

# The Art of R Programming

Norman Matloff

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# Preface

This book is for those who wish to *write code* in R, as opposed to those who use R mainly for a sequence of separate, discrete statistical operations, plotting a histogram here, performing a regression analysis there. The reader's level of programming background may range from professional to novice to "took a programming course in college," but the key is that the reader wishes to write R code. Typical examples of our intended audience might be:

- Analysts employed by, say, a hospital or government agency, who produce statistical reports on a regular basis, and need to develop production programs for this purpose.
- Academic researchers developing statistical methodology that is either new or combines existing methods into an integrated procedure that needs to be codified for usage by the general research community.
- Specialists in marketing, litigation support, journalism, publishing and so on who need to develop sophisticated graphical presentations of data.
- Professional programmers who have been working in other languages, but whose employers have now assigned them to projects involving statistical analysis.
- Students in statistical computing courses.

Accordingly, this book is not a compendium of the myriad types of statistical methodologies available in the wonderful R package. It really is about programming. It covers programming-related topics missing from most other books on R, and places a programming "spin" on even the basic subjects. Examples include:

- Rather than limiting examples to two or three lines of code of an artificial nature, throughout the book there are sections titled "Extended Example," consisting of real applications. In this manner, the reader not only learns how individual R constructs work, but also how to put them together into a useful program. In many cases, there is discussion of design alternatives, i.e. "Why did we do it this way?"

- The material is written with programmer sensibilities in mind. In presenting material on data frames, for instance, not only is it stated that a data frame is an R list, but also later the programming implications of that relationship are pointed out. Comparisons of R to other languages are brought in when useful.
- For programming in any language, debugging plays a key role. Only a few R books even touch this topic, and those that do limit coverage to the mechanics of R's debugging facilities. But here, an entire chapter is devoted to debugging, and the book's Extended Example theme again comes into play, with worked-out examples of debugging actual programs.
- With multicore computers common today even in the home, and with an increasing number of R applications involving very large amounts of computation, parallel processing has become a major issue for R programmers. Thus there is a chapter on this aspect, again presenting not just the mechanics but also with Extended Examples.
- For similar reasons, there is a separate chapter on speeding up R code.
- Concerning the interface of R to other languages, such as C and Python, again there is emphasis on Extended Examples, as well as tips on debugging in such situations.

I come to the R party from a somewhat unusual route. The early years of my career were spent as a statistics professor, teaching and doing research in statistical methodology. Later I moved to computer science, where I have spent most of my career, teaching and doing research in computer networks, Web traffic, disk systems and various other fields. Much of my computer science teaching and research has involved statistics. Thus I have both the point of view of a "hard core" computer scientist and as an applied statistician and statistics researcher. Hopefully this blend has enhanced to value of this book.

# Chapter 1

## Why R?

### 1.1 What Is R?

R is a scripting language for statistical data manipulation and analysis. It was inspired by, and is mostly compatible with, the statistical language S developed by AT&T. The name S, obviously standing for statistics, was an allusion to another programming language developed at AT&T with a one-letter name, C. S later was sold to a small firm, which added a GUI interface and named the result S-Plus.

R has become more popular than S/S-Plus, both because it's free and because more people are contributing to it. R is sometimes called “GNU S.”

### 1.2 Why Use R for Your Statistical Work?

Why use anything else? As the Cantonese say, *yauh peng*, *yauh leng*—“both inexpensive and beautiful.”

Its virtues:

- a public-domain implementation of the widely-regarded S statistical language; R/S is the de facto standard among professional statisticians
- comparable, and often superior, in power to commercial products in most senses
- available for Windows, Macs, Linux
- in addition to enabling statistical operations, it's a general programming language, so that you can automate your analyses and create new functions
- object-oriented and functional programming structure

- your data sets are saved between sessions, so you don't have to reload each time
- open-software nature means it's easy to get help from the user community, and lots of new functions get contributed by users, many of which are prominent statisticians

I should warn you that one submits commands to R via text, rather than mouse clicks in a Graphical User Interface (GUI). If you can't live without GUIs, you should consider using one of the free GUIs that have been developed for R, e.g. R Commander or JGR. (See Chapter 17.) Note that R definitely does have graphics—tons of it. But the graphics are for the output, e.g. plots, not for the input.

Though the terms *object-oriented* and *functional programming* may pique the interests of computer scientists, they are actually quite relevant to anyone who uses R.

The term *object-oriented* can be explained by example, say statistical regression. When you perform a regression analysis with other statistical packages, say SAS or SPSS, you get a mountain of output. By contrast, if you call the **lm()** regression function in R, the function returns an object containing all the results—estimated coefficients, their standard errors, residuals, etc. You then pick and choose which parts of that object to extract, as you wish.

R is *polymorphic*, which means that the same function can be applied to different types of objects, with results tailored to the different object types. Such a function is called a *generic function*.<sup>1</sup> Consider for instance the **plot()** function. If you apply it to a simple list of numbers, you get a simple plot of them, but if you apply it to the output of a regression analysis, you get a set of plots of various aspects of the regression output. This is nice, since it means that you, as a user, have fewer commands to remember! For instance, you know that you can use the **plot()** function on just about any object produced by R.

The fact that R is a programming language rather than a collection of discrete commands means that you can combine several commands, each one using the output of the last, with the resulting combination being quite powerful and extremely flexible. (Linux users will recognize the similarity to shell pipe commands.)

For example, consider this (compound) command

```
nrow(subset(x03, z==1))
```

First the **subset()** function would take the data frame **x03**, and cull out all those records for which the variable **z** has the value 1. The resulting new frame would be fed into **nrow()**, the function that counts the number of rows in a frame. The net effect would be to report a count of **z = 1** in the original frame.

R has many functional programming features. Roughly speaking, these allow one to apply the same function to all elements of a vector, or all rows or columns of a matrix or data frame, in a single operation. The advantages are important:

- Clearer, more compact code.

---

<sup>1</sup>In C++, this is called a *virtual function*.



- Potentially much faster execution speed.
- Less debugging (since you write less code).
- Easier transition to parallel programming.

A common theme in R programming is the avoidance of writing explicit loops. Instead, one exploits R's functional programming and other features, which do the loops internally. They are much more efficient, which can make a huge timing difference when running R on large data sets.



## Chapter 2

# Getting Started

In this chapter you'll get a quick introduction to R—how to invoke it, what it can do, what files it uses and so on.

### 2.1 How to Run R

R has two modes, *interactive* and *batch*. The former is the typical one used.

#### 2.1.1 Interactive Mode

You start R by typing “R” on the command line in Linux or on a Mac, or in a Windows Run window. You'll get a greeting, and then the R prompt, `>`.

You can then execute R commands, as you'll see in the quick sample session discussed in Section 2.2. Or, you may have your own R code which you want to execute, say in a file `z.r`. You could issue the command

```
> source("z.r")
```

which would execute the contents of that file. Note by the way that the contents of that file may well just be a function you've written, say `f()`. In that case, “executing” the file would mean simply that the R interpreter reads in the function and stores the function's definition in memory. You could then execute the function itself by calling it from the R command line, e.g.

```
> f(12)
```

### 2.1.2 Running R in Batch Mode

Sometimes it's preferable to automate the process of running R. For example, we may wish to run an R script that generates a graph output file, and not have to bother with manually running R. Here's how it could be done. Consider the file `z.r`, which produces a histogram and saves it to a PDF file:

```
pdf("xh.pdf") # set graphical output file
hist(rnorm(100)) # generate 100 N(0,1) variates and plot their histogram
dev.off() # close the file
```

Don't worry about the details; the information in the comments (marked with #) suffices here.

We could run it automatically by simply typing

```
R CMD BATCH --vanilla < z.r
```

The `--vanilla` option tells R not to load any startup file information, and not to save any.

## 2.2 A First R Example Session (5 Minutes)

We start R from our shell command line, and get the greeting message and the `>` prompt:

```
R : Copyright 2005, The R Foundation for Statistical Computing
Version 2.1.1 (2005-06-20), ISBN 3-900051-07-0
...
Type 'q()' to quit R.
>
```

Now let's make a simple data set, a *vector* in R parlance, consisting of the numbers 1, 2 and 4, and name it `x`:

```
> x <- c(1,2,4)
```

The standard assignment operator in R is `<-`. However, there are also `->`, `=` and even the `assign()` function.

The “c” stands for “concatenate.” Here we are concatenating the numbers 1, 2 and 4. Or more precisely, we are concatenating three one-element vectors consisting of those numbers. This is because any object is considered a one-element vector.

Thus we can also do, for instance,

```
> q <- c(x, x, 8)
```

which would set **q** to (1,2,4,1,2,4,8).

Since “seeing is believing,” go ahead and confirm that the data is really in **x**; to print the vector to the screen, simply type its name. If you type any variable name, or more generally an expression, while in interactive mode, R will print out the value of that variable or expression. (Python programmers will find this feature familiar.) For example,

```
> x
[1] 1 2 4
```

Yep, sure enough, **x** consists of the numbers 1, 2 and 4.

The “[1]” here means in this row of output, the first item is item 1 of that output. If there were say, two rows of output with six items per row, the second row would be labeled [7]. Our output in this case consists of only one row, but this notation helps users read voluminous output consisting of many rows.

Again, in interactive mode, one can always print an object in R by simply typing its name, so let’s print out the third element of **x**:

```
> x[3]
[1] 4
```

We might as well find the mean and standard deviation:

```
> mean(x)
[1] 2.333333
> sd(x)
[1] 1.527525
```

Note that this is again an example of R’s interactive mode feature in which typing an expression results in printing the expression’s value. In the first instance above, our expression is “mean(x),” which does have a value—the return value of the function. Thus the value is printed automatically, without our having to, say, call R’s **print()** function.

If we had wanted to save the mean in a variable instead of just printing it to the screen, we could do, say,

```
> y <- mean(x)
```

Again, since you are learning, let’s confirm that **y** really does contain the mean of **x**:

```
> y
[1] 2.333333
```

As noted earlier, we use **#** to write comments.

```
> y # print out y
[1] 2.333333
```

These of course are especially useful when writing programs, but they are useful for interactive use too, since R does record your commands (see Section 2.7). The comments then help you remember what you were doing when you later read that record.

As the last example in this quick introduction to R, let's work with one of R's internal datasets, which it uses for demos. You can get a list of these datasets by typing

```
> data()
```

One of the datasets is **Nile**, containing data on the flow of the Nile River. Let's again find the mean and standard deviation,

```
> mean(Nile)
[1] 919.35
> sd(Nile)
[1] 169.2275
```

and also plot a histogram of the data:

```
> hist(Nile)
```

A window pops up with the histogram in it, as seen in Figure 2.1. This one is bare-bones simple, but R has all kinds of bells and whistles you can use optionally. For instance, you can change the number of bins by specifying the `breaks` variable; `hist(z,breaks=12)` would draw a histogram of the data `z` with 12 bins. You can make nicer labels, etc. When you become more familiar with R, you'll be able to construct complex color graphics of striking beauty.

Well, that's the end of this first 5-minute introduction. We leave by calling the quit function (or optionally by hitting `ctrl-d` in Linux):

```
> q()
Save workspace image? [y/n/c]: n
```

That last question asked whether we want to save our variables, etc., so that we can resume work later on. If we answer `y`, then the next time we run R, all those objects will automatically be loaded. This is a very important feature, especially when working with large or numerous datasets; see more in Section 2.7.

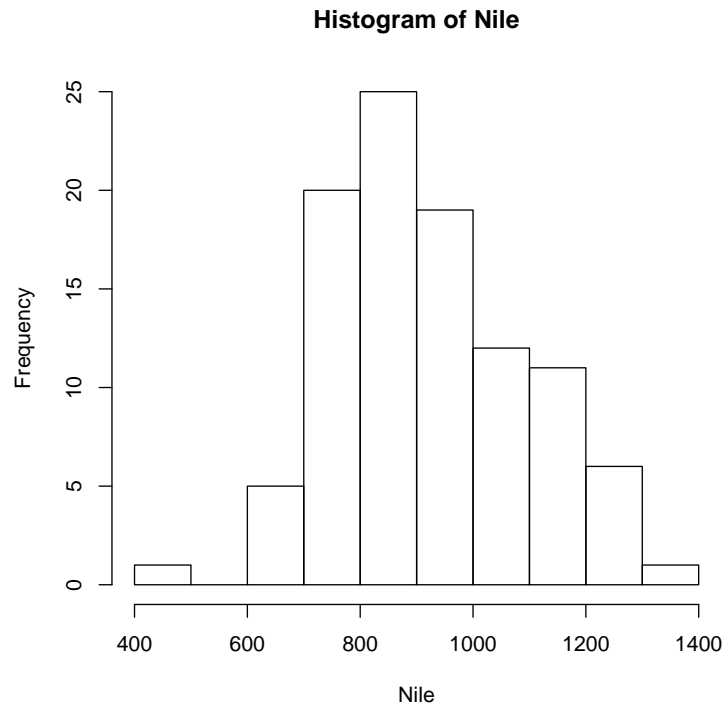


Figure 2.1: Nile data

## 2.3 Functions: a Short Programming Example

In the following example, we first define a function `oddcount()` while in R's interactive mode. (Normally we would compose the function using a text editor, but in this quick-and-dirty example, we enter it line by line in interactive mode.) We then call the function on a couple of test cases. The function is supposed to count the number of odd numbers in its argument vector.

```
# comment: counts the number of odd integers in x
> oddcount <- function(x) {
+   k <- 0
+   for (n in x) {
+     if (n %% 2 == 1) k <- k+1
+   }
+   return(k)
+ }
> oddcount(c(1,3,5))
[1] 3
> oddcount(c(1,2,3,7,9))
[1] 4
```

Here is what happened: We first told R that we would define a function `oddcount()` of one argument `x`. The left brace demarcates the start of the body of the function. We wrote one R statement per line. Since we were still in the body of the function, R reminded us of that by using `+` as its prompt<sup>1</sup> instead of the usual `>`. After we finally entered a right brace to end the function body, R resumed the `>` prompt.

Note that arguments in R functions are read-only, in that a copy of the argument is made to a local variable, and changes to the latter don't affect the original variable. Thus changes to the original variable are typically made by reassigning the return value of the function.

If one feels comfortable using global variables, a global can be written to from within a function, using R's superassignment operator, `<<-`.

For instance:

```
> w <- 5
> addone <- function(x)
+   x <- x+1
> addone(w) # formal argument x is only a local copy of w
> w # so w doesn't change
[1] 5
> addone <- function(x) return(x+1)
> w <- addone(w)
> w
[1] 6
> addone <- function() w <<- w+1 # use of superassignment op
> addone()
> w
[1] 7
```

Further details will be discussed in Chapter 8.

R also makes frequent use of **default arguments**. In the (partial) function definition

```
function(x, y=2)
```

`y` will be initialized to 2 if the programmer does not specify `y` in the call.

## 2.4 Preview of Some Important R Data Structures

Here we browse through some of the most frequently-used R data structures. This will give you a better overview of R before diving into the details, and will also allow usage of these structures in examples without having “forward references.”

---

<sup>1</sup>Actually, this is a line continuation character.



### 2.4.1 Vectors

The vector type is the R workhorse. It's hard to imagine R code, or even an R interactive session, that doesn't involve vectors.

Our examples of vectors in the preceding sessions will suffice for now.

### 2.4.2 Matrices

A matrix corresponds to the mathematical concept, i.e. a rectangular array. Technically, it is a vector, with two attributes added—the numbers of rows and columns.

Here is some sample code:

```
> m <- rbind(c(1,4),c(2,2))
> m
  [,1] [,2]
[1,]  1   4
[2,]  2   2
> m %*% c(1,1)
  [,1]
[1,]  5
[2,]  4
```

First we used the **rbind()** (“row bind”) function to build a matrix from two vectors, storing the result in **m**. We then typed that latter name, to confirm that we produced the intended matrix. Finally, we multiplied the vector (5,4) by **m**. In order to get matrix multiplication of the mathematical type, we used the **%\*%** operator.

### 2.4.3 Lists

An R list is analogous to a C struct, i.e. a container whose contents can be items of diverse data types.

A common usage is to package the return values of elaborate statistical functions. For example, the **lm()** (“linear model”) function performs regression analysis, computing not only the estimated coefficient but also residuals, hypothesis test statistics and so on. These are packaged into a list, thus enabling a single return value.

List members, which in C are delimited with periods, are indicated with dollar signs in R. Thus **x\$u** is the **u** component in the list **x**.

### 2.4.4 Data Frames

A typical data set contains data of diverse types, e.g. numerical and character string. So, while a data set of, say, *n* observations of *r* variables has the “look and feel” of a matrix, it does not qualify as such in R.

Instead, we have the R data frame.

A data frame is technically a list, with each component being a vector corresponding to a column in our data “matrix.” The designers of R have set things up so that many matrix operations can also be applied to data frames.

## 2.5 Startup Files

If there are R commands you would like to have executed at the beginning of every R session, you can place them in a file **.Rprofile** either in your home directory or in the directory from which you are running R. The latter directory is searched for such a file first, which allows you to customize for a particular project.

Other information on startup files is available by querying R’s online help facility:

```
> ?.Rprofile
```

## 2.6 Extended Example: Regression Analysis of Exam Grades

For our second introductory example, we walk through a brief statistical regression analysis. There won’t be much actual programming in this example, but it will illustrate usage of some of the data types from the last section, will introduce R’s style of object-oriented programming, and will and serve as the basis for several of our programming examples in subsequent chapters.

Here I have a file, **ExamsQuiz.txt** of grades from a class I taught. The first few lines are

```
2    3.3    4
3.3  2      3.7
4    4.3    4
2.3  0      3.3
...
```

The numbers correspond to letter grades on a four-point scale, so that 3.3, for instance, is a B+. Each line contains the data for one student, consisting of the midterm examination grade, final examination grade, and the average quiz grade. One might be interested in seeing how well the midterm and quiz grades predict the student’s grade on the final examination.

Let’s first read in the file:

```
> examsquiz <- read.table("ExamsQuiz.txt",header=F)
```

Our file had no header line, i.e. no line naming each of the variables, so we specified **header=F**, an example of the default arguments mentioned in Section 2.3. Actually, the default value of that argument is FALSE

anyway, as can be checked by R's online help facility for `read.table()`. Thus we didn't need to specify the **header** argument, but it's clearer if we do.

So, our data is now in `examsquiz`, an R object of class “**data.frame**”:

```
> class(examsquiz)
[1] "data.frame"
```

Just to check that the file was read in correctly, let's take a look at the first few rows:

```
> head(examsquiz)
  V1 V2 V3
1 2.0 3.3 4.0
2 3.3 2.0 3.7
3 4.0 4.3 4.0
4 2.3 0.0 3.3
5 2.3 1.0 3.3
6 3.3 3.7 4.0
```

Lacking a header for the data, R named the columns V1, V2 and V3. Row numbers appear on the left.

Let's try to predict Exam 2 from Exam 1:

```
lma <- lm(examsquiz[,2] ~ examsquiz[,1])
```

The `lm()` (“linear model”) function call here instructs R to fit the prediction equation

$$\text{predicted Exam 2} = \beta_0 + \beta_1 \text{Exam 1} \quad (2.1)$$

using least squares. Note that Exam 1, being stored in column of our data frame, is referred to collectively as `examsquiz[,1]`. Here the lack of the first subscript, i.e. row number, means that we are referring to the entire column, and similarly for Exam 2.

The results are returned in the object we've named `lma` of class “**lm**”. We can see the various components of that object by calling `attributes()`:

```
> attributes(lma)
$names
 [1] "coefficients" "residuals"      "effects"        "rank"
 [5] "fitted.values" "assign"         "qr"            "df.residual"
 [9] "xlevels"      "call"          "terms"         "model"

$class
[1] "lm"
```

For instance, the estimated values of the  $\beta_i$  are stored in `lma$coefficients`. As usual, we can print them, by typing the name, and by the way save some typing by abbreviating:

```
> lma$coef
(Intercept) examsquiz[, 1]
1.1205209      0.5899803
```

Since `lma$coef` is a vector, printing it is simple. But consider what happens when we print the object `lma` itself:

```
> lma
Call:
lm(formula = examsquiz[, 2] ~ examsquiz[, 1])

Coefficients:
(Intercept) examsquiz[, 1]
1.121      0.590
```

How did R know to print only these items, and not the other components of `lma`? The answer is that the **generic function** used for any printing, `print()`, actually hands off the work to a print function that has been declared to be the one associated with objects of class “`lm`”. This function is named `print.lm()`, and illustrates the concept of polymorphism we introduced briefly in Chapter 1. We’ll see the details in Chapter 12.

We can get a more detailed printout of the contents of `lma` by calling `summary()`, another generic function, which in this case triggers a call to `summary.lm()` behind the scenes:

```
> summary(lma)

Call:
lm(formula = examsquiz[, 2] ~ examsquiz[, 1])

Residuals:
    Min       1Q   Median       3Q      Max
-3.4804 -0.1239  0.3426  0.7261  1.2225

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    1.1205     0.6375   1.758  0.08709 .
examsquiz[, 1] 0.5900     0.2030   2.907  0.00614 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 1.092 on 37 degrees of freedom
Multiple R-squared:  0.1859,    Adjusted R-squared:  0.1639
F-statistic: 8.449 on 1 and 37 DF,  p-value: 0.006137
```

A number of other generic functions are defined for this class. See the online help for `lm()` for details.

To predict Exam 2 from both Exam 1 and the Quiz score, we would use the ‘+’ notation:

```
> lmb <- lm(examsquiz[,2] ~ examsquiz[,1] + examsquiz[,3])
```

(There is also a ‘\*’ notation, not covered here.)

## 2.7 Session Data

As you proceed through an interactive R session, R will record the commands you submit. And as you long as you answer yes to the question “Save workspace image?” put to you when you quit the session, R will save all the objects you created in that session, and restore them in your next session. You thus do not have to recreate the objects again from scratch if you wish to continue work from before.

The saved workspace file is named **.Rdata**, and is located either in the directory from which you invoked this R session (Linux) or in the R installation directory (Windows). Note that that means that in Windows, if you use R from various different directories, each save operation will overwrite the last. That makes Linux more convenient, but note that the file can be quite voluminous, so be sure to delete it if you are no longer working on that particular project.

You can also save the image yourself, to whatever file you wish, by calling **save.image()**. You can restore the workspace from that file later on by calling **load()**.



## Chapter 3

# Vectors

The fundamental data type in R is, without question, the *vector*. You'll learn all about vectors in this chapter.

### 3.1 Scalars, Vectors, Arrays and Matrices

Remember, objects are actually considered one-element vectors. So, there is really no such thing as a scalar.

Vector elements must all have the same *mode*, which can be **integer**, **numeric** (floating-point number), **character** (string), **logical** (boolean), **complex**, **object**, etc.

Vectors indices begin at 1. Note that vectors are stored like arrays in C, i.e. contiguously, and thus one cannot insert or delete elements, *a la* Python. If you wish to do this, use a list instead.

A variable might not have a value, a situation designated as **NA**. This is like **None** in Python and **undefined** in Perl, though its origin is different. In statistical datasets, one often encounters missing data, i.e. observations for which the values are missing. In many of R's statistical functions, we can instruct the function to skip over any missing values.

Arrays and matrices are actually vectors too, as you'll see; they merely have extra attributes, e.g. in the matrix case the numbers of rows and columns. Keep in mind that since arrays and matrices are vectors, that means that everything we say about vectors applies to them too.

One can obtain the length of a vector by using the function of the same name, e.g.

```
> x <- c(1, 2, 4)
> length(x)
[1] 3
```

## 3.2 “Declarations”

You must warn R ahead of time that you intend a variable to be one of the vector/array types. For instance, say we wish **y** to be a two-component vector with values 5 and 12. If you try

```
> y[1] <- 5
> y[2] <- 12
```

the first command (and the second) will be rejected, but

```
> y <- vector(length=2)
> y[1] <- 5
> y[2] <- 12
```

works, as does

```
> y <- c(5,12)
```

The latter is OK because the right-hand side is a vector type, so we are binding **y** to an already-existent vector.

## 3.3 Generating Useful Vectors with “:”, `seq()` and `rep()`

Note the `:` operator:

```
> 5:8
[1] 5 6 7 8
> 5:1
[1] 5 4 3 2 1
```

Beware of the operator precedence:

```
> i <- 2
> 1:i-1
[1] 0 1
> 1:(i-1)
[1] 1
```

The `seq()` (“sequence”) generates an arithmetic sequence, e.g.:



```
> seq(5,8)
[1] 5 6 7 8
> seq(12,30,3)
[1] 12 15 18 21 24 27 30
> seq(1.1,2,length=10)
[1] 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
```

Though it may seem innocuous, the `seq()` function provides foundation for many R operations. See examples in Sections 10.8 and Section 13.16.

The `rep()` ("repeat") function allows us to conveniently put the same constant into long vectors. The call form is `rep(z,k)`, which creates a vector of  $k \times \text{length}(z)$  elements, each equal to `z`. For example:

```
> x <- rep(8,4)
> x
[1] 8 8 8 8

> rep(1:3,2)
[1] 1 2 3 1 2 3
```

## 3.4 Vector Arithmetic and Logical Operations

You can add vectors, e.g.

```
> x <- c(1,2,4)
> x + c(5,0,-1)
[1] 6 2 3
```

You may be surprised at what happens when we multiply them:

```
> x * c(5,0,-1)
[1] 5 0 -4
```

As you can see, the multiplication was elementwise. This is due to the functional programming nature of R.

The `any()` and `all()` functions are handy:

```
> x <- 1:10
> if (any(x > 8)) print("yes")
[1] "yes"
> if (any(x > 88)) print("yes")
> if (all(x > 88)) print("yes")
> if (all(x > 0)) print("yes")
[1] "yes"
```

### 3.5 Recycling

When applying an operation to two vectors which requires them to be the same length, the shorter one will be *recycled*, i.e. repeated, until it is long enough to match the longer one, e.g.

```
> c(1,2,4) + c(6,0,9,20,22)
[1] 7 2 13 21 24
Warning message:
longer object length
  is not a multiple of shorter object length in: c(1, 2, 4) + c(6,
  0, 9, 20, 22)
```

Here's a more subtle example:

```
> x
  [,1] [,2]
[1,]  1  4
[2,]  2  5
[3,]  3  6
> x+c(1,2)
  [,1] [,2]
[1,]  2  6
[2,]  4  6
[3,]  4  8
```

What happened here is that  $\mathbf{x}$ , as a 3x2 matrix, is also a six-element vector, which in R is stored column-by-column. We added a two-element vector to it, so our addend had to be repeated twice to make six elements. So, we were adding  $c(1,2,1,2,1,2)$  to  $\mathbf{x}$ .

### 3.6 Vector Indexing

You can also do *indexing* of vectors, picking out elements with specific indices, e.g.

```
> y <- c(1.2,3.9,0.4,0.12)
> y[c(1,3)]
[1] 1.2 0.4
> y[2:3]
[1] 3.9 0.4
```

Note carefully that duplicates are definitely allowed, e.g.

```
> x <- c(4,2,17,5)
> y <- x[c(1,1,3)]
> y
[1] 4 4 17
```

Negative subscripts mean that we want to exclude the given elements in our output:

```
> z <- c(5,12,13)
> z[-1] # exclude element 1
[1] 12 13
> z[-1:-2]
[1] 13
```

In such contexts, it is often useful to use the `length()` function:

```
> z <- c(5,12,13)
> z[1:length(z)-1]
[1] 5 12
```

Note that this is more general than using `z[1:2]`. In a program with general-length vectors, we could use this pattern to exclude the last element of a vector.

Here is a more involved example of this principle. Suppose we have a sequence of numbers for which we want to find successive differences, i.e. the difference between each number and its predecessor. Here's how we could do it:

```
> x <- c(12,15,8,11,24)
> y <- x[-1] - x[-length(x)]
> y
[1] 3 -7 3 13
```

Here we want to find the numbers  $15-12 = 3$ ,  $8-15 = -7$ , etc. The expression `x[-1]` gave us the vector (15,8,11,24) and `x[-length(x)]` gave us (12,15,8,11). Subtracting these two vectors then gave us the differences we wanted.

**Make careful note of the above example. This is the “R way of doing things.” By taking advantage of R’s vector operations, we came up with a solution which avoids loops. This is clean, compact and likely much faster when our vectors are long. We often use R’s functional programming features to these ends as well.**

## 3.7 Vector Element Names

The elements of a vector can optionally be given names. For instance:

```
> x <- c(1,2,4)
> names(x)
NULL
> names(x) <- c("a","b","ab")
> names(x)
[1] "a" "b" "ab"
> x
  a  b ab
1  2  4
```

We can remove the names from a vector by assigning NULL:

```
> names(x) <- NULL
> x
[1] 1 2 4
```

We can even reference elements of the vector by name, e.g.

```
> x <- c(1, 2, 4)
> names(x) <- c("a", "b", "ab")
> x["b"]
b
2
```

## 3.8 Elementwise Operations on Vectors

Suppose we have a function  $f()$  that we wish to apply to all elements of a vector  $\mathbf{x}$ . In many cases, we can accomplish this by simply calling  $f()$  on  $\mathbf{x}$  itself.

### 3.8.1 Vectorized Functions

As we saw in Section 3.4, many operations are **vectorized**, such as  $+$  and  $>$ :

```
> u <- c(5, 2, 8)
> v <- c(1, 3, 9)
> u+v
[1] 6 5 17
> u > v
[1] TRUE FALSE FALSE
```

The key point is that if an R function uses vectorized operations, it too is vectorized, i.e. it can be applied to vectors in an elementwise fashion. For instance:

```
> w <- function(x) return(x+1)
> w(u)
[1] 6 3 9
```

Here  $w()$  uses  $+$ , which is vectorized, so  $w()$  is vectorized as well.

The function can have auxiliary arguments:

```
> f
function(x,c) return((x+c)^2)
> f(1:3,0)
[1] 1 4 9
> f(1:3,1)
[1] 4 9 16
```

Even the transcendental functions are vectorized:

```
> sqrt(1:9)
[1] 1.000000 1.414214 1.732051 2.000000 2.236068 2.449490 2.645751 2.828427
[9] 3.000000
```

This applies to many of R's built-in functions. For instance, let's apply the function for rounding to the nearest integer to an example vector **y**:

```
> y <- c(1.2, 3.9, 0.4)
> z <- round(y)
> z
[1] 1 4 0
```

The point is that the **round()** function was applied individually to each element in the vector **y**. In fact, in

```
> round(1.2)
[1] 1
```

the operation still works, because the number 1.2 is actually considered to be a vector that happens to consist of a single element 1.2.

Here we used the built-in function **round()**, but you can do the same thing with functions that you write yourself.

Note that the functions can also have extra arguments, e.g.

```
> f <- function(elt, s) return(elt+s)
> y <- c(1, 2, 4)
> f(y, 1)
[1] 2 3 5
```

As seen above, even operators such as **+** are really functions. For example, the reason why elementwise addition of 4 works here,

```
> y <- c(12, 5, 13)
> y+4
[1] 16 9 17
```

is that the **+** is actually considered a function! Look at it here:

```
> '+'(y, 4)
[1] 16 9 17
```

### 3.8.2 The Case of Vector-Valued Functions

The above operations work with vector-valued functions too. However, since the return value is in essence a matrix, it needs to be converted. A better option is to use **sapply()**, discussed in Section 4.10.2.

### 3.8.3 Elementwise Operations in Nonvectorizable Settings

Even if a function that you want to apply to all elements of a vector is not vectorizable, you can still avoid writing a loop, e.g. avoid writing

```
lv <- length(v)
outvec <- vector(length=lv)
for (i in 1:lv) {
  outvec[i] <- f(v[i])
}
```

by treating the vector as a matrix and using the **apply()** function (Section 4.10):

```
outvec <- apply(as.matrix(v),1,f)
```

The call to **as.matrix()** will return a matrix whose sole column is **v**.

This may not save you much time if you are running R on just one machine, but if you are using, for instance, the snow package (see Section 17.3), with **parApply()** instead of **apply()**, it could be well worth doing.

## 3.9 Filtering

Another idea borrowed from functional programming is filtering, which is one of the most common operations in R.

For example:

```
> z <- c(5,2,-3,8)
> w <- z[z*z > 8]
> w
[1] 5 -3 8
```

Here is what happened above: We asked R to find the indices of all the elements of **z** whose squares were greater than 8, then use those indices in an indexing operation on **z**, then finally assign the result to **w**.

Look at it done piece-by-piece:

```
> z <- c(5,2,-3,8)
> z
[1] 5 2 -3 8
> z*z > 8
[1] TRUE FALSE TRUE TRUE
```

Evaluation of the expression  $z*z > 8$  gave us a vector of booleans! Let's go further:

```
> z[c(TRUE,FALSE,TRUE,TRUE)]
[1] 5 -3 8
```

This example will place things into even sharper focus:

```
> z <- c(5,2,-3,8)
> j <- z*z > 8
> j
[1] TRUE FALSE TRUE TRUE
> y <- c(1,2,30,5)
> y[j]
[1] 1 30 5
```

We may just want to find the positions within  $z$  at which the condition occurs. We can do this using **which()**:

```
> which(z*z > 8)
[1] 1 3 4
```

Here's an extension of an example in Section 3.6:

```
# x is an array of numbers, mostly in nondecreasing order, but with some
# violations of that order nviol() returns the number of indices i for
# which x[i+1] < x[i]

nviol <- function(x) {
  diff <- x[-1]-x[1:(length(x)-1)]
  return(length(which(diff < 0)))
}
```

I noted in Section 1.2 that using the **nrow()** function in conjunction with filtering provides a way to obtain a count of records satisfying various conditions. If you just want the count and don't want to create a new table, you should use this approach.

You can also use this to selectively change elements of a vector, e.g.

```
> x <- c(1,3,8,2)
> x[x > 3] <- 0
> x
[1] 1 3 0 2
```

### 3.10 Combining Elementwise Operations and Filtering, with the `ifelse()` Function

The form is

```
ifelse(b,u,v)
```

where `b` is a boolean vector, and `u` and `v` are vectors.

The return value is a vector, element `i` of which is `u[i]` if `b[i]` is true, or `v[i]` if `b[i]` is false. This is pretty abstract, so let's go right to an example:

```
> x <- 1:10
> y <- ifelse(x %% 2 == 0,5,12)
> y
[1] 12  5 12  5 12  5 12  5 12  5
```

Here we wish to produce a vector in which there is a 5 wherever `x` is even, with a 12 wherever `x` is odd. So, the first argument is `c(F,T,F,T,F,T,F,T)`. The second argument, 5, is treated as `c(5,5,5,5,5,5,5,5,5,5)` by recycling, and similarly for the third argument.

Here is another example, in which we have explicit vectors.

```
> x <- c(5,2,9,12)
> ifelse(x > 6,2*x,3*x)
[1] 15  6 18 24
```

The advantage of `ifelse()` over the standard if-then-else is that it is vectorized. Thus it's potentially much faster.

### 3.11 Extended Example: Recoding an Abalone Data Set

Due to the vector nature of the arguments, one can nest `ifelse()` operations. In the following example, involving an abalone data set, gender is coded as 'M', 'F' or 'I', the last meaning infant. We wish to recode those characters as 1, 2 or 3:

```
> g <- c("M","F","F","I","M")
> ifelse(g == "M",1,ifelse(g == "F",2,3))
[1] 1 2 2 3 1
```

The inner call to `ifelse()`, which of course is evaluated first, produces a vector of 2s and 3s, with the 2s corresponding to female cases, and 3s being for males and infants. The outer call results in 1s for the males, in which cases the 3s are ignored.



Remember, the vectors involved could be columns in matrices, and this is a very common scenario. Say our abalone data is stored in the matrix **ab**, with gender in the first column. Then if we wish to recode as above, we could do it this way:

```
> ab[,1] <- ifelse(ab[,1] == "M",1,ifelse(ab[,1] == "F",2,3))
```



## Chapter 4

# Matrices

A matrix is a vector with two additional attributes, the number of rows and number of columns.

### 4.1 General Operations

Multidimensional vectors in R are called *arrays*. A two-dimensional array is also called a *matrix*, and is eligible for the usual matrix mathematical operations.

Matrix row and column subscripts begin with 1, so for instance the upper-left corner of the matrix **a** is denoted **a[1,1]**. The internal linear storage of a matrix is in *column-major order*, meaning that first all of column 1 is stored, then all of column 2, etc.

One of the ways to create a matrix is via the **matrix()** function, e.g.

```
> y <- matrix(c(1,2,3,4),nrow=2,ncol=2)
> y
  [,1] [,2]
[1,] 1   3
[2,] 2   4
```

Here we concatenated what we intended as the first column, the numbers 1 and 2, with what we intended as the second column, 3 and 4. That was our data in linear form, and then we specified the number of rows and columns. The fact that R uses column-major order then determined where these four numbers were put.

Though internal storage of a matrix is in column-major order, we can use the **byrow** argument in **matrix()** to TRUE in order to specify that the data we are using to fill a matrix be interpreted as being in row-major order. For example:

```
> m <- matrix(c(1,2,3,4,5,6),nrow=3)
> m
```

```

      [,1] [,2]
[1,]    1    4
[2,]    2    5
[3,]    3    6
> m <- matrix(c(1,2,3,4,5,6),nrow=2,byrow=T)
> m
      [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6

```

(‘T’ is an abbreviation for “TRUE”).

Since we specified the matrix entries in the above example, we would not have needed to specify **ncol**; just **nrow** would be enough. For instance:

```

> y <- matrix(c(1,2,3,4),nrow=2)
> y
      [,1] [,2]
[1,]    1    3
[2,]    2    4

```

Note that when we then printed out **y**, R showed us its notation for rows and columns. For instance, **[,2]** means column 2, as can be seen in this check:

```

> y[,2]
[1] 3 4

```

Another way we could have built **y** would have been to specify elements individually:

```

> y <- matrix(nrow=2,ncol=2)
> y[1,1] = 1
> y[2,1] = 2
> y[1,2] = 3
> y[2,2] = 4
> y
      [,1] [,2]
[1,]    1    3
[2,]    2    4

```

We can perform various operations on matrices, e.g. matrix multiplication, matrix scalar multiplication and matrix addition:

```

> y %*% y # ordinary matrix multiplication
      [,1] [,2]
[1,]    7   15
[2,]   10   22
> 3*y
      [,1] [,2]
[1,]    3    9
[2,]    6   12

```

```
> y+y
  [,1] [,2]
[1,] 2    6
[2,] 4    8
```

For linear algebra operations on matrices, see Section 10.4.

Again, keep in mind—and when possible, exploit—the notion of recycling (Section 3.5. For instance:

```
> x <- 1:2
> y <- c(1,3,4,10)
> x*y
[1] 1 6 4 20
```

Since  $\mathbf{x}$  was shorter than  $\mathbf{y}$ , it was recycled to the four-element vector  $\mathbf{c}(1,2,1,2)$ , then multiplied elementwise with  $\mathbf{y}$ .

## 4.2 Matrix Indexing

The same operations we discussed in Section 3.6 apply to matrices. For instance:

```
> z
  [,1] [,2] [,3]
[1,] 1    1    1
[2,] 2    1    0
[3,] 3    0    1
[4,] 4    0    0
> z[,c(2,3)]
  [,1] [,2]
[1,] 1    1
[2,] 1    0
[3,] 0    1
[4,] 0    0
```

Here's another example:

```
> y <- matrix(c(11,21,31,12,22,32),nrow=3,ncol=2)
> y
  [,1] [,2]
[1,] 11   12
[2,] 21   22
[3,] 31   32
> y[2:3,]
  [,1] [,2]
[1,] 21   22
[2,] 31   32
> y[2:3,2]
[1] 22 32
```

You can copy a smaller matrix to a slice of a larger one:

```
> y
      [,1] [,2]
[1,]    1    4
[2,]    2    5
[3,]    3    6
> y[2:3,] <- matrix(c(1,1,8,12),nrow=2)
> y
      [,1] [,2]
[1,]    1    4
[2,]    1    8
[3,]    1   12

> x <- matrix(nrow=3,ncol=3)
> x[2:3,2:3] <- cbind(4:5,2:3)
> x
      [,1] [,2] [,3]
[1,]   NA   NA   NA
[2,]   NA    4    2
[3,]   NA    5    3
```

### 4.3 Matrix Row and Column Mean Functions

The function `mean()` applies only to vectors, not matrices. If one does call this function with a matrix argument, the mean of all of its elements is computed, not multiple means row-by-row or column-by-column, since a matrix is a vector.

The functions `rowMeans()` and `colMeans()` return vectors containing the means of the rows and columns. There are also corresponding functions `rowSums()` and `colSums()`.

### 4.4 Matrix Row and Column Names

The natural way to refer to rows and columns in a matrix is, of course, via the row and column numbers. However, optionally one can give alternate names to these entities.

For example:

```
> z <- matrix(c(1,2,3,4),nrow=2)
> z
      [,1] [,2]
[1,]    1    3
[2,]    2    4
> colnames(z)
NULL
> colnames(z) <- c("a","b")
> z
      a b
[1,] 1 3
[2,] 2 4
```

```
[1,] 1 3
[2,] 2 4
> colnames(z)
[1] "a" "b"
> z[,"a"]
[1] 1 2
```

As you see here, these names can then be used to reference specific columns. The function `rownames()` works similarly.

This feature is usually less important when writing R code for general application, but can be very useful when analyzing a specific data set. An example of this is seen in Section 4.5.

## 4.5 Extended Example: Preliminary Analysis of Automobile Data

Here we look at one of R's built-in data sets, named `mtcars`, automobile data collected back in 1974. The help file for this data set is invoked as usual via

```
> ?mtcars
```

while the data set itself, being in the form of a data frame, is accessed simply by its name.

There are data on 11 variables, as the help file tells us:

```
[, 1] mpg   Miles/(US) gallon
[, 2] cyl   Number of cylinders
[, 3] disp  Displacement (cu.in.)
[, 4] hp    Gross horsepower
[, 5] drat  Rear axle ratio
[, 6] wt    Weight (lb/1000)
[, 7] qsec  1/4 mile time
[, 8] vs    V/S
[, 9] am    Transmission (0 = automatic, 1 = manual)
[,10] gear  Number of forward gears
[,11] carb  Number of carburetors
```

Since this chapter concerns matrix objects, let us first change it to a matrix. This is not really necessary in this case, as the matrix indexing operations we've covered here do apply to data frames too, but it's important to understand that these are two different classes. Here is how we do the conversion:

```
> class(mtcars)
[1] "data.frame"
> mtc <- mtcars
> class(mtc)
[1] "data.frame"
```

Let's take a look at the first few records, i.e. the first few rows:

```
> head(mtc)
      mpg cyl disp  hp drat   wt  qsec vs am gear carb
Mazda RX4           21.0   6  160 110 3.90 2.620 16.46 0  1   4   4
Mazda RX4 Wag       21.0   6  160 110 3.90 2.875 17.02 0  1   4   4
Datsun 710           22.8   4  108  93 3.85 2.320 18.61 1  1   4   1
Hornet 4 Drive       21.4   6  258 110 3.08 3.215 19.44 1  0   3   1
Hornet Sportabout   18.7   8  360 175 3.15 3.440 17.02 0  0   3   2
Valiant              18.1   6  225 105 2.76 3.460 20.22 1  0   3   1
```

You can see that the matrix has been given row names corresponding to the car names. The columns have names too.

Let's find the overall average mile-per-gallon figure:

```
> mean(mtc[,1])
[1] 20.09062
```

Now let's break it down by number of cylinders:

```
> mean(mtc[mtc[,2] == 4,1])
[1] 26.66364
> mean(mtc[mtc[,2] == 6,1])
[1] 19.74286
> mean(mtc[mtc[,2] == 8,1])
[1] 15.1
```

Or, more compactly from a programming point of view:

```
> for (ncyl in c(4,6,8)) print(mean(mtc[mtc[,2] == ncyl,1]))
[1] 26.66364
[1] 19.74286
[1] 15.1
```

As explained earlier, here the expression `mtc[,2] == ncyl` returns a boolean vector, with TRUE components corresponding to the rows in `mtc` that satisfy `mtc[,2] == ncyl`. The expression `mtc[mtc[,2] == ncyl,1]` yields a submatrix consisting of those rows of `mtc`, in which we look at column 1.

How many have more than 200 horsepower? Which are they?

```
> nrow(mtc[mtc[,4] > 200,])
[1] 7
> rownames(mtc[mtc[,4] > 200,])
[1] "Duster 360"          "Cadillac Fleetwood" "Lincoln Continental"
[4] "Chrysler Imperial"  "Camaro Z28"         "Ford Pantera L"
[7] "Maserati Bora"
```

As can be seen in the first command above, the `nrow()` function is a handy way to find the count of the number of rows satisfying a certain condition. In the second command, we extracted a submatrix corresponding to the given condition, and then asked for the names of the rows of that submatrix—giving us the names of the cars satisfying the condition.



## 4.6 Dimension Reduction: a Bug or a Feature?

In the world of statistics, dimension reduction is a good thing, with many statistical procedures aimed to do it well. If we are working with, say, 10 variables, and can reduce that number to three, we're happy.

However, in R there is something else that might merit the name “dimension reduction. Say we have a four-row matrix, and extract a row from it:

```
> z <- matrix(1:8,nrow=4)
> z
      [,1] [,2]
[1,]    1    5
[2,]    2    6
[3,]    3    7
[4,]    4    8
> r <- z[2,]
> r
[1] 2 6
```

This seems innocuous, but note the format in which R has displayed **r**. It's a vector format, not a matrix format. In other words, **r** is a vector of length 2, rather than a 1x2 matrix. We can confirm this:

```
> attributes(z)
$dim
[1] 4 2

> attributes(r)
NULL
```

This seems natural, but in many cases it will cause trouble in programs that do a lot of matrix operations. You may find that your code works fine in general, but fails in a special case. Say for instance that your code extracts a submatrix from a given matrix, and then does some matrix operations on the submatrix. If the submatrix has only one row, R will make it a vector, which could ruin your computation.

Fortunately, R has a way to suppress this dimension reduction, with the **drop** argument. For example:

```
> r <- z[2,, drop=F]
> r
      [,1] [,2]
[1,]    2    6
> dim(r)
[1] 1 2
```

Ah, now **r** is a 1x2 matrix.

See Sections 4.8 for an example in which **drop** is used.

If you have a vector which you wish to be treated as a matrix, use **as.matrix()**:

```

> u
[1] 1 2 3
> v <- as.matrix(u)
> attributes(u)
NULL
> attributes(v)
$dim
[1] 3 1

```

## 4.7 Adding/Deleting Elements of Vectors and Matrices

Technically, vectors and matrices are of fixed length and dimensions. However, they can be reassigned, etc. Consider:

```

> x <- c(12,5,13,16,8)
> x <- c(x,20) # append 20
> x
[1] 12 5 13 16 8 20
> x <- c(x[1:3],20,x[4:6]) # insert 20
> x
[1] 12 5 13 20 16 8 20 # delete elements 2 through 4
> x <- x[-2:-4]
> x
[1] 12 16 8 20

```

The **rbind()** and **cbind()** functions enable one to add rows or columns to a matrix.

For example:

```

> one
[1] 1 1 1 1
> z
  [,1] [,2] [,3]
[1,] 1   1   1
[2,] 2   1   0
[3,] 3   0   1
[4,] 4   0   0
> cbind(one,z)
[1,]1 1 1 1
[2,]1 2 1 0
[3,]1 3 0 1
[4,]1 4 0 0

```

You can also use these functions as a quick way to create small matrices:

```

> q <- cbind(c(1,2),c(3,4))
> q
  [,1] [,2]
[1,] 1   3
[2,] 2   4

```

We can delete rows or columns in the same manner as shown for vectors above, e.g.:

```
> m <- matrix(1:6,nrow=3)
> m
      [,1] [,2]
[1,]    1    4
[2,]    2    5
[3,]    3    6
> m <- m[c(1,3),]
> m
      [,1] [,2]
[1,]    1    4
[2,]    3    6
```

## 4.8 Extended Example: Discrete-Event Simulation in R

*Discrete-event simulation* (DES) is widely used in business, industry and government. The term “discrete event” refers to the fact that the state of the system changes only at discrete times, rather than changing continuously. A typical example would involve a queuing system, say people lining up to use an ATM machine. The number of people in the queue increases only when someone arrives, and decreases only when a person finishes an ATM transaction, both of which occur only at discrete times.

It is not assumed here that the reader has prior background in DES. For our purposes here, the main ingredient to understand is the *event list*, which will now be explained.

Central to DES operation is maintenance of the *event list*, a list of scheduled events. Since the earliest event must always be handled next, the event list is usually implemented as some kind of priority queue. The main loop of the simulation repeatedly iterates, in each iteration pulling the earliest event off of the event list, updating the simulated time to reflect the occurrence of that event, and reacting to this event. The latter action will typically result in the creation of new events. R’s lack of pointer variables means that we must write code for maintaining the event list in a nontraditional way, but on the other hand it will also lead to some conveniences too.

One of the oldest approaches to write DES code is the *event-oriented paradigm*. Here the code to handle the occurrence of one event sets up another event. In the case of an arrival to a queue, the code may then set up a service event (or, if there are queued jobs, it will add this job to the queue). As an example to guide our thinking, consider the ATM situation, and suppose we store the event list as a simple vector.

At time 0, the queue is empty. The simulation code randomly generates the time of the first arrival, say 2.3. At this point the event list is simply (2.3). This event is pulled off the list, simulated time is updated to 2.3, and we react to the arrival event as follows: The queue for the ATM is empty, so we start the service, by randomly generating the service time; say it is 1.2 time units. Then the completion of service will occur at simulated time  $2.3+1.2 = 3.5$ , so we add this event to the event list, which will now consist of (3.5). We will also generate the time to the next arrival, say 0.6, which means the arrival will occur at time 2.9. Now the event list consists of (2.9,3.5).

As will be detailed below, our example code here is hardly optimal, and the reader is invited to improve it. It does, however, serve to illustrate a number of the issues we have discussed in this chapter. The code consists of some generally-applicable library functions, such as `schedevnt()` and `mainloop()`, and a sample application of those library functions. The latter simulates an  $M/M/1$  queue, i.e. a single-server queue in which both interarrival time and service time are exponentially distributed.

```

1 # DES.r: R routines for discrete-event simulation (DES), with an example
2
3 # each event will be represented by a vector; the first component will
4 # be the time the event is to occur; the second component will be the
5 # numerical code for the programmer-defined event type; the programmer
6 # may add application-specific components
7
8 # a list named "sim" holds the events list and other information; for
9 # convenience, sim has been stored as a global variable; some functions
10 # have side effects
11
12 # create "sim"
13 newsim <- function(numfields) {
14   sim <- list()
15   sim$currtime <- 0.0 # current simulated time
16   sim$evnts <- NULL # event list
17 }
18
19 # insert event evnt into event list
20 insevnt <- function(evnt) {
21   if (is.null(sim$evnts)) {
22     sim$evnts <- matrix(evnt,nrow=1)
23     return()
24   }
25   # find insertion point
26   inspt <- binsearch(sim$evnts[,1],evnt[1])
27   # now "insert"
28   if (inspt > 1) e <- rbind(sim$evnts[1:(inspt-1),],evnt)
29   nrse <- nrow(sim$evnts)
30   if (inspt <= nrse)
31     e <- rbind(evnt, sim$evnts[inspt:nrse,])
32   sim$evnts <- e
33 }
34
35 # schedule new event; evnttime is the time at which the event is to
36 # occur; evnttype is the event type; and appfields are the values of the
37 # programmer-defined fields, if any
38 schedevnt <- function(evnttime,evnttype,appfields=NULL) {
39   evnt <- c(evnttime,evnttype,appfields)
40   insevnt(evnt)
41 }
42
43 # start to process next event (second half done by application
44 # programmer via call to reactevnt())
45 getnextevnt <- function() {
46   head <- sim$evnts[1,]
47   # delete head
48   if (nrow(sim$evnts) == 1) sim$evnts <- NULL else
49     sim$evnts <- sim$evnts[-1,,drop=F]
50   return(head)
51 }
52

```

```

53 # main loop of the simulation
54 mainloop <- function(maxsimtime) {
55   while(sim$curtime < maxsimtime) {
56     head <- getnextevnt()
57     sim$curtime <<- head[1] # update current simulated time
58     reactevnt(head) # process this event (programmer-supplied ftn)
59   }
60 }
61
62 # binary search of insertion point of y in the sorted vector x; returns
63 # the position in x before which y should be inserted, with the value
64 # length(x)+1 if y is larger than x[length(x)]
65 binsearch <- function(x,y) {
66   n <- length(x)
67   lo <- 1
68   hi <- n
69   while(lo+1 < hi) {
70     mid <- floor((lo+hi)/2)
71     if (y == x[mid]) return(mid)
72     if (y < x[mid]) hi <- mid else lo <- mid
73   }
74   if (y <= x[lo]) return(lo)
75   if (y < x[hi]) return(hi)
76   return(hi+1)
77 }
78
79 # application: M/M/1 queue, arrival rate 0.5, service rate 1.0
80
81 # globals
82 # rates
83 arvrate <- 0.5
84 srbrate <- 1.0
85 # event types
86 arvtype <- 1
87 srbdonetype <- 2
88 # initialize server queue
89 srvq <- NULL # will just consist of arrival times of queued jobs
90 # statistics
91 njobsdone <- NULL # jobs done so far
92 totwait <- NULL # total wait time so far
93
94 # event processing function required by general DES code above
95 reactevnt <- function(head) {
96   if (head[2] == arvtype) { # arrival
97     # if server free, start service, else add to queue
98     if (length(srvq) == 0) {
99       srvq <<- head[3]
100      srbdonetime <- sim$curtime + rexp(1,srbrate)
101      schedevnt(srbdonetime,srbdonetype,head[3])
102     } else srvq <<- c(srvq,head[3])
103     # generate next arrival
104     arvtime <- sim$curtime + rexp(1,arvrate)
105     schedevnt(arvtime,arvtype,arvtime)
106   } else { # service done
107     # process job that just finished
108     # do accounting
109     njobsdone <<- njobsdone + 1
110     totwait <<- totwait + sim$curtime - head[3]
111     # remove from queue
112     srvq <<- srvq[-1]

```

```

113     # more still in the queue?
114     if (length(srvq) > 0) {
115         # schedule new service
116         srvdonetime <- sim$currtime + rexp(1,srvrate)
117         schedevnt (srvdonetime,srvdonetype,srvq[1])
118     }
119 }
120 }
121
122 mmlsim <- function() {
123     srvq <<- vector(length=0)
124     njobsdone <<- 0
125     totwait <<- 0.0
126     # create simulation, 1 extra field (arrival time)
127     newsim(1)
128     # get things going, with first arrival event
129     arvtime <- rexp(1,rate=arvrate)
130     schedevnt (arvtime,arvtype,arvtime)
131     mainloop(100000.0)
132     return(totwait/njobsdone)
133 }

```

The simulation state, consisting of the current simulated time and the event list, have been placed in an R list, **sim**. This was done out of a desire to encapsulate the information, which in R typically means using a list.

This list **sim** has been made a global variable, for convenience and clarity. This has led to the use of R's superassignment operator `<<-`, with associated side effects. For instance in **mainloop()**, the line

```
sim$currtime <<- head[1] # update current simulated time
```

changes a global directly, while **sim\$evnts** is changed indirectly via the call

```
head <- getnextevnt()
```

If one has objections to use of globals, this could be changed, though without pointers it could be done only in a limited manner.

As noted, a key issue in writing a DES library is the event list. It has been implemented here as a matrix, **sim\$evnts**. Each row of the matrix corresponds to one scheduled event, with information on the event time, the event type (say arrival or service completion) and any application-specific data the programmer wishes to add. The rows of the matrix are in ascending order of event time, which is contained in the first column.

The main potential advantage of using a matrix as our structure here is that it enables us to maintain the event list in ascending order by time via a binary search operation by event time in that first column. This is done in the line

```
inspt <- binsearch(sim$evnts[,1],evnt[1])
```

in `insevt()`. Here we wish to insert a newly-created event into the event list, and the fact that we are working with a vector enables the use of a fast binary search.

However, looks are somewhat deceiving here. Though for an event set of size  $n$ , the search will be of time order  $O(\log n)$ , we still need  $O(n)$  to reassign the matrix, in the code

```
if (inspt > 1) e <- rbind(sim$evnts[1:(inspt-1)],,evnt)
nrse <- nrow(sim$evnts)
if (inspt <= nrse)
  e <- rbind(evnt, sim$evnts[inspt:nrse,])
sim$evnts <- e
```

Again, this exemplifies the effects of lack of pointers. Here is a situation in which it may be useful to write some code in C/C++ and then interface to R, which is discussed in Chapter 16.

This code above is a good example of the use of `rbind()`. We use the function to build up the new version of `sim$evnts` with our new event inserted, row by row. Recall that in this matrix, earlier events are stored in rows above later events. We first use `rbind()` to put together our new event with the existing events that are earlier than it, if any:

```
if (inspt > 1) e <- rbind(sim$evnts[1:(inspt-1)],,evnt)
```

Then, unless our new event is later than all the existing ones, we tack on the events that are later than it:

```
nrse <- nrow(sim$evnts)
if (inspt <= nrse)
  e <- rbind(evnt, sim$evnts[inspt:nrse,])
```

There are a couple of items worth mentioning in the line

```
sim$evnts <- sim$evnts[-1,,drop=F]
```

First, note that the negative-subscript feature of vector operations, in which the indices indicate which elements to skip, applies to matrices too. Here we are in essence deleting the first row of `sim$evnts`.

Second, here is an example of the need for `drop`. If there are just two events, the deletion will leave us with only one. Without `drop`, the assignment would then change `sim$evnts` from a matrix to a vector, causing problems in subsequent code that assumes it is a matrix.

The DES library code we've written above requires that the user provide a function `reactevnt()` that takes the proper actions for each event. In our M/M/1 queue example here, we've defined two types of events—arrival and service completion. Our function `reactevnt()` must then supply code to execute for each of these two events. As mentioned earlier, for an arrival event, we must add the new job to the queue, and if the server is idle, schedule a service event for this job. If a service completion event occurs, our code updates

the statistics and then checks the queue; if there are still jobs there, the first has a service completion event scheduled for it.

In this example, there is just one piece of application-specific data that we add to events, which is each job's arrival time. This is needed in order to calculate total wait time.

## 4.9 Filtering on Matrices

Filtering can be done with matrices too. Note that one must be careful with the syntax. For instance:

```
> x
      x
[1,] 1 2
[2,] 2 3
[3,] 3 4
> x[x[,2] >= 3,]
      x
[1,] 2 3
[2,] 3 4
```

Again, let's dissect this:

```
> j <- x[,2] >= 3
> j
[1] FALSE TRUE TRUE
> x[j,]
      x
[1,] 2 3
[2,] 3 4
```

Here is another example:

```
> m <- matrix(c(1,2,3,4,5,6),nrow=3)
> m
      [,1] [,2]
[1,]    1    4
[2,]    2    5
[3,]    3    6
> m[m[,1] > 1,]
      [,1] [,2]
[1,]    2    5
[2,]    3    6
> m[m[,1] > 1 & m[,2] > 5,]
[1] 3 6
```



## 4.10 Applying the Same Function to All Rows or Columns of a Matrix

### 4.10.1 The `apply()` Function

The arguments of `apply()` are the matrix/data frame to be applied to, the dimension—1 if the function applies to rows, 2 for columns—and the function to be applied.

For example, here we apply the built-in R function `mean()` to each column of a matrix `z`.

```
> z
      [,1] [,2]
[1,]    1    4
[2,]    2    5
[3,]    3    6
> apply(z,2,mean)
[1] 2 5
```

Here is an example of working on rows, using our own function:

```
> f <- function(x) x/c(2,8)
> y <- apply(z,1,f)
> y
      [,1] [,2] [,3]
[1,] 0.5 1.000 1.50
[2,] 0.5 0.625 0.75
```

You might be surprised that the size of the result here is 2 x 3 rather than 3 x 2. If the function to be applied returns a vector of `k` components, the result of `apply()` will have `k` rows. You can use the matrix transpose function `t()` to change it.

As you can see, the function to be applied needs at least one argument, which will play the role of one row or column in the array. In some cases, you will need additional arguments, which you can place following the function name in your call to `apply()`.

For instance, suppose we have a matrix of 1s and 0s, and want to create a vector as follows: For each row of the matrix, the corresponding element of the vector will be either 1 or 0, depending on whether the majority of the first `c` elements in that row are 1 or 0. Here `c` will be a parameter which we may wish to vary. We could do this:

```
> copymaj <- function(rw,c) {
+   maj <- sum(rw[1:c]) / c
+   return(ifelse(maj > 0.5,1,0))
+ }
> x <- matrix(c(1,1,1,0, 0,1,0,1, 1,1,0,1, 1,1,1,1, 0,0,1,0),nrow=4)
> x
      [,1] [,2] [,3] [,4] [,5]
[1,]    1    0    1    1    0
[2,]    1    1    1    1    0
```

```

[3,] 1 0 0 1 1
[4,] 0 1 1 1 0
> apply(x,1,copymaj,3)
[1] 1 1 0 1
> apply(x,1,copymaj,2)
[1] 0 1 0 0

```

Here the values 3 and 2 form the actual arguments for the formal argument **c** in **copymaj()**.

So, the general form of **apply** is

```
apply(m,dimcode,f,fargs)
```

where **m** is the matrix, **dimcode** is 1 or 2, according to whether we will operate on rows or columns, **f** is the function to be applied, and **fargs** is an optional list of arguments to be supplied to **f**.

If **f()** is only to be executed here, and if **fargs** consists of variables visible here, we might consider “inlining” it by defining it just before the call to **apply()**, as described in Section 9.5. In that case **fargs** would not be necessary.

*Note carefully that in writing **f()** itself, its first argument must be a vector that will be supplied by the caller as a row or column of **m**.*

As R moves closer and closer to parallel processing, functions like **apply()** will become more and more important. For example, the **clusterApply()** function in the snow package gives R some parallel processing capability, by distributing the submatrix data to various network nodes, with each one basically running **apply()** on its submatrix, and then collect the results. See Section 17.3.

### 4.10.2 The **sapply()** Function

If we call a vectorized function whose return value is a vector, the result is, in essence, a matrix.

```

z12 <- function(z) return(c(z,z^2))
x <- 1:8
> z12(x)
[1] 1 2 3 4 5 6 7 8 1 4 9 16 25 36 49 64
> matrix(z12(x),ncol=2)
      [,1] [,2]
[1,] 1 1
[2,] 2 4
[3,] 3 9
[4,] 4 16
[5,] 5 25
[6,] 6 36
[7,] 7 49
[8,] 8 64

```

We can streamline things using **sapply()**:

```
> z12 <- function(z) return(c(z, z^2))
> sapply(1:8, z12)
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
[1,]    1    2    3    4    5    6    7    8
[2,]    1    4    9   16   25   36   49   64
```

## 4.11 Digging a Little Deeper on the Vector/Matrix Distinction

It was stated at the outset of this chapter that

A matrix is a vector with two additional attributes, the number of rows and number of columns.

Let's look at this a bit more closely:

```
> z <- matrix(1:8, nrow=4)
> z
      [,1] [,2]
[1,]    1    5
[2,]    2    6
[3,]    3    7
[4,]    4    8
```

Looks fine. But **z** is still a vector, so that for instance we can query its length:

```
> length(z)
[1] 8
```

But as a matrix, **z** is a bit more than a vector:

```
> class(z)
[1] "matrix"
> attributes(z)
$dim
[1] 4 2
```

In other words, there actually is a **matrix** class, in the object-oriented programming sense. We'll cover OOP in Chapter 12, but for now it will suffice to say that R classes use a dollar sign to denote members of a class, just like C++, Python and so on use a period. So, we see that the **matrix** class has one attribute, named **dim**, which is a vector containing the numbers of rows and columns in the matrix.

You can also obtain **dim** via the **dim()** function:

```
> dim(z)
[1] 4 2
```

The numbers of rows and columns are obtainable individually via the **nrow()** and **ncol()** functions:

```
> nrow(z)
[1] 4
> ncol(z)
[1] 2
```

These just piggyback on **dim()**, as you can see by inspecting the code (functions, as objects, can be printed in interactive mode by simply typing their names), e.g.

```
> nrow
function (x)
dim(x)[1]
```

This calls **dim()**, then extracts element 1 from the resulting vector.

These functions are useful when you are writing a general-purpose library function whose argument is a matrix. By being able to sense the number of rows and columns in your code, you alleviate the caller of the burden of supplying that information as two additional arguments.

# Chapter 5

## Lists

R's **list** structure is similar to a C **struct**. It plays an important role in R, with data frames, object oriented programming and so on, as we will see later.

### 5.1 Creation

As an example, consider an employee database. Suppose for each employee we store name, salary and a boolean indicating union membership. We could initialize our database to be empty if we wish:

```
j <- list()
```

Or we could create a list and enter our first employee, Joe, this way:

```
j <- list(name="Joe", salary=55000, union=T)
```

We could print **j** out:

```
> j
$name
[1] "Joe"

$salary
[1] 55000

$union
[1] TRUE
```

Actually, the element names, e.g. “salary,” are optional. One could alternatively do this:

```

> jalt <- list("Joe", 55000, T)
> jalt
[[1]]
[1] "Joe"

[[2]]
[1] 55000

[[3]]
[1] TRUE

```

Here we refer to **jalt**'s elements as 1, 2 and 3 (which we can also do for **j** above).

## 5.2 List Tags and Values, and the `unlist()` Function

If the elements in a list do have names, e.g. with **name**, **salary** and **union** for **j** above, these names are called **tags**. The value associated with a tag is indeed called its **value**.

You can obtain the tags via **names()**:

```

> names(j)
[1] "name" "salary" "union"

```

To obtain the values, use **unlist()**:

```

> ulj <- unlist(j)
> ulj
  name salary union
"Joe" "55000" "TRUE"
> class(ulj)
[1] "character"

```

The return value of **unlist()** is a vector, in this case a vector of mode character, i.e. a vector of character strings.

## 5.3 Issues of Mode Precedence

Let's look at this a bit more closely:

```

> x
$abc
[1] 2

$de
[1] 5

```

Here the list **x** has two elements, with **x\$abc = 2** and **x\$de = 5**. Just for practice, let's call **names()**:

```
> names(x)
[1] "abc" "de"
```

Now let's try **unlist()**:

```
> ulx <- unlist(x)
> ulx
abc de
  2  5
> class(ulx)
[1] "numeric"
```

So again **unlist()** returned a vector, but R noticed that all the values were numeric, so it gave **ulx** that mode. By contrast, with **ulj** above, though one of the values was numeric, R was forced to take the “least common denominator,” and make the vector of mode character.

This sounds like some kind of precedence structure, and it is. As R's help for **unlist()** states,

Where possible the list elements are coerced to a common mode during the unlisting, and so the result often ends up as a character vector. Vectors will be coerced to the highest type of the components in the hierarchy NULL < raw < logical < integer < real < complex < character < list < expression: pairlists are treated as lists.

But there is something else to deal with here. Though **ulx** is a vector and not a list, R did give each of the elements a name. We can remove them by settings their names to NULL, seen in Section 3.7:

```
> names(ulx) <- NULL
> ulx
[1] 2 5
```

## 5.4 Accessing List Elements

The **\$** symbol is used to designate named elements of a list, but also **[[ ]]** works for referencing a single element and **[ ]** works for a group of them:

```
> j
$name
[1] "Joe"

$salary
[1] 55000
```

```

$union
[1] TRUE

> j[[1]]
[1] "Joe"

> j[2:3]
$salary
[1] 55000

$union
[1] TRUE

```

Note that `[[ ]]` returns a value, while `[ ]` returns a sublist.

## 5.5 Adding/Deleting List Elements

One can dynamically add and delete elements:

```

> z <- list(a="abc",b=12)
> z
$a
[1] "abc"

$b
[1] 12

> z$c = 1
> z
$a
[1] "abc"

$b
[1] 12

$c
[1] 1

> z$1] <- NULL # delete element 1
> z
$b
[1] 12

$c
[1] 1

[[3]]
[1] 1 2
> if (is.null(z$d)) print("it's not there") # testing existence
[1] "it's not there"
> y[[2]] <- 8 # can "skip" elements
> y
[[1]]

```



```
NULL
[[2]]
[1] 8
```

## 5.6 Indexing of Lists

To do indexing of a list, use [ ] instead of [[ ]]:

```
z[2:3]
$c
[1] 1

[[2]]
[1] 1 2
```

Names of list elements can be abbreviated to whatever extent is possible without causing ambiguity, e.g.

```
> j$sa1
[1] 55000
```

One common use is to package return values for functions that return more than one piece of information. Say for instance the function `f()` returns a matrix `m` and a vector `v`. Then one could write

```
return(list(mat=m, vec=v))
```

at the end of the function, and then have the caller access these items like this:

```
l <- f()
m <- l$mat
v <- l$vec
```

This is typical form for functions in the R library.

## 5.7 Extended Example: Managing Breakpoints in a Debugging Session

R's built-in debugging system is primitive but usable. We'll discuss the details in Chapter 14, but here is an overview sufficient to follow our example of list programming below.

The main library function of interest to us here is `trace()`. Say we are debugging `f()` in our own code. By typing

```
trace("f",browser,at=3)
```

we instruct R to insert a call to R's built-in function **browser()** in a temporary version of our code for **f()**. The action of **browser()** is to enter debug mode, in which we can single-step through the code, and so on.

In effect, the call to **browser()** serves as breakpoint. The **at** argument in **trace()** indicates where in the code to insert the breakpoint, in terms of a step number. The latter is similar to a line number but is actually a statement number. Comments are excluded, and loops and the like count as single statements.

If we are debugging several functions at once, it is hard to manage all our breakpoints. The goal of the code below is to facilitate this process. Our function **bk()** will add the breakpoints specified in the argument **toadd**, and delete those in **todel**.

```
# package to manage breakpoints with trace(), browser()

# breaks will be a list, indexed by quoted the names of the functions being
# debugged; breaks[["f"]] is then a vector of current breakpoints of
# f(); note that these are "step numbers" in R terminology, differing
# from line numbers within the function in that comments don't count and
# blocks count as just one step
breaks <- list()

# set a breakpoint using trace()
# note: quotedftnname must be a quoted string!
setbp <- function(quotedftnname,stepnums)
  trace(quotedftnname,browser,at=stepnums)

# bk() changes the current breakpoint set of the function quotedftnname,
# adding breakpoints at the steps specified by toadd, and deleting those
# specified by todel; if wipe is TRUE, then all breakpoints for this
# function are deleted, overriding toadd and todel, and calling
# untrace() on the function; example:
#   bk("g",c(5,8),12)
# will add breakpoints at steps 5 and 8 of g() while deleting the one at 12
bk <- function(quotedftnname,toadd=NULL,todel=NULL,wipe=F) {
  if (wipe) {
    breaks[[quotedftnname]] <- NULL
    untrace(quotedftnname)
    return()
  }
  # since each call to trace() nullifies previous such actions, we must
  # update our breakpoint list, then call trace() with the new list
  if (!is.null(toadd))
    breaks[[quotedftnname]] <- c(breaks[[quotedftnname]],toadd)
  if (!is.null(todel))
    breaks[[quotedftnname]] <- setdiff(breaks[[quotedftnname]],todel)
  setbp(quotedftnname,breaks[[quotedftnname]])
}

# list step numbers (for use as "at" in trace())
lssns <- function(ftnname)
  as.list(body(ftnname))
```

The most salient point here is that the main data structure, **breaks**, has been implemented as a list. There is one member of the list for each function being debugged. An alternative would be to store this data in a

matrix, say with each column storing the breakpoints for a particular function. We could call R's `colnames()` function to name the columns of our matrix according to our function names, e.g. "f" in the example above, and then access the columns using these names. But since each function will typically have a different number of breakpoints, a matrix implementation would be wasteful in terms of space.

Our indexing of the list is also via strings representing our function names and the list `[[ ]]` operator. Fortunately, `trace()` allows the function to be specified in either quoted or unquoted form; we need the former.

Each list member is a vector, showing the step numbers for the current breakpoints of the given function.

Note the use of `as.list()` near the end of the code. A function is an object, consisting of statements. applying `as.list()` to any object will create a list containing one element for each fundamental unit in the object (obviously depending on the nature of the object). In the case of a function, this will be a list consisting of one list member per statement. In this manner we can determine the statement numbers within a function, and thus determine the value(s) we wish for the `at` argument in `trace()`.

For example:

```
> f
function(x,y) {
  x <- x + y
  return(x)
}
> as.list(body(f))
[[1]]
`{`

[[2]]
x <- x + y

[[3]]
return(x)
```

So, for example the statement

```
x <- x + y
```

will be step 2. If we want to put a breakpoint in front of this statement, our call will be

```
bk("f", toadd=2)
```

## 5.8 Applying the Same Function to All Elements of a List

The analog of `apply()` for lists is `lapply()`. It applies the given function to all elements of the specified list. For example:

```
> lapply(list(1:3, 25:27), median)
[[1]]
[1] 2

[[2]]
[1] 26
```

In this example the list was created only as a temporary measure, so we should convert back to numeric:

```
> as.numeric(lapply(list(1:3, 25:27), median))
[1] 2 26
```

## 5.9 Size of a List

You can obtain the number of elements in a list via **length()**:

```
> length(j)
[1] 3
```

## 5.10 Recursive Lists

Lists can be recursive, i.e. you can have lists within lists. For instance:

```
> b <- list(u = 5, v = 12)
> c <- list(w = 13)
> a <- list(b, c)
> a
[[1]]
[[1]]$u
[1] 5

[[1]]$v
[1] 12

[[2]]
[[2]]$w
[1] 13

> length(a)
[1] 2
```

So, **a** is now a two-element list, with each element itself being a list.

## Chapter 6

# Data Frames

On an intuitive level, a *data frame* is like a matrix, with a rows-and-columns structure. However, it differs from a matrix in that each column may have a different mode. For instance, one column may be numbers and another column might be character strings.

On a technical level, a data frame is a list of equal-length vectors. Each column is one element of the list.

### 6.1 Continuation of Our Earlier Session

Recall our course examination data set in Section 2.6. There we didn't have a header, but I've added one now, so that the first few records in the file now are

```
"Exam 1" "Exam 2" Quiz
2 3.3 4
3.3 2 3.7
4 4.3 4
2.3 0 3.3
2.3 1 3.3
3.3 3.7 4
3 3.7 3.3
2.7 1 3.3
4 3.3 4
3.7 3.7 4
4.3 4.3 4
```

As you can see, each line contains the three test scores for one student. This is the classical “two-dimensional file” notion, i.e. each line in our file contains the data for one observation in a statistical dataset. The idea of a data frame is to encapsulate such data, along with variable names into one object.

Note that I have separated fields here by spaces. Other delimiters may be specified, notably commas for Excel output. As mentioned, I've specified the variable names in the first record. Names with embedded spaces must be quoted.

Suppose the second exam score for the first student had been missing. Then we would have typed

```
2.0 NA 4.0
```

in that line of the exams file. In any subsequent statistical analyses, R would do its best to cope with the missing data, in the obvious manners. In some situations, We have to set the option **na.rm=T**.) If for instance we had wanted to find the mean score on Exam 2, calling R's function **mean()** would skip that first student in find the mean.

The first few rows in our R object **examsquiz** now look like this:

```
> head(examsquiz)
  Exam.1 Exam.2 Quiz
1    2.0    3.3  4.0
2    3.3    2.0  3.7
3    4.0    4.3  4.0
4    2.3    0.0  3.3
5    2.3    1.0  3.3
6    3.3    3.7  4.0
```

In R, the components of an object are accessed via the \$ operator. Since a data frame is a list of vectors, then for example the vector of all the Exam 1 scores is **examsquiz\$Exam.1**, as we confirm here:

```
> examsquiz$Exam.1
 [1] 2.0 3.3 4.0 2.3 2.3 3.3 3.0 2.7 4.0 3.7 4.3 3.0 3.0 4.0 1.0 4.3 3.3 1.7 4.3
[20] 2.3 3.3 4.0 3.3 3.0 4.3 3.0 2.0 3.0 4.0 3.7 2.7 3.0 2.0 2.0 1.7 3.3 3.7 2.3
[39] 1.7
```

The [1] means that items 1-19 start here, the [20] means that items 20-38 start here, etc.

This book focuses on programming, a context in which the names of data frame columns are less often used. Generically, we can always refer to a column in “matrix column” manner, such as:

```
> examsquiz$Exam.1
 [1] 2.0 3.3 4.0 2.3 2.3 3.3 3.0 2.7 4.0 3.7 4.3 3.0 3.0 4.0 1.0 4.3 3.3 1.7 4.3
[20] 2.3 3.3 4.0 3.3 3.0 4.3 3.0 2.0 3.0 4.0 3.7 2.7 3.0 2.0 2.0 1.7 3.3 3.7 2.3
[39] 1.7
> examsquiz[2,1]
 [1] 3.3
```

In that second command, we've printed the element of the second row, first column, i.e. the Exam 2 score for the second student.

## 6.2 Matrix-Like Operations

Many matrix operations can also be used on data frames.

### 6.2.1 rowMeans() and colMeans()

```
> colMeans(examsquiz)
  Exam.1 Exam.2 Quiz
3.020513 2.902564 3.569231
```

### 6.2.2 rbind() and cbind()

The **rbind()** and **cbind()** matrix functions introduced in Section 4.7 work here too.

We can also create new columns from old ones, e.g. we can add a variable which is the difference between Exams 1 and 2:

```
> eq <- cbind(examsquiz, examsquiz$Exam.2 - examsquiz$Exam.1)
> class(eq)
[1] "data.frame"
> head(eq)
  Exam.1 Exam.2 Quiz examsquiz$Exam.2 - examsquiz$Exam.1
1  2.0  3.3  4.0                                1.3
2  3.3  2.0  3.7                                -1.3
3  4.0  4.3  4.0                                 0.3
4  2.3  0.0  3.3                                -2.3
5  2.3  1.0  3.3                                -1.3
6  3.3  3.7  4.0                                 0.4
```

The new name is rather unwieldy, but we could change it, using the **names()** function.

### 6.2.3 Indexing, Filtering and apply()

One can also refer to the rows and columns of a data frame using two-dimensional array notation, including indexing. For instance, in our example data frame **examsquiz** here:

- `examsquiz[2,3]` would refer to the third score for the second student
- `examsquiz[2,]` would refer to the set of all scores for the second student
- `examsquiz[c(1,2,5),]` would refer to the set of all scores for the first, second and fifth students
- `examsquiz[10:13,]` would refer to the set of all scores for the tenth through thirteenth students
- `examsquiz[-2,]` would refer to the set of all scores for all students except the second

Filtering works the same way in data frames as in matrices. This is a key operation on these kinds of objects. An extended example follows, in Section 6.3.

You can use **apply()** on data frames, as R will coerce them into matrices.

### 6.3 Extended Example: Data Preparation in a Statistical Study

In a study of engineers and programmers sponsored by U.S. employers for permanent residency, I considered the question, “How many of these workers are ‘the best and the brightest,’ i.e. people of extraordinary ability?”<sup>1</sup>

The government data is limited. The (admittedly imperfect) way to determine whether a worker is of extraordinary ability was to look at the ratio of actual salary to the government prevailing wage for that job and location. If that ratio is substantially higher than 1.0 (by law it cannot be less than 1.0), one can reasonably assume that this worker has a high level of talent.

I used R to prepare and analyze the data, and will present excerpts of my preparation code here. First, I read in the data file:

```
all2006 <- read.csv("2006.csv",header=T,as.is=T)
```

The function `read.csv()` is essentially identical to `read.table()`, except that the input data are in the Comma Separated Value format exported by spreadsheets, which is the way the dataset was prepared by the Department of Labor (DOL). We now have a data frame, `all2006`, consisting of all the data for the year 2006.

Some wages in the dataset were reported in hourly, rather than yearly, terms. This indicates possible part-time job status, which I wanted to exclude. A look at the documentation for the dataset showed that column 17 was the pay period, i.e. yearly, hourly, etc., while columns 15 and 19 contained the actual salary and prevailing wage. So, I did some filtering:

```
all2006 <- all2006[all2006[,17]=="Year",]
all2006 <- all2006[all2006[,15]> 20000,]
all2006 <- all2006[all2006[,19]> 200,]
```

I also needed to create a new column for the ratio between actual wage and prevailing wage:

```
all2006 <- cbind(all2006,all2006[,15]/all2006[,19])
```

This new variable is in column 25. Since I knew I would be calculating the median in this column for many subsets of the data, I defined a function to do this work:

```
medrat <- function(dataframe) {
  return(median(dataframe[,25],na.rm=T))
}
```

---

<sup>1</sup>N. Matloff, *New Insights from the Dept. of Labor PERM Labor Certification Database*, Sloan Foundation West Coast Program on Science and Engineering Workers conference, January 18, 2008.



Note the need to exclude NA values, which are common in government datasets.

In addition, I wanted to analyze the talent patterns at particular companies, using the company name stored in column 7:

```
makecorp <- function(corpname) {
  t <- all2006[all2006[,7]==corpname,]
  return(t)
}
goog2006 <- makecorp("GOOGLE INC.")
medrat(goog2006)
ms2006 <- makecorp("MICROSOFT CORPORATION")
medrat(ms2006)
...
```

I also wanted to analyze by occupation. DOL has a code number for each job title, stored in column 14, which I used to create the corresponding subsets of the original dataset:

```
makeocc <- function(df,lowerbd,upperbd) {
  return(df[df[,14]>=lowerbd & df[,14]<=upperbd,])
}
# for 2007 data
prg2007 <- makeocc(all2007,"15-1021.00","15-1052.00") # programmers
se2007 <- makeocc(all2007,"15-1030.00","15-1039.00") # s.w. engineers
enr2007 <- makeocc(all2007,"17-2000.00","17-2999.00") # other engineers
...
```

One more example: I wanted to analyze by nationality, which is in column 24. Note my use of looping over a vector of mode character:

```
mainnatnames <- c("CHINA", "INDIA", "CANADA", "GERMANY", "UNITED KINGDOM")
natwithindf <- function(natlist,df) {
  for (nat in natlist) {
    tmpdf <- df[df[,24]==nat,]
    mr <- medrat(tmpdf)
    cat(nat,": ",mr,"\n")
  }
}
natwithindf(mainnatnames,prg2007)
natwithindf(mainnatnames,se2007)
...
```

## 6.4 Creating a New Data Frame from Scratch

We saw above how to create a data frame by reading from a data file. We can also create a data frame directly, using the function `data.frame()`.

For example,

```
> z <- data.frame(cbind(c(1,2),c(3,4)))
> z
  X1 X2
1  1  3
2  2  4
```

Note again the use of the **cbind()** function.

We can also coerce a matrix to a data frame, e.g.

```
> x <- matrix(c(1,2,3,4),nrow=2,ncol=2)
> x
  [,1] [,2]
[1,]  1   3
[2,]  2   4
> y <- data.frame(x)
> y
  X1 X2
1  1  3
2  2  4
```

As you can see, the column names will be X1, X2, ... However, you can change them, e.g.

```
> z
  X1 X2
1  1  3
2  2  4
> names(z) <- c("col 1","col 2")
> z
  col 1 col 2
1  1     3
2  2     4
```

## 6.5 Converting a List to a Data Frame

For printing, statistical calculations and so on, you may wish to convert a list to a data frame. Here's straightforward code to do it:

```
# converts a list lst to a data frame, which is the return value
wrtlst <- function(lst) {
  frm <- data.frame()
  rw <- 1
  for (key in names(lst)) {
    frm[rw,1] <- key
    frm[rw,2] <- lst[key]
    rw <- rw+1
  }
  return(frm)
}
```

But if our list has named tags and has only numeric values, the following code may run faster:

```
# converts a list lst that has only numeric values to a data frame,
# which is the return value of the function
lsttodf <- function(lst) {
  n <- length(lst)
  # create the data frame, using the default column names
  frm <- data.frame(V1=character(n),V2=numeric(n))
  frm[,1] <- names(lst)
  frm[,2] <- as.numeric(unlist(lst))
  return(frm)
}
```

## 6.6 The Factor Factor

If your table does have a variable, i.e. a column, in character mode, you probably should set **as.is=T** in your call to **read.table()**, so that this variable stays a vector rather than a factor. Otherwise even your numeric columns will become factors, which could cause problems as seen in our example below.

The same is true for creating a data frame via a call to **data.frame()**. Consider:

```
> d <- data.frame(cbind(c(0,5,12,13),c("xyz","ab","yabc",NA)))
> d
  X1 X2
1  0 xyz
2  5  ab
3 12 yabc
4 13 <NA>
> d[1,1] <- 3
Warning message:
In `[<-factor`(`*tmp*`, iseq, value = 3) :
  invalid factor level, NAs generated
> d
  X1 X2
1 <NA> xyz
2  5  ab
3 12 yabc
4 13 <NA>
```

Why didn't **d[1,1]** change? Well, since the second column was a character vector, **data.frame()** treated it as a factor, and thus “demoted” the first column to factor status too. Then when we tried to assign the value 3 to **d[1,1]**, R told us that 3 was not one of the official values (**levels** for this factor).

This can be avoided by setting **stringsAsFactors** to false:

```
> d <- data.frame(cbind(c(0,5,12,13),c("xyz","ab","yabc",NA)), stringsAsFactors=F)
> d
  X1 X2
1  0 xyz
2  5  ab
```

```
3 12 yabc
4 13 <NA>
> d[1,1] <- 3
> d
  X1 X2
1  3 xyz
2  5  ab
3 12 yabc
4 13 <NA>
```

See Section 11.3.

## Chapter 7

# Factors and Tables

Consider the data frame, say in a file **ct.dat**,

```
"VoteX" "VoteLastTime"  
"Yes" "Yes"  
"Yes" "No"  
"No" "No"  
"Not Sure" "Yes"  
"No" "No"
```

where in the usual statistical fashion each row represents one subject under study. In this case, say we have asked five people (a) "Do you plan to vote for Candidate X?" and (b) "Did you vote in the last election?" (The first line in the file is a header.)

Let's read in the file:

```
> ct <- read.table("ct.dat",header=T)  
> ct  
      VoteX VoteLastTime  
1      Yes           Yes  
2      Yes           No  
3      No            No  
4 Not Sure          Yes  
5      No            No
```

We can use the **table()** function to convert this data to contingency table format, i.e. a display of the counts of the various combinations of the two variables:

```
> cttab <- table(ct)  
> cttab  
      VoteLastTime  
VoteX   No Yes  
No      2  0  
Not Sure 0  1  
Yes     1  1
```

The 2 in the upper-left corner of the table shows that we had, for example, two people who said No to (a) and No to (b). The 1 in the middle-right indicates that one person answered Not Sure to (a) and Yes to (b).

We can in turn change this to a data frame—not the original one, but a data-frame version of the contingency table:

```
> ctdf <- as.data.frame(cttab)
> ctdf
  VoteX VoteLastTime Freq
1     No           No    2
2 Not Sure           No    0
3     Yes           No    1
4     No           Yes    0
5 Not Sure           Yes    1
6     Yes           Yes    1
```

Note that in our original data frame, the two columns are called *factors* in R. A factor is basically a vector of mode **character**, intended to represent values of a categorical variable, such as the `ct$VoteX` variable above. The **factor** class includes a component **levels**, which in the case of `ct$VoteX` are Yes, No and Not Sure.

Of course, all of the above would still work if our original data frame `ct` we had three factors, or more.

We get counts on a single factor in isolation as well, e.g.

```
> y <- factor(c("a", "b", "a", "a", "b"))
> z <- table(y)
> z
y
a b
3 2
> as.vector(z)
[1] 3 2
```

Note the use here of `as.vector()` to extract only the counts.

Among other things, this gives us an easy way to determine what proportion of items satisfy a certain condition. For example:

```
> x <- c(5, 12, 13, 3, 4, 5)
> table(x == 5)

FALSE  TRUE
   4     2
```

So our answer is 2/6.

The function `table()` is often used with `cut()`. To explain what the latter does, first consider the call

```
y <- cut(x, b, labels=F)
```

where **x** is a vector of observations, and **b** defines bins, which are the semi-open intervals **(b[1],b[2])**, **(b[2],b[3])**,.... Then **y[j]** will be the index **i** such that **x[j]** falls into bin **i**.

For instance,

```
> cut(1:8,c(0,4,7,8),labels=F)
[1] 1 1 1 1 2 2 2 3
```

The function **cut()** has many, many other options but in our context here, the point is that we can pipe the output of **cut()** into **tabe()**, thus getting counts of the numbers of observations in each bin.

```
bincounts <- function(x,b) {
  y <- cut(x,b)
  return(as.vector(table(y)))
}
```





## Chapter 8

# R Programming Structures

R is a full programming language, similar to scripting languages such as Perl and Python. One can define functions, use constructs such as loops and conditionals, etc.

R is also block-structured, in a manner similar to those of the above languages, as well as C. Blocks are delineated by braces, though they are optional if the block consists of just a single statement.

## 8.1 Control Statements

### 8.1.1 Loops

#### Basic Structure

In our function `oddcount()` in Section 2.3, the line

```
for (n in x) {
```

will be instantly recognized by Python programmers. It of course means that there will be one iteration of the loop for each component of the vector `x`, with `n` taking on the values of those components. In other words, in the first iteration, `n = x[1]`, in the second iteration `n = x[2]`, etc. For example:

```
> x <- c(5,12,13)
> for (n in x) print(n^2)
[1] 25
[1] 144
[1] 169
```

C-style looping with `while` and `repeat` are also available, complete with `break`:

```
> i <- 1
> while(1) {
+   i <- i+4
+   if (i > 10) break
+ }
> i
[1] 13
```

Of course, **break** can be used with **for** too.

Another useful statement is **next**, which instructs the interpreter to go to the next iteration of the loop. Usage of this construct often allows one to avoid using complexly nested if-then-else statements, which make the code confusing. See Section 10.9 for an example.

### Looping Over Nonvector Sets

The **for** construct works on any vector, regardless of mode. One can loop over a vector of file names, for instance. Say we have files **x** and **y** with contents

```
1
2
3
4
5
6
```

and

```
5
12
13
```

Then this loop prints each of them:

```
> for (fn in c("x","y")) print(scan(fn))
Read 6 items
[1] 1 2 3 4 5 6
Read 3 items
[1] 5 12 13
```

R does not directly support iteration over nonvector sets, but there are indirect yet easy ways to accomplish it. One way would be to use **lapply()**, as shown in Section 5.8. Another would be to use **get()**, as in the following example. Here we have two matrices, **u** and **v**, containing statistical data, and we wish to apply R linear regression function **lm()** to each of them:

```

> u
      [,1] [,2]
[1,]    1    1
[2,]    2    2
[3,]    3    4
> v
      [,1] [,2]
[1,]    8   15
[2,]   12   10
[3,]   20    2
> for (m in c("u","v")) {
+   z <- get(m)
+   print(lm(z[,2] ~ z[,1]))
+ }

Call:
lm(formula = z[, 2] ~ z[, 1])

Coefficients:
(Intercept)      z[, 1]
   -0.6667      1.5000

Call:
lm(formula = z[, 2] ~ z[, 1])

Coefficients:
(Intercept)      z[, 1]
   23.286      -1.071

```

The reader is welcome to make his/her own refinements here.

### 8.1.2 If-Else

The syntax for if-else is like this:

```

> if (r == 4) {
+   x <- 1
+   y <- 2
+ } else {
+   x <- 3
+   y <- 4
+ }

```

**Note that the braces are necessary, even for single-statement bodies for the if and else, and the newlines are important too.** For instance, the left brace before the **else** is what the parser uses to tell that this is an **if-else** rather than just an **if**; this would be easy for the parser to handle in batch mode but not in interactive mode. However, if you place the **else** on the same line as the **if**, this problem will not occur.

See also the **ifelse()** function discussed in Section 3.10.

## 8.2 Arithmetic and Boolean Operators and Values

```

x + y          addition
x - y          subtraction
x * y          multiplication
x / y          division
x ^ y          exponentiation
x %% y         modular arithmetic
x %/% y        integer division
x == y         test for equality
x <= y         test for less-than-or-equal
x >= y         test for greater-than-or-equal
x && y         boolean and for scalars
x || y         boolean or for scalars
x & y          boolean and for vectors (vector x,y,result)
x | y          boolean or for vectors (vector x,y,result)
!x             boolean negation

```

The boolean values are TRUE and FALSE. They can be abbreviated to T and F, but must be capitalized. These values change to 1 and 0 in arithmetic expressions, e.g.

```

> 1 < 2
[1] TRUE
> (1 < 2) * (3 < 4)
[1] 1
> (1 < 2) * (3 < 4) * (5 < 1)
[1] 0
> (1 < 2) == TRUE
[1] TRUE
> (1 < 2) == 1
[1] TRUE

```

You can invent your own operators! Just write a function whose name begins and ends with %. An example is given in Section 10.7.

## 8.3 Type Conversions

The `str()` function converts an object to string form, e.g.

```

> x <- c(1,2,4)
> class(x)
[1] "numeric"
> str(x)
num [1:3] 1 2 4

```

There is a generic function `as()` which does conversions, e.g.

```

> x <- c(1,2,4)
> y <- as.character(x)

```

```
> y
[1] "1" "2" "4"
> as.numeric(y)
[1] 1 2 4
> q <- as.list(x)
> q
[[1]]
[1] 1

[[2]]
[1] 2

[[3]]
[1] 4

> r <- as.numeric(q)
> r
[1] 1 2 4
```

You can see all of this family by typing

```
> methods(as)
```

The **unclass()** function converts a class object to an ordinary list.



## Chapter 9

# R Functions

In terms of syntax and operation, R functions are similar to those of C, Python and so on. However, as will be seen, R's lack of pointer variables does change the manner in which we write functions.

### 9.1 Functions Are Objects

Note that functions are first-class objects, of the class **function** of course. They thus can be used for the most part just like, say, a vector. This is seen in the syntax of function creation:

```
> g <- function(x) {  
+   return(x+1)  
+ }
```

Here **function()** is a built-in R function whose job is to create functions! What it does, technically, is create objects of the **function** class, just as **list()** creates objects of the **list** class.

On the right-hand side above, there are really two arguments to **function()**, the first of which is **x** and the second is "return(x+1)". These will be used by **function()** to create the desired function, which it then assigns to **g**.

Recall that when using R in interactive mode, simply typing the name of an object results in printing that object to the screen. Functions are no exception, since again they are objects just like anything else:

```
> g  
function(x) {  
  return(x+1)  
}
```

This is handy if you're using a function that you've written but have forgotten what its arguments are, for instance. It's also useful if you are not quite sure what an R library function does; by looking at the code

you may understand it better.

Similarly, we can assign functions, use them as arguments to other functions, and so on:

```
> f1 <- function(a,b) return(a+b)
> f2 <- function(a,b) return(a-b)
> f <- f1
> f(3,2)
[1] 5
> f <- f2
> f(3,2)
[1] 1
> g <- function(h,a,b) h(a,b)
> g(f1,3,2)
[1] 5
> g(f2,3,2)
[1] 1
```

The return value of `function()` is a function, even if you don't assign it to a variable. Thus you can create *anonymous* functions, familiar to Python programmers. Modifying the above example, for instance, we have:

```
> g <- function(h,a,b) h(a,b)
> g(function(x,y) return(x*y),2,3)
[1] 6
```

Here, the expression

```
function(x,y) return(x*y)
```

created the specified function, which then played the role of `g()`'s formal argument `h` in the call to `g()`.

## 9.2 Return Values

The return value of a function can be any R object.

Normally `return()` is explicitly called. However, lacking this, the last value computed will be returned by default. For instance, in the `oddcount()` example in Section 2.3, we could simply write

```
oddcount <- function(x) {
  k <- 0
  for (n in x) {
    if (n %% 2 == 1) k <- k+1
  }
}
```

Again, the return value can be any R object. Here for instance a function is returned:



```

> g <- function() {
+   t <- function(x) return(x^2)
+   return(t)
+ }
> g()
function(x) return(x^2)
<environment: 0x8aafde8>

```

If your function has multiple return values, place them in a list or other container. We'll discuss this more in Section 9.3.3.

## 9.3 Functions Have (Almost) No Side Effects

Yet another influence of the functional programming philosophy is that functions do not change their arguments, i.e. there are no *side effects*.<sup>1</sup> Consider, for instance, this:

```

> x <- c(4,1,3)
> y <- sort(x)
> y
[1] 1 3 4
> x
[1] 4 1 3

```

The point is that **x** didn't change. We'll discuss the details, and implications for programming style, in the next few subsections.

### 9.3.1 Locals, Globals and Arguments

Say a variable **z** appearing within a function has the same name as a variable that is global to it.<sup>2</sup> Then it will be treated as local, except that its initial value will be that of the global. Subsequent assignment to it within the function will not change the value of the global. (An exception to this arises with the superassignment operator. See Section 9.3.2.)

The same is true in the case in which **z** is a formal argument to the function. It's initial value will be that of the actual argument, but subsequent changes to it will not affect the actual argument.

For example:

```

> u <- 1
> v <- 8
> g <- function(x) {

```

---

<sup>1</sup>There are two exceptions to this. One is the superassignment operator, to be discussed in Section 9.3.2. The other is that a function might do a plot, write to a disk file, etc., actions that the R literature refers to as side effects.

<sup>2</sup>The phrasing here has been chosen to recognize the fact that one can define functions within functions. See Section 9.5.

```

+   x <- x + 1
+   u <- u + x
+   return(u)
+ }
> g(v)
[1] 10
> u
[1] 1
> v
[1] 8

```

Neither **u** nor **v** changed.

### 9.3.2 Writing to Globals Using the Superassignment Operator

If you do want to write to global variables (or more precisely, to variables one level higher than the current scope), you can use the **superassignment** operator, `<<-`. For example,

```

> two <- function(u) {
+   u <<- 2*u
+   y <<- 2*y
+   z <- 2*z
+ }
> x <- 1
> y <- 2
> z <- 3
> two(x)
> x
[1] 1
> y
[1] 4
> z
[1] 3

```

### 9.3.3 Strategy in Dealing with Lack of Pointers

Suppose you have a function whose goal is to change several variables lying outside of it. As we have seen, we cannot simply take the approaches available to us in, say, C: We cannot pass pointers to these variables as arguments to our function, nor can we place the variables in a **struct** and pass a pointer to it as an argument.

So, what can we do? The key is to reassign the function's return value. For instance, recall our earlier example:

```

> x <- c(4, 1, 3)
> y <- sort(x) # x does not change

```

The simple solution here is to reassign the sorted vector to **x**:

```
> x <- c(4,1,3)
> x <- sort(x) # x is now sorted
```

What if we have several variables to change? A solution is to gather them together into a list, call the function with this list as an argument, then reassign to the original list.

As a simple example, suppose we wish the function **addone()** to add 1 to its two arguments. We can accomplish it this way:

```
> addone <- function(lst) {
+   lst$u <- lst$u + 1
+   lst$v <- lst$v + 1
+   return(lst)
+ }
> uvlst <- list()
> uvlst$u <- 1
> uvlst$v <- 8
> uvlst
$u
[1] 1

$v
[1] 8

> uvlst <- addone(uvlst)
> uvlst
$u
[1] 2

$v
[1] 9
```

See Section 10.9 for an example in which this strategy is employed.

## 9.4 Default Values for Arguments

In Section 6.1, we read in a dataset from a file **exams**:

```
> testscores <- read.table("exams",header=TRUE)
```

The parameter **header=TRUE** told R that we did have a header line, so R should not count that first line in the file as data.

This is an example of the use of *named arguments*. The function **read.table()** has a number of arguments, some of which are optional, which means that we must specify which arguments we are using, by using their names, e.g. **header=TRUE** above. (Again, Python programmers will find this familiar.) The ones you don't specify all have default values.

## 9.5 Functions Defined Within Functions

Since functions are objects, it is perfectly valid to define one function within the body of another. The rules of scope still apply:

```
> f <- function(x) {
+   v <- 1
+   g <- function(y) return((u+v+y)^2)
+   gu <- g(u)
+   print(gu)
+ }
> u <- 6
> f()
[1] 169
```

Here the global variable **u** was visible within the body of **f()**, including in the construction of **g()**. Similarly, the variable **v**, while local to **f()**, is global to **g()**, since the latter is defined within **f()**.

Now, is it desirable to do such a thing? The answer is yes! If **g()** is to be used only within **f()**, the placing within **f()** is consistent with the principle of *encapsulation*, and thus a Good Thing. Even if you are generally reluctant to use global variables, you may find that this approach is clear and clean as long as **g()** consists of only a line or two.

Remember that if you wish the function to write to a variable which is global to the function in scope, you must use the superassignment operator.

On the other hand, if the definition of **g()** were more than a few lines, placing it within the body of **f()** might be distracting, but for short functions this works well. An example appears in Section 10.5.

## 9.6 Writing Your Own Binary Operations

You can invent your own operations! Just write a function whose name begins and ends with **%**, with two arguments of a certain type, and a return value of that type.

For example, here's a binary operation that adds double the second operand to the first:

```
> "%a2b%" <- function(a,b) return(a+2*b)
> 3 %a2b% 5
[1] 13
```

A less trivial example is given in Section 10.7.

## 9.7 Editing Functions

Also, a nice implication of the fact that functions are objects is that you can edit functions from within R's interactive mode. Most R programmers do their code editing in a separate window, but for a small, quick change, the `edit()` function is sometimes handy.

For instance, I could change the function `f1()` by typing

```
> f1 <- edit(f1)
```

This would open the default editor on the code for `f1`, which I could then edit and assign back to `f1`.

The editor invoked will depend on R's internal options variable `editor`. In Unix-class systems, R will set this from your `EDITOR` or `VISUAL` environment variable, or you can set it yourself, e.g.

```
> options(editor="/usr/bin/vim")
```

See the online documentation if you have any problems.

Note that in this example I am saving the revision back to the same function. Note too that when I do so, I am making an assignment, and thus the R interpreter will compile the code; if I have any errors, the assignment will not be done. I can recover by the command

```
> x <- edit()
```

Warning: Apparently comments are not preserved.



## Chapter 10

# Doing Math in R

R contains built-in functions for your favorite math operations, and of course for statistical distributions.

### 10.1 Math Functions

The usual `exp()`, `log()`, `log10()`, `sqrt()`, `abs()` etc. are available, as well as `min()`, `which.min()` (returns the index for the smallest element), `max()`, `which.max()`, `pmin()`, `pmax()`, `sum()`, `prod()` (for products of multiple factors), `round()`, `floor()`, `ceiling()`, `sort()` etc. The function `factorial()` computes its namesake, so that for instance `factorial(3)` is 6.

Note that the function `min()` returns a scalar even when applied to a vector. By contrast, if `pmin()` is applied to two or more vectors, it returns a vector of the elementwise minima. For example:

```
> z
     [,1] [,2]
[1,]    1    2
[2,]    5    3
[3,]    6    2
> min(z[,1], z[,2])
[1] 1
> pmin(z[,1], z[,2])
[1] 1 3 2
```

Also, some special math functions, described when you invoke `help()` with the argument `Arithmetic`.

Function minimization/maximization can be done via `nlm()` and `optim()`.

R also has some calculus capabilities, e.g.

```
> D(expression(exp(x^2)), "x") # derivative
exp(x^2) * (2 * x)
```

```
> integrate(function(x) x^2,0,1)
0.3333333 with absolute error < 3.7e-15
```

There are R packages such as **odesolve** for differential equations, **ryacas** to interface R with the Yacas symbolic math system, and so on.

## 10.2 Functions for Statistical Distributions

R has functions available for various aspects of most of the famous statistical distributions. Prefix the name by **d** for the density, **p** for the cdf, **q** for quantiles and **r** for simulation. The suffix of the name indicates the distribution, such as *norm*, *unif*, *chisq*, *binom*, *exp*, etc.

For example for the chi-square distribution:

```
> mean(rchisq(1000,different=2))  find mean of 1000 chi-square(2) variates
[1] 1.938179
> qchisq(0.95,1)  find 95th percentile of chi-square(2)
[1] 3.841459
```

An example of the use of **rnorm()**, to generate random normally-distributed variates, as well as one for **rbinom()** for binomial/Bernoulli random variates. The function **dnorm()** gives the normal density, **pnorm()** gives the normal CDF, and **qnorm()** gives the normal quantiles.

The **d**-series, for density, gives the probability mass function in the case of discrete distributions. The first argument is a vector indicating at which points we wish to find the values of the pmf. For instance, here is how we would find the probabilities of 0, 1 or 2 heads in 3 tosses of a coin:

```
> dbinom(0:2,3,0.5)
[1] 0.125 0.375 0.375
```

See the online help pages for details, e.g. by typing

```
> help(pnorm)
```

## 10.3 Sorting

Ordinary numerical sorting of a vector can be done via **sort()**.

```
> x <- c(13,5,12,5)
> sort(x)
[1] 5 5 12 13
```



If one wants the inverse, use `order()`. For example:

```
> order(x)
[1] 2 4 3 1
```

Here is what `order()`'s output means: The 2 means that `x[2]` is the smallest in `x`; the 4 means that `x[4]` is the second-smallest, etc.

You can use `order()`, together with indexing, to sort data frames. For instance:

```
> y <- read.table("y")
> y
      V1 V2
1  def  2
2   ab  5
3 zzzz  1
> r <- order(y$V2)
> r
[1] 3 1 2
> z <- y[r,]
> z
      V1 V2
3 zzzz  1
1  def  2
2   ab  5
```

What happened here? We called `order()` on the second column of `y`, yielding a vector telling us which numbers from that column should go before which if we were to sort them. The 3 in this vector tells us that `x[3,2]` is the smallest number; the 1 tells us that `x[1,2]` is the second-smallest; and the 2 tells us that `x[2,2]` is the third-smallest.

We then used indexing (Section 6.2.3) to produce the frame sorted by column 2, storing it in `z`.

## 10.4 Linear Algebra Operations on Vectors and Matrices

Multiplying a vector by a scalar works directly, as seen earlier. For example,

```
> y
[1] 1 3 4 10
> 2*y
[1] 2 6 8 20
```

If you wish to compute the inner product (“dot product”) of two vectors, use `crossprod()`. Note that the name is a misnomer, as the function does not compute vector cross product.

For matrix multiplication in the mathematical sense, the operator to use is `%*%`, not `*`. Note also that a vector is considered a one-row matrix, not a one-column matrix, and thus is suitable as the left factor in a matrix product, but not directly usable as the right factor.

The function `solve()` will solve systems of linear equations, and even find matrix inverses. For example:

```
> a <- matrix(c(1,1,-1,1),nrow=2,ncol=2)
> b <- c(2,4)
> solve(a,b)
[1] 3 1
> solve(a)
      [,1] [,2]
[1,]  0.5  0.5
[2,] -0.5  0.5
```

Use `t()` for matrix transpose, `qr()` for QR decomposition, `chol()` for Cholesky, and `det()` for determinants.

Use `eigen()` to compute eigenvalues and eigenvectors, though if the matrix in question is a covariance matrix, the R function `prcomp()` may be preferable. The function `diag()` extracts the diagonal of a square matrix, useful for obtaining variances from a covariance matrix.

## 10.5 Extended Example: A Function to Find the Sample Covariance Matrix

The R function `var()` computes the sample variance of a set of scalar observations, but R seems to lack the analog for the vector-valued case. For sample data  $X_{ij}$ ,  $i = 1, \dots, n$  and  $j = 1, \dots, r$ , i.e.  $n$  observations of an  $r$ -dimensional column vector  $X$ , the sample covariance matrix is

$$\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(X_i - \bar{X})^T \quad (10.1)$$

(Some authors would divide by  $n-1$  instead of  $n$ .) Here  $X_i$  is the column vector  $(X_{i1}, \dots, X_{ir})^T$ ,  $\bar{X} = \sum_{i=1}^n X_i/n$  and  $T$  denotes matrix transpose. Keep in mind that the summands in (10.1) are  $r \times r$  matrices.

The function below calculates this quantity:

```
# finds sample covariance matrix of the data x, the latter arranged one
# observation per row; if zeromean = T, it is assumed that the
# population mean is known to be 0; assumes ncol(x) > 1 (otherwise use
# var())
sampcov <- function(x,zeromean=F) {
  if (!zeromean) {
    sampmean <- colMeans(x)
    # subtract the sample mean from each observation
    sampmeanmat <- rep(sampmean,nrow(x))
    sampmeanmat <- matrix(sampmeanmat,nrow=nrow(x),byrow=T)
    x <- x - sampmeanmat
  }
  yyt <- function(y) return(y %*% t(y))
  # result of apply() will have ncol(x)^2 rows, nrow(x) columns
  xyyt <- apply(x,1,yyt)
  rmeans <- rowMeans(xyyt)
  return(as.matrix(rmeans,nrow=ncol(xyyt)))
}
```

Note the default value for the argument **z**. As explained in the comments, in situations in which we know that the population mean is 0, we would use that value instead of the sample mean in our computation of the sample covariance matrix. But otherwise we must compute the sample mean first, and then subtract it from each observation. Let's look at the details.

First, remember that since we are working with vector-valued observations, the sample mean is a vector too. Since each observation is stored in one row of our data matrix, the call

```
sampmean <- colMeans(z)
```

gives us exactly what we need.

Now we must subtract that mean from each of the observations. This of course could be done via a loop, but in R we try to avoid loops. Instead, we make good use of R's powerful operations as follows:

```
sampmeanmat <- rep(sampmean, nrow(x))
sampmeanmat <- matrix(sampmeanmat, nrow=nrow(x), byrow=T)
x <- x - sampmeanmat
```

The code first makes  $n$  copies of the sample mean, then forms a matrix from them, one copy per row. (Note the value T for **byrow**.) We then subtract the result from our observation matrix **x**, thus subtracting the sample mean from each observation. Let's call the results "centered" observations.

Now we must compute vector-times-vector-transpose for each centered observation. This suggests using **apply()**, so we need a function to be applied; **yyt()** is that function. We do have to take care here, as the function is nonscalar, with implications on the dimensions of the output of **apply()**, as warned in the comment. The vector-times-vector-transpose value for the  $i^{th}$  observation will be stored in the  $i^{th}$  column of the output. However, it will be stored as a vector of length  $r^2$ , rather than an  $r \times r$  matrix. So, after the call to **rowMeans()** to do the summation and division by  $n$  in (10.1), we need to convert back to matrix form before returning:

```
return(as.matrix(rmeans, nrow=ncol(xytt)))
```

So, how might we test this code? We could make up a small data set and compute its sample covariance matrix by hand. An alternative would be to simulate a large sample from a distribution with known covariance matrix, which is what we'll do here:

```
> x <- matrix(rnorm(2000), ncol=2)
```

This is 1000 randomly generated pairs of independent  $N(0,1)$  random variables. Since they are independent with variance 1, the population covariance matrix here is the identity matrix,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (10.2)$$

Since the sample size of 1000 is fairly large, the sample covariance matrix should be pretty close to its population counterpart. Let's check that our code produces this result:

```
> sampcov(x)
           [,1]
[1,] 0.98156922
[2,] 0.01316139
[3,] 0.01316139
[4,] 1.01864366
```

Confirmed.

Actually, the code above can be made more compact, and probably faster, by using R's `sweep()` function. We replace

```
# subtract the sample mean from each observation
sampmeanmat <- rep(sampmean,nrow(x))
sampmeanmat <- matrix(sampmeanmat,nrow=nrow(x),byrow=T)
x <- x - sampmeanmat
```

by

```
# subtract the sample mean from each observation
x <- sweep(x,2,sampmean)
```

This call tells R to subtract the  $i^{th}$  element of `sampmean` from the  $i^{th}$  element of each row of `x`, exactly what we want.

## 10.6 Extended Example: Finding Stationary Distributions of Markov Chains

A Markov chain is a random process in which we move among various states, in a “memoryless” fashion whose definition need not concern us here. We assume the states to be finite in number. The state could be the number of jobs in a queue, the amount of items stored in inventory and so on. Let  $p_{ij}$  denote the probability of moving from state  $i$  to state  $j$  during a time step. We are interested calculating  $\pi_i$ , the long-run proportion of time spent at state  $i$ , over all states  $i$ .

Let  $P$  denote the transition matrix, whose  $i^{th}$  row,  $j^{th}$  column element is  $p_{ij}$ , and let the vector  $\pi$  consist of the long-run probabilities  $\pi_i$ . Then it can be shown that  $\pi$  must satisfy

$$\pi = \pi P \tag{10.3}$$

or

$$(I - P^T)\pi = 0 \tag{10.4}$$

Note that there is also the constraint

$$\sum_i \pi_i = 1 \quad (10.5)$$

One of the equations in the system is redundant. We thus eliminate one of them, say by removing the last row of I-P in (10.4). To reflect This can be used to calculate the  $\pi_i$ . It turns out that one of the equations in the system is redundant. We thus eliminate one of them, say by removing the last row of I-P in (10.4).

To reflect (10.5), which in matrix form is

$$\mathbf{1}_n^T \pi = 1 \quad (10.6)$$

where  $\mathbf{1}_n$  is a column vector of all 1s, we replace the removed row in I-P by a row of all 1s, and in the right-hand side of (10.4) we replace the last 0 by a 1. We can then solve the system.

All this can be done with R's `solve()` function:

```

1 findpi1 <- function(p) {
2   n <- nrow(p)
3   imp <- diag(n) - t(p) # I-P
4   imp[n,] <- rep(1,n) # insert row of 1s
5   rhs <- c(rep(0,n-1),1) # form right-hand side
6   pivec <- solve(imp,rhs) # solve the system
7   return(pivec)
8 }
```

Or one can note from (10.3) that  $\pi$  is a left eigenvector of P with eigenvalue 1, so one can use R's `eigen()` function. It can be proven that if P satisfies certain conditions which hold commonly in applications (irreducibility and aperiodicity), every eigenvalue other than 1 is smaller than 1 in absolute value (so we can speak of *the* eigenvalue 1), and the eigenvector corresponding to 1 has all components real.

Since  $\pi$  is a left eigenvector, the argument in the call must be P transpose rather than P. In addition, since an eigenvector is only unique up to scalar multiplication, we must deal with the fact that the return value of `eigen()` may have negative components, and will likely not satisfy (10.5). Here is the code:

```

1 findpi2 <- function(p) {
2   n <- nrow(p)
3   # find first eigenvector of P transpose
4   pivec <- eigen(t(p))$vectors[,1]
5   # guaranteed to be real, but could be negative
6   if (pivec[1] < 0) pivec <- -pivec
7   # normalize to sum to 1
8   pivec <- pivec / sum(pivec)
9   return(pivec)
10 }
```

## 10.7 Set Operations

There are set operations, e.g.

```
> x <- c(1,2,5)
> y <- c(5,1,8,9)
> union(x,y)
[1] 1 2 5 8 9
> intersect(x,y)
[1] 1 5
> setdiff(x,y)
[1] 2
> setdiff(y,x)
[1] 8 9
> setequal(x,y)
[1] FALSE
> setequal(x,c(1,2,5))
[1] TRUE
> 2 %in% x # note that plain "in" doesn't work
[1] TRUE
> 2 %in% y
[1] FALSE
```

Recall from Section 9.6 that you can write your own binary operations. Here is an operation for the symmetric difference between two sets (i.e. all the elements in exactly one of the two operand sets):

```
> "%sdf%" <- function(a,b) {
+   sdfxy <- setdiff(x,y)
+   sdfyx <- setdiff(y,x)
+   return(union(sdfxy,sdfyx))
+ }
> x "%sdf%" y
[1] 2 8 9
```

The function **combn** generates combinations:

```
> c32 <- combn(1:,3,2)
> c32
      [,1] [,2] [,3]
[1,]    1    1    2
[2,]    2    3    3
> class(c32)
[1] "matrix"
```

The function also allows the user to specify a function to be called by **combn()** on each combination.

## 10.8 Simulation Programming in R

Here is a simple example, which finds  $E[\max(X, Y)]$  for independent  $N(0,1)$  random variables  $X$  and  $Y$ :

```
# MaxNorm.r

sum <- 0
nreps <- 100000
for (i in 1:nreps) {
  xy <- rnorm(2) # generate 2 N(0,1)s
  sum <- sum + max(xy)
}
print(sum/nreps)
```

### 10.8.1 Built-In Random Variate Generators

R has functions to generate variates from a number of different distributions. For example, **rbinom()** generates binomial or Bernoulli random variates.<sup>1</sup> If we wanted to, say, find the probability of getting at least 4 heads out of 5 tosses of a coin, we could do this:

```
> x <- rbinom(100000, 5, 0.5)
> length(x[x >= 4])/100000
[1] 0.18791
```

There are also **rnorm()** for the normal distribution, **rexp()** for the exponential, **runif()** for the uniform, **rgamma()** for the gamma, **rpois()** for the Poisson and so on.

### 10.8.2 Obtaining the Same Random Stream in Repeated Runs

By default, R will generate a different random number stream from run to run of a program. If you want the same stream each time, call **set.seed()**, e.g.

```
> set.seed(8888) # or your favorite number as an argument
```

## 10.9 Extended Example: a Combinatorial Simulation

Consider the following probability problem:

Three committees, of sizes 3, 4 and 5, are chosen from 20 people. What is the probability that persons A and B are chosen for the same committee?

This problem is not hard to solve analytically, but we may wish to check solution using simulation, and in any case, writing the code will illustrate how R's set operations can come in handy in combinatorial settings. Here is the code:

---

<sup>1</sup>A sequence of independent 0-1 valued random variables with the same probability of 1 for each is called **Bernoulli**.

```

1 # Committee.r, combinatorial computation example: Three committees, of
2 # sizes 3, 4 and 5, are chosen from 20 people; what is the probability
3 # that persons A and B are chosen for the same committee?
4
5 # number the committee members from 1 to 20, with A and B being 1 and 2
6
7 sim <- function(nreps) {
8   commdata <- list() # will store all our info about the 3 committees
9   commdata$countabsamecomm <- 0
10  for (rep in 1:nreps) {
11    commdata$whosleft <- 1:20 # who's left to choose from
12    commdata$numabchosen <- 0 # number among A, B chosen so far
13    # choose committee 1
14    commdata <- choosecomm(commdata,5)
15    # if A or B already chosen, no need to look at the other comms.
16    if (commdata$numabchosen > 0) next
17    # choose committee 2
18    commdata <- choosecomm(commdata,4)
19    if (commdata$numabchosen > 0) next
20    # choose committee 3
21    commdata <- choosecomm(commdata,3)
22  }
23  print(commdata$countabsamecomm/nreps)
24 }
25
26 choosecomm <- function(comdat,comsize) {
27   # choose committee
28   committee <- sample(comdat$whosleft,comsize)
29   # count how many of A and B were chosen
30   comdat$numabchosen <- length(intersect(1:2,committee))
31   if (comdat$numabchosen == 2)
32     comdat$countabsamecomm <- comdat$countabsamecomm + 1
33   # delete chosen committee from the set of people we now have to choose from
34   comdat$whosleft <- setdiff(comdat$whosleft,committee)
35   return(comdat) # R (usually) has no side effects
36 }

```



# Chapter 11

## Input/Output

### 11.1 Reading from the Keyboard

You can use `scan()`:

```
> z <- scan()
1: 12 5
3: 2
4:
Read 3 items
> z
[1] 12 5 2
```

Use `readline()` to input a line from the keyboard as a string:

```
> w <- readline()
abc de f
> w
[1] "abc de f"
```

### 11.2 Printing to the Screen

In interactive mode, one can print the value of a variable or expression by simply typing the variable name or expression. In batch mode, one can use the `print()` function, e.g.

```
print(x)
```

The argument may be an object.

It's a little better to use **cat()** instead of **print()**, as the latter can print only one expression and its output is numbered, which may be a nuisance to us. E.g.

```
> print("abc")
[1] "abc"
> cat("abc\n")
abc
```

The arguments to **cat()** will be printed out with intervening spaces, for instance

```
> x <- 12
> cat(x, "abc", "de\n")
12 abc de
```

If you don't want the spaces, use separate calls to **cat()**:

```
> z <- function(a,b) {
+   cat(a)
+   cat(b, "\n")
+ }
> z("abc", "de")
abcde
```

### 11.3 Reading a Matrix or Data Frame From a File

The function **read.table()** was discussed in Section 6.1. Here is a bit more on it.

- The default value of **header** is **FALSE**, so if we don't have a header, we need not say so.
- By default, character strings are treated as R factors. To turn this “feature” off, include the argument **as.is=T** in your call to **read.table()**.
- If you have a spreadsheet export file, i.e. of type **.csv** in which the fields are separated by commas instead of spaces, use **read.csv()** instead of **read.table()**. There is also **read.xls** to read core spreadsheet files.
- Note that if you read in a matrix via **read.table()**, the resulting object will be a data frame, even if all the entries are numeric. You may need it as a matrix, in which case do a followup call to **as.matrix()**.

There appears to be no good way of reading in a matrix from a file. One can use **read.table()** and then convert. A simpler way is to use **scan()** to read in the matrix row by row, making sure to use the **byrow** option in the function **matrix()**. For instance, say the matrix **x** is

```
1 0 1
1 1 1
1 1 0
1 1 0
0 0 1
```

We can read it into a matrix this way:

```
> x <- matrix(scan("x"), nrow=5, byrow=T)
```

## 11.4 Reading a File One Line at a Time

You can use `readLines()` for this. We need to create a *connection* first, by calling `file()`.

For example, suppose we have a file `z`, with contents

```
1 3
1 4
2 6
```

Then we can do this:

```
> c <- file("z", "r")
> readLines(c, n=1)
[1] "1 3"
> readLines(c, n=1)
[1] "1 4"
> readLines(c, n=1)
[1] "2 6"
```

To read the entire file in one fell swoop, set `n` to a negative value.

If `readLines()` encounters the end of the file, it returns a null string.

## 11.5 Writing to a File

### 11.5.1 Writing a Table to a File

The function `write.table()` works very much like `read.table()`, in this case writing a data frame instead of reading one.

In the case of writing a matrix, to a file, just state that you want no row or column names, e.g.

```
> write.table(xc, "xcnew", row.names=F, col.names=F)
```

### 11.5.2 Writing to a Text File Using `cat()`

(The point of the word *text* in the title of this section is that, for instance, the number 12 will be written as the ASCII characters ‘1’ and ‘2’, as with `printf()` with `%d` format in C—as opposed to the bits 000...00001100.)

The function `cat()` can be used to write to a file, one part at a time. For example:

```
> cat("abc\n", file="u")
> cat("de\n", file="u", append=T)
```

The file is saved after each operation, so at this point the file on disk really does look like

```
abc
de
```

One can write multiple fields. For instance

```
> cat(file="v", 1, 2, "xyz\n")
```

would produce a file `v` consisting of a single line,

```
1 2 xyz
```

### 11.5.3 Writing a List to a File

You have various options here. One would be to convert the list to a data frame, as in Section 6.5, and then call `write.table()`.

### 11.5.4 Writing to a File One Line at a Time

Use `writeLines()`. See Section 11.4.

## 11.6 Directories, Access Permissions, Etc.

R has a variety of functions for dealing with directories, file access permissions and the like.

Here is a short example. Say our current working directory contains files `x` and `y`, as well as a subdirectory `z`. Suppose the contents of `x` is

```
12
5
13
```

and **y** contains

```
3
4
5
```

The following code sums up all the numbers in the non-directory files here:

```
tot <- 0
ndatafiles <- 0
for (d in dir()) {
  if (!file.info(d)$isdir) {
    ndatafiles <- ndatafiles + 1
    x <- scan(d,quiet=T)
    tot <- tot + sum(x)
  }
}
cat("there were",ndatafiles,"nondirectory files, with a sum of",tot,"\n")
```

Type

```
> ?files
```

to get more information on permissions etc.

The functions **getwd()** and **setwd()** can be used to determine or change the current working directory.

The “..” notation for parent directory in Linux systems does work on them.

## 11.7 Accessing Files on Remote Machines Via URLs

Functions such as **read.table()**, **scan()** and so on accept file names as arguments. For example, I placed a file **z** on my Web page, with the contents

```
1 2
3 4
```

and accessed it from within R running on my home machine:

```
> z <- read.table("http://heather.cs.ucdavis.edu/~matloff/z")
> z
  V1 V2
1  1  2
2  3  4
```

You can also read the file one line at a time, as in Section 11.4.

## 11.8 Extended Example: Monitoring a Remote Web Site

## Chapter 12

# Object-Oriented Programming

R definitely has object-oriented programming (OOP) themes. These are in many sense different from the paradigm of, say, Java, but there are two key themes:

- Everything in R is an object.
- R is polymorphic, i.e. the same function call leads to different operations for object of different classes.

This chapter the OOP aspects of R.

### 12.1 Managing Your Objects

#### 12.1.1 Listing Your Objects with the `ls()` Function

The `ls()` command will list all of your current objects.

A useful named argument is `pattern`, which enables wild cards. For example:

```
> ls()
 [1] "acc"      "acc05"    "binomci"  "cmeans"  "divorg"   "dv"
 [7] "fit"      "g"        "genxc"    "genxnt"  "j"        "lo"
[13] "out1"     "out1.100" "out1.25"  "out1.50" "out1.75"  "out2"
[19] "out2.100" "out2.25"  "out2.50"  "out2.75" "par.set"  "prpdf"
[25] "ratbootci" "simonn"   "vecprod"  "x"       "zout"
"zout.100"
[31] "zout.125" "zout3"    "zout5"    "zout.50" "zout.75"
> ls(pattern="ut")
 [1] "out1"     "out1.100" "out1.25"  "out1.50"  "out1.75"  "out2"
 [7] "out2.100" "out2.25"  "out2.50"  "out2.75"  "zout"      "zout.100"
[13] "zout.125" "zout3"    "zout5"    "zout.50"  "zout.75"
```

### 12.1.2 Removing Specified Objects with the `rm()` Function

To remove objects you no longer need, use `rm()`. For instance,

```
> rm(a,b,x,y,z,uuu)
```

would remove the objects **a**, **b** and so on.

One of the named arguments of `rm()` is `list`, which makes it easier to remove multiple objects. For example,

```
> rm(list = ls())
```

would assign all of your objects to `list`, thus removing everything. If you make use of `ls()`'s `pattern` argument this tool becomes even more powerful, e.g.

```
> ls()
[1] "doexpt"          "notebookline"      "nreps"             "numcorrectcis"
[5] "numnotebooklines" "numrules"          "observationpt"     "prop"
[9] "r"              "rad"              "radius"            "rep"
[13] "s"              "s2"              "sim"              "waits"
[17] "wbar"          "x"              "y"              "z"
> ls(pattern="notebook")
[1] "notebookline"      "numnotebooklines"
> rm(list=ls(pattern="notebook"))
> ls()
[1] "doexpt"          "nreps"             "numcorrectcis"    "numrules"
[5] "observationpt"  "prop"             "r"                "rad"
[9] "radius"        "rep"              "s"                "s2"
[13] "sim"           "waits"           "wbar"            "x"
[17] "y"            "z"
```

Here we had two objects whose names included the string “notebook,” then asked to remove them, which was confirmed by the second call to `ls()`.

### 12.1.3 Saving a Collection of Objects with the `save()` Function

Calling `save()` on a collection of objects will write them to disk for later retrieval by `load()`.

### 12.1.4 Listing the Characteristics of an Object with the `names()`, `attributes()` and `class()` Functions

An object consists of a gathering of various kinds of information, with each kind being called an *attribute*. The `names()` function will tell us the names of the attributes of the given object. For a data frame, for example, these will be the names of the columns. For a regression object, these will be **coefficients**, **residuals** and so on. Calling the `attributes()` function will give you all this, plus the class of the object itself. To just get the class, call `class()`.



### 12.1.5 The exists() Function

The function `exists()` returns TRUE or FALSE, depending on whether the argument exists. Be sure to quote the argument, e.g.

```
> exists("acc")
[1] TRUE
```

shows that the object `acc` exists.

### 12.1.6 Accessing an Object Via Strings

The call `get("u")` will return the object `u`. An example appears on page 68.

## 12.2 Generic Functions

As mentioned in Chapter 2, R is polymorphic, in the sense that the same function, can have different operation for different classes.<sup>1</sup> One can apply `plot()`, for example, to many types of objects, getting an appropriate plot for each. The same is true for `print()` and `summary()`.

In this manner, we get a uniform interface to different classes. So, when someone develops a new R class for others to use, we can try to apply, say, `summary()` and reasonably expect it to work. This of course means that the person who wrote the class, knowing the R idiom, would have had the foresight of writing such a function in the class, knowing that people would expect one.

The functions above are known as *generic functions*. The actual function executed will be determined by the class of the object on which you are calling the function.

For example, let's look at a simple regression analysis (introduced in Section 2.6):

```
> x <- c(1,2,3)
> y <- c(1,3,8)
> lmout <- lm(y ~ x)
> class(lmout)
[1] "lm"
> lmout
```

```
Call:
lm(formula = y ~ x)
```

```
Coefficients:
(Intercept)          x
      -3.0           3.5
```

---

<sup>1</sup>Technically, it is not the same function, but rather different functions initially invoked via the same name. This will become clear below.

Note that we printed out the object **lmout**. (Remember, by simply typing the name of an object in interactive mode, the object is printed.) What happened then was that the R interpreter saw that **lmout** was an object of class **"lm"** (the quotation marks are part of the class name), and thus instead of calling **print()**, it called **print.lm()**, a special print method in the **"lm"** class.

In fact, we can take a look at that method:

```
> print.lm
function (x, digits = max(3, getOption("digits") - 3), ...)
{
  cat("\nCall:\n", deparse(x$call), "\n\n", sep = "")
  if (length(coef(x)) > 0) {
    cat("Coefficients:\n")
    print.default(format(coef(x), digits = digits), print.gap = 2,
                  quote = FALSE)
  }
  else cat("No coefficients\n")
  cat("\n")
  invisible(x)
}
<environment: namespace:stats>
```

Don't worry about the details here; our main point is that the printing was dependent on context, with a different print function being called for each different class.

You can see all the implementations of a given generic method by calling **methods()**, e.g.

```
> methods(print)
 [1] print.acf*           print.anova
 [3] print.aov*           print.aovlist*
 [5] print.ar*            print.Arima*
 [7] print.arima0*        print.AsIs
 [9] print.Bibtex*        print.by
 ...
```

You can see all the generic methods this way:

```
> methods(class="default")
 ...
```

## 12.3 Writing Classes

A class is named via a quoted string:

```
> class(3)
[1] "numeric"
> class(list(3,TRUE))
[1] "list"
```

```
> lmout <- lm(y ~ x)
> class(lmout)
[1] "lm"
```

If a class is derived from a parent class, the name of the derived class will be a vector consisting of two strings, first one for the derived class and then one for the parent.

Methods are implemented as generic functions. The name of a method is formed by concatenating the function name with a period and the class name, e.g. **print.lm()**.

The class of an object is stored in its **class** attribute.

### 12.3.1 Old-Style Classes

Older R functions use a cobbled-together structure for classes, referred to as S3. Under this approach, a class instance is created by forming a list, with the elements of the list being the member variables of the class. (Readers who know Perl may recognize this *ad hoc* nature in Perl's own OOP system.) The **"class"** attribute is set by hand by using the **attr()** or **class()** function, and then various generic functions are defined.

For instance, continuing our employee example from Section 5.1, we could write

```
> j <- list(name="Joe", salary=55000, union=T)
> class(j) <- "employee"
> attributes(j) # let's check
$names
[1] "name" "salary" "union"

$class
[1] "employee"
```

Now we will write a generic function for this class. First, though, let's see what happens when we call the default **print()**:

```
> j
$name
[1] "Joe"

$salary
[1] 55000

$union
[1] TRUE

> j[[1]]
[1] "Joe"
```

Now let's write our own function.

```
print.employee <- function(wrkr) {
  cat(wrkr$name, "\n")
  cat("salary", wrkr$salary, "\n")
  cat("union member", wrkr$union, "\n")
}
```

Test it:

```
> j
Joe
salary 55000
union member TRUE
```

What happened here is that the R interpreter, seeing that we wish to print `j`, checked to see which class `j` is an object of. That class is “**employee**”, so R invoked the version of `print()` for that class, `print.employee()`.

### 12.3.2 Extended Example: A Class for Storing Upper-Triangular Matrices

Here is a more involved example, in which we will write an R class “**ut**” for upper-triangular matrices. Recall that these are square matrices whose elements below the diagonal are 0s, such as

$$\begin{pmatrix} 1 & 5 & 12 \\ 0 & 6 & 9 \\ 0 & 0 & 2 \end{pmatrix}$$

If the matrix is large, having such a class will save storage space (though at the expense of a little extra access time).

The component `mat` of this class will store the matrix. To save on storage space, only the diagonal and above-diagonal elements will be stored, in column-major order. Storage for the above matrix, for instance, would consist of the vector (1,5,6,12,9,2).

We have included a component `ix` in this class, to show where in `mat` the various columns begin. For the above case, `ix` would be `c(1,2,4)`, meaning that column 1 begins at `mat[1]`, column 2 begins at `mat[2]` and column 3 begins at `mat[4]`. This allows for handy access to individual elements or columns of the matrix.

The function `ut()` below creates an instance of this class. Its argument `inmat` is in full matrix format, i.e. including the 0s.

```
# ut.r, compact storage of upper-triangular matrices

# create an object of class "ut" from the full matrix (0s included)
# inmat
ut <- function(inmat) {
  n <- nrow(inmat)
  rtn <- list() # start to build the object
```

```

class(rtrn) <- "ut"
rtrn$mat <- vector(length=sumltoi(n))
rtrn$ix <- sumltoi(0:(n-1)) + 1
for (i in 1:n) {
  # store column i
  ixi <- rtrn$ix[i]
  rtrn$mat[ixi:(ixi+i-1)] <- inmat[1:i,i]
}
return(rtrn)
}

# returns 1+...+i
sumltoi <- function(i) return(i*(i+1)/2)

# uncompress utmat to a full matrix
expandut <- function(utmat) {
  n <- length(utmat$ix) # numbers of rows and cols of matrix
  fullmat <- matrix(nrow=n,ncol=n)
  for (j in 1:n) {
    # fill j-th column
    start <- utmat$ix[j]
    fin <- start + j - 1
    abovediagj <- utmat$mat[start:fin] # above-diagonal part of col j
    fullmat[,j] <- c(abovediagj,rep(0,n-j))
  }
  return(fullmat)
}

# print matrix
print.ut <- function(utmat)
  print(expandut(utmat))

# multiply one ut matrix by another, returning another ut instance;
# implement as a binary operation
"%mut%" <- function(utmat1,utmat2) {
  n <- length(utmat1$ix) # numbers of rows and cols of matrix
  utprod <- ut(matrix(0,nrow=n,ncol=n))
  for (i in 1:n) {
    # let a[i] and bi denote columns i of utmat1 and utmat2,
    # respectively; column i of product is equal to
    # bi[1]*a[1] + ... + bi[i]*a[i]
    # index of start of column i in utmat2
    startbi <- utmat2$ix[i]
    prodcoli <- rep(0,i)
    for (j in 1:i) {
      startaj <- utmat1$ix[j]
      bielement <- utmat2$mat[startbi+j-1]
      prodcoli[1:j] <- prodcoli[1:j] +
        bielement * utmat1$mat[startaj:(startaj+j-1)]
    }
    startprodcoli <- sumltoi(i-1)+1
    utprod$mat[startbi:(startbi+i-1)] <- prodcoli
  }
  return(utprod)
}

# examples:
utm1 <- ut(rbind(1:2,c(0,2)))
utm2 <- ut(rbind(3:2,c(0,1)))
utp <- utm1 %mut% utm2

```

```

print(utm1)
print(utm2)
print(utp)
utm1 <- ut(rbind(1:3,0:2,c(0,0,5)))
utm2 <- ut(rbind(4:2,0:2,c(0,0,1)))
utp <- utm1 %mut% utm2
print(utm1)
print(utm2)
print(utp)

```

Note that the second call to **sum1toi()** in **ut()**,

```
rtrn$ix <- sum1toi(0:(n-1)) + 1
```

is applied to a vector, saving us a loop. Note too the use of recycling in

```
utprod <- ut(matrix(0,nrow=n,ncol=n))
```

though **rep()** could be used too.

We have included a function **expandut()** to convert a compressed matrix back to full matrix form. We use this in **print.ut()** our generic function **print()** for this class.

Finally, we include a matrix-multiply function. Since this is a binary operation, we take advantage of the fact that R accommodates user-defined binary operations, as described in Section 9.6, and implement our matrix-multiply function as **%mut%**. Here are the details:

The code here has been written around the fact that R stores matrices in column-major order. As mentioned in the comments, column *i* of the product can be expressed as a linear combination of the columns of the first factor. This is computed in the loop:

```

for (j in 1:i) {
  startaj <- utmat1$ix[j]
  bielement <- utmat2$mat[startbi+j-1]
  prodcoli[1:j] <- prodcoli[1:j] +
    bielement * utmat1$mat[startaj:(startaj+j-1)]
}

```

Note that since each column of that first factor contains 0s near the bottom, we have expressions like **prodcoli[1:j]** above instead of **prodcoli[,j]**. This way we avoid wasting time by multiplying 0s.

### 12.3.3 New-Style Classes

Here one creates the class by calling **setClass()**. Continuing our employee example, we could write

```
> setClass("employee",
+   representation(
+     name="character",
+     salary="numeric",
+     union="logical")
+ )
[1] "employee"
```

Now, let's create an instance of this class, for Joe, using `new()`:

```
> joe <- new("employee", name="Joe", salary=55000, union=T)
> joe
An object of class employee
Slot "name":
[1] "Joe"

Slot "salary":
[1] 55000

Slot "union":
[1] TRUE
```

Note that the member variables are called *slots*. We reference them via the `@` symbol, e.g.

```
> joe@salary
[1] 55000
```

The `slot()` function can also be used.

To define a generic function on a class, use `setMethod()`. Let's do that for our class **"employee"** here. We'll implement the `show()` function. To see what this function does, consider our command above,

```
> joe
```

As we know, in R, when we type the name of a variable while in interactive mode, the value of the variable is printed out:

```
> joe
An object of class employee
Slot "name":
[1] "Joe"

Slot "salary":
[1] 55000

Slot "union":
[1] TRUE
```

The action here is that `show()` is called.<sup>2</sup> In fact, we would get the same output here by typing

---

<sup>2</sup>The function `show()` has precedence over `print()`.

```
> show(joe)
```

Let's override that, with the following code (which is in a separate file, and brought in using `source()`):

```
setMethod("show", "employee",
  function(object) {
    inorout <- ifelse(object@union, "is", "is not")
    cat(object@name, "has a salary of", object@salary,
        "and", inorout, "in the union", "\n")
  }
)
```

The first argument gives the name of the generic function which we will override, with the second argument giving the class name. We then define the new function.

Let's try it out:

```
> joe
Joe has a salary of 55000 and is in the union
```

## 12.4 Extended Example: a Procedure for Polynomial Regression

Consider a statistical regression setting, with one predictor variable. Since any statistical model is merely an approximation, one can in principle get better and better models by fitting polynomials of higher and higher degree. However, at some point this becomes overfitting, so that the prediction of new, future data actually deteriorates for degrees higher than some value.

The class “**polyreg**” below aims to deal with this issue. It fits polynomials of various degrees, but assesses fits via cross-validation to reduce the risk of overfitting.

```
1 # Poly.r: S3 class for polynomial regression
2
3 # polyfit(x,maxdeg) fits all polynomials up to degree maxdeg; y is
4 # vector for response variable, x for predictor; creates an object of
5 # class "polyreg", consisting of outputs from the various regression
6 # models, plus the original data
7 polyfit <- function(y,x,maxdeg) {
8   pwrs <- powers(x,maxdeg) # form powers of predictor variable
9   lmout <- list() # start to build class
10  class(lmout) <- "polyreg" # create a new class
11  for (i in 1:maxdeg) {
12    lmo <- lm(y ~ pwrs[,1:i])
13    # extend the lm class here, with the cross-validated predictions
14    lmo$fitted.xvvalues <- lvoneout(y,pwrs[,1:i,drop=F])
15    lmout[[i]] <- lmo
16  }
17  lmout$x <- x
18  lmout$y <- y
```



```

19     return(lmout)
20 }
21
22 # generic print() for an object fits of class "polyreg": print
23 # cross-validated mean-squared prediction errors
24 print.polyreg <- function(fits) {
25     maxdeg <- length(fits) - 2 # count lm() outputs only, not $x and $y
26     n <- length(fits$y)
27     tbl <- matrix(nrow=maxdeg,ncol=1)
28     cat("mean squared prediction errors, by degree\n")
29     colnames(tbl) <- "MSPE"
30     for (i in 1:maxdeg) {
31         fi <- fits[[i]]
32         errs <- fits$y - fi$fitted.xvvalues
33         spe <- crossprod(errs,errs) # sum of squared prediction errors
34         tbl[i,1] <- spe/n
35     }
36     print(tbl)
37 }
38
39 # forms matrix of powers of the vector x, through degree dg
40 powers <- function(x,dg) {
41     pw <- matrix(x,nrow=length(x))
42     prod <- x
43     for (i in 2:dg) {
44         prod <- prod * x
45         pw <- cbind(pw,prod)
46     }
47     return(pw)
48 }
49
50 # finds cross-validated predicted values; could be made much faster via
51 # matrix-update methods
52 lvoneout <- function(y,xmat) {
53     n <- length(y)
54     predy <- vector(length=n)
55     for (i in 1:n) {
56         # regress, leaving out i-th observation
57         lmo <- lm(y[-i] ~ xmat[-i,])
58         betahat <- as.vector(lmo$coef)
59         # the 1 accommodates the constant term
60         predy[i] <- betahat %*% c(1,xmat[i,])
61     }
62     return(predy)
63 }
64
65 # polynomial function of x, coefficients cfs
66 poly <- function(x,cfs) {
67     val <- cfs[1]
68     prod <- 1
69     dg <- length(cfs) - 1
70     for (i in 1:dg) {
71         prod <- prod * x
72         val <- val + cfs[i+1] * prod
73     }
74 }

```

Here is an example of usage:

```

> n <- 60
> x <- (1:n)/n
> y <- vector(length=n)
> for (i in 1:n) y[i] <- sin((3*pi/2)*x[i]) + x[i]^2 + rnorm(1,mean=0,sd=0.5)
> dg <- 15
> lmo <- polyfit(y,x,dg)

```

We generated some simulated data, then created a **polyreg** object with fitted polynomial models through degree 15.

The main work is done by the function **polyfit()**, which creates an object of class “**polyreg**”. The object consists of the objects returned by the R regression fitter **lm()**, as well as copies of our original data.

Note the line

```
lmo$fitted.xvvalues <- lvsoneout(y,pwrs[,1:i,drop=F])
```

Here **lmo** is an object returned by **lm()**, but we are adding an extra component to it, **xvvalues**. In this way, we can achieve the notion of inheritance of object-oriented programming.

We have a generic function for **print()**. As explained in Section 12.2, we define it as **print.polyreg()**. In Section 13.5, we will add another generic function, **plot.polyreg()**.

We chose to have the **print()** function display the mean squared prediction error for each fitted polynomial:

```

> lmo
mean squared prediction errors, by degree
      MSPE
[1,] 0.4439931
[2,] 0.3071160
[3,] 0.2970378
[4,] 0.2941694
[5,] 0.3097453
[6,] 0.3235018
[7,] 0.2908504
[8,] 0.2912780
[9,] 0.3149614
[10,] 0.2911685
[11,] 0.2961925
[12,] 0.3338770
[13,]      NA
[14,]      NA
[15,]      NA

```

Starting with degree 13, fits were numerically unstable in this case, so R refused to perform them, resulting in NA values for the mean squared prediction errors. In computing prediction errors, we used cross validation, the “leaving one out method” in the form used here, predicting each observation from all the others. To implement this, we take advantage of R’s use of negative subscripts:

```
lmo <- lm(y[-i] ~ xmat[-i,])
```

As mentioned in the comment in the code, a much faster implementation would make use of matrix-inverse update methods.



## Chapter 13

# Graphics

**R has a very rich set of graphics facilities.** The top-level R home page, <http://www.r-project.org/>, has some colorful examples, and there is a very nice display of examples in the R Graph Gallery, <http://addictedtor.free.fr/graphiques>. An entire book, *R Graphics* by Paul Murrell (Chapman and Hall, 2005), is devoted to the subject.

Our coverage here is not extensive, but it will give the reader enough foundation to work the basics and learn more. We will cover mainly R's *base* or *traditional* graphics package, with some examples from others, including the **trellis** package.

### 13.1 The Workhorse of R Base Graphics, the `plot()` Function

This **`plot()`** function forms the foundation for much of R's base graphing operations, serving as the vehicle for producing many different kinds of graphs.

As mentioned in Section 12.2, **`plot()`** is a generic function, i.e. a placeholder for a family of functions. The function that actually gets called will depend on the class of the object on which it is called. Let's see what happens for example when we call **`plot()`** with an X vector and a Y vector, which are interpreted as a set of pairs in the (X,Y) plane.

```
> plot(c(1,2,3), c(1,2,4))
```

will cause a window to pop up, seen here in Figure 13.1, plotting the points (1,1), (2,2) and (3,4).

This is a very plain Jane graph, of course. We'll discuss some of the fancy bells and whistles later.

The points in the graph will be symbolized by empty circles. If you want a different character type, specify a value for the named argument **`pch`** ("point character").

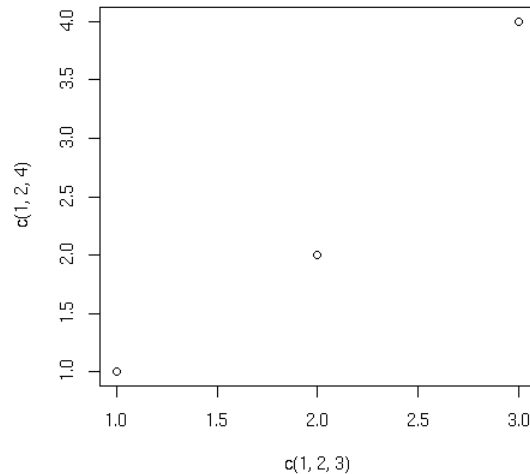


Figure 13.1:

As noted in Section 13.2, one typically builds a graph through a succession of several commands. So, as a base, we might first draw an empty graph, with only axes. For instance,

```
> plot(c(-3,3), c(-1,5), type = "n", xlab="x", ylab="y")
```

draws axes labeled “x” and “y”, the horizontal one ranging from  $x = -3$  to  $x = 3$ , and the vertical one ranging from  $y = -1$  to  $y = 5$ . The argument **type=“n”** means that there is nothing in the graph itself.

## 13.2 Plotting Multiple Curves on the Same Graph

The **plot()** function works in stages, i.e. you can build up a graph in stages by issuing more and more commands, each of which adds to the graph. For instance, consider the following:

```
> x <- c(1,2,3)
> y <- c(1,3,8)
> plot(x,y)
> lmout <- lm(y ~ x)
> abline(lmout)
```

The call to **plot()** will graph the three points as in our example above. At this point the graph will simply show the three points, along with the X and Y axes with hash marks.

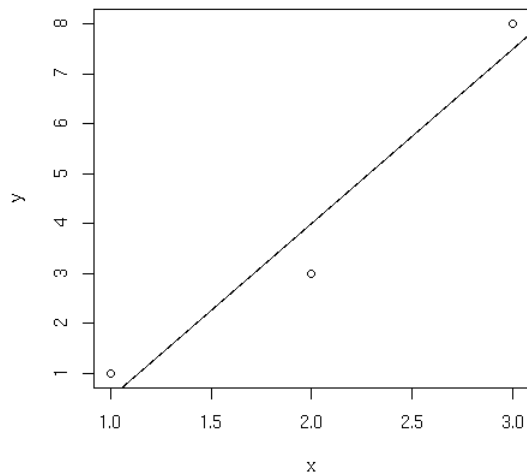
The call to **abline()** then adds a line to the current graph. Now, which line is this? As we know from Section 2.6, the result of the call to the linear regression function **lm()** is a class instance containing the slope and intercept of the fitted line, as well as various other quantities that won't concern us here. We've assigned that class instance to **lmout**. The slope and intercept will now be in **lmout\$coefficients**.

Now, what happens when we call **abline()**? This is simply a function that draws a straight line, with the function's arguments being treated as the intercept and slope of the line. For instance, the call **abline(c(2,1))** would draw the line

$$y = 1 \cdot x + 2$$

on whatever graph we've built up so far.

But actually, even **abline()** is a generic function, and since we are invoking it on the output of **lm()**, this version of the function knows that the slope and intercept it needs will be in **lmout\$coefficients**, and it plots that line. Note again that it superimposes this line onto the current graph—the one which currently graphs the three points. In other words, the new graph will show both the points and the line, as seen in Figure 13.2.



### 13.3 Starting a New Graph While Keeping the Old Ones

Each time you call **plot()** (directly or indirectly), the current graph window will be replaced by the new one. If you don't want that to happen, you can on Linux systems call **X11()**. There are similar calls for other platforms.

## 13.4 The lines() Function

Though there are many options, the two basic arguments to `lines()` are a vector of  $X$  values and a vector of  $Y$  values. These are interpreted as  $(X,Y)$  pairs representing points to be added to the current graph, with lines connecting the points.

For instance, if `x` and `y` are the vectors  $(1.5,2.5)$  and  $(3,)$ , then the call

```
> lines(c(1.5,2.5),c(3,3))
```

would add a line from  $(1.5,3)$  to  $(2.5,3)$  to the present graph.

If you want the lines “connecting the dots” but don’t want the dots themselves, include `type="l"` in your call to `lines()`, or to `plot()`:

```
> plot(x,y,type="l")
```

You can use the `lty` parameter in `plot()` to specify the type of line, e.g solid, dashed, etc. Type

```
> help(par)
```

to see the various types and their codes.

## 13.5 Extended Example: More on the Polynomial Regression Example

In Section 12.4, we defined a class `polyreg` that facilitates fitting of polynomial regression models. Our code there included a generic `print()` function. Let’s now add a generic `plot()`:

```
1 # Poly.r: S3 class for polynomial regression
2
3 # polyfit(x,maxdeg) fits all polynomials up to degree maxdeg; y is
4 # vector for response variable, x for predictor; creates an object of
5 # class "polyreg", consisting of outputs from the various regression
6 # models, plus the original data
7 polyfit <- function(y,x,maxdeg) {
8   pwrs <- powers(x,maxdeg) # form powers of predictor variable
9   lmout <- list() # start to build class
10  class(lmout) <- "polyreg" # create a new class
11  for (i in 1:maxdeg) {
12    lmo <- lm(y ~ pwrs[,1:i])
13    # extend the lm class here, with the cross-validated predictions
14    lmo$fitted.xvvalues <- lvoneout(y,pwrs[,1:i,drop=F])
15    lmout[[i]] <- lmo
16  }
17  lmout$x <- x
```



```

18   lmout$y <- y
19   return(lmout)
20 }
21
22 # generic print() for an object fits of class "polyreg": print
23 # cross-validated mean-squared prediction errors
24 print.polyreg <- function(fits) {
25   maxdeg <- length(fits) - 2 # count lm() outputs only, not $x and $y
26   n <- length(fits$y)
27   tbl <- matrix(nrow=maxdeg,ncol=1)
28   cat("mean squared prediction errors, by degree\n")
29   colnames(tbl) <- "MSPE"
30   for (i in 1:maxdeg) {
31     fi <- fits[[i]]
32     errs <- fits$y - fi$fitted.xvvalues
33     spe <- crossprod(errs,errs) # sum of squared prediction errors
34     tbl[i,1] <- spe/n
35   }
36   print(tbl)
37 }
38
39 # generic plot(); plots fits against raw data
40 plot.polyreg <- function(fits) {
41   plot(fits$x,fits$y,xlab="X",ylab="Y") # plot data points as background
42   maxdg <- length(fits) - 2
43   cols <- c("red","green","blue")
44   dg <- curvecount <- 1
45   while (dg < maxdg) {
46     prompt <- paste("RETURN for XV fit for degree",dg,"or type degree",
47                   "or q for quit ")
48     rl <- readline(prompt)
49     dg <- if (rl == "") dg else if (rl != "q") as.integer(rl) else break
50     lines(fits$x,fits[[dg]]$fitted.values,col=cols[curvecount%%3 + 1])
51     dg <- dg + 1
52     curvecount <- curvecount + 1
53   }
54 }
55
56 # forms matrix of powers of the vector x, through degree dg
57 powers <- function(x,dg) {
58   pw <- matrix(x,nrow=length(x))
59   prod <- x
60   for (i in 2:dg) {
61     prod <- prod * x
62     pw <- cbind(pw,prod)
63   }
64   return(pw)
65 }
66
67 # finds cross-validated predicted values; could be made much faster via
68 # matrix-update methods
69 lvoneout <- function(y,xmat) {
70   n <- length(y)
71   predy <- vector(length=n)
72   for (i in 1:n) {
73     # regress, leaving out i-th observation
74     lmo <- lm(y[-i] ~ xmat[-i,])
75     betahat <- as.vector(lmo$coef)
76     # the 1 accommodates the constant term
77     predy[i] <- betahat %*% c(1,xmat[i,])

```

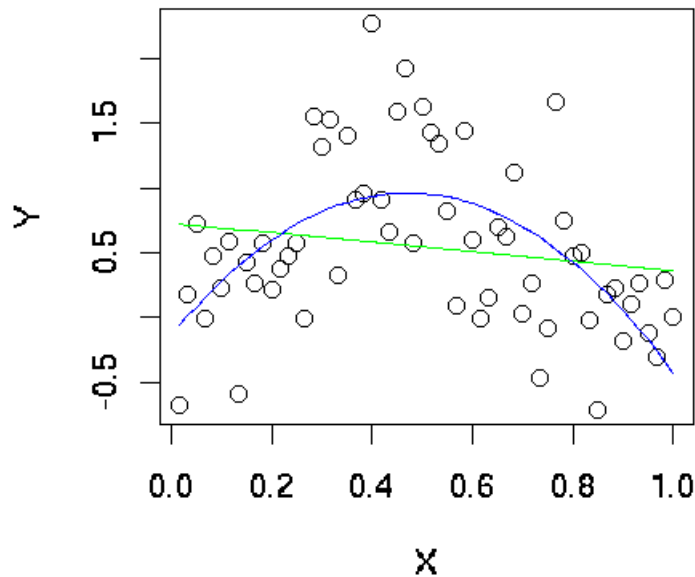
```
78     }
79     return(predy)
80 }
81
82 # polynomial function of x, coefficients cfs
83 poly <- function(x,cfs) {
84     val <- cfs[1]
85     prod <- 1
86     dg <- length(cfs) - 1
87     for (i in 1:dg) {
88         prod <- prod * x
89         val <- val + cfs[i+1] * prod
90     }
91 }
```

As before, our generic function takes the name of the class, here **plot.polyreg()**.

We use the R string function **paste()** to assemble a prompt, offering the user a choice of plotting the next fitted polynomial, or one of a different degree, or quitting. The prompt appears in the interactive R window in which we issued the **plot()** call. So for instance after taking the default choice twice, the command window looks like this

```
> plot(lmo)
RETURN for XV fit for degree 1 or type degree or q for quit
RETURN for XV fit for degree 2 or type degree or q for quit
RETURN for XV fit for degree 3 or type degree or q for quit
```

while the plot window looks like 13.5.



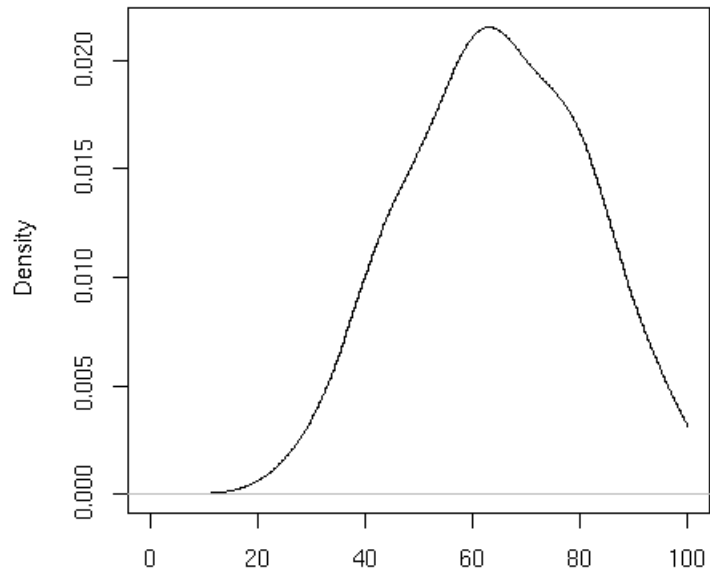
In order to better visually distinguish the various fitted curves, we alternative colors.

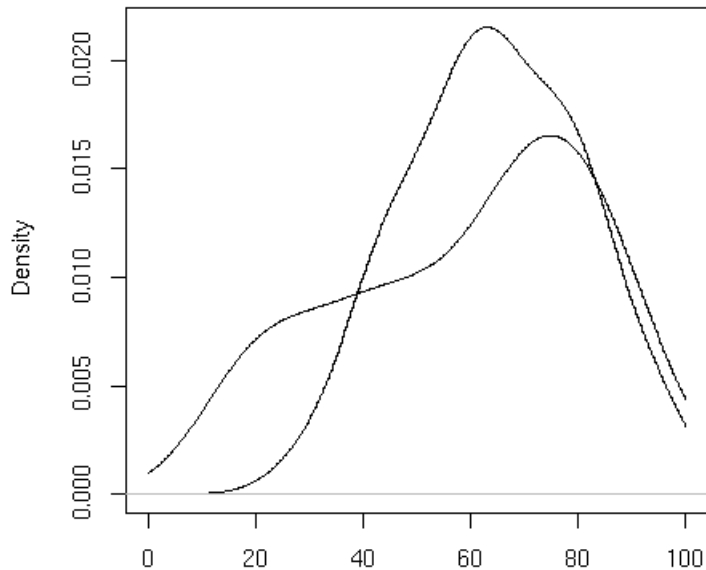
### 13.6 Extended Example: Two Density Estimates on the Same Graph

Let's plot nonparametric density estimates (these are basically smoothed histograms) for two sets of examination scores in the same graph. We use the function `density()` to generate the estimates. Here are the commands we issue:

```
> d1 = density(testscores$Exam1, from=0, to=100)
> d2 = density(testscores$Exam2, from=0, to=100)
> plot(d1, main="", xlab="")
> lines(d2)
```

Here's what we did: First, we computed nonparametric density estimates from the two variables, saving them in objects `d1` and `d2` for later use. We then called `plot()` to draw the curve for Exam 1, at which point the plot looked like Figure 13.6. We then called `lines()` to add Exam 2's curve to the graph, producing Figure 13.6





Note that we asked R to have blank labels for the figure as a whole and for the X axis. Otherwise, R would have gotten such labels from **d1**, which would have been specific to Exam 1.

Note too that we had to plot Exam 1 first. The scores there were less diverse, so the density estimate was narrower and taller. Had we plotted Exam 2 first, with its shorter curve, Exam 1's curve would then have been too tall for the plot window.

The call to **plot()** both initiates the plot and draws the first curve. (Without specifying `type="l"`, only the points would have been plotted.) The call to **lines()** then adds the second curve.

## 13.7 Adding Points

The **points()** function adds a set of (x,y)-points, with labels for each, to the currently displayed graph. For instance, in our first example, Section 2.2, the command

```
points(testscores$Exam1, testscores$Exam3, pch="+")
```

would superimpose onto the current graph the points of the exam scores from that example, using "+" signs to mark them.

As with most of the other graphics functions, there are lots of options, e.g. point color, background color, etc.

## 13.8 The legend() Function

A nice function is **legend()**, which is used to add a legend to a multivariate graph. For instance,

```
> legend(2000,31162,legend="CS",lty=1)
```

would place a legend at the point (2000,31162) in the graph, with a little line of type 1 and label of "CS". Try it!

## 13.9 Adding Text: the text() and mtext() Functions

Use the **text()** function to place some text anywhere in the current graph. For example,

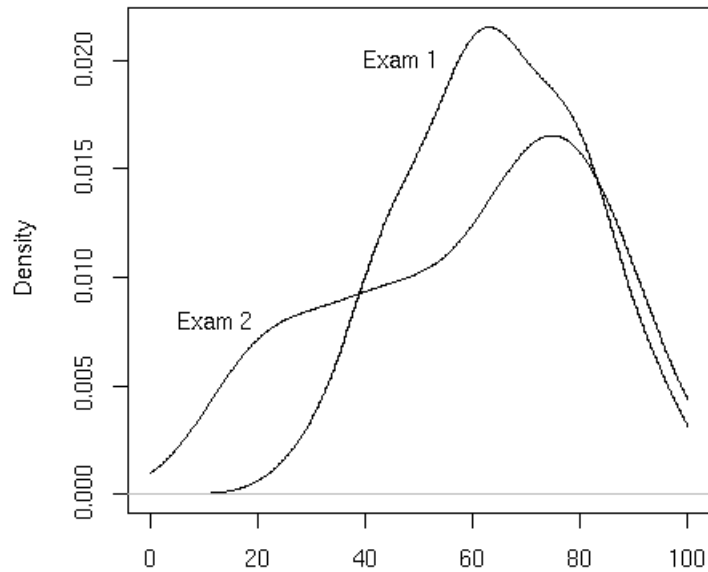
```
text(2.5,4,"abc")
```

would write the text "abc" at the point (2.5,4) in the graph. The center of the string, in this case "b", would go at that point.

To see a more practical example, let's add some labels to the curves in Section 13.6 (assuming the plot is our current one, so we can add to it):

```
> text(46.7,0.02,"Exam 1")  
> text(12.3,0.008,"Exam 2")
```

The result is shown in Figure 13.9.



In order to get a certain string placed exactly where you want it, you may need to engage in some trial and error. R has no “undo” command (though alternatives will be discussed in Section 13.11).

But you may find the `locator()` function to be a much quicker way to go. See Section 13.10.

To add text in the margins, use `mtext()`.

## 13.10 Pinpointing Locations: the `locator()` Function

Typing

```
locator(1)
```

will tell R that you will click in 1 place in the graph. Once you do so, R will tell you the exact coordinates of the point you clicked on. Call `locator(2)` to get the locations of 2 places, etc. (Warning: Make sure to include the argument.)

You can combine this, for example, with `text()`, e.g.

```
> text(locator(1), "nv=75")
```

## 13.11 Replaying a Plot

R has no “undo” command. However, if you suspect you may need to undo your next step of a graph, you can save it using **recordPlot()**, and then later restore it with **replayPlot()**.

Less formally but more conveniently, you can put all the commands you’re using to build up a graph in a file, and then use **source()**, or cut-and-paste with the mouse, to execute them. If you change one command, you can then redo the whole graph by sourcing or copying-and-pasting your file.

In the example in Section 13.6, for instance, we could create file named **examplot.r**, with the following contents:

```
d1 = density(testscores$Exam1, from=0, to=100)
d2 = density(testscores$Exam2, from=0, to=100)
plot(d1, main="", xlab="")
lines(d2)
text(46.7, 0.02, "Exam 1")
text(12.3, 0.008, "Exam 2")
```

If we decided that the label for Exam 1 was a bit far to the right, we can edit the file, and then either execute

```
> source("examplot")
```

or do the copy-and-paste.

## 13.12 Changing Character Sizes: the **cex** Option

The **cex** (“character expand”) function allows you to expand or shrink characters within a graph, very useful. You can use it as a named parameter in various graphing functions.

```
text(2.5, 4, "abc", cex = 1.5)
```

would print the same text as in our earlier example, but with characters 1.5 times normal size.

## 13.13 Operations on Axes

You may wish to have the ranges on the X- and Y-axes of your plot to be broader or narrower than the default. You can do this by specifying the **xlim** and/or **ylim** parameters in your call to **plot()** or **points()**. For example, **ylim=c(0,90000)** would specify a range on the Y-axis of 0 to 90000.

This is especially useful if you will be displaying several curves in the same graph. Note that if you do not specify **xlim** and/or **ylim**, then draw the largest curve first, so there is room for all of them.



## 13.14 The polygon() Function

You can use **polygon()** to draw arbitrary polygonal objects, with shading etc. For example, the following code draws the graph of the function  $f(x) = 1 - e^{-x}$ , then adds a rectangle that approximates the area under the curve from  $x = 1.2$  to  $x = 1.4$ :

```
> f <- function(x) return(1-exp(-x))
> curve(f,0,2)
> polygon(c(1.2,1.4,1.4,1.2),c(0,0,f(1.3),f(1.3)),col="gray")
```

In the call to **polygon()** here, the first argument is the set of X coordinates for the rectangle, while the second argument specifies the Y coordinates. The third argument specifies that the rectangle should be shaded in gray; instead we could have, for instance, used the **density** argument for striping.

## 13.15 Smoothing Points: the lowess() Function

Just plotting a cloud of points, whether connected or not, may turn out to be just an uninformative mess. In many cases, it is better to smooth out the data by fitting a nonparametric regression estimator such as **lowess()**:

```
plot(lowess(x,y))
```

The call **lowess(x,y)** returns the pairs of points on the regression curve, and then **plot()** plots them. Of course, we could get both the cloud and the smoothed curve:

```
plot(x,y)
lines(lowess(x,y))
```

## 13.16 Graphing Explicit Functions

Say you wanted to plot the function  $g(t) = (t^2 + 1)^{0.5}$  for  $t$  between 0 and 5. You could use the following R code:

```
g <- function(t) { return (t^2+1)^0.5 } # define g()
x <- seq(0,5,length=10000) # x = [0.0004, 0.0008, 0.0012, ..., 5]
y <- g(x) # y = [g(0.0004), g(0.0008), g(0.0012), ..., g(5)]
plot(x,y,type="l")
```

But you could avoid some work on your part here by using the **curve()** function, which basically uses the same method:

```
> curve((x^2+1)^0.5,0,5)
```

If you were adding this curve to an existing plot, you would use the **add** argument:

```
> curve((x^2+1)^0.5,0,5,add=T)
```

The optional argument **n** has the default value 101, meaning that the function will be evaluated at 101 equally-spaced points in the specified range of X.

You can also use **plot()**:

```
> f <- function(x) return((x^2+1)^0.5)
> plot(f,0,5) # the argument must be a function name
```

Here the call **plot()** leads to calling **plot.function()**, the generic function for the **function** class.

## 13.17 Extended Example: Magnifying a Portion of a Curve

We can use **curve()** to graph a function, as seen above. However, after graphing it, we may wish to “zoom in” on one portion of the curve. We could do this by simply calling **curve()** again, on the same function but with restricted X range, but suppose we wish to display the original plot and the closeup one in the same picture. Here we will develop a function, to be named **inset()**, to do this.

In order to avoid redoing the work that **curve()** did in plotting the original graph, we will slightly modify its code, to save its work, via a return value. We can do that by taking advantage of the fact that one can easily inspect the code of R functions written in R (as opposed to the fundamental R functions written in C):

```
> curve
function (expr, from = NULL, to = NULL, n = 101, add = FALSE,
  type = "l", ylab = NULL, log = NULL, xlim = NULL, ...)
{
  sexpr <- substitute(expr)
  if (is.name(sexpr)) {
# ...lots of lines omitted here...
  x <- if (lg != "" && "x" %in% strsplit(lg, NULL)[[1]]) {
    if (any(c(from, to) <= 0))
      stop("'from' and 'to' must be > 0 with log=\"x\"")
    exp(seq.int(log(from), log(to), length.out = n))
  }
  else seq.int(from, to, length.out = n)
  y <- eval(expr, envir = list(x = x), enclos = parent.frame())
  if (add)
    lines(x, y, type = type, ...)
  else plot(x, y, type = type, ylab = ylab, xlim = xlim, log = lg,
    ...)
}
```

The code forms vectors **x** and **y**, consisting of the X and Y coordinates of the curve to be plotted, at **n** equally-spaced points in the range of X. Since we'll make use of those in **inset()**, let's modify the above code to return **x** and **y**, giving the name **crv()** to the modified **curve()**:

```
> crv
function (expr, from = NULL, to = NULL, n = 101, add = FALSE,
         type = "l", ylab = NULL, log = NULL, xlim = NULL, ...)
{
  sexpr <- substitute(expr)
  if (is.name(sexpr)) {
# ...lots of lines omitted here...
    x <- if (lg != "" && "x" %in% strsplit(lg, NULL)[[1]]) {
      if (any(c(from, to) <= 0))
        stop("'from' and 'to' must be > 0 with log=\"x\"")
      exp(seq.int(log(from), log(to), length.out = n))
    }
    else seq.int(from, to, length.out = n)
  }
  y <- eval(expr, envir = list(x = x), enclos = parent.frame())
  if (add)
    lines(x, y, type = type, ...)
  else plot(x, y, type = type, ylab = ylab, xlim = xlim, log = lg,
           ...)
  return(list(x=x,y=y)) # this is the only modification
}
```

So, here is our **inset()** function:

```
# savexy: list consisting of x and y vectors returned by crv()
# x1,y1,x2,y2: coordinates of rectangular region to be magnified
# x3,y3,x4,y4: coordinates of inset region
inset <- function(savexy,x1,y1,x2,y2,x3,y3,x4,y4) {
  rect(x1,y1,x2,y2) # draw rectangle around region to be magnified
  rect(x3,y3,x4,y4) # draw rectangle around the inset
  # get vectors of coordinates of previously plotted points
  savex <- savexy$x
  savey <- savexy$y
  # get subscripts of xi our range to be magnified
  n <- length(savex)
  xvalsinrange <- which(savex >= x1 & savex <= x2)
  yvalsforthosex <- savey[xvalsinrange]
  # check that our first box contains the entire curve for that X range
  if (any(yvalsforthosex < y1 | yvalsforthosex > y2)) {
    print("Y value outside first box")
    return()
  }
  # record some differences
  x2mnsx1 <- x2 - x1
  x4mnsx3 <- x4 - x3
  y2mnsy1 <- y2 - y1
  y4mnsy3 <- y4 - y3
  # for the i-th point in the original curve, the function plotpt() will
  # calculate the position of this point in the inset curve
  plotpt <- function(i) {
    newx <- x3 + ((savex[i] - x1)/x2mnsx1) * x4mnsx3
    newy <- y3 + ((savey[i] - y1)/y2mnsy1) * y4mnsy3
    return(c(newx,newy))
  }
}
```

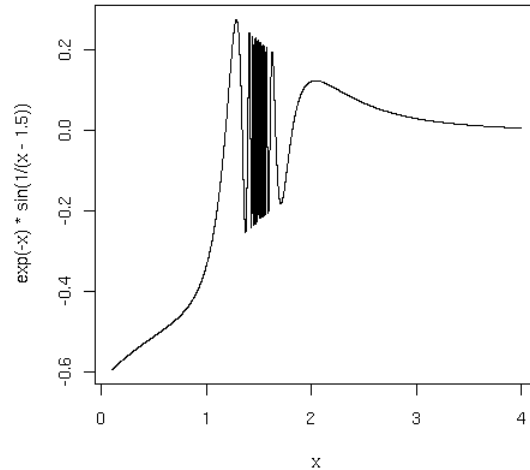


Figure 13.2:

```

}
newxy <- sapply(xvalsinrange,plotpt)
lines(newxy[1,],newxy[2,])
}

```

Let's try it out:

```

png("inset.png")
xyout <- crv(exp(-x)*sin(1/(x-1.5)),0.1,4,n=5001)
inset(xyout,1.3,-0.3,1.47,0.3, 2.5,-0.3,4,-0.1)
dev.off()

```

The resulting plot looks like Figure 13.2:

## 13.18 Graphical Devices and Saving Graphs to Files

R has the notion of a graphics device. The default device is the screen. If we want to have a graph saved to a file, we must set up another device. For example, if we wish to save as a PDF file, we do something like the following. (Warning: This is actually too tedious an approach, and a shortcut will be presented later on. But the reader should go through this “long way” once, to understand the principles.)

```
> pdf("d12.pdf")
```

This opens a file, which we have chosen here to call **d12.pdf**. We now have two devices open, as we can confirm:

```
> dev.list()
X11 pdf
 2  3
```

The screen is named X11 when R runs on Linux; it is device number 2 here. Our PDF file is device number 3. Our active device is now the PDF file:

```
> dev.cur()
pdf
 3
```

All graphics output will now go to this file instead of to the screen. But what if we wish to save what's already on the screen? We could re-establish the screen as the current device, then copy it to the PDF device, 3:

```
> dev.set(2)
X11
 2
> dev.copy(which=3)
pdf
 3
```

Note carefully that the PDF file is not usable until we close it, which we do as follows:

```
> dev.set(3)
pdf
 3
> dev.off()
X11
 2
```

(We could also close the device by exiting R, though it's probably better to proactively close.)

The above set of operations to print a graph can become tedious, but there is a shortcut:

```
> dev.print(device=pdf, "d12.pdf")
X11
 2
```

This opens the PDF file **d12.pdf**, copies the X11 graph to it, closes the file, and resets X11 as the active device.

## 13.19 3-Dimensional Plots

There are a number of functions to plot data in three dimensions, such as **persp()** and **wireframe()**, which draw surfaces, and **cloud()**, which draws three-dimensional scatter plots. There are many more.

For **wireframe()** and **cloud()**, one loads the **lattice** library. Here is an example:

```
> a <- 1:10
> b <- 1:15
> eg <- expand.grid(x=a,y=b)
> eg$z <- eg$x^2 + eg$x * eg$y
> wireframe(z ~ x+y, eg)
```

The call to **expand.grid()** creates a data frame, consisting of two columns named **x** and **y**, combining all the values of the two inputs. Here **a** and **b** had 10 and 15 values, respectively, so the resulting data frame will have 150 rows.

We then added a third column, named **z**, as a function of the first two columns. Our call to **wireframe()** then creates the graph. Note that **z**, **x** and **y** of course refer to names of columns in **eg**.

All the points would be connected as a surface (like connecting points by lines in two dimensions). With **cloud()**, though, the points would just be isolated.

For **wireframe()**, the (X,Y) pairs must form a rectangular grid, though not necessarily evenly spaced.

Note that the data frame that is input to **wireframe()** need not have been created by **expand.grid()**.

By the way, these functions have many different options. A nice one for **wireframe()**, for instance, is **shade=T**, which makes it all easier to see.

## Chapter 14

# Debugging

The R base package includes a number of debugging facilities. They are nowhere near what a good debugging tool offers, but with skillful usage they can be effective.

A much more functional debugging package is available for R, of course called **debug**. I will discuss this in Section 14.4.

### 14.1 The `debug()` Function

One of the tools R offers for debugging your R code is the built-in function **debug()**. It works in a manner similar to C debuggers such as GDB.

#### 14.1.1 Setting Breakpoints

Say for example we suspect that our bug is in the function **f()**. We enable debugging by typing

```
> debug(f)
```

This will set a breakpoint at the beginning of **f()**.

To turn off this kind of debugging for a function **f()**, type

```
> undebug(f)
```

Note that if you simultaneously have a separate window open in which you are editing your source code, and you had executed **debug(f)**, **then if** you reload using **source()**, the effect is that of calling **undebug(f)**.

If we wish to set breakpoints at the line level, we insert a line

```
browser()
```

before line at which we wish to break.

You can make a breakpoint set in this fashion conditional by placing it within an **if** statement, e.g.

```
if (k == 6) browser()
```

You may wish to add an argument named, say, **dbg**, to most of your functions, with **dbg = 1** meaning that you wish to debug that part of the code. The above then may look like

```
if (dbg && k == 6) browser()
```

### 14.1.2 Stepping through Our Code

When you execute your code and hit a breakpoint, you enter the debugger, termed the *browser* in R. The command prompt will now be something like

```
Browse[1]
```

instead of just `>`. Then you can invoke various debugging operations, such as:

- **n or Enter:** You can single-step through the code by hitting the Enter key. (If it is a line-level breakpoint, you must hit `n` the first time, then Enter after that.)
- **c:** You can skip to the end of the “current context” (a loop or a function) by typing `c`.
- **where:** You can get a stack report by typing `where`.
- **Q:** You can return to the `>` prompt, i.e. exit the debugger, by typing `Q`.
- All normal R operations and functions are still available to you. So for instance to query the value of a variable, just type its name, as you would in ordinary interactive usage of R. If the variable’s name is one of the **debug()** commands, though, say `c`, you’ll need to do something like **print(c)** to print it out.

## 14.2 Automating Actions with the `trace()` Function

The **trace()** function is quite flexible and powerful, though it takes some initial effort to learn. I will discuss some of the simpler usage forms here.

The call



```
> trace(f,t)
```

would instruct R to call the function **t()** every time we enter the function **f()**. For instance, say we wish to set a breakpoint at the beginning of the function **gy()**. We could do this by the command

```
> trace(gy,browser)
```

This would have the same effect as placing the command **browser()** in our source code for **gy()**, but would be quicker and more convenient than inserting such a line, saving the file and rerunning **source()** to load in the new version of the file.

It would also be quicker and more convenient to undo, by simply running

```
> untrace(gy)
```

You can turn tracing on or off globally by calling **tracingState()**, with the argument **TRUE** to turn it on, **FALSE** to turn it off. Recall too that these boolean constants in R can be abbreviated **T** and **F**.

## 14.3 Performing Checks After a Crash with the **traceback()** and **debugger()** Functions

Say your R code crashes when you are not running the debugger. There is still a debugging tool available to you after the fact: You can do a “post mortem” by simply calling **traceback()**.

You can get a lot more if you set R up to dump frames on a crash:

```
> options(error=dump.frames)
```

If you’ve done this, then after a crash run

```
> debugger()
```

You will then be presented with a choice of levels of function calls to look at. For each one that you choose, you can take a look at the values of the variables there. After browsing through one level, you can return to the **debugger()** main menu by hitting **n**.

## 14.4 The debug Package

The **debug** package provides a more usable debugging interface than R's built-in facilities do. It features a pop-up window in which you can watch your progress as you step through your source code, gives you the ability to easily set breakpoints, etc.

It requires another package, **mvbutils**, and the Tcl/Tk scripting and graphics system. The latter is commonly included in Linux distributions, and is freely downloadable for all the major platforms. It suffers from a less-than-perfect display, but is definitely worthwhile, much better than R's built-in debugging tools.

### 14.4.1 Installation

Choose an installation directory, say **/MyR**. Then install **mvbutils** and **debug**:

```
> install.packages("mvbutils", "/MyR")
> install.packages("debug", "/MyR")
```

For R version 2.5.0, I found that a bug in R caused the **debug** package to fail. I then installed the patched version of 2.5.0, and **debug** worked fine. On one machine, I encountered a Tcl/Tk problem when I tried to load **debug**. I fixed that (I was on a Linux system) by setting the environment variable, in my case by typing

```
% setenv TCL_LIBRARY /usr/share/tcl8.4
```

### 14.4.2 Path Issues

Each time you wish to use **debug**, load it by executing

```
> .libPaths("/MyR")
> library(debug)
```

Or, place these in an R startup file, say **.Rprofile** in the directory in which you want these commands to run automatically.

Or, create a file **.Renviron** in your home directory, consisting of the line

```
R_LIBS=~ /MyR
```

### 14.4.3 Usage

Now you are ready to debug. Here are the main points:

- Breakpoints are first set at the function level. Say you have a function **f()** at which you wish to break. Then type

```
> mtrace(f)
```

Do this for each function at which you want a breakpoint.

- Then go ahead and start your program. (I'm assuming that your program itself consists of a function.) Execution will pause at **f()**, and a window will pop up, showing the source code for that function. The current line will be highlighted in green. Back in the R interactive window, you'll see a prompt **D(1)>**.
- At this point, you can single-step through your code by repeatedly hitting the Enter key. You can print the values of variables as you usually do in R's interactive mode.
- You can set finer breakpoints, at the line level, using **bp()**. Once you are in **f()**, for instance, to set a breakpoint at line 12 in that function type

```
D(1)> bp(12)
```

- To set a conditional breakpoint, say at line 12 with the condition **k == 5**, issue **bp(12,k==5)**.
- To avoid single-stepping, issue **go()**, which will execute continuously until the next breakpoint.
- To set a temporary breakpoint at line **n**, issue **go(n)**.
- To restart execution of the function, issue **skip(1)**.
- If there is an execution error, the offending line will be highlighted.
- To cancel all **mtrace()** breaks, issue **mtrace.off()**. To cancel one for a particular function **f()**, issue **mtrace(f,tracing=F)**.
- To cancel a breakpoint, say at line 12, issue **bp(12,F)**.
- To quit, issue **qqq()**.
- For more details, see the extensive online help, e.g. by typing

```
D(1)> ?bp
```

## 14.5 Ensuring Consistency with the set.seed() Function

If you're doing anything with random numbers, you'll need to be able to reproduce the same stream of numbers each time you run your program during the debugging session. To do this, type

```
> set.seed(8888) # or your favorite number as an argument
```

## 14.6 Syntax and Runtime Errors

The most common syntax errors will be lack of matching parentheses, brackets or braces. When you encounter a syntax error, this is the first thing you should check and double-check. I highly recommend that you use a text editor, say Vim, that does parenthesis matching and syntax coloring for R.

Beware that often when you get a message saying there is a syntax error on a certain line, the error may well be elsewhere. This can occur with any language, but R seems especially prone to it.

If it just isn't obvious to you where your syntax error is, I recommend selectively commenting-out some of your code, thus enabling you to better pinpoint the location of the syntax problem.

If during a run you get a message

```
could not find function "evaluator"
```

and a particular function call is cited, it means that the interpreter cannot find that function. You may have forgotten to load a library or source a code file.

You may sometimes get messages like,

```
There were 50 or more warnings (use warnings() to see the first 50)
```

These should be heeded; run **warnings()**, as suggested. The problem could range from nonconvergence of an algorithm to misspecification of a matrix argument to a function. In many cases, the program output may be invalid, though it may well be fine too, say with a message “fitted probabilities numerically 0 or 1 occurred in: glm...”

## 14.7 Extended Example: A Full Debugging Session

## Chapter 15

# Writing Fast R Code

R is an interpreted language. Many of the commands are written in C and thus do run in machine code, but other commands, and of course your own R code, are pure R. thus interpreted. Thus there is the risk that your R application may run more slowly than you need. What can be done to remedy this? Here are the main tools available to you:

- (a) Optimize your R code, through vectorization and other approaches.
- (b) Write the key, CPU-intensive parts of your code in a compiled language such as C/C++.
- (c) Write your code in some form of parallel R.

Approach (a) will be covered in this chapter, while (b) and (c) are covered in Chapters 16 and 17.

### 15.1 Optimization Tools

Optimizing R code involves the following and more:

- Vectorization.
- Understanding R's functional programming nature, and the way R uses memory.
- Making use of some of R's optimized utility functions, such as **row/colSums()**, **outer()** and the various **\*apply()** functions.

### 15.1.1 The Dreaded for Loop

Here we exploit the fact that R is fundamentally a vector-oriented language, which often allows us to avoid writing explicit loops. For example, if **x** and **y** are vectors of equal lengths, then writing

```
z <- x + y
```

will be not only more compact, but also more importantly, it will be faster than

```
for (i in 1:length(x)) z[i] <- x[i] + y[i]
```

Let's do a quick timing comparison:

```
> x <- runif(1000000)
> y <- runif(1000000)
> system.time(z <- x + y)
  user  system elapsed
 0.056   0.012   0.071
> system.time(for (i in 1:length(x)) z[i] <- x[i] + y[i])
  user  system elapsed
23.305   0.040  23.498
```

What a difference! The nonloop version was over 300 times faster in elapsed time. While it must be noted that timings may vary from one run to another—a second run of the loop version had elapsed time of 22.958—it is clear that in some cases “de-looping” R code can really pay off.

It's worth discussing some of the sources of slowdown in the loop version. What may not be obvious to programmers coming to R from other languages is that there are numerous function calls involved here:

- Though syntactically the loop looks innocuous, **for()** is in fact a function.
- The colon, “:” looks even more innocuous, but it's a function too.
- Likewise, each of vector subscript operations represents a function call, to “[” for the two reads and to “[;-” in the case of the write.

Function calls can be expensive, as they involve setting up stack frames and the like. Moreover, the generality of R functions such as **for()** comes with the price of considerable overhead. The “[” operation can have especially high overhead, as we will see in Section 15.4.

By contrast, if we were to write this in C, there would be no function calls. Indeed that is essentially what happens in our first code snippet above, though we do have a call to “+”. Thus the first version of the code is much faster.

One type of vectorization is vector filtering. For instance, let's rewrite our function **oddcnt()** in Section 2.3:

## 15.2. EXTENDED EXAMPLE: ACHIEVING BETTER SPEED IN MONTE CARLO SIMULATION 137

```
> oddcount <- function(x) return(length(which(x%%2==1)))
```

There is no explicit loop here, and even though R will internally loop through the array, this will be done in native machine code. Again, the anticipated speedup does occur:

```
> x <- sample(1:1000000,100000,replace=T)
> system.time(oddcount(x))
  user system elapsed
0.020  0.008  0.026
> system.time(
+   {
+   c <- 0
+   for (i in 1:length(x))
+     if (x[i] %% 2 == 1) c <- c+1
+   return(c)
+ }
+ )
  user system elapsed
0.368  0.000  0.368
```

You might wonder whether it matters in this case, since even the loop version of the code took much less than a second to run. But if this code had been part of an enclosing loop, with many iterations, the difference could be quite important indeed.

By the way, note that multistatement code can be fed into `system.time()`, by enclosing the code with braces, thus rendering it a single expression.

Examples of other vectorized function that may be useful good speedup are `ifelse()`, `which()`, `where()`, `any()` and `all()`, and in the matrix case, `rowSums()`, `colSums()`.

In “all possible combinations” types of settings, `outer()`, `lower.tri()`, `upper.tri()` or `expand.grid()` may be just what you need.

Though `apply()` eliminates an explicit loop, it is actually implemented in R rather than C, and thus will usually not speed up your code. However, the other “apply” functions, e.g. `lapply()` can be very helpful.

## 15.2 Extended Example: Achieving Better Speed in Monte Carlo Simulation

In some applications, simulation code can run for hours, days or even months, so speedup methods are of high interest.

To start, let’s consider the code in Section 10.8:

```
# MaxNorm.r
sum <- 0
nreps <- 100000
```

```

for (i in 1:nreps) {
  xy <- rnorm(2) # generate 2 N(0,1)s
  sum <- sum + max(xy)
}
print(sum/nreps)

```

Here's a revision, hopefully faster:

```

# MaxNorm2.r
nreps <- 100000
xymat <- matrix(rnorm(2*nreps), ncol=2)
maxs <- pmax(xymat[,1], xymat[,2])
print(mean(maxs))

```

In this code, we generate all the random variates at once, storing them in a matrix **xymat**, with one (X,Y) pair per row:

```
xymat <- matrix(rnorm(2*nreps), ncol=2)
```

We then find all the  $\max(X,Y)$  values, storing those values in **maxs**, and then simply call **mean()**. It's easier to program, and we believe it will be faster. Let's check that:

```

> system.time(source("MaxNorm.r"))
[1] 0.5667599
   user  system elapsed
 1.700   0.004   1.722
> system.time(source("MaxNorm2.r"))
[1] 0.5649281
   user  system elapsed
 0.132   0.008   0.143

```

The speedup is dramatic.

Note by the way that we achieved an increase in speed at the expense of using more memory, by keeping our random numbers in an array instead of generating and discarding them one at a time. The time/space tradeoff is a common one in the computing world, and in the R world in particular.

We attained an excellent speedup in the example above, but it was misleadingly easy. Let us look at a slightly more complicated example, though still a rather simple problem, a classical exercise from elementary probability courses. Urn 1 contains 10 blue marbles and eight blue ones. In Urn 2 the mixture is six blue and six yellow. We draw a marble at random from Urn 1 and transfer it to Urn 2, and then draw a marble at random from Urn 2. What is the probability that that second marble is blue? This quantity is easy to find analytically, but we'll use simulation. Here is the straightforward way:

```

sim <- function(nreps) {
  nb1 = 10 # 10 blue marbles in Urn 1
  n1 <- 18 # number of marbles in Urn 1 at 1st pick

```



## 15.2. EXTENDED EXAMPLE: ACHIEVING BETTER SPEED IN MONTE CARLO SIMULATION 139

```
n2 <- 13 # number of marbles in Urn 2 at 2nd pick
count <- 0
for (i in 1:nreps) {
  nb2 = 6 # 6 blue marbles orig. in Urn 2
  # pick from Urn 1 and put in Urn 2; is it blue?
  if (runif(1) < nb1/n1) nb2 <- nb2 + 1
  # pick from Urn 2; is it blue?
  if (runif(1) < nb2/n2) count <- count + 1
}
return(count/nreps) # est. P(pick blue from Urn 2)
}
```

Let's look at vectorizing this simulation:

```
# Marbles2.r

sim <- function(nreps) {
  nb1 = 10 # 10 blue marbles in Urn 1
  nb2 = 6 # 6 blue marbles orig. in Urn 2
  n1 <- 18 # number of marbles in Urn 1 at 1st pick
  n2 <- 13 # number of marbles in Urn 2 at 2nd pick
  u <- matrix(c(runif(2*nreps)),nrow=nreps,ncol=2)
  # set up the condition vector for ifelse()
  cndtn <- u[,1] <= nb1/n1 & u[,2] <= (nb2+1)/n2 |
    u[,1] > nb1/n1 & u[,2] <= nb2/n2
  z <- ifelse(cndtn,1,0)
  return(mean(z)) # est. P(pick blue from Urn 2)
}

print(sim(10000))
```

The key point here is to vectorize using **ifelse()**. The main work for this is done in the statement

```
cndtn <- u[,1] <= nb1/n1 & u[,2] <= (nb2+1)/n2 |
  u[,1] > nb1/n1 & u[,2] <= nb2/n2
```

And sure enough, this brings quite an improvement:

```
> system.time(source("Marbles.r"))
[1] 0.5024
  user system elapsed
 0.240  0.000  0.242
> system.time(source("Marbles2.r"))
[1] 0.5011
  user system elapsed
 0.012  0.000  0.015
```

In principle, the **ifelse()** approach we took to speedup here could be applied to many other Monte Carlo simulations. However, it's clear that the analog of the statement above that statement **cndtn** would quickly become quite complex even for some seemingly simple applications.

Moreover, the approach would not work in "infinite-stage" situations, meaning an unlimited number of time steps; here we are considering the marble example as being two-stage, with two columns to the matrix **u**.

### 15.3 Extended Example: Generating a Powers Matrix

Recall in Section 12.4, we needed to generate a matrix of powers of our predictor variable. We used the code

```
# forms matrix of powers of the vector x, through degree dg
powers <- function(x,dg) {
  pw <- matrix(x,nrow=length(x))
  prod <- x
  for (i in 2:dg) {
    prod <- prod * x
    pw <- cbind(pw,prod)
  }
  return(pw)
}
```

Here the line

```
prod <- prod * x
```

is vectorized, so the above code should be faster than

```
# forms matrix of powers of the vector x, through degree dg
powersalt1 <- function(x,dg) {
  pw <- matrix(x,nrow=length(x))
  prod <- x
  for (i in 2:dg) {
    for (j in 1:length(x)) prod[j] <- prod[j] * x[j]
    pw <- cbind(pw,prod)
  }
  return(pw)
}
```

And indeed, **powers()** is a lot faster:

```
> x <- runif(100000)
> system.time(powers(x,8))
  user  system elapsed 
0.184   0.040   0.224 
> system.time(powersalt1(x,8))
  user  system elapsed 
16.005   0.008  16.052
```

And yet, **powers()** still does contain a loop. Furthermore, the statement

```
pw <- cbind(pw,prod)
```

is another red flag, as it means reallocating space for the expanded matrix. This in fact may be the reason that `powers()` used more system time than did `powersalt1()`. Can we do better?

It would seem that this setting is perfect for `outer()`:

```
powersalt2 <- function(x,dg) return(outer(x,1:dg,"^"))
```

However, the results are disappointing:

```
> system.time(powersalt2(x,8))
  user  system elapsed
0.468   0.040   0.506
```

The version using `outer()` did better in different setting, though:

```
> x <- runif(100000)
> system.time(powersalt2(x,12))
  user  system elapsed
0.540   0.048   0.588
> system.time(powers(x,12))
  user  system elapsed
0.408   0.068   0.477
```

## 15.4 Functional Programming and Memory Issues

Most R operations are implemented as functions, a trait that can have performance implications.

As an example, consider the innocuous-looking statement

```
z[3] <- 8
```

Let's see where R has `z` in memory, before and after the above statement is executed:

```
> z <- 1:10
> tracemem(z)
[1] "<0x8908278>"
> z[3] <- 8
tracemem[0x8908278 -> 0x8800eb0]:
> tracemem(z)
[1] "<0x8800eb0>"
```

So, `z` was originally at the memory address `0x8908278`, then was copied to `0x8800eb0`, and ultimately that copy was the new `z`. What happened?

As noted in Chapter 9, this assignment is more complex than it seems. It is actually implemented via the replacement function “[<-”, through the call and *assignment*

```
z <- "[<-"(z,3,value=8)
```

In other words, even though we are ostensibly changing just one element of the array, *the entire vector is reassigned*. This, as seen above, would mean that the entire vector is copied, which can take up a lot of time for long vectors—especially if the ostensible vector *element* assignment is part of a loop, as in the code we saw earlier,

```
for (i in 1:length(x)) z[i] <- x[i] + y[i]
```

In R parlance, this illustrates that R (usually) uses a *copy-on-change* policy. For example, if we execute

```
> y <- z
```

then at first **y** refers to the same memory location as **z**. But if **z** is changed, it is transferred to a separate location, so that **y** doesn't change.

So, sources of slowdown can be subtle. Yet, there are further subtleties to deal with. For example, R doesn't exhibit the location-change behavior in the following setting:

```
> z <- runif(10)
> tracemem(z)
[1] "<0x88c3258>"
> z[3] <- 8
> tracemem(z)
[1] "<0x88c3258>"
```

Thus though one should be vigilant about location change, on the other hand we can't assume it.

## 15.5 Extended Example: Avoiding Memory Copy

This example, though artificial, will illustrate the memory-copy issues discussed in Section 15.4.

Suppose we have a large number of unrelated vectors, and among other things, we wish to set the third element of each to 8. We could store the vectors in a matrix, one vector per row. But since they are unrelated, we may consider storing them in a list. This would have the potential advantage of being able to use **lapply()**, which as was pointed out earlier, is a true loop-avoider, in contrast to **apply()**. Let's try it out:

```
> m <- 5000
> n <- 1000
>
> z <- list()
> for (i in 1:m) z[[i]] <- sample(1:10,n,replace=T)
> print(system.time({
```

```

+   for (i in 1:m) z[[i]][3] <- 8
+ )))
  user system elapsed
0.544  0.004  0.546
>
> z <- matrix(sample(1:10,m*n,replace=T),nrow=m)
> print(system.time({
+   for (i in 1:m) z[i,3] <- 8
+ )))
  user system elapsed
0.180  0.068  0.248

```

Except (again) for system time, the matrix formulation did better. The reason is that in the list version, we were encountering the memory-copy problem in each iteration of the loop. But in the matrix version, we encountered it only once. (Remember, matrices are special cases of vectors, so that the discussion on vectors in Section 15.4 does apply.)

The reader has undoubtedly noticed an improvement in the matrix code—vectorization! Let's try it:

```

> z <- matrix(sample(1:10,m*n,replace=T),nrow=m)
> print(system.time({
+   z[,3] <- 8
+ }))
  user system elapsed
0.100  0.048  0.149

```

Ah yes. But what about using **lapply()** on the list version?

```

>
> set3 <- function(lv) {
+   lv[3] <- 8
+   return(lv)
+ }
> z <- list()
> for (i in 1:m) z[[i]] <- sample(1:10,n,replace=T)
> print(system.time({
+   lapply(z,set3)
+ }))
  user system elapsed
0.284  0.008  0.292

```

Nice try, and often **lapply()** works, but not in this case.



## Chapter 16

# Interfacing R to Other Languages

### 16.1 Writing C/C++ Functions to be Called from R

You may wish to write your own C/C++ functions to be called from R, as they may run much faster than if you wrote them in R. (Recall, though, that you may get a lot of speed out of R if you avoid using loops.) The SWIG package can be used for this; see <http://www.swig.org>. MAY USE SWIG, .C OR BOTH. THINK IT OVER.

### 16.2 Extended Example: Speeding Up Discrete-Event Simulation

### 16.3 Using R from Python

Python is an elegant and powerful language, but lacks built-in facilities for statistical and data manipulation, two areas in which R excels. Thus an interface between the two languages would be highly useful; RPy is probably the most popular of these. RPy is a Python module that allows access to R from Python. For extra efficiency, it can be used in conjunction with NumPy.

You can build the module from the source, available from <http://rpy.sourceforge.net>, or download a prebuilt version. If you are running Ubuntu, simply type

```
sudo apt-get install python-rpy
```

To load RPy from Python (whether in Python interactive mode or from code), execute

```
from rpy import *
```

This will load a variable `r`, which is a Python class instance.

Running R from Python is in principle quite simple. For instance, running

```
>>> r.hist(r.rnorm(100))
```

from the Python prompt will call the R function `rnorm()` to produce 100 standard normal variates, and then input those values into R's histogram function, `hist()`. As you can see, R names are prefixed by `r`, reflecting the fact that Python wrappers for R functions are members of the class instance `r`.<sup>1</sup>

By the way, note that the above code will, if not refined, produce ugly output, with your (possibly voluminous!) data appearing as the graph title and the X-axis label. You can avoid this by writing, for example,

```
>>> r.hist(r.rnorm(100),main='',xlab='')
```

RPy syntax is sometimes less simple than the above examples would lead us to believe. The problem is that there may be a clash of R and Python syntax. Consider for instance a call to the R linear model function `lm()`. In our example, we will predict `b` from `a`:

```
>>> a = [5,12,13]
>>> b = [10,28,30]
>>> lmout = r.lm('v2 ~ v1',data=r.data_frame(v1=a,v2=b))
```

This is somewhat more complex than it would have been if done directly in R. What are the issues here?

First, since Python syntax does not include the tilde character, we needed to specify the model formula via a string. Since this is done in R anyway, this is not a major departure.

Second, we needed a data frame to contain our data. We created one using R's `data.frame()` function, but note that again due to syntax clash issues, RPy converts periods in function names to underscores, so we need to call `r.data_frame()`. Note that in this call, we named the columns of our data frame `v1` and `v2`, and then used these in our model formula.

The output object is a Python dictionary, as can be seen:

```
>>> lmout
{'qr': {'pivot': [1, 2], 'qr': array([[ -1.73205081, -17.32050808],
 [ 0.57735027, -6.164414  ],
 [ 0.57735027,  0.78355007]]), 'qraux': [1.5773502691896257, 1.6213286481208891], 'rank': 2, 'tol': 9.999
```

You should recognize the various attributes of `lm()` objects there. For example, the coefficients of the fitted regression line, which would be contained in `lmout$coefficients` if this were done in R, are here in Python as `lmout['coefficients']`. So, we can access those coefficients accordingly, e.g.

<sup>1</sup>They are loaded dynamically, as you use them.



## 16.4. EXTENDED EXAMPLE: ACCESSING R STATISTICS AND GRAPHICS FROM A PYTHON NETWORK MONITOR

```
>>> lmout['coefficients']
{'v1': 2.5263157894736841, '(Intercept)': -2.5964912280701729}
>>> lmout['coefficients']['v1']
2.5263157894736841
```

One can also submit R commands to work on variables in R's namespace, using the function `r()`. This is convenient if there are many syntax clashes. Here is how we could run the `wireframe()` example in Section 13.19 in RPy:

```
>>> r.library('lattice')
>>> r.assign('a',a)
>>> r.assign('b',b)
>>> r('g <- expand.grid(a,b)')
>>> r('g$Var3 <- g$Var1^2 + g$Var1 * g$Var2')
>>> r('wireframe(Var3 ~ Var1+Var2,g)')
>>> r('plot(wireframe(Var3 ~ Var1+Var2,g))')
```

We first used `r.assign()` to copy a variable from Python's namespace to R's. We then ran `expand.grid()` (with a period in the name instead of an underscore, since we are running in R's namespace), assigning the result to `g`. Again, the latter is in R's namespace.

Note that the call to `wireframe()` did not automatically display the plot, so we needed to call `plot()`.

The official documentation for RPY is at <http://rpy.sourceforge.net/rpy/doc/rpy.pdf>, with a useful presentation available at <http://www.daimi.au.dk/~besen/TBiB2007/lecture-notes/rpy.html>.

## 16.4 Extended Example: Accessing R Statistics and Graphics from a Python Network Monitor Program



# Chapter 17

## Parallel R

Since many R users have very large computational needs, various tools for some kind of parallel operation of R have been devised. Thus this chapter is devoted to parallel R.

### 17.1 Overview of Parallel Processing Hardware and Software Issues

Many a novice in parallel processing has with great anticipation written up parallel code for some application, only to find that the parallel version actually ran more slowly than the serial one. Accordingly, understanding the nature of parallel processing hardware and software is crucial to success in the parallel world. In this section you'll get a solid grounding in the basics.

#### 17.1.1 A Brief History of Parallel Hardware

The early parallel machines were mostly **shared-memory multiprocessor** systems, developed in the 1960s. In such a machine, multiple processors (CPUs) are physically connected to the same memory (RAM). A memory request for, say, address 200, issued by a processor would access the same physical location, regardless of which processor it comes from.

This was a successful model, and indeed, large-scale systems of that type are sold today, used by businesses such as large banks. But the prices run in the six- and seven-figure range, so it was natural that cheaper, if less powerful, alternatives would be sought.

The main class of alternatives is that of **message-passing** systems. Here the different computational units are actually separate computers. The term “message passing” means that the computers must transfer data among each other by sending messages through a network, rather than using shared memory for joint data storage. Each computer has its own separate memory, and address 200 on one is separate from address 200 on another.

The first commonly used message-passing systems were **hypercubes**, introduced in the 1980s. In a  $d$ -dimensional hypercube, each computer was connected via fast I/O to  $d$  others, and the total system size is  $s^d$ . In the case  $d = 3$ , the system has a cubic shape, hence the name.

Since hypercubes were made of cheap, off-the-shelf components, they were much cheaper than multiprocessors systems, but still required some effort to build, and thus were not quite at the commodity level needed for true cost savings. Thus in the 1990s the parallel processing community turned to **networks of workstations** (NOWs). The key point with a NOW is that almost any institution—businesses of at least medium size, university departments and so on—already has one. Any set of PCs connected to a network will work (though one can purchase special high-performance networks, discussed below).

More recently, the advent of cheap, commodity **multicore** processor chips has changed the picture quite a lot. It had always been the view of many (though hardly all) in the parallel processing community that programming on shared-memory machines is easier and clearer than in message-passing systems, and now multicore PCs are common even in the home. This has led to a resurgence of interest in shared-memory programming.

Since NOWs today typically are made up of multicore-equipped computers, hybrid shared-memory/message-passing paradigms are now common.

In another direction, there is **general programming on graphics processor units** (GPGPU). If your PC has a high-end graphics card, that device is actually a parallel processing system in its own right, and is thus capable of very fast computation. Though GPUs are designed for the specific task of doing graphics computations, such computations involve, for instance, matrix operations, and thus can be used for fast computing in many non-graphics contexts. The R package `gputools` is available for some operations, providing one has a sufficiently sophisticated graphics card.

### 17.1.2 Parallel Processing Software

Here we will introduce three popular software systems for parallel programming.

The most common approach today to programming shared-memory systems is through **threading**. Though many variations exist, typically threads are set up to be processes in the operating system sense, which act independently except for sharing their global variables. Interthread communication is done through that sharing, as well as wait/signal operations and the like.

Threaded code is usually written in C, C++ or FORTRAN. R can take advantage of threading via calls to threaded code written in those languages. An example of this will be presented in Section ??.

A higher-level approach to shared-memory programming is the popular OpenMP package, again taking the form of APIs from C, C++ or FORTRAN. While OpenMP is typically implemented on top of a thread system, it alleviates the programmer of the burden of dealing with the bothersome details of threaded programming. For example, in OpenMP, one can easily parallelize a **for** loop, without having to deal explicitly with threads, locks et cetera. Again, this is accessible from R via calls to those other languages, as we will see in Section ??.

In the message-passing realm, the most popular package today is the Message Passing Interface, MPI, again implemented as APIs for C, C++ or FORTRAN. Here the programmer explicitly constructs strings that serve as messages, and sends/receives them via MPI calls. In R, the package Rmpi provides a nice interface to MPI, as we will see in Section 17.2.

For R, higher-level message-passing packages have been developed, so as to alleviate the programmer from the burden of setting up and managing messages. These will be presented in this chapter as well.

It should be noted that message-passing software systems such as MPI can be used on shared-memory hardware. The programmer still writes in the message-passing paradigm, but internally MPI can exploit the underlying shared-memory system for greater efficiency.

The opposite situation, writing shared-memory code on message-passing hardware, occurs as well. In **software distributed shared-memory** (SDSM) systems, the computers' virtual memory hardware is used to send updates of the values of shared variables across the system. A popular system of this type at the C/C++ level is Treadmarks, developed at Rice University. My Rdsm package does this for R.

### 17.1.3 Performance Issues

As indicated earlier, obtaining a good speedup on a parallel system is not always easy. Indeed, a naive attempt at parallelizing an application may result in code that actually is slower than the serial, i.e. nonparallel version. We will discuss some of the major obstacles in this section.

We must first bring in some terminology, involving the notion of **granularity**. If a given application's work can be broken into large independent tasks whose work can be done independently, we describe it as **coarse-grained parallel**. Otherwise, it is **fine-grained parallel**. The key word here is **independent**, because it means that different computational entities—different threads on a shared-memory machine, different computers in a NOW, and so on—can work on the tasks without communicating with each other. This is vital, as communication between the computational entities can really slow things down. In other words, applications which fine-grained granularity are especially sensitive to communications delays.

And what are the elements of those communications delays? First, there is **bandwidth**, the number of bits per second that can be dispatched per unit time. The word “dispatched” here is key; bandwidth simply the number of bits that “go out the door,” i.e. leave the source, per unit time, and has no relation to the amount of time needed for a bit to travel from its source to its destination. That latter quantity is called **latency**.

There is also the **contention** problem, in which bits from different transmissions contend with each other for the same resource.

You may find it helpful to think of a toll bridge, with toll booths at the entrance to the bridge, our “source” here. The destination is the opposite end of the bridge, so latency is the time it takes for a car to travel from one end of the bridge to the other. This will be a function of the length of the bridge, and the speed at which cars travel on it.

The bandwidth here is the number of cars we can get through the toll booths per unit time. This can be

increased by, for instance, automating the toll collection, and by widening the bridge to have more lane and more toll booths.

The contention problem would arise, for example, if there were six approach lanes to the bridge, but they funneled into four toll booths. Traffic congestion on the bridge would also produce contention, though in complicated ways we won't discuss here.

### Shared-Memory Hardware

The big communications issue here is memory contention. Consider for instance a multiprocessor system in which the processors and memory are all attached to the same bus. The bus becomes a serialization point for the computation, as only one memory request can be transmitted along the bus to memory at a time. The multicore situation is similar, but has an additional element of serialization in the processor chip's interface to the bus.

If the system has more than a handful of processors, the situation gets even worse, as now some kind of multistage interconnection network (MIN) is used for communication. This allows more memory requests to be transmitted in parallel, thus increasing bandwidth, but now latency increases too, and contention for the MIN routing switches arises as well.

To ameliorate the bandwidth problem in bus-based systems, and the latency problem in MIN systems, system designers place local caches at each processor, to avoid having to communicate in the first place. But that creates a problem of its own, that of **cache coherency**. And that in turn creates communications delays, as follows.

Each time a processor writes to its local cache, the latter must inform the other caches, which either update their copies of the location or cache block that was written, or mark their copies invalid. In the latter case, a read will result in an update. Thus a memory access can result in quite a lot of communication traffic between the caches, causing substantial delay.

In order to get good performance, programmers must work around these problems, for example minimizing the number of writes the program does. More subtly, they must try to avoid the **false sharing** problem. Consider for instance an invalidate coherency protocol, and suppose variables **x** and **y** are in the same cache block. Then a write to **x** at one cache will make the copy of **y** invalid at the other caches, even though those other copies are correct. The next read to **y** will then result in coherency traffic, harming performance.

Thus the programmer must taken into account various memory issues, such as the fact that R stores matrices in column-major order.

### Message-Passing Hardware

Here the underlying network is the obvious bottleneck. Even on high-bandwidth network hardware, end-to-end latencies will be on the order of milliseconds, an eternity on CPU scales.

Less obvious, though, is the latency due to software. The TCP/IP protocol that forms the basis of the Internet incurs a lot of overhead. It is a layered protocol, with the various layers referred to as the **network protocol stack** in the operating system. Here there are major issues of time-consuming copying of messages between layers, and between the OS and the application program.

Special network hardware such as Myrinet and Infiniband address both the hardware and software latency issues. Short of using special networks, though, the programmer can take actions to minimize the latency problems, such as using some of the functions offered in MPI. These include group operations, such as broadcast, scatter/gather and reduce, as well as nonblocking I/O.

## 17.2 Rmpi

The Rmpi package provides R programmers a nice interface to MPI.

### 17.2.1 Usage

Of course, you must have MPI installed first. Some popular implementations that work well with Rmpi are MPICH2, LAM and OpenMPI (not to be confused with OpenMP); all of these allow one to “boot” up MPI prior to loading Rmpi, which is important.

Note that Rmpi requires that your MPI library consist of position-independent code, so you may have to rebuild MPI, even if your system already has it. In your build, add the **-fPIC** compiler flag. You also may need to enable shared libraries.

First start up MPI. Since the Rmpi paradigm uses a master process to control one or more workers, you should set up one more MPI process than your desired degree of parallelism.

Load in Rmpi, via the usual **library()** call. If you get an error message that the MPI library is not found, set the proper environment variable, e.g. LD\_LIBRARY\_PATH for Unix-family systems.

Then start Rmpi:

```
> mpi.spawn.Rslaves(nslaves=k)
```

where k is the desired number of worker processes.

This will start R on k of the machines in the group you started MPI on. Note that I say “machines” here, but on a multicore system, we might have all the spawned processes on the same machine. (If the latter is the case, be sure that you are using your implementation of MPI properly to accomplish this.)

The first time you do this, try this test:

```
mpi.remote.exec(paste("I am", mpi.comm.rank(), "of", mpi.comm.size()))
```

which should result in all of the spawned processes checking in.

The available functions are similar to those of MPI, such as

- **mpi.comm.rank()**: Returns the rank of the process, i.e. its ID number, that executes it.
- **mpi.comm.size()**: Returns the number of MPI processes, including the master that spawned the other processes. The master will be rank 0.
- **mpi.send()**, **mpi.recv()**: The MPI send/receive operations.
- **mpi.bcast()**, **mpi.scatter()**, **mpi.gather()**, **mpi.reduce()**: The MPI broadcast, scatter and gather operations.

These functions are mainly interfaces to the corresponding MPI functions. However, one major difference from MPI of Rmpi is that coding in the latter is dominated by a master/worker paradigm. The master must send code to the workers, using **pi.bcast.Robj2slave()** then send a command ordering the workers to execute one or more of the functions in that code, via **mpi.bcast.cmd()**.

### 17.2.2 Extended Example: Mini-quicksort

Here's an example, a "mini-quicksort." The master process splits the given vector into low and high piles as in the usual quicksort, then sends each pile to a worker process to sort. After receiving the sorted piles, the master concatenates them into the sorted version of the original vector.

```
# simple Rmpi example: Quicksort on a 2-node cluster

# sort x and return the sorted vector; x is assumed to consist of
# distinct elements
qs <- function(x) {
  if (length(x) < 3) return(sort(x))
  pivot <- median(x)
  mpi.bcast.cmd(sortchunk())
  # separate into the 2 piles
  xsmaller <- x[x < pivot]
  lxs <- length(xsmaller)
  xlarger <- x[x > pivot]
  lxl <- length(xlarger)
  # warn workers of vector lengths
  mpi.send(lxs, type=1, dest=1, tag=0)
  mpi.send(lxl, type=1, dest=2, tag=0)
  # send the vectors
  mpi.send(xsmaller, type=2, dest=1, tag=1)
  mpi.send(xlarger, type=2, dest=2, tag=1)
  # order the workers to sort their chunks
  # receive the results
  anytag <- mpi.any.tag()
  xsmaller <- mpi.recv(double(lxs), type=2, source=1, tag=anytag)
  xlarger <- mpi.recv(double(lxl), type=2, source=2, tag=anytag)
```



```

# piece all together
return(c(xsmaller,pivot,xlarger))
}

sortchunk <- function() {
  anytag <- mpi.any.tag()
  # receive vector length from master
  lxc <- mpi.recv(integer(1),type=1,source=0,tag=anytag)
  # receive vector from master
  xc <- mpi.recv(double(lxc),type=2,source=0,tag=anytag)
  mpi.send(sort(xc),type=2,dest=0,tag=2)
}

library(Rmpi) # load Rmpi
pi.spawn.Rslaves(nslaves=2) # start the worker processes

# send all the workers the code they need to run
mpi.bcast.Robj2slave(sortchunk)

# test case
x <- runif(20)
print(qs(x))

```

The general program flow is clear from the comments, but some details need to be explained. First, consider this statement:

```
mpi.send(lxs,type=1,dest=1,tag=0)
```

Here **type=2** refers to data of mode **double**, **dest=1** means we wish to send to the worker having ID 1, and **tag=0** means that we are declaring this message to be of message type 0.

Message types are programmer-defined, and allow the programmer to have a receiver wait for a particular kind of message. We do not need to do so in our code here, and thus set **tag=anytag** in all our receive operations.

Having two separate actions here, one to send the length of the vector and the next to send the vector itself, seems unnecessary. What could we do instead?

One alternative would be to allow enough room in the first arguments of our **mpi.recv()** calls for the longest vector we anticipate having. That of course would be rather constraining, and it might slow things down as well, e.g. increasing virtual memory paging activity.

However, just sending two separate messages for a vector and its length would seem to be rather at odds with the object-oriented nature of R. Why not simply send the vector as an object, which would include not only the vector but information about its length? Indeed, this could be done, via **mpi.send.Robj()**:

```

qsa <- function(x) {
  if (length(x) < 3) return(sort(x))
  mpi.bcast.cmd(sortchunka())
  pivot <- median(x)
  xsmaller <- x[x < pivot]

```

```

xlarger <- x[x > pivot]
mpi.send.Robj(xsmaller, dest=1, tag=1)
mpi.send.Robj(xlarger, dest=2, tag=1)
anytag <- mpi.any.tag()
xsmaller <- mpi.recv.Robj(source=1, tag=anytag)
xlarger <- mpi.recv.Robj(source=2, tag=anytag)
return(c(xsmaller, pivot, xlarger))
}

sortchunka <- function() {
  anytag <- mpi.any.tag()
  xc <- mpi.recv.Robj(source=0, tag=anytag)
  mpi.send.Robj(sort(xc), dest=0, tag=2)
}

```

So for example

```
mpi.send.Robj(xsmaller, dest=1, tag=1)
```

sends **xsmaller** as an object, by first calling **serialize()** to convert the object to a character string. This of course exacts a time penalty. Let's sample how much it is.

We ran the above code on a quad-core machine. Note that whenever it is active, i.e. not waiting to receive, the master is using one core, while each active worker process is using a core. Also, even though there is no physical network involved in this case, messages do have to traverse the network protocol stack, with all its copying.

```

> x <- runif(100000000)
> system.time(qs(x))
  user  system elapsed
14.928   5.486  47.332
> system.time(qsa(x))
  user  system elapsed
24.204  21.879  71.293

```

Yes, there is a quite a difference, thus worth analyzing in detail. First, user time here is the amount of time the master process (**system.time()** was invoked there, so the reported times are for that process) spent in its own code. Here this means primarily the forming of the two work piles, plus the serialization work. The latter must have taken about 4.6 seconds each, an eternity!

There is an even wider discrepancy in the system times. The conversion to character form makes the size of the vector much larger, and thus takes longer to transmit. Note carefully what that means. As noted earlier, there is much delay incurred in traversing the protocol stack. And if we had had a network involved here, things would have been even worse.

So, while object-oriented code may be considered by some to make for good programming style, it can really slow things down, and the use of ordinary **mpi.send()** and **mpi.recv()** was superior here to the object version.

However, we should not get too smug, as we have not entirely evaded communications problems. Look at what happens if we don't take a parallel approach at all:

```
> system.time(sort(x))
   user  system elapsed 
41.663   0.776  42.445
```

So, our non-object parallel program was still inferior to an entirely serial approach. In Section ??, we'll present an example with a happier ending.

## 17.3 The snow Package

Rmpi is a great interface of R to MPI, but MPI itself is rather burdensome to the programmer, who must take care of many different details. Thus a higher-level package was developed, snow, that has more of a feel of "parallel R."

The snow package runs on top of Rmpi (or Rpvm), or directly via network sockets. The latter means one does not need MPI installed, though one might get better performance from Rmpi in some applications on some platforms that MPI has been tailored to.

The snow package aims to maintain "the look and feel of R" while providing a way to do parallel computations in R. For instance, just as the ordinary R function **apply()** applies the same function to all rows of a matrix, the snow function **parApply()** does that in parallel, across multiple machines; different machines will work on different sets of rows.

In snow, there is a master process that farms out work to worker clients, who eventually return their results to the master. Communication is only between master and workers. Thus snow is not suitable for all parallel applications, and Rmpi might be better in some cases. Suppose for instance one wishes to do a parallel sort the Odd/Even Transposition method. Here pairs of machines repeatedly exchange data, so snow is not the platform of choice. Of course, one can also run Rmpi and snow simultaneously.

### 17.3.1 Starting snow

After loading snow, one then sets up a cluster, by calling the snow function **makeCluster()**, e.g.

```
> cls <- makeCluster(type="SOCK", spec=c("pc48","pc49"))
```

Here we've indicated that we wish snow to run on TCP/IP sockets that it creates itself, rather than going through MPI, and that we wish the workers to be pc48 and pc49. To run instead over Rmpi, we would type

```
> clm <- makeCluster(type="MPI", spec=2)
```

indicating that we wish the workers to be taken to be the first two of our MPI processes, created earlier.

Note that while the above R code sets up worker nodes at the machines named **pc48** and **pc49**, these are in addition to the master node, which is the machine on which that R code is executed

There are various other optional arguments. One you may find useful is **outfile**, which records the result of the call in the file **outfile**. This can be helpful if the call fails.

### 17.3.2 Overview of Available Functions

Let's look at a simple example of multiplication of a vector by a matrix. We set up a test matrix:

```
> a <- matrix(c(1,2,3,4,5,6,7,8,9,10,11,12),nrow=6)
> a
      [,1] [,2]
[1,]    1    7
[2,]    2    8
[3,]    3    9
[4,]    4   10
[5,]    5   11
[6,]    6   12
```

We will multiply the vector  $(1, 1)^T$  (T meaning transpose) by our matrix **a**, first without parallelism:

```
> apply(a,1,"%*%",c(1,1))
[1]  8 10 12 14 16 18
```

To review your R, note that this applies the function **dot()** to each row (indicated by the 1, with 2 meaning column) of **a** playing the role of the first argument to **dot()**, and with **c(1,1)** playing the role of the second argument.

Now let's do this in parallel, across our two machines in our cluster **cls**:

```
> parApply(cls,a,1,"%*%",c(1,1))
[1]  8 10 12 14 16 18
```

The function **clusterCall(cls,f,args)** applies the given function **f()** at each worker node in the cluster **cls**, using the arguments provided in **args**.

The function **clusterExport(cls,varlist)** copies the variables in the list **varlist** to each worker in the cluster **cls**. You can use this to avoid constant shipping of large data sets from the master to the workers; you just do so once, using **clusterExport()** on the corresponding variables, and then access those variables as global. For instance:

```
> z <- function() return(x)
> x <- 5
```

```

> y <- 12
> clusterExport(cls, list("x", "y"))
> clusterCall(cls, z)
[[1]]
[1] 5

[[2]]
[1] 5

```

The function **clusterEvalQ(cls,expression)** runs **expression** at each worker node in **cls**. Continuing the above example, we have

```

> clusterEvalQ(cls, x <- x+1)
[[1]]
[1] 6

[[2]]
[1] 6

> clusterCall(cls, z)
[[1]]
[1] 6

[[2]]
[1] 6

> x
[1] 5

```

Note that **x** still has its original version back at the master.

The function **clusterApply(cls,individualargs,f,commonargsgohere)** runs **f()** at each worker node in **cls**, with arguments as follows. The first argument to **f()** for worker  $i$  is the  $i^{th}$  element of the list **individualargs**, i.e. **individualargs[[i]]**, and optionally one can give additional arguments for **f()** following **f()** in the argument list for **clusterApply()**.

Here for instance is how we can assign an ID to each worker node, like MPI **rank**:<sup>1</sup>

```

> myid <- 0
> clusterExport(cls, "myid")
> setid <- function(i) {myid <- i} # note superassignment operator
> clusterApply(cls, 1:2, setid)
[[1]]
[1] 1

[[2]]
[1] 2

> clusterCall(cls, function() {return(myid)})
[[1]]
[1] 1

```

---

<sup>1</sup>I don't see a provision in snow itself that does this.

```
[[2]]
[1] 2
```

Recall that the way snow works is to have a master node, the one from which you invoke functions like **parApply()**, and then a cluster of worker nodes. Suppose the function you specify as an argument to **parApply()** is **f()**, and that **f()** calls **g()**. Then **f()** itself (its code) is passed to the cluster nodes, but **g()** is not. Therefore you must first pass **g()** to the cluster nodes via a call to **clusterExport()**.

Don't forget to stop your clusters before exiting R, by calling **stopCluster(clustername)**.

There are various other useful snow functions. See the user's manual for details.

### 17.3.3 More Snow Examples

In the first example, we do a kind of one-level Quicksort, breaking the array into piles, Quicksort-style, then having each cluster node work on its pile, then consolidate. We assume a two-node cluster.

```
# sorts the array x on the cluster cls
qs <- function(cls,x) {
  pvt <- x[1] # pivot
  chunks <- list()
  chunks[[1]] <- x[x <= pvt] # low pile
  chunks[[2]] <- x[x > pvt] # high pile
  # now parcel out the piles to the clusters, which sort the piles
  rcvd <- clusterApply(cls, chunks, sort)
  lx <- length(x)
  lc1 <- length(rcvd[[1]])
  lc2 <- length(rcvd[[2]])
  y <- vector(length=lx)
  if (lc1 > 0) y[1:lc1] <- rcvd[[1]]
  if (lc2 > 0) y[(lc1+1):lx] <- rcvd[[2]]
  return(y)
}
```

The second example implements the Shearsort sorting algorithm. Here one imagines the nodes laid out as an  $n \times n$  matrix (they may really have such a configuration). Here is the pseudocode:

```
for i = 1 to  $\log_2(n^2) + 1$ 
  if i is odd
    sort each even row in descending order
    sort each odd row in ascending order
  else
    sort each column in ascending order
```

And here is the Snow code:

```
is <- function(cls, dm) {
  n <- nrow(dm)
```

```

numsteps <- ceiling(log2(n*n)) + 1
for (step in 1:numsteps) {
  if (step %% 2 == 1) {
    # attach a row ID to each row, so will know odd/even
    augdm <- cbind(1:n,dm)
    # parcel out to the cluster members for sorting
    dm <- parApply(cls,augdm,1,augsort)
    dm <- t(dm) # recall need to transpose after apply()
  } else dm <- parApply(cls,dm,2,sort)
}
return(dm)
}

augsort <- function(augdmrow) {
  nelt <- length(augdmrow)
  if (augdmrow[1] %% 2 == 0) {
    return(sort(augdmrow[2:nelt],decreasing=T))
  } else return(sort(augdmrow[2:nelt]))
}

```

### 17.3.4 Parallel Simulation, Including the Bootstrap

If you wish to use snow on simulation code, including bootstrapping, you need to make sure that the random number streams at each cluster node are independent. Indeed, just think of what would happen if you just take the default random number seed—you'll get identical results at all the nodes, which certainly would defeat the purpose of parallel operation!

The careful way to do this is to install the R package **rlcuyer**. Then, before running a simulation, call **clusterSetupRNG()**, which in its simplest form consists simply of

```
clusterSetupRNG(cl)
```

for the cluster **cl**.

### 17.3.5 Example

Here we estimate the prediction error rate using “leaving one out” cross-validation in logistic regression.

```

# working on the cluster cls, find prediction error rate for the data
# matrix dm, with the response variable having index resp and the
# predictors having indices prdids
prerr <- function(cls,dm,resp,prdids) {
  # set up an artificial matrix for parApply()
  nr <- nrow(dm)
  loopmat <- matrix(1:nr,nrow=nr)
  # parcel out the rows of loopmat to the various members of the
  # cluster cls; for each row, they will apply delone() with arguments
  # being that row, dm, resp and prdids; the return vector consists
  # of 1s and 0s, 1 meaning that our prediction was wrong
  errs <- parApply(cls,loopmat,1,delone,dm,resp,prdids)
}

```

```

    return(sum(errs)/nr)
  }

# temporarily delete row delrow from the data matrix dm, with the
# response variable having index resp and the predictors having indices
# prdids
delone <- function(delrow,dm,resp,prdids) {
  # get all indices except delrow
  therest <- abl(1,delrow,nrow(dm))
  # fit the model
  lmout <- glm(dm[therest,resp] ~ dm[therest,prdids], family=binomial)
  # predict the deleted row
  cf <- as.numeric(lmout$coef)
  predvalue <- logit(dm[delrow,prdids],cf)
  if (predvalue > 0.5) {predvalue <- 1}
  else {predvalue <- 0}
  return(abs(dm[delrow,resp]-predvalue))
}

# "allbutone": returns c(a,a+1,...,b-1,b+1,...c)
abl <- function(a,b,c) {
  if (a == b) return((b+1):c)
  if (b == c) return(a:(b-1))
  return(c(a:(b-1), (b+1):c))
}

# finds the value of the logistic function at a given x for a given set
# of coefficients b
logit <- function(x,b) {
  lin <- c(1,x) %**% b
  return(1/(1+exp(-lin)))
}

```

### 17.3.6 To Learn More about snow

I recommend the following Web pages:

- <http://cran.cnr.berkeley.edu/web/packages/snow/index.html>  
CRAN page for snow; the package and the manual are here.
- <http://www.bepress.com/cgi/viewcontent.cgi?article=1016&context=uwbiostat>  
A research paper.
- <http://www.cs.uiowa.edu/~luke/R/cluster/cluster.html>  
Brief intro by the author.
- <http://www.sfu.ca/~sblay/R/snow.html#clusterCall>  
Examples, short but useful.



## **17.4 Extended Example: Computation-Intensive Variable Selection in Regression**



## Chapter 18

# String Manipulation

R has a number of string manipulation utilities, the need for which arise more frequently in statistical contexts than one might guess.

### 18.1 Some of the Main Functions

- **grep()**: Searches for a substring, like the Linux command of the same name.
- **nchar()**: Finds the length of a string.
- **paste()**: Assembles a string from parts.
- **sprintf()**: Assembles a string from parts.
- **substr()**: Extracts a substring.
- **strsplit()**: Splits a string into substrings.

For example, suppose we wish to test for a specified suffix in a file name:

```
# tests whether the file name fn has the suffix suff,  
# e.g. "abc" in "x.abc"  
testsuffix <- function(fn,suff) {  
  ncf <- nchar(fn) # nchar() gives the string length  
  dotpos <- ncf - nchar(suff) + 1 # dot would start here if there is one  
  # now check that suff is at the end of the name  
  return(substr(fn,dotpos,ncf)==suff)  
}
```

Note that **grep()** searches for a given string in a vector of strings, returning the indices of the strings in which the pattern is found. That makes it perfect for row selection, e.g.

```

> d <- data.frame(cbind(c(0,5,12,13),x))
> d
  V1    x
1  0 xyz
2  5 yabc
3 12  abc
4 13 <NA>
> dabc <- d[grep("ab",d[,2]),]
> dabc
  V1    x
2  5 yabc
3 12  abc

> s <- strsplit("a b"," ")
> s
[[1]]
[1] "a" "b"

> s[1]
[[1]]
[1] "a" "b"

> s[[1]]
[1] "a" "b"
> s[[1]][1]
[1] "a"

```

## 18.2 Extended Example: Forming File Names

Suppose I wish to create five files, **q1.pdf** through **q5.pdf** consisting of histograms of 100 random  $N(0,i^2)$  variates. I could execute the code<sup>1</sup>

```

for (i in 1:5) {
  fname <- paste("q",i, ".pdf")
  pdf(fname)
  hist(rnorm(100, sd=i))
  dev.off()
}

```

The **paste()** function concatenates the string "q" with the string form of **i**. For example, when  $i = 2$ , the variable **fname** will be "q 2".

But that wouldn't quite work, as it would give me filenames like "q 2.pdf". On Linux systems, filenames with embedded spaces create headaches.

One solution would be to use the **sep** argument:

```

for (i in 1:5) {
  fname <- paste("q",i, ".pdf", sep=" ")
}

```

<sup>1</sup>The main point here is the string manipulation, creating the file names **fname**. Don't worry about the graphics operations, or check Section 13.18 for details.

```
pdf(fname)
hist(rnorm(100, sd=i))
dev.off()
}
```

Here we used an empty string for the separator.

Or, we could use a function borrowed from C:

```
for (i in 1:5) {
  fname <- sprintf("q%d.pdf", i)
  pdf(fname)
  hist(rnorm(100, sd=i))
  dev.off()
}
```

Since even many C programmers are unaware of the **sprintf()** function, some explanation is needed. This function works just like **printf()**, except that it “prints” to a string, not to the screen. Here we are “printing” to the string **fname**. What are we printing? The function says to first print “q”, then print the character version of **i**, then print “.pdf”. When **i** = 2, for instance, we print “z2.pdf” to **fname**.

For floating-point quantities, note also the difference between **%f** and **%g** formats:

```
> sprintf("abc%fdef", 1.5)
[1] "abc1.500000def"
> sprintf("abc%gdef", 1.5)
[1] "abc1.5def"
```

## 18.3 Extended Example: Data Cleaning

*%* input SampleResample



## Chapter 19

# Installation: R Base, New Packages

### 19.1 Installing/Updating R

#### 19.1.1 Installation

There are precompiled binaries for Windows, Linux and MacOS X at the R home page, [www.r-project.org](http://www.r-project.org). Just click on Download (CRAN).

Or if you have Fedora Linux, just type

```
$ yum install R
```

In Ubuntu Linux, do:

```
$ sudo apt-get install r-base
```

On Linux machines, you can compile the source yourself by using the usual

```
configure  
make  
make install
```

sequence. If you want to install to a nonstandard directory, say `/a/b/c`, run **configure** as

```
configure --prefix=/a/b/c
```

#### 19.1.2 Updating

Use `updatepackages()`, either for specific packages, or if no argument is specified, then all R packages.

## 19.2 Packages (Libraries)

### 19.2.1 Basic Notions

R uses packages to store groups of related pieces of software.<sup>1</sup> The libraries are visible as subdirectories of your library directory in your R installation tree, e.g. `/usr/lib/R/library`. The ones automatically loaded when you start R include the **base** subdirectory, but in order to save memory and time, R does not automatically load all the packages. You can check which packages are currently loaded by typing

```
> .path.package()
```

### 19.2.2 Loading a Package from Your Hard Drive

If you need a package which is in your R installation but not loaded into memory yet, you must request it. For instance, suppose you wish to generate multivariate normal random vectors. The function `mvrnorm()` in the package MASS does this. So, load the library:

```
> library(MASS)
```

Then `mvrnorm()` will now be ready to use. (As will be its documentation. Before you loaded MASS, “`help(mvrnorm)`” would have given an error message).

### 19.2.3 Downloading a Package from the Web

However, the package you want may not be in your R installation. One of the big advantages of open-source software is that people love to share. Thus people all over the world have written their own special-purpose R packages, placing them in the CRAN repository and elsewhere.

#### Using `install.package()`

One way to install a package is, not surprisingly, to use the `install.packages()` function.

As an example, suppose you wish to use the `mvtnorm` package, which computes multivariate normal cdf's and other quantities. Choose a directory in which you wish to install the package (and maybe others in the future), say `/a/b/c`. Then at the R prompt, type

```
> install.packages("mvtnorm", "/a/b/c/")
```

---

<sup>1</sup>This is one of the very few differences between R and S. In S, packages are called *libraries*, and many of the functions which deal with them are different from those in R.



This will cause R to automatically go to CRAN, download the package, compile it, and load it into a new directory `/a/b/c/mvtnorm`.

You do have to tell R where to find that package, though, which you can do via the `.libPaths()` function:

```
> .libPaths("/a/b/c/")
```

This will add that new directory to the ones R was already using. If you use that directory often enough, you may wish to add that call to `.libPaths()` in your `.Rprofile` startup file in your home directory.

A call to `.libPaths()`, without an argument, will show you a list of all the places R will currently look at for loading a package when requested.

### Using “R CMD INSTALL”

Sometimes one needs to install “by hand,” to do modifications needed to make a particular R package work on your system. The following example will show how I did so in one particular instance, and will serve as a case study on how to “scramble” if ordinary methods don’t work.

I wanted to install a package `Rmpi` on our department’s instructional machines, in the directory `/home/matloff/R`. I tried using `install.packages()` first, but found that the automated process could not find the MPI library on our machines. The problem was that R was looking for those files in `/usr/local/lam`, whereas I knew they were in `/usr/local/LAM`.

So, I downloaded the `Rmpi` files, in the packed form `Rmpi.0.5-3.tar.gz`. I unpacked that file in my directory `/tmp`, producing a directory `/tmp/Rmpi`.

Now if there had been no problem, I could have just typed

```
% R CMD INSTALL -l /home/matloff/R Rmpi
```

from within the `/tmp` directory. That command would install the package contained in `/tmp/Rmpi`, placing it in `/home/matloff/R`. This would have been an alternative to calling `install.packages()`.

But as noted, I had to deal with a problem. Within the `/tmp/Rmpi` directory there was a `configure` file, so I ran

```
% configure --help
```

on my Linux command line. It told me that I could specify the location of my MPI files to `configure` as follows:

```
% configure --with-mpi=/usr/local/LAM
```

This is if one runs **configure** directly, but I ran it via R:

```
% R CMD INSTALL -l /home/matloff/R Rmpi --configure-args=--with-mpi=/usr/local/LAM
```

Well, that seemed to work, in the sense that R did install the package, but it also noted that it had a problem with the threads library on our machines. Sure enough, when I tried to load **Rmpi**, I got a runtime error, saying that a certain threads function wasn't there.

I knew that our threads library was fine, so I went into **configure** file and commented-out two lines:

```
# if test $ac_cv_lib_pthread_main = yes; then
    MPI_LIBS="$MPI_LIBS -lpthread"
# fi
```

In other words, I forced it to use what I knew (or was fairly sure) would work. I then reran “R CMD INSTALL,” and the package then loaded with no problem.

## 19.2.4 Documentation

You can get a list of functions in a package by calling **library()** with the **help** argument, e.g.

```
> library(help=mvrnorm)
```

for help on the **mvrnorm** package.

## 19.2.5 Built-in Data Sets

R includes a few real data sets, for use in teaching, research or in testing software. Type the following:

```
> library(utils)
> help(data)
```

Here **data** is contained within the **utils** package. We load that package, and use **help()** to see what's in it, in this case various data sets. We can load any of them but typing its name, e.g.

```
> LakeHuron
```

## Chapter 20

# User Interfaces

Though some may feel that “real programmers use the command line,” others prefer more sophisticated interfaces. We discuss two here.

### 20.1 Using R from emacs

There is a very popular package which allows one to run R (and some other statistical packages) from within emacs, ESS. I personally do not use it, but it clearly has some powerful features for those who wish to put in a bit of time to learn the package. As described in the R FAQ, ESS offers R users:

R support contains code for editing R source code (syntactic indentation and highlighting of source code, partial evaluations of code, loading and error-checking of code, and source code revision maintenance) and documentation (syntactic indentation and highlighting of source code, sending examples to running ESS process, and previewing), interacting with an inferior R process from within Emacs (command-line editing, searchable command history, command-line completion of R object and file names, quick access to object and search lists, transcript recording, and an interface to the help system), and transcript manipulation (recording and saving transcript files, manipulating and editing saved transcripts, and re-evaluating commands from transcript files).

### 20.2 GUIs for R

As seen above, one submits commands to R via text, rather than mouse clicks in a Graphical User Interface (GUI). If you can't live without GUIs, you should consider using one of the free GUIs that have been developed for R, e.g. R Commander (<http://socserv.mcmaster.ca/jfox/Misc/Rcmdr/>), JGR

(<http://stats.math.uni-augsburg.de/JGR/>), or the Eclipse plug-in StatEt (<http://jgr.markushelbig.org/JGR.html>).

# Chapter 21

## To Learn More

There is a plethora of resources one can draw upon to learn more about R.

### 21.1 R's Internal Help Facilities

#### 21.1.1 The `help()` and `example()` Functions

For online help, invoke `help()`. For example, to get information on the `seq()` function, type

```
> help(seq)
```

or better,

```
> ?seq
```

Each of the help entries comes with examples. One really nice feature is that the `example()` function will actually run thus examples for you. For instance:

```
?seq> example(seq)

seq> seq(0, 1, length=11)
[1] 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

seq> seq(rnorm(20))
[1] 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

seq> seq(1, 9, by = 2) # match
[1] 1 3 5 7 9
```

```
seq> seq(1, 9, by = pi)# stay below
[1] 1.000000 4.141593 7.283185

seq> seq(1, 6, by = 3)
[1] 1 4
...
```

Imagine how useful this is for graphics! To get a quick and very nice example, the reader is urged to run the following **RIGHT NOW**:

```
> example(persp)
```

### 21.1.2 If You Don't Know Quite What You're Looking for

You can use the function `help.search()` to do a “Google”-style search through R's documentation in order to determine which function will play a desired role. For instance, in Section 19 above, we needed a function to generate random variates from multivariate normal distributions. To determine what function, if any, does this, we could type

```
> help.search("multivariate normal")
```

getting a response which contains this excerpt:

```
mvrnorm(MASS)          Simulate from a Multivariate Normal
                        Distribution
```

This tells us that the function `mvrnorm()` will do the job, and it is in the package **MASS**.

You can also go to the place in your R directory tree where the base or other package is stored. For Linux, for instance, that is likely `/usr/lib/R/library/base` or some similar location. The file **CONTENTS** in that directory gives brief descriptions of each entity.

## 21.2 Help on the Web

### 21.2.1 General Introductions

- <http://cran.r-project.org/doc/manuals/R-intro.html>, is the R Project's own introduction.
- <http://personality-project.org/r/r.guide.html>, by Prof. Wm. Revelle of the Dept. of Psychology of Northwestern University; especially good for material on multivariate statistics and structural equation modeling.

- <http://www.math.csi.cuny.edu/Statistics/R/simpleR/index.html>: a rough form of John Verzani's book, *simpleR*; nice coverage of various statistical procedures.
- [http://zoonek2.free.fr/UNIX/48\\_R/all.html](http://zoonek2.free.fr/UNIX/48_R/all.html): A large general reference by Vincent Zoonekynd; really excellent with as wide a statistics coverage as I've seen anywhere.
- <http://wwwmaths.anu.edu.au/~johnm/r/usingR.pdf>: A draft of John Maindonald's book; he also has scripts, data etc. on his full site <http://wwwmaths.anu.edu.au/~johnm/r/>.
- <http://www2.uwindsor.ca/~hlynka/HlynkaIntroToR.pdf>: A nice short first introduction by M. Hlynka of the University of Windsor.
- <http://www.math.ilstu.edu/dhkim/Rstuff/Rtutor.html>: A general tutorial but with lots of graphics and good examples from real data sets, very nice job, by Prof. Dong-Yun Kim of the Dept. of Math. at Illinois State University.
- <http://www.ling.uni-potsdam.de/~vasishth/VasishthFS/vasishthFS.pdf>: A draft of an R-simulation based textbook on statistics by Shravan Vasishth.
- <http://www.medepi.net/epir/index.html>: A set of tutorials by Dr. Tomas Aragon. Though they are aimed at an epidemiologist readership, there is much material here. Chapter 2, "Working with R Data Objects," is definitely readable by general audiences.
- <http://cran.stat.ucla.edu/doc/contrib/Robinson-icebreaker.pdf>: *icebreakR*, a general tutorial by Prof. Andrew Robinson, excellent.

### 21.2.2 Especially for Reference

- <http://www.mayin.org/ajayshah/KB/R/index.html>: *R by Example*, a quick handy chart on how to do various tasks in R, nicely categorized.
- <http://cran.r-project.org/doc/contrib/Short-refcard.pdf>: R reference card, 4 pages, very handy.
- <http://www.stats.uwo.ca/computing/R/mcleod/default.htm>: A.I. McLeod's *R Lexicon*.

### 21.2.3 Especially for Programmers

- [http://zoonek2.free.fr/UNIX/48\\_R/02.html](http://zoonek2.free.fr/UNIX/48_R/02.html): The programming section of Zoonekynd's tutorial; includes some material on OOP.
- <http://cran.r-project.org/doc/FAQ/R-FAQ.html>: R FAQ, mainly aimed at programmers.

- <http://bayes.math.montana.edu/Rweb/Rnotes/R.html>: Reference manual, by several prominent people in the R/S community.
- <http://wiki.r-project.org/rwiki/doku.php?id=tips:tips>: Tips on miscellaneous R tasks that may not have immediately obvious solutions.

### 21.2.4 Especially for Graphics

There are many extensive Web tutorials on this, including:

- [http://www.sph.umich.edu/~nichols/biostat\\_bbag-march2001.pdf](http://www.sph.umich.edu/~nichols/biostat_bbag-march2001.pdf): A slide-show tutorial on R graphics, very nice, easy to follow, by M. Nelson of Esperion Therapeutics.
- [http://zoonek2.free.fr/UNIX/48\\_R/02.html](http://zoonek2.free.fr/UNIX/48_R/02.html): The graphics section of Zoonekynd's tutorial. Lots of stuff here.
- <http://www.math.ilstu.edu/dhkim/Rstuff/Rtutor.html>: Again by Prof. Dong-Yun Kim of the Dept. of Math. at Illinois State University. Extensive material on use of color.
- <http://wwwmaths.anu.edu.au/~johnm/r/usingR.pdf>: A draft of John Maindonald's book; he also has scripts, data etc. on his full site <http://wwwmaths.anu.edu.au/~johnm/r/>. Graphics used heavily throughout, but see especially Chapters 3 and 4, which are devoted to this topic.
- [http://www.public.iastate.edu/%7emervyn/stat579/r\\_class6\\_f05.pdf](http://www.public.iastate.edu/%7emervyn/stat579/r_class6_f05.pdf). Prof. Marasinghe's section on graphics.
- <http://addictedtor.free.fr/graphiques/>: The R Graphics Gallery, a collection of graphics examples with the source code.
- <http://www.stat.auckland.ac.nz/~paul/RGraphics/rgraphics.html>: Web page for the book, *R Graphics*, by Paul Murrell, Chapman and Hall, 2005; Chapters 1, 4 and 5 are available there, plus all the figures from the book and the R code which generated them.
- <http://www.biostat.harvard.edu/~carey/CompMeth/StatVis/dem.pdf>: By Vince Carey of the Harvard Biostatistics Dept. Lots of pictures, but not much explanation.

### 21.2.5 For Specific Statistical Topics

- *Practical Regression and Anova using R*, by Julian Faraway (online book!), <http://www.cran.r-project.org/doc/contrib/Faraway-PRA.pdf>.



### 21.2.6 Web Search for R Topics

Lots of resources here.

- Various R search engines are listed on the R home page; <http://www.r-project.org>. Click on Search.
- You can search the R site itself by invoking the function **RSiteSearch()** from within R. It will interact with you via your Web browser.
- I use RSeek, <http://www.rseek.org> a lot.
- I also use [finzi.psych.upenn.edu](http://finzi.psych.upenn.edu).