Outline

1. Polarizers and Retarders
2. Polarimeters
3. Scattering Polarization
4. Zeeman Effect
5. Hanle Effect

Polarization Summary

- polarization is an intrinsic property of light
- polarization properties and intensity of light can be described by 4 parameters:
  - *coherent* Jones calculus
  - *incoherent* Stokes/Mueller calculus
- degree of polarization is the fraction of the intensity that is fully polarized
- typical values for degree of polarization:
  - 45 degree reflection off aluminum mirror: 5%
  - clear blue sky: up to 75%
  - 45 degree reflection off glass: 90%
  - LCD screen: 100%
  - solar scattering polarization: 1% to 0.001%
  - exoplanet signal: 0.001%

Polarizers

- polarizer: optical element that produces polarized light from unpolarized input light
- linear, circular, or in general elliptical polarizer, depending on type of transmitted polarization
- linear polarizers by far the most common
- large variety of polarizers

Jones Matrix for Linear Polarizers

- Jones matrix for linear polarizer:

$$\mathrm{J}_p = \begin{pmatrix} p_x & 0 \\ 0 & p_y \end{pmatrix}$$

- $0 \leq p_x \leq 1$ and $0 \leq p_y \leq 1$, real: transmission factors for x, y-components of electric field: $E'_x = p_x E_x, \quad E'_y = p_y E_y$
- $p_x = 1,\ p_y = 0$: linear polarizer in $+Q$ direction
- $p_x = 0,\ p_y = 1$: linear polarizer in $-Q$ direction
- $p_x = p_y$: neutral density filter

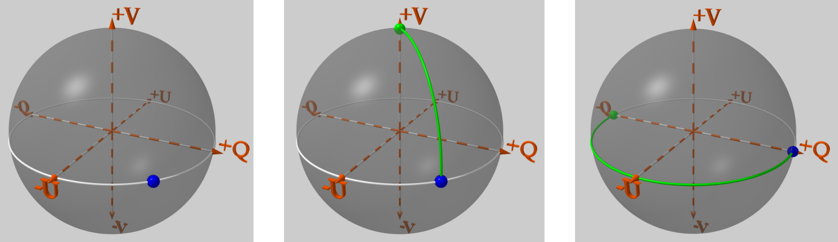
Mueller Matrix for Linear Polarizers

$$\mathrm{M}_p = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Mueller Matrix for Ideal Linear Polarizer at Angle θ

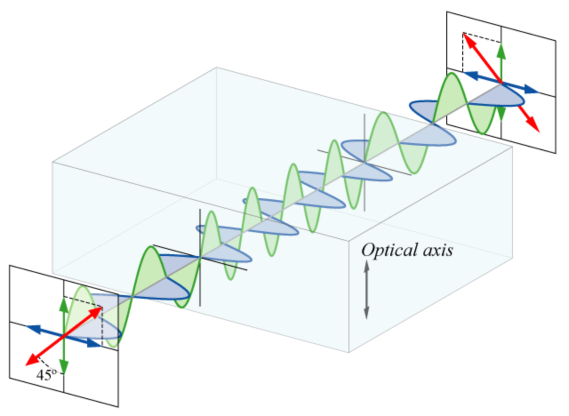
$$\mathrm{M}_{\mathrm{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Poincare Sphere



- polarizer is a point on the Poincaré sphere
- transmitted intensity: $\cos^2(l/2)$, l is arch length of great circle between incoming polarization and polarizer on Poincaré sphere

Retarders



en.wikipedia.org/wiki/Wave_plate

General Retarders or Wave Plates

- retarder: retards (delays) phase of one electric field component with respect to the orthogonal component

Retarder Properties

- does not change intensity or degree of polarization
- characterized by two (not identical, not trivial) Stokes vectors of incoming light that are not changed by retarder $\Rightarrow$ *eigenvectors* of retarder
- depending on polarization described by eigenvectors, retarder is
  - *linear retarder*
  - *circular retarder*
  - *elliptical retarder*
- linear, circular retarders are special cases of elliptical retarders
- circular retarders sometimes called *rotators* since they rotate the orientation of linearly polarized light
- linear retarders by far the most common type of retarder

Jones Matrix for Linear Retarders

- linear retarder with fast axis at 0° characterized by Jones matrix

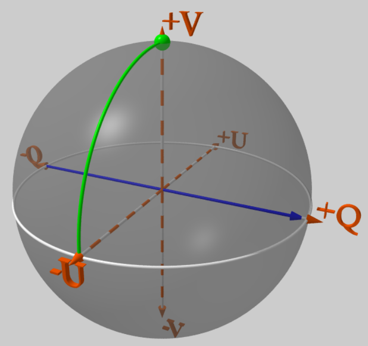
$$\mathsf{J}_r(\delta) = \begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathsf{J}_r(\delta) = \begin{pmatrix} e^{i\frac{\delta}{2}} & 0 \\ 0 & e^{-i\frac{\delta}{2}} \end{pmatrix}$$

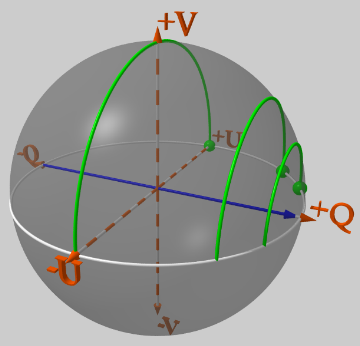
- δ: phase shift between two linear polarization components (in radians)
- absolute phase does not matter

Mueller Matrix for Linear Retarder

$$\mathsf{M}_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

Retarders on the Poincaré Sphere





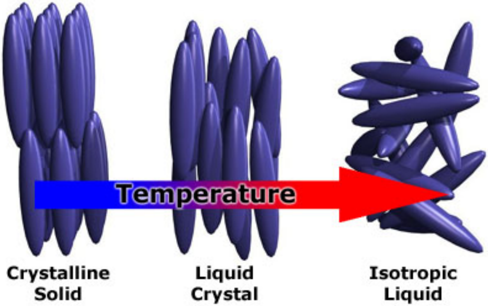
- retarder eigenvector (fast axis) in Poincaré sphere
- points on sphere are rotated around retarder axis by amount of retardation

Variable Retarders

Introduction

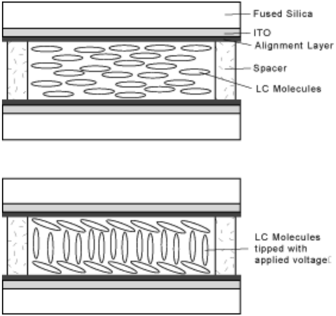
- sensitive polarimeters requires retarders whose properties (retardance, fast axis orientation) can be varied quickly (*modulated*)
- retardance changes (change of birefringence):
  - liquid crystals
  - Faraday, Kerr, Pockels cells
  - piezo-elastic modulators (PEM)
- fast axis orientation changes (change of *c*-axis direction):
  - rotating fixed retarder
  - ferro-electric liquid crystals (FLC)

Liquid Crystals



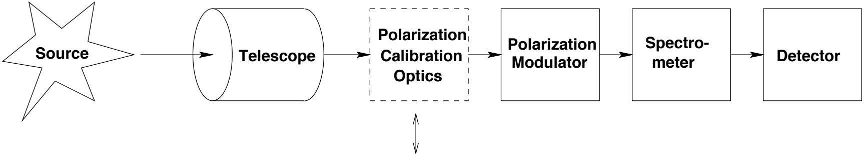
- liquid crystals: fluids with elongated molecules
- at high temperatures: liquid crystal is isotropic
- at lower temperature: molecules become ordered in orientation and sometimes also space in one or more dimensions
- liquid crystals can line up parallel or perpendicular to external electrical field

Liquid Crystal Retarders



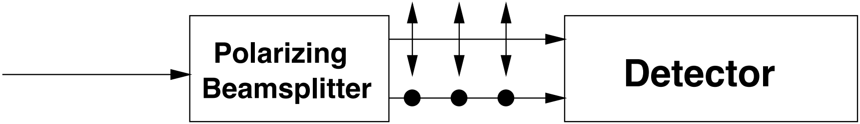
- dielectric constant anisotropy often large $\Rightarrow$ very responsive to changes in applied electric field
- birefringence δn can be very large (larger than typical crystal birefringence)
- liquid crystal layer only a few μm thick
- birefringence shows strong temperature dependence

General Polarimeters



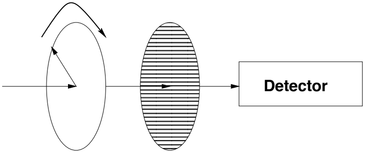
- polarimeters: optical elements (e.g. retarders, polarizers) that change polarization state of incoming light in controlled way
- detectors always measure only intensities
- intensity measurements combined to retrieve polarization state of incoming light
- polarimeters vary by polarization modulation scheme
- polarimeter should also include polarization calibration optics

Polarizing Beam-Splitter Polarimeter



- simple linear polarimeter: polarizing beam-splitter producing 2 beams corresponding to 2 orthogonal linear polarization states
- full linear polarization information from rotating assembly
- *spatial modulation*: simultaneous measurements of two (or more) Stokes parameters

Rotating Waveplate Polarimeter



- rotating retarder, fixed linear polarizer
- measured intensity is function of retardance δ, position angle θ
- only terms in θ lead to modulated signal
- *temporal modulation*: sequential measurements of $I\pm$ one or more Stokes parameters

Comparison of Temporal and Spatial Modulation Schemes

Modulation	Advantages	Disadvantages
temporal	negligible effects of flat field and optical aberrations	influence of seeing if modulation is slow
	potentially high polarimetric sensitivity	limited read-out rate of array detectors
spatial	off-the-shelf array detectors	requires up to four times larger sensor
	high photon collection efficiency	influence of flat field
	allows post-facto reconstruction	influence of differential aberrations

schemes rather complementary ⇒ modern, sensitive polarimeters use both to combine advantages and minimize disadvantages

SOLIS Vector-Spectro-Magnetograph (VSM)

Science Goals

Provide unique observations to understand

- the **solar activity cycle**
- **sudden energy releases** in the solar atmosphere (flares, coronal mass ejections)
- solar **irradiance changes** and relationship to global change

Magnetic field

- Line-of-sight component of photospheric magnetic field: Averaged over 2 Mm^2, sensitivity = 1 gauss, zero point stable to 0.1 gauss, time for a full disk map = 15 minutes
- Transverse component of the photospheric magnetic field: Same parameters as line-of-sight component except sensitivity $\geq$ 20 gauss.

Science Requirements

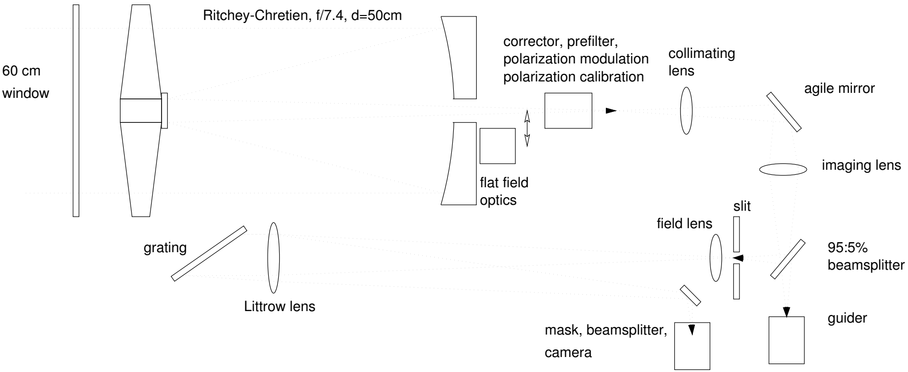
Parameter	Specification
angular element	$1''.125$ by $1''.125$
angular coverage	$2048''$ by $2048''$
geometric accuracy	$<0''.5$ rms after remapping
motion in RA	$\pm0.25^\circ$ for flat-fielding
scan rate in Dec	0.2-5.0 s/$''$
timing accuracy	better than 1 ms
spectral resolution	200,000
wavelength ranges	630.2 $\pm$0.1 nm
polarimetry	630.2 nm: I,Q,U,V
polarimetric sensitivity	0.0002 per pixel in 0.5 s
polarimetric accuracy	0.001
image stabilization	$>$40 Hz to improve spatial resolution

Design Challenges

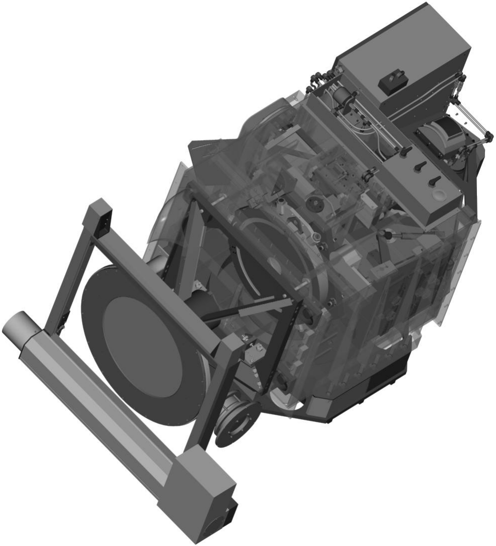
- compact instrument no longer than 2.5 m
- athermal optical design that is stable at varying ambient temperatures
- high guiding accuracy of better than $0\overset{\prime\prime}{.}5$ rms
- low instrumental polarization of less than $1 \cdot 10^{-3}$
- large wavelength range (630 to 1090 nm) with constant magnification
- high spectral resolution of 200,000
- highest possible throughput
- high energy densities of up to 20 W/cm^2
- high data rate of up to 300 MByte/s

Concept in Proposal

Vector Spectromagnetograph

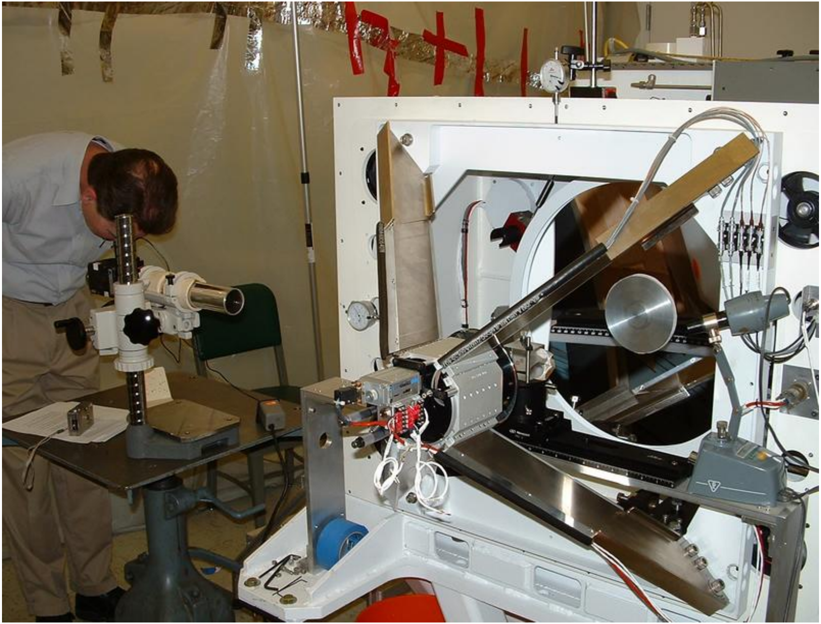


On the Computer



From the Welders

Aligning the Optics

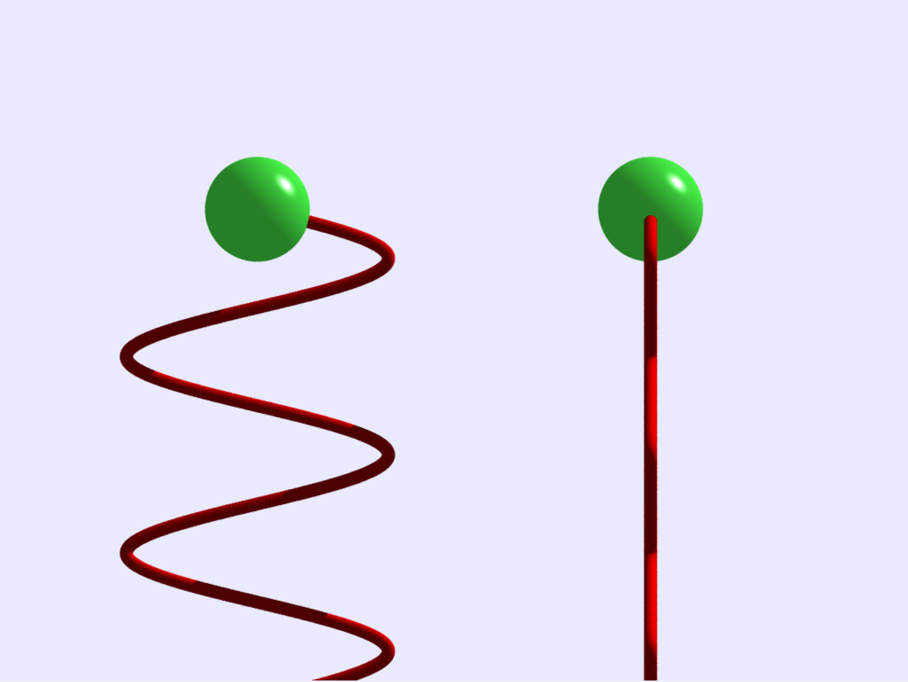


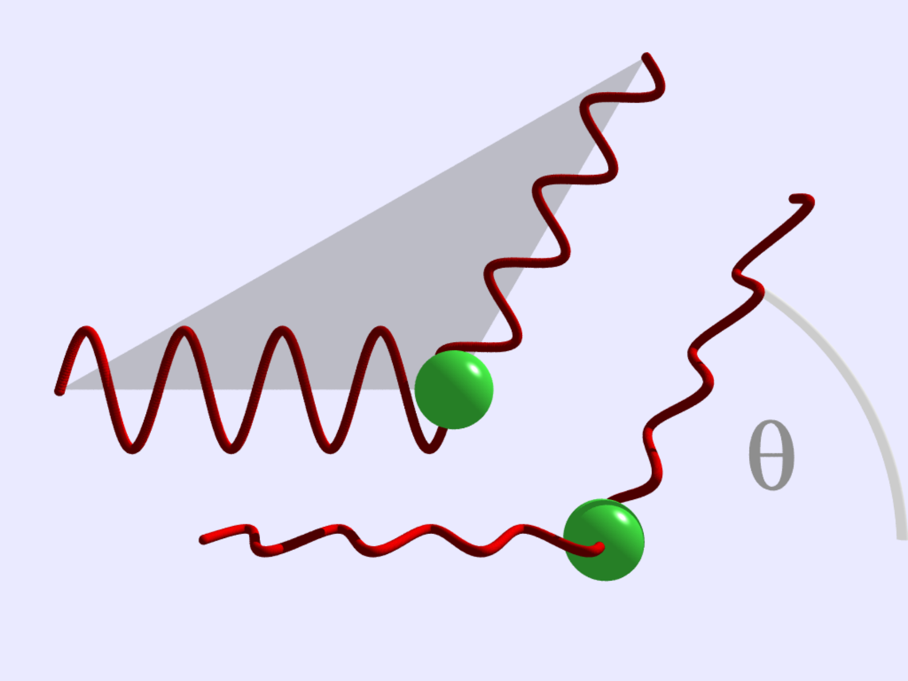
Ready for Science

Scattering Polarization

Single Particle Scattering

- light is absorbed and re-emitted
- if light has low enough energy, no energy transfered to electron, but photon changes direction $\Rightarrow$ elastic scattering
- for high enough energy, photon transfers energy onto electron $\Rightarrow$ inelastic (Compton) scattering
- Thomson scattering on free electrons
- Rayleigh scattering on bound electrons
- based on very basic physics, scattered light is linearly polarized

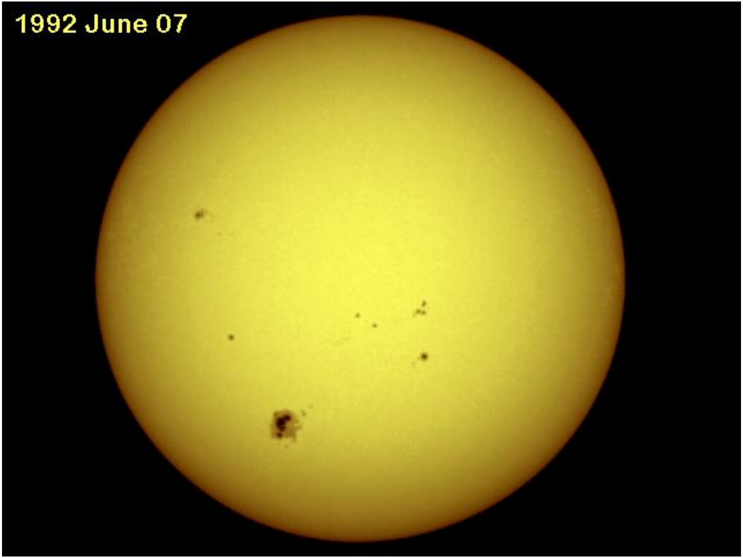



θ

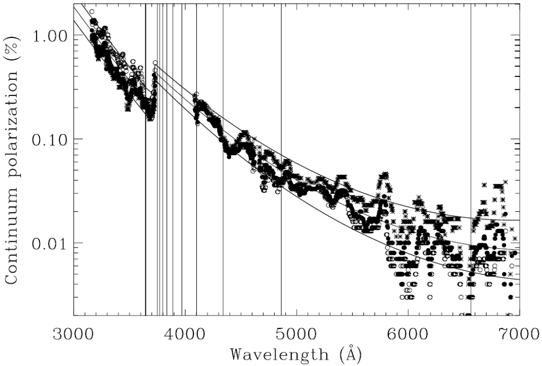
Polarization as a Function of Scattering Angle

- same variation of polarization with scattering angle applies to Thomson and Rayleigh scattering
- scattering angle θ
- projection of amplitudes:
  - 1 for polarization direction perpendicular to scattering plane
  - $\cos\theta$ for linear polarization in scattering plane
- intensities = amplitudes squared
- ratio of $+Q$ to $-Q$ is $\cos^2\theta$ (to 1)
- total scattered intensity (unpolarized = averaged over all polarization states) proportional to $\frac{1}{2}\left(1+\cos^2\theta\right)$

Limb Darkening



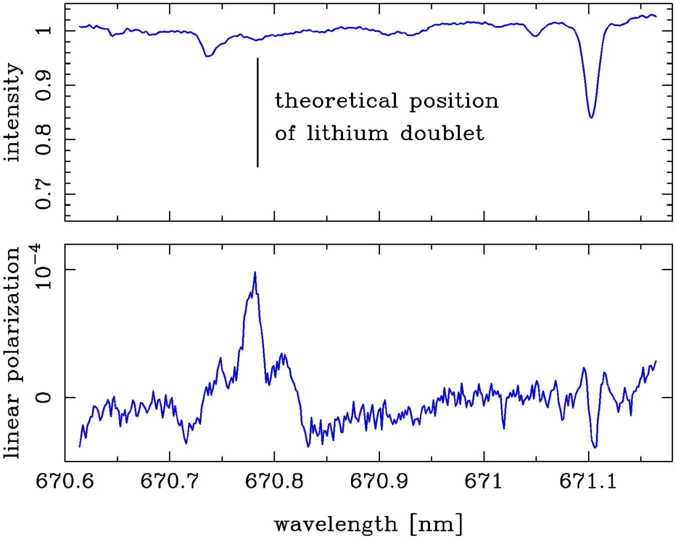
Solar Continuum Scattering Polarization



Stenflo 2005

- due to anisotropy of the radiation field
- anisotropy due to limb darkening
- limb darkening due to decreasing temperature with height
- last scattering approximation without radiative transfer

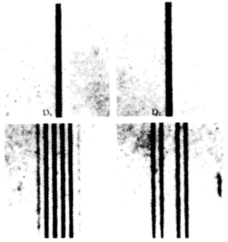
Solar Spectral Line Scattering Polarization



resonance lines exhibit “large” scattering polarization signals

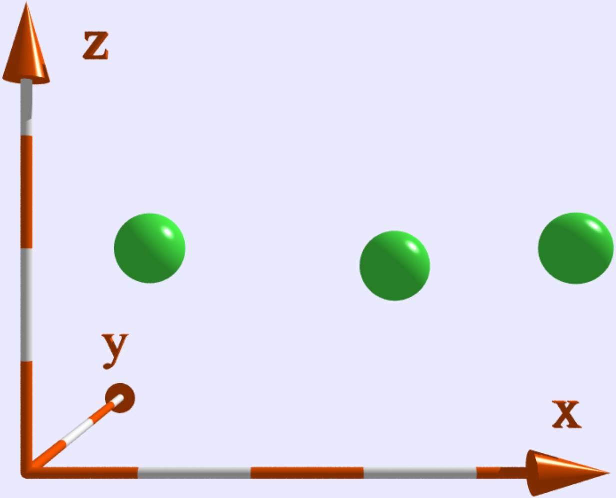
Zeeman Effect

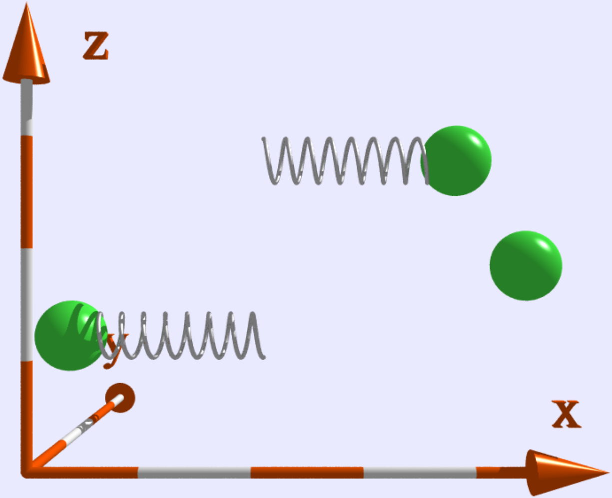
photos.aip.org/

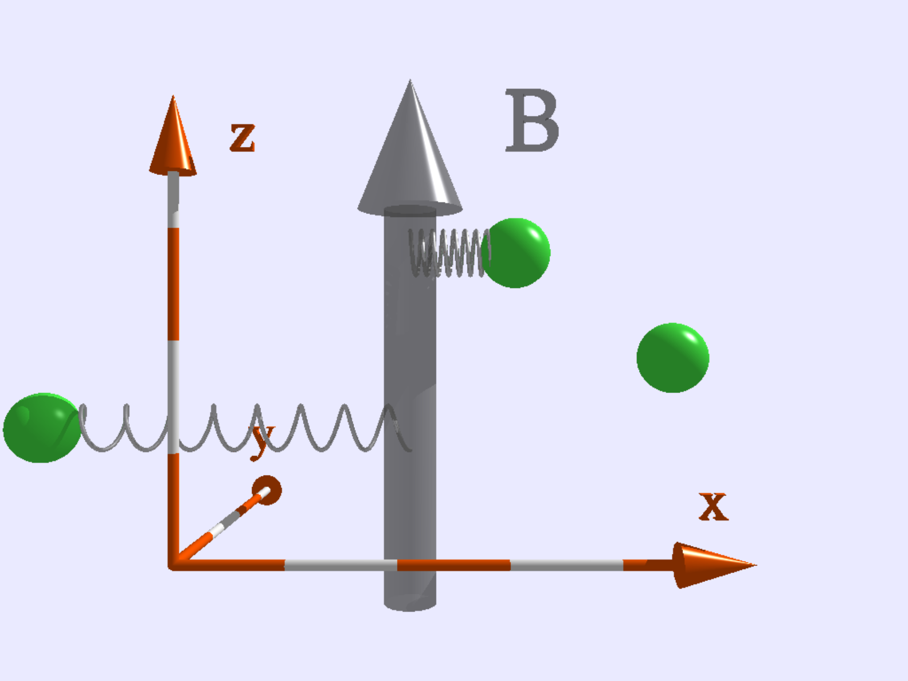


Splitting/Polarization of Spectral Lines

- discovered in 1896 by Dutch physicist Pieter Zeeman
- different spectral lines show different splitting patterns
- splitting proportional to magnetic field
- split components are polarized
- *normal Zeeman effect* with 3 components explained by H.A.Lorentz using classical physics
- splitting of sodium D doublet could not be explained by classical physics (*anomalous Zeeman effect*)
- quantum theory and electron's intrinsic spin led to satisfactory explanation


z
y
x


z
y
x


z
B
y
x

Quantum-Mechanical Hamiltionian

- classical interaction of magnetic dipol moment $\vec{\mu}$ and magnetic field given by magnetic potential energy

$$U = -\vec{\mu} \cdot \vec{B}$$

  $\vec{\mu}$ the magnetic moment and $\vec{B}$ the magnetic field vector
- magnetic moment of electron due to orbit and spin
- Hamiltonian for quantum mechanics

$$H = H_0 + H_1 = H_0 + \frac{e}{2mc}\left(\vec{L} + 2\vec{S}\right)\vec{B}$$

H_0 Hamiltonian of atom without magnetic field
H_1 Hamiltonian component due to magnetic field
e charge of electron
m electron rest mass
$\vec{L}$ the orbital angular momentum operator
$\vec{S}$ the spin operator

Energy States in a Magnetic Field

- energy state $\langle E_{NLSJ}|$ characterized by
  - main quantum number N of energy state
  - $L(L+1)$, the eigenvalue of $\vec{L}^2$
  - $S(S+1)$, the eigenvalue of $\vec{S}^2$
  - $J(J+1)$, the eigenvalue of $\vec{J}^2$,
    $\vec{J} = \vec{L} + \vec{S}$ being the total angular momentum
  - M, the eigenvalue of J_z in the state $\langle NLSJM|$
- for the magnetic field in the z-direction, the change in energy is given by

$$\Delta E_{NLSJ}(M) = \langle NLSJM|H_1|NLSJM\rangle$$

The Landé g Factor

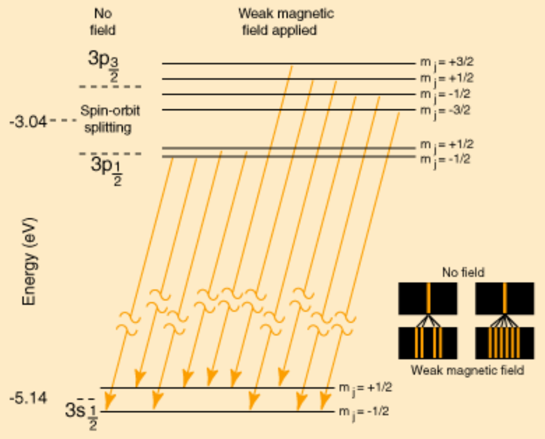
- based on pure mathematics (group theory, Wiegner Eckart theorem), one obtains

$$\Delta E_{NLSJ}(M) = \mu_0 g_L B M$$

with $\mu_0 = \frac{e\hbar}{2m}$ the Bohr magneton, and g_L the Landé g-factor

- in LS coupling where B sufficiently small compared to spin-orbit splitting field

$$g_L = 1 + \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)}$$



hyperphysics.phy-astr.gsu.edu/hbase/quantum/sodzee.html

Spectral Lines - Transitions between Energy States

- spectral lines are due to transitions between energy states:

  lower level with $2J_l + 1$ sublevels M_l

  upper level with $2J_u + 1$ sublevels M_u

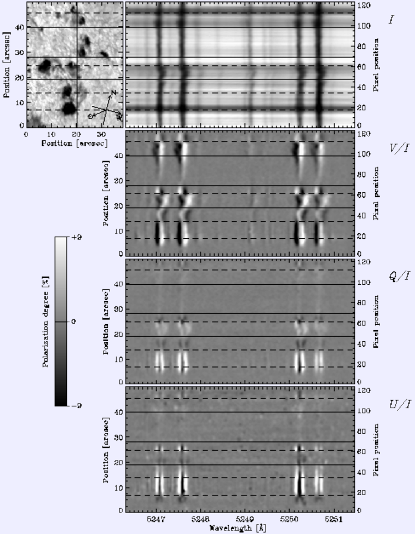
- not all transitions occur

Selection rule

- not all transitions between two levels are allowed
- assuming dipole radiation, quantum mechanics gives us the *selection rules*:
  - $L_u - L_l = \Delta L = \pm 1$
  - $M_u - M_l = \Delta M = 0, \pm 1$
  - $M_u = 0$ to $M_l = 0$ is forbidden for $J_u - J_l = 0$
- total angular momentum conservation: photon always carries $J_{\mathrm{photon}} = 1$
- *normal Zeeman effect*: line splits into three components because
  - Landé g-factors of upper and lower levels are identical
  - $J_u = 1$ to $J_l = 0$ transition
- *anomalous Zeeman effect* in all other cases

Effective Landé Factor and Polarized Components

- each component can be assigned an effective Landé g-factor, corresponding to how much the component shifts in wavelength for a given field strength
- components are also grouped according to the linear polarization direction for a magnetic field perpendicular to the line of sight
  - π components are polarized parallel to the magnetic field (**p**i for *parallel*)
  - σ components are polarized perpendicular to the magnetic field (**s**igma for German *senkrecht*)
- for a field parallel to the line of sight, the π-components are not visible, and the σ components are circularly polarized

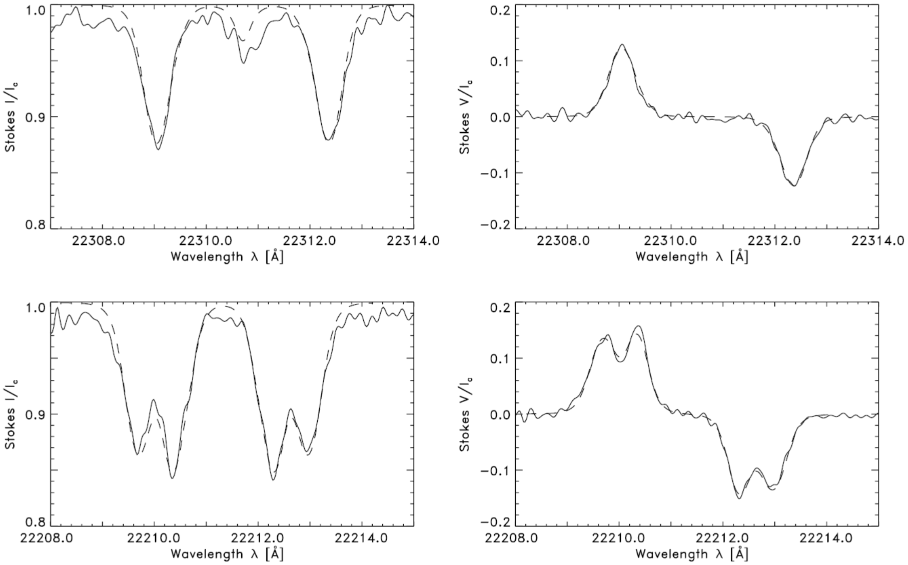


Bernasconi et al. 1998

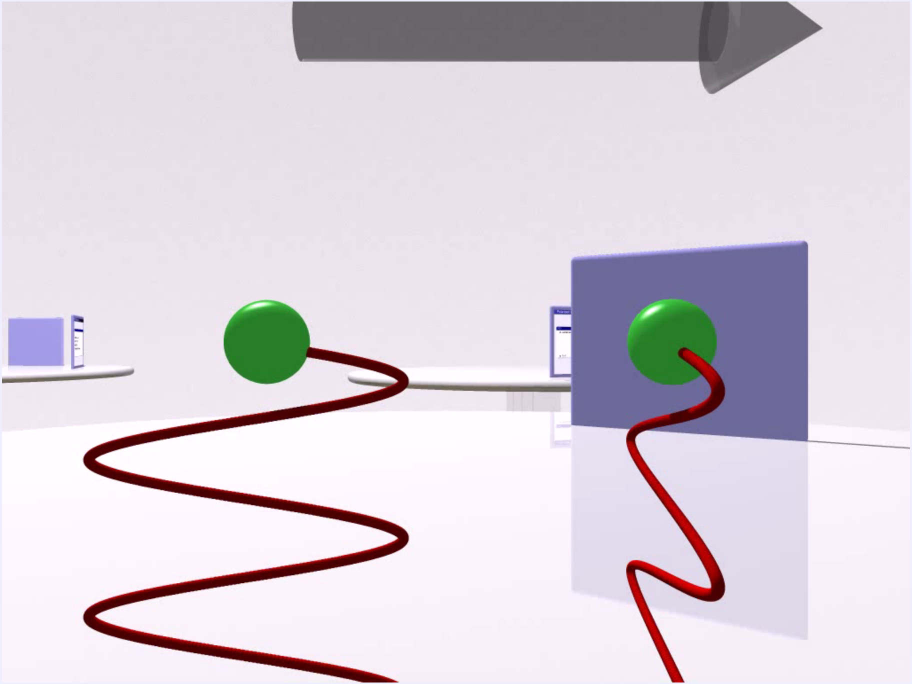
Zeeman Effect in Solar Physics

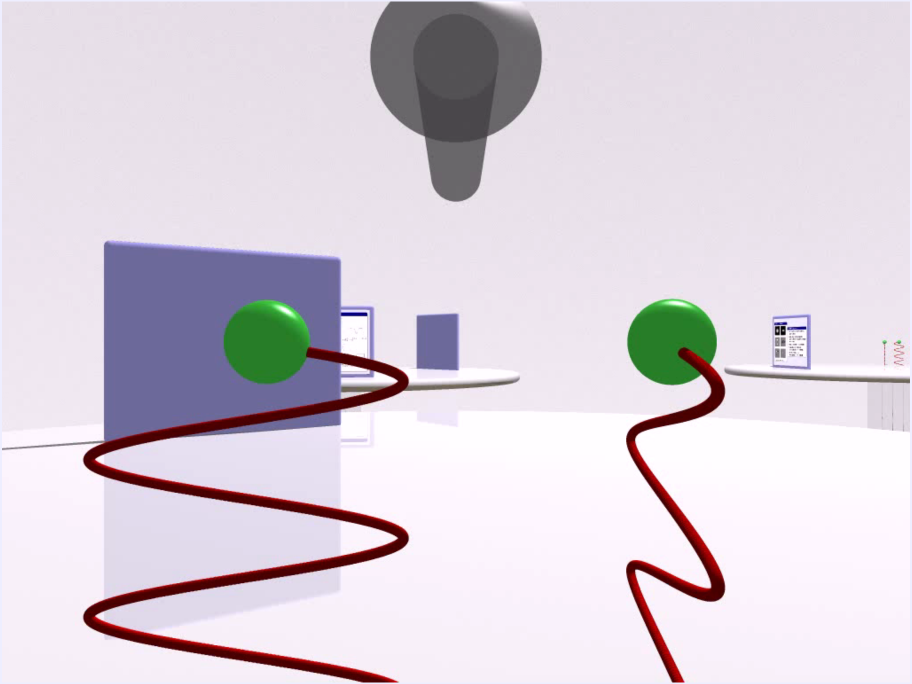
- discovered in sunspots by G.E.Hale in 1908
- splitting small except for in sunspots
- much of intensity profile due to non-magnetic area ⇒ filling factor
- a lot of strong fields outside of sunspots
- full Stokes polarization measurements are key to determine solar magnetic fields
- 180 degree ambiguity

Fully Split Titanium Lines at 2.2μm

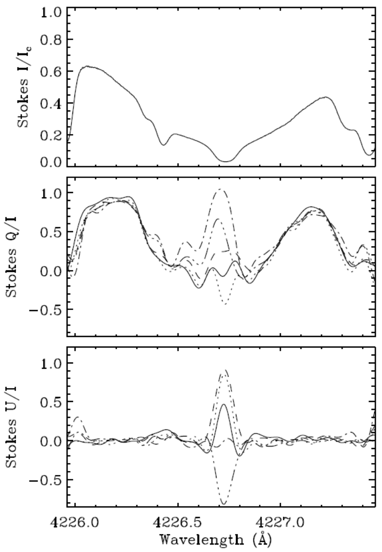


Rüedi et al. 1998





Hanle Effect



Bianda et al. 1998

Depolarization and Rotation

- scattering polarization modified by magnetic field
- precession around magnetic field depolarizes and rotates polarization
- sensitive $\sim 10^3$ times smaller field strengths that Zeeman effect
- measureable effects even for isotropic field vector orientations