## APPENDIX C

## AUTOTRANSFORMERS

The autotransformer designed by J. C. Martin and I. Smith ${ }^{1}$ is shown schematically in Figs. C. 1 a, b, c, d. It consists of $0.002-$ to 0.005 -inch copper foil cut to the shape indicated in Fig. C. 1a and attached to a sheet of polyethylene or mylar insulation. As indicated in Fig. C. 1b absorbent (toilet) paper of about the same thickness as the copper is inserted around the edges of the copper to fill the remaining space. The sandwich is then wound on an insulating cylinder (e.g., a cylinder made of lucite) which has an outer radius and length appropriate for the particular transformer being constructed. As indicated in Figs. C. 1c and C.1d, a second insulating cylinder is added after the winding is completed, and the two cylinders are sealed at the bottom by an insulating plate. The result is put into an evacuated chamber for several hours and is then vacuum impregnated with copper sulfate solution which has previously been outgassed. The copper sulfate* has the important function of distributing the field smoothly at the edges of the copper winding so that breakdown through the insulating sheets does not occur. The tapered windings are intended to optimize the flux linkage from primary to secondary.

The arrangement used for generating a high voltage pulse is shown in Fig. C. 2a. The primary inputs of the transformer (C and D of Figs. C. la-d) are connected across a capacitor $C_{p r i m}$ at a voltage $V_{\text {in }}$ by closing a switch $S$. The outputs of the secondary, $A$ and $B$, are applied to the load $C_{\text {sec }}$, which might, for example, be a Blumlein. Alternate arrangements are shown in Figs. C.2b and C.2c. The transformer of Fig. C. 2 b is constructed by cutting off the pattern of Fig. C. 1a just left of the point C and by using C as one of the secondary as well as one of the primary leads. The transformer of Fig. C. 2c is constructed by winding the original sandwich back upon itself just after the point $D$ and by using $D$ as both a primary and a secondary lead.
*The resistivity of the copper sulfate solution should be of the order $10^{4} \mathrm{ohm}-\mathrm{cm}$.


FIG. C.1--Layout and construction details for the autotransformer developed by J. C. Martin and I. Smith of AWRE.


An equivalent circuit for the autotransformer is shown in Fig. C. 3. The leakage inductance $L_{\text {leak }}$ in $n H$ can be calculated from the total thickness of the windings $\Delta$, the minimum and maximum widths of the windings $b_{\min }$ and $b_{\max }$, and the radius of the transformer $r$ :

$$
\begin{equation*}
\mathrm{L}_{\text {leak }}=\mathrm{p} \frac{36 \Delta \mathrm{r}}{\left(2 \mathrm{~b}_{\max }+\mathrm{b}_{\min }\right)} \mathrm{nH} \tag{C-1a}
\end{equation*}
$$

The parameter $p$ has a value of $1 / 2$ for the circuit of Fig. C. 2a, 2. 0 for the circuit of Fig. C. 2b, and 1.0 for the circuit of Fig. C. 2c. All dimensions are in cm . The primary inductance is given by

$$
\begin{equation*}
\mathrm{L}_{\mathrm{prim}}=\frac{40 \mathrm{r}^{2}}{\mathrm{r}+\mathrm{b}_{\max }} \mathrm{nH} \tag{C-1b}
\end{equation*}
$$

Neglecting $\mathrm{L}_{\mathrm{sec}} / \mathrm{N}^{2}$, the ratio of output to input voltages is given by

$$
\begin{equation*}
\mathrm{V}_{\text {out }} / \mathrm{V}_{\text {in }}=\left[\frac{\mathrm{L}_{\text {prim }}-\mathrm{L}_{\text {leak }}}{\mathrm{L}_{\text {prim }}+\mathrm{L}_{\text {leads }}+\mathrm{L}_{\text {cap }}+\mathrm{L}_{\text {gap }}}\right][\mathrm{N}]\left[\frac{2 \mathrm{C}_{\text {prim }}}{\mathrm{C}_{\text {prim }}+\mathrm{N}^{2} \mathrm{C}_{\text {sec }}}\right] \tag{C-1c}
\end{equation*}
$$

where $N$ is the turns ratio. The final factor, $2 C_{\text {prim }} /\left(C_{\text {prim }}+N^{2} C_{\text {sec }}\right)$, is the "ringing efficiency"; the first factor is an expression for the loss in voltage due to the various inductances. The rise time of the output pulse in nsec is given by

$$
\begin{aligned}
T & =\pi\left(L_{\text {total }} \cdot C_{\text {total }}\right)^{1 / 2} \text { nsec } \\
& =\pi\left[\left(\mathrm{L}_{\text {gap }}+\mathrm{L}_{\text {cap }}+\mathrm{L}_{\text {leads }}+\mathrm{L}_{\text {leak }}\right)\left(\frac{\mathrm{C}_{\text {prim }} \mathrm{N}^{2} \mathrm{C}_{\text {sec }}}{\mathrm{C}_{\text {prim }}+\mathrm{N}^{2} \mathrm{C}_{\text {sec }}}\right)\right]^{1 / 2} \text { nsec (C-1d) }
\end{aligned}
$$

where $L$ is expressed in $n H$ and $C$ in $n F$. It is assumed throughout that $2 r$ is greater than $\mathrm{b}_{\text {max }}$ and that r is much greater than $\Delta$.

Some calculations based upon Eqs. (C-1a-d) are plotted in Figs. C. 4 and C. 5. In Fig. C. 4 , the rise time $T$ is given as a function of $b_{\text {max }}$ for various values of $r$ and for all three transformer configurations. The turns ratio N , typically in the range 16-20, was adjusted to satisfy the requirement $\mathrm{V}_{\text {out }} / \mathrm{V}_{\mathrm{in}}=800 \mathrm{kV} / 50 \mathrm{kV}=16$, where $V_{\text {out }}$ was the required output voltage assumed for the two-meter chamber and where $V_{i n}$ was the voltage rating of two ESC $259^{*}$ capacitors in series. Our intention in this case was to minimize the rise time for the given output voltage. The fixed parameter values were:

$$
\begin{aligned}
\mathrm{C}_{\text {prim }} & =500 \mathrm{nF}, & \mathrm{~L}_{\text {leads }} & =6.0 \mathrm{nH}, \\
\mathrm{~V}_{\text {in }} & =50 \mathrm{kV}, & \mathrm{C}_{\text {sec }} & =0.9 \mathrm{nF}, \\
\mathrm{~L}_{\text {cap }} & =1.0 \mathrm{nH}, & \Delta & =1.5 \mathrm{~cm}, \\
\mathrm{~L}_{\text {gap }} & =4.0 \mathrm{nH}, & \text { Input energy } & =624 \text { joules } .
\end{aligned}
$$

As indicated in Fig. C. 4 , the center-tap transformer is preferred; however, it is inconvenient in practice since the input leads attain roughly half the output voltage during the pulse. The rise time decreases slowly with transformer radius (as long as $2 \pi \mathrm{Nr} \ll \mathrm{TC}$ ), and it is fairly close to minimum for $\mathrm{b}_{\max } \geq 20 \mathrm{~cm}$.

In Fig. C.5, we have plotted $V_{\text {out }}$ versus $T$ for various values of $r$ and $b_{\max }$ in the center top configuration with four ESC 295 capacitors in series. In this case, the number of turns was chosen to maximize $V_{\text {out }}$; the resulting value was $\mathrm{N}=16$ at every point. The fixed parameters were:

$$
\begin{aligned}
\mathrm{C}_{\text {prim }} & =250 \mathrm{nF}, & \mathrm{~L}_{\text {leads }} & =6.0 \mathrm{nH}, \\
\mathrm{~V}_{\text {in }} & =100 \mathrm{kV}, & \mathrm{C}_{\text {sec }} & =0.9 \mathrm{nF}, \\
\mathrm{~L}_{\text {cap }} & =2.0 \mathrm{nH}, & \Delta & =3.0 \mathrm{~cm}, \\
\mathrm{~L}_{\text {gap }} & =4.0 \mathrm{nH}, & \text { Input energy } & =1240 \text { joules. }
\end{aligned}
$$

[^0]
FIG. C.4--Calculated rise time in nsec for $\mathrm{V}_{\text {out }} / \mathrm{V}_{\mathrm{in}}=800 \mathrm{kV} / 50 \mathrm{kV}$ versus $\mathrm{b}_{\text {max }}$, the maximum width of the tapered autotransformer winding. Calculations were made for the three circuit configurations indicated and for the transformer radii shown with each curve. Additional parameter values are included in the text.


FIG. C. $5-$-Calculated output voltage maximized with respect to the number of turns in a center-tap autotransformer versus the output rise time in $\mu \mathrm{sec}$. The maximum width $\mathrm{b}_{\text {max }}$ of the tapered winding, and the radius $r$ identify the different curves. Additional parameter values are included in the text.

Figure C. 5 illustrates that the output voltage has a maximum as a function of $T$ as well as $b_{\text {max }}$ and $N$, and that ultimately the rise time can be shortened only by sacrificing output voltage. Since the transformer would be followed in our application by a pulse-shaping network, the rise time is important only in that it adds to the delay in applying the final pulse to the streamer chamber.

## REFERENCE

1. J. C. Martin and I. Smith, private communication.

[^0]:    * Tobe Deutschmann Laboratories, Canton, Massachusetts

