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Radiation from a Charge in a Gravitational Field

Amos Harpaz & Noam Soker

Department of Physics, University of Haifa at Oranim, Tivon 36006, ISRAEL
phr89ah@vmsa.technion.ac.il
soker@physics.technion.ac.il

ABASTRACT

When an electric charge is supported at rest in a static gravitational field, its electric field is not supported with the charge, and it falls freely in the gravitational field. Drawing the electric field lines continuously in time, we find that they always emerge from the charge, but the electric field is curved and there is a stress force between the freely falling (curved) field and the static charge. The charge radiates and the work done by the gravitational field to overcome the stress force is the source for the energy radiated by the supported (static) charge. *A static charge in a gravitational field radiates, as predicted by the principle of equivalence.* This mechanism is similar to the one applied to an electric charge accelerated in a free space. In this case, the electric field is not accelerated with the charge. The electric field is curved, and there is a stress force between the charge and its field. The work done in overcoming the stress force is the source of the energy radiated by the accelerated charge.

key words: Principle of Equivalence, Curved Electric Field

1. Introduction

The validity of the principle of equivalence (POE) to the case of radiation from an electric charge in a gravitational field (GF) is a long-standing problem (refs. [1], [2], and references cited therein). Specifically it is discussed in connection with two cases: (1) Does an electric charge, freely falling in a gravitational field radiate? (2) Does a charge supported at rest in a gravitational field radiate? Using plainly the POE one may conclude that a freely falling charge in a GF will not radiate because its situation is equivalent to that of a free charge in empty space, and a charge supported at rest in a GF (chracterized by an acceleration, g), will radiate because its situation is equivalent to that of a charge accelerated in free space with an acceleration g. The common approach in the physical society is the opposite one - it is believed that a static observer in a gravitational field will find that a freely falling charge in a GF does radiate, while a charge supported at rest in a GF does not radiate [1]. It is also concluded that the validity of the POE is limited, and it is not a general principle.

However, this approach led to several contradictions, which in turn, led people to conclude that the ability to observe a radiation depends on the relative accelration between the charge and the observer: an observer falling freely in parallel to a freely falling charge will not observe radiation, while a static observer in the same field will observe radiation. In the same way, a static observer located in a lab where a charge is supported at rest in a GF will not observe radiation, while a freely falling observer, passing by the same charge, will observe radiation (ref [1], pp 218).

The electromagnetic radiation is defined as a relative phenomenon, that depends on the relative acceleration between the observer and the charge. In the following, we analyze the process that leads to the creation of radiation. We demand that radiation as a process of energy transfer is a physical event (which is an objective phenomenon), and we come to the conclusion that a freely falling charge does not radiate, and a charge supported at rest in a GF does radiate. These conclusions are in accord with the POE.

In §2 we present the problems concerned with the energy carried by the radiation and the non-existence of the radiation reaction force in certain cases. In §3 we present a freely falling system of reference as the preferred system to work in, and in §4 we calculate the energy carried by the radiation from the supported charge in a GF, using the work done to overcome the stress force of the field. We conclude in §5.

2. The Problem

Treating radiation as a relative phenomenon leads to contradictions, because radiation transffers energy from one system to another. If the energy carried by the radiation is absorbed in some system and causes there a certain change, like an excitation of a higher energy level, this absorption must be observed by any observer, even if he does not have the means to observe directly the flow of the energy. If a static observer observes radiation from a freely falling charge, he also must be able to identify the source of the energy for this radiation. An observer falling freely in parallel to the charge, must observe this source of energy, even if he cannot observe directly the radiation that carries the energy. Similar contradictions arise for the case of a charge supported at rest in a GF, where a static observer does not observe the radiation, and a freely falling observer does. We find that treating radiation as a relative phenomenon leads to contradictions concerning both the source of the energy carried by the radiation, and the phenomena that may be caused in absorbing the radiation. The emmitance of radiation is a physical event that cannot be transformed away by a coordinate transformation (see [3]).

There is another difficulty with the common approach - it is generally believed that when radiation is created by an accelration, a radiation reaction force is created, which contradicts the force that creates the acceleration. The work done by the external force to overcome the reaction force, is considered as the source of the energy carried by the radiation. However, when the velocity of the charge is low ($v \ll c$), the radiation is emitted mainly in a plane which is perpendicular to the direction of motion ([4] pp. 663 and [7]). No momentum is imparted to the accelerated charge by the radiation, and no radiation reaction force exists [6]. The source of the energy carried by the radiation should be looked for elsewhere.

3. A Freely Falling System of Reference

According to Jackson [4], a radiation exists whenever an electric charge is accelerated. However, a question should be raised to what system of reference this acceleration is related. Without stating it explicitly, Jackson refers to an inertial system of reference. Ordinarily, when general relativity is considered, the inertial system of reference should be replaced by a freely falling system of reference, characterized by a set of geodesics that covers this system. The "absolute acceleration" of a charge supported at rest in a gravitational field does not vanish, where absolute acceleration is the covariant time derivative of the four velocity of the charged particle. A general relativistic criterion for the existence of radiation, is the non-vanishing of the absolute acceleration. A regular acceleration is related to the system of geodesics that covers the local space. The preferred system of reference to work in is the system characterized by local geodesics, and freely falling objects - particles and fields - follow these geodesics. The electric field of a charge is an independent physical entity. Once it is induced on space, its behaviour is determined by the properties of space. When the charge is accelerated by an external (non-gravitational) force, the electric field of the charge is not accelerated, and a relative acceleration exists between the charge and its field. As was shown by Fulton and Rohrlich [6], the electric field of the charge is curved. There is a stress force between the charge and its curved field, and, as shown in [5], this force gives rise to radiation.

A neutral particle and a similar charged particle will fall with the same acceleration. It was shown that the key feature for the creation of radiation is not the relative acceleration between the charge and the observer, but rather the relative acceleration between the charge and its own electric field.

A freely falling charge in a uniform GF follows a geodetic line in this system, and it is not subject to any external force. The electric field of the charge follows similar geodesics. The charge and its field both are located in the same frame of reference, and in that frame their relative situation is similar to the one existing between a static charge and its field in a free space. No relative acceleration exists between the charge and its electric field, and we conclude that a freely falling charge does not radiate.

The creation of radiation by a uniformly accelrated charge was analyzed ([5],[7]), and it was shown that the electric field of the accelrated charge is curved, and there exists a stress force between the charge and its (curved) field. The stress force F_s, is given by: $F_s = E^2/4\pi R_c$, where R_c is the radius of curvature, whose value close to the point charge is: $R_c = c^2/(a \sin\theta)$, where a is the acceleration, and θ is the angle between the direction of the acceleration and the initial direction of the field. By calculating the stress force and the work performed to overcome this force, it is shown that for a uniformly accelerated charge and for very low velocities, the power supplied by the accelerating (external) force to overcome the the stress force, equals the power radiated by the accelerated charge according to Larmor formula [5]. It is concluded that the work done in overcoming the stress force is the source of the energy carried by the radiation, and this work is done by the external force that imparts the acceleration to the charge, in addition to the work it does in creating the kinetic energy of the charge.

4. A Charge Supported in a Homogenous Gravitaional Field

The electric field of a charge supported at rest in the lab against GF seems static, but it is not. The electric field, which is an independent physical entity, is not supported with the charge, and it falls freely in the gravitaional field. There is a relative acceleration between the charge and its electric field, the field is curved (both in the lab system and in the freely falling system), and a stress force exists between the charge and its field. The (freely falling) electric field follows the system of refernec characterized by the geodesics. To calculate the fields of the supported charge in the freely falling geodetic system, we adopt the results given by Rohrlich [8]. Let us assign primes to the variables calculated in the freely falling system, S'.

According to Rohrlich, the field equations of the supported charge, in S' are:

$$E'_\rho = \frac{8e\alpha^2\rho' z'}{\xi'^3} \tag{1}$$

$$E'_z = \frac{-4e\alpha^2}{\xi'^3}[z'^2_p + \rho'^2 - z'^2] \tag{2}$$

$$B'_\phi = \frac{8e\alpha^2\rho' ct'}{\xi'^3} \tag{3}$$

$$E'_\phi = B'_\rho = B'_z = 0 \tag{4}$$

where

$$\xi'^2 = [z'^2_p - \rho'^2 - z'^2]^2 + (2\alpha\rho')^2 \tag{5}$$

where we used for the particle location: $z'^2_p = \alpha^2 + (ct')^2$, and $\alpha = c^2/g$ is the particle location at $t' = 0$. Certainly, the Poynting vector does not vanish in this system.

Using transformations given by Rohrlich [8] we can calculate the electromagnetic fields in the lab system. It follows (as can be expected), that the magnetic filed vanishes in this system, and the Poynting vector vanishes as well. This led Rohrlich to conclude that a charge supported at rest in a gravitational field does not radiate. However, we know that a Poynting vector is not an invariant [9], and we demand that the existence of radiation must be represented by a non-vanishing Poynting vector in the frame of reference characterized by the local geodesics, S', and *in this system the Poynting vector does not vanish.*

The situation is not static, and the electric field exists in a steady state. The pattern of the electric field remains constant, but the field itself does not. As we emphasized earlier, the electric field is a property of the space on which it was induced, and its behaviour is determined by this space. The electric field is detached from the supported charge, and it is not supported against gravity as the charge is. Hence the electric field falls in a free fall, and it has an acceleration g relative to the supported charge. In the freely falling system, which also has an acceleration g relative to the supported charge, the charge is accelerated upward with an acceleration g.

It was also shown by Rohrlich [8], that in the system characterized by the geodesics, a magnetic field does exist, and it comes out that the Poynting vector does not vanish. We conclude that a charge supported at rest in a gravitational field does radiate. In Figure 1 we present the curved elcetric field lines calculated for an electric charge supported at rest in a uniform homogenous GF, characterized by an acceleration g. The field is similar to the one calculated by Singal [10], for a uniformly accelerated charge.

The curved electric field gives rise to a stress force, and we calculate the work done in overcoming this force in a way similar to that used in [5] for the uniformly

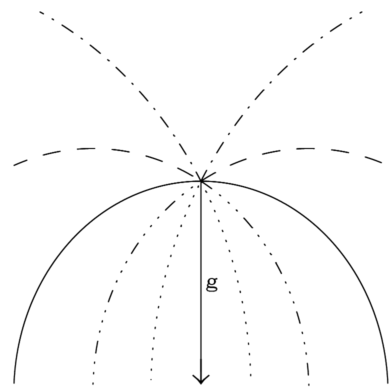

Figure 1: A curved electric field of a charge supported in a uniform homogenous gravitational field.

accelerated charge, where the calculations are carried now in the (flat) freely falling system of reference.

For the sake of convenience we ommit now the primes. We shall sum over the stress force of the field, f_s, and calculate the work done against this force. In order to sum over f_s, we have to integrate over a sphere whose center is located on the charge. Naturally, such an integration involves a divergence (at the center). To avoid such a divergence, we take as the lower limit of the integration a small distance from the center, $r = c\Delta t$, (where Δt is infinitsimal), and later we demand that $\Delta t \to 0$. We calculate the work done by the stress force in the volume defined by $c\Delta t < r < r_{up}$, where $c^2/g \gg r_{up} \gg c\Delta t$. These calculations are performed in the geodetic system (the system of reference defined by the geodesics), which momentarily coincides with the frame of reference of the charge at the charge location, at time $t = 0$.

The force per unit volume due to the electric stress is $f_s = E^2/(4\pi R_c)$, where E is the electric field, and R_c is the radius of curvature of the field lines. The radius of curvature is: $R_c \simeq c^2/(g \sin\theta)$, where θ is the angle between the initial direction of the electric field line and the direction of the acceleration g of the charge, as seen in the geodetic system. The force per unit volume due to the electric stress is

$$f_s(r) = \frac{E^2(r)}{4\pi R_c} = \frac{g\sin\theta}{c^2}\frac{e^2}{4\pi r^4}, \qquad (6)$$

where in the second equality we have substituted for the electric field $E = e/r^2$, which is a good approximation in weak graviatational fields [8]. The stress force is perpendicular to the direction of the field lines, so that the component of the stress force along the acceleration g is $-f_s(r)\sin\phi$, where ϕ is the angle between the local field line and the acceleration. For very short intervals (where the direction of the field lines did not change much from their original direction) $\phi \sim \theta$, and we can write: $-f_s(r)\sin\phi \simeq -f_s(r)\sin\theta = \frac{-g\sin^2\theta}{c^2}\frac{e^2}{4\pi r^4}$. The dependence of this force on θ is similar to the dependence of the radiation distribution of an accelerated charge at zero velocity on θ. Integration of this force over a spherical shell extending from $r = c\Delta t$ to $r_{\rm up}$ (where $c^2/g \gg r_{\rm up} \gg c\Delta t$), yields the total force due to the stress

$$F_s(t) = 2\pi\int_{c\Delta t}^{r_{\rm up}} r^2 dr \int_0^\pi \sin\theta d\theta[-f_s(r)\sin\theta] = -\frac{2}{3}\frac{g}{c^2}\frac{e^2}{c\Delta t}\left(1-\frac{c\Delta t}{r_{\rm up}}\right). \quad (7)$$

Clearly the second term in the parenthesis can be neglected. The power created in overcoming the electric stress force is:

$$P_s = -F_s v = -F_s g\Delta t, \quad (8)$$

where we substituted $v = g\Delta t$, and v is the charge velocity in the geodetic system, at time $t = \Delta t$. Substituting for F_s we obtain (at the limit $\Delta t \rightarrow 0$):

$$P_s(t) = \frac{2}{3}\frac{g^2 e^2}{c^3} \quad (9)$$

which is equivalent to the power radiated by an accelerated charged particle (Larmor formula), where the acceleration is replaced by g. Thus we find that the work done against the stress force, supplies the energy carried by the radiation.

Who is performing this work or, what is the source of the energy of the radiation?

The charge is supported by a solid object, which is static in the GF. This solid objet must be rigidly connected to the source of the GF. Otherwise, it will fall in the GF, together with the "supported" charge. This means that actually, the supporting object is part of the object that creates the GF.

As we already mentioned, the charge is static and no work is done by the GF that acts on the charge. However, the electric field of the charge is not static, and it falls in a free fall in the GF. If there was no interaction between the electric field and the charge that induced the field, the field would have follow a geodetic line and no work would have been needed to keep it following the geodetic line. But the field is curved, and a stress force is implied. The interaction between the curved field and the supported charge creates a force that contradicts the free fall. In order to overcome this force and cause the electric field to follow the geodetic lines, a work should be done on the electric field, and this work is done by the GF. This work is the source of the energy carried by the radiation. It comes out that the energy carried away by the radiation is supplied by the GF, that loses this energy.

5. Conclusions

It is found that the “naive” conclusion from the principle of equivalence - that a freely falling charge does not radiate, and a charge supported at rest in a gravitational field does radiate - is a correct conclusion, and one should look for rdiation whenever a relative acceleration exists between an electric charge and its electric field. The electric field which falls freely in the gravitational field is accelerated relative to the static charge. The field is curved, and the work done in overcoming the stress force created in the curved field, is the source of the energy carried by the radiation. This work is done by the gravitational field on the electric field, and the energy carried by the radiation is created in the expence of the gravitational energy of the system.

Motz [11] suggested that the huge radiation emerging from quasars may be created by charges located in the strong gravitational fields close to the surface of the quasars. Although the current expalnation for this phenomenon is different, radiation from charges located in strong gravitational fields can still play a role in certain cosmological phenomena.

We conclude that we find both the mechanism that creates the radiation emitted by a charge supported in a GF, and the source of the energy carried by this radiation.

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