sigma_{T_1} \sigma_{T_2} \ldots \sigma_{T_n},$$

where $\sigma_{T_1}, \sigma_{T_2}, \ldots, \sigma_{T_n}$ are the dynamic pressure loss coefficients in the corresponding cylindrical elements.

It is wellknown that in order to determine σ_T it is essential to know M and T^* at the inlet to each cylindrical section and the quantity of heat Q

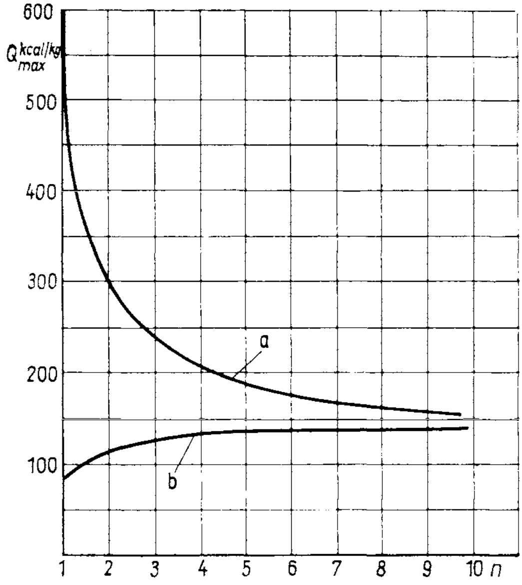

FIG. 75. Maximum preheating in a chamber, determined by the method of substitution of the variable cross-section by n-cylindrical sections.

a — $Q_{\max}$ determined with respect to circumscribed cylindrical sections;

b — $Q_{\max}$ determined with respect to inscribed cylindrical sections.

supplied to the section. By dividing the combustion chamber into n cylindrical sections, and knowing the law of heat supply along the combustion chamber and also the total quantity of heat absorbed by the gas Q_{Σ}, the quantity of heat supplied to each cylindrical element can be determined. In the case of a linear law (longitudinal) of heat supply and a uniform length of the sections

$$Q_n = \frac{Q_{\Sigma}}{n}.$$

In the case of the circumscribed conical chamber, the Mach number $M_{2(1)} = M_{2T}$ and $T^*_{2(1)} = T^*_{2T}$, for the first section. Knowing Q_n, the flow parameters can be calculated by the expression presented above at the end of the 1st section necessary for determining σ_{T_1}. For conversion to the calculation for the 2nd section it is necessary to know $M_{2(2)}$ and $T^*_{2(2)}$.

Assuming that transition from section F_{n_1} to section F_{n_2} takes place adiabatically, we obtain $T^*_{3(1)} = T^*_{2(2)}$, and $M_{2(2)}$ is found from the expression

$$\frac{\left(1+\frac{k-1}{2}M^2_{3(1)}\right)^{\frac{k+1}{2(k-1)}}}{M_{3(1)}} = \frac{\left(1+\frac{k-1}{2}M^2_{2(2)}\right)^{\frac{k+1}{2(k-1)}}}{M_{2(2)}}\frac{F_{n_2}}{F_{n_1}}. \quad \text{(VI.14)}$$

Assuming that $M_{3(1)}$ and $M_{2(2)}$ for a small change of area approximate to one another, it can be assumed, with a sufficient degree of accuracy, that

$$\frac{1+\frac{k+1}{4}M^2_{3(1)}}{M_{3(1)}} = \frac{1+\frac{k+1}{4}M^2_{2(2)}}{M_{2(2)}}\frac{F_{n_2}}{F_{n_1}}. \quad \text{(VI.15)}$$

$M_{2(2)}$ is determined graphically from the expressions presented.

Thus, the initial data are obtained for calculating the second section and, correspondingly, for all the remaining sections.

4. RELATIONSHIP BETWEEN AERODYNAMIC AND THERMODYNAMIC LOSSES

The supply of heat usually leads to an increase in aerodynamic losses (σ_{aero}) in the combustion chamber. Actually, with increase of temperature the velocity is increased and the streamline character of units located in the combustion zone is changed. Intrinsic non-uniformities of velocity and temperature arise. The thermodynamic losses themselves cannot be accurately determined by the scheme of calculation discussed above in consequence of the fact that heat is given up to the gas moving with different local velocities depending upon the layout of the combustion chamber and upon the non-uniform supply of heat throughout the chamber.

The extremely complex nature of the process does not make it possible at the present time to derive a method of calculating the dynamic pressure losses, which should reflect accurately the actual picture of the flow.

However, there exists a relatively simple method of calculating $\sigma_{\text{c. c.}}$, based on the principle of the independence of the aerodynamic losses σ_{aero} on the thermodynamic losses σ_T.

The considerations which serve as the basis of such an assumption can be formulated in the following manner:

There are many units in the modern combustion chamber which determine a considerable portion of the aerodynamic losses (swirlers and inlets to the flame-tubes of a turbo-jet engine, the fuel manifold, precombustion chamber, part of the stabilizing equipment of a ram-jet engine), located in the cold portion of the combustion chamber and, consequently, losses by them should be invariable, whether there is a combustion process or not (for non-variable flow conditions).

Assuming that the thermodynamic losses are chiefly determined by the one-dimensional scheme adopted previously, we arrive at the fact that

$$\sigma_{c.c.} = \sigma_{aero}\,\sigma_T\,,$$

where (1) $\sigma_{aero} = 1 - \xi \frac{k}{2} M_2^2$;

ξ is the aerodynamic coefficient for the chamber, determined by cold flow tests;

M_2 is the Mach number, relative to any characteristic section of the chamber;

(2) σ_T is a coefficient defining the thermodynamic losses.

Thus, we can arbitrarily represent the chamber in the form of two consecutive sections: "aerodynamic" and "thermal".

Owing to the fact that reduction of the dynamic pressure occurs in the aerodynamic section, the Mach number M_{2T} at the inlet to the thermal portion is changed — it is increased the more for a given flow relative to M_2 the smaller σ_{aero}.

In the increase of the Mach number M_{2T}, the effect of the aerodynamic losses on the thermodynamic losses is also included.

M_{2T} can be determined as a function of σ_{aero} from the expression

$$M_{2T} = \frac{M_2}{\sigma_{aero}} = \frac{M_2}{1 - \xi \frac{k}{2} M_2^2}\,, \tag{VI.16}$$

where M_2 is the mean Mach number which should exist at the entrance to the "thermal" portion in the absence of aerodynamic losses.

By way of an example we shall determine $\sigma_{c.c.}$ for a cylindrical chamber, taking into account, and without taking into account, the correction for the Mach number M_2 because of aerodynamic losses, for the following initial data:

$$M_2 = 0{\cdot}2; \qquad \tau = \frac{Q}{p\,T_{2T}^*} = 3{\cdot}0 \qquad \text{and} \qquad \xi = 4\,.$$

(1) $M_{2T} = M_2 = 0{\cdot}2$;

$$\sigma_{aero} = 1 - \xi \frac{k}{2} M_2^2 = 1 - 4\times 0{\cdot}7\times 0{\cdot}04 = 0{\cdot}888\,.$$

According to Fig. 73 we find for $M = 0{\cdot}2$ and $\xi = 3{\cdot}0$ that

$$\sigma_T = 0{\cdot}890\,.$$

Whence

$$\sigma_{c.c.} = \sigma_{aero}\,\sigma_T = 0{\cdot}888\times 0{\cdot}890 = 0{\cdot}79\,.$$

(2) $M_{2T} \neq M_2$.

$$\sigma_{aero} = 0{\cdot}888.$$

$$M_{2T} = \frac{M_2}{\sigma_{aero}} = \frac{0{\cdot}2}{0{\cdot}888} = 0{\cdot}225\,.$$

According to Fig. 73 we find for $M_{2T} = 0{\cdot}225$ and $\xi = 3{\cdot}0$, that

$$\sigma_T = 0{\cdot}842\,.$$

Whence

$$\sigma_{\text{c.c.}} = 0{\cdot}888 \times 0{\cdot}842 = 0{\cdot}745\,.$$

The limiting preheatings as a result of this are changed even more significantly: thus, for $M_{2T} = 0{\cdot}2$, $\tau_{\max} \approx 4{\cdot}3$, and for $M_{2T} = 0{\cdot}225$, $\tau_{\max} \approx 3{\cdot}3$.

5. DETERMINATION OF LOSSES IN A COMBUSTION CHAMBER

We shall acquaint ourselves briefly with a few methods for determining experimentally the losses in a combustion chamber.

(a) *Combustion efficiency*

The combustion efficiency ξ_z, as mentioned above, is defined by relationship (VI.1)

$$\xi_z = \frac{Q_1}{Q_2}\,.$$

It can be seen from expression (VI.1) that in order to determine ξ_z it is necessary to know the quantity of heat actually released as a result of combustion Q_1, and the quantity of heat supplied by the fuel Q_2. The value of Q_2 can always be determined by direct calculation, knowing the fuel supply and its calorific value. The determination of Q_1 presents the principal difficulty (and sometimes it is a very considerable one).

In principle, Q_1 can be determined by a variety of methods: by the chemical composition of the combustion products, by direct measurement of temperature rise and by various gas-dynamic methods necessitating a knowledge of the fuel supply and also of the static and dynamic pressures in certain sections of the engine. Gravimetric methods are also possible based on the change of "thrust" of the chamber etc. The drawback of certain gas-dynamic and "gravimetric" methods consists in the fact that in order to determine ξ_z, one is obliged to eliminate such a doubtful value as the aerodynamic losses in a "hot" chamber.

It should also be noted that all the numerical methods for determining ξ_z, with the exception of the chemical one, automatically include in them the losses through the wall of the combustion chamber to the surrounding medium.

Methods for experimentally determining the characteristics of a combustion chamber are analysed in appropriate courses, consequently we shall limit ourselves here to comments concerning the various methods.

The very concept of ξ_z indicates that the determination of ξ_z can be settled either by the direct or indirect determination of the "weighted mean temperature" of the gas †.

Actually, we can write

$$\xi_z = \frac{Q_1}{Q_2} = \frac{c_{p_s}(T^*_{3\,\text{actual}} - T^*_2)}{c_{p_s}(T^*_{3\,\text{theor}} - T^*_2)} = \frac{\Delta T^*_{\text{actual}}}{\Delta T^*_{\text{theor}}}, \qquad \text{(VI.17)}$$

where T^*_2 is the initial temperature of the combustible mixture;

$T^*_{3\,\text{actual}}$ is the temperature at the end of the chamber;

$T^*_{3\,\text{theor}}$ is the temperature which the gas should have as a result of complete combustion;

$T^*_{\text{theor}} = T^*_{3\,\text{theor}} - T^*_2$ is easily calculated with respect to known values of the excess air coefficient α (i.e., with respect to a known air-fuel ratio), the initial temperature T^*_2, the calorific value H_u and the composition of the fuel.††

Actually, as is well known, ΔT^* can be calculated by one of the expressions (dependent upon which is known: α_Σ or G_f kg fuel/kg air sec):

$$T^*_{\text{theor}} = \frac{H'_u}{c_{p_{\text{av}}}(1 + \alpha L_0)}$$

or

$$T^*_{\text{theor}} = \frac{G_f H'_u}{c_{p_{\text{av}}}(1 + G_f)}, \qquad \text{(VI.18)}$$

where H'_u is the calorific value of the fuel, reduced to a temperature of T^*_2;

$c_{p_{\text{av}}}$ is the mean specific heat of the combustion products within the temperature range $T^*_{3\,\text{theor}} - T^*_2$;

α and L_0 are the excess air coefficient and the theoretical quantity of air required for the complete combustion of 1 kg of fuel;

G_f is the fuel supply per 1 kg of air.

---

† The only exception is the chemical analysis method which permits Q_1 and Q_2 to be determined indirectly from the composition of the combustion products and from the chemical analysis of the original products. For example, the formulae for the chemical analysis method enables ξ_z to be determined approximately as a function of the concentration by volume in a sample, $(CO_2)_{\text{actual}}$ and the quantity $(CO_2)_{\text{theor}}$ which should be present in the sample as a result of complete combustion. The quantity $(CO_2)_{\text{theor}}$ is determined from the ultimate analysis of the fuel.

The formulae, determining exact values of ξ_z contain not only CO_2 but also other chemical compounds; however, the principle of the determination of ξ_z is unchanged.

†† The temperature and composition of the fuel are necessary for an accurate determination of the specific heat of the combustion products.

H'_u and H_u are often used, where H_u is the calorific value of the fuel at a standard temperature (0 °C). In this case, for large values of T_2^* (600–700 °K) an error of up to 1·5–2·0 per cent is possible in determining $\Delta T^*_{\text{theor}}$.

It is likewise easy to determine the temperature prior to combustion. It can be assumed that the temperature of the gas stream at the beginning of the chamber is uniform throughout. Consequently no difficulty is presented in accurately measuring the mean value of T_2^* by means of a thermocouple, resistance thermometer or any other method.

It is a more complex matter to determine T_3^*. The difficulties of determining T_3^* are connected, in the first place, with the fact that the gas stream on discharging from the combustion chamber is essentially non-uniform with respect to temperature, density and velocity and, secondly, the temperatures themselves of the gas stream are high, which makes their direct measurement difficult.

T_3^* can be directly measured at a specific point of the gas stream by means of cooled thermocouples in which there are no radiation losses (for example, suction thermocouples), by certain optical methods, and also by means of chemical analysis of the combustion products.

Obviously, the measurement of T_3^* at different points of the discharge section of the chamber still does not enable the "weighted mean temperature" of the gas to be determined which is essential for determining the combustion efficiency.

In order to explain how one should proceed to determine the weighted mean temperature, let us imagine that in the section 1—1 (Fig. 76), in which we desire to determine the weighted mean temperature, there exists non-uniformity of temperature, density and velocity.

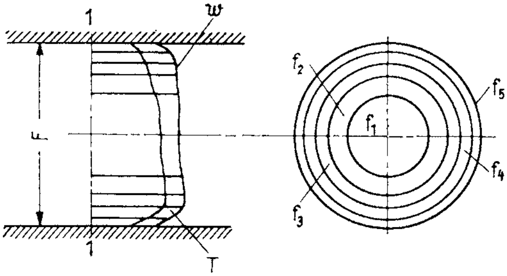

FIG. 76. Determination of the mean parameters of the gas stream ($F = \sum f_i$).
1—1 measurement-section.

The characteristic distribution of velocities on discharge from the combustion chamber is depicted in Fig. 77.

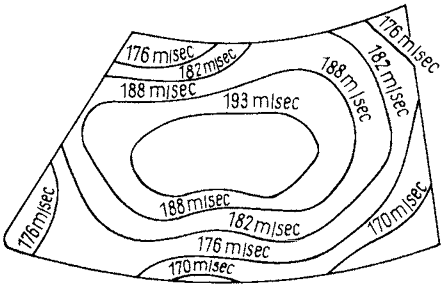

FIG. 77. Characteristic distribution of velocities on discharge from the combustion chamber.

Dividing the area of the section into n equal parts, we shall assume that within the limits of each n-th section the flow is identical with respect to all parameters. It is obvious that this supposition will be more justified the greater is n and the less is the non-uniformity of the flow. By measuring in each n-th section one and the same property T_3^*, the weighted mean temperature can be found from the following formula†:

$$T_3^* = \frac{G_1\,T_{31}^* + G_2\,T_{32}^* + \cdots + G_n\,T_{3n}^*}{\sum_1^n G_i} = \frac{\sum_1^n G_i\,T_{3i}^*}{\sum_1^n G_i}, \qquad \text{(VI.19)}$$

where G_1, G_2 etc., are the flow rates of gas through corresponding sections.

It can be seen from the expression obtained that for an exact determination of ξ_z, a knowledge of the gas flow rates through each of the sections is required in addition to the measurement of the temperature at different points of the section.

As is wellknown, the flow rate through the section f_i can be determined from the expression

$$G_i = \sqrt{\frac{g\,k}{R}}\,\frac{p_i^*}{\sqrt{(T_{3i}^*)}}\,\frac{M_i}{\left(1 + \frac{k-1}{2}\,M_i^2\right)^{\frac{k+1}{2(k-1)}}}\,f_i \ \text{kg/sec}, \qquad \text{(VI.20)}$$

where M_i is a known function of the ratio $\frac{p^*}{p}$

$$M^2 = \frac{2}{k-1}\left[\left(\frac{p^*}{p}\right)^{\frac{k-1}{k}} - 1\right];$$

p_i^* and p_i are the dynamic and static pressures respectively;
T_{3i}^* is the overall temperature.

---

† Assuming that the specific heat in each section is identical.

The dynamic and static pressures in the gas stream are determined by means of cooled pitot tubes, or simply by total head tubes. In the latter case, the static pressure is measured at the wall and it is assumed that p changes throughout only slightly.

It should be noted that such an assumption for combustion chambers is not justified in the majority of cases.

However, the essential fact is that for M_i, close to unity, $\frac{p^*}{p} \rightarrow \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$ and the value of the expression

$$\frac{M_i}{\left(1 + \frac{k-1}{2} M_i^2\right)^{\frac{k+1}{2(k-1)}}} = f\left(\frac{p^*}{p}\right)$$

changes extremely "sluggishly", and consequently a relatively large error in p^*/p gives a small error in the determination of G_i.

Measurement of the static pressure in this region is more justifiable only along the wall.

Determination with respect to "areas", as is frequently done in the case of a non-uniform flow distribution leads to a considerable error.

The determination with respect to areas is accurate only when $G_1 \approx G_2 \approx G_3$ etc. The formula for T_3^* takes the following form:

$$T_3^* = \frac{G_i\,(T_{31}^* + T_{32}^* + T_{33}^* + \cdots + T_{3n}^*)}{\sum\limits_1^n G_i} = \frac{\sum\limits_1^n T_{3i}}{n}. \tag{VI.21}$$

If the parts into which we have divided the section are unequal ($f_1 \neq f_2$ etc.), but the distribution of the flow through each section is uniform, then we obtain

$$T_3^* = \frac{f_1\,T_{31}^* + f_2\,T_{32} + \cdots}{f_1 + f_2 + f_3 + \cdots} = \frac{\sum\limits_1^n f_i\,T_{3i}^*}{F}. \tag{VI.22}$$

The remarks made are general and they should always be taken into account when it is required to determine weighted mean values of the various parameters of the gas stream, the calculation of which is connected with the non-uniformity of flow distribution through the section.

For example, in dertermining ξ_z by means of chemical analysis we can find ξ_{zi} with respect to the composition of the combustion products at the place of measurement. Obviously, in order to determine ξ_z it is essential to know which part of the total supply corresponds to the value obtained for ξ_{zi}.

The expression for ξ_z will be similar to the expression for T_3^*:

$$\xi_z = \frac{\sum\limits_1^n G_i\,(\xi_z)\,i}{G},$$

and for a uniform supply through the section, corresponding to the cases chosen above,

$$\xi_z = \frac{\sum\limits_1^n (\xi_{zi})}{n} \quad \text{or} \quad \xi_z = \frac{\sum\limits_1^n f_i(\xi_{zi})}{F}.$$

Gas-dynamic methods of determining ξ_z lead to the determination of T_3^* by an indirect method.

(1) The method of static pressures can be applied to a cylindrical combustion chamber. Having determined the drop in static pressure Δp through the chamber by an indirect method of measurement, it is possible to determine $\frac{\Delta T^*}{T^*}$ and thus to find $T_3^* = T_2^* + \Delta T^*$, using the relationship $\frac{\Delta p}{p} = f\left(M_2, \frac{\Delta T^*}{T^*}\right)$, derived previously.

This method is convenient in that for determining T_3^* it is necessary in the first place to eliminate from the overall Δp the value Δp_{aero} of the aerodynamic losses created in the combustion chamber and, secondly, to assume that the flow in the chamber is identical with respect to all parameters.

(2) The flow method can be used, knowing the flow distribution through the chamber. From the flow rate equation

$$G = \sqrt{\frac{g\,k}{R}}\,\frac{p_3^*}{\sqrt{T_3^*}}\,\frac{M_3}{\left(1 + \frac{k-1}{2}M_3^2\right)^{\frac{k-1}{2(k+1)}}}\,f$$

we shall have

$$T_3^* = \left[\sqrt{\frac{g\,k}{R}}\,p_3^*\,\frac{M_3}{\left(1 + \frac{k-1}{2}M_3^2\right)^{\frac{k-1}{2(k+1)}}}\,f\right]^2 \frac{1}{G^2}, \qquad \text{(VI.23)}$$

where p_3^* is the mean dynamic pressure in the section, with respect to which T_3 is determined;

M_3 is the average Mach number in section 3, equal to $f\left(\frac{p_3^*}{p_3}\right)$.

The method presented gives low accuracy in the region of low M; for approach of M to the velocity of sound the accuracy will be increased as a consequence of the slow change of $\frac{M}{\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k+1}{2(k-1)}}}$.

If T_3^* is determined directly in the throat and f_{throat} is less than the exhaust section of the chamber, then for the condition such that transition to the throat is achieved by smooth flow lines, and the immediate approach to the throat is achieved by curves with large radii of curvature, it is entirely reasonable to suppose that the flow in the throat will be sufficiently uniform with respect to the static and dynamic pressures.

Hence it is not recomended to carry out measurements in the throat in view of the fact that the instruments inserted change its area and distort the flow, T_3^* can be determined with respect to the static pressure determined at the wall.

We can obtain the expression for T_3^* by replacing the value for p^* in expression (VI.23) by the static pressure $p\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$ and substituting $M = 1{\cdot}0$.

Finally we shall have

$$T_3^* = \left[\sqrt{\frac{g\,k}{R}}\sqrt{\frac{k+1}{2}}\,p_{\text{throat}}\,f_{\text{throat}}\right]^2 \frac{1}{G^2} = \frac{G\,(k+1)\,k}{2\,R}\left(\frac{p_{\text{throat}}\,f_{\text{throat}}}{G}\right)^2. \tag{VI.24}$$

(b) *Dynamic Pressure Losses*

As is well known, dynamic pressure losses in a combustion chamber are determined by the coefficient $\sigma_{\text{c.c.}} = \frac{p_3^*}{p_2^*}$. In order to find the value of the dynamic pressure loss coefficient it is necessary to know the dynamic pressure on entry into and on discharge from the combustion chamber.

The dynamic pressure in a section does not remain constant, but its value varies from point to point proportionally to the dynamic pressure loss itself. Consequently, the measurement of this parameter presents certain difficulties.

The measurement of the dynamic pressure at different points of the measurement section is carried out by means of a total head tube traversing the section and connected at frequent intervals with the recording equipment. The pressure from this tube is usually led to a U-shaped liquid manometer, the readings of which are recorded for the experiment. The determination of the dynamic pressure in the section at the intake is considerably simplified, as also the measurement of the temperature, if the measurement section is preceded by a sufficiently long duct in which the distribution of high-velocity is smoothed; moreover, this enables the static pressure through the section to be assumed constant. In this case one of the total head tubes is installed and the pressure measured by it is assumed to be the mean. In those cases when it is necessary to cut down the time of the experiment it is advantageous to install in the measurement section a total head "comb" (Fig. 78), representing an assembly of total head tubes incorporated into one manifold tube.

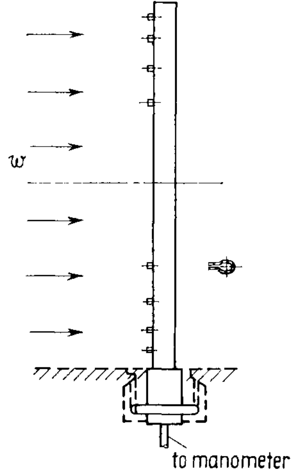

FIG. 78. "Comb" for measurement of dynamic pressure in a gas stream.

The tubes in the total head comb are arranged at definite distances from each other such that they should be at the mid-point of sections with equi-dimensional areas (which is convenient for working).

The method of averaging the dynamic pressure is similar to the method of averaging of local temperatures discussed above.

Through the measurement section flows a quite definite quantity of fluid (air or gas). At each point throughout the section the flow velocity and the temperature are different, and consequently the mass of fluid flowing at various points is unequal. In order to average the pressure in the section it is divided up into a certain number of sections, n. At the mid-point of each section the value of the dynamic pressure is measured or is located on a diagram, and it is assumed that this value is constant in every section.

This is done for each section. As a result, the mean value of the dynamic pressure is obtained.

Depending upon whether or not local flows are measured, the mean value of p^*_{av} can be obtained with respect to flow or to area.

In the first case

$$p^*_{av} = \sum_1^n p_i G_i \Big/ \sum_1^n G_i ,$$

and in the second case

$$p^*_{av} = \sum_1^n p_i F_i \Big/ \sum_1^n F_i .$$

The measurement of dynamic pressure by means of the comb at the exit from the combustion chamber is not always convenient since the high temperatures of the gas necessitate the use of cooled instruments (which complicates their design) or instruments manufactured from special materials. All methods of determining σ^* and T^* by a gas-dynamic method are sound for uniform static pressure fields (p), which can be obtained by the setting-up of flow-smoothing adapters at the beginning of the combustion chamber and at the exit from it.

Convergent adapters (nozzles) for smoothing the gas stream do not offer high resistances and consequently they may also be used for obtaining the characteristics of combustion chambers.

CHAPTER VII

COMBUSTION CHAMBERS OF GAS TURBINE ENGINES

1. SIMPLIFIED DIAGRAM AND WORKING PRINCIPLE OF THE COMBUSTION CHAMBERS OF GAS TURBINE ENGINES

THE theoretical problems discussed in the foregoing sections explain the elements of the operating processes taking place in any combustion systems whatsoever. In passing on to a consideration of the working principle of the combustion chambers of jet propulsion engines and the processes taking place in them, we shall acquaint ourselves in the first instance with the simplified diagram of a combustion chamber (Fig. 79) which nowadays is fundamental for gas turbine engines.

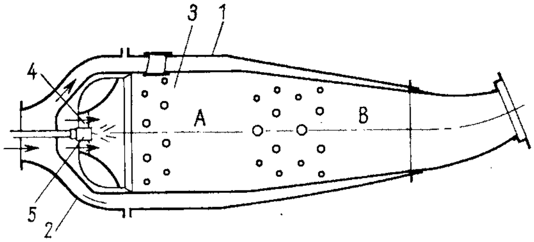

FIG. 79. Diagram of the combustion chamber of a gas turbine engine. 1 — Outer casing; 2 — Inlet diffuser; 3 — Flame tube; 4 — Inlet assembly; 5 — Injector.

The combustion chamber consists of the outer casing 1, the inlet diffuser 2 and the flame tube 3.

In the front portion of the chamber is installed the inlet assembly 4, in the centre of which is located the injector 5. By means of the injector the fuel, supplied to the chamber, is atomized into finely divided particles.

Combustion of the fuel takes place mainly in the forward portion of the flame-tube (zone A). In the second portion of the flame-tube (zone B) mixing of the combustion products with air takes place, with the aim of reducing their temperature. This zone is called the mixing zone. It should be noted that it is not possible to draw strict limits between the zones, since processes of fuel burnout also take place in the mixing zone. Consequently, division into specific zones is of a somewhat arbitrary nature.

The air entering the combustion chamber is divided into two streams. The primary air enters directly into the central region of the flame-tube through the front assembly and the first series of air ports and ensures intense combustion of the fuel. The secondary air is usually admitted to the second half of the flame-tube through a series of openings or stub pipes and is destined to reduce the temperature of the combustion products prior to the nozzle assembly of the turbine. The secondary air comprises the main bulk of the air.

Installed in the upstream portion of the flame-tube, the inlet assembly 4 serves to form a constantly-operating ignition source and, in addition, causes the air stream to become turbulent which contributes to the intensification of the combustion process.

Combustion chambers can be divided, according to their structural layout, into tubular (individual), cannular, annular and tubo-annular combustion chambers.

The Tubular Combustion Chamber

The tubular or individual combustion chamber is widely used nowadays, since it has a number of definite advantages.

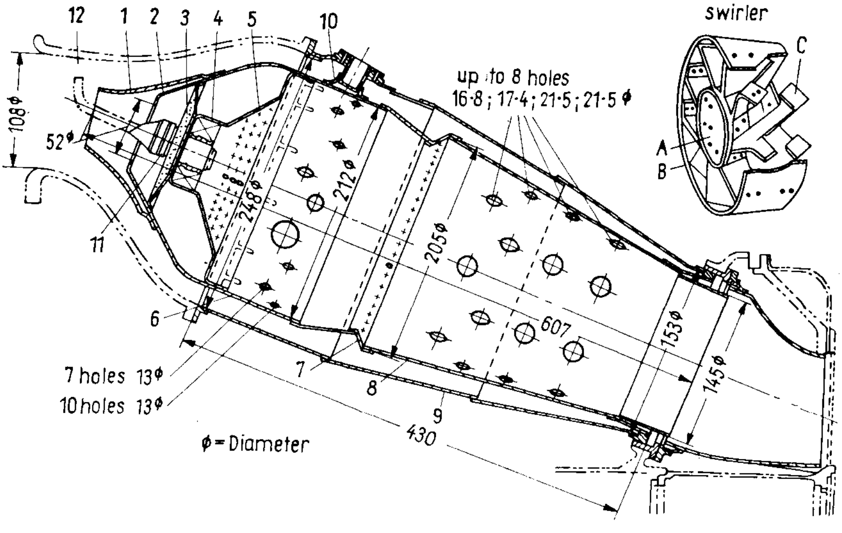

FIG. 80. Tubular combustion chamber. Φ = diameter.
1 — "Snout"; 2 — Housing; 3 — Perforated diaphragm; 4 — Vaned swirler; 5 — Colander or dome; 6 — Flame-tube; 7 — Perforated mixing ring; 8 — End section of flame-tube; 9 — Outer casing; 10 — Centring plug; 11 — Fuel injector; 12 — Bracing sleeve; A — Inner collar; B — Swirl vane; C — Outer collar.

Figure 80 shows the tubular combustion chamber of a gas turbine engine. The chamber has a conical outer casing 9, to which is attached a bracing sleeve 12, cast from an aluminium alloy and which serves to attach the combustion chamber to the body of the ducting.

Inside the outer casing is installed the flame-tube 6, fabricated from heat-resistant sheet material in order to withstand the high temperatures. The flame-tube is retained in position by three centring plugs 10, two of which are used simultaneously as interconnector tubes between adjacent chambers for equalizing the pressure. In addition, the interconnectors play a special role in starting the engine which will be mentioned below.

Onto the front of the flame-tube is welded an intake nozzle ("snout") 1, having a fully-defined intake area calculated such that the requisite quantity of air is admitted through the inlet assembly.

The area of the intake, as also its very position in the tube section, is chosen by experiment when developing the combustion chamber, dependent upon the nature of the combustion process and the air inlet velocity range of the chamber.

In the colander is installed a vaned swirler 4. The vanes of the swirler, set at an angle, twist the stream of air flowing through it. In the centre is installed a swirl injector 11, which feeds the fuel into the combustion chamber.

The flame-tube has a number of zones on its surface with holes of different diameters for admitting air.

Figure 80 shows the dimensions of the holes in the flame-tube and their number in each zone. On the cylindrical section there are two rows of holes with a diameter of 13 mm — seven holes in the first row and ten in the second row. The flame-tube has a conical section, in the wall of which is located a number of small-diameter holes. These holes serve to create an air layer near the wall of the flame-tube with the object of cooling it. In the conical portion of the flame-tube are distributed rows of holes for admitting the secondary air. Their number in each row is constant and their diameter is gradually increased from 16·8 to 21·5 mm.

It should be noted that this arrangement of the holes in the flame-tube is not completely fixed. Figures 81 and 82 show the flame-tubes of the combustion chambers of certain engines. It can be seen from the figures that the arrangement and size of the holes of the flame-tubes described are different for different engines.

The determination of the requisite number and sizes of holes, ensuring a satisfactory course of the combustion process, can be carried out for each chamber by calculation with a subsequent check on the test bench. However, the necessity for gradually increasing the diameter of the holes in the upstream portion of the flame-tube arises directly from ideas concerning the course of the process of combustion and of mixture formation.

Amongst the merits of combustion chambers of the type under consideration should be noted the comparative ease of operation, since in order to replace one of its chambers it is not necessary to dismantle the engine.

Fig. 81. Flame tubes of combustions chambers.

Fig. 82. Flame tubes of combustion chambers.

In addition, for developing and testing a tubular (individual) chamber, compressor equipment with large air flow deliveries is not required in contrast to an annular combustion chamber for the testing of which test rigs delivering air supplies of around 60–100 kg/sec are needed.

The principal drawback of the tubular combustion chamber is the probability of increasing the mid-section of an engine by installing a chamber of this type, especially in those cases when the engine has an axial compressor with a relatively small pressure change.

The Cannular Combustion Chamber

This type of combustion chamber differs structurally from the previous one in that a number of tubular chambers, joined together with connecting tubes and having a common air duct, are enclosed in one common body. Figure 83 shows the cannular combustion chamber of an engine.

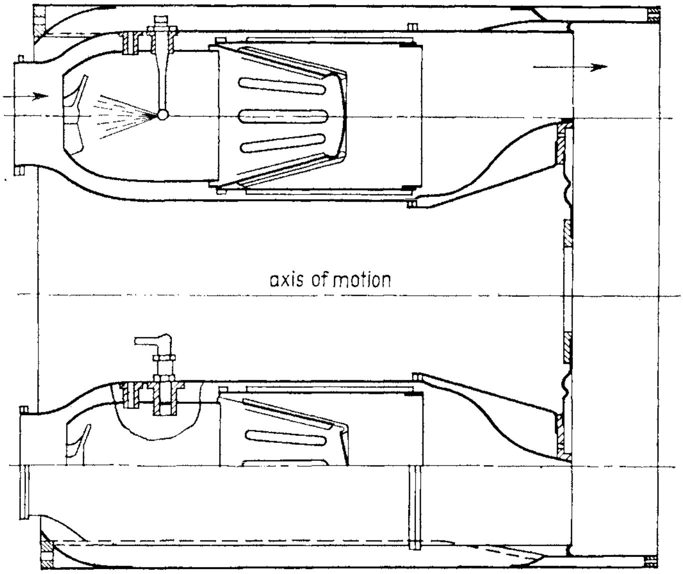

FIG. 83. Cannular combustion chamber.

The casing of the combustion chamber is made from sheet material. At the extremities are spot-welded massive flanges to which are fixed the neighbouring units of the engine. In addition to the flanges, there is a rigid fin on the casing to which is attached a trunnion for securing the engine to the fuselage of the aircraft. Thus, the body of the combustion chamber fulfils the function of a power unit which is dependent upon the power layout of the engine.

Each chamber has an outer casing 1 (Fig. 84), inside which is installed coaxially a pre-combustion chamber 3. A swirler 4 is installed at the entry to the pre-combustion chamber.

The fuel injector 9 is located at the centre of the pre-combustion chamber. The fuel is supplied to the injector under a pressure of 50–55 kg/cm^2 under operating conditions.

At the extremity of the pre-combustion chamber is located a slotted mixer in the form of a truncated cone, on the surface of which are disposed the slots 7.

Near the end of the cone (to the sloping walls forming it) a flame deflector 6 is welded.

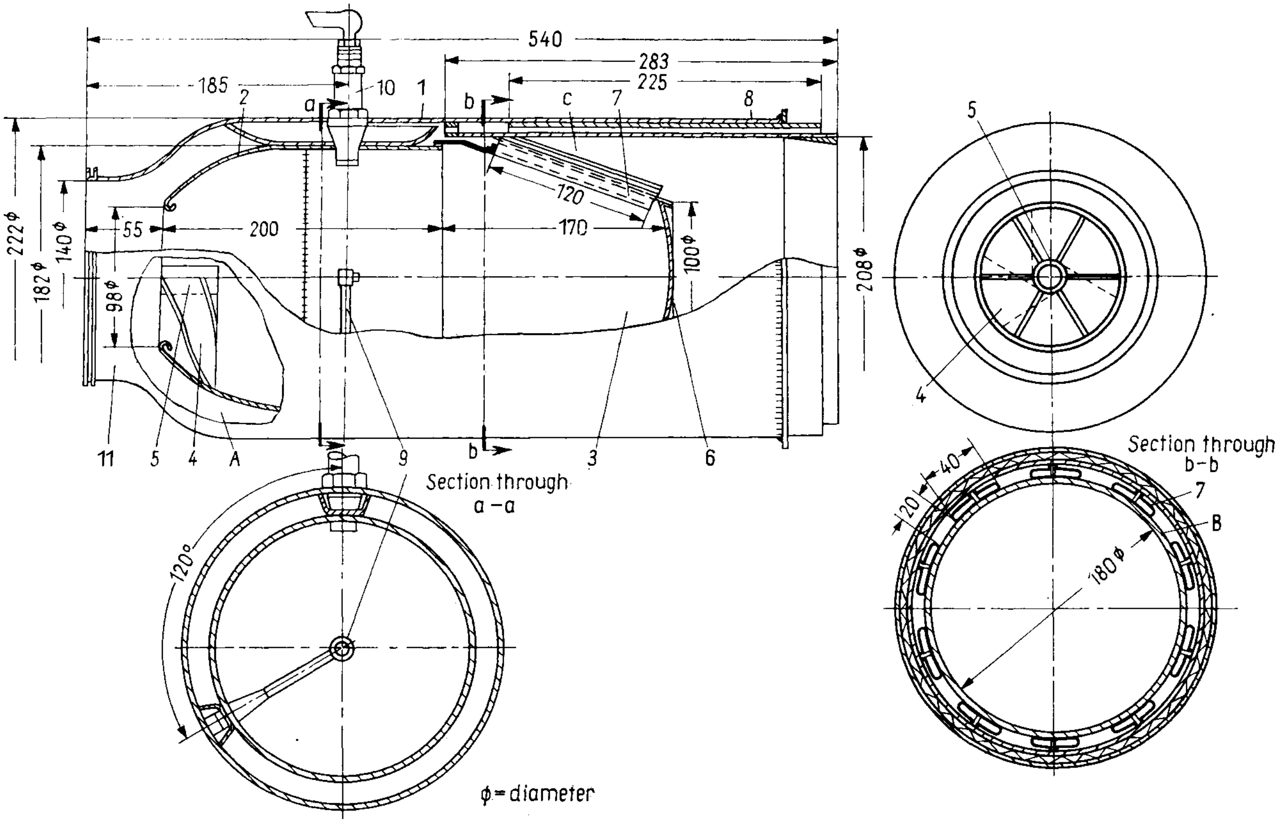

Fig. 84. Tubular combustion chamber.

1 — Chamber housing; 2 — Intake snout; 3 — Pre-combustion chamber; 4 — Swirler; 5 — Swirler duct; 6 — Flame deflector; 7 — Inclined cone with slots; 8 — Screen for chamber housing; 9 — Injector; 10 — Igniter plug; 11 — Throat of air intake to chamber; A — Annular duct; B — Slots; C — Cylindrical space.

The combustion products, possessing high temperatures, discharge from the pre-combustion chamber through the orifices of the slot mixer into the cylindrical portion of the chamber where they are mixed with the secondary air and partially burned fuel. The design of the chamber ensures a rapid mixing of the combustion products with the air.

The combustion chamber under discussion has considerable aerodynamic pressure losses which correspond to a fall in the design dynamic pressure for the compressor by approximately 8–10 per cent.

The Annular Combustion Chamber

The annular combustion chamber has a single flame tube, formed by two cylindrical walls arranged concentrically. At the intake to the flame tube a whole series of swirlers or simply of poorly streamlined housings for the injectors is arranged round the periphery. The number of such units amounts to 15–20.

The supply of secondary air flowing between the outer and the inner casing and the walls of the flame tube to the combustion products is done with the object of intensifying the burning process and reducing the temperature of the gases, just as in other types of chamber through holes and, in certain designs — through stub tubes or chutes arranged in one or several rings.

By way of an example let us consider the annular combustion chamber of an engine (Fig. 85).

In this chamber the primary air reaches the combustion zone through sixteen jets, in the centre of which is located a swirl injector.

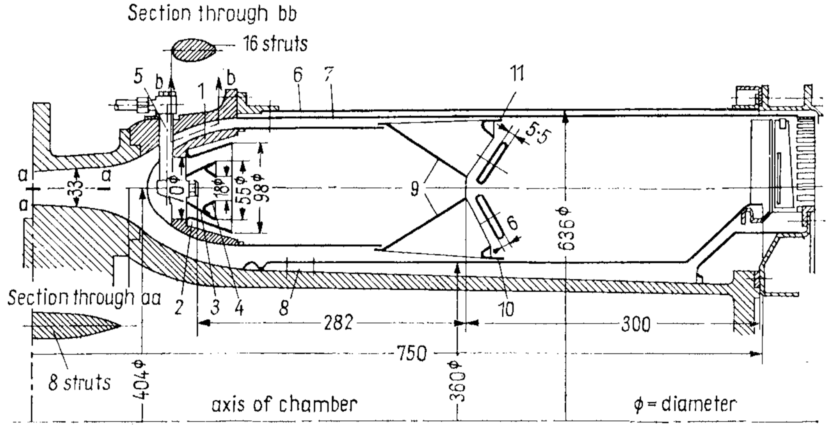

FIG. 85. Section of an annular combustion chamber.
1 — Body of combustion chamber; 2 — Outer cone of combustion chamber; 3 — Inner cone of combustion chamber; 4 — Deflector; 5 — Injector; 6 — Chamber casing; 7 — Outer jacket; 8 — Inner jacket; 9 — Mixing chutes; 10 and 11 — Annular slots for admitting cooling air.

Admission of secondary air is achieved through special mixing chutes 9 arranged uniformly round the periphery and through the annular slots 10 and 11. A small portion of the air is led through an annular gap prior to the turbine, cooling the wall of the combustion chamber.

In the chamber being discussed, these mixers are arranged in one ring, but they can be arranged, however, in several rings. The inclination of the mixer chutes, their lengths and straight-through cross-sections are selected on the basis of test data and are verified by experiment.

It should be noted that the drawback of an annular combustion chamber is the necessity for dismantling the engine to replace or maintain the chamber, and also the difficulty of carrying out development work, since it is necessary to have test equipment with a large air supply amounting to 60–100 kg/sec and more.

However, an annular combustion chamber also has substantial advantages, viz: it has relatively small dimensions and weight for large supplies of air. The diameter of an engine with an annular combustion chamber can be considerably less than one with a combustion chamber of the tubular type.

Moreover, starting of the engine is made relatively easy, particularly at high altitudes, since combustion of the fuel takes place in a single volume and does not require interconnecting tubes which do not always guarantee transference of the flame from chamber to chamber.

The Tubo-annular Combustion Chamber

The trend towards a combustion chamber without increased engine diameter (mid-section), which is reliable in operation and easy to develop, has led to the appearance of the tubo-annular combustion chamber.

The tubo-annular combustion chamber represents a combination of features of both the tubular and annular combustion chambers.

A combustion chamber of this type consists of an external and an internal cylindrical shell forming an annular space in which a number of separate flame tubes is installed.

Testing and developing of this chamber can be freely carried out on normal-sized rigs with air supplies of 10–12 kg/sec.

The flame tubes (cans) of such a combustion chamber may be tested quite separately or in groups, consisting usually of three cans. In this case the conditions for the air supply to the middle flame-tube will be similar to the conditions existing in the actual chamber, since the effect of the adjacent flame tubes here is automatically taken into account.

An additional advantage of the tubo-annular combustion chamber, as compared with an annular chamber, is the more uniform distribution of the air and fuel in the combustion zone and of mixing, because of the symmetrical construction.

The structural layouts of the combustion chambers discussed do not exhaust all the possible cases. However, it should be borne in mind that the principle of the combustion process in all cases remains the same, and only the design of the various units is changed.

Cooling of Combustion Chambers

Particular attention during design should be paid to the problem of cooling combustion chambers. A number of the units of a combustion chamber, such as the flame-tube, interconnector tubes, etc. are under the action of the stream of high temperature gases, as a result of which many materials lose their durability. Consequently, with the object of increasing the life of the combustion chamber, these units are made from heat-resistant materials and, in addition, they are cooled to a temperature not exceeding ~800 °C.

Cooling of the flame tube is achieved by two methods simultaneously. In the first, the secondary air flow, passing through the annular space between the outer casing and the flame tube, cools its walls on the outside surface. However, this is inadequate, since the consequence of having non-uniformity of distribution of the temperature of the gas stream close to the walls can give rise to local overheating, which may lead to burning or to a sharp increase of thermal stresses, leading in its turn to the formation of cracks in the flame tube. Consequently, cooling of the flame tube is achieved additionally by jets of air which "scrub" the walls of the flame-tube on the internal surface. For this purpose, a portion of the secondary air is admitted to the flame tube through special holes disposed around the periphery at certain places along the flame tube. Figure 86 shows part of the flame tube of a combustion chamber, on the surface of which are shoulders 1 (See Fig. 86a) with many small diameter holes.

The jets of air passing through these holes or ports are directed along the walls. The air "shroud" formed in this manner on the inner surface of the flame tube reduces the flow of heat to the wall.

In addition to the ports in special regions, a number of protuberances may be made, shown diagrammatically in Fig. 86b. The direction of the air flow is shown by the arrows.

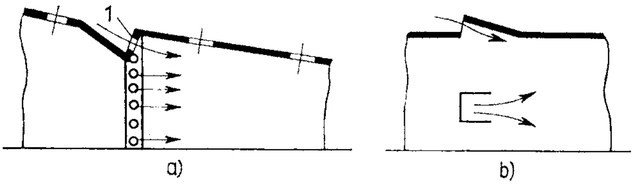

FIG. 86. Ports in the flame tube for cooling the walls.

It should be noted that in designing a new combustion chamber it is difficult to determine beforehand the number of shoulders or ports in the flame tube and their disposition.

The final adjustment of cooling takes place, together with refinement of the process of mixture formation and the process of combustion, at the time of testing of the chamber.

It should also be noted that the walls of the flame tube are also cooled by the air directly participating in the process of combustion and of mixing.

The jets passing through, which frequently "scrub" the zone adjacent to the wall, reduce the temperature of the gas, which should be particularly effective in the mixing zone, where combustion is already practically completed.

2. ELEMENTS OF THE OPERATING PROCESS OF COMBUSTION CHAMBERS. NATURE OF THE AIRFLOW IN A COMBUSTION CHAMBER

The air flow from the last stage of the compressor has a relatively high velocity ($w > 100$ m/sec).

To achieve combustion of the fuel for such mean flow velocities would lead to very considerable dynamic pressure losses or an inadmissibly long combustion chamber; in the latter case the operating range of the chamber with respect to α would also be contracted, and the difficulties associated with a sufficiently high combustion efficiency would be considerably increased.

Consequently, a diffuser is usually introduced into the upstream portion of the combustion chamber†, i.e., an expansion duct in which the mean flow velocity of the air is reduced to 60–70 m/sec.

As already mentioned above, the bulk of the air passing through the combustion chamber can be divided into two streams — the primary and the secondary stream, of which the first participates directly in the combustion process, and the second serves mainly for reducing the temperature of the combustion products.

The ratio between the primary and secondary air streams is a quite definite value and is chosen from the following considerations.

It is well known that the velocity of combustion for the majority of fuels, including mixtures of gasoline and kerosene with air, has its maximum value for a mixture composition close to stoichiometric. Consequently, the quantity (relative and absolute) of the primary air is determined by the requirement to ensure the requisite mixture composition in the combustion zone ($\alpha \approx 1-1{\cdot}4$).

† Since the presence of the diffuser after the compressor is dictated by the needs of the combustion process, then it should be considered as part of the combustion chamber, the more so in that structurally it adds weight to the latter.

In modern gas-turbine engines the temperature of the gas discharging from the combustion chamber under conditions close to normal is of the order of 800–900 °C, which corresponds to a volume excess air factor of $\alpha = 4{\cdot}5-3{\cdot}7$. Naturally the values given for α will be reduced as higher temperature engines become developed.

The relationship between the maximum possible temperature of the gases and the excess air factor, for an inlet air temperature of 250°, is shown in Fig. 87.

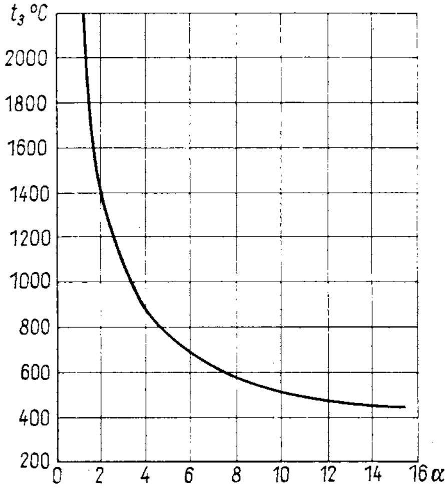

FIG. 87. Variation of temperature of the gases leaving the combustion chamber, as a function of the excess air factor α. $H_u = 10{,}250$ kcal/kg; $t_2 = 250$ °C; $\xi_z = 1$.

Thus, the primary air in the combustion chambers of modern engines amounts to approximately 30–35 per cent of the total air.

By reducing α relative to the values mentioned above, the optimum ratio of primary to secondary air is also correspondingly changed. Thus, for $\alpha = 2{\cdot}5$, the relative amount of primary air may be as great as 45 per cent or more.

The characteristic special features of the gas-dynamic flow pattern for all modern combustion chambers is the presence near the axis of the flame tube of a zone of reverse flow.

It can be seen from Fig. 88, that along the axis of the chamber near the inlet assembly (section *aa*) reversal of the direction of flow occurs. As a consequence of this, part of the combustion products, possessing a high temperature, returns upstream to the base of the fuel-air flame; by this means, ignition of fresh mixture is ensured and the subsequent development of the combustion process.

Let us consider in more detail the phenomena which lead to the formation of the reversal zone.

If a badly-streamlined body is inserted into the air stream, for example a circular plate (Fig. 89), then a zone having a reduced static pressure is created behind it. As a consequence of the reduced pressure, the air will flow into this zone from a space situated some distance behind the bluff body. The direction of this motion will be opposed to the direction of the main flow. Thus, a reversal zone is formed because of the bluff body.

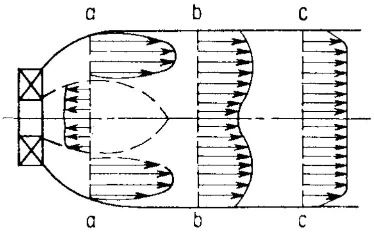

FIG. 88. Distribution of axial velocities along the cross-section of a combustion chamber.

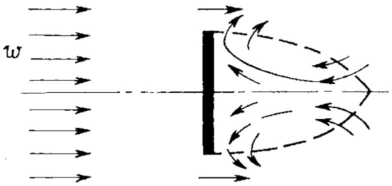

FIG. 89. Streamlines for a plate, with a flow of fluid (air).

The dimensions of the reversal zone (diameter) in a given case will be determined by the dimensions of the badly-streamlined body — the stabilizer.

A second possible method of creating a reversal zone is by swirling the flow. This method is, however, used primarily in the combustion chambers of gas-turbine engines. The presence in the combustion chamber of the development of a reversal zone has a favourable effect on the operating process. This requirement is usually solved complexly by means of a number of devices. In particular, the front portion of the flame tube is made divergent (Fig. 90). The stream of air issuing from the swirler is directed away from the axial direction. In this case the size of the reversal zone is increased. However, as a result of the duct opening out through a large angle, the stream is observed to break away from the wall and form a vortex region, as shown in Fig. 91.

The presence of vortices near the wall of the flame tube is undesirable, since as a result of this, conditions arise for carbon deposition and local overheating, and also the aerodynamic pressure losses are increased. Ports in the front portion of the flame tube (Fig. 92) contribute to the elimination of this boundary vortex zone.

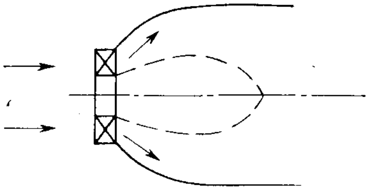

FIG. 90. Diagram of the flow of air in a combustion chamber with a divergent flame tube.

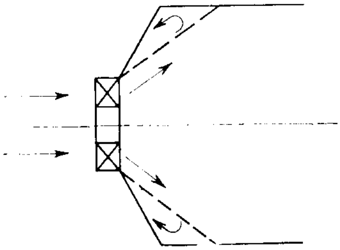

FIG. 91. Diagram of the formation of vortices at the periphery of the flame tube.

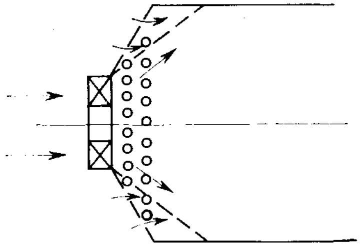

FIG. 92. Diagram of the flame tube of a combustion chamber with supplementary admission of air through ports in the dome.

Swirl of the stream plays a special role in increasing the reversal zone (Fig. 93).

For this purpose, a vaned impeller is used in the majority of modern combustion chambers.

The magnitude of the swirl is best for a setting of the blades of the impeller within the limits $\varphi = 45$–$60°$.

Increase of swirl of the stream leads to an increase in the diameter of the reversal zone. In addition, the amount of the gases returned by the reverse flow is increased, which has a favourable effect on the development of the combustion process.

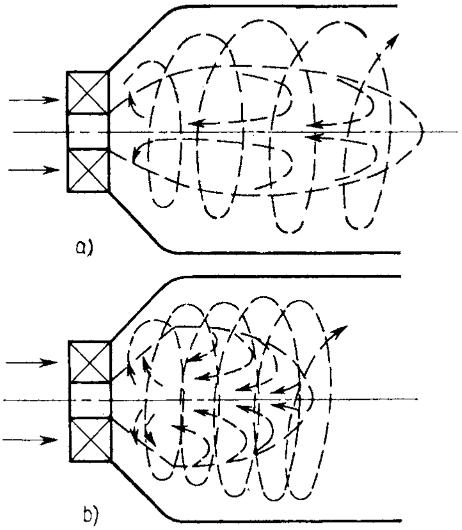

FIG. 93. Effect of swirling the air stream in a combustion chamber (by means of a vaned impeller) on the size of the reversal zone.

In case (b) the angle of set of the impeller blades is greater than in case (a). However, increase of swirl of the stream also leads to an increase in the aerodynamic pressure losses.

For angles of setting of the impeller vanes greater than 60°, a considerable increase in aerodynamic pressure losses is observed for a relatively small improvement of the combustion process.

It should be borne in mind that the size of the reversal zone depends further on a whole series of other factors, in particular on the distribution of the ports in the wall of the flame tube, and also on the temperature of the gases.

The disposition of the ports for supplying air close to the inlet assembly leads to a reduction of the spread of the reversal zone, since as a result of this the jets contract, as it were, the reversal zone (Fig. 94).

The penetration of the jet into the reversal zone leads to a certain reduction of the temperature of the gases and to a deterioration of stabilization of the combustion process.

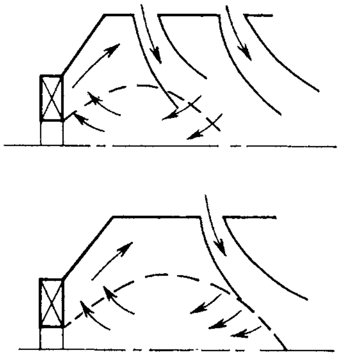

FIG. 94. Effect of distribution of ports in the lateral walls of the flame tube on the size of the reversal zone.

By increasing the temperature of combustion, the size of the reversal zone is also reduced, as a consequence of the reduction in density of the gases in the primary stream.

The jets of air passing through the wall of the flame tube exert a considerable influence on the course of the combustion process. In the first place the jets of air passing through these ports carry additional oxidant (oxygen) to the hot fuel-air mixture, ensuring the local concentrations of fuel most essential for combustion. In the second place, the air jets give rise to a certain turbulence of the flow, and to some extent may themselves behave as stabilizers, since as a result of their streamlines (particularly jets of large diameter) local eddies or vortices are created.

The air jets, passing through the ports in the mixing portion of the flame tube, reduce the temperature of the combustion products. The distribution and quantity of the ports can influence the temperature profile of the flow of gases prior to the turbine.

The ports in the mixing region of the flame tube have a relatively large diameter, which ensures penetration of the jet to a great depth and, thereby, a relatively uniform reduction in temperature of the combustion products.

It should be noted that the distribution of the air in the chamber, and therefore the size and number of the ports for admitting the air to the inside of the flame tube, have a considerable influence on the characteristics of the combustion chamber.

It is possible to determine approximately the flow pattern in a combustion chamber by applying the theory of turbulent flow (the theory of turbulent flow is given in the works of G. N. Abramovich).

The method of aerodynamic calculation of a combustion chamber, which enables the distribution of the air along the chamber to be determined, with deep penetration of the jets and loss of dynamic pressure, is given in section 7 of the present chapter.

3. MIXTURE FORMATION IN THE COMBUSTION CHAMBER OF A GAS TURBINE ENGINE

It was mentioned earlier that the process of combustion can proceed in a proper manner only for a definite composition of the fuel-air mixture. The production of a combustible fuel-air mixture within a wide range of variation of the operating regimes of the chamber is a complex problem. The fuel is fed in, in the majority of cases, by swirl injectors installed in the dome of the combustion chamber, into the wake of stabilizers.

With a vaned impeller the injectors are located on the axis of the impeller. The fuel discharges from the injector, forming a fine conical sheet which, at a certain distance from the injector disrupts and is broken down into individual droplets.

Investigations have shown that 1 cm^3 of fuel is disintegrated into finely-divided droplets, the number of which amounts to ten millions, and their diameter varies within the range 200 to 10 micron.

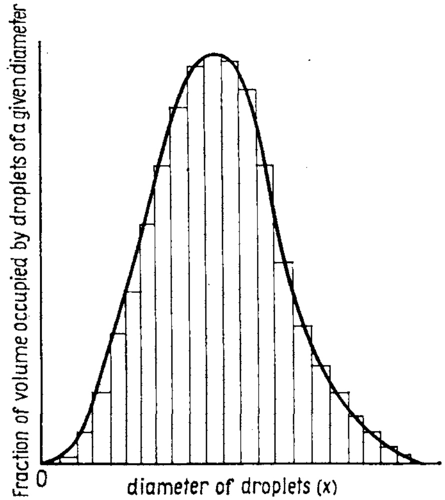

FIG. 95. Distribution of droplets according to sizes.

It can be seen from Fig. 95 that the maximum fraction of the volume of fuel occupied by droplets of a specified diameter corresponds to a certain mean value of the diameter.

A droplet of fuel entering the combustion zone is gradually vaporized, whereupon the fuel vapour forms a vapour zone around the liquid nucleus.

The fuel vapour, having mixed with the surrounding air, burns.

Vaporization of the droplet takes place on account of the heat transferred to the liquid nucleus by the combustion products.

It is obvious that the finely divided droplets are rapidly vaporized and burn at a comparatively short distance from the injectors. However, coarse droplets can traverse a considerable distance, often equal to the length of the chamber, before they burn completely. Therefore, with a large number of coarse droplets, a sharp reduction in the combustion efficiency may be observed.

It should be noted however, that extremely fine atomization of the fuel can lead to deterioration of mixture formation and consequently to a reduction of combustion efficiency and to a narrowing of the limits of stable operation, etc. In this case, as a result of very fine sub-division and supply of fuel, into the gas stream, the particles are immediately carried away. The fuel, here, is dispersed within a limited volume in which an insufficiently rich mixture is produced. The diffusion processes may then prove to be inadequate for ensuring the optimum concentrations in the combustion zone.

The usual swirl injector gives a spectrum of droplet sizes. The most finely-divided droplets are carried away immediately by the stream whilst the coarse droplets maintain their individuality of motion to a great distance and consequently penetrate the air stream to a great depth. As a result of this the fuel is dispersed throughout a greater volume of the chamber.

Every chamber has its optimum atomization spectrum, directly associated with its aerodynamics — distribution and direction of the air flows. A combustion chamber designed for a supply of liquid fuel with a given swirl injector may operate badly with another injector which provides a finer spray or with a supply of fuel in the vapour phase. Conversely also, for a chamber designed for a supply of fuel in the vapour phase, there may be deterioration of its operation as a result of supplying it with liquid fuel.

Swirl injectors are used, in the majority of cases, as fuel atomizers in combustion chambers. The fineness of atomization of the fuel, the flow through the injectors and the angle of atomization depend on its construction and on the pressure of the fuel inside the injector, or more accurately, on the difference $\Delta p = p_i - p_{c.c.}$, where p_i is the fuel pressure, inside the injector and $p_{c.c.}$ is the pressure in the combustion chamber. Fuel atomization and other characteristics of the fuel jet depend also on the magnitude of the absolute pressure in the combustion chamber $p_{c.c.}$ Nowadays, there exists a theory for the swirl injector, enabling the liquid flow to be calculated and the spray angle of the fuel cone to be calculated (See G. N. Abramovich, *Applied Gas Dynamics*, GTTI, 1953).

A single-channel swirl injector is shown in Fig. 96.

The fuel, on admission from the manifold through the pipeline 1, passes through the filter 3 to the swirler 6.

The swirler has a number of tangential ports which give rise to swirling of the fuel. The jet of fuel issues from the injector through the nozzle 9 in the form of a conical sheet which is broken down into droplets at a comparatively short distance from the injector orifice.

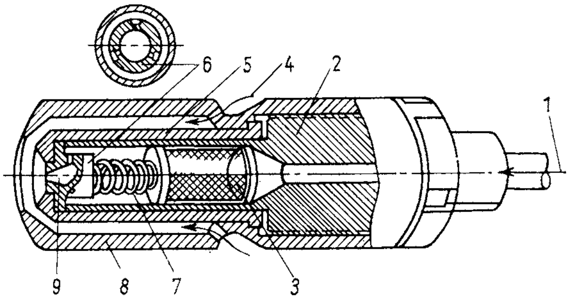

FIG. 96. Single-channel swirl injector.
1 — Supply line; 2 — Body of injector; 3 — Filter; 4 — Supply of cooling air; 5 — Locking ring; 6 — Swirler; 7 — Spring; 8 — Outer cap of injector; 9 — Orifice of injector.

The swirl injector brings about satisfactory atomization of the fuel only within definite limits of Δp. For small fuel flow, when Δp should be small, the atomization becomes unsatisfactory, which is immediately manifested in the form of deterioration of operation of the chamber at low throughputs, under high altitude conditions, etc.

In order to eliminate this drawback and to ensure satisfactory fuel atomization over a wide range of chamber operating regimes, twin-channel swirl injectors are used (Fig. 97).

A twin-channel injector has two swirlers, 3 and 4, designed for different fuel feeds. The fuel is admitted to the swirlers through two pipelines. For operating the engine at high speeds and low flight altitude the fuel is admitted through both channels of the injector. For low-load conditions or for operating at high altitude, the fuel is admitted through only one of the two channels. This system of operation of the injector enables the pressure difference Δp to be maintained, ensuring good atomization of the fuel over a large range of operating regimes of the engine.

The dispersion of the fuel throughout the chamber also exerts an influence on the process of combustion.

The fuel, on issuing from the injector, is broken down into droplets under the action of centrifugal forces and forms a hollow cone. Further, the cone of fuel is widened under the action of the radial velocity component.

On looking at the dispersion of the fuel along the chamber, regions can be seen with particularly large concentrations of fuel (Fig. 98).

The line OT joining points with maximum values of local fuel concentrations is determined by the path taken by the main quantity of fuel in the combustion chamber.

The excess air factor through the section varies inversely with the concentrations, i.e. the minimum values of α_{loc} correspond to the maximum values of fuel concentration.

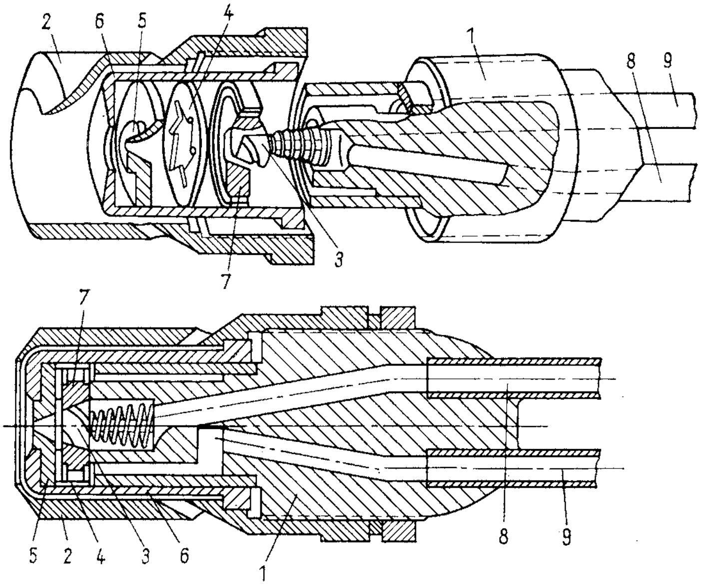

FIG. 97. Twin-channel swirl injector.
1 — Body of injector; 2 — Cap; 3 — Conical swirler; 4 — Flat swirler; 5 — Orifice; 6 — Locking bush; 7 — Ring; 8 — Starting channel; 9 — Main channel.

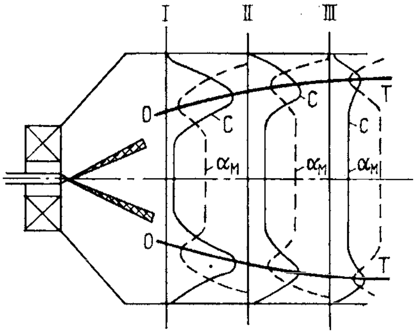

FIG. 98. Distribution of local values of fuel concentration and excess air coefficient in the chamber.

Mixture formation is intimately connected with the nature of the supply of air to the combustion zone.

The fuel jet consists of liquid and vapour phases of the fuel, so that the relative quantity of the latter is increased in the direction of the jet.

Hence it follows that not all the fuel is prepared immediately for combustion. Assuming that it is necessary to create local values of α within fairly narrow limits for combustion, we should feed air to the fuel jet in strict conformity with the quantity of fuel prepared for combustion.

The supply of air to the fuel jet, according to the extent of preparation of the fuel for combustion in each chamber of a turbo-jet engine, is accomplished through the ports in the dome and in the walls of the flame tube.

As a result of this, local values of the excess air factor necessary for combustion are maintained in the combustion zone.

With an unsatisfactory arrangement of mixture formation the combustion process deteriorates sharply. In particular, a bad arrangement of the processes of mixture formation and of combustion can often be judged according to the discharge of flame from the combustion chamber towards the turbine.

In addition to the atomization system, including the injectors and the fuel supply lines at high pressure, vaporizing devices are used in a number of combustion chambers.

One of the features of a vaporizing system is that by its use it is not necessary to aim for fine atomization of the fuel. In this case it is not necessary to have large fuel pressure drops at the injectors.

The fuel, at a relatively low pressure, is sprayed into vaporizing tubes, around which flow the combustion products (Fig. 99).

A certain measured quantity of air is admitted into these same tubes, which ensures vaporization of the fuel and prevents it cracking. From the vaporizing tube the rich fuel–air mixture ($\alpha = 0{\cdot}25 - 0{\cdot}3$) discharges in a direction opposite from the direction of the main stream. The fuel–air mixture is diluted in the front portion of the chamber by an additional quantity of air, which ensures obtaining a combustible concentration.

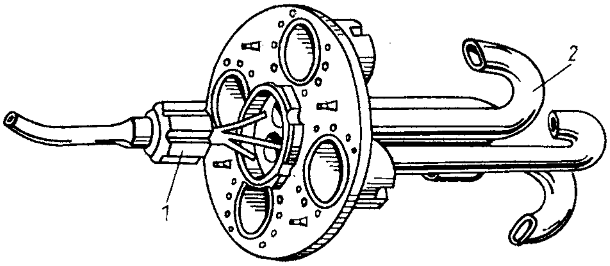

FIG. 99. Elements of the vaporizer of a combustion chamber.
1 — Injector; 2 — Vaporizing tubes.

The vaporizing tubes are installed in several groups in a separate flame tube. Figure 100 shows a tubular chamber with the vaporizer unit discussed above.

The use of the vaporizing unit is not only encountered in tubular combustion chambers. Individual annular combustion chambers with similar

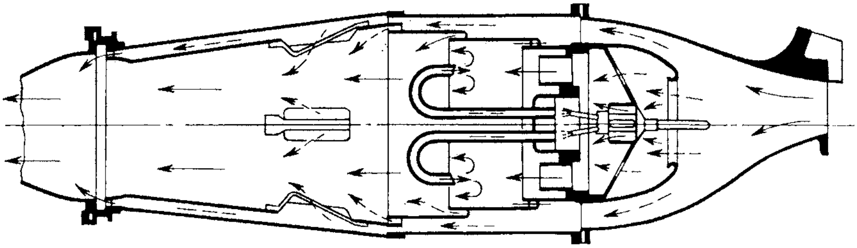

FIG. 100. Combustion chamber of the vaporizing type.

units are known. However, the possibility of considerable carbon deposition in the vaporizing tubes is not ruled out, which is a serious disadvantage of this type of fuel supply. A further disadvantage of a vaporizing chamber is the necessity of having a special starting system for the fuel delivery.

4. IGNITION OF FUEL IN THE CHAMBER

For the initial ignition of the fuel-air mixture in the combustion chamber of a gas-turbine engine, it is necessary to transfer a certain quantity of heat to it, sufficient to ignite the combustible mixture primarily in preparation for combustion.

When the engine is operating, ignition of the incoming fuel-air mixture takes place continuously because of the heat of the combustion products.

An electric spark plug is usually used as a source of heat for the initial ignition.

In certain early designs of combustion chambers the igniter plug was located in the wall of the chamber and it ignited the combustible mixture formed in the combustion chamber as a result of spraying the fuel through the main injector. Such a system of ignition did not guarantee reliable starting of the engine, chiefly because of the inadequate power of the electric spark and the poor atomization of the fuel at the small pressures inside the injector under starting conditions.

Re-lighting at high altitude was made particularly difficult and consequently such a starting unit found a very limited practical application.

The ignition unit shown in Fig. 101 is a development of the unit discussed earlier, and consists of a starting injector designed for small fuel flows, and an ignition plug.

The ignition unit is installed in the dome of the combustion chamber in a stagnant zone, where relatively low flow velocities occur. This device also has not found wide application since it did not guarantee reliable starting.

An ignition device of the jet type meets the requirements most satisfactorily. One of the possible layouts of such a device is shown in Fig. 102.

This unit consists of a starting injector 2, an electromagnetic valve 1 which admits the fuel to the injector only at the instant of starting, and an igniter plug 3. The starting injector and the igniter plug are assembled in one housing, the lower portion of which leads to the flame tube. Below, on each side, the housing has a number of holes through which the secondary air is admitted directly to the starting injector and the ignition plug. Thus, the ignition unit represents a small combustion chamber. At the instant of starting, a jet of flame is formed in the ignition unit, which penetrates into the depth of the chamber and ignites the main fuel.

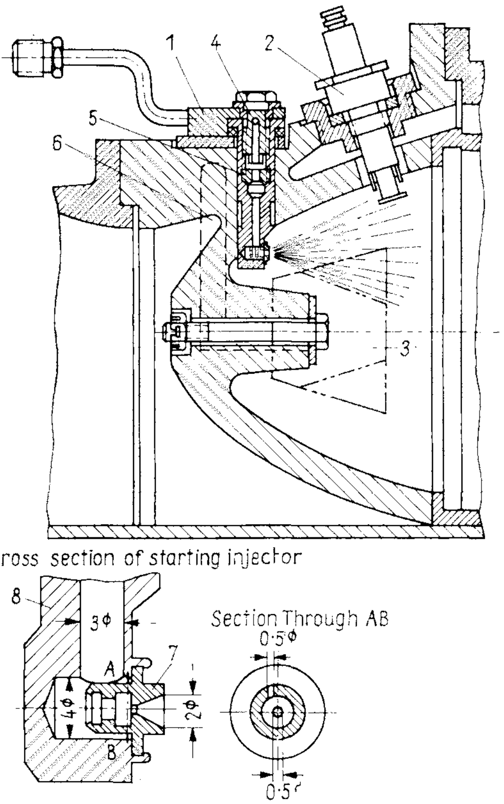

FIG. 101. Combustion chamber starting unit.
1 — Starting injector; 2 — Igniter plug; 3 — Combustion chamber stabilizer; 4 — Filter; 5 — Gauze filter; 6 — Main injector; 7 — Orifice of starting injector; 8 — Body of starting injector.

In view of the relatively large heat flow and the directed action of the jet, similar units guarantee starting of the engine under the various flight conditions.

Starting an engine with an annular combustion chamber is somewhat more simple than starting an engine with tubular chambers, since the propagation of the flame from the starting jet takes place throughout the whole volume almost instantaneously.

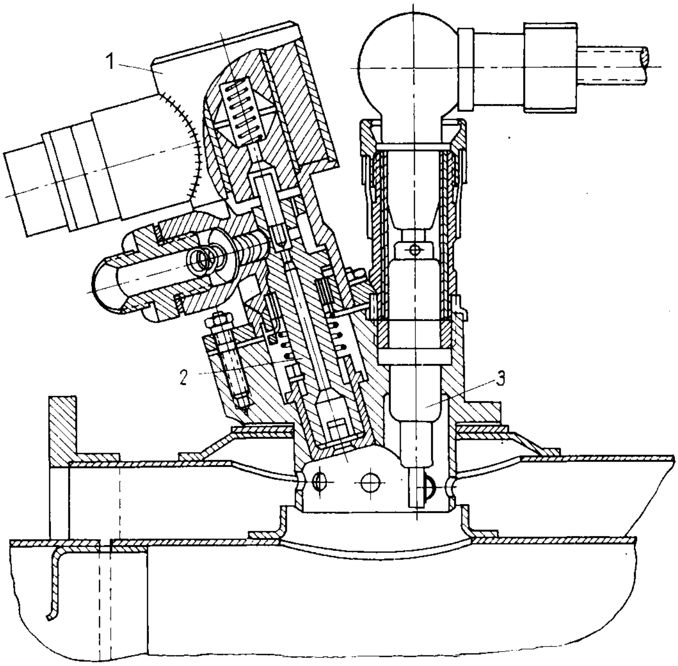

FIG. 102. Combustion chamber ignition unit.
1 — Electromagnetic valve; 2 — Starting injector; 3 — Igniter plug.

With tubular combustion chambers, the starting ignition units are usually installed in two chambers. Ignition of the fuel in adjacent chambers takes place by means of the flame penetrating into them through the interconnector tubes. The diameter and length of the interconnector tubes also exerts a significant influence on starting. Reduction of the diameter of the interconnector tube leads to a deterioration in starting, particularly under high altitude conditions.

Re-lighting an engine at altitude is usually made difficult because of the reduction of pressure of the air in the chamber and the reduction of the power of the igniting jet.

In order to ensure reliable re-lighting at altitude, one should always aim at increasing the power of the starting unit, making it as far as possible independent of the degree of reduction of pressure in the compressor.

5. DESCRIPTION OF THE OPERATING PROCESS IN THE COMBUSTION CHAMBER OF A GAS TURBINE ENGINE

In the present section is given the most simple qualitative explanation of the operating process in the combustion chamber of a gas turbine engine; in particular, the role of interaction of the air and fuel jets is explained.

Arguments are developed for the combustion chamber of a gas turbine engine with fuel atomization by a swirl injector.

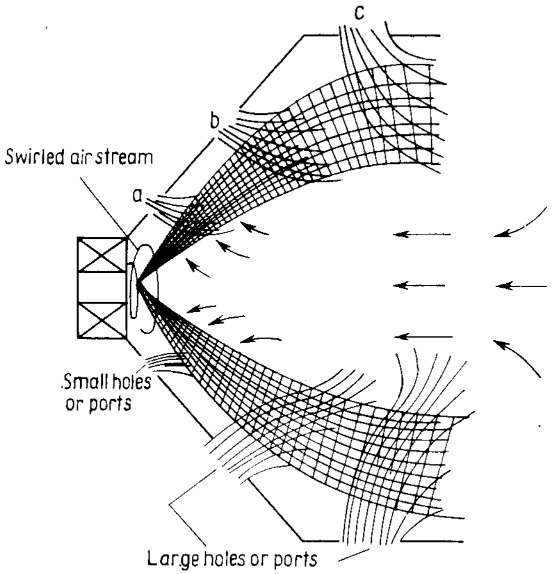

FIG. 103. Schematic diagram of the direction of motion of the air and fuel.

Figure 103 shows a diagram of the fuel and air jets entering the holes of the dome and of the initial part of the flame tube. The density of the cross-hatching arbitrarily characterizes the extent of the fuel vaporization in a given cross-section of the fuel jet.

The jets of air through the ports *a*, *b*, etc., are fed to the fuel jet according to the extent of preparation (vaporization) of the fuel for combustion and the burnout of the oxygen in the air which passed through the swirler and the foregoing ports.

This gradual supply of air to the fuel jet ensures that the optimum concentrations of fuel (capable of burning) and oxidant ($\alpha_m \approx 1$) for combustion are obtained at every point of the combustion zone.

With the expansion of the fuel-air jet the depth of penetration of the air through the ports into the flame tube needs to be increased; this is accomplished by increasing their diameter.

A reduction or an increase in the diameter of the ports, respectively, leads to enrichment or weakening of the flame and to an appropriate change in the penetration of cold air into the reversal zone.

In the first case carbon deposition on the walls of the chamber and the passage of unburned fuel into the reversal zone is possible, and in the second case chilling of the gas by cold air may occur.

Just as in the first case, so also in the second case a deterioration in the operation of the combustion chamber will be observed.

As a result of insufficiently sound control of mixture formation, several characteristic events may be observed.

(1) The air is fed to the fuel torch in quantities corresponding to local values of $\alpha_{loc} \gg 1{\cdot}0$ (with respect to the fuel prepared for combustion), and a certain portion of it gets into the reversal zone, as a result of which the temperature is reduced.

In consequence of this the operating range of the combustion chamber is decreased with respect to α to the side of lean flameout. Moreover, the combustion efficiency in such a chamber may deteriorate.

(2) The fuel torch is arranged such that part of the unburnt fuel gets into the reversal zone; at the same time the local values of α in the combustion zone remain, on an average, large.

This circumstance can occur when the air is not completely mixed with the fuel torch owing to the small dimensions of the ports a, b, etc. (for a sufficiently larger number of them) or as a result of a small angle of spread. As a result of this, a considerable quantity of unburnt fuel may accumulate in the reversal zone. Here, the air getting into the reversal zone may improve the characteristics of the chamber with respect to combustion efficiency and limits of operating stability.

(3) The air is fed to the fuel torch in quantities corresponding to $\alpha_{loc} \ll 1{\cdot}0$. If the concentrations of fuel as a result of this are such that the mixture still remains combustible, then owing to the large quantity of unburnt fuel subjected to the action of the high temperature, there arises a real danger of considerable carbon deposition.

It can be expected that in order to attain a high combustion efficiency it is necessary to increase the length of the combustion chamber.

As already mentioned, modern combustion chambers operate over a wide range with respect to the overall excess air coefficient α_0 $(\alpha_0 2 - 200)$. This is possible only in the case when local concentrations of fuel in the combustion chamber, as a result of considerable variation in the air supply, are such that the mixture still remains combustible.

By increasing α_0 the fuel supply pressure is reduced, the cone angle is reduced and the fuel is dispersed over a smaller volume of the chamber.

This circumstance may prove to be a decisive factor for operating the chamber on weak mixtures. Actually, the distribution of the fuel in a small volume, under these conditions, enables the local excess air factor α_{100} to be maintained within the limits necessary for combustion, which also contributes in the case quoted towards stabilization of the flame.

Naturally, the combustion efficiency falls as a result of operating the chamber on weak mixtures.

Similar comments may also be made for chambers with spray dispersion of the fuel, in which the role of the stabilizing device is played by a bluff body, and also for combustion chambers operating on vaporized fuel.

As mentioned above, the process of combustion is mainly completed in the first half of the flame tube. The process in the second half is mainly that of mixing of the air with the combustion products in order to reduce their temperature.

Holes in the walls of the flame tube serve to supply the air. These holes often have larger diameters than the holes supplying air to the combustion zone. This is explained by the fact that for cooling the combustion products it is essential to have a large depth of penetration of the secondary air stream.

The size and disposition of the holes should be ascertained by obtaining the desired temperature profile upstream of the turbine, bearing in mind for this that the depth of penetration of the air jets for given conditions is defined by their dimensions.

The desire to reduce the temperature of the walls at the largest diameter of the flame tube may lead to the fact that the smallest holes in the mixing zone will be found at the beginning.

It is clear from what has been said that the distribution of fuel throughout the chamber, the depth of its penetration, etc., exert a large influence on the operation of the chamber.

In the absence of data concerning the pressure difference between the air in the outer duct and the gas in the flame tube, and data concerning the air flow rate as a function of the diameter and shape of the holes, it can be roughly assumed that the supply of air through the holes in the flame tube is proportional to the area of these holes. By taking advantage of this fact, or by experimental data, it is possible in designing the chamber to select the arrangement, shape and sizes of the holes in accordance with the desired air flow distribution.

6. CHARACTERISTICS OF COMBUSTION CHAMBERS

In the foregoing sections the layout of combustion chambers and the elements of the operating processes in them have been considered. Combustion chambers function over a wide range of operating conditions for turbo-jet engines.

As a result of this, the supply of air through the chamber will vary, and consequently also the air velocity, the inlet temperature to the combustion chamber, the fuel supply, pressure, and other parameters.

The relationships showing how the basic dimensions are changed — (combustion efficiency ξ_z, α_{av}, $\sigma_{c.c.}$) — which determine the operation of the chamber at various operating conditions, are usually called the characteristics of a combustion chamber.

The determination of the characteristics of a combustion chamber is carried out by an experimental method, since to compute analytically the course of the operating process in a combustion chamber does not appear to be possible at present.

For rating a combustion chamber, the dependence of the combustion efficiency on the basic parameters defining the operating condition is most frequently used — the characteristics with respect to flameout, starting, etc.

Below we shall consider the characteristics of a chamber with respect to combustion efficiency and flameout characteristics.

The relationships characterizing the variation of aerodynamic losses in a combustion chamber were discussed in Chapter V, "Losses in Combustion Chambers".

Characteristics of Combustion Chambers with respect to combustion efficiency

Combustion efficiency is one of the basic parameters defining the overall efficiency of the operating process in a combustion chamber.

Experiments and theory shows that the combustion efficiency is not constant for every combustion chamber but varies, depending upon the temperature and velocity of the inlet air, the composition of the mixture, the pressure in the chamber and other factors [$\xi_z = f(\alpha\ p_2, T_2, w_2)$].

The dependence of ξ_z on the parameters mentioned is determined, to a large extent, by the structural layout of the chamber, by the characteristics of the fuel supply system, etc.

The variation of combustion efficiency with respect to mixture composition $\xi_z = f(\alpha)$ is of considerable interest (Fig. 104).

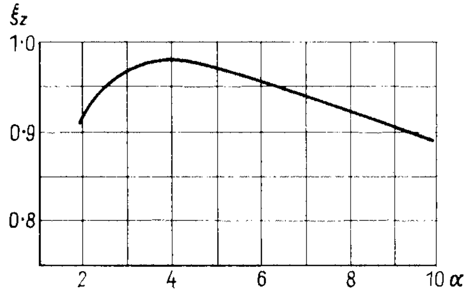

FIG. 104. Characteristics of a combustion chamber with respect to combustion efficiency.

It can be seen from Fig. 104 that the maximum value for the combustion efficiency for the combustion chambers of modern engines occurs at an overall excess air coefficient of 3·5–4·5, which corresponds to cruising conditions of the given chamber, and is determined by the nominal value of the temperature of the gases prior to the turbine.

By reducing the fuel supply to obtain a weak mixture, the pressure of the fuel inside the injector is reduced, which has an effect on the quality of atomization. In this case the size of the droplets increases. The process of mixture formation and preparation of the mixture for combustion is prolonged and part of the fuel in certain cases, not having time to burn, will be carried out of the combustion chamber by the gas stream. The combustion efficiency will be reduced according to the extent of increase of weakness of the mixture.

On the other hand, rich mixtures relative to design conditions lead to the optimum distribution of the fuel being disturbed, the flame becomes longer and frequently does not lie within the allotted dimensions of the chamber but emerges from its end. Combustion of the fuel takes place outside the chamber and the combustion efficiency within the limits of the chamber is reduced.

It should be noted that rich mixtures usually lead to a more abrupt reduction of combustion efficiency than is observed for weak mixtures (see Fig. 104).

By varying the air distribution along the flame tube it is possible to obtain a steadier variation of combustion efficiency and also to attain the maximum value for combustion efficiency with a richer or a weaker mixture composition.

Figure 105 shows the characteristics of a chamber of the vaporization type with respect to combustion efficiency. Comparison of the characteristics presented for chambers with a different fuel supply system shows that the combustion efficiency of a vaporizer combustion chamber varies more steadily, and for weak mixtures in the case quoted, has a higher value than for a combustion chamber with the usual fuel supply system.

As already mentioned (see Chapter V), variation of the air pressure in the chamber has a substantial effect on the combustion process. Reduction of the pressure, as a rule, worsens the characteristics of a chamber with respect to combustion efficiency, especially for pressures less than 1 atm.

In a combustion chamber with the usual fuel supply system, a reduction in combustion efficiency is also observed when pressure is reduced.

An increase in the temperature of the air entering the combustion chamber has a favourable effect on the increase in the intensity of combustion and correspondingly improves the characteristics of the combustion chamber.

Figure 107 shows experimental data with respect to variation of the combustion efficiency as a function of the inlet air temperature.

Finally, it should be noted that the type of fuel can exert an influence on the chamber characteristics with respect to combustion efficiency since the chemical composition of the fuel, volatility, viscosity, etc., obviously affects ξ_z.

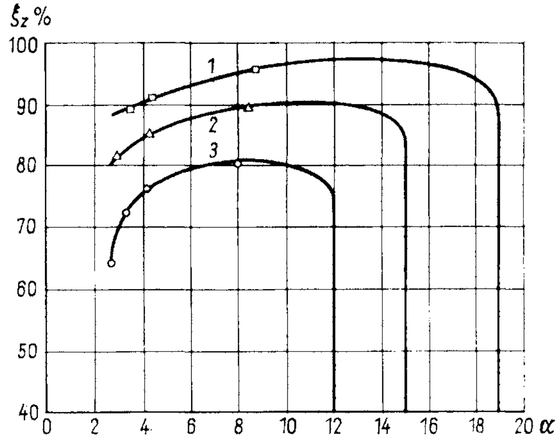

FIG. 105. Characteristics of a chamber of the vaporization type with respect to combustion efficiency (t_2 = 130 °C, w_2 = 25 m/sec).
1 — p_2 = 660 mm Hg. 2 — p_2 = 457 mm Hg. 3 — p_2 = 254 mm Hg.

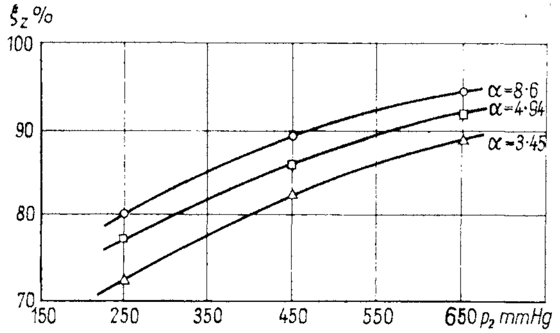

FIG. 106. Effect of air inlet pressure on the combustion efficiency for t_2 = 130 °C, w_2 = 25 m/sec.

Figure 108 depicts the variation of combustion efficiency as a function of engine speed for various hydrocarbon fuels (A–F).

The shape of the curves in Fig. 108 shows that at high operating speeds of the engine the physical properties of the fuel have but little effect on the combustion efficiency.

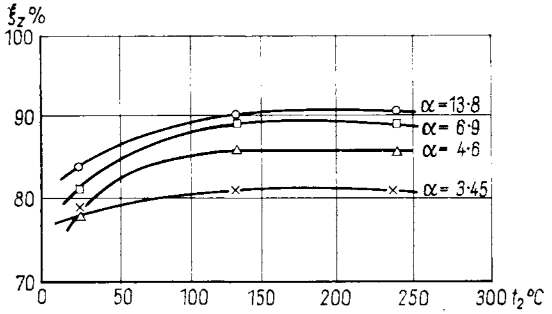

FIG. 107. Effect of chamber inlet air temperature on combustion efficiency.

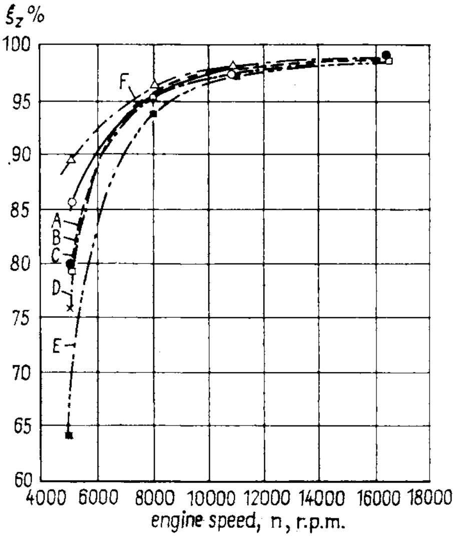

FIG. 108. Variation of combustion efficiency as a function of engine speed for various fuels.

However, at low engine speeds (which are characterized by small air supplies through the chamber and poor fuel atomization), the combustion efficiency for the lighter fuels with low boiling points and small values of the C/H ratio (Fuel *F*) attains a higher value than for fuels with high boiling points and a high C/H ratio (Fuel *E*).

Experiments with different fuels show that the choice of an appropriate design of combustion chamber can lead to a minimum sensitivity towards fuel type.

Extinction Characteristics of a Combustion Chamber

As already mentioned above, one of the fundamental requirements set forth for combustion chambers is the maintenance of stable combustion over a wide range of operating conditions.

The composition of the mixture, and the pressure and velocity of the air flow exert an influence on the combustion process and, in the first place, on its stability. By weakening or enriching the mixture the moment comes when combustion ceases in the combustion chamber.

The phenomena of flame extinction in the combustion chamber as a result of changing the composition of the mixture are called the weak and rich stability limits respectively.

In order to study the phenomenon of flame extinction it is necessary to examine the concentration of fuel, velocity of the air, etc., immediately within the combustion zone. However, for plotting the overall operating characteristics of a combustion chamber, it is convenient to use as parameters not the local but the mean values for the velocity (or supply of air for a given p), the overall excess air coefficient, etc.

The relationship

$$w_{\mathrm{av}} = f(\alpha_0)\,,$$

where w_{av} is the value of the velocity at which the combustion chamber is extinguished, relative to one of the characteristic cross-sections of the combustion chamber $\left(w_{\mathrm{av}} = \frac{G_{\mathrm{air}}}{\gamma_{\mathrm{air}} F_{\mathrm{c.c.}}}\right)$;

α_0 is the total excess air coefficient determined with respect to the total supply of fuel and to the total air supply at which the chamber is extinguished,

is called the extinction characteristic of the combustion chamber.

The extinction characteristics, typical in form for many practicable combustion chambers of gas-turbine engines, are presented below.

In Fig. 109 are shown curves defining the extinction limits as a function of the overall excess air coefficient α_0 and the supply of air (mean velocity of the air in the chamber). All the points lying between the curves for rich (a) and weak (b) extinction correspond to conditions under which the combustion process proceeds stably. It can be seen from Fig. 109 that an

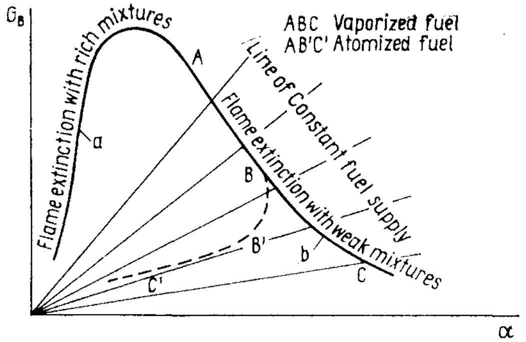

FIG. 109. Extinction characteristics of a combustion chamber.

increase in the air velocity (air supply) in the combustion chamber sharply narrows the range of stability of operation with respect to the excess air coefficient.

The significant deterioration of the extinction characteristics as a result of increasing w_{av} is explained by the fact that with increase of w_{av} the local velocities in the combustion zone are increased, and the time of preparation of the fuel for combustion is reduced. Reliable ignition of the combustion mixture, commencing with a certain value of w_{av}, requires a substantial increase of thermal energy for the primary source of ignition (reversal zone), which with the given combustion arrangement does not take place, as a consequence of which the flame is also extinguished. This explains why the limits of operating stability of the chamber with respect to w_{av} are narrowed, whereupon α_0 is increased at the rich extinction limit and is reduced at the weak extinction limit.

In addition to the efficiency of the stabilizing device, the mixture composition and the velocities of the gas stream, and the pressure and temperature of the air at the intake to the combustion chamber exert an influence on the extinction characteristics of the chamber.

The limits of stability for variations of pressure and temperature are different. Experimental data obtained by studying flame extinction show that a reduction of pressure to $p_2 \approx 1-2$ atm does not lead in practice to a variation of the range of stability of combustion.

However, at low pressures ($p_2 \ll 1$), the range of operating stability may be narrowed very considerably, particularly for weak mixtures (see Table 11).

TABLE 11

Pressure atm	Velocity m/sec	Limiting values of excess air coefficient
0·2	140	1·31 – 0·69
1·1	120	1·68 – 0·60
2·9	120	1·68 – 0·61

By increasing the air temperature, the combustion process and the combustion stability are improved, especially in the weak mixture region.

In the rich mixture region, as experimental data shows, the limit of stable operation is changed less markedly.

Investigations carried out with combustion chambers have also shown that the installation of a vaporizing burner improves the extinction characteristics and as a result of this the value of the excess air coefficient for weak extinction is particularly increased.

According to data in the literature, the combustion chambers of a number of modern engines will tolerate very weak mixtures ($\alpha_{\text{weak lim}} \approx 150\text{–}200$).

It should be noted that cases can occur when combustion proceeds so badly in the combustion chamber of a gas-turbine engine that normal operation of the engine cannot be ensured.

This phenomenon is observed mainly at low pressures in the combustion chamber, corresponding, for example, to flight at high altitude and operation of the engine at low speeds.

The fact of the matter is that to sustain operation of the engine at a given speed, completely defined values of the temperature of the gases $T_{3_{\mathrm{req}}}$ are required (requisite temperature) prior to the turbine (Fig. 110). Further, if the combustion efficiency in the chamber be so inadequate that the maximum temperature of the gas $T_{3_{\mathrm{adeq}}}$† (for any value of α_0) cannot be attained and cannot exceed the requisite temperature $T_{3_{\mathrm{req}}}$, then it is obvious that normal operation of the engine will be impossible.

There is a region in which $T_{3_{\mathrm{adeq}}} \leq T_{\mathrm{req}}$, as a rule, in all combustion chambers.

For different chambers this region is characterized by definite values of p_2 and T_2 or by the engine speed, velocity and flight altitude correspondingly.

Figure 110 shows the mutual variations of $T_{3_{\mathrm{adeq}}}$ and $T_{3_{\mathrm{req}}}$ with the engine speed, for $T_2 = \mathrm{const}$. It is obvious that the entire region lying to the left of the intersection of the curves $T_{3_{\mathrm{adeq}}} = f(n)$ and $T_{3_{\mathrm{req}}} = f(n)$ is a non-operating region.

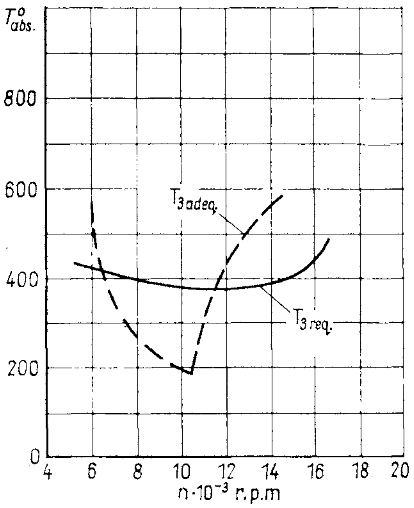

FIG. 110. Determination of the range of stable operation.

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† If $T_{3_{\mathrm{adeq}}} \leq T_{3_{\mathrm{req}}}$, then speeding-up of the engine is excluded and the regime itself will not be stable.

Such a deterioration in the operation of the chamber is explained in the majority of cases by the unfavourable effect of low pressures on the combustion process, and in certain cases by the disruption of the process of mixture formation.

7. AERODYNAMIC RATING OF A COMBUSTION CHAMBER

Correct distribution of the air throughout the combustion chamber, as mentioned above, ensures an intense development of the combustion process and optimum characteristics for the chamber. The aerodynamic rating makes it possible, in designing combustion chambers, to determine the quantity and size of the holes in the walls of the flame tube to produce a selected law of distribution of the secondary air and reduction of aerodynamic pressure loss, (expressing the overall aerodynamic resistance of the combustion chamber).

The following initial data should be given for an aerodynamical calculation:

(1) Basic volume dimensions of the combustion chamber and the type of combustion chamber;

(2) The parameters for the air on entry into the combustion chamber (pressure p_2^*, temperature T_2^*, and the air flow rate G_a);

(3) The excess air coefficient α_0 and the calorific value of the fuel H_u or the temperature of the gases prior to the turbine, T_3.

Proceeding from the experimental data, it is necessary, before commencing the calculation, to choose a law for the supply of secondary air in the flame tube and a law of penetration of the supply of secondary air along the flame tube. For this, a choice is also made of an approximate law of heat release.

It should be borne in mind that there is an optimum distribution for the secondary air along the flame tube and for the depth of penetration of the stream.

It is wellknown that the distribution of the secondary air along the flame tube exerts a considerable influence on the characteristics of the chamber with respect to the combustion efficiency and extinction characteristics.

Two laws of supply of secondary air are depicted in Fig. 111 for comparison, differing from each other in that the length in which the supply of air is achieved is different. For a secondary air supply in the smallest length, the maximum value of ξ_z is displaced to the side of richer mixtures (Fig. 112, curve 2).

For an extended air supply in the forward zone of the combustion chamber (curve 1 in Fig. 112) the process of heat release is developed relatively slowly. Enrichment of the mixture for this law of air supply leads to discharge of the flame jet beyond the limits of the flame tube and to a sharper reduction in the curve for ξ_z. With earlier secondary air supply the combustion process proceeds more intensely, which leads to more efficient combustion of the fuel for lower values of the excess air coefficient.

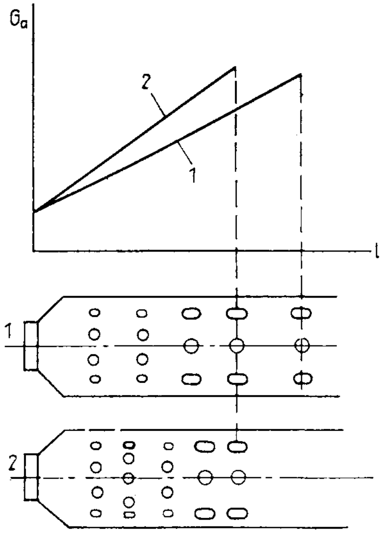

FIG. 111. Comparison of laws of air supply in a combustion chamber.

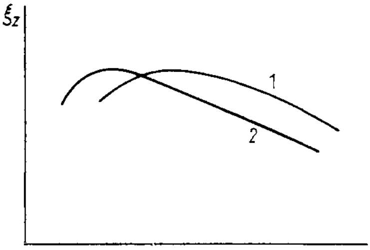

FIG. 112. Variation of combustion efficiency as a function of α for different laws of air supply.

A secondary air supply which will satisfy a specified law may be accomplished with a different depth of penetration of the stream. By increasing the depth of penetration of the jets in the initial section of the flame tube up to $H \approx (0{\cdot}6\text{–}0{\cdot}7)\, R$, the combustion efficiency is increased, but the stability of the combustion process deteriorates.

It is essential to make a choice of the air supply law and the depth of penetration of the jet, arising from the above-mentioned considerations and from experimentation by scientific research organization and design departments built up for the development of various designs of combustion chamber.

Let us consider the diagram for aerodynamic rating as applied to a particular combustion chamber (Fig. 113).

First of all we shall determine the air supply through the swirler $G_{a_{\text{prim}}}$ resulting from the value of α_{prim}. The value of α_{prim} can be set within the limits $\alpha_{\text{prim}} = 0{\cdot}3$–$0{\cdot}4$.

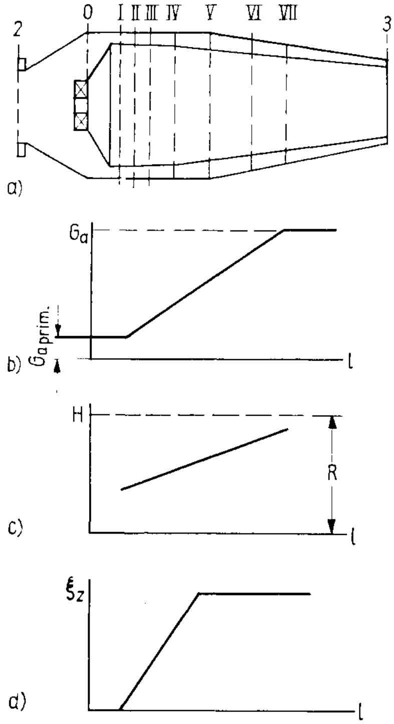

FIG. 113. Rating of a combustion chamber.

The approximate nature of the law of supply of secondary air to the combustion chamber is shown in Fig. 113b.

We can assign an approximate change of depth of penetration of the jet into the flame tube of the combustion chamber based on experimental data, and we can depict it graphically (Fig. 113c).

The variation of combustion efficiency ξ_z can be given directly, commencing at a distance of 40–50 mm from the swirler and attaining the value of $\xi_z = 0{\cdot}95$–$0{\cdot}98$ at a distance l (see Fig. 113d). The distance l corresponds to the length in which the value of the excess air coefficient amounts to 2–2·2.

Having assigned the requisite data, let us further carry out the calculation in this sequence.

(1) Calculation of the parameters of the air throughout the secondary air duct.

We mark off a number of sections along the combustion chamber into which the secondary air will be supplied (Fig. 113a; sections I, II, III, etc.).

The gas parameters in section I are calculated according to general aerodynamic and gas-dynamic formulae. The straight-through section for the secondary air, between the wall of the surrounding jacket and the flame tube in the initial portion of the can, is chosen to have an air velocity of approximately 70–80 m/sec. The values given for the velocity of the air flow ensure sufficiently good cooling of the walls of the flame tube for small aerodynamic pressure loss.

The diffuser is usually located within sections 2–1. The losses in it dependent upon the impact are estimated by taking into account the impact loss coefficient.

$$\Delta p^*_{\mathrm{imp}_{2-\mathrm{I}}} = k\frac{\varrho_2 w_2^2}{2}\left[1-\frac{F_2}{F_\mathrm{I}}\right]^2,$$

where k is the impact loss coefficient;
ϱ_2 is the air density on entry into the combustion chamber;
w_2 is the air velocity on entry into the combustion chamber;
F_2 is the area of the intake sleeve;
F_1 is the area of the duct in section I.

The value of the coefficient k is chosen relative to the angle of curvature of the equivalent circular diffuser in accordance with the graph shown in Fig. 114.

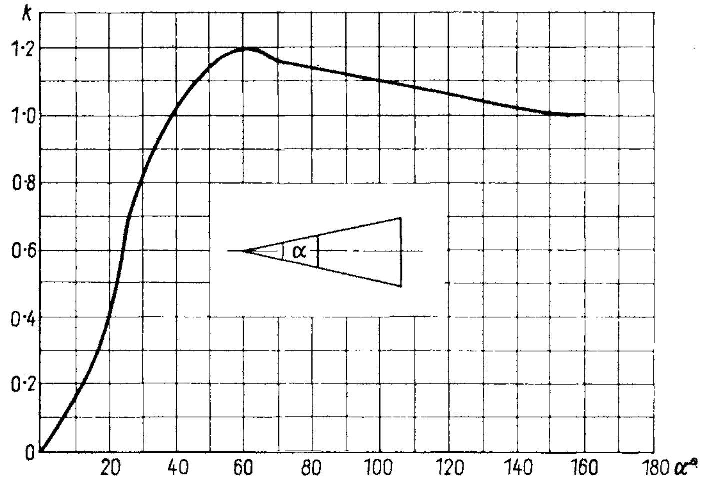

FIG. 114. Dependence of impact coefficient on the angle of a diffuser of circular cross-section.

In addition to the impact losses in section 2–I (See Fig. 113), losses of pressure by friction occur, determined according to the equation

$$\Delta p^*_{\text{friction}_{2-\mathrm{I}}} = \xi \frac{l}{r} \frac{\varrho_2 w_2^2}{2},$$

where r is the aerodynamic radius $r = \frac{F}{\Omega}$;

F is the area of the duct;

Ω is the perimeter of the wall of the duct in the section being considered;

l is the length of the section being calculated;

ξ is the aerodynamic drag coefficient.

It is helpful to the calculation to assign two or three variants of change of cross-section of the diffuser and to select the optimum case. A directly equivalent diffuser, profiled according to the law $\frac{\mathrm{d}p}{\mathrm{d}x} = \text{const.}$, can be recommended as the variants.

Let us further carry out the calculation for an annular duct charged with gases in all the marked section (Fig. 113a).

The dynamic pressure in section I is equal to

$$p_{\mathrm{I}}^* = p_2^* - \Delta p^*_{\text{imp}} - \Delta p^*_{\text{friction}}.$$

The static pressure in the section is easily determined, knowing the dynamic pressure and the velocity of the air in the section

$$p_{\mathrm{I}}^* = p_{\mathrm{I}} + \frac{\gamma w_{\mathrm{I}}^2}{2g}.$$

We shall assume in the calculation that γ in the secondary air duct does not change ($\gamma = \text{const.}$).

We determine the static pressure in the second section from the following relationship

$$p_{\mathrm{I}} + \frac{\gamma w_{\mathrm{I}}^2}{2g} = p_{\mathrm{II}} + \frac{\gamma w_{\mathrm{II}}^2}{2g} + \Delta p^*_{\text{friction}_{\mathrm{I}-\mathrm{II}}}.$$

The velocity of the air in the section is equal to

$$w_{\mathrm{II}} = \frac{G_{a_{\mathrm{II}}}}{F_{\mathrm{II}}},$$

where $G_{a_{\mathrm{II}}}$ is the air supply through the secondary air duct in section II.

$$G_{a_{\mathrm{II}}} = G_a - G_{a_{\text{prim}}} - \Delta G_{a_{\mathrm{I}}},$$

where $G_{a_{\mathrm{I}}}$ is the quantity of air entering the flame tube through the holes in section I;

F_{II} is the area in section II, assigned by a sketch of the composite combustion chamber.

Calculation of the static pressure in subsequent sections is carried out in the same manner.

(2) Calculation of the mean gas temperature along the flame tube.

We shall use the equation for the enthalpy, in the absence of work performed by the gas, for the calculation:

$$Q = i_2 - i_1 + A\frac{w_2^2 - w_1^2}{2g}.$$

In view of the smallness of the third term it can be neglected. Hence, from consideration of the thermal balance for the section between $i - 1$ and section i (Fig. 115) we have

$$t_i = \frac{c_{p\,i-2}\, t_{i-2} \sum\limits_{\text{first}}^{i-2} G + c_{p_a} t_a G_{i-1} + \Delta x\, G_f H_u}{c_{p_i} \sum\limits_{\text{first}}^{i-1} G},$$

where t_i is the mean temperature in section i;

Δx is the fraction of burnt fuel in the section under consideration, in accordance with the specified law of heat release (see Fig. 113d).

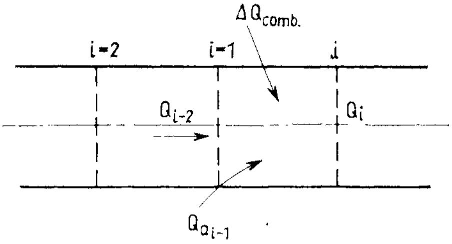

FIG. 115. Determination of the mean temperature in a section.

(3) Calculation of the variation of static pressure along the flame tube.

The static pressure of the air beyond the swirler can be found from the flow expression:

$$G_{\text{prim}} = \mu_{\text{flow}} F_{\text{flow}} \sqrt{2 g \gamma \Delta p},$$

where $\Delta p = p_0 - p_{01}$;

μ_{flow} is the flow factor, chosen according to experimental data.

In the flame tube we have the case of gas flow for variable area of straight-through section F, variable mass of gas G and for a supply of heat Q.

In order to calculate the static pressure between the sections being considered, the equation of reverse reactions can be used which, for the final section, is written down as

$$(M^2 - 1)\frac{\Delta p}{p} = k\left[M^2\left(\frac{\Delta G}{G} - \frac{\Delta F}{F}\right) + \frac{g}{A\,a^2}M^2\,(k-1)\,\Delta Q\right];$$

$$p_i = p_{i-1} \pm \Delta p\,,$$

whence

$$\Delta p = \frac{k\,p}{(M^2-1)}\left[M^2\left(\frac{\Delta G}{G} - \frac{\Delta F}{F}\right) + \frac{g}{A\,a^2}M^2\,(k-1)\,\Delta Q\right],$$

where $\frac{\Delta G}{G}$ is the relative variation of supply in the portion between sections $i - 1$ and i;

$\frac{\Delta F}{F}$ is the relative variation of area of the duct in the section under discussion;

$\frac{\Delta p}{p}$ is the relative variation of pressure in the section under discussion;

ΔQ is the quantity of heat supplied.

(4) Calculation of the variation of velocity along the flame tube.

Having used the analogous equation of reverse reactions for the variation of velocity in the final section, we find

$$\Delta w = \frac{1}{(M^2-1)\,w}\left[\frac{\Delta F}{F} - \frac{\Delta G}{G} - g\frac{k-1}{A\,a^2}\Delta Q\right];$$

then

$$w_i = w_{i-1} + \Delta w\,.$$

(5) Calculation of the holes in the walls of the flame tube.

From data obtained by us, the drop in pressure between the secondary duct and the flame tube in each of the individual sections is known, and the mean flow velocity in the flame tube is also known.

The velocity of the air in the holes of section i is equal to

$$w_{0\,i} = \sqrt{2\,g\frac{\Delta p}{\gamma}}\,.$$

The diameter of the hole which ensures the selected depth of penetration of the jet for the flow parameters as found, can be found from the equation

$$H = d\left[0{\cdot}3 + 0{\cdot}415\frac{w_0}{w_{\text{long}}}\right]\left(\frac{l}{d}\right)^{0{\cdot}63},$$

where H is the depth of penetration of the jet (Fig. 116);

d is the diameter of the orifice;

w_0 is the velocity of the air through the hole;

w_{long} is the longitudinal velocity of the stream;

l is the flow length.

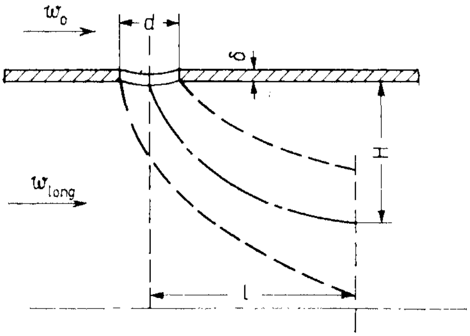

FIG. 116. Determination of depth of penetration of the air jet.

The flow factor μ for the hole depends on the ratio of the velocities w_0/w_{long} and upon δ/d.

$$\mu = f\left(\frac{w_0}{w_{\text{long}}} \frac{\delta}{d}\right),$$

where δ is the thickness of the wall of the flame tube.

The value of μ can be selected according to data presented in Fig. 117.

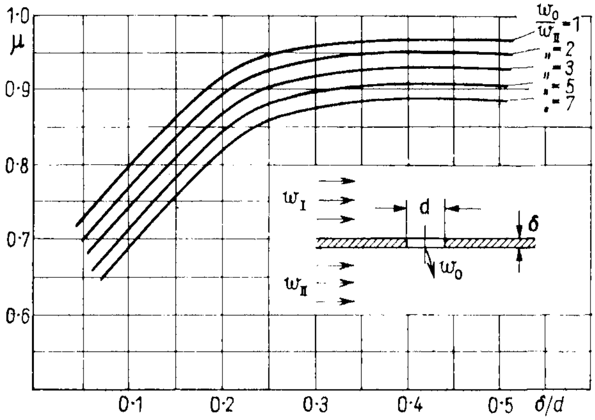

FIG. 117. Variation of flow factor μ as a function of the value $\frac{\delta}{d}$.

Then the flow of gas through one hole will be equal to

$$q_{\text{flow}_i} = \mu f_{\text{flow}} \sqrt{2 g \gamma \Delta p}.$$

The number of holes in a given section n will be equal to

$$n = \frac{\Delta G_{a_i}}{q_{\text{flow}_i}}.$$

The number n obtained is then rounded off to a whole number.

In a similar manner the calculation is carried out for all sections.

Knowing the parameters of the gas flow in the flame tube (w, p and T), the dynamic pressure in the final section p_3^* can easily be found, which enables the loss of dynamic pressure to be determined

$$\sigma_{\text{c.c.}} = \frac{p_3^*}{p_2^*} = 1 - \frac{\Delta p^*}{p_2^*},$$

and the aerodynamic drag coefficient

$$\xi_{\text{c.c.}} = \frac{\Delta p^*}{\frac{\gamma w_2^2}{2}}.$$

8. DEFECTS OF COMBUSTION CHAMBERS

Defects in the operation of combustion chambers, encountered as a result of their development and operating, may be caused either by unsatisfactory arrangement of the processes of mixture formation and combustion (bad mixture formation and insufficient thermal energy in the reversal zone, high flow velocities, etc., which lead to thermal losses as a result of considerable combustion loss, narrowing of the limits of operating stability of the chamber with respect to α, difficulties in starting etc.), or by inadequate mechanical strength of the walls of the casing and of the flame tube.

In the present section, we shall consider purely operating defects†, to which belong carbon deposition and inadequate heat resistance of the chamber structure.

One of the intrinsic defects in the operation of a combustion chamber is carbon deposition on the walls of the flame tube.

Usually, carbon deposition in a combustion chamber is accompanied by a smoky discharge and by a low value of the heat release factor. The cause of carbon deposition is the presence, near the walls, of stagnant zones as a result of local enrichment of the combustible mixture.

Carbon deposition occurs frequently on the injectors, and this disturbs the process of mixture formation and, consequently, disturbs the normal operation of the combustion chamber.

In spite of the fact that carbon deposition is an extremely detrimental phenomenon, it has, until now, been studied quite inadequately.

There are only isolated data in the literature which explain the appearance of carbon on the walls of the combustion chamber.

Investigations carried out in this sphere have established the principal factors affecting carbon deposition. The following are related to them,

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† These defects are manifested during the course of a relatively prolonged period of operation of the chamber.

primarily: mixture composition (ratio of weight of air to weight of fuel), atomization of the fuel, chemical composition, dynamic pressure of the incoming air, flow pattern in the primary zone.

It has been noted that as a result of bad atomization of the fuel carbon deposition is increased according to the extent of increase of the droplet size. The point of deposition of carbon is usually shifted close to the exit from the combustion chamber, depending on the extent of the increase in dimensions of the droplets.

The dynamic pressure of the stream entering the chamber is dependent, in the first place, upon the air flow through the swirler. An increase of dynamic pressure up to a certain value leads to a reduction of carbon deposition, obviously as a result of change of local fuel concentrations in the inlet portion of the combustion chamber, and an increase in the extent of turbulent flow, which leads to more intense and more complete combustion.

The chemical composition of the fuel also exerts a considerable influence on carbon deposition.

Experiments show that kerosenes containing aromatic hydrocarbons are more prone to carbon deposition than are kerosenes containing hydrocarbons of the naphthene or paraffin series. The data presented in Table 12 enables the tendency of a fuel towards carbon deposition to be assessed as a function of its chemical composition.

Table 12.

Kerosenes	Chemical composition of fuel in per cent			Carbon deposition in per cent against a standard fuel
	Aromatics	Naphthenes	Paraffins	
With added Aromatics	75.2	11.4	13.4	700
Standard	19.4	39.2	41.4	100
Naphthene base	1.4	51.9	46.7	70
Paraffin base	2.6	5.2	92.2	10

From the data presented in Table 12 it is obvious that the kerosene with added aromatics gives rise to the most substantial carbon deposition.

The effect of mixture composition† on the amount of carbon deposited on the walls of one of the test chambers can be assessed by the data presented in Table 13.

Table 13.

Ratio of air to fuel	10:1	20:1	30:1	40:1	45:1
Carbon deposition in chamber, g/hr	70	35	15	5	2

† The question here is a matter of the overall estimate of the ratio between air and fuel for all chambers.

Carbon deposition is increased according to the extent of enrichment of the mixture and, conversely, it is decreased by weakening of the mixture.

Carbon deposition is a detrimental phenomenon and indicates, in the first instance, an incorrect combustion stetting and, in certain cases, a tendency of the chamber towards burning and cracking.

Analysis of the damage to the flame tubes and casing shows that in the majority of cases it arises as a result of local burning of the material from which the chamber is fabricated. Burning of the walls of the flame tube is associated not only with cooling, but with the disposition of the flame. For example, with a flame located close to the wall, cooling is made difficult.

The surface temperature of the flame tube may vary considerably around the periphery and along the chamber.

As a result of large temperature differences over the surface of the flame tube, stresses are set up in the material which lead to buckling or to the formation of cracks.

The defects mentioned frequently do not appear at once, but only in the course of prolonged operation of the combustion chamber.

Work on the refining and investigation of the combustion process in the chamber, and also the utilization of heat-resistant alloys have made it possible to increase the life of a combustion chamber up to several hundreds of hours.

Nowadays, the lifetime of particular combustion chambers amounts to 300–700 hr.

CHAPTER VIII

COMBUSTION CHAMBERS OF RAM-JET ENGINES

1. GENERAL OBSERVATIONS

TYPICAL layouts for combustion chambers of ram-jet engines are depicted in Figs. 118 and 119, which show the basic and structural diagrams of ram-jet engines as a whole.

The combustion chamber of a ram-jet engine comprises several elements, the need for which arises from the fact that they should ensure good completeness and stability of the combustion process.

The pre-combustion chamber, or the primary chamber *a* (see Fig. 118) is intended for the formation of a flame core, by means of which ignition of the main bulk of the fuel–air mixture takes place†. Ignition of the combustible mixture in the primary chamber itself can be achieved by an igniter plug *c*, since the flow velocity in the primary chamber is small, and therefore the energy of the spark is adequate for ignition.

It is advantageous to supply the fuel to the primary chamber through a special starting jet *b*. The presence of a separate starting fuel system improves the operation of the primary chamber, making it independent of the main fuel supply. As a result of the presence of a special starting fuel system, starting of the chamber is improved (starting is made "smoother") — owing to the fact that in the primary chamber, independent of the operating conditions of the engine (α_{Σ}), the most intensive combustion process can be substained — and the operation of the chamber as a whole is improved, i.e. the combustion efficiency is improved and the limits of stable operation are extended.

The baffles or stabilizing devices serve to increase the completeness and stability of combustion by creating local, continuously-operating strong sources of ignition. The stabilizing devices in the chamber of a ram-jet engine usually consist of a distinct combination of bluff bodies. Reverse flows of hot combustion products are formed behind the stabilizers, which

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† Chambers of ram-jet engines can exist without the special primary chamber (see Fig. 121), the role of which in this case is fulfilled properly by stabilizing devices.

are subsequently mixed in around the periphery of the bluff body with the fresh fuel–air mixture and ignite it.

These stabilizing devices, made in the form of "conical" rings (see Fig. 128a), individual cones, joined conical rings (see Fig. 122), flat gratings of angularly-symmetrical shape — "gutter" — the apices of which are aligned contrary to the flow (see Fig. 119), systems of radially distributed gutters (see Fig. 120), etc., are widely used in the combustion chambers of ram-jet engines.

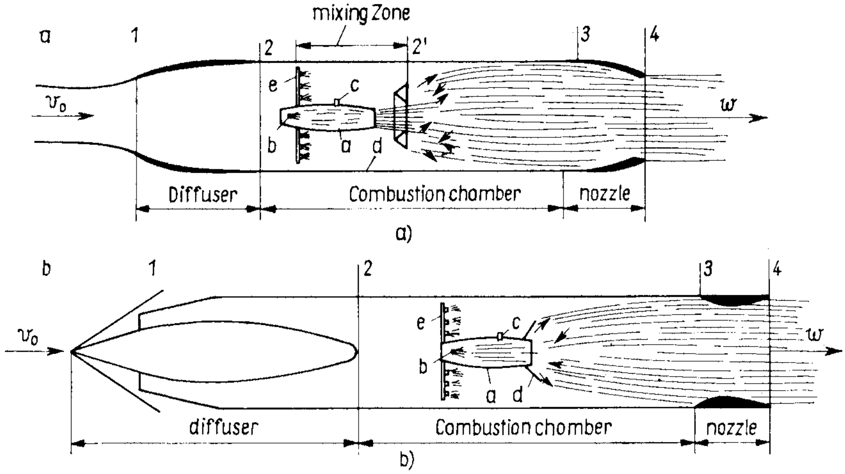

FIG. 118. Schematic diagram of a ram-jet engine.
(a) Subsonic ram-jet engine; (b) Supersonic ram-jet engine.

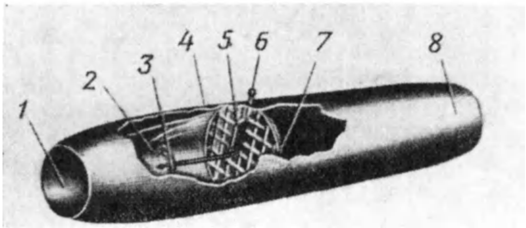

FIG. 119. Diagram of subsonic ram-jet engine.
1 — Air intake; 2 — Diffuser; 3 — Fuel injectors; 4 — Engine housing; 5 — Starting device; 6 — Fuel supply; 7 — Flame stabilizer; 8 — Exhaust nozzle.

The fuels in ram-jet engines are usually hydrocarbon fuels — gasoline, kerosene, etc.

The fuel is fed into the chamber through injectors linked by a common fuel manifold as shown in Figs. 118–121.

The quantity and type of injectors in the manifold may be very varied. The best arrangement for the fuel manifold relative to the stabilizing devices is established by experiment†. The possibility is not excluded of an arrangement of injectors in the direct proximity of the stabilizer, as shown in Fig. 121. Such an arrangement of injectors is particularly advantageous if the chamber, with respect to operating conditions, should operate over a wide range of variation of the overall excess air coefficient α.

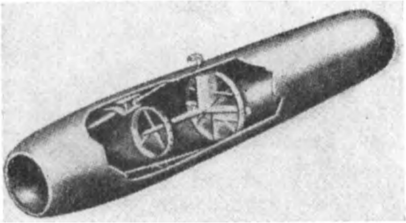

FIG. 120. Subsonic ram-jet engine.

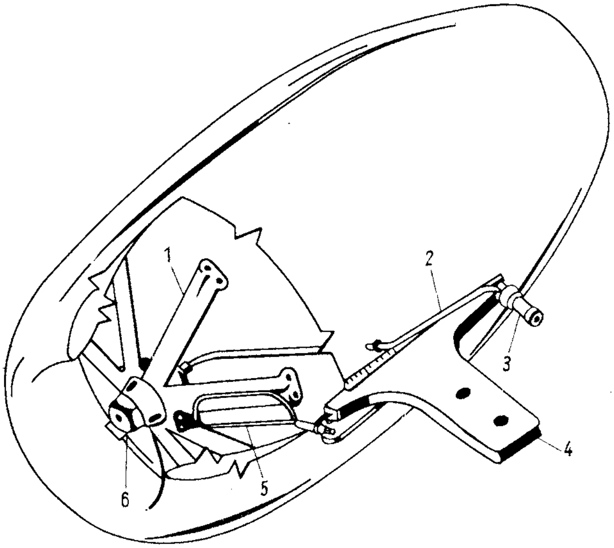

FIG. 121. Flame stabilizer and fuel feed system.
1 — Flame stabilizer; 2 — Fuel pipeline; 3 — Fuel pipeline inlet; 4 — Engine bracket; 5 — Fuel supply for starting system; 6 — Injector.

With respect to the operating process, the combustion chambers of ram-jet engines are distinguished from the chambers of turbo-jet engines by a higher mean flow velocity†, (w up to 100 m/sec) of the gas, a lower value of

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† One has in mind the velocity of motion related to the maximum cross-section of the combustion chamber, for the gas parameters prior to combustion.

α_{Σ} ($\alpha_{\Sigma} < 3{\cdot}0$) and a narrower range of operating stability with respect to α_{Σ} ($\Delta\alpha_{\Sigma} = 1$–$2$) this, in particular, makes it possible to feed the fuel uniformly through that section of the chamber which is set aside for combustion.

It should be noted that in order that combustion should proceed most efficiently†, the excess air coefficient in the combustion zone should be close to 1·0. The excess air should be led outside the combustion zone and it should be mixed with the combustion products after the combustion process has been mainly completed.

In view of the high temperature of the combustion, which attains 2000–2500 °K, the combustion chamber of a ram-jet engine as a rule has a cooling system in order to safeguard the combustion chamber units from the effect of the high temperature which exists in its "hot" portion.

The excess air, led beyond the combustion zone, is also used for cooling the units of the combustion chamber, including also the outer casing of the chamber.

Schematically, the combustion chamber of a ram-jet engine, taking into account the by-passing of the excess air and cooling of the chamber, can be represented in the following manner. The air passing through the chamber is divided into two parts, the first part of which is led into the annular duct formed by the outer casing and the flame tube of the chamber, and the second part passes through the flame tube and participates directly in combustion. The cooling air flows through the gap formed by the flame-tube and the outer casing of the chamber (see Fig. 122).

Finally, depending upon the layout of the chamber, the air passing into the annular duct may partially penetrate into the combustion zone through

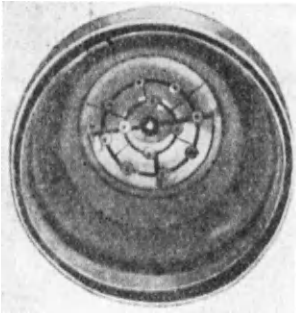

FIG. 122.
Chamber of a ram-jet engine
(view from nozzle side).

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† By the efficiency of the combustion process is understood the completion of the combustion for given flow parameters (velocity, temperature, pressure) in the minimum length and the attainment of the maximum range of operating stability.

holes in the flame tube and participate in the combustion, but its main purpose consists in the safeguarding of the outer casing of the chamber and the flame tube from the effects of the high temperature and to enable the requisite value of α to be established in the combustion zone.

The air issuing from the annular duct is mixed with the combustion products either in the combustion chamber or (partially) in the nozzle, where it can also be utilized for cooling the walls of the nozzle.

As can be seen from the layouts considered, the combustion chambers of ram-jet engines are substantially different in their construction from the construction of the combustion chambers of turbo-jet engines described earlier.

First and foremost it should be recalled that the combustion chambers of ram-jet engines have a considerably lower aerodynamic pressure loss coefficient ($\xi \leq 3.0$), which is compulsory for the combustion chambers of ram-jet engines (particularly for the chambers of ram-jet engines designed for subsonic flight velocity).

If the chambers of turbo-jet engines are used in ram-jet engines, then the aerodynamic losses which will occur in this case will lead to a strong deterioration of the engine characteristics.

Thus, for example, the specific characteristics (specific thrust, specific consumption) of a subsonic ram-jet engine may deteriorate by more than 10–20 per cent and correspondingly for a supersonic engine by more than 5–8 per cent.

It is possible to reduce the losses by reducing the velocity, i.e., by increasing the diameter of the chamber, which, in the majority of cases is inadmissible.

The lower aerodynamic pressure loss impairs the fuel burnout along the combustion chamber, and this will lead to the fact that the combustion chambers of ram-jet engines have a considerably greater length than the chambers of turbo-jet engines.

The difficulty in the practical feasibility of combustion chambers consists in the fact that measures which are effective in achieving one of the requirements set forth for the chamber, contradict the measures ensuring the fulfilment of other requirements.

Thus, for example, in order to attain a high combustion efficiency it is beneficial to increase the aerodynamic pressure drop of the chamber, its diameter and length.

Consequently, the combustion chamber of any engine represents a more or less successful compromise between different contradictory requirements.

It should be noted that the information found in the literature concerning the design of combustion chambers for ram-jet engines is very scanty and consequently opinions concerning their construction are only of an extremely general nature.

We shall pass on to a consideration of the characteristics of combustion chambers and the effect on them of the main components of the chamber.

2. CHARACTERISTICS OF A COMBUSTION CHAMBER

The operation of a combustion chamber is characterized by certain general properties which are defined by operating requirements. The following may be pointed out as primary characteristics of combustion chambers: extinction characteristics; characteristics with respect to combustion efficiency; starting characteristics.

Extinction Characteristics

We shall assume by the extinction characteristics of a combustion chamber the relationship $w_{av} = f(\alpha_{\Sigma\,av}, p_2, t_2)$ where w_{av} is the mean mass velocity in any characteristic section of the chamber, at which combustion ceases and $\alpha_{\Sigma av}$, p_2, t_2 are the corresponding values of α_Σ, pressure p_2 and air temperature t_2 in the chamber.

As is wellknown from the theory of combustion, cessation of combustion may occur in the case when local concentrations of fuel-mixture in the combustion zone (particularly in the region of ignition) become "hot", but the strength of the ignition source is inadequate for igniting the mixture.

A typical picture of the extinction characteristics is shown in Fig. 123†.

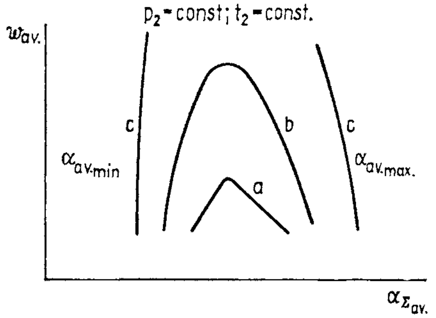

FIG. 123. Extinction characteristics of the combustion chamber of a ram-jet engine.

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† Clearly, different combustion chambers may have the same diverse operating values for α_Σ depending upon the requirements set for the operating conditions of the engine. Consequently, the extinction characteristic for different combustion chambers will be displaced relatively along the axis α_Σ. The "combination" of extinction characteristics in Fig. 123 is prompted by the desire to show the nature of the variation of $\alpha_{\Sigma av}$, with respect to w_{av} and to enable the limit of the operating range of the chamber to be assessed relatively.

Depending upon the design of the combustion chamber, the dimensions of the stabilizing devices etc., the characteristics *a*, *b* or *c* may hold, from which the best with respect to range of operating stability is the characteristic *c*.

It is obvious that the operating zone of the chamber is represented by the area enclosed within the line $w_{av} = f(\alpha_\Sigma)$.

The pressure, temperature and turbulence of the air admitted also exert an influence on the extinction characteristics of a combustion chamber.

By reducing the pressure and the temperature of the air, the limits of operating stability $\Delta\alpha_\Sigma = (\alpha_{av})_{max} - (\alpha_{av})_{min}$ are narrowed, as shown in Fig. 124.

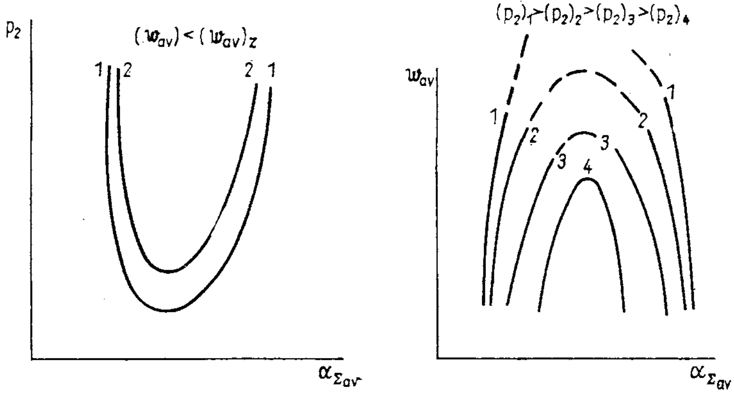

FIG. 124. Effect of pressure and velocity of the air on entry into the combustion chamber on the extinction characteristics.

At the same time, a region of substantial reduction of the range of operating stability is observed for $p_2 < 1$ atm. If $p_2 > 1$ atm, then the effect of pressure is said to be relatively small.

Reduction of the air temperature can also lead to narrowing of the limits of operating stability.

In a quantitative relationship the effect of p_2 and t_2 will be manifested differently for different combustion chambers. In particular, the greater the drag of the combustion stabilizing system†, the more stable will be the operation of the chamber for approximately the same conditions.

The extinction characteristics of combustion chambers are determined by experiment: in order to obtain them it is necessary to start up the chamber and vary the fuel supply (or the air) until extinction does not set in.

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† Naturally, this consideration only applies completely to efficiently designed chambers.

At the instant of extinction it is obviously necessary to know the supply of fuel and air (in order to determine $\alpha_{\Sigma \text{av}}$), and also the parameters of the air prior to combustion, p_2 and t_2.

The principal devices for stabilizing combustion in the chambers of ram-jet engines (and the reheat chambers of gas-turbine engines) are badly streamlined bodies and, to a certain extent, the flame of the primary combustion chamber.

In view of the importance of the problem, we shall consider the operation of a badly streamlined body (bluff body) as a flame stabilizer.

It is wellknown from theory that a stationary combustion process is feasible only in the case when a continuously operating source of ignition is formed behind it, which is shown schematically in Fig. 125.

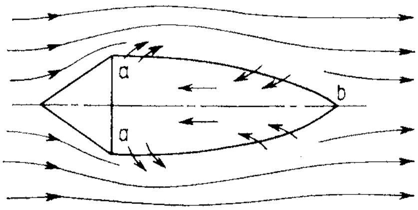

FIG. 125. Schematic diagram of the flow behind a stabilizer (along the line *ab* the axial velocities are equal to zero).

On combustion, the combustion products getting into the recirculation zone are mixed in, on discharging, with the fuel–air stream and ignite it.

The thermal state of the recirculation zone and the volume between the zone and the main flow play a decisive role in the operation of the stabilizing device (in the given case a bluff body).

Theory and experiment indicate that the operation of the stabilizer depends on the composition of the combustible mixture α_{av} in the region of the stabilizer, the velocity and degree of turbulence of the surrounding stabilizer flow, the dimensions of the stabilizer and, finally, on the temperature and pressure of the fuel–air stream.

A typical picture, characterizing the operation of stabilizers, is shown in Fig. 126, where the extinction characteristics are presented for similar stabilizers of different dimensions.

It can be seen from Fig. 126 that an increase in the flow velocity leads to a narrowing of the operating range of the stabilizer with respect to α_{av}, and at the same time the effect of velocity is greater, the smaller are the dimensions of the stabilizer.

Commencing with certain dimensions *d*, the range of stable operation depends only relatively slightly on the velocity of the surrounding stream, which is most marked in the region of rich flameout (lines 1 and 2 in

Fig. 126). At the present time the theory of stabilization is inadequately developed. Consequently, without giving an account of the work carried out in this sphere, we shall limit ourselves to a short explanation of the qualitative aspects of the problem arising from a simplified (one of the possible) diagram of the phenomenon. The ignition of a certain moving volume of gas can be represented in the following manner. We shall introduce the volume of gas into which the hot combustion products have penetrated due to the presence of turbulent pulsations (in the stabilizer zone).

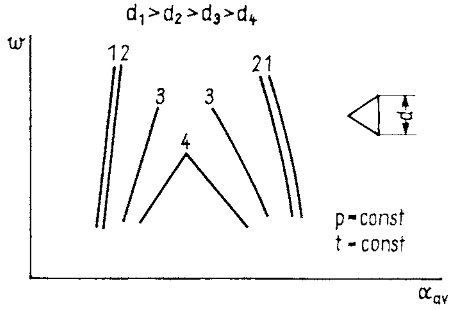

FIG. 126. Effect of size of stabilizer on the extinction characteristics.

If, as a result of this, the quantity of heat brought in was sufficient to increase the temperature of this volume to such a value at which a rapid development of a chemical reaction for given conditions is possible (for example, apart from the supply of heat, there is also its removal), then ignition takes place, and in the converse case — there is no ignition.

It is obvious that one of the principal factors determining ignition is the quantity of heat Q_M, generated by the zone per unit mass GM of the main stream (here, that part of the primary stream is taken into which the interchange from the zone occurs immediately).

Obviously, Q_M can be expressed by the following formula:

$$Q_M = \frac{G_l\, T_{\text{zone}}\, c_p}{w\, h_1\, \gamma_1}\ \text{cal/kg}, \qquad \text{(VIII.1)}$$

where $G_l = \frac{G}{l}$ is the quantity of gas carried away to the perimeter of the stabilizer, having passed from the zone into the main stream in unit time;

T_{zone} is the temperature of the gas in the zone;

w is the flow velocity at the rim of the stabilizer;

h_1 is the transverse dimension which determines the width of the jet over which the mass transfer is directly spread, in some characteristic section behind the stabilizer (see Fig. 127);

γ and c_p are the specific gravity and specific heat of the gas in the main stream.

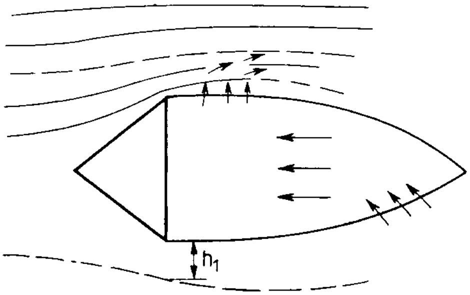

FIG. 127. Flow diagram of the mass transfer of the gas behind a stabilizer.

The temperature in the zone for a given mixture composition is determined by the interval of time τ_{res} (residence time) which elapses from the instant of ignition to the instant of exit of a gas particle from the zone and its joining up with the main stream†.

If it be assumed that the transfer for a given velocity is proportional to a first approximation to the volume of the zone, then the specific transfer G_l, i.e. the mass transfer relative to unit length of the perimeter of the stabilizer, will vary for conical stabilizers as the cube of the diameter, and for long flat stabilizers it will vary as the square of the width of the stabilizer h (see Fig. 128), since it can be assumed approximately that the length of the zone is proportional to d or h.

If it be further supposed that the transfer is proportional to the surface area of the zone, then G_l increases proportionally to d or h, respectively, with increase of the stabilizer dimensions (diameter or width).

It is natural also to suppose that τ_{res} increases with increase in size of the stabilizer and decreases with increase in velocity.

Thus, it can be assumed approximately that firstly, with increase in the dimensions of the stabilizer the specific quantitative transfer between the zone and the main stream G_l is increased, and secondly, the conditions are created for obtaining maximum temperature in the zone, corresponding to a given α_{stab}, since the time τ_{res} increases, as a consequence of which

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† In the considerations presented we have in mind certain mean values characterizing the process as a whole.

a high combustion efficiency can be obtained. Hence, it is possible to draw a general conclusion that an increase in the dimensions should have a favourable effect on the extinction characteristics of the stabilizer on account of the increase of Q.

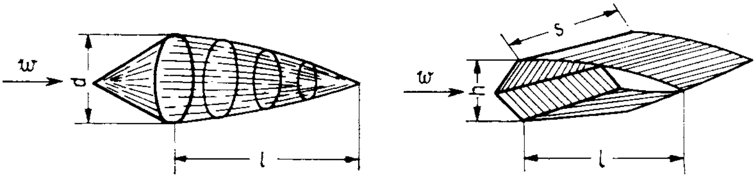

FIG. 128. Determination of the magnitude of the mass transfer.

The effect of increasing Q_M with variation of the velocity can be established from the following considerations: by increasing the velocity the magnitude of G_l will also be increased; however, if it be assumed that $G_l \sim w$, then the ratio G_l/w, entering into expression (VIII.1), is unaltered. The temperature T_{zone} may be reduced because of the reduction of τ_{res}, which in turn leads to a reduction of Q_M.

Moreover, by increasing the velocity (or by reducing the size of the stabilizer) the time of contact τ_c of the hot mixture with the surface of the zone is shortened. If τ_c is less than the time required for the reaction to develop, then ignition (if it is found to be generally possible) takes place outside the zone, which will no longer be "fed" by the hot combustion products from the main stream. This regime (when the flame is displaced towards the end of the zone or breaks away from it), as experiment shows, corresponds to cessation of combustion, i.e. the formation of a stable flame becomes impossible.

Thus, with increase in velocity, narrowing of the limits of operating stability of the stabilizer can be expected, which is also observed in practice (see Figs. 123–126).

For a stabilizer of small dimensions, the relationship $\tau_{\text{res}} < \tau_{\text{comb}}$ is characteristic, where τ_{comb} is the time required for the combustion reaction to develop (all the preliminary processes and especially the combustion process). Since τ_{comb} depends to a marked degree on α_{stab}, then it can be assumed that the range of operation (if such is only generally observed) of the stabilizer will be extremely narrow and close to the value for α_{stab} at which the combustion velocity attains the maximum limit.

As a result of increasing the velocity, in consequence of a reduction of τ_c and τ_{res}, the range of operating stability will continue to become narrower until at a certain value of $w_{\max}$ stable operation becomes generally impossible.

With increase of size of the stabilizer, G_l, τ_c and τ_{res} increase, which widens the zone of stable operation with respect to α_{stab} and w. At the same time, again the value of α_{stab}, corresponding to the value of w_{max}, will determine the maximum velocity for stable combustion.

It is quite possible to represent the case when, starting with a certain size (d or h) $\tau_{\text{res}} \gg \tau_{\text{comb}}$ and T_{zone} attains its maximum value, but the value of Q_M and τ_c will ensure full and reliable ignition of the mixture over a wide range of velocities. As a result of this it can be anticipated that the extinction characteristics of the stabilizer will not be dependent on w.

The extinction characteristics of "large" stabilizers with respect to weak and rich extinction are determined here only by the concentration limits, i.e. by kinetic factors, and represent two parallel lines in the coordinates of w and α_{stab}.

The considerations presented are completely confirmed in practice: the extinction characteristics obtained by experiments for "large" stabilizers are, in fact, represented by two almost vertical lines, concentrated in the region of one and the same values of α_{stab} (for example, lines 1 and 2 in Fig. 126).

Finally, we shall investigate the effect of temperature and pressure on the extinction characteristics.

The effect of the temperature of the fuel–air stream flowing round the stabilizer consists in the fact that with increase of temperature of the combustible mixture T_1, the value of G_l, necessary for ignition, is reduced, and the time τ_{comb} is shortened.

In order to explain what has been said we shall analyse the simplest example.

Let us assume that for the ignition of 1 kg of fuel–air mixture under specific conditions it is necessary to increase its temperature from T_1 to T_{ig}.

Increase of temperature of the mixture occurs on account of the mixing in with it of G kg of combustion products having a temperature of $T_1 + \Delta T$.

From the equation for thermal balance

$$c_{p1}\, T_1 + G\, c_p\, (T_1 + \Delta T) = (1 + G)\, c_{p_{ig}}\, T_{ig}\ ;$$

assuming for simplicity that $c_{p_1} = c_p = c_{p_{ig}}$, we obtain

$$G = \frac{T_{ig} - T_1}{(T_1 + \Delta T) - T_{ig}} = \frac{1}{\dfrac{\Delta T}{T_{ig} - T_1} - 1}. \qquad \text{(VIII.2)}$$

Assuming that ΔT is invariable for change of T_1 (which in our case implies neglecting the effect on τ_{comb} of the initial temperature T_1 and $\alpha_{\text{stab}} = \text{const.}$), we can see immediately from the expression obtained that according to the extent of approximation of T_1 to T_{ig}, G is decreased, to say nothing of the additional effect of T_1 on the increment ΔT.

Thus, in these cases when the thermal strength of the opposed zone is inadequate for supporting combustion in consequence, for example, of the small mass transferred, the increase in T_1 may lead to an extension of the limits of stable operation of the stabilizer.

For "large" stabilizers the effect of temperature may only be felt in that the initial temperature influences the concentration limits of ignition. From the formula given for G and from general considerations it is obvious that according to the extent of the increase of T_1 the necessity for stabilizing devices gradually diminishes. The design of the combustion chambers in this case should be simplified, and their pressure losses will be reduced. The effect of the pressure p on the extinction characteristics of the stabilizer cannot possibly be assessed until completion of the installation. The effect of p, it seems is connected with the increase in τ_{comb} as a result of reduction of the pressure, and also with the change of the conditions of heat exchange between the elementary volume to which heat is supplied from the recirculation zone, and the entire mass of the gas.

The effect of the stabilizer on the combustion process is not limited by the arrangements for igniting the mixture.

An increase in the flow turbulence in the stabilizer wake leads to an increase in the velocity of propagation of the flame, and consequently to an overall intensification of the combustion process.

In practice this leads to a shortening of the length of the flame jet and to a corresponding reduction of the length of the combustion chamber.

Finally, it should be noted that the extinction characteristics of the stabilizer under combustion chamber conditions may be other than in open flow because of the effect of the flame from the primary chamber which ensures an additional supply of heat to the fuel–air mixture, and also to that part of the recirculation zone of the stabilizer (stabilizers), which is adjacent to it.

On the basis of what has been said, it is easy also to represent the significance of the primary chamber, which not only serves for the initial ignition of the fuel–air mixture, but also improves the operation of the stabilizer, extending the limits of operating stability of the chamber, particularly in the region of weak extinctions, and contributes to a more intense development of the process along the chamber.

From what has been said, it is easier to understand why for approximately equal conditions a given chamber possesses a greater range of stable operation and a better intensity if it possesses a greater pressure loss.

Actually, stability is attained by increasing the dimensions of the stabilizers, by increasing the flame of the primary chamber (and consequently also by increasing the dimensions of the primary chamber) and its pressure loss. In order to increase the combustion efficiency in a relatively small length it is necessary to increase the number of stabilizing units, such as units which ensure ignition and which increase the rapidity of flame propagation.

It should be noted that the extinction characteristics of stabilizers, with a correction for the effect of the flame of the primary chamber, do not always define a rich limit in the combustion chamber, which sets in more rapidly (at high α_{Σ}) than the stabilizers permit. The causes of this phenomenon may be very varied, but they all lead to the appearance of unstable oscillatory processes in the system, which may either develop substantially without leading to cessation of combustion, or may lead to flame extinction at the instant of their origination.

Without dealing with all the possible causes leading to an oscillatory process, we shall consider one which is, it appears to us, of considerable importance for chambers, with continuously formed flames, which in essence are the combustion chambers of ram-jet engines and reheat chambers.

Oscillatory processes, which lead to extinction or abnormal "pulsating" operation of the chamber, will be observed at all times when conditions arise requiring reorganization of the flow (for example, change of overall flow as well as local, etc.).

One of these conditions may prove to be the appearance in the combustion chamber of local sonic zones, which "choke" the flow in certain regions and give rise to a re-distribution of the flows throughout the chamber, which leads to the disappearance of the sonic zone at the place of its origination but causes its appearance at some other place.

When the local sonic zones encompass a considerable proportion of the whole stream, then the disturbances associated with reorganization of the flow may be considerable, and may lead either to marked pulsations or directly to cessation of combustion.

The simplest example of this is the process $f =$ const., analysed in detail above, for the discussion of which it was assumed that heat is supplied uniformly throughout the entire cross-section.

It has been shown that, for a certain value of relative preheating τ_{crit}, sonic velocity is attained, and further supply of heat becomes impossible without reducing the flow (more accurately, the Mach number M_2) prior to the supply of heat.

We shall introduce the combustion chamber of a ram-jet engine, in which fuel is supplied in a quantity ensuring that the preheating $\tau > \tau_{\text{crit}}$.

Since there is sufficient oxygen in the chamber, then a combustion process will take place, but the flow through the engine is practically stopped because of the fact that the combustion will take place, as it were, in a "semi-closed" volume and will be accompanied by a corresponding increase in pressure; the combustion products commence to discharge through the diffuser as well as through the nozzle.

Combustion is stopped by the burnout of the combustible mixture, the pressure in the chamber is reduced (and may become considerably lower than it was initially), the supply of air is re-established (and, if $M_0 < 1{\cdot}0$, then it is possible that part of the air will be admitted through the nozzle) and the phenomenon is then repeated in the entire volume, providing that conditions can be guarunteed for the ignition of the fresh fuel-air mixture.

In continuous flame chambers, which are in essence the chambers of ram-jet engines and reheat chambers, local sonic zones, "choking" the flow, may make their appearance considerably earlier than the attaining of τ_{crit}.

The simplest example, indicating the possibility of "choking" by a cold stream in continuous flame chambers is the case shown in Fig. 129.

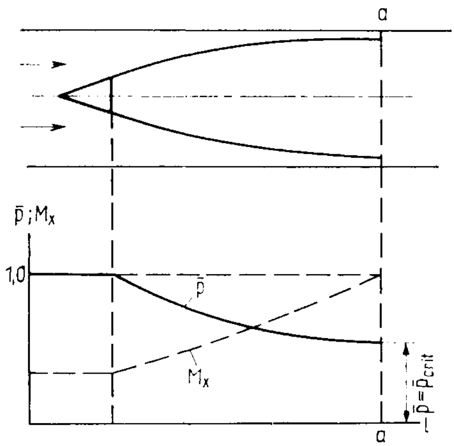

FIG. 129. Consideration of the phenomenon of the attainment of critical flow velocity in a continuous flame chamber.

It is wellknown that the supply of heat in a cylindrical chamber is accompanied by an increase in velocity and a corresponding reduction of the static pressure.

At the same time the static pressure may be reduced, as a result of this, by more than a factor of two. If it is assumed for simplicity that there is no heat exchange between the "hot" stream, then sonic velocity is attained by the "cold" stream in the section *aa*, in which $p_1^*/p \approx 1{\cdot}84$, and further supply of heat becomes impossible without re-organization of the flow. The phenomenon which originates as a result of this will be similar to the phenomenon described above.

In multi-jet chambers with a considerable number of stabilizers, similar phenomena of "choking" may make their appearance at different places in the chamber, as a result of which fluctuations of all the flow parameters arise, amongst them also α_{stab}, which may lead to cessation of combustion.

Such, in general outline, is one of the causes which may displace the limits of "rich extinction" to the side of large values of $(\alpha_{\Sigma})_{\text{av}}$ in comparison with $(\alpha_{\Sigma})_{\text{av}}$ with respect to the stabilizer.

It should be noted that the requirements set forth for the extinction characteristics of the chamber of a ram-jet engine, on account of the narrowness of the range of operation with respect to α_{Σ}, as a rule is lower than the requirements set forth for the chamber of a gas-turbine engine.

This fact simplifies the fuel supply system. In particular, in a number of cases it is possible to arrange for supplementary mixture formation by means of a common fuel manifold, giving rise to approximately uniform distribution of fuel concentrations throughout the flame region of the combustion chamber.

If it is further necessary to extend substantially the operating range of the combustion chamber of a ram-jet engine with respect to α_{Σ}, then the principle should be applied which permits the creation of local concentrations in the region of the stabilizers, only slightly dependent upon a change of α_{Σ} or to proceed by way of the building of a cannular chamber, a part of which may not operate beyond conditions greater than α_{Σ}.

Characteristics of a Combustion Chamber with respect to combustion efficiency

It was pointed out that the completeness of combustion is assessed as the so-called combustion efficiency ξ_z, representing the ratio

$$\xi_z = \frac{Q_{\text{act}}}{Q_{\text{theor}}} = \frac{\text{Actual heat released}}{\text{Heat supplied by the fuel}}.$$

The coefficient ξ_z characterizes the efficiency of the combustion process in the chamber and is one of its fundamental operating characteristics.

The effect of ξ_z on the engine characteristics can be assessed from various points of view. For example, by assuming a constant fuel supply, the conclusion can be reached that a change of ξ_z affects the thrust (by changing the temperature in the chamber) and the efficiency (C_{sp}) characteristics of the engine.

It is more convenient to assume that ξ_z has no effect on the thrust characteristics of the engine, which, for approximately similar conditions, depends only on the combustion temperature T_3^*, and affects only the fuel supply which is necessary to obtain the specified temperature T_3^*.

In this case ξ_z determines only the specific fuel consumption (efficiency) of the engine:

$$C_{\text{sp}} = C_{\text{sp}0}\, \xi_z\,,$$

where C_{sp0} is the specific fuel consumption for a given value of T_3^* on the assumption that $\xi_z = 1{\cdot}0$.

In Fig. 130, typical chamber characteristics are shown with respect to the combustion efficiency as a function of α_Σ, w, p and the temperature of the fuel t_f.

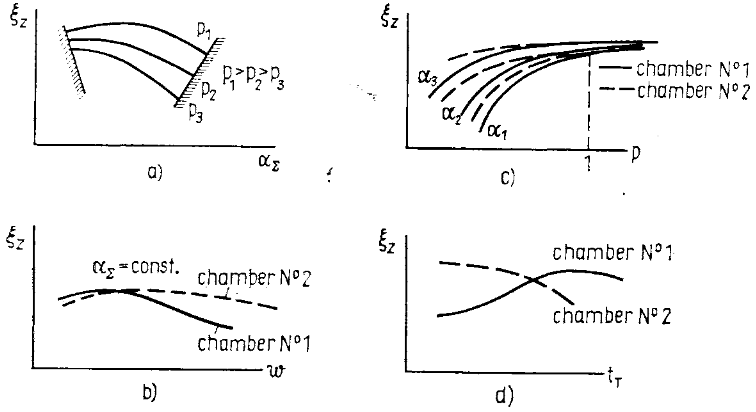

FIG. 130. Chamber characteristics with respect to combustion efficiency.

The relationship $\xi_z = f(\alpha_\Sigma)$ has a sloping maximum. Change of ξ_z is inherent, since it cannot be expected that for all operating values of α_Σ those factors will be maintained unchanged which have an influence on combustion, as for example, the quality of the mixture formation α_{comb} on the stabilizers, local values of α in the flame core, local flow velocity in the chamber, etc.

It is obvious that $(\xi_z)_{max}$ corresponds to the most favourable combination of all the factors indicated.

By means of a reasonable distribution of the air throughout the combustion chamber, in conjunction with the fuel supply system, it is always possible to ensure that $(\xi_z)_{max}$ will lie in the main operating portion of the chamber with respect to α_Σ. By these same measures, the nature of the relationship $\xi_z = f(\alpha_\Sigma)$ can be changed, making it, for example, more sloping.

An increase of velocity w in the chamber, as a rule, leads to a reduction of ξ_z; it is only in the region of small (in relation to the velocity under the main operating conditions) velocities that another picture can be observed (see Fig. 130).

The reduction in ξ_z by an increase in velocity is connected with the shortening of the residence time of the products of incomplete combustion in the chamber, and also with the change of operating regime of the stabilizing units†.

The form of the relationship $\xi_z = f(w)$, just as the extinction characteristics, depends markedly on the design of the chamber (see the curves presented in Fig. 130b).

The pressure at which combustion takes place exerts a considerable influence on ξ_z. It can be seen from Fig. 130c that ξ_z commences to decrease significantly at $p < 1{\cdot}0$ atm.

The extent of the effect of p is different for different chambers, but every chamber has its characteristic value p', starting at which ξ_z falls abruptly (see Fig. 130c).

In general it should be borne in mind that for small pressures p, the combustion process becomes extremely sensitive to change of any parameter having an effect on it.

In order to obtain the relationship of the characteristic $\xi_z = f(\alpha_\Sigma, p_2)$, of practical importance, experimental data are necessary. For approximate estimates, the form of the function $\xi_z = f(\alpha_\Sigma, p_2)$ can be represented by the expression

$$\frac{\xi_z}{(\xi_z)_{\max}} = \varphi_1(p) + \varphi_2(\alpha_\Sigma, p) ,$$

where ξ_z and $(\xi_z)_{\max}$ are the instantaneous and maximum values, respectively, of the combustion efficiency.

The function $\varphi_1(p)$ determines the effect of p for the most advantageous value of α_Σ, and the function $\varphi_2(\alpha_\Sigma, p)$ refers to the correction for the effect of mixture composition, taking into account the change of pressure.

An extremely rough estimate of the nature of the change of ξ_z relative to p (for $p < 1$) can be derived according to the empirical formula

$$\xi_z = 1 - \left[\frac{1}{a\,e^{np}} + (\bar{\alpha}^2 - 1)\,(b - c\,p^2)\right], \qquad \text{(VIII.3)}$$

where p is the pressure in the combustion chamber in atm, $\bar{\alpha}$ is the ratio of the instantaneous excess air coefficient to its most advantageous value. For $p > 1$ atm, the invariable value of $p = 1$ atm should be substituted in the formula, which expresses arbitrarily the independence of the combustion efficiency on the pressure for $p > 1$. The formula as shown can be applied only for $\bar{\alpha} > 0$.

The coefficients a, n, b and c should be defined on the basis of experimental data.

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† It was shown above that with increase of velocity, a reduction in Q_M may be observed, at the expense of T_{zone} and h_1. This leads to the fact that the combustion process will develop relatively weakly along the combustion chamber and the quantity of combustion products is increased in the flame.

Finally, if the experimental points are adequate for plotting $\xi_z = f(p_2, \alpha_\Sigma)$ within all the intervals of change of p_2 and α_Σ of interest to us, then the necessity for applying formula (VIII.3) falls off.

But if the experimental points are inadequate and they do not embrace the entire range of variation of α_Σ and p_2, then the use of formula (VIII.3) becomes expedient.

In the absence of experimental data, one can take for the purpose of an extremely rough estimate: $a = 2{\cdot}0$; $n = 3{\cdot}5$; $b = 0{\cdot}055$ and $c = 0{\cdot}04$.

The values of ξ_z calculated according to the formula are presented in Fig. 131. Attention should also be paid to the effect of the temperature of the fuel on ξ_z. The case should be included here when some preheating of the fuel promotes a better mixture formation and gives a considerable increase of combustion efficiency (see Fig. 130d).

However, owing to the fact that the extent of the effect of preheating of the fuel is determined to a considerable extent by the individual peculiarities of the different chambers, the effect of t_f on ξ_z may be extremely different (see Fig. 130).

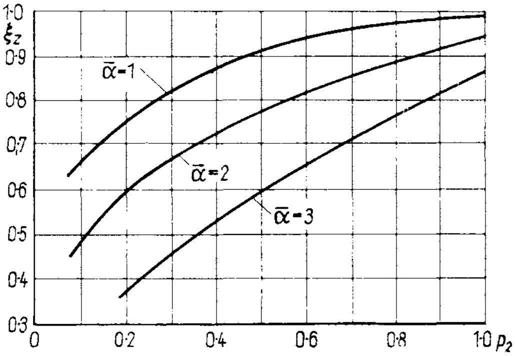

FIG. 131. Variation of combustion efficiency versus chamber inlet pressure.

Moreover, the effect of t_f on ξ_z is determined also by the properties of the fuel itself. A test of the operation of stationary combustion chambers operating on fuels having a high viscosity, indicate that preheating of the fuel considerably facilitates the combustion process, since in the contrary case it is difficult to attain good mixture formation.

Increase in the air temperature exerts a favourable effect on ξ_z: as a rule with increase of T_a, ξ_z also increases. It is natural that the extent of this effect depends, to a great extent, on the design of each specific chamber and, in particular, upon the dimensions and number of stabilizers installed in the chamber, the thermal strength of the flame of the primary chamber, and so on.

In view of the inadequacy of experimental data, it is impossible to quote typical figures with authenticity, which characterize the effect on ξ_z of the basic parameters (w, p_2, t_2, α_Σ etc.); consequently we shall confine ourselves to the comment that ξ_z for the combustion chambers of ram-jet engines is somewhat lower than in the chambers of gas-turbine engines, and for ground-level conditions (with respect to pressure this implies that $p \geq 1$ atm) ξ_z exceeds 0·9. The relatively small value of ξ_z is explained first by the fact that the combustion chambers of ram-jet engines have less pressure loss than the chambers of gas-turbine engines.

Starting Characteristics

By starting characteristics of a combustion chamber we shall understand the relationship $\alpha_{\Sigma\,\text{start}} = f(w, p_2 \text{ etc.})$, where $\alpha_{\Sigma\,\text{start}}$ represents that maximum or minimum value of α_Σ for which, under specific conditions (w, p_2, etc.) starting of the chamber is still possible.

As should also be expected, the starting characteristics call to mind the extinction characteristics, but as a rule, however, they are considerably narrower.

The extinction and starting characteristics of a chamber are shown in Fig. 132.

It can be seen from Fig. 132 that the starting region occupies a considerable portion of the operating range of the chamber.

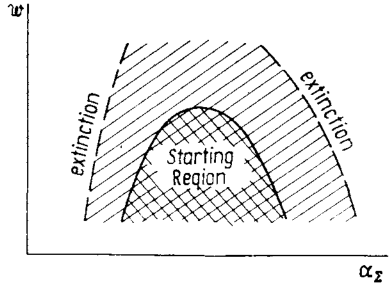

FIG. 132. Extinction and starting characteristics of a combustion chamber.

The single hatched portion of Fig. 132 indicates the region in which the combustion chamber can still operate but cannot be started.

A significant effect on the starting characteristics is exerted by p_2, as a result of the reduction of which the starting region may be greatly narrowed or even be completely eliminated.

The effect of the remaining factors on the starting characteristics is similar to their effect on the extinction characteristics and consequently we shall not consider this problem separately.

Also, just as in the case of the extinction characteristics, the starting characteristics of a chamber are improved with increase of thermal strength of the flame from the primary chamber and with increase of the dimensions of the stabilizer.

3. THERMAL AND GAS-DYNAMIC LOSSES. COMBUSTION LOSSES

The losses from incomplete combustion, as already shown above, are determined by the combustion efficiency ξ_z which was analysed in detail in sections 2 of the present chapter.

Heat Losses through the Walls

It is obvious that the heat losses through the walls of the chamber have an effect on the characteristics of the engine, just like combustion losses.

It has already been noted that many methods intended for determining the combustion efficiency take into account the losses of heat through the walls in the value of ξ_z itself.

The losses of heat through the walls are usually small. For rating an engine, they are usually not calculated separately, but are included in ξ_z. The combustion chambers of ram jet engines may constitute a special exception, since they operate with an excess air coefficient close to unity, i.e. at high temperatures.

However, in this case, these losses will be small — usually less than 1 per cent — with a sensible design for the cooling system.

The losses of heat through the walls attain a maximum value in chambers which do not have internal cooling (see Fig. 118) and which are cooled only by the external air flow.

Internal cooling is always essential in those cases when the combustion chamber is installed in the fuselage or in a special pod and cannot be cooled by the external air flow.

It should be borne in mind that the design of an internal cooling system, by reducing the heat losses, leads to an increase in the pressure loss of the combustion chamber.

The latter remark is, in certain cases, somewhat oversimplified, since it is necessary to assume that in reasonably designed combustion chambers there may not exist an independent system of internal cooling†. For

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† Combustion chambers with a high combustion temperature constitute an exception, requiring a large supply of air specially for cooling the walls of the chamber and of the nozzle.

example, one of the purposes of the flame tube in the chamber of a gas-turbine engine is the protection of the surrounding casing of the chamber from the direct action of the flame, but the second and main purpose of the flame tube is to ensure a supply of air reaching the operating space of the combustion and mixing zone, without which normal operation of the chamber would be impossible. The flame tube in the chamber of a gas-turbine engine together with the holes through which the air passes, considerably increases the aerodynamic drag of the combustion chamber.

The air passing between the flame tube and the surrounding walls of the chamber, performs two functions simultaneously: it cools the flame tube and the outer casing of the chamber and also, in passing through the holes into the flame tube, controls the processes of combustion and of mixing.

Naturally, in this case it is difficult to say accurately what fraction of the pressure losses should be ascribed to cooling and what fraction to the control of the operating process.

If required, the losses of heat to the surrounding medium can be calculated by well-known formulae for heat transfer, knowing the dimensions of the chamber, the thickness and the material of the walls, and also the temperature and velocity of the gas in the chamber and of the cooling outside air.

Gas-dynamic Losses

Friction of the gas against the surface of the chamber, the formation of vortices as a result of flow around the various units, mixing of the various air streams, their motion together with different velocities, etc., leads to the fact that the dynamic pressure of the gas at the end of the combustion chamber p_3^* is less than the dynamic pressure p_2^* at the beginning of the chamber.

Obviously, the greater p_3^*, the higher, for approximately equal conditions, is the working capability of the gas. For ram-jet engines.this implies an increase in velocity of the discharging stream, issuing from the nozzle, and consequently also, in the specific thrust of the engine R_{sp}. The efficiency of the engine $C_{sp} = G_f/R_{sp}$ as a result of this is also improved.

Gas-dynamic losses and their effect on the basic parameters of a ram-jet engine have been disucussed above in detail in Chapter VI. Consequently, this problem will be omitted from the discussion of combustion chambers of ram-jet engines.

CHAPTER IX

REHEAT COMBUSTION CHAMBERS

IN ORDER to reduce the length of the take-off run of an aircraft, and also for rapid climb or for other manoeuvres, it is essential to have an excess of thrust in comparison with the thrust required to establish flight. Naturally, this requirement should not be met by the use of powerful extra-large engines, since this may considerably worsen the flight qualities of the aircraft under other flight conditions. Moreover, for flights at high supersonic velocity ($M \approx 2{\cdot}0 - 3{\cdot}0$), large thrusts are required with minimum engine dimensions and weights.

Calculations show that one of the methods of obtaining high thrusts for permissible weight and sizes of engine is the combustion of a supplementary quantity of fuel downstream of the turbine of the engine. For this, a special combustion chamber is installed at the rear of the turbine, the so-called reheat chamber. A diagram of a gas-turbine engine with a reheat chamber is shown in Fig. 133.

As a result of the supplementary heating of the working substance behind the turbine, the velocity of discharge of gases from the nozzle is increased, and consequently also the thrust of the engine.

The method of engine reheat, as discussed, is fairly extensively used nowadays, and an increase in thrust on take-off of approximately 30 per cent is achieved and at high flight velocities of up to 100 per cent and more, with satisfactory efficiency.

1. REQUIREMENTS PUT FORWARD FOR REHEAT CHAMBERS

A whole series of requirements is put forward for a reheat chamber, which must be satisfied in designing and developing the chamber.

It has been pointed out above that the combustion chambers of jet propulsion engines should have the minimum possible pressure loss, the maximum combustion efficiency and should ensure stable combustion over a wide range of operating conditions, which is particularly difficult to guarantee for climbing to altitude.

Similar requirements can also be put forward for reheat chambers. It should be remembered however, that certain requirements have their own special features.

In particular, extremely rigid requirements are put forward for reheat chambers (for the same reasons as for the combustion chambers of ram-jet engines) in relation to pressure loss. The pressure loss coefficient of reheat chambers amounts to $> \xi_{\text{c.r.}}4$ instead of $\xi_{\text{c.c.}} = 8$–10 for the main combustion chambers of turbo-jet engines. This is explained by the fact that owing to the high velocity of the gases in a reheat chamber, as a result of the large value of ξ, the pressure losses considerably reduce the dynamic pressure, which has a marked effect on the engine efficiency. For a non-operating reheat chamber the presence of pressure loss in the reheat chamber causes an approximately 4 per cent greater specific fuel consumption of the engine in comparison with the corresponding value for an engine without a reheat chamber.

The combustion efficiency of the fuel also plays an extremely important role in modern reheat combustion chambers, since a relatively small reduction of the coefficient ξ_z worsens the efficiency of the engine as a whole. The combustion process in reheat chambers is controlled by means of stabilizers which have a lower pressure loss than the inlet units in main combustion chambers. Under these conditions with large flight velocities and for the presence of stabilizers with small pressure losses) the obtaining of a high combustion efficiency is ensured mainly by the use of a reheat chamber of considerable length.

It should be noted that the high initial temperature is an extremely opportune factor. In satisfactory

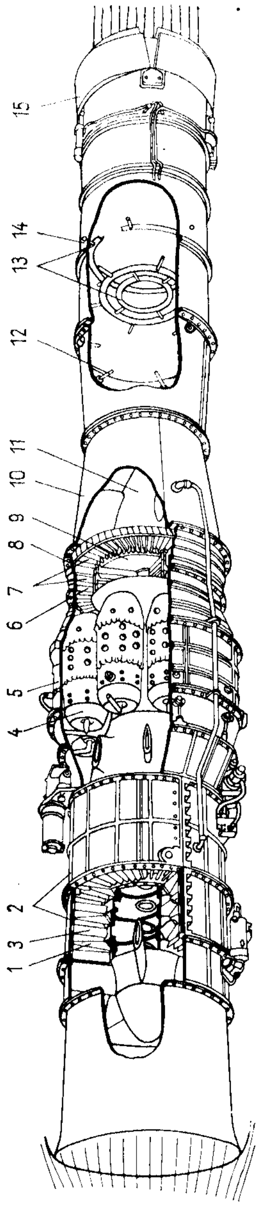

FIG. 133. Diagram of a gas-turbine engine with reheat chamber.

1 — Rotor blades of the compressor; 2 — Compressor casing; 3 — Stator blades; 4 — Flame tube of combustion chamber; 5 — Casing of combustion chamber; 6 — Turbine blades; 7 — Rotor blades of the turbine; 8 — Casing of turbine; 9 — Turbine disc; 10 — Outer cone; 11 — Inner cone; 12 — Injectors for injecting fuel into the reheat chamber; 13 — Flame stabilizer (staggered); 14 — Casing of reheat chamber; 15 — Variable nozzle.

combustion chambers the value of the combustion efficiency fluctuates within the limits $\xi_z = 0{\cdot}85$–$0{\cdot}95$ depending upon the operating conditions.

A no less important requirement for a reheat chamber is to guarantee a stable combustion process within the range of operating conditions.

In addition, a number of the usual requirements are put forward for a reheat chamber, such as, for example: dependable lighting-up under any flight conditions, smoothness of increase of engine thrust on lighting-up the chamber, low weight, small diameter and length.

2. BASIC LAYOUT OF A REHEAT CHAMBER

Let us consider the basic layout of a reheat chamber which is typical of the majority of manufactured chambers (Fig. 134).

Immediately behind the turbine of the engine is located the diffuser section 1, which is designed to reduce the velocity of the gases immediately on entry into the combustion chamber. The extent of the diffusion in this section is selected from conditions for obtaining minimum losses and optimum size or a specified value for the velocity on entry into the chamber.

Behind the diffuser section is located the reheat chamber proper, 4. At the beginning of the chamber are installed the fuel supply units 2, often made in the form of a ring manifold on which is installed a number of swirl injectors.

The combustion process takes place directly behind the stabilizing unit 3. At the exit from the combustion chamber is located the variable nozzle 5, controlled by a special mechanism 6.

In comparing the basic layout of a reheat chamber with the combustion chamber of a ram-jet engine discussed earlier, we can see that there are many of the usual components in these chambers.

However, the operating process in a reheat chamber takes place under special conditions, which are dependent upon the parameters of the working substance on entry into the chamber.

After combustion of the fuel in the main combustion chamber, a sufficient quantity of oxygen (approximately 75 per cent of the quantity of oxygen present in the air) remains in the combustion products to ensure combustion of the supplementary fuel with a sufficiently high combustion efficiency ($\xi_z = 0{\cdot}5$–$0{\cdot}95$).

The dynamic pressure of the gases on entry into the reheat chamber under conditions of take-off and for flight at high velocities are quite high, as a result of which stable combustion of the fuel can be guaranteed. However, on lighting-up the reheat chamber in flight at high altitude and low velocity, the pressure of the gases in the reheat chamber may still prove to be so low that it may be impossible to achieve stable combustion and reliable ignition for a specified chamber design.

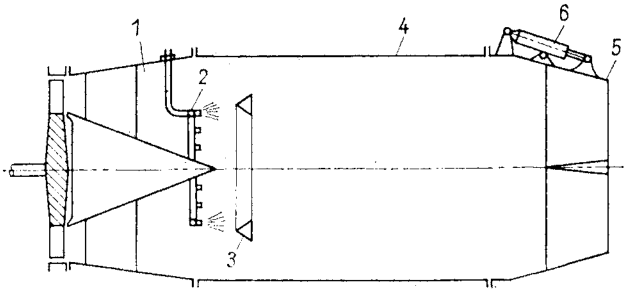

FIG. 134. Basic layout of a reheat chamber.
1 — Diffuser; 2 — Fuel manifold; 3 — Stabilizing device; 4 — Reheat chamber; 5 — Variable nozzle; 6 — Mechanism for controlling nozzle.

The data presented in Table 14 shows the variation in the magnitude of the dynamic pressure at the inlet to the reheat chamber with altitude and flight velocity, near the engine with a pressure ratio of $\pi^*_{c.c.} = 6$. It should be noted particularly that the pressure in the combustion chamber on climbing to altitude falls (for $M_0 = \text{const}$).

TABLE 14. Variation of combustion efficiency at the inlet to the reheat chamber of an engine with $T_3 = 1200$ °K and $\pi^*_{c.c.} = 6$.

Altitude, *m*	Flight Mach number				
	0	1·0	1·5	2·0	2·5
0	2·56	3·87	5·82	9·72	15·95
8000	1·225	1·765	2·58	4·14	6·58
11,000	0·888	1·27	1·84	2·94	4·66
20,000	0·213	0·306	0·442	0·702	1·1105

The temperature of the gas admitted has no less a significance on the development of the combustion process in the chamber. Earlier, when discussing the kinetics of chemical reactions (see Chapter I), the effect of temperature on the velocity of a chemical reaction was described. At the same time, the conclusion was drawn that increase of the temperature of the fuel-air mixture by diluting it with the hot combustion products sharply increases the reaction velocity regardless of the reduction of the concentration of the reactants. The processes in a reheat chamber take place considerably more intensely than in the main chamber of a gas-turbine engine as a consequence of the increase in temperature.

The temperature of the gases behind the turbine at different flight conditions ($n =$ const.) differ to a lesser degree than the pressure of the gas. The flow velocity of the gases at the inlet to the reheat chamber is of particular significance in considering the gas parameters.

3. LAYOUT AND OPERATION OF THE COMPONENTS OF A REHEAT CHAMBER

Let us consider in more detail the layout and operation of the various components of a reheat chamber (in the order corresponding to the order of their installation in the engine).

The Diffuser

On discharging from the turbine, the gases have a high velocity, attaining 400 m/sec and more ($M_{\text{inlet}} \approx 0{\cdot}9$)†. It is clear that for such flow velocities, it is extremely difficult to obtain stable combustion over the operating range of fuel flows.

Consequently, with the aim of improving the stabilization of the combustion processes and in order to reduce the pressure losses, it is necessary to reduce the velocity of the gases and this is achieved by means of the diffuser.

For modern reheat chambers, the magnitude of the intake velocity lies within the limits 120–180 m/sec. The velocity, and consequently also the transverse dimensions of the diffuser at the inlet to the reheat chamber are frequently defined from design drawings, for example, resulting in the fact that the transverse dimension of the combustion system should not be greater than the mid-section of the engine. Naturally, as a result of this, it is necessary to take into account the possible losses, which are smaller, the smaller the velocity in the combustion chamber.

It is also necessary to take into account the fact that a significant reduction of velocity below the limits specified earlier involves, as a result, an increase in the diametrical dimensions of the chamber, and consequently, an increase of weight and of the mid-section of the engine. The latter circumstance is highly undesirable and the aim at all times is to avoid it. Moreover, the point of installation and the size of the stabilizing device exerts an effect on the magnitude of the losses, about which more will be said below.

In Fig. 135 are presented data from theoretical calculations and from experiments for $M_{\text{inlet}} = 0{\cdot}27$, characterizing the gain in thrust versus

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† It should also be remembered, that for M close to unity it is not possible, in general, to have any substantial preheating in a cylindrical combustion chamber on account of the possibility of advent of critical conditions.

extent of preheating. The data presented in Fig. 135 show that the thermal losses may reduce considerably the gain in thrust.

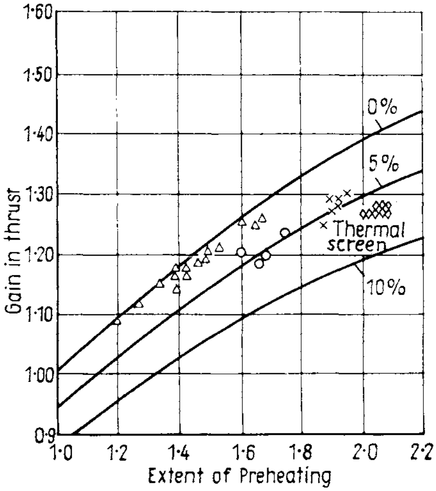

FIG. 135. Change of gain in thrust for various thermal losses versus extent of preheating.

Components of the Fuel Supply System

Different fuel supply systems are encountered in reheat chambers in use. The most prevalent of them are the following:

(1) The fuel is fed into the gas flow by means of swirl injectors located on a circular fuel manifold, which is installed directly in the gas flow, coaxially with the stabilizer. The injectors are distributed uniformly around the periphery and their number and the cross-section of the swirler are determined for each individual case from the required quality of atomization, the fuel flow and the uniformity of the fuel supply required throughout the chamber. With this system of installation of the fuel manifold (Fig.136) there can be no fear that fuel, which has seeped through joints, may give rise to a fire hazard, since it burns immediately in the chamber.

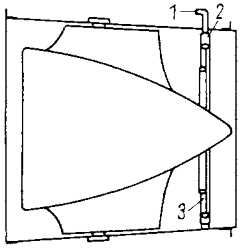

FIG. 136. Location of injectors and ring manifold in the gas flow. 1 — Fuel supply; 2 — Fuel manifold; 3 — Swirl injectors.

(2) The fuel is supplied by means of radially-disposed injectors which deliver the fuel from a manifold situated outside the chamber (Fig. 137) or from a manifold situated inside the fairing of the turbine disc (Fig. 138).

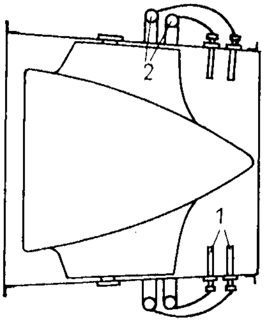

FIG. 137. Arrangement of manifold outside chamber.
1 — Fuel injectors;
2 — Fuel manifold.

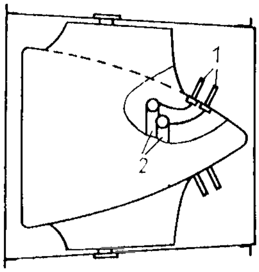

FIG. 138. Arrangement of manifold inside fairing of turbine disc.
1 — Fuel injectors;
2 — Fuel manifold.

With an external arrangement of the fuel manifolds, leakage from the system may lead to the occurrence of a fire hazard, and with an internal arrangement it may lead to buckling of the components of the fairing of the turbine disc by heating as a result of combustion of leaking fuel.

In order that combustion of the fuel should be complete and rapid, it is not necessary to guarantee at once good mixing of the fuel with the gases. This is achieved by proper distribution of the injectors, good quality of atomization and turbulence of the flow.

As is wellknown, an increase in temperature leads to a widening of the concentration limits of flammability of the mixture.

The atomized fuel should be rapidly mixed with the gases, vaporized and thermally prepared for combustion. In this case the combustion process will take place intensively and with minimum thermal losses (with a high combustion efficiency).

A droplet of fuel with a diameter of a few microns, as calculations have shown, may be completely vaporized at a distance of 100 mm from the injector in the conditions in a reheat chamber. This value is a characteristic for reheat chambers, since it defines the siting of the stabilizer behind the fuel injectors. It should also be taken into consideration that under certain conditions the size of a droplet may be considerably increased as a consequence of the reduction in pressure of the fuel prior to the injector. Minimum fuel consumption is usually observed for flight at high altitude and for the minimum possible velocity, whilst for flight near the ground at high velocity the fuel consumption increases, which leads to an increase of pressure in the fuel system. Tests carried out on one of the engines have shown that for flight at an altitude of approximately 15,000 m the minimum fuel pressure was equal to 7 kg/cm^2, whilst near the ground for flight at a velocity corresponding to $M_0 = 0{\cdot}9$ the required pressure was increased to 64 kg/cm^2, i.e., by more than a factor of 9.

Stabilizing Devices

The prepared combustible mixture proceeds to the stabilizing devices behind which combustion takes place.

In considering the main combustion chambers of gas-turbine engines and the combustion chambers of ram-jet engines, process of flame stabilization was chosen by means of the reversal zone which is formed behind a bluff body. This same principle of flame stabilization is also used in reheat chambers.

The system of stabilizers is situated at a certain distance from the injectors, at which the fuel would be successfully mixed with the oxidant and prepared for combustion.

In choosing the point of installation for the stabilizer it is necessary to take into consideration the conditions of mixture formation, since the mixture should proceed to the stabilizing unit prepared for combustion.

It has already been mentioned earlier that during the time of motion from the injectors to the stabilizer the greater proportion of the fuel droplets are vaporized. Moreover, preheating of the mixture shortens the induction period and thereby promotes intensification of the combustion process.

The high initial temperature in a reheat chamber makes it possible to somewhat reduce blocking of the cross-section by the stabilizer, which in its turn leads to a relative reduction of the pressure loss.

Amongst the rather numerous designs of stabilizers for reheat chambers, just as for the chambers of ram-jet engines, the most prevalent is the ring V-gutter, depicted diagrammatically in Fig. 139.

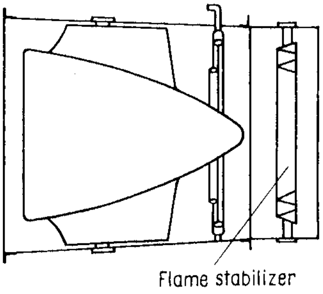

FIG. 139. Diagrammatic picture of the layout of a ring stabilizer.

Frequently, a whole series of radial spokes are attached to the ring stabilizer, behind which recirculation zones are also formed and stabilization of the combustion process is achieved, and also transfer of the flame from one ring stabilizer to another.

Figure 140 shows a flame stabilizer, on the surface of which are supplementary vanes for causing turbulence in the combustion zone. Since these vanes are relatively small in height, they do not give rise to a sharp increase in pressure losses or of the tangential velocity component.

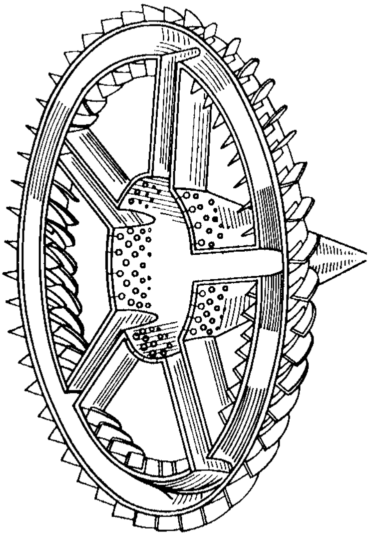

FIG. 140. Flame stabilizer.

Besides ring stabilizers, the fairing of the turbine disc can be used to create recirculation zones by trimming it to a certain length, as shown diagrammatically in Fig. 141.

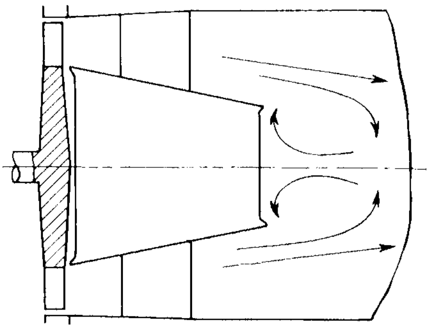

FIG. 141. Arrangement for creating recirculating zones behind the fairing of the turbine disc.

In a number of cases, several ring stabilizers are installed with the object of shortening the length of the chamber.

Experiments which have been carried out show that with such complex stabilizing systems, consisting of several rings, the losses depend to a considerable extent on their mutual disposition.

In Fig. 142 are presented the results of an experimental investigation. For an arrangement of three ring stabilizers in a single plane, the losses in pressure were greater than for a similar arrangement of two ring stabilizers.

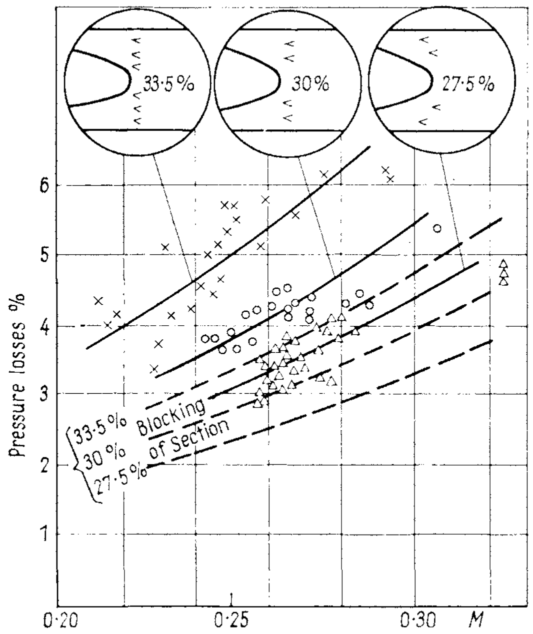

Fig. 142. Comparison of calculated stabilizer characteristics with experimental data for various Mach numbers upstream of stabilizer. (The variation of the loss of dynamic pressure as a result of blowing through cold air is shown dotted.)

The reason for the increase of the losses in the case of the arrangement of three stabilizing rings in a single plane in relation to a similar system consisting of two rings, is obvious. The pressure losses will always be greater the greater the blocking of the duct cross-section, since in this case the flow velocities will be greater.

By shifting the disposition of the stabilizers the losses may be reduced in relation to their arrangement in a single plane. Such a staggered arrangement of the stabilizers (Fig. 143) also exerts a decided influence on the stability of the combustion process. From the data given in Fig. 142, it can be visualized to what extent it is important to make a sensible choice of position for the stabilizers and the value of the velocity at the chamber intake.

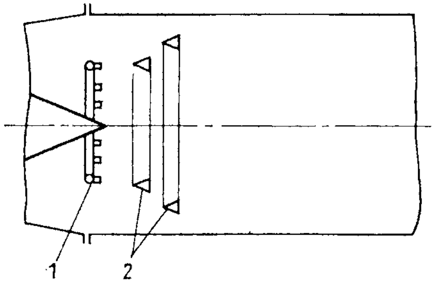

FIG. 143. Layout of a staggered stabilizing unit.
1 — Fuel manifold; 2 — Staggered stabilizer.

The stabilizing unit, in the first place, exerts an influence on the stability of the combustion. If Q_M in the recirculation zone becomes small (which may be observed for weak or over-rich operating mixtures), then the primary fuel will not be ignited and extinction of the flame will ensue.

Besides stabilization by means of a bluff body, flame stabilization by means of a powerful flame core may be introduced, the control of which is

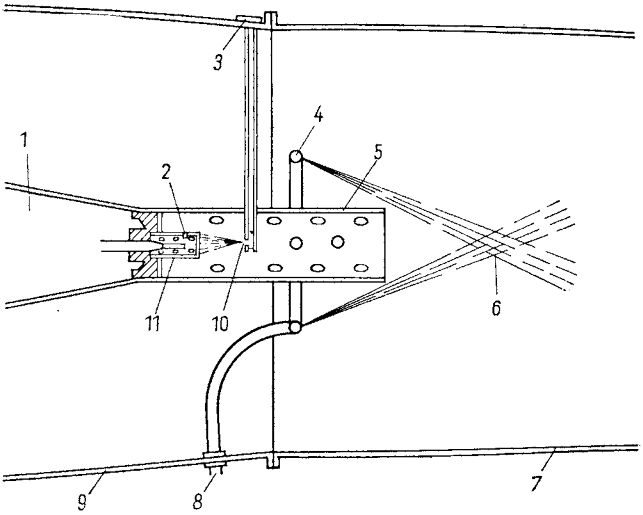

FIG. 144. Precombustion chamber for stabilizing the combustion process in a reheat chamber.
1 — Fairing; 2 — Spark plug; 3 — Fuel supply for small injector; 4 — Fuel manifold; 5 — Combustion chamber; 6 — Atomized fuel; 7 — Jet pipe; 8 — Reheat fuel supply; 9 — Cone; 10 — Small injector; 11 — Casing.

possible in the following manner. At the end of the fairing of the turbine disc is installed a small precombustion chamber (Fig. 144) in which are installed an injector and an electric igniter plug. The flame in the precombustion chamber is stabilized by the stagnant zone which is formed there. The flame core, projected from the precombustion chamber carries with it a quantity of heat sufficient for ensuring the process of combustion of the main fuel.

The initial ignition of the fuel in the reheat chamber is possible as a result of the auto-ignition of the fuel as if from an extraneous source.

By injecting the fuel directly into the reheat chamber at the usual initial temperatures ($T_4 \leq 950$ °K), auto-ignition is inadequately stable. Consequently, ignition of the fuel in this case takes place from the extraneous source.

An electric spark plug is used as the ignition device, situated on the stabilizer or in the vicinity of it.

Initial ignition of the fuel by means of auto-ignition is possible only in the case when the distance from the point of injection of the fuel to the stabilizer is chosen such that the time of travel of the fuel jet is not less than the induction period.

By using this arrangement it is possible to achieve injection of the starting fuel prior to the turbine. Then ignition takes place in the immediate vicinity of the stabilizers. The primary fuel is ignited from the originating source of the flame. It should be noted, however, that the second method has quite a limited application, since it is not known to what extent it ensures reliable starting of the reheat chamber under different engine operating conditions. The application of this method is limited also because of the possible ignition of the fuel in the turbine ducts.

Chamber Housing

In designing a reheat chamber it is essential to choose the shape of the combustion chamber and its length.

The shape of the combustion chamber is determined from the conditions for obtaining acceptable losses with the least possible diameter.

A reheat chamber can be represented as cylindrical, convergent and divergent (Fig. 145, a, b, c).

In Chapter VI, in analysing the losses arising as a result of the supply of heat, it was mentioned that the supply of heat in a convergent duct may considerably limit the total preheating and may lead to significant losses.

If one compares a cylindrical and a divergent reheat chamber, then in respect to loss of pressure the divergent chamber has an obvious advantage. However, the increase of diameter is extremely undesirable.

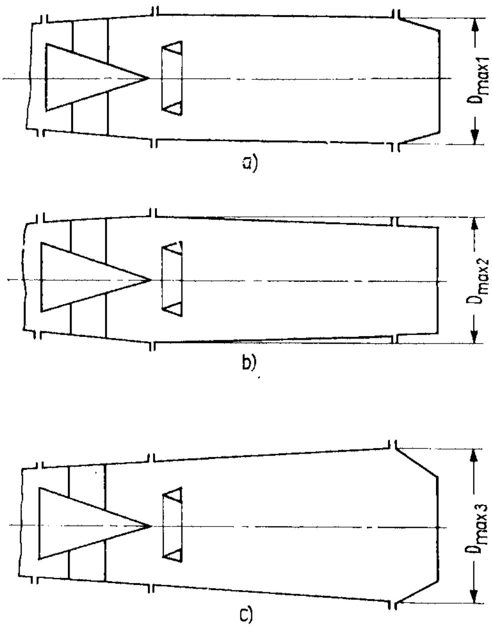

FIG. 145. Diagram of reheat chambers.

Figure 146 shows a reheat chamber which at first diverges and then converges somewhat.

The length of reheat chambers varies from 1·5 to 2 m and is determined entirely by the combustion process in the chamber, the velocity of the gases, and the initial temperature.

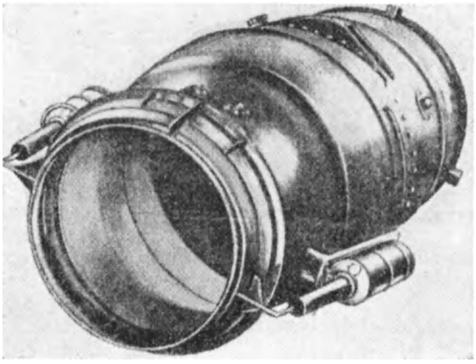

FIG. 146. Reheat chamber.

The temperature of the gases in a reheat chamber amounts to 1700 °C and above. In order that the life of the walls of the chamber should not be reduced it is essential to cool them. For this purpose, the wall of a reheat chamber is made double. Between the inner wall, fabricated from heat-resistant alloy, and the outer casing, cooling air passes.

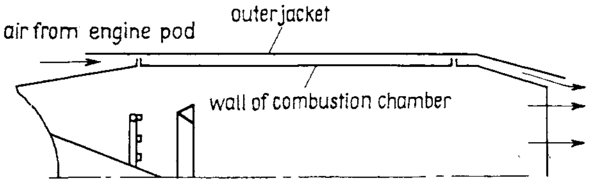

FIG. 147. Diagram of the cooling of a reheat chamber.

The cooling air may be admitted either under pressure (on account of the pressure difference), or under the action of the suction arising as a result of the ejection of the gas jet (Fig. 147).

Variable Jet Pipe

The variable jet pipe is attached directly to the combustion chamber. The necessity of locating the variable nozzle at the exit from the reheat chamber is dependent upon the fact that the supplementary heating of the gases with a constant cross-section nozzle leads to an increase of pressure behind the turbine which reduces the pressure drop at the turbine. In order that the speed of rotation should be maintained constant as a result of this, it is necessary to increase the temperature of the gases prior to the turbine, which is extremely undesirable. In order to eliminate the possibility of an increase in temperature of the gases prior to the turbine, it is neces-

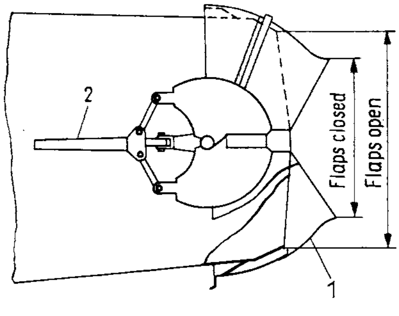

FIG. 148. Variable nozzle.
1 — Nozzle flap; 2 — Rod controlling position of flaps.

sary to increase the straight through cross-section of the nozzle with the lighting-up of the reheat chamber.

The creation of a design for a variable supersonic nozzle is an extremely complex problem. The designs brought out nowadays do not completely satisfy all the requirements put forward for them. Figure 148 shows a subsonic nozzle, the straight through area of which is varied depending upon the position of two flaps, situated on each side of the nozzle and controlled by means of a special hydraulic mechanism.

The number of flaps in the various designs varies from two to eight or more. Figure 149 shows a similar nozzle design with four flaps.

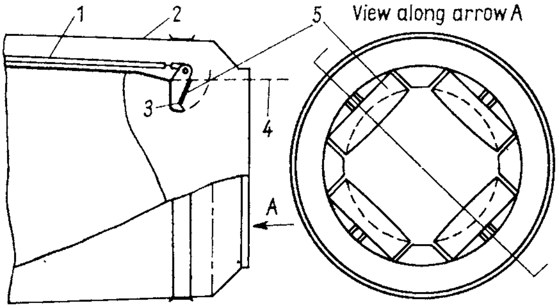

FIG. 149. Adjustable nozzle with four flaps.
1 — Control rod; 2 — Cooling jacket; 3 — Flaps in "closed" position; 4 — Flaps in "open" position; 5 — Nozzle flaps.

One of main drawbacks of the designs discussed is that they do not permit the area of the straight through section of the nozzle to be varied smoothly.

Such nozzle designs are usually called variable but more correctly they should be considered as "two positional" since they are adjustable only in two extreme positions ("open" and "closed").

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