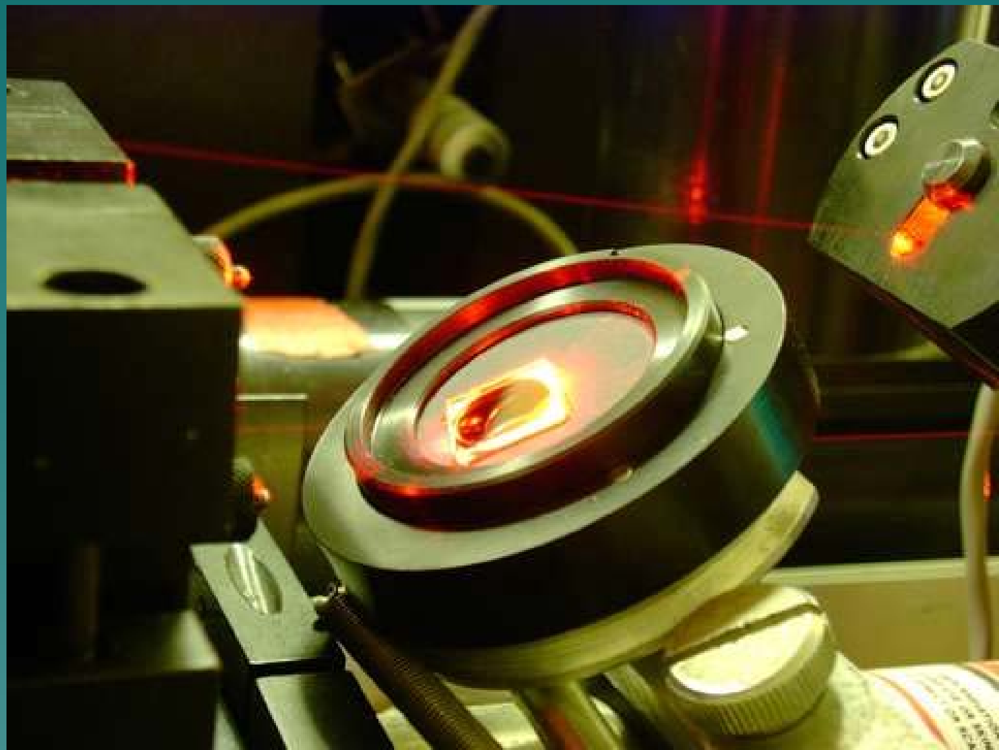
Birefringent Filter



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Phy 271, Quantum Optics, Spring 2004

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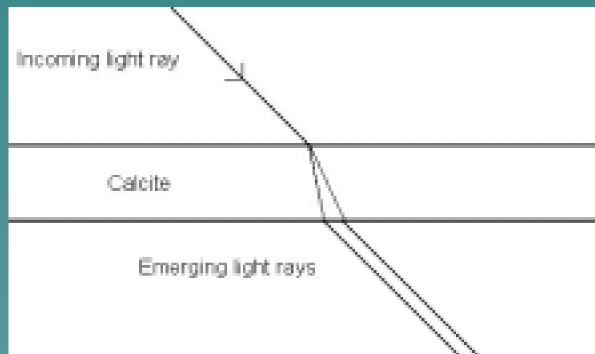
Outline

- Part 1: Birefringence
- Part 2: Birefringent Filter

Birefringent Crystal

- In 1969, Erasmus Bartholius described the double refraction observed in calcite.
- When a beam of ordinary unpolarized light is incident on a birefringent crystal, there will be, in addition to the reflected beam, two refracted beams in place of the usual single one observed.
- Other examples of birefringent crystals: quartz, ruby, ice, sapphire, sulfur……



Double/Triple Refraction

- In an anisotropic medium:

$$D_i = \sum_j \varepsilon_{ij} E_j$$

- The dielectric permittivity tensor ε_{ij} can be diagonalized so that:

$$D_1 = \varepsilon_{11} E_1 = \varepsilon_1 E_1, D_2 = \varepsilon_{22} E_2 = \varepsilon_2 E_2, D_3 = \varepsilon_{33} E_3 = \varepsilon_3 E_3.$$

- The corresponding refractive indices:

$$n_1 = (\frac{\varepsilon_1}{\varepsilon_0})^{1/2}, n_2 = (\frac{\varepsilon_2}{\varepsilon_0})^{1/2}, n_3 = (\frac{\varepsilon_3}{\varepsilon_0})^{1/2}$$

- Isotropic crystal (glass): $n_1 = n_2 = n_3$

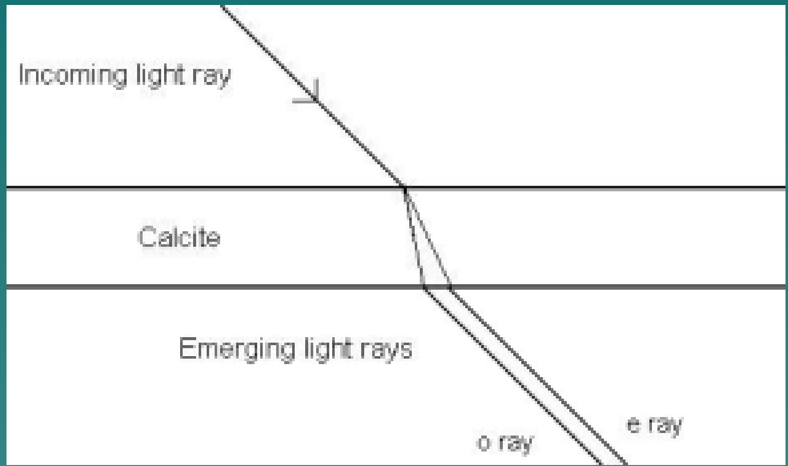
  Uniaxial crystal (calcite): $n_1 = n_2 \neq n_3$

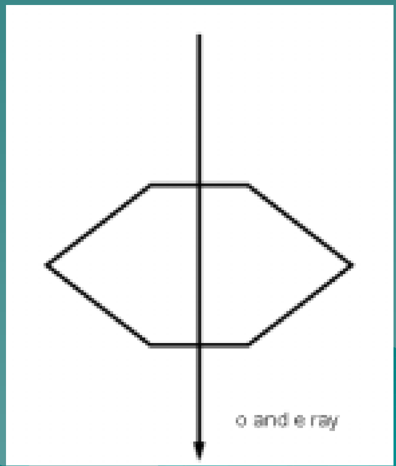
  Biaxial crystal: (sapphire) $n_1 \neq n_2 \neq n_3$

Ordinary Ray and Extraordinary Ray

- In uniaxial crystal, $n_1 = n_2 = n_o$ (ordinary refraction index); $n_3 = n_e$ (extraordinary refraction index)
- If $n_e < n_o$: negative crystal (eg. Calcite)

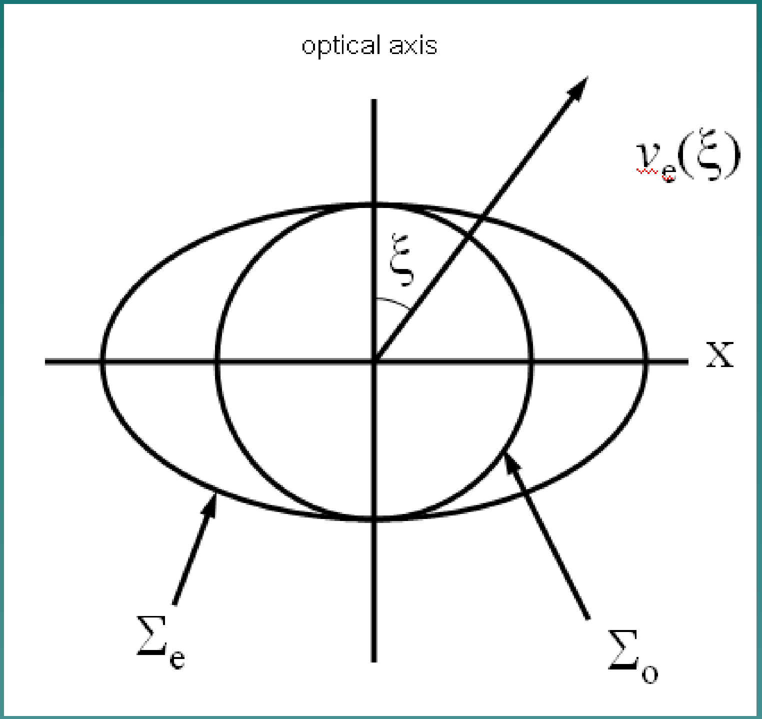
  If $n_o < n_e$: positive crystal (eg. Quartz)
- Define the two refracted rays as ordinary ray and extraordinary ray.
- There exists an "optical axis" in the crystal. If the incoming ray is parallel to that axis, o ray and e ray won't be separated.





Huygens' wavelets of o & e rays

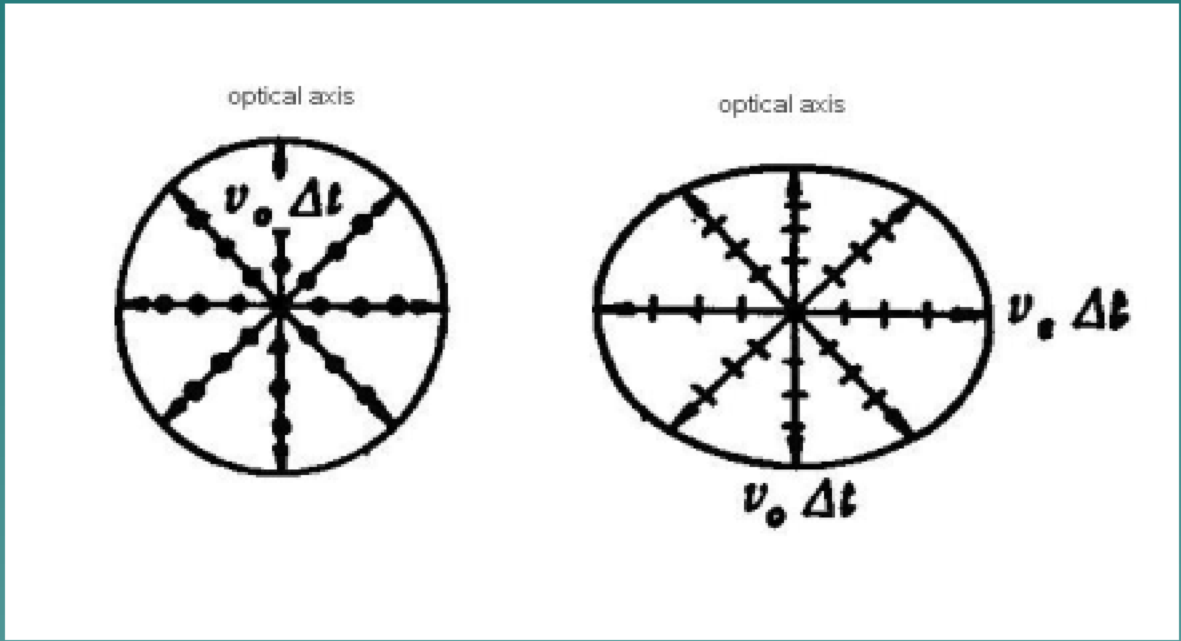
- O wavelets spread at a same speed in all different directions, forming a spherical wave front.
- E wavelets, however, spread at different speeds in different directions, and form an ellipsoidal wave front.
- The two wave fronts Σ_e & Σ_o are tangential at the direction of the optical axis. (same speed along optical axis)



Negative Crystal

Polarization of o&e rays

- Which part of the incoming light is o ray, which part is e ray?
- We can tell them from their directions of polarization.

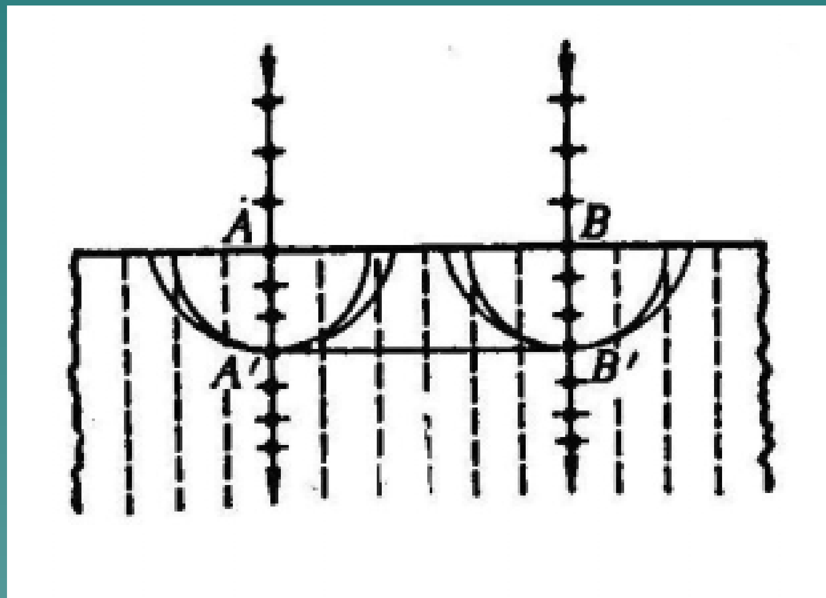


$\vec{E}_o \perp \vec{n}_{OA}$

$\vec{E}_e // \vec{n}_{OA}$

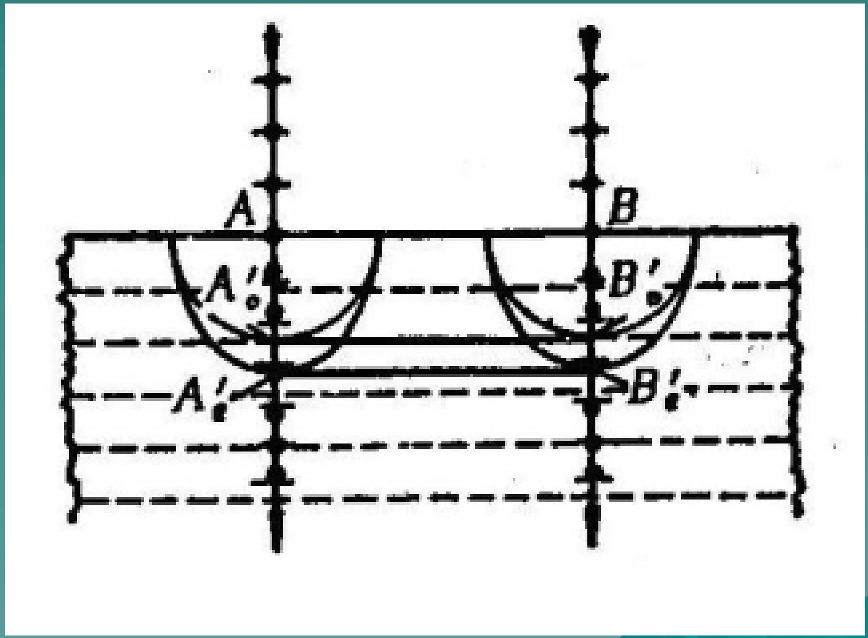
Example 1

- Incoming light normal to the surface.
- Optical axis normal to the surface.
- No double refraction.



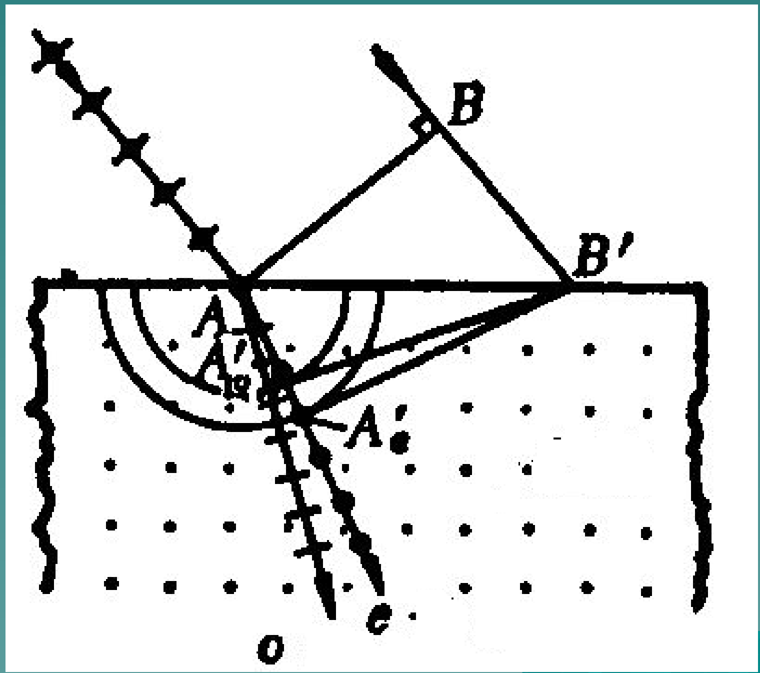
Example 2

- Incoming light normal to the surface
- Optical Axis parallel to the surface
- No explicit double refraction can be seen, but the e ray and the o ray will travel at different speed after passing the crystal.



Example 3

- Light incoming with an angle to the surface
- Optical axis normal to the incident plane
- Double refraction happen, but still in the incident plane



Conclusion of Part 1

- In the most general cases, e-ray can be out of the incident plate.
- Different travel speed of o-ray and e-ray will cause their phases to be different after passing through the birefringent crystal
- This property can be used in one part of the laser system: Birefringent Filter.

Part II Birefringent Filter

Outline:

- Two classical designs
- Lyot Filter
- Application

Two classical designs

- Lyot filters:

  1933, Bernard Lyot, French astronomer

  1938, Y. Ohman, constructed first Lyot filter for solar observations

- Solc filters:

  1955, Ivan Solc, Czech inventor of birefringent polarizing filters

Two Classical Designs

- Lyot filters:

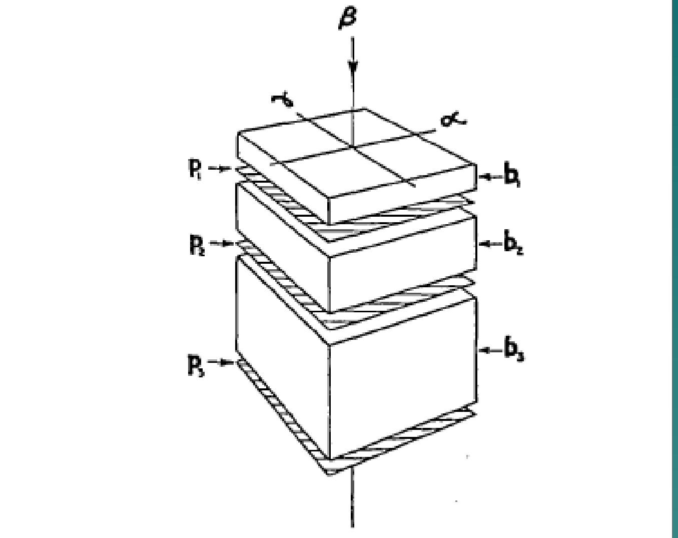
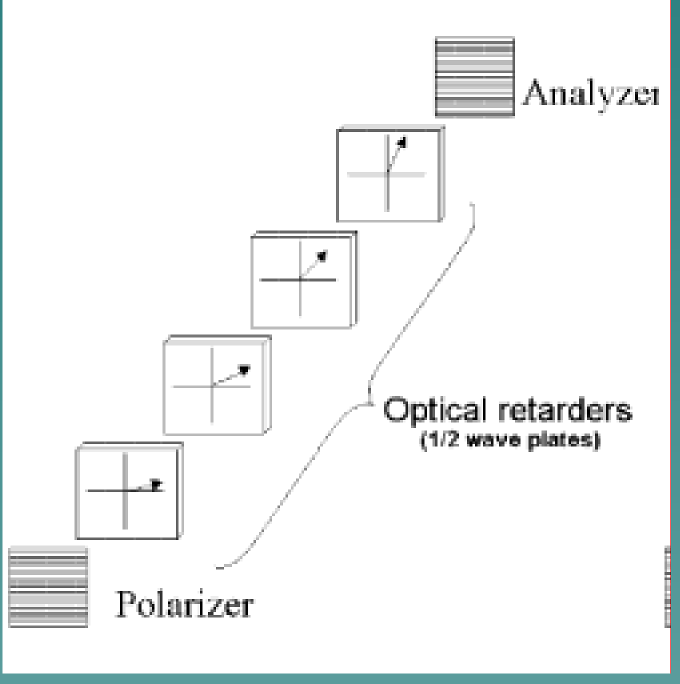
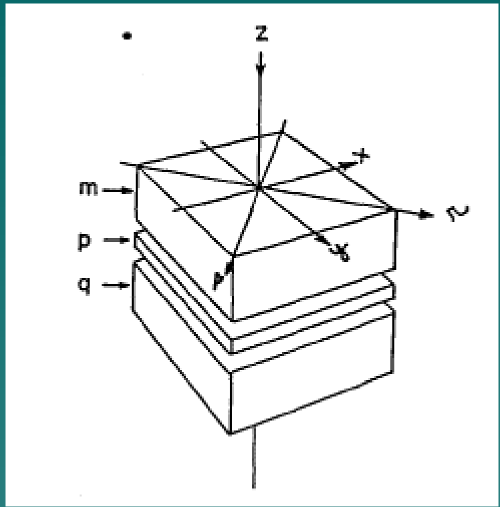


FIG. 1. Birefringent filter of three elements.

- Solc filters:





- Phase Shift:

$$\Delta\phi = \frac{2\pi}{\lambda}(n_e - n_o)d$$

- The intensity of the transmitted beam:

$$T = Cos^2(\frac{\Delta\phi}{2})$$

- Thickness $d_1 = d$
  Phase Shift
  Transmission

$$\Delta\phi_1 = \frac{2\pi}{\lambda}(n_e - n_o)d_1 = 2x$$

$$T_1 = Cos^2\left(\frac{\Delta\phi_1}{2}\right) = Cos^2 x$$

- Thickness $d_2 = 2d$
  Phase Shift
  Transmission

$$\Delta\phi_2 = 4x$$

$$T_2 = Cos^2 2x$$

- Transmission of the two plates:

$$T = T_1 T_2 = Cos^2 x Cos^2 2x$$

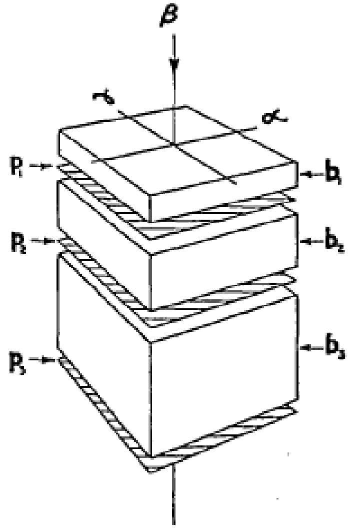


FIG. 1. Birefringent filter of three elements.

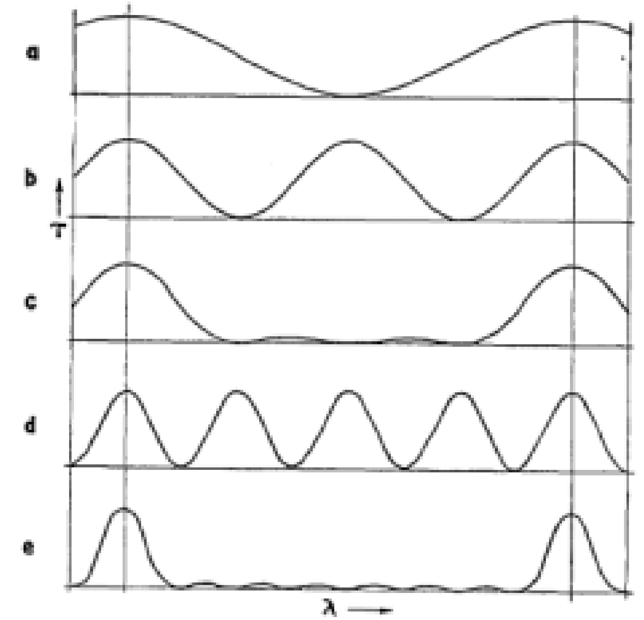
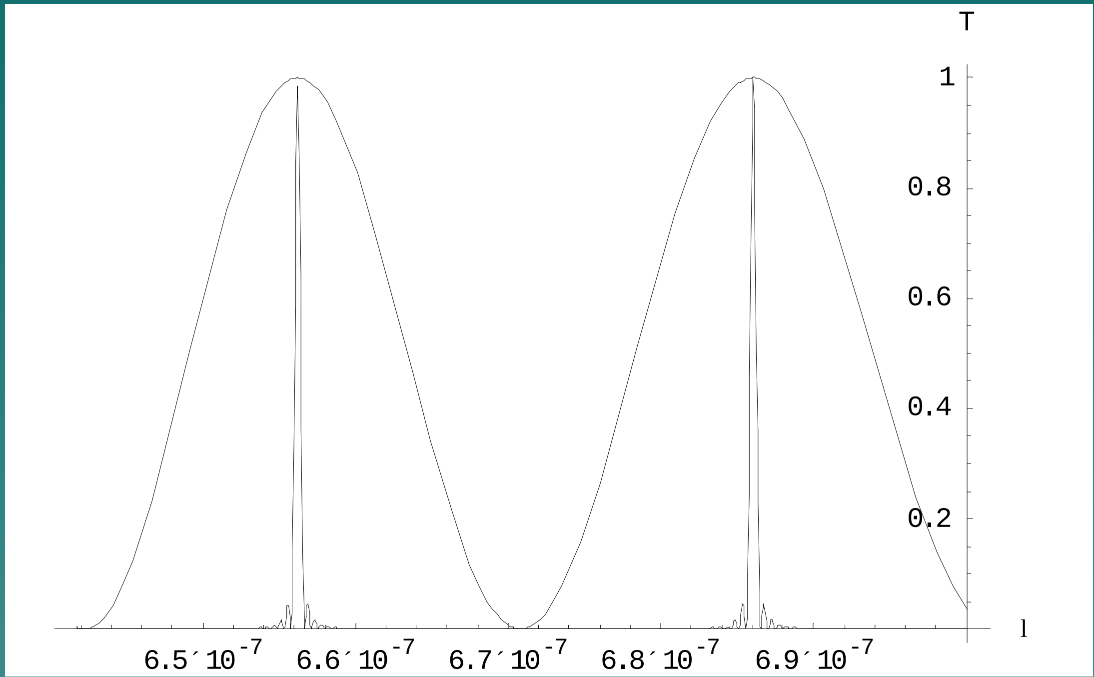


FIG. 2. Transmission curves for elements of Fig. 1. (a) b_1p_1; (b) b_2p_2; (c) $b_1p_1b_2p_2$; (d) b_3p_3; (e) $b_1p_1b_2p_2b_3p_3$.

$$\Delta\nu_{FSR} = \frac{c}{d(n_e - n_o)}$$

$$\Delta\nu_{FWHM} \approx \frac{c}{2^N d(n_e - n_o)}$$

$$T = T_1 T_2 \ldots T_{N-1} = Cos^2(x)Cos^2(2x)\ldots Cos^2(2^{N-1}x)$$



$$n_o = 1.5416$$

$$n_e = 1.5506$$

$$d_1 = 1.677\, mm$$

$$d_6 = 53.658\, mm$$

$$\lambda = 6563\, A$$

$$\Delta\lambda_{FWHM} = 4.1 A$$

A simple filter of six quartz plates, measure H_α line of hydrogen in the Solar Corona

Other Applications

- Birefringent tuning in dye lasers
- Generating Full color in a liquid Crystal Display

Summary

- Use birefringent media between polarizers
- Narrow transmission to a desired bandwidth
- Applications

Reference

- A. Yariv, P. Yeh, *Optical Waves in Crystals*. New York: Wiley, 1984
- J.W. Evans, "The Birefringent Filter," *Journal of the Optical Society of America*, V39, N3, 1949
- S. Saeed et al, " *A method of generating Full Color in a Liquid Crystal Display using Birefringent Filters*"