Polarization Optics Tutorial:
Polarizers, Waveplates, Rotators, and Lyot Filters

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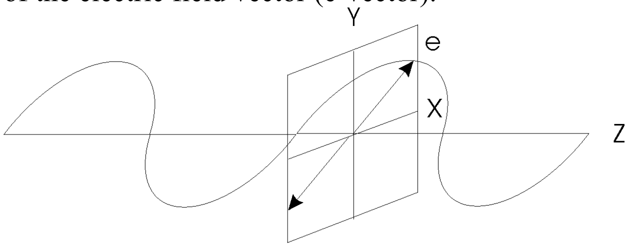
Introduction

We will discuss polarized light; some of the optical components used to control, direct, and modify it; and applications. This tutorial is intended to be an accessible introduction to technicians, buyers, and generalists; and to be useful to engineers, designers, and quality assurance personnel.

What is Light?

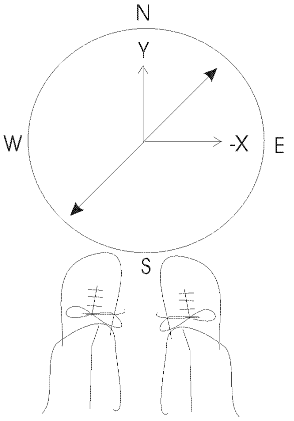
For our purposes here we will treat light as a transversely oscillating electromagnetic wave propagating from a source, and we will consider only the electrical field. As with all waves, amplitude, frequency and phase are essential parts of the description. Because the electric field vibration is transverse to the direction of propagation, we also need to account for the orientation of the electric field vector (e-vector).

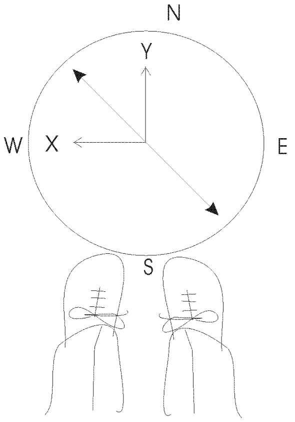


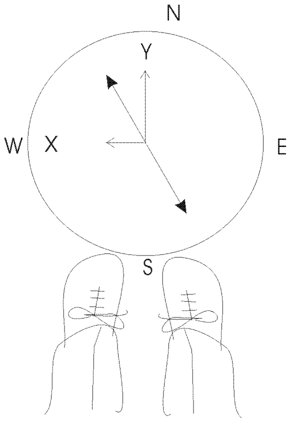
What is polarization?

Polarization is the state of the e-vector orientation. We may use an XYZ coordinate system in which Z is the direction of propagation. Since light is a transverse wave, the polarization state can be analyzed by projecting the e-vector onto arbitrary orthogonal axes called X and Y, then evaluating these projected components. Viewed across time, the relationship of X and Y projections may be fully or partly disordered, and any ordered portion will have a phase and amplitude relation between X and Y components.

It will be helpful here to imagine light propagating vertically.







Linear polarization can be projected in a North-South plane (N-S) and an East-West plane (E-W). As mentioned, X and Y are entirely arbitrary (so long as they are perpendicular to each other and the direction of propagation) so we may assign them to be N and E here at our convenience. If this light is polarized in a plane going NE-SW, then projections along N-S and E-W contain equal amplitudes in phase. By shifting the phase between the N-S and E-W components by half a wavelength, we obtain NW-SE. Then by adjusting the amplitudes we can obtain any compass point.

The possible states of polarization, and how to describe them

A brief glossary:

Unpolarized*:* A completely disordered, chaotic orientation of the e-vector over time. At any instant there is no relation of the e-vector to prior or future orientation.
Partial polarization*:* A statistical preference for one polarization state over others. Much light in nature is partially polarized.
Linear polarization*:* The e-vector oscillates within a plane that is constant over time. This plane is called the *plane of polarization.*
Circular polarization*:* The phase difference between equal X and Y projections is one quarter of a wavelength. In the example of the wave propagating vertically downward, the e-vector encountering the table could point N, then E, then S, then W. This is called left circular, because it is counterclockwise when looking *into* the beam. It is useful to visualize a threaded rod pushing forward along its length without rotating; the threads representing the e-vector. At any stationary plane, the e-vector describes a circle over time. At any instant the e-vector describes a helix through space, but the entire helix is in constant motion forward. The case of right circular polarization could be visualized as a N-W-S-E progression of the threads.
Elliptical polarization*:* This is a more general case than circular polarization, in which there is a phase difference between the two components. Elliptical polarization is the result when the components are either equal with non-quarter-wave phase difference, or unequal with any non-zero phase difference
Random polarization: Due to the nature of certain lasers, random polarization is distinct from unpolarized. Randomly polarized lasers usually emit a linearly or elliptically polarized beam whose orientation changes at random but finite intervals, usually to an orthogonal state.

Mathematical Representations

The Stokes parameters describe polarization states including partially or completely unpolarized light. The Stokes parameters are expressed in vector notation (although they are not the usual sort of vector). The Mueller matrices describe the action of optical elements upon the Stokes parameters. The strength of this approach is its ability to describe natural light states. Its weakness is its inefficient notation when dealing with coherent light.

The Stokes parameters are:

$S_0 = 2I_o$	Incident irradiance
$S_1 = 2I_1 - 2I_0$	Degree of horizontal (+) or vertical (-) polarization
$S_2 = 2I_2 - 2I_0$	Degree of polarization at 45° (+ or -)
$S_3 = 2I_3 - 2I_0$	Degree of right circular (+) or left circular (-) polarization

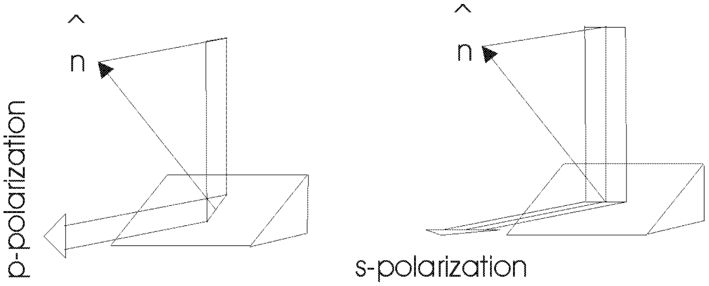
The Jones vector only addresses fully polarized states, and directly corresponds to the e-field vector. The Jones matrices describe the action of optical elements upon the Jones vectors. The strength of this approach is its economy and its ability to deal with coherent light states. Its weakness is that it is only applicable to polarized light.

The Jones vector consists of the elements $E_x(t)$ and $E_y(t)$, instantaneous projections of the e-vector onto x and y planes.

Representations of the various polarization states and optical elements using these notations are available in many excellent textbooks including Optics by Eugene Hecht.

The relation between plane of incidence and plane of polarization: S and P

The *plane of polarization* contains the e-vector and the propagation direction. Any optical components in the path are not considered in this definition. The *plane of incidence* includes the propagation direction of the light and the surface normal of the component and is arbitrary for normal incidence. Polarization is not considered in this definition. The relationship between these two planes is of critical importance.



What we call 'P' and 'S' polarization refers to the relation between these two planes. Let's return to the example of vertical propagation, N-S linear. We place a mirror in the beam path tilted so that the beam will reflect horizontally North or South. The beam is then P polarized *with respect to the mirror*. P polarization is defined as the condition where the plane of incidence includes the plane of polarization. If we arrange for the mirror to reflect the same beam horizontally East or West, the beam is considered S (senkrecht) polarized with respect to the mirror. Those more familiar with the English language than with German may find it helpful to remember these cases as ones where the e-vector "Pokes" the mirror in its vibrations, or "Slips" side to side on the mirror. You may also imagine the orientation of a flat stone thrown onto water and use P for "Plunge" and S for "Skip".

Interaction of polarized light with matter

Interaction with surfaces

Normal incidence surface reflection and transmission through isotropic materials are unaffected by polarization. At non-normal incidence S polarization will almost always reflect more than P polarization, and with increasing angles will reach 100%.

At Brewster's angle, defined as the angle whose tangent is equal to the ratio of the indices on either side of a surface, P polarized reflection goes to zero, and in the absence of absorption, transmission reaches 100%. For the case of a substrate with n = 1.5 in air, $\phi_B = \text{atan } 1.5/1 \cong 56°19'$. As the angle of incidence increases beyond ϕ_B, P reflection also increases to 100%.

When light is incident upon a higher index material, 100% reflection is approached at grazing incidence. Using Snell's law for refraction we can obtain the corresponding angle of internal propagation, known as the critical angle. When light encounters the surface from the high index side, 100% reflection for P and S is attained at the critical angle and above. This phenomenon is known as total internal reflection (TIR).

While pure P or S polarization will remain P or S upon reflection or transmission, any other incident polarization state will exhibit changes. It is always preferable to choose P polarization for oblique transmission, S polarization for oblique reflection, and near normal incidence for preserving any other polarization state.

Interaction with birefringent materials

The transmission of polarized light through birefringent materials, while done at the speed of light by Nature, is rather more tedious for mortals to explain. Accordingly we will limit this discussion to crystal quartz in the case of normal incidence and the optic axis in the surface plane. For a complete description of birefringence refer to one of the many fine texts including Hecht, Yariv & Yeh, or Born and Wolf.

When light propagates through a medium such as glass its speed is reduced by the ratio $v/c = 1/n$, where c is the speed of light in vacuum and n is the index of refraction of the medium. In uniaxial birefringent materials such as crystal quartz, there are two indices n_o and n_e ("ordinary" and "extraordinary"). n_o applies to light polarized perpendicular to the optic axis of the crystal while n_e applies to light polarized along the optic axis. These indices are a function of wavelength and temperature. n_o and n_e differ by an amount sufficient to produce phase shifts between polarization components, as utilized in waveplates.

Rotation (Optical Activity)

When light propagates along the optic axis of crystal quartz, its plane of polarization is rotated. Certain other materials, including organic liquids such as corn syrup, also exhibit rotation. The degree of rotation is independent of the initial orientation, and is a strong function of wavelength; both useful properties exploited by polarization rotators.

Polarizing Optics and Applications

Linear Polarizers

A linear polarizer transmits only the portion of incident light that is projected along its pass axis, regardless of the incident light's degree or state of polarization. This portion can be anywhere from nearly 100% of the incident light to very nearly zero. Depending on the type of polarizer, the remainder can be reflected, refracted or absorbed: Plastic sheet polarizers reject the unwanted component by absorption, and typically transmit less than 75% even along the pass axis. Wire grid polarizers reflect and transmit orthogonal linear polarization states, and can work in strongly converging beams across a wide wavelength range, but have low extinction ratios especially at shorter wavelengths approaching the dimension of the grid spacing. Thin film polarizers separate the portions into reflected and transmitted beams, usually with better than 98% efficiency, but work well only within a limited spectral and angular range. Crystal polarizers either reflect or refract the rejected portion, without significant absorption of either portion, and can achieve extinction ratios of 10^{-6}:1 over a broad spectral range, but only over a small range of incident angles. Crystal polarizers come in many forms, each with unique characteristics.

The thin film polarizer plate is simple and inexpensive, consisting of a plane parallel glass plate with a coating on one side. It has high transmittance for P polarization, high power handling capacity and a high extinction ratio. The plate is designed for oblique incidence, usually at

Brewster's angle. One surface receives a thin film polarizer coating. The transmitted light is laterally displaced by about 0.43 times the plate's thickness for glass, but undeviated in direction.

The thin film polarizing beamsplitter prism offers wider spectral bandwidth than the thin film polarizer plate. The transmitted light is not displaced or deviated. The cube style design reflects the S polarized light at 90° to the incoming beam. Deflection angles other than 90°, while somewhat less convenient in system layout and alignment, offer considerable performance advantages. Prisms with optically contacted or air-gap interfaces achieve much higher power handling capabilities than those with cemented interfaces.

Waveplates: General

Waveplates, also known as *plate retarders*, change the relative phase between two components of polarization. As we'll see, this can be very useful. A common configuration of waveplate is a plane-parallel plate of a birefringent crystal, typically quartz. Mica, magnesium fluoride, and sapphire have also found niche uses.

A crystal quartz waveplate is made with the optic axis in the plane of the surface. It is oriented so that incident polarized light may be resolved into components projected along the optic axis and perpendicular to it. These two components will experience a relative phase shift (retardation) proportional to the thickness of the plate. When the fractional part of this retardation is a non-zero value, the waveplate will transform polarizations from one state to another. Note that a waveplate does not polarize light – it only modifies the state of polarized light. Further, a waveplate cannot produce a relative phase shift unless it sees components both parallel and perpendicular to its optic axis.

Retardation equals birefringence times thickness. Birefringence ($n_o - n_e$) varies with temperature and wavelength. The thickness of a true zero order quartz waveplate with 0.25 waves retardance anywhere in the visible spectrum (0.4 to 0.7 μm wavelength) is between 10 and 20 μm. Such a part is difficult to manufacture, handle, and mount. The practical and standard solution to this problem is to create "multiple order" waveplates of approximately 1 mm thickness so that the retardation will be M + 0.25 where M is an integer in the range of ~ 12 to 24 waves. The actual order number is determined by wavelength, manufacturing considerations, and customer requirements. The integer part of the retardation has no direct effect on the performance of a waveplate. However, because the integer part constitutes most of the overall retardation, it increases the sensitivity of the part's effective retardation to temperature or wavelength variation.

The optical paths along and across the optic axis vary separately with the magnitude and orientation of the incident light. Thus the field of view is strictly limited for thicker crystal waveplates. In collimated space, that feature can be utilized for angle-tuning.

Waveplates: Zero order

A zero order waveplate may be constructed of two multiple order plates whose retardation differs by the desired amount. The two plates are aligned with their axes crossed. The resulting assembly has the temperature and wavelength variation of a true zero order plate, which is a small fraction of the multiple order plate. For example, 0.25/12.25 = 1/49.

Optically contacted waveplates work well for moderate temperature excursions, but can delaminate (especially in larger dimensions) with temperature extremes suffered even once. Cemented zero's are more durable in extreme excursions; but have limited UV transmittance, can have higher beam deviation, and are applicable only to very low power applications. For highest power and more adverse environments, an air-gap waveplate is preferred despite its higher cost.

Waveplates: Order number

NOTE: There are some unfortunate conventions in use for defining the "order" of a waveplate. A 0.25 wave retarder is generally considered a "zero order quarter wave plate". One might think that a 1.25 wave retarder would therefore be a "first order quarter wave plate". By one convention, though, it is considered a 3rd order (the 1st order being 0.25 and the 2nd being 0.75). One might wonder why the second order is not 0.5 waves. The answer is that a 0.5 wave retarder has a different function than a 0.25 or a 1.25 wave retarder. This answer is still insufficient because 0.25 and 0.75 wave plates produce opposite handedness, a functional difference that is significant in some situations. The situation becomes hopeless with retarders such as $(M+0.6)\lambda$.

To avoid misunderstanding, refer to retarders with <1λ retardation as zero order waveplates, and those with >1λ retardation as multiple order waveplates. When the order of retardation is critical, it is safest to specify the total retardation (i.e. 12.25λ) rather than "nth order".

Waveplates: Dual wavelength

A multiple order waveplate will exhibit useful values of retardation at more than one wavelength. Its retardation values will match particular targets more or less accurately according to the desired wavelengths and the multiple chosen. (A particularly nice combination, for example, is 6.000 waves at 632.8 nm which is also 7.2504 waves at 532 nm, achieved in a single quartz plate of about 420 microns thickness.) It is sometimes even possible to design a plate that is useful for three wavelengths. For some wavelength combinations, however, useful matches may not occur at a reasonable thickness ($0.1 \text{ mm} < t < \sim 4 \text{ mm}$). Good matches at popular wavelengths are listed in several vendor catalogs.

Waveplates: Achromatic

The retardation of quartz waveplates, expressed in length units such as nm or μm, varies only weakly with wavelength. A zero-order λ/2 waveplate at 500 nm wavelength has a retardation of 500/2 = 250 nm. At 400 nm wavelength, it still has a retardation of *approximately* 250 nm (actually 258.16 nm) – but at 400 nm that is not nearly a half-wave: (258.16 nm)/(400 nm/λ) = 0.6454 λ.

One way to achieve a broader useful spectral range is with an achromatic waveplate. This is analogous to an achromatic lens. It uses two materials of different retardances and different dispersions of retardance to balance each other. Magnesium fluoride and crystal quartz make one such useful combination.

Waveplates: Some Applications

DESIRED RESULT	METHOD
Change linear polarization to circular	Insert a $\lambda/4$ or $3\lambda/4$ plate with axis at 45° to the input polarization.
Change circular polarization to linear	Insert a $\lambda/4$ or $3\lambda/4$ plate with axis at 45° to the desired output polarization.
Rotate linear polarization to a more desirable orientation	Insert a $\lambda/2$ plate with axis at ½ the desired rotation.
Change the handedness of circular polarization	Insert a $\lambda/2$ plate – orientation unimportant.
Isolate (eliminate or re-route) a reflected beam	Insert a linear polarizer and $\lambda/4$ plate before the reflector. After the reflection, the second pass of the $\lambda/4$ plate restores linear polarization, this time orthogonal to the linear polarizer.
Isolate one of two wavelengths	Insert a linear polarizer and combination waveplate before reflector. $\delta_a = M\lambda_a$ or $(M+1/2)\lambda_a$ $\delta_b = (N+1/4)\lambda_b$ or $(N+3/4)\lambda_b$ Where M and N are integers.
Isolate two wavelengths simultaneously	Insert a linear polarizer and combination waveplate before reflector. $\delta_a = (M+1/4)\lambda_a$ or $(M+3/4)\lambda_a$ $\delta_b = (N+1/4)\lambda_b$ or $(N+3/4)\lambda_b$ Where M and N are integers.
Eliminate small amounts of ellipticity	Insert a full wave plate ($M\lambda$) in beam with axis at 45° to major axis of ellipse. Tilting it along its fast or slow axis will insert compensating ellipticity.
Compensate ellipticity due to prism or mirror reflections	Either use the approach above, or for more significant ellipticity insert a waveplate designed to compensate.
Eliminate variable ellipticity in articulated arms	Start with circular polarization. Attach $\lambda/4$ plate to input side of bend oriented to restore S polarization on reflector, and $\lambda/4$ plate to output of bend to circularize polarization between articulations.
Rotate one wavelength relative to another	Insert a combination waveplate: $\delta_a = M\lambda_a$ $\delta_b = (N+1/2)\lambda_b$ Where M and N are integers. Rotate plate one half the desired rotation of λ_b

Prism retarders

Total internal reflections – and oblique external reflections – cause a relative phase shift between the polarization components. Prism designs such as the Fresnel Rhomb, the Mooney Rhomb, and others exploit this to make $\lambda/4$ and $\lambda/2$ retarders. These prisms are zero-order, broadband, highly achromatic, and nearly invariant with temperature. Their primary disadvantage is their length, which can be twenty or more times their aperture.

Polymer retarders

A relatively new class of retarders is based on the birefringence of thin layers of polymers on a glass substrate. These devices are thin, zero-order, they offer an extremely wide field of view, and are available at reasonable cost in larger apertures than crystal or prism types. Achromatic designs can be made from stacks of such polymers. Liquid crystal designs can be spectrally tuned by applying a voltage differential. These technologies are only available from a few companies. Their primary disadvantages are in power density handling and limited temperature range. The power handling may be offset in some cases through the larger apertures available.

Rotators

A crystal quartz polarization rotator is made so that light propagates along the optic axis. Crystal quartz is optically active for light propagating along its axis, rotating the input polarization proportionally to the optical path. Its rotatory power is a strong function of wavelength, varying nearly as the inverse square of the wavelength. This property may be useful in such applications as wavelength selection and separation. A practical feature of rotators is that unlike half wave plates their rotation is independent of orientation. Another feature is that in contrast to quarter wave plates their action is undone upon a second transmission in the reverse direction. Thus it can be made cumulative upon repeated passes only in the same direction, as in ring lasers. They are usually single plate zero order devices and are therefore relatively insensitive to temperature. For infrared wavelengths the required thickness may be significant (25 mm or more), causing difficulties from an accumulation of material defects.

Birefringent filter plates (Lyot filters)

A birefringent filter plate is similar to an uncoated multiple order waveplate in construction. The optic axis is again in the plane of the surface, and at an angle to the plane of input polarization. For laser applications, the plate is tilted to Brewster's angle so that P polarization suffers no loss on the first surface. Upon reaching the rear surface, the polarization state has been transformed by the plate's retardance. For certain wavelengths, the polarization state is restored to P and again no loss is suffered. Slight rotation of the plate in its plane causes the projections and relative path length to change, thereby changing the wavelengths selected. Used intracavity, this can be sufficient to force single mode operation. Note that wavelength selectivity is a function of the entire system and not a property of the filter plate alone.

Birefringent filter stacks

When using birefringent filter plates intracavity, the laser medium may have a gain bandwidth that exceeds the free spectral range of a given plate. Another plate of ½ or ¼ the thickness of the first will quench alternate peaks. Units with 2, 3 or 4 plates of appropriate thickness are rigidly assembled to achieve the desired gain profile. Once again, the appropriate number and thicknesses of the plates are affected by the entire laser system. Custom assemblies are typically made to customer specifications. These excellent references are offered to assist in that design process:

Bloom	Journal of the Optical Society of America	V64	p447	1974
Evans	Journal of the Optical Society of America	V39	p229	1949
Walther & Hull	Applied Physics Letters	V17	p239	1970
Preuss & Gole	Applied Optics	V19#5	p702	1980
Wang & Yao	Applied Optics	V31#22	p4505	1992