

# Pressure Vessel Design

The Direct Route

Josef L. Zeman

In cooperation with  
Franz Rauscher • Sebastian Schindler

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**Josef L. Zeman**  
*In cooperation with*  
*Franz Rauscher • Sebastian Schindler*



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*It is the mark of an instructed mind to rest assured  
with that degree of precision that the nature of the subject admits,  
and not seek exactness  
when only an approximation of the truth is possible.  
Aristotle*

## Foreword

Twelve years after the first draft on the new approach in Design by Analysis was published by CEN TC 54 WG C, seven years after the adoption of the legal basis for its usage in the design of pressure vessels, the so-called Pressure Equipment Directive (PED) [1], five years after the issue of the Design-by-Analysis Manual [3], a handbook based on the draft of this new approach, five years after the coming into force of the PED and the approval of the harmonized standard EN 13445 Parts 1 through 5 [2] on unfired pressure vessels, seems to be the right time for a comprehensive, consolidated compendium related to this new approach, which is now called Direct Route in Design by Analysis, and which is laid down in the normative Annex B of EN 13445: Unfired Pressure Vessels, Part 3: Design.

This book had already been planned long ago, as a continuation of my basic textbook on the fundamental principles of the structural design of pressure vessels [4], in German. Discussions at international conferences, experience in international research groups, and the numerous publications on this topic [5–22], have convinced me that a publication in English is the best vehicle to achieve the desired objective – the promotion of this new and promising approach in the design of pressure vessel components.

Most admissibility checks of the structural design of pressure vessels are based on the concept of Design by Formulae (DBF), which involves relatively simple calculations to arrive at required thicknesses of components, or cross-sectional dimensions, via more or less simple formulae or diagrams, and by usage of the concept of the nominal design stress, also called allowable stress, allowable working stress, or design stress intensity. Most of the space of design codes is devoted to this concept, and this concept is still part of the culture and state of the art in pressure vessel structural design. The great benefit of the DBF approach is still its simplicity, only in the recent past the formulae and calculations in DBF have become more and more elaborate, pretending accuracy that is often not there.

The DBF approach is limited to specific geometries and geometric details, and involves strict adherence to specific rules delineated in the standards, adherence to strict restrictions with regard to the range of validity of the formulae, and strict adherence to the relevant material, manufacturing, and testing requirements. If, for example, specified manufacturing tolerances, which are usually based just on good workmanship concepts, are exceeded, this approach cannot be used without additional proof of the admissibility, and this proof is, in general, not possible within the approach.

The DBF approach is also limited with regard to the actions for which formulae are provided. Often awkward and inefficient rules have to be applied to incorporate e.g. environmental actions, agreed upon rules based on the state of the art in other fields of engineering technology.

In the determination of the nominal design stress, the DBF approach employs only one safety factor for normal operating load cases and one for testing load cases, this approach lacks, therefore, the flexibility to adjust safety margins according to differences in the dispersion of actions, the likelihood of combinations of actions, the consequences of failure, and the uncertainty of the analysis.

The Direct Route in Design by Analysis, on the other hand, is very flexible, allows for any combination of actions, any geometries and geometrical details, addresses directly the creativity of the designer, and is, possibly, restricted only by material and non-destructive testing requirements.

This book is intended as a support of the Direct Route in Design by Analysis as laid down in EN 13445, Part 3: Design, Annex B. It is intended as a reference book for this new approach, by providing background information on the underlying principles, basic ideas, and presuppositions. Examples are included to familiarize the reader with the details of this approach, but also to highlight problems, solutions, and information gained by means of the diverse procedures used.

This book is intended as a guidebook for the Direct Route in Design by Analysis: This Direct Route is new, very general, with very wide application range; terms and concepts are used in a very general context; new ideas, new terms, and definitions have been introduced, old and familiar designations used in a new, unfamiliar sense, with more general definitions – a guidebook in this new territory of design of pressure vessels is considered essential.

Design check specific chapters include introductory sections, with a description of the design check's background and associated phenomena, as a guide in the execution of the design check's investigations.

These design check specific chapters are concluded by dedicated, typical examples, which are intended as illustrations of the design checks' principles and application rules, to elucidate their applications, but also to indicate the possibilities of knowledge gain on the design and its behaviour.

Being dedicated to the advancement of the Direct Route in Design by Analysis as laid down in Annex B of EN 13445-3 [2], the scope of this book is limited to that of the standard:

Design, construction, inspection, and testing of unfired pressure vessels made of sufficiently ductile steels and steel castings.

The definition of pressure vessels is the one of the PED [1], encompassing vessels designed and built to contain fluids under pressure with a maximum allowable pressure PS greater than 0.5 bar. Excluded from the scope are, for example, transportable equipment, pipelines to and from installations, and items specifically designed for nuclear use, the failure of which may cause a release of radioactivity.

Sufficient ductility is already defined, as a general rule, in the PED, and detailed in Part 2 of the EN 13445.

The minimum elongation after fracture in any direction shall be  $\geq 14\%$ . The specified minimum impact energy measured on a Charpy-V-notch impact test specimen (EN 10045-1) shall be  $\geq 27$  J for ferritic and 1.5–5% Ni-alloy-steels, and  $\geq 40$  J for steels of material groups 8, 9.3, and 10, at a test temperature in accordance with Annex B (requirements for prevention of brittle fracture) of EN 13445-2, but not higher than 20°C. For the determination of this test temperature for the impact testing of base metals, of heat-affected zones (including the fusion line), and of weld metals, Annex B of EN 13445-2 provides two alternatives, one based on long-standing practice and one on fracture mechanics. This test temperature is lower, equal to, or higher than the minimum metal temperature, depending on the material, the relevant thickness, the stress level, and whether welds have been post weld heat treated or not.

The first method for the determination of this test temperature has been developed from operating experience, it is applicable to all metallic materials in the scope of this EN 13445-2, but is limited to material group-related thicknesses.

The second method is based on fracture mechanics and operating experience. This method encompasses a wider range of thicknesses than the first method, but is restricted to ferritic steels – C, C-Mn, and fine grain steels – and 1.5–5% Ni-alloy-steels, all with a specified minimum yield strength of 460 MPa maximum.

As an alternative, in the third method, requirements for a (pure) fracture mechanics analysis are given in the Annex B of EN 13445-2. This method is quite general, is applicable also in cases not covered by any of the other two methods, and also for deviations from the requirements of the other two methods.

Furthermore, it is required that the chemical composition of steels intended for welding shall be limited to specified (material group dependent) values, and if subsequent manufacturing processes, including welding, may affect base material properties, the changes in material properties are to be taken into account in the

specification of the base material requirements, if the changes are to be considered detrimental with regard to safety.

The book is, like the standard, also limited with regard to the rate of change of actions – slow enough such that velocity dependence of material properties can be ignored.

Being dedicated to the Direct Route in Design by Analysis as specified in EN 13445-3, the book deals solely with the design proper, as part of the manufacturing, inspection, and testing process, before vessels are placed on the market and put into service – the Direct Route to Design by Analysis is not intended for in-service analyses.

The Direct Route in Design by Analysis is part of EN 13445-3, which in turn is one part of a series of five parts, all dedicated to various aspects of design, construction, inspection, and testing of unfired pressure vessels. Therefore, the book is based on the presupposition that all relevant requirements of all other parts apply.

Usage of the Direct Route in Design by Analysis requires, for the time being, the involvement of an appropriate independent body:

*Due to the advanced methods applied, until sufficient in-house experience can be demonstrated, the involvement of an independent body, appropriately qualified in the field of DBA, is required in the assessment of the design (calculations) and the potential definition of particular NDT requirements (EN 13445-3).*

No standard and no handbook can encompass all the details encountered in practical applications. This book is in this respect no exception, but the overall objective was to present, as far as possible, all the technical background information to allow for the required interpretation in all the cases not dealt with in detail.

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## On the Use of this Book

In many discussions, with colleagues and with students, I realized that many misunderstandings and problems are related to different, or just fuzzy usage of designations and definitions of terms. Even frequently used terms, like shakedown, structural stress/strain, are used and defined differently in the very same context by experts of different “schools”.

At the same time, terms and definitions that are appropriate for simple calculations of simple structures under simple loads are carried over to modern detailed analyses of complex structures subject to complex actions, where these outdated terms and definitions are not appropriate (anymore), because they do not possess the required flexibility.

Therefore, emphasis is on definitions and on usage of designations as general and as clear as possible without a too strong interference with easy reading. To keep the main text clear, most of the terms and definitions are put together in Section 2.2. A definition can then be found via the subject index, where the relevant number of the page containing the definition is in bold face.

Implicit definitions and definitions of more local usage are given in the text, with the designations in bold face.

Direct, unchanged or only editorially changed, citations from legal documents and from harmonized standards are in italics, followed by an abbreviated designation of the source.

The whole book follows strictly the Pressure Equipment Directive (PED) [1] and Annex B of EN 13445-3 [2], without any intentional deviation, but, in cases of doubt, it is the text of the PED and of this EN 13445-3 that is decisive. There are a few cases where the text of Annex B of EN 13445 is not explicit enough, where sufficient understanding of the basic ideas is required to obtain correct results. A typical example is the design check for deformation weakening GPD load cases, where the ideas, specifications and requirements of different design checks are to be combined to obtain the desired results. The relevant chapters here have already been adapted, such that this “interpretation” is not required. Deviations from the fatigue clause in EN 13445-3, Clause 18, and complementary requirements to Clause 18, indicated in the standard only as possible alternatives, are clearly stated as recommended alternatives.



To keep load case-specific chapters and sub-chapters self-contained, repetitions are unavoidable, are even considered helpful.

Design check-related chapters are complemented by examples, and all examples are collected in a separate annex, to make the main text better readable and more compact. The numbering in each of the examples differs from the one in the main text, inasmuch as it has been chosen to resemble those used in actual admissibility checks: The first digit refers to the design check and the second to the load case. The style chosen for many of the examples is that suitable for a design report. The development of finite element method (FEM) software had a distressing side effect: FEM input listings have become a rare species, despite their advantages in reporting, in following input changes, and in checking of results. To provide a good example, most examples are complemented by input listings. But the listings are included also to allow for easy experimentation with the mathematical (FEM) models – the surest way to the understanding of the Direct Route is to apply it.

To ease the usage for German speakers, translations of designations into German are given (in italics) in the subject index.

## Chapter 1

# Introduction

Within the European Union the coming into force of common national laws in the pressure equipment field, all based on the very same legal act of the European Parliament and the European Council, the so-called Pressure Equipment Directive (PED) [1] created a serious need for corresponding complementary standards harmonized at a European level, adopted by the European Committee for Standardization (CEN) at the request of the European Commission.

The PED requires certain types of pressure equipment brought onto the European Market to comply with the so-called essential safety requirements (ESR), in order to ensure the required safety of pressure equipment. Compliance with the requirements of a relevant harmonized standard provides for a product the presumption of conformity with the ESR that the standard addresses. The harmonized standards need not be used, they are only one means of demonstrating compliance with the ESR of the PED, but they are the only means that provide directly the presumption of conformity with the ESR.

This need for a harmonized standard created a unique chance and challenge: The chance for a new approach to Design by Analysis (DBA), using all the knowledge in engineering mechanics – theoretical as well as practical – and all the experience with numerical methods and with commercially available hard- and software, used in simulations of the behaviour of structures under various actions.

Work on this new approach, called Direct Route in Design by Analysis (DBA-DR), started in 1992, the first sketch of a draft dates October 1992. The draft went through (informal) enquiries repeatedly and formed the basis of an EU-research project, which rendered proposals for changes and a handbook [3] with numerous examples, input listings, etc.

This new approach, DBA-DR, is now laid down in a normative annex, Annex B of Part 3: Design, of the harmonized standard EN 13445: Unfired Pressure Vessels [2]. The relevant parts of this standard were approved on 23 May 2002: Part 1: General, Part 2: Materials, Part 3: Design, Part 4: Fabrication, Part 5: Inspection and Testing.

Since then, this new approach has been used in numerous industrial applications and research projects; numerous papers deal with this approach and are dedicated to it directly [5–21].

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Industrial applications and investigated examples have shown that this Direct Route is a major step forward in DBA. This new approach is sound, gives the designer (and the user) not only the presumption of conformity to the ESR of the PED, but also, at the same time, much insight into the behaviour of components and the safety margins against failure modes.

Furthermore, the DBA-DR has shown to be of great help in the determination of safety-critical points and of critical actions. Therefore, this new approach can lead, and has already led, to design improvements and to improved in-service inspection periods and dedicated in-service inspection procedures.

Applications have also pointed out one basic problem: The growing gap between analysis software capabilities on one side and the expertise of the users on the other.

Some of the software tools are so easy to use that little thinking is required to obtain fantastically looking, colourful pictures of stress distributions, and many users tend to believe their results are correct because they look so good and convincing. Wrong results look usually as good as correct ones.

The Direct Route has made DBA easier to use in the design process, more straightforward and logical in the design decisions, but technical knowledge of engineering principles and careful analysis of results is still a prerequisite of good workmanship. It is still the analyst who has to decide on the model, the geometry, and the boundary conditions. It is practically always necessary to use part models, and the decision on the boundaries and the boundary conditions is a very critical one, requiring thought, and, possibly, additional investigations.

A good DBA still requires from the analyst

- good workmanship with regard to the tools used,
- knowledge of the basic engineering principles and the phenomena involved,
- fantasy and creativity with regard to the selection of the models used,
- fair knowledge of the legal requirements pertaining to design,
- fair knowledge of manufacturing and testing procedures, and especially
- extreme carefulness in each step, from the design specification to the design report.

## Chapter 2

# General

### 2.1. General on the Direct Route in Design by Analysis

The direct route in design by analysis (DBA-DR) is a modern, advanced approach to check the admissibility of pressure vessel designs. This approach is included, as a normative annex, in the Harmonized Standard EN 13445, and, therefore, conformity of a design to the requirements of this approach as specified in the standard implies presumption of conformity to the relevant essential safety requirements (ESR) of Annex I of the pressure equipment directive (PED) [1].

This approach may be used

- as an alternative to the “usual” design by formulae (DBF) route,
- as a complement to the DBF route, for
  - cases not covered by this DBF route,
  - cases involving superposition of actions, e.g. wind, snow, earthquake, piping forces, forces imposed by attached equipment,
  - cases where DBA is explicitly required, e.g. by authorities in major hazard, or environmentally sensitive situations, and
  - cases where manufacturing tolerances specified in the standard are exceeded.

As an alternative to the DBF route, DBA-DR may be used even in cases within the scope of the DBF route and within the scope of the formulae specified there. As a complement to the DBF route, DBA-DR may be used in all cases outside the scope of DBF formulae, and in cases not covered in the DBF approach. It may be used in cases of superposition of various actions, where the DBF route is not specific enough or leads to overly conservative results. The DBA-DR approach gives much insight into the behaviour of components and their safety, and shows critical design details and safety critical points, and, therefore, cases where authorities require (additionally) a DBA-DR investigation are not uncommon.

If specified tolerance limits are exceeded while manufacturing, the DBF route must not be used without additional proof of admissibility of the deviation – DBA-DR is a very convenient, admissible tool in such cases.

As a modern, efficient method for designing reliable pressure vessels for longer service, the DBA-DR takes into account that the “usual” materials in pressure vessel technology are ductile, that plastic flow does not necessarily limit the usability, and

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that onset of plastic flow is not a failure mode. Limited plastic flow in testing and in normal operating load cases is admissible, even if it may occur repeatedly. It is taken into account explicitly in constitutive laws of design models used, and in the plasticity correction within the check against cyclic fatigue damage.

Because of the importance of the possibility of plastic deformation in efficient pressure vessel design, and because DBA-DR is especially dedicated to “standard” pressure vessels materials, this approach is, in the standard and in this work, for the time being restricted to vessels made of sufficiently ductile steels and steel castings and at operating temperatures below the creep regime. The extension to vessels made of other sufficiently ductile materials and operating temperatures below the creep regime is straightforward, and the extension to vessels operating in the creep regime is under discussion.

The DBA-DR deals with pressure vessel failure modes directly, in the so-called design checks. These design checks are named after the main failure mode they deal with, but some design checks also deal with other failure modes, other than the main name-giving failure mode. In these design checks the response of specific design models under the influence of specific design actions with respect to specific limit states or specific response modes is investigated.

These design checks should not be confused with simulations of the structure’s behaviour. Although they give much insight into the structure’s behaviour, design checks are neither simulations of the structure’s behaviour, nor are they intended to be simulations. The behaviour investigated or checked is the behaviour of the design model. The analysis of this behaviour gives us information about the likely behaviour of the real structure, but should never be confused with that.

The purpose of a design check is not to simulate the behaviour of a real structure, but to check the safety of a design with regard to the failure mode(s), the design check deals with. If a design fulfills the requirements of a design check for specific actions, it is considered to be sufficiently safe for these actions with respect to the failure mode(s) the design check deals with.

If, for a given design and specified actions, all requirements of all required design checks are fulfilled, this design is considered to be sufficiently safe with regard to the specified actions, and with respect to the safety level required by the PED [1], i.e. by the law in all the European Union’s member states.

In other words, the safety of a component against failure under the influence of specified actions is assessed by analysis of responses of design models to corresponding design actions, the results of the analysis being compared with specified limits or specified response modes, which assure sufficient safety of the design of the component against the specified actions, as required by the PED, and if complemented by the relevant material, manufacturing and testing requirements of the standard.

Logically consistent in different design checks different design models with different geometries and different constitutive laws are used.

It is not surprising that design models may predict for specific actions response modes that do not exist at all in the corresponding real structures under the very same actions. For example, there are materials with specific hardening behaviour, for which the design model of the progressive plastic deformation design check predicts, for a specific cyclic action, progressive plastic deformation, but the real structure shakes down to alternating plasticity, i.e. ratchetting does not exist at all for this type of cyclic action. Nevertheless, the requirements of this design check are still justified, because the response of the real structure may result in large deformations and/or plastic deformations of a magnitude not taken into account in other design checks, i.e. violating presuppositions of other design checks.

The linear-elastic ideal-plastic constitutive law used in some design models, and also the usage of geometrically non-linear relations in the case of some actions and structures makes one powerful tool of linear theory unavailable – linear superposition. For these non-linear cases linear superposition of responses to single actions cannot be used to obtain the response of a multi-action load case; each load case may require an individual calculation.

Some design checks are specified as obligatory, but in some cases it may be necessary to investigate additional design checks. For example, leakage at flanges may be a problem, and it may then be necessary to check a design against leakage (as an ultimate or serviceability limit, depending on the hazard).

In each design check the investigation of several load cases may be required. It is the responsibility of the manufacturer to specify, in writing, the relevant load cases, possibly with the help and information from the user. It is also the responsibility of the manufacturer to prepare, possibly with the help and information by the user, load case specifications for all relevant load cases, for all combinations of actions that can occur coincidentally under reasonably foreseeable conditions.

For reference purposes, it is advisable to identify each load case (specification) by an abbreviation of the designation of the load case class, e.g. NOLC for normal operating load case, SLC for special load case and ELC for exceptional load case, followed by a serial number, e.g. NOLC 4 for the fourth normal operating load case.

To allow for an easy, straightforward combination of pressure action with other actions, such as environmental ones or actions from attached parts, and to give the flexibility expected from a modern standard, to be able to adjust safety margins to the differences in (stochastic) variation of actions, the likelihood of action combinations, the consequences of failure, the differences of structural behaviour and consequences in different failure modes, and to the uncertainties in analyses, a multiple safety factor format was introduced in DBA-DR, using different partial

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safety factors for different actions, for different combinations of actions, for different design checks, for different load cases, and for different materials.

The partial safety factors laid down in the standard are not based on probabilistic investigations or decision theory under uncertainty. The partial safety factors for pressure and material strength parameters result from a (modified) calibration with respect to the DBF results.

Values for other actions are aligned to those of Eurocode 3 [22,23]. For environmental actions – wind, snow, earthquake – country-specific data, i.e. values specified in relevant regional codes, are to be used if they are larger than the ones specified in the standard, but consistency with the corresponding characteristic values must be checked, so that the overall safety is maintained.

## 2.2. General Terms and Definitions

### 2.2.1. Failure-Related Terms

**Failure:** Failure of a structure is an event, the transition from a normal working state, where the structure meets its intended requirements, to a failed state, where it does not meet its requirements.

Failure of any structure cannot be predicted exactly, deterministically – but it can only be characterized by the stochastic properties of the structure and the actions the structure is subjected to.

**Failure modes:** Failure mode is a term used in the classification of failures of structures, via a simplifying assumption that failure of a structure can occur only in a finite number of modes – it is a description of the way a failure occurs. Failure modes can be regarded as discretizations of a more general and possibly continuous set of failures.

**Limit states:** *A limit state is a structural condition beyond which the design performance requirements of a component are not satisfied. Limit states are classified into ultimate limit states and serviceability limit states (Eurocode 3, EN 13445-3 Annex B).*

In the literature the term limit state is used for (real) limit states in real and virtual structures, where these may relate to unrestricted plastic flow, plastic collapse, burst, ultimate action, (functional) displacement limits, etc., and it is used also for the, possibly different, limit states in models, where these may relate to strain limitations, displacement limitations, limitations of combinations of stress resultants, limit analysis loads, etc.

With the exception of this section, the term is used in this book exclusively for model limit states.

**Elastic limit states:** An elastic limit state is a structural condition associated with the onset of plastic deformation. This term is usually used in connection with monotonic actions, and it relates to virtual structures, usually with zero initial stress distribution.

The value of a monotonic action that corresponds to the onset of plastic deformation is called **elastic limit action**.

**Ultimate limit states:** An ultimate limit state is a structural condition (of the component or vessel) associated with burst, collapse or with other forms of structural failure, which may endanger the safety of people.

*Ultimate limit states include failure by gross plastic deformation, rupture caused by fatigue, collapse by instability of the vessel or part of it, loss of equilibrium of the vessel or any part of it, considered as a rigid body, or overturning or displacement and leakage which affects safety. Some states prior to collapse which, for simplicity, are considered in the place of collapse itself are also classified and treated as ultimate limit states (Eurocode 3, EN 13445-3 Annex B).*

The term relates to real or virtual structures.

**Serviceability limit states:** *A serviceability limit state is a structural condition (of the component or vessel) beyond which service criteria specified for the component are no longer met. Serviceability limit states include deformation or deflection which adversely affects the use of the vessel (including the proper functioning of machines or services), or causes damage to structural or non-structural elements and leakage which affects efficient use of the vessel but does not compromise safety nor causes an unacceptable environmental hazard. Depending on the hazard, leakage may create either an ultimate or a serviceability limit state (Eurocode 3, EN 13445-3 Annex B).*

The term relates to real or to virtual structures.

**Reliability:** Reliability is the probability that a structure does not fail over its expected lifetime under specified conditions and subjected to specified actions.

Reliability is the complement of failure probability.

**Unrestricted plastic flow:** Unrestricted plastic flow is a phenomenon occurring in tests on certain types of real structures, made of mild steel, where large deformations – considerably greater than the deformations in the elastic range – occur with little or no increase in load. This behaviour is caused by the development of plastic flow in the structure to such an extent that the remaining elastic material plays a relatively insignificant role in sustaining the load, the structure begins to deform under constant or nearly constant load. This phenomenon is also called **unstable gross plastic yielding**.

The **unrestricted plastic flow load** is the load when unrestricted plastic flow sets in.

This term relates to real structures, with actual (strain hardening) constitutive laws, and includes effects of geometry changes due to large deformations.



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In the literature this load is also called **plastic collapse load** [24] or **plastic load** [25].

At this load, significant plastic deformation occurs for the structure as a whole, the plastic region has grown to a sufficient extent that the surrounding elastic regions no longer prevent overall plastic deformation from occurring, but this load is, in general, not equal to the ultimate load of the structure.

**Ultimate loads, ultimate actions:** The ultimate load and the ultimate action are the maximum load and the maximum action a real structure can carry in a single monotonic and quasi-static application. The burst pressure of cylindrical or spherical vessels is a typical example of an ultimate action.

Because the ultimate strength of ductile materials is greater than their yield strength, the ultimate pressure is greater than the unrestricted plastic flow pressure.

This term relates to real structures.

**Gross plastic deformation:** Gross plastic deformation is a failure mode related to a single monotonic application of an action that is attended by extensive gross plastic deformation, by unrestricted plastic flow followed by ductile fracture, i.e. unstable gross section yielding (unstable material flow instability) or unstable crack growth, and/or brittle fracture. The related action at the onset of gross plastic deformation is an ultimate action, and burst and collapse are typical examples.

**Progressive plastic deformation:** Progressive plastic deformation is a response mode of a structure or of a model subjected to cyclic actions, referring to a deformation pattern where deformation increments over consecutive action cycles are neither zero nor tend to zero.

This phenomenon is also called **ratchetting** and **inadaptation** – the structure does not shake down under the cyclic action.

Progressive plastic deformation eventually leads to failure of the structure, and, therefore, is a failure mode related to cyclic actions. In the literature the failure mode is also called **incremental collapse**. This designation is not used in this book – it is rather misleading in general, but especially in cases of progressive plastic deformation in the absence of mechanical actions, i.e. in cases where there is no direct transition to the instantaneous collapse situation.

**Shakedown:** Shakedown is a response mode of a structure or of a model subjected to cyclic actions, referring to a deformation pattern where, after a finite or infinite number of action cycles, stress and strain become cyclic and deformation increments over consecutive cycles vanish, i.e. progressive plastic deformation is absent.

This term encompasses elastic shakedown and elastic–plastic shakedown, and is also called **adaptation**.

**Elastic shakedown:** This term refers to shakedown to purely elastic behaviour, i.e. the response of the structure becomes eventually elastic, after a finite or infinite number of action cycles.

**Elastic–plastic shakedown:** This term refers to shakedown to elastic–plastic behaviour, i.e. after a finite or infinite number of action cycles, stress and strain fields become cyclic, deformation increments over consecutive action cycles vanish, but in each cycle plastic deformations occur, strain increments change signs in every cycle and cancel each other out within the cycle. This response mode is also called **alternating plasticity**.

**Cyclic fatigue:** Cyclic fatigue is a phenomenon in structures subject to cyclic actions involving progressive localized damage, with cracks and crack propagation. Cracks may initiate in originally undamaged areas and propagate afterwards, and already existing cracks and crack-like defects propagate. The process eventually leads to a reduction of cross-sectional areas to such an extent that rupture occurs under an action of a magnitude that has been withstood satisfactorily before. The final fracture may be ductile or brittle.

Since cyclic fatigue eventually leads to failure of the structure, it is a failure mode related to cyclic actions.

**Instability:** Instability of a structure, often called buckling, is a failure mode related to single application of monotonically increasing actions whereby the initially stable deformation mode becomes unstable, and the structure seeks another, stable, deformation mode, which differs not only quantitatively but also qualitatively from the initial deformation mode. Two modes prevail: bifurcation instability and snap-through, the latter also called limit point buckling. The critical points on the action–deflection paths where the pre-buckling modes become unstable are singular points – called bifurcation points and limit points, respectively.

**Bifurcation instability:** Bifurcation instability or **bifurcation buckling**, often called classical buckling, is an instability mode where the transition from the initial, pre-buckling deformation pattern to the qualitatively different one is continuous and the structure passes from its unbuckled state continuously to an infinitesimally close buckled state.

For actions close to the critical value, at which this transition occurs, more than one equilibrium path in an action–deflection plot of the structure exist, each one corresponding to states of the structure in equilibrium with the considered actions, and these equilibrium paths emanate from the same point, referred to as bifurcation point, and the corresponding action referred to as bifurcation action.

Typical examples of bifurcation instability are:

- (classical) buckling of straight columns (Euler buckling),
- (classical) buckling of flat plates in compression, and
- buckling of cylindrical shells under axial forces or external pressure.

**Snap-through, limit point buckling:** Snap-through or limit point buckling is an instability mode where the transition from the initial, pre-buckling deformation

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pattern to the qualitatively different one is discontinuous – the new, non-adjacent state is attained in a jump-type, discontinuous transition, with dynamic effects. The critical action, where the structure becomes unstable, corresponds to a maximum in the action-deflection plot.

Typical examples of snap-through instability problems are

- arches, with restrained ends under diverse forces creating compressive stresses,
- shallow spherical domes and caps under external pressure,
- kinking of Venetian blind slats in bending, and
- flattening instability of pipe bends in bending.

**Interactive buckling:** Buckling with at least two critical points for different buckling patterns occurring at, or near, the same action value is called interactive buckling. Interactive buckling occurs frequently in structures with multiple symmetry, and also in structures that are optimized with regard to different buckling modes, e.g. externally pressurized cylindrical shells with optimized stiffeners.

**Stable structural state:** A structural state in equilibrium with a certain action is called stable, if the deformation caused by a small perturbation, e.g. a small imposed initial displacement, a small imposed additional force, or a small imperfection of the structure's perfect geometry, remains bounded, and if a specific measure of this perturbation caused deformation does not exceed any arbitrarily small value if only the perturbation is small enough but different from zero. Otherwise the equilibrium state is called unstable.

**Stability of a structure:** Stability of a structure is the quality of the structure being in stable equilibrium under a specified action.

**Loss of static equilibrium:** Loss of static equilibrium is a failure mode related to a rigid-body movement of the structure, and it includes overturning and global displacement of a structure like a rigid body. This failure mode is different from all the others, because it is in general not related to internal pressure, which gives pressure equipment its name and is its deciding hazard.

### 2.2.2. Action-Related Terms

**Actions:** *Actions are imposed thermo-mechanical influences which cause stress and/or strain in a structure, e.g. imposed pressures, imposed forces, imposed displacements, and imposed temperatures (EN 13445 Annex B).*

The term action encompasses also combinations of single actions. Other actions – mechanical, physical, chemical, or biological actions – not encompassed by this definition, may have an influence on the safety of a structure, but in DBA only

those are considered that cause stress and/or strain. Covered by the definition are, for example, self-weight, pressure, and imposed surface loadings, temperature changes, displacements imposed on the structure at connections or foundations, e.g. displacement due to temperature changes or settlement.

Depending on their variation in time and their probability of occurrence, actions are classified into:

- permanent actions,
- pressure, temperature, and actions related deterministically to pressure and/or temperature,
- variable actions other than pressure, temperature, and deterministically related actions, and
- exceptional actions.

Examples of permanent actions are self-weight of the structures themselves, self-weight of associated fittings, ancillaries, and fixed equipment.

Examples of variable actions are imposed displacements, wind and snow loads, earthquake excitations, all of a magnitude anticipated to occur under reasonably foreseeable conditions such that their consideration is required (by law).

Pressure and temperature are variable actions, but they have, very often, special characteristics with regard to their variation in time, to random properties, etc. Therefore they are classified in a special class. Temperature changes have a dual role in that they may cause stress in the structure and also change its material properties especially the strength related ones.

**Exceptional actions:** Exceptional actions are variable actions of very low probability of occurrence, actions that require, should they occur, the safe shutdown and/or inspection of the vessel.

Characteristics of exceptional actions include their very low probability of occurrence, and the fact that they are not anticipated to occur under reasonably foreseeable conditions, and, therefore, need not be included (by law) in the normal design considerations.

Examples of exceptional actions are pressure acting on a secondary containment after failure of the primary one, pressure due to an internal explosion, and wind or earthquake excitation, all of which have such a (very low) probability that they need not be anticipated to occur under reasonably foreseeable conditions and, therefore, need not be included in the design considerations required (by the requirements of the PED). Rather, these are included voluntarily or by force of other legal requirements.

**Monotonic actions:** An action is called monotonically increasing if

- in case of a single scalar action, the magnitude of the action increases consistently, i.e. its rate of change is always positive and

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- in case of a single vectorial action or a combination of single actions, when all components increase consistently, i.e. all rates of change of the individual magnitudes are always positive.

### Cyclic actions:

- An action is called cyclic if the action states repeat themselves in a regular sequence.
- An action  $A_i(\tau), i=1, \dots, n$ , is called cyclic if it can be described as a periodic function of time, i.e. if for any time  $\tau$

$$A_i(\tau) = A_i(\tau + T), \quad i=1, \dots, n, \quad (2.1)$$

where  $T$  is the cycle period, a constant scalar of dimension time.

- An action is called **single-amplitude cyclic** if it is cyclic and within one cycle there is only one maximum and one minimum.
- An action is called **multi-amplitude cyclic** if it is cyclic and within one cycle there are at least two maxima and two minima; an action is called a **variable amplitude action** if it is multi-amplitude cyclic or fluctuating but non-cyclic.
- A cyclic action is called a **shakedown action** if the model shakes down under the action.

**Load cases:** A load case is a combination of coincident actions. Load cases are classified into *NOLCs*, *SLCs* and *ELCs* (EN 13445-3, Annex B).

**Normal operating load cases:** *NOLCs* are those acting on the pressure vessel during normal operation, including start-up and shutdown (EN 13445-3). *NOLCs* are load cases where normal conditions apply (EN 13445-3 Annex B).

*NOLCs* include start-up, shutdown, and normal operation as specified for the vessel to perform its intended functions, but also include operating excursions, initiation and recovery from upset conditions that must be considered in the design.

**Special load cases:** *SLCs* are load cases where conditions for testing, construction, erection, or repair apply (EN 13445-3 Annex B).

**Exceptional load cases:** *ELCs* are those corresponding to events of very low-occurrence probability requiring the safe shutdown and inspection of the vessel or plant (EN 13445-3).

*ELCs* are load cases related to exceptional actions. Occurrence of an event related to an *ELC* requires action by the user – shutdown and/or inspection. *ELCs* are included in design investigations usually in cases of major hazards, to provide assurance that no gross loss of structural integrity will result.

**Multi-action load case:** Multi-action load cases are load cases dealing with combinations of single actions.

**Upset conditions:** Upset conditions are deviations of moderate frequency from normal start-up and shutdown conditions, and from normal operation, anticipated to occur under reasonably foreseeable conditions, and, therefore, have to be included in design considerations, to achieve a reasonable capability to withstand these conditions without operational impairment.

Upset conditions include those transients that result from single operator error or control malfunction, transients caused by a fault in the component requiring its isolation from the system; they include abnormal incidents not resulting in a forced shutdown, and those which do not require repair of structural damage.

Upset conditions, therefore, may include pressure transients that result in opening of safety valves and where the momentary pressure surge is limited to a value below 110% of the maximum allowable pressure.

**Pressure** (in bar or in MPa): *Pressure means pressure relative to atmospheric pressure, i.e. gauge pressure. As a consequence, vacuum is designated by a negative value (PED).*

**Calculation pressure** ( $PC$  or  $p_c$  in MPa): *The calculation pressure is the differential pressure used for the purpose of calculations of a component (EN 764-1, EN 13445-3).*

Calculation pressure is a designation used in the DBF approach.

**Calculation temperature** ( $TC$  or  $t_c$  in °C): *The calculation temperature is the temperature used for the purpose of calculations of a component (EN 764-1, EN 13445-3).*

Calculation temperature is a designation used in the DBF approach.

**Design pressure** ( $PD$  or  $p_d$  in bar or MPa): *The design pressure is the pressure at the top of each chamber of the pressure equipment chosen for the derivation of the calculation pressure of each component (EN 764-1, EN 13445-3).*

**Design temperature** ( $TD$  or  $t_d$  in °C): *The design temperature is the temperature chosen for the derivation of the calculation temperature of each component (EN 764-1, EN 13445-3).*

**Design mechanical loads:** Design mechanical loads are combinations of forces and moments chosen for the derivation of forces and moments used in DBF calculations in conjunction with design pressure and design temperature.

**Operating pressure** ( $P_o$  or  $p_o$  in bar or MPa): *The operating pressure is the fluid pressure, which occurs under specified operating conditions (EN 764-1).*

**Operating temperature** ( $T_o$  or  $t_o$  in °C): *The operating temperature is the fluid temperature, which occurs under specified operating conditions (EN 764-1).*

**Maximum permissible pressure or rating pressure** ( $PR$  or  $p_r$  in bar or MPa): *The maximum permissible pressure is the pressure obtained with the analysis thickness at the calculation temperature for a given component from the DBF (EN 13445-3)*

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**Test pressure** ( $PT$  or  $p_t$  in bar or MPa): *The test pressure is the pressure the equipment is subjected to for test purposes (EN 764-1, EN 13445-3).*

**Test temperature** ( $TT$  or  $t_t$  in °C): *The test temperature is the temperature at which the pressure test of the pressure equipment is carried out (EN 764-1, EN 13445-3).*

**Maximum allowable pressure** (in bar): *Maximum allowable pressure means the maximum pressure for which the equipment is designed, as specified by the manufacturer. It is defined at a location specified by the manufacturer. This must be the location of connection of protective and/or limiting devices or the top of the equipment or, if not appropriate, any point specified (PED).*

*The maximum allowable pressure is the maximum pressure for which the pressure vessel is designed as specified by the manufacturer (EN 13445-1).*

**Maximum/minimum allowable temperature** ( $TS$  in °C): *Maximum/minimum allowable temperature means the maximum/minimum temperature for which the equipment is designed, as specified by the manufacturer (PED, EN 13445-1).*

In general, this term refers to fluid temperatures in specified reference points, e.g. the mean fluid temperature in the inlet or outlet nozzle, whichever is higher.

For simplicity, this term is, in simple cases of uniform temperature distributions with negligible temperature transients and gradients, also used for the mean metal temperature of the structure.

To avoid confusion, and to take into account that in DBA the determination of the temperature distributions is frequently part of the analysis, the term maximum/minimum allowable temperature is used here only in its basic meaning – as fluid temperature in a specified reference point.

**Maximum allowable loads:** Maximum allowable loads are imposed forces, imposed moments, and combinations thereof, acting at specified points, loads for which the equipment is designed, as specified by the manufacturer.

The manufacturer may specify these forces and moments to take into account actions from other parts that can occur under reasonably foreseeable conditions at e.g. supports, attachments and, piping joints. This term encompasses only actions, but not reactions (at constraints).

**Maximum allowable actions:** Maximum allowable action is a term used for the combination of maximum or minimum allowable pressure, maximum or minimum allowable temperature and maximum allowable loads for which the equipment is designed, as specified by the manufacturer, to take into account actions which can occur coincidentally under reasonably foreseeable conditions.

In the action space, the set of maximum allowable actions defines the domain of allowable actions – the **design domain**.

Usually, this set consists of  $n$ -tuples of related maximum and minimum allowable action values, e.g. maximum allowable pressure, related maximum allowable temperature, related maximum allowable forces, related maximum allowable moments, etc.

Fig. 2.1 shows an example with two actions: pressure and temperature. The design domain is, in this example, given by the set of action pairs  $(PS1^+, TS1^+)$ ,  $(PS2^+, TS2^+)$ ,  $(PS_-, TS^+)$ ,  $(PS3_-, TS3^+)$ ,  $(PS3_-, TS3_-)$ ,  $(PS3^+, TS3_-)$ .

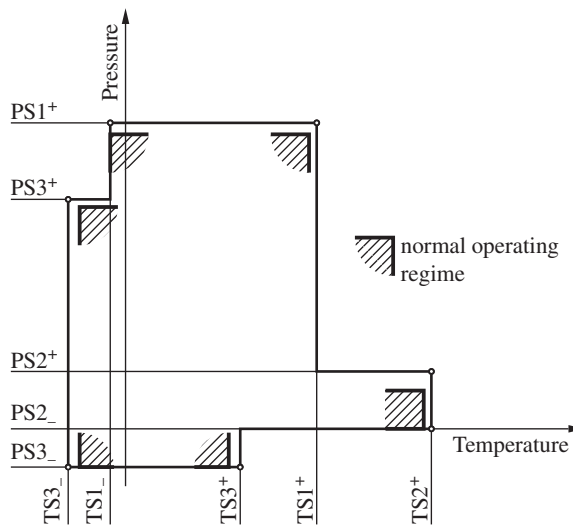


Figure 2.1: Design domain.

### 2.2.3. Model-Related Terms

**Vessels:** *Vessel means a housing designed and built to contain fluids under pressure including its direct attachments up to the coupling point connecting it to other equipment. A vessel may be composed of more than one chamber (PED).*

**Components:** *A component is a part of pressure equipment or assembly, which can be considered as an individual item for the calculation (EN 764-1, EN 13445-2).*

**Chambers:** *A chamber is a single fluid space within a unit of pressure equipment (EN 764-1, EN 13445-1).*

**Structures:** *A structure is an organized combination of connected parts designed to provide some measure of rigidity (Eurocode 3, ISO 6707, Part 1). It*



is a combination of all load-carrying parts relevant to a component, e.g. the whole vessel, its load carrying attachments, supports, and foundations (EN 13445-3 Annex B).

**Real structures:** The term real structure refers to an actual, real, existing structure.

**Geometrical imperfections:** Geometrical imperfections are deviations of the geometry of real or virtual structures from the nominal or an ideal one, e.g. out-of-roundness (ovality), buckles, axial misalignment, angular distortion (peaking, roof-topping at welds), angular misalignment of nozzles and local thinning.

**Virtual structures:** The term virtual structure refers to a structure as specified by design and manufacturing drawings, material lists, directly or indirectly referred to standards (for materials, tolerances of pre-products, shape deviations, allowed manufacturing deviations or defects, etc.) [3].

**Physical models or analysis models:** A physical model, quite often also called analysis model, is a model of a structure that is deduced from the real or virtual structure by an abstraction or idealization process with regard to geometry, boundaries and boundary conditions, constitutive laws, etc. This idealization quite often requires assumptions on material properties or even constitutive laws that are unknown and even not determinable for the real structure – the real constitutive law of base metal, “zones” of weldments, the real deviation from the ideal geometry, and so on [3].

**Mathematical models:** A mathematical model of a structure is a mathematical description of the physical model using the principles of mechanics. In case of finite element analysis this mathematical model is obtained using the FEA software [3].

**Kinematic boundary conditions:** Kinematic boundary conditions are boundary conditions with prescribed displacements (or displacement gradients), including zero displacement, i.e. restraint of displacement. In points with kinematic boundary conditions, **reactions** – surface tractions caused by the restraint of displacement – will in general occur. In this book only kinematic boundary conditions are considered that can be expressed by equations, presupposing that kinematic boundary conditions expressed by inequalities are dealt with via a proper choice of the former type of boundary conditions and a check of the results for agreement with the inequalities.

**Dynamic boundary conditions:** Dynamic boundary conditions are boundary conditions with prescribed imposed surface tractions and forces (or moments).

**Supports:** Supports are said to be **statically determinate** if the global equilibrium equations are sufficient to determine the required reactions at the supports, i.e. the contact pressures at the separation between support and foundation or the corresponding resultants – resultant forces and resultant moments. Therefore, supports where only the resultants of the reactions can be determined by means of the global

equilibrium equations, but not the contact pressure themselves, are encompassed by this definition if the resultants are sufficient for the relevant design check.

**Kinematic relations:** For reasons of brevity and readability, the kinematic relations between strain components and displacement gradients are in the following called kinematic relations. In the design models non-linear as well as linearized kinematic relations are used.

**Quasi-static models:** A mathematical model is called quasi-static if dynamic effects are neglected, i.e. if acceleration related terms are absent.

**Limit analysis models:** The limit analysis model of a structure is the idealized mathematical model used in limit analysis investigations with linear-elastic ideal-plastic constitutive law, linear kinematic relations (between strain and displacements), and equilibrium conditions for the undeformed structure.

**Limit analysis actions:** The limit analysis action is the action for which, in a limit analysis model of the structure, deformations increase without limit while the action is held constant. This term relates to mathematical models of the structure. In the literature this limit analysis action is also called **limit load** [24,25] and **theoretical limit load**.

**Safe actions:** In gross plastic deformation design checks, an action is called safe if it is not larger than the corresponding limit analysis action or, in other words, if it is enclosed by the set of limit analysis actions.

In structural stability design checks, an action is called safe if it is not larger than the smallest bifurcation action or snap-through action, depending on the instability mode.

**Design models:** *The design model is a physical or mathematical model of the structure used in determining the effects of actions (EN 13445-3 Annex B).*

The term design model is used exclusively for models used in design checks and the details are discussed in Section 2.2.6.

**Local structural perturbation source:** A geometrical detail, a deviation of a (more regular) geometry, or a local change in material properties is called a local structural perturbation source if it affects the stress or strain distribution only through a fraction of the thickness, if it is associated solely with localized types of deformation or strain and has no significant non-local effect.

Examples are small fillet radii, small attachments, small bores, and welds.

In the literature a local structural perturbation source is also called **local structural discontinuity**. To avoid confusion with discontinuity in the mathematical sense, the term local structural perturbation source is used here throughout.

**Stress-concentration-free models:** *A stress-concentration-free model of a structure is an equivalent idealized model of the structure without local stress/strain raisers (EN 13445-3 Annex B).*

A stress-concentration-free model of a structure, or of a more refined model, is an equivalent idealized model of the structure, or of the model, without local structural perturbation sources but with all other aspects of the models being the same.

**Elastic stress fields:** For brevity and readability, stress fields determined with (unbounded) linear-elastic models are called here elastic stress fields.

#### 2.2.4. *Thickness-Related Terms*

**Actual thickness:** The actual thickness is the thickness of the real structure.

**Analysis thickness** ( $e_a$  in mm): *The analysis thickness is the effective thickness available to resist the actions in corroded condition (EN 13445-3).*

The analysis thickness is the thickness used in a physical model for a specific design check and load case. It differs depending on the analysis situation, i.e.:

- In design situations that deal with the virtual structure, the effective thickness is derived from the nominal thickness by subtracting the sum of the thickness tolerance and the corrosion or erosion allowance. However the case of (cyclic) fatigue design checks is an exception, where subtraction of only half the corrosion or erosion allowance suffices, and this thickness is then called fatigue analysis thickness (see Section 7.3).
- In re-analysis situations that deal with the real structure, the effective thickness is derived from the actual thickness by subtracting the corrosion or erosion allowance, or of a part of this allowance (see above indent and Section 7.3).

**Nominal thickness** ( $e_n$  in mm): *The nominal thickness is the thickness specified on the drawing (EN 13445-3).*

**Thickness tolerance allowance** ( $\delta$  in mm or %): The thickness tolerance allowance is the absolute value of the admissible negative tolerance of the nominal thickness. The admissible negative tolerance may be as per material standard, e.g. EN 10216, EN 10217, EN 10029, as per standards of pre-products, e.g. standards for dished ends, or it may be specified on the drawing as tolerance for possible thinning in manufacturing processes, e.g. by adding MW (minimum wall thickness) to the thickness value; in this case  $\delta = 0$ .

**Corrosion or erosion allowance** ( $c$  in mm): The corrosion or erosion allowance is the allowance specified on the drawing for possible corrosion or erosion in service.

**Fatigue relevant thickness:** This term is used in fatigue design checks, and is defined as the shortest distance from a specific critical point on one surface to any point on any other surface of the design model, the shortest length of a critical crack to break through.

**Fatigue analysis thickness:** The fatigue analysis thickness is the thickness used in models of the fatigue design checks. This thickness is obtained by subtracting from the difference of the nominal thickness and the relevant tolerance allowance, or from the actual thickness, not the whole of the corrosion allowance, as in the determination of the analysis thickness, but only half of the corrosion allowance, i.e. these fatigue analysis thicknesses are larger by half of the corrosion allowance than the analysis thicknesses used, e.g. in the GPD-DC.

### 2.2.5. Response-Related Terms

**Stationary responses:** A response is called stationary if it is independent of time.

**Quasi-stationary responses:** A response is called quasi-stationary if its derivative with respect to time is constant. This notion is used especially in thermal stress problems owing to fluids with temperatures increasing or decreasing at a constant rate – after a sufficiently long time, or after the initial condition influenced response has decayed sufficiently, the response can be treated like a stationary one.

**Total stresses/strains:** The total stress or strain is the stress or strain inclusive of all concentration effects, or the stress or strain in a design model with local structural perturbation sources.

**Structural stresses/strains:** Structural stress/strain is the stress/strain in a stress-concentration-free model of the structure. Structural stress/strain includes the effects of gross structural details, e.g. branch connections, cone–cylinder intersections, vessel–end junctions, thickness discontinuities, presence of attachments, deviations from (ideal) design shape with global effect, but it excludes effects of local structural perturbation sources, such as effects due to small fillet radii, weld toe details, weld profile irregularities, small (partial penetration) bores, and local temperature field details.

**Equivalent linear stress distributions:** The linear stress distribution of a stress component along an evaluation line is called the equivalent linear stress distribution if it is equivalent to the actual (non-linear) stress distribution of this component with regard to stress resultants – resultant forces and moments. **Evaluation lines** are lines connecting one point on one surface of the structure with one point on the opposite surface, and are usually straight lines through hot spots normal to the mean surface of a shell or plate.

**Theoretical stress concentration factors:** In cases of uniaxial stress states the theoretical stress concentration factor is defined as the ratio of the total stress to the structural stress in a linear-elastic model. In case of multi-axial stress states the

theoretical stress concentration factor is defined as the ratio of the corresponding equivalent stresses. This theoretical stress concentration factor, used in the fatigue design checks, relates the total stress to the structural stress, and therefore considers only the stress concentration due to local structural perturbation sources, or over the thickness non-linearly distributed thermal stresses, but not stress concentrations due to global perturbations. These theoretical stress concentration factors must not be confused with the often used, but in principle different, (structural) stress concentration factors based on conveniently chosen reference stresses, e.g. nominal stresses that are used for the convenient description of the influence of global effects, such as effects of global geometric changes.

**Thermal stresses:** Thermal stresses are stresses in a structure caused by changes in the structure's temperature distribution, stresses due to constraints of thermal expansion or contraction, non-uniform temperature distributions or inhomogeneous thermal expansion properties. They are self-stresses, and are self-equilibrating if there are no external constraints, or if the reactions at external constraints vanish.

**Residual stresses:** Residual stresses are stresses in a solid material after any kind of non-elastic treatment, such as plastic deformation, heating, cooling, recrystallization, and phase transformation [26]. They are self-stresses.

**Self-stresses:** A stress field is called a self-stress field if it is statically admissible for vanishing imposed forces, in the interior of the structure and at each point of the surface with dynamic boundary conditions. Reactions, i.e. surface tractions in points with kinematic boundary conditions, need not be zero, but must be in (global) equilibrium with imposed actions.

To avoid further confusion: This designation, used consistently here, is called self-equilibrating stress field in EN 13445-3. Unfortunately the two designations are not clearly distinguished in the literature, causing much confusion.

**Statically admissible stresses:** A stress field is called statically admissible if it fulfils the equilibrium conditions at each point of the structure and the boundary conditions at each point of the surface where imposed tractions are prescribed.

**Self-equilibrating stresses:** A stress field is called self-equilibrating if it is statically admissible for zero external forces – zero imposed forces in the interior of the structure and zero surface tractions – for imposed tractions at points where tractions are prescribed and for reactions at points where kinematic boundary conditions are prescribed. Self-equilibrating stresses are self-stresses, but self-stresses are not necessarily self-equilibrating. Various definitions exist in the relevant literature for the notions self-stress, self-equilibrating stress, eigenstress, and residual stress. These designations are often used synonymously, but with different meanings in different sources, depending on the approach and the topic (see e.g. [4, 26–39]).

**Elastic follow-up:** In the relevant literature, there are at least two different phenomena called elastic follow-up:

- For structures operating in the creep regime, the designation elastic follow-up is used for the phenomenon where after an action cycle, from a stationary action to the very same stationary action, creep still continues and inelastic strain accumulates. This designation is usually used for the inelastic strain increase at zero stress in creep tensile tests with specimens uniaxially stressed from zero to a constant value and then back to zero.
- In displacement-controlled cases, the designation elastic follow-up is used for the phenomenon of disproportionately large inelastic strain accumulation in weaker regions of a structure due to elastic strain redistribution of other, stronger regions of the structure. The inelastic strain concentration may be due to plasticity or creep in these regions, because of larger stresses and/or higher temperatures. The displacement control may be due to imposed displacements or due to restrained thermal displacements.

In this book the designation elastic follow-up encompasses both phenomena, but only the second one is really of importance.

### 2.2.6. Design Check-Related Terms

**Design checks:** *A design check of a component is an investigation of the component's safety under the influence of specified combinations of actions with respect to specified limit states (EN 13445-3 Annex B).*

A design check is an investigation of a component's safety under the influence of specified actions with respect to specified failure modes. The component's safety is evaluated in investigations of the fulfilment of design check's principles by the responses of design models subjected to design actions, i.e. by the effects of design actions on the design models.

The component is safe with respect to a specific failure mode if the effects of the combinations of design actions on the design model are admissible with respect to the requirements of the design check that corresponds to the failure mode, requirements that are specified in the design check's principle.

*For each relevant failure mode, relevant to the scope of the standard, there corresponds a single design check. Each design check represents one or more failure modes (EN 13445-3 Annex B). Design checks are named after the main failure mode they deal with (EN 13445-3 Annex B). In general, each design check comprises various load cases (EN 13445-3 Annex B).*

**Design models:** *The design model is a physical or mathematical model of the structure used in determining the effects of actions (EN 13445-3 Annex B).*

The geometry of the design model depends on the design check – depending on the design check it may be necessary to include local structural perturbation sources, or it may be admissible to use a stress-concentration-free model.

The constitutive law of the design model depends on the design check and, of course, on the material of the structure. Depending on the design check, the constitutive laws are linear or linear-elastic ideal-plastic.

Depending on the structure, the actions considered, and the design check, mathematical design models may be geometrically linear, or it may be necessary to use **geometrically non-linear relations**, i.e. non-linear kinematic relations and equilibrium conditions applied on the deformed structure.

**Response modes:** Response mode is a term used in the classification of the response of models to specified actions, and it encompasses gross plastic deformation, progressive plastic deformation, shakedown, cyclic fatigue (damage), structural instability, static equilibrium, leak tightness, excessive local strains, etc.

**Effects:** *An effect is the response (e.g. stress, strain, displacement, resultant force or moment, equivalent stress resultant) of a component to a specific action or combination of actions (EN 13445-3 Annex B).*

The term effect relates in the following only to model responses, and encompasses not only (model) states, such as states of stress, strain, displacement, stress resultants, but also response functions (of space and time) and response modes. Furthermore, in the static equilibrium design check (Chapter 8), the notion effect applies to reactions as well, i.e. to action caused forces, contact pressures, and moments, at points of the models with kinematic boundary conditions.

**Characteristic values of actions:** *A characteristic value of an action is a representative value, which takes into account the variation of an action (EN 13445-3 Annex B).*

A characteristic value of a single scalar action of a component of a vectorial action, or of a combination of actions, is a value representative of an extreme value of the magnitude of the action or the component of the action that takes into account the statistical variation of the extreme value and/or properties of limiting devices or is given by natural limits, if any. If the considered extreme value is a maximum the characteristic value is called upper characteristic value; and if the considered extreme value is a minimum, it is called lower characteristic value.

Characteristic values may be statistical mean values of the extreme values, specified (upper or lower) percentiles, reasonably foreseeable extreme values, values corresponding to natural limits, or, in case of exceptional actions, they may be individually agreed upon and specified values. They may be space-dependent, and depend not only on the action, but also on the design check and the load case.

For example, a value of an action that must be taken into consideration in NOLCs because it can occur under reasonably foreseeable conditions whereby the specified number of occurrences is low; such a value must be included in determining the characteristic value in design checks dealing with gross plastic deformation, structural instability, etc., but not in design checks dealing with fatigue. Or, as another example, the value of the test pressure must be taken into account in determining the characteristic value for testing load cases, but not for NOLCs. The case of exceptional actions is even more obvious: they must be included in ELCs, but not in others.

In multi-action load cases the interdependency of the encompassed single actions may be taken into consideration – it may be necessary to specify more than one combination of characteristic values, more than one load case. For example, it may be that in one type of normal operation a high pressure occurs at moderate temperatures, and in a different type of normal operation moderate pressures occur at high temperatures, but the case of high pressures at high temperatures need not be considered in NOLCs.

**Characteristic functions of actions:** In some cases, the effect of an action depends on the time-dependency of the action. For example, a thermal stress that depends on the time-dependency of thermal transients (of the fluid). In these cases it is necessary to determine and specify characteristic functions of time for these actions in specific design checks, e.g. design checks dealing with fatigue, with progressive plastic deformation, or with structural instability.

In other cases the order of cyclic or repeatedly occurring action states is of importance, and not the instantaneous rate of change. In such cases, the characteristic functions of these actions are defined as functions of a time-like parameter, which defines this order of action states.

*In both cases, realistic assessment of these functions is quite often crucial: The characteristic function shall represent an “upper bound estimate” of the time-dependent action to be expected under reasonably foreseeable conditions during the full design life – in a statistical sense like for the characteristic values (EN 13445-3 Annex B). For different design checks different characteristic functions may be specified (EN 13445-3 Annex B).*

Characteristic functions of actions depend on time or a time-like parameter, but may also be space-dependent.

**Design values of actions:** The design value of a single scalar action or a component of a vectorial action, or of a combination of actions is the value of the action or component of the action to be used in a design check. This design value is given by the product of the relevant characteristic value and the relevant partial safety factor.

**Design functions of actions:** The design function of an action is the function (of time or a time-like parameter) to be used in a design check. This design function



is given by the product of the relevant characteristic function and the relevant partial safety factor.

**Design values of material strength parameters:** The design value of a material strength parameter is the value of the material strength parameter to be used in a design model. This design value is given by the quotient of the characteristic value of the relevant material strength parameter, as specified in the design check, and the relevant partial safety factor.

*In the determination of these characteristic values, the minimum material strength data shall be used which apply to the materials in the final fabricated condition, and which must conform to the minimum values of the technical documentation prepared in accordance with EN 13445-5, Clause 5. The minimum values, specified for the delivery condition, may be used unless heat treatment is known to lead to lower values, in which case these lower values shall be used. If the weld metal gives lower strength values after fabrication, these shall be used (EN 13445-3).*

**Design values of buckling strengths:** The design value of the buckling strength is the value to be used in the instability design check. This design value is given by the quotient of the value of the buckling strength, determined for the relevant design model, and the relevant partial safety factor.

**Partial safety factors:** *The partial safety factor is a factor which is applied to a characteristic value of an action or a material (strength) parameter in order to obtain the corresponding design value (EN 13445-3 Annex B).*

- Partial safety factors of actions are numbers by which characteristic values of actions are multiplied to obtain the design values of the actions.
- Partial safety factors of material strength parameters are numbers by which characteristic values of material strength parameters are divided to obtain the design values of the material strength parameters.
- Partial safety factors of resistances are numbers by which limit values of resistances of models are divided to obtain the design values of the resistances.
- Partial safety factors of actions are also numbers by which characteristic functions of actions are multiplied in order to obtain the corresponding design functions.
- Partial safety factors of actions depend on the action, the design check, and the load case.
- Partial safety factors of material strength parameters depend on the material type, the design check, and the load case.
- Partial safety factors take account of
  - the possibility of non-conservative deviation of actions from the characteristic values (or functions),
  - the uncertainty in the constitutive laws of the models,
  - the uncertainty in any stochastic model of the action,

- whether the action has a favourable or an unfavourable effect: for example, in one load case the weight of a component might be opposing the governing action, e.g. pressure, and, therefore, has a favourable effect. In another, the weight might be acting with the pressure and so the weight has an unfavourable effect. In the two corresponding load cases, the partial safety factor of this weight would have a different value. If the governing action is not obvious, separate load cases are required, and
- the possibility of non-conservative deviations of material strength parameters from their characteristic values (of the material strength parameters).

**Coefficients of variation:** The coefficient of variation is a measure of the statistical dispersion of a random quantity; it is given by the ratio of the standard deviation of the quantity to its mean value.

**Combination factors:** The combination factor is applied to design values of variable actions with stochastic properties if these actions are combined with pressure, or if two or more of these actions are included in one multi-action load case. These combination factors are applied to account for the likelihood that extreme values of these stochastic actions and pressure occur coincidentally.

**Load case specifications:** A load case specification is a load case-related list of actions, their characteristic values, and, possibly, characteristic functions, for which the equipment is designed, as specified by the manufacturer, together with the partial safety factors used. For cyclic actions, the load case specifications contain the number of design cycles, and, for components designed for operation in the creep regime, they contain the design life. To each load case there corresponds one load case specification. All characteristic values listed in one load case specification are considered to be possibly attained coincidentally, and coincidentally with values of the listed characteristic functions at any time.

The partial safety factors used should be specified and included in the load case specification. Whenever the choice follows uniquely from the specification of partial safety factors in EN 13445-3 Annex B, a simple reference to this Annex B suffices. Partial safety factors depend also on the design check – it is usually required to list different values for the different relevant design checks in a load case specification.

Design values of actions and design functions of actions may be used instead of characteristic values and characteristic functions of actions. The complete set of load case specification is a part of the design specification, to be included in the technical documentation.

**Principles:** This designation is used in design checks for

- general or definitive statements for which no alternative exist unless specifically stated otherwise and

- requirements and design models for which no alternative exists unless specifically stated otherwise.

**Application rules:** This designation is used in design checks for generally recognized rules that follow and satisfy the principle's requirements. Alternative rules, different from the specified application rules, may be used, provided that it is shown that the alternative rule accords with the relevant principle and is at least equivalent with regard to reliability, serviceability, and durability.

## 2.3. General on Characteristic Values and Characteristic Functions of Actions

### 2.3.1. Requirements in the Pressure Equipment Directive

The definition of characteristic values of actions given in the preceding section is quite general, and indicates the stochastic nature of (all) actions, and can easily be misinterpreted to give freedom of choice in specifications that is actually not there.

The definitions and their use in specifying characteristic values of actions must stay within the context of the legal framework – the PED [1] – and must take into consideration the state of the rules of technology, as e.g. specified in the DBF part of the harmonized standard [2].

Requirements of the legal framework must be adhered to, requirements from other relevant standards or from other parts of EN 13445 must be taken into consideration, and deviations that result in smaller wall thickness must be justified.

The most important requirements are laid down in the PED [1], in Annex I Sections 1 and 2:

- Pressure equipment must be designed, manufactured and checked, and if applicable equipped and installed, in such a way as to ensure its safety when put into service in accordance with the manufacturer's instructions, or in reasonably foreseeable conditions (PED Annex I, 1.1).*
- The pressure equipment must be properly designed taking all relevant factors into account in order to ensure that the equipment will be safe throughout its intended life (PED Annex I, 2.1).*
- The design must incorporate appropriate safety coefficients using comprehensive methods which are known to incorporate adequate safety margins against all relevant failure modes in a consistent manner (PED Annex I, 2.1).*
- Various loadings, which can occur at the same time must be considered, taking into account the probability of their simultaneous occurrence (PED Annex I, 2.2.1).*

- (e) *The pressure equipment must be designed for loadings appropriate to its intended use and other reasonably foreseeable operating conditions (PED Annex I, 2.2.1).*
- (f) *The calculation pressure must not be less than the maximum allowable pressure and must take into account static head and dynamic fluid pressures and the decomposition of unstable fluids (PED Annex I, 2.2.3 (b), 1st indent).*
- (g) *The calculation temperatures must allow for appropriate safety margins (PED Annex I, 2.2.3 (b), 2nd indent).*
- (h) *The design must take appropriate account of all possible combinations of temperature and pressure which might arise under reasonably foreseeable operating conditions for the equipment (PED Annex I, 2.2.3 (b), 3rd indent).*
- (i) *Where, under reasonably foreseeable conditions, the allowable limits could be exceeded, the pressure equipment must be fitted with, or provisions made for the fitting of, suitable protective devices, unless the equipment is intended to be protected by other devices within an assembly (PED Annex I, 2.10).*
- (j) *Pressure limiting devices must be so designed that the pressure will not permanently exceed the maximum allowable pressure; however a short duration pressure surge in keeping with the specifications laid down in 7.3 is allowable, where appropriate (PED Annex I, 2.11.2).*

Section 7 of Annex I of the PED gives specific quantitative requirements that apply as a general rule. Where these rules are not applied, the manufacturer must demonstrate that appropriate measures have been taken to achieve an equivalent overall level of safety. These requirements are obviously laid down keeping in mind the DBF approach as the “standard” approach.

The requirements of Section 7 of Annex I most relevant for the present section are:

- (k) *The momentary pressure surge referred to in Section 2.11.2 must be kept to 10% of the maximum allowable pressure (PED Annex I, 7.3),*

and the (not very expertly formulated) requirements in 7.1.2 of Annex I for “permissible general membrane stresses for predominantly static loads and for temperatures outside the range in which creep is significant” for ferritic steels including normalized (normalized rolled) steels and excluding fine-grained steels and specially heat-treated steels, for austenitic steels, for cast steels, for aluminium and aluminium alloys excluding precipitation hardening alloys.

These requirements in 7.1.2 of Annex I are indications of the expected level of safety via the specification of requirements for determining the allowable stresses for some specified materials. **Allowable stress** is a term used in the PED for the stresses to be used in the DBF approach, a stress that is called nominal design stress in EN 13445. Since the formulae for determining the allowable stresses are given, the corresponding global safety factors can be deduced.

### 2.3.2. *Consequences from the PED Requirements*

The definitions of maximum allowable pressure and maximum/minimum allowable temperature are definitions in the PED, fixed by law, required to be provided in legal documents and required to be used in marking and labelling.

The definition of minimum allowable pressure can be deduced directly, without doubt, via a consequent extension of the legal definition of the one for the maximum allowable pressure. Maximum/minimum allowable pressure and maximum/minimum allowable temperature are values to be specified by the manufacturer, as values for which the equipment is designed or has been designed. The definitions of maximum allowable loads, and of maximum allowable actions, are straightforward, are consequent extensions of the definition of maximum allowable pressure, already indicated in the PED (see Section 2.3.1 (d) and (e)).

Taking into account the definitions, given in Section 2.2, and the requirements of the PED cited in the preceding section, the following requirements on characteristic values can be deduced:

- (a) For design checks that deal with gross plastic deformation, with structural stability, or with progressive plastic deformation, the load cases for normal operating conditions must include an upper characteristic value for pressure that, at the reference point of the maximum allowable pressure, is not smaller than the maximum allowable pressure, and must include an upper characteristic value for the temperature that, at the reference point of the maximum allowable temperature, is not smaller than the maximum allowable temperature.
- (b) If, for any reason, a maximum design pressure is specified, this maximum design pressure must not be smaller than the maximum allowable pressure, and then the upper characteristic value of pressure mentioned in (a) above must not be smaller than the specified maximum design pressure.
- (c) For the design checks and load cases mentioned in (a) above, it may be necessary to include load cases with combinations of upper characteristic values of temperature and lower characteristic values of pressure.
- (d) For the design checks and load cases referred to in (c) above, the lower characteristic value of pressure must not be larger than the minimum allowable pressure or the minimum design pressure. If this minimum pressure is negative this means that the absolute value of the (negative) characteristic value must not be smaller than the absolute value of the (negative) minimum allowable pressure or minimum design pressure. The minimum design pressure must not be larger than the minimum allowable pressure.

- (e) In cases of multi-chamber vessels it may be necessary to include load cases with combinations of upper characteristic values of pressure and lower ones in the various chambers, if it cannot be ensured that extreme combinations cannot occur under reasonably foreseeable operating conditions.
- (f) The interdependency of single actions in a load case may be taken into consideration, taking into account the probability of their simultaneous occurrence – e.g. more than one pair of upper characteristic value of pressure and upper characteristic value of temperature may be specified, each pair resulting in at least one load case.

On the other hand, the upper characteristic value of pressure need not be higher than the corresponding maximum allowable pressure. Short duration pressure surges need not be taken into account in the specification of the upper characteristic value of pressure – these pressure surges are taken into account in the relevant partial safety factors.

It follows that in cases where a pressure limiting device is not required, i.e. where under reasonably foreseeable conditions the pressure cannot exceed the maximum allowable pressure, the maximum allowable pressure is an upper limit of pressure, and smaller partial safety factors for pressure may be justified, smaller than in cases where short pressure excursions are to be included in the load case specification.

Annex B of EN 13445-3 adheres to this approach, but is sometimes not specific enough. As an example, in the sub-clause on characteristic values and characteristic functions of actions:

*The upper characteristic value of the pressure may be based on the maximum allowable pressure, the pressure accumulation at a pressure relief device when the pressure relief device starts to discharge, the pressure increase over the maximum allowable pressure need not be taken into account (EN 13445-3 Annex B, B.6.2).*

For actions other than pressure and temperature the requirements in the PED are less explicit, and refer to “loadings appropriate to the intended use and other reasonably foreseeable operating conditions” (see Section 2.3.1(e), and “various loadings which can occur at the same time must be considered, taking into account the probability of their simultaneous occurrence” (see Section 2.3.1(d)).

Annex B of EN 13445-3 fills this lack of detail by giving general to specific requirements for determining the characteristic values of other types of actions, in sub-clause B.6.2. These requirements are quite flexible, and even allow the usage of highest and lowest credible values, but unfortunately only for permanent actions for which a statistical approach is not possible. Characteristic values for forces and moments from attached piping are not specifically addressed, and characteristic functions are addressed only in the general way stated in the definition of characteristic functions of actions.

## 2.4. General on Design Models and Constitutive Laws

### 2.4.1. *General on Design Models*

Design checks are related to specific failure modes and, therefore, in the design models for different design checks

- different geometrical idealizations may be required,
- different constitutive laws may be required, and
- different kinematic, equilibrium, and boundary conditions may be required.

The geometry of design models has to be based on the nominal values of individual dimensions, with the exception of thicknesses, for which analysis thicknesses have to be used. In determining the analysis thickness, whether the whole of the corrosion or erosion allowance or only part of it has to be subtracted depends on the specific design check, and is dealt with in the design check-specific chapters (see e.g. Section 7.3).

Whether cladding has to be, may be, or must not be included in the models depends on the specific design check, and is dealt with in the design check specific sections.

Geometric deviations from the nominal, ideal shape have to be included in all design models if the allowed values specified in Part 4 of EN 13445 are exceeded.

Other geometric deviations, allowed according to Part 4 of EN 13445, have to be included in some design models, e.g. design models of the instability design check. The values of the deviations may be already known ones or maximum permissible ones – allowed by direct or indirect reference to Part 4 of EN 13445, or by explicit specification on the drawings.

Almost always part models have to be used, and then care is required that the models include all the parts of the structure which are necessary to capture possible elastic follow-up effects.

With regard to geometrical idealizations, in some design models a stress-concentration-free model suffices and in others local structural perturbation sources have to be included.

In some design checks, structural stresses or strains have to be determined. Some models may give these directly, e.g. some finite element models using shell or beam elements. In cases where the model does not give structural stresses and strains directly, e.g. because local structural perturbation sources are kept in the model or are introduced by the geometrical idealization, the required value of the structural stress or strain in a critical point (hot spot) may be

- determined by quadratic extrapolation along the surface with surface pivot points at distances  $0.4e$ ,  $0.9e$  and  $1.4e$  from the critical point, where  $e$  is the (fatigue) relevant thickness of the structure at the critical point,
- replaced by the total stress or strain in any model that deviates from the stress-concentration-free model only in local structural perturbation sources. Of course, total stress or strain in the model investigated may always be used instead of the extrapolated values, with the total stress or strain being determined by the software's solution procedure in the hot spot, or by extrapolation into the hot spot within the element, or total stress or strain in the element integration points.

The extrapolation line is not specified in the standard, only the distances of the pivot points are. Depending on the design check, the related failure mode may be crack initiation or crack propagation of an existing crack. The extrapolation line then recommended is the intersection of the model's surface with the plane containing the normal to the considered incipient crack plane and the surface normal at the hot spot. Frequently, the extrapolation line is given by the normal to the considered incipient crack plane at the hot spot.

For the quadratic extrapolation, denoting the quantity of interest – the structural stress or strain component – by  $y$ , and the corresponding one in the pivot point  $P_i$  by  $y_i$ , the extrapolated value of  $y$  in the hot spot is given by

$$y_0 = 2.52y_1 - 2.24y_2 + 0.72y_3, \quad (2.2)$$

where  $P_1$  is the pivot point nearest to the critical one,  $P_2$  the next, and  $P_3$  the farthest point.

The idealization process in modelling may introduce new local structural perturbation sources or may modify existing ones. In the example of the flat end to cylinder connection shown in Fig. 2.2, the total stress considered in the detailed model depends strongly on the (notch) radius at the weld toe and a sharp corner, i.e. a vanishing weld toe radius, may result in a stress and strain singularity.

Use of structural stress and strain, on the other hand, ideally eliminates this dependence on local structural perturbation sources – results of the extrapolation are practically independent of local geometry details and fairly insensitive to mesh size.

Finite element models with plate and shell elements, on the other hand, may in this example give large stress and strain values at surface points of the model perpendicular to the intersection of the mid-surfaces, values that are not relevant. In such cases the values corresponding to the evaluation cross-sections, shown in the figure, may be used.



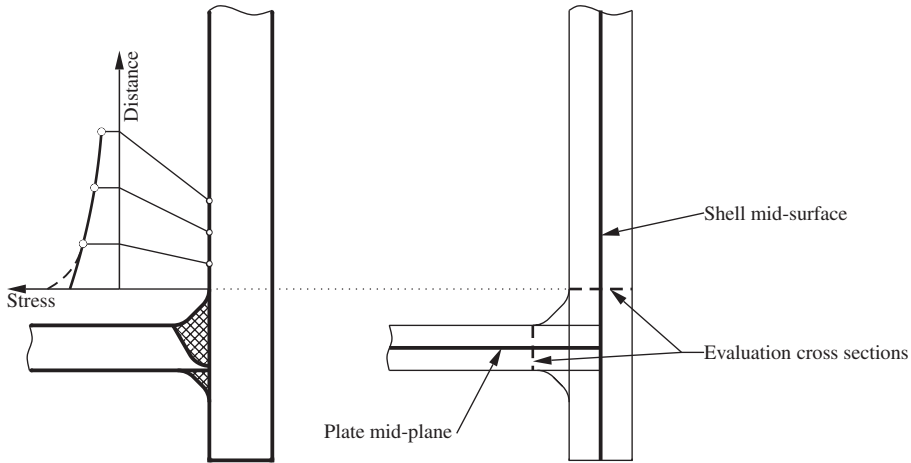


Figure 2.2: Modelling of local geometry details.

**Other characteristics:** With regard to geometric relations and boundary conditions, in some design models **geometrically non-linear relations** are required, i.e. non-linear kinematic relations and equilibrium conditions (between stress and surface traction components) applied on the deformed structure, and, if it can be shown to be accurate enough, **second-order-theory** may be used, i.e. linear kinematic relations and equilibrium conditions applied on the deformed structure. In other design models **first-order-theory**, i.e. **geometrically linear theory**, must be used, i.e. linearized kinematic relations and equilibrium conditions are applied on the undeformed structure.

For all design models, sufficiently slow rates of change of actions are presupposed such that quasi-static models can always be used – all used design models are quasi-static.

For all design models sufficiently small deformation gradients are presupposed such that differences in the various definitions of stress and strain tensors are unimportant and the “usual” definitions of the classical theory of infinitesimal deformations may be used.

In all design models isotropic constitutive laws are used, and all design models are homogeneous, with the exception of points of material change.

In some design checks an initial (weightless) stress state is required – in these design checks a stress-free initial state has to be used, with one exception: In the fatigue design check, residual stresses due to plastic deformation in the hydraulic (or pneumatic) test of the final assessment may be taken into account.

### 2.4.2. General on Constitutive Laws

An actual structure is a very complex body with complex material behaviour, already fabrication steps produce residual stresses and also anisotropic material behaviour. Real materials show essentially non-isotropic behaviour, especially with regard to initial and subsequent yield limits, however defined. It takes considerable effort to produce a structural material that is initially isotropic in its inelastic response.

In view of the practically infinite complexity of the inelastic behaviour of real materials, and keeping in mind that the design models are used solely in design checks, isotropic constitutive laws are used in all design models, i.e. constitutive laws that are independent of material orientation.

In the various design models two types of constitutive laws are used:

- linear-elastic constitutive laws and
- linear-elastic ideal-plastic constitutive laws, with specific yield condition and associated flow law.

**Linear-elastic constitutive laws:** In the linear-elastic models the classical Hooke's law of elasticity holds, i.e. an (unbounded) linear relationship exists between stress and strain, which is in cases of thermal stress problems generalized to include the strain due to temperature changes. The elastic constants, the coefficient of linear thermal expansion, thermal diffusivity, conductivity, and specific thermal capacity (for temperature calculations) are to be taken from EN 13445-3 Annex O, at a temperature specified for the relevant design check.

Whether the coefficient of linear thermal expansion between the reference temperature of the strain-free state and the relevant metal temperature is used or the differential coefficient of linear expansion depends on the software procedure. Usually, principal values of the spherical thermal strain tensor in the form  $\beta \cdot \Delta t$  are used, with the temperature difference  $\Delta t = t - t_{ref}$ , between the current temperature  $t$  and the reference temperature of the strain-free state  $t_{ref}$ . In this case, in which the temperature difference referred to is the temperature in the initial (strain-free) state, the coefficient of linear thermal expansion between the current temperature and the reference temperature is to be used for  $\beta$ , and not the differential coefficient of linear thermal expansion.

**Linear-elastic ideal-plastic constitutive laws:** In the elastic-plastic models the constitutive laws

- for initial monotonic loading (from the initially stress-free state),
  - for monotonic unloading from plastic flow, and
  - for subsequent monotonic reloading, after unloading,
- are linear-elastic, as described above.

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The limit of the region of elastic behaviour, the **yield limit**, is specified by the so-called **yield condition**, given, for the ideal-plastic constitutive law independent of previous deformation, by an equation of the form

$$f(\sigma_{ij}, R) = 0, \quad (2.3)$$

where  $\sigma_{ij}$  are the components of the stress tensor, and  $R$  is the so-called **yield stress**. In the design models the yield stress is given by the design value of the material strength parameter:

$$R = RM_d. \quad (2.4)$$

The scalar-valued function  $f$  is called **yield function**. This yield function is convex, and is defined in such a way that  $f < 0$  corresponds to elastic behaviour. The yield stress  $R$  may vary from point to point of the structure, either because of material inhomogeneities, e.g. dissimilar base materials or at weld junctions, or because of non-uniform temperature distributions. For a stationary temperature at a specific point of the structure the yield stress is also stationary.

Ideal-plastic behaviour requires that during plastic deformation the stress state (in a specific point of the structure) must always satisfy the yield condition, and that there is neither hardening nor softening. In a uniaxial stress-strain diagram this behaviour is given by parallel lines for elastic loading, unloading, and reloading, and a horizontal straight line, at constant stress  $R$ , for plastic flow, see Fig. 2.3.

For an infinitesimal change of the stress state (at a specific point of the structure), with infinitesimal changes of the components  $d\sigma_{ij}$  and corresponding infinitesimal change of the yield function  $df$ ,  $df > 0$  is usually called **loading** (at the considered point),  $df < 0$  **unloading**, and  $df = 0$  **neutral loading**; and  $f < 0$ ,  $df > 0$  corresponds to loading in the elastic regime,  $f = 0$ ,  $df = 0$  to plastic deformation,  $f = 0$ ,  $df < 0$  to unloading from the plastic regime, and  $f < 0$ ,  $df < 0$  to unloading in the elastic regime. These designations, as defined and used here, relate to changes of the values of the yield function in a point of the structure. In the literature they are used, quite frequently and especially in connection with simple textbook examples, also for changes in the response of structures to actions resulting in changes of the integral of the values of the yield function over the whole structure or a whole part of it.

In all the elastic-plastic design models used, **associated flow rules** apply for plastic deformations: In all stress states with plastic deformation and where the yield function is smooth, the yield function serves as potential function for the infinitesimal increments of plastic strain:

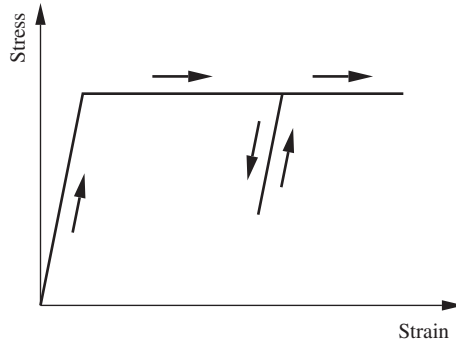


Figure 2.3: Linear-elastic ideal-plastic model, uniaxial stress state.

$$d\varepsilon_{ij}^P = d\lambda \left( \frac{\partial f}{\partial \sigma_{ij}} \right), \quad (2.5)$$

where  $d\lambda$  is the infinitesimal change of a scalar proportionality factor, the **plastic multiplier**. This relationship for the infinitesimal increment of plastic strain is also called **normality rule**.

In all elastic-plastic design models plastic incompressibility applies for plastic deformation – non-vanishing volume dilatation is caused only by elastic stresses or temperature changes.

It is helpful to visualize stress states as points in a nine-dimensional Euclidean space whose coordinates are the stress components. The yield condition  $f=0$  defines in this space a surface, called the **yield surface**. Stress points inside the yield surface correspond to elastic behaviour, and no plastic deformations occur and all incremental deformations are linear-elastic. Plastic deformations occur as long as stress points are in the yield surface. For plastic flow to continue, a stress point must remain in the yield surface.

If plastic strain states are visualized in the same nine-dimensional Euclidean space, the normality rule renders a nice presentation of the infinitesimal increment of plastic strain or of the plastic strain rate – its direction coincides with the normal to the yield surface in the corresponding stress point.

The deviatoric map is a useful tool in the visualization of stress paths – the development of the stress state as a function of time – in specific points, lines or surfaces, vis-à-vis onset of plastic deformation. It is also useful in the presentation, discussion, and checking of results involving plastic deformation, and in the identification of critical stress ranges, if these are related to equivalent stress. Almost indispensable in the visualization, presentation, and plausibility checking of

results, are the colour or contour plots of the so-called equivalent stress, as a function of space and at specific instants.

**Deviatoric map:** The deviatoric map, or the deviatoric projection, a tool in plasticity theory for visualizing stress states vis-à-vis yield conditions, is, in principle, the projection of the stress point in the (three-dimensional, Cartesian) space of principal stresses – with coordinates in the directions of the unit vectors  $e_1, e_2, e_3$  equal to the magnitudes of the three principal stresses – onto the deviatoric plane, the plane which is normal to the hydrostatic axis given by equal principal stresses. Using this deviatoric projection as a tool, this projection point can be obtained simply by vector addition of  $\sigma_1 e_1^d, \sigma_2 e_2^d$  and  $\sigma_3 e_3^d$ , with an arbitrary (fixed) scale, in an isometric plot of the unit vectors  $e_1^d, e_2^d, e_3^d$ , having the three axes equally inclined and drawn to equal scale, see Fig. 2.4 and [8].

In this deviatoric map yield limits are represented by closed and convex curves.

Of course, in this mapping of the nine-dimensional stress state onto the two-dimensional deviatoric map much information of the stress state is lost: The orientation of the principal stress axes relative to the basic material coordinate system in the corresponding point of the structure, and also the value of the mean stress. Starting from a stress point in the deviatoric map, only the values of the principal stresses except for a common additive term, the mean stress, can be recovered.

Despite this loss of information, the deviatoric map is for models with mean stress independent yield limits a very informative tool in the visualization of the development of plastic deformation, the response of design models to monotonic and cyclic actions, in the visualization of stress paths of cyclic or recurring actions for (plausibility) checks of response modes [17], and, in the easy determination of critical equivalent stress ranges required, e.g. in fatigue design checks.

Use of compatibility ratios of principal stresses, instead of principal stresses, is recommended, especially in cases of yield stresses that depend on space and/or time.

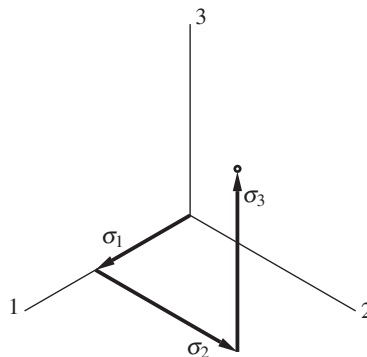


Figure 2.4: Deviatoric map.

*Note:* When plotting stress paths in the deviatoric map, it is of paramount importance to keep the initially chosen identity of principal stress axes over the whole history. FEM software and mathematical routines usually designate the principal stresses according to their values in the order  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ , without indicating that this may mean a change of axes. Therefore, one principal stress axis that corresponds initially to  $\sigma_1$  may, in the course of the action history, become the axis of the middle or even the smallest principal strain, and the corresponding value of the FEM output may then be  $\sigma_2$  or  $\sigma_3$ . In a plot of the history of principal strains this is easily recognizable, but, admittedly, there are cases where identification is not easy, and may require time-consuming separate investigations. In an automatic plot of the stress path in the deviatoric map, wrong identity of principal axes, if uncorrected, may lead to non-explainable behaviour. If identification is achieved, then renumbering of the principal stresses after the crossing point of the principal stresses provides a simple remedy (see Annex E.5.2).

**Equivalent stresses:** The equivalent stress is a non-negative scalar, with the dimension of stress that is equivalent to the stress state (with six independent components) vis-à-vis the yield function.

For all elastic-plastic design models of the standard the yield condition can be expressed in the form

$$f_{eq}(\sigma_{ij}) - R = 0. \quad (2.6)$$

With the equivalent stress  $\sigma_{eq}$ , defined by

$$\sigma_{eq} = f_{eq}(\sigma_{ij}), \quad (2.7)$$

the yield condition can be written in the simple (and instructive) form as

$$\sigma_{eq} = R. \quad (2.8)$$

As a useful reminder:

- the yield function can be negative, zero, and positive; negative values correspond to stress states in the elastic regime; a value of zero corresponds to plastic deformation; positive values are, strictly speaking, not possible in ideal-plastic results, but correspond to mistakes or numerical inaccuracies,
- the equivalent stress can only be non-negative,
- in the deviatoric map stress points outside the yield limit are not possible for ideal-plastic results, those for (unbounded) linear-elastic models are, and
- loading and unloading does not refer to the change of actions, but solely to the change of the value of the yield function, or the change of the value of the equivalent stress. A change of actions may result in loading at points of one part

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of the structure and in neutral loading, or even unloading, at points of another. A change of actions may result in one and the same point in loading with regard to Mises' yield condition, but in unloading or neutral loading with regard to Tresca's.

**Yield conditions:** In the elastic-plastic design models two yield conditions are used: Tresca's yield condition and Mises' yield condition. Both yield conditions are isotropic and independent of the mean stress.

**Tresca's yield condition:** In Tresca's yield condition the yield function  $f(\sigma_{ij}, R)$  is given by

$$\text{Max}[|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|] - R, \quad (2.9)$$

where  $\sigma_1, \sigma_2, \sigma_3$  are the principal stresses of the stress tensor  $\sigma_{ij}$ .

The equivalent stress  $\sigma_{eqT}$  is given by

$$\sigma_{eqT} = \text{Max}[|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|], \quad (2.10)$$

Tresca's yield condition corresponds to the maximum shear stress hypothesis, that the onset of plastic deformation depends on the value of the maximum shear stress.

**Mises' yield condition:** In Mises' yield condition the yield function  $f(\sigma_{ij}, R)$  is given by

$$(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2) - 2R^2, \quad (2.11)$$

or

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - 2R^2, \quad (2.12)$$

or

$$J_2 - \frac{R^2}{3}, \quad (2.13)$$

where, as above,  $\sigma_1, \sigma_2$  and  $\sigma_3$  are the principal stresses, and  $J_2$  is the second invariant of the stress deviator.

The equivalent stress is given by

$$\sigma_{eqM} = + \sqrt{[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)]/2}, \quad (2.14)$$

or

$$\sigma_{eqM} = + \sqrt{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]/2}. \quad (2.15)$$

The right-hand side of Equation (2.14) is called **Mises' equivalent stress function**, and denoted here by  $MEQ [\sigma_{ij}]$ . The same function is also used in the definition of Mises' equivalent strain, defined as  $MEQ [\epsilon_{ij}]$ .

Mises' yield condition corresponds to the maximum distortion energy hypothesis or the maximum octahedral shear stress hypothesis, that the onset of plastic deformation depends on the value of the maximum distortion energy or the maximum octahedral shear stress.

**Octahedral stresses:** Octahedral shear stress and octahedral normal stress are shear stress and normal stress, respectively, acting at the octahedral planes, i.e. the (eight) planes with equal angles to the three principal stress directions.

The value of the **octahedral shear stress**  $\tau_o$ , used in one of the interpretations of Mises' equivalent stress, is given by

$$\tau_o = \sqrt{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] / 9}, \quad (2.16)$$

where  $\sigma_i$ ,  $i=1, 2, 3$ , are the principal stresses of the stress tensor  $\sigma_{ij}$ .

The value of the **octahedral normal stress**  $\sigma_{no}$ , used in the mean stress correction of fatigue design curves of unwelded regions (Section 7.6.5), is given by

$$\sigma_{no} = (\sigma_1 + \sigma_2 + \sigma_3). \quad (2.17)$$

**Yield limit in the deviatoric map:** In the deviatoric map Mises' yield limit is represented by a concentric circle of radius  $R$ , and Tresca's yield limit by a concentric and regular hexagon. If both yield limits agree for the uniaxial case, the hexagon is inscribed in the circle (see Fig. 2.5).

Fig. 2.4 shows well that Tresca's yield condition always predicts onset of plastic deformation not later than Mises', the maximum deviation for a proportional increase of stress, i.e. for a radial stress path is  $R \cdot (2/\sqrt{3} - 1)$ , approximately 15.5%.

The loci of stress states of constant equivalent stress are represented by concentric circles if related to Mises' yield function, and by concentric and regular hexagons if related to Tresca's; neutral loading is represented by a stress path, possibly degenerated to a point, on a circle if related to Mises' yield function, and on a concentric and regular hexagon if related to Tresca's.

In a deviatoric map of stress to yield ratios, Mises' yield limit is a circle of radius equal to unity, and Tresca's yield limit the largest concentric, regular, and inscribed hexagon.

**Compatibility with yield condition:** A stress field is said to be compatible with a yield condition if the corresponding equivalent stress is nowhere larger than the corresponding yield stress.



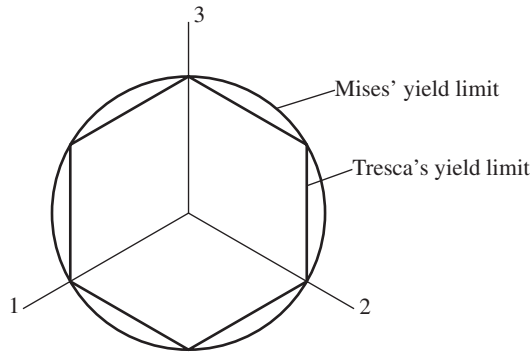


Figure 2.5: Mises' and Tresca's yield limit in deviatoric map.

**Compatibility ratios:** The quotients of the values of principal stresses at a specific instant and the corresponding, relevant yield stress at the same instant are called compatibility ratios of the principal stresses. These compatibility ratios of principal stresses are conveniently used while plotting stress paths in deviatoric maps, especially in cases where the yield stresses depend on time.

The quotients of equivalent stresses at specific instants and the corresponding, relevant yield stresses at the same instants are called compatibility ratios of the stress states. The compatibility ratio of a stress state is a scalar measure of the stress state vis-à-vis onset of plastic deformation – standardized such that the onset of plastic flow or plastic flow in a perfectly plastic model, corresponds to unity. These compatibility ratios are conveniently used while plotting stress distributions in models, especially in cases where the yield stress is space-dependent, be it due to different materials or due to space-dependent temperatures.

**Generalized stresses and local limit states:** The yield functions, defined above, serve not only as conditions for the onset of yielding, but may also be used as a measure of the closeness of a stress state to the onset of yielding *at a specific point*.

The analogous functions for full plasticity *in a cross-section or along a line* through a section thickness, however defined, are called **local technical limit state functions** or **local limit state functions in generalized stresses**. These functions can serve as conditions for full plasticity in a cross-section or along a line through the section thickness, but are especially suitable as a measure of the degree of plasticity (in a cross-section or along a line through a section thickness).

Generalized variables are introduced in structural analysis of shells and plates to describe static and kinematic quantities in order to reduce complicated three-dimensional problems to simpler two- or one-dimensional ones, by means of kinematic hypotheses, such as Kirchhoff's hypothesis.

For shells and plates the **generalized stresses** are the stress resultants, defined, for example, by

$$n_{\alpha\beta} = \int_{-e/2}^{e/2} \sigma_{\alpha\beta} dz, \quad m_{\alpha\beta} = \int_{-e/2}^{e/2} z \sigma_{\alpha\beta} dz, \quad q_\alpha = \int_{-e/2}^{e/2} \sigma_{z\alpha} dz, \quad (2.18)$$

where  $e$  is the thickness at the cross-section,  $z$  the coordinate in thickness direction (normal to the middle surface), and  $\alpha, \beta = 1, 2$  indicate directions in the middle surface.

The **local technical limit state function** defines the locus of all combinations of generalized stresses that correspond to full plasticity (at a specific cross-section or along the line in thickness direction) for a specific linear-elastic ideal-plastic constitutive law.

A reasonably good, recommended approximation for relatively thin shells and plates with Mises' yield condition is given by Ivanov's (second) yield function [40], defined by

$$Iv^2 = \bar{Q}_n + \left[ \bar{Q}_m + \sqrt{\bar{Q}_m^2 + 4\bar{Q}_{mn}^2} \right] / 2 - \left[ \frac{\bar{Q}_n \cdot \bar{Q}_m - \bar{Q}_{mn}^2}{\bar{Q}_n + 0.48 \bar{Q}_m} \right] / 4 \quad (2.19)$$

with

$$\bar{Q}_n = [n_1^2 + n_2^2 - n_1 \cdot n_2 + 3n_{12}^2] / (R \cdot e)^2, \quad (2.20a)$$

$$\bar{Q}_m = [m_1^2 + m_2^2 - m_1 \cdot m_2 + 3m_{12}^2] / (R \cdot e^2 / 4)^2, \quad (2.20b)$$

$$\bar{Q}_{mn} = [m_1 \cdot n_1 + m_2 \cdot n_2 - (m_1 \cdot n_2 + m_2 \cdot n_1) / 2 + 3m_{12} \cdot n_{12}] / [(R \cdot e)(R \cdot e^2 / 4)], \quad (2.20c)$$

and where  $Iv=1$  corresponds to full plasticity, i.e. onset of unconstrained plastic flow (provided the surrounding material allows for that). In the derivation of Ivanov's yield function, shear has been ignored – Ivanov's yield function fails when shear cannot be neglected.

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## Chapter 3

# Design Checks and Load Cases

### 3.1. Design Checks

In the design of components with calculation temperatures below the creep range, the following five design checks have to be considered:

- Gross Plastic Deformation Design Check (GPD-DC),
- Progressive Plastic Deformation Design Check (PD-DC),
- Stability Design Check (S-DC),
- Fatigue Design Check (F-DC), and
- Static Equilibrium Design Check (SE-DC).

In the design of components with calculation temperatures in the creep range, the following three design checks have to be considered additionally:

- Creep Rupture Design Check (CR-DC),
- Excessive Creep Strain Design Check (ECS-DC), and
- Creep Fatigue Interaction Design Check (CFI-DC).

While work on these design checks in the sub-group Design Criteria of TC 54 WG C and the enquiry procedure has been completed, the required discussion is ongoing and the voting procedure is as yet far away. Therefore, this book is limited to load cases with calculation temperatures below the creep range.

The range of calculation temperatures for which creep related design checks are not required is given by the calculation temperatures for which the 0.2%-proof stress is not larger than both of the following two values,

- the product of 1.2 and the creep rupture strength at calculation temperature and for the relevant lifetime,
- the product of 1.5 and the 1% creep strain strength at calculation temperature and for the relevant lifetime.

The designations creep rupture strength and 1% creep strain strength refer to mean values, as specified in the material standard, for which a scatter band of experimental results of  $\pm 20\%$  is assumed. For larger scatter bands, 1.25 times the minimum band values should be used instead of mean values.

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That a design check has to be considered does not mean that calculations are actually required – it may be that the failure mode corresponding to the design check is not relevant, or that another design check makes the considered one superfluous – however, the design check has to be addressed in the design documentation.

It is the duty of the designer to ascertain whether additional design checks are required, and that all relevant design checks – the obligatory and the possibly required additional ones – have been addressed. For example, especially for materials with high ductility, design checks related to failure modes of excessive local deformation, i.e. excessive local deformation at mechanical joints (with possible leakage problems), or violation of service restraints might be required.

As a checklist, in support of this activity, the list of failure modes and limit states given in EN 13445-3 Annex B is reproduced in Table 3.1.

Table 3.1: Classification of failure modes and limit states

Failure mode	Action type				
	Short-term		Long-term		Cyclic
	Single application	Multiple application	Single application	Multiple application	
Brittle fracture	U				
Ductile rupture <sup>3</sup>	U				
Excessive deformation 1 <sup>4</sup>	S, U <sup>1</sup>				
Excessive deformation 2 <sup>5</sup>	U				
Excessive deformation 3 <sup>6</sup>	S				
Excessive local strain <sup>7</sup>	U				
Instability <sup>8</sup>	U, S <sup>2</sup>				
Progressive plastic deformation <sup>9</sup>		U			
Alternating plasticity <sup>10</sup>		U			
Creep rupture			U		
Creep-excessive deformation 1 <sup>11</sup>			S, U <sup>1</sup>		
Creep-excessive deformation 2 <sup>12</sup>			U		
Creep-excessive deformation 3 <sup>13</sup>			S		
Creep instability			U, S <sup>2</sup>		
Erosion, corrosion			S		
Environmentally assisted cracking <sup>14</sup>			U		

(Continued)

Table 3.1: (Continued)

Failure mode	Action type				
	Short-term		Long-term		Cyclic
	Single application	Multiple application	Single application	Multiple application	
Creep-excessive deformation 1 <sup>11</sup>				S, U <sup>1</sup>	
Creep-excessive deformation 2 <sup>12</sup>				U	
Creep-excessive deformation 3 <sup>13</sup>				S	
Creep instability				U, S <sup>2</sup>	
Erosion, corrosion				S	
Environmentally assisted cracking <sup>14</sup>				U	
Fatigue					U
Environmentally assisted fatigue					U

Note: U indicates ultimate limit state and S serviceability limit state.

1. In case of risk due to leakage of content (toxic, inflammable, steam, etc.).
2. In case of sufficient post-instability load-carrying capacity.
3. Unstable gross plastic yielding or unstable crack growth.
4. Excessive deformation at mechanical joints.
5. Excessive deformation resulting in unacceptable transfer of load.
6. Excessive deformation related to service restraints.
7. Resulting in crack formation or ductile tearing by exhaustion of material ductility.
8. Elastic, plastic, or elastic-plastic.
9. Progressive plastic deformation (or ratchetting).
10. Alternating plasticity.
11. Creep-excessive deformation at mechanical joints.
12. Creep-excessive deformation resulting in unacceptable transfer of load.
13. Creep-excessive deformation related to service restraints.
14. Stress corrosion cracking (SCC), hydrogen-induced cracking (HIC), and stress orientated hydrogen-induced cracking (SOHIC).

## 3.2. Load Cases

The preparation of a design specification is a very responsible task, and it is the responsibility of the user, or his/her designated agent, to provide sufficient details, but, nevertheless, the final responsibility lies with the manufacturer:

*The manufacturer is under the obligation to analyse the hazards in order to identify those which apply to his equipment on account of pressure; he must then design and construct it taking account of his analysis (PED Annex I, 3).*

*Pressure equipment must be designed, manufactured and checked, and if applicable equipped and installed, in such a way as to ensure its safety when put into service in accordance with the manufacturer's instructions, or in reasonably foreseeable conditions (PED Annex I, 1.1).*

*Where the potential for misuse is known or can be clearly foreseen, the pressure equipment must be designed to prevent danger from such misuse or, if that is not possible, adequate warning given that the pressure equipment must not be used in that way (PED Annex I, 1.3).*

It is the manufacturer's responsibility

- to identify all actions that can affect the safety of the pressure vessel under reasonably foreseeable conditions;
- to have these actions – together with their appropriate characteristic values, characteristic functions, and used partial safety factors – grouped together into sets of actions that can occur coincidentally under reasonably foreseeable conditions, i.e. to have the load case specifications in writing;
- to have all specified exceptional load cases included in the list of load cases and in the technical documentation.

Because of the fact that

- Design by Formulae is still the “usual” route in design – a route in which the maximum allowable pressure together with the maximum allowable temperature governs the design, or the most critical pair of maximum allowable pressure and maximum allowable temperature, and a route in which investigations of fatigue and influence of “other” loads are still rare, and treated separately, and
- most of the experience is with linear problems, where, again, investigation of the most critical combination of values of actions suffices. It is worth noting that in Design by Analysis, the history of actions is quite often of importance, that the notion of load cases involves also time-dependent actions, e.g. in thermal stress problems, and that it involves actions that depend on a parameter related monotonically to time, a parameter which determines the order of actions states, e.g. in fatigue and in PD-DC.

Visualization in the **action space** – the space of all actions to be considered – gives a clearer picture:

- to each combination of action values there corresponds one point,
- to each combination of extreme values of actions specified to be considered coincidentally there corresponds one point, the vertex of a domain of all combinations of actions for which the component is to be designed, called **design domain**,

and the union set of all cuboids with corners given by these vertices defines this design domain uniquely,

- to each to-be-considered combination of characteristic values of actions there corresponds one point, and all of these points coincide with the vertices of the design domain,
- to each combination of characteristic values and characteristic functions there corresponds one curve, possibly degenerated into a straight line.
- load cases are represented, in this action space, with their characteristic values and/or characteristic functions, by points or by curves – the points are identical with the vertices of the design domain, and the curves are within the design domain,
- it is advisable to consider at least two different design domains – one for normal operating load cases and one for testing load cases,
- for exceptional load cases, if any, there may be a third design domain.

In general, load cases cannot be set up without taking into account the DCs:

- the GPD-DC and the instability design checks require sets of load cases that include all vertices of the design domain,
- if, in fortunately rare cases, non-stationary thermal stresses have an influence on the results in these design checks, then load cases with temperature functions (of time) may be additionally required,
- the PD-DCs require functions of actions for to-be-considered cycles of variable actions, functions which include vertices of the design domain related to variable actions, functions of a time-like parameter or, in case of thermal stresses, of time,
- the F-DCs require functions similar to the ones in the PD-DCs, but these functions are, in general, specific for these design checks, and need not include the vertices of the design domain,
- for the SE-DCs, in general, specific load cases corresponding to a sub-set of the vertices of the design domain are to be set up.

Of course, when considering a specific design check, some load cases can be eliminated, because the design check for these load cases is necessarily fulfilled if it is fulfilled for any of the other not eliminated load cases.

Great care is required in such an elimination process:

- correct elimination will reduce time and effort required in the admissibility check, but
- wrong elimination may lead to a design with insufficient safety margins, even to an unsafe design.

In case of slightest doubt, a load case should be kept and not eliminated – naïve intuition is a poor guide and time constraint a bad companion in design, especially in Design by Analysis.



Specific hints on the elimination possibilities are included in the various design check specific chapters.

Requirements that follow from the (legal) requirements of the PED for the determination of characteristic values of actions have already been discussed in Section 2.3.1, additional ones from EN 13445-3 Annex B are given in Table 3.2, and in the chapters dedicated to specific design checks.

For many load cases, even the most important ones, the characteristic values of actions are pre-given, by legal or contractual requirements. In these cases, the table serves only as a reminder of the randomness of actions, and that the characteristic values are, in general, by no means extreme values.

Table 3.2: Characteristic values for different types of action

Action	Coefficient of variation	Symbol	Characteristic value
Permanent	$\leq 0.1$ <sup>1</sup>	$G_k$ <sup>2</sup>	Mean of extreme values
Permanent	$> 0.1$ <sup>3</sup>	$G_{k,sup}$ <sup>2</sup>	Upper limit with 95% probability of not being exceeded <sup>4</sup>
		$G_{k,inf}$ <sup>2</sup>	Lower limit with 95% probability of being exceeded <sup>4</sup>
Variable	$\leq 0.1$ <sup>1</sup>	$Q_k$ <sup>2</sup>	Mean of extreme values
Variable	$> 0.1$	$Q_k$ <sup>2</sup>	97 percentile of extreme value in given period <sup>5</sup>
Exceptional Pressures and temperatures	—	—	Shall be individually specified
		$P_{sup}$	Reasonably foreseeable highest pressure
		$T_{sup}$	Reasonably foreseeable highest temperature
		$P_{inf}$	Reasonably foreseeable lowest pressure <sup>6</sup>
		$T_{inf}$	Reasonably foreseeable lowest temperature

Note:

1. The mean of the extreme values may also be used when the difference between the reasonably foreseeable largest value and the smallest one is not greater than 20% of their arithmetic mean value.
2. The subscript k indicates that there are usually several actions in a load case and they are individually numbered.
3. Also applies where the actions are likely to vary during the life of the vessel (e.g. some superimposed permanent loads).
4. If a statistical approach is not possible, the largest and smallest credible values may be used.
5. For variable actions, which are bounded, the limit values may be used as characteristic values.
6. This value is usually either zero or  $-1.0$  (for vacuum conditions).

For example, maximum allowable pressure and maximum allowable temperature are, per definitions, the maximum pressure and the coincident maximum temperature for which the equipment is designed, as specified by the manufacturer, and EN 13445-3 Annex B indicates that the upper characteristic value of pressure may be based on the value of the maximum allowable pressure, both referred to the same reference point, and that the pressure accumulation over the maximum allowable pressure after pressure relief devices start to discharge need not be taken into account. As a matter of fact, the partial safety factors (for pressure and the material strength parameter) were calibrated on the basis that the characteristic value of pressure in the GPD-DC is equal to the maximum allowable pressure. Therefore, an upper characteristic value of pressure in normal operating load cases equal to the maximum allowable pressure, both with the same reference point, is in agreement with the requirements, and is the normal choice.

It follows that pressure can exceed in normal operation its upper characteristic value for normal operating load cases, just as it can exceed its maximum allowable pressure.

Similar reasoning leads to the normal choice of the related upper characteristic values of temperature equal to the related maximum allowable temperatures.

For the self-weight of the structure and of non-structural parts, EN 13445-3 Annex B allows the usage of results based on nominal dimensions and mean unit mass.

For wind, snow, and earthquake actions, the standard allows the usage of country-specific data, from relevant regional codes. In these cases, the table may be useful, if in these regional codes values and safety factors are not based on the partial safety factor format which is used in the Direct Route in Design by Analysis, and where, therefore, adjustments are required.

The table is necessary for the specification of characteristic values of actions other than pressure, temperature, and self-weight, and for the specification of characteristic functions of actions. Especially the result of the F-DC is, because of the inherently strong non-linear relationship between stress range and allowed number of corresponding cycles, very sensitive to the choice of the characteristic functions – characteristic functions need not envelop the functions to be expected under reasonably foreseeable but extreme conditions, and they need not be enveloping functions. These functions should be based on a realistic assessment of upper bound functions, if possible in a statistical sense, in order to obtain reasonably economic results that are still safe.

It is appropriate to indicate in the load case specification specifically those actions, or parts of actions, that are limited by a natural law, e.g. pressure under vacuum conditions, because the partial safety factor format acknowledges this limitation – the partial safety factors are smaller for such limited actions.

Partial safety factors of actions that have to be included in the load case specification are discussed in the chapters dedicated to the specific design checks.

### 3.3. Procedure

#### 3.3.1. *Step 1: Setting Up of Load Case Specifications List*

In a systematic approach to the whole process of checking a design, the first task is the identification of the safety relevant actions and the safety relevant combination of actions that have to be considered because they can occur under reasonably foreseeable conditions, or have been specified (for other reasons) in the order.

Next, the characteristic values of actions and the characteristic functions of actions have to be specified, after having taken into consideration related failure modes, and, usually, also taking into account related design check requirements. For example, for the action pressure, there may be different upper characteristic values for the GPD-DC, the F-DC, and the creep design check.

The final task in the setting-up of load case specifications then is the specification of the relevant partial safety factors for these load cases, taking into account the relevant design checks.

The final product of this process is the list of load case specifications, with all the action-related values and functions, and all the partial safety factors, related to specific design checks.

Whether one includes in one load case different (design check related) sub-cases or specifies different load cases, each one related to only one design check, is up to the discretion of the designer, and will very likely depend on the complexity of the load cases.

It seems to be advantageous to use the sub-case approach, with appropriately chosen load case identifier designations, e.g. NOLC 3.2 for the second sub-case of the normal operating load case 3. It is good practice to relate the sub-case identifier to the order number of the design check, e.g. 1 for the GPD-DC, 2 for the PD-DC, and so on.

The decision as to whether the consideration of the obligatory design check suffices, or whether other design checks are required, should also be part of this first step.

Frequently, this step is not followed through completely, leaving gaps in the (ideal) load case specifications, sometimes with data related to legal requirements, e.g. maximum allowable pressure, maximum allowable temperature.

#### 3.3.2. *Step 2: Setting Up of Design Check Table*

In all but the simplest cases, it is advisable to prepare a design check table, i.e. a table of the required design checks and the related load cases. This table follows directly from the load case specifications list generated in Step 1. As an example,

Table 3.3: Design check matrix

Load case	Design check				
	GPD-DC	PD-DC	F-DC	S-DC	SE-DC
NOLC 1.1	✓	—	—	—	—
NOLC 1.4	—	—	—	✓	—
NOLC 2	—	—	—	✓	—
NOLC 3.4	—	—	—	✓	—
NOLC 3.5	—	—	—	—	✓
NOLC 4.4	—	—	—	✓	—
NOLC 4.5	—	—	—	—	✓
NOLC 5.2	—	✓	—	—	—
NOLC 5.3	—	—	✓	—	—
TLC 6.1	✓	—	—	—	—
TLC 6.4	—	—	—	✓	—

see Table 3.3 for a storage vessel with skirt, under internal pressure, wind, and operating at ambient temperature.

In this table, load case identifier 1 refers to the fully filled vessel, maximum internal pressure (and no wind); load case identifier 2 to the empty vessel, vacuum (and no wind); load case identifier 3 to the empty vessel, vacuum and wind action; load case identifier 4 to the fully filled vessel, maximum internal pressure and wind; load case identifier 5 to operational fluctuations of pressure and liquid height, and, finally, load case identifier 6 to the hydraulic test. Load cases 1 through 4, and 6, correspond to vertices in the two design domains, one for normal operation and one for testing.

This table shows quite clearly how complex the design check table can become if more than two actions have to be considered:

In the case of  $n$  independent actions, independent in the sense that for any action only one maximum exists and not two or more in dependence of regimes of other actions, in this simplest case  $2^n$  different combinations of extreme values of the actions exist, i.e. the design domain is specified by  $2^n$  vertices.

In the case considered here, of the 8 combinations of the maxima and minima of the three actions – pressure, hydrostatic head of fluid, wind – only 4 are included in the matrix. Not included are the four combinations:

- maximum pressure, empty vessel, wind,
- maximum pressure, empty vessel, no wind,
- vacuum, fully filled vessel, wind,
- vacuum, fully filled vessel, no wind.

This is because the corresponding design checks are encompassed by the ones in the table.

Load cases NOLC 1.1, 1.2, and NOLC 2 could have been deleted as well, but pressure for a pressure vessel is a very special action, and it is good practice to keep load cases that deal with pressure and temperature only – for comparison with DBF results and plausibility checks.

### ***3.3.3. Step 3: Setting Up of Design Models***

Design models are design check and load case specific, details are given in the design check specific chapters.

The geometry of models is, in general, only design check specific, i.e. it is the same for all load cases of a design check.

The type of the design models' constitutive laws is likewise design check specific, but the material parameters depend, in general, on the load case.

The kind of mathematical approximation – first-order theory, second-order theory, or geometrical non-linearity – depends in most cases only on the design check, but it may also depend on the load case.

### ***3.3.4. Step 4: Execution of Design Checks***

In this step the responses of design models to design actions is investigated and checked for compatibility with the design checks' requirements, i.e. checked whether they fulfil the requirements of the design checks' principles, or the requirements of application rules.

This step is the most interesting one, from a theoretical point of view, but especially from the designer's point of view, since it provides much insight into the component behaviour in service.

Details are given in the design check specific chapters.

Most of the design checks are specified as principles, and application rules are given additionally.

Usage of the principle is the most straightforward route and gives the most information, but the route via one of the application rules is frequently a useful, convenient short cut.

The geometry, the type of constitutive law, and the kind of mathematical approximation, being frequently the same for all load cases of one design check, it is good practice to perform all load case investigations of one design check, i.e. all investigations for all load cases relevant for one design check, consecutively,

and then proceed to the next design check. Even if material parameters differ for the various load cases, and also if the mathematical approximation is not the same, the advantage of the similarity in the models can be used in this way.

It is good practice to conclude each investigation of a load case with a definite statement on the satisfaction of the design check requirements.

### ***3.3.5. Step 5: Final Conclusion***

It is good practice to finish the whole admissibility check by a definite, signed statement that for all required design checks the design check principles are fulfilled for all the relevant load cases stated in the design specification.

This may be seen as unnecessary red tape, but experience has shown the value of such a step – a deviation only noted in a detailed investigation is easily forgotten afterwards and necessary correction steps may not be carried out.

## **3.4. Example**

An example with a very formalistic deduction of a design check table can be found in Annex E.3.

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## Chapter 4

# Gross Plastic Deformation Design Check (GPD-DC)

### 4.1. Introduction

The Gross Plastic Deformation Design Check (GPD-DC) deals with

- the main failure mode gross plastic deformation and
- the secondary failure mode excessive local strain,

with regard to

- short-time response, i.e. excluding creep effects, and
- single monotonic application of actions.

Figs. 4.1a–d show four types of such responses of real structures made of real materials. Fig. 4.1a shows a load versus displacement curve that is typical for experimental results with tensile bars and trusses subjected to axial forces at ambient temperatures, or results for cylindrical or spherical shells under internal pressure, made of mild steel or any other material with pronounced yield point at the testing temperature.

When the monotonically increasing load reaches the unrestricted plastic flow load  $A_{up}$  the rate of change becomes zero, or nearly zero, but the structure can still carry larger loads, up to the ultimate load. The increase in load carrying capacity is mainly due to the strain hardening of the material, possibly enhanced by geometric effects.

A limit analysis model of the structure with yield limit equal to the yield strength of its material, will show a load versus displacement diagram similar to the one shown in Fig. 4.2 – a limit analysis model with the appropriate yield limit will, in this case, be a reasonably good model for the determination of the unrestricted plastic flow load.

Figs. 4.1b show two load versus displacement curves typical for experimental results

- with materials without pronounced yield point at the test temperature but with strong hardening,
- with structures and loads for which deformation has a strong positive effect, thus contributing markedly to the improvement of the load carrying capacity of the structure, e.g. not too thin-walled dished ends under internal pressure,



- with structures and loads where growth of plastified zones and/or newly plastified zones contribute markedly to the increase of the load carrying capacity of the structure, e.g. flat circular ends with pronounced diaphragm effect.

Compared with the response shown in Fig. 4.1a, there is, in this case, no plateau (of unrestricted plastic flow) and the sharp bend in the load versus displacement

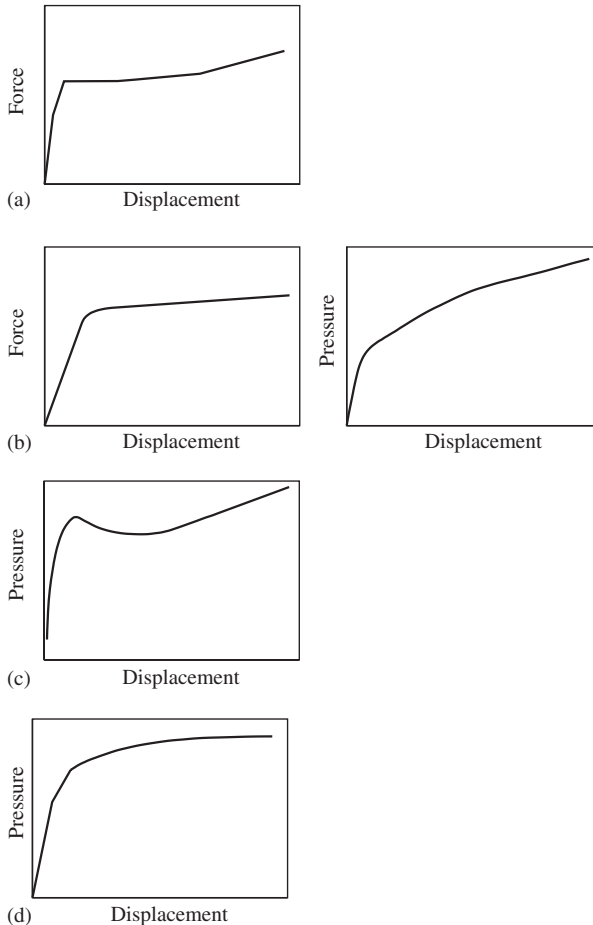


Figure 4.1: (a) Load versus displacement curve, type 1: Three-bar truss made of mild steel [120]; (b-1) Load versus displacement curves, type 2: Uniformly loaded, simply supported beam, I-shaped cross section, made of mild steel [120]; (b-2) Load versus displacement curve, type 2: Dished end, internal pressure, made of mild steel [121]; (c) Load versus displacement curve, type 3: Post buckling of a cylindrical shell under torsion [53]; (d) Load versus displacement curve, type 4: Three-bar truss made of aluminium [120].

curve at the unrestricted plastic flow load is missing due to the absence of a pronounced yield point, a marked hardening, or a positive effect of deformation.

In this case, a limit analysis model of the structure will not be a good simulation model, but still can be a reasonably good design model, possibly a highly conservative one.

Fig. 4.1c shows a load versus displacement curve that is typical for experimental results with **deformation-weakening**, where deformation due to the applied action has a detrimental effect on the unrestricted plastic flow load. Examples are

- instability problems, e.g. cylindrical, spherical, and conical shells, or dished ends, under external pressure, or beams and cylindrical shells under axial compressive loading,
- cases where deformation has a negative, weakening effect on the structure's carrying capacity, e.g. nozzles in cylindrical shells under transverse moments, nozzles in cylindrical shells under axial compressive forces, bends under curvature increasing moments, cylindrical shells with out-of-roundness or peaking under external pressure.

The response is sensitive to initial stresses, often highly sensitive to initial deformations and to initial deviations from the ideal geometry, and often strongly dependent on kinematic boundary conditions.

In this case, a limit analysis model of the structure is neither suitable as a simulation model nor as a design model. A design model, to be suitable in cases of such a behaviour, must at least include second-order theory, preferably also non-linear kinematic relations.

Fig. 4.1d shows a load versus displacement curve which is typical for experimental results with structures made of a material without pronounced yield point and with actions resulting in stress states without a marked re-distribution during loading, e.g. statically determinate trusses, or cylindrical and spherical shells under internal pressure. For a monotonically increasing action the ultimate load will be reached asymptotically, or, quite often, after a finite displacement, and the rate of change varies frequently monotonically.

Despite the similarity of the load versus displacement curve of this type and the one of a limit analysis model, see Fig. 4.2, a limit analysis model will, in this case, not be a good simulation model – for a reasonably good simulation, the yield strength of the limit analysis model has to be adjusted or calibrated, to a value somewhere between a proof stress and the ultimate strength of the material, obtained with the usual tension test specimen, depending on the structure and the action.

Nevertheless, in this case a limit analysis model may be a reasonably good design model, with a yield strength based on the 0.2% (or 1%) proof stress specified

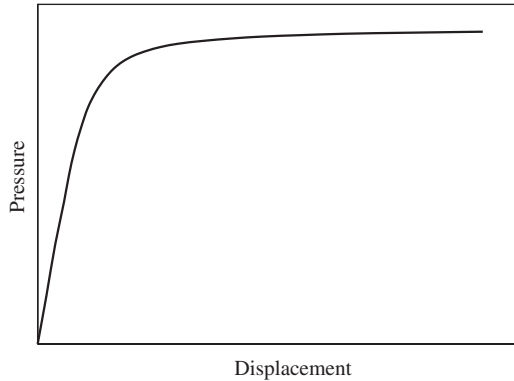


Figure 4.2: Load versus displacement curve, limit analysis model: Flat end to cylinder connection with relief groove [3].

in the material standards. Very likely, such a design model is highly conservative, neglecting totally the (safety) margin due to hardening of the real material.

Fig. 4.2 shows a load versus displacement curve that is typical for the response of limit analysis models to actions without instability effects and without displacement weakening. For a monotonically increasing action the ultimate load is reached asymptotically, or after a finite displacement, and strains are usually small up to loads very close to the ultimate load.

In this case, the response of limit analysis models to monotonically increasing actions is insensitive to initial stresses and fairly insensitive to initial deformations and initial deviations from the perfect geometry. The limit analysis load itself is independent of initial stresses, of initial deformations, and of the action history, but may depend on initial deviations from the perfect geometry, if these deviations are non-local structural perturbation sources.

## 4.2. Procedure

The GPD-DC can be seen as an investigation of the capacity of the structure to carry safely all of the to-be considered states of actions. As such, the design check encompasses all action states in the design domain and deals with all load cases that correspond to the vertices of the design domain. In cases of deformation-weakening, i.e. in cases where deformation decreases the load carrying capacity of the structure, additional design checks, as described in the instability design check specific Chapter 6, are required. If in such cases non-stationary thermal stresses influence the result, additional load cases with characteristic time-dependent temperature (functions) are required.

From this point of view, the GPD-DC procedure can be summarized as follows:

- the gross plastic deformation check deals with all load cases that correspond to the vertices of the design domain;
- of these load cases some can be eliminated, need no specific investigation because the investigation result is encompassed by any of those for the non-eliminated load cases;
- for all of the remaining load cases the checks shall be performed as specified in this chapter; and
- these proper checks are investigations as to whether the design models can carry the design actions of the load cases, with specifically limited structural strains.

For deformation-weakening load cases additional investigations are required. If, in any of these deformation-weakening load cases, non-stationary thermal stresses possibility influence the result, then at least one additional load case with characteristic time-dependent temperature is required. For these (additional) load cases only the additional investigations, as described for these deformation-weakening load cases, are required if the results for allowable design actions obviously encompass the corresponding ones of the usual GPD-DCs. In case of doubt both the checks are required – the models differ not only in the geometric non-linearity, but also in the initial conditions and the additional consideration of thermal effects, if any.

The following sections (of this chapter) deal solely with the investigations with respect to the failure modes gross plastic deformation and excessive local strains, required for all (non-eliminated) load cases, without and with deformation-weakening. The additional investigations required for deformation-weakening load cases are dealt with in Chapter 6 – the behaviour of the required geometrically non-linear design models is similar to that of the design models for buckling load cases, and often includes buckling.

### **4.3. Design Models**

All design models used in the investigations discussed here may be stress-concentration-free models.

In the modelling of the stress-concentration-free geometry, in principle, material is to be removed – adding of material requires justification.

For clad components the following applies:

- Structural strength may be attributed to cladding only in the case of integrally bonded type, and by the agreement of the parties concerned.
- The nominal face of the cladding is to be used as the surface of the pressure application.

- A pressure correction may be required in cases of finite element models with shell elements if pressure is (in the software) applied at the centroidal surfaces of the elements and not on their actual surfaces. A pressure correction is required if cladding is not included in the model. In both cases the correction factor is given by the ratio of the (infinitesimal) areas of the surfaces on which pressure is applied – actually and in the finite element model.

All of the design models are limit analysis models, using

- first-order theory,
- linear-elastic ideal-plastic constitutive laws,
- Tresca's yield condition and associated flow rule,
- specified material and temperature-dependent yield stress given by the specified design value of the material strength parameter.

Tresca's yield condition has been chosen for calibration purposes – the results of the maximum admissible internal pressure of sufficiently long, closed cylindrical and of spherical shells according to DBF and to the GPD-DC should agree. Or, in other words: The formulae given in the DBF section of the standard for cylindrical and spherical shells are based on the limit analysis pressure for Tresca's yield condition. Consequently, the very same yield condition, i.e. Tresca's yield condition, was specified in the GPD-DC, in order not to favour DBA for these simple geometries, for which sufficient experience with the results according to the DBF approach exists. Usage of Mises' yield condition would result in thinner closed cylindrical shells in cases where gross plastic deformation is the dominant, decisive failure mode, without any convincing argument.

Another reason for the choice of Tresca's yield condition has been the wish for results that are conservative compared to experimental ones.

Nevertheless, Mises' yield condition may be used instead of Tresca's, but then the specified yield stress, specified for Tresca's yield condition, has to be multiplied by  $\sqrt{3}/2$  in order to obtain the yield stress to be used with Mises' yield condition.

Usage of Mises' yield condition may be necessary because of software or numerical problems related to Tresca's yield condition. Routines with Mises' yield condition are much faster than routines with Tresca's – usage of Mises' yield condition is recommended for first trials and for non-critical checks.

The yield stresses used in the design models – the design values of the material strength parameter – are obtained by division of the relevant material strength parameter  $RM$  by the relevant partial safety factor  $\gamma_R$ . The values to be used are given in the following two tables, Table 4.1 for normal operating load cases and Table 4.2 for testing load cases. For exceptional load cases, the values for the relevant material strength parameters are given by those for normal operating load cases,

Table 4.1:  $RM$  and  $\gamma_R$  for normal operating load cases

Material	$RM$	$\gamma_R$
Ferritic steel <sup>a</sup>	$R_{eH}$ or $R_{p0.2/t}$	1.25 for $\frac{R_{p0.2/t}}{R_{m/20}} \leq 0.8$
		$1.5625 \left( \frac{R_{p0.2/t}}{R_{m/20}} \right)$ otherwise
Austenitic steel with $30\% \geq A_5 < 35\%$	$R_{p1.0/t}$	1.25
Austenitic steel with $A_5 \geq 35\%$	$R_{p1.0/t}$	1.0 for $\frac{R_{p1.0/t}}{R_{m/t}} \leq 0.4$
		$\frac{2.5R_{p1.0/t}}{R_{m/t}}$ for $0.4 < \frac{R_{p1.0/t}}{R_{m/t}} \leq 0.5$
		1.25 for $\frac{R_{p1.0/t}}{R_{m/t}} > 0.5$
Steel castings	$R_{p0.2/t}$	$19/12$ for $\frac{R_{p0.2/t}}{R_{m/20}} \leq 19/24$
		$\frac{2R_{p0.2/t}}{R_{m/20}}$ otherwise

<sup>a</sup>Steel other than austenitic steel with  $A_5 \geq 30\%$  and steel castings.

i.e. by those in Table 4.1, and the partial safety factors are to be agreed by the parties concerned, but must not be smaller than those for testing load cases, given in Table 4.2.

For the values of the material strength parameters, see also the definition of the design value of the material strength parameter, it is common practice to use the minimum values specified in the material standards.

For the reference temperature for the determination of temperature-dependent material strength parameters, a temperature not less than the maximum metal temperature of the load case is to be used. This reference temperature may be (chosen) space independent, or space dependent. In the first case, the (chosen) value must not be smaller than the maximum calculation temperature in any point of the considered model, in the second the reference temperature in each point must not be smaller than the calculation temperature in this point. It is common practice to

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Table 4.2:  $RM$  and  $\gamma_R$  for testing load cases

Material	$RM^a$	$\gamma_R$
Ferritic steel <sup>b</sup>	$R_{eH}$ or $R_{p0.2}$	1.5
Austenitic steel with $30\% \leq A_5 < 35\%$	$R_{p1.0}$	1.5
Austenitic steel with $A_5 \geq 35\%$	$R_{p1.0}$	1.05 for $\frac{R_{p1.0}}{R_m} \leq 0.525$  $\frac{2.0R_{p1.0}}{R_m}$ otherwise
Steel castings	$R_{p0.2}$	1.33

<sup>a</sup>Values for  $RM$  are for the test temperature.

<sup>b</sup>Steel other than austenitic steel with  $A_5 \geq 30\%$  and steel castings.

use, in this latter case, the stationary result of the (numerical) temperature calculation for the load case directly.

The results of the investigations are insensitive to the used values for the material parameters for the linear-elastic regime – modulus of elasticity and Poisson's ratio; in many cases these results are even independent from these material parameters.

Taking this into account, and also for simplicity, the material parameters to be used for the linear-elastic regime are specified such that they can be used unchanged in thermal stress problems involving temperature changes from ambient to the maximum characteristic temperature of the load case considered:

- The reference temperature  $t^*_E$  for the determination of the temperature-dependent modulus of elasticity shall not be less than

$$t^*_E = 0.75t^*_{RM} + 5K \quad (4.1)$$

with  $t^*_E$  and  $t^*_{RM}$  in °C, and where  $t^*_{RM}$  is the reference temperature for the determination of the material strength parameter discussed in the preceding paragraph. The corresponding modulus of elasticity is to be determined using Annex O of EN 13445-3.

- For Poisson's ratio the value 0.3 is specified (for the elastic regime).

The summand in the equation for the reference temperature, Equation (4.1), is not a temperature margin, introduced for safety reasons, but is the contribution from ambient temperature, taken to be 20°C.

The design models ensure unique results with regard to safe actions, and the response of the models to a given action history is unique in terms of the stress history (but not necessarily in terms of the strain and displacement histories) [40], p. 253.

#### 4.4. Design Values of Actions

The design values of the actions, to be considered in this (part of the) design check, are given by the product of their characteristic values and the relevant partial safety factors, given in Table 4.3 for normal operating load cases and in Table 4.4 for testing load cases. The partial safety factors for exceptional load cases are equal to unity.

Formally, the temperature distribution in a structure is the effect of the temperature action. This is one of the reasons for the inclusion of the corresponding partial safety factors for temperature in these tables, for normal operating and for testing load cases. Another reason is the following one:

The temperature (distribution) in the structure is taken into account in these investigations via the temperature dependence of the material strength parameters,

Table 4.3: Partial safety factors for actions and normal operating load cases

Action	Condition	Partial safety factor
Permanent <sup>a</sup>	For actions with an unfavourable effect	$\gamma_G = 1.2$
Permanent <sup>a</sup>	For actions with a favourable effect	$\gamma_G = 0.8$
Variable <sup>a</sup>	For unbounded variable actions	$\gamma_Q = 1.5$
Variable <sup>a</sup>	For bounded variable actions and limit values	$\gamma_Q = 1.0$
Pressure	For actions without a natural limit	$\gamma_P = 1.2$
Pressure	For actions with a natural limit, e.g. vacuum	$\gamma_P = 1.0$
Temperature		$\gamma_T = 1.0$
Displacement		$\gamma_D = 1.0$

<sup>a</sup>Other than pressure, temperature and displacement.

Table 4.4: Partial safety factors for actions and testing load cases

Action	Condition	Partial safety factor
Permanent <sup>a</sup>	For actions with an unfavourable effect	$\gamma_G = 1.2$
Permanent <sup>a</sup>	For actions with a favourable effect	$\gamma_G = 0.8$
Pressure		$\gamma_P = 1.0$
Temperature		$\gamma_T = 1.0$
Displacement		$\gamma_D = 1.0$

<sup>a</sup>Other than pressure, temperature and displacement.



and in general, formally, thermal stresses need not be taken into consideration: In a pure limit analysis approach thermal stresses need not be taken into account – the limit analysis action is independent of thermal stresses, because thermal stresses are self-stresses. The (structural) strain-limiting requirement in the principle changes this (simple) picture: There are (bad design) cases where strains due to thermal stresses may become too large to be neglected in this design check. Typical examples are temperature-induced forces at nozzles due to restrained displacement at anchor (fix) points of piping attached to vessels, forces which can result in non-negligible strains in the nozzle to shell region. Quite often, these piping reactions are determined in piping analyses assuming total restraint of the piping at the nozzles. This approach usually leads to overly large reactions, and, thus, to overly large strains. Although not required in the standard, it is, in such cases, recommended to include temperature in the load cases, and to include, at the same time, relevant parts of the piping in the models. Alternatively, in the piping analyses appropriately flexible supports at the nozzles can be used. As a warning note: Only the thermal stress-related strains have to be considered in the strain-limiting requirement, and not the strains due to thermal expansion,  $\beta\Delta t$ , with the coefficient of linear thermal expansion  $\beta$ .

Analogously, the partial safety factors for imposed displacements have been included here in these tables, whereas none are given in the standard.

For wind, for snow, and for earthquake actions, country-specific data, i.e. values specified in relevant regional codes, may be used, but their usage should be accompanied by a check of the (characteristic) values used such that the overall safety margins are maintained.

Taking into account the fact that the coincident occurrence of values of non-correlated variable actions near the actions' characteristic values is small if one of these actions has stochastic properties, the design values of stochastic variable actions, like wind, snow, earthquake, may be multiplied by a combination factor, if these stochastic actions are combined, in multiple-action load cases, with pressure and/or at least one other stochastic action.

If only a part of the pressure is subjected to a natural limit, e.g. the part of the pressure that corresponds to a static head, then, in the determination of the design value of pressure, this part may be multiplied by the partial safety factor 1.0 and the other part by 1.2.

For checking the completeness of the load case specifications with regard to this GPD-DC, the following combination rules may be used:

- all permanent actions must occur in every load case,
- each pressure action must be combined with the most unfavourable variable action,
- each pressure action must be combined with the relevant sum of variable actions,

- specifications for combination of actions in relevant regional codes (on environmental actions) may be used, and
- favourable variable actions, including pressure, must not be considered.

In cases of “normal” standard hydraulic tests, as specified in EN 13445-5, i.e. hydraulic tests without stability problems and with negligible actions other than pressure, the GPD-DC for testing load cases is, in general, not required. Only in cases where the influence of the static head in the test situation cannot be neglected compared with that of the test pressure is a GPD-related testing load case required.

## 4.5. The Principle

Using the preceding specifications for design models, Section 4.3, and design values of actions, Section 4.4, the GPD-DC’s principle can be stated quite simply:

The design values of actions, of all relevant load cases, shall be carried by the relevant design models with maximum absolute values of principal structural strains not exceeding 5% in normal operating load cases, and 7% in testing load cases, for a stress-free and weightless initial state, and for proportional increase of all actions except temperature.

For exceptional load cases the strain limitation does not apply.

Temperature has a special role in this principle: In general, thermal stresses need not to be taken into consideration (see Section 4.4), and then temperature enters the investigations only via the temperature-dependent material strength parameters (see Section 4.3). Therefore, temperature has been excluded from the proportional increase of actions, and the material strength parameters do not vary during the increase of the (other) actions.

Without the strain-limiting requirement the necessary investigations were limit analysis investigations, and the theorems of limit analysis would then apply, e.g. that

- the problem of the determination of limit analysis actions has a unique answer,
- the results, i.e. the values of the limit analysis actions, are independent of initial conditions,
- the results are independent of action paths,
- the results are independent of the material parameters in the linear-elastic regime,
- the set of safe actions is convex, given that the relevant characteristic value of the material strength parameter is a convex function of temperature, and
- if, for a given action, any statically admissible stress field can be found which is compatible with the relevant yield condition, then the action is a safe one.

All of these theorems are important in the applications.

The first one gives the designer confidence: For a suitably set up model, with appropriate initial and boundary conditions, there is one, and only one, solution, despite the non-linearity of the design model owing to the non-linear constitutive law.

The second one tells the designer that the unavoidable initial residual stresses owing to manufacturing processes can be ignored.

The third one is of importance especially in the design stage: The, in general unknown, actual action histories are unimportant, and the load case specifications can be simplified.

The fourth one allows for simple approximations for these parameters.

The fifth one justifies the investigation of only load cases corresponding to the vertices of the design domain.

The sixth one, the **lower bound theorem of limit analysis theory**, is the basis for many approximation procedures and rough checks, and it is the reason for neglecting thermal stresses in these investigations.

For values of actions close to the limit analysis action values problems with numerical stability are quite common; these problems are almost always a nuisance, requiring often a complete restart.

Therefore, the strain-limiting requirement had been introduced,

- to avoid numerical instability problems for design action values close to the limit analysis action values,
- to create a unique break-up point for the calculation, such that the result does not depend on the patience of the designer, nor on the computing power, but also
- to encompass the failure mode excessive local strain, which is important in cases of strain concentrations.

The values of the strain limits are the result of a compromise – large enough to allow for results close to the limit analysis actions, and small enough to avoid numerical instabilities and to cover the failure mode excessive local strains safely. They are seen to be by far small enough for the sufficiently ductile materials considered – with an elongation after rupture not less than 14%. Discussions have shown that it is necessary to remind designers that these strain limits are limits for the effects in the design models – with design material strength parameters – and that they apply for design values of actions, i.e. the actual strains for characteristic values of actions in the actual structure are in general much smaller than these specified limits.

The specification of limits on structural strains allows for simpler FE-models: For shell and plate-type structures shell and plate FE-elements can be used.

Because of the introduction of the strain-limiting requirement some of the above-listed advantages are lost. The results are still unique, but they depend, in principle, on the (in general unknown) initial conditions and the (in general unknown) action paths.

Results with the strain limitation are, in general, close to the corresponding limit analysis results, and quite often the strain-limiting requirement is not governing, the results being then identical. Therefore, the initial conditions and the action paths have been specified quite pragmatically: as simple as possible, to allow for easy pre-processing approaches.

Results with strain limitation being close to limit analysis results, often identical, convexity of the set of the limited strain limit actions may be assumed, is probably provable, and therefore the investigation of load cases corresponding to the vertices in the design domain suffices.

## 4.6. Application Rule

The (only) application rule reads:

*If it can be shown that any lower bound limit value of the action, determined with the design model specified in the principle, is reached without violation of the strain limit, the principle is fulfilled if the design value of the action does not exceed that lower bound value (EN 13445-3 Annex B).*

The validity of this application rule is obvious, it has nevertheless been included in the standard to draw the user's attention to this.

This application rule may be used in cases for which upper bounds on the structural strain in the limit analysis result can be given and such an upper bound is not larger than the strain limit of the principle. It is usually used in conjunction with the lower bound limit analysis theorem.

## 4.7. Examples

Examples can be found in Annex E.4.

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## Chapter 5

# Progressive Plastic Deformation Design Check (PD-DC)

### 5.1. Introduction

Progressive plastic deformation design checks (PD-DC) deal with the failure mode progressive plastic deformation, also called ratchetting and inadaptation, with regard to cyclic application of actions and short-time response, i.e. excluding creep effects. This design check often uncovers clearly singular features of structural behaviour under the influence of repeated actions.

Three basic types of responses of structures to cyclic actions are shown in Figs. 5.1–5.3.

Figs. 5.1a–c show examples of shakedown to linear-elastic behaviour.

Fig. 5.1a shows an example of a shell with a local structural perturbation source subjected to one single-amplitude cyclic proportional combination of actions, an example where the structure shakes down to linear-elastic behaviour in one action cycle. The stress path at the critical point is shown in the deviatoric map, with the shakedown trajectory shown as the thick line. From the initially stress-free state, represented by the point 0 in the origin, a proportional increase of actions up to their maximum value results for a linear-elastic model in a linear-stress path, up to  $S_1^{el}$  in the deviatoric map. The stress path for the linear-elastic ideal-plastic model deviates from this straight line at the intersection with the circle, representing the yield limit, remains at the circle and ends in  $S_1^{el}$ . Unloading is linear-elastic; the deloading path is parallel to the first loading path, ends in  $S_2$  for total removal of all actions. The length of the straight line  $0-S_1^{el}$  is equal to that of  $S_1^{el}-S_2$ . The reloading stress path ends in  $S_1^{pl}$  – the structure has shaken down to linear-elastic behaviour.

Fig. 5.1b shows an example of an infinitely long strip subjected to a constant bending moment and a single-amplitude cyclic axial force, an example with the response mode elastic shakedown with adaptation after infinitely many cycles, and with one-sided plastic deformation. The stress trajectory plotted for the critical point shows again shakedown – in this case the trajectory converges towards the shakedown trajectory, shown again as a thick line. During unloading at the critical point, for which the stress path has been plotted, there is a short loading phase at

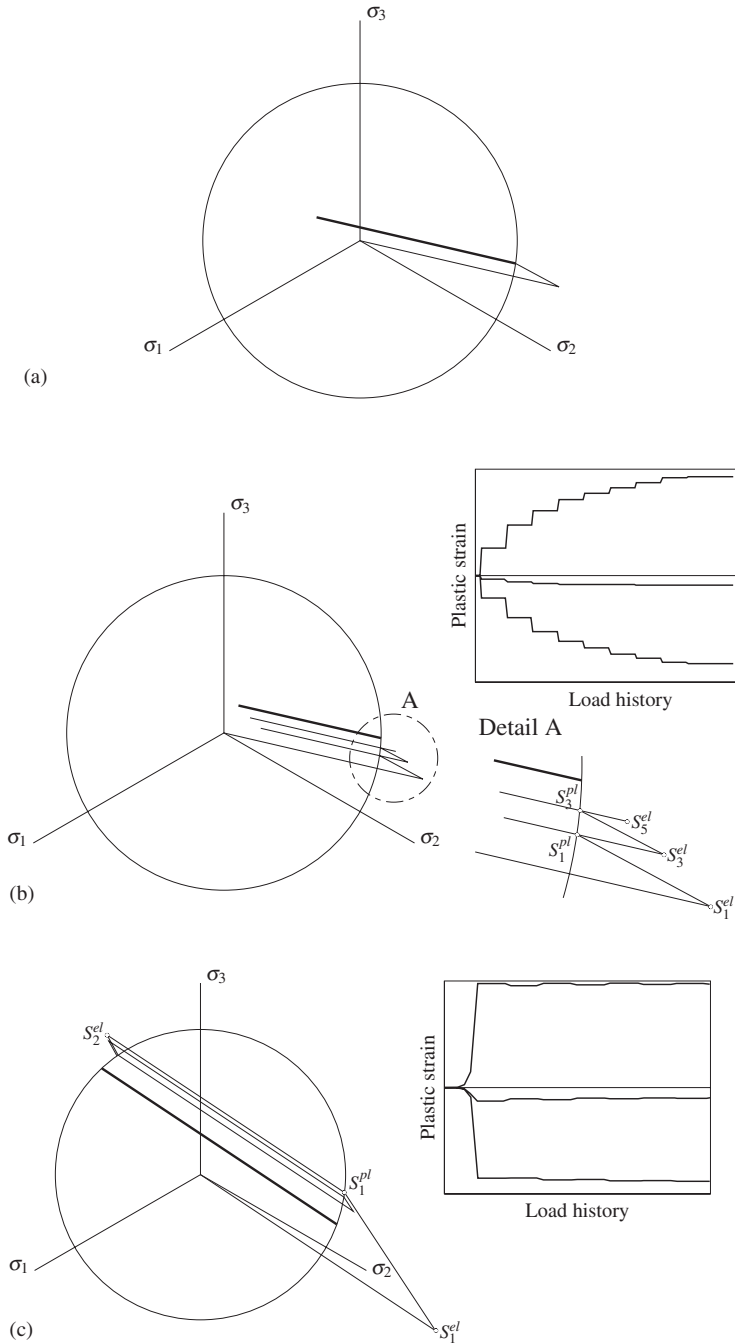


Figure 5.1: (a) Elastic shakedown after one action cycle; (b) Elastic shakedown after infinitely many cycles with one-sided plasticity; (c) Elastic shakedown after infinitely many cycles with two-sided plastic deformation.

another point, and this loading influences the unloading at the critical point. The picture on the left is the presentation of the stress path for the critical point and an initially stress-free state, for a model with linear-elastic ideal-plastic constitutive law and Mises' yield condition, in a deviatoric map. In contradistinction to the preceding case, there is, during the decrease of the action, purely elastic unloading at the point for which the stress path is plotted and a short plastic deformation phase at another point of the structure. This plastic deformation phase influences the elastic unloading at the other point, and the length of the deloading paths are not necessarily equal. Reloading results again in plastic deformation, and the stress path ends in  $S_3^{pl}$ . Unloading is at this point purely linear-elastic, and ends in  $S_4$ , and so forth. For this action cycle it takes the structure infinitely many cycles to shake down to linear-elastic behaviour. The diagram on the top right is a plot of the principal plastic strains versus the action steps.

Fig. 5.1c shows an example of a cylinder-cone intersection subjected to cyclic internal pressure, an example of elastic shakedown after infinitely many cycles and two-sided plastic deformation at the critical point. The stress cycles at the critical point, on the inside of the intersection, converge again asymptotically to the shakedown cycle. In this example there is, at the point considered and in every action cycle, plastic deformation at the maximum pressure and also (reverse) plastic deformation at the minimum pressure with plastic strain increments converging asymptotically to 0.

Fig. 5.2 shows another example with the same model and type of action as in Fig. 5.1b, but an example with the response mode alternating plasticity or elastic-plastic shakedown is considered – after adaptation has taken place the response shows pure alternating plasticity, and in each cycle plastic deformation occurs but the plastic

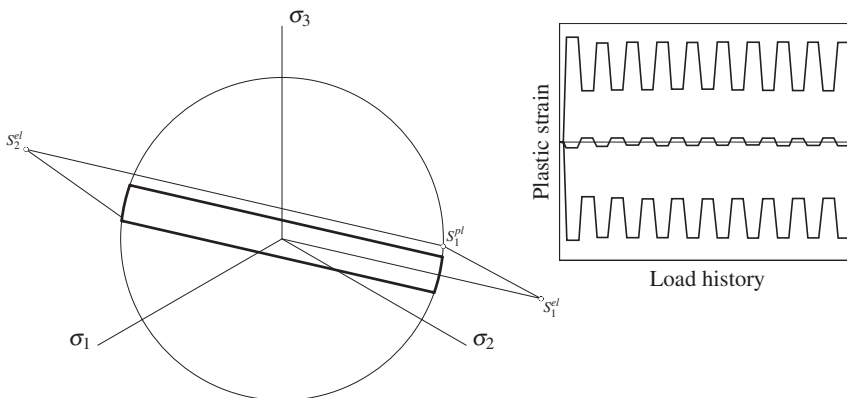


Figure 5.2: Elastic-plastic shakedown (alternating plasticity).



strain increments cancel out each other within each cycle. The picture on the left is the presentation of the stress path of a critical point in a deviatoric map, for the same model as in Fig. 5.1a. After a few action cycles the stress path becomes cyclic, but plastic deformation does not cease. The diagram on the right is again a plot of principal strains versus the action steps after adaptation strains are cyclic.

Fig. 5.3 shows, for the same model and type of action, an example with the response mode progressive plastic deformation, or ratchetting – adaptation does not occur, in each action cycle the plastic strain increment is different from 0, and deformation increases. The picture on the left is again the presentation of the stress path of a critical point in a deviatoric map – the stress path converges, but to a cycle which indicates plastic strain increments resulting in non-vanishing increments per cycle. The diagram on the right is again a plot of the principal strains versus the action steps – (plastic) deformation clearly increases in each cycle and does not converge to a (finite) limit value.

It is long known that in case of different repeated actions, low-cycle fatigue can not only cause structural failure for actions well below the values of the ultimate actions, but also may cause accumulation of plastic deformation, resulting in excessive deformations of the structure, excessive with regard to service requirements.

In other well-known codes and standards, and also EN 13445-3 Annex C, as safeguard against the relevant failure mode, requirements based on elastic shakedown concepts are used, usually in the form of the so-called 3f-criterion – a necessary but not sufficient condition for shakedown to linear-elastic behaviour in linear-elastic ideal-plastic models.

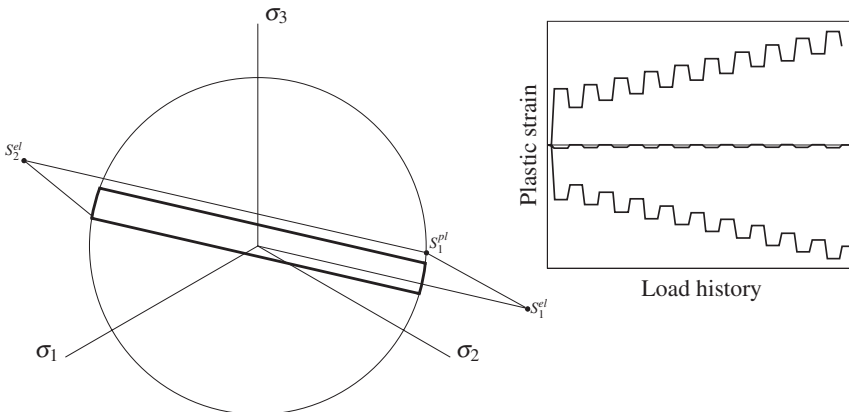


Figure 5.3: Progressive plastic deformation (ratchetting).

Use of 3f-criterion requires great care, and is therefore not recommended – fulfillment of the condition does, in general, not ensure elastic shakedown.

Elastic shakedown, on the other hand, does ensure that progressive plastic deformation does not occur – the proof of elastic shakedown is used in the Direct Route as an application rule.

Requiring of a design that it shakes down to elastic behaviour under the relevant action cycles may be appropriate in designs where high-cycle fatigue is a governing failure mode, e.g. in machine design. In the design of pressure equipment it is usually unnecessarily restrictive, because in cases of large cyclic stresses the number of specified action cycles is frequently small enough such that the repeated plastic deformation can be tolerated.

In other words, alternating plasticity is a response mode that is related to low-cycle fatigue and is appropriately dealt with in the fatigue design check. The critical response mode is progressive plastic deformation. Progressive plastic deformation as a response mode may lead to incremental collapse, as a failure mode. Progressive plastic deformation may also lead to excessive deformations, possibly resulting in impairment of serviceability, but it can also lead to deformations large enough to violate presuppositions of other investigations, such as displacements small enough to justify calculations based on first-order theory, or on the validity of theorems or experimental results based on first-order theory.

The responses shown in the figures correspond to the pure phenomena, and are obtained with idealized models with linear-elastic ideal-plastic constitutive law.

Responses of real structures differ considerably due to the following reasons:

- Owing to the influence of the cyclic actions the yield surface is, relative to the initial one, displaced, deformed, and its diameter is increased or decreased. Even in a simple cyclic tensile test with cyclic uniaxial stress of constant amplitude the stress–strain curve changes asymptotically from the (initial) monotonic curve to the stabilized cycle with hardening or, possibly, softening, [41,42].
- Materials with pronounced yield point at ambient temperatures lose their previously horizontal platform at higher temperatures but still well below the creep regime.
- Combination modes are evidently also possible.

Nevertheless, the PD-DC is considered to be a very valuable complement to the other design checks – it deals with repeated actions and contributes to the legitimacy of the other checks. Furthermore, this design check quite often renders valuable insight into the behaviour of the structure under repeated actions, uncovers clearly singular features of structural behaviour under the influence of repeated actions.

Unfortunately, and contrary to the gross plastic deformation design check (GPD-DC) where the principle can be and usually is used directly, direct application of

the PD-DC's principle is frequently not feasible. Numerical calculations of the responses to quite many action cycles frequently render strain increments of the order of the numerical errors, say  $10^{-5}$  per action cycle, and a decision as to whether there is progressive plastic deformation or whether the resulting values for strain increments over cycles are due to numerical inaccuracies is frequently not possible.

In addition, at present, no directly applicable, generally valid theorem against progressive plastic deformation is known. There are theorems in the literature [32,33,41], but all lack generality, and are restricted to special deformation patterns. As a consequence, the PD-DC often results in a trial and error procedure with the various application rules.

To understand the behaviour of the responses of the models used in this design check to actions that are periodic, two theorems are of importance:

- the theorem on the existence of steady cycles, and
- the theorem on the uniqueness of stresses in the steady cycle.

Both theorems are valid for materials for which the normality rule applies. The first theorem states that the stress and strain rate response of the models to actions that are periodic gradually stabilizes to remain unchanged in consecutive cycles [32, p. 508]. The second theorem states that the stress distribution in the steady cycle does not depend on the initial state of the model, and that it is unique in those regions in which the plastic strain rates are non-vanishing in the steady cycle [32, p. 509].

For, with regard to progressive plastic deformation, non-critical cases for which shakedown to linear-elastic behaviour can be proven, the shakedown application rule provides a simple, straightforward procedure via the usage of Melan's shakedown theorem, or, even more straightforward, the extended shakedown theorem.

**Melan's shakedown theorem** is usually used in its original (classical) formulation, restricted in applicability by the presupposition of a time-invariant constitutive law:

A first-order-theory linear-elastic ideal-plastic model with associated flow law will shake down under a cyclic action if a time-invariant self-stress field can be found such that the sum of this self-stress field and the cyclically varying elastic stress field for the given cyclic action is compatible with the yield condition, i.e. the equivalent stress nowhere and at no time exceeds the yield stress.

There follow two obvious, but useful corollaries:

1. If a first-order linear-elastic ideal-plastic model with associated flow law shakes down to linear-elastic behaviour under a cyclic action for any self-stress field then it will shake down for all self-stress fields.
2. Whether a first-order theory linear-elastic ideal-plastic model with associated flow law shakes down or not to linear-elastic behaviour under a cyclic action does not depend on the initial (residual) stress distribution.

The above-mentioned presupposition for Melan's shakedown theorem of a time-invariant constitutive law has importance with regard to two effects:

- the temperature dependence of the elastic material parameters of the (unbounded) linear-elastic constitutive law and
- the temperature dependence of the yield stress, used in the check of the compatibility with the yield condition, or in the comparison of the equivalent stress with the yield stress.

Both effects may be of importance in cases of cyclic temperatures – cyclic temperatures render cyclic elastic material parameters and cyclic yield stresses.

Fortunately, for temperatures below the creep regime, the temperature dependence of elastic material parameters is small, and the use of time-invariant elastic material parameters renders good approximate results – this effect does not limit the applicability of the theorem. An extension of the theorem for temperature-dependent, time-varying elastic material properties exists [32, p. 54; 43], but is hardly used.

The temperature dependence of the yield stress, on the other hand, is significant in pressure vessel applications. Fortunately, the theorem, as stated above, can easily be extended to include this temperature dependence:

**Extended Shakedown Theorem:** A first-order linear-elastic ideal-plastic model with associated flow law will shake down under a cyclic action if a time-invariant self-stress field can be found such that the sum of this stress field and the cyclically varying elastic stress field for the given cyclic action is at each time compatible with the time-dependent yield condition at the same time, i.e. in Melan's shakedown theorem the time-independent yield stress has to be replaced by the temperature-dependent, and thus time-dependent, yield stress [33, p. 125ff; 32, pp. 54, 74], or, in other words, the equivalent stress of the superposition shall nowhere and at no time exceed the yield stress on the very same point and at the very same time.

The choice of the time-invariant self-stress field is arbitrary, quite often a very good, near-optimal or optimal self-stress field can be obtained by the difference of the stress fields for the same maximum action but for two different constitutive laws. Such stress fields are usually already known – the stress field for the linear-elastic ideal-plastic model of the GPD-DC and the stress field in the linear-elastic regime scaled up to correspond to the very same action.

That this difference is a self-stress field is evident, and this procedure can be used conveniently for each load case in the GPD-DC. Linear superposition of these self-stress fields results again in a new self-stress field.

Optimization of the superposition factors and appropriate scaling of the self-stress field can be visualized well by means of the deviatoric map. The deviatoric

map can also be used to visualize the superposition of the time-invariant self-stress field and the cyclic stress field required by Melan's shakedown theorem, in critical points of the model [3,8,9,11,19].

It is recommended always to check superpositions also via the plots of the equivalent stresses, at critical times.

## 5.2. Procedure

The GPD-DC, as discussed in the preceding chapter, deals with extreme values of actions, which occur or can occur under reasonably foreseeable normal operating conditions, during testing or under exceptional conditions. These extreme action values correspond to points in the design domains.

In the PD-DC, on the other hand, cyclic functions of actions of time or a time-like parameter are considered. These cyclic functions correspond to closed action paths in the design domain. If in a specific load case only mechanical actions are considered, then only the order of consecutive action states during a cycle is of importance, and the action cycles can be described by cyclic characteristic functions of a time-like parameter. Of course, these cyclic characteristic functions may be specified, and are usually specified by periodic functions of time. If, on the other hand, thermal stresses are to be included, then the whole temperature history is of importance, and the cyclic characteristic functions have to be specified by periodic functions of time. Corresponding cyclic action paths may frequently be visualized in an extended design domain, one that includes an instantaneous rate of temperature change, additionally. This visualization is especially useful in cases where more than one cyclic action path is required in the design specification to model conservatively a specific process cycle, e.g. one fast and one slow cycle encompassing one planned, intended periodic process cycle.

The PD-DC procedure can be summarized as follows:

- The PD-DC deals with all load cases that include cyclic characteristic functions of actions, usually specified by periodic functions of time, and that are specified to occur often enough to require their inclusion in this design check.
- In load cases with non-negligible non-stationary thermal stresses, the cyclic characteristic functions of actions have to be specified by periodic functions of time.
- Each combination of values of actions that corresponds to a vertex of the design domain and that is specified as (possibly) occurring repeatedly, often enough to require its inclusion in this design check, has to be on at least one of the specified cyclic characteristic functions of actions. The whole set of these vertices of the design domain, on all of the specified cyclic characteristic functions of actions, is a

subset of the set of vertices dealt with in the GPD-DC. This subset may also include vertices corresponding to load cases that have been eliminated in the GPD-DC procedure (see Section 4.2).

- For all of these load cases with cyclic actions, the investigation shall be performed as specified in this chapter. Investigation using the design check's principle is frequently not conclusive, and is therefore not recommended. In load cases with non-negligible cyclic thermal stresses, a trial and error procedure involving some of the application rules is usually required, but no general procedure for the selection can be given. In cases without thermal stresses and without stresses induced by prescribed cyclic displacements, the Application Rule 4 renders an easy solution procedure.

### **5.3. Design Models**

The basic design models specified in the principle differ from the design models specified in Section 4.3 by the following essential points:

- The geometry of the design models depends on the used application rule:
  - In Application Rule 1 a stress-concentration-free geometry may be used. Like in the GPD-DC, in the modelling process material may be removed but not added.
  - In Application Rules 2 and 3 the standard requires detailed models, models with relevant local structural perturbation sources. The decision as to which of the local structural perturbation sources are relevant, i.e. have to be included in a model, is not easy. As a general rule, in case of doubt, a perturbation source should be included, and, if a perturbation source is excluded, in the modelling process material may be removed but not added.
- Cladding has to be included in the models with the exception of integrally bonded cladding with nominal thickness not exceeding 10% of the component thickness at the same place, inclusive of the cladding thickness – in this exceptional case the influence of cladding may be neglected, i.e. the model based on the base metal geometry.
- Mises' yield condition and associated flow rule may be used without correction of the design yield stress.

All other requirements specified in Section 4.3 remain and are as follows:

- analysis thicknesses are to be used
- limit analysis models are to be used with
  - first-order theory,

- linear-elastic ideal-plastic constitutive laws and associated flow rules, and
- specified material and temperature-dependent yield stresses, given by the specified design values of the material strength parameter.

But the relevant partial safety factors are equal to unity, and the yield stresses used in the design models are equal to the relevant material strength parameters. Under normal operating load cases, for these material strength parameters to be used as design values of the yield stress in the design models, the same relations as for the design models of the GPD-DC, given in Table 4.1, apply, but with different reference temperatures. In the PD-DC, the reference temperature for the material strength parameters is given by the (time- and space-dependent) metal temperature, e.g. the one obtained in a numerical temperature calculation procedure. As alternative, a time-independent (but space-dependent) temperature may be used that is at each point not less than the weighted mean cycle temperature  $t_{\text{ref}}^*$  given by  $0.75t_{c,\text{max}} + 0.25t_{c,\text{min}}$ , where  $t_{c,\text{max}}$  and  $t_{c,\text{min}}$  are the maximum and minimum calculated metal temperature, respectively, at this point and during the whole action cycle. Instead of the (time- and space-dependent) metal temperature, this time-independent and space-dependent temperature may also be used as reference temperature for determining the other material parameters, such as modulus of elasticity, coefficient of linear thermal expansion, thermal conductivity, thermal diffusivity, for which Annex O of EN 13445-3 applies. A Poisson's ratio of 0.3 may be used in the elastic regime. For determining the design values of the material strength parameter, the above mentioned usage of the time-dependent metal temperature as reference temperature is recommended – this approach is usually quite straightforward, and also closer to the Extended Shakedown Theorem than the alternative, which is a rough approximation for large temperature differences between  $t_{c,\text{max}}$  and  $t_{c,\text{min}}$ .

Testing load cases and exceptional load cases being exempted from this design check, design models for these load cases are not required.

Mises' yield condition is specified, because it was considered to be safe enough for this design check – safety margins, due to hardening and non-linear geometric relations, have been considered to balance any non-conservativity in the yield condition.

*Note:* The decision as to which local structural perturbation sources are to be included in the detailed model is not always easy. Frequently one has to decide which specific weld details, e.g. in the design stage only via weld procedure tests statistically known weld surface irregularities, have to be included, or which geometric imperfections. The decision is even more complicated in cases of singularities introduced by the modelling process itself, in the “simplification” of

the model geometry. The answer to this problem is contained in the Technical Shakedown Application Rule. Details that may significantly affect the stress distribution over more than 10% of a cross-section have to be included, and those that affect the stress distribution significantly only over less than 10% can be neglected. If in the region only one surface is affected the limit shifts to 20%. If this requirement for exclusion is fulfilled, then the local structural perturbation source may be omitted in the model, or the model smoothed. Fortunately, in most of the cases involving modelling-related points, or lines of singularity, the singularity may simply be ignored.

## 5.4. Design Functions of Actions

The PD-DC deals with the responses of design models to cyclic design actions. These cyclic design actions are combinations of time-independent functions and cyclic functions of time. The time-independent functions are given by the characteristic values of permanent actions and by the time-independent sections of variable actions, including pressure and temperature. The cyclic functions are given by the cyclic sections of characteristic functions of variable actions, including pressure and temperature.

It is important that the characteristic functions are indeed representative of the corresponding action. For temperature actions, it is especially important that the characteristic functions envelop not only the trajectories of reasonably foreseeable re-occurring actions in the action space, but are also representative with regard to the speed of change, i.e. they should also envelop (closely) the corresponding trajectories in the action-time space. In case of doubt, it is recommended to specify two characteristic functions, a fast and a slow one to encompass the worst case. Some theorems, useful in the selection of design functions of actions, are given in Annex A.

Most of the combination rules are similar to those of the GPD-DC:

- all permanent actions must be included in every load case,
- each pressure and temperature action must be combined with the most unfavourable variable action,
- each pressure and temperature action must be combined with the relevant sum of variable actions, and
- favourable variable actions must not be considered.

In contradistinction to the GPD-DC, wind, snow, and earthquake actions need not be considered unless required explicitly in the design specification.



## 5.5. The Principle

Using the preceding specifications for design models (Section 5.3) and design functions of actions (Section 5.4), the PD-DC's principle can be stated quite simply:

*On repeated application of the cyclic design actions on the relevant design models progressive plastic deformation shall not occur (EN 13445-3 Annex B).*

As already mentioned in Section 5.1, direct application of the principle is hardly ever feasible – a generally valid and usable theorem for progressive plastic deformation is not known, and numerical inaccuracies prevent often a numerical approach.

Therefore, a trial and error procedure with the various application rules is usually the only way out.

## 5.6. Application Rules

The application rules specified in the following are new, to be included in one of the next issues of the standard, having achieved sufficient agreement in the CEN voting procedure.

There are four application rules available, each one with advantages in special situations. A general recipe for the selection of the appropriate rule cannot be given, and the following remarks may be of help:

If a load case encompasses only mechanical actions, and not thermal stresses or stresses induced by imposed displacements, then Application Rule 4 (Section 5.6.4) is the appropriate one, giving the result without further investigations.

In non-critical cases, in which the whole detailed model shakes down to linear-elastic behaviour, Application Rule 2: Shakedown is the usually used application rule, under the usage of Melan's shakedown theorem, or the extended shakedown theorem. But only application of any of the shakedown theorems for a load case will show whether the load case is a non-critical one.

Application Rule 1: Technical adaptation is a straightforward, purely numerical approach, but because of the required multiple repetition of the action cycles, the computer time is often excessive, especially if extrapolation from a few cycles to the specified number is not satisfactory.

Application Rule 3: Technical shakedown is based on the results of many numerical calculations. It can be the last choice if the other application rules fail or are too cumbersome.

**Application Rule 1: Technical Adaptation (TA).** *The principle is fulfilled if it can be shown that the maximum absolute value of the principal structural strains is, after application of the specified number of all design action cycles on the design model, less than 5%. If the number is not specified, a reasonably large number, but at least 500, shall be assumed (EN 13445-3 Annex B).*

As already outlined in the introduction to this chapter (in Section 5.1), fatigue due to repeated plastic deformation is appropriately dealt with in fatigue design checks, and only progressive plastic deformation must be dealt with in separate design checks. Strictly speaking, even progressive plastic deformation is per se not necessarily a failure mode – it can be tolerated as long as the resulting deformations and strains due to specified application cycles are limited sufficiently. This application rule is a straightforward transposition of this idea.

Since the strain limitation applies for structural strains, stress-concentration-free models may be used, and shell and plate elements for shell- and plate-type structures. Otherwise the design models agree with those of the principle.

Despite this possibility of using simpler models, computing time is often excessive, even if extrapolation in cycles is used.

**Application Rule 2: Shakedown (SD).** *The principle is fulfilled if the detailed design model of the principle, with local structural sources of disturbance, shakes down to linear-elastic behaviour under the action cycles considered (EN13445-3 Annex B).*

This application rule is based on the fact that whenever a model shakes down under action cycles to linear-elastic behaviour, then neither alternating plasticity nor progressive plastic deformation can occur.

Together with Melan's shakedown theorem or the extended shakedown theorem, this application rule is a very convenient, handy, informative tool, with one disadvantage: It is unnecessarily restrictive, by uniting both inadaptation modes as unsafe – progressive plastic deformation and alternating plasticity.

With regard to the decision which local structural perturbation sources have to be included in the design model see the note at the end of Section 5.3.

**Application Rule 3: Technical shakedown (TSD).** *The principle is fulfilled if the following conditions are satisfied:*

- (a) *The equivalent stress-concentration-free model, or any model which deviates from the model with local structural sources of disturbance solely in these sources of disturbance, shakes down to linear-elastic behaviour under the action cycles considered.*
- (b) *For the (detailed) model with local structural sources of disturbance any time-invariant self-stress field can be found such that the sum of this stress field and the cyclic stress field determined with the (unbounded) linear-constitutive law for the cyclic action considered is compatible with the relevant yield condition continuously in a core of the structure which encompasses at least 80% of every wall thickness (EN 13445-3 Annex B).*

*Note:* The designation local structural source of disturbance used in the standard is synonymous to the designation local structural perturbation sources used here.

This application rule is less restrictive than Application Rule 2.

The first condition may be replaced by the requirement that under the action cycles considered **technical shakedown**, also called shakedown in generalized stresses, of the whole structure can be shown. In this approach the generalized shakedown theorem, also called shakedown theorem in generalized stresses, is usually used:

This **generalized shakedown theorem** [33,4] reads: If a time-invariant field of generalized self-stresses can be found such that the sum of this field and the cyclically varying (elastic) generalized stress field, determined for the given cyclic action with the (unbounded) linear constitutive law, is compatible with the local technical limit state condition, e.g. the Ivanov function  $I_v$  nowhere exceeds unity, then progressive plastic deformation cannot occur.

This “theorem” is not generally valid: it is valid only for special (plastic) deformation modes [33,41], and is not valid, for example, if a decrease of an action after plastic deformation in a cross-section does not result in unloading at each point of the cross-section. Therefore, the second condition has been added to catch the deformation modes not considered in the proof of the technical shakedown theorem.

This second requirement is based on ideas proposed in the United States of America and in Japan [43–47], and checked by numerous falsification trials in Austria.

It may be that one of the two requirements already ensures fulfillment of the principle, but both have been included as a conservative measure and for the time being. Very likely the first requirement suffices if the design value of the yield stress is determined with a partial safety factor  $\gamma_R = 1.25$  (instead of 1.0).

**Application Rule 4: Technical Shakedown for Mechanical Actions.** *The principle is fulfilled for all action cycles within the range of actions allowable according to the Gross Plastic Deformation Design Check (EN 13445-3 Annex B).*

This application rule is restricted to structures made of essentially the same materials, and to load cases with mechanical actions only, i.e. to load cases without thermal stresses and without stresses induced by prescribed displacements. It may also be used for load cases with prescribed displacements that can be converted into load cases with prescribed forces via global equilibrium conditions, e.g. load cases with prescribed vanishing vertical displacements at brackets.

If applicable, this application rule gives results quickly and without additional effort.

## 5.7. Examples

Examples can be found in Annex E.5.

## Chapter 6

# Stability Design Check (S-DC)

### 6.1. Introduction

In the gross plastic deformation design check (GPD-DC) and the progressive plastic deformation design check (PD-DC) the design models used give confidence in results – the results for well-posed problems are unique, they are fairly insensitive to initial residual stresses, initial geometric imperfections, material inhomogeneities, action histories and to perturbations of action values, and quite often they are even independent of these disturbances.

Additionally, in the PD-DC the underlying failure mode is not related to a sudden failure in a single application of an action, but with progressing deformation due to cyclic actions, allowing the timely detection of failure by appropriate in-service inspections.

In the S-DC, discussed in this chapter, this confidence cushion does not exist – non-uniqueness is an essential feature of this design check and the used design models, and imperfection sensitivity as well as sensitivity to initial residual stresses to the action histories is a quite frequent property of real structures and of design models.

To reflect the behaviour of real structures appropriately, models with non-linear kinematic relations and second-order-theory are required. The second requirement, the obligation to apply second-order-theory, i.e. equilibrium conditions for the deformed structure, leads to another feature of the models not encountered in first-order theory models: Pressure is displacement dependent, and it acts normal to the surfaces of structures. In second-order-theory models, pressure is normal to the deformed surfaces, and its value may also depend on displacements. This can decrease results for critical pressures in the stability checks quite considerably, compared with the results for models with assumed deformation-independent pressure. For example, for the sufficiently long cylinder under external lateral pressure, the result for pressure normal to the (displaced) surface is near the bifurcation point 25% smaller than the result for displacement-independent radial pressure, even for small displacements and small strain, and even in an elastic case (see Fig. 6.1 and [49,50]).

In this case, calculation with displacement-independent pressure is (unconservatively) erroneous by 25%. On the other hand, in the better known Beck's problem, the Euler column clamped at one end and with tangential follower

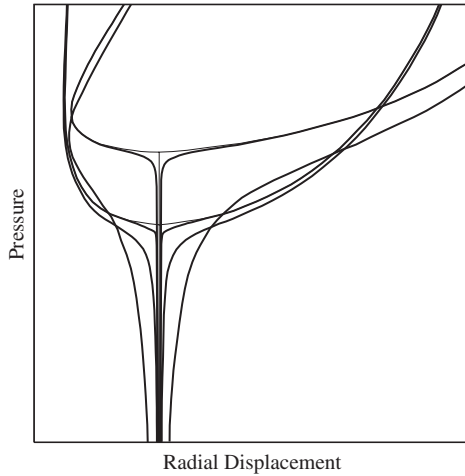


Figure 6.1: Pressure–displacement paths: long (infinitely) cylinder under external pressure; with pressure normal to displaced surface, and with displacement-independent pressure [50, p. 66].

force at the other, the result for the follower (tangential) force is larger than the corresponding one for the displacement-independent force by a factor of approximately 8 [51] – use of the Euler buckling load or a calculation with displacement-independent force were by far too conservative.

*Note:* In the treatment of Beck’s problem a dynamic design model is required, whereas in the other examples mentioned static models suffice.

The problems related to displacement-dependent pressure in finite element analyses are discussed in [52] in detail. The following conclusion can be made: With the exception of cases where the pressure obviously requires a dynamic model, e.g. in cases of flutter, quasistatic models are sufficient in all cases of pressure vessel design. This pressure, which acts normal to the displaced surface, is to be taken into account in all models used in S-DCs. Admittedly, for the infinitely long cylinder, this effect is for buckling patterns with many buckles smaller than those mentioned above and may even become negligible.

Geometric non-linearity, imperfection sensitivity, and sensitivity to residual stresses result in a multitude of different buckling phenomena, to be taken into account in the design models to allow for their occurrence.

For example, a perfect model of a circular cylindrical homogeneous shell of uniform temperature and subject to lateral external pressure will deform initially in a circular-symmetric mode, but at larger pressures this circular-symmetric state becomes unstable, the model does not deform circular-symmetrically anymore.

Figs. 6.2–6.4 show pressure–deflection paths for three different imperfect cylinders; short (Fig. 6.2), medium long (Fig. 6.3), and for a relatively long one (Fig. 6.4).

For the perfect cylinders, the pressure–deflection paths are shown by the fine lines. These figures show that the initial pressure–deflection paths bifurcate at specific pressures, the bifurcation point pressures, and for higher pressures the states are unstable. The figures also show that at pressures below the bifurcation point pressure, states are unstable, and indicate that for very short cylinders, shorter than the short one depicted here, the bifurcation point state may be stable.

In cases where the state at the bifurcation point pressure is unstable, states at pressures below the bifurcation point pressure but above the minimum pressure in the bifurcating pressure–deflection path are stable only for small perturbations and unstable for larger ones.

The paths for the imperfect cylinders, shown as thick lines, are monotonically increasing or pass through a maximum and a minimum, often called upper and lower critical point, respectively.

Experimental results, reported in [53, Section 3.4], also show jumps from circular symmetric states to states without circular symmetry, in loading and vice versa in unloading, indicating clearly that the lower critical point pressures are design-relevant, although not necessarily design-decisive – the perturbation required for the jump may be large enough, such that the circular-symmetric state, being stable for the small perturbation, is safe enough.

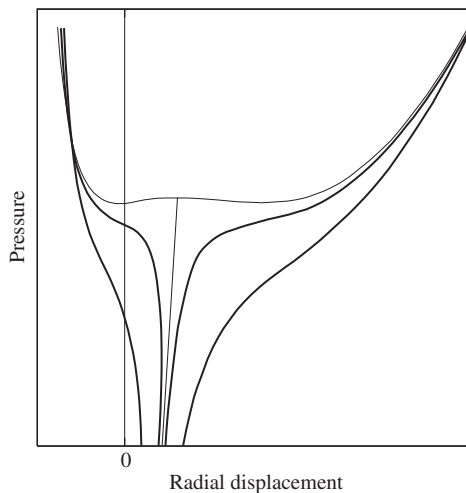


Figure 6.2: Pressure–deflection paths of short, elastic, circular cylinders under external pressure;  $L = 4.6 \sqrt{Re}$ ,  $R/e = 405$  [53, p. 312].

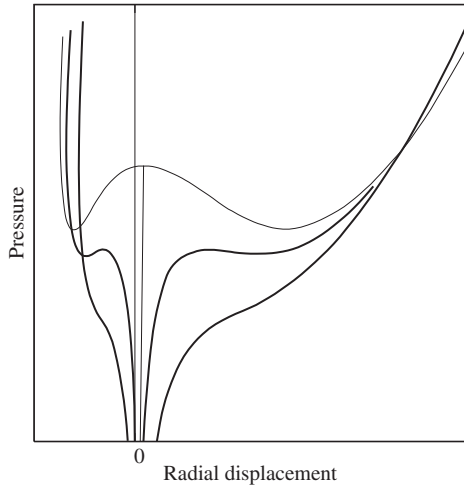


Figure 6.3: Pressure–deflection paths of short, elastic, circular cylinders under external pressure;  $L = 10.2 \sqrt{Re}$ ,  $R/e = 405$  [53, p. 312].

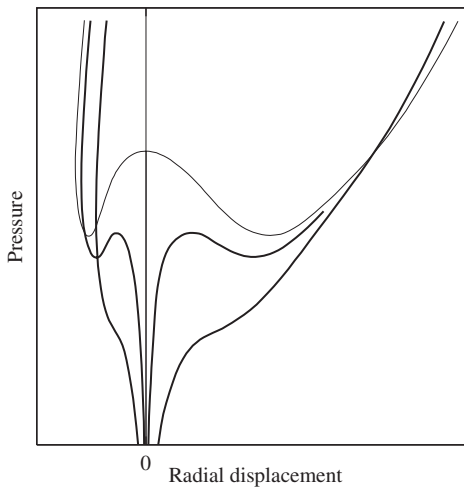


Figure 6.4: Pressure–deflection paths of elastic circular cylinders of moderate length under external pressure;  $L = 22.9 \sqrt{Re}$ ,  $R/e = 405$  [53, p. 314].

In all of these results for circular cylindrical shells under lateral external pressure, the buckling modes are symmetric with regard to the mid-plane between the two end-planes, see e.g. the post-buckling modes in Fig. 6.5, for a long cylinder.

The above results indicate that for all cylinders with the exception of extremely short ones, post-buckling deformation patterns resulting in minimal critical lower point pressures are different from the patterns resulting in minimal bifurcation point pressures for perfect shells, or minimal upper limit point pressures for imperfect shells. In other words, the bifurcation pattern corresponding to the lowest bifurcation point pressure is, in general, not the critical post-buckling pattern, but the pattern which corresponds to the design relevant minimal lower limit point pressure, giving the minimal equilibrium pressure after buckling (see also [48]).

Circular cylindrical shells under axial compressive forces show this change in post-buckling deformation patterns even more pronounced. Fig. 6.6 shows experimental load–deflection curves for this case, and Fig. 6.7 shows the observed deformation patterns – an asymmetric one, with regard to the mid-plane.

In Figs. 6.6 and 6.7, first buckling took place distinctly, but was associated with a sudden drop in force carrying capacity and snapping into an asymmetric deformation pattern with 12 circumferential buckles. Increase of the imposed, but now lower, force resulted in another buckling, with sudden force drop into another asymmetric deformation pattern, but now with 11 buckles, and so on. Symmetric

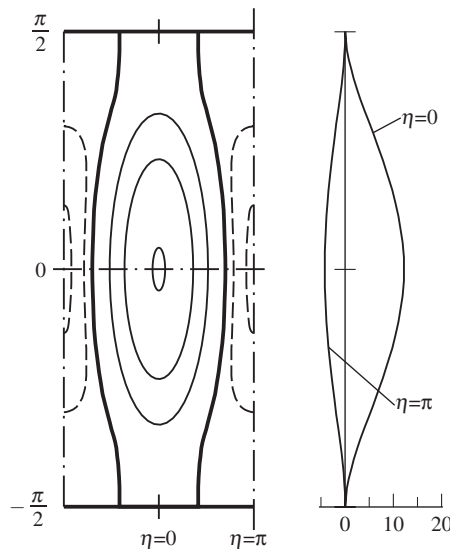


Figure 6.5: Post-buckling deformation pattern of a long circular cylinder under lateral external pressure;  $L = 32.4 \sqrt{Re}$ ,  $R/e = 405$ [53, p. 213].



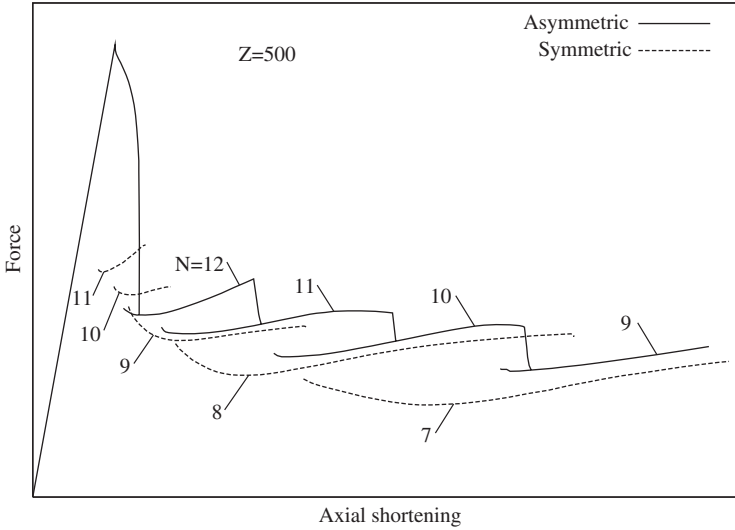


Figure 6.6: Force-shortening paths of a long circular cylinder under axial compression;  $L = 22.9 \sqrt{Re}$ ,  $R/e = 405$  [53, p. 230].

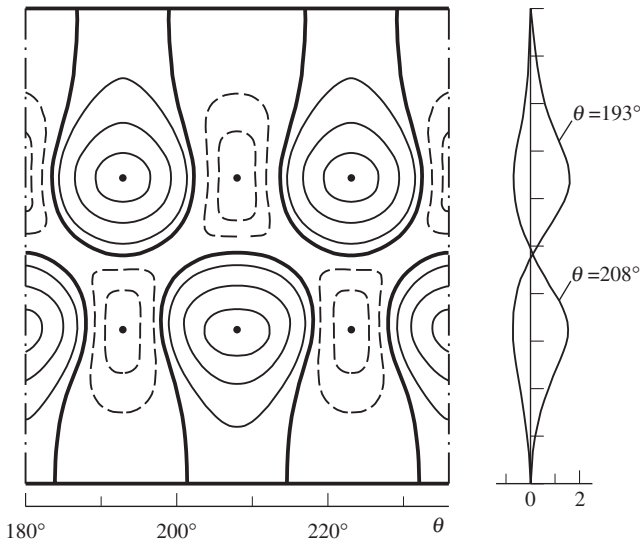


Figure 6.7: Post-buckling deformation pattern observed on long circular cylinder under axial compression;  $L = 22.9 \sqrt{Re}$ ,  $R/e = 405$  [53, p. 230].

deformation patterns were obtained by applying appropriate finger tip pressure at the shell wall; at the same time the wave number increased by 1.

For short shells, with  $L \leq 4.6 \sqrt{Re}$ , only symmetric post-buckling patterns were observed, and for the longer shells with  $L \geq 22.9 \sqrt{Re}$  asymmetric patterns occurred spontaneously.

For an extremely short shell with  $L = 4.6 \sqrt{Re}$  no force reduction was observed in buckling, but a local buckle appeared first, and then propagated circumferentially, with increase in force and end shortening, until the whole circumference was covered with uniformly distributed waves [53, p. 226].

The experimental results plotted in Fig. 6.6 depend, of course, on the testing equipment. They indicate a very strong sensitivity to geometric imperfections, not only with regard to the values of the imperfection, however defined, but also with regard to the imperfection patterns. This can be seen clearly in analytical results [53, p. 353ff]. Figs. 6.8a and b, show analytical results for a shell of medium length: The initial pre-buckling path and post-buckling equilibrium curves for various symmetric post-buckling patterns are plotted on the left and asymmetric ones on the right. For the symmetric patterns the lowest bifurcation point force corresponds to a pattern with 18 waves in circumferential direction; for the asymmetric patterns, it corresponds to a pattern with 17 waves and with slightly larger force. Neither of these two patterns agree with those related to the so-called characteristic post-buckling pattern, proposed in [54] as design-decisive, and defined as the

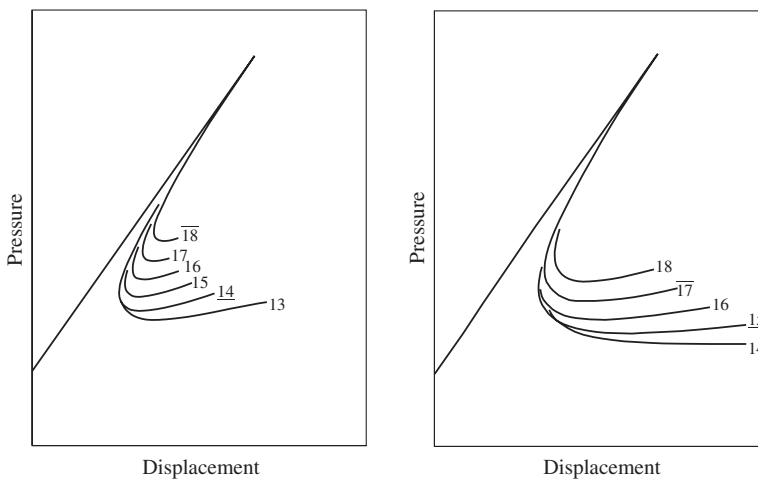


Figure 6.8: Force-shortening relations for a circular cylindrical shell of medium length under axial force.  $N$ , wave number;  $L = 14.4 \sqrt{Re}$ ,  $R/e = 405$  [53, p. 353].  
 Left: Symmetric patterns; right: asymmetric patterns.

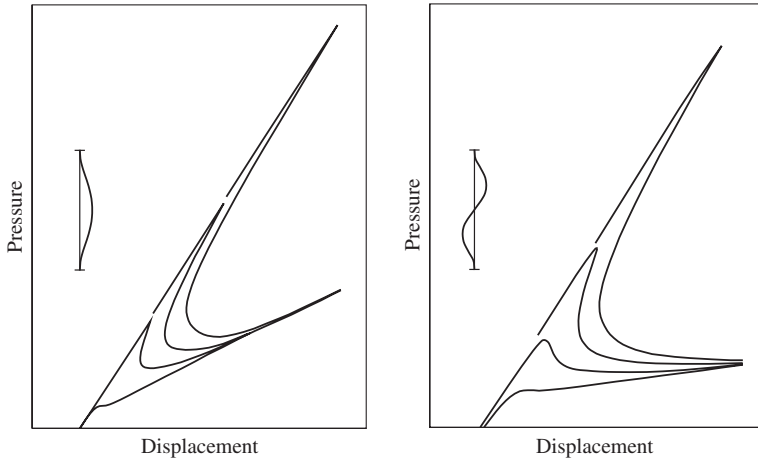


Figure 6.9: Force-shortening paths of a circular cylindrical shell of medium length under axial force;  $\bar{w}_0 = \max_0 / e$ ,  $L = 14.4 \sqrt{Re}$ ,  $R/e = 405$  [53, p. 356].  
Left: symmetric patterns; right: asymmetric patterns.

equilibrium pattern having the minimum edge shortening. They also do not agree with the experimentally observed ones (shown in Fig. 6.6 as dashed lines). The post-buckling pattern will depend on the testing equipment but is, in general, different from the bifurcation pattern.

From the highly unstable post-buckling equilibrium paths, shown in Fig. 6.8, it is obvious that small imperfections will result in considerable reductions in the critical forces. Fig. 6.9 shows this extreme imperfection sensitivity with regard to the values of the imperfection, and the ratio of the maximum deviation and the wall thickness for patterns with 14 symmetric waves and for patterns with 15 asymmetric ones, respectively.

Fig. 6.9 shows that the asymmetric patterns are associated with smaller forces; for larger imperfections, upper and lower critical points disappear, and the force-shortening paths become monotonically increasing curves without critical points. In Fig. 6.9, this change in behaviour occurs for the asymmetric pattern at a maximum initial deviation from the perfect shape of approximately 60% of the wall thickness, and the smallest upper critical force is approximately 40% of the bifurcation point force (of the perfect shell).

The geometric imperfection sensitivity plots (the upper and the lower critical force vs. the maximum initial deviation from the perfect shape) in Fig. 6.10 show, for a symmetric post-buckling pattern with 14 waves in circumferential direction (lines a, c) and the asymmetric one with 15 waves (lines b, d) respectively, the very

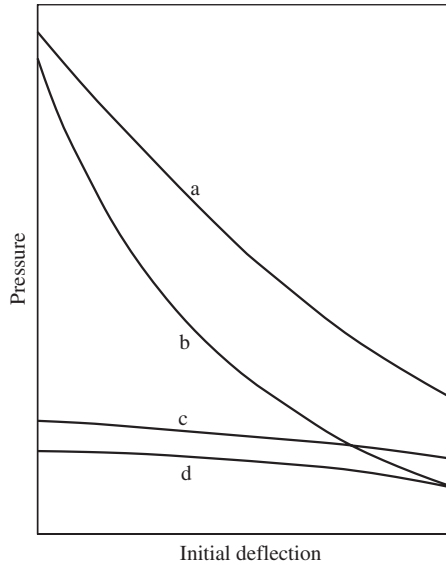


Figure 6.10: Upper and lower critical point force vs. maximum initial imperfection circular of a cylinder of medium length under axial force;  
 $L = 14.4 \sqrt{Re}$ ,  $R/e = 405$ .

strong imperfection sensitivity of the circular cylindrical shell under compressive axial force, and the dramatic reduction in carrying capacity, quite clearly.

The spherical shell under external pressure exhibits equally dramatic imperfection sensitivity. Figs. 6.11 and 6.12 show numerical results for the upper critical point pressure vs. the maximum initial deviation from the perfect shape; Fig. 6.12 (right) shows the initial circular symmetric deviations used in this investigation [56–58].

The parameter  $\lambda = (48 \cdot (1 - \mu^2))^{1/4} \cdot \sqrt{H/e}$  in Fig. 6.11 is a measure of the assumed size of the deviation. The dashed line is a reasonable envelope of the festoon curve (for integer values of  $\lambda$ ).

Fig. 6.11 can be used as a very practical tool in determining the critical imperfection size for a given imperfection depth  $\bar{w}_0$ . For example, for a spherical shell of 1400 mm diameter, 5 mm thickness, and an imperfection depth  $w_0 / e = 1.0$ , Fig. 6.11 gives an approximate value of  $\lambda = 4$ , which renders  $H = 12.1$  mm, and thus a critical imperfection diameter of 259.2 mm.

Fig. 6.13 shows the pressure–deformation paths for the spherical shell subject to external pressure.

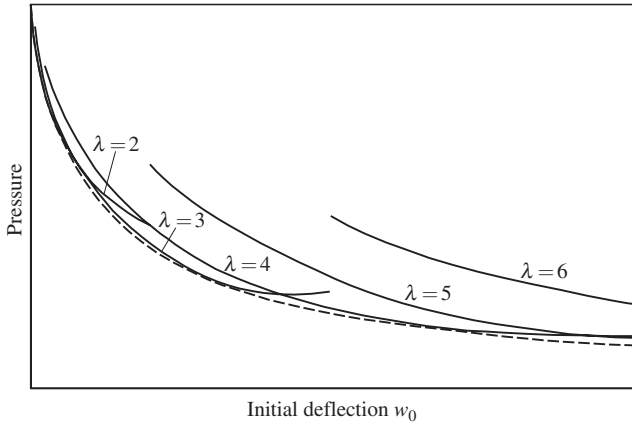


Figure 6.11: Upper critical point pressure of a spherical shell under external pressure vs. initial imperfection;  $\bar{w}_0 = w_0/e$ ,  $\lambda = 257\sqrt{H/e}$  [56, p. 73].

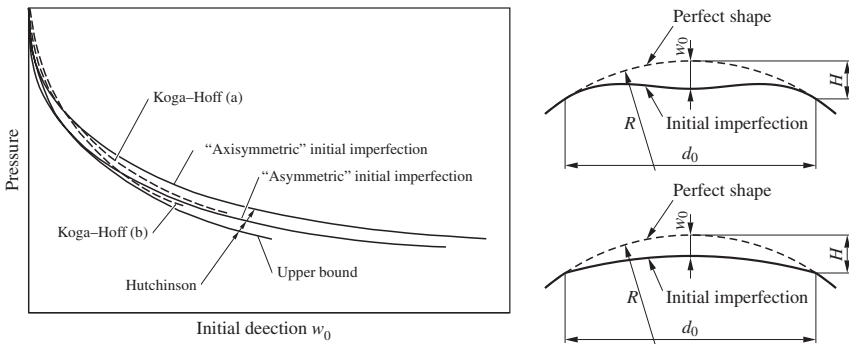


Figure 6.12: Spherical shell under external pressure;  $\bar{w}_0 = w_0/e$  [56, p. 71; 58]. Left: upper critical point pressure; right: assumed initial imperfection shape.

For all but extremely small imperfection depths, and for the spherical shell – in contrast to the cylindrical one – practically the same results for the upper and the lower limit point pressure, respectively, are obtained, irrespective of whether only circular symmetric imperfections and post-buckling deformations are used or also asymmetric imperfections and deformations [56, p. 74].

Some special spherical caps, sufficiently large and with very special hypothetical boundary conditions, buckle exactly like full spherical shells, but practically all caps used in pressure vessel design behave less dramatically. Boundary conditions or the transition conditions to adjacent parts, such as knuckles, flanges, cylindrical

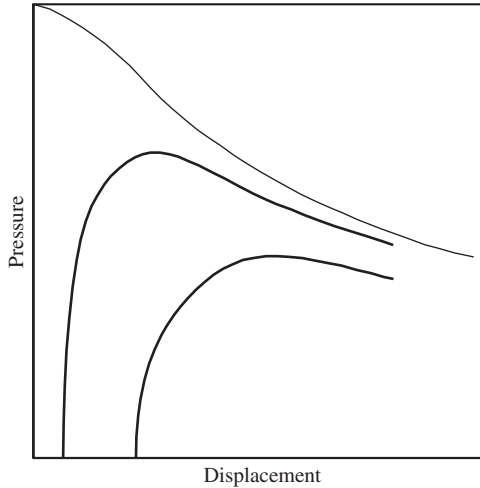


Figure 6.13: Pressure–deformation paths of spherical shells under external pressure [56, p. 208].

shells, cannot be fulfilled by pure membrane stress states, but result in bending stresses, and, therefore, even geometrically perfect caps exhibit bending stresses before buckling, and geometrically perfect spherical caps behave as if they had initial imperfections, with more clearly defined and observable limit points than the full spherical shells [56, p. 74ff].

Similar effects occur also with structures where the perfect structures with appropriate perfect boundary conditions exhibit bifurcation buckling: Boundary conditions, adjacent to other parts of the structure, and offset of axes or middle surfaces, may result in bending stresses, and, therefore, the perfect structure behaves like a geometrically imperfect one, and buckling, if it occurs at all, will be limit point buckling. Additionally, buckling load–deflection paths may become asymmetric, i.e. results for positive initial deformations are different from those for negative ones.

Interactive buckling is an instability failure mode that has to be considered quite often in pressure vessel design. Interactive buckling is often related to a very strong imperfection sensitivity [59,60, p. 314; 61], and if it is the result of multiple symmetry, symmetry-breaking imperfections can render a complicated imperfection sensitivity, with an infinite variety of effects [60, p. 316]. Symmetry-breaking imperfections are also frequently introduced into finite element models of perfectly circular symmetric problems via routine usage of automatic meshing routines.

All the analytical results shown above are for purely linear-elastic buckling. Plasticity does not change the phenomena very much, especially so for imperfect structures (see, e.g. [62,63]).

An extensive literature on buckling phenomena exists; the selection [49–67] and, with emphasis on steel constructions, the guide issued by the Structural Stability Research Council [68], is biased towards those which give insight into the various phenomena.

An important field in pressure vessel buckling or structural buckling is treated quite poorly in the literature on structural buckling. Thermal stress buckling and buckling influenced by thermal effects [69–73]. Temperature changes in a structure result in deformation and/or stresses. Temperature-related deformations act like initial geometric imperfections, and perfect structures behave like geometrically imperfect ones. Thermal stresses, on the other hand, can result in buckling on their own, bifurcation or limit point buckling, and they can enhance or diminish buckling due to other actions, often changing symmetric buckling load–deflection paths into asymmetric ones.

Like other deformation-controlled buckling, thermal stress buckling is accompanied by less critical action–deformation behaviour, although jump phenomena do exist.

In the design models used in the principle, characteristic values of material parameters – modulus of elasticity, Poisson’s ratio, and material strength parameters – are to be used as design values, i.e. no partial safety factors need be applied. Partial safety factors are to be applied on the actions and on the calculated buckling strengths. The partial safety factors to be used have been calibrated with respect to DBF results for simple geometries.

Imperfection sensitivity, plasticity and thermal effects are dealt with in the stability design checks directly, using linear-elastic ideal-plastic geometrically imperfect models with thermal effects. The phenomenon that buckling patterns change during buckling and post-buckling, and that the pattern at the upper critical or bifurcation point may be different to the one at the smallest lower critical point, if any, is dealt with only indirectly, in a pragmatic approach. This is done considering the behaviour of an imperfect structure with initial deformation corresponding to the buckling patterns at the bifurcation or upper limit points of the geometrically perfect structure and calibrated with respect to the allowable geometric fabrication tolerance limits, and presuming that this will give a reasonable approximation to the lower critical point values of the imperfect structure.

As already discussed in Sections 4.1 and 4.2, there are also non-buckling cases where resulting deformation has a weakening effect, where the carrying capacity is detrimentally influenced by ensuing deformation and where the results are highly imperfection-sensitive. For these cases, the failure modes may be gross plastic deformation, excessive local strains, and, possibly, buckling. The usual gross plastic deformation design checks (Chapter 4) are for these cases not safe. For the corresponding load cases, called here **deformation-weakening GPD-DC load cases**, additional checks as described in the following are required. These

checks are usually in addition to the relevant gross plastic deformation design checks, but in many load cases it may make the relevant gross plastic deformation investigations superfluous if it is obvious that the relevant design model (for the deformation-weakening GPD load case) results in allowable design actions that obviously encompass the corresponding ones of the usual GPD design check, described in Section 4.3 (see also Section 4.1).

In these deformation-weakening GPD load cases, initial deformation patterns that correspond to the initial elastic deformation patterns are to be used, instead of the buckling patterns used in buckling load cases.

## 6.2. Procedure

If thermal effects do not exist or can be neglected, then the procedure in the checks discussed here is practically the same as in the GPD design checks, discussed in Chapter 4.2:

- The design checks can be seen as investigations of the capacity of the structure to carry safely all of them to be considered states of actions.
- The design checks encompass all action states in the design domain, they deal with all load cases which correspond to vertices of the design domain related to buckling problems or problems with deformation-weakening.
- Of these load cases some can be eliminated, because the investigation result is encompassed by any of those for the non-eliminated ones.
- For all of the remaining load cases the checks shall be performed as specified in this chapter.
- These proper checks are investigations as to whether the design models can carry the design actions of the load cases with specifically limited structural strains.

If thermal effects do exist and cannot be neglected, these effects have to be considered in the investigations, and proper design functions of temperatures are required in the load cases.

The following sections deal with the investigations required for all (non-eliminated) buckling and deformation-weakening load cases that correspond to vertices of the design domain, complemented with design functions of temperature if thermal effects exist and cannot be ignored.

## 6.3. Design Models

All design models used in the investigations discussed here may be stress-concentration-free models. In the modelling of the stress-concentration-free



geometry, in principle, material may be removed and adding of material requires justification. Integrally bonded cladding has to be considered with respect to both thermal analysis and stress analysis. Structural strength may be attributed to the cladding only in case of integrally bonded type and by agreement of the parties concerned. For clad components the nominal face of the cladding is to be used as surface of the pressure application.

A pressure correction may be required in cases of finite element models with shell elements, if pressure is (in the software) applied at the centroidal surfaces of the elements and not on their actual surfaces. A pressure correction is required if cladding is not included in the model. In both cases the correction factor is given by the ratio of the (infinitesimal) areas of the surfaces on which pressure is applied – actually and in the finite element model. If thermal effects do exist and cannot be ignored, cladding has to be included in the model.

All of the design models used are geometrically non-linear with

- non-linear kinematic relations and second-order-theory
- linear-elastic ideal-plastic constitutive laws
- Mises' yield condition and associated flow rule
- specified material and temperature-dependent yield stress, given by the specified design value of the material strength parameter (see Sections 6.3.2 and 6.3.3 respectively)
- stress-free initial state
- specified initial deviations from the perfect geometry (see Sections 6.3.2 and 6.3.3).

Second-order theory with linear kinematic relations may be used if known to render sufficiently accurate results. Mises' yield condition was chosen here for convenience – in general plasticity effects are small anyway.

The yield stresses to be used in the design models – the design values of the material strength parameters – are given by the relevant material strength parameters  $RM$ . The values to be used are the same as for the GPD design check and are given in Table 4.1 for normal operating load cases and exceptional load cases, and in Table 4.2 for testing load cases. These values may be used directly as design values, i.e. no division by partial safety factors is required.

For the values of the material strength parameters, see also the definition of the design value of the material strength parameter, it is common practice to use the minimum values specified in the material standards.

Contrary to the GPD design check, where the results are insensitive to or even independent of used values for the material parameters in the linear-elastic regime, the results for buckling load cases depend (directly) on the values of the parameters modulus of elasticity and Poisson's ratio, and the results for

deformation-weakening load cases may depend on these values. Therefore, it is required that for the (design) modulus of elasticity the temperature-dependent values of Annex O of EN 13445-3 are used, and Poisson's ratio of 0.3 is used in the elastic regime.

For the reference temperature,  $t_{RM}^*$  for the determination of the material strength parameters,  $t_E^*$  for the determining all other material parameters, such as modulus of elasticity, Poisson's ratio, coefficient of linear thermal expansion, coefficient of thermal conductivity, a temperature not less than the maximum calculated metal temperature of the load case shall be used. This reference temperature may be (chosen) space-independent or space-dependent. In the first case, the (chosen) value must not be smaller than the maximum calculated metal temperature in any point of the considered model. In the second case, the reference temperature at each point must not be smaller than the calculated metal temperature at this point. It is common practice to use, in this second case, the stationary result of the (numerical) temperature calculation for the load case directly.

In load cases involving non-stationary thermal stresses, the non-stationary results of the (numerical) temperature field calculation for the load case are to be used as reference temperature for determining the material parameters.

In all design models initial deviations of the perfect shape of the structure are to be incorporated. The initial deviations to be used are given as follows:

- in buckling load cases, by the deformation patterns of the perfect structure at the bifurcation points or the upper limit points,
- in deformation-weakening GPD load cases, by the initial elastic deformation patterns of the perfect structure,
- in both cases with the deviations calibrated with respect to the maximum permissible geometric fabrication tolerances, as specified directly in the Technical Documentation, usually the drawings, or indirectly by reference to EN 134455, or by reference to any other technical specification.

In some buckling load cases neither the initial imperfection corresponding to the calibrated (classical) deformation pattern for the bifurcation point with the lowest action value nor the one for the limit point with the lowest action value is the critical imperfection with regard to this stability design check. The usually large initial imperfections and the effect of plastic flow quite often render results for different initial imperfection shapes that are reasonably close together. For critical cases it is recommended to apply the following procedure:

- Determination of the first deformation shapes of the perfect structure at bifurcation or limit points, calibrated, using the software in the usual way, with maximum deviation from the perfect shape of the structure equal to unity.

- Determination of the initial imperfection given by the linear superposition of these first deformation shapes and calibrated such that the maximum deviation from the perfect shape corresponds to the maximum permissible fabrication tolerances. The required number of deformation shapes depends on the density of the bifurcation or limit point values; 10 seems to be a reasonable value for almost all cases.
- Performance of the design check with this initial imperfection, with investigation of the development of the deformation pattern, and, if the evolving pattern differs substantially from the initial one, then
- Performance of another design check run, but now with an initial imperfection given by the final pattern of the preceding run, calibrated such that the maximum deviation from the perfect shape corresponds to the maximum permissible fabrication tolerances.

#### 6.4. Design Values and Functions of Actions

The design values of actions and the combination rules are the same as in the GPD design check (Section 4.4). Of course, only those load cases are relevant where instability is a failure mode to be considered. Contrary to the GPD-DC, where thermal stresses and thermal deformations may be neglected, these thermal effects are in this design check to be taken into account. The relevant partial safety factor for temperature actions, equal to unity, is already included in Table 4.3. Also, contrary to the GPD-DC, checks for testing load cases are necessary.

#### 6.5. The Principle

Using the specifications for design models, mentioned earlier (Section 6.3) and design values of actions (Section 6.4), the principle of the stability design check can be stated as follows: For all relevant load cases, the product of the design values of the actions and the relevant partial safety factors  $\gamma_R$  shall be carried by the relevant design models with maximum absolute value of principal structural strains not exceeding 5% under normal operating load cases, and 7% in testing and exceptional load cases, and for proportional increase of all actions except temperature, which shall be increased first, or simply applied as initial condition. The design value of the buckling strength is given by the quotient of the design model's buckling strength and the corresponding partial safety factor  $\gamma_R$ .

In load cases where thermal effects are included, design functions for temperature are to be used. In load cases without thermal effects temperature is included

in the investigations only via the temperature-dependent material parameters, via the reference temperatures  $t_{RM}^*$  and  $t_E^*$  (see Section 6.3).

The partial safety factors are as follows:

- In buckling load cases:
  - 1.25 for normal operating load cases provided that (external) standard pressure tests as called for in EN 13445 5 are carried out;
  - 1.5 for normal operating load cases without such a standard pressure test;
  - 1.1 in testing load cases.
- In deformation-weakening load cases:
  - for normal operating load cases the values given in Table 4.1 for gross plastic deformation load cases,
  - for testing load cases the values given in Table 4.2, for gross plastic deformation load cases.
- For exceptional load cases partial safety factors are to be agreed by the parties concerned, but must not be smaller than 1.1.

## 6.6. Application Rules

There are two application rules for this design check in Annex B of EN 13445. The first rule deals with experimental results, and will not be discussed here. The second one states that fulfilment of the requirements of clause 8 of EN 134453 (the DBF requirements) suffices as stability design check for pressure action, and no discussion seems to be required here.

## 6.7. Examples

Examples can be found in Annex E.6.

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## Chapter 7

# Cyclic Fatigue Design Check (F-DC)

### 7.1. Introduction

#### 7.1.1. General Remarks to the F-DC

The principle of this design check in Annex B of EN 13445-3 is very general: *The design value of the accumulated fatigue damage index  $D_d$  for cyclic fatigue, obtained for all the (cyclic) design functions of pressure/temperature and variable actions, shall not exceed 1 (EN 13445-3, Annex B).*

This requirement is given meaning by the corresponding application rule, which refers to clause 18, the fatigue clause in the DBF section of EN 13445-3: *Fulfillment of the requirements given in clause 18 suffices as a check against fatigue failure (EN 13445-3, Annex B).*

Therefore, the cyclic fatigue requirements given in this clause 18 are dealt with in the following, with the incorporation of the requirements for the consideration of cladding given in the fatigue sub-clause of Annex B of EN 13445-3. Of course, emphasis is on the usage as a design check within the DBA approach. Clause 18 of EN 13445-3 not being incorporated in Annex B, the annex containing all the other requirements for the Direct Route in Design by Analysis, and having seen so many misuses, it seems to be necessary to repeat the warning with regard to design checks: Clause 18 is, in its intention and planned usage, a design check – it is not meant to give rules for a simulation of the fatigue behaviour of real structural parts, it is not meant to give an approximation of the fatigue behaviour, nor has it been developed for usage in on-line determination of fatigue damage. That clause 18 can be used, and has been used, for these purposes successfully in combination with great care and expertise does not change that.

The fatigue analysis according to clause 18 looks very complicated and confusing, and it easily leads to mistakes in manual calculations, if the standard has to be consulted for details. In actual applications it usually is straightforward and uncomplicated, especially if supported by simple computer programs, which take care of the various variants. Clause 18 looks complicated, because so many different cases and possibilities had to be included, in a general presentation of the requirements.

The approach to cyclic fatigue investigation in clause 18 is a very modern one, taking account of the fact that welded regions show a different cyclic fatigue behaviour than unwelded regions:

- In unwelded regions, a large proportion of the cyclic life is required for crack initiation and only a short proportion for crack propagation till breakthrough of the crack, or till rupture.
- In welded regions, in contrast, the existence of microcracks (crack-like weld defects of microscopic scale) has to be taken into consideration – in a conservative approach and in a statistical evaluation of experimental results a very short or even non-existent crack initiation phase has to be taken into account, with the main proportion of the cyclic life determined by the crack propagation phase.
- In welded regions many of the influencing factors on cyclic life are not, or not sufficiently, known, at least not in the design stage:
  - local surface notches at the weld, like weld bead roughness, weld ripples, local undercut, local shrinkage grooves, local root concavity, welding stop/start craters,
  - material properties in the various weld “zones”,
  - residual stresses, and
  - internal defects.

This contrasting behaviour is taken into account by usage of

- different **design fatigue curves**, i.e. curves representing the relevant reference stress range versus the allowable number of cycles till failure – break-through or initiation of technical cracks, with
  - different reference stress ranges,
  - different corrections for the various other influences, other than the reference stress ranges,
  - different fatigue design curves for welded regions in dependence of the weld details and the orientation of main principal stresses, via different fatigue classes, corresponding to weld details, possible crack initiation spots and possible crack propagation directions, and orientation of main principal structural stresses,
  - different fatigue design curves for unwelded regions in dependence of the base material’s ultimate strength.

For damage accumulation, due to different single-amplitude cyclic actions or different sub-cycles of multi-amplitude cyclic actions, linear accumulation is prescribed in the standard, i.e. the usage of the **Palmgren-Miner cumulative damage rule**, commonly called **Miner rule**, and for the determination of the relevant sub-cycles of multi-amplitude cyclic actions the reservoir cycle counting method is prescribed.

Warning note 1: Plain material may contain weld repairs. Where weld repairs cannot be excluded, the corresponding regions have to be assessed as welded regions, like clad material.

Warning note 2: The basic databases for the design fatigue curves and for some of the correction factors encompass only results of tests in air, the design fatigue curves and the correction factors do not allow for any influence of environment. Corrosive environment has, in general, a strong detrimental effect on fatigue behaviour – crack initiation and crack growth under cyclic actions can be substantially accelerated, i.e. in environmentally enhanced fatigue cracks can occur at lower stress range values, cracks can occur earlier, and crack propagation rates can be much larger. An environment may produce negligible uniform corrosion under constant actions, but result – at crevices, gaps, or cracks – in intensive localized attack and in strongly enhanced corrosion fatigue under cyclic actions. Non-detected cracks at final testing and cracks initiated by fatigue in service can provide ideal sites for such accelerated corrosive attack – in the cracks corrosive reactions may be set up resulting in accelerated crack propagation. Especially prone to such corrosion fatigue effects are welded regions, because of the possibility of non-detected microcracks and of unfavourable weld residual stresses, which can, in some materials and environments, lead to stress corrosion cracking with possible synergetic interaction with cyclic fatigue. As with other aspects of corrosion and fatigue, attention to (corrosion resistant) material selection and to detail design, with regard to stress concentrations, gaps, and crevices, where corrosion fatigue is possible, is essential – the designer should always be conscious of the risk of stress–corrosion cracking and corrosion fatigue.

Clause 18 of EN 13445-3 gives only requirements with regard to the conservation of the magnetite layer of parts made from non-austenitic steels in contact with water at temperatures exceeding 200°C.

In the context of warning note 2, the general presupposition for the application of the fatigue calculation specified in EN 13445-3 is of importance: Fatigue critical regions are accessible for inspection and non-destructive testing, and instructions for appropriate maintenance (and dedicated inspection) are established and included in the operating instructions. If fatigue critical regions are not accessible for inspection and non-destructive testing, the design shall ensure that in these regions fatigue relevant stress ranges are not larger than the respective endurance limits.

Where corrosion fatigue has to be taken into account and effective protection of the material from the corrosive environment cannot be assured, special correction factors should be used, correction factors based on experiment or testing. In setting up of experiments and tests great care and expertise is required, since temperature, medium velocity, and, quite frequently, small traces of substances or impurities can significantly affect rate and form of corrosion. Relevant testing standards can be found in



the extensive lists of the American Society for Testing and Materials (ASTM), the European Committee for Standardization (CEN), the European Federation of Corrosion (EFC), the International Organization for Standardization (ISO), and the National Association of Corrosion Engineers (NACE), to name only the most widely used ones, in alphabetical order. Relevant fatigue test factors, required for the evaluation of test results, are given in sub-clause 18.10.3 of EN 13445-3.

Increase of in-service inspection frequency is a valuable and useful supplement, especially in cases where sufficient reliable experience is not yet available.

### 7.1.2. General Remarks to the F-DC of Unwelded Region

The design fatigue curves of clause 18 for unwelded regions, shown in Fig. 7.1, have been deduced from a fairly extensive database of experimental results with single-amplitude push-pull and bending tests on polished specimens, see [72] and the literature cited therein. The main proportion of the results relate to rupture, but it had been found that results for technical crack initiation are within the scatterband of those for rupture – for the mainly small unnotched specimens rupture occurred shortly after technical crack initiation, and the number of cycles to technical crack initiation was approximately 80% of those to rupture. It had been considered permissible to use all the results, for cycles to rupture and for cycles to technical crack initiation in these

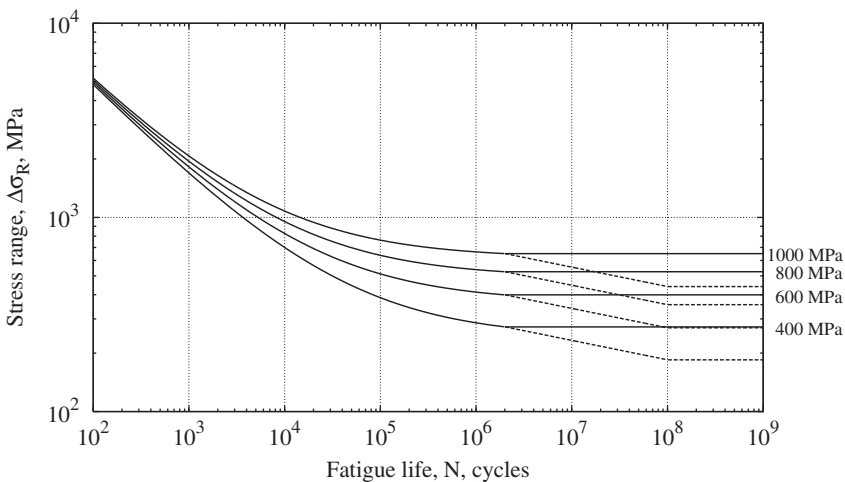


Figure 7.1: Design fatigue curves for unwelded regions of rolled and forged steels – equivalent total stress range vs. allowable number of cycles, with tensile strength  $R_m$  as parameter.

test specimens, for the deduction of design fatigue curves till initiation of technical cracks in structures, with the usually larger dimensions.

In this evaluation of experimental results, the well-founded dependence of endurance limits of ultimate strength [75–78] had been conservatively taken into account. Re-evaluation during the preparation of the standard of additional test results on unnotched specimens in the data collection [79] had been used to check the long-used fatigue curves [74]. This re-evaluation had shown that the usage of these mean value curves is still adequate, including the usage for austenitic steels and for high strength steels with ultimate strengths up to 1000 MPa.

In the determination of fatigue design curves safety factors had been applied to the mean curves of the experimental results, based on the assumption of a normal distribution of the results. For the safety factor in cycle numbers the value 10 had been chosen, a value which corresponds to a failure probability of 0.01% – a reasonable value in pressure vessel design, taking into account that pressure vessels of higher risk are, in general, subject per law to mandatory periodic in-service inspections, with, for pressure vessels with risk of fatigue failure, periods depending on the calculated allowable (cyclic) lifetime. For the safety factor in stresses the value 1.63 had been chosen in the region of endurance stresses, corresponding to the same failure probability of 0.01% [74], Annex 1.

Smoothing and slight adaptation of the resulting curves reduced this safety factor in stresses in the region of endurance stresses to values between 1.5 and 1.57, and increased the failure probability in this region to approximately 0.1% [74] Annex 1, a value still considered to be reasonable in pressure vessel design, because of the usual legal in-service inspection requirements, discussed above.

The test results for most steels indicate for single-amplitude cycles abrupt changes in slope at approximately  $10^7$  (test) cycles. This change in slope is frequently called knee point, and the corresponding stress range is called **endurance limit, fatigue limit**, or more correctly **single-amplitude endurance limit, or single-amplitude fatigue limit**: For stress ranges slightly smaller than this threshold value single-amplitude cycle tests show, in a statistical sense, no sign of crack propagation, even after the application of more than  $10^7$  cycles.

Experiments have shown that multi-amplitude cycles that combine single-amplitude cycles with stress ranges above the endurance limit with other single-amplitude cycles with stress ranges below the endurance limit show greater damage than the single-amplitude stress cycles with stress ranges above the endurance limit alone, i.e. that single-amplitude cycles with stress ranges smaller than the single-amplitude endurance limit do contribute to fatigue damage if combined with single-amplitude cycles with stress ranges larger than the single-amplitude endurance limit.

To take this experimentally observed fact into account, the single-amplitude design curves have been extended fictitiously for multi-amplitude cycles smaller

than the single-amplitude endurance limit, with a shallower curve as suggested e.g. by Haibach [105].

Unfortunately, the design fatigue curves give, for historical reasons, the functional relationship between number of cycles and stress ranges:

In the early days of experimental investigations into fatigue problems, emphasis was on fatigue problems involving cyclic stresses well below the yield strengths, or 0.2%-proof stresses, of the used materials. In this part of the high-cycle fatigue regime there is a one-to-one relationship between stress and strain, and stress was then considered to be the relevant response parameter in design, with material strength parameters being determined in tensile tests.

This representation of fatigue curves, common in engineering applications, hides the fact that not the range of the stress cycles, but the range, or the amplitude, of the strain cycles were the appropriate variables, that strain is the essential physical cause of fatigue damage. This representation leads, in the low-cycle fatigue regime, to fatigue curves reaching into stress ranges well above the ultimate strengths of the materials – the results of the strain-controlled tests have been converted to stresses by usage of linear-elastic relationships also in the plastic regime, and, therefore, the stresses in the plastic regime are fictitious stresses.

To summarize the test conditions:

The resulting design fatigue curves (see Fig. 7.1) have been determined by means of test results for

- unwelded, polished, small, perfect, tensile or bending specimens, made of ferritic or austenitic steels,
- specimens subjected to uniaxial constant amplitude cyclic stress states,
- tests under laboratory conditions, in air and at ambient temperature, and
- tests with stress or strain control, with measured strains converted into stresses by usage of (unbounded) linear-elastic constitutive laws.

The design fatigue curves have been fictitiously extended in the high cycle regime to allow for the incorporation of the damage contribution of sub-cycles of multi-amplitude cycles with sub-cycle stress ranges below the single-amplitude endurance limit.

Therefore, all of the influences on cyclic fatigue not accounted for in the tests and the deduction of the design fatigue curves have to be accounted for in the design model – in the determination of the equivalent stress range, by application of appropriate correction factors to the equivalent stress range and the fatigue design curves; or these influences have to be excluded from the scope of the design check.

The used basic database does not include results of specimens with stress concentrations. Therefore, the design models have to include local structural perturbation sources and the total stress ranges are to be used with the fatigue design curves.

All results in the used database are for tests with uniaxial stress states. For multi-axial stress states clause 18 of EN 13445-3 allows the usage of Tresca's or Mises' equivalent stress. Investigations of results with multi-axial stress states have shown that both give, in the statistical sense, sufficiently conservative results. But it is not the range of the equivalent stresses that is to be used, but the range of equivalent stress of stress differences. For the frequent case of stress cycles between two states, the equivalent stress of the difference of the two stress states is the range to be used in the further investigation. For multi-amplitude stress histories one extreme stress state shall be selected as "starting state" and the history of the equivalent stress of the difference of the stress components of the other states and the "starting state" be determined. The process is to be repeated with different "starting states", and the "starting state" which leads to the largest fatigue damage indicator is the decisive one.

Fig. 7.2 shows experimental results from tests with typical pressure vessels with failure due to cyclic pressure in unwelded regions. Also shown are corresponding fatigue curves, derived from the fatigue design curves by application of the correction factors for surface condition and for mean stress. The lower fatigue design curve is the one for steel with ultimate strength of 410 MPa (and yield strength of 265 MPa). These values are typical for the majority of the used materials [81]. The upper curve is the one for 720 MPa ultimate strength and with 520 MPa 0.2%-proof stress, representative for the high-strength materials used in the tests. Unfortunately, the surface condition of the test vessels in the failure region is no longer known. Therefore, a reasonable value for the likely surface roughness  $R_z$  of

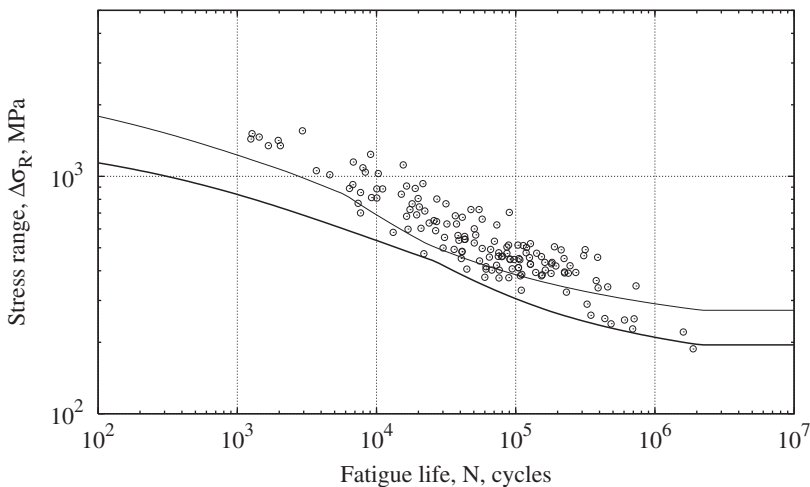


Figure 7.2: Comparison of test data with design fatigue curves, corrected for mean stress and surface condition influences, and with test results from the collection in [80] and from [81].

200  $\mu\text{m}$  has been used, a value specified in clause 18 for untreated rolled (or extruded) surfaces. For the mean stress correction, a mean stress value of half the allowable stress range has been used in all cases not requiring plasticity correction – the vessels had been subjected to pressure cycles from a very small value up to a maximum value. In the low-cycle fatigue regime the curves include an additional correction, given by the additional application of the plasticity correction factors for the yield strengths of 260 and 520 MPa, respectively. A detailed numerical comparison of the number of cycles achieved in the experiments with the corresponding allowable ones according to the clause 18 procedures showed that of the 145 experimental results only one is (marginally) smaller than the allowable one, i.e. the clause 18 procedure can be considered to render appropriate results, results that are statistically sufficiently safe.

The progressive plastic deformation design check does not require shakedown to linear-elastic behaviour, and, therefore, alternating plasticity is a response mode to be taken into account in the F-DC. Because of the requirement of the progressive plastic deformation design check, confined cyclic plasticity can be assumed, with the deformation controlled by the regions remaining elastic. In a numerical approach, of interest here, models with linear-elastic constitutive laws are suggested in the standard. These models will underestimate total strains in such cases of alternating plasticity due to local structural perturbation sources, and correction of the calculated stress ranges is required, by means of the so-called plasticity correction factor.

Usage of linear-elastic models may also require corrections of the mean stress, to allow for the adaptation of the mean stress if in an action cycle plastic deformation occurs.

As an alternative to the linear-elastic models, models with non-linear elastic–plastic constitutive laws may be used, with conversion of calculated strains into stresses afterwards, using linear-elastic relationships like in the deduction of fatigue curves from experimental results mentioned above. In this approach, stabilized cyclic stress–strain relationships are to be used, e.g. those collected in [79] or the rough approximation given in [82], BR-E30, or monotonic stress–strain relationships with corrected action ranges, assuming the validity of the Masing rule [42], see e.g. [83–85]. No correction is required in these approaches with non-linear elastic–plastic models and conversion of resulting strains linear-elastically into stresses. In this alternative the required equivalent stress range is determined directly, without any plasticity correction, and the equivalent mean stress is given by the equivalent stress of the mean state of the two extreme stress states resulting in the maximum range.

Results of tests with notched specimen have shown that in general not all of the total stress range, if relevant corrected for alternating plasticity effects, is effective for the cyclic fatigue life, but only a part, which depends essentially on the notch sensitivity of the material and the stress gradient. To take this influence, which is

especially pronounced in cases of large theoretical stress concentration factors and large stress gradients, into account, the effective stress concentration factor  $K_f$ , derived from test results, was introduced, which may be used to reflect better the effective influence of local stress concentrations. This **effective stress concentration factor** is defined as the ratio of the equivalent total stress of specimens without a notch to that of notched specimens, whereby both result in the same number of fatigue cycles. This effective stress concentration factor depends in general on the shape of the notch, on the type of loading – tensile, bending, torsion, etc. –, on the stress gradient, on the material, especially the hardening behaviour, and on the magnitude of the equivalent stress, but it does not depend on the material, provided that it is homogenous and isotropic. The effective stress concentration factor is never larger than the theoretical one, and, hence, need not be applied – the usage of the total stress range, if relevant corrected for alternating plasticity, is safe. In the standard, in a pragmatic approach, only a rough, simple approximation based on experimental results was chosen [86], for a more detailed one see [82], BR-E11.

The effective equivalent total stress range, to be used in the design fatigue curves, is given by the product of the effective stress concentration factor and the range of the equivalent structural stress, if relevant corrected for alternating plasticity.

The two correction factors discussed above, the plasticity correction factor and the effective stress concentration factor, are incorporated in the F-DC to be applied on the equivalent stress range, to modify this stress range (into the effective equivalent stress range) for application with the design fatigue curves.

The following four correction factors are incorporated to be applied, at least in principle, on the design fatigue curves, to modify these curves, to adapt them to take into account influences on the fatigue life not taken into account in the determination of these design fatigue curves:

- The surface finish correction factor, to take into account the influence of the specified or real surface finish of the structural part, if different from the polished surfaces of the test specimens, see also the note at this chapter's end.
- The thickness correction factor, to take into account the influence of the thickness of the structural part larger than the thicknesses of the test specimen.
- The mean stress correction factor, to take into account the influence of mean stresses different from zero, the value used in the evaluation of the test results.
- The temperature correction factor, to take into account the influence of temperatures different from the ambient temperature of the tests. This correction factor, which is also used for welded region, is for steels other than austenitic ones, larger than required for the compensation of the temperature dependence of the elastic properties – for these steels it does take account of other temperature effects on the fatigue test results as well, whereas for austenitic steels it just reflects the temperature dependence of the modulus of elasticity with temperature [74],

Annex 1. The increase of the fatigue resistance of ferritic steels, especially of unalloyed steels, observed in the temperature range of approximately 250–350°C (region of blue-brittleness sensitivity) has not been taken into account [74], Annex 1.

### **7.1.3. General Remarks to the F-DC of Welded Regions**

The design fatigue curves of clause 18 for welded regions were deduced, like their pendants for unwelded regions, from a fairly extensive database of experimental results, but unlike these pendants from results of tests with real welded structural parts, as fabricated, with all the usual geometrical and structural imperfections due to material production and manufacture, imperfections such as

- allowed local surface notches at the weld, like weld bead roughness, weld ripples, local undercut, local shrinkage grooves, local root concavity, and welding stop/start craters,
- different metallurgical properties in the weld “zones” – base metal, weld metal, and heat affected zone,
- residual stresses due to fabrication and welding,
- admissible internal defects, and
- local geometric imperfections, like linear and angular misalignment, within the (specified) tolerance limits (corresponding to good workmanship).

Work on first international recommendations for this new approach, which was used in clause 18 of EN 13445-3, started in 1979 in the Technical Committee 6 “Fatigue” of the European Convention for Constructional Steelwork (ECCS), resulted in a first edition of relevant recommendations [88], and created a common basis for the interpretation of fatigue test results and the principles for cyclic F-DCs of welded regions.

This approach is also incorporated in Part 1 of Eurocode 3 [89] and in diverse design recommendations by The International Institute of Welding [90–95], see also [96,97].

This approach is based on the recognition of the stochastic character of stress concentrations at welds and of the stochastic character of crack propagation in welded regions.

The database results—range of principal structural stress vs. cycles to failure—are for break-through of the cracks, from the crack initiation side to any other surface of the part.

The results have shown that linear regression lines in double logarithmic plots can be used as fairly good approximations of the mean values of test results for

parts with similar weld details and orientations of the principal structural stresses. The test results have also shown a sharp bend at the endurance limit for single-amplitude loading, and the test results have also shown that the influence of the mean value of the stress due to the applied forces is not identifiable, remains hidden in the scatter of the test results, and is, therefore, included in the fatigue design curves of welded regions.

Furthermore, the test results have shown that post weld heat treatment does not decrease weld residual stresses sufficiently to justify usage of different design fatigue curves for welded regions with post weld heat treatment, and the test results have shown that differences in materials and material strengths, in the scope of the standard, have no statistically significant influence on the cyclic fatigue life of welded regions, in contrast to unwelded regions where the dependence of the fatigue life from the ultimate strength has been shown. This different behaviour is the result of the different dependencies of the crack initiation phase and the crack propagation phase – the number of cycles for crack initiation depends on the material’s ultimate strength, but the number of cycles in the crack propagation phase does not. Thus, different fatigue design curves for different materials or different material strength parameters are used for unwelded regions, but for welded regions are neither required nor justifiable.

The design fatigue curves chosen in EN 13445-3 clause 18 are shown in Fig. 7.3. To be able to take into account weld details, location of hot spot, and orientation of

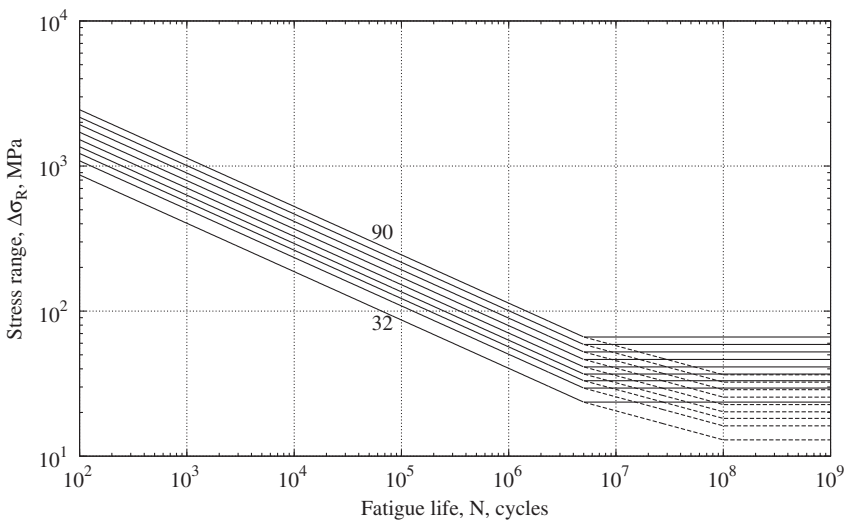


Figure 7.3: Design fatigue curves for welded regions. FAT Classes [32,40,45,50,56,63,71,80,90].



principal stresses, 10 different design fatigue curves, corresponding to 10 different fatigue classes, have been chosen and labelled **Fatigue Class**, or simply **Class** or **FAT Class**, followed by a number which is equal to the value of the curve at 2 million allowable cycles. Test results covered the range of 10 000 to 10 million cycles. To include low cycle fatigue, a linear extrapolation in the double logarithmic plot down to 100 allowable cycles was chosen and is considered to be conservative.

The full lines in Fig. 7.3 apply for single-amplitude cyclic actions, with more or less arbitrarily chosen transition to the endurance region at 5 million cycles. The broken lines apply for sub-cycles of multi-amplitude cyclic actions, extended fictitiously below the single-amplitude endurance limit for the same reason as for unwelded regions, see Section 7.1.2.

The assignment of the various cases of weld details, location of hot spot, and orientation of principal stresses, relates to fatigue classes, and is given by tables compiled in Annex P of EN 13445-3. Table 7.1 is an example of such a table.

The design fatigue curves are considered to be three standard deviations below the mean lines, for all the weld details and orientation of principal structural stresses encompassed in the corresponding fatigue class. This corresponds to a probability of failure of approximately 0.135%.

The design fatigue curves were evaluated on the basis of the range of principal structural stresses in the point of crack initiation normal and parallel to the weld joint, with extrapolation of strain gauge measurements into this point of crack initiation, linear as well as quadratic extrapolation [92, 94].

This fact requires that, in the F-DCs of welded regions, the very same stress ranges are used, i.e. the ranges of principal structural stresses normal and parallel to the weld joint direction at all the points of likely crack initiation, the so-called critical points or **hot spots**, whereby in the determination of the structural stresses the very same extrapolation as in the test result evaluation is to be applied. This extrapolation can be omitted by usage of stress-concentration-free models if these lead to sufficiently good approximations of structural stresses. The pivot points used in the quadratic extrapolation are shown in Fig. 7.4.

The standard also allows the usage of stress ranges of equivalent structural stress differences, but then different fatigue classes are to be used – the equivalent stress does not have a direction, and, therefore, the most detrimental one was used in the determination of the fatigue class.





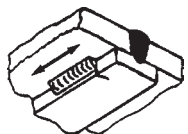
Usage of the basic approach, the usage of principal structural stress ranges as relevant stress ranges, is strongly recommended – it is more straightforward, is less prone to mistakes, avoids penalization in the fatigue class determination, and it is on the safe side (in cases where the equivalent stress range is smaller than the maximum principal stress range). Therefore, in the following, solely the principal structural stress range approach is dealt with.

Table 7.1: Example of fatigue class tables (this example consists of the first two pages of the tables in EN 13445-3:2002 (E) Issue 9 (2004–2002) Annex P). For the Direct Route in Design by Analysis only testing group 1 is admissible

Table P.1: Seam welds

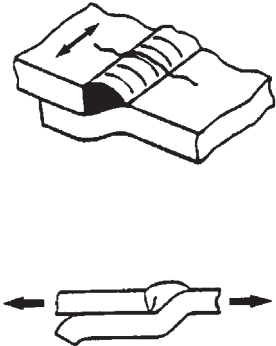
Detail no.	Joint type	Sketch of detail	Comments	Class	
				Testing group 1 or 2	Testing group 3
1.1	Full penetration butt weld flush ground, including weld repairs		Weld to be proved free from surface-breaking flaws and significant sub-surface flaws (see EN 13445-5) by non-destructive testing	90 <sup>a</sup>	71 <sup>a</sup>
				90	71
1.2	Full penetration butt weld made from both sides or from one side on to consumable insert or temporary non-fusible backing		Weld to be proved free from significant flaws (see EN 13445-5) by non-destructive testing	80 <sup>b</sup>	63 <sup>b</sup>
				80 <sup>b</sup>	63 <sup>b</sup>
1.3			Weld to be proved free from significant flaws by non-destructive testing (see EN 13445-5)	80 <sup>b</sup>	63 <sup>b</sup>
				80	63
1.4			Weld to be proved free from significant flaws (see EN 13445-5) by non-destructive testing	$\alpha \leq 30^\circ$	80
				$\alpha > 30^\circ$	71
				80	71

Table 1 (Continued)

Detail no.	Joint type	Sketch of detail	Comments	Class	
				Testing group 1 or 2	Testing group 3
1.5	Full penetration butt welds made from one side without backing		Weld to be proved to be full penetration and free from significant flaws (see EN 13445-5) by non-destructive testing	80	71
			If full penetration can be assured If inside cannot be visually inspected	63 <sup>b</sup> 40 <sup>b</sup>	40 <sup>b</sup> 40 <sup>b</sup>
1.6	Full penetration butt welds made from one side onto permanent backing	(1.6a)	Circumferential seams only (see 5.7)		
			Backing strip to be continuous and, if attached by welding, tack welds to be ground out or buried in main butt weld, or continuous fillet welds are permitted	63	63
			Minimum throat = shell thickness. Weld root pass shall be inspected to ensure full fusion to backing Single pass weld	56 40	40 40
	(1.6b)	Circumferential seams only (see 5.7)			
			Backing strip attached with discontinuous fillet weld	63 <sup>a</sup>	63 <sup>a</sup>

(Continued)

Table 1 (Continued)

Detail no.	Joint type	Sketch of detail	Comments	Class	
				Testing group 1 or 2	Testing group 3
1.7	Joggle joint		Circumferential seams only (see 5.7)	63 <sup>a</sup>	63 <sup>a</sup>
			Minimum throat = shell thickness		
			Weld root pass shall be inspected to ensure full fusion	56	40
			Single pass weld	40	40

<sup>a</sup>Use  $f_e$  instead of  $f_{ew}$ ;

<sup>b</sup>Effect of misalignment to be included in calculated stress, see 18.10.4.

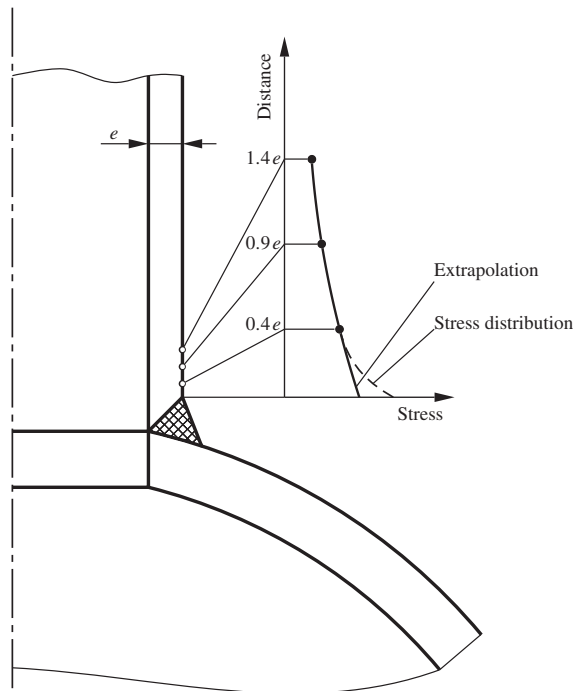


Figure 7.4: Pivot points for the quadratic extrapolation of stresses into the crack initiation point (hot spot).

To summarize the influences taken account of in the evaluation procedure of the test results: In the design fatigue curves incorporated are the influences of

- the stress concentrations due to the surface irregularities and material inhomogeneities of the welds themselves,
- the welding residual stresses,
- the mean stress resulting from the applied actions,
- thickness of parts up to 25 mm, and
- different materials – steels and steel castings admissible according to EN 13445-2.

Not taken into account in the design fatigue curves, and, therefore, to be taken into account in the design models or in the fatigue damage calculation procedure are the influences of

- alternating plasticity,
- temperature, if the calculation temperature differs from the ambient temperature, and
- thickness of the relevant part, from the hot spot to any other side of the part, greater than 25 mm.

Whereas the temperature correction factor for welded regions is the same as the one for unwelded regions, the thickness correction factor for welded regions differs from the one for unwelded regions.

Geometric deviations from the ideal shape at the weld joints, like misalignment of middle lines, peaking, and ovality, are partially included in the tests, but to account for the differences in civil engineering constructions and pressure vessels, a different classification was chosen in EN 13445-3, and, consequently, in the models of the F-DC some of these deviations are to be included, or taken into account via relevant stress concentration factors in the fatigue damage calculation procedure, even if these imperfections are permissible according to EN 13445-4. The details for which these imperfections are to be incorporated in the models, mainly butt welds with main principal structural stress orientation normal to the weld joint, are specified in the tables for the fatigue class selection.

## 7.2. Procedure

The relations in clause 18 of EN 13445-3 were deduced for single-amplitude stress and strain cycles. The transformation of specified multi-amplitude action cycles into relevant single-amplitude stress cycles can become quite complicated and cumbersome, especially in cases involving thermal stresses and changes in the directions of principal stresses in the cycles.

Fortunately, the investigations being part of design checks, this transformation is in the majority of cases quite straightforward and obvious. The specified usage of linear-elastic models and of linear damage accumulation simplifies the whole procedure further. Care should be taken if geometric deviations from the ideal geometry are included in geometric non-linear models and new non-linearities thereby introduced.

The whole procedure of determining the fatigue damage indicator can be split up into six consecutive steps:

Step 1: Setting up of the cyclic design actions – design values at specific instants of time, or of a time-like parameter, and, if required, cyclic design functions of time.

Step 2: Determination of the stress cycles, the stress responses of the (linear-elastic) models.

Step 3: Determination of all critical points of the structure and of all cycles of the relevant stresses there – of equivalent stresses in unwelded regions and of main principal stresses (approximately) normal and parallel to weld joint direction in welded regions.

Step 4: Transformation of the (usually variable amplitude) stress cycles obtained in Step 3 into single-amplitude cycles of relevant stresses – relevant stress ranges and cycle numbers.

Step 5: Determination of the fatigue damage index for each of the constant amplitude stress cycles obtained in Step 4.

Step 6: Linear superposition of all of the fatigue damage indices obtained in Step 5 to determine the cumulative fatigue damage index.

All of these steps are addressed in the following sections, but not consecutively, in order to improve readability and understanding of the concepts. Wherever differences for welded and unwelded regions require different procedures, these are dealt with in different sections.

## 7.3. Design Models

### 7.3.1. Requirements with Regard to Welded Regions

As input into the design fatigue curves for welded regions principal structural stresses are required. The models used for the determination may, therefore, be stress-concentration-free models. If detailed models with local structural perturbation sources are used, or if stress/strain singularities are introduced by the modelling, structural stresses should be determined. Mises' equivalent stresses may be used, but this option is not suggested and is not discussed here.

As output of FEM calculations principal structural stresses are required, at each relevant hot spot on the surface of the structure, including weld surfaces. At each hot spot at the root of directly loaded fillet welds or partial penetration welds either the components of the stress vector at the weld throat plane or the average stress normal to the weld throat plane and the average shear stress in the weld throat plane are required, with averages along the weld throat line, see Section 7.8.

### ***7.3.2. Requirements with Regard to Unwelded Regions***

As input into the design fatigue curves for unwelded regions total stresses are required, taking into account all local structural perturbation sources. Therefore, the models used for the determination have to be detailed models, i.e. must include all relevant local structural perturbation sources. To mention it again: a convenient output of general software packages is equivalent stresses, but, in general, these equivalent stresses are not the proper output, are not relevant for the ensuing steps of the F-DC, see Section 7.1.2. All of the stress components are required to allow for the determination of the ranges of equivalent stresses of total stress differences relevant for the following steps.

If plasticity correction is required and the procedure of the standard is used, then the determination of equivalent linearly distributed stresses and of structural stresses at the relevant hot spot is necessary – for the latter the extrapolation of (already known) total stresses into the hot spot is recommended. The procedure in the standard is not satisfactory, and a different, conservative procedure is recommended here (see Section 7.6.1) and then this problematic determination of equivalent linear stresses along a possibly difficult to define evaluation line is not required.

### ***7.3.3. General Requirements with Regard to Welded and Unwelded Regions***

The design fatigue curves are highly non-linear and small changes in the stress range may result in large changes in the allowable number of cycles. It is therefore necessary to use fairly realistic stress ranges. This requires not only realistic characteristic values or characteristic functions of actions (see Section 7.4) but also realistic models, especially realistic thicknesses. Usage of analysis thicknesses, with the subtraction of the whole corrosion or erosion allowance, is not required. It is good engineering practice to use fatigue analysis thicknesses. Of course, analysis thicknesses may always be used instead of fatigue analysis thickness.

Linear-elastic constitutive laws are to be used. Only in the (fortunately rare) cases where plasticity correction is required, elastic–plastic constitutive laws may be used to determine the plasticity correction factor, see Sections 7.1.2 and 7.6.1.

The standard leaves the determination of the material parameters of the linear-elastic constitutive law, like modulus of elasticity, Poisson’s ratio, coefficient of linear thermal expansion, and coefficient of thermal conductivity, at the discretion of the designer. The following procedure is recommended:

For Poisson’s ratio the value 0.3 is used. For all other material parameters of the linear-elastic constitutive law the temperature-dependent values of Annex O of EN 13445-3 are used.

As reference temperature, a temperature not less than the maximum calculated metal temperature is used. This reference temperature may be chosen as space-independent or space-dependent. In the first case, the value must not be smaller than the maximum calculated metal temperature in the considered (part of the) model, with the maximum for all points and all times of the considered cycle. In the second case, the value must not be smaller than the maximum calculated metal temperature in the same point, with the maximum over all times of the considered cycle. In this case it is common practice to use the stationary result of the (numerical) temperature calculation for the considered cycle directly.

Geometrically linear models may be used in all cases. Usage of geometrically non-linear models is strongly recommended for cases of deformation-hardening—for these cases the relevant stress ranges may be substantially smaller and the allowable number of cycles remarkably increased [98-101].

In principle, geometric imperfections have to be included in the design models. In most of the load cases without deformation-weakening their effect will be small, and the much simpler models without geometric imperfections may be used. In deformation-weakening load cases on the other hand, geometric imperfections do have in general a non-negligible effect, and are, therefore, always to be included in the design models.

Cladding is to be considered with respect to both thermal analysis and stress analysis. However, when the cladding is of the integrally bonded type and the nominal cladding thickness is not more than 10% of the respective total nominal thickness of the structural part, the presence of the cladding may be neglected, i.e. the model based on the base metal geometry, and only pressure correction applied, as described in Section 6.3.

For clad surfaces the requirements of welded regions apply, with the correction factors of the base metal, and it is recommended to use conservatively as incipient crack plane direction, the one that leads to the largest stress range  $\Delta\sigma_{\perp}$ . The standard does not specify a FAT Class for clad surfaces, but 90 is a very likely value that all of the parties concerned can agree to.



## 7.4. Design Values and Design Functions of Actions

The (cyclic) F-DC deals with load cases that include cyclic characteristic functions of actions, usually specified as periodic functions of time or a time-like parameter, load cases that are specified because they are likely to occur often enough to require inclusion in this design check. In load cases with non-negligible and non-stationary thermal stresses, the cyclic characteristic functions of actions have to be specified by functions of time, because time derivatives of temperature are of importance as well.

As already mentioned in Section 7.3.3, the design fatigue curves are highly non-linear, and hence fairly realistic stress ranges are required as input to the fatigue damage determination. Therefore, great care is required in the specification of the characteristic functions of actions to be used in this design check. Unlike the GPD-DC and the I-DC, it is of paramount importance to specify in the (cyclic) F-DC characteristic values and characteristic functions of actions that indeed represent realistically those to be considered. It is not required to specify characteristic values and functions that envelop realistically all those extreme values and functions that can occur under reasonably foreseeable conditions. In general, mean values of the extremes and mean functions of the extreme cyclic functions suffice – in a carefully drafted specification, the distribution of the extremes, and of the extreme functions, is usually skewed to smaller values, and the design fatigue curves, on the other hand, amplify the larger values, and it can be assumed that these two effects usually balance each other approximately.

Safety margins are already incorporated in the design fatigue curves, and, therefore, application of partial safety factors is not required, the design values and the design functions of actions coincide with characteristic values and characteristic functions.

To each characteristic function the relevant number of cycles has to be specified, and the characteristic values, the characteristic functions, and the used number of cycles specified in the Design Documentation.

## 7.5. The Principle

The principle of this design check reads:

*The design value of the damage indicator  $D_d$ , obtained for the design functions of pressure/temperature and variable actions, shall not exceed 1 (EN 13445-3 Annex B).*

As already mentioned in the beginning of this chapter, this principle is given meaning only by the application rule, it has to be read in conjunction with the application rule that states: *Fulfillment of the requirements given in clause 18 suffices as a check against fatigue failure (EN 13445-3 Annex B).*

## 7.6. Correction Factors for Unwelded Regions

### 7.6.1. Plasticity Correction Factor

Plasticity corrections are, in the usual procedure, required because linear-elastic constitutive laws are used in the determination of the relevant stress ranges. The standard requires this plasticity correction only if  $\Delta\sigma_{eq,l}$ , the range of Mises' equivalent stress function of the differences of the equivalent linear stresses at the considered hot spot, exceeds twice the relevant yield strength at the relevant reference temperature  $t^*$ ,  $R_{p0.2/t^*}$  for ferritic steels and  $R_{p1.0/t^*}$  for austenitic ones. Here  $t^*$  denotes the weighted mean cycle temperature (of the considered action cycle at the considered point of the model), defined by

$$t^* = 0.25 \cdot t_{c, \min} + 0.75 \cdot t_{c, \max}, \quad (7.1)$$

where  $t_{c, \max}$  and  $t_{c, \min}$  are maximum and minimum calculated metal temperatures of the considered action cycle at the considered point of the model.

The (approximate) plasticity correction in EN 13445-3, Clause 18, is not really satisfactory. There exist cases of large stress concentrations with alternating plasticity but in which  $\Delta\sigma_{eq,l} < 2 \cdot R_{p0.2/t^*}$  or  $2 \cdot R_{p1.0/t^*}$  respectively. In these not unusual cases a plasticity correction should be necessary, but is not according to the standard's procedure.

Usage of the procedure outlined in the following is recommended instead. This procedure is, compared with the standard's procedure, conservative, i.e. on the safe side, may, therefore, always be used in permissibility checks according to the standard. Furthermore, this procedure agrees with the basic document [74].

In this route, plasticity correction is required if  $\Delta\sigma_{eq}$ , the range of Mises' equivalent stress function of the differences of the relevant total stresses at the considered hot spot, exceeds twice the relevant yield strength at the relevant reference temperature  $t^*$ ,  $R_{p1.0/t^*}$  for austenitic steels, and  $R_{p0.2/t^*}$  for other steels. Here  $t^*$  corresponds to the weighted mean cycle temperature defined by Equation (7.1).

For the special case of thermal stresses with non-linear distribution over evaluation lines that render the fatigue relevant thicknesses, the plasticity correction factor is denoted by  $k_v$  and is given by

$$k_v = \text{Max} [1.0; 0.7 / (0.5 + 0.4 \cdot R_{p1.0/t^*} / \Delta\sigma_{eq})] \quad (7.2a)$$

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for austenitic steels, and by

$$k_v = \text{Max} [1.0; 0.7 / (0.5 + 0.4 \cdot R_{p0.2/t^*} / \Delta\sigma_{eq})] \quad (7.2b)$$

for steels other than austenitic ones. This (questionable) correction factor is to be applied to parts of stresses and stress ranges that correspond to the non-linearly distributed parts of the thermal stresses.

In the general case, for all other stresses and parts of stresses, the plasticity correction factor is denoted by  $k_e$ , and is given by

$$k_e = 1 + 0.4 \cdot (0.5 \cdot \Delta\sigma_{eq} / R_{p1.0/t^*} - 1) \quad (7.3a)$$

for austenitic steels, and for all other steels by

$$k_e = 1.0 + A_0 \cdot (0.5 \cdot \Delta\sigma_{eq} / R_{p0.2/t^*} - 1) \quad (7.3b)$$

with

$$\begin{aligned} A_0 &= 0.4 \text{ for } R_m \leq 500 \text{ MPa,} \\ &= 0.4 + (R_m - 500) / 3000 \text{ for } 500 < R_m \leq 800 \text{ MPa,} \\ &= 0.5 \text{ for } 800 < R_m \leq 1000 \text{ MPa.} \end{aligned}$$

This correction factor is to be applied to parts of stresses and stress ranges that do not correspond to the non-linearly distributed parts of thermal stresses.

In action cycles without non-linearly distributed thermal stresses, a principal stress or a stress range corrected for plasticity effects is given by

$$\sigma_{pc} = k_e \cdot \sigma \quad (7.4)$$

and

$$\Delta\sigma_{pc} = k_e \cdot \Delta\sigma. \quad (7.5)$$

In action cycles with non-linearly distributed thermal stresses, a principal stress is given by

$$\sigma_{pc} = (k_e \cdot p_{gc} + k_v \cdot p_{sc}) \cdot \sigma \quad (7.6)$$

with

$$p_{gc} = \Delta\sigma_{eq,l} / \Delta\sigma_{eq}, \quad (7.7a)$$

$$p_{sc} = 1 - p_{gc}, \quad (7.7b)$$

and with  $\Delta\sigma_{eq}$  denoting the range of Mises' equivalent stress of the differences of the total stresses at the hot spot, see 7.3.2, and  $\Delta\sigma_{eq,l}$  the corresponding range at the hot spot for the equivalent linear thermal stress distribution. In the determination of  $\Delta\sigma_{eq}$  the sum of all stresses is to be used, independently of the type of stresses. The relevant stress range is, in action cycles with non-linearly distributed displacement controlled stresses, given by

$$\Delta\sigma_{pc} = MEQ[\Delta[k_e \cdot \sigma_{ij,gc} + k_v \cdot \sigma_{ij,sc}]], \quad (7.8)$$

where  $\sigma_{ij,gc}$  and  $\sigma_{ij,sc}$  are the parts of the relevant stress, for the general and the special case respectively, and where for  $MEQ$  and  $\Delta\sigma_{ij}$  the definitions of Section 7.14 apply.

If the stress range  $\Delta\sigma_{eq}$  of the considered cycle is smaller than  $2R_{p1.0/t^*}$  for austenitic steels and  $2R_{p0.2/t^*}$  for all other steels, respectively, no plasticity correction is required, i.e.  $k_e = 1.0$  and  $k_v = 1.0$ .

The plasticity correction factor for the general case may be used also in the special cases of non-linearly distributed thermal stresses.

In all thermal stress cases, and in cases of doubt, it is strongly recommended to determine the plasticity correction factor by FEA, as the quotient of results of Mises' equivalent strains for two different models – one with a constitutive law that approximates the (real) cyclic one of the material in the range of (small) plastic strains sufficiently well, and one being the already used one with the (unbounded) linear-elastic constitutive law. In this approach the following steps are required:

- determination of the components of the relevant strain differences in the hot spot in the model with the non-linear cyclic constitutive law,
- determination of the corresponding Mises' equivalent strain  $\Delta\varepsilon_{eq,c}$ , and usage of the corresponding stress range  $E \cdot \Delta\varepsilon_{eq,c}$  directly, see 7.1.2, or
- determination of the components of the relevant strain differences for the linear-elastic model  $\Delta\sigma_{eq,le}$ , and
- determination of the plasticity correction factor given by the quotient values of Mises' equivalent strain determined in the second and fourth step.

### 7.6.2. Effective Stress Concentration Factor

This correction factor is required to take into account the fatigue sensitivity of the material and the influence of the stress gradient, conveniently represented by the theoretical stress concentration factor, see also Section 7.1.2.

This fatigue relevant effective stress concentration factor is given by

$$K_f = 1 + 1.5(K_t - 1)/(1 + 0.5 \cdot \text{Max} [1; \Delta\sigma_{total, eq} / \Delta\sigma_D]), \quad (7.9)$$

where  $\Delta\sigma_D$  is the endurance limit for single-amplitude stress cycles,  $\Delta\sigma_{total, eq}$  the range of the equivalent stress of the total stress, if applicable corrected for plasticity effects, i.e. by application of the plasticity correction factor given in the preceding Section 7.6.1, and where  $K_t$  is the **theoretical stress concentration factor**, defined as the ratio of the equivalent total stress  $\sigma_{total, eq}$  and the equivalent structural stress  $\sigma_{struc, eq}$ :

$$K_t = \sigma_{total, eq} / \sigma_{struc, eq} \quad (7.10)$$

This theoretical stress concentration factor characterizes the influence of local structural perturbation sources, and in the definition the equivalent total stress and the equivalent structural stress are the values at the considered hot spot and for the same action values, and both determined with models with the same linear-elastic constitutive law.

As a warning of an unfortunately often occurring mistake: This theoretical stress concentration factor, used in the F-DCs, must not be confused with the theoretically or experimentally determined (shape) factors for the determination or comparison of stresses due to changes of global geometric parameter. Unfortunately these shape factors are sometimes called also stress concentration factors. These shape factors are usually taken from literature and were derived from e.g. experiments or analytical results, using some kind of reference stresses, conveniently chosen reference stresses but which are by definition different from the structural stresses at the hot spot. Theoretical stress concentration factor  $K_t$  and effective stress concentration factor  $K_f$  are different from 1.0 only if the considered hot spot is within a region influenced by a local structural perturbation source.

To calculate the stress range relevant in the fatigue calculation, the effective stress concentration factor  $K_f$  has to be applied on the equivalent structural stress range, or  $K_f / K_t$  applied on the equivalent total stress range.

### 7.6.3. Surface Finish Correction Factor

This correction factor is required to take into account the influence of surface conditions on the fatigue design life. This factor, denoted by  $f_s$ , depends on the surface texture, characterized by the roughness-height index  $R_z$ , in  $\mu\text{m}$ , on the ultimate strength  $R_m$  of the material and, unfortunately, on the number of allowed cycles  $N$ . It is specified by

$$f_s = F_s^{0.1 \ln N - 0.465} \text{ for } N \leq 2000\,000, \quad (7.11a)$$

$$= F_s \text{ for } N > 2000\,000, \quad (7.11b)$$

with

$$F_s = 1 - 0.056(\ln R_z)^{0.64} \ln R_m + 0.289(\ln R_z)^{0.53}. \quad (7.12)$$

In clause 18 of the standard values for the roughness-height index  $R_z$  are specified as:

$$\begin{aligned} R_z &= 200 \mu\text{m} \text{ for surfaces of rolled and extruded parts,} \\ &= 50 \mu\text{m} \text{ for machined surfaces,} \\ &= 10 \mu\text{m} \text{ for ground surfaces that are free from notches,} \end{aligned}$$

and  $f_s = 1$  is allowed for polished surfaces with  $R_z < 6\mu\text{m}$ . In a note it is indicated that Equations (7.11) are not applicable for (untreated surfaces of) deep drawn components and forgings. The approximation for  $F_s$  given in the literature [102] for untreated surfaces of deep drawn components and forgings

$$F_s = 0.25 + 0.75(1 - R_m / 1500)^{1.8}, \quad (7.13)$$

which is based on the graphical presentation of test results in [96], p. 78, and [103], is still under consideration in the standard's working groups but may be used safely.

### 7.6.4. Thickness Correction Factor

This correction factor is required to take into account the influence of thickness on the fatigue design life. This correction factor, denoted by  $f_e$ , depends on the fatigue

relevant thickness at the considered hot spot,  $e_n$ , and unfortunately it depends also on the number of allowed cycles  $N$ . It is specified by

$$f_e = 1.0 \text{ for } e_n \leq 25, \quad (7.14a)$$

$$= F_e^{(0.1 \ln N - 0.465)} \text{ for } e_n > 25 \text{ and } N \leq 2000\,000, \quad (7.14b)$$

$$= F_e \text{ for } e_n > 25 \text{ and } N > 2000\,000, \quad (7.14c)$$

with

$$F_e = (25/e_n)^{0.182} \text{ for } 25 < e_n \leq 150, \quad (7.15a)$$

$$= (1/6)^{0.182} \text{ for } e_n > 150, \quad (7.15b)$$

and with the nominal thickness  $e_n$  in mm.

### 7.6.5. Mean Stress Correction Factor

This correction factor is required to take into account the influence of mean stresses on the fatigue design life. Compared with the design life for zero mean stress, which is specified for the design fatigue curves, positive mean stresses decrease the fatigue design life and negative mean stresses increase it.

This mean stress correction factor, denoted by  $f_m$ , depends for  $N \leq 2 \times 10^6$  cycles on the material's ultimate strength  $R_m$  and on the quotient of the equivalent mean stress  $\bar{\sigma}_{eq}$  and the value of the relevant fatigue design curve  $\Delta\sigma_R$  at the considered number of cycles  $N$ . In this range of number of cycles  $f_m$  is given by

$$f_m = [1 - (2\bar{\sigma}_{eq} / \Delta\sigma_R)M(2 + M)/(1 + M)] \quad \text{for } -R_{p0.2/t^*} \leq \bar{\sigma}_{eq} \leq 0.5\Delta\sigma_R/(1 + M), \quad (7.16a)$$

and

$$f_m = (1 + M/3)/(1 + M) - M(2\bar{\sigma}_{eq} / \Delta\sigma_R)/3 \quad \text{otherwise,} \quad (7.16b)$$

with

$$M = 0.00035R_m - 0.1. \quad (7.16c)$$

For  $N \geq 2 \times 10^6$  cycles,  $f_m$  is independent of the number of cycles and depends on the material's ultimate strength and on the quotient of the equivalent mean stress and  $\Delta\sigma_D$ , the value of the relevant fatigue design curve for  $2 \times 10^6$  cycles (see Table 7.3). In this range of number of cycles  $f_m$  is given by

$$f_m = [1 - (2\bar{\sigma}_{eq} / \Delta\sigma_D)M(2 + M)/(1 + M)] \quad \text{for } -R_{p0.2/t^*} \leq \bar{\sigma}_{eq} \leq 0.5\Delta\sigma_D/(1 + M) \quad (7.17a)$$

and

$$f_m = (1 + M/3) / (1 + M) - M(2\bar{\sigma}_{eq} / \Delta\sigma_D) / 3 \quad \text{otherwise,} \quad (7.17b)$$

with  $M$  as above.

In case of plasticity effects on the mean stress, the value of the equivalent mean stress  $\bar{\sigma}_{eq}$  used in the above equations has to be the corrected one, see Section 7.14.

### 7.6.6. Temperature Correction Factor

This correction factor is required to take into account the influence of temperatures of the structure above 100°C on the fatigue design life.

This correction factor, denoted by  $f_{t^*}$ , depends on the material (type) and on the weighted mean cycle temperature  $t^*$ , defined by Equation (7.1) in Section 7.6.1. For  $t^* \leq 100^\circ\text{C}$ , this temperature correction factor is given by

$$f_{t^*} = 1.0 \text{ for } t^* \leq 100^\circ\text{C}, \quad (7.18a)$$

and for higher temperatures by

$$f_{t^*} = 1.043 - 4.3 \cdot 10^{-4}t^* \quad \text{for austenitic steels, and} \quad (7.18b)$$

$$f_{t^*} = 1.03 - 1.5 \cdot 10^{-4}t^* - 1.5 \cdot 10^{-6}t^{*2} \quad \text{for other steels.} \quad (7.18c)$$



## 7.7. Correction Factors for Welded Regions

As already mentioned in Section 7.7.3, there are less influences not accounted for in the fatigue design curves for welded regions than in those for unwelded regions, and, therefore, less correction factors are required.

### 7.7.1. Plasticity Correction Factor

In the standard procedure this correction factor is required if  $\Delta\sigma_{eq,l}$ , the equivalent stress range corresponding to the variation of the equivalent linear distribution at the considered hot spot, exceeds twice the relevant yield strength,  $R_{p0.2/t^*}$  or  $R_{p1.0/t^*}$  respectively, at the relevant weighted mean cycle temperature  $t^*$ , see 7.6.6. This factor is for welded regions the same one as for unwelded regions, given in Section 7.6.1, but is to be applied on the principal structural stresses.

Like for the plasticity correction factor for unwelded regions, it is recommended to deviate from the standard's procedure and to replace  $\Delta\sigma_{eq,l}$  by the stress range relevant to the region, i.e. by  $\Delta\sigma_{eq\perp}$  and  $\Delta\sigma_{eq\parallel}$  respectively, the ranges of the principal structural stress normal and parallel to the weld joint direction. Contrary to the procedures for unwelded regions, the differences between the two procedures are for welded regions minimal to zero.

### 7.7.2. Thickness Correction Factor

This correction factor, denoted by  $f_{ew}$ , is required to take into account the influence of the fatigue relevant thickness on the design fatigue life. Unlike its pendant for unwelded regions, this factor depends only on the relevant thickness  $e_n$ , in mm, and is, in general, specified by

$$f_{ew} = 1.0 \text{ for } e_n \leq 25 \text{ mm}, \quad (7.19a)$$

$$= (25 / e_n)^{0.25} \text{ for } 25 < e_n \leq 150 \text{ mm}, \quad (7.19b)$$

$$= (1 / 6)^{0.25} \text{ for } e_n > 150 \text{ mm}. \quad (7.19c)$$

There are cases where no thickness correction is required, or where the thickness correction factor for unwelded regions has to be used. These deviations from

the generally valid Equations (7.19) are detailed in Annex P of EN 13445-3, the annex for the classification of weld details (with the principal stress approach).

### 7.7.3. Temperature Correction Factor

This correction factor, required to take into account the influence of temperatures of the structure above 100°C on the fatigue life. It is for welded regions the same one as for unwelded regions, given in Section 7.6.6.

## 7.8. Design Fatigue Curves

### 7.8.1. Design Fatigue Curves for Welded Regions

The design fatigue curves for welded regions, shown in Fig. 7.3, can be described as follows:

For  $\Delta\sigma_R \geq \Delta\sigma_D$

$$N = C_1 / \Delta\sigma_R^3, \quad (7.20a)$$

where  $N$  is the number of allowed cycles,  $\Delta\sigma_D$  the **endurance stress range** (for single-amplitude stress cycles), and  $\Delta\sigma_R$  the relevant stress range.

For single-amplitude stress cycles  $N$  is equal to infinity if  $\Delta\sigma_R \leq \Delta\sigma_D$ .

For single-amplitude sub-cycles of multi-amplitude cycles with sub-cycle stress range  $\Delta\sigma > \Delta\sigma_D$ , the design fatigue curves are the same as given by Equation (7.20a).

For multi-amplitude stress cycles with at least one single-amplitude sub-cycle stress range  $\Delta\sigma > \Delta\sigma_D$ , the design fatigue curves for sub-cycles with  $\Delta\sigma \leq \Delta\sigma_D$  can be described by

$$N = C_2 / \Delta\sigma_R^5 \quad \text{for } \Delta\sigma_D > \Delta\sigma_R > \Delta\sigma_{cur}, \quad (7.20b)$$

$$= \infty \quad \text{for } \Delta\sigma_R \leq \Delta\sigma_{cur}. \quad (7.20c)$$

The coefficients  $C_1$ ,  $C_2$ , the values of the endurance stress range (for single-amplitude stress cycles)  $\Delta\sigma_D$  and the **cut-off stress range**  $\Delta\sigma_{cur}$  are given in Table 7.2.

Endurance stress range  $\Delta\sigma_D$  and cut-off stress range  $\Delta\sigma_{cut}$  correspond to the values of the respective design fatigue curves for  $N$  equal to 5 million and 100 million allowed cycles, respectively. These coefficients and stress values depend on the respective fatigue class.

Table 7.2: Coefficients and values of the fatigue design curves for welded regions

Class	$C_1$	$C_2$	$\Delta\sigma_D$ in MPa	$\Delta\sigma_{cut}$ in MPa
100	$2.00 \cdot 10^{12}$	$1.09 \cdot 10^{16}$	74	40
90	$1.46 \cdot 10^{12}$	$6.41 \cdot 10^{15}$	66	36
80	$1.02 \cdot 10^{12}$	$3.56 \cdot 10^{15}$	59	32
71	$7.16 \cdot 10^{11}$	$1.96 \cdot 10^{15}$	52	29
63	$5.00 \cdot 10^{11}$	$1.08 \cdot 10^{15}$	46	26
56	$3.51 \cdot 10^{11}$	$5.98 \cdot 10^{14}$	41	23
50	$2.50 \cdot 10^{11}$	$3.39 \cdot 10^{14}$	37	20
45	$1.82 \cdot 10^{11}$	$2.00 \cdot 10^{14}$	33	18
40	$1.28 \cdot 10^{11}$	$1.11 \cdot 10^{14}$	29.5	16
32	$6.55 \cdot 10^{10}$	$3.64 \cdot 10^{13}$	24	13

### 7.8.2. Design Fatigue Curves for Unwelded Regions

The design fatigue curves for unwelded regions, shown in Fig. 7.1, can be described as follows:

For  $\Delta\sigma_R \geq \Delta\sigma_D$

$$N = [4.6 \cdot 10^4 / (\Delta\sigma_R - 0.63 \cdot R_m + 11.5)]^2, \quad (7.21a)$$

where  $N$  is the number of allowed cycles,  $\Delta\sigma_D$  the endurance stress range (for single-amplitude stress cycles), and  $\Delta\sigma_R$  the relevant stress range.

For single-amplitude stress cycles  $N$  is equal to infinity if  $\Delta\sigma_R \leq \Delta\sigma_D$ .

For single-amplitude sub-cycles of multi-amplitude cycles with sub-cycle stress range  $\Delta\sigma \leq \Delta\sigma_D$ , the design fatigue curves are the same as given by Equation (7.21a).

For multi-amplitude stress cycles with at least one single-amplitude sub-cycle stress range  $\Delta\sigma \leq \Delta\sigma_D$ , the fatigue design curves for sub-cycles with  $\Delta\sigma \leq \Delta\sigma_D$  can be described by

$$N = [(2.69R_m + 89.72) / \Delta\sigma_R]^{10} \text{ for } \Delta\sigma_D \geq \Delta\sigma_R > \Delta\sigma_{cut}, \quad (7.21b)$$

$$= \infty \text{ for } \Delta\sigma_R \leq \Delta\sigma_{cut} \quad (7.21c)$$

The values of the endurance stress range and the cut-off stress range,  $\Delta\sigma_D$  and  $\Delta\sigma_{cut}$  respectively, depend on the material's ultimate strength  $R_m$  and are given for specific values of  $R_m$  in Table 7.3. They correspond to the values of the design fatigue curves for  $N$  equal to 2 million and 100 million cycles, respectively.

Table 7.3: Values of endurance and cut-off stress range in MPa for unwelded regions

	$R_m$ in MPa			
	400	600	800	1000
$\Delta\sigma_D$	273	399	525	651
$\Delta\sigma_R$	184	270	355	440

## 7.9. Cycle Counting

### 7.9.1. General

The design fatigue curves give the allowed number of cycles for a given relevant stress range of single-amplitude cycles.

The design functions of actions, on the other hand, are in general periodic variable-amplitude functions of time, or of a time-like parameter, or even non-periodic at all, see e.g. Fig. 7.5.

The responses of the models to these design functions of actions reflect the characteristics of the design function of actions – the corresponding relevant stresses are either periodic multi-amplitude scalar functions of time, or of a time-like parameter, or non-periodic scalar functions.

An important task in the determination of the fatigue damage index is, therefore, to deduce from this (multi-amplitude) stress history a sequence of single-amplitude stress cycles that, applied separately without interaction, result in the same fatigue damage (index) as the original stress history.

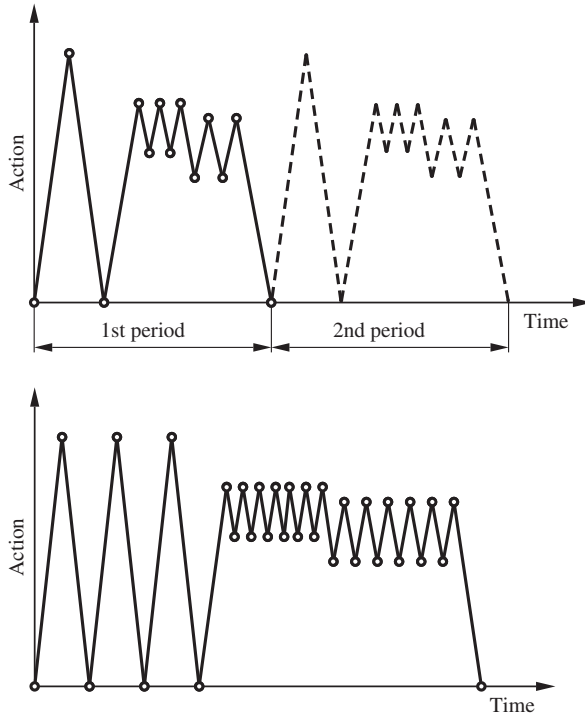


Figure 7.5: Examples of design histories of actions. Top: Periodic action. Bottom: Non-periodic action.

For this task a number of alternative procedures have been proposed, are in use, and can be found in standards, handbooks, and other literature, e.g. [96,104–114]. For complicated stress functions, and especially for on-line calculations of used design life, the **Rainflow Cycle Counting Method**, developed by Matsuishi and Endo, and by de Jonge, is recommended, being amenable to mathematical algorithms. This method is considered by some researchers to be the optimum counting procedure for on-line fatigue damage calculations. In clause 18 the **Reservoir Cycle Counting Method** is specified, being quite convenient and easy to apply in the design stage. This method is specified as an alternative to an all but the simplest cases unusable Simplified Cycle Counting Method. For periodic stress histories Rainflow and Reservoir Cycle Counting lead to the same results.

In the Rainflow and Reservoir Cycle Counting Method the largest stress range is correctly counted first, and both consider practical combinations of minima and maxima of the stress history. The Rainflow Cycle Counting Method is more difficult to apply manually, is therefore more prone to errors than the Reservoir Cycle Counting Method [114].

### 7.9.2. The Reservoir Cycle Counting Method

This cycle counting method can be explained quite clearly by means of a vivid graphic representation.

The maxima and minima of the relevant stress history are plotted versus time, or the time-like parameter, and connected by straight lines, see Fig. 7.6. In the case of a periodic stress history only one period from one absolute maximum to the maximum after the period is required. In the case of a non-periodic stress history the part of the stress history before its maximum is added at the end, in order to create an artificial period from the (old) maximum to the thereby created new maximum, see Fig. 7.7.

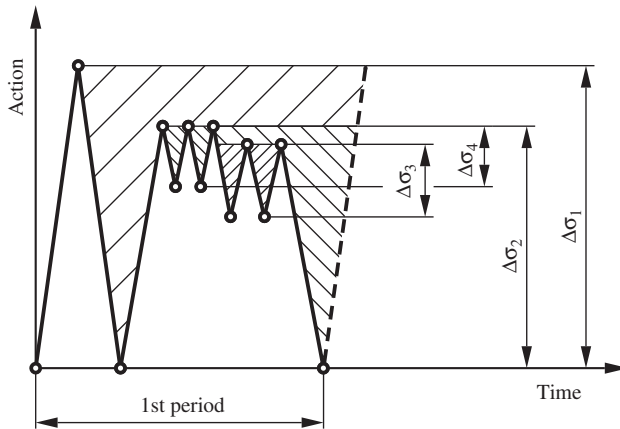


Figure 7.6: Reservoir cycle counting method for periodic stress history.

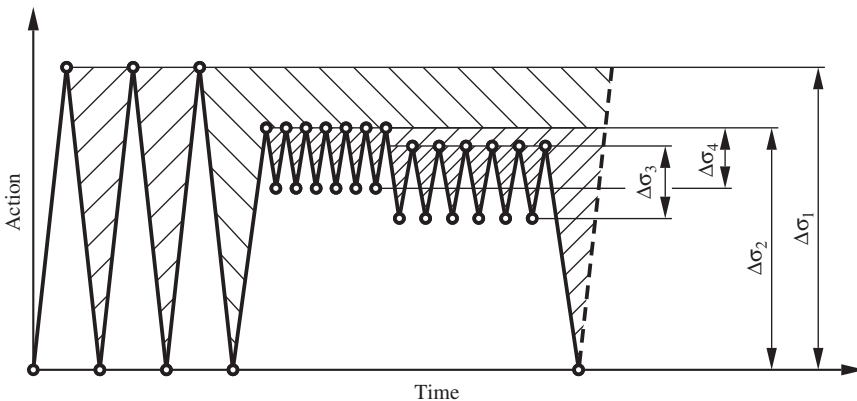


Figure 7.7: Reservoir cycle counting method for non-periodic stress history.

The profile from one maximum to the next is visualized as reservoir walls, with peaks and troughs, and the reservoir between the two maximum peaks filled with water.

The largest depth of the whole reservoir, between the highest peak and the lowest trough, corresponds to the first stress range, to be listed as the stress range of one cycle. The reservoir is then considered to be drained at the lowest trough, leaving the water that cannot escape trapped in troughs. If there is more than one lowest point the drainage may be from any one of them.

The whole procedure is then repeated again and again with the next respective lowest trough, the next minimum, until all water is drained out.

The procedure results in a list of individual stress ranges, if necessary conveniently ordered in descending order afterwards.

In the case of stress histories for hot spots in unwelded regions, the stress components and the temperature at the instants of time, or time-like parameter, at which the maximum and the minimum of the stress range occurs have to be included in the list of individual stress ranges.

In the case of stress histories for hot spots in welded regions, only the stress ranges and the temperatures matter, and cycles for which stress range and temperatures agree may be grouped together to cycles of equal stress range.

## 7.10. Fatigue Damage Accumulation

With any of the above-mentioned cycle counting methods, multi-amplitude stress histories are reduced to a sequence of single-amplitude stress cycles with cycles of equal relevant parameters grouped conveniently together. For each of these groups of single-amplitude stress cycles the allowable number of cycles can then be determined. The quotient of the number of cycles  $n_i$  of a specific group and the corresponding number of allowable cycles is defined as the **fatigue damage index** (for this group of single-amplitude cycles).

For the determination of the **cumulative fatigue damage index** the frequently used **Palmgren–Miner Rule** is prescribed. This rule, often simply called Miner Rule, is based on a linear damage hypothesis, was first proposed by A. Palmgren in 1924 [115] and popularized by M.A. Miner in 1945 [116]. It states that the cumulative fatigue damage index  $D_d$  of a sequence of  $k$  groups of constant amplitude stress cycles, of equal relevant characteristics, is given by

$$D_d = n_1 / N_1 + n_2 / N_2 + \dots + n_i / N_i + \dots + n_k / N_k, \quad (7.22)$$

where  $n_i$  is the number of single cycles in group  $i$  and  $N_i$  the number of allowed cycles corresponding to the parameters of these stress cycles in group  $i$ .

This rule takes no account of the effect of the order of occurrences of the constant amplitude cycles, but it is very simple, and it is considered to render fair approximations in usual cases, i.e. in cases with strongly alternating individual stress cycles without any especially pronounced cycle order.

## 7.11. General Remarks to the Methodology

Unlike the design checks discussed before this design check, where individual, separate load cases are to be investigated in a design check, in the F-DC the whole cyclic design life has to be investigated in one encompassing check, and, therefore, the whole cyclic design life has to be specified in the design specification, in the form of cyclic actions as a continuous function of time, or as a function of a discrete series of a time-like parameter, used to specify the order of occurrences of action states, see also Section 7.4.

The stresses at the hot spots due to this cyclic action function are required for the ensuing evaluation – the determination of the cyclic fatigue damage index. These stresses are to be determined with the linear-elastic models discussed in Section 7.3.

Depending on the location of the considered hot spot, different stress functions are required:

- (a) For welded regions the design fatigue curves depend on the fatigue class, and the fatigue class in turn depends on the weld details and on the considered fatigue crack propagation, characterized by the incipient crack plane. In the case of hot spots on the surface of the structure, inclusive of weld surfaces, in general two planes of likely incipient cracks have to be considered – one parallel to the weld joint and one normal. In both cases the required stress function is given by the respective values of the principal structural stress (in the hot spot) closest to the normal of the considered incipient crack plane.
- (b) In the case of hot spots at the root of directly loaded fillet welds, or of partial penetration welds, the plane to be considered is the **weld throat plane**, the plane through the weld's root that is defined by the straight line in weld joint direction and the straight line that renders the minimum thickness in the weld cross-section. This minimum thickness is called **weld throat thickness**, and the line **weld throat line**. In this case the required stress functions are the functions of the three components of the stress vector at the hot spot that acts on the weld throat plane. Usage of averaged components is allowed, averaged over the



weld throat line. Usage of the values at the hot spot of the equivalent linear distribution is suggested if large bending stresses exist along the weld throat, and this approach is recommended and used solely in the following. Admittedly, the weld throat plane does in general not coincide with the incipient crack plane, which in general coincides with any of the tangential planes of the joined parts, but the stresses acting on the weld throat plane were chosen as reference stresses in the classification of cracks emanating from the root of directly loaded fillet welds and partial penetration welds, and, therefore, these stresses have to be used in the design check for these cases as well.

- (c) For unwelded regions, equivalent total stress ranges, or, more clearly, ranges of the equivalent total stress functions, equivalent to the differences of the components of total stress at different instants, are required for the determination of the fatigue damage index. Therefore, functions of all total stress components at the considered hot spots are required as output of the design models. Furthermore, if plasticity correction is required and if effective stress concentration factors are to be used, the functions of all structural stress components at the considered hot spot are required as well.

## 7.12. Methodology for Welded Regions and Surface Hot Spots

This case of considered hot spots on the surface of welded regions is the simplest, and it is relatively easy and straightforward.

At each surface hot spot in a welded region, the response of the model to the design function of actions is required, in the form of a sequence of extreme values, i.e. maxima and minima of the respective principal structural stress closest to the normal of the considered incipient crack plane, together with the temperature at the corresponding instants (of time or the time-like parameter). In most cases the relevant, to be considered incipient crack plane is parallel to the weld joint, because this crack plane corresponds to a lower fatigue class. In some cases a plane normal to the weld joint is the relevant one, and in a few cases both planes, parallel and normal to the weld, have to be considered.

From this, in general multi-amplitude sequence of stress values with related temperature values, a sequence of constant amplitude cycles is then deduced by means of any of the cycle counting methods discussed in Section 7.9, a sequence that has been conveniently grouped such that each cycle in a group has the same stress range and the same maximum and minimum temperature. This sequence can be written conveniently in the form  $(\Delta\sigma, t^*, n)_m$  where  $m$  is the group identifier,  $\Delta\sigma$  stress range, and  $n$  the number of cycles in the group. The weighted mean cycle temperature  $t^*$  is given by Equations (7.18) in Section 7.6.6, with  $t_{c,\max}$  and  $t_{c,\min}$  the

highest and lowest metal temperatures in all the extreme value points in the sub-cycle with stress range  $\Delta\sigma$ .

The next step is the correction of the stress range for possible plastic deformation, for fatigue relevant thicknesses over 25 mm, and for weighted mean cycle temperatures above 100°C. The result of this step is a sequence of groups of corrected stress ranges and related number of occurrences  $(\Delta\sigma_{cor}, t^*, n)_m$ , where

$$\Delta\sigma_{cor} = \Delta\sigma \cdot k / (f_{t^*} \cdot f_{ew}), \quad (7.23)$$

is the corrected stress range, corrected for direct usage in the design fatigue curves, and with  $k$  denoting here the plasticity correction factor, see Section 7.6.1, either  $k_e$  given by Equations (7.3), or  $k_v$  given by Equations (7.2), or a combination of both, see Equation (7.6).

For each stress range  $\Delta\sigma_{cor,m}$  the corresponding allowable number of cycles  $N_m$  is to be determined next, by usage of the equations for the relevant design fatigue curve, see Section 7.8.1. This number of allowable cycles depends on  $\Delta\sigma_{cor,m}$  and the fatigue class FAT, and for  $\Delta\sigma_{cor,m} < \Delta\sigma_D$  it depends also on the maximum stress range of the other groups,  $Max [\Delta\sigma_{cor,l}]$  over all  $l \neq m$ .

The cumulative damage index is then given by

$$\sum n_m / N_m \quad (7.24)$$

at this hot spot and for the considered incipient crack plane.

If both crack planes have to be considered, the relevant cumulative fatigue index at the considered hot spot is given by the larger of the two individual ones.

### 7.13. Methodology for Welded Regions and Internal Hot Spots

The procedure for these cases of hot spots at the root of directly loaded fillet or partial penetration welds is already more complex, inasmuch as not one single scalar function is to be dealt with, as in the cases discussed in the preceding section, but three concurrent scalar functions – the three components of the stress vector acting in the hot spot on the weld throat plane. From these three functions  $\sigma_{a,i}(\tau)$ ,  $i = 1, 2, 3$ , a single scalar function denoted by  $\sigma_a(\tau)$  has to be deduced that is somehow representative of the changes of the stress vector.

The chosen function is specified by

$$\sigma_a(\tau) = \sqrt{[\sigma_{a,1}(\tau) - \sigma_{a,1}(\tau_0)]^2 + [\sigma_{a,2}(\tau) - \sigma_{a,2}(\tau_0)]^2 + [\sigma_{a,3}(\tau) - \sigma_{a,3}(\tau_0)]^2}, \quad (7.25)$$

where  $\tau_0$  is a properly chosen time, chosen such that the resulting function  $\sigma_a(\tau)$  is the most detrimental one, i.e. resulting in the largest fatigue damage index. There follows, that in general a trial and error procedure is required to find the most detrimental (pivot) time  $\tau_0$ . Fortunately in many applications the choice of  $\tau_0$  is quite obvious and no repetitions of the whole procedure are required.

The required value of the weighted mean cycle temperature  $t^*$  is defined exactly as in Section 7.2.

The ensuing steps are the same as in Section 7.12:

- Deduction of the sequence of single-amplitude cycles with common weighted mean cycle temperature  $(\Delta\sigma_a, t^*, n)_m$ .
- Correction of stress range for possible plastic deformation, for fatigue relevant thicknesses over 25 mm, and for weighted cycle temperatures above 100°C,

$$\Delta\sigma_{cor} = \Delta\sigma_a \cdot k / (f_{t^*} \cdot f_{ew}), \quad (7.26)$$

resulting in a sequence of groups  $(\Delta\sigma_{cor}, t^*, n)_m$ .

- Determination of the corresponding numbers of allowed cycles  $N_m$ .
- Summation of individual fatigue damage contributions  $n_m / N_m$  to obtain the cumulative fatigue damage index  $D_d$ .

## 7.14. Methodology for Unwelded Regions

The procedure for these cases of hot spots in unwelded regions is similar to the preceding one, but with an even more complex equation for the deduction of the single scalar function of time  $\tau$ , denoted here by  $\sigma_{eq\Delta}(\tau)$ , a scalar function that is representative of the changes of the six total stress components  $\sigma_{ij}(\tau)$  at the hot spot. The procedure is additionally complicated by the need for the determination of a relevant mean stress for the sub-cycles.

The fatigue relevant scalar stress function  $\sigma_{eq\Delta}(\tau)$ , the function to be used in the determination of the equivalent stress ranges, is specified to be given by the Mises' equivalent stress function  $MEQ[\sigma_{ij}]$  of the differences  $\Delta\sigma_{ij}$  of the total stress components  $\sigma_{ij}$  at the instants  $\tau$  and  $\tau_0$ . With

$$\Delta\sigma_{ij} = \sigma_{ij}(\tau) - \sigma_{ij}(\tau_0), \quad i, j = 1, 2, 3, \quad (7.27)$$

the function is given by

$$\sigma_{eq\Delta}(\tau) = MEQ [\Delta\sigma_{ij}] = \sqrt{[(\Delta\sigma_{11} - \Delta\sigma_{22})^2 + (\Delta\sigma_{22} - \Delta\sigma_{33})^2 + (\Delta\sigma_{33} - \Delta\sigma_{11})^2 + 6 \cdot (\Delta\sigma_{12}^2 + \Delta\sigma_{23}^2 + \Delta\sigma_{31}^2)]/2} \quad (7.28)$$

see also Equation (2.14) in Section 2.4.2. In this equation  $\tau_0$  is, as in the preceding section, a properly chosen time, chosen such that the resulting function  $\sigma_{eq\Delta}(\tau)$  is the most detrimental one.

In these cases of hot spots in unwelded regions, weighted mean cycle temperatures and equivalent mean stresses of the single-amplitude sub-cycles, determined with any of the cycle counting methods, are also required. The temperature  $t^*$  is defined as in Section 7.12, whereby, for the determination of the temperature, the temperature values at all instants of extreme values of the function  $\sigma_{eq\Delta}(\tau)$  are required. For the determination of the relevant equivalent mean stress in each of these sub-cycles, the values of all stress components at all of these instants of extreme values of  $\sigma_{eq\Delta}(\tau)$  are also required.

This **equivalent mean stress**  $\bar{\sigma}_{eq}$  can be obtained for each sub-cycle as follows:

Denoting the stress components at these instants (of maximum and minimum  $\sigma_{eq\Delta}(\tau)$  in a sub-cycle) by  $\sigma_{ij, \max}$  and  $\sigma_{ij, \min}$ , respectively, and the mean value of each of them by  $\bar{\sigma}_{ij}$ , i.e.

$$\bar{\sigma}_{ij} = (\sigma_{ij, \max} + \sigma_{ij, \min})/2, \quad i, j = 1, 2, 3, \quad (7.29)$$

the equivalent mean stress  $\bar{\sigma}_{eq}$  for each sub-cycle is specified by

$$\bar{\sigma}_{eq} = I_1(\bar{\sigma}_{ij}) = \bar{\sigma}_{11} + \bar{\sigma}_{22} + \bar{\sigma}_{33}, \quad (7.30)$$

and the result for uniaxial stress states agrees with the value used in the determination of the mean stress correction factor, see Section 7.6.5.

Using the octahedral stress representation, the equivalent stress range  $\Delta\sigma_{eq}(\tau)$  of a sub-cycle and the corresponding equivalent mean stress are related to the octahedral shear stress of the stress differences at  $\tau$  and  $\tau_0$  and the octahedral normal stress of the mean values of the stress components at  $\tau$  and  $\tau_0$ .

This approach, explicitly allowed but not detailed in clause 18, is in strict analogy to the approach detailed in clause 18, where the equivalent stress range is specified to be given by Tresca's equivalent stress function and the equivalent mean stress by the maximum normal stress acting at the plane of maximum shear stress.

A comparison of the results obtained with this approach, which is based on Mises' yield hypothesis, and the results obtained with the approach that is detailed in clause 18, which is based on Tresca's yield hypothesis, is quite instructive and interesting.

Compared with the clause 18 results, the results with this approach for the equivalent stress ranges are never larger, but, on the other hand, the absolute values of the equivalent mean stresses are never smaller – the advantage given by the possible decrease of the equivalent stress range is frequently compensated by the possible increase of the equivalent mean stress, i.e. the corrected stress range is frequently nearly unaffected by the choice of the approach.

The models used in the determination of  $\sigma_{eq\Delta}(\tau)$  are linear-elastic ones, i.e. do not take into account plasticity effects. The effect of plastic deformations on the equivalent stress ranges is taken care of by application of the plasticity correction factor, see Section 7.6.1. The effect of plastic deformations on the equivalent mean stress of each sub-cycle has to be taken care of in a part of the cycle-counting procedure, by correcting the mean values  $\bar{\sigma}_{ij}$  of the stress components, Equation (7.29), before the determination of the equivalent mean stress  $\bar{\sigma}_{eq}$ , Equation (7.30).

Without saying so, the respective procedure contained in clause 18 gives reasonably valid results only for the special cases in which, in the space of stress components, the straight line through  $\sigma_{ij}(\tau_{\max})$  and  $\sigma_{ij}(\tau_{\min})$  of the considered sub-cycle goes through the origin, or passes the origin reasonably closely.

The procedure outlined here has a much wider range of applicability and encompasses the clause 18 procedure as a special case.

Recognizing that in almost all F-DCs repeated plastic deformation is confined plasticity, this procedure is based on the idea of confined plasticity, where plastic flow occurs in a relatively small region that is enclosed by material which remains elastic, and where the strain in the region of plastic deformation is essentially controlled by the strain in the elastic region.

If  $MEQ[\sigma_{ij, \max}] \leq R_p$ , where  $R_p$  is equal to  $R_{p1.0/t^*}$  for austenitic steels and equal to  $R_{p0.2/t^*}$  for all other steels, no correction is required:

$$\bar{\sigma}_{ijcor} = \bar{\sigma}_{ij}. \quad (7.31)$$

If  $MEQ[\sigma_{ij, \max}] > R_p$ , different sub-cases have to be considered:

- sub-case 1:  $MEQ[\bar{\sigma}_{ij}] \leq R_p$ ,
- sub-case 2:  $MEQ[\bar{\sigma}_{ij}] > R_p$ .

In sub-case 1, the intersections of the straight line in the stress space that connects  $\sigma_{ij, \max}$  and  $\sigma_{ij, \min}$  with Mises' yield surface has to be determined first.

The straight line is given by

$$\bar{\sigma}_{ij} + \alpha \cdot \Delta\sigma_{ij} \quad (7.32)$$

with  $\bar{\sigma}_{ij}$  as per Equation (7.29) and  $\Delta\sigma_{ij}$  as per Equation (7.27). Denoting the value of  $\alpha$  at the intersection on the side of  $\sigma_{ij, \max}$  by  $\alpha_{\max}$ , and on the side of  $\sigma_{ij, \min}$  by  $\alpha_{\min}$ , one has

$$MEQ[\bar{\sigma}_{ij} + \alpha_{\max} \cdot \Delta\sigma_{ij}] = R_p \quad (7.33a)$$

and

$$MEQ[\bar{\sigma}_{ij} - \alpha_{\min} \cdot \Delta\sigma_{ij}] = R_p, \quad (7.33b)$$

with  $\alpha_{\max}$  and  $\alpha_{\min}$  positive. These Equations (7.33) define  $\alpha_{\max}$  and  $\alpha_{\min}$ , and serve for their determination.

The route branches then in

- sub-case 1.1:  $\alpha_{\max} + \alpha_{\min} \geq 1$ ,
- sub-case 1.2:  $\alpha_{\max} + \alpha_{\min} < 1$ .

In sub-case 1.1 the corrected mean values are given by

$$\bar{\sigma}_{ijcor} = \bar{\sigma}_{ij} + \Delta\sigma_{ij} \cdot (\alpha_{\max} - \alpha_{\min})/2, \quad (7.34a)$$

and in sub-case 1.2 by

$$\bar{\sigma}_{ijcor} = \bar{\sigma}_{ij} + (\alpha_{\max} - 0.5) \cdot \Delta\sigma_{ij}. \quad (7.34b)$$

In sub-case 2, there are on the side of  $\sigma_{ij, \max}$  two intersections of the straight connecting line, given by Equations (7.32) with Mises' yield surface. Denoting the values of  $\alpha$  at the intersections now by  $\alpha_{\min 1}$  and  $\alpha_{\min 2}$ , with  $\alpha_{\min 1} < \alpha_{\min 2}$ , they can be determined by means of the equation

$$MEQ[\bar{\sigma}_{ij} - \alpha_{\min 1,2} \Delta \cdot \sigma_{ij}] = R_p, \quad (7.35)$$

with  $R_p$  as above.

Again the route branches in

- sub-case 2.1:  $\alpha_{\min 1} + \alpha_{\min 2} \geq 1$ ,
- sub-case 2.2:  $\alpha_{\min 1} + \alpha_{\min 2} < 1$ .

In sub-case 2.1 the corrected mean values are given by

$$\bar{\sigma}_{ijcor} = \bar{\sigma}_{ij} - \Delta\sigma_{ij} \cdot (\alpha_{\min 1} + \alpha_{\min 2})/2, \quad (7.36a)$$

and in sub-case 2.2 given by

$$\bar{\sigma}_{ijcor} = \bar{\sigma}_{ij} - (0.5 + \alpha_{\min 1}) \cdot \Delta\sigma_{ij}. \quad (7.36b)$$

For these corrected mean stress components the equivalent mean stress has to be determined, using Equation (7.30).

This procedure for correction of mean stresses for plasticity effects is not only applicable for cases of alternating plasticity in sub-cycles, but also in cases of order effects, where plastic deformation in one sub-cycle renders residual stresses favourable for following sub-cycles, and where this order can be reliably specified because it is reliably ascertained, e.g. the prescribed pressure test before going into operation.

After the cycle counting procedure with, for each sub-cycle, determination of the equivalent stress range, the weighted mean cycle temperature and the equivalent mean stress possibly corrected for plasticity effects, the sequence of single-amplitude cycles can then be conveniently grouped and written in the form  $(\Delta\sigma_{eq}, t^*, \bar{\sigma}_{eq}, n)_m$ .

The steps of the whole methodology are again analogous to those in Section 7.12:

- Deduction of the sequence of constant amplitude cycles with common weighted mean cycle temperature and common equivalent mean stress  $(\Delta\sigma_{eq}, t^*, \bar{\sigma}_{eq}, n)_m$ .
- Correction for effects of possible plastic deformation of the equivalent mean stresses  $\bar{\sigma}_{eq}$  and, afterwards, of the equivalent stress ranges  $\Delta\sigma_{eq}$ .
- Correction of this (corrected) equivalent stress ranges  $\Delta\sigma_{eq}$  for fatigue relevant thicknesses over 25 mm, for weighted mean cycle temperatures above 100°C, and here also for possible equivalent mean stresses different from zero, for surface conditions, and for fatigue sensitivity:

$$\Delta\sigma_{cor} = \Delta\sigma_{\alpha} \cdot k \cdot (K_f / K_t) / (f_{t^*} \cdot f_e \cdot f_s \cdot f_m), \quad (7.37)$$

resulting in a sequence of groups  $(\Delta\sigma_{cor}, t^*, \bar{\sigma}_{cor}, n)_m$ . The value of the mean stress correction factor  $f_m$  has to be determined for the corrected mean stress  $\bar{\sigma}_{cor}$ , corrected for possible one- or two-sided plastic deformation, see above. The various correction factors are discussed in Sections 7.6.1 through 7.6.6, for  $k$ , see also Section 7.12, the discussion after Equation (7.23).

- Determination of the corresponding numbers of allowed cycles  $N_m$  for each group.
- Summation of individual fatigue damage contributions  $n_m / N_m$ , to obtain the cumulative fatigue damage index  $D_d$ .

## 7.15. Examples

Examples can be found in Annex E.7.

**Note:** Whereas the standard's requirements for weld regions, dealt with here in sub-chapter 7.7, 7.8.1, and 7.12, apply to all kinds of (sufficiently ductile) steels – rolled, wrought, deep drawing, and cast–, the requirements for unwelded regions do not apply for steel castings. Useable correction factors for steel castings can be found, for instance, in [87]. Moreover, fatigue of bolting is dealt with in the standard in a separate sub-clause, sub-clause 18.17, with different, but simple and straightforward requirements not discussed here.



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## Chapter 8

# Static Equilibrium Design Check (SE-DC)

### 8.1. Introduction

This design check deals with the global movement of structures, i.e. with the failure modes overturning, uplifting, and global displacement. In practically all cases uplifting is already considered to be an ultimate failure mode, making the check against overturning superfluous.

This design check is so closely linked to the GPD-DC that in many codes, especially civil engineering ones, both design checks are combined in one more general design check for a comparison of design effects of actions with resistances of the structures, resistances against GPD, overturning, uplifting, and global displacement.

In the Direct Route in DBA this design check is kept separately, because in the overwhelming majority of cases in pressure vessel design this design check is very different from the other design checks discussed above, inasmuch as the structure can be considered as a rigid body and the whole design check reduced to the usual traditional check of supports [117–119].

Only in cases in which reactions at supports – forces, contact pressures, and moments – cannot be determined by means of global equilibrium conditions together with simple conservative assumptions are non-rigid models required. And even then usually part models suffice for the determination of these reactions, which then can be used in the design check of the whole structure considered as a rigid body. In the majority of these cases the required information can be directly obtained from results of the GPD-DC, often the required reactions themselves.

Therefore, the discussion in this chapter is concise. The usual checks of the most important supports are discussed in more detail by means of examples in Annex E.8.

The principle of this design check requires the comparison of stabilizing design effects of actions with destabilizing design effects. This makes sense only if the notion effect refers also to reactions at supports, i.e. to forces, to contact pressures, and to moments, at the boundary between foundation and base rings or base plates of supports.

## 8.2. Procedure

The idea of this design check is expressed in the principle quite clearly, but in applications interpretations are quite often required – the principle is not always as straightforward as it reads.

Depending on the type of support and on the considered failure mode – uplifting or global displacement – the procedures differ in detail, but essentially the procedure in the various load cases consists in general of the following two steps:

- Determination of appropriate resultants of the contact pressures at the separation between parts of the support and the foundation.
- Comparison of these resultants with allowable ones, and, if applicable, check the compatibility of the model's boundary conditions with the real ones.

If the support of a structure is statically determinate the required resultants can be determined by means of the global equilibrium equations. If the support is statically indeterminate, i.e. if the reactions at the various support points cannot be determined by means of (global) equilibrium conditions alone, and if, therefore, temperature changes may render changes in the reactions, additional flexibility calculations are required, or a carryover of results from the GPD-DC.

For the determination of the allowable resultants the usual, traditional procedures for the support calculations apply, see e.g. [23, 117–119] and Annex E.8. When using allowable values from other codes or standards, like allowable pressures for foundation material, or nominal design stresses for anchors, or coefficients for static friction, the inherent safety factors may have to be adjusted – in the design actions partial safety factors are already incorporated.

Checks for compatibility of model boundary conditions are required if the design model and the physical model differ in the boundary conditions; for example, if in the design model two-sided boundary conditions are used, e.g. vanishing vertical displacement, but in the physical model the corresponding boundary conditions are one-sided, e.g. no downward displacement is possible but uplifting is.

## 8.3. Design Models

Special models are not required if supports are statically determinate. In these cases the results for the relevant reactions can be carried over from the corresponding load cases of the GPD-DC, or can be determined directly by means of global equilibrium equations, plus, if required, usual assumptions on the distribution of the reactions, see, for example [23] sub-clause 6.11 and Annex L, and the examples given in annex E.8 of this book.

In case of statically indeterminate supports, the reactions can still be carried over from the corresponding GPD-DC load cases, but with care because:

- temperature changes may result in non-vanishing reactions and these temperature-related reactions have to be taken into account here,
- the reactions in the GPD-DC load cases have been determined with linear-elastic ideal-plastic models with design yield stresses that are smaller than the characteristic values, and, hence, their usage can lead to unconservative results. It is recommended to use reactions determined with a linear-elastic constitutive law, or scaled-up reactions from GPD-DC load cases for action values where the structure is still purely elastic, scaled-up such that values correspond to the required design values.

## 8.4. Design Values of Actions

The requirements for design values and design load cases are very similar, and often identical to the ones in the GPD-DC (Section 4.4) with one important exception: pressure.

In the overwhelming majority of cases the internal (space independent) pressure and the space-independent external pressure have no effect on the static equilibrium (SE). Therefore, in all of these cases, pressure, the important name-giving action in pressure vessel design, need not be considered, and, therefore, all the combination rules of the GPD-DC, given in Section 4.4, that refer to pressure do not apply.

Internal pressure may have to be considered in cases of attached piping with rigid fixture and expansion bellow.

Internal pressure due to liquid weight should be incorporated via the weight directly.

Wind pressure can be incorporated via its global resultant force, usually prescribed directly in the national codes for wind. In the GPD-DC, partial safety factors for variable actions are not required for testing load cases, and are therefore not specified. In contrast, at least in the check of the maximum pressure at the base to foundation interface, a sub-check of the SE-DC, the inclusion of the wind action is recommended also for testing load cases, and for the partial safety factors of wind actions the usage of those for normal operating load cases.

Otherwise the requirements of the GPD-DC (Section 4.4) for design actions can be carried over:

- Partial safety factors for actions are the ones given in Tables 4.3 and 4.4.
- If for characteristic values country-specific data are used, it may be necessary to adapt partial safety factors for actions in order to maintain the overall safety required.

- Permanent actions have to be represented by appropriate design values, depending on whether the stabilizing and destabilizing effects result from a stabilizing or destabilizing part of the action.
- Self-weights of structural or non-structural elements are to be incorporated as separate permanent actions.
- Actions with stabilizing effect have to be represented by lower characteristic values and actions with destabilizing effect by upper characteristic values.

For the combination rules the ones of the GPD-DC(Section 4.4) can be carried over analogously:

- Actions with stabilizing effect may be included in combinations only if they can be assumed to be present reliably.
- Variable actions have to be considered when they increase the destabilizing effect but omitted when they increase the stabilizing effect.
- Account shall be taken of the possibility that elements might be omitted or removed.
- Where the uncertainty of geometry significantly affects static equilibrium, the corresponding dimension has to be incorporated by the most unfavourable value that it can attain under reasonably foreseeable conditions.

Temperature has to be incorporated in load cases only if the support of the structure is statically indeterminate.

Pre-stress at anchors or at pipe supports, if any, has to be incorporated in load cases as a permanent action. For anchors, the design value of the material strength parameter is obtained by division of the relevant material strength parameter by the relevant partial safety factor . The values to be used are the same as in the GPD-DC, Section 4.3, e.g. Table 4.1 for normal operating load cases and Table 4.2 for testing load cases.

Special care is required in the setting up of load cases – an action may be favourable for uplifting but unfavourable for the contact pressure at the separation between base of support and foundation. Therefore, it may be necessary to consider one action with different design values in one and the same load case. In these cases it is recommended to consider within one design check different sub-checks:

Two for overturning – for uplifting and for maximum contact pressure – and one for rigid body displacement.

## 8.5. The Principle

The principle, taken from Eurocode 3 [23], simply states that in all relevant load cases *the design effect of the destabilizing actions shall be smaller than the design effect of the stabilizing actions (EN 13445-3, Annex B).*

As already mentioned in the preceding sections, this principle can be used directly in the majority of cases with regard to uplifting and overturning, but may need adaptation with regard to cases of rigid body displacement of structures, where static friction forces contribute to the structures' resistances against such displacements.

In these cases the static friction forces have to be incorporated by their design values, determined by usage of design values of the coefficient of static friction, given by the quotient of their lower characteristic values and the partial safety factor  $\gamma_R = 1.2$ .

## **8.6. Examples**

Examples can be found in Annex E.8.

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## Epilogue

The Direct Route to Design by Analysis, as laid down in EN 13445-3 Annex B, is a major step forward towards a more rational, informative design process for unfired pressure vessel components.

This approach provides much insight into the behaviour of the components and, especially, into the safety margins against individual failure modes.

This approach offers advantages with respect to design improvements, and also with respect to in-service inspections, in-service inspection procedures, and the specification of in-service inspection intervals, including risk-based inspection approaches.

The presently existing restriction to unfired pressure vessels made of sufficiently ductile steels is not really restricting – the present standard encompasses the majority of unfired pressure vessel and the extension to other sufficiently ductile materials is straightforward.

The presently existing restriction to unfired pressure vessels operating below the creep regime will cease to exist in the near future – the draft of the design checks for unfired pressure vessels operating in the creep regime has already passed the first enquiry stage.

There may exist theoretically more attractive, more rational approaches, based on total probabilistic concepts, on the concept of reliability of structures [123], but these have still a long way to go to gain general acceptance in the pressure vessel industry, with its variety of designs and phenomena. Moreover, the partial safety factor concept, used in the Direct Route to Design by Analysis, is flexible and open enough for incorporation of some of these (improved) probabilistic concepts – the concept of reliability may serve the framework for a more rational, better allocation of the partial safety factors.

There are, in this approach, still a few details that are not really solved satisfactorily, details worthy of being dealt with, theoretically as well as experimentally – contributions are welcome.

We do hope that this book will be of help in the dissemination of this very promising design route for unfired pressure vessels.



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# Annex A: Useful Shakedown Theorems

The following theorems from [41,32,4] are useful in the setting up of the load case specification lists, see Section 3.3.1, and they are also useful in pre-checks in the context of the Progressive Plastic Deformation Design Check, in pre-checks whether detailed checks or detailed investigations are required. The theorems are proven, and therefore valid, for shakedown to linear-elastic behaviour.

## Theorem 1

Each cyclic action for which the corresponding elastic stress field is enveloped by the envelope of the elastic stress field of a shakedown action is also a shakedown action.

## Theorem 2

A cyclic action can be a shakedown action only if for each point of the model a time-invariant stress  $\sigma_{ij}^o$  can be found such that the superposition of this stress with the cyclic elastic stress due to the cyclic action is always compatible with the yield condition. The time-invariant stress  $\sigma_{ij}^o$  may be different for each point of the structure.

In cases in which, in each point of the model and during the whole action cycle, the yield stress is invariant, this means that it must be possible to shift all of the cyclic elastic stress trajectories (in the stress-space) due to the cyclic action translatorially such that they are enveloped by the (invariant) yield surface. The translatorial shift may be different for each point of the model.

In the more general cases in which, in each point of the model and during the cycle, the yield stress may vary (cyclically), e.g. due to temperature changes, this means that it must be possible to shift all of the trajectories of the compatibility ratios of the cyclic elastic stresses translatorially such that they are enveloped by the yield surface for the yield stress compatibility ratio equal to unity.

Note: This theorem renders simple checks whether a cyclic action can be a shakedown action. It is a necessary condition for shakedown, i.e. its fulfilment does not mean that the model will indeed shake down. Only if the stress field  $\sigma_{ij}^o$  is a self-stress field is shakedown assured (by Melan's shakedown theorem and the extended shakedown theorem, respectively). This theorem results in easy checks

via plotting of critical elastic stress trajectories or the trajectories of the compatibility ratios in the deviatoric map, together with a plot of the yield limit.

### **Corollary 1 to Theorem 2**

This corollary applies to cases in which, in each point of the model during the whole action cycle, the yield stress is invariant: A cyclic action can be a shakedown action only if in the deviatoric map for each point of the model the diameter of the corresponding cyclic elastic stress trajectory due to the cyclic action is not larger than two times the yield stress.

### **Corollary 2 to Theorem 2**

This corollary applies to the more general cases in which, in each point of the model during the whole action cycle, the yield stress may vary (cyclically): A cyclic action can be a shakedown action only if in the deviatoric map for each point of the model the distance between points of the trajectory of the compatibility ratio of the corresponding cyclic elastic stress path due to the cyclic action is not larger than 2.0.

### **Theorem 3**

This theorem applies to cyclic actions with zero or negligible non-stationary thermal stresses.

If the cyclic action that encompasses all corners of an action domain that define the domain is a shakedown action, then each cyclic action in this action domain is also a shakedown action.

### **Theorem 4**

This theorem applies to cyclic actions with zero or negligible non-stationary thermal stresses.

Each cyclic action in the smallest convex enveloping domain of a shakedown action is also a shakedown action. The enveloping surface of this smallest convex enveloping domain is characterized by the fact that each point is on a secant of the shakedown trajectory (see Fig. A.1).

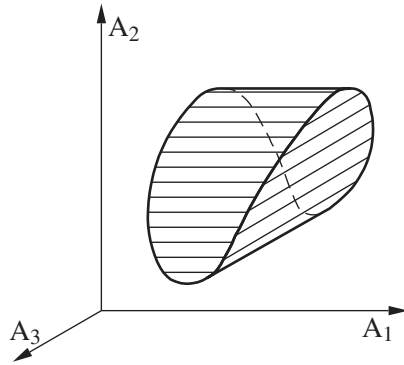


Figure A.1: Smallest convex enveloping surface.

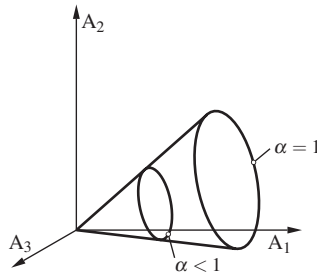


Figure A.2: Shakedown trajectories in the context of Theorem 5.

**Theorem 5**

If a cyclic action  $A_i(\tau)$ ,  $i = 1, 2, \dots, n$ , is a shakedown trajectory, then the cyclic action  $\alpha \cdot A_i(\tau)$ ,  $i = 1, 2, \dots, n$ , with  $\alpha < 1$  is also a shakedown trajectory (see Fig. A.2).

**Theorem 6**

The following two theorems apply in cases of zero or negligible non-stationary thermal stresses and actions of the form

$$A_i(\tau) = \alpha_i \cdot A(\tau) \tag{A.1}$$

where  $A(\tau)$  is a single-amplitude cyclic function that cycles between  $A_{\min}$  and  $A_{\max}$ . In the action space these actions are represented by portions of straight lines through the origin. Both the theorems are necessary conditions for shakedown.

### **Theorem 6a**

An action of the above-mentioned type can be a shakedown action if and only if  $A_{\max} - A_{\min} \leq 2A_e$ , where  $A_e$  corresponds to the elastic limit action with zero initial stress state.

### **Theorem 6b**

An action of the above-mentioned type can be a shakedown action if and only if in each point of the model  $\bar{\sigma}_{ij}^e \cdot (A_{\max} - A_{\min})/2$  is compatible with the yield condition, where  $\bar{\sigma}_{ij}^e$  is the stress distribution in the equivalent linear-elastic model for the action  $A_i = \alpha_i$ .

# Annex E: Examples

## Annex E.3: Example of a design check table

In the following example, an extremely formalistic but instructive approach to the setting up of a design check table is shown.

### *E.3.1: Design check table of a jacketed autoclave*

The jacketed autoclave is sketched in Fig. E.3-1, and the process cycle is shown in Fig. E.3-2.

The pressure and temperature increases in Chamber 1, denoted by  $p_{1,op}$  and  $t_{1,op}$ , are planned and controlled to occur concurrently with those in Chamber 2, denoted by  $p_{2,op}$  and  $t_{2,op}$ , respectively. The concurrent pressurization in both rooms is not ensured, and, therefore, the operating pressures (and temperatures) have to be considered to be independent. After the process period at high temperatures, the heating medium in both chambers can be blown out, and replaced by cold water in Chamber 2. The cooling period in Chamber 1 is planned to end at ambient conditions. Negative pressures are not planned, but cannot be excluded either – condensation can occur under reasonably foreseeable conditions. For Chamber 2, condensation, and thus negative pressure, is planned as a possible alternative process. Consequently, the four process parameters  $p_{1,op}$ ,  $t_{1,op}$ ,  $p_{2,op}$ , and  $t_{2,op}$  are considered for the design check table to be independent.

Table E.3-1 is the decision table for these four independent actions under normal operating load cases. A plus sign in the first four rows refers to an upper characteristic value, and the corresponding value itself depends on the load case, i.e. in principle it may be different from column to column. A minus sign refers to a lower characteristic value, and the corresponding value itself depends, again, on the load case. Rows 4 and 5 indicate load cases that seem to require consideration. The designations refer to load case designations, and those in parentheses refer to load cases that are formally not required by the standard. The stars in Row 6 indicate combinations of values to be considered in the specification of the cyclic action of the PD-DC that pass through the relevant corners of the design domain, or through the neighbourhood of these corners.

Each of the two chambers is provided with one safety valve, with set pressures above the maximum operating pressure, and with sufficient margins for pressure

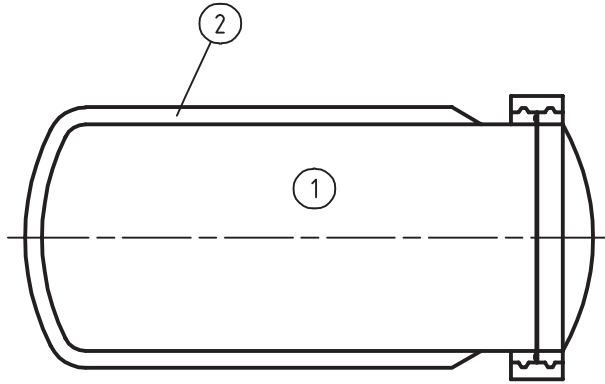


Figure E.3-1: Jacketed autoclave.

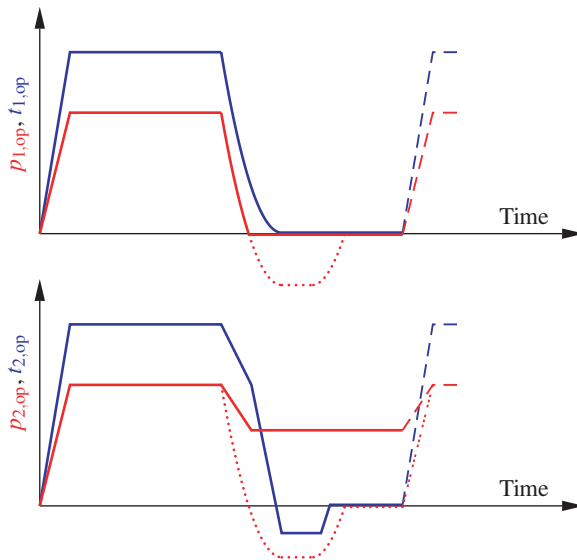


Figure E.3-2: Process cycle.

excursions. The set pressures must not be larger than the corresponding maximum allowable pressures. In agreement with common practice,  $P_{1set} = PS1^+$  and  $P_{2set} = PS2^+$  have been chosen, whereby in this example,  $PS1^+ = PS2^+ = 2.6 \text{ bar} = 0.26 \text{ MPa}$ , and, taking the vapor–pressure relationship into account, consequently  $TS1^+ = TS2^+ = 140^\circ\text{C}$  follows.

These values,  $PS1^+$ ,  $TS1^+$ ,  $PS2^+$ ,  $TS2^+$ , are also the upper characteristic values for pressure and temperature under normal operating load cases of the GPD-DC and the I-DC. The corresponding lower characteristic values are

$$PS1_- = -1 \text{ bar} = -0.1 \text{ MPa}; \quad TS1_- = RT$$

$$PS2_- = -1 \text{ bar} = -0.1 \text{ MPa}; \quad TS2_- = 0^\circ\text{C}$$

where RT denotes ambient temperature ( $20^\circ\text{C}$ ).

The 16 combinations of the four upper and four lower characteristic values, as given by the 16 columns in the decision table, are the corners of the design domain, of which the 12 in the condensed decision table (Table E.3-2) may be design relevant, i.e. have to be considered in the various gross plastic deformation and stability design checks, indicated in Rows 4 and 5 of the full decision table.

It is acknowledged that not all of the resulting load cases indeed require detailed investigations, but it is not obvious which ones could be deleted.

The load case specifications related to GPD design checks are listed in Table E.3-3. Load cases NOLC 1.1 through NOLC 1.3 deal with GPD of the outer shell and the end of the jacket, and with the closure. Load cases NOLC 1.4 through

Table E.3-1: Decision table for example E.3.1

$p_1$	+	+	+	+	+	+	+	+	-	-	-
$p_2$	+	+	+	+	-	-	-	-	+	+	+
$t_1$	+	+	-	-	+	+	-	-	+	+	-
$t_2$	+	-	+	-	+	-	+	-	+	-	+
1	1.1	(1.2)	(1.3)		1.4	(1.5)	(1.6)				
3					3.1	3.2	3.3		3.4	3.5	3.6
2	*	*	*		*	*	*		*	*	*

Table E.3-2: Considered decision table

$p_1$	+	+	+	+	+	+	-	-	-	-	-	-
$p_2$	+	+	+	-	-	-	+	+	+	-	-	-
$t_1$	+	+	-	+	+	-	+	+	-	+	+	-
$t_2$	+	-	+	+	-	+	+	-	+	+	-	+
1	1.1	1.2	1.3	1.4	1.5	1.6						
3				3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9



Table E.3-3: Load cases related to the GPD-DC

Load case	Characteristic values <sup>a</sup> <i>p</i> (MPa) and <i>t</i> (°C)				Partial safety factors				Design values <i>p</i> (MPa) and <i>t</i> (°C)			
	<i>p</i> <sub>1</sub>	<i>p</i> <sub>2</sub>	<i>t</i> <sub>1</sub>	<i>t</i> <sub>2</sub>	<i>p</i> <sub>1</sub>	<i>p</i> <sub>2</sub>	<i>t</i> <sub>1</sub>	<i>t</i> <sub>2</sub>	<i>p</i> <sub>1</sub>	<i>p</i> <sub>2</sub>	<i>t</i> <sub>1</sub>	<i>t</i> <sub>2</sub>
NOLC 1.1	+, u	+, u	+	+								
	0.26	0.26	140	140	1.2	1.2	1.0	1.0	0.312	0.312	140	140
NOLC 1.2 <sup>b</sup>	+, u	+, u	+	-								
	0.26	0.26	140	0	1.2	1.2	1.0	1.0	0.312	0.312	140	0
NOLC 1.3 <sup>b</sup>	+, u	+, u	-	+								
	0.26	0.26	RT	140	1.2	1.2	1.0	1.0	0.312	0.312	RT	140
NOLC 1.4	+, u	-, u, l	+	+								
	0.26	-0.1	140	140	1.2	1.0	1.0	1.0	0.312	-0.1	140	140
NOLC 1.5 <sup>b</sup>	+, u	-, u, l	+	-								
	0.26	-0.1	140	0	1.2	1.0	1.0	1.0	0.312	-0.1	140	0
NOLC 1.6 <sup>b</sup>	+, u	-, u, l	-	+								
	0.26	-0.1	RT	140	1.2	1.0	1.0	1.0	0.312	-0.1	RT	140

<sup>a</sup>*p* is expressed in MPa and *t* in °C in all the tables.

<sup>b</sup>Consideration of these load cases is formally not required by the standard – thermal stresses may be neglected in GPD design checks and the design values of the yield stresses are not larger than the ones in NOLC 1.2 or NOLC 1.4, respectively.

NOLC 1.6 deal with GPD of the end and the cylindrical shell of the inner chamber, and possibly with the closure.

The plus and minus sign indicate upper and lower characteristic values, and the designations u, f, and l, refer to unfavourable, favourable, and limited actions, respectively.

In Table E.3-4, the load case specifications related to the instability design checks are listed. The load cases NOLC 3.1 through NOLC 3.3 and NOLC 3.7 through NOLC 3.9 deal with buckling of the jacket’s cylindrical shell and the end, load cases NOLC 3.4 through NOLC 3.6 with buckling of the inner cylindrical shell and the end, and load cases NOLC 3.4 through NOLC 3.9 with buckling of the closure, including possible influences of jacket pressure and temperature.

These S-DC-related load case specifications of Table E.3-4 are to be complemented by appropriate characteristic (and design) functions of the respective temperature transitions into the specified states. The same applies to the cyclic action for the PD-DC, but there these characteristic functions are less critical than for the S-DC load cases, which are related to spontaneous failure.

Table E.3-4: Load cases related to the S-DC

Load case	Characteristic values $p, t$				Partial safety factors				Design values $p, t$			
	$p_1$	$p_2$	$t_1$	$t_2$	$p_1$	$p_2$	$t_1$	$t_2$	$p_1$	$p_2$	$t_1$	$t_2$
NOLC 3.1	+, u	-, u, l	+	+	1.2	1.0	1.0	1.0	0.312	-0.1	140	140
NOLC 3.2	+, u	-, u, l	+	-	1.2	1.0	1.0	1.0	0.312	-0.1	140	0
NOLC 3.3	+, u	-, u, l	-	+	1.2	1.0	1.0	1.0	0.312	-0.1	RT	140
NOLC 3.4	-, u, l	+, u	+	+	1.0	1.2	1.0	1.0	-0.1	0.312	140	140
NOLC 3.5	-, u, l	+, u	+	-	1.0	1.2	1.0	1.0	-0.1	0.313	140	0
NOLC 3.6	-, u, l	+, u	-	+	1.0	1.2	1.0	1.0	-0.1	0.312	RT	140
NOLC 3.7	-, u, l	-, u, l	+	+	1.0	1.0	1.0	1.0	-0.1	-0.1	140	140
NOLC 3.8	-, u, l	-, u, l	+	-	1.0	1.0	1.0	1.0	-0.1	-0.1	140	0
NOLC 3.9	-, u, l	-, u, l	+	+	1.0	1.0	1.0	1.0	-0.1	-0.1	RT	140

## Annex E.4: Examples of Gross Plastic Deformation Design Checks

The examples in this part of Annex E are intended to illustrate the procedures of the gross plastic deformation design check, and to indicate modelling problems and solutions. Therefore, all examples deal only with individual load cases that are related to the GPD-DC, without consideration of completeness of the design checks.

The first example deals with a clad pressure vessel, for which in a pure DBF approach the nozzle moment is not permissible. The second deals with variants of the transition region of a cylindrical shell to a hemispherical one, illustrating the insight into a structure's behaviour gained by DBA investigations. The third example, a typical header of an air cooler (of rectangular cross-section), illustrates the safety margin due to plastification, and also the modelling problems of tubesheet to tube connections.

The fourth deals with a clad pressure vessel, with in a pure DBF approach non-admissible nozzle moment.

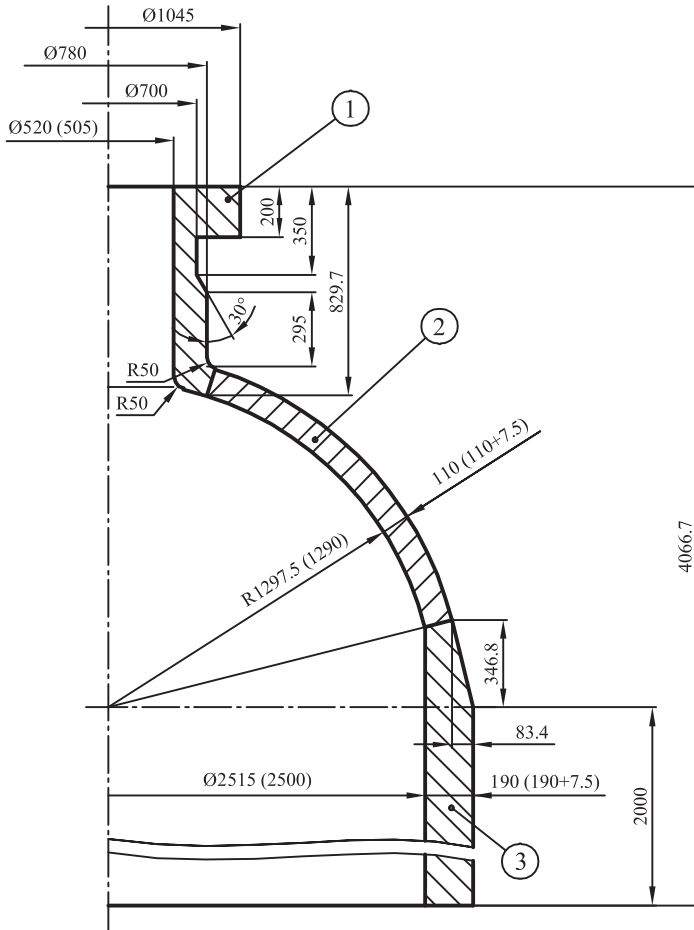


Figure E.4.1-1: Hydrogenation reactor.

**E.4.1: GPD-DC of a hydrocracking reactor**

This example deals with one GPD load case of the upper part of a hydrocracking reactor shown in Fig. E.4.1-1. The objective is to check the admissibility of the design under pressure and the specified piping reactions at the nozzle, which are not admissible according to DBF. Therefore, only the upper part of the vessel is investigated, and, taking into account the symmetry of structure and actions, only one-half of the structure's upper part is modelled. The load case corresponds to one corner of the design domain, relates to the state of maximum pressure, maximum temperature, and reactions at the nozzle corresponding to the hot state.

The geometry data without parentheses in Fig. E.4.1-1 are those of the design model, while those in parentheses are the nominal ones of the design drawing if different from the ones of the design model.<sup>1</sup>

#### Excerpt from the design data specification

- Maximum allowable pressure:  $PS = 180 \text{ bar} = 18 \text{ MPa}$
- Maximum allowable temperature:  $TS = 400^\circ\text{C}$
- Maximum allowable actions at upper nozzle (upper face of vessel flange):
  - Maximum allowable force:<sup>2</sup>
    - Hot:  $FS_z = 90 \text{ kN}$
    - Cold:  $FS_z = 126 \text{ kN}$
  - Maximum allowable bending moment:<sup>2</sup>
    - Hot:  $MS_x = 296 \text{ kNm}$
    - Cold:  $MS_x = 88 \text{ kNm}$
- Number of operational cycles:  $N = 100 \text{ cycles}$
- Material:
  - Main shell and ends: EN 10222-2 11CrMo9-10
  - Nozzle forgings: EN 10222-2 11CrMo9-10
  - Cladding: 1.4571 (2 layers with together 7.5 mm nominal thickness)
- Thickness allowances:
  - Corrosion/erosion:  $c = 0$
  - Tolerances of main shell, end, and forging:  $\delta_e = 0$
- Insulation thickness: 350 mm

#### (Considered) Design Load Case

##### NOLC 1.1:

- (Upper) characteristic value of pressure:  $p_c^+ = 18 \text{ MPa}$
- (Upper) characteristic value of temperature:  $t_c^+ = 400^\circ\text{C}$
- (Upper) characteristic value of upper nozzle force:  $F_{z,c}^+ = 90 \text{ kN}$ , with 126 kN due to weights and  $-36 \text{ kN}$  due to temperature changes
- (Upper) characteristic value of upper nozzle moment:  $M_{x,c}^+ = 296 \text{ kN m}$ , with 88 kN m due to weights and 208 kN m due to temperature changes

<sup>1</sup> In the GPD-DC, no structural strength shall be attributed to the cladding. Therefore, with a nominal diameter of the main shell's cladding face of 2500 mm and a nominal cladding thickness of 7.5 mm, an inner diameter of the model's main shell of 2515 mm results. For the hemispherical ends, with a nominal radius of the ends' cladding faces of 1290 mm, an inner radius of the model's ends of 1297.5 results. For the nozzles with a nominal diameter of the cladding face of 505 mm, an inner diameter of the model's nozzles of 520 mm results. For the main cylindrical shell, the hemispherical ends, and the forged reinforcements, zero corrosion/erosion allowance and zero thickness tolerance allowance are specified, i.e. the model thicknesses of these parts are equal to the nominal thicknesses (of the base metal).

<sup>2</sup> Force and moment occur concurrently, with temperature change related parts changing proportionally. Force and moment in the cold state are due to weights, the changes to the hot state are due to the temperature change, from cold to hot, i.e. from RT to 400°C.

The corresponding load case specification, with derivation details, is given in Table E.4.1-1.

The relevant material parameters are given in Table E.4.1-2. The reference temperature for determining the modulus of elasticity is  $t_{rE}$ , and for determining the strength parameter is  $t_{rRM}$ .

For reasons of computing time, a model with Mises' yield condition and, therefore, the design yield stress  $RM_{d,red}$  is used first. The input listing of the FEM run is given in Annex L.4.1. To apply force and moment at the upper nozzle, all nodes of the upper face of the nozzle flange are coupled together by a rigid region, and one-half of the force and one-half of the moment applied, and additionally one-half of the upwardly acting pressure resultant  $F_r = pD_{in}^2\pi/4$ . The displacement in axial direction of all points of the design model's lower end is restrained to zero, as well as the displacement of all points in the meridional plane normal to this

Table E.4.1-1: Load case specification

NOLC 1.1			
Action	Characteristic value	Partial safety factor	Design value
Pressure $p_d$ (MPa)	18 <sup>b</sup>	1.2	21.6 <sup>b</sup>
Temperature $t_d$ (°C)	400	1.0	400
Nozzle force $F_{z,d}$ (kN) <sup>a</sup>	126	1.2	(151.2)
	-36	1.0	(-36.0)
Nozzle moment $M_{x,d}$ (kNm) <sup>a</sup>	88	1.2	115.2
	208	1.0	(105.6)
			(208)
			313.6

<sup>a</sup>The values in the first row correspond to the permanent part, due to weight. The values in the second row correspond to the temperature change related part. The value in the third row is the design value, given by the sum of the design values of the two parts.

Nozzle force and moment occur concurrently, and with temperature change related parts changing proportionally. Therefore, the temperature change related part of the force is taken into account, although it has a favourable effect.

<sup>b</sup>The nominal face of the cladding shall be the surface on which the pressure acts. In the model the pressure acts on the inner face of the model. Therefore, the pressure used in the model is different from the given calculation pressure. The correction factors are smaller than unity and usually so close to unity that this correction is usually neglected. The correction factors are given by the ratio of the corresponding infinitesimal surface elements, on the nominal cladding face and on the model inner surface, respectively. In the present example they are:

- For the main shell  $(2500/2)/(2515/2) = 0.994$ .
- For the hemispherical ends  $1290^2/1297.5^2 = 0.988$ .
- For the nozzle  $(520/2)/(535/2) = 0.972$ .

Following common practice, these corrections are neglected here.

Table E.4.1-2: Material parameters

NOLC 1.1											
Part	Material group	$t_{rE}$ (°C)	$t_{rRM}$ (°C)	$E$ (GPa)	$\mu_{el}$ –	$R_m$ (MPa)	$R_p^{0.2t_{rRM}}$ (MPa)	(4)/(3) –	$\gamma_R$ –	$RM_d$ (MPa)	$RM_{d,red}$ (MPa)
				(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
										= (4)/(6)	
1, 2, 3	5.2	305	400	190	0.3	520	195.0	0.38	1.25	156	135

Note: Because of the insulation, a reference temperature  $t_{rRM}$  for determining the material parameters equal to the characteristic value of temperature,  $t_c^+=400^\circ\text{C}$ , has been chosen for all parts of the model. For determining the modulus of elasticity the reference temperature  $t_{rE}$ , with  $t_{rE}=0.75t_{\max}+0.25t_{\min}=0.75t_c^++5$  has been chosen.

Values for  $E$  and  $\mu_{el}$  are from EN 13445-3, Annex O; values for material strength parameters are from the referenced material standards, EN 10222-2 and EN 10028-2, respectively.

In the design models either Mises' yield condition is used together with the reduced design value of the yield stress  $RM_{d,red}=RM_d\sqrt{3}/2$  or Tresca's yield condition with the (non-reduced) design value of the yield stress  $RM_d$ .

plane. To prevent a still possible rigid-body displacement of the model, the displacement in the third Cartesian direction of all structural points of the model's lower end symmetry line are restrained to zero. In this run, pressure, nozzle force, and nozzle moment are increased proportionally from zero to the design values, temperature and material properties are constant, and the initial state is stress-free. In this approach, with Mises' yield condition and reduced yield stress, the end point of the action path, given by the design values of the actions, is not reached: Fig. E.4.1-2 shows the development of the maximum absolute value of the principal strains up to the largest values of the actions with a convergent solution, and Fig. E.4.1-3 shows the distribution of Mises' equivalent stress for these action values, given by 92.3% of the design values. Therefore, with this model the requirements of the design check's principle are, for this load case, not fulfilled.

### Plausibility check and remedy:

As a comparison, and also as a (rough) check of the results, the theoretical limit analysis results of idealized parts of the structure can be calculated. These results (see e.g. [4], [138], [140]) correspond to the theoretical carrying capacity of the corresponding models of these idealized parts without the strain-limiting requirement of the design check's principle.

For the closed cylindrical shell with Tresca's yield condition and the yield stress given by  $RM_d$ , the limit analysis pressure is

$$p_{laT}=RM_d \cdot \ln(D_o/D_i)=156 \cdot \ln(2895/2515)=21.95 \text{ MPa.}$$

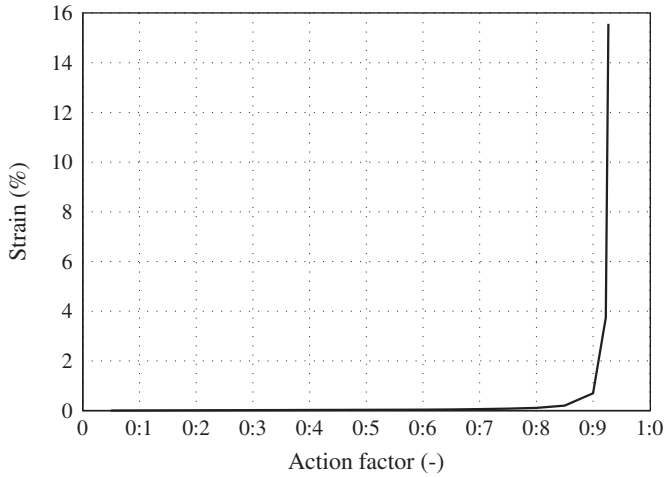


Figure E.4.1-2: Maximum absolute value of principal strains vs. action ratio.

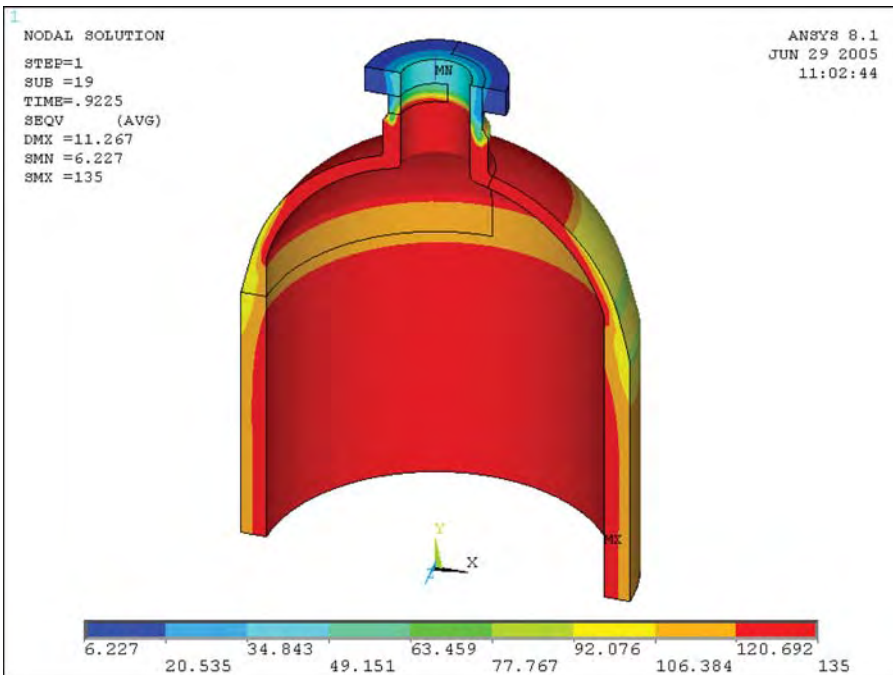


Figure E.4.1-3: Mises' equivalent stress distribution.

For the closed cylindrical shell with Mises' yield condition and the yield stress as required by the principle, i.e. a yield stress given by  $RM_{d,red}$ , the corresponding limit analysis pressure is

$$p_{laM} = (2/\sqrt{3}) \cdot RM_{d,red} \cdot \ln(D_o/D_i) = 21.95 \text{ MPa},$$

i.e. the same as for Tresca's yield condition, a result due to the fully plastic stress states in the closed cylindrical shell that renders the largest difference in Mises' and Tresca's equivalent stresses.

For the spherical shell the limit analysis pressure is independent of the kind of yield condition:

$$p_{la} = 2 \cdot RM \cdot \ln(R_o/R_i).$$

Therefore, for Tresca's yield condition and the yield stress  $RM_d$  one has

$$p_{laT} = 2 \cdot RM_d \cdot \ln(1407.5/1297.5) = 25.39 \text{ MPa},$$

and for Mises' yield condition and the then required yield stress  $RM_{d,red}$

$$p_{laM} = 2 \cdot RM_{d,red} \cdot \ln(1407.5/1297.5) = 21.99 \text{ MPa},$$

with the resulting difference due to the fully plastic stress states in the spherical shell with equal Mises' and Tresca's equivalent stresses, and the difference resulting from the difference in the yield stresses to be used according to the principle.

These results indicate that the usage of the basic approach, with Tresca's yield condition and yield stress  $RM_d$ , can provide the required remedy. Therefore, the check has been repeated with Tresca's yield condition and the (non-reduced) yield stress  $RM_d$ .

The calculation time for this model, with Tresca's yield condition, is much larger than that for the former model, with Mises' yield condition, but the result is worth the effort:

The design model with Tresca's yield condition and yield stress  $RM_d$  can carry the design action. Fig. E.4.1-4 shows Tresca's equivalent stress distribution for the design values of the actions. Fig. E.4.1-5 shows for this model the development of the maximum absolute value of the principal strains for proportional increase of actions from zero to the design values. The figure shows that the maximum absolute value of the principal strains for the design values of actions, given by the action ratio 1.0, is less than 5% – the GPD-DC requirements are fulfilled for this NOLC.

Therefore:

**NOLC 1.1: O.K.**



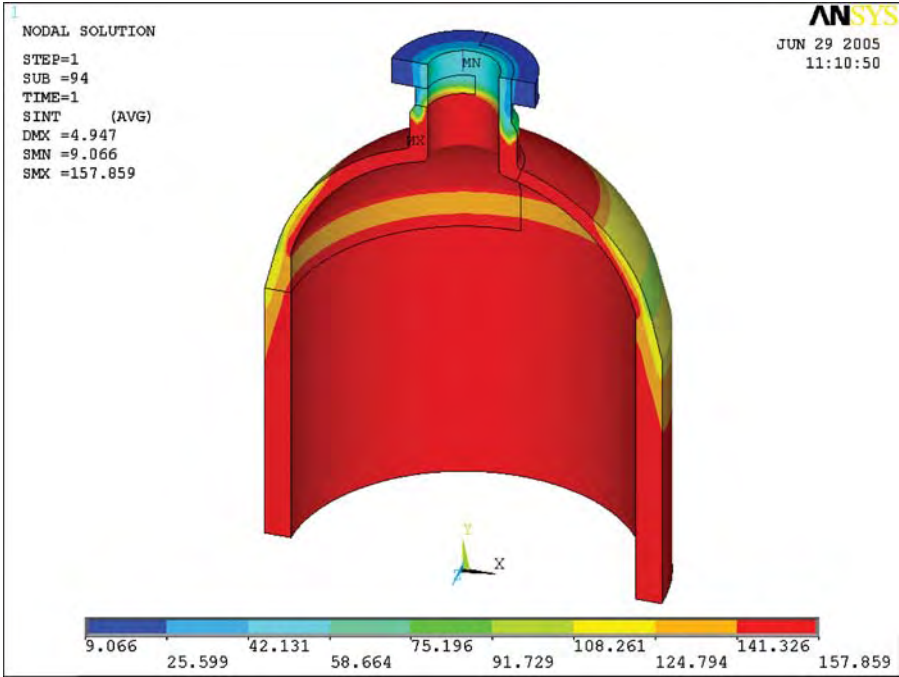


Figure E.4.1-4: Tresca's equivalent stress distribution.

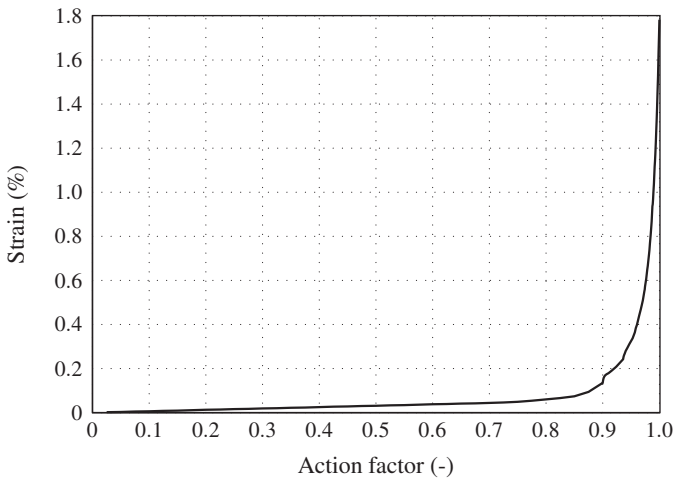


Figure E.4.1-5: Maximum absolute value of principal strains vs. action ratio.

Remarks: In the strain-critical region the model can be considered to be stress-concentration-free – principal total strains are practically equal to the principal structural strains, which are required in the design check's principle.

The gain in carrying capacity, via the usage of Tresca's yield condition and non-reduced yield stress, is quite typical for models of spherical shells, and also for cases with structures that allow adaptation of the stress distribution to the yield condition [122], but, of course, the gain can never be larger than 15.5%.

#### ***E.4.2: Detailed investigation of the transition of a cylindrical to a hemispherical shell***

In the preceding example, the thickness difference between the cylindrical and the hemispherical shell is too large to allow for the required taper of 1:4 in the cylindrical shell only, and, therefore, some of the usual design options for the transition are not possible, e.g.

- a smooth transition of the inner surfaces (which furthermore results in an overly large misalignment of the middle surfaces),
- a hemispherical shell without straight flange welded to the cylindrical shell without (planned) misalignment of middle surfaces.

Therefore, four variants of the transition, allowable according to the misalignment and taper requirements of EN 13445-4, are compared with each other and with two additional variants with non-permissible misalignments. These additional variants are incorporated because the tolerance limits for misalignment of middle surfaces in EN 13445-4 are for the present example so restrictive that a DBA-DR investigation for misalignment deviation is frequently required, an investigation also explicitly foreseen in the standard for such deviations.

**Detail geometry 1:** With the exception of the nozzle, which has been omitted from the model for the investigations in this example, the models used in these investigations are the same as the one used in the preceding example. Details of the model geometry for detail geometry 1 are shown in Fig. E.4.2-1. The distribution of Mises' equivalent stress for the largest pressure for which the requirements of the GPD-DC are fulfilled is shown in Fig. E.4.2-2 and the corresponding distribution for the same pressure but a linear-elastic constitutive law in Fig. E.4.2-3. Like in the preceding example, the model is considered to be stress-concentration-free – despite the jump in the meridian tangent at the intersections of spherical surfaces and cylindrical or conical ones, extrapolations into the hot spot result in practically the same stresses and strains as the corresponding total ones directly from the FEM output.

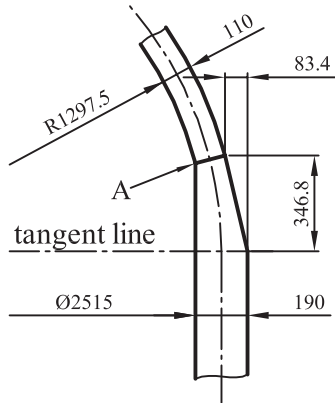


Figure E.4.2-1: Detail geometry 1.

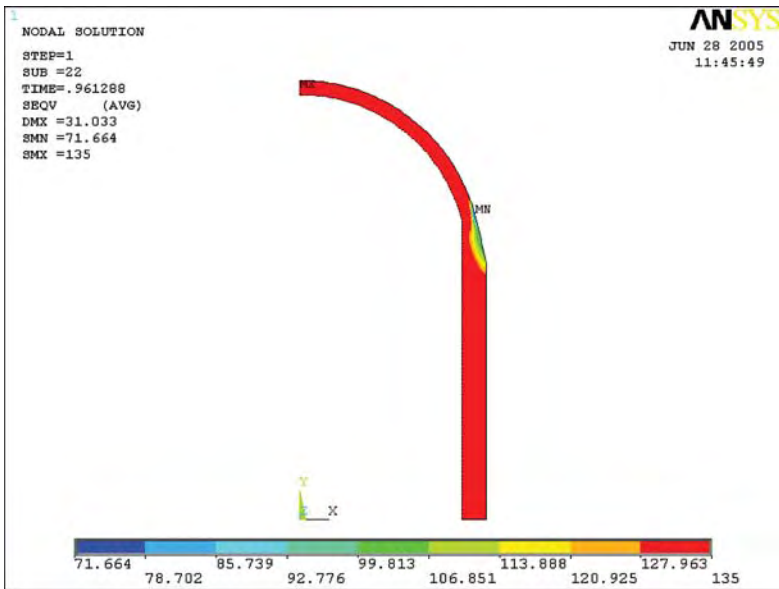


Figure E.4.2-2: Maximum equivalent stress distribution. GPD-DC design model: detail geometry 1.

Maximum permissible pressure, and the results obtained with a linear-elastic model for the maximum equivalent stress in the region outside of the butt welding affected zone of the butt weld and for the maximum principal stress normal to the

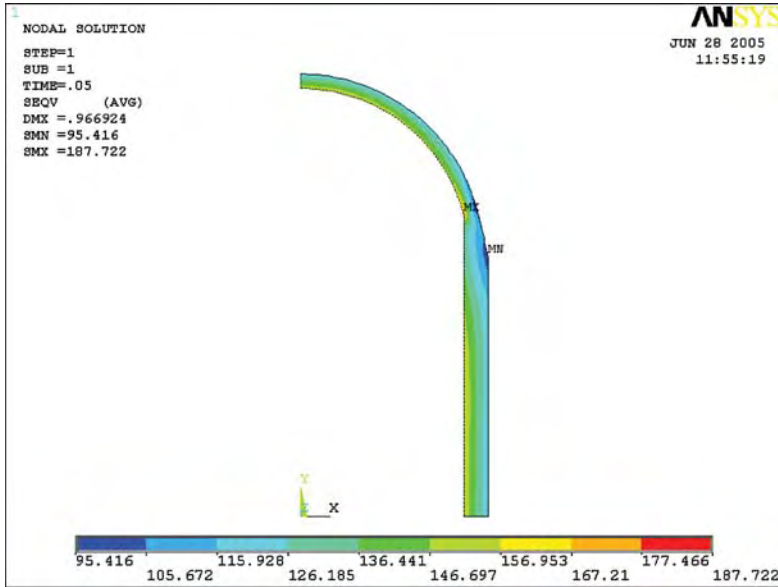


Figure E.4.2-3: Maximum equivalent stress distribution. Linear-elastic model: detail geometry 1.

weld joint direction in the butt weld region,<sup>3</sup> are listed in Table E.4.2-1, together with the corresponding results of the other geometry variants. This table is situated on page 180.

*Note:* The value 131.5 MPa of the maximum principal stress at the lower end of the model’s cylindrical shell inner surface agrees well with the corresponding theoretical value for the (closed) infinitely long cylindrical shell

$$\sigma_c(D) = p[(r_o/r_i)^2 + 1]/[(r_o/r_i)^2] = 131.8 \text{ MPa}$$

The value 129.0 MPa of the maximum Mises equivalent stress at the same point also agrees well with the corresponding theoretical value of the closed, infinitely long cylindrical shell

$$\sigma_{eqM}(D) = MEQ(\sigma_c, \sigma_t, \sigma_r) = 130.1 \text{ MPa}$$

with

$$\sigma_t = p/[(r_o/r_i)^2 - 1] = 56.7 \text{ MPa}, \quad \sigma_r = -p = -18.4 \text{ MPa}.$$

<sup>3</sup> The welded region, i.e. the region relevantly influenced by the welding, is assumed to extend 1/10 of the weld joint thickness, i.e. 11 mm, beyond the weld preparation surfaces.

Table E.4.2-1: Listing of results

Model	$P_{\text{GPD max}}$ (MPa)	$\sigma_{\text{eq,max}}$ (MPa)	$\sigma_{\perp,\text{max}}$ (MPa)
Detail geometry 1	18.42	156.4 (156.4) in A	158.6 (158.6) in A
Detail geometry 2	18.46	147.9 (147.5) in A	144.2 (143.8) in A
Detail geometry 3	18.31	164.4 (165.4) in A	171.5 (172.5) in A
Detail geometry 4	18.37	151.2 (151.6) in A	148.2 (148.7) in A
Detail geometry 5	18.45	133.2 (133.0) in B	129.6 (129.4) in C
Detail geometry 6	18.02	186.8 (190.9) in A	204.1 (208.6) in A

$P_{\text{GPD max}}$ , maximum permissible pressure according to the GPD-DC.

$\sigma_{\text{eq,max}}$ , maximum Mises' equivalent stress corresponding to  $P_{\text{GPD max}}$ .

$\sigma_{\perp,\text{max}}$ , maximum principal stress normal to direction of butt weld joint in butt welded region corresponding to  $P_{\text{GPD max}}$ .

The maximum principal stress outside of the butt weld region is for all geometries on the inside of the cylindrical shell, at the clad surface, at the model's lower end, with principal stress direction circumferentially. This value is the same for all geometries, and is equal to 131.5 MPa for  $P_{\text{GPD max}}$  of detail geometry 1. The maximum Mises equivalent stress there is also the same for all geometries, and is given by 129.0 MPa. The values in parentheses are for  $P_{\text{GPD max}}$  of detail geometry 1.

**Detail geometry 2:** The geometry is similar to the one above, but with a (permissible) offset of middle surfaces (of 8 mm) due to smaller hemisphere radii (see Fig. E.4.2-4). The distribution of the Mises equivalent stress for the largest pressure that fulfills the requirements of the GPD-DC is shown in Fig. E.4.2-5 and the corresponding distribution for the same pressure but a linear-elastic constitutive law is shown in Fig. E.4.2-6.

Maximum permissible pressure, and the results obtained with a linear-elastic model for the maximum equivalent stress in the region outside of the butt welding affected zone and for the maximum principal stress normal to the weld joint direction in the butt welded region<sup>3</sup> are listed in Table E.4.2-1.

**Detail geometry 3:** Like the preceding example, this example is similar to the one for detail geometry 1, but now with an (permissible) offset (of 8 mm) due to a larger hemisphere radii (see Fig. E.4.2-7). The results are shown in Figs. E.4.2-8 and E.4.2-9, and listed in Table E.4.2-1.

**Detail geometry 4:** This example is similar to the one with detail geometry 1, but with the shortest permissible taper of 1:4, on inside and outside of the cylindrical shell (see Fig. E.4.2-10). The results are shown in Figs. E.4.2-11 and E.4.2-12, and listed in Table E.4.2-1.

**Detail geometry 5:** This example is similar to the one of detail geometry 2, but with a larger offset of middle surfaces of 30 mm (see Fig. E.4.2-13), an offset not permissible according to the DBF requirements. Results are shown in Figs. E.4.2-14 and E.4.2-15, and listed in Table E.4.2-1.

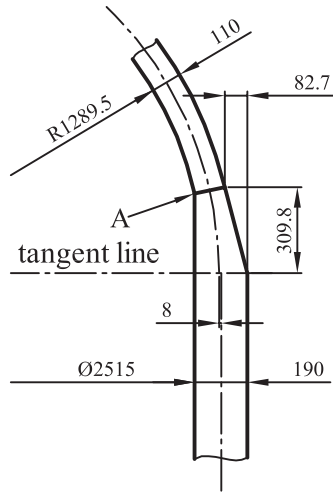


Figure E.4.2-4: Detail geometry 2.

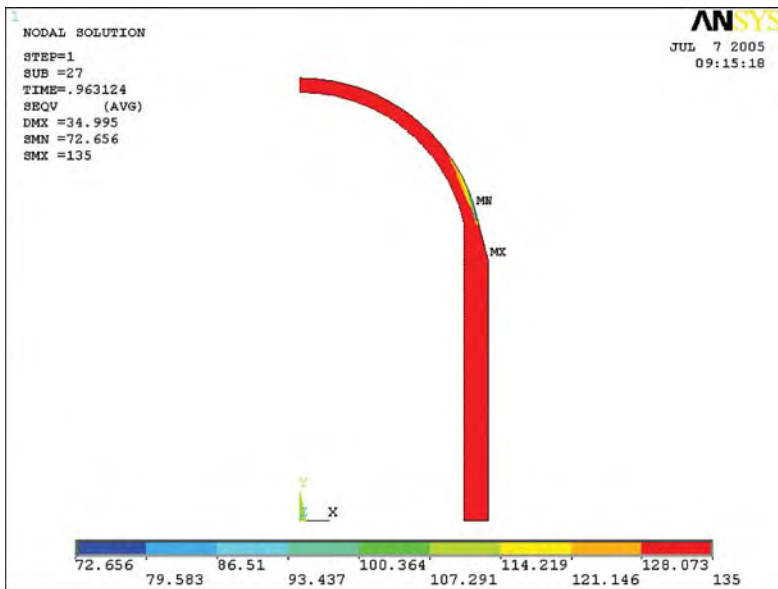


Figure E.4.2-5: Maximum equivalent stress distribution. GPD-DC design model: detail geometry 2.

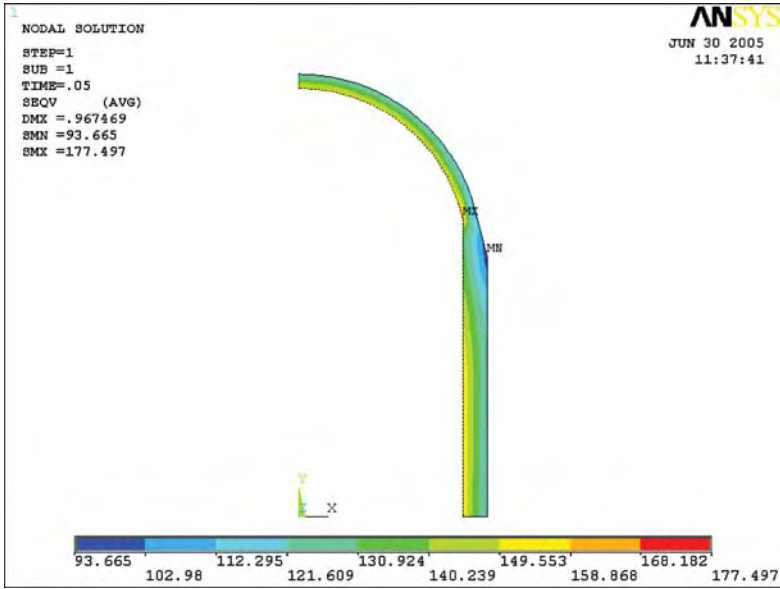


Figure E.4.2-6: Maximum equivalent stress distribution. Linear-elastic model: detail geometry 2.

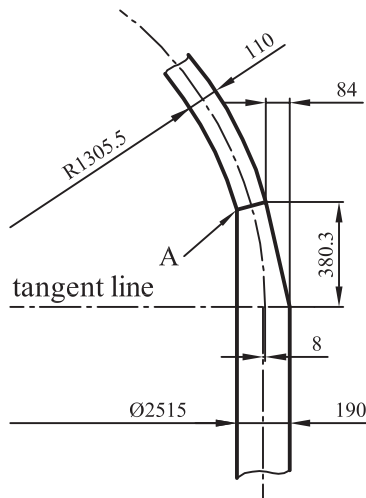


Figure E.4.2-7: Detail geometry 3.

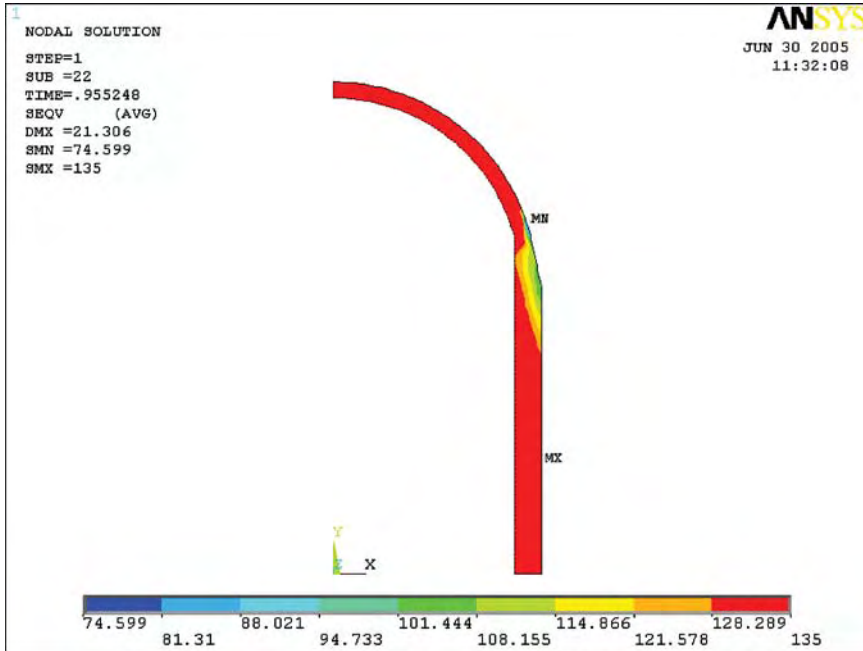


Figure E.4.2-8: Maximum equivalent stress distribution. GPD-DC design model: detail geometry 3.

**Detail geometry 6:** This example is similar to the one with detail geometry 3, but with a larger offset of middle surfaces of 30 mm (see Fig. E.4.2-16), an offset not permissible according to the DBF requirements. Results are shown in Figs. E.4.2-17 and E.4.2-18, and listed in Table E.4.2-1, page 180.

**Conclusions:**

The results are quite interesting and instructive:

- In none of the detail geometries is the transition critical with regard to gross plastic deformation, not even in the cases with quite large offset of middle surfaces, and, especially surprising, also not in the case with taper of the cylindrical shell extending below the tangent line – the additional material in the transitional part of the hemisphere compensates the removed material in the transitional part of the cylinder, and the effect of the bending moment is small enough and decays rapidly enough.



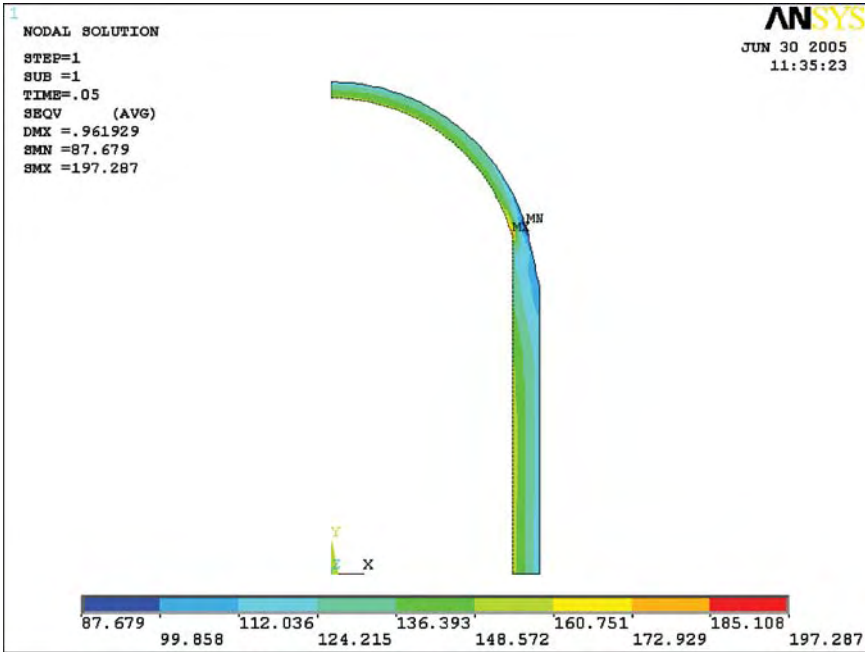


Figure E.4.2-9: Maximum equivalent stress distribution: Linear-elastic model: detail geometry 3.

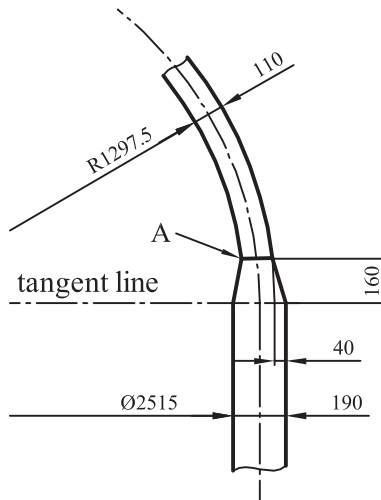


Figure E.4.2-10: Detail geometry 4.

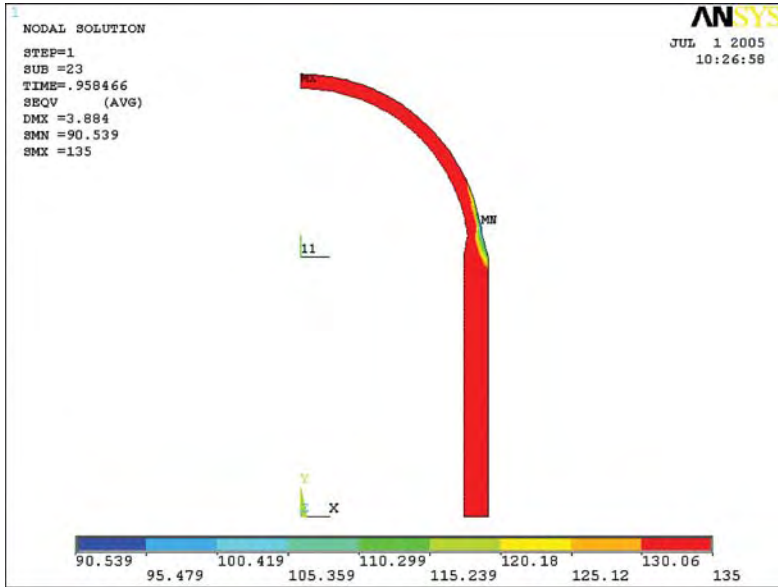


Figure E.4.2-11: Maximum equivalent stress. GPD-DC model: detail geometry 4.

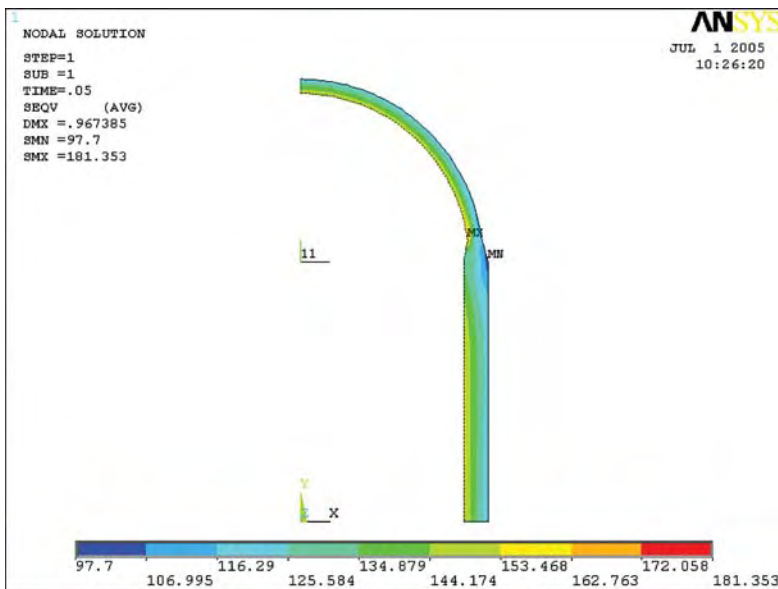


Figure E.4.2-12: Maximum equivalent stress. Linear-elastic model: detail geometry 4.

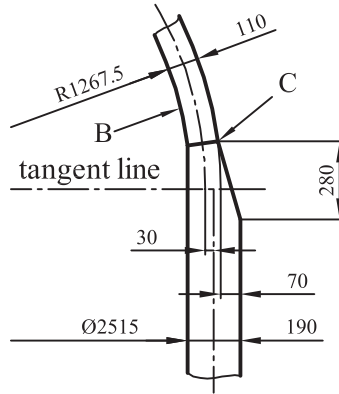


Figure E.4.2-13: Detail geometry 5.

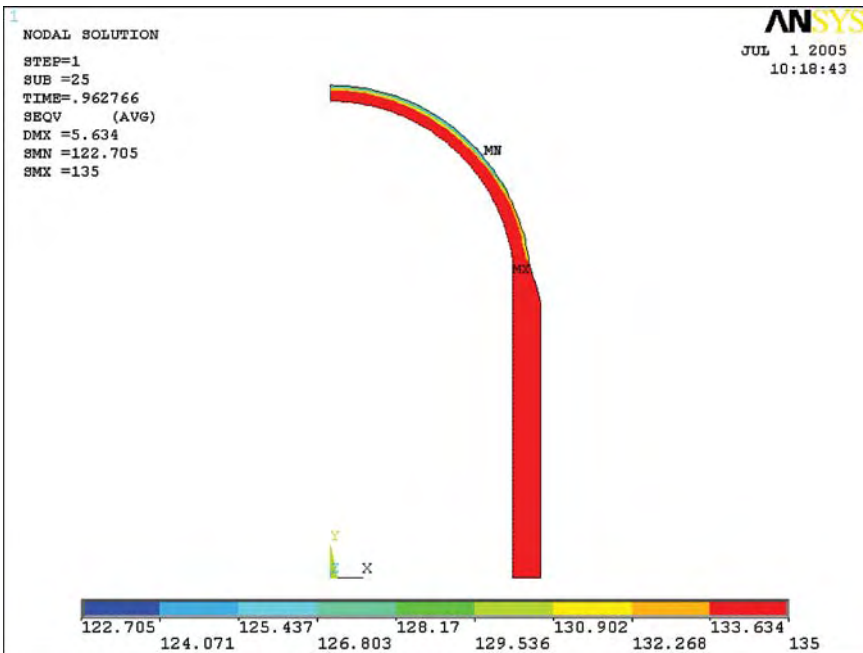


Figure E.4.2-14: Maximum equivalent stress. GPD-DC model: detail geometry 5.

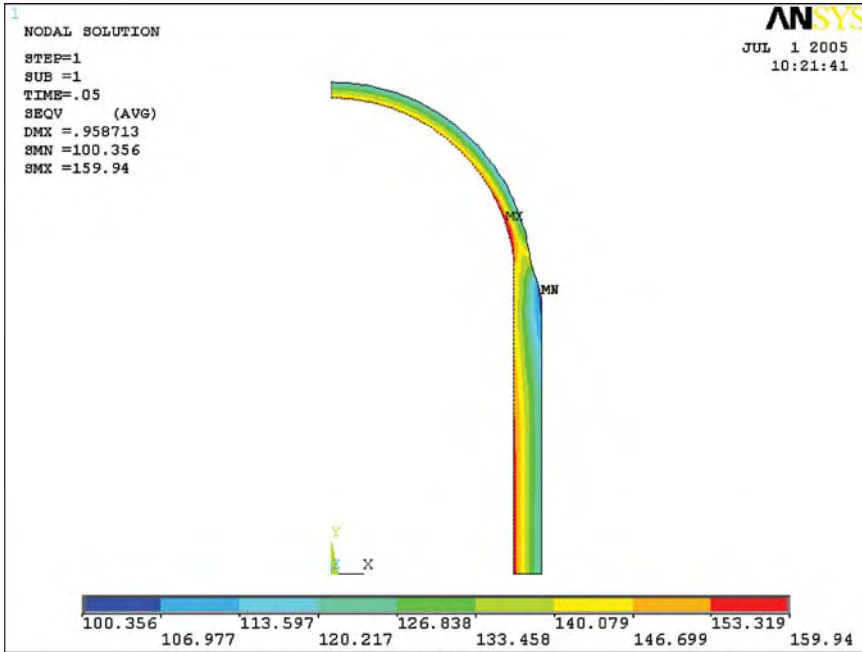


Figure E.4.2-15: Maximum equivalent stress. Linear-elastic model: detail geometry 5.

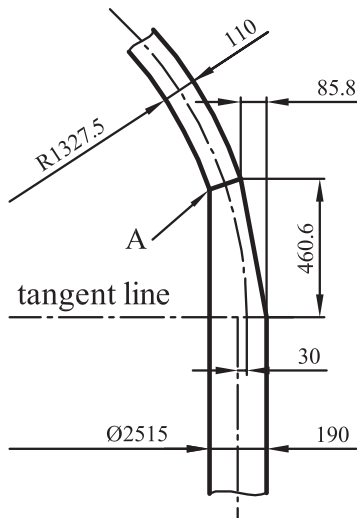


Figure E.4.2-16: Detail geometry 6.

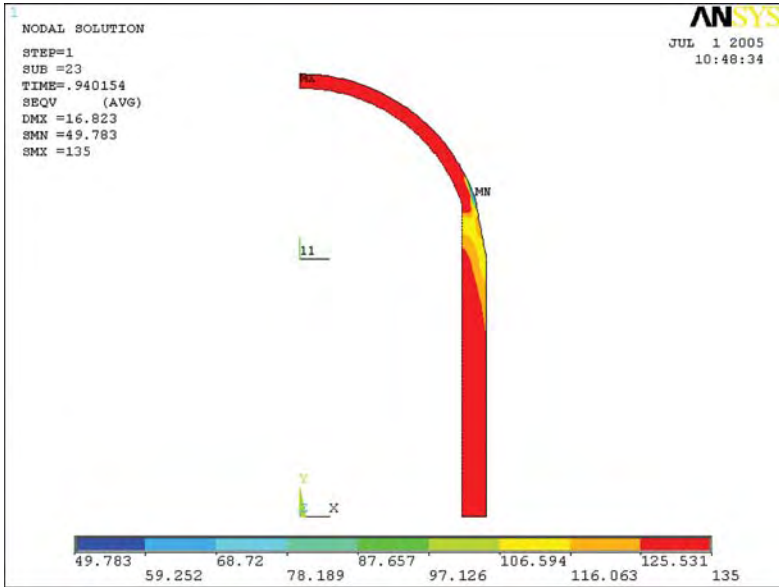


Figure E.4.2-17: Maximum equivalent stress. GPD-DC model: detail geometry 6.

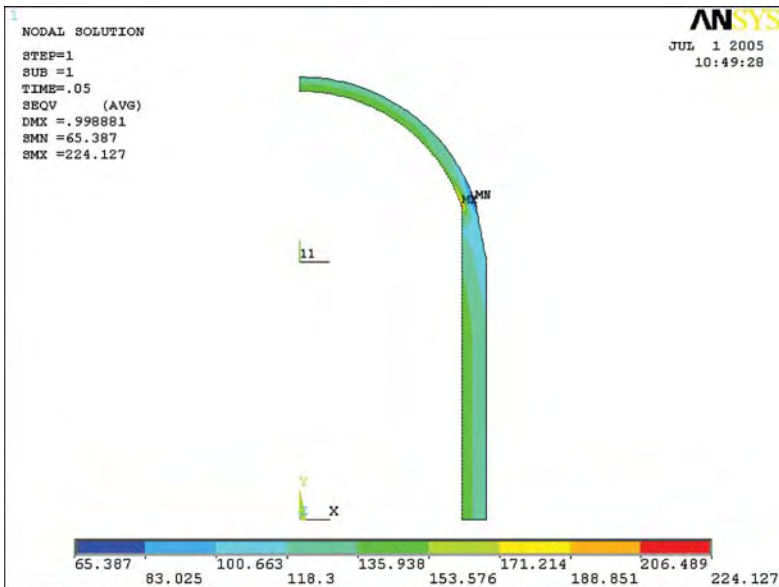


Figure E.4.2-18: Maximum equivalent stress. Linear-elastic model: detail geometry 6.

- The differences in the maximum permissible pressures (according to the GPD-DC) of the various geometries are quite small.
- The thicknesses of the two shells are almost optimal with regard to GPD-DC models with Mises' yield condition and reduced yield stress (see also the comparison with the theoretical results with limit analysis models given above). As a result, the critical region is either the apex of the spherical shell or the lower end of the cylindrical one, depending on the radius of the spherical shell, which differs for different geometries due to the middle surface offset:
- The critical region is the apex for detail geometries 1, 3, 4, 6, in which the mean radius of the spherical shell is equal to or larger than the one of the cylindrical shell. Models with Tresca's yield condition and non-reduced yield stress shift the critical region to the lower end of the cylindrical shell, but the change in the permissible pressure is minimal, the maximum permissible pressure being limited by that of the cylindrical shell, given by the result for detail geometries 2 and 5.
- The critical region is the lower end of the cylindrical shell for the other detail geometries, in which the mean radius of the spherical shell is smaller than the one for the cylindrical shell.
- For all geometries but one is the strain limitation not governing – only for detail geometry 6 is the strain at the apex limiting the maximum permissible pressure according to the GPD-DC.
- In all cases except one is the line of the maximum equivalent stress identical to that of the maximum principal stress normal to the weld joint direction, and is given by the intersection of the inner surfaces of the spherical and the cylindrical shell. For detail geometry 5 is the line of the maximum principal stress normal to the weld joint region given by the intersection of the outer surfaces of the spherical shell and the tapered section, and the line of the maximum equivalent stress is above the intersection of the inner surfaces of the spherical and the cylindrical shell, i.e. in the unwelded region and well above the welded region.
- With regard to fatigue in the butt welded region, governed by the maximum principal stress normal to the weld joint direction, detail geometry 5 is the best and detail geometry 6 is the worst, with differences mainly due to the different sphere radii (see also Example E.7.1).

### ***E.4.3: GPD-DC of an air cooler header***

This example deals with the air cooler header shown in Fig. E.4.3-1. For a feasibility study, a GPD-DC is to be performed on the part model shown in

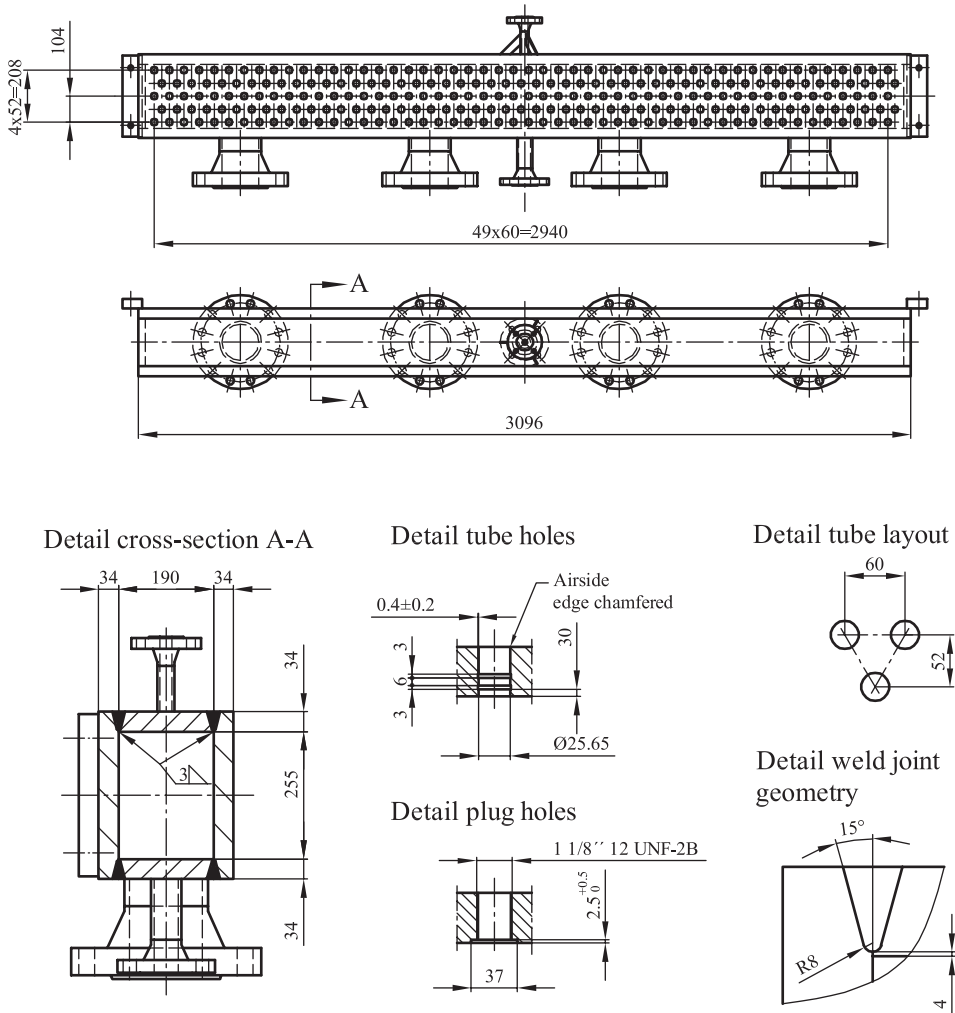


Figure E.4.3-1: Geometry of header.

Fig. E.4.3-2, a slice of the middle portion of the header, for the basic normal operating load case with maximum allowable pressure and maximum allowable temperature. In the model the effects of the finite length of the header, i.e. the effect of the front plates and also the effect of the nozzles in the side-plates are neglected. Given that the nozzles can be made strong enough such that they are not governing the design check, the chosen model is conservative. The finite element model is shown in Fig. E.4.3-3.

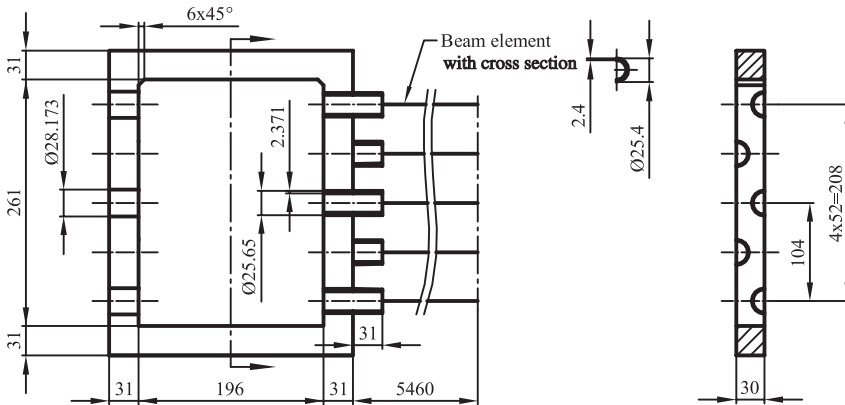


Figure E.4.3-2: Geometry of investigated model.

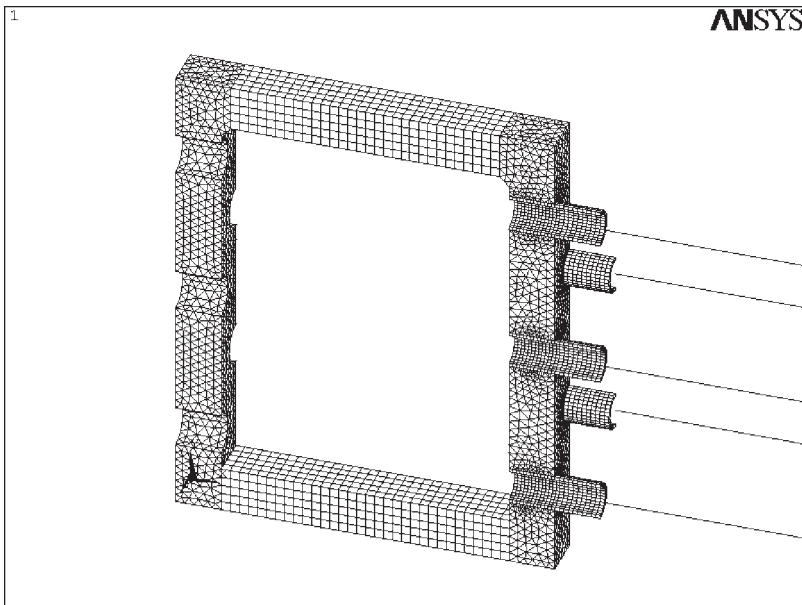


Figure E.4.3-3: Finite element model.

### Excerpt from the design data specification

- Maximum allowable pressure:  $PS = 72 \text{ bar} = 7.2 \text{ MPa}$
- Maximum allowable temperature:  $TS = 120^\circ\text{C}$
- Number of operational cycles:  $N = 1000 \text{ cycles}$



- Materials:
  - Plates: EN 10028-3 P355NL1
  - Tubes: EN 10216-4 P215
  - Nozzles: EN 10222-4 P355QH1
- Thickness allowances:
  - Corrosion/erosion allowance:  $c = 3$  mm, with the exception of the heat exchanger tubes, for which  $c = 0$  applies
  - Tolerances allowance:  $\delta_e = 0$

**(Considered) Design Load Case**

**NOLC 1.1:**

- (Upper) characteristic value of pressure:  $p_c^+ = 7.2$  MPa
- (Upper) characteristic value of temperature:  $t_c^+ = 120^\circ\text{C}$

The corresponding load case specification, with derivation details, is given in Table E.4.3-1.

In this load case the design check is performed strictly in accordance with the requirements in Annex B of EN 13445-3, i.e. thermal stresses are not included in the investigation.

The relevant material parameters, with derivation details, are given in Table E.4.3-2. The reference temperature for the determination of the modulus of elasticity is  $t_{rE}$  and for the determination of the strength parameter is  $t_{rRM}$ .

The input listing of the FEM run is given in Annex L.4.3. In the model, the displacement  $u_z$  of all points of the front face, given by  $z = 30$  mm, are restrained to zero, and all points of the end face given by  $z = -30$  mm, are coupled to prevent warping of this face. The pressure forces on the plugs are incorporated to the model via evenly distributed shear stresses on the bore surfaces, neglecting the influence of the threads. Within the tubesheet thickness the corresponding tubes and the tubesheet points are connected – an acceptable model of the expanded tube connection with two grooves if the required friction forces without support by the grooves are admissible. Only part of the tubes are modelled three-dimensionally; the rest is replaced by an equivalent beam, with cross-sectional area equal to one-half of the full tube cross-sectional area and bending rigidity equal to one-half of

Table E.4.3-1: Load case specification

NOLC 1.1			
Action	Characteristic value	Partial safety factor	Design value
Pressure $p_d$ (MPa)	7.2	1.2	8.64
Temperature $t_d$ ( $^\circ\text{C}$ )	120	1.0	120

Table E.4.3-2: Material parameters

NOLC 1.1											
Part	Material group	$t_{rE}$ (°C)	$t_{rRM}$ (°C)	$E$ (GPa)	$\mu_{el}$ –	$R_m$ (MPa)	$R_{p0.2/t_{rRM}}$ (MPa)	$(5)/(4)$ –	$\gamma_R$ –	$RM_d$ (MPa)	$RM_{d,red}$ (MPa)
1, 2, 3	1.2	95	120	206	0.3	490	296	0.60	1.25	236.8	205.1
4	1.1	95	120	206	0.3	360	182	0.50	1.25	145.6	126.1

For simplicity, a maximum metal temperature equal to the upper characteristic value of temperature  $t_c^+ = 120^\circ\text{C}$  is chosen, and, therefore,  $t_{rRM} = 120^\circ\text{C}$ .

For the determination of the modulus of elasticity the reference temperature  $t_{rE}$ , with  $t_{rE} = 0.75t_{\max} + 0.25t_{\min} = 0.75t_c^+ + 5$ , has been chosen.

Values for  $E$  and  $\mu_{el}$  are from EN 13445-3, Annex O; values for material strength parameters are from the referenced material standards, EN 10222-2 and EN 10028-2, respectively.

In the design models Mises' yield condition is used together with the reduced design value of the yield stress  $RM_{d,red} = RM_d \sqrt{3}/2$ .

For the tube material hot tensile properties are not given in the material standard. An interpolated value has been chosen, with interpolation between the values for EN 10216-2 P195GH and P235GH. Verification by hot tensile test at  $120^\circ\text{C}$  (or higher) is required.

the ones of the full tube. Points of these beams' left ends act as master points to which all of the (half) tube end points are connected. The displacement of all points of these beams' right ends, corresponding to the middle of the whole tube lengths, is constrained to zero in the tubes' and the header's axial directions, so are the rotations, taking into account the symmetry of structure and actions.

The results of the FEM run show that the model can carry the design pressure: Fig. E.4.3-4 shows on the left Mises' equivalent stress distribution in the four plates for the design value of pressure, and on the right the compatibility ratio distribution, i.e. the ratio of the equivalent stress and the corresponding yield stress. Fig. E.4.3-5 shows the corresponding distributions for the (half) tubes.

In this example the plots of the compatibility ratios are quite instructive, because of the different yield stresses of the two different materials. Fig. E.4.3-6 shows the development of the maximum absolute value of principal total strains for proportional increase of pressure from zero to its design value, given by the action ratio 1.0, as per FEM output for the chosen mesh. Total strains may always be used instead of structural ones, which are required in the strain-limiting requirement of the GPD-DC's principle, but with this model usage of total strains requires additional investigation. For example, the corner at the root of the one-sided weld joining the plug-plate and the lower side-plate creates a stress/strain singularity in a linear-elastic model – the finer the mesh the larger total stresses and total strains. Fig. E.4.3-7 shows the transition of the principal strain in the direction of the tubes

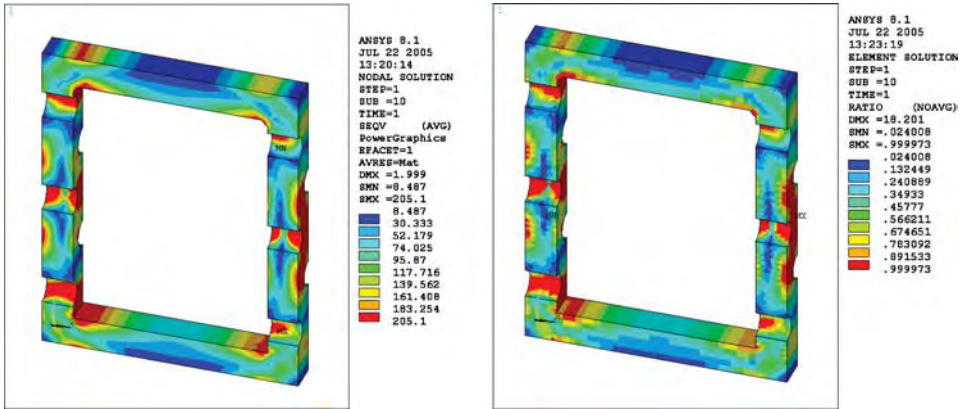


Figure E.4.3-4: Stress distribution in plates. Left, Mises' equivalent stress; right, compatibility ratio.

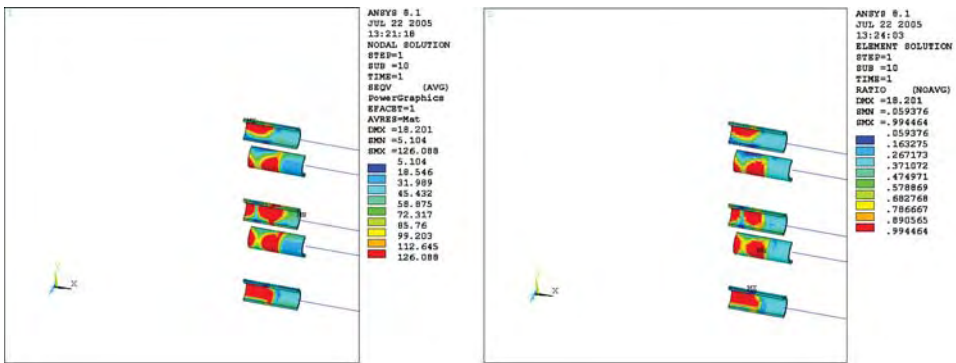


Figure E.4.3-5: Stress distribution in tubes. Left, Mises' equivalent stress; right, compatibility ratio.

in the inner surface of the lower plate for the linear-elastic ideal-plastic model, for which there is no singularity but a strong strain concentration. The dashed lines are transitions along three lines of the principal total strain in the direction of the tubes. The full lines are extrapolations into the corner point, and the pivot points  $P_1$ ,  $P_2$ , and  $P_3$ , are shown as circles on the abscissa. This figure shows that the structural strains are much smaller than the total ones, and, therefore, usage of total strains is, as far as this corner line is concerned, very conservative. Total strains being smaller than required by the principle, usage of total strains is still a suitable approach. Especially so, since the strain critical point is on the inside of a

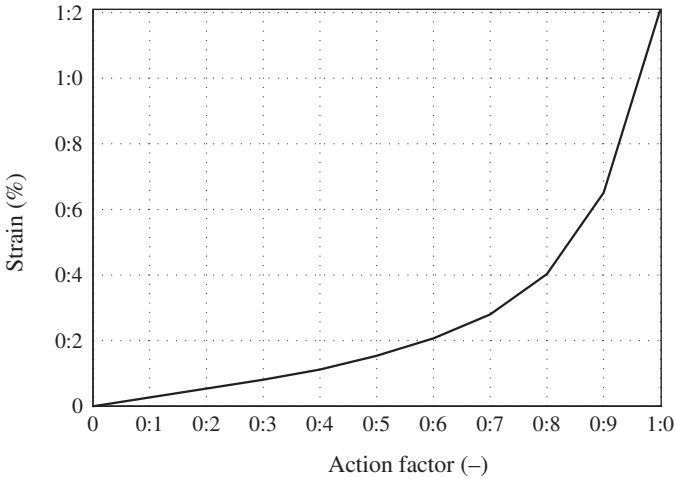


Figure E.4.3-6: Maximum absolute value of principal strains vs. action ratio.

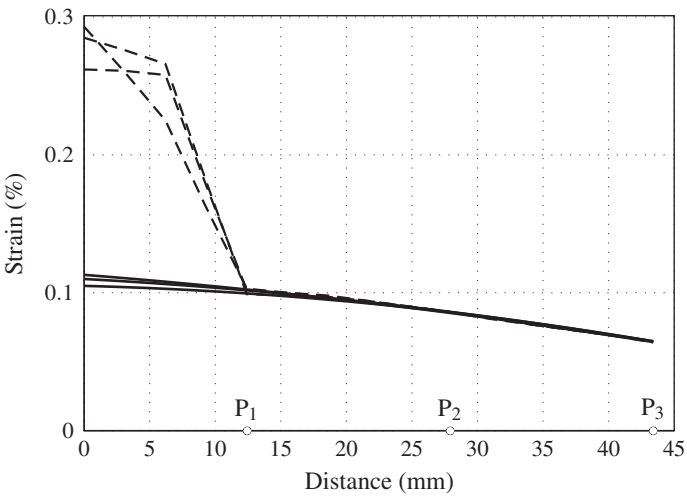


Figure E.4.3-7: Comparison of FEM output with extrapolations into the corner.

plug hole, where no singularity exists. The maximum absolute value of the principal total strains is less than 5%, and, hence, the GPD-DC requirements are for this NOLC fulfilled.

Therefore:

**NOLC 1.1: O.K.**

Remark: In this example, the possibility incorporated in the standard of using the large stress re-distributions in DBA-DR during plastic flow leads to great economic advantages. It allows much smaller plate thicknesses than the DBF approach, resulting in much smaller weld thicknesses, i.e. less welding, and, furthermore, post weld heat treatment is not required.

**Plausibility Check:**

With regard to gross plastic deformation, the weakest part of the header is obviously the plug wall, with two likely failure modes:

- failure of the plug wall in three horizontal planes with one plane through the middle row of plug holes and two planes through the outermost plug holes, and
- failure of the plug wall in three horizontal planes with one plane through the middle row of plug holes and two in extension of the inner surfaces of top and bottom plate of the header.

In the following, it is shown that the limit analysis model given by a cross-section slice of the header of 60 mm depth, with front and end plane through the centre of three plug holes, can carry the design value of pressure, given by 8.64 MPa.

For both failure modes the mean normal stress in the three failure planes is statically determined, and given by

$$\bar{\sigma}_{n,n} = (8.64 \cdot 60 \cdot 196/2)/(31(60-28.173)) = 51.5 \text{ MPa}$$

for the plane through the centre of the plug holes, and by

$$\bar{\sigma}_{n,c} = (8.64 \cdot 60 \cdot 196/2)/(31 \cdot 60) = 27.3 \text{ MPa}$$

for planes not containing plug holes.

The resultant bending moments, resulting from the normal stresses in the three failure planes in the plug wall side of the slice are statically undetermined, a moment distribution symmetrical to the central failure plane is obviously optimal, a moment distribution given by

$$M_{01} + q \cdot x^2/2$$

with  $x$  the distance from the central failure plane,  $M_o$  the resulting moment at  $x \equiv 0$ , and

$$q = 8.64 \cdot 60 = 518.4 \text{ N mm}^{-1}.$$

For the first failure mode the optimal distribution is shown in Fig. E.4.3-8 on the left, with equal moments in all three failure planes given by

$$M_{01} = q \cdot 104^2 / 4 = 1.402 \cdot 10^6 \text{ N mm.}$$

In all three failure planes this mean moment is accompanied by equal resulting normal forces, given by

$$N_o = \bar{\sigma}_{n,n} \cdot 31 \cdot (60 - 28.173) = 5.080 \cdot 10^4 \text{ N.}$$

The corresponding standardized values, standardized with respect to fully plastic moment and fully plastic normal force, are then

$$\bar{M}_{01} = M_{01} / (RM_{d,red} \cdot (60 - 28.173) \cdot 31^2 / 4) = 0.894,$$

$$\bar{N}_o = N_o / (RM_{d,red} \cdot (60 - 28.173) \cdot 31) = 0.251,$$

with  $RM_{d,red} = 205.1 \text{ MPa}$ .

The combination rule for bending moment and normal force is known and renders

$$\bar{M}_{01} + \bar{N}_o^2 = 0.957,$$

less than 1.0, but already close to the limit (see Fig. E.4.3-6).

To illustrate usage and accuracy of the Ivanov function (see Section 2.4.3):

$$\bar{Q}_n = \bar{N}_o^2, \quad \bar{Q}_m = \bar{M}_{01}^2, \quad \bar{Q}_{mn} = \bar{M}_{01} \cdot \bar{N}_o$$

and, thus,

$$I_V^2 = 0.9206 \rightarrow I_V = 0.960,$$

which is very close to the exact combination value obtained above.

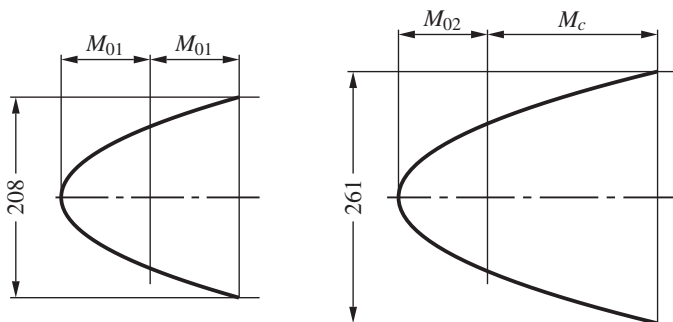


Figure E.4.3-8: Moment distributions.

For the assumed uniaxial stress distribution there is no difference in the results due to the underlying yield condition, and, therefore, the non-reduced yield stress could have been used. The reduced yield stress has been used to allow for direct comparison with the FEM results, for which Mises' yield condition with reduced yield stress has been used.

For the second failure mode the choice of the optimal resultant moment distribution is not so obvious. A statically admissible distribution is shown in Fig. E.4.3-8 on the right, with

$$M_c = q \cdot 261^2/8 - M_{02},$$

and the statically undetermined moment in the middle  $M_{02}$ .

A reasonable choice for  $M_{02}$  is the maximum moment compatible with the yield condition

$$M_{02} = (1 - \bar{N}_0^2) \cdot RM_{d,red} \cdot (60 - 28.173) \cdot 31^2/4 = 1.5405 \cdot 10^6 \text{ N mm.}$$

There follows that  $M_c = 2.874 \cdot 10^6 \text{ N mm}$  and  $\bar{M}_c = M_c / (RM_{d,red} \cdot 60 \cdot 31^2/4) = 0.972$ .

This moment is to be combined with the resultant normal force

$$N_c = q \cdot 196/2 = 5.0803 \cdot 10^4 \text{ N,}$$

with resulting standardized value  $\bar{N}_c = 0.133$ .

Contrary to the stresses in failure planes through the centre of plug holes, stresses which can be considered to be uniaxial, in this failure plane longitudinal stresses do exist and should be incorporated in the investigation.

The resulting normal force, in longitudinal direction of the header, can be neglected and the resulting bending moment can be approximated by  $0.5 \cdot M_c = 1.437 \cdot 10^6 \text{ N mm}$ , with standardized value 0.486.

Then

$$\bar{Q}_n = \bar{N}_c^2, \bar{Q}_m = \bar{M}_c^2 + (0.5 \bar{M}_c)^2 - 0.5 \bar{M}_c^2 = 0.7086,$$

$$\bar{Q}_{mn} = \bar{M}_c \cdot \bar{N}_c - 0.5 \cdot \bar{M}_c \cdot \bar{N}_c / 2 = 0.0970,$$

and finally

$$I_V^2 = 0.7372 \rightarrow I_V = 0.859.$$

The results show that the first considered failure plane is the critical one.

**Recommended normal operating load case:** This load case investigation is supplementary to the preceding one – NOLC 1.1. It deals with the same actions as NOLC 1.1, but here, in this (recommended) load case, thermal stresses are included, as recommended in Section 4.4. As a rough approximation of the rather complicated heat transfer boundary conditions, uniform surface temperatures have been prescribed, based on simple heat transfer calculations for simple objects:

- at all tube side model surfaces 100°C,
- at all tubes outer model surfaces 70°C, and
- at all header outer model surfaces 40°C.

The resulting stationary temperature distribution, determined with the coefficient of thermal conductivity given in Table E.4.3-3, is shown in Fig. E.4.3-9.

#### (Considered) Design Load Case

**NOLC 1.2:** The load case specification is the same as the one for NOLC 1.1, given in Table E.4.3-1. To allow for an easy comparison of the investigations with and without thermal stresses, the same reference temperatures for determining the material parameters as in NOLC 1.1, given in Table E.4.3-2, are used here as well, i.e. without taking advantage of the possibility of using values based on the calculated metal temperature distribution. For the determination of the stationary temperature distribution and the corresponding stationary thermal stresses, temperature-dependent values of the coefficient of thermal conductivity and the coefficient of linear thermal expansion have been used, values which were determined, in the software, by linear interpolation between the pivot point values given in Table E.4.3-3.

Table E.4.3-3: Material parameters

NOLC 1.2							
Part	Material group	$t_{ref1}$ (°C)	$\lambda$ (W/(mK))	$\beta$ (1/K)	$t_{ref2}$ (°C)	$\lambda$ (W/(mK))	$\beta$ (1/K)
1, 2, 3	1.2	20	49.5	$11.30 \cdot 10^{-6}$	120	47.7	$12.04 \cdot 10^{-6}$
4	1.1	20	55.2	$11.30 \cdot 10^{-6}$	120	52.6	$12.04 \cdot 10^{-6}$

Values for  $t_{rE}$ ,  $E$ ,  $\mu_{el}$ , and  $RM_{d,red}$  are the same as given in Table E.4.3-2.

$\lambda$  is the coefficient of thermal conductivity;  $\beta$  the coefficient of linear thermal expansion and  $t_{ref1}$ ,  $t_{ref2}$  the pivot temperatures for the linear interpolation of  $\lambda$ ,  $\beta$ , and  $RM_{d,red}$ .



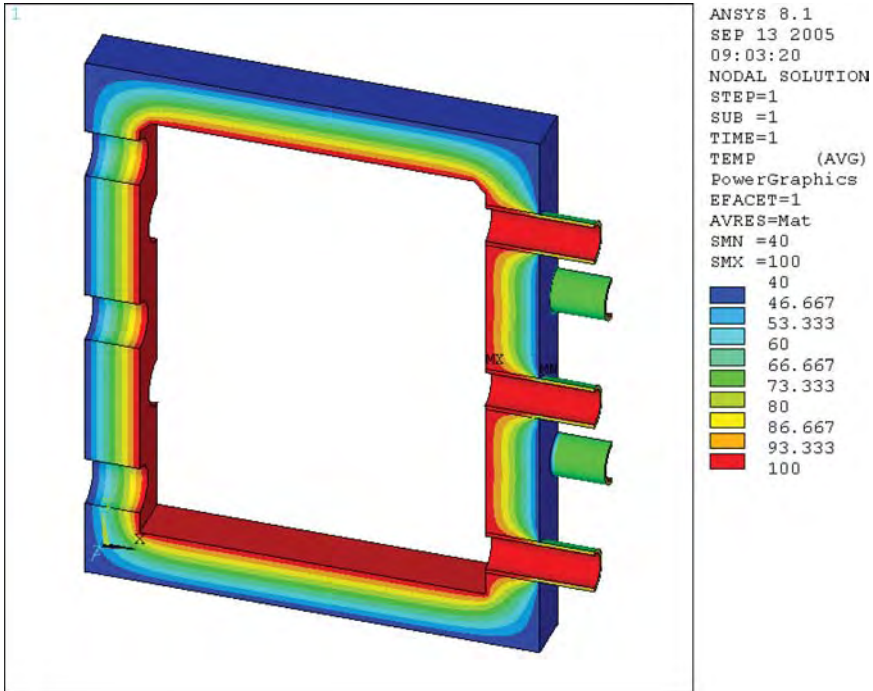


Figure E.4.3-9: Temperature distribution.

Fig. E.4.3-10 shows the development of the maximum absolute value of principal total strains for two different cases:

- upper full line for proportional increase of temperature from zero to the calculated stationary distribution, for values of the time parameter from  $-1.0$  to  $0$ , followed by proportional increase of pressure from zero to its design value, for values of the action ratio from  $0$  to  $1.0$ , and
- lower full line for synchronous increase of pressure from zero to its design value and thermal stresses from zero to the stationary distribution corresponding to the calculated stationary temperature distribution.

Also included in this figure, as dashed line, is the curve for proportional increase of pressure without thermal stresses, the curve already shown in Fig. E.4.3-5.

The model can carry the design value of pressure and the maximum absolute value of the principal total strains is less than  $5\%$ , and, therefore, the GPD-DC requirements for this NOLC that includes the thermal stresses are also fulfilled.

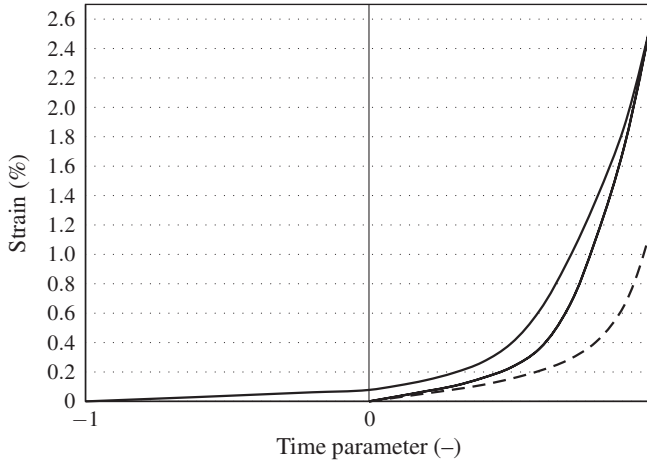


Figure E.4.3-10: Maximum absolute value of principal total strains vs. time parameter.

Therefore:

**NOLC 1.2: O.K.**

*Note:* Comparison of Fig. E.4.3-10 with Fig. E.4.3-6 shows clearly the influence of thermal stresses on the principal total strains in this GPD-DC:

- For monotonic increase of actions the result is near the strain limit practically independent of the action path, and
- thermal stress influence on strains is noticeable, the effect on the limit action may be relevant, and the recommended design check may be design-decisive.

#### ***E.4.4: GPD-DC of a nozzle in hemispherical end***

In this example the hemispherical end with nozzle shown in Fig. 4.4-1 is investigated. The dimensions without parentheses are those of the model and those in parentheses are nominal ones as per design drawing, if different from the ones of the design model. The lines separating different parts are demarcation lines in the model and not weld edge preparation lines or weld joint lines.

#### **Excerpt from the design data specification**

- Maximum allowable pressure:  $PS = 80 \text{ bar} = 8.0 \text{ MPa}$
- Maximum allowable temperature:  $TS = 295^\circ\text{C}$
- Insulation thickness: 150 mm

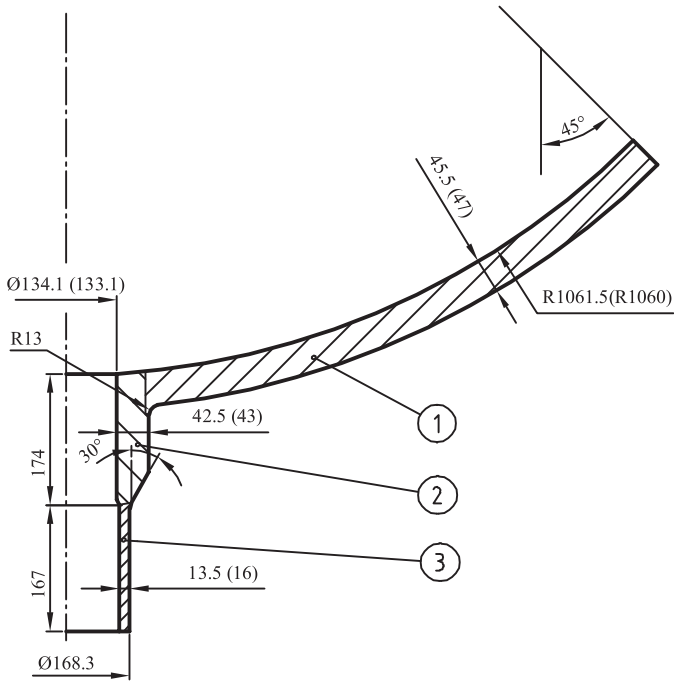


Figure E.4.4-1: Nozzle geometry.

Start-up and shutdown slow enough such that non-stationary thermal stresses are never larger than stationary ones.

- Injection temperature of cold medium:  $TIS = 60^\circ\text{C}$   
 Injection temperature initially controlled, with transition slow enough such that non-stationary thermal stresses are never larger than stationary ones.<sup>4</sup>
- Operational cycles:  
 One operational design cycle encompasses:
  - Start-up with pressure and temperature synchronously.
  - 500 cold medium injection at constant pressure and vessel content temperature, long enough such that stationary state is reached.
  - Shutdown with pressure and temperature synchronously

<sup>4</sup> Non-stationary calculations of temperature and elastic thermal stress fields have shown that even a short ramp of the medium temperature transition with transition time in the order of seconds suffices.

- Materials:
  - Hemispherical shell: EN 10028-2 10CrMo9-10 + NT
  - Nozzle reinforcement: EN 10216-2 11CrMo9-10 + QT
  - Nozzle: EN 10216-2 P265GH
- Thickness allowances:
  - Corrosion/erosion:  $c = 0.5$  mm
  - Tolerances:
    - Hemispherical shell:  $\delta_e = 1$  mm
    - Nozzle reinforcement:  $\delta_e = 0$  mm (surfaces machined)
    - Nozzle: Acc. to EN 10216-2:  $\delta_e = 0.125 \cdot 16.0 = 2.0$  mm

**(Considered) Design load cases:**

**NOLC 1.1:** Stationary state/maximum pressure/maximum temperature/no injection

- (Upper) characteristic value of pressure:  $p_c^+ = 8.0$  MPa
- (Upper) characteristic value of temperature:  $t_c^+ = 295^\circ\text{C}$

The corresponding load case specification, with derivation details, is given in Table E.4.4-1.

The relevant material parameters are given in Table E.4.4-2. The reference temperature for determining the modulus of elasticity is  $t_{rE}$  and for determining the material strength parameters is  $t_{rRM}$ .

To use the same model also in the second load case, with space-dependent temperature, the temperature-dependent design values  $RM_{d,red}$  given in Table E.4.4-3 are used as pivot points in the input listing, for the piecewise linear function  $RM_{d,red}$  of  $t_d$ , required in the FEM model.

In this example, a different approach was used in the setting up of the model geometry, using the interactive software commands. The listing of the check points is given in Annex L.4.5. A rotational-symmetric model is used, with meridional displacements at all points of the wide end boundary of the spherical shell restrained to zero. At the lower end boundary of the pipe all points are coupled to a master node to which the pressure load is applied. In this problem, with large

Table E.4.4-1: Load case specification

NOLC 1.1			
Action	Characteristic value	Partial safety factor	Design value
Pressure $p_d$ (MPa)	8.0	1.2	9.6
Temperature $t_d$ ( $^\circ\text{C}$ )	295	1.0	295

Table E.4.4-2: Material parameters

NOLC 1.1											
Part	Material group	$t_{rE}$ (°C)	$t_{rRM}$ (°C)	$E$ (GPa)	$\mu_{el}$ –	$R_m$ (MPa)	$R_{p0.2/t_{rRM}}$ (MPa)	(4)/(3) –	$\gamma_R$ –	$RM_d$ (MPa)	$RM_{d,red}$ (MPa)
				(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
										=(4)/(6)	
1	5.2	226.25	295	196.6	0.3	480	221.6	0.46	1.25	177.3	153.5
2	5.2	226.25	295	196.6	0.3	540	289.7	0.54	1.25	231.8	200.7
3	1.11	226.25	295	196.6	0.3	410	155.7	0.38	1.25	124.6	107.9

Table E.4.4-3: Pivot points of  $RM_{d,red}$  (MPa) vs.  $t_d$

Position No.	Temperature (°C)					
	20	100	150	200	250	300
1	201	173	165	161	157	153
2	246	224	216	211	205	200
3	184	157	148	133	118	107

thermal stresses, progressive plastic deformation and fatigue are the critical failure modes, and the PD-DC and the F-DC are the important design checks. In the PD-DC, Mises' yield condition and in the F-DC an (unbounded) linear-elastic constitutive law are prescribed, and, therefore and to save computing time, instead of Tresca's yield condition, required by the GPD-DC's principle, Mises' yield condition is used here, in the GPD-DC, with, of course, the reduced design value for the yield stress. Metal temperatures are constant and equal to  $t_d$ ; the pressure is increased from zero to the design value  $p_d$ . The FEM results show that the design model can carry the design pressure – Fig. E.4.4-2 shows the distribution of the compatibility ratio,<sup>5</sup> i.e. the ratio of Mises' equivalent stress and the design value of the yield stress at the design value of the pressure, and Fig. E.4.4-3 shows the

<sup>5</sup> Use of the usual plotting routines can lead to irritating results. In the postprocessing routines usually the values of relevant quantities in the nodal points are used. These values are obtained in the solution routine via extrapolation or copying from calculation values in the integration points. Extrapolation is the usually used option. In linear-elastic ideal-plastic analyses extrapolation leads to stress fields that may not everywhere be compatible with the relevant yield condition, whereas the stresses in the integration points, in which they are evaluated, are. This phenomenon is especially irritating in the load cases considered here, with different materials and high temperature and thermal stress gradients. Therefore a different plotting procedure has been used here. Values are copied from the integration points to the nodal points – ANSYS-command (ERESX,NO). This procedure leads to compatible stress fields. The evaluation of compatibility ratios has been performed at the mid-points of the elements and, therefore, the uniform color of each element corresponds to the value at its mid-point.

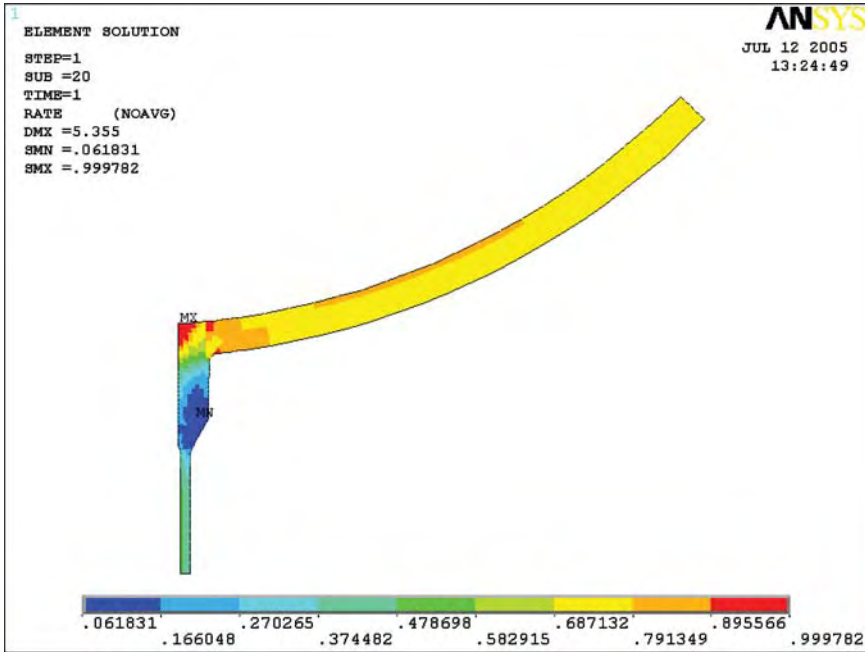


Figure E.4.4-2: Compatibility ratio distribution.

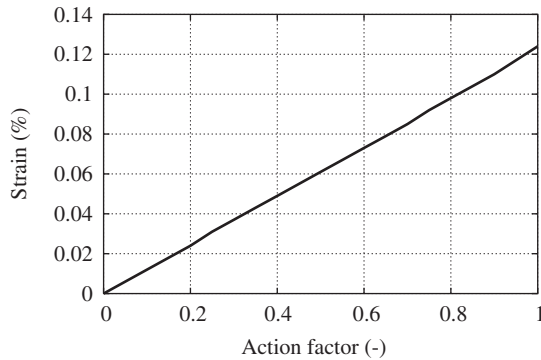


Figure E.4.4-3: Maximum absolute value of principal strains vs. action ratio.

development of the maximum absolute value of the principal total strains as a function of the pressure increase form zero to the design value. In the critical regions the model can be considered as stress-concentration-free. Fig. E.4.4-3 shows that the maximum absolute value of the principal total strains at the design pressure is less than 5% – the requirements of the GPD-DC are fulfilled.

Therefore:

**NOLC 1.2: O.K.**

**NOLC 1.2:** Stationary state/maximum pressure/maximum temperature/cold injection

- (Upper) characteristic value of pressure:  $p_c^+ = 8.0$  MPa
- (Upper) characteristic value of temperature:  $t_c^+ = 295^\circ\text{C}$   
Cold injection with characteristic value of temperature  $t_{ic-} = 60^\circ\text{C}$

The load case specification for pressure is the same as above, and the characteristic values and design values of temperature are given by the FEM results of the temperature calculation for the medium temperature  $t_c^+$  at points in the inner surface of the hemispherical shell and  $t_{ic-}$  at points in the inner surface of the nozzle. The chosen coefficients of surface heat transfer are  $h_s = 1.16$  kW/(m<sup>2</sup>·K) =  $1.16 \cdot 10^{-3}$  W mm<sup>-2</sup> K<sup>-1</sup> for the inner hemispherical shell surface,  $h_n = 10.8$  kW/(m<sup>2</sup>·K) =  $10.8 \cdot 10^{-3}$  W mm<sup>-2</sup> K<sup>-1</sup> for the inner nozzle surface, and perfect insulation at the outer surfaces. The temperature-dependent pivot values of  $RM_{d,red}$  are given again in Table E.4.4-3, and the pivot values for the material's thermal conductivity, from EN 13445-3 Annex O, in Table E.4.4-4. The modulus of elasticity and the coefficient of linear thermal expansion of the model are independent of time and space, and are given by their values for the reference temperature  $t_{rE}$ , as per Table E.4.4-2.

This load case can be considered as (another) corner of the design space for normal operating load cases. Formally, for this GPD load case a detailed investigation is not required, because thermal stresses need not be considered in GPD design checks and the temperatures in this load case are not higher than the corresponding ones of NOLC 1.1, and, therefore, the design values of the yield stresses never higher than the corresponding ones of this NOLC 1.1, i.e. the GPD-DC of NOLC 1.2 is formally encompassed by that of NOLC 1.1.

To follow the recommendation to investigate the thermal stress influences also in the GPD design checks, this NOLC 1.2 is dealt with in detail here as well.

Table E.4.4-4: Pivot points for thermal conductivity (K<sup>-1</sup>)

Part No.	Material group	Temperature (°C)		
		20	155	295
1, 2	5.2	$37.09 \cdot 10^{-6}$	$37.29 \cdot 10^{-6}$	$36.44 \cdot 10^{-6}$
3	1.1	$55.22 \cdot 10^{-6}$	$51.59 \cdot 10^{-6}$	$47.32 \cdot 10^{-6}$

To allow for comparison, three sub-cases are considered:

- NOLC 1.2a: Proportional increase of pressure, no thermal stresses, time-independent design temperature distribution, and time-independent distribution of  $RM_{d,red}$  corresponding to the stationary temperature distribution for cold injection.
- NOLC 1.2b: Increase of pressure from zero to its design value, and proportional increase of temperature and thermal stresses from ambient and zero stress state, respectively, to their design value distributions, corresponding to the stationary distributions for cold injection. To ease input work, temperature-dependent values of  $RM_{d,red}$ , with pivot points given in Table E.4.4-3, are used.
- NOLC 1.2c: Increase of pressure from zero to its design value, time-independent temperature, and thermal stress distribution equal to their design value distributions, corresponding to the stationary distributions for cold injection.

**NOLC 1.2a:** To be able to show the influence of thermal stresses on the results, the increase of pressure was not stopped at the design value, but continued to increase until the strain limit or the design carrying capacity had been reached.

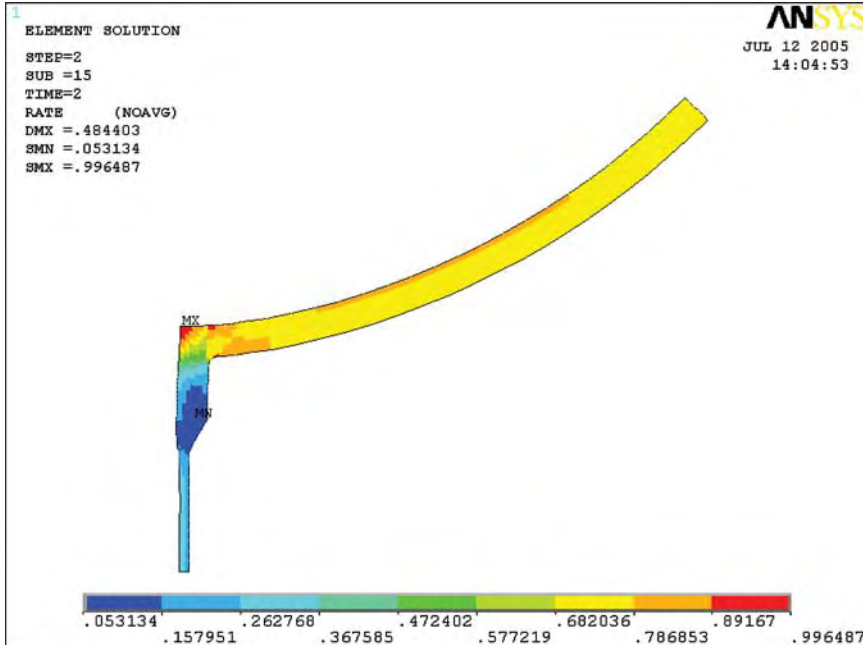


Figure E.4.4-4: Compatibility ratio distribution.



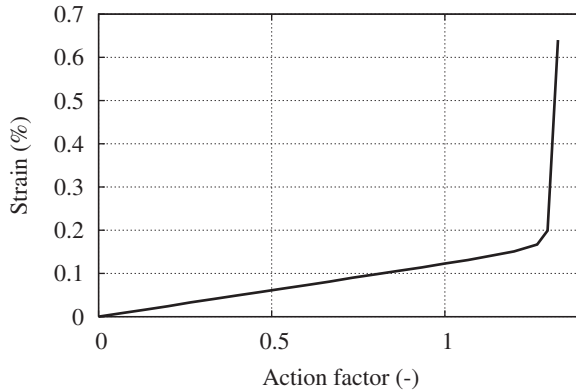


Figure E.4.4-5: Maximum absolute value of principal strains vs. action ratio.

Fig. E.4.4-4 shows the distribution of the compatibility ratio for a pressure value equal to its design value, and Fig. E.4.4-5 the development of the maximum absolute value of the principal total strains as a function of the action ratio, standardized such that a value of 1.0 corresponds to the design value. The last step for which a convergent solution was obtained corresponds to an action ratio larger than 1.33. Pressure corresponding to action ratios over 1.33 cannot be carried by the design model, and the maximum absolute value of principal total strains for an action ratio of 1.33 is still below 5% – the strain limitation is not governing.

**NOLC 1.2b:** The procedure corresponds to the one of the preceding sub-case, but in this load case thermal stresses are considered, and pressure and thermal stresses increased proportionally from zero up to an action ratio of 1.0, and then pressure only is increased further and thermal stresses kept at the values achieved at an action ratio of 1.0. Fig. E.4.4-6 shows the distribution of the compatibility ratio for a pressure value equal to its design value, and Fig. E.4.4-7 shows the development of the maximum absolute value of principal total strains as a function of the action ratio. The kink in the curve is the result of the thermal stresses, which are time-invariant for action ratios above 1.0. The last step for which a convergent solution had been obtained corresponds to an action ratio (of pressure) of 1.33, the same value as in NOLC 1.2a. Pressures corresponding to action ratios of over 1.33 cannot be carried by the design model and the maximum absolute value of principal total strains for an action ratio of 1.33 is still well below 5% – the strain limitation is, like in NOLC 1.2a, not governing.

**NOLC 1.2c:** The procedure is like the one of the preceding sub-case, but now thermal stresses are time-invariant, equal to the values corresponding to the stationary temperature distribution for cold injection, and pressure is increased from zero to a value where either the strain limit or the carrying capacity of the design model

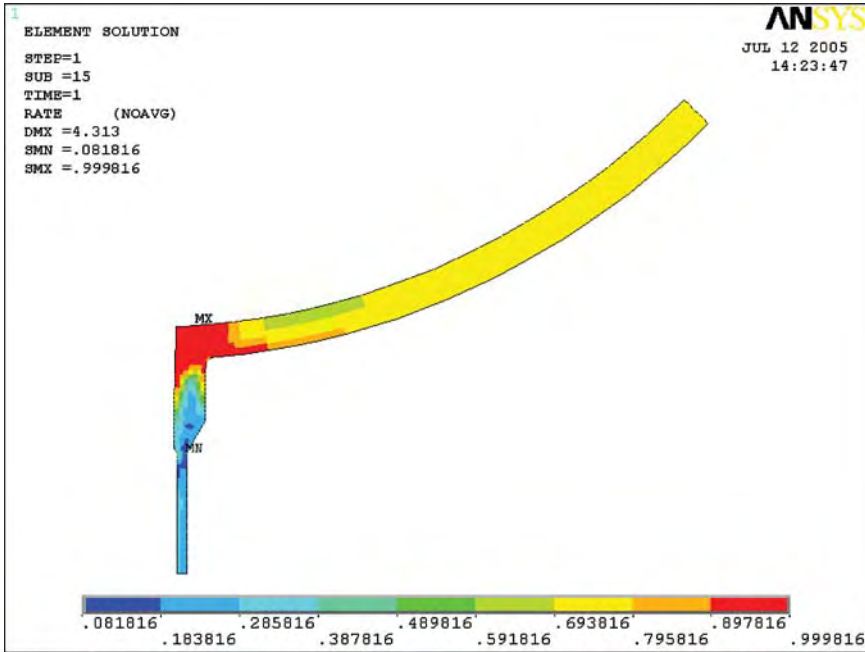


Figure E.4.4-6: Compatibility ratio distribution.

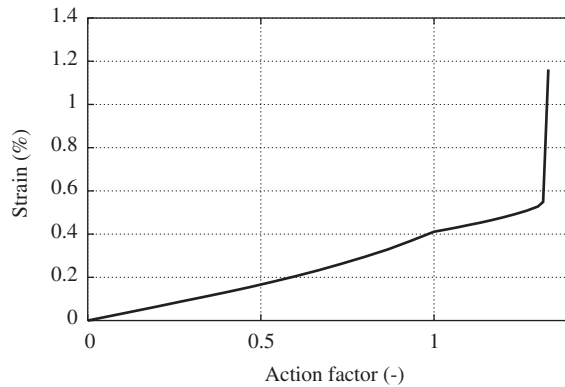


Figure E.4.4-7: Maximum absolute value of total principal strains vs. action ratio.

is reached. The action ratio is again standardized such that the value of 1.0 corresponds to the design value. Fig. E.4.4-8 shows the distribution of the compatibility ratio for a pressure equal to its design value, and Fig. E.4.4-9 the development of the maximum absolute value of principal total strains as a function of the action ratio.

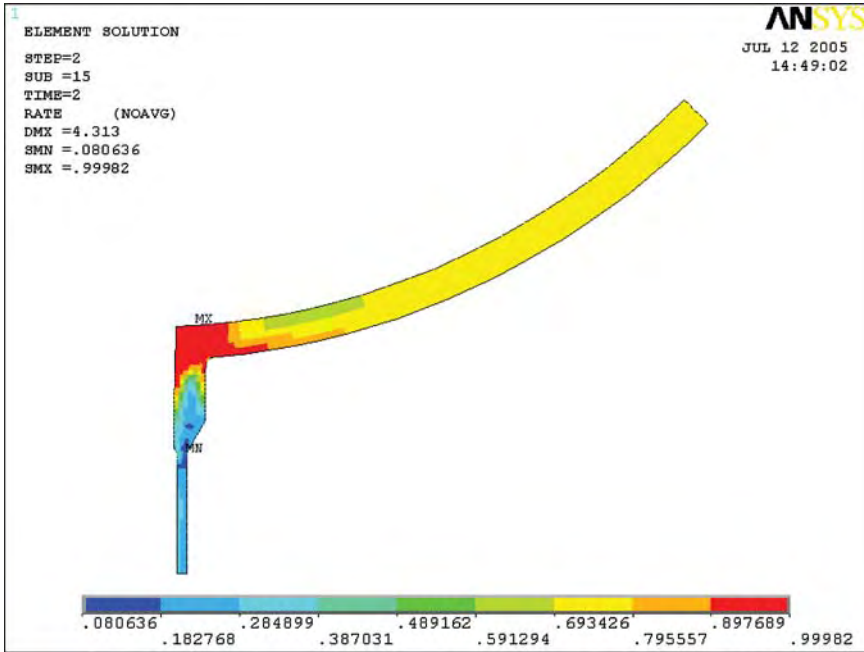


Figure E.4.4-8: Compatibility ratio distribution.

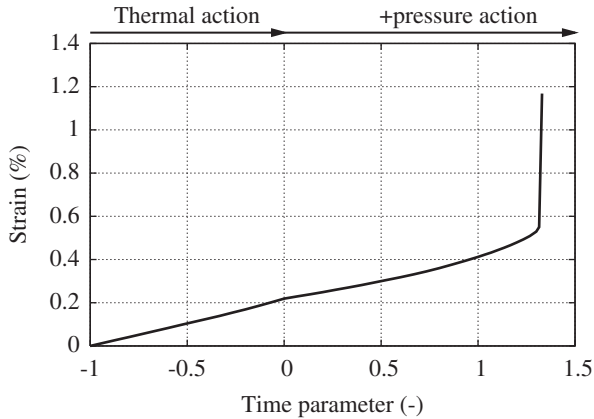


Figure E.4.4-9: Maximum absolute value of principal strains vs. time parameter.

Negative values on the abscissa are not values that correspond to action values as per definition, but they represent time. The history in this part of the abscissa corresponds to the application of temperature and to the increase of the corresponding thermal stresses. Positive action ratios correspond to increase of pressure.

**Conclusion:** The influence of thermal stresses on the result is zero, and so is the influence of the action path into the limit action, a result of the fact that the strain limitation in the design checks principle is not governing. The difference in strains in the results without and with thermal stresses is small in comparison with the strain limit of 5%.

## Annex E.5: Examples of Progressive Plastic Deformation Design Checks

Like in the preceding part E.4, the examples in this part are intended to illustrate the procedure, modelling problems, and solutions, in one specific design check, here the progressive plastic deformation design check, and all examples deal, therefore, only with individual load cases that are related to this PD-DC.

The first example, related to the first one in Annex E.4, E.4.1, is a complement to the NOLC 1.1 investigated there. The second example is a complement to the NOLC 1.1 investigated in Section E.4.3, and the third example is a complement to the investigation in Section E.4.4.

### *E.5.1: PD-DC of a hydrocracking reactor*

This example deals with one PD load case of the upper part of the hydrocracking reactor, discussed in Section E.4.1, with geometry shown in Fig. E.4.1-1. With the cladding being integrally bonded and its nominal thickness being smaller than 10% of the related nominal component thickness, cladding is neglected in this load case, like in NOLC 1.1 of Section E.4.1.

Temperature changes are specified to be slow enough such that non-stationary thermal stresses need not be taken into consideration, but, nevertheless, nozzle forces and bending moment have to be considered as being independent of the temperature of the vessel.

Consequently, one cyclic action trajectory to be considered, in the following denoted by NOLC 2.1, has to pass through the zero action point and the following four action points, see also Table E.5.1-1, page 218.

- NOLC 2.1: A1:

$$p_d = PS = 18 \text{ MPa}, t_d = TS = 400^\circ\text{C},$$

$$F_{z,d} = FS_z = 90 \text{ kN}, M_{x,d} = MS_x = 428 \text{ kN m}$$

- NOLC 2.1: A2:

$$p_d = PS = 18 \text{ MPa}, t_d = TS = 400^\circ\text{C}$$

$$F_{z,d} = FS_z = 126 \text{ kN}, M_{x,d} = MS_x = 88 \text{ kN m}$$

- NOLC 2.1: A3:

$$p_d = P_- = 0, t_d = T_- = \text{ambient}(20^\circ\text{C})$$

$$F_{z,d} = FS_z = 90 \text{ kN}, M_{x,d} = MS_x = 428 \text{ kN m}$$

- NOLC 2.1: A4:

$$p_d = P_- = 0, t_d = T_- = \text{ambient}(20^\circ\text{C})$$

$$F_{z,d} = FS_z = 126 \text{ kN}, M_{x,d} = MS_x = 88 \text{ kN m}.$$

According to Theorem 3 in Annex A: If for any cyclic action that passes through these five action points shakedown to linear-elastic behaviour can be proven, then the model will shake down to linear-elastic behaviour for each cyclic action in the action domain specified by these five action points as corner points.

Mises' equivalent stress distributions in the elastic model with  $E$  and  $\mu_{el}$  as in Table E.4.1-2 and due to actions A1 through A4 are shown in Figs. E.5.1-1–E.5.1-4.

The equivalent stresses for the four action points are nowhere larger than the design value of the yield stress for the weighted mean cycle temperature, i.e. in all of the action points are the stress fields compatible with the yield condition with the yield stress based on the weighted mean cycle temperature. Therefore, according to the variant of the principle, which uses the time-independent yield stress corresponding to this weighted mean cycle temperature, and by virtue of Theorem

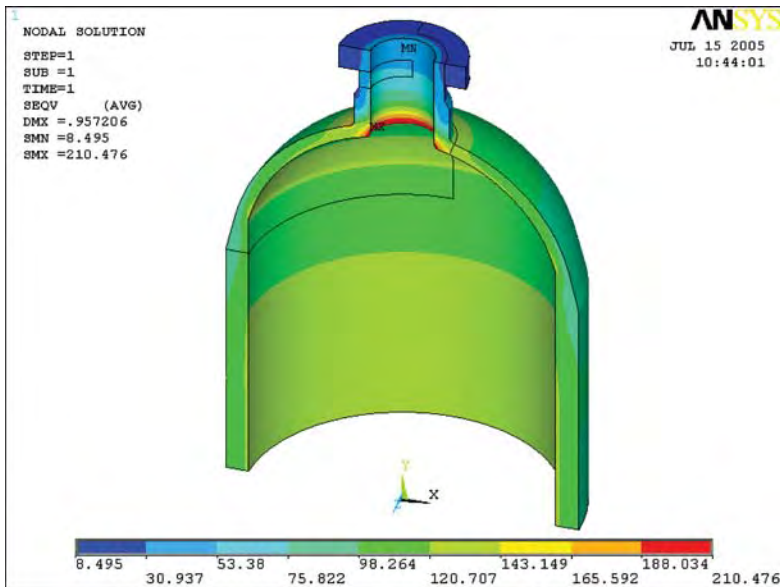


Figure E.5.1-1: Mises' equivalent stress: action A1: elastic model.

3 of Annex A, the design model shakes down to linear-elastic behaviour for each action cycle in the action domain specified by these five action points.

Nevertheless, the recommended approach, which uses time-dependent yield stresses is applied in the following as well: The stress points  $S_{A1}$  through  $S_{A4}$  in the deviatoric map, shown in Fig. E.5.1-5, correspond to the stresses in the presumably critical structural point denoted by MX in Fig. E.5.1-1.

The lines connecting the origin with these stress points represent the cyclic stress path  $0-S_{A1}-0-S_{A2}-0-S_{A3}-0-S_{A4}-0$ , due to the cyclic action  $0-A1-0-A2-0-A3-0-A4-0$ , and the full circle corresponds to Mises' yield limit with  $RM_d=195$  MPa for this load case and  $t_d=400^\circ\text{C}$ , the dashed one to Mises' yield limit with  $RM_{d,mct}=218.5$  MPa for this load case and the weighted mean cycle temperature. The stress points  $S_{A1}$  and  $S_{A2}$  are close to each other, so are  $S_{A3}$  and  $S_{A4}$ , and these are close to the origin, all an indication that the stresses due to nozzle force and moment are small. None of the four stress points  $S_{A1}$  through  $S_{A4}$  are outside of the dashed cycle, i.e. the stresses represented by these four stress points are all compatible with the corresponding yield condition, as discussed above. But  $S_{A1}$  and  $S_{A2}$  are outside of the full circle, indicating plastic flow, which can also be seen in Figs. E.5.1-1 and E.5.1-2 – Mises' equivalent stresses in excess of 195 MPa occur. The largest distance between the points 0,  $S_{A1}$ ,  $S_{A2}$ ,  $S_{A3}$ , and  $S_{A4}$  is smaller than  $2 \cdot RM_d$  and the

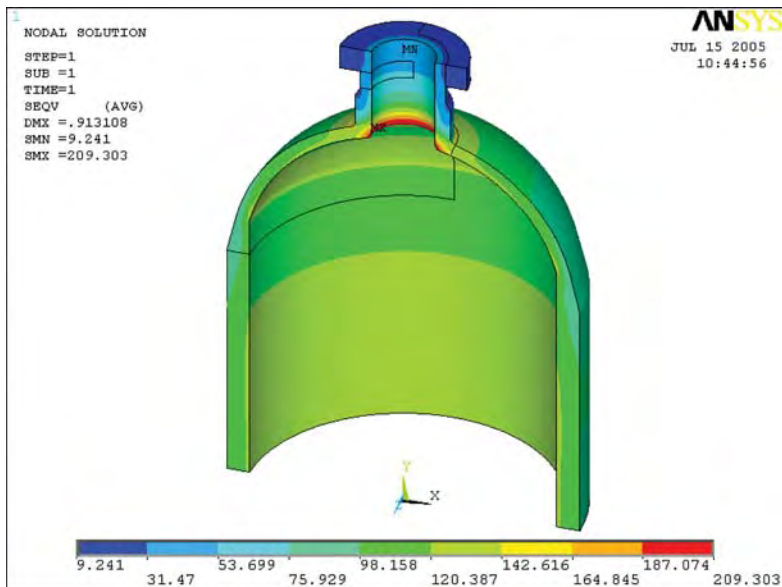


Figure E.5.1-2: Mises' equivalent stress: action A2: elastic model.

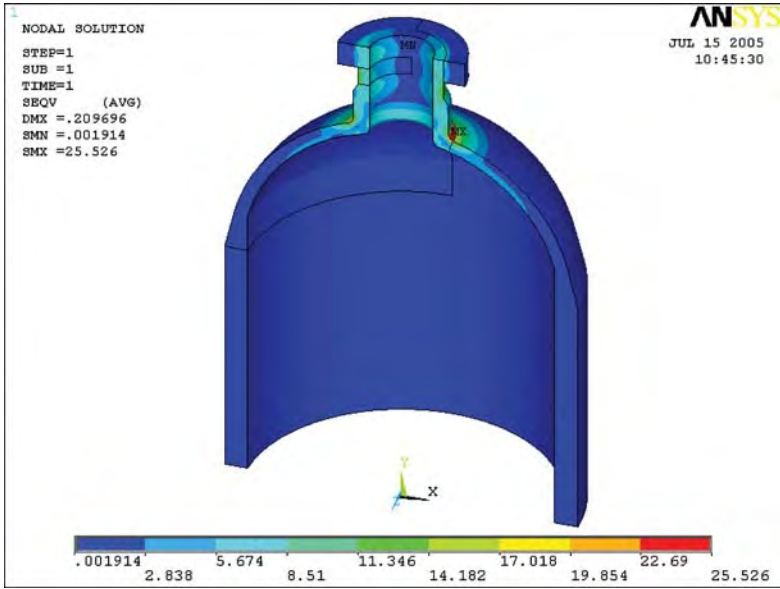


Figure E.5.1-3: Mises' equivalent stress: action A3: elastic model.

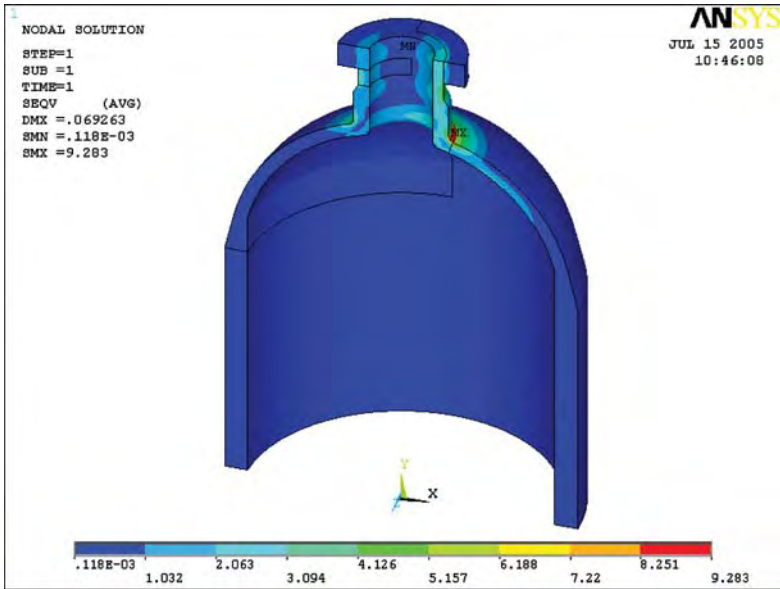


Figure E.5.1-4: Mises' equivalent stress: action A4: elastic model.

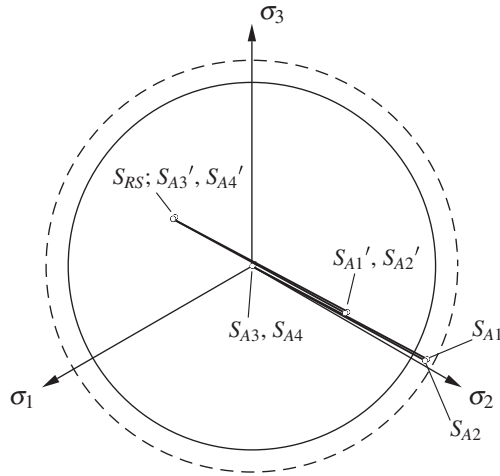


Figure E.5.1-5: Deviatoric map.

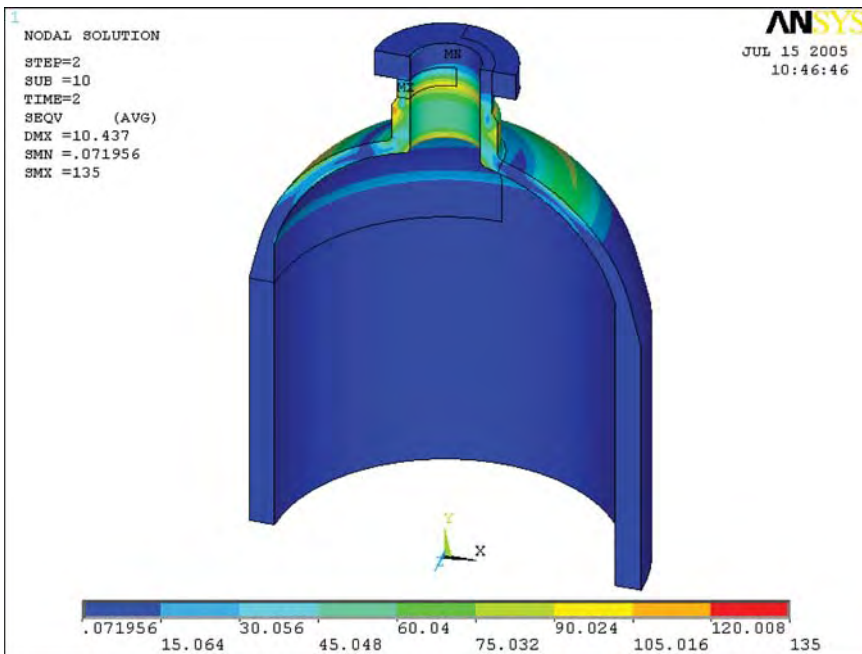


Figure E.5.1-6: Mises' equivalent stress: residual stress (RS).



whole stress path can easily be shifted translatorially into the circle, indicating the possibility of shakedown to linear-elastic behaviour and of the proof via usage of Melan's shakedown theorem. To follow this idea, a residual stress distribution was determined with the design model used in NOLC 1.1 (Section E.4.1) with Mises' yield condition and reduced yield stress and the cyclic action from zero actions towards the design values of NOLC 1.1 (Table E.4.1-2) but only up to action values with convergent solution, i.e. an action ratio of 0.923 (see Section E.4.1) and then back to zero actions. Mises' equivalent stress distribution of this residual stress is shown in Fig. E.5.1-6.

The stress point in the deviatoric map, Fig. E.5.1-5, denoted by  $S_{RS}$ , represents the residual stress in the structural point MX.

In the deviatoric map (Fig. E.5.1-5) the superposition at the critical point of this residual stress, which is a self-stress, and the cyclic stress path  $0-S_{A1}-0-S_{A2}-0-S_{A3}-0-S_{A4}-0$  is represented by the stress path  $S_{RS}-S_{A1}'-S_{RS}-S_{A2}'-S_{RS}-S_{A3}'-S_{RS}-S_{A4}'-S_{RS}$ . The translatorial shift of the two stress paths corresponds to the vector from the origin to  $S_{RS}$ . The whole stress path obtained by this superposition is inside of the corresponding yield limit (circle), an indication that shakedown to linear-elastic behaviour is possible. Mises' equivalent stress distributions of the four superpositions are shown in Figs. E.5.1-6–E.5.1-9.

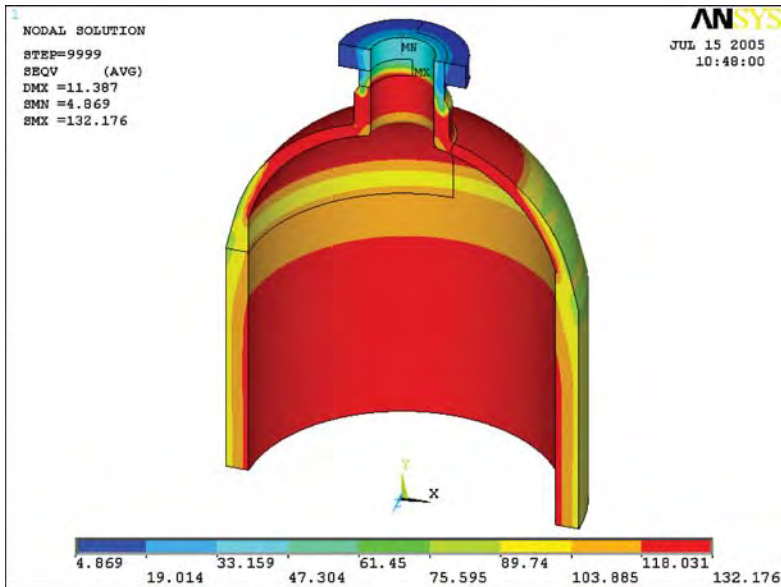


Figure E.5.1-7: Mises equivalent stress: superposition of RS and A1 stress.

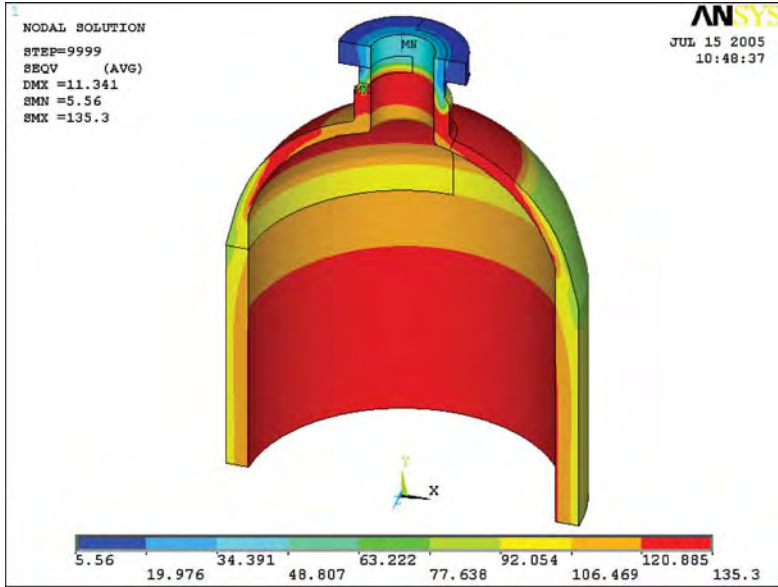


Figure E.5.1-8: Mises' equivalent stress : superposition of RS and A2 stress.

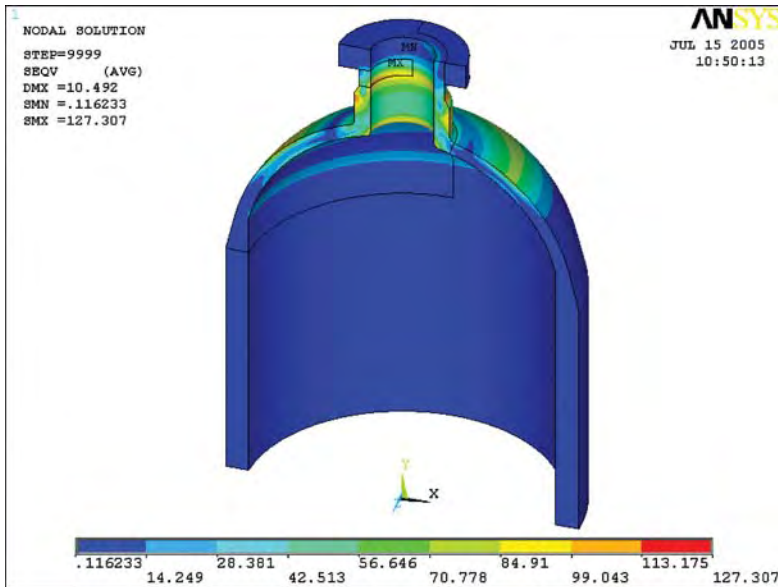


Figure E.5.1-9: Mises' equivalent stress : superposition of RS and A3 stress.

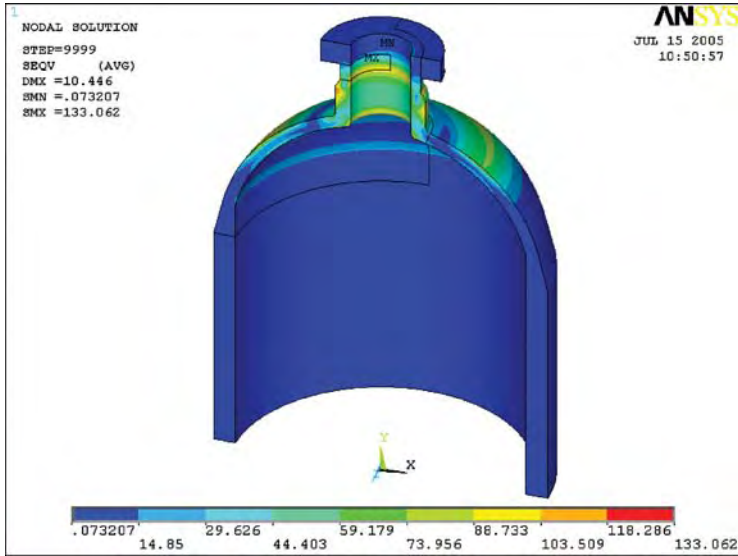


Figure E.5.1-10: Mises' equivalent stress: superposition of RS and A4 stress.

Table E.5.1-1: Design values at action points

Load case point	Design values				
	$p_d$ (MPa)	$t_d$ (°C)	$F_{z,d}$ (kN)	$M_{x,d}$ (kNm)	$RM_d$ (MPa)
NOLC 2.1: A1	18.0	400	90	296	$RM_{d,A1} = 195^a$
NOLC 2.1: A2	18.0	400	126	88	$RM_{d,A2} = 195^b$
NOLC 2.1: A3	0	20	90	296	$RM_{d,A3} = 310^b$
NOLC 2.1: A4	0	20	126	88	$RM_{d,A4} = 310^b$

<sup>a</sup> $RM_d = R_{p0.2/t_{rRM}}$  (see Table E.4.1-2). The value of  $RM_d$  for the weighted mean cycle temperature  $t_{rE} = 305^\circ\text{C}$  is  $RM_{d,mcl} = 218.5$  MPa.

<sup>b</sup> $RM_d = R_{eH} = 310$  MPa according to EN 10222-2

Mises' equivalent stress distributions in Figs. E.5.1-7 and E.5.1-8 do not exceed  $RM_d = 195$  MPa, the yield stress corresponding to  $400^\circ\text{C}$ , and those in Figs. E.5.1-9 and E.5.1-10 do not exceed  $RM_d = 310$  MPa, the yield stress corresponding to  $20^\circ\text{C}$ . According to the extended shakedown theorem the model shakes down to linear-elastic behaviour under the cyclic action 0-A1-0-A2-0-A3-0-A4-0, and, according to Theorem 3 in Annex A, it also shakes down to linear-elastic behaviour under each cyclic action within the design regime specified by the corner points A1, A2, A3, and A4.

Therefore, this approach leads to the same result. Hence: **NOLC 2.1: O.K.**

### E.5.2: PD-DC of an air cooler header

This example deals with one PD load case of the air cooler header, discussed in Section E.4.3, with geometry shown in Fig. E.4.3-1 and model geometry shown in Fig. E.4.3-2.

Temperature changes are specified to be slow enough such that non-stationary thermal stresses need not be taken into consideration. The cyclic action considered is given by proportional increase of the actions pressure and temperature from zero to the action point A and back, whereby the design values in the action point A are given by the (upper) characteristic values of NOLC 1.1 of Section E.4.3, i.e. defined by NOLC 2.1:A:  $p_d=7.2$  MPa,  $t_d=120^\circ\text{C}$ , including thermal stresses. The stationary temperature distribution used for the determination of the thermal stresses is the one discussed in Section E.4.3 in connection with NOLC 1.2.

To illustrate the options in the PD-DC open to the designer, a fictitious load case is discussed first: cyclic actions as given above, with cyclic pressure and temperature but without thermal stresses.

In the first option a time-invariant model is used, with material parameters determined for the weighted mean cycle temperature as time-independent reference temperature

$$t_{rE}=t_{rRM}=t_{ref}=0.75 \cdot 120 + 0.25 \cdot 20 = 95^\circ\text{C}$$

see Section 5.3, where instead of  $t_{c,max}$  the design value of temperature for action point NOLC 2.1:A, is used, which is not smaller than  $t_{c,max}$ .

The used design values of the material parameters in the model are shown in Table E.5.2-1.

Fig. E.5.2-1 shows the distribution of Mises' equivalent stress for a pressure of 1.44 MPa, which corresponds to 20% of the design value, a pressure for which all regions are still elastic.

The maximum relevant equivalent stress occurs at Point B, with a value of 94 MPa and with a compatibility ratio of  $94.0/307.2 = 0.306$ . Point A, at the tube inside, is the point with maximum compatibility ratio. The Mises equivalent stress in this point is 88.7 MPa and the compatibility ratio  $88.7/188.0 = 0.472$ . The stresses in the corner, designated in the figure by MX, are ignored, since they are due to the modelling related singularity and a minor smoothing would bring the values below that at Point B.

For the design value of pressure the compatibility ratio at Point B is given by  $5 \cdot 0.306 = 1.53$ , and at Point A by  $5 \cdot 0.472 = 2.36$ . The latter value being greater than 2, shakedown to linear-elastic behaviour cannot occur there, whereas the

Table E.5.2-1: Material parameters

(NOLC 2.1) <sup>a</sup>					
Part	Material group	$t_{ref}(^{\circ}\text{C})$	$E$ (GPa)	$\mu_{el}$	$RM_d(\text{MPa})$
1, 2, 3	1.2	95	206	0.3	307.2
4	1.1	95	206	0.3	188.0

In this option a time-invariant reference temperature for the design value of the yield stress and the other material parameters is used.

For the tube material hot tensile properties are not given in the standard. An interpolated value has been chosen, with interpolation between the values for EN 10216-2 P195GH and P235GH.

Verification by hot tensile test at 95°C (or higher) is required.

<sup>a</sup>Fictitious load case.

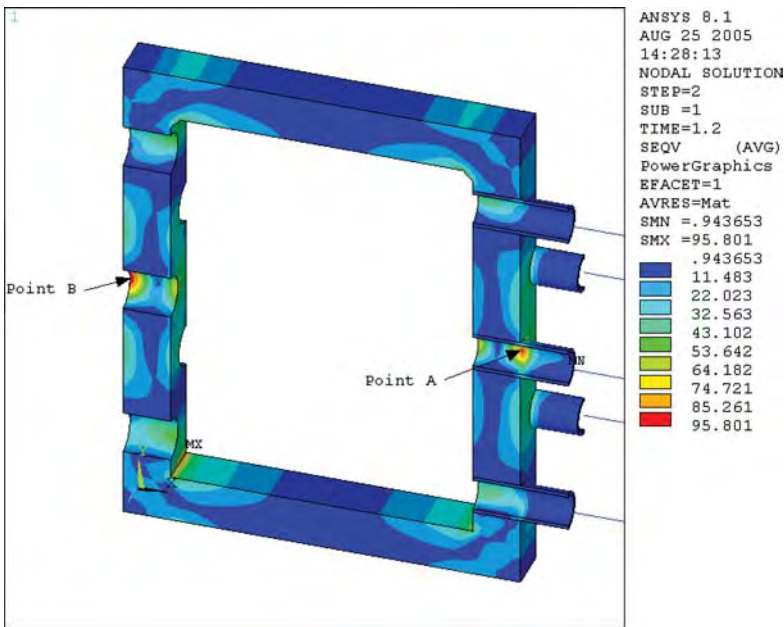


Figure E.5.2-1: Mises' equivalent stress distribution for 1.44 MPa pressure.

corresponding value at Point B shows that in the header proper shakedown to linear-elastic behaviour is possible.

Fig. E.5.2-2 shows the stress path of Point A in the deviatoric map. Subscripts in the stress point designations refer to the action step, superscript *el* refers to linear-elastic results, and superscript *pl* to the end point of linear-elastic ideal-plastic

results. The stress path fluctuates around the origin, after 10 cycles the path goes practically through the origin – an indication of shakedown to alternating plasticity [17]. The same conclusion can be drawn from the development of the principal total strains at Point A, as shown in Fig. E.5.2-3.

In the first option the design model is time-independent, and can thus be used with Melan’s shakedown theorem and conveniently investigated with usual plotting of stress paths in the deviatoric map.

In the second option the design model is time-dependent, insofar as the design value of the material strength parameter is temperature-dependent, and, hence, varies within the action cycle. Therefore, this option requires the use of the extended shakedown theorem. Usual plotting of stress paths in the deviatoric map is often confusing and prone to errors, but plotting of compatibility ratios, i.e. of the quotients of  $\sigma_i$  at the various instants of time and the corresponding design values of the yield stress at the same instants, can be used well. Table E.5.2-2 shows the material parameters used in this option.

Fig. E.5.2-4 shows in the deviatoric map the path of the stress at Point A for the second action cycle, in terms of the compatibility ratios.

The corresponding figure for option 1 is Fig. E.5.2-2. Both figures are quite instructive and lead to the same conclusion. The required load case, which includes

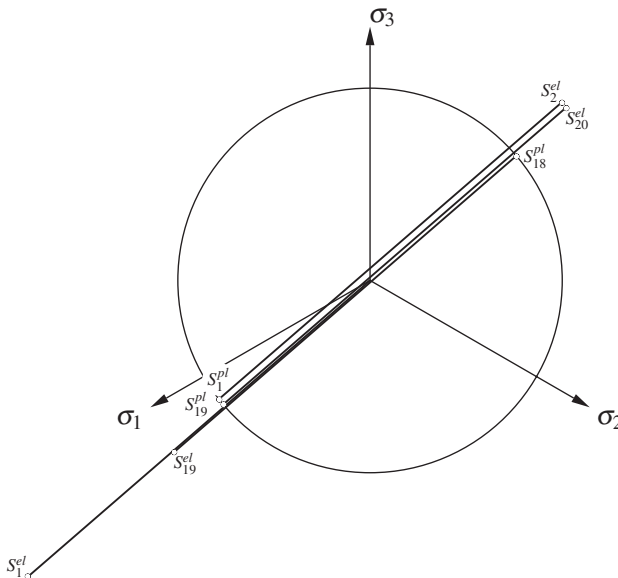


Figure E.5.2-2: Deviatoric map: Point A.

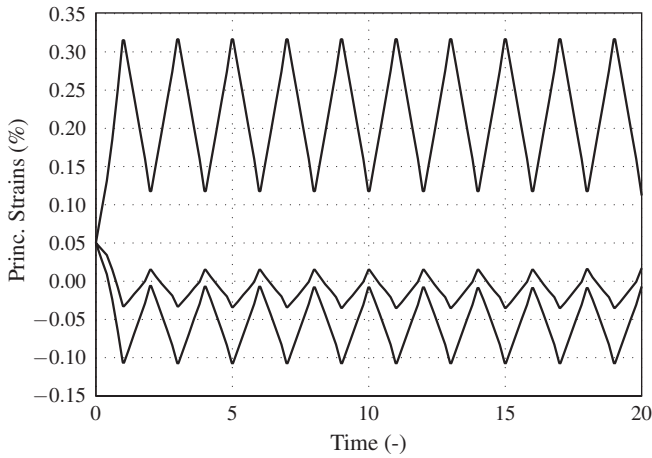


Figure E.5.2-3: Principal strains vs. time parameter.

Table E.5.2-2: Material parameters

NOLC 2.1								
Part	Material group	$t_{rE}$ (°C)	$E$ (GPa)	$\mu_{el}$	$t_{rRM,A}$ (°C)	$RM_{d,A}$ (MPa)	$t_{rRM,0}$ (°C)	$RM_{d,0}$ (MPa)
1, 2, 3	1.2	95	206	0.3	120	296.0	20	355
4	1.1	95	206	0.3	120	182.0	20	215

$t_{rRM,A}$  is the reference temperature for  $RM_{d,A}$ , the design value of the yield stress at action point A; and  $t_{rRM,0}$  the reference temperature for  $RM_{d,0}$ , the design value of the yield stress at action point 0. For the tube material hot tensile properties are not given in the standard. An interpolated value has been chosen, with interpolation between the values for EN 10216-2 P195GH and P235GH. Verification by hot tensile test at 120°C (or higher) is required.

the (existing) thermal stresses and is specified in the beginning of this Section E.5.2, is discussed next. Fig. E.5.2-5 shows the distribution of Mises' equivalent stress for a pressure of 1.44 MPa, which corresponds to 20% of the design value, and for thermal stresses equal to 20% of the stationary ones due to the stationary temperature distribution discussed in Section E.4.3 and at the beginning of this Section E.5.2. For these action values all regions are still elastic.

The maximum relevant stress occurs are point B, with a value of 121.7 MPa and a compatibility ratio of 0.395. Point A, at the tube inside, is the point with the

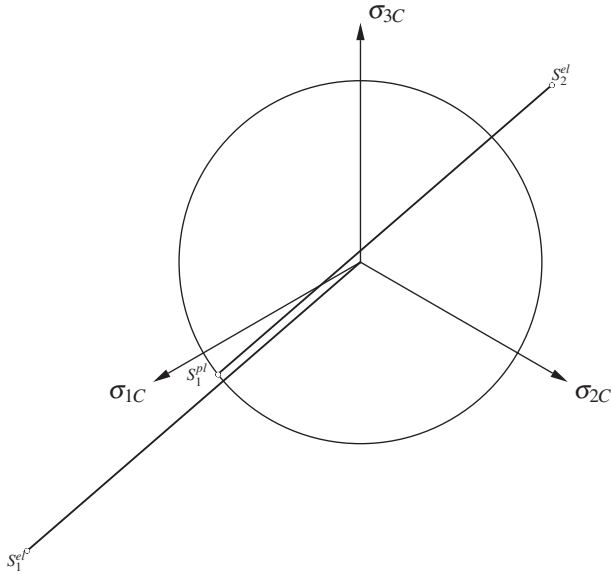


Figure E.5.2-4: Deviatoric map: compatibility ratios: Point A.

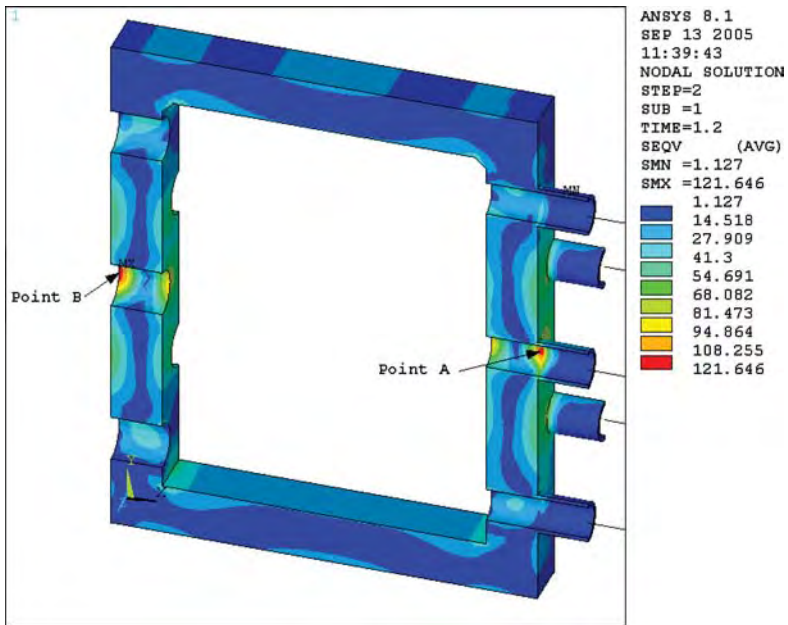


Figure E.5.2-5: Mises' equivalent stress distribution.



maximum compatibility ratio. The Mises equivalent stress at this point is 111.8 MPa and the compatibility ratio is 0.593. For the design values of the actions the compatibility ratio at Point B is given by  $5 \cdot 0.395 = 1.98$ , and at Point A by  $5 \cdot 0.593 = 2.96$ . The latter value being greater than 2, shakedown to linear-elastic behaviour cannot occur at Point A, whereas the corresponding value at Point B shows that in the header proper shakedown to linear-elastic behaviour is possible.

The behaviour in the neighbourhood of the critical point, Point A, is investigated further in the following. Fig. E.5.2-6 shows the stress path at Point A for the first cycle in form of the compatibility ratios in the deviatoric map.

The behaviour is similar to the preceding one, shown in Fig. E.5.2-4, indicating alternating plasticity. This indication is supported by the Figs. E.5.2-7 and E.5.2-8.

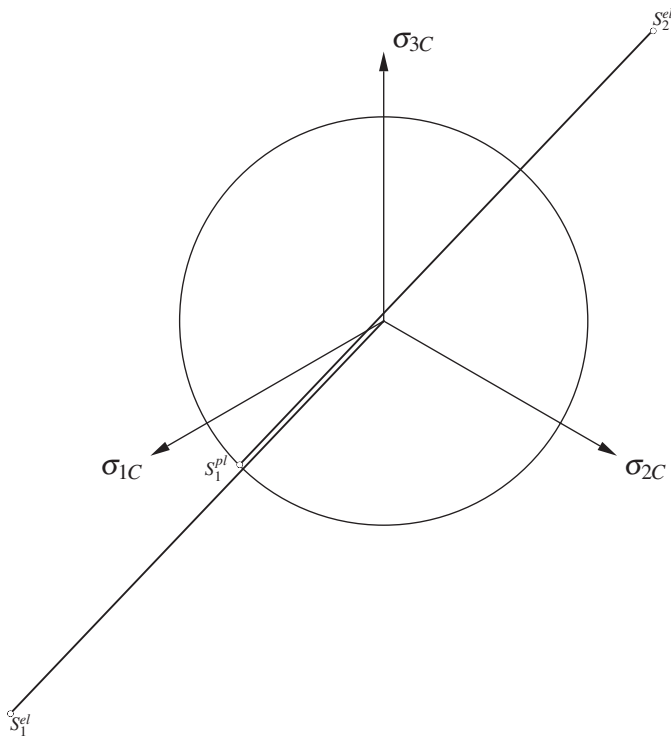


Figure E.5.2-6: Deviatoric map: compatibility ratios: Point A.

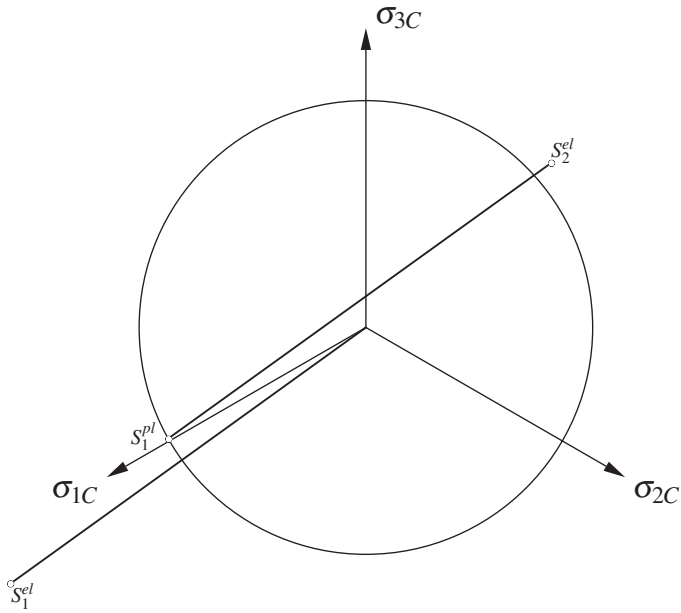


Figure E.5.2-7: Deviatoric map: compatibility ratios: Point  $A_{t,o}$ .

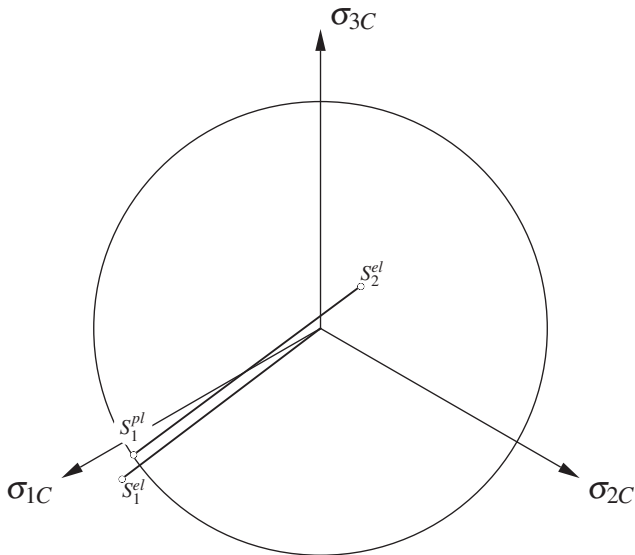


Figure E.5.2-8: Deviatoric map: compatibility ratios: Point  $A_{p,o}$ .

Fig. E.5.2-7 shows the corresponding stress path at a point opposite to Point A at the outside surface of the tube, and Fig. E.5.2-8 the corresponding stress path at the same point but as a point of the tubesheet.

Fig. E.5.2-7 shows for this point on the tube's outer surface much less plastication than Fig. E.5.2-6 for Point A, and Fig. E.5.2-8 shows that in the point of the header tubesheet shakedown to linear-elastic behaviour occurs after the first action cycle.

Therefore:

**NOLC 2.1: O.K.**

Remark: In this example keeping the identity of initially chosen principal stress axes, for plotting of stress paths in the deviatoric map, has been especially difficult. Quite often the principal stresses as per FEM output changed their order according to their values, and detailed comparison of the development of principal stresses and of stress components vs. time was necessary to determine the identity of the principal stress axes.

### ***E.5.3: PD-DC of a nozzle in hemispherical end***

This example deals with one PD load case of the nozzle detail discussed in Section E.4.4, with model geometry shown in Fig. E.4.4-1. Temperature changes are specified to be slow enough such that non-stationary thermal stresses need not be taken into consideration.

The cyclic action considered is given by proportional increase of the actions pressure and temperature from zero to the action point A1, one cold injection corresponding to action point A2, return to A1, and then back to zero. The design values in the action points are defined as follows:<sup>6</sup>

- NOLC 2.1: A1:
  - $p_d = PS = 8.0 \text{ MPa}$ ,  $t_d = TS = 295^\circ\text{C}$
- NOLC 2.1: A2:
  - $p_d = PS = 8.0 \text{ MPa}$ ,  $t_d = TS = 295^\circ\text{C}$
  - cold injection  $t_{i,d} = TIS = 60^\circ\text{C}$ , with heat transfer boundary conditions and coefficients as per NOLC 1.2 of Section E.4.4.
- NOLC 2.1: A3 = A1:
  - $p_d = PS = 8.0 \text{ MPa}$ ,  $t_d = TS = 295^\circ\text{C}$

The equivalent elastic stress distributions for action points A1 and A2 are shown in Figs. E.5.3-1 and E.5.3-2, respectively. The results are for a linear-elastic model

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<sup>6</sup>All partial safety factors are equal to 1.0.

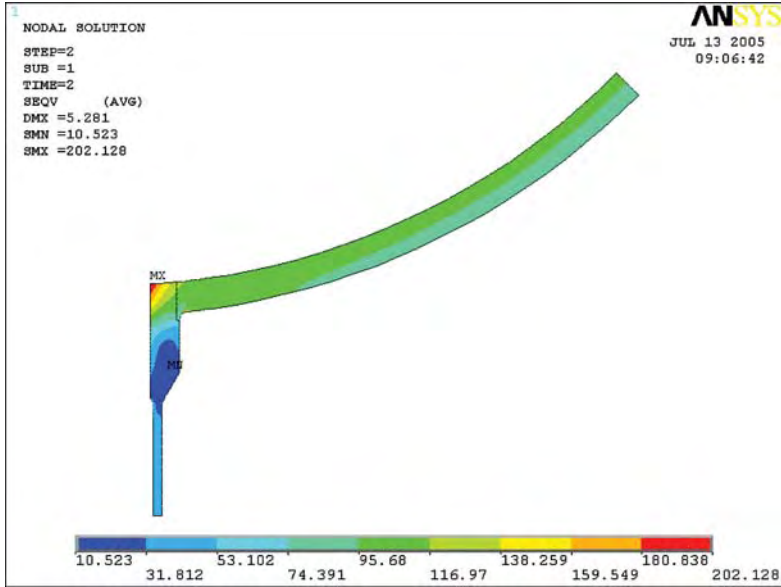


Figure E.5.3-1: Mises' equivalent stress: action A1: elastic model.

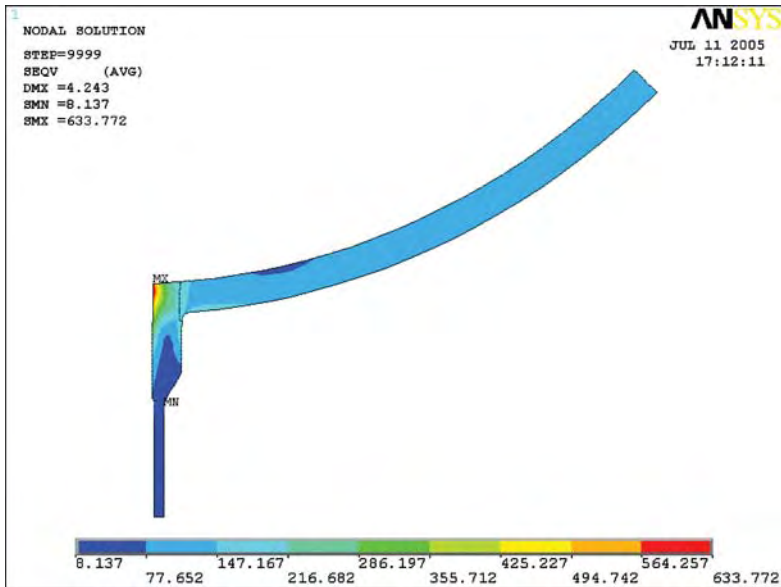


Figure E.5.3-2: Mises' equivalent stress: action A2: elastic model.

Table E.5.3-1: Pivot points of  $RM_d$  (MPa) vs.  $t_d$

Position No.	Temperature (°C)					
	20	100	150	200	250	300
1	290	249	238	232	227	221
2	355	323	312	304	296	289
3	265	241	223	205	188	173

Partial safety factors being equal to 1.0, all values are upper yield strength values and proof stress values, respectively, as per material standards.

with modulus of elasticity, Poisson’s ratio, thermal conductivity, and heat transfer boundary conditions as in Section E.4.4.

The maximum equivalent stress is equal to 642.3 MPa, larger than  $2 \cdot RM_{d,mct} = 599.6$  MPa, with  $t_{mct} = t_{rE} = 226.25^\circ\text{C}$  and  $RM_{d,mct} = 299.8$  MPa at the critical point.

But this maximum equivalent stress is smaller than  $RM_{dA2} + RM_{d0}$  at the critical point, with <sup>7</sup>  $RM_{dA2} > 289.7$  MPa,  $RM_{d0} = 355$  MPa, and thus  $RM_{dA2} + RM_{d0} = 644.7$  MPa. These comparisons show that the requirement of Melan’s shakedown theorem cannot be met, but the one of the extended shakedown theorem can be met (see also Theorem 2 in Annex A, with corollaries).

To obtain a suitable self-stress field for usage in the extended shakedown theorem, the model with linear-elastic ideal-plastic constitutive law has been subjected to one action cycle, 0-A1-A2-A1-0, with linear-elastic material parameters as above, and with yield stress values equal to the ones obtained in the software processor for the corresponding metal temperatures via linear interpolation between the pivot points of  $RM_d$  as in Table. E.5.3-1.

The corresponding distribution of  $RM_d$ , for the actions 0, A1, and A2, are shown in Figs. E.5.3-3 through E.5.3-5.<sup>8</sup>

The compatibility ratio distribution of this residual stress, in this model after this cyclic action, is shown in Fig. E.5.3-6.

<sup>7</sup>The value on the right is for  $295^\circ\text{C}$ , a temperature which is with certainty not smaller than the metal temperatures for action A2.

<sup>8</sup>Use of the usual plotting of the following inequality routines can lead to irritating results. In the postprocessing routines usually the values of the relevant quantities in the nodal points are used. These values are obtained in the solution routine via extrapolation or copying from calculation values in the integration points. Extrapolation is the usually used option, sometimes the default one. Use of this option can lead to irritating results – in this load case with large temperature and thermal stress gradients. Therefore, as in E.4.4, a different procedure has been used here: Integration point values copied into the nodal points, evaluation of diverse quantities in mid-points of the elements, and corresponding coloring of each element according to this mid-point value.

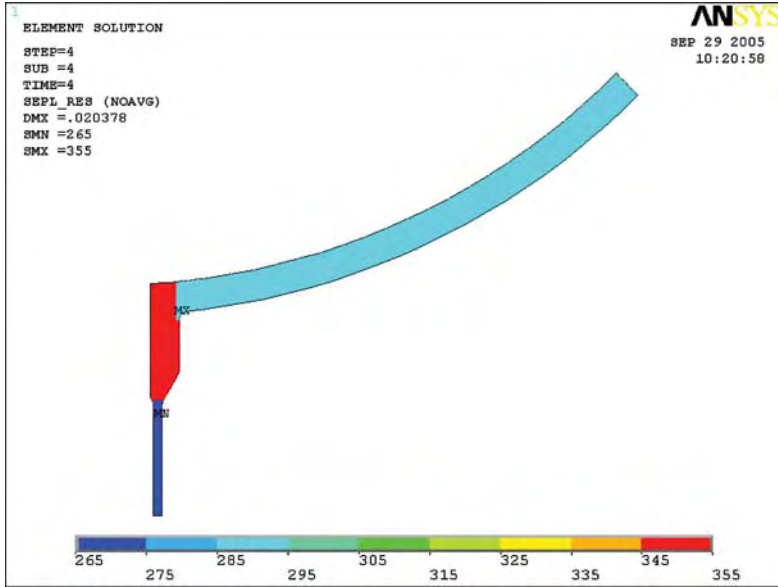


Figure E.5.3-3: Distribution of  $RM_d$  for action 0: metal temperatures equal to ambient temperature ( $20^{\circ}\text{C}$ ).

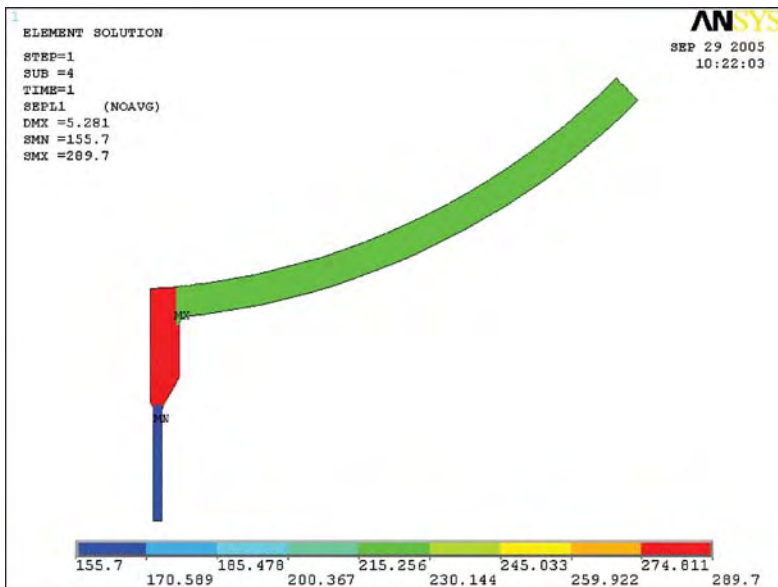


Figure E.5.3-4: Distribution of  $RM_d$  for action A1: metal temperatures equal to  $T_S=295^{\circ}\text{C}$ .

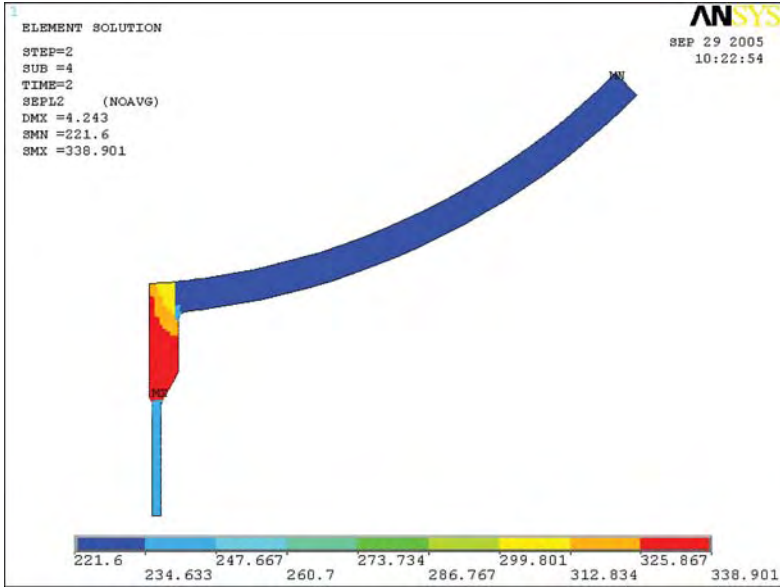


Figure E.5.3-5: Distribution of  $RM_d$  for action A2: metal temperatures equal to stationary temperature distribution for cold injection at  $t_d=TS=295^\circ\text{C}$ .

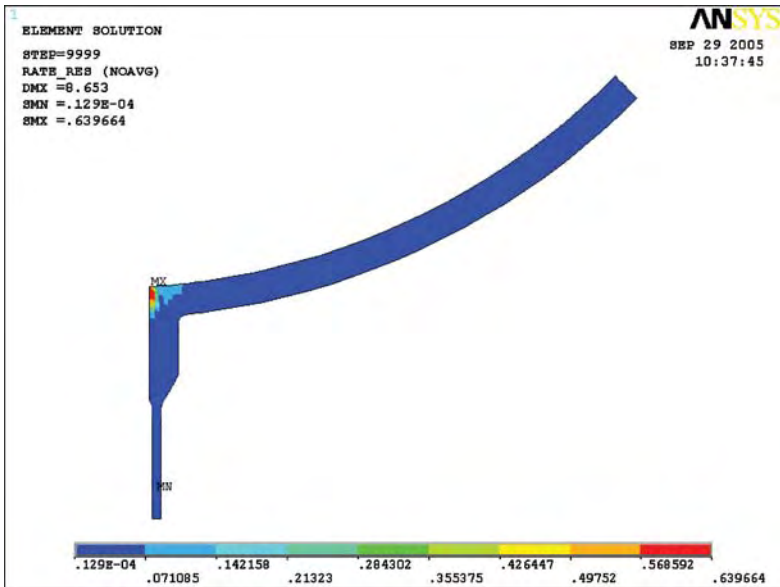


Figure E.5.3-6: Compatibility ratio: residual stress (RS).

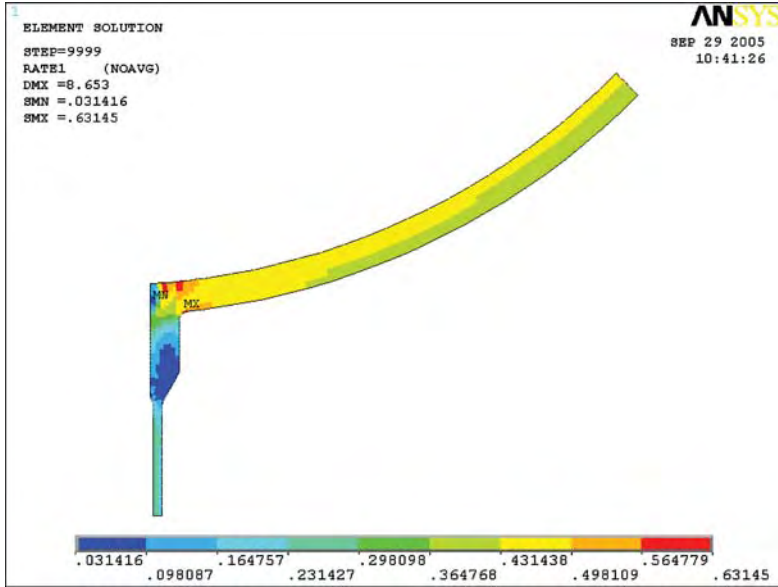


Figure E.5.3-7: Compatibility ratio:superposition of RS and A1 stress.

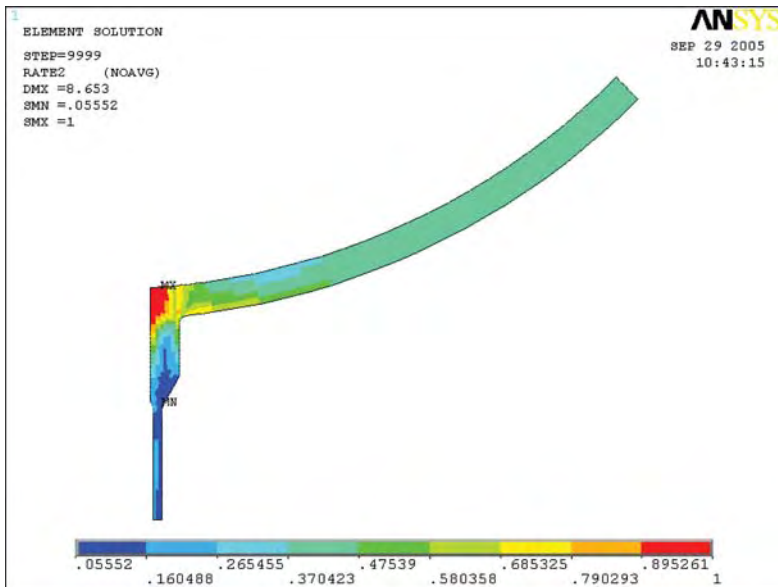


Figure E.5.3-8: Compatibility ratio: superposition of RS and A2 stress.



The compatibility ratio distributions of the superpositions of this residual stress, which of course is a self-stress, and the elastic stress distributions according to actions A1 and A2 are shown in Figs. E.5.3-7 and E.5.3-8, respectively.

All the three figures have a maximum value not exceeding 1.0, i.e. the extended shakedown theorem is fulfilled. The model shakes down to linear-elastic behaviour under the cyclic action 0-A1-A2-A1-0, and, therefore, shakes down to linear-elastic behaviour under each cyclic action in the design domain specified by the actions 0, A1, A2 as corner points.

Therefore:

**NOLC 2.1: O.K.**

## **Annex E.6: Examples of Stability Design Checks**

Like in the preceding annexes, the following examples are intended to illustrate the procedure, modelling, problems, and solutions, in one specific design check, here the stability design check. Therefore, these examples deal solely with individual load cases that are related to this design check.

### ***E.6.1: First S-DC of a jacketed stirring vessel***

This example deals with one load case of a dished lower end of a jacketed stirring vessel, with geometry shown in Fig. E.6.1-1.

The geometry shown in this figure is the one of the model.

#### **Excerpt from the design data specification:**

- Maximum/minimum allowable pressure:<sup>9</sup>  
 $PS = 4/-1$  bar for the inner vessel;  $PS = 4$  bar for the jacket
- Maximum allowable temperature:<sup>9</sup>  
 $TS = 152^{\circ}\text{C}$  for inner vessel and jacket
- Material:  
 EN 10028-7: X6CrNiTi18-10
- Thickness allowances:  
 Corrosion/erosion:  $c = 0$ ,  
 Tolerances:  $\delta_e = 0$
- Fabrication tolerances:  
 According to EN 13445-4
- Insulation:  
 Yes

---

<sup>9</sup>For the inner vessel maximum allowable pressure and maximum allowable temperature correspond approximately to saturated vapour of the medium contained, the minimum allowable pressure takes into account condensation.

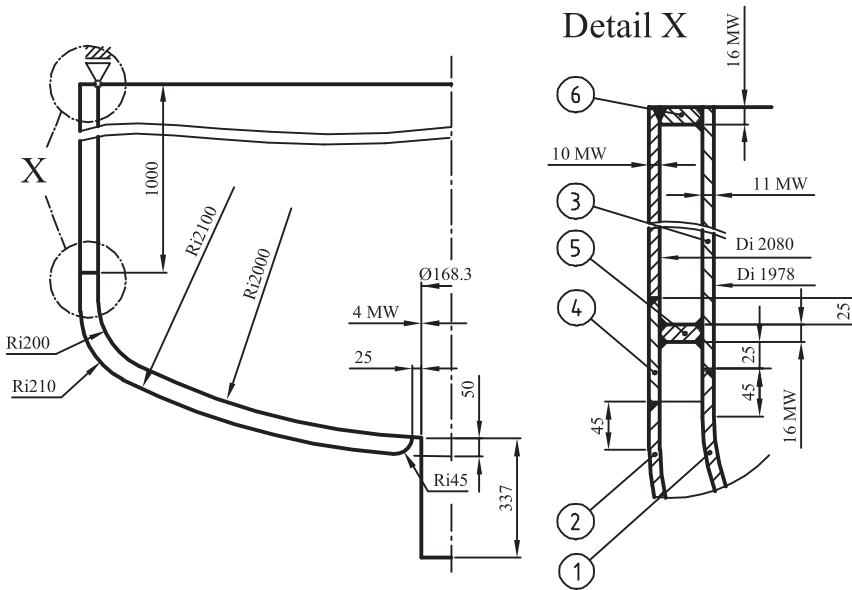


Figure E.6.1-1: Geometry of dished end.

Thermal transients are slow enough such that non-stationary thermal stresses can be neglected

**(Considered) Design Load Case**

**NOLC 3.1:**

- (Relevant) characteristic values of pressure:
  - Inner vessel:  $p_{c,i^-} = 0$
  - Jacket:  $p_{c,j^+} = 0.42$  MPa of which 0.02 MPa due to static head
- (Relevant) characteristic values of temperature:
  - Inner vessel:  $t_{c,i^+} = 152^\circ\text{C}$
  - Jacket:  $t_{c,j^-} = 20^\circ\text{C}$
- Heat transfer coefficients:

$$h_i = \left\{ \begin{array}{l} 14.4 \text{ kW}(\text{m}^{-2} \cdot \text{K}) \text{ for inner vessel side} \\ 1.16 \text{ kW}(\text{m}^{-2} \cdot \text{K}) \text{ for jacket side} \end{array} \right\}$$

ideal insulation on outside of jacket

The corresponding load case specification, with derivation details, is given in Table E.6.1-1.

In this load case thermal stresses have to be included in the investigation.

### 234 Pressure Vessel Design: The Direct Route

A sufficiently good approximation of the temperature distribution in the inner vessel walls is the one for the infinite straight plate and medium temperatures on both sides of 152 and 20°C, respectively. This approximation renders a linear temperature distribution over the thickness. For the 11 mm thick plate this approximation results in metal surface temperatures of 146.4 and 89.8°C, respectively. For the outer shell and the annular rings a uniform temperature of 20°C may be assumed conveniently.

The relevant material parameters at pivot points are given in Table E.6.1-2.

For an admissible design, the model must be able to carry the  $\gamma_R$ -fold of the design values of actions, i.e. in this load case, not only the  $\gamma_R$ -fold of the design values of pressure, but also the  $\gamma_R$ -fold of the thermal stresses, due to the  $\gamma_R$ -fold of the design values of temperature. In the design model the design values of the yield stress are specified as the values corresponding to the design temperature,

Table E.6.1-1: Load case specification

NOLC 6.1			
Action	Characteristic value	Partial safety factor	Design value
Inner vessel pressure $p_{d,i}$ (MPa)	0		0
Jacket pressure $p_{d,j}$ (MPa) <sup>a</sup>	0.4	1.2	(0.48)
	0.02	1.0	(0.02)
			0.50
Inner vessel temperature $t_{d,i}$ (°C)	152	1.0	152
Jacket temperature $t_{d,j}$ (°C)	20	1.0	20

<sup>a</sup>The values in the first row correspond to the pressure proper at the reference point and the values in the second row to the pressure due to static head, relative to the reference point.

Table E.6.1-2: Material parameters at pivot points

NOLC 6.1: $\gamma_R=1.25$						
Part	Material group	$t_p$ (°C)	$E_{tp}$ (GPa)	$\beta_{tp}$ (K <sup>-1</sup> )	$R_{p1.0/tp}$ (MPa)	$RM_{d/tp}$ (MPa)
1-5	8.1	20	200.0	$15.3 \cdot 10^{-6}$	240.0	240.0
1-5	8.1	160	188.1	$16.3 \cdot 10^{-6}$	194.0	194.0

For determining the partial safety factor it is assumed that the (external) pressure test as called for in EN 13445-5 is to be carried out.

$E_{tp}$ ,  $\beta_{tp}$ ,  $R_{p1.0/tp}$  and  $RM_{d/tp}$  are the values of  $E, \beta, R_{p1.0}$ , and  $RM_d$  at the pivot temperature  $t_p$ . Poisson's ratio is equal to 0.3 in the elastic regime.

and, therefore, in a design model with temperature-dependent yield stress the  $\gamma_R$ -fold of thermal stresses cannot be determined simply via the  $\gamma_R$ -fold of temperature if there is plastic flow. To overcome this problem, the values of the coefficient of linear thermal expansion have been multiplied in the input artificially by  $\gamma_R$ , and, thus, at the design temperature thermal stresses correspond to temperatures given by the  $\gamma_R$ -fold of the design temperature, in a model with correct yield stress design values.

Like in preceding examples involving thermal stresses, temperature has been increased to its design value first and then the pressure up to the  $\gamma_R$ -fold of its design value. In this investigation, the used design model corresponds to the description in Section 6.3 for critical cases. To obtain the required initial imperfections of the inner dished end of the model geometry, a classical (eigenvalue) buckling calculation of the perfect structure with linear-elastic constitutive law has been performed. The so obtained buckling shapes of the first buckling modes are superposed and the superposition scaled, with regard to the maximum permissible irregularity in profile according to EN 13445-4, 5.4.4, and the result is then used for the determination of the initial imperfect geometry. In this case the maximum permissible local irregularity, according to EN 13445-6, 5.4.4, is given by 14.8 mm. This classical buckling calculation has been performed for pressure action on the outer surface of the inner dished end only, without thermal stresses and a scaled superposition of the first 10 modes chosen as initial imperfection.

To include the critical buckling modes that are not rotational-symmetric, nor necessarily symmetric to any meridional plane, an FE model of the whole end has been used.

The model with this imperfect geometry, used in the succeeding non-linear FEM analysis, has been created by using of the macro IMPER (see Annex L.6.1). With this macro the imperfect geometry is created. In this procedure the FEM mesh is detached from the geometric lines and areas that had been used to create the model of the perfect structure.

Two buckling modes of the classical (eigenvalue) buckling analysis of the inner vessel's dished end are shown in Fig. E.6.1-2, the first one on the left and the tenth on the right.

Mises' equivalent stress distributions of the non-linear model, with maximum permissible imperfection and for actions given by the  $\gamma_R$ -fold of the design values, are shown in Fig. E.6.1-3, those at the inside surface of the inner dished end on the left and those at the outside surface on the right. The corresponding distributions of the maximum of the absolute values of the principal total strains are shown in Fig. E.6.1-4, again those at the inner surface on the left and those at the outer surface on the right.

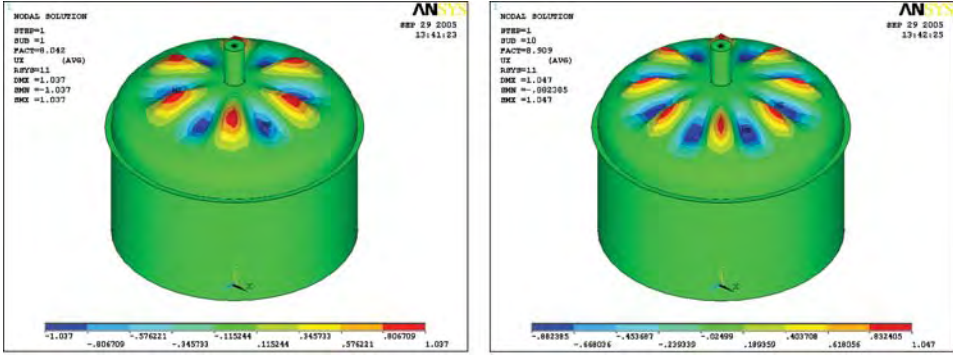


Figure E.6.1-2: Classical buckling modes.

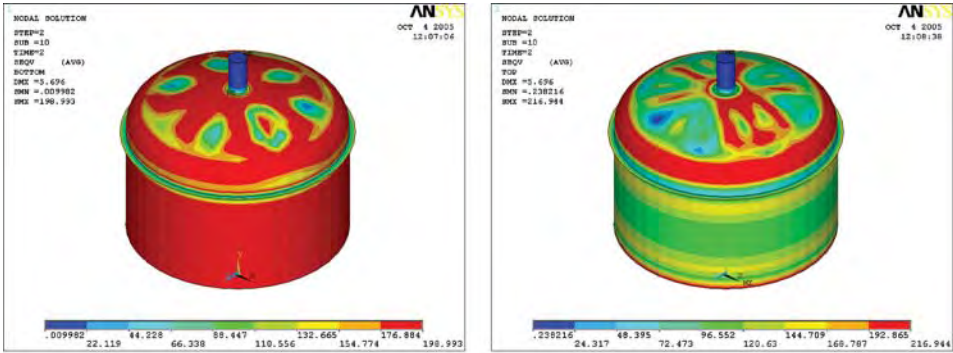


Figure E.6.1-3: Mises' equivalent stress for  $\gamma_R$ -fold of design actions.

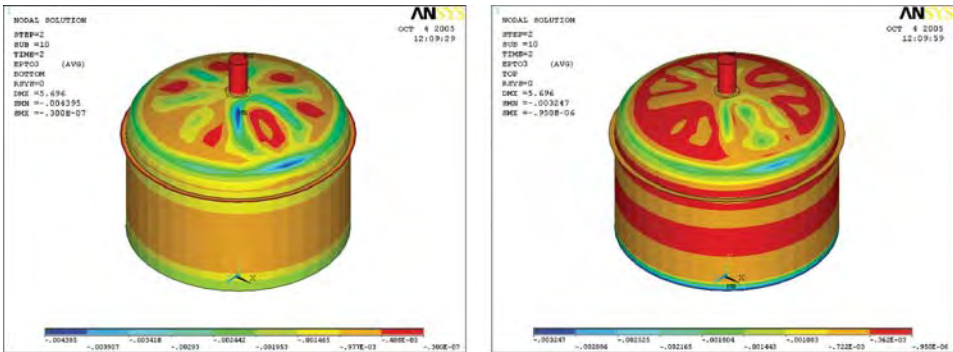


Figure E.6.1-4: Maximum principal total strain for  $\gamma_R$ -fold of design actions.

The model can carry the  $\gamma_R$ -fold of the design values of the actions with maximum absolute value of principal total strains less than 5%.

Therefore:

**NOLC 3.1.: O.K.**

### *E.6.2: Second S-DC of a jacketed stirring vessel*

This example deals with another load case of the lower end described in the preceding example.

#### **(Considered) Design Load Case**

##### **NOLC 3.2:**

- (Relevant) characteristic values of pressure:
  - Inner vessel:  $p_{c,i} = -0.1$  MPa
  - Jacket:  $p_{c,j}^+ = 0.42$  MPa  
 this pressure acts in the lower chamber of the design model's jacket only and 0.02 MPa of this pressure are due to static head, and the pressure in the upper chamber of the design model is zero
- (Relevant) characteristic values of temperature:
  - Inner vessel and jacket:  $t_{c,i} = t_{c,j} = 20^\circ\text{C}$

The corresponding load case specification, with derivation details, is given in Table E.6.2-1.

In this load case there are no thermal stresses, but the pressure difference is maximal.

Table E.6.2-1: Load case specification

Action	NOLC 6.2		
	Characteristic value	Partial safety factor	Design value
Inner vessel pressure $p_{d,i}$ (MPa)	-0.1	1.0	-0.1
Jacket pressure $p_{d,j}$ (MPa) <sup>a</sup>	0.4	1.2	(0.48)
	0.02	1.0	(0.02)
			0.50
Inner vessel temperature $t_{d,i}$ ( $^\circ\text{C}$ )	20	1.0	20
Jacket temperature $t_{d,j}$ ( $^\circ\text{C}$ )	20	1.0	20

<sup>a</sup>The values in the first row correspond to the pressure proper, at the reference point, the values in the second row to the pressure due to static head, relative to the reference point. The specified pressure acts only in the lower chamber of the jacket, the pressure in the upper chamber is zero.

The relevant material parameters are given in Table E.6.2-2. The reference temperature for determining the material strength parameters, the modulus of elasticity and Poisson’s ratio is  $t_{rRM}=t_{rE}=t_r=20^{\circ}\text{C}$ , respectively.

The procedure used in this example and the initial imperfection used are similar to that used in the preceding example, E.6.1.

Mises’ equivalent stress distributions, in the model with maximum permissible imperfection and for the  $\gamma_R$ -fold of the design values of the actions, are shown in Fig. E.6.2-1, those at the inside surface of the inner dished end on the left and at the outside surface on the right. The corresponding distributions of the maximum of the absolute values of the principal total strains are shown in Fig. E.6.2-2; those at the inside surface on the left and at the outside surface on the right.

The model can carry the  $\gamma_R$ -fold of the design actions with maximum absolute value of principal total strains less than 5%.

Therefore:

**NOLC 3.2: O.K.**

Table E.6.2-2: Material parameters

NOLC 6.2						
Part	Material group	$E$ (GPa)	$R_m$ (MPa)	$R_{p1.0/tr}$ (MPa)	$\gamma_R$ –	$RM_d$ (MPa)
		(1)	(2)	(3)	(4)	(5)
1–6	8.1	200.0	500.0	240.0	1.25	240.0

Poisson’s ratio is 0.3.

For determining the partial safety factor  $\gamma_R$ , it is assumed that the (external) pressure test as called for in EN 13445-5 is to be carried out.

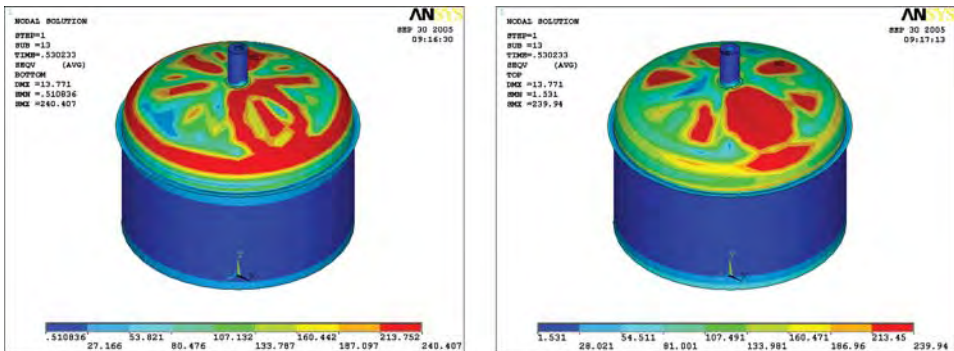


Figure E.6.2-1: Mises’ equivalent stress for  $\gamma_R$ -fold of design actions.

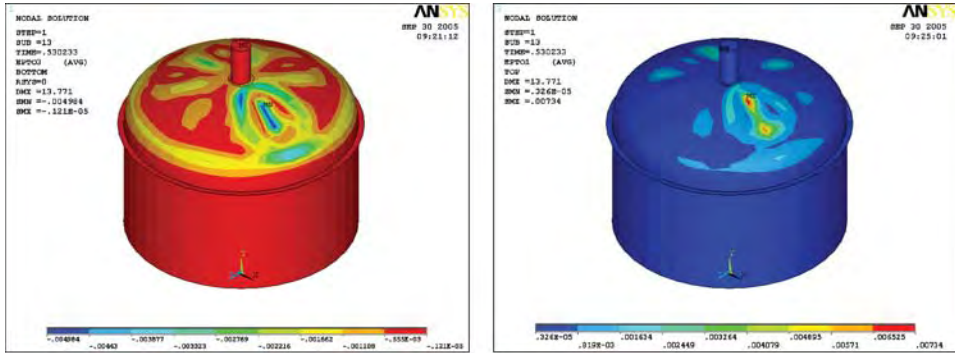


Figure E.6.2-2: Maximum principal total strain for  $\gamma_R$ -fold of design actions.

*Note 1:* For both load cases the procedure recommended in sub-clause 6.3 for critical cases has been used. For a comparison of results for different initial imperfections the better suited of the two load cases is the second one, i.e. NOLC 3.2: In this load case there is only one action, there are no thermal stresses and the design yield stress is time-independent. As a result, the increase of action to values higher than the  $\gamma_R$ -fold is straightforward. Therefore, in this load case NOLC 3.2, pressure was increased until the model ceased to carry any further increase. For the initial imperfection given by the scaled superposition of the first 10 classical buckling shapes the model can carry a pressure of 0.795 MPa. The corresponding action factor is  $0.795/(0.5+0.1)=1.325$ , i.e. 6% greater than the required one, given by  $\gamma_R=1.25$ . The maximum absolute value of the principal strains at this pressure is smaller than the limit of 5%. With the initial imperfection given by the scaled result of this investigation, i.e. the buckling shape that corresponds to the maximum pressure the model can carry, the model can carry a pressure of 0.765 MPa, i.e. with an action factor of 1.275, which is smaller than the preceding value, but still 2% larger than the required one. As in the preceding case, the maximum absolute value of the principal strains is smaller than the limit of 5%. Both pressure values are over 12% smaller than the corresponding value for the DBF approach in EN 13445-3 (sub-clause 8.8), albeit for different permissible shape deviations.

*Note 2:* The investigations showed also that approximately 96% of the axial resultant force due to pressure on the jacket's dished end has to be carried by the outer connection of the jacket and the inner shell, and only approximately 4% by the inner connection, near the nozzle. This fact, which, of course, will also show and be taken care of in the corresponding GPD design checks, can be easily overlooked in a DBF design.



## Annex E.7: Examples of Cyclic Fatigue Design Checks

Like in the preceding parts of Annex E, E.4 and E.5, the examples in this part are intended to illustrate the procedure in one specific design check, here the cyclic fatigue design check, and the examples deal, therefore, solely with individual load cases that are related to this F-DC.

The first example is a complement to the comparison of design details in Section E.4.2. The second is a complement to the investigations for the air cooler header considered in Sections E.4.3 and E.5.2.

### *E.7.1: F-DC of a cylindrical to hemispherical shell transition*

This example of an F-DC with hot spots in welded regions deals with the transition of the cylindrical shell to the hemispherical end as given by detail geometry 5 considered in Section E.4.2, and by the comparison of its cyclic fatigue life with that of the transition as given by detail geometry 1, the nominal one. For this example of an F-DC, the structures are assumed to be without a cladding and the model thicknesses (of E.4.2 and E.4.1) are assumed to be fatigue analysis thicknesses.

For the F-DC linear-elastic stress-concentration-free models are used. Therefore, the results obtained with the models used and described in Sections E.4.2 and E.4.1 can be carried over into this F-DC.

Results for diverse stresses at various points due to the maximum permissible pressure  $P_{GPD_{max}}$  of detail geometry 1 are listed in Table E.4.2-1. In the investigations discussed here, a cyclic action NOLC 4.1, from action point 0 to action point A and vice versa, is considered, with

- NOLC 4.1: 0:  $p_d=0$ ,  $t_d=$  ambient (20°C), stress-free,
- NOLC 4.1: A:  $p_d=165.8$  bar,<sup>10</sup>  $t_d=TS=400^\circ\text{C}$ ,


whereby temperature transients are sufficiently slow such that non-stationary thermal stresses can be neglected.

For detail geometry 5 the value of the maximum principal stress due to action NOLC 4.1:A is 118.4 MPa. This maximum principal stress occurs at a point, denoted in the following by D, at the inner surface of the lower end of the cylindrical shell, with first principal stress axis in circumferential direction. For a circumferential weld in the (undisturbed) cylindrical shell this maximum principal stress acts parallel to the weld joint direction, and the corresponding FAT Class is 80. In the butt weld region, the maximum principal stress range acts normal to the weld direction and occurs Point C. Its value is 116.5 MPa, slightly smaller than the one at Point D.

---

<sup>10</sup>This value corresponds to 90% of  $P_{GPD_{max}}$  of detail geometry 1, a value considered to be a realistically representative upper characteristic value in the sense of the F-DC.

Table E.7.1-1: Fatigue life calculation/Detail Geometry 5/Point C

 DBA : DR F-DC	<b>Fatigue design check</b> <i>welded material – ferritic steel</i>	F-DC WM/FS Page 1
<b>Object:</b> Example E.7.1		<b>Load case No.:</b> NOLC 4.1
<b>Point:</b> C		<b>Direction<sup>a</sup>:</b> ⊥

**Input data**

$t_{c,max} = 400^{\circ}\text{C}$ $t_{c,min} = 20^{\circ}\text{C}$ $t^* = 0.75t_{c,max} + 0.25t_{c,min} = 305^{\circ}\text{C}$ $R_m = 520 \text{ MPa}$ $R_{p0.2t^*} = 218.5 \text{ MPa}$ $e_n = 110 \text{ mm}$	FAT class <sup>b</sup> : 80 (EN 13445-3, Annex P) Stress range at $5 \cdot 10^6$ cycles: (EN13445-3, Table 18-7) $\Delta\sigma_D = 59 \text{ MPa}$ Constants <sup>b</sup> of $\Delta\sigma_R - N$ curve: (EN 13445-3, Table 18-7) $m = 3: C = 1.02 \cdot 10^{12}$ $m = 5: C = 3.56 \cdot 10^{15}$
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**Stress data**

$\Delta\sigma_{struc} = 116.5 \text{ MPa}$	relevant <sup>c</sup> structural principal stress range
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**18.8 Plasticity correction factor**

General case <sup>d</sup>	Special case <sup>e</sup>
If $\Delta\sigma_{eq} > 2R_{p0.2t^*}$ $k_e = 1 + A_0 [0.5(\Delta\sigma_{eq,l}/R_{p0.2t^*}) - 1]$ $A_0 = \begin{cases} 0.4 & \text{for } R_m \leq 500 \text{ MPa} \\ 0.4 + (R_m - 500)/3000 & \text{for } 500 \leq R_m \leq 800 \text{ MPa} \\ 0.5 & \text{for } 800 \leq R_m \leq 800 \text{ MPa} \end{cases}$ $A_0 = \quad, k_e = \quad, \Delta\sigma = k_e \Delta\sigma_{eq} = \quad \text{MPa}$ Else $\Delta\sigma = \Delta\sigma_{struc} = 116.5 \text{ MPa}$	If $\Delta\sigma_{eq} > 2R_{p0.2t^*}$ $k_v = \text{Max}[1.0; 0.7/(0.5 + 0.4 \cdot R_{p0.2t^*}/\Delta\sigma_{eq,l})]$ $k_v = \quad, \Delta\sigma = k_v \Delta\sigma_{eq} = \quad \text{MPa}$ Else $\Delta\sigma = \Delta\sigma_{struc} = \quad \text{MPa}$
$\Delta\sigma = 116.5 \text{ MPa}$	

*Note:* If stresses of both cases are to be considered, the correction has to be made on each component of the stress tensors, with  $k_e$  and  $k_v$  calculated with the above formulas with  $\Delta\sigma_{eq}$  for the full equivalent stress range, force and displacement controlled parts. The factor  $k_e$  is applied to the force controlled parts and  $k_v$  to the displacement controlled parts of the stress tensors, then both tensors are added and the new stress range is calculated. The plasticity correction factor of the general case may be used for the special case as well.

<sup>a</sup>Direction of normal to the investigated incipient crack plane relative to the weld joint direction, ⊥ or //.


<sup>b</sup>With respect to determined FAT class.

<sup>c</sup>Range of structural principal stress closest to the investigated direction, see footnote<sup>a</sup>.

<sup>d</sup>Stresses other than those of the special case.

<sup>e</sup>Non-linearly distributed parts of thermal stresses, evaluated in the plane approximating the assumed crack surface.

Table E.7.1-1: Continued

 <b>A &amp; B</b>	DBA : DR F-DC	<b>Fatigue design check</b> <b>welded material – ferritic steel</b>	F-DC WM/FS Page 2
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**18.10.6.2 Temperature correction factor  $f_{t^*}$**

For  $t^* > 100^\circ\text{C}$        $f_{t^*} = 1.03 - 1.5 \cdot 10^{-4} t^* - 1.5 \cdot 10^{-6} t^{*2} = 0.8447$   
 Else  $f_{t^*} = 1$   
 $f_{t^*} = 0.8447$

**18.11.1.2 Thickness correction factor  $f_{ew}$**

Case 1: $e_n \leq 25 \text{ mm}$  $f_{ew} = 1$	Case 2: $25 \text{ mm} \leq e_n \leq 150 \text{ mm}$ $f_{ew} = (25/e_n)^{0.25} = 0.6905$ $f_{ew} = 0.6905$	Case 3: $e_n \geq 150 \text{ mm}$ $f_{ew} = 0.6389$
--	--	--

**18.11.2.1 Overall correction factor  $f_w$**

$$f_w = f_{ew} f_{t^*} = 0.5832$$

$$\Delta\sigma_{cor} = \Delta\sigma / f_w = 199.8 \text{ MPa}$$

**18.11.3 Allowable number of cycles**

Case 1: $\Delta\sigma_{cor} > \Delta\sigma_D$ :  $m = 3$ $C = 1.02 \times 10^{12}$ $N = C / (\Delta\sigma_{cor})^m = 127960$	Case 2: $\Delta\sigma_{cor} < \Delta\sigma_D$ and at least one other sub-cycle with $\Delta\sigma_{cor} > \Delta\sigma_D$ : $m = 5$ $C =$ $N = C / (\Delta\sigma_{cor})^m =$ $N = 127960 \text{ cycles}$	Case 3: $\Delta\sigma_{cor} < \Delta\sigma_D$ and all other sub-cycles with $\Delta\sigma_{cor} > \Delta\sigma_D$ :  $N = \infty$
--	--	---

**Remarks:**

For detail geometry 1 the maximum principal stress due to action NOLC 4.1:A occurs in the butt weld region, at point A, results from the principal stress normal to the weld joint direction and its value is 142.7 MPa. At Point D the maximum principal stress is the same as for detail geometry 5.

Table E.7.1-2: Some results of fatigue life calculations

Detail geometry hot spot	$\Delta\sigma$ (MPa)	FAT class	$N$ (cycles) <sup>a</sup>
Detail geometry 5 Point C	116.5	80	127,960
Detail geometry 5 Point D	118.4	80 <sup>b</sup>	121,900
Detail geometry 1 Point A	142.7	80	69,630
Detail geometry 1 Point D	118.4	80 <sup>b</sup>	121,900

<sup>a</sup>Allowable number of cycles according to the F-DC.

<sup>b</sup>For assumed circumferential weld.

All these hot spots are in welded regions of ferritic material. The detailed calculation for the fatigue life at Point C of detail geometry 5 is given in the fatigue calculation form in Table E.7.1-1, a form convenient for reporting and checking. Some results for the allowable number of cycles NOLC 7.1 for detail geometries 1 and 5 are listed in Table E.7.1-2.

**Conclusion:** As far as (full) pressure cycle fatigue is concerned, detail geometry 5, with misalignment not permissible by the (routine) DBF approach is much better than the (ideal) detail geometry 1. The difference in allowable number of cycles is quite remarkable. For detail geometry 5, the circumferential welds in the undisturbed cylindrical shell and the transition weld between cylindrical shell and hemispherical ends are almost equally critical. For the circumferential weld incipient crack planes normal to the weld joint direction are critical, for the transition joint incipient crack planes in weld joint direction. For detail geometry 1 the transition weld is the critical one, with critical incipient crack planes in weld joint direction.

### ***E.7.2: F-DC of an air cooler header***

This example of an F-DC with a hot spot in an unwelded region deals with the air cooler header considered in Sections E.4.3 and E.5.2.

In the PD-DC, discussed in E.5.2, the action NOLC 2.1:A resulted in the linear-elastic model in a fairly large equivalent stress and a compatibility ratio greater than 1.0 at one point in the inner surface of the tubes, near the outer face of the tubesheet. The allowable number of action cycles in this hot spot is determined in the following.

The considered action cycle NOLC 4.1 passes through the action points 0-A1-A2-A3-0 with

- NOLC 4.1: 0:  $p_d = 0$ ,  $t_d = \text{ambient (20°C)}$ , stress-free,
- NOLC 4.1: A1:  $p_d = 64.8 \text{ bar}$ ,<sup>11</sup>  $t_d = \text{ambient}$ ,
- NOLC 4.1: A2:  $p_d = 64.8 \text{ bar}$ ,  $t_d = TS = 120^\circ\text{C}$ ,
- NOLC 4.1: A3:  $p_d = 0$ ,  $t_d = TS = 120^\circ\text{C}$ ,

whereby temperature transients are sufficiently slow such that non-stationary thermal stresses can be neglected. The header is not insulated, and, therefore, (stationary) thermal stresses are to be included in the investigation. As a rough approximation, the temperature distribution determined in E.4.3 NOLC 1.2 (see Fig. E.4.3-8) has been used here as well.

The resulting elastic stresses in the hot spot due to the various actions, as per FEM output, are

$$\sigma_{ij}^{(A1)} = \begin{bmatrix} -84.6 & -0.2 & 19.8 \\ 0.2 & 323.3 & 0.0 \\ -19.8 & 0.0 & 17.1 \end{bmatrix}, \quad \sigma_{ij}^{(A2)} = \begin{bmatrix} -219.3 & -0.2 & -20.7 \\ 0.2 & 383.8 & 0.0 \\ -20.7 & 0.0 & 30.6 \end{bmatrix}$$

$$\sigma_{ij}^{(A3)} = \sigma_{ij}^{(A2)} - \sigma_{ij}^{(A1)}.$$

The relevant stress range of the resulting stress cycle can be guessed, but, because of the change in orientation of principal axes, the more complicated approach outlined in Section 7.14 is used here. The fatigue relevant functions  $\sigma_{\text{eq}\Delta}(\tau)$ , given by Equation (7.28), are shown in Fig. E.7.2-1.

The curve denoted by 0 corresponds to the reference parameter  $\tau$  at the zero action, the one denoted by A1 to the reference parameter at action A1. The maximum stress range is 526.4 MPa, resulting from the stress difference for the action A2 and the zero action. The corresponding mean stress, defined by Equation (7.29), is given by

$$\bar{\sigma}_{ij} = \begin{bmatrix} -109.7 & -0.1 & 10.4 \\ 0.1 & 191.9 & 0.0 \\ -10.4 & 0.0 & 15.3 \end{bmatrix}.$$

The detailed calculation for the fatigue life at this hot spot is given in the fatigue calculation form in Table E.7.2-1. In this case of a hot spot in an unwelded region, this calculation form is much less useful for the calculation itself – some correction factors depend on the number of cycles, and, therefore, iteration is

<sup>11</sup>This value corresponds to 90% of  $PS$ , a value considered to be a realistically representative upper characteristic value in the sense of the F-DC.

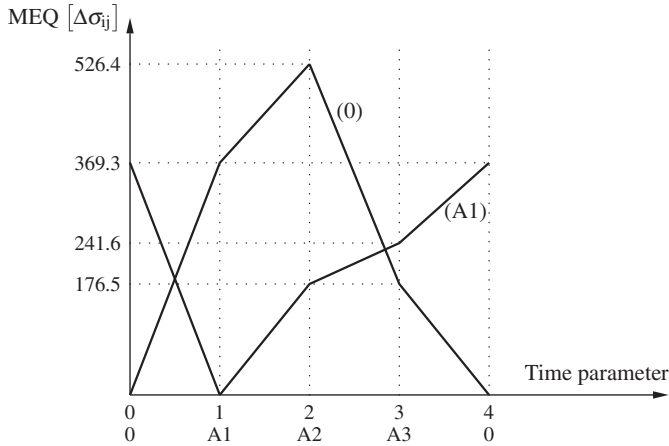


Figure E.7.2-1: Fatigue relevant functions.

required. The calculation form is still convenient for reporting the results obtained by means of short and simple programmes, and is useful for the checking of results. The allowable number of cycles is 9425.

As illustration of the approach and of its possible advantages, the detailed procedure for the determination of the plasticity correction factor, as outlined at the end of Section 7.6.1, is used in the following.

The strain tensor at the hot spot of the linear-elastic model for action A1, already known from the FEM calculations performed, is given by

$$\varepsilon_{ij}^{(l)} = \begin{bmatrix} -166.8 & -0.1 & 13.1 \\ 0.1 & 213.8 & 0.0 \\ -13.1 & 0.0 & -9.1 \end{bmatrix} \cdot 10^{-5}$$

and the Mises equivalent strain by  $MEQ[\varepsilon_{ij}^{(l)}] = 332.0 \cdot 10^{-5}$ .

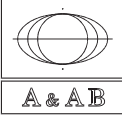
The PD-DC check has shown that the tubesheet shakes down to linear-elastic behaviour, whereas the tubes shake down to alternating plasticity. Therefore, taking into account the expected small plastic strain range, a linear-elastic ideal-plastic constitutive law is considered for the non-linear model with yield stress values given by

$$R_{p0,2/t^*} = \begin{cases} 193.6 \text{ MPa for the tubes} \\ 316.8 \text{ MPa for the plates} \end{cases}$$

These values were chosen as approximate values for the required cyclic constitutive law.

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Table E.7.2-1: Fatigue life calculation/Tube hot spot

	DBA : DR F-DC	<b>Fatigue design check</b> <i>unwelded material – ferritic steel</i>	F-DC UWM/FS Page 1
<b>Object:</b> Example E.7.2		<b>Load case:</b> NOLC 4.1	<b>Point:</b> MX in tubes
<b>Input data</b>		<b>Fatigue curve data</b>	
$t_{c,max} = 100^{\circ}\text{C}$ $t_{c,min} = 20^{\circ}\text{C}$ $t^* = 0.75t_{c,max} + 0.25t_{c,min} = 80^{\circ}\text{C}$ $R_m = 360 \text{ MPa}$ $R_{p0.2/t^*} = 193.6 \text{ MPa}$ $e_n = 2.4 \text{ mm}$ $R_z = 200 \mu\text{m}$ roughness-height index (EN 13445-3, Table 18-8)		Endurance stress range (for $2 \cdot 10^6$ cycles) $\Delta\sigma_D = 0.63R_m + 21.0 = 247.8 \text{ MPa}$ Iteration <b>initial value</b> for number of cycles $N = 9425$ Allowable stress range for $N$ cycles $\Delta\sigma_R = 0.63R_m - 11.5 + 46000/\sqrt{N} = 761.7 \text{ MPa}$	
<b>Stress data</b>			
$\Delta\sigma_{eq,t} = 526.4 \text{ MPa}$ equivalent total stress range <sup>a</sup>  $\bar{\sigma}_{eq} = 0 \text{ MPa}$ equivalent mean of total stress <sup>b</sup>		$\Delta\sigma_{struct} = 526.4 \text{ MPa}$ structural equivalent stress range <sup>c</sup>	
<b>Theoretical elastic stress concentration factor <math>K_t</math></b>		<b>Effective stress concentration factor <math>K_{eff}</math></b>	
$K_t = \Delta\sigma_{eq,t}/\Delta\sigma_{struct} = 1.0$		$K_{eff} = 1 + \frac{1.5(K_t - 1)}{1 + 0.5 \text{ Max}[1; K_t \Delta\sigma_{struct,eq}/\Delta\sigma_D]} = 1.0$	
<b>18.8 Plasticity correction factor</b>			
<b>General case<sup>d</sup></b>		<b>Special case<sup>e</sup></b>	
If $\Delta\sigma_{eq,t} > 2R_{p0.2/t^*}$ $k_e = 1 + A_0[0.5(\Delta\sigma_{eq,t}/R_{p0.2/t^*}) - 1]$ $A_0 = \begin{cases} 0.4 & \text{for } R_m \leq 500 \text{ MPa} \\ 0.4 + (R_m - 500)/3000 & \text{for } 500 \leq R_m \leq 800 \text{ MPa} \\ 0.5 & \text{for } 800 \leq R_m \leq 1000 \text{ MPa} \end{cases}$ $A_0 = 0.4, k_e = 1.144, \Delta\sigma = k_e \Delta\sigma_{eq,t} = 602.2 \text{ MPa}$ Else $\Delta\sigma = \Delta\sigma_{eq,t} = \text{MPa}$		If $\Delta\sigma_{eq,t} > 2R_{p0.2/t^*}$ $k_v = \text{Max}[1.0; 0.7/(0.5 + 0.4 \cdot R_{p0.2/t^*}/\Delta\sigma_{eq,t})]$ $k_v = \text{MPa}, \Delta\sigma = k_v \Delta\sigma_{eq,t} = \text{MPa}$ Else $\Delta\sigma = \Delta\sigma_{eq,t} = \text{MPa}$	
		$\Delta\sigma = 602.2 \text{ MPa}$	

*Note:* If stresses of both cases are to be considered, the correction has to be made on each component of the stress tensors, with  $k_e$  and  $k_v$  calculated with the above formulas with  $\Delta\sigma_{eq,t}$  for the full equivalent stress range, force and displacement controlled parts. The factor  $k_e$  is applied to the force controlled parts and the factor  $k_v$  to the displacement controlled parts of the stress tensors, then both tensors are added and the new stress range is calculated. The plasticity correction factor of the general case may be used for the special case as well.

<sup>a</sup>Range of Mises' equivalent stress of the differences of the total stresses.

<sup>b</sup>Equivalent mean stress, if required corrected for plastification effects.

<sup>c</sup>Range of Mises' equivalent stress of the differences of the structural stresses.

<sup>d</sup>Stresses other than those of the special case.

<sup>e</sup>Non-linearly distributed parts of thermal stresses, evaluated in the plane approximating the assumed crack surface.





The non-linear FEM calculation resulted in a difference of the strain tensors, in the hot spot and for actions A2 and 0 (after A3), of

$$\Delta \varepsilon_{ij}^{(nl)} = \begin{bmatrix} 165.2 & 0.0 & 33.5 \\ 0.0 & -24.3 & 0.0 \\ -33.5 & 0.0 & -43.4 \end{bmatrix} \cdot 10^5,$$

with resulting Mises' equivalent strain

$$MEQ[\Delta \varepsilon_{ij}^{(nl)}] = 367.8 \cdot 10^{-5}.$$

Therefore, the plasticity correction factor, obtained via this detailed procedure is given by

$$k_e = MEQ[\varepsilon_{ij}^{(nl)}] / MEQ[\varepsilon_{ij}^{(l)}] = 1.108,$$

which is slightly smaller than 1.144, the value obtained above in the standard approach. Use of this plasticity correction factor renders an allowable number of cycles of 10 270, i.e. 9% larger than the result with the procedure using the plasticity correction factor obtained with Equation (7.3b).

*Note 1:* There are cases where the route for determining the plasticity correction factor via FEM models is favourable and, therefore, may provide a remedy under critical situations.

*Note 2:* It may be good engineering practice to use tubes of softer material than the tubesheet material, in order to ease the tube expansion procedure, but too large a difference is risky – there may be alternating plasticity in the tubes, resulting in strongly reduced cyclic life, and there may be sliding friction, with the risk of fretting corrosion.

## **Annex E.8: Examples of Static Equilibrium Design Checks**

The main purpose of the examples given here is not to discuss routine procedures in the admissibility check of supporting structures, but to discuss the usage of the partial safety factor concept in the context of these routine procedures.

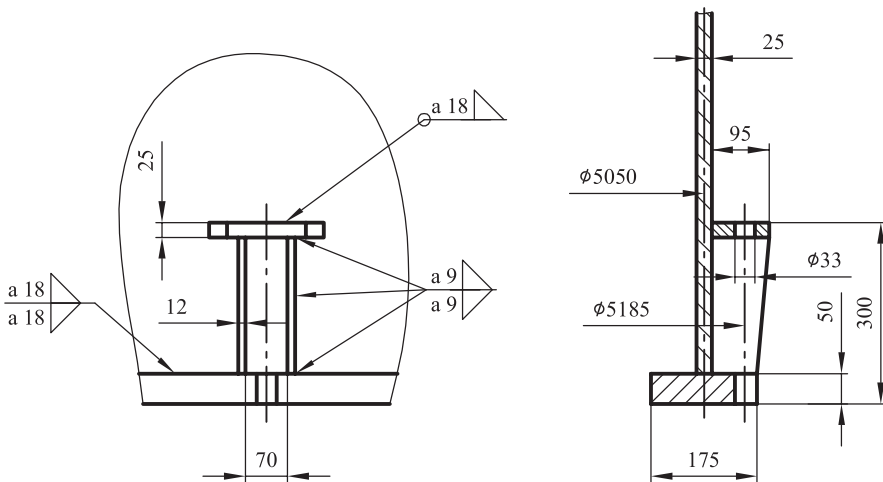
### ***E.8.1: SE-DC of a skirt supported heavy reactor column***

In this example two load cases, each with three sub-cases, are considered:

- NOLC 5.1: Operating weight + Wind
- TLC 5.1: Self-weight + Fluid weight + Wind

**Design Data:**

- Geometry of lower end of the skirt and base ring: see Fig. E.8.1.
- Material of base ring: Steel EN 10028 P265GH
- Material of anchor bolts: Steel EN 10025 S235JRG2
- Material of foundation: Structural concrete C25/30
- Self-weight:<sup>12</sup>  $F_{SW}=3.48\text{MN}$
- Operating weight:<sup>13</sup>  $F_O=3.6\text{MN}$
- Fluid weight during hydraulic test:  $F_F=2.68\text{MN}$
- Wind:<sup>14</sup>
  - Resultant (horizontal) force according to the (local) code for wind action:  $F_W=53.49\text{ kN}$ .
  - Resultant moment with respect to the centre point of the base (ring):  $M_W=0.6096\text{ MN m}$



a-measure of fillet not smaller than 0,7 times the thinner of the joined parts

Figure E.8.1: Skirt geometry of reactor column.

<sup>12</sup>Including internals, the whole skirt and accessories.

<sup>13</sup>The same value for the upper and the lower characteristic values is chosen – the main contribution is from self-weight.

<sup>14</sup>According to the (local) code for wind action the specified values are considered to be upper characteristic values.

### 8.1.1: NOLC 5.1/U

This load case deals with uplifting – weight is favourable, but wind action is unfavourable.

Design value of the operating weight:

$$F_{Od1} = F_O \cdot \gamma_{Gf} = 3.6 \cdot 0.8 = 2.88 \text{ MN}$$

Design value of the wind moment

$$M_{Wd1} = M_W \cdot \gamma_Q = 0.6096 \cdot 1.5 = 0.9144 \text{ MNm}$$

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to operating weight:

$$n_{lO1} = F_{Od1} / (D_m \pi) = 181.5 \text{ Nmm}^{-1}$$

where  $D_m = 5050$  mm is the mean shell diameter.

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to wind:

$$\text{Maximum: } n_{lW1} = M_{Wd1} / (D_m^2 \pi / 4) = 45.7 \text{ Nmm}^{-1}$$

**Uplifting check:**

$$n_{lO1} > n_{lW1} \rightarrow$$

**NOLC 5.1/U: O.K.**  
(no anchoring required)

### 8.1.2: NOLC 5.1/P

This design check deals with the maximum contact pressure at the base ring to foundation interface – operating weight and wind act unfavourably.

Design value of the operating weight:

$$F_{Od2} = F_O \cdot \gamma_{Gu} = 3.6 \cdot 1.2 = 4.32 \text{ MN}$$

Design value of the wind moment (see NOLC 5.1/U):

$$M_{Wd2} = 0.9144 \text{ MN m}$$

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to the operating weight:

$$n_{lO2} = F_{Od2} / (D_m \pi) = 272.3 \text{ N mm}^{-1}$$

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to the wind moment:

$$\text{Maximum: } n_{IW2} = M_{Wd2} / (D_m^2 \pi / 4) = 45.7 \text{ Nmm}^{-1}$$

**Maximum pressure at the base ring to foundation interface:**<sup>15,16</sup>

$$p_{\max} = (n_{IO2} + n_{IW2}) / b_{BR} = 317.9 / 175 = 1.8 \text{ MPa}$$

where  $b_{BR} = 175 \text{ mm}$  is the base ring width.

$$\text{Maximum pressure}^{15} = 1.8 \text{ MPa}$$

**Maximum stress in the base ring:**<sup>17</sup>

Maximum moment:

$$M_{BR} = p_{\max} \cdot b_{BR}^2 / 2 = 5109 \text{ N}$$

where  $b_{BR} = 75 \text{ mm}$  is the externally projecting width.

Maximum stress:<sup>17</sup>

$$\sigma_{BR} = 6 \cdot m_{BR} / e_{BRa}^2 = 12.3 \text{ MPa}$$

where  $e_{BRa} = 48.5 \text{ mm}$  is the analysis thickness of the base ring:

$$e_{BRa} = e_{BR} - \delta_e - c = 50 - 1.0 - 0.5 = 48.5 \text{ mm},$$

with the absolute value of the possible negative thickness tolerance  $\delta_e = 1.0 \text{ mm}$  according to EN 10029, Table 1, Tolerance Class A, and the (chosen) corrosion allowance  $c = 0.5 \text{ mm}$ .

Stress check in base ring:  $\sigma_{BR} < RM_{dBR} = 245 / 1.25 = 196 \text{ MPa} \rightarrow$

**NOLC 5.1/P: O.K.**

<sup>15</sup>Value to be forwarded to civil engineering design department.

<sup>16</sup>The external projecting width of the base ring  $b_{BR} = 75 \text{ mm}$  is smaller than the one permissible in the calculation (see [23], Annex L):

$$w_p = e_{BRa} \sqrt{RM_{dBR} / (3\beta_j k_j RM_{dc})} = 117.6 \text{ mm}$$

where  $e_{BRa} = e_{BR} - \delta_e - c = 48.5 \text{ mm}$  denotes the analysis thickness of the base ring, with the absolute value of the possible negative thickness tolerance  $\delta_e = 1.0 \text{ mm}$  according to EN 10029, Table 1, Tolerance Class A, and the (chosen) corrosion allowance  $c = 0.5 \text{ mm}$ ;  $RM_{dBR}$  denotes the design value of the base ring material ( $245 / 1.25 = 196.0 \text{ MPa}$ ),  $RM_{dc}$  the characteristic cylinder compressive strength of the structural concrete foundation ( $25 / 1.5 = +6.7 \text{ MPa}$ ),  $\beta_j$  the separation factor equal to  $3/3$ , and  $k_j$  the concentration factor for which the value 1.0 is conservatively assumed, in accordance with [23] Annex L. Therefore, a uniform contact pressure distribution across the base ring width may be assumed.

<sup>17</sup>Owing to insufficient ductility of concrete the maximum stress is calculated for a linear-elastic constitutive law, and limited by the design value  $RM_{dBR}$ .

### 8.1.3: NOLC 5.1/D

This design check deals with the failure mode rigid body displacement – operating weight acts favourably, and wind unfavourably.

Design value of the operating weight:

$$F_{Od3} = F_{Od1} = 2.88 \text{ MN}$$

Design value of the wind force:

$$F_{Wd3} = F_W \gamma_Q = 53.49 \cdot 1.5 = 80.24 \text{ kN}$$

Design value of the reaction, i.e. design value of the static friction force:

$$F_{Rd} = F_{Od3} \mu_{sd} = 2.88 \cdot 0.5 = 1.44 \text{ MN}$$

with the design value of the coefficient of static friction<sup>18</sup>  $\mu_{sd}$ .

Rigid body displacement check:

$$F_{Wd3} < F_{Rd} \rightarrow$$

**NOLC 5.1/D: O.K.**

### 8.1.4: TLC 5.1/U

This testing load case deals with uplifting – weight acts favourably, and wind unfavourably.

Design value of the self-weight:

$$F_{SWd4} = F_{SW} \gamma_{Gf} = 3.48 \cdot 0.8 = 2.78 \text{ MN}$$

Design value of the fluid weight:

$$F_{Fd4} = F_F \gamma_{Gf} = 2.68 \cdot 0.8 = 2.14 \text{ MN}$$

Design value of the wind moment

$$M_{Wd4} = M_{Wd1} = 0.9144 \text{ MNm}$$

---

<sup>18</sup>A value of  $\mu_{sd} = \mu_{sc} / \gamma_{\mu} = 0.6 / 1.2 = 0.5$  may be used for steel on concrete according to EN 1337-1:2000: Structural bearings. Part 1: General design rules.

**Uplifting check:** The sum of the design values of favourable actions is larger than in NOLC 1/U and the design value of the unfavourable wind moment is equal to that in NOLC 1/U → Uplifting check is encompassed by NOLC 1/U →

**NOLC 5.1/U: O.K.**

### 8.1.5: TLC 5.1/P

This testing load case deals with the maximum contact pressure at the base ring to foundation interface – self-weight, fluid weight and wind act unfavourably.

Design value of the self-weight:

$$F_{SWd5} = F_{SW} \cdot \gamma_{Gu} = 3.48 \cdot 1.2 = 4.18 \text{ MN}$$

Design value of the fluid weight:

$$F_{Fd5} = F_F \cdot \gamma_{Gu} = 2.68 \cdot 1.2 = 3.22 \text{ MN}$$

Design value of the sum:

$$F_{Vd5} = 7.39 \text{ MN}$$

Design value of the wind moment<sup>14</sup>

$$M_{Wd5} = M_{Wd1} = 0.9144 \text{ MNm}$$

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to the vertical force  $F_{Vd5}$ :

$$n_{IF5} = F_{Vd5} / (D_m \pi) = 465.8 \text{ MNm}^{-1}$$

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to the wind moment:

$$\text{Maximum: } n_{IW5} = M_{Wd5} / (D_m^2 \pi / 4) = 45.7 \text{ MNm}^{-1}$$

**Maximum pressure at the base ring to foundation interface:**

$$p_{\max} = (n_{IF5} + n_{IW5}) / b_{BR} = 511.5 / 175 = 2.9 \text{ MPa}$$

$$\text{Maximum pressure}^{15} = 2.9 \text{ MPa}$$

**Maximum stress in base ring:**<sup>16</sup>

$$\text{Maximum moment: } M_{BR} = p_{\max} \cdot b_{BR}^2 / 2 = 8220 \text{ N}$$

$$\text{Maximum bending stress: } \sigma_{BR} = 6 \cdot m_{BR} / e_{BRa}^2 = 20.9 \text{ MPa}$$

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Stress check in base ring  $\sigma_{BR} < RM_{BR}/\gamma_R = 245/1.05 = 233.3 \text{ MPa} \rightarrow$   
**NOLC 5.1/P: O.K.**

### 8.1.6: TLC 5.1/D

This load case deals with rigid body displacement – weight acts favourably, wind unfavourably.

Design value of the self-weight:

$$F_{SWd6} = F_{SWd4} = 2.78 \text{ MN}$$

Design value of the fluid weight:

$$F_{Fd6} = F_{Fd4} = 2.14 \text{ MN}$$

Design value of the wind force

$$F_{Wd6} = F_{Wd3}$$

Design value of the reaction, i.e. design value of the static friction force

$$F_{Rd} = (F_{SWd6} + F_{Fd6}) \cdot \mu_{sd} = 2.46 \text{ MN}$$

Rigid body displacement check:

$$F_{Wd6} < F_{Rd} \rightarrow$$

**TLC 5.1/D: O.K.**

### *E.8.2: SE-DC of a skirt supported light pressure vessel*

This example is complementary to the preceding one: With the exception of the skirt thickness, the thickness of the chairs' top plates, and the base ring thickness, the same skirt geometry applies, but the operating weight is much smaller. Only one load case, with three sub-cases, is considered – the load case that is critical for the bolts:

NOLC 1: Operating weight + wind

#### **Design Data:**

The geometry of the lower end of the skirt and of the base ring is as in the preceding example, but with thickness of the skirt 12 mm, the thickness of the top plate of

chairs 16 mm, and the thickness of the base ring 20 mm. The mean diameter of the skirt is still 5050 mm (see Fig. E.8.2).

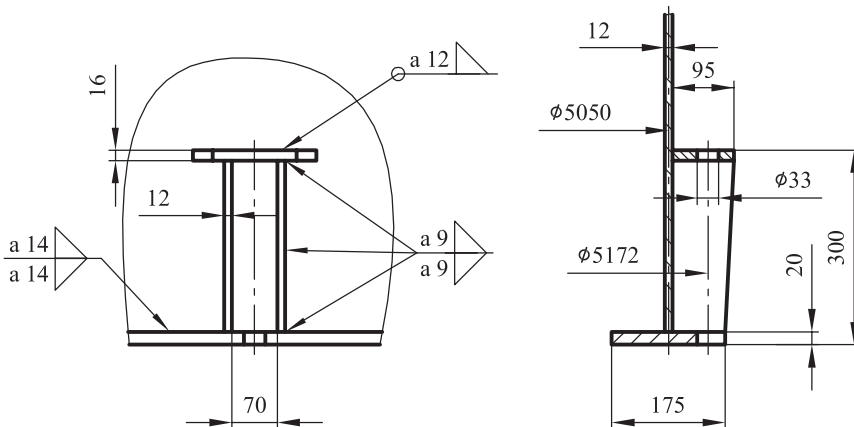
- Material of base ring: Steel EN 10028 P265GH
- Material of anchor bolts: Steel EN 10025 S235JRG2
- Material of foundation: Structural concrete C25/30
- Operating weight:<sup>19</sup> $F_O=0.36\text{MN}$
- Wind:<sup>20</sup>
  - Resultant (horizontal) force according to the (local) code for wind action:  
 $F_W=53.49\text{ kN}$ .
  - Resultant moment with respect to the centre point of the base (ring):  
 $M_W=0.6096\text{ MNm}$

### 8.2.1: NOLC 5.1/U

This design check deals with uplifting – weight acts favourably and wind acts unfavourably.

Design value of the operating weight <sup>21</sup>:

$$F_{Odl}=F_O \cdot \gamma_{Gf}=0.36 \cdot 0.8=0.288\text{ MN}$$



a-measure of fillet not smaller than 0,7 times the thinner of the joined parts

Figure E.8.2: Skirt geometry details.

<sup>19</sup>The same value for the upper and the lower characteristic values is chosen – the main contribution is from self-weight.

<sup>20</sup>According to the (local) code for wind action the specified values are considered to be upper characteristic values.

<sup>21</sup>The same value for the upper and the lower characteristic values is chosen – the main contribution is from self-weight.



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Design value of the wind moment:

$$M_{wd1} = M_w \cdot \gamma_Q = 0.6096 \cdot 1.5 = 0.9144 \text{ MNm}$$

Longitudinal membrane force in the skirt at the junction to the base ring due to operating weight:

$$n_{1O1} = F_{Od1} / (D_m \pi) = 18.15 \text{ Nmm}^{-1}$$

where  $D_m = 5050$  mm is the mean shell diameter.

Longitudinal membrane force in the skirt at the junction to the base ring due to wind:

$$n_{1W1} = M_{wd1} / (D_m^2 \pi / 4) = 45.65 \text{ Nmm}^{-1}$$

Because of  $n_{1O1} < n_{1W1}$ , anchor bolts are required.

Required design value of the bolt force to fulfill the principle's requirement:

$$F_{Bdreq} > (n_{1W1} \cdot n_{1O1}) \cdot D_m \pi / N = 36.6 \text{ kN}$$

with the number of bolts  $N = 12$ .

Lower characteristic value of the bolt force:

$$F_{Bcinf} = F_{Bdreq} / \gamma_{Gf} > 36.6 / 0.8 = 45.8 \text{ kN}$$

### **Pretensioning of bolts required:**<sup>22,23</sup>

Required nominal bolt pretensioning force:

$$F_{B0req} > F_{Bcinf} > 45.8 \text{ kN}$$

Taking into account the scatter values  $\varepsilon_-$  and  $\varepsilon_+$  of pretensioning with torque wrench given in Annex G of EN 13445-3, a value of the pretensioning bolt force  $F_{B0} = 60$  kN is chosen. This chosen value is slightly larger than the required one, given by

$$(1 + \varepsilon_-) \cdot F_{Bcinf} = 1.2 \cdot 45.8 = 55.0 \text{ kN}$$

<sup>22</sup>Fact and value of chosen pre-tensioning bolt force to be forwarded to civil engineering design department.

<sup>23</sup>The value given below is a (conservative) lower bound, valid for an infinitely stiff foundation, see also the comment after Section 8.2.

The lower characteristic value of the pretensioning bolt force is then given by

$$F_{Bcinf} = F_{B0} / (1 + \varepsilon_-) = 60 / 1.2 = 50 \text{ kN.}$$

The design value of the pretensioning bolt force, given by

$$F_{Bd} = F_{Bcinf} \cdot \gamma_{Gf} = 50 \cdot 0.8 = 40 \text{ kN,}$$

is then larger than the required bolt force  $F_{Bdreq} = 36.6 \text{ kN}$ , and the upper characteristic value of the pretensioning bolt force is then given by

$$F_{Bcsup} = (1 + \varepsilon_+) \cdot F_{B0} = 1.2 \cdot F_{B0} = 72.0 \text{ kN.}$$

### Check of bolt stress:

$$\sigma_B = F_{Bcsup} \cdot \gamma_{Gu} / A_B = 72.0 \cdot 1.2 / 570.6 = 154.0 \text{ MPa} < R_{m,dB} = 225 / 1.25 = 180 \text{ MPa} \rightarrow \mathbf{O.K.}$$

for bolt material EN 10025 S235JRG2, stress relevant cross-section of  $M30$  bolts  $A_B = 560.7 \text{ mm}^2$  and  $\gamma_{Gu} = 1.2$ .

### Check of top plate:

Design value of the maximum moment in the top plate<sup>24</sup>:

$$M_{TPd} = F_{Bcsup} \cdot \gamma_{Gu} \cdot (b_r + e_r - w_F) / 4 = 72.0 \cdot 36 / 4 = 648 \text{ kNm}$$

where  $b_r + e_r = 70 + 12 = 82 \text{ mm}$  is the mean distance of the ribs, and  $w_F = 46 \text{ mm}$  the width across the flats of the  $M30$  bolts.

Design value of the resistance (for material P265GH):

$$R_{dMTP} = R_{m,d} \cdot (b_{TP} - d_{bTP}) \cdot e_{TPd}^2 / 4 = (245 / 1.25) \cdot (95 - 33) \cdot 14.9^2 / 4 = 674 \text{ kNm}$$

where  $b_{TP} = 95 \text{ mm}$  is the width of the top plate,  $d_{bTP} = 33 \text{ mm}$  the bore in the top plate, and  $e_{TPd} = 14.9 \text{ mm}$  the analysis thickness of the top plate,

$$e_{TPd} = e_{TP} - \gamma_e - c = 16 - 0.6 - 0.5 = 14.9 \text{ mm}$$

with the absolute value of the negative possible tolerance  $\delta_e = 0.6 \text{ mm}$  according to EN 10029, Table 1, Tolerance Class A, and the (chosen) corrosion allowance  $c = 0.5 \text{ mm}$ .

<sup>24</sup>Conservatively considered as simply supported beam, i.e. bending support by ribs and skirt neglected.

Check of top plate:

$$M_{TPd} = 648 < R_{dMTP} = 674 \text{ kNmm} \rightarrow \text{O.K.}$$

**Check of anchor chairs' ribs:**<sup>25</sup>

Design value of compressive force

$$F_{dr} = F_{Bcsup} \cdot \gamma_{Gu} / 2 = 36 \text{ kN}$$

Design value of resistance:

$$F_{dres} = \chi \cdot \beta_A \cdot A \cdot R \cdot M / \gamma_R = 119.3 \text{ kN}$$

with A denoting the used (analysis) cross-section of the rib of analysis thickness 11.0 mm, with  $\delta_e = 0.5$  mm according to EN 10029, Table 1, Tolerance Class A and the (chosen) corrosion allowance  $c = 0.5$  mm,

$$A = 75 \cdot 11.0 = 825.0 \text{ mm}^2,$$

and with

$RM = 245$  MPa, for the rib material EN 10028 P265GH,

$\gamma_R = 1.25$ , the relevant partial safety factor,

$\beta_A = 1.0$ , a factor, here 1.0, for a part of Class 1,

$l_{rb} = 0.7(300 - 16/2 - 20/2) = 197.4$  mm, the relevant buckling length,

$i = \sqrt{I/A} = 3.18$  mm, the relevant radius of gyration of the cross-section,

$\lambda = l_{rb}/i = 62.2$ , the slenderness,

$\lambda_s = \pi \cdot \sqrt{E/RM} = 93.9 \cdot \sqrt{235/RM} = 91.7$ , the reference slenderness for the rib material – Steel EN 10028 P265GH,

$\bar{\lambda} = \lambda/\lambda_s = 0.678$ , the relative slenderness, and

$\chi = 0.7381$ , the reduction factor for buckling according to [23], sub-clause 5.5, relevant buckling line  $c$ .

---

<sup>25</sup>Conservatively calculated as buckling columns, clamped at the lower end and upper end pivoted, in accordance With [23], Chapter 5.5.

Check of ribs:

$$F_{dr} = 36 < F_{drres} = 119.3 \text{ kN}$$

Total sub-load case result:

**NOLC 5.1/U:O.K.**

### 8.2.2: NOLC 5.1/P

This design check deals with the maximum contact pressure at the base ring to foundation interface – operating weight, pretensioning force, and wind act unfavourably.

Upper characteristic value of the pretensioning force per bolt, carried over from NOLC 1/U:

$$F_{Bcsup} = 72.0 \text{ kN}$$

Design value of the pretensioning bolt force:

$$F_{Bod2} = F_{Bsup} \cdot \gamma_{Gu} = 72.0 \cdot 1.2 = 86.4 \text{ kN}$$

Design value of the operating weight:

$$F_{Od2} = F_O \cdot \gamma_{Gu} = 360 \cdot 1.2 = 432 \text{ kN}$$

Design value of the wind moment:

$$M_{Wd2} = M_{Wd1} = 0.9144 \text{ MNm}$$

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to operating weight:

$$n_{1O2} = (F_{Od2}/D_m \pi) = 27.2 \text{ Nmm}^{-1}$$

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to pretensioning:

$$n_{1B02} = N \cdot F_{Bod2}/(D_m \pi) = 65.4 \text{ Nmm}^{-1}$$

Resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to wind action:

$$\text{Maximum: } n_{1W2} = M_{Wd2}/(D_m^2 \pi/4) = 45.7 \text{ Nmm}^{-1}$$

**Maximum pressure at the base ring to foundation interface:**<sup>26,27</sup>

$$p_{\max} = (n_{IO2} + n_{IB02} + n_{IW2}) / e_{BR\text{ eff}} = 1.3 \text{ MPa}$$

with the effective base ring with<sup>27</sup>  $e_{BR\text{ eff}} = 12 + 2 \cdot 45.8 = 103.6 \text{ mm}$

$$\text{Maximum pressure}^{26} = 1.3 \text{ MPa}$$

**Maximum stress in base ring:**

$$\text{Maximum moment}^{27}: m_{BR} = p_{\max} \cdot w_p^2 / 2 = 1.40 \text{ kN}$$

$$\text{Maximum bending stress}^{28} \sigma_{BR} = 6 \cdot m_{BR} / e_{BRa}^2 = 23.5 \text{ MPa}$$

where  $e_{BRa} = 18.9 \text{ mm}$  is the analysis thickness of the base ring

$$e_{BRa} = e_{BR} - \delta_e - c = 20 - 0.6 - 0.5 = 18.9 \text{ mm},$$

with the absolute value of the possible negative thickness tolerance  $\delta_e = 0.6 \text{ mm}$  according to EN 10029, Table 1, Tolerance Class A, and the (chosen) corrosion allowance  $c = 0.5 \text{ mm}$ .

Stress check in base ring:  $\sigma_{BR} < RM_{BR} / \gamma_R = 245 / 1.25 = 196 \text{ MPa} \rightarrow$  **O.K.**  
**NOLC 5.1/P:OK.**

**Comment on the approximation used above:**

The procedure used above is based on the presupposition of an infinitely stiff foundation, the influence of the flexibility of the prestressed parts, i.e. foundation, base ring, part of skirt, is neglected. This simple procedure is recommended for non-critical cases. For critical cases the following approach, which takes this flexibility into account, is recommended.

<sup>26</sup>Value to be forwarded to civil engineering design department.

<sup>27</sup>External and internal projecting width of the base ring,  $b_{BRe}$  and  $b_{BRi}$ , are larger than the width permissible to be used in the calculation:

$$w_p = e_{BRa} \sqrt{RM_{dBR} / (3\beta_j k_j RM_{dc})} = 45.8 \text{ mm},$$

where  $e_{BRa} = e_{BR} - \delta_e - c = 18.9 \text{ mm}$  denotes the analysis thickness of the base ring, with the absolute value of the possible negative thickness tolerance  $\delta_e = 0.6 \text{ mm}$  according to EN 10029 Table 1, Tolerance Class A, and the (chosen) corrosion allowance  $c = 0.5 \text{ mm}$ ;  $RM_{dBR}$  denotes the design value of the base ring material ( $245 / 1.25 = 196.0 \text{ MPa}$ ),  $RM_{dc}$  the characteristic cylinder compressive strength of the structural concrete foundation ( $25 / 1.5 = 16.7 \text{ MPa}$ ),  $\beta_j$  the separation factor 2/3, and  $k_j$  the concentration factor (1.0 in accordance with [23] Annex L conservatively assumed). Therefore, a constant contact pressure over the effective width equal to skirt thickness plus  $2c$  is used.

<sup>28</sup>Owing to insufficient ductility of concrete the maximum stress is calculated for a linear-elastic constitutive law, and limited by the design value  $RM_{dBR}$

The flexibility factor  $\beta_F$  of the prestressed parts, i.e. of those parts that are under compression in pretensioning, is defined by

$$\delta_F = \beta_F \cdot F,$$

where  $\delta_F$  is the shortening of the prestressed parts under the arbitrary pretensioning force  $F$ . The flexibility factor  $\beta_B$  of the prestressing parts, i.e. of those parts that are under tension in pretensioning, is defined by

$$\delta_B = \beta_B \cdot F,$$

where  $\delta_B$  is the elongation of the prestressing parts under the arbitrary pretensioning force  $F$ .

The required pretensioning force (per bolt) is, in this more elaborate approach, given by

$$F_{B0} = (1 - \beta) \cdot F_{Breq},$$

with

$$\beta = \beta_F / (\beta_F + \beta_B).$$

The maximum pressure at the base ring to foundation interface is, in this approach, given by

$$p_{\max} = [n_{IO1d} + n_{IB0d} + (1 - \beta)(n_{IO2d} + n_{IWd})] / b_{BR \text{ eff}}$$

where  $n_{IO1d}$  is the resultant longitudinal force (per unit length) in the skirt at the junction to the base ring due to weight already acting before pretensioning and  $n_{IO2d}$  the one due to the weight added after pretensioning of the anchor bolts. For the usual design details with steel bolts and concrete foundation  $\beta = 0.14$  may be used.

### 8.2.3: NOLC 5.1/D

This load case deals with rigid body displacement – pretensioning force and weight act favourably and wind acts unfavourably.

Design value of the total pretensioning bolt force:

$$F_{Bd3} = N \cdot F_{B \text{ inf}} \cdot \gamma_{Gf} = 12 \cdot 50 \cdot 0.8 = 480.0 \text{ kN}$$

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Design value of the operating weight:

$$F_{Od3} = F_{Od1} = 288.0 \text{ kN}$$

Design value of the wind force:

$$F_{Wd3} = F_W \cdot \gamma_Q = 53.49 \cdot 1.5 = 80.2 \text{ kN}$$

Design value of the reaction, i.e. design value of the static friction force:<sup>29</sup>

$$F_{Rd} = (F_{Od3} + F_{Bd3}) \cdot \mu_{sd} = 384.0 \text{ kN}$$

### Rigid body displacement check

$$F_{Wd3} < F_{Rd} \rightarrow$$

**NOLC 5.1/D: O.K.**

### *E.8.3: SE-DC of a leg supported vertical storage tank*

In this example of a vertical storage vessel, which is fairly large for leg supports, only one load case, with three sub-cases, is considered:

NOLC 1: Operating, weight + wind

#### Design Data:

- Leg geometry: see Figs. E.8.3-1 and E.8.3-2
- Material of legs: EN 10025 S235JRG2
- Material of base plates: EN 10025 S235JRG2
- Material of foundation: Structural concrete C35/45
- Minimum operating weight<sup>30</sup> = Self weight:<sup>30</sup>  $F_{SW} = 407.6 \text{ kN}$
- Maximum operating weight<sup>30</sup>:  $F_{Osup} = 543.3 \text{ kN}$
- Self weight of one leg:  $F_{SWL} = 910 \text{ N}$
- Wind<sup>31, 32</sup>
  - Resultant (horizontal) force according to the (local) code for wind action:  
 $F_W = 5.75 \text{ kN}$
  - Resultant moment with respect to the centre point of the base:  
 $M_W = 28.76 \text{ kN m}$

<sup>29</sup>A value of  $\mu_{sd} = \mu_{sc} / \gamma_{\mu} = 0.6 / 1.2 = 0.5$  may be used for steel on concrete according to EN 1337-1:2000: Structural bearings. Part 1: General design rules.

<sup>30</sup>Including internals, legs, and accessories.

<sup>31</sup>According to the (local) code for wind action the specified values are considered to be upper characteristic values.

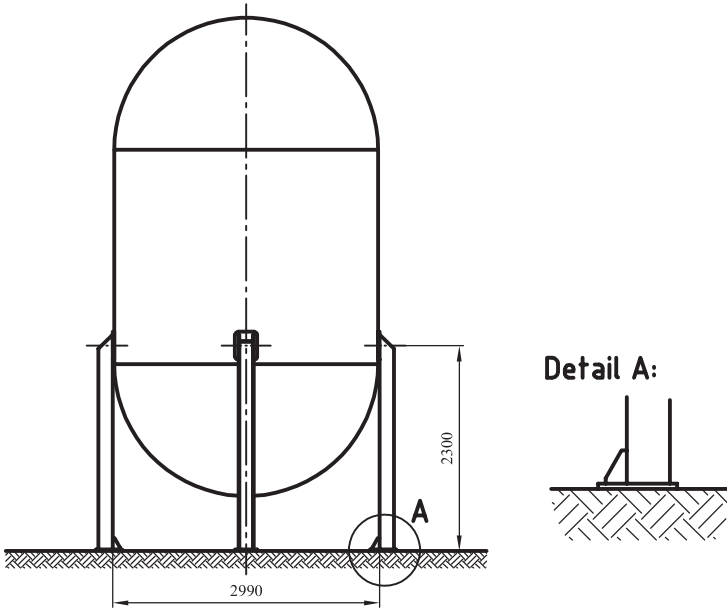


Figure E.8.3-1: Vertical storage tank.

- Pipe thrust<sup>32</sup>:
  - Horizontal force:  $F_p = 22.25 \text{ kN}$
  - Lever arm with respect to the centre point of the base:  $a_p = 6095 \text{ mm}$ ,
  - Moment with respect to the centre point of the base:  $M_p = 135.61 \text{ kNm}$

**8.3.1: NOLC 5.1/U**

This load case deals with uplifting – weight acts favourably, but wind and pipe thrust act unfavourably.

Design value of the operating weight:

$$F_{Oa1} = F_{sw} \cdot \gamma_{Gf} = 407.6 \cdot 0.8 = 326.1 \text{ kN}$$

Design value of the wind moment with respect to centre point of the base:

$$M_{Wd1} = M_w \cdot \gamma_Q = 28.76 \cdot 1.5 = 43.14 \text{ kNm}$$

<sup>32</sup>Conservatively, both actions, wind and pipe thrust, are considered to act independently and in the same direction, but, because of the determination of the piper thrust actions via a piping analysis with the pipe to pressure vessel nozzle modelled as clamped support, the pipe thrust action is treated as bounded variable action.



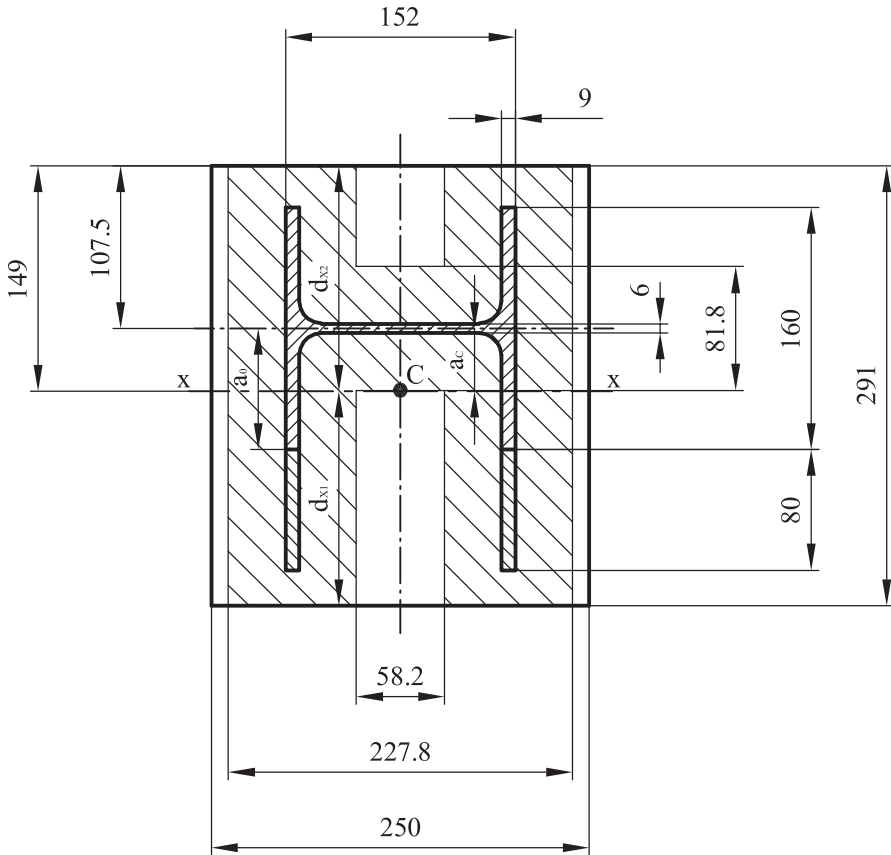


Figure E.8.3-2: Effective contact pressure bearing area and leg cross-section.

Design value of pipe thrust moment<sup>32</sup> with respect to centre point of the base:

$$M_{Pd1} = M_P \cdot \gamma_{Qb} = 135.61 \cdot 1.0 = 135.61 \text{ kNm}$$

Design value of the vertical leg force due to wind and pipe thrust:<sup>33</sup>

Maximum:  $F_{LdWP1} = 4 \cdot (M_{Wd1} + M_{Pd1}) / (D_{LC} \cdot N) = 59.8 \text{ kN}$  with the number of legs  $N=4$ , and the diameter  $D_{LC}=2990 \text{ mm}$  of the circle through the assumed position of the leg reaction forces (see Fig. E.8.3).

<sup>33</sup> In the used model (of the supporting legs) the legs are considered to be columns clamped at the upper ends, at the reinforcing plates welded to the shell or at the shell, and with reaction forces and a (limited) reaction moment acting at the intersection points of the legs' axes with the base plate to foundation interfaces. These reactions are determined by means of (global) equilibrium equations complemented by simplifying assumptions:

Design value of the vertical leg force due to operating weight:

$$F_{LdO1} = F_{Od1} / N = 81.5 \text{ kN}$$

Uplifting check:

$$F_{LdO1} > F_{LdWP1} \rightarrow$$

**NOLC 8.1/U: O.K.**

### 8.3.2: NOLC 5.1/P

This load case deals with the maximum contact pressure at the leg base plates to foundation interfaces – operating weight, wind, and pipe thrust act unfavourably.

Design value of the operating weight:

$$F_{Od2} = F_{Osup} \cdot \gamma_{Gu} = 543.3 \cdot 1.2 = 6520 \text{ kN}$$

Design value of the wind moment with respect to centre point of the base:

$$M_{Wd2} = M_{Wd1} = 43.14 \text{ kNm}$$

---

<sup>33</sup> (continued)

The vertical reaction forces are assumed to be proportional to their distance from the axis of the global moment of imposed forces with respect to the centre point 0 of the leg axes circle, and the moment of all these vertical reaction forces must be in equilibrium with the moment of all imposed forces, again with respect to the same reference point. This results in  $F_{Lv \max} = 4 \cdot M / (D_{LC} \cdot N)$ , where  $F_{Lv \max}$  is the largest of all vertical leg forces,  $M$  is the (global) moment of all imposed forces,  $N$  the number of legs, and  $D_{LC}$  is the diameter of the circle through the vertical reaction forces, see Figure E.8.4.

The ratio of the horizontal reaction force of any leg to the sum of all horizontal reaction forces (in all legs) is assumed to be equal to the quotient of the moment of inertia of the cross-section of the considered leg, about the axis perpendicular to the direction of the force and through the leg's axis, to the sum of all these moments of inertia of the cross-section of all legs.

Part of the moment due to the offset of the legs' axes and the mean shell surface is assumed to be carried by a linearly distributed contact pressure distributions between base plates and foundation: Due to the offset of leg axes to the shell of the vessel, the transfer of vertical forces, from the vessel to the legs, results in each leg in a bending moment at the upper end. Part of that moment will be transferred to the leg, the remaining part transferred to the vessel shell. It is assumed that the part of the moment transferred by the leg is given by the product of the vertical reaction force (in the leg) and the distance  $a_0$  of the innermost fibre of the leg cross-section from the centroid of the effective contact pressure bearing area, see Figure E.8.4. Slightly different variants of this assumption are in use [117–119], which assign a larger or smaller portion of the offset moment to the base plate to foundation interface. The (usual) assumption used here results in reasonable base plate sizes and leg to base plate designs. In all variants no other moment is assigned to the base plate to foundation interface.

Design value of the pipe thrust moment with respect to the center point of the base:

$$M_{Pd2} = M_{Pd1} = 135.61 \text{ kNm}$$

Effective contact pressure bearing area of the legs' base plates<sup>34</sup>

Leg cross-section: IPB1 160 (see Fig. E.8.3-2)

Effective contact pressure bearing area: see Fig. E.8.3-2:

$$A = 54114 \text{ mm}^2$$

Moment of inertia of the effective contact pressure bearing area about the axis  $x-x$  through the centroid:

$$I_{xx} = 3.5118 \cdot 10^8 \text{ mm}^4$$

Distances from the centroid:

$$d_{x1} = 149.0 \text{ mm}, \quad d_{x2} = 142.0 \text{ mm}, \quad a_c = 37.0 \text{ mm}$$

Design value of the vertical leg force due to operating weight:

$$F_{Ld02} = F_{Od2} / N = 651.2 / 4 = 163.0 \text{ kN}$$

Design value of the vertical leg force due to wind and pipe thrust:

$$F_{LdWP2} = F_{LdWP1} = 59.8 \text{ kN}$$

Maximum design value of the vertical leg force:

$$F_{Ld2} = F_{Ld02} + F_{LdWP2} = 222.8 \text{ kN}$$

---

<sup>34</sup>The width of projecting parts of the base plates, projecting over the leg column cross-section, are laterally larger than the width permissible to be used in the calculation according to [23], Annex L:

$$w_p = e_{BRa} \cdot \sqrt{RM_{dBR} / (3\beta_j k_j RM_{dc})} = 37.9 \text{ mm},$$

where  $RM_{dBP}$  denotes the design value of the base plate material ( $235/1.25 = 188.0 \text{ MPa}$ ),  $RM_{dc}$  the characteristic cylinder compressive strength of the structural concrete foundation ( $35/1.5 = 23.3 \text{ MPa}$ ),  $\beta_j$  the separation factor  $2/3$ , and  $k_j$  the concentration factor (1.0 in accordance with [23] Annex L conservatively assumed), and  $e_{BRa} = e_{BP} - \delta_c - c = 20.0 - 0.6 - 0.5 = 18.9 \text{ mm}$  the analysis thickness of the base plates, with  $\delta_c = 0.5 \text{ mm}$  being the absolute value of the possible negative tolerance according to EN 10029, and  $c = 0.5$  the (chosen) corrosion allowance. Therefore, the effective contact pressure area, permissible to be used in the calculation according to [23] Annex L, is used. This area, based on the leg column cross-section extended by  $w_p$ , is shown in Figure E.8.4 together with the leg cross-section and the actual base plate dimensions.

Average contact pressure at the leg's base plate to foundation interface:<sup>34</sup>

$$p_{ave} = F_{Ld2}/A = 4.1 \text{ MPa}$$

Moment in legs due to the offset of the leg axes from the shell mean surface:

$$\text{Offset: } a_0 = 80 \text{ mm}$$

$$\text{Moment: } M = (F_{Ld2} - F_{SWL}) \cdot a_0 = 17.75 \text{ MN mm}$$

Moment with respect to the centroid of the effective contact pressure bearing area:

$$\text{Lever arm: } a_0 - a_c = 43.0 \text{ mm}$$

$$\text{Moment: } M_C = (F_{Ld2} - F_{SWL})(a_0 - a_c) - F_{SWL}a_c = 9.51 \text{ MN mm}$$

Maximum contact pressure due to  $M_C$ :

$$p_{b1} = M_C \cdot d_{x1}/I_{xx} = 4.0 \text{ MPa}$$

**Maximum contact pressure:**<sup>34,35</sup>

$$p_{\max} = p_{ave} + p_{b1} = 8.1 \text{ MPa}$$

Maximum contact pressure<sup>35</sup> = 8.1 MPa

**Check of stresses in base plates:**

Moment in critical cross-section 1-1, see Fig. E.8.4:

$$\text{Maximum: } m_{1-1} = p_{\max} \cdot w_p^2/2 = 5817 \text{ N}$$

Maximum stress:

$$\sigma_b = m_{1-1} \cdot 6/e_{BP}^2 = 98 \text{ MPa},$$

where  $e_{BP} = 18.9 \text{ mm}$  is the analysis thickness of the base plates and

$$\sigma_b = 98 \text{ MPa} < RM_d = 235/1.25 = 188 \text{ MPa} \rightarrow$$

**O.K.**

---

<sup>35</sup>Value to be forwarded to civil engineering design department.

**Check of minimum contact pressure:**

The operating weight acts in this check favourably, and wind and pipe thrust act unfavourably.

Design value of the operating weight:

$$F'_{Od2} = F_{SW} \cdot \gamma_{Gf} = 407.6 \cdot 0.8 = 326.1 \text{ kN}$$

Design value of the vertical leg force due to operating weight:

$$F'_{Ld2} = F'_{Od2} / N = 81.5 \text{ kN}$$

Maximum design value of the leg force:

$$F'_{Ld2} = F'_{Ld2} + F_{LdWP2} = 141.3 \text{ kN}$$

Average contact pressure at the leg's base plate to foundation interface<sup>34</sup>:

$$p'_{av} = F'_{Ld2} / A = 2.6 \text{ MPa}$$

Moment with respect to the centroid of the effective contact pressure bearing area:  
Moment:

$$M'_c = (F'_{Ld2} - F_{SWL})(a_0 - a_c) - F_{SWL} \cdot a_c = 6.00 \text{ MNmm},$$

Minimum contact pressure due to  $M'_c$ :

$$p'_{b2} = -M'_c \cdot d_{x2} / I_{xx} = -2.4 \text{ MPa}$$

**Minimum contact pressure check:**

$$p_{\min} = p'_{ave} + p'_{b2} = 2.6 - 2.4 = 0.2 \text{ MPa} > 0 \rightarrow \text{O.K.}$$

Total sub-load case result:

**NOLC 5.1/P : O.K.**

**8.3.3: NOLC 5.1/D**

This load case deals with the failure mode rigid body displacement – the operating weight acts favourably, and wind and pipe thrust act unfavourably

Design value of the operating weight

$$F_{Od3} = F_{SW} \cdot \gamma_{Gf} = 407.6 \cdot 0.8 = 326.1 \text{ kN}$$

Design value of the wind force:

$$F_{Wd3} = F_W \cdot \gamma_Q = 5.75 \cdot 1.5 = 8.6 \text{ kN}$$

Design value of the pipe thrust<sup>32</sup>:

$$F_{Pd3} = F_p \cdot \gamma_{Qb} = 22.25 \cdot 1.0 = 22.2$$

Design value of the reaction, i.e. design value of the static friction force<sup>36</sup>

$$F_{Rd} = F_{Od3} \cdot \mu_{sd} = 326.1 \cdot 0.5 = 163.0 \text{ kN}$$

with the design value of the coefficient of static friction <sup>36</sup>  $\mu_{sd}$ .

### Rigid body displacement check:

$$F_{Wd3} + F_{Pd3} = 30.8 \text{ kN} < F_{Rd} = 163.0 \text{ kN} \rightarrow$$

**NOLC 5.1/D:O.K.**

---

<sup>36</sup>A value of  $\mu_{sd} = \mu_{sc} / \gamma_{\mu} = 0.6 / 1.2 = 0.5$  may be used for steel on concrete according to EN 1337-1:2000: Structural bearings. Part 1: General design rules.

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# Annex L: ANSYS Input Listings

This annex contains ANSYS input listings and other lists useful for checking some of the examples dealt with in Annex E. The section numbering refers to the example designations in Annex E, e.g. Section L.5.2 deals with Example 5.2 dealt with in Annex E.5.2.

## L.4.1: GPD-DC of a hydrocracking reactor

```

1 /filnam,Model,1
2 /PREP7
3 *set,pi,acos(-1)
4 *afun,deg
5 !geometry cylindrical shell
6 *set,riz,2515/2
7 *set,raz,2515/2+190
8 *set,hz,2000
9 csys,0
10 k,1,riz,0,0
11 k,2,raz,0,0
12 k,3,raz,hz,0
13 k,4,riz,hz,0

14 !geometry hemispherical end
15 *set,rik,1297.5
16 *set,rak,1297.5+110
17 local,11,1,hz,
18 k,5,rak,acos(riz/rik),0
19 k,6,rik,acos(riz/rik),0

20 !geometry nozzle
21 *set,ris,520/2
22 *set,ras,520/2+130
23 *set,rar,520/2+90
24 *set,hsv,295
25 *set,hsr,150
26 *set,rf,50
27 k,7,rak,acos(ras/rak),0
28 k,8,rik,acos(ris/rik),0
29 l,5,7
30 l,8,6
31 csys,0
32 k,9,ras,hz+(sqrt(rak**2-
33 ras**2))+hsv
k,10,ris,hz+(sqrt(rak**2-
ras**2))+hsv
34 k,11,rar,hz+(sqrt(rak**2-
35 ras**2))+hsv+(ras-rar)/tan(30)
k,12,ris,hz+(sqrt(rak**2-
36 ras**2))+hsv+(ras-rar)/tan(30)
k,13,rar,hz+(sqrt(rak**2-
37 ras**2))+hsv+(ras-
rar)/tan(30)+hsr
k,14,ris,hz+(sqrt(rak**2-
38 ras**2))+hsv+(ras-
rar)/tan(30)+hsr
39 l,7,9
40 l,10,8
41 LFILLT,1,3,rf,0
42 LFILLT,2,4,rf,0

43 !geometry flange
44 *set,hf,200
45 *set,raf,1045/2
46 *set,rda,700/2
47 k,19,raf,hz+(sqrt(rak**2-
48 ras**2))+hsv+(ras-
rar)/tan(30)+hsr
k,20,raf,hz+(sqrt(rak**2-
49 ras**2))+hsv+(ras-
rar)/tan(30)+hsr+hf
k,21,rda,hz+(sqrt(rak**2-
50 ras**2))+hsv+(ras-
rar)/tan(30)+hsr+hf
49 k,22,ris,hz+(sqrt(rak**2-
51 ras**2))+hsv+(ras-
rar)/tan(30)+hsr+hf

52 a,1,2,3,4
53 a,4,3,5,6
54 a,10,9,11,12
55 a,12,11,13,14
56 LANG,2,15,90,,0.1

```



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```

55 LANG,4,16,90,,0.7
56 al,1,22,21,12
57 al,5,24,23,6,2,22
58 al,3,14,4,24
59 a,14,13,21,22
60 a,13,19,20,21

61 csys,0
62 k,100,0,0,0
63 k,101,0,1,0
64 VROTAT,all,,,,,,,,100,101,180,2

65 !defining elements
66 et,1,plane42,,,,,,,,
67 et,2,solid45,,,,,,,,

68 !material properties
69 !material 1 cylindrical shell
   & end
70 mp,ex,1,191915 !E-Modul
   305°C
71 mp,nuxy,1,0.3
72 TBDE,BKIN,1,,,
73 TB,BKIN,1,2,2,1
74 !TBTEMP,400
75 TBDATA,,135,0,,,,

76 !material 2 nozzle
77 mp,ex,2,191915 !E-Modul
   305°C
78 mp,nuxy,2,0.3
79 TBDE,BKIN,2,,,
80 TB,BKIN,2,2,2,1
81 TBDATA,,135,0,,,,

82 !material assignment
83 VSEL,s,volu,,1,2,1
84 VSEL,a,volu,,5,9,4
85 VSEL,a,volu,,10,13,3
86 VATT,1,,2,0
87 allsel
88 VSEL,u,volu,,1,2,1
89 VSEL,u,volu,,5,9,4
90 VSEL,u,volu,,10,13,3
91 VATT,2,,2,0
92 allsel

93 !meshing
94 lesize,5,,,11
95 lesize,2,,,4
96 lesize,23,,,4
97 lesize,6,,,3
98 lesize,7,,,5

99 lesize,9,,,5
100 lesize,12,,,5
101 lesize,14,,,5
102 lesize,16,,,5
103 lesize,19,,,5
104 lesize,25,,,5
105 lesize,27,,,5
106 lesize,28,,,5
107 lesize,22,,,5
108 lesize,24,,,5
109 lesize,26,,,5
110 lesize,29,,,5
111 lesize,30,,,5
112 lesize,3,,,10
113 lesize,4,,,10
114 lesize,15,,,5
115 lesize,17,,,5
116 lesize,18,,,5
117 lesize,20,,,5
118 lesize,1,,,25
119 lesize,21,,,25
120 lesize,11,,,8
121 lesize,13,,,8
122 lesize,8,,,30
123 lesize,10,,,30

124 lsel,s,line,,2,6,4
125 lsel,a,line,,23,23,0

126 lccat,all
127 allsel

128 asel,s,area,,1,9,1
129 amesh,all
130 allsel

131 EXTOPT,ACLEAR,1
132 EXTOPT,VSWE,AUTO,0
133 EXTOPT,ESIZE,22,0

134 VSWEEP,1,1,14
135 VSWEEP,10,14,53
136 VSWEEP,2,2,18
137 VSWEEP,11,18,57
138 VSWEEP,5,5,31
139 VSWEEP,14,31,70
140 VSWEEP,6,6,37
141 VSWEEP,15,37,76
142 VSWEEP,7,7,40
143 VSWEEP,16,40,79
144 VSWEEP,3,3,23
145 VSWEEP,12,23,62

```

```

146 VSWEEP,4,4,27
147 VSWEEP,13,27,66
148 VSWEEP,8,8,44
149 VSWEEP,17,44,83
150 VSWEEP,9,9,48
151 VSWEEP,18,48,87

152 !defining beam element
153 ET,3,BEAM189,
154 save
155 finish

156 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
157 !creating cross section for
    beam element
158 /filnam,Beamsection,1
159 /clear,
160 /PREP7
161 local,11,1,,,,,180,,,,
162 *set,ri,520/2
163 *set,ra,1045/2
164 k,1,ri,0
165 k,2,ra,0
166 k,3,ra,180
167 k,4,ri,180
168 a,1,2,3,4

169 et,1,mesh200,7
170 esize,ra-ri
171 mshkey,1
172 amesh,all
173 secwrite,beamsection,sect,,1
174 save
175 finish
176 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
177 !!!!!!!!!
178 /filnam,GPD_Mises,1
179 RESUME, GPD_Mises,db
180 /PREP7
181 SECTYPE,1,Beam,mesh,hrohr0,
182 SECREAD,beamsection,,mesh
183 SECOFFSET,origin,
184 k,200,0,hz+(sqrt(rak**2-
    ras**2))+ hsv+(ras-
    rar)/tan(30)+hsr+hf,0
185 k,201,0,hz+(sqrt(rak**2-
    ras**2))+ hsv+(ras-
    rar)/tan(30)+hsr+hf+200,0
186 l,200,201
187 latt,1,1,3

188 lesize,136,,,1
189 lmesh,136
190 allsel
191
192 !boundary conditions
193 csys,0
194 nrotat,all
195 nsel,s,loc,y,0,,,
196 d,all,uy,0
197 nsel,s,loc,z,0,,,
198 d,all,uz,0
199 nsel,s,loc,y,0
200 nsel,r,loc,x,0
201 d,all,ux,0
202 *set,h,hz+(sqrt(rak**2-
    ras**2))+ hsv+(ras-
    rar)/tan(30)+hsr+hf
203 nsel,s,loc,x,0
204 nsel,r,loc,z,0,
205 nsel,r,loc,y,h
206 d,all,rotx,0
207 d,all,roty,0
208 !rigid region!
209 csys,0
210 *set,h,hz+(sqrt(rak**2-
    ras**2))+ hsv+(ras-
    rar)/tan(30)+hsr+hf
211 nsel,s,loc,y,h,h+1,,
212 nsel,r,loc,x,0
213 nsel,r,loc,z,0
214 *get,ncerig,node,,num,max
215 nsel,s,loc,y,h,h+1,,
216 csys,5
217 nsel,r,loc,x,0,rda
218 csys,0
219 CERIG,ncerig,all,uy
220 nsel,s,loc,y,h,h+1,,
221 nsel,r,loc,x,0
222 csys,5
223 nsel,r,loc,x,0,rda
224 csys,0
225 CERIG,ncerig,all,ux
226
227 allsel
228 save
229 Finish

230 /solu
231 antype,0,new
232 eresx,no !copy integration
    point results to the nodes for
    all elements
    
```

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```

233
234 !!!!!!!!!!!!!
235 !!actions!!
236 !!!!!!!!!!!!!
237 *set,Pr,21.6
238 *set,Ms,313600000
239 *set,Fs,115200
240
241 !pressure
242 asel,s,area,,13,17,4
243 asel,a,area,,22,26,4
244 asel,a,area,,30,34,4
245 asel,a,area,,35,36,1
246 asel,a,area,,39,43,4
247 asel,a,area,,52,56,4
248 asel,a,area,,61,65,4
249 asel,a,area,,69,73,4

250 asel,a,area,,74,75,1
251 asel,a,area,,78,82,4
252 NSLA,s,1
253 sf,all,pres,Pr,
254 allsel

255 !nozzle force and moment
256 F,ncerig,FY,Pr*ris**2*pi/2-
    Fs/2,,,
257 F,ncerig,MZ,-Ms/2,,,

258 time,1
259 deltim,0.05,0.0001,0.05
260 neqit,15
261 outres,all,all
262 solv
263 finish

```

## L.4.2: GPD-DC of details of cylindrical shell to hemispherical end

```

1 /PREP7
2 *set,pi,acos(-1)
3 *afun,deg
4 !geometry cylindrical shell
5 *set,riz,2515/2
6 *set,raz,2515/2+190
7 *set,hz,2000
8 csys,0
9 k,1,riz,0,0
10 k,2,raz,0,0
11 k,3,raz,hz,0
12 k,4,riz,hz,0

13 !geometry hemispherical end
14 *set,rik,1297.5
15 *set,rak,1297.5+110
16 local,11,1,,hz,
17 k,5,rak,acos(riz/rik),0
18 k,6,rik,acos(riz/rik),0
19 k,7,rik,90,0
20 k,8,rak,90,0

21 a,6,5,8,7
22 csys,0
23 a,1,2,3,4
24 a,3,5,6,4

25 et,1,plane82,0,0,1,
    !plane82 2d, 8-node element!
26 !material data!

27 mp,ex,1,191915 !material
    properties
28 mp,nuxy,1,0.3
29 TBDE,BKIN,1,,,
30 TB,BKIN,1,2,2,1
31 TB,DATA,,135,0,,,,

32 esize,30 !meshing!
33 amap,3,3,5,6,4
34 amap,1,6,5,8,7
35 amap,2,1,2,3,4

36 csys,0
37 nrotat,all !boundary
    conditions!
38 nsel,s,loc,y,0
39 d,all,uy,0
40 allsel
41 nsel,s,loc,x,0
42 d,all,ux,0
43 allsel
44 save

45 /solu
46 antype,0,new
47 eresx,no !copy inte-
    gration point results to the
    nodes for all elements
48 *set,p,23
49 lsel,s,line,,4,8,4

```

```

50  lsel,a,line,,10
51  NSLL,S,1
52  sf,all,pres,p
53  allsel
54  csys,0
55  time,1

56  deltim,0.05,0.0001,0.05
57  neqit,15
58  outres,all,all
59  save
60  solv
61  finish
    
```

### L.4.3: GPD-DC of an air cooler header

**Input file: NOLC 1.1: LC without thermal stresses**

```

1  /filnam,Model,1
2  /PREP7

3  *set,pi,acos(-1)
4  *afun,deg

5  a=190+2*3
6  b=255+2*3
7  sa=34-3
8  sb=34-3
9  T=30
10 aw=6

11 !geometry plates
12 k,1,0,0,0
13 k,2,sb,0,0
14 k,3,sb,sa,0
15 k,4,sb,sa+b-aw,0
16 k,5,sb+aw,sa+b,0
17 k,6,sb+aw,2*sa+b,0
18 k,7,0,2*sa+b,0
19 k,8,sb+a,0,0
20 k,9,2*sb+a,0,0
21 k,10,2*sb+a,2*sa+b,0
22 k,11,sb+a-aw,2*sa+b,0
23 k,12,sb+a-aw,sa+b,0
24 k,13,sb+a,sa+b-aw,0
25 k,14,sb+a,sa,0

26 a,1,2,3,4,5,6,7
27 a,2,8,14,3
28 a,8,9,10,11,12,13,14
29 a,11,6,5,12

30 vext,1,4,1,0,0,T,

31 bore !see macro bore.mac

32 !defining elements
33 et,1,solid95,,,,,,

34 !material properties plates
35 mp,ex,1,206000
36 mp,nuxy,1,0.3
37 TBDE,BKIN,1,,,
38 TB,BKIN,1,1,,1
39 TBDATA,,205.1,0,,,,

40 !material properties tubes
41 mp,ex,3,206000
42 mp,nuxy,3,0.3
43 TBDE,BKIN,3,,,
44 TB,BKIN,3,1,,1
45 TBDATA,,126.1,0,,,,

46 !meshing
47 cmsel,s,tube,volu
48 Vatt,3,,,
49 esize,2.4
50 vsweep,all
51 allsel
52 cmsel,u,tube,volu
53 Vatt,1,,,
54 esize,sa/5
55 vsweep,2
56 vsweep,4
57 MSHKEY,0
58 MSHAPE,1,3d
59 VMESH,13,15,2

60 tube !see macro tube.mac

61 !boundary conditions
62 csys,0
63 nsel,s,loc,z,0,,,
64 nsel,r,loc,x,0,2*sb+a+lr,,
65 d,all,uz,0
66 nsel,s,loc,y,sa+b/2,,,
67 nsel,r,loc,x,sb+a,,,
    
```

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```

68 d,all,uy,0
69 nsel,s,loc,z,T,,,
70 nsel,r,loc,x,0,2*sb+a+lr,,
71 cp,1,uz,all
72 allsel
73 nsel,s,loc,x,2*sb+a+lr,
74 d,all,ux,0
75 d,all,rotx,0
76 d,all,roty,0
77 d,all,rotz,0
78 allsel

79 finish
80 save

81 !solution
82 /SOLU
83 !
84 eresx,no !copy integration
    point results to the nodes for
    all elements
85 !
86 *set,p,7.2*1.2
87 SFA,8,1,PRES,p
88 SFA,9,1,PRES,p
89 SFA,16,1,PRES,p
90 SFA,22,1,PRES,p
91 SFA,26,1,PRES,p
92 SFA,27,1,PRES,p
93 SFA,31,1,PRES,p
94 SFA,34,1,PRES,p
95 SFA,38,1,PRES,p
96 SFA,41,1,PRES,p
97 SFA,44,1,PRES,p
98 SFA,49,1,PRES,p
99 SFA,51,1,PRES,p
100 SFA,54,1,PRES,p
101 SFA,61,1,PRES,p
102 SFA,64,1,PRES,p
103 SFA,69,1,PRES,p
104 SFA,71,1,PRES,p
105 SFA,74,1,PRES,p
106 SFA,95,1,PRES,p
107 SFA,100,1,PRES,p
108 plugforce !see macro
    plugforce.mac
109 allsel
110 time,1
111 deltim,0.1,0.001,0.1
112 outres,all,all
113 save
114 solv

```

### Macro: bore.mac

```

1 x=0
2 y=sa+b/2-52
3 z=0
4 wplane,-1,x,y,z,x,y,z-
    1,x,y+1,z
5 cyl4,0,0,28.173/2,,,,sb
6 wplane,-
    1,x+sb+a,y,z,x+sb+a,y,z-1,
    x+sb+a,y+1,z
7 cyl4,0,0,25.65/2,90,25.65/2-
    2.371,270,sb
8 x=0
9 y=sa+b/2+52
10 z=0
11 wplane,-1,x,y,z,x,y,z-
    1,x,y+1,z
12 cyl4,0,0,28.173/2,,,,sb
13 wplane,-
    1,x+sb+a,y,z,x+sb+a,y,z-1,
    x+sb+a,y+1,z
14 cyl4,0,0,25.65/2,90,25.65/2-
    2.371,270,sb
15 x=0
16 y=sa+b/2-104
17 z=T
18 wplane,-1,x,y,z,x,y,z-
    1,x,y+1,z
19 cyl4,0,0,28.173/2,,,,sb
20 wplane,-
    1,x+sb+a,y,z,x+sb+a,y,z-1,
    x+sb+a,y+1,z
21 cyl4,0,0,25.65/2,-90,25.65/2-
    2.371,90,sb
22 x=0
23 y=sa+b/2+104
24 z=T
25 wplane,-1,x,y,z,x,y,z-
    1,x,y+1,z
26 cyl4,0,0,28.173/2,,,,sb
27 wplane,-
    1,x+sb+a,y,z,x+sb+a,y,z-1,
    x+sb+a,y+1,z
28 cyl4,0,0,25.65/2,-90,25.65/2-
    2.371,90,sb
29 x=0
30 y=sa+b/2
31 z=T
32 wplane,-1,x,y,z,x,y,z-
    1,x,y+1,z

```

```

33  cyl4,0,0,28.173/2,,,,sb
34  wplane,-
    1,x+sb+a,y,z,x+sb+a,y,z-
    1,x+sb+a,y+1,z
35  cyl4,0,0,25.65/2,-90,25.65/2-
    2.371,90,sb

36  Vsel,s,volu,,5,13,2
37  cm,bohrung,volu
38  Vsel,s,volu,,6,14,2
39  cm,tube,volu
40  Vsel,s,volu,,1,4,1
41  cm,rahmen,volu
42  allsel
43  vsbv,rahmen,bohrung,,,delete
44  allsel
45  vsbv,rahmen,tube,,,keep
46  vsel,s,volu,,1
47  vsel,a,volu,,5,11,2
48  vdele,all,,,1
    
```

### Macro: tube.mac

```

1  !creating cross sections for
    beam elements
2  save
3  finish
4  /filnam,beam1,1
5  /clear,
6  /PREP7
7  local,11,1,,,,180,,,,
8  *set,ra,25.4/2
9  *set,ri,ra-2.4
10 k,1,ri,0
11 k,2,ra,0
12 k,3,ra,180
13 k,4,ri,180
14 a,1,2,3,4
15 et,1,mesh200,7
16 esize,ra-ri
17 mshkey,1
18 amesh,all
19 secwrite,beam1,sect,,1
20 finish

21 /filnam,beam2,1
22 /clear,
23 /PREP7
24 local,11,1,,,,0,,,,
25 *set,ra,25.4/2
    
```

```

26 *set,ri,ra-2.4
27 k,1,ri,0
28 k,2,ra,0
29 k,3,ra,180
30 k,4,ri,180
31 a,1,2,3,4
32 et,1,mesh200,7
33 esize,ra-ri
34 mshkey,1
35 amesh,all
36 secwrite,beam2,sect,,1
37 finish

38 !modelling tubes
39 /filnam,Model,1
40 RESUME,Model,db
41 /PREP7
42 *set,lr,5460
43 ET,3,BEAM189,
44 SECTYPE,1,Beam,mesh,hrohr0,
45 SECREAD,beam1,,,mesh
46 SECOFFSET,origin,

47 numcmp,kp
48 numcmp,line

49 x=2*sb+a
50 y=sa+b/2
51 z=T
52 csys,0
53 k,,x+sb,y,z
54 k,,x+lr,y,z
55 *get,nkp,kp,,num,max
56 l,nkp-1,nkp
57 *get,nline,line,,num,max
58 lsel,s,line,,nline
59 lesize,all,,,3
60 latt,3,,3,,,1
61 lmesh,nline
62 nsel,s,loc,z,z,,,
63 nsel,r,loc,y,y,,,
64 nsel,r,loc,x,x+sb,,,
65 *get,ncerig,node,,num,max
66 local,20,1,x,y,z,0,0,90
67 asel,s,loc,x,0,25.65/2
68 asel,r,loc,z,0
69 local,21,0,x,y,z,0,0,90
70 extopt,esize,10
71 mat,3
72 vext,all,,,0,0,sb,25.4/25.65,
    25.4/25.65,
73 csys,20
    
```

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```

74 nsel,s,loc,x,0,25.65/2
75 nsel,r,loc,z,sb,
76 CERIG,ncerig,all,ux,
77 csys,21
78 nsel,r,loc,y,0,,,
79 CERIG,ncerig,all,uy,
80 allsel

81 x=2*sb+a
82 y=sa+b/2-104
83 z=T
84 csys,0
85 k,,x+sb,y,z
86 k,,x+lr,y,z
87 *get,nkp,kp,,num,max
88 l,nkp-1,nkp
89 *get,nline,line,,num,max
90 lsel,s,line,,nline
91 lesize,all,,,3
92 latt,3,,3,,,1
93 lmesh,nline
94 nsel,s,loc,z,z,,,
95 nsel,r,loc,y,y,,,
96 nsel,r,loc,x,x+sb,,,
97 *get,ncerig,node,,num,max
98 local,20,1,x,y,z,0,0,90
99 asel,s,loc,x,0,25.65/2
100 asel,r,loc,z,0
101 local,21,0,x,y,z,0,0,90
102 extopt,esize,10
103 mat,3
104 vext,all,,,0,0,sb,25.4/25.65,
25.4/25.65,
105 csys,20
106 nsel,s,loc,x,0,25.65/2
107 nsel,r,loc,z,sb,
108 CERIG,ncerig,all,ux,
109 csys,21
110 nsel,r,loc,y,0,,,
111 CERIG,ncerig,all,uy,
112 allsel

113 x=2*sb+a
114 y=sa+b/2+104
115 z=T
116 csys,0
117 k,,x+sb,y,z
118 k,,x+lr,y,z
119 *get,nkp,kp,,num,max
120 l,nkp-1,nkp
121 *get,nline,line,,num,max
122 lsel,s,line,,nline

123 lesize,all,,,3
124 latt,3,,3,,,1
125 lmesh,nline
126 nsel,s,loc,z,z,,,
127 nsel,r,loc,y,y,,,
128 nsel,r,loc,x,x+sb,,,
129 *get,ncerig,node,,num,max
130 local,20,1,x,y,z,0,0,90
131 asel,s,loc,x,0,25.65/2
132 asel,r,loc,z,0
133 local,21,0,x,y,z,0,0,90
134 extopt,esize,10
135 mat,3
136 vext,all,,,0,0,sb,25.4/25.65,
25.4/25.65,
137 csys,20
138 nsel,s,loc,x,0,25.65/2
139 nsel,r,loc,z,sb,
140 CERIG,ncerig,all,ux,
141 csys,21
142 nsel,r,loc,y,0,,,
143 CERIG,ncerig,all,uy,
144 allsel

145 SECTYPE,2,Beam,mesh,hrohr0,
146 SECREAD,beam2,,,mesh
147 SECOFFSET,origin,
148 x=2*sb+a
149 y=sa+b/2-52
150 z=0
151 csys,0
152 k,,x+sb,y,z
153 k,,x+lr,y,z
154 *get,nkp,kp,,num,max
155 l,nkp-1,nkp
156 *get,nline,line,,num,max
157 lsel,s,line,,nline
158 lesize,all,,,3
159 latt,3,,3,,,2
160 lmesh,nline
161 nsel,s,loc,z,z,,,
162 nsel,r,loc,y,y,,,
163 nsel,r,loc,x,x+sb,,,
164 *get,ncerig,node,,num,max
165 local,20,1,x,y,z,0,0,90
166 asel,s,loc,x,0,25.65/2
167 asel,r,loc,z,0
168 local,21,0,x,y,z,0,0,90
169 extopt,esize,10
170 mat,3
171 vext,all,,,0,0,sb,25.4/25.65,
25.4/25.65,

```

```

172 csys,20
173 nsel,s,loc,x,0,25.65/2
174 nsel,r,loc,z,sb,
175 CERIG,ncerig,all,ux,
176 csys,21
177 nsel,r,loc,y,0,,,
178 CERIG,ncerig,all,uy,
179 allsel

180 x=2*sb+a
181 y=sa+b/2+52
182 z=0
183 csys,0
184 k,,x+sb,y,z
185 k,,x+lr,y,z
186 *get,nkp,kp,,num,max
187 l,nkp-1,nkp
188 *get,nline,line,,num,max
189 lsel,s,line,,nline
190 lesize,all,,,3
191 latt,3,,3,,,2
192 lmesh,nline
193 nsel,s,loc,z,z,,,
194 nsel,r,loc,y,y,,,
195 nsel,r,loc,x,x+sb,,,
196 *get,ncerig,node,,num,max
197 local,20,1,x,y,z,0,0,90
198 asel,s,loc,x,0,25.65/2
199 asel,r,loc,z,0
200 local,21,0,x,y,z,0,0,90
201 extopt,esize,10
202 mat,3
203 vext,all,,,0,0,sb,25.4/25.65,
25.4/25.65,
204 csys,20
205 nsel,s,loc,x,0,25.65/2
206 nsel,r,loc,z,sb,
207 CERIG,ncerig,all,ux,
208 csys,21
209 nsel,r,loc,y,0,,,
210 CERIG,ncerig,all,uy,
211 allsel

7 nsel,r,loc,z,0,sb
8 *get,nnode,node,,count,
9 csys,0
10 f,all,fx,-
p*28.173**2*pi/4/2/nnode

11 x=0
12 y=sa+b/2-104
13 z=T
14 csys,0
15 local,20,1,x,y,z,0,0,90
16 nsel,s,loc,x,0,28.173/2
17 nsel,r,loc,z,0,sb
18 *get,nnode,node,,count,
19 csys,0
20 f,all,fx,-
p*28.173**2*pi/4/2/nnode

21 x=0
22 y=sa+b/2+104
23 z=T
24 csys,0
25 local,20,1,x,y,z,0,0,90
26 nsel,s,loc,x,0,28.173/2
27 nsel,r,loc,z,0,sb
28 *get,nnode,node,,count,
29 csys,0
30 f,all,fx,-
p*28.173**2*pi/4/2/nnode

31 x=0
32 y=sa+b/2+52
33 z=0
34 csys,0
35 local,20,1,x,y,z,0,0,90
36 nsel,s,loc,x,0,28.173/2
37 nsel,r,loc,z,0,sb
38 *get,nnode,node,,count,
39 csys,0
40 f,all,fx,-
p*28.173**2*pi/4/2/nnode

41 x=0
42 y=sa+b/2-52
43 z=0
44 csys,0
45 local,20,1,x,y,z,0,0,90
46 nsel,s,loc,x,0,28.173/2
47 nsel,r,loc,z,0,sb
48 *get,nnode,node,,count,
49 csys,0
50 f,all,fx,-
p*28.173**2*pi/4/2/nnode
    
```

### Macro: plugforce.mac

```

1 x=0
2 y=sa+b/2
3 z=T
4 csys,0
5 local,20,1,x,y,z,0,0,90
6 nsel,s,loc,x,0,28.173/2
    
```



## 280 Pressure Vessel Design: The Direct Route

### Input file: Recommended NOLC 1.2: LC with thermal stresses

```

1 /filnam,Model,1
2 /PREP7
3 *set,pi,acos(-1)
4 *afun,deg

5 a=190+2*3
6 b=255+2*3
7 sa=34-3
8 sb=34-3
9 T=30
10 aw=6

11 !geometry plates
12 k,1,0,0,0
13 k,2,sb,0,0
14 k,3,sb,sa,0
15 k,4,sb,sa+b-aw,0
16 k,5,sb+aw,sa+b,0
17 k,6,sb+aw,2*sa+b,0
18 k,7,0,2*sa+b,0
19 k,8,sb+a,0,0
20 k,9,2*sb+a,0,0
21 k,10,2*sb+a,2*sa+b,0
22 k,11,sb+a-aw,2*sa+b,0
23 k,12,sb+a-aw,sa+b,0
24 k,13,sb+a,sa+b-aw,0
25 k,14,sb+a,sa,0

26 a,1,2,3,4,5,6,7
27 a,2,8,14,3
28 a,8,9,10,11,12,13,14
29 a,11,6,5,12

30 vext,1,4,1,0,0,T,

31 bore !see macro bore.mac

32 !defining elements
33 et,1,solid95,,,,,,

34 !material properties plates
35 mp,ex,1,206000
36 mp,nuxy,1,0.3
37 TBDE,BKIN,1,,,
38 TB,BKIN,1,1,,1
39 TBDATA,,205.1,0,,,,
40 mptemp,1,20,120
41 UIMP,1,REFT,,,20

42 MPDATA,ALPX,1,,11.30E-6,12.04E-
!thermal expansion!
43 MPDATA,kxx,1,,49.50,47.70
!thermal conductivity!
44 !material properties tubes
45 mp,ex,3,206000
46 mp,nuxy,3,0.3
47 TBDE,BKIN,3,,,,
48 TB,BKIN,3,1,,1
49 TBDATA,,126.1,0,,,,
50 mptemp,1,20,120
51 UIMP,3,REFT,,,20
52 MPDATA,ALPX,3,,11.30E-
6,12.04E-6
53 MPDATA,kxx,3,,55.2,52.6

54 !meshing
55 cmsel,s,tube,volu
56 Vatt,3,,,
57 esize,2.4
58 vsweep,all
59 allsel
60 cmsel,u,tube,volu
61 Vatt,1,,,
62 esize,sa/5
63 vsweep,2
64 vsweep,4
65 MSHKEY,0
66 MSHAPE,1,3d
67 VMESH,13,15,2

68 tube !see macro tube.mac

69 !boundary conditions
70 csys,0
71 nsel,s,loc,z,0,,,
72 nsel,r,loc,x,0,2*sb+a+lr,,
73 d,all,uz,0
74 nsel,s,loc,y,sa+b/2,,,
75 nsel,r,loc,x,sb+a,,,
76 d,all,uy,0
77 nsel,s,loc,z,T,,,
78 nsel,r,loc,x,0,2*sb+a+lr,,
79 cp,1,uz,all
80 allsel
81 nsel,s,loc,x,2*sb+a+lr,,
82 d,all,ux,0
83 d,all,rotx,0
84 d,all,roty,0
85 d,all,rotz,0
86 allsel

```

```

87 finish
88 save

89 /filnam,Model_therm,1

90 /PREP7
91 lclear,all
92 etchg,stm !change element type
    from struct to thermal

93 finish

94 /SOLU
95 ANTYPE,0
96 !
97 eresx,no !Copy inte-
    gration point results to the
    nodes for all elements
98 !
99 asel,s,area,,6
100 asel,a,area,,11
101 asel,a,area,,14
102 asel,a,area,,18
103 asel,a,area,,20
104 asel,a,area,,25
105 asel,a,area,,70
106 asel,a,area,,94
107 da,all,temp,40

108 asel,s,area,,3
109 asel,a,area,,19
110 asel,a,area,,30
111 asel,a,area,,47
112 asel,a,area,,98
113 da,all,temp,70

114 asel,s,area,,9
115 asel,a,area,,16
116 asel,a,area,,22
117 asel,a,area,,26
118 asel,a,area,,31
119 asel,a,area,,41
120 asel,a,area,,51
121 asel,a,area,,61
122 asel,a,area,,69
123 asel,a,area,,71
124 asel,a,area,,95
125 asel,a,area,,8
126 asel,a,area,,27
127 asel,a,area,,34
128 asel,a,area,,38

129 asel,a,area,,44
130 asel,a,area,,49
131 asel,a,area,,54
132 asel,a,area,,64
133 asel,a,area,,74
134 asel,a,area,,100
135 da,all,temp,100
136 allsel

137 time,1
138 nsubst,1
139 save
140 solv
141 finish

142 /filnam,Model,1
143 RESUME,Model,db
144 /SOLU
145 ANTYPE,0
146 !
147 eresx,no !Copy
    integration point results
    to the nodes for all
    elements
148 !
149 outres, strs,all
150 outres, epel,all
151 outres, eppl,all

152 LDREAD,TEMP,1,1,,
    'Model_therm','rst',
153 allsel
154 time,1
155 deltim,0.2,0.001,0.2
156 save
157 solv

158 *set,p,7.2*1.2
159 SFA,8,1,PRES,p
160 SFA,9,1,PRES,p
161 SFA,16,1,PRES,p
162 SFA,22,1,PRES,p
163 SFA,26,1,PRES,p
164 SFA,27,1,PRES,p
165 SFA,31,1,PRES,p
166 SFA,34,1,PRES,p
167 SFA,38,1,PRES,p
168 SFA,41,1,PRES,p
169 SFA,44,1,PRES,p
170 SFA,49,1,PRES,p
171 SFA,51,1,PRES,p
172 SFA,54,1,PRES,p

```

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```

173 SFA, 61, 1, PRES, p
174 SFA, 64, 1, PRES, p
175 SFA, 69, 1, PRES, p
176 SFA, 71, 1, PRES, p
177 SFA, 74, 1, PRES, p
178 SFA, 95, 1, PRES, p
179 SFA, 100, 1, PRES, p
180 plugforce !see macro plug-
    force.mac
181 allsel
182 time, 2
183 deltim, 0.1, 0.001, 0.1
184 save
185 solv
    
```

**L.4.4: GPD-DC of a nozzle in hemispherical end**

In this example, instead of the usual listings of the input, a listing of the key points is given. Fig. L.4.4 shows the key points used, and the list after the figure shows the corresponding data.

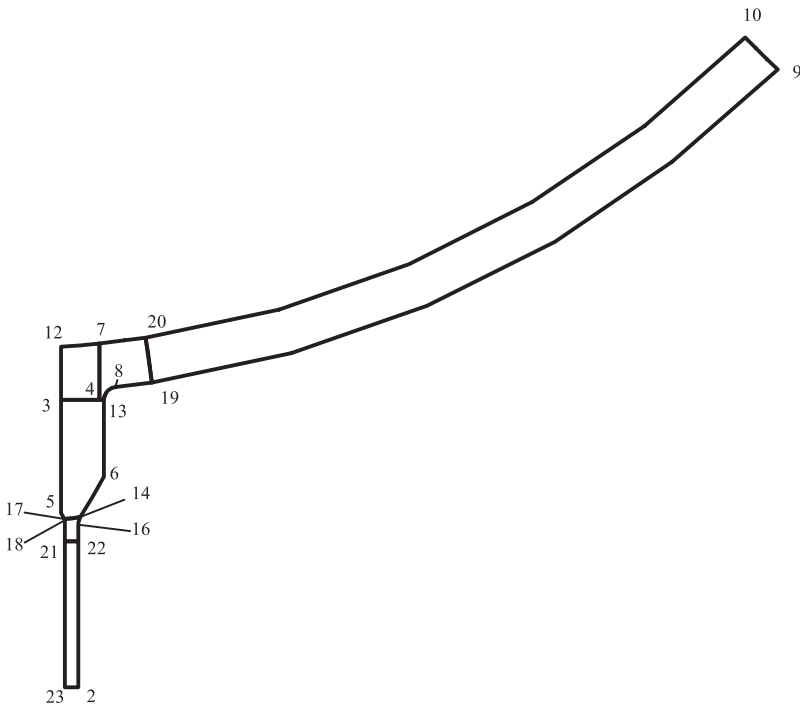


Figure L.4.4: Key point designations.



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```

20 k,9,2*sb+a,0,0
21 k,10,2*sb+a,2*sa+b,0
22 k,11,sb+a-aw,2*sa+b,0
23 k,12,sb+a-aw,sa+b,0
24 k,13,sb+a,sa+b-aw,0
25 k,14,sb+a,sa,0

26 a,1,2,3,4,5,6,7
27 a,2,8,14,3
28 a,8,9,10,11,12,13,14
29 a,11,6,5,12

30 vext,1,4,1,0,0,T,

31 bore !see macro bore.mac

32 !defining elements
33 et,1,solid95,,,,,,

34 !material properties plates
35 mp,ex,1,206000
36 mp,nuxy,1,0.3
37 TBDE,BKIN,1,,,
38 TB,BKIN,1,1,,1
39 TBDATA,,307.2,0,,,,

40 !material properties tubes
41 mp,ex,3,206000
42 mp,nuxy,3,0.3
43 TBDE,BKIN,3,,,
44 TB,BKIN,3,1,,1
45 TBDATA,,187.9,0,,,,

46 !meshing
47 cmsel,s,tube,volu
48 Vatt,3,,,
49 esize,2.4
50 vsweep,all
51 allsel
52 cmsel,u,tube,volu
53 Vatt,1,,,
54 esize,sa/5
55 vsweep,2
56 vsweep,4
57 MSHKEY,0
58 MSHAPE,1,3d
59 VMESH,13,15,2

60 tube !see macro tube.mac

61 !boundary conditions
62 csys,0

63 nsel,s,loc,z,0,,,
64 nsel,r,loc,x,0,2*sb+a+lr,,
65 d,all,uz,0
66 nsel,s,loc,y,sa+b/2,,,
67 nsel,r,loc,x,sb+a,,,
68 d,all,uy,0
69 nsel,s,loc,z,T,,,
70 nsel,r,loc,x,0,2*sb+a+lr,,
71 cp,1,uz,all
72 allsel
73 nsel,s,loc,x,2*sb+a+lr,
74 d,all,ux,0
75 d,all,rotx,0
76 d,all,roty,0
77 d,all,rotz,0
78 allsel

79 finish
80 save

81 !solution
82 /SOLU
83 !
84 eresx,no !copy inte-
      gration point results to the
      nodes for all elements
85 !
86 *set,p,7.2
87 SFA,8,1,PRES,p
88 SFA,9,1,PRES,p
89 SFA,16,1,PRES,p
90 SFA,22,1,PRES,p
91 SFA,26,1,PRES,p
92 SFA,27,1,PRES,p
93 SFA,31,1,PRES,p
94 SFA,34,1,PRES,p
95 SFA,38,1,PRES,p
96 SFA,41,1,PRES,p
97 SFA,44,1,PRES,p
98 SFA,49,1,PRES,p
99 SFA,51,1,PRES,p
100 SFA,54,1,PRES,p
101 SFA,61,1,PRES,p
102 SFA,64,1,PRES,p
103 SFA,69,1,PRES,p
104 SFA,71,1,PRES,p
105 SFA,74,1,PRES,p
106 SFA,95,1,PRES,p
107 SFA,100,1,PRES,p
108 plugforce !see macro
      plugforce.mac
109 allsel

```

```

110 time,1
111 deltim,0.2,0.001,0.2
112 save
113 solv

114 *set,p,0
115 SFA,8,1,PRES,p
116 SFA,9,1,PRES,p
117 SFA,16,1,PRES,p
118 SFA,22,1,PRES,p
119 SFA,26,1,PRES,p
120 SFA,27,1,PRES,p
121 SFA,31,1,PRES,p
122 SFA,34,1,PRES,p
123 SFA,38,1,PRES,p
124 SFA,41,1,PRES,p
125 SFA,44,1,PRES,p
126 SFA,49,1,PRES,p
127 SFA,51,1,PRES,p
128 SFA,54,1,PRES,p
129 SFA,61,1,PRES,p
130 SFA,64,1,PRES,p
131 SFA,69,1,PRES,p
132 SFA,71,1,PRES,p
133 SFA,74,1,PRES,p
134 SFA,95,1,PRES,p
135 SFA,100,1,PRES,p
136 plugforce !see macro plug-
    force.mac
137 allsel
138 time,2
139 deltim,0.2,0.001,0.2
140 save
141 solv

11 !geometry plates
12 k,1,0,0,0
13 k,2,sb,0,0
14 k,3,sb,sa,0
15 k,4,sb,sa+b-aw,0
16 k,5,sb+aw,sa+b,0
17 k,6,sb+aw,2*sa+b,0
18 k,7,0,2*sa+b,0
19 k,8,sb+a,0,0
20 k,9,2*sb+a,0,0
21 k,10,2*sb+a,2*sa+b,0
22 k,11,sb+a-aw,2*sa+b,0
23 k,12,sb+a-aw,sa+b,0
24 k,13,sb+a,sa+b-aw,0
25 k,14,sb+a,sa,0

26 a,1,2,3,4,5,6,7
27 a,2,8,14,3
28 a,8,9,10,11,12,13,14
29 a,11,6,5,12

30 vext,1,4,1,0,0,T,

31 bore !see macro bore.mac

32 !defining elements
33 et,1,solid95,,,,,,,,

34 !material properties plates
35 mptemp,1,20,120
36 mpdata,ex,1,1,206000,206000
37 mpdata,nuxy,1,1,0.3,0.3
38 TBDE,BKIN,1,,,,
39 TB,BKIN,1,2,1,1
40 TBTEMP,20
41 TBDATA,,355,0,,,,
42 TBTEMP,120
43 TBDATA,,296,0,,,,

44 !material properties tubes
45 mptemp,1,20,120
46 mpdata,ex,3,1,206000,206000
47 mpdata,nuxy,3,1,0.3,0.3
48 TBDE,BKIN,3,,,,
49 TB,BKIN,3,2,1,1
50 TBTEMP,20
51 TBDATA,,215,,,,
52 TBTEMP,120
53 TBDATA,,182,,,,

54 !meshing
55 cmsel,s,tube,volu
    
```

**Input file: LC without thermal stresses / Option 2: Temperature-dependent material strength parameter**

```

1 /filnam,Model,1
2 /PREP7

3 *set,pi,acos(-1)
4 *afun,deg

5 a=190+2*3
6 b=255+2*3
7 sa=34-3
8 sb=34-3
9 T=30
10 aw=6
    
```

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```

56 Vatt,3,,,
57 esize,2.4
58 vsweep,all
59 allsel
60 cmsel,u,tube,volu
61 Vatt,1,,,
62 esize,sa/5
63 vsweep,2
64 vsweep,4
65 MSHKEY,0
66 MSHAPE,1,3d
67 VMESH,13,15,2

68 tube      !see macro tube.mac

69 !boundary conditions
70 csys,0
71 nsel,s,loc,z,0,,,
72 nsel,r,loc,x,0,2*sb+a+lr,,
73 d,all,uz,0
74 nsel,s,loc,y,sa+b/2,,,
75 nsel,r,loc,x,sb+a,,,
76 d,all,uy,0
77 nsel,s,loc,z,T,,,
78 nsel,r,loc,x,0,2*sb+a+lr,,
79 cp,1,uz,all
80 allsel
81 nsel,s,loc,x,2*sb+a+lr,
82 d,all,ux,0
83 d,all,rotx,0
84 d,all,roty,0
85 d,all,rotz,0
86 allsel

87 finish
88 save

89 !solution
90 /SOLU
91 !
92 eresx,no      !copy inte-
    gration point results to the
    nodes for all elements
93 !
94 tunif,120
95 *set,p,7.2
96 SFA,8,1,PRES,p
97 SFA,9,1,PRES,p
98 SFA,16,1,PRES,p
99 SFA,22,1,PRES,p
100 SFA,26,1,PRES,p
101 SFA,27,1,PRES,p
102 SFA,31,1,PRES,p

103 SFA,34,1,PRES,p
104 SFA,38,1,PRES,p
105 SFA,41,1,PRES,p
106 SFA,44,1,PRES,p
107 SFA,49,1,PRES,p
108 SFA,51,1,PRES,p
109 SFA,54,1,PRES,p
110 SFA,61,1,PRES,p
111 SFA,64,1,PRES,p
112 SFA,69,1,PRES,p
113 SFA,71,1,PRES,p
114 SFA,74,1,PRES,p
115 SFA,95,1,PRES,p
116 SFA,100,1,PRES,p
117 plugforce      !see macro
    plugforce.mac
118 allsel
119 time,1
120 deltim,0.2,0.001,0.2
121 save
122 solv

123 tunif,20
124 *set,p,0
125 SFA,8,1,PRES,p
126 SFA,9,1,PRES,p
127 SFA,16,1,PRES,p
128 SFA,22,1,PRES,p
129 SFA,26,1,PRES,p
130 SFA,27,1,PRES,p
131 SFA,31,1,PRES,p
132 SFA,34,1,PRES,p
133 SFA,38,1,PRES,p
134 SFA,41,1,PRES,p
135 SFA,44,1,PRES,p
136 SFA,49,1,PRES,p
137 SFA,51,1,PRES,p
138 SFA,54,1,PRES,p
139 SFA,61,1,PRES,p
140 SFA,64,1,PRES,p
141 SFA,69,1,PRES,p
142 SFA,71,1,PRES,p
143 SFA,74,1,PRES,p
144 SFA,95,1,PRES,p
145 SFA,100,1,PRES,p
146 plugforce !see macro
    plugforce.mac
147 allsel
148 time,2
149 deltim,0.2,0.001,0.2
150 save
151 solv

```

**Input file: LC with thermal stresses  
& temperature dependent material  
parameters**

```

1  /filnam,Model,1
2  /PREP7

3  *set,pi,acos(-1)
4  *afun,deg

5  a=190+2*3
6  b=255+2*3
7  sa=34-3
8  sb=34-3
9  T=30
10 aw=6

11 !geometry plates
12 k,1,0,0,0
13 k,2,sb,0,0
14 k,3,sb,sa,0
15 k,4,sb,sa+b-aw,0
16 k,5,sb+aw,sa+b,0
17 k,6,sb+aw,2*sa+b,0
18 k,7,0,2*sa+b,0
19 k,8,sb+a,0,0
20 k,9,2*sb+a,0,0
21 k,10,2*sb+a,2*sa+b,0
22 k,11,sb+a-aw,2*sa+b,0
23 k,12,sb+a-aw,sa+b,0
24 k,13,sb+a,sa+b-aw,0
25 k,14,sb+a,sa,0

26 a,1,2,3,4,5,6,7
27 a,2,8,14,3
28 a,8,9,10,11,12,13,14
29 a,11,6,5,12

30 vext,1,4,1,0,0,T,

31 bore !see macro bore.mac

32 !defining elements
33 et,1,solid95,,,,,,,,

34 tofst,273
35 !material properties plates
36 mptemp,1,20,120
37 mpdata,ex,1,1,206000,206000
38 mpdata,nuxy,1,1,0.3,0.3
39 UIMP,1,REFT,,,20

40 MPDATA,ALPX,1,,11.30E-
41 6,12.04E-6
42 !thermal expansion!
43 MPDATA,kxx,1,,49.50,47.70
44 !thermal conductivity!

42 TBDE,BKIN,1,,,
43 TB,BKIN,1,2,1,1
44 TBTEMP,20
45 TBDATA,,355,0,,,,
46 TBTEMP,120
47 TBDATA,,296,0,,,,

48 !material properties tubes
49 mptemp,1,20,120
50 mpdata,ex,3,1,206000,206000
51 mpdata,nuxy,3,1,0.3,0.3
52 UIMP,3,REFT,,,20
53 MPDATA,ALPX,3,,11.30E-
54 6,12.04E-6
55 MPDATA,kxx,3,,55.2,52.6

55 TBDE,BKIN,3,,,
56 TB,BKIN,3,2,1,1
57 TBTEMP,20
58 TBDATA,,215,,,,
59 TBTEMP,120
60 TBDATA,,182,,,,

61 !meshing
62 cmsel,s,tube,volu
63 Vatt,3,,,
64 esize,2.4
65 vsweep,all
66 allsel
67 cmsel,u,tube,volu
68 Vatt,1,,,
69 esize,sa/5
70 vsweep,2
71 vsweep,4
72 MSHKEY,0
73 MSHAPE,1,3d
74 VMESH,13,15,2

75 tube !see macro tube.mac

76 !boundary conditions
77 csys,0
78 nsel,s,loc,z,0,,,
79 nsel,r,loc,x,0,2*sb+a+lr,,
80 d,all,uz,0
81 nsel,s,loc,y,sa+b/2,,,
    
```



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```

82  nsel,r,loc,x,sb+a,,
83  d,all,uy,0
84  nsel,s,loc,z,T,,
85  nsel,r,loc,x,0,2*sb+a+lr,,
86  cp,1,uz,all
87  allsel
88  nsel,s,loc,x,2*sb+a+lr,
89  d,all,ux,0
90  !d,all,uy,0
91  !d,all,uz,0
92  d,all,rotx,0
93  d,all,roty,0
94  d,all,rotz,0
95  allsel

96  finish
97  save

98  /filnam,Model,1

99  /PREP7
100 lclear,all
101 etchg,slt !change element
    type from struct to thermal

102 finish

103 /SOLU
104 ANTYPE,0
105 !
106 eresx,no          !Copy
    integration point results to
    the nodes for all elements
107 !
108 asel,s,area,,6
109 asel,a,area,,11
110 asel,a,area,,14
111 asel,a,area,,18
112 asel,a,area,,20
113 asel,a,area,,25
114 asel,a,area,,70
115 asel,a,area,,94
116 da,all,temp,40

117 asel,s,area,,3
118 asel,a,area,,19
119 asel,a,area,,30
120 asel,a,area,,47
121 asel,a,area,,98
122 da,all,temp,70

123 asel,s,area,,9
124 asel,a,area,,16
125 asel,a,area,,22
126 asel,a,area,,26
127 asel,a,area,,31
128 asel,a,area,,41
129 asel,a,area,,51
130 asel,a,area,,61
131 asel,a,area,,69
132 asel,a,area,,71
133 asel,a,area,,95
134 asel,a,area,,8
135 asel,a,area,,27
136 asel,a,area,,34
137 asel,a,area,,38
138 asel,a,area,,44
139 asel,a,area,,49
140 asel,a,area,,54
141 asel,a,area,,64
142 asel,a,area,,74
143 asel,a,area,,100
144 da,all,temp,100
145 allsel

146 time,1
147 nsubst,1
148 save
149 solv
150 finish

151 /filnam,Model,1
152 RESUME,Model,db
153 /SOLU
154 ANTYPE,0
155 !
156 eresx,no !Copy integration
    point results to the nodes
    for all elements
157 !
158 outres, strs,all
159 outres, epel,all
160 outres, eppl,all

161 LDREAD,TEMP,1,1,,
    'Model_therm','rst',
162 *set,p,7.2
163 SFA,8,1,PRES,p
164 SFA,9,1,PRES,p
165 SFA,16,1,PRES,p
166 SFA,22,1,PRES,p
167 SFA,26,1,PRES,p

```

```

168 SFA,27,1,PRES,p
169 SFA,31,1,PRES,p
170 SFA,34,1,PRES,p
171 SFA,38,1,PRES,p
172 SFA,41,1,PRES,p
173 SFA,44,1,PRES,p
174 SFA,49,1,PRES,p
175 SFA,51,1,PRES,p
176 SFA,54,1,PRES,p
177 SFA,61,1,PRES,p
178 SFA,64,1,PRES,p
179 SFA,69,1,PRES,p
180 SFA,71,1,PRES,p
181 SFA,74,1,PRES,p
182 SFA,95,1,PRES,p
183 SFA,100,1,PRES,p
184 plugforce !see macro
    plugforce.mac
185 allsel
186 time,1
187 deltim,0.2,0.001,0.2
188 save
189 solv

190 bf,all,temp,20
191 *set,p,0
192 SFA,8,1,PRES,p

193 SFA,9,1,PRES,p
194 SFA,16,1,PRES,p
195 SFA,22,1,PRES,p
196 SFA,26,1,PRES,p
197 SFA,27,1,PRES,p
198 SFA,31,1,PRES,p
199 SFA,34,1,PRES,p
200 SFA,38,1,PRES,p
201 SFA,41,1,PRES,p
202 SFA,44,1,PRES,p
203 SFA,49,1,PRES,p
204 SFA,51,1,PRES,p
205 SFA,54,1,PRES,p
206 SFA,61,1,PRES,p
207 SFA,64,1,PRES,p
208 SFA,69,1,PRES,p
209 SFA,71,1,PRES,p
210 SFA,74,1,PRES,p
211 SFA,95,1,PRES,p
212 SFA,100,1,PRES,p
213 plugforce !see macro
    plugforce.mac
214 allsel
215 time,2
216 deltim,0.2,0.001,0.2
217 save
218 solv
    
```

**L.5.3: PD-DC of a nozzle in hemispherical end:** The model with key points as per L.4.4 can be used.

### L.6.1: First S-DC of a jacketed autoclave

#### Input file (macro vessel.mac)

```

1 /FILENAME,vessel,1
2 !!!inner shell
3 *afun,deg
4 *set,ei,11
5 *set,rki,2000+ei/2
6 *set,rzi,1978/2+ei/2
7 *set,rfi,200+ei/2

8 xfi=rzi-rfi
9 yfi=sqrt((rki-rfi)**2-(rzi-
    rfi)**2)
10 alphai=acos(yfi/(rki-rfi))

11 /VUP,1,y
12 /PREP7
13 local,11,2,0,0,0,0,-90,0
14 k,1,0,0,0
15 csys,0
16 k,2,0,2*168.3+sqrt
    ((168.3/2)**2+rki**2)
17 k,3,168.3/2,2*168.3+sqrt
    ((168.3/2)**2+rki**2)
18 csys,11
19 k,4,rki,0,90-asin(168.3/2/rki)
20 k,5,rki,0,90-
    asin((168.3/2+25)/rki)
21 k,6,rki,0,90-alphai
22 local,12,1,xfi,yfi,
    
```

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```

23 k,7,rfi,0,0
24 csys,0
25 k,8,rzi,yfi-78
26 k,9,rzi,yfi-78-1000

27 1,2,3,3,1
28 1,3,4,10,1
29 csys,11
30 1,4,5,1,1
31 1,5,6,15,1
32 csys,12
33 1,6,7,4,1
34 csys,0
35 1,7,8,2,1
36 1,8,9,10,1

37 !!!outer shell
38 *set, eo, 10
39 *set, rko, 2100+eo/2
40 *set, rzo, 2080/2+eo/2
41 *set, rfo, 210+eo/2

42 xfo=rzo-rfo
43 yfo=sqrt((rko-rfo)**2-(rzo-
rfo)**2)
44 alphao=acos(yfo/(rko-rfo))

45 local,21,2,0,rki+50-rko,0,0,-
90,0
46 k,10,rko,0,85.45227
47 k,11,rko,0,90-alphao
48 local,22,1,xfo,yfo,
49 k,12,rfo,0,0
50 csys,0
51 k,13,rzo,yfi-78
52 k,14,rzo,yfi-78-1000

53 local,23,1,159.11480,
1999.25894,
54 1,5,10,3,1
55 csys,21
56 1,10,11,15,1
57 csys,22
58 1,11,12,4,1
59 csys,0
60 1,12,13,2,1
61 1,13,14,10,1
62 1,8,13
63 1,9,14

64 lsel,s,line,,1,7
65 arotat,all,,,,,2,1,360,40

66 lsel,s,line,,8,12
67 arotat,all,,,,,2,1,360,40
68 allsel
69 arotat,13,,,,,2,1,360,40
70 arotat,14,,,,,2,1,360,40

71 ET,1,SHELL93
72 R,1,ei, , , , , ,
73 R,2,eo, , , , , ,
74 R,3,4, , , , , ,
75 R,4,16, , , , , ,
76 UIMP,1,EX, , , 200000,
77 UIMP,1,NUXY, , , 0.3,

78 csys,11
79 lsel,s,loc,z,90-alphai,
80 lsel,a,loc,z,90-
asin(168.3/2/rki)
81 lsel,a,loc,z,90-
asin((168.3/2+25)/rki)
82 LESIZE,all,,,1,,,,,

83 csys,21
84 lsel,s,loc,z,90-alphao,
85 lsel,a,loc,z,85.45227
86 LESIZE,all,,,1,,,,,

87 csys,0
88 lsel,s,loc,y,yfi,
89 lsel,a,loc,y,yfi-78,
90 lsel,a,loc,y,yfi-78-1000,
91 LESIZE,all,,,1,,,,,
92 lsel,s,loc,y,yfo,
93 LESIZE,all,,,1,,,,,
94 allsel

95 csys,5
96 asel,s,loc,x,0,168.3/2
97 aatt,1,3,1
98 allsel
99 asel,u,loc,x,0,168.3/2
100 asel,r,area,,1,280
101 aatt,1,1,1

102 asel,s,area,,281,480
103 aatt,1,2,1

104 asel,s,area,,481,560
105 aatt,1,4,1
106 allsel
107 amesh,all
108 nummrg,all

```

```

109  csys,5
110  asel,s,loc,x,0,168.3/2+25
111  ESLA,s
112  CM,nozzle,elem

113  asel,s,area,,1,280
114  asel,u,loc,x,0,168.3/2+25
115  asel,u,loc,z,0,yfi-78
116  ESLA,s
117  CM,innerhead,elem

118  asel,s,area,,281,560
119  asel,u,loc,z,0,yfi-78
120  ESLA,s
121  CM,outerhead,elem

122  asel,s,loc,z,yfi-78
123  ESLA,s
124  CM,ring,elem

125  asel,s,area,,1,280
126  asel,r,loc,z,0,yfi-78-1
127  ESLA,s
128  CM,innerzyl,elem

129  cmsel,s,nozzle
130  cmsel,a,innerhead
131  cmsel,a,innerzyl
132  cmsel,a,ring
133  cm,innen,elem
134  allsel
135  csys,0

136  !boundary conditions!
137  csys,5
138  nsel,s,loc,z,yfi-78-1000
139  nsel,r,loc,x,rzi
140  nrotat,all
141  d,all,uz,0
142  d,all,uy,o
143  allsel
144  save
145  FINISH

146  !!!eigenvalue (linear) buck-
    ling analysis
147  /assign,rst,linear,rst
148  /solu
149  antype,0,new
150  pstres,on
151  cmsel,s,nozzle
152  cmsel,a,innerzyl

153  sfe,all,1,pres,,-0.1*1.25
154  cmsel,s,innerhead
155  sfe,all,1,pres,,-0.6*1.25
156  cmsel,s,outerhead
157  sfe,all,1,pres,,0.5*1.25
158  cmsel,s,ring
159  sfe,all,1,pres,,-0.5*1.25
160  allsel
161  solv
162  finish

163  /assign,rst,modes,rst
164  /solu
165  antype,buckle
166  bucopt,lanb,30,
167  outpr,nsol,all
168  save
169  solv
170  finish

171  /solu
172  expass,on
173  mexpand,
174  outres,all,all
175  save
176  solve
177  finish

Nonlinear buckling analysis (macro  
imperTH.mac; files vessel.db and  
modes.rst created by vessel.mac are  
required.)

1  resume,vessel,db
2  csys,0
3  /post1
4  rsys,0
5  file,modes,rst

6  nmodes=10

7  *do,m,1,nmodes,1
8  LCDEF,m,1,m,
9  *enddo

10 LCZERO
11 *do,m,1,nmodes,1
12 LCOPER,add,m
13 *enddo

```

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```

14  RAPPND,2,2
15  set,2,1
16  *set,kappa,14.8/11

17  nsort,u,sum,,1
18  *get,umx,sort,,max
19  *get,iumx,sort,,imax
20  umul=kappa*11/umx
21  finish

22  /FILENAME,imperTH,1
23  /prep7
24  mptemp,1,20,160
25  mpdata,ex,1,1,200000,188100
26  mpdata,nuxy,1,1,0.3,0.3
27  UIMP,1,REFT,,20
28  MPDATA,ALPX,1,,15.3E-
    6*1.25,16.3E-6*1.25
    !thermal expansion!

29  TBDE,BKIN,1,,
30  TB,BKIN,1,2,1,1
31  TBTEMP,20
32  TBDATA,,240,0,,,,
33  TBTEMP,160
34  TBDATA,,194,0,,,,

35  upgeom,umul,2,1,modes,rst,
36  eplot
37  save
38  finish

39  /assign,rst,modeTH,rst
40  /solu
41  antype,0,new
42  eresx,no !Copy integration
    point results to the nodes for
    all elements
43  nlgeom,on
44  outres,all,all

45  csys,5
46  asel,s,area,,281,560
47  asel,r,loc,z,yfi-78-
    1000+1,yfi-78-1

48  ESLA,s
49  CM,outerzyl,elem

50  asel,s,area,,281,560
51  asel,r,loc,z,yfi-78-1000
52  ESLA,s
53  CM,ring2,elem
54  csys,0
55  allsel

56  sfedele,all,all,all
57  tunif,20
58  cmsel,s,innerzyl
59  cmsel,a,innerhead
60  BFE,all,TEMP,1,146.4,
    146.4,146.4,146.4
61  BFE,all,TEMP,5,89.8,89.8,
    89.8,89.8
62  cmsel,s,nozzle
63  BFE,all,TEMP,1,152,152,152,
    152
64  BFE,all,TEMP,5,152,152,152,
    152

65  allsel
66  deltim,0.1,0.001,0.1
67  save
68  solv

69  cmsel,s,innerzyl
70  sfe,all,1,pres,,-0.5*1.25
71  cmsel,s,innerhead
72  sfe,all,1,pres,,-0.5*1.25
73  cmsel,s,outerhead
74  sfe,all,1,pres,,0.5*1.25
75  cmsel,s,outerzyl
76  sfe,all,1,pres,,0.5*1.25
77  cmsel,s,ring2
78  sfe,all,1,pres,,-0.5*1.25

79  allsel
80  deltim,0.1,0.001,0.1
81  save
82  solv
83  finish

```

### L.6.2: Second S-DC of a jacketed autoclave

**Nonlinear buckling analysis** (macro imper.mac; files vessel.db and modes.rst created by vessel.mac from

Annex L.6.1 are required. Eigenvalue (linear) buckling analysis is the same as for Example E.6.1)

```

1  resume,vessel,db
2  csys,0
3  /post1
4  rsys,0
5  file,modes,rst

6  nmodes=10

7  *do,m,1,nmodes,1
8  LCDEF,m,1,m,
9  *enddo

10 LCZERO
11 *do,m,1,nmodes,1
12 LCOPER,add,m
13 *enddo

14 RAPEND,2,2
15 set,2,1

16 *set,kappa,14.8/11

17 nsort,u,sum,,1
18 *get,umx,sort,,max
19 *get,iux,sort,,imax
20 umul=kappa*11/umx
21 finish

22 /FILENAME,imper,1
23 /prep7
24 TBDE,BKIN,1,,

25 TB,BKIN,1,1,,1
26 TBDATA,,240,0,,,,
27 upgeom,umul,2,1,modes,rst,
28 eplot
29 save
30 finish

31 /assign,rst,modeNL,rst
32 /solu
33 antype,0,new
34 eresx,no !Copy integration
    point results to the nodes for
    all elements
35 nlgeom,on
36 outres,all,all
37 cmsel,s,nozzle
38 cmsel,a,innerzyl
39 sfe,all,1,pres,,-0.1*1.25*2
40 cmsel,s,innerhead
41 sfe,all,1,pres,,-0.6*1.25*2
42 cmsel,s,outerhead
43 sfe,all,1,pres,,0.5*1.25*2
44 cmsel,s,Ring
45 sfe,all,1,pres,,-0.5*1.25*2
46 allsel
47 nsubst,20
48 arclen,on,1,
49 arctrm,u,30
50 save
51 solv
52 finish
    
```

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