Exchange Rate Economics

Where Do We Stand?

edited by Paul De Grauwe

CESifo Seminar Series

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CESifo Seminar Series in Economic Policy

The book is part of the CESifo Seminar Series in Economic Policy, which aims to cover topical policy issues in economics from a largely European perspective. The books in this series are the products of the papers presented and discussed at seminars hosted by CESifo, an international research network of renowned economists supported jointly by the Center for Economic Studies at Ludwig-Maximilians University, Munich, and the Ifo Institute for Economic Research. All publications in this series have been carefully selected and refereed by members of the CESifo research network.

Hans-Werner Sinn

Introduction

Like the movements of the major exchange rates, exchange rate economics has gone through long cycles. In the 1970s during the early stage of the postwar experience with floating exchange rates, economists enthusiastically proposed simple models to explain and to predict exchange rates. These models were all based on simple analytical tools. One strand of literature used the quantity theory of money and purchasing power parity, describing the long-run equilibrium relation of money, prices, and the exchange rate, and some simple assumptions about price inertia in the short run. The most celebrated model in this vein undoubtedly was the Dornbusch model (Dornbusch 1976). Another strand of literature started from the portfolio balance model and added a dynamics linking the supply of net foreign assets to the current account (Kouri 1976; Branson 1977).

During a conference on flexible exchange rates in Stockholm in 1975 there was a strong feeling among the participants that major theoretical breakthroughs in exchange rate modeling had been achieved. The feeling of optimism, even elation, that was present was not very different from the feelings of elation during a speculative bubble in financial markets.

The theoretical bubble burst in the early 1980s, when Meese and Rogoff published their well-known empirical evaluation of the existing exchange rate models (Meese and Rogoff 1983). The results were devastating for all the existing theoretical models. These models appeared to have no predictive power compared to a simple alternative model, the random walk. Despite the fact that occasionally some researchers claimed to have found models that would outperform the random walk (e.g., Mark 1995), it appeared that these positive results were very sensitive to the sample periods selected in these studies (Faust et al. 2001). This conclusion is confirmed by chapter 8 of this book in which Yin-Wong Cheung, Menzie Chinn, and Antonio Garcia Pascual analyze a larger spectrum of economic models of the exchange rates than in the original Meese and Rogoff studies, confirming that none of these models outperform the random walk.

It has often been noted that economic models tend to withstand the test against the random walk better when used for long-term predictions (see Mark 1995). This was sometimes interpreted to mean that the economic models of the exchange rates were not that bad after all. But this was only superficially so. The truth is that the Meese-Rogoff empirical evaluation loads the dice against the random walk model. The reason is that when out of sample forecasts of the exchange rates are made with the economic models, the realized values of the exogenous variables are used, while the forecasts with the random walk model do not have this information. As the horizon of the forecasts increases, the handicap of the random walk forecasts (as compared to the forecasts with the economic models) increases. Thus much of the superior predictive performance of economic models over longer horizons is due to a statistical construction favoring these models.

After the intellectual crash of the early 1980s triggered by the Meese and Rogoff empirical studies, theoretical modeling of exchange rates came to a virtual standstill for a decade. Few economists dared to develop exchange rate models, let alone test these models with empirical data. This lasted until the early 1990s when a turnaround was in the making. This turnaround came about as a result of several new developments.

First, new theoretical insights were gained about the microstructure of the financial markets. These insights were first applied in stock markets, and later introduced in the analysis of the foreign exchange markets. Pioneering work in this area was done by Richard Lyons (Lyons 1999). This led to a flourishing new literature that concentrated on the question of how information is transmitted in the market when agents have private information. This literature was a major breakthrough compared to the previous one in which representative agents use the same public information. It led to exciting new insights into the functioning of the foreign exchange market. The first two chapters of this book testify for this. The first chapter by Evans and Lyons uses insights from the microstructure literature and comes to the conclusion that the portfolio balance theory is surprisingly alive, that there are economically meaningful effects arising from the imperfect substitutability between domestic and foreign assets even in a world of highly integrated financial markets. The authors conclude that this has important implications for the ability of the monetary authorities to intervene successfully in the foreign exchange markets.

The second chapter is in the same vein. It analyzes the importance of trading flows and finds that the effects of these flows differ as between the type of agents who initiate these flows. This suggests that heterogeneous expectations are important in the understanding of the dynamics in the foreign exchange markets.

Another equally important theoretical development occurred in the 1990s and gave a new boost to the theoretical analysis of the exchange rate. This is the new open economy macroeconomics pioneered by Obstfeld and Rogoff in the mid-1990s (Obstfeld and Rogoff 1996). This theoretical development started from the idea that macroeconomic analysis should be firmly grounded on a microeconomic foundation. This led to macroeconomic models in which all decisions of agents are based on explicit utility maximization in a multi-period setup. Any assumption deviating from this paradigm was branded as an intolerable ad hoc assumption. A new fundamentalism took over the profession and led to a large literature in which the implications of this paradigm were analyzed.

It also led to a large literature analyzing the exchange rate, an example of which is to be found in chapter 3 of this book. In this chapter Michael Moore and Maurice Roche present a micro-founded macro model explaining the volatility of the exchange rate in such a framework. Not surprisingly, in such a world of fully informed rational agents the high volatility of the nominal exchange rate must be based on real exchange rate variability. The authors identify the source of this variability in the variability of the marginal rate of substitution between home and foreign goods, which in turn arises from an externality in habit persistence.

There is no doubt that by its insistence on logical consistency and intellectual rigor, the new open economy macroeconomics provides new avenues of sophisticated research opportunities for young economic graduates. Up to now, however, this research has not led to the formulation of many empirical propositions that could lead to a refutation of these models. As a result it is still unclear whether this approach has a sufficiently strong scientific foundation. After all, the success of a theory should be judged by its capacity to stand empirical tests, and not by its logical consistency or its intellectual rigour.

Scepticism about the ability of the rational expectations–fully informed agent paradigm has led researchers into other directions. One such direction recognizes that agents use different information sets, and thus not all can be rational in the sense of using all available information. Note that this is also implicit in the microstructure literature that was discussed earlier. Such a world of heterogeneous agents creates a rich dynamics of exchange rate movements, as is shown in the chapter of De Grauwe and Grimaldi. In this chapter, chartists and fundamentalists interact and create a dynamics that in many respects resembles the dynamics observed in the foreign exchange market (systematic disconnection of the exchange rate from its fundamental, excess volatility, fat tails, volatility clustering). Similar results are found in chapter 6 where Volker Böhm and Tomoo Kikuchi analyze the connection between the business cycle fluctuation and the fluctuations in the exchange rate.

Much remains to be done in the modeling of the foreign exchange markets. This is very clear from the empirical studies collected in this volume. Chapter 4 written by Mark Taylor documents the strong nonlinearities that exist in the dynamic adjustment of the real exchange rate toward its equilibrium value. The author suggests that these nonlinearities can only be understood by introducing transactions costs into our models. These transactions costs create a band of inaction of the arbitrage opportunities in the goods markets. As a result the real exchange rate will react in a nonlinear way to the size of the shocks; namely the speed of adjustment of the real exchange rate toward its equilibrium value increases with the size of the initial disturbance.

Econometric techniques have not stood still. New and powerful techniques have been developed allowing researchers to devise better empirical tests. These techniques have also influenced the empirical analysis of the exchange rates. Several chapters in this book use these state of the art econometric techniques to subject the exchange rates to an empirical analysis.

In chapter 7 Hans-Werner Sinn and Frank Westermann subject the dollar/DM and the dollar/euro exchange rates to an empirical analysis. Using a modified portfolio balance model that takes into account the link with the money market, they come to the conclusion that the depreciation of the euro during the period 1999 to 2001 and its subsequent recovery was very much influenced by the shifts in the demand for marks in central and eastern Europe.

The last two chapters contain a similar quest for underlying fundamentals of the exchange rates. In chapter 8 Camarero, Ordonez, and

Tamarit use dynamic panel data econometrics to measure the importance of a number of fundamental economic variables. The authors come to the conclusion that these fundamental economic variables contain useful information to understand the movements of the exchange rate. The extent to which these fundamental economic variables can be used for predictive purposes remains an open question, however.

In the last chapter Cumperayot adds another dimension to the analysis. She argues persuasively that in order to explain the movements of the exchange rates, not only the traditional macroeconomic variables such as the money stocks, inflation, and output matter. Macroeconomic uncertainty is of equal importance. Therefore the author uses measures of macroeconomic uncertainty and finds that variations in this uncertainty explains a significant part of the fluctuations of the exchange rate around its fundamentals.

The chapters of this book reflect the very divergent paradigms now in use in the economics profession. Some chapters are grounded on the paradigm of the representative and fully informed rational agent. Other chapters rely on a paradigm of heterogeneity of agents who use different and incomplete sets of information. These differences in the fundamental paradigms lead to different insights and heated discussions among their proponents.

These differences may also lead to the impression that macro and monetary analysis is in a state of crisis. To a certain extent this is also the case. At the same time the competition between these different paradigms is a source of new debates and insights that hopefully will lead to a new synthesis allowing us to better understand and predict the movements in the exchange rate.

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Exchange Rate Economics

1 Are Different-Currency Assets Imperfect Substitutes?

Martin D. D. Evans and
Richard K. Lyons

The idea that different-currency assets are imperfect substitutes occupies an important place within exchange rate economics.[1] It is still invoked, for example, for why sterilized intervention can be effective. And theoretical work continues to rely on this assumption.[2] Yet supportive empirical evidence is scant.[3] This chapter addresses the gap between theory and empirics. We test imperfect substitutability in a new, more powerful way and find it strongly supported.

Empirical work on imperfect substitutability in foreign exchange falls into two groups: (1) tests using measures of asset supply and (2) tests using measures of central bank asset demand. We address the demand side, but we examine demand by the public broadly rather than focusing on demand by central banks. Under floating rates, changing public demand has no direct effect on monetary fundamentals, current or future. This provides an opportunity to test for price effects from imperfect substitutability. Because data on public trades became available only recently (due to the advent of electronic trading), this strategy is feasible for the first time.

The discriminating power of our approach arises from avoiding difficulties inherent in past approaches. The asset-supply approach, for example, has low power because measuring supplies is notoriously difficult. First, one must determine which measure of supply is the most appropriate. (There is considerable debate in the literature about this issue; e.g., see Golub 1989.) Then, for any given measure, the consistency of data across countries is a concern. Finally, these data are available only at lower frequencies (e.g., quarterly or monthly) and are rather slow-moving, making it difficult to separate the effects of changing supply from the many other forces moving exchange rates.

The central bank demand approach—an "event study" approach—may also have limited statistical power because central bank trades in major markets are relatively few and are small relative to public trading. For example, the average US intervention reported by Dominguez and Frankel (1993b) is only $200 million, or roughly one-thousandth of the daily spot volume in either of the two largest markets. (Since then, US intervention has been larger, typically in the $300 million to $1.5 billion range, but market volume has been higher too; see Edison 1998.) Studies using this latter approach are more successful in finding portfolio balance effects (e.g., Loopesko 1984; Dominguez 1990; Dominguez and Frankel 1993a). Nevertheless, results using this approach are not exclusively positive (e.g., Rogoff 1984) and the extent these event studies pertain to price effects from portfolio shifts in the broader market is not clear.

The "micro portfolio balance model" we develop embeds both Walrasian features (as in the traditional portfolio balance approach) and features more familiar to models from microstructure finance. Regarding the latter, the model clarifies the role played by order flow in conveying information about shifts in traders' asset demands.[4] Beyond this clarification, two analytical results in particular are important guideposts for our empirical analysis: (1) order flow's effect on price is persistent as long as public demand for foreign currency is less than perfectly elastic (even when beliefs about future interest rates are held constant), and (2) in the special case where central bank trades are sterilized, conducted anonymously, and convey no policy signal, the price impact of these trades is indistinguishable from that of public trades.[5] The latter result links our analysis directly to intervention operations of this type.

We establish three main results. First, testable implications of our model are borne out: we find strong evidence of price effects from imperfect substitutability. The portfolio balance approach—with its rich past but lack of recent attention—may warrant some fresh consideration. Second, we provide a precise estimate of the immediate price impact of trades: 0.44 percent per $1 billion (of which about 80 percent persists indefinitely). Our third result speaks to intervention policy. (As noted above, our price impact estimate is applicable to central bank trades as long as they are sterilized, secret, and provide no signal.) Estimates suggest that central bank intervention of this type is most effective at times when the flow of macroeconomic news is strong.

The remainder of the chapter is in five sections. Section 1.1 introduces our trading-theoretic approach to measuring price impact. Section 1.2

presents our micro portfolio balance model. Section 1.3 describes the data. Section 1.4 presents model estimates and discusses their implications (e.g., for central bank intervention). Section 1.5 concludes.

1.1 A Trading-Theoretic Approach to Imperfect Substitutability

This section links the traditional macroeconomic approach to exchange rates to microeconomic theories of asset trading. This is useful for two main reasons. First, theories of asset trading provide greater resolution on how trades affect price. By greater resolution, we mean that individual channels within the macro approach can be broken into separate subchannels. These subchannels are themselves empirically identifiable. Second, a trading-theoretic approach establishes that most channels through which trades—including intervention—affect price involve information asymmetry. Impounding dispersed information in price is an important function of the trading process (which our model is designed to capture).

Within macroeconomics, central bank (CB) currency demand affects price through two channels: imperfect substitutability and asymmetric information. Distinct modeling approaches are used to examine these two channels. For the first channel, imperfect substitutability, macro analysis is based on the portfolio balance approach. Models within this approach are most useful for analyzing intervention that is sterilized and conveys no information (signal) about future monetary policy. Macro analysis of the second channel, asymmetric information, is based on the monetary approach. These models are most useful for analyzing intervention that conveys information about current policy (unsterilized intervention) or future policy (sterilized intervention with signaling). This channel captures the CB's superior information about its own policy intentions. Let us examine these two macro channels within a trading-theoretic approach.

1.1.1 Imperfect Substitutability

In contrast to macro models, which address imperfect substitutability at the marketwide level only, theories of asset trade address imperfect substitutability at two levels. The first level is the dealer level. Dealers—being risk averse—need to be compensated for holding positions they would not otherwise hold. This requires a temporary risk premium, which takes the form of a price-level adjustment. This price adjustment is temporary because this risk premium is not necessary once positions are shared with the wider market. In trading-theoretic models, price effects from this channel are termed “inventory effects.” These effects dissipate quickly in most markets because full risk sharing occurs rapidly (e.g., within a day).[6]

Within trading models, imperfect substitutability also operates at a second level, the marketwide level. At this level the market as a whole—being risk averse—needs to be compensated for holding positions it would not otherwise hold.[7] This too induces a risk premium, which elicits a price-level adjustment. Unlike price adjustment at the first level of imperfect substitutability, price adjustment at this second level is persistent (because risk is fully shared at this level). This is precisely the price adjustment that macro models refer to as a portfolio balance effect.

The first of these two levels of imperfect substitutability is not present within the macro approach. Indeed, use of the term imperfect substitutability within that approach refers to the second level only. The logic among macroeconomists for addressing only the second level is that effects from the first level are presumed fleeting enough to be negligible at longer horizons. This is of course an empirical question—one that our trading-theoretic approach allows us to address in a rigorous way. Moreover our modeling of this channel provides a more disciplined way to understand why part of intervention’s effect on price is fleeting (and what determines the duration of this part of the effect).

Below we test empirically whether either or both of these two levels of imperfect substitutability are present. If the first level is present—the dealer level—then *FX* trades should have an impact on the exchange rate, but the effect should be temporary. We term this effect a “temporary portfolio balance channel.” If the second level is present—the marketwide level—then trades should have persistent impact. We term this effect a “persistent portfolio balance channel.”

1.1.2 Asymmetric Payoff Information

Theories of asset trading provide a third channel through which trades affect price—asymmetric payoff information (e.g., see Kyle 1985; Glosten and Milgrom 1985).[8] If trades convey future payoff information (sometimes referred to as “fundamentals” in exchange rate economics), then they will have a second persistent effect on price beyond the persistent portfolio balance effect noted above. (For example, in equity markets managers of firms have inside information about earnings, and their trades can convey this information.) Unsterilized interventions are an example of currency trades that convey payoff information (i.e., information about current interest rates). Another example is sterilized intervention that signals future interest rate changes.

In foreign-exchange markets, however, trades by market participants other than central banks (the public) do not in general convey payoff information: under floating-rate regimes, public trades have no direct effect on monetary fundamentals (money supplies, interest rates, and by extension, future price levels).[9] For these trades, then, the payoff-information channel is not operative. This presents an opportunity to use public trades to test for the presence of the two types of portfolio balance effect.

1.2 A Micro Portfolio Balance Model

The model is designed to show how the trading process reveals information contained in order flow. At a micro level, it is the flow of orders between dealers that is particularly important: public trades are not observable marketwide but are subsequently reflected in interdealer trades, which *are* observed marketwide. Once observed, this information is impounded in price. This information is of two types, corresponding to the two portfolio balance effects outlined in the previous section: information about temporary portfolio balance effects and information about persistent portfolio balance effects.

To understand these different portfolio balance effects, consider the model's basic structure. At the beginning of each day, the public and central bank place orders in the foreign exchange market. (These orders are stochastic and are not publicly observed.) Initially dealers take the other side of these trades—shifting their portfolios accordingly. To compensate the (risk-averse) dealers for the risk they bear, an intraday risk premium arises, producing a temporary portfolio balance effect on price. The size of this price effect depends on the size of the realized order flow. This is the first of the two information types conveyed by order flow.

To understand the second, first note that at the end of each day, dealers pass intraday positions on to the public (consistent with empirical findings that dealers end their trading day with no position; see Lyons 1995 and Bjonnes and Rime 2003). Because the public's (nonstochastic) demand at the end of the day is not perfectly elastic—that is, different-currency assets are imperfect substitutes in the macro sense—beginning-of-day orders have portfolio balance effects that persist beyond the day. Thus the price impact of these risky positions is not diversified away even when they are shared marketwide.[10] The size of this price effect too is a function of the size of the beginning-of-day order flow. This is the second of the two information types conveyed by order flow.

1.2.1 Specifics

Consider an infinitely lived, pure-exchange economy with two assets, one riskless and one risky, the latter representing foreign exchange.[11] Each day, foreign exchange earns a payoff R, publicly observed, which is composed of a series of random increments:

$$R_t = \sum_{i=1}^{t} \Delta R_i. \tag{1}$$

The increments ΔR are iid normal, $\mathrm{N}(0, \sigma_R^2)$. We interpret the increments as the flow of public macroeconomic information (e.g., interest rate changes).

The foreign exchange market has three participant types: dealers, customers, and a central bank. The N dealers are indexed by i. There is a continuum of customers (the public), indexed by $z \in [0, 1]$. Dealers and customers all have identical negative exponential utility defined over periodic wealth. Central bank trades are described below.

Within each day t there are four rounds of trading:

Round 1: Dealers trade with the central bank and public.

Round 2: Dealers trade among themselves (to share inventory risk).

Round 3: R_t is realized and dealers trade among themselves a second time.

Round 4: Dealers trade again with the public (to share risk more broadly).

The timing of events within each day is shown in figure 1.1, which also introduces our notation.

1.2.2 Central Bank Trades

To accommodate analysis of intervention, we include trades by a central bank. The intervention we consider is of a particular type, equivalent in its features to public trades: intervention that is sterilized, secret

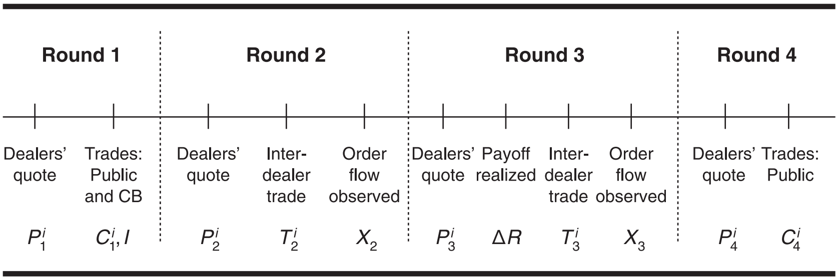

Figure 1.1
Daily timing

(anonymous and unannounced), and conveys no signal of future monetary policy.[12]

More specifically, each day, one dealer is selected at random to receive an order from the central bank. To maintain anonymity, the CB order is routed to the selected dealer via an agent. Let I_t denote the intervention on day t, where $I_t < 0$ denotes a CB sale (dealer purchase). The central bank order arrives with the public orders at the end of round 1. The CB trade is distributed normally: $I_t \sim \mathrm{N}(0, \sigma_I^2)$.[13] Because the CB trade is sterilized and conveys no signal, I_t and the daily interest increments ΔR_t are uncorrelated (at all leads and lags). Secret intervention insures that only the dealer who receives the CB trade observes its size (though not its source). A CB trade is, under these circumstances, indistinguishable from other customer orders.[14]

1.2.3 *Trading Round 1*

At the beginning of each day t, each dealer simultaneously and independently quotes a scalar price to the public and central bank.[15] We denote this round-1 price of dealer i as P_1^i. (We suppress unnecessary notation for day t; as we will see, it is the within-day rounds—the subscripts—that capture the model's economics.) This price is conditioned on all information available to dealer i.

Each dealer then receives from the public a net customer order, C_1^i, that is executed at his quoted price P_1^i; $C_1^i < 0$ denotes a net customer sale (dealer i purchase). Each of these N customer-order realizations is distributed normally, $C_1^i \sim \mathrm{N}(0, \sigma_C^2)$. They are uncorrelated across dealers and uncorrelated with the payoff R. These orders represent portfolio shifts by the public, for example, coming from changing hedging demands, changing transactional demands, or changing risk preferences. Their realizations are not publicly observed. At the time the customer orders are received, one dealer also receives the intervention trade.

1.2.4 *Trading Round 2*

Round 2 is the first of two interdealer trading rounds. Each dealer simultaneously and independently quotes a scalar price to other dealers at which he agrees to buy and sell (any amount), denoted P_2^i. These interdealer quotes are observable and available to all dealers in the market. Each dealer then simultaneously and independently trades on other dealers' quotes. Orders at a given price are split evenly between dealers quoting that price.

Let T_2^i denote the net interdealer trade initiated by dealer i in round two. At the close of round 2, all agents observe a noisy signal of interdealer order flow from that period:

$$X_2 = \sum_{i=1}^{N} T_2^i + \nu, \tag{2}$$

where $\nu \sim \mathrm{N}(0, \sigma_\nu^2)$, independently across days. The model's difference in transparency across trade types corresponds well to institutional reality: customer–dealer trades in major foreign-exchange markets (round 1) are not generally observable, whereas interdealer trades do generate signals of order flow that can be observed publicly.[16]

1.2.5 *Trading Round 3*

Round 3 is the second of the two interdealer trading rounds. At the outset of round 3 the payoff increment ΔR_t is realized and the daily payoff R_t is paid (both observable publicly). As in round 2, each dealer then simultaneously and independently quotes a scalar price to other dealers at which he agrees to buy and sell (any amount), denoted P_3^i. These interdealer quotes are observable and available to all dealers in the market. Each dealer then simultaneously and independently trades on other dealers' quotes. Orders at a given price are split evenly between dealers quoting that price.

Let T_3^i denote the net interdealer trade initiated by dealer i in round 3. At the close of round 3, all agents observe interdealer order flow from that period:

$$X_3 = \sum_{i=1}^{N} T_3^i. \tag{3}$$

Note that this round-3 order flow is observed without noise, unlike the noisy order flow signal observed in round 2 (equation 2). The idea here is a natural one: dealers' beliefs about random customer demands in round 1 become more precise over successive interdealer trading rounds (these beliefs are due to learning from interdealer trades). Of course, the observation process is not noiseless. We use this more extreme assumption for technical convenience. If dealer updating were Bayesian in round 3, as it is in the round 2, then prices set in round 4 will introduce some noise in the risk sharing between dealers and the nondealer public (described below). This noise would not alter the basic economics of the model, nor the basic structure of the model's solution as presented in proposition 1 below.

1.2.6 Trading Round 4

In round 4, dealers share overnight risk with the nondealer public. Unlike round 1, the public's trading in round 4 is nonstochastic. Initially each dealer simultaneously and independently quotes a scalar price P_4^i at which he agrees to buy and sell any amount. These quotes are observable and available to the public.

The mass of customers on the interval $[0, 1]$ is large (in a convergence sense) relative to the N dealers. This implies that the dealers' capacity for bearing overnight risk is small relative to the public's capacity. By this assumption, dealers set prices optimally such that the public willingly absorbs dealer inventory imbalances, and each dealer ends the day with no net position (which is common practice among actual spot foreign-exchange dealers). These round-4 prices are conditioned on the interdealer order flow X_3, described in equation (3). We will see that this interdealer order flow informs dealers of the size of the total position that the public needs to absorb to bring the dealers back to a position of zero.

To determine the round-4 price—the price at which the public willingly absorbs the dealers' aggregate position—dealers need to know (1) the size of that aggregate position and (2) the risk-bearing capacity of the public. We assume the latter is less than infinite. Specifically, given negative exponential utility, the public's total demand for foreign exchange in round 4 of day t, denoted C_4, is proportional to the expected return on foreign exchange conditional on public information:

$$C_4 = \gamma(E[P_{4,t+1} + R_{t+1} \mid \Omega_{4,t}] - P_{4,t}), \tag{4}$$

where the positive coefficient γ captures the aggregate risk-bearing capacity of the public ($\gamma = \infty$ is infinitely elastic demand), and $\Omega_{4,t}$ includes all public information available for trading in round 4 of day t.

1.2.7 Equilibrium

The dealer's problem is defined over six choice variables, the four scalar quotes P_1^i, P_2^i, P_3^i, and P_4^i, and the two dealer's interdealer trades T_2^i and T_3^i. Appendix A provides details of the model's solution. Here we provide some intuition.

Consider the four quotes P_1^i, P_2^i, P_3^i, and P_4^i. No arbitrage ensures that at any given time all dealers will quote a common price: quotes are executable by multiple counterparties, so any difference across dealers would provide an arbitrage opportunity. Hereafter we write P_1, P_2, P_3, and P_4 in lieu of P_1^i, P_2^i, P_3^i, and P_4^i. It must also be the case that if all dealers quote a common price, then that price must be conditioned on common information only. Common information arises at three points: at the end of round 2 (order flow X_2), at the beginning of round 3 (payoff R), and at the end of round 3 (order flow X_3). The price for round-4 trading, P_4, reflects the information in all three of these sources.

The following optimal quoting rules specify when the common-information variables $(X_2, X_3, \Delta R)$ are impounded in price. These quoting rules describe a linear, Bayes-Nash equilibrium.

Proposition 1 Dealers in our micro portfolio balance model choose the following quoting rules, where the parameters $\lambda_2, \lambda_3, \delta$, and ϕ are all positive:

$$P_2 - P_1 = 0,$$

$$P_3 - P_2 = \lambda_2 X_2,$$

$$P_4 - P_3 = \lambda_3 X_3 + \delta \Delta R - \phi(P_3 - P_2).$$

For intuition on these quoting rules, note that the price change from round 1 to round 2 is zero because no additional public information is observed from round-1 trading (neither customer trades nor the CB trade are publicly observed). The change in price from round 2 to round 3, $\lambda_2 X_2$, is driven by public observation of the interdealer order flow X_2. X_2 here serves as an information aggregator. Specifically, it aggregates dispersed information about privately observed trades of the public and CB. The value $\lambda_2 X_2$ is the price adjustment required for market clearing—it is a risk premium that induces dealers to absorb the round-1 flow from the public and CB (that round-1 flow equaling $\sum_i C_1^i + I$). The price change from round 3 to round 4 includes both pieces of public information that arise in that interval: X_3 and ΔR. The second round of interdealer flow X_3 conveys additional information about round-1 flow (from the public and CB) because it does not include noise. The payoff increment ΔR will persist into the future, and therefore must be discounted into today's price. The third component of the price change from round 3 to 4 is dissipation of a temporary portfolio balance effect that arose between rounds 2 and 3. Specifically, part of the risk premium that $\lambda_2 X_2$ represents is a temporary premium that induces dealers to hold risky positions intraday. The end-of-day price P_4 does not include this because dealers hold no positions overnight.

The persistent portion of the portfolio balance effect arises in this model because interdealer order flow informs dealers about the portfolio shift ($\sum_i C_1^i + I$) that must be absorbed at day's end by the public. If the end-of-day public demand were perfectly elastic, order flow would still convey information about the portfolio shift, but the shift would not affect the end-of-day price. This persistent portfolio balance effect is the same in the model regardless of whether the initial order flow came from the public or the central bank. Thus CB trades of the type we consider here have the same effect on price as a customer order of the same size—both induce the same portfolio shift at day's end by the public.

1.3 Empirical Analysis

1.3.1 Data

The dataset contains time-stamped, tick-by-tick observations on actual transactions for the largest spot market—DM/\$—over a four-month period, May 1 to August 31, 1996. These data are the same as those used by Evans (2002), and the reader is referred to that paper for additional detail. The data were collected from the Reuters Dealing 2000-1 system via an electronic feed customized for the purpose. Dealing 2000-1 is the most widely used electronic dealing system. According to Reuters, over 90 percent of the world's direct interdealer transactions take place through the system.[17] All trades on this system take the form of bilateral electronic conversations. The conversation is initiated when a dealer uses the system to call another dealer to request a quote. Users are expected to provide a fast two-way quote with a tight spread, which is in turn dealt or declined quickly (i.e., within seconds). To settle disputes, Reuters keeps a temporary record of all bilateral conversations. This record is the source of our data. (Reuters was unable to provide the identity of the trading partners for confidentiality reasons.)

For every trade executed on D2000-1, our data set includes a time-stamped record of the transaction price and a bought/sold indicator. The bought/sold indicator allows us to sign trades for measuring order flow. This is a major advantage: we do not have to use the noisy algorithms used elsewhere in the literature for signing trades. One drawback is that it is not possible to identify the size of individual transactions. For model estimation, order flow is therefore measured as the difference between the *number* of buyer-initiated and seller-initiated trades.[18]

The variables in our empirical model are measured hourly. We take the spot rate, as the last purchase-transaction price (DM/$) in hour h, P_h. (With roughly 1 million transactions per day, the last purchase transaction is generally within a few seconds of the end of the hour. Using purchase transactions eliminates bid-ask bounce.) Order flow, X_h, is the difference between the number of buyer- and seller-initiated trades (in hundred thousands, negative sign denotes net dollar sales) during hour h. We also make use of three further variables to measure the state of the market: trading intensity, N_h, measured by the gross number of trades during hour h; price dispersion, σ_h, measured by the standard deviation of all transactions prices during hour h, and the number of macroeconomic announcements, A_h. These announcements comprise all those reported over the Reuter's News service that relate to macroeconomic data for the United States or Germany. The source is Olsen Associates (Zurich) (for details, see, e.g., Andersen and Bollerslev 1998).

Although trading can take place on the D2000-1 system 24 hours a day, 7 days a week, the vast majority of transactions in the DM/$ take place between 6 am and 6 pm, London time, Monday through Friday. Although the results we report below are based on this subsample, they are quite similar to results based on the 24-hour trading day (as noted below). This subsample still leaves us with vast number of trades, providing us with considerable power to test for effects from portfolio balance.

1.3.2 *The Empirical Model*

Our model is specified with each day split into four trading rounds. We now develop an empirical implementation for examining the model's implications in hourly data.

Let $\rho_j(y_h)$ denote the probability that the market will move from round j to $j+1$ between the end of hours h and $h+1$, when the state of the market at the end of hour h is y_h.[19] Given these transition probabilities, the probability that the market will be in round j at the end of hour h, $\pi_j(Y_{h-1})$, is defined recursively as

$$\pi_j(Y_{h-1}) = \rho_{j-1}(y_{h-1})\pi_{j-1}(Y_{h-2}) + [1 - \rho_j(y_{h-1})]\pi_j(Y_{h-2}), \tag{5}$$

where $Y_h = \{y_h, y_{h-1}, \ldots\}$ denotes current and past states of the market.

According to proposition 1, prices change when the market moves from rounds 2 to 3, and from rounds 3 to 4. Let ΔP_h and ΔR_h respectively denote the change in price and the flow of macroeconomic information between the end of hours $h-1$ and h. With the aid of the probabilities $\rho_j(y_{h-1})$ and $\pi_j(Y_{h-1})$, we can derive the probability distribution of hourly price changes as shown in table 1.1.

Rows II and III of table 1.1 identify the price change associated with the market moving into round 3 and into round 4 between the end of hours $h-1$ and h respectively. In the former case, the price change is proportional to order flow during the hour. In the latter, the price change depends on order flow and macroeconomic information during the hour, and a lagged price change ΔP_{h-k}, for $k > 0$. The length of the lag k equals the number of hours the market spends in round-3 trading before moving to round 4. The probabilities in the right-hand column are complicated functions of $\rho_j(y_{h-l})$ for $j = 1, 2, 3, 4$, and $l > 0$ and so depend on the past states of the market, $Y_{h-1} = \{y_{h-1}, y_{h-2}, \ldots\}$ (see appendix B for details). In the special case where the probability of moving from round 3 to round 4, $\rho_3(y_{h-1})$, equals one, k must also equal one, and the probabilities simplify to

Table 1.1
Distribution of hourly price changes

	ΔP_h: Hourly price change	Probability
I	0	$\theta_I(Y_{h-1})$
II	$\lambda_2 X_h$	$\theta_{II}(Y_{h-1})$
III	$\lambda_3 X_h - \phi \Delta P_{h-k} + \delta \Delta R_h$	$\theta_{III,k}(Y_{h-1})$

$$\theta_I(Y_{h-1}) = 1 - \theta_{II}(Y_{h-1}) - \theta_{III,1}(Y_{h-1}),$$

$$\theta_{II}(Y_{h-1}) = \rho_2(y_{h-1})\pi_2(Y_{h-1}),$$

$$\theta_{III,1}(Y_{h-1}) = \rho_2(y_{h-2})\pi_2(Y_{h-2}).$$

Our empirical model is derived from the distribution of hourly price changes. Specifically, let $\Omega_h = \{X_h, Y_{h-1}, \Delta P_{h-1}, \Delta P_{h-2}, \ldots\}$ denote the information set spanned by current order flow, past states of the market, and past hourly price changes. The observed hourly price change can be written as

$$\Delta P_h = E[\Delta P_h \mid \Omega_h] + \eta_h, \tag{6}$$

where η_h is the expectational error in hour h. Since the flow of macroeconomic information in hour h, ΔR_h, is orthogonal to Ω_h, this error includes ΔR_h. To complete the empirical model, we need the conditional expectation from the distribution of hourly price changes. For the special case noted above where $\rho_3(y_{h-1}) = 1$, this expectation is given by

$$E[\Delta P_h \mid \Omega_h] = \beta_1(Y_{h-1})X_h + \beta_2(Y_{h-1})\Delta P_{h-1} \tag{7}$$

with $\beta_1(Y_{h-1}) = \lambda_2\theta_{II}(Y_{h-1}) + \lambda_3\theta_{III,1}(Y_{h-1})$ and $\beta_2(Y_{h-1}) = -\phi\theta_{III,1}(Y_{h-1})$. Hourly price-change dynamics can therefore be represented by

$$\Delta P_h = \beta_1(Y_{h-1})X_h + \beta_2(Y_{h-1})\Delta P_{h-1} + \eta_h. \tag{8}$$

In the more general case where $\rho_3(y_{h-1}) \leq 1$, the equation for price changes contains more than one lag of past price changes on the right-hand side (see appendix B for details). These lags are not statistically significant in our data. We therefore focus attention on equation (8), which takes the form of a regression with state-dependent coefficients.

1.3.3 Causality

A common critique of empirical models along the lines of equation (8) is based on the following alternative hypothesis: public information causes positively correlated adjustment in both price and order flow, with no causal relationship between price and order flow themselves. For example, macroeconomic news that is positive for the dollar causes the DM price of a dollar to go up and causes a relative increase in transactions initiated by dollar buyers. (This alternative hypothesis is distinct from the reverse causality hypothesis under which price increases cause buyer-initiated transactions—i.e., positive feedback trading. Evans and Lyons 2002b reject the hypothesis that positive feedback trading accounts for the positive correlation between interdealer *FX* order flow and price changes.)

Though intuitively appealing, this hypothesis of correlation without causation is inconsistent with rational expectations. As long as expectations are rational, public news does not produce the positive concurrent correlation between order flow and price changes that one finds empirically. The reason is because—under rational expectations—public information is impounded in price instantaneously. At the new price, which embeds all the public information, there is no longer motivation for dollar buying relative to dollar selling. True, the change in price level may induce trading (i.e., unsigned volume), due perhaps to portfolio rebalancing, but one would not expect good news for the dollar to produce positive order flow on average (a relative increase in transactions initiated by dollar buyers).

Consider the possibility that all market participants do not interpret public macro news the same way (in terms of its implication for the exchange rate). This is a departure from traditional modeling of public information in exchange rate economics. Under this scenario, price-setting market-makers who need to clear the market need to determine the interpretations of other market participants (which they cannot know a priori, by assumption). How might they learn them? The answer from microstructure theory is that they learn from the sequence of submitted orders over time. In this case, price instantaneously adjusts to the market-maker's rational expectation of the mean market interpretation, and then goes through a period of gradual adjustment to the sequence of transacted orders. Thus, in this (again, nontraditional) setting, causality in part goes directly from public news to price and in part goes from public news to order flow to price. Though catalyzed by public information, it is not the case that there is no causal relationship between price and order flow.

1.4 Results and Implications

Estimation of our micro portfolio balance model allows us to answer three key questions. First, is there support for portfolio balance in the data? Though existing negative results have led to the view that portfolio balance theory is moribund, past work may suffer from low power (as noted in the introduction). Second, do trades have both temporary and persistent portfolio balance effects? Third, does the price impact of trades depend on the state of the market? This last question is central to identifying states in which intervention is most effective.

1.4.1 Model Estimates

Our estimation strategy proceeds in two stages. First, we estimate a constant-coefficient version of equation (8) and test for state dependency in the coefficients. As we will see, the coefficients in this model accord with portfolio balance predictions in terms of sign and significance. The estimated coefficients also accord with our model in that they are indeed state dependent. This latter result motivates the second stage of our strategy, namely, estimation of the precise nature of this state dependency (using nonparametric kernel regressions).

Table 1.2 presents results from the first stage of our estimation: the constant-coefficient model. Both contemporaneous order flow X_h and lagged price change ΔP_{h-1}—the two core variables in our model—have the predicted signs and are significant. (Though constants do not arise in our derivation, for robustness we also estimate the model with constants; they are insignificant.) A coefficient on order flow X_h of 0.26 translates into price impact of about 0.44 percent per $1 billion.[20] (The magnitude is similar when we use log price change as the dependent variable, as can be seen in table 1.4 of the appendix.) A coefficient on lagged price change of -0.2 implies that $1/1.2$, or 83 percent of the impact effect of order flow persists indefinitely. Thus we are finding evidence of both types of portfolio effect noted in row I: the temporary portfolio balance channel and the persistent portfolio balance channel. Though the temporary channel is clearly present, the permanent channel accounts for the lion's share of order flow's price effect (it is also, we would argue, the more important economically).

As pointed out by the referee, the temporary channel implies profitable trading strategies, at least at high frequencies, so it is useful to consider just how profitable this would be given our estimates and given realistic transaction costs. (Of course, the reason these temporary price effects arise in the model is that they represent compensation to dealers for bearing intraday risk, i.e., they are risk premia. Hence profitability is not the only criterion for judging their realism. Nevertheless, if the implied profits are large, then the idea that they represent a premium for bearing risk becomes less tenable.) As a back-of-the-envelope

Table 1.2
Estimates of micro portfolio balance model (constant coefficients), $\Delta P_h = \beta_1 X_h + \beta_2 \Delta P_{h-1} + \eta_h$

					Diagnostics		
	X_h	ΔP_{h-1}	X_{h-1}	ΔP_{h-2}	R^2	Serial	Hetero
I	0.258	−0.203			0.212	0.437	0.071
	(13.205)	(4.103)				0.287	0.020
II	0.225				0.173	<0.001	0.070
	(12.297)					<0.001	0.011
III		−0.061			0.003	0.150	0.271
		(1.359)				0.311	0.016
IV	0.234		−0.041		0.180	<0.001	0.067
	(12.083)		(2.551)			<0.001	0.009
V	0.258	−0.202	−0.001		0.212	0.205	0.071
	(13.016)	(3.857)	(0.081)			0.282	0.020
VI	0.260	−0.200		0.044	0.220	0.823	0.086
	(13.213)	(3.878)		(0.729)		0.381	0.028

State variables	$\beta_1(\cdot)$	$\beta_2(\cdot)$
Tests for state-dependency, p-values		
τ: Time of day	<0.001	0.652
N_h: Number of trades	0.002	0.051
σ_h: Standard deviation of prices	<0.001	0.239
A_h: Number of announcements	0.176	0.006

Note: OLS estimates are based on hourly observations from 6:00 to 18:00 BTS from May 1 to August 31, 1996, excluding weekends; t-statistics are shown in parentheses and are calculated with asymptotic standard errors corrected for heteroskedasticity. ΔP_h is the hourly change in the spot exchange rate (DM/\$). X_h is the hourly interdealer order flow, measured contemporaneously with ΔP_h (negative for net dollar sales, in thousands). The Serial column presents the p-value of a chi-squared LM test for first-order (top row) and fifth-order (bottom row) serial correlation in the residuals. The Hetero column presents the p-value of a chi-squared LM test for first-order (top row) and fifth-order (bottom row) ARCH in the residuals. The lower panel presents the p-values for a heteroskedasticity-consistent Wald test of the null hypothesis that $\beta_i(\cdot)$ does not vary with the state variable shown in the left-hand column. σ_h is the standard deviation of all the transactions prices, N_h is the number of transactions, and A_h is the number of macroeconomic announcements, all during hour h. τ is a vector of three dummy variables, $[\tau_1 \ \tau_2 \ \tau_3]$. τ_1 equals one for hours between 6:00 am and 7:59 am, and zero otherwise; τ_2 equals one for hours between 8:00 am and 11:59 am, and zero otherwise; and τ_3 equals one for hours between 12:00 pm and 1:59 pm, and zero otherwise.

calculation, note that order flow here is explaining about 20 percent of hourly returns. Given that about 20 percent of these flow-driven effects are dissipating, this implies that about 4 percent of total hourly returns are predictable. Now the standard deviation of hourly DM/$ returns is on the order of *20 basis points*, calculated from the annual standard deviation of roughly 10 percent divided by the square root of 220×12, where 220 is the number of trading days per year and 12 is the number of trading hours in the active part of the DM/$ trading day (assumes temporal independence, which is close enough to being correct for this purpose). Four percent of 20 basis points is about 1 basis point, which is on par with the inside bid-offer spread for interdealer trading in this market. Hence, the implied profitability from maintaining risky positions that arise naturally from market-making is just small enough to prevent the lowest-transaction-cost players from executing incremental round-trip transactions (even if they were risk neutral).

Rows II through VI of table 1.2 illustrate the sensitivity of the estimates to various departures from our derived specification. Note from row III, for example, that returns are not negatively autocorrelated unconditionally; only when order flow is included does the negative relationship emerge. Note too from rows V and VI that when both our core variables are included, lags of these variables do not enter significantly (which accords with the model). Row IV shows that lagged order flow proxies for lagged price change, albeit imperfectly, when the latter is excluded (as one would expect). The reduced explanatory power of the proxy may arise for two reasons: our order flow measure does not capture all order flow in the market (e.g., it does not include brokered interdealer flow) and the transparency of order flow in practice is lower than the transparency of price.

The bottom panel of table 1.2 presents results from the state dependency tests. These tests address whether the two core coefficients are affected by variables describing the state of the market, which include time of day, the number of trades, the volatility of prices, and the number of macroeconomic announcements (the latter three measured over the previous hour). The results of these tests are clear: the coefficients are indeed state dependent.

To measure the effects of changing state variables, we turn to the second stage of our estimation: nonparametric kernel estimation (see Bierens 1983; Robinson 1983). This is a flexible and intuitive means of estimating precisely how the state variables affect the two core coefficients. These regressions take the form

$$\Delta P_h = m(z_h) + \eta_h,$$

where $m(\cdot)$ is an arbitrary fixed but unknown nonlinear function of the vector $z_h = [X_h \ \Delta P_{h-1} \ \sigma_{h-1} \ N_{h-1} \ A_{h-1}]$, which includes the two core variables and three state variables. The error η_h is iid mean zero. Intuitively this method estimates $m(\cdot)$ by taking a weighted average of the ΔP's from other observations with similar z's. The weights are normally distributed, attaching greater weight to the most similar z's. (See appendix B for a formal definition of the weighting procedure.) The derivatives of the estimated kernel $\hat{m}(z_h)$ with respect to our two core variables can then be regressed on the state variables, providing a precise measure of how they vary by state.[21]

Table 1.3 presents these dependencies of the two core coefficients on movements in the state variables. The first panel presents the effects of state variables on the derivative of the kernel with respect to order flow X_h. For example, the positive and significant coefficient on the number of announcements A_{h-1} implies that the price impact of order flow is 0.024 higher for each additional macro announcement (for perspective, recall that the unconditional estimate of that price-impact coefficient in table 1.2 is 0.26). Note too from this first panel that there does not appear to be nonlinearity in the order-flow/price-change relation (per the insignificant impact of X_h on the derivative of the estimated kernel with respect to X_h, reported in column 3).

The effects of the state variables are more broadly present in the case of the lagged price-change coefficient (panel two of table 1.3). Column 4, for example, shows that there is some nonlinearity in this case: the larger the lagged price change, the greater the negative autocorrelation in price (i.e., the greater the transitory portion of portfolio balance effects). Columns 5, 6, and 7 show additional state dependence, in these cases due to volatility, trading activity (numbers of trades), and announcements. The negative (and significant) coefficient on announcements in this case implies that the greater the number of announcements, the greater the negative autocorrelation in price.

The third panel of table 1.3 shows the effects of the state variables on the volatility (absolute change) of the estimated residual from the kernel regressions. The most striking result in this panel is the strong intraday seasonality in this unexplained volatility. This is evident from the estimated coefficients on τ_h (the number of hours since midnight at the end of hour h) and τ_h^2 in columns 8 and 9. These estimates indicate a U-shape in volatility over the trading day, consistent with findings

Table 1.3
Nonparametric (kernel) regressions, $\Delta P_h = m(z_h) + \eta_h$

Dependent variable (1)	Constant (2)	z_h: X_h (3)	ΔP_{h-1} (4)	σ_{h-1} (5)	N_{h-1} (6)	A_{h-1} (7)	τ_h (8)	τ_h^2 (9)	Diagnostics: R^2	Serial	Hetero
$\hat{m}_x(z_h)$	0.134	<0.001	<0.001	−0.014	0.001	0.024			0.071	0.866	0.556
	(15.249)	(0.342)	(0.997)	(1.225)	(0.264)	(3.788)				0.231	0.315
	0.077			−0.013	−0.001	0.023	0.010	<0.001	0.071	0.943	0.531
	(1.989)			(0.977)	(0.261)	(3.558)	(1.493)	(1.366)		0.198	0.323
	0.136			−0.018	0.001	0.024			0.069	0.968	0.536
	(14.719)			(1.432)	(0.352)	(3.803)				0.225	0.313
$\hat{m}_{\Delta p}(z_h)$	−0.050	0.001	−0.002	−0.072	0.016	−0.028			0.066	0.116	<0.001
	(3.120)	(1.552)	(2.851)	(3.287)	(2.560)	(2.626)				0.230	<0.001
	−0.005			−0.055	0.018	−0.026	−0.007	<0.001	0.044	0.294	<0.001
	(0.072)			(1.915)	(2.490)	(2.340)	(0.545)	(0.201)		0.147	<0.001
	−0.061			−0.052	0.014	−0.027			0.038	0.279	<0.001
	(3.662)			(1.958)	(2.215)	(2.534)				0.195	<0.001
$\|\hat{\eta}_h\|$	4.488	−0.011	−0.027	13.780	−1.096	0.566			0.184	0.973	0.999
	(5.042)	(0.662)	(1.048)	(9.757)	(3.145)	(1.207)				0.222	0.217
	29.214			12.201	−0.297	0.946	−4.458	0.176	0.200	0.493	0.661
	(4.952)			(7.049)	(0.737)	(2.089)	(4.436)	(4.364)		0.209	0.232
	31.046			11.698		0.910	−4.807	0.189	0.200	0.414	0.590
	(5.815)			(7.442)		(2.073)	(5.326)	(5.126)		0.249	0.255

Note: In the kernel regression, z_h is the vector of five conditioning variables and $\hat{m}(z_h)$ is the nonparametric estimate of $E[\Delta P_h \mid z_h]$. $\hat{m}_x(z_h)$ and $\hat{m}_{\Delta p}(z_h)$ are the derivatives of the estimated kernel with respect to X_h and ΔP_{h-1} respectively. See table 1.2 for other variable definitions. Estimated using OLS, with t-statistics shown in parentheses (calculated with standard errors corrected for heteroskedasticity). τ_h is the number of hours since midnight at the end of hour h. The Serial column presents the p-value of a chi-squared LM test for first-order (top row) and fifth-order (bottom row) residual serial correlation. The Hetero column presents the p-value of a chi-squared LM test for first-order (top row) and fifth-order (bottom row) ARCH in the residuals.

elsewhere (e.g., Andersen and Bollerslev 1998). In addition the second and third rows of panel three indicate that conditional heteroskedasticity is tied to the flow of announcements.

1.4.2 Implications for Intervention

What are the implications of these empirical results for the efficacy of intervention? First, and foremost, our results indicate that order flow has a significant price impact under normal conditions. Recall from the theoretical model that this happens only when public demand for foreign currency is less than perfectly elastic. The data provide solid evidence of imperfect substitutability, a necessary condition for the efficacy of intervention that is sterilized, secret, and conveys no policy signal.

Our results also provide a guide to the size of an intervention's price impact. From table 1.2, \$1 billion of net dollar purchases increases the DM price of dollars by 0.44 percent, with about 80 percent of this persisting indefinitely. When linking this estimate to intervention, however, the finding needs to be interpreted with care: a dollar of net interdealer flow is not equivalent to a dollar of public flow. To take an extreme case, a dealer trading with a central bank could decide to retain the resulting inventory indefinitely, so that intervention has no impact whatsoever on interdealer order flow. However, this isn't an optimal strategy for the risk-averse dealers in our model, and it is doubtful that it would be optimal more generally. Moreover empirical evidence in Lyons (1995) and Bjonnes and Rime (2003) indicates that dealers unwind inventory positions rather quickly. Only under extreme and counterfactual assumptions would interdealer order flow be unaffected by an intervention trade.

It is also possible for intervention's effect on interdealer flow to be magnified. For example, suppose the inventory position caused by the CB's trade is quickly passed from dealer to dealer over a succession of trading rounds, a phenomenon market participants refer to as "hot potato" trading (Lyons 1997). This process could amplify the effects on interdealer order flow considerably, thereby amplifying the subsequent impact on price. With this possibility of hot potato trading, then, it is appropriate to consider the 0.44 percent per \$1 billion estimate as a lower bound.

The model estimates may also shed light on whether intervention can help maintain an "orderly market." Theory provides little guidance along these lines because—without market failure of some kind—it is unclear why markets should be any less orderly than required by market efficiency. Nevertheless, central banks do cite the maintenance of orderly markets as a distinct objective when articulating their policies. As an empirical matter, then, it is useful to consider whether intervention trades might affect exchange rates differently depending on whether the market is "orderly" or "disorderly."

Estimates in table 1.3 suggest that the price impact of order flow is not significantly affected by variables commonly associated with disorder, namely the volatility of transaction prices σ_h and the intensity of trading (proxied by the number of trades per unit time, N_h). This suggests that—at least in terms of mean effects—intervention retains its efficacy even during times of higher volatility and trading intensity. The one state variable that clearly affects price impact is the flow (number) of macroeconomic announcements. In this case a stronger flow of announcements is associated with greater price impact.

1.5 Conclusion

In this chapter we measure portfolio adjustment where it actually occurs—within the trading process. The resulting flow of transacted orders provides a powerful means of testing for imperfect substitutability. Until recently this strategy was not feasible due to data limitations. The advent of electronic trading, and the data it provides, has made it feasible for the first time.

Our analysis provides three main results. First, our model's implications are borne out: we find strong evidence of price effects from imperfect substitutability, both temporary and persistent. This contrasts with a common belief that these effects (from intervention or otherwise) are too small to be detectable. Not only are they detectable, they are also economically significant, leading us to conclude that portfolio balance theory is more applicable than many believe. The second result pertains to the economic significance noted above. Specifically, we establish the (unconditional) price impact of trades of about 0.5 percent per $1 billion. With gross flows in the largest spot markets at about $200 billion per day, this level of price impact is potentially quite important.[22] Our third result clarifies how this unconditional price impact varies with the state of the market. The most important state variable for the size of the price-impact coefficient is the flow of macroeconomic announcements. (It may be, for example, that order flow is the variable that market participants use to resolve uncertainty about how these announcements are interpreted.) Whatever the reason, our estimates imply that trades have the most price impact when the flow of macroeconomic news is strong. This applies to intervention trades as well, provided they are sterilized, secret, and provide no policy signal (i.e., as long as they mimic private trades).

Finally, we offer some thoughts on future application of our trading-theoretic approach to intervention. Market data now coming available allow precise tracking of how the market absorbs actual CB trades, and any information in them. CB's with precise knowledge of their own trades—announcements, timing, stealth level, and so on—can estimate the impact of these various "parameter" settings. Consider, for example, the type of data used by Payne (1999), which includes the order book of an electronic interdealer broker. A CB with these data, over a sufficiently large number of intervention trades, can learn exactly how the "book" is affected, including the process of price adjustment, liquidity provision on both sides, and transaction activity. It is something like a doctor who has a patient ingest blue dye to determine how it passes through the system. The whole process becomes transparent. Such is the future of empirical work on this topic.

Appendix A

Model Solution

Each dealer determines quotes and speculative demand by maximizing a negative exponential utility function defined over periodic (daily) wealth. Within a given day t, let W_j^i denote the end of round j wealth of dealer i. By this convention, W_0^i denotes wealth at the end of day $t-1$. (We suppress notation to reflect the day t where clarity permits.) With this notation, and normalizing the gross return on the riskless asset to one, we can define the dealers' problem over the six choice variables described in section 1.2, namely the four scalar quotes P_j^i, one for each round j, and the two outgoing interdealer trades, T_2^i and T_3^i:

$$\underset{\{P_1^i, P_2^i, P_3^i, P_4^i, T_2^i, T_3^i\}}{\text{Max}} \quad E[-\exp(-\theta W_4^i \,|\, \Omega^i)] \tag{A1}$$

subject to

$$\begin{aligned} W_4^i = W_0^i &+ C_1^i(P_1^i - \tilde{P}_2^i) + \tilde{T}_2^i(P_2^i - \tilde{P}_3^i) + \tilde{T}_3^i(P_3^i - P_4^i) \\ &+ (T_2^i - C_1^i)(\tilde{P}_3^i - \tilde{P}_2^i) + (T_3^i + T_2^i - C_1^i - \tilde{T}_2^i)(P_4^i - \tilde{P}_3^i). \end{aligned}$$

Dealer i's wealth over the four-round trading day is affected by positions taken two ways: incoming random orders and outgoing deliberate orders. The incoming random orders include the public order C_1^i and the incoming interdealer orders $\tilde{T}_2^i$ and $\tilde{T}_3^i$ (the tilde distinguishes incoming interdealer orders and prices from outgoing). The outgoing deliberate orders are the two interdealer trades T_2^i and T_3^i. $\tilde{P}_j^i$ denotes an incoming interdealer quote received by dealer i in round j. For example, the second term in the budget constraint reflects the position from the public order C_1^i received in round 1 at dealer i's own quote P_1^i and subsequently unwound at the incoming interdealer quote $\tilde{P}_2^i$ in round 2. (Recall that the sign of dealer i's position is opposite that of C_1^i, so a falling price is good for dealer i if the public order is a buy, i.e., positive. The dealer's speculative positioning based on information in C_1^i is reflected in the final two terms of the budget constraint.) Terms 3 and 4 reflect the incoming random dealer orders and are analogous.

Terms 5 and 6 of the budget constraint reflect the dealer's speculative and hedging demands. The outgoing interdealer trade in round 2 has three components:

$$T_2^i = C_1^i + D_2^i + E[\tilde{T}_2^i|\Omega_{2T}^i], \tag{A2}$$

where D_2^i is dealer i's speculative demand in round 2, and $E[\tilde{T}_2^i|\Omega_{2T}^i]$ is the dealer's hedge against incoming orders from other dealers (this term is zero in equilibrium given the distribution of the C_1^i's). The dealer's total demand can be written as follows:

$$T_2^i - C_1^i = D_2^i + E[\tilde{T}_2^i|\Omega_{2T}^i],$$

which corresponds to the position in term five of the budget constraint. The sixth term in the budget constraint is analogous: the dealer's total demand in round 3 is his total trade in round 3 (T_3^i) plus his total demand in round 2 $(T_2^i - C_1^i)$ less the random interdealer order he received in round 2 $(\tilde{T}_2^i)$.

The conditioning information Ω^i at each decision node (4 quotes and 2 outgoing orders) is summarized below (see also the daily timing in figure 1.1).

$$\Omega_{1P}^i = \{_{k=1}^{t-1}\{\Delta R_k, X_{2k}, X_{3k}, P_{1k}, P_{2k}, P_{3k}, P_{4k}\}\},$$

$$\Omega_{2P}^i = \{\Omega_{1P}^i, P_{1t}, C_1^i\},$$

$$\Omega_{2T}^i = \{\Omega_{2P}^i, P_{2t}\},$$

$$\Omega_{3P}^i = \{\Omega_{2T}^i, X_{2t}\},$$

$\Omega^i_{3T} = \{\Omega^i_{3P}, P_{3t}, \Delta R_t\}$,

$\Omega^i_{4P} = \{\Omega^i_{3T}, X_{3t}\}$.

At this stage it is necessary to treat each of the prices in these information sets as a vector that contains the price of each individual dealer i (though in equilibrium each of these prices is a scalar, as shown below).

Equilibrium

The equilibrium concept we use is Bayesian-Nash equilibrium, or BNE. Under BNE, Bayes's rule is used to update beliefs and strategies are sequentially rational given beliefs.

To solve for the symmetric BNE, first consider optimal quoting strategies.

Proposition A1 A quoting strategy is consistent with symmetric BNE only if quotes within any single trading round are common across dealers.

Proposition A2 A quoting strategy is consistent with symmetric BNE only if $P_1 = P_2$, and these prices are equal to the final round price P_4 from the previous day.

Proposition A3 A quoting strategy is consistent with symmetric BNE only if the common round-3 quote is

$P_3 = P_2 + \lambda_2 X_2$,

where the constant λ_2 is strictly positive and X_2 denotes the signal of round-2 interdealer order flow.

Proposition A4 A quoting strategy is consistent with symmetric BNE only if the common round-four quote is

$P_4 = P_3 + \lambda_3 X_3 + \delta \Delta R - \phi(P_3 - P_2)$,

where the constants λ_3, δ, and ϕ are strictly positive and X_3 denotes round-3 interdealer order flow.

Propositions A1 through A4

The proof of proposition A1 is straightforward. All dealers must post the same quote in any given trading round to eliminate risk-free arbitrage. (Recall from section 1.2 that all quotes are scalar prices at which the dealer agrees to buy/sell any amount, and trading with multiple partners is feasible.)

The proof of proposition A2 is straightforward as well. Common prices require that quotes depend only on information that is commonly observed. In round 1, this includes the previous day's round-4 price. Because there is no new information commonly observed between the round-4 and round-2 quotings on the following day, the round-4 price is not updated. (Recall that public trading in round 4 is a deterministic function of round-4 prices and therefore conveys no information.) Thus dealers' round-2 quotes are not conditioned on individual realizations of C_1^i.

Propositions A3 and A4 require equations that pin down the levels of the four prices. As shown above, these equations are necessarily functions of public information. Naturally they also embed the equilibrium trading rules of dealers and customers. The equations are the following:

$$E[C_1 + I \mid \Omega_{1P}] + E[ND_2^i(P_1) \mid \Omega_{1P}] = 0, \tag{A3}$$

$$E[C_1 + I \mid \Omega_{2P}] + E[ND_2^i(P_2) \mid \Omega_{2P}] = 0, \tag{A4}$$

$$E[C_1 + I \mid \Omega_{3P}] + E[ND_3^i(P_3) \mid \Omega_{3P}] = 0, \tag{A5}$$

$$E[C_1 + I \mid \Omega_{4P}] + E[C_4(P_4) \mid \Omega_{4P}] = 0, \tag{A6}$$

where C_1 denotes the sum of C_1^i over all N dealers. The first three equations state that for each round j ($j = 1, 2, 3$), at price P_j dealers willingly absorb the estimated demand from customers and the central bank (realized at the beginning of the day but not observed publicly). The fourth equation states that at price P_4 the public willingly absorbs the estimated demand from customers and the central bank. These equations pin down equilibrium prices because any price other than that which satisfies each generates irreconcilable demands in interdealer trading in rounds 2 and 3 (e.g., if price is too low, all dealers know that, on average, dealers are trying to buy from other dealers, which is inconsistent with rational expectations; see Lyons 1997 for a detailed treatment in another model within the simultaneous trade approach).

From these equations, $P_2 - P_1 = 0$ follows directly from two facts: (1) the expected value of $C_1 + I$ conditional on public information Ω_{1P} or Ω_{2P} is zero and (2) expected dealer demand D_2^i is also zero at this public information unbiased price. To be more precise, this statement *postulates* that the dealer's demand D_2^i has this property; derivation of the optimal trading rule shows that this is the case.

That $P_3 - P_2 = \lambda_2 X_2$ with $\lambda_2 > 0$ follows from two facts: (1) interdealer order flow X_2 is the only public information revealed in this interval and (2) X_2 is positively correlated with—and therefore provides information about—the morning portfolio shift $C_1 + I$. The positive correlation arises because each of the dealer orders T_2^i of which X_2 is composed is proportional to the C_1^i received by that dealer (and proportional to $C_1^i + I$ in the case of the dealer receiving the central bank order, per proposition A5 below). A positive expected $C_1 + I$ induces an increase in price because it implies that dealers—having taken the other side of these trades—are short and need to be induced to hold this short position with an expected downward drift in price.

The exact size of this downward drift in price depends on where price is expected to settle at the end of the day. Per proposition A4, $P_4 - P_3 = \lambda_3 X_3 + \delta\Delta R - \phi(P_3 - P_2)$. This price change depends positively on the two pieces of public information revealed in this interval, X_3 and ΔR.[23] The logic behind the positive X_3 effect is the same as that behind the positive X_2 effect in round 2: a positive average T_3^i implies that the market's estimate of $C_1 + I$ from X_2 was too low; absorption of the additional short position requires price increase. (That a positive average T_3^i implies this is clear from the derivation of T_3^i.) The term $\delta\Delta R$ is the perpetuity value of the change in the daily payoff R_t. Finally, the drift term $-\phi(P_3 - P_2)$ is the equilibrium compensation to dealers for having to absorb the morning portfolio shift through the interval in which ΔR (and the associated price risk) is realized. This is an intraday price effect that dissipates by the end of the day.

Equilibrium Trading Strategies

An implication of common interdealer quotes is that in rounds two and three each dealer receives a share $1/(N-1)$ of every other dealer's interdealer trade. These orders correspond to the position disturbances $\tilde{T}_2^i$ and $\tilde{T}_3^i$ in the dealer's problem in equation (A1).

Given the quoting strategy described in propositions 1 through 4, the following trading strategy is optimal and corresponds to symmetric linear equilibrium:

Proposition A5 The trading strategy profiles

$$T_2^i = \alpha C_1^i$$

for dealers not receiving the central bank order and

$$T_2^i = \alpha(C_1^i + I)$$

for the one dealer receiving the central bank order, with $\alpha > 0$, conform to Bayesian-Nash equilibrium.

Proposition A6 The trading strategy profiles

$$T_3^i = \kappa_1 C_1^i + \kappa_2 X_2 + \kappa_2 \tilde{T}_2^i$$

for dealers not receiving the central bank order and

$$T_3^i = \kappa_1 (C_1^i + I) + \kappa_2 X_2 + \kappa_3 \tilde{T}_2^i$$

for the one dealer receiving the central bank order conform to Bayesian-Nash equilibrium.

Sketch of Proofs for Propositions A5 and A6

As noted above, because returns are independent across periods, with an unchanging stochastic structure, the dealers' problem collapses to a series of independent trading problems, one for each day. Because there is a finite number (N) of dealers, however, each dealer acts strategically in the sense that his speculative demand depends on the impact his trade will have on subsequent prices.

Propositions A5 and A6 are special cases of the analysis in Lyons (1997), which is also set in the context of a simultaneous-trade game with two interdealer trading rounds. Accordingly we refer readers to that analysis for details on the derivation of optimal trading rules in this setting.[24] Two differences warrant note here. First, the Lyons (1997) analysis also includes private and public signals (denote s_i and s in that chapter) that are not present in our specification, meaning they equal zero. Second, our model includes a central bank trade. However, because the central bank trade is sterilized, secret, and provides no monetary policy signal, the dealer receiving that trade treats it the same as any other customer trade.

Appendix B

Kernel Regression

To examine whether there is state dependency in the relation between aggregate order flow and price changes we consider nonparametric regressions of the form

$$\Delta p_h = m(z_h) + \eta_h,$$

where $m(\cdot)$ is an arbitrary fixed but unknown nonlinear function of the variables in the vector z_h, and η_h is a mean zero iid error. An estimate of the $m(\cdot)$ function is estimated by kernel regression as

$$m(z_h) = \frac{\sum_{j=0,\, j\neq h}^{H} K_{\vartheta}(z_h - z_j)\Delta p_j}{\sum_{j=0,\, j\neq h}^{H} K_{\vartheta}(z_h - z_j)},$$

where the kernel function $K(u) \geq 0$, $\int K(u)\,du = 1$, and $K_b(u) = b^{-1}K(u/b)$, where b is the bandwidth parameter. In this application we use the multivariate Gaussian kernel $K(u) = (2\pi)^{-d/2}\exp(-u'u/2)$, where $d = \dim(u)$. The bandwidth parameter, b, is chosen by cross-validation. That is to say, b minimizes

$$\frac{1}{H}\sum_{h}^{H}(\Delta p_h - m(z_h))^2 w_h,$$

where w_h is a weighting function that cuts off 5 percent of the data at each end of the data interval as in Hardle (1990, p. 162). For the regressions in table 1.3, z_h contains $\{X_h, \Delta P_{h-1}, \sigma_{h-1}, N_{h-1}, A_{h-1}\}$. We follow the common practice of including the standardized value of each of these variables in the Gaussian kernel (i.e., each element of z_h is divided by its sample standard deviation).

Asymptotic theory for kernel regressions in the time series context appear in Bierens (1983) and Robinson (1983). Robinson shows that consistency and asymptotic normality of the estimator can be established when the data satisfy α-mixing with mixing coefficients $\alpha(k)$ that obey the condition $H\sum_{h}^{\infty}\alpha(k)^{1-2/\delta} = O(1)$ and $E|\Delta p_h|^{\delta} < \infty, \delta > 2$.

The Empirical Model

In the general case where $\rho_3(y_t^{h-1}) \leq 1$, the distribution of hourly price changes is

$$\Delta P_h = \begin{cases} 0, & \theta_I(Y_{h-1}), \\ \lambda_2 X_h, & \theta_{II}(Y_{h-1}), \\ \lambda_3 X_h - \phi \Delta P_{h-k} + \delta \Delta R_h, & \theta_{III,k}(Y_{h-1}), \end{cases}$$

where

$$\theta_I(Y_{h-1}) = \pi_1(Y_{h-1}) + (1 - \rho_2(y_{h-1}))\pi_2(Y_{h-1}) + \pi_4(Y_{h-1})$$
$$\quad + (1 - \rho_3(y_{h-1}))\pi_3(Y_{h-1}),$$

$$\theta_{II}(Y_{h-1}) = \rho_2(y_{h-1})\pi_2(Y_{h-1}),$$

$$\vdots$$

$$\theta_{III,k}(Y_{h-1}) = \rho_3(y_{h-1}) \prod_{j=2}^{k} (1 - \rho_3(y_{h-j}))\rho_2(y_{h-k-1})\pi_2(Y_{h-k-1})$$

The expected change in prices is given by

$$E[\Delta P_h \mid \Omega_h] = \beta_0(Y_{h-1})\Delta X_h + \sum_{i=1}^{\infty} \beta_i(Y_{h-1})\Delta P_{h-i},$$

where

$$\beta_0(Y_{h-1}) = \lambda_2 \theta_{II}(Y_{h-1}) + \lambda_3 \sum_{k=1}^{\infty} \theta_{III,k}(Y_{h-1})$$

and

$$\beta_k(Y_{h-1}) = -\phi \theta_{III,k}(Y_{h-1}).$$

Appendix C

Log Price Changes

Our model derivation implies a dependent variable in the form of changes in the level of price, rather than changes in the log of price (despite the latter being customary in exchange rate economics). For robustness, we estimate the model using log changes. The results corresponding to tables 1.2 and 1.3 appear as tables 1.4 and 1.5.

Table 1.4
Estimates of the micro portfolio balance model: Log price changes, $\Delta p_h = \beta_1 X_h + \beta_2 \Delta p_{h-1} + \eta_h$

				Diagnostics		
	X_h	Δp_{h-1}	X_{h-1}	R^2	Serial	Hetero
I	0.171	−0.204		0.211	0.430	0.074
	(13.098)	(4.053)			0.252	0.022
II	0.149			0.171	<0.001	0.074
	(12.210)				<0.001	0.012
III		−0.062		0.003	0.156	0.276
		(1.380)			0.303	0.018
IV	0.155		−0.027	0.179	<0.001	0.070
	(11.993)		(2.542)		<0.001	0.010
V	0.171	−0.203	−0.001	0.211	0.199	0.074
	(12.907)	(3.810)	(0.082)		0.247	0.022

State variables	$\beta_1(\cdot)$	$\beta_2(\cdot)$
Tests for state-dependency, p-values		
τ: Time of day	<0.001	0.675
N_h: Number of trades	0.002	0.051
σ_h: Standard deviation of prices	<0.001	0.241
A_h: Number of announcements	0.183	0.007

Note: OLS estimates are based on hourly observations from 6:00 to 18:00 BTS from May 1 to August 31, 1996, excluding weekends; t-statistics are shown in parentheses and are calculated with asymptotic standard errors corrected for heteroskedasticity. Δp_h is the hourly change in the log spot exchange rate (DM/\$). X_h is the hourly interdealer order flow, measured contemporaneously with Δp_h (negative for net dollar sales, in thousands). The serial column presents the p-value of a chi-squared LM test for first-order (top row) and fifth-order (bottom row) serial correlation in the residuals. The Hetero column presents the p-value of a chi-squared LM test for first-order (top row) and fifth-order (bottom row) ARCH in the residuals. The lower panel presents the p-values for a heteroskedasticity-consistent Wald test of the null hypothesis that $\beta_i(\cdot)$ does not vary with the state variable shown in the left-hand column. σ_h is the standard deviation of all the transactions prices, N_h is the number of transactions, and A_h is the number of macroeconomic announcements, all during hour h. τ is a vector of three dummy variables, $[\tau_1 \ \tau_2 \ \tau_3]$. τ_1 equals one for hours between 6:00 am and 7:59 am, and zero otherwise; τ_2 equals one for hours between 8:00 am and 11:59 am, and zero otherwise; and τ_3 equals one for hours between 12:00 pm and 1:59 pm, and zero otherwise.

Table 1.5
Nonparametric (kernel) regressions: Log price changes, $\Delta p_h = m(z_h) + \eta_h$

Dependent variable	Constant	z_h: X_h	Δp_{h-1}	σ_{h-1}	N_{h-1}	A_{h-1}	τ_h	τ_h^2	Diagnostics: R^2	Serial	Hetero
$\hat{m}_x(z_h)$	0.089	<0.001	<0.001	−0.009	0.001	0.016			0.069	0.873	0.577
	(15.213)	(0.326)	(0.966)	(1.215)	(0.284)	(3.741)				0.238	0.340
	0.051			−0.009	−0.001	0.015	0.007	0.000	0.069	0.948	0.552
	(1.980)			(0.969)	(0.246)	(3.515)	(1.487)	(1.356)		0.204	0.347
	0.090	−0.012	0.001	0.016					0.067	0.971	0.557
	(14.707)	(1.421)	(0.369)	(3.757)						0.233	0.338
$\hat{m}_{\Delta p}(z_h)$	−0.049	0.001	−0.002	−0.073	0.016	−0.030			0.069	0.116	<0.001
	(3.073)	(1.542)	(2.879)	(3.359)	(2.540)	(2.749)				0.184	<0.001
	−0.010			−0.054	0.018	−0.027	−0.006	0.000	0.046	0.300	<0.001
	(0.155)			(1.915)	(2.404)	(2.476)	(0.545)	(0.201)		0.121	<0.001
	−0.059	−0.052	0.014	−0.029					0.041	0.286	<0.001
	(3.653)	(1.973)	(2.189)	(2.656)						0.158	<0.001
$\lvert\hat{\eta}_h\rvert$	2.914	−0.008	−0.028	9.262	−0.732	0.378			0.186	0.926	0.978
	(4.859)	(0.697)	(1.051)	(9.660)	(3.143)	(1.201)				0.165	0.216
	19.213			8.232	−0.204	0.629	−2.937	0.116	0.202	0.440	0.633
	(4.878)			(6.977)	(0.754)	(2.072)	(4.381)	(4.301)		0.158	0.232
	20.472			7.887		0.604	−3.177	0.125	0.201	0.364	0.559
	(5.761)			(7.374)		(2.053)	(5.290)	(5.078)		0.192	0.255

Note: In the kernel regression, z_h is the vector of conditioning variables and $\hat{m}(z_h)$ is the nonparametric estimate of $E[\Delta p_h \mid z_h]$, where Δp_h is the hourly change in the log spot exchange rate (DM/\$). $\hat{m}_x(z_h)$ and $\hat{m}_{\Delta p}(z_h)$ are the derivatives of the estimated kernel with respect to X_h and Δp_{h-1} respectively. Estimated using OLS, with t-statistics shown in parentheses (calculated with standard errors corrected for heteroskedasticity). τ_h is the number of hours since midnight at the end of hour h. (See table 1.2 for other variable definitions.) The serial column presents the p-value of a chi-squared LM test for first-order (top row) and fifth-order (bottom row) residual serial correlation. The Hetero column presents the p-value of a chi-squared LM test for first-order (top row) and fifth-order (bottom row) ARCH in the residuals.

Notes

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1. Theory on imperfect substitutability is centered in work on portfolio balance models. See Kouri (1976), Branson (1977), Girton and Henderson (1977), Allen and Kenen (1980), Dooley and Isard (1982), and the survey by Branson and Henderson (1985).

2. See, for example, Cavallo, Perri, Roubini, Kisselev (2002), Martin and Rey (2002), and Sinn and Westermann (2001).

3. See Branson et al. (1977), Frankel (1982a, 1982b), Dooley and Isard (1982), Backus (1984), Lewis (1988), and the surveys by Lewis (1995) and Taylor (1995), among many others.

4. Order flow is not synonymous with trading volume. Order flow is a concept from microstructure finance that refers to *signed* volume. Trades can be signed in these microstructure models depending on whether the "aggressor" is buying or selling. (The dealer posting the quote is the passive side of the trade.) For example, a sale of 10 units by a trader acting on a dealer's quotes is order flow of −10. (Order flow is undefined in rational expectations models of trading because all transactions are symmetric in that setting—an aggressor cannot be identified.)

5. For more on how central banks intervene secretly, for example, see Hung (1997) and Dominguez and Frankel (1993b). Along the spectrum of intervention transparency, from secret to announced (in every detail), we are considering here only the secret end.

6. For theoretical work on inventory effects on price, see Amihud and Mendelson (1980), Ho and Stoll (1983), and Vogler (1997) among many others. Empirical work on inventory effects in FX include Lyons (1995), Yao (1998), and Bjonnes and Rime (2003).

7. Relevant theory includes rational expectations models like Grossman and Stiglitz (1980) and Kodres and Pritsker (1998). Though not a fully rational model, another recent paper that includes price effects from marketwide imperfect substitutability is Kyle and Xiong (2001). Empirical work on imperfect substitutability across stocks at the marketwide level includes Scholes (1972), Shleifer (1986), Bagwell (1992), and Kaul et al. (2000), among others.

8. The word "payoffs" in trading-theoretic models refers to the cash flows that accrue to the security's holder (e.g., dividends in the case of a stock).

9. Of course, if the floating rate is not pure, but is instead managed by the central bank via changes in monetary fundamentals, then an *indirect* payoff channel arises, in the form of a monetary policy reaction function (i.e., private trades, via their impact on the current exchange rate, do indeed correlate with future monetary policy). As an empirical matter, it is generally believed that monetary responses to the DM/$ exchange rate were not significant at the time of our sample (mid-1996). Nevertheless, that these policy responses were not exactly zero prevents us from concluding that the payoff-related effects of private trades were exactly zero.

10. Note that the size of the order flows the DM/\$ spot market needs to absorb are on average more than 10,000 times those absorbed in a representative US stock (e.g., the average daily volume on individual NYSE stocks in 1998 was about \$9 million, whereas the average daily volume in DM/\$ spot was about \$300 billion).

11. The structure of the model is similar to that of Evans and Lyons (2002a). Other papers that have adopted the model developed here include Rime (2000).

12. The model can be extended to accommodate all the main intervention types and the main channels through which intervention can be effective. (For example, sterilized versus unsterilized intervention can be modeled by admitting correlation between CB trades and the current periodic payoff R_t. A signaling channel can be introduced by admitting correlation between CB trades and future R_{t+1}.)

13. We treat the CB trade as having an expected value of zero. This should be viewed as a normalization around an "expected" intervention trade, should any nonzero expectation exist.

14. That intervention in practice often does not take this form in no way negates the fact that this intervention strategy is indeed available.

15. Introducing a bid-offer spread (or price schedule) in round one to endogenize the number of dealers is a straightforward—but distracting—extension of our model. The simultaneous-move nature of the model is in the spirit of simultaneous-move games more generally (vs. sequential-move games).

16. The screens of interdealer brokers (e.g., EBS) are an important source of these interdealer order-flow signals.

17. Interdealer transactions account for about two-thirds of total trading in major spot markets. This two-thirds from interdealer trading breaks into two transaction types—direct and brokered. Direct trading accounted for about half of interdealer trade and brokered trading accounted for the other half. For more detail on the Reuters Dealing 2000-1 System, see Lyons (2001) and Evans (2002).

18. This is common in the literature; for example, see Hasbrouck (1991). See also Jones et al. (1994) for analysis suggesting that trade size conveys no additional information (beyond that conveyed by the number of buys minus sells).

19. We assume that the market remains in a round for a minimum of one hour. Dropping this assumption complicates the calculations needed to find the distribution of hourly price changes, but it does not alter the basic structure of the empirical model; see appendix B for details.

20. Order flow X_h is measured as the net number of dollar purchases (in 10,000s). With an average trade size in the sample of \$3.9 million, this implies that \$39 billion of positive order flow raises price by 0.26 DM/\$, which is 17 percent of the average spot rate of 1.5 DM/\$. Dividing 17 percent by \$39 billion yields approximately 0.44 percent per \$1 billion. This estimate is slightly lower than that reported in Evans and Lyons (2002a) for the price impact of order flow in *daily* data. The difference stems from order flow being positively autocorrelated at the hourly frequency, so that swings in order flow persistently move prices from one hour to the next. (See Evans 2002 for more on the dynamics of this measure of order flow.) It is also slightly less than the 8 basis points per \$100 million reported by Dominguez and Frankel (1993b).

21. Including the state variables in estimating the kernel $m(z_h)$ means that we are not restricting the partial derivatives of the kernel with respect to these variables to equal zero. Imposing that restriction in estimating the kernel would have transformed our subsequent tests on the kernel derivatives into joint tests.

22. An immediate example of this fact's value is its ability to help us understand why portfolio balance effects from sterilized intervention are so hard to detect: the average intervention of $200 million reported by Dominguez and Frankel (1993b) translates into an exchange rate movement of only 0.10 percent, an amount easily swamped by movements due to other factors.

23. Interdealer order flow X_3 is observed without noise, which means it reveals the value of $C_1 + I$ fully. The price in period 4 must therefore adjust such that equation (A6) is satisfied exactly.

24. Available from the authors on request.

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2 Volume and Volatility in the Foreign Exchange Market: Does It Matter Who You Are?

Geir H. Bjønnes, Dagfinn Rime, and Haakon O. Aa. Solheim

In this chapter we study the relationship between volume and volatility in the market for foreign exchange (FX) using a unique data set from the Swedish krona (SEK) market. The data are based on daily reporting from a number of primary dealers (market making banks), both Swedish and foreign, and covers as much as 95 percent of all currency trading in Swedish krona. Each primary dealer reports their total purchases and sales in five different instruments: (1) spot, (2) outright forwards, (3) short swaps ("tomorrow-next"), (4) FX swaps, and (5) options.[1]

Studies from a number of different market settings suggest that there is a positive relationship between volatility and volume (see Karpoff 1987). Due to the lack of data there are few studies of the FX market, and those that include actual volume data have only had access to a limited part of total volume. The studies conducted by Goodhart and Figliuoli (1991) and Bollerslev and Domowitz (1993) both use the frequency of indicative quotes on the Reuters FXFX-screen as a proxy for volume. Grammatikos and Saunders (1986) and Jorion (1996) use the number of futures contracts traded at the CBOE. Wei (1994) and Hartmann (1999) use the Bank of Japan's data set on brokered transactions in the Tokyo JPY/USD market. Galati (2000) uses data provided by the BIS on actual trading volume for seven developing countries. In general, these studies suggest a positive relationship between volatility and volume consistent with evidence from other markets. Compared with previous studies our data set has the following advantages: (1) It covers the entire market for the Swedish krona, (2) FX volume is separated into different instruments, and (3) FX volume is reported individually by each primary dealer.

An important question is why the volume-volatility relationship arises. Three central contributions on the theory of volume and volatility are Clark (1973), Epps and Epps (1976), and Tauchen and Pitts (1983). Clark (1973) introduces the mixture of distribution hypothesis, where the correlation between volume and volatility arises due to the arrival of new information that drives both exchange rate changes and volume. Epps and Epps (1976) provide a second, and complementary, explanation. They argue that the volume-volatility relationship is due to disagreement between traders when they revise their reservation prices. More heterogeneous beliefs should cause more volatility.

Tauchen and Pitts (1983) provide a model that combines these two features. They point out that volume might change over time for different reasons. There might be an increase in the number of traders, new information may arrive or there may be heterogeneous beliefs between different traders. A trend in volume due to an increase in the number of traders should lead to lower volatility due to higher liquidity.

Foster and Viswanathan (1990) and Shalen (1993) present models where the dispersion of beliefs creates both more price variability and excess volume. Shalen (1993) argues that uninformed traders increase volatility because they cannot differentiate liquidity demand from fundamental value change. The market microstructure literature (e.g., Glosten and Milgrom 1985) emphasizes the role of heterogeneous beliefs in the pricing process.

In this chapter we make three contributions. First, we document a positive relationship between volume and volatility using data that covers almost all currency trading in SEK. Although a positive volume–volatility relationship is documented for the FX market in previous studies, this is to our knowledge the first time such a relationship has been documented for one of the ten largest currencies using such an extensive set of volume data.[2]

Second, we are able to separate total volume into different instruments. The standard assumption is that the spot market should be the important market for determining the exchange rate. However, previous studies have used data from both the spot market and the forward market. We show that it is indeed the spot volume that is most important. However, we also find some indications that option volume is correlated with spot exchange rate volatility.

Last, but maybe most important, we examine the role of heterogeneity in explaining volatility. This is possible since we have the volume of each of the reporting banks. That means that we have aggregates of volume that are actually observable in the market, although only to the reporting bank. This is truly private information. Since large banks have more customer orders and thus see more order flows, these banks are potentially better informed than smaller banks (Lyons 2001). It is also likely that the composition of their order flows is different. Large banks may, for instance, have a larger proportion of financial customers than smaller banks (Lyons 2001; Fan and Lyons 2003). Another distinction that may matter is that between Swedish and foreign banks. All foreign reporting banks are large in the FX market, but they are not among the largest in the market for the Swedish krona.

Our results suggest that trading with large banks tend to have the strongest impact on volatility. This is especially the case in periods of high volatility. These results suggest that private information may be important in understanding the relationship between volume and volatility. Controlling for size, there is also evidence that trading by Swedish banks is more correlated with volatility than trading by foreign banks. Thus we conclude that large Swedish banks have the highest correlation with volatility.

Studies from other market settings also suggest that heterogeneity among market players may be important to understanding volatility (e.g., see Grinblatt and Keloharju 2001). Bessembinder and Seguin (1993) and Daigler and Wiley (1999), both studying futures markets, document the importance of different types of traders for explaining the volume–volatility relationship. Daigler and Wiley (1999) find that trade "speculators," namely traders located outside the actual market, tends to be more correlated with volatility than trade by investors in the market. Since these "outsiders" may be interpreted as noise traders, this result is different from ours.

The chapter is organized as follows. In section 2.2, we give a detailed presentation of our data. In section 2.3, we present the results. In section 2.4, we make some concluding observations.

2.1 Data

We start by describing our volume data. We then present the macro variables (control variables) applied in the analysis.

2.1.1 Volume Data

Sveriges Riksbank (the central bank of Sweden) receives *daily* reports from a number of Swedish and foreign banks (currently 10) on their *buying and selling* of *five different instruments*. The reported series is an

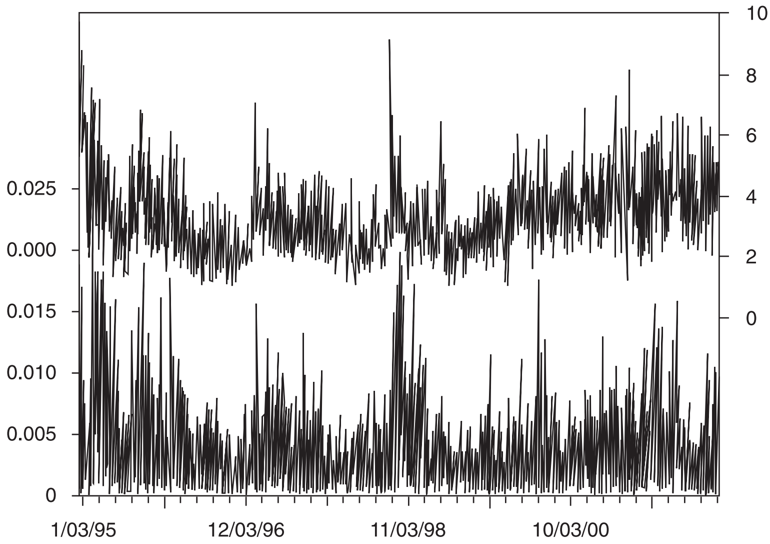

Figure 2.1
Gross spot volume and absolute changes in SEK/EUR. Upper line shows gross spot volume, measured in 10 billion SEK. Lower line shows absolute changes in the log of SEK/EUR

aggregate of Swedish krona (SEK) trading against all other currencies, measured in krona, and covers 90 to 95 percent of all worldwide trading in SEK. Close to 100 percent of all interbank trading and 80 to 90 percent of customer trading is made in SEK/EUR. In our analysis, we will therefore focus on the SEK/EUR exchange rate.

Aggregate volume information is not available to the market. FX markets are organized as multiple dealer markets and have low transparency. The specific reporter will only know her own volume and a noisy signal on aggregate volume that is received through brokers. Reporting banks do obtain some statistical summaries of volume aggregates from the Riksbank, but only with a considerable lag. The data set used in this chapter is not available to market participants.

The data set stretches from January 1, 1995, to June 28, 2002. Figure 2.1 shows the total gross volume in the spot market and the absolute returns in the exchange rate. There seems to be a relationship between volume and volatility, especially in periods of high volatility like 1996–97 and in the fall of 1998. We also note that there is no clear trend in the two series.

Table 2.1
Importance of different instruments

	Spot	For-ward	Short swap	Swap	Option
Percentage of total volume	0.27	0.06	0.37	0.27	0.03
Mean	3.14	0.73	4.72	3.53	0.41
Median	3.02	0.60	4.09	2.94	0.22
Standard deviation	1.32	0.53	2.70	2.29	0.54
Skewness	0.43	1.62	0.73	1.21	3.09
Kurtosis	4.39	7.39	3.25	4.68	15.49
Correlation					
Forward	0.58	1			
Short swap	0.40	0.59	1		
Swap	0.49	0.64	0.77	1	
Option	0.42	0.55	0.57	0.56	1

Note: The sample ranges from January 1995 to June 2002. The summary statistics of volume in the SEK market are divided by the various instruments. Short swap is a liquidity instrument with settlement within 7 days. All numbers are calculated on a daily basis. Volume is measured in units of 10 billion SEK.

The five instruments are spot, forwards, options, short swaps, and standard swaps. Short swaps are mainly used as a liquidity control instrument when cash with delivery in less than two days is required (the time of a standard spot transaction). Table 2.1 gives an indication of the relative usage of the different instruments. As a percentage of total volume in the market, short swaps is the largest category, followed by spot trading. Forward and option trading make up much smaller parts of total market volume.

The reporting banks are not named. However, we can distinguish Swedish banks from foreign banks and branches of foreign banks located in Sweden. The reporters are the main market-makers in the SEK market. At most, there are 15 reporting banks active in the market. In total, 19 banks are represented in our data.

For confidentiality reasons we cannot display detailed information on the size of each bank. Two of the banks are clearly bigger than the others. These are Swedish banks. Their market share averages 44 percent, and does not vary much over the sample period. Other Swedish banks have a market share of 20 percent. The average market share of foreign reporters is 25 percent, while the market share of branches of foreign banks is 11 percent.

Table 2.2
Concentration of primary dealers

	Large	Medium	Small
Percentage of total spot volume	0.45	0.43	0.12
Mean	1.42	1.39	0.39
Standard deviation	0.62	0.58	0.28
Skewness	1.35	0.59	1.61
Kurtosis	8.00	3.61	11.81
Correlation			
Medium	0.67	1	
Small	0.42	0.61	1

Note: The sample ranges from January 1995 to June 2002. We divide the primary dealers into three groups. Large banks are the two largest primary dealers. Medium banks are the next seven largest banks. The remaining reporters in our sample are small banks, 10 in total. The table shows summary statistics of volume in the SEK market. All numbers are calculated on a daily basis. Volume is measured in units of 10 billion SEK.

We split our banks two ways. First we split by size. The two largest banks are categorized as "large banks." Of the remaining 17 banks, we find seven banks that have an approximately equal trading volume (5–10 percent of total volume). These are categorized as "medium-sized banks." The remaining banks are regarded as "small." The group of small banks will include some banks that are in the sample for only short periods of time. The aggregate of small banks as a percentage of total volume is, however, relatively constant over the sample period. The average daily trading volume in the spot market is about 700 million SEK for the large banks, 200 million SEK for the medium-sized banks, and 40 million for the small banks. Some statistical properties are reported in table 2.2.

The banks are also split by nationality. We then look at Swedish banks and foreign banks situated outside Sweden but registered as reporters in the SEK market.

2.1.2 Macro Data

In the volatility regressions we use both the absolute value of changes and squared value of changes in the exchange rate measured from close to close in the Swedish market.[3] This is the most relevant exchange rate because most of the volume reported is carried out before the Swedish market closes. The reports are sent to the Riksbank right after the close. For the period prior to January 1, 1999, we use SEK/DEM. The exchange rate is indexed to EUR equivalent terms (SEK/DEM $\times$ 1.95583). Before 1999, DEM played the role now taken by EUR.

Figure 2.2 shows the exchange rate together with the 10-year bond spread between Sweden and Germany. From 1990 until November 1992 the SEK was pegged to the ECU. In November 1992, Sweden experienced a speculative attack, and the SEK was allowed to float and has been floating since then. Sweden introduced an inflation target in 1993. The current target is set by law at 2 percent, with a band of ± 1 percent. Sveriges Riksbank has no obligation to intervene in the foreign exchange market. A dummy is included to control for days with interventions.

The krona appreciated sharply during 1995 and early 1996. A period of depreciation then followed the Russian moratorium in August 1998. Further there was strong depreciationary pressure during 2000 and 2001. Over the period as a whole, the exchange rate has moved within a range of 27 percent from top to bottom. The standard deviation of daily changes over the period has been about 0.45 percent, with a maximum daily return of 2.0 percent. The bond spread gives an indication of the credibility of the inflation target and of macroeconomic developments in Sweden. It has fallen from nearly 4 percent in 1995 to a current spread fluctuating around zero.

According to the statistics from the BIS 2001 survey of the foreign exchange market, the Swedish krona is the eighth largest currency in the world. However, SEK is still a small currency compared to EUR, USD, or JPY. An interesting question is to which extent the volatility in the SEK/EUR market is reflection of volatility in the relative price of SEK to EUR and to which extent it is the result of volatility in EUR on a broader scale. A movement in the USD/EUR rate might, for example, be expected to trigger expectations of a similar movement in the SEK/EUR rate. There is evidence of some correlation between the two series. The correlation over the period from January 1995 to June 2002 is 0.29. We include changes in USD/EUR in the regressions below, as a proxy of general volatility in the foreign exchange market.

2.1.3 *Expected versus Unexpected Volume*

As we pointed out above, Tauchen and Pitts (1983) differentiate between an increase in volume due to an increase in the number of

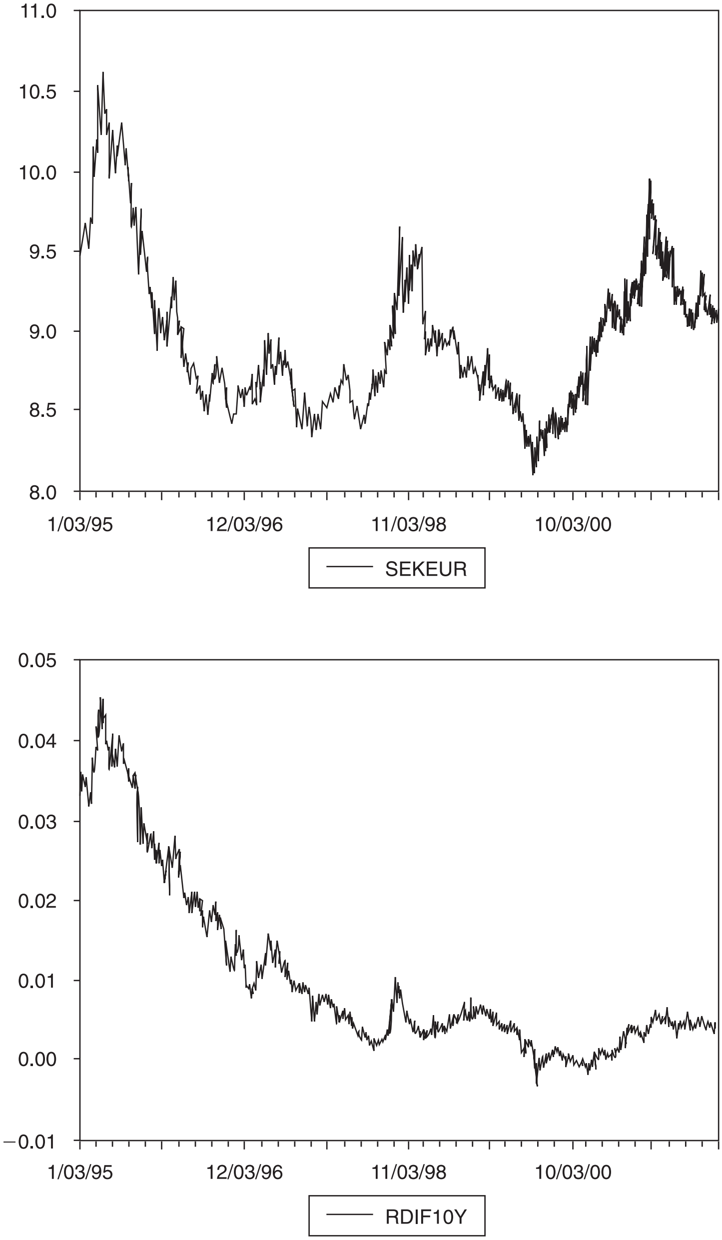

Figure 2.2
Log of the SEK/EUR exchange rate and the difference between Swedish and German ten-year bonds

traders and an increase in volume due to, for example, new information. An increase in volume due to an increase in the number of traders can be interpreted as "expected volume." Expected volume should primarily increase liquidity, and should have little or negative impact on volatility. Bessembinder and Seguin (1992) and Hartmann (1999) document the importance of unexpected volume in explaining the volume–volatility relationship.

The standard method to distinguish between expected and unexpected volume is to identify systematic time series behavior in the volume data, namely using an ARIMA model. Using stationarity tests like the augmented Dickey-Fuller or the Phillips-Perron, we find no evidence of nonstationarity. However, when we estimate an ARMA model on the volume series, the AR root tends to be close to or outside the unit circle. At the same time we find that the MA coefficient is close to -1.

Similar observations have been made by Hartmann (1999). Hartmann has volume data reported from Tokyo-based brokers, covering trading in JPY/USD over the period from 1986 to 1994. He reports that the series are stationary according to standard tests, but the AR roots have a unit root and the MA is close to -1. According to Hartmann, the fact that the MA is close to -1 might distort the stationarity tests. He therefore argues that one should treat the series as nonstationary.

Hartmann (1999) argues that an ARIMA$(9, 1, 1)$ gives the best fit for his data. However, repeated tests on our sample do not seem to give any firm evidence of improvement when we move beyond an ARMA$(2, 2)$. We have run regressions using a number of different ARIMA specifications, and these do not seem to influence the results. Nor does it have any effect whether we use the level or the first difference in these regressions. We therefore choose to use a model that is as simple as possible.

Further Hartmann argues that an ARCH(3) process removes ARCH/GARCH effects from his series. This feature can also be replicated in our data. However, again we find no improvement from using a GARCH$(3, 0)$ rather than the more standard GARCH$(1, 1)$. We therefore choose to use a GARCH$(1, 1)$.

To the ARMA$(2, 2)$ model we add a constant and dummies for each of first four days of the week. Chang, Pinegar, and Schachter (1997) document that there tend to be weekday patterns in volume data. Harris and Raviv (1993) have a model that predicts an increase in the volume on Mondays, as the dispersion of beliefs is higher after a period of closed markets. Foster and Viswanathan (1990) predict that volume on Mondays will be lower than on Tuesdays because private information accrues over weekends while public information does not. We find strong evidence in support of lower volume on Mondays, and some evidence in support of higher volume on Wednesdays. Our results are in accordance with Foster and Viswanathan (1990). The results of the regressions are reported in table 2.12 in the appendix.

Our model of expected volume has a reasonable fit. For most series we find an R^2 between 30 and 60 percent. We use the fitted values as "expected," and the residual as "unexpected."

2.2 Results

In all our regressions a measure of volatility will be the dependent variable. We use two different measures. The first is absolute return, and the second is squared return. The second measure puts more emphasis on large changes than the first.

In the regressions we need to control for volatility that is expected, and hence cannot be driven by new information or revisions in beliefs. To control for the expected volatility, all reported regressions are estimated using a GARCH$(1,1)$-M, meaning we include the squared root of the variance term in the regression as an estimate of conditional volatility.

We also take into account that volatility might be driven by the same underlying macro variables. It is therefore reasonable to include macro variables. These include absolute changes in the log of the USD/EUR, the log of a German stock index (DAX30), the log of a Swedish stock index (OMX16) and the 10-year and 3-month interest rate differential between Sweden and Germany. When the dependent variable is squared returns, these variables are included as squared changes. We also include a specific dummy that takes the value 1 in every period where Sveriges Riksbank reports an intervention. It is a notable result that this dummy is significant and positive in most regressions reported.

Theory suggests that it is unexpected volume that should be positively correlated with volatility. We estimate expected volume using ARMA$(2,2)$ models. The residual from these models is defined as unexpected volume. Using generated regressors might bias the parameter estimates. All results should therefore be interpreted with care. We do, however, find that the results for the volume terms are stable with regard to choice of estimation methods.[4] Further, the important issue in our discussion is the comparison of volume from different groups—not the coefficient of volume itself. We have no reason to believe that a possible bias in the volume coefficient should be different between different groups.

The rest of this section provides results regressing volatility on volume in different instruments and volume from different reporters or groups of reporters.

2.2.1 Instruments

The most common approach to estimating the volume–volatility relationship would be to regress the volatility of spot exchange rates on some measure of spot volume. A reasonable a priori assumption is that a volume–volatility relationship for the spot exchange rate should be dominated by transactions in the spot market. Lyons (2001) describes the spot market as the driving force of the FX market. By comparison, a swap transaction has no "order flow" effect, as it is just two opposing transactions being made at the same time.

However, volume in other instruments than spot may reflect the arrival of new information or a dispersion of beliefs, and thereby also be informative about spot *volatility*. For instance, customers may take speculative positions by trading in forward contracts. In this case the information effect might primarily be picked up by the forward volume, although this forward trading will trigger trading in the interbank spot market when the dealers try to off-load the effect on their inventories. Option volume may also reflect changes in beliefs about the true spot volatility, potentially due to new information. It may thus be interesting to see whether other instruments can also explain volatility.

Table 2.3 reports the estimations of volatility (absolute changes) on the volume for each of the five instruments. In the table we focus only on the effect of expected and unexpected volume, although the regressions also include macro variables and predicted volatility. We see that the effect of expected volume is not significantly different from zero in four of the regressions. In the only regression with a significant coefficient on expected volume (short swaps), the coefficient is significantly positive and not negative. Theory predicts that the coefficient should be negative rather than positive, since more expected volume from, for example, an increase in the number of dealers would typically mean higher liquidity.

Table 2.3
Estimating $|\Delta \log(SEK/EUR)|$

	Spot		Forward		Short swap		Swap		Options	
Unexpected	0.0008	9.31**	0.0004	2.38*	0.0000	1.34	0.0001	3.11**	0.0009	4.31**
Expected	0.0001	0.75	0.0000	0.28	0.0001	2.33*	0.0000	0.25	−0.0001	−0.75
R^2-adj	0.25		0.19		0.19		0.20		0.21	
DW-stat	2.01		2.03		2.00		2.00		2.00	
R^2-adj[c]	0.20		0.20		0.20		0.20		0.20	
R^2-adj[d]	0.12		−0.02		−0.02		−0.02		−0.01	

Note: The sample ranges from January 1995 to June 2002 with daily observations. The GARCH$(1,1)$ regressions are on the absolute value of changes in SEK/EUR. We only report results for the volume variables. Estimation includes the squared root of the conditional variance (ARCH-in-mean) and the following macro variable information (all as absolute changes): log(USD/EUR), German stock index, Swedish stock index, oil price, 10-year and 3-month interest differential between Sweden and Germany, and a dummy that takes the value 1 on days when Sveriges Riksbank reports an intervention. Volume is measured in units of 10 billion SEK. ** = significant at the 1 percent level; * = significant at the 5 percent level.

a. Expected volume is the fit of an ARMA$(2,2)$ model.
b. Unexpected volume is the residual of the expected estimation.
c. R^2-adjusted with macro variables and the conditional variance but no volume.
d. R^2-adjusted in a regression including only expected and unexpected volume.

For unexpected volumes we find positive and significant coefficients in four of the five regressions. As expected we see that spot volumes have the highest explanatory power. The table rows R^2-adj (b) and R^2-adj (c) report values for the regression only including macro variables and predicted volatility, and for the regression only including expected and unexpected volume, respectively. The table clearly shows that it is only the unexpected spot volumes that have an independent contribution to the explanatory power.

Table 2.4 reports regressions that include the unexpected volume of all instruments. Since expected volume does not seem to be important in the single regressions presented in table 2.3, we only include unexpected volumes. We report regressions on both absolute changes and squared changes in the exchange rate. The results are qualitatively similar, although the explanatory power is a little less when using squared returns. We find that only spot trading enters with a significant and positive value at the five percent level in both regressions. The coefficient on option volume is significantly larger than zero at the 5 percent level in the regression with absolute returns as the dependent variable, but only at the 10 percent level in the regression with squared returns as dependent variable. For forward, swap, and short swap trading the coefficients are actually negative, however, not significantly different from zero except in one case. Short swaps are primarily liquidity instruments, while ordinary swaps are more interest rate related instruments. It is much harder to think about information releases that might trigger swap volume instead of spot volume, while still having implications for spot exchange rate, than it is with, for example, options. From our results we find it natural to focus on spot volumes only in later regressions.

The size effects of the parameter values in table 2.4 are not obvious. To give an indication of size effects, we perform an illustrative exercise in table 2.5. One standard deviation of absolute returns is 0.3 percent. If we multiply the standard deviation of the conditional volatility term (0.0008 or 0.08 percent) with the parameter value of 1.01 (in the case of the regression on absolute changes reported in table 2.4), we obtain 0.0008 (0.08 percent). Compared with the standard deviation of the absolute changes in SEK/EUR, we see that this variable is economically significant. A similar procedure for unexpected spot volumes gives a number of 0.00084 (or 0.084 percent). This indicates that the coefficient on unexpected volume is also economically significant. Interestingly we see that the coefficient on absolute changes in USD/EUR is not so

Table 2.4
Estimating volume and volatility

	Absolute change		Squared change	
SQR(GARCH)[a]	1.01	12.05**	0.54	1.39
Constant	0.00	−2.14*	0.00	−0.49
Spot	0.0009	8.32**	9.73E-06	6.54**
Forward	−0.0003	−2.01*	−3.61E-06	−0.72
Short swap	−0.0001	−1.46	−1.43E-06	−1.03
Swap	0.0000	−0.46	−7.39E-07	−0.48
Options	0.0004	1.96*	6.54E-06	1.73
log(USD/EUR)	0.05	3.30**	0.05	3.43**
log(DAX30)	0.03	4.03**	0.01	4.62**
log(OMX16)	0.01	1.79	0.00	0.83
log(oil)	0.00	0.89	0.00	0.22
$(r^{SWE} - r^{GER})_{10Y}$	0.96	4.78**	13.50	14.57**
$(r^{SWE} - r^{GER})_{3M}$	0.39	1.44	7.42	3.85**
INT[b]	0.00	2.68**	0.00	1.93
Variance equation				
Constant	0.00	5.20**	0.00	0.11
ARCH(1)	0.05	5.50**	0.15	5.70**
GARCH(1)	0.94	97.55**	0.60	32.66**
R^2	0.26		0.20	
DW-statistic	2.01		1.84	

Note: The sample ranges from January 1995 to June 2002 with daily observations. We estimate the absolute value and the squared value of Δ log $SEK(EUR)$ on unexpected volume (the residual of an ARMA$(2,2)$) and macro variables. The model is estimated using a GARCH$(1,1)$. Volume is measured in units of 10 billion SEK. Significance is indicated by * (5 percent) and ** (1 percent). All macro variables are included as absolute changes in the regression on absolute changes, and squared changes in the regression on squared changes.

a. SQR(GARCH) is the squared conditional variance (ARCH-in-mean).

b. INT is a dummy that takes the value 1 on days when Sveriges Riksbank report an intervention.

Table 2.5
Relative effects on volatility

	Standard deviation	Parameter	Predicted effect[a]	Percentage of FX-volatility[b]
Absolute change in SEK/EUR	0.0031			
Unexpected spot	0.9334	0.0009	0.00084	26.85
Absolute change in USD/EUR	0.0043	0.05	0.0002	6.99
Absolute change in RDIF10	0.0004	0.96	0.0004	12.76
SQR(GARCH)[c]	0.0008	1.01	0.0008	24.95

Note: The sample ranges from January 1995 to June 2002. All parameters are collected from table 2.4.
a. Predicted effect is the predicted effect of a change of one standard deviation (multiply the standard deviation with parameter).
b. Percentage of FX volatility is the ratio of the predicted effect over the standard deviation of absolute returns in the SEK/EUR (measured in percent).
c. SQR(GARCH) is the squared conditional variance (ARCH-in-mean).

significant economically. A change of one standard deviation in the variable multiplied by the coefficient gives a value of only 0.0002 (or 0.02 percent). Thus volatility in the most important currency pair (i.e., USD/EUR) is not a very important driver of volatility in SEK/EUR.

2.2.2 *Reporters*

Recent research from the microstructure approach to foreign exchange indicates that traders have different strategies and information (e.g., see Lyons 1995; Bjønnes and Rime 2004). It is also reasonable to assume that different banks will focus on specific types of trading strategies (Cheung and Chinn 2001). However, banks are mostly unwilling to reveal their explicit strategies, so this is an area where few results have been published.

We have bank-specific volumes and can therefore test for differential impact from banks on volatility directly. A priori it is not obvious that different reporters should be correlated differently with volatility. If the increase in number of transactions is due to the arrival of public information only, we should expect a simultaneous increase in trading from all reporters. However, if the dispersion of beliefs (different dealers interpret information differently) is important, or if different dealers are asymmetrically informed, then the trading volume of some reporters might be more closely correlated with volatility than the volume of other reporters.

Table 2.6
Estimating $|\Delta \log(SEK/EUR)|$

	Large[a]		Medium[b]		Small[c]	
Unexpected[d]	0.0016	8.39**	0.0014	8.18**	0.0027	5.62**
Expected[e]	0.0006	3.02**	0.0001	0.87	−0.0001	−0.48
R^2-adj	0.25		0.23		0.22	
DW-stat	2.01		2.01		2.03	

Note: The sample ranges from January 1995 to December 2001. The GARCH(1, 1) regressions are on the absolute value of changes in SEK/EUR. We only report results for the volume variables.

a. Large banks are the two largest reporting banks in the sample.

b. Medium banks are the seven following banks.

c. Small banks are remaining reporters.

d. Unexpected volume is the residual of the expected estimation.

e. Expected volume is the fit of an ARMA(2, 2) model.

The issue of the size of the bank can be tested more thoroughly. In table 2.6 we have estimated the relationship by grouping reporting banks into three categories, small, medium, and large, according to size of volume. Aggregated, the two banks included in "large banks" on average control 45 percent of daily spot trading. In "medium-sized banks" we include seven banks that on average control 43 percent of trading in the spot market. "Small banks" are the remaining banks.

We see that all groups have a significant effect on volume. In fact the coefficient is clearly larger for small than for large banks. However, the R^2-adjs are highest for the regression with large banks. A clear picture emerges from table 2.7. We see that the regression with only volume from large banks as the independent variable explains 15 percent of FX volatility, while the regression with medium banks explains only 6 percent. Note that average total volume for medium banks is roughly similar to the total volume of large banks. The regression with only small banks explains only 2 percent. The difference in explanatory power is considerable, especially when considering that interdealer trades increase the correlation between the volumes of different groups.

In table 2.8 we report regressions including unexpected volume from all three categories. When we run this regression on absolute changes in SEK/EUR, we find that all groups are significant.

An interesting result becomes visible when we repeat the same regression on squared changes. In this case only the volume of large banks is significant. Squared changes do, of course, put more weight

Table 2.7
Adjusted R^2

	Macro[a]	Volume[b]	Macro and volume[c]
Absolute changes			
Large	0.20	0.15	0.25
Medium	0.20	0.06	0.23
Small	0.20	0.02	0.22
Squared changes			
Large	0.16	0.13	0.22
Medium	0.16	0.06	0.18
Small	0.16	0.03	0.17

Note: The sample ranges from January 1995 to June 2002. The adjusted R^2 is from three separate estimations: macro, volume, and macro and volume combined.
a. Estimations include the squared root of the conditional variance and the following macro variable information (as absolute changes in regression on absolute changes, as squared changes in regression on squared changes): log(USD/EUR), German stock index, Swedish stock index, oil price, 10 year and 3 month interest differential between Sweden and Germany, and a dummy that takes the value 1 on days when Sveriges Riksbank reports an intervention.
b. Estimations include expected and unexpected volume of the specified group. The volume is measured in units of 10 billion SEK.
c. Estimations are identical to those in table 2.6.

on extreme observations than absolute changes. This result seems to indicate that when volatility is truly high, trading tends to coalesce around the largest banks.

A second indication of the importance of large banks can be found in table 2.9. Here we compare the effect of volume from each of the three groups for the regression with absolute change in SEK/EUR. By comparing predicted effects of a one standard deviation change in the independent variables, we see that the effect of large banks is much stronger than the effect of medium and small banks.

In table 2.10 we test whether differences in nationality matter. To be able to compare banks of similar size, we exclude the two largest (Swedish) banks. By also excluding branches of foreign banks located in Sweden, we think that the difference between Swedish and foreign banks should be as clear as possible. The group of Swedish banks (excluding the two largest) covers on average 20 percent of total volume, while the group of foreign banks covers 25 percent. In the regression with absolute changes in SEK/EUR as the dependent variable, we see that the coefficients of unexpected volumes are significantly positive for both Swedish and foreign banks. However, the size

Table 2.8
Estimating volume and volatility: Banks divided by size

	Absolute change		Squared change	
SQR(GARCH)[a]	0.98	11.78**	0.52	1.32
Constant	0.00	−1.45	0.00	−0.42
Large[b]	0.0011	5.08**	1.41E-05	4.07**
Medium[c]	0.0005	2.71**	2.38E-06	0.49
Small[d]	0.0012	3.10**	1.12E-05	1.22
log(USD/EUR)	0.05	3.30**	0.05	3.40**
log(DAX30)	0.03	4.06**	0.01	4.48**
log(OMX16)	0.01	1.60	0.00	0.78
log(oil)	0.00	0.84	0.00	0.17
$(r^{SWE} - r^{GER})_{10Y}$	0.87	4.32**	13.39	14.31**
$(r^{SWE} - r^{GER})_{3M}$	0.23	0.86	6.92	3.58**
INT[e]	0.00	2.75**	0.00	1.93
Variance equation				
Constant	0.00	4.72**	0.00	0.11
ARCH(1)	0.05	5.43**	0.15	5.58**
GARCH(1)	0.94	93.43**	0.60	30.93**
R^2	0.26		0.20	
DW-stat	2.02		1.84	

Note: The sample ranges from January 1995 to June 2002, with daily observations. We estimate the absolute value and the squared value of Δ log $SEK(EUR)$ on unexpected volume (the residual of an ARMA$(2, 2)$) and macro variables. The model is estimated using a GARCH$(1, 1)$. Volume is measured in units of 10 billion SEK. Significance is indicated by * (5 percent) and ** (1 percent). All macro variables are included as absolute changes in the regression on absolute changes, and squared changes in the regression on squared changes.

a. SQR(GARCH) is the squared conditional variance (ARCH-in-mean).

b. Large banks are the two largest reporting banks in the sample.

c. Medium banks are the seven following banks.

d. Small banks are the remaining reporters.

e. INT is a dummy that takes the value 1 on days when Sveriges Riksbank reports an intervention.

Table 2.9
Relative effects on volatility

	Standard deviation	Parameter	Predicted effect[a]	Percentage of FX-volatility[b]
Absolute change in SEK/EUR	0.0031			
Large	0.4948	0.0011	0.0005	17.09
Medium	0.4266	0.0005	0.0002	6.97
Small	0.1875	0.0012	0.0002	7.09
Absolute change in USD/EUR	0.0043	0.05	0.0002	6.97
Absolute change in RDIF10	0.0004	0.87	0.0004	11.62
SQR(GARCH)	0.0008	0.98	0.0007	24.26

Note: The sample ranges from January 1995 to June 2002. All parameters are collected from table 2.8.
a. The predicted effect is the predicted effect of a change of one standard deviation (multiply the standard deviation by the parameter).
b. The percentage of FX volatility is the ratio of the predicted effect over the standard deviation of absolute returns in the SEK/EUR (measured in percent).

of the coefficient is almost twice the size for Swedish banks. When considering the regression with squared changes in SEK/EUR, the picture becomes even clearer. The coefficient on Swedish banks is highly significant, while the coefficient on foreign banks is insignificant.

We also test whether Swedish banks of different size (large vs. small and medium-sized banks) had different effects (table 2.11). The results again suggest that size is important. To sum up, size is important when explaining volatility. This indicates that private information may be an important driver of FX volatility in SEK/EUR. The finding that Swedish banks (controlling for size) are more important when explaining volatility than foreign banks may suggest that volatility in SEK/EUR is primarily related to economic conditions in Sweden.

2.3 Conclusion

The literature on volume and volatility asks one primary question: Why does the relationship arise? If everyone has the same expectations, and all groups behave similarly, the effect should be caused by more trading due to the arrival of new information. However, all rational agents should have the same opportunity to take advantage of the new information, and heterogeneity should be of less importance. On the other hand, if the volume–volatility relationship is the result of dispersion of beliefs or asymmetric information, then heterogeneity is certainly a central feature in the analysis.

Table 2.10
Estimating volume and volatility

	Absolute change		Squared change	
SQR(GARCH)[a]	1.05	11.94**	0.53	1.30
Constant	0.00	−3.11**	0.00	−0.49
Swedish[b]	0.0019	5.31**	1.95E-05	4.06**
Foreigners[c]	0.0010	5.09**	7.81E-06	1.39
log(USD/EUR)	0.06	3.87**	0.05	3.61**
log(DAX30)	0.03	4.28**	0.01	4.76**
log(OMX16)	0.01	1.79	0.00	0.83
log(oil)	0.00	1.15	0.00	0.28
$(r^{SWE} - r^{GER})_{10Y}$	1.01	4.72**	14.29	15.42**
$(r^{SWE} - r^{GER})_{3M}$	0.30	1.13	7.50	3.67**
INT[d]	0.00	3.14**	0.00	2.28*
Variance equation				
Constant	0.00	6.52**	0.00	0.11
ARCH(1)	0.05	5.13**	0.15	5.45**
GARCH(1)	0.94	86.46**	0.60	33.98**
R^2	0.23		0.18	
DW-stat	2.02		1.86	

Note: The sample ranges from January 1995 to June 2002, with daily observations. We estimate the absolute value and the squared value of d(log(SEK(EUR)) on unexpected volume (the residual of an ARMA$(2,2)$) and macro variables. The model is estimated using a GARCH$(1,1)$. Volume is measured in units of 10 billion SEK. Significance is indicated by * (5 percent) and ** (1 percent). All macro variables are included as absolute changes in the regression on absolute changes, and squared changes in the regression on squared changes.
a. SQR(GARCH) is the squared root of the conditional variance (ARCH-in-mean).
b. These are all Swedish reporters with the exception of the two largest banks.
c. These are all foreign reporters. "Swedish" makes up approximately 20 percent of total volume, and "foreigners" approximately 25 percent of total volume.
d. INT is a dummy that takes the value 1 on days when Sveriges Riksbank reports an intervention.

In this chapter we review evidence from a unique set of volume data from the Swedish FX market, covering five and half years of daily data. The Swedish market is a small market compared with, for example, the USD/EUR or USD/JPY market. However, SEK/EUR is among the 10 most traded currency crosses in the world, and the market is well developed with high liquidity. For this market we find evidence to indicate that different agents have different effects on the volume–volatility relationship. In particular, we find that it is the volume of the largest banks that is most important. In the SEK market these banks

Table 2.11
Estimating volume and volatility

	Absolute change		Squared change	
SQR(GARCH)[a]	0.95	6.90**	0.51	1.31
Constant	0.00	−0.91	0.00	−0.41
Large Swedish[b]	0.0013	12.74**	1.60E-05	4.35**
Other Swedish[c]	0.0007	3.25**	3.78E-06	0.55
log(USD/EUR)	0.06	5.33**	0.05	3.44**
log(DAX30)	0.03	5.38**	0.01	4.42**
log(OMX16)	0.01	2.36*	0.00	0.81
log(oil)	0.00	1.00	0.00	0.16
$(r^{SWE} - r^{GER})_{10Y}$	0.85	7.38**	13.34	14.26**
$(r^{SWE} - r^{GER})_{3M}$	0.14	0.88	6.95	3.54**
INT[d]	0.00	5.33**	0.00	1.93
Variance equation				
Constant	0.00	2.14*	0.00	0.11
ARCH(1)	0.05	10.14**	0.15	5.60**
GARCH(1)	0.95	216.55**	0.60	30.93**
R^2	0.25		0.20	
DW-stat	2.01		1.84	

Note: The sample ranges from January 1995 to June 2002, with daily observations. We estimate the absolute value and the squared value of d(log(SEK(EUR)) on unexpected volume (the residual of an ARMA$(2, 2)$) and macro variables. The model is estimated using a GARCH$(1, 1)$. Volume is measured in units of 10 billion SEK. Significance is indicated by * (5 percent) and ** (1 percent). All macro variables are included as absolute changes in the regression on absolute changes, and squared changes in the regression on squared changes.
a. SQR(GARCH) is the squared root of conditional variance (ARCH-in-mean).
b. These are the two largest banks.
c. These are other Swedish banks. “Large Swedish” make up approximately 45 percent of total volume, and “other Swedish” approximately 20 percent of total volume.
d. INT is a dummy that takes the value 1 on days when Sveriges Riksbank reports an intervention.

are Swedish banks. There is reason to believe that the large Swedish banks are relatively well informed. This is in contrast with the findings of Daigler and Wiley (1999) from future markets that it is the volume of the least informed traders that creates the volume–volatility relationship. While the Daigler and Wiley result is about noise traders, our result is one about information advantage. We also find that Swedish banks are more important when explaining volatility than foreign banks, even when controlling for size. This suggests that volatility in SEK/EUR is primarily related to economic conditions in Sweden.

Table 2.12
Estimating an ARMA(2, 2) process on volume

	Total spot		Large banks		Small banks	
Constant	2.91	10.15**	1.29	17.34**	0.34	2.89**
Monday	−0.39	−7.59**	−0.19	−7.47**	−0.04	−3.85**
Tuesday	0.12	2.23*	0.04	1.42	−0.01	−0.63
Wednesday	0.23	4.03**	0.10	3.33**	0.01	1.51
Thursday	0.14	2.58**	0.05	1.60	0.02	1.77
ZERO	−3.20	−23.32**	−1.34	−29.61**	−0.41	−10.89**
AR(1)	1.58	22.90**	1.52	16.45**	1.37	7.52**
AR(2)	−0.58	−8.56**	−0.53	−5.86**	−0.37	−2.06*
MA(1)	−1.25	−15.83**	−1.26	−12.41**	−1.11	−5.91**
MA(2)	0.29	4.11**	0.30	3.37**	0.19	1.20
Variance equation						
Constant	0.05	2.04*	0.01	2.31*	0.00	0.56
ARCH(1)	0.06	3.38**	0.06	2.60**	0.03	5.35**
GARCH(1)	0.87	19.65**	0.89	24.04**	0.97	158.82**
R^2	0.49		0.38		0.55	
DW-stat	1.92		1.91		2.03	

Note: The period ranges from January 1995 to June 2002. The model is estimated to differentiate expected and unexpected volume. We treat the fit of the model as expected, and the residual as unexpected. The model is estimated using an GARCH(1, 1).

Notes

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1. A short swap is a contract to be delivered within two days, such as before a spot contract.

2. According to the BIS (2002), the Swedish krona is the eighth most traded currency. The Swedish krona is, for example, larger than the emerging markets studied in Galati (2000).

3. Other potential measures for volatility is intraday high–low or implied volatility from option prices. However, such data are not available for the SEK/EUR market.

4. We have also used GMM and simple OLS regressions. There is no indication that this affects any of the results. Recursive regressions reveal that parameter stability in the volume parameters reported is good.

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3 A Neoclassical Explanation of Nominal Exchange Rate Volatility

Michael J. Moore and Maurice J. Roche

Economists are quite clear about what determines exchange rates. The exchange rate emerges from the relative supply and demand of domestic and foreign money. Exchange rates should depend on output and money, both domestic and foreign.[1] The trouble is that the data consistently reject parsimonious models based on this view. International economics finds it hard to transcend the point made by Meese and Rogoff (1983) that it is hard to beat an atheoretic random walk model of exchange rates. Mark (1995) has argued that the lack of forecastability of nominal exchange rates is a purely short-term problem but this is hotly contested by Faust, Rogers, and Wright (2001). The extent of exchange rate volatility is particularly difficult to explain. From the point of view of conventional exchange rate models, either money or income has to display variability that it obviously does not possess.[2]

The most common explanation, emanating from the Dornbusch overshooting model, is that "excessive" exchange volatility is caused by the presence of sticky prices in the goods market or sticky wages. The current incarnation of this view is provided by the "new open economy macroeconomics" (see Lane 2001). This literature has been shy of exposing itself to the data (e.g., see Obstfeld and Rogoff 2001; Benigno and Benigno 2001). When it does, its success is mixed: Bergin (2001) admits that it does a poor job of forecasting nominal exchange rates. Chari, McGrattan, and Kehoe (2002) are apparently more successful, but they require goods' prices to be fixed for implausibly long periods.

Recently a number of researchers have emphasized the importance of microstructural approaches to exchange rate determination. The manifesto for this is to be found in Flood and Rose (1999). Evans and Lyons (2002) provides its most important result: that exchange rates are determined by order flow. De Grauwe and Grimaldi (2001) is broadly in this tradition: they argue that exchange rate volatility is caused by the activities of irrational noise traders. The main limitation of this literature is that it is generally partial rather than general equilibrium in its approach.

A third strategy is largely atheoretical. It argues that the primary failure of the existing literature is its reliance on linear modeling. Kilian and Taylor (2003), for example, argue that an ESTAR model helps to beat the random walk model.[3] In addition Taylor and Peel (2000), Clarida et al. (2001), and Taylor (2001) all emphasize the importance of nonlinearities in nominal exchange rate modeling.

Our approach is ambitious. We claim to be able to explain nominal exchange rate volatility in a neoclassical model (no sticky prices) exchange economy with rational agents. We also show that the model matches the data. The key innovation is that we introduce a habit persistence externality that generates a nonlinear adjustment path in nominal exchange rates. In this respect we are closest to the research program of Mark Taylor and others, as outlined in the previous paragraph.

Our specific modeling strategy is to extend Campbell and Cochrane (1999) preferences to both a monetary and an international setting.[4] With this specification there is an aggregate consumption externality (e.g., see Abel 1990; Duesenberry 1949), and utility is time inseparable because of habit persistence. The utility function depends not only on the consumption of home and foreign goods but also on the surplus of consumption over an externally generated habit that is both volatile and persistent. This makes the marginal rate of substitution between home and foreign goods volatile enough to explain the variability in nominal exchange rates since prices are pinned down by the modest volatility of the money stock. We use the exchange rate equation from the model to forecast the dollar–sterling exchange rate at 4-quarter to 12-quarter horizons. The model produces superior forecasts (in root mean squared error terms) relative to those generated from the monetary or random walk models.

The plan of the chapter is as follows. First, the model is developed. In section 3.2, we discuss the data. The model is calibrated in section 3.3. The results are presented in section 3.4. In section 3.5, we make some concluding remarks.

3.1 The Model

The basic structure of the model is the well-known Lucas (1982) two-country, two-good, two-money representative agent story. In this model the nominal exchange rate is equated with the relative price weighted intratemporal marginal rate of substitution between domestic and foreign goods and can be written as

$$S_t = \frac{(\delta U/\delta C_{it}^2)/P_t^2}{(\delta U/\delta C_{it}^1)/P_t^1}, \tag{1}$$

where S_t is the domestic price of foreign currency at time t, U is instantaneous utility, C_{it}^j is the consumption of goods and services of country j by the household of country i at time t, and P_t^j is the price of country j goods in terms of country j money. It is obvious that volatile and persistent nominal exchange rates will primarily depend on the properties of the intratemporal marginal rate of substitution and not necessarily on the time series properties of consumption.

We will take the following simple model to illustrate the effects of introducing the habit externality on the time series properties of nominal exchange rates. Households in both countries are assumed to maximize the discounted expected value of lifetime utility.[5] We will consider two cases. The first case we label the *standard model*, where utility depends on consumptions only. The second case assumes that households also have habits in domestic and foreign goods. We label this case the *habit model*. The expected lifetime utility function is given as

$$\begin{aligned} &E_0 \sum_{t=0}^{\infty} \beta^t U(C_{it}^1 - H_{it}^1, C_{it}^2 - H_{it}^2) \\ &\quad = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_{it}^1 - H_{it}^1)^{(1-\gamma)}}{1-\gamma} + \frac{(C_{it}^2 - H_{it}^2)^{(1-\gamma)}}{1-\gamma} \right), \qquad i = 1, 2, \end{aligned} \tag{2}$$

where β is the discount factor, γ is a parameter that governs the curvature of the utility function,[6] and H_{it}^j is the subsistence consumption (or habit) of goods and services of country j by the household of country i. Note that if $H_{it}^j = 0$, $\forall i, j$, in equation (2) collapses to standard addilog preferences with no habit. Habit persistence takes the form of an aggregate consumption externality, namely "Keeping up with the Joneses'" effects along the lines of Duesenberry (1949) and Abel (1990). Recent work in the economics of happiness literature suggests that relative income is an important factor in individual's levels of satisfaction; for example, see Oswald and Clarke (1996) and Oswald (1997).

We reparameterize the utility function in equation (2) in terms of X_{it}^{j}, the surplus consumption ratio of goods and services of country j by the household of country i:

$$X_{it}^{j} = \frac{C_{it}^{j} - H_{it}^{j}}{C_{it}^{j}}, \qquad i = 1, 2, \quad j = 1, 2. \tag{3}$$

When $C_{it}^{j} = H_{it}^{j}$, $X_{it}^{j} = 0$, this is the worst possible state. By contrast, as C_{it}^{j} rises, the surplus consumption ratio converges on unity.

We assume that consumption and money growth follow simple AR(1) processes:

$$\mu_t^j = (1 - \rho_c^j)\bar{\mu}^j + \rho_c^j \mu_{t-1}^j + v_t^j, \quad v_t^j \sim N(0, \sigma_{c^j}^2), \qquad j = 1, 2, \tag{4}$$

and

$$\pi_t^j = (1 - \rho_m^j)\bar{\pi}^j + \rho_m^j \pi_{t-1}^j + u_t^j, \quad u_t^j \sim N(0, \sigma_{m^j}^2), \qquad j = 1, 2, \tag{5}$$

respectively. The unconditional means of consumption and money growth in country j are defined as $\bar{\mu}^j$ and $\bar{\pi}^j$ respectively. The variances of shocks to consumption and money growth in country j are defined as $\sigma_{c^j}^2$ and $\sigma_{m^j}^2$ respectively. The first-order autocorrelation coefficients of consumption and money growth in country j are defined as ρ_c^j and ρ_m^j respectively. We make the following simplifying assumptions:

1. The unconditional mean of consumption growth may differ from the unconditional mean of money growth but the parameters are the same across countries. That is,

$$\bar{\mu}^1 = \bar{\mu}^2 = \bar{\mu} \quad \text{and} \quad \bar{\pi}^1 = \bar{\pi}^2 = \bar{\pi}.$$

2. The variance of shocks to consumption growth may differ from the variance of shocks to money growth but the parameters are the same across countries. In symbols,

$$\sigma_{c^1}^2 = \sigma_{c^2}^2 = \sigma_c^2 \quad \text{and} \quad \sigma_{m^1}^2 = \sigma_{m^2}^2 = \sigma_m^2.$$

3. The covariances between all shocks are zero.[7]

4. The first-order autocorrelation coefficient of consumption growth may differ from that for money growth, but the parameters are the same across countries. Again,

$$\rho_c^1 = \rho_c^2 = \rho_c \quad \text{and} \quad \rho_m^1 = \rho_m^2 = \rho_m.$$

From this point we also assume that the habit is common across countries for a given good.[8]

We closely follow Campbell and Cochrane (1999) by assuming that the log of the surplus consumption ratios evolve as follows:

$$x_t^j = (1-\phi)\bar{x}^j + \phi x_{t-1}^j + \lambda(x_{t-1}^j)(v_t^j), \qquad j = 1, 2, \tag{6}$$

where $\phi < 1$, is the habit persistence parameter, $\bar{x}^j$ is the steady state value for the logarithm of the surplus consumption ratio for good j and v_t^j is the shock to consumption growth in good j. The function $\lambda(x_t^j)$ describes the sensitivity of the future log surplus consumption ratio to endowment innovations. It depends nonlinearly on the current log surplus consumption ratio. The form of the sensitivity function $\lambda(x_t^j)$ is

$$\lambda(x_t^j) = \begin{cases} \dfrac{\sqrt{1-2(x_t^j - \bar{x}^j)}}{\bar{X}^j} - 1 & \text{for } x_t^j \leq x_{\max}^j, \\ 0 & \text{for } x_t^j > x_{\max}^j, \end{cases} \tag{7}$$

where

$$x_{\max}^j = \bar{x}^j + \frac{1-(\bar{X}^j)^2}{2}.$$

$\bar{X}^j$ is the steady state value of the surplus consumption ratio for good j and is defined as

$$\bar{X}^j = \sigma_c \sqrt{\frac{\gamma}{1-\phi}}, \qquad j = 1, 2. \tag{8}$$

There are a couple of advantages to specifying the habit along the lines of equations (6) through (8). First, the habit is predetermined at the steady state. This means that it takes time for the consumption externality to affect an individual agents habit. The second advantage avoids a possible difficulty with the first. The habit is not predetermined *outside of* the steady state, but if it were, a sufficiently low realization of consumption would mean that habit exceeded current consumption. The arguments of the utility functions in equation (2) become negative. Our habit specification prevents this by ensuring that the habit moves nonnegatively with consumption everywhere. These two features are illustrated in detail in Campbell and Cochrane (1999). Most important, for the problems that we are addressing, the form of the external habit in equations (6) through (8) guarantees that the intratemporal marginal rate of substitution in (1) is both volatile and persistent.

The rest of the model is as follows: The agent in the goods market faces the following cash-in-advance constraint

$$M_{it}^{j} \geq P_t^j C_{it}^j, \qquad i = 1, 2, \quad j = 1, 2, \tag{9}$$

where M_{it}^{j} is the amount of money of country j held by the household of country i for transactions in the goods market at time t. At the end of period t (or the beginning of period $t + 1$), the domestic households holding of domestic currency

$$M_{1t+1}^{1} \geq P_t^1 C_t^1 + B_{1t}^1 \tag{10}$$

are made up of proceeds from the sale of the endowment and the redemption of nominal discount bonds, B_{it}^{j}. A domestic household's holding of foreign currency is

$$M_{1t+1}^{2} \geq B_{1t}^{2}. \tag{11}$$

Analogously the foreign households holding of foreign currency is

$$M_{2t+1}^{2} \geq P_t^2 C_t^2 + B_{2t}^2, \tag{12}$$

and of domestic currency is

$$M_{2t+1}^{1} \geq B_{2t}^{1}. \tag{13}$$

The only role for the government is to have a central bank that engages in open market operations. In each period the central bank of each country changes the money stock by issuing one-period discount bonds. The bonds are redeemed at the end of period t (or the beginning of period $t + 1$). Equilibrium in the goods market is given by

$$C_t^j = C_{1t}^j + C_{2t}^j, \qquad j = 1, 2. \tag{14}$$

Equilibrium in the money market given by

$$M_t^j = M_{1t}^j + M_{2t}^j, \qquad j = 1, 2. \tag{15}$$

Each household maximizes equation (2) subject to equations (3) through (13).[9] Like Lucas (1982) we assume that there is perfect international risk pooling in equilibrium. Recent work by Brandt, Cochrane, and Santa-Clara (2001) suggests that international risk sharing is very high. With perfect risk sharing the equilibrium consumption of each good equals half of the current endowment, namely $C_{it}^j = 0.5C_t^i$, where C_t^j is the endowment of the jth country at time t.[10] The solution to the household maximization problem is conventional and the expression for the nominal exchange rate can be found readily from equation (1) and the utility function (2) as

$$S_t = \frac{(C_t^2 X_t^2)^{-\gamma}/P_t^2}{(C_t^1 X_t^1)^{-\gamma}/P_t^1}. \tag{16}$$

Recalling that we use lowercase letters to indicate the log of a variable, we have

$$s_t = (\gamma - 1)(c_t^1 - c_t^2) + \gamma(x_t^1 - x_t^2) + (m_t^1 - m_t^2). \tag{17}$$

Equation (17) is the main result of this chapter. In the standard model where there are no habits in the utility function, the x_t^j terms are not present. In this case the curvature parameter needs to be very high to generate the required volatility in the nominal exchange rate.[11] This requirement implies an implausible degree of risk aversion.

In our model, high levels of γ are not needed to mimic the observed level of volatility. Instead, it is the highly volatile differential between the home and foreign surplus consumption ratio that delivers the result. The reason why linear exchange rate determination models such as

$$s_t = \beta_0 + \beta_1(c_t^1 - c_t^2) + \beta_2(m_t^1 - m_t^2)$$

(where the β_i are parameters) are so poor is that the differential in the log surplus consumption ratio is omitted. The surplus consumption ratios are of course related to (real) fundamentals but the relationship is highly nonlinear. The surplus consumption ratios are also "near-integrated," which is why their contribution to volatility is so significant. In effect the habit persistence makes rapid adjustment in quantities expensive in utility terms. Because of this the price (the exchange rate) adjusts instead.

3.2 The Data

There have been many studies documenting properties of bilateral exchange rates between the United States and European countries (e.g., see Chari, Kehoe, and McGrattan 2002). The series used in this chapter are constructed from raw data for the United States and the 15 countries of the European Union. The data are collected by the Organisation for Economic Cooperation and Development (OECD) and the International Monetary Fund (IMF) and are available from Datastream. The data are quarterly and cover the floating period 1973:1 to 1998:4. The nominal exchange rates, S_t, are defined as the dollar price of one unit of foreign exchange. In addition to presenting basic statistics on bilateral exchange rates vis-à-vis the United States, we calculate a trade-weighted European nominal exchange rate using the following formula:

$$I_t = I_{t-1} \prod_{j=1}^{n} \left(\frac{S_{j,t}}{S_{j,t-1}} \right)^{W_{j,t}}, \tag{18}$$

where I_t is the nominal index and $W_{j,t}$ is the weight of currency j at time t in the total competitiveness index for the US dollar. The base period (1973:1 in our data set) is assumed to take on the value equal to 100. The weights are those employed by the Board of Governors of the Federal Reserve System in their trade-weighted exchange rate indexes and are available from their website.[12] For a thorough discussion on the construction of these specific indices see Leahy (1998). In addition Coughlin and Pollard (1996) present a detailed investigation of the issues involved in the construction of commonly used trade-weighted indexes.

We detrend the logged exchange rates using both first-difference and Hodrick-Prescott filters and present the standard deviation and first-order autocorrelation coefficient in table 3.1. The stylized facts are very similar to those reported in the literature.[13] The Hodrick-Prescott (first-difference) filtered nominal exchange rates are very volatile and persistent with standard deviations of around 8% (6% for first-difference) and AR(1) coefficients of around 0.8 (0.1 for first-difference).

3.3 Calibration

We present the baseline parameterization in table 3.2 that we use in section 3.4 to simulate a "quarterly economy." The parameters of the exogenous endowment and money growth rate processes are taken from the literature. Campbell and Cochrane (1999) use seasonally adjusted US real consumption expenditure on nondurables and services per capita to proxy for endowments. Based on their table 3.1 they estimated the unconditional mean of consumption growth to be 0.4725

Table 3.1
Properties of exchange rates and consumer price indexes in the data

	Standard deviation		AR(1) coefficient	
	First-differenced	Hodrick-Prescott filtered	First-differenced	Hodrick-Prescott filtered
Austria	6.16	8.60	0.05	0.77
Belgium	6.26	9.47	0.11	0.81
Denmark	5.92	8.50	0.11	0.78
Finland	5.03	8.49	0.21	0.84
France	5.92	8.96	0.13	0.81
Germany	6.28	8.84	0.06	0.78
Greece	5.28	7.08	0.07	0.75
Ireland	5.64	8.74	0.16	0.81
Italy	5.76	8.91	0.17	0.81
Luxembourg	6.26	9.47	0.11	0.81
Netherlands	6.16	8.80	0.08	0.78
Portugal	5.93	8.63	0.18	0.81
Spain	5.56	9.03	0.19	0.84
Sweden	5.75	8.64	0.15	0.80
United Kingdom	5.41	8.45	0.15	0.81
EU aggregate	5.36	8.19	0.13	0.81

Note: The statistics are based on logged quarterly data for the period 1973:1 to 1998:4. The statistics for the European Union are trade-weighted aggregates of all countries in the table with the exception of Denmark and Greece.

Table 3.2
Baseline parameterization

	Consumption growth	Money growth
Unconditional mean	0.4725%	1.80%
Standard deviation of shock	0.75%	0.64%
AR(1) coefficient	0.00	0.35
Curvature of the utility function γ	—	0.10
Persistence of the log surplus–consumption ratio ϕ	—	0.97

percent per quarter, the standard deviation of shocks to consumption growth to be 0.75 percent per quarter and the first-order autocorrelation coefficient to be zero. We use these parameters in our baseline. Christiano (1991), who assumes cash in advance, uses US money base as his measure of money. Using data on seasonally adjusted US monetary base from the database of the Federal Reserve Bank of St. Louis for the period 1973:1 to 1998:4 we estimated the unconditional mean of money base growth to be 1.4 percent per quarter, the standard deviation of shocks to money base growth to be 0.64 percent per quarter and the first-order autocorrelation coefficient to be 0.35. We use these parameters in our baseline.

The curvature of the utility function parameter, γ, and the AR(1) coefficient of the log of the surplus consumption ratio, ϕ, have major effects in the habit persistence model. In the baseline parameterization we set $\gamma = 0.1$. The value of γ is low compared to Campbell and Cochrane (1999): their lowest value for this parameter is 0.7. However, even with $\gamma = 0.1$, it is worth remembering that in the habit model local curvature of the utility function in the steady state is $\gamma/\bar{X} = 7.3$. In Campbell and Cochrane (1999) this varies between 19 and 33 depending on what value they assign to γ. The AR(1) coefficient of log surplus consumption is set equal to 0.97, a value used in Campbell and Cochrane (1999).[14]

We perform sensitivity analysis and examine how the results change when we vary six key parameters. As mentioned above, γ and ϕ have major effects in the habit persistence model. In our first sensitivity experiment, we allow γ to change from 0.1 to 1.4, and in our second sensitivity experiment, we allow ϕ to change from 0.91 to 0.99. In our third experiment, we vary both parameters in the same ranges along a two-dimensional plane.

Changing the unconditional mean of consumption growth does not affect our results at all, and thus we keep the baseline value for all experiments. In our fourth sensitivity experiment, we allow σ_c to change from 0.55 to 0.85 percent, and in our fifth experiment, we allow ρ_c to change from -0.1 to 0.4.

Changing the unconditional mean of money growth also does not affect our results at all and thus we keep the baseline value for all experiments. In our sixth sensitivity experiment, we allow σ_m to change from 0.7 to 1.1 percent and in our seventh, and final experiment, we allow ρ_m to change from 0.25 to 0.75.

Table 3.3
Properties of nominal exchange rates in the simulated models

	Standard deviation		AR(1) coefficient	
	First-differenced	Hodrick-Prescott filtered	First-differenced	Hodrick-Prescott filtered
Habit model	6.66	8.49	0.003	0.66
	(0.005)	(0.008)	(0.016)	(0.012)
Standard model	1.35	1.96	0.17	0.77
	(0.0001)	(0.0003)	(0.010)	(0.006)

Note: The means of each detrended series in the habit and standard models based on 100 simulations are reported with the associated standard error in parenthesis.

3.4 The Results

3.4.1 Simulated Models

We simulated the habit model using (4) through (8) and generate the nominal exchange rate using (17). We replicate each experiment 100 times generating 604 observations for each series. The first 500 observations are discarded leaving a sample size of 104 and is the typical sample size used in section 3.3. The simulated data are detrended using the first-difference and Hodrick-Prescott filters. The results from the baseline parameterizations for the moments of interest are contained in table 3.3 where we present the mean of the simulated moment and its standard error (in parenthesis) for both the habit and standard models.

It is obvious that the volatility of the nominal exchange rate in the standard model is very low. This simply confirms what has already been noted elsewhere in the literature. By contrast, the new habit model comes satisfyingly close to describing the data. For the first-difference filtered series, the nominal exchange rate has an average standard deviation of 6.66 percent, which is somewhat above the 5.03 to 6.28 percent range we reported for the data in table 3.1. The nominal exchange rate has an average AR(1) coefficient of 0.003, which is slightly below the 0.05 to 0.21 range that we observe in the data.[15] For the Hodrick-Prescott filtered series, the nominal exchange rate has an average standard deviation of 8.49 percent that is in the 7.08 to 9.47 percent range we reported for the data in table 3.1. The nominal exchange rate has an average AR(1) coefficient of 0.66 that is somewhat lower than the 0.75 to 0.84 range that we observe in the data but still exhibit high levels of persistence.

The reason for our results is evident from equation . The Hodrick-Prescott filtered consumption and money differentials have standard deviations of 1.4 and 1.5 percent respectively. Thus, ignoring covariance terms, we find that the standard deviation of the nominal exchange rate is low in the standard model where the x_t^j terms are not present. The Hodrick-Prescott filtered surplus consumption differential has a standard deviation of 87 percent and the coefficient multiplying this in is $\gamma = 0.1$. Thus the x_t^j terms account for most of the volatility found in the spot exchange rate.

A question relates to the sensitivity of our results to our parameter assumptions. Figures 3.1 to 3.7 contain the results of a number of the seven experiments that were flagged in section 3.3 for the Hodrick-Prescott filtered series only. As γ changes from 0.1 to 1.4 the standard deviation of the nominal exchange rate increases dramatically and the AR(1) coefficient hardly changes. These results are presented in figure 3.1. On the other hand, figure 3.2 demonstrates that as ϕ changes from 0.91 to 0.99 the standard deviation of the nominal exchange rate falls and the AR(1) coefficient increases slightly. Thus there is a delicate balancing act between choosing γ and ϕ. This is highlighted in figure 3.3 where we plot a surface allowing γ to change from 0.1 to 1.4 and ϕ to change from 0.96 to 0.99. The model produces volatility in the nominal

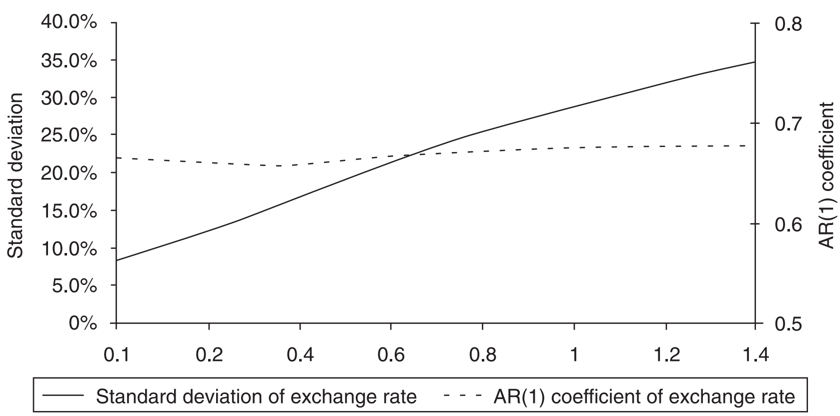

Figure 3.1
Changing the curvature of the utility function

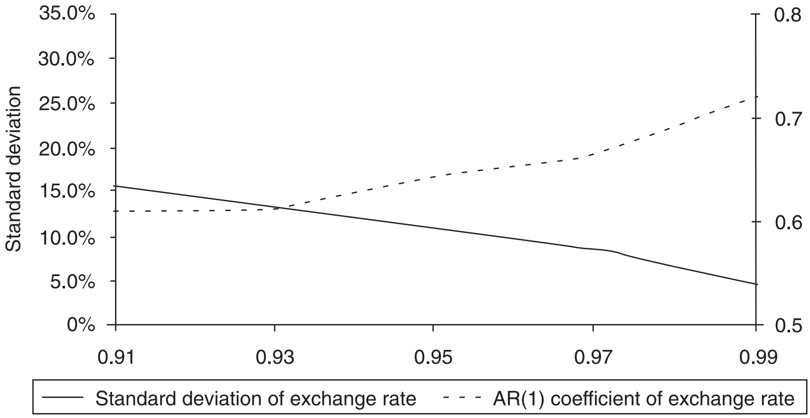

Figure 3.2
Changing the persistence of surplus consumption

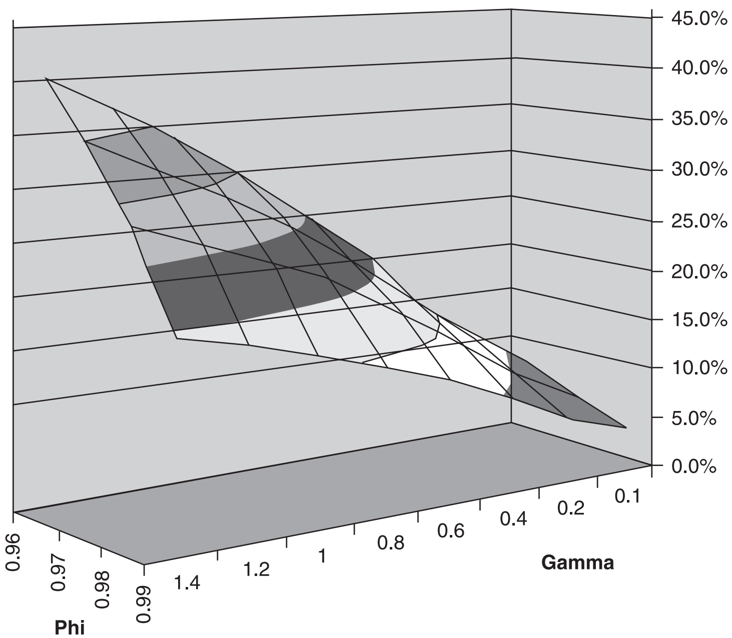

Figure 3.3
Standard deviation of nominal exchange rate

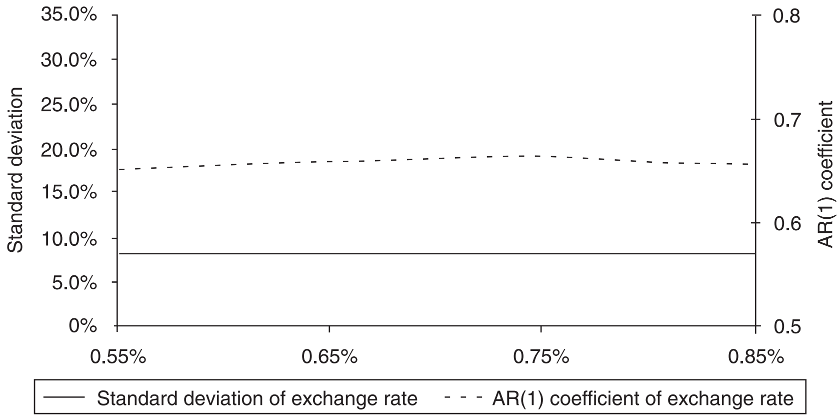

Figure 3.4
Changing the size of the shock to consumption growth

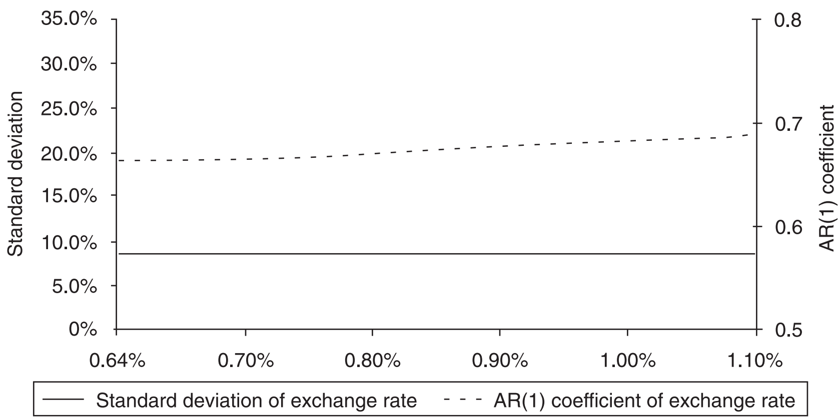

Figure 3.5
Changing the size of the shock to money growth

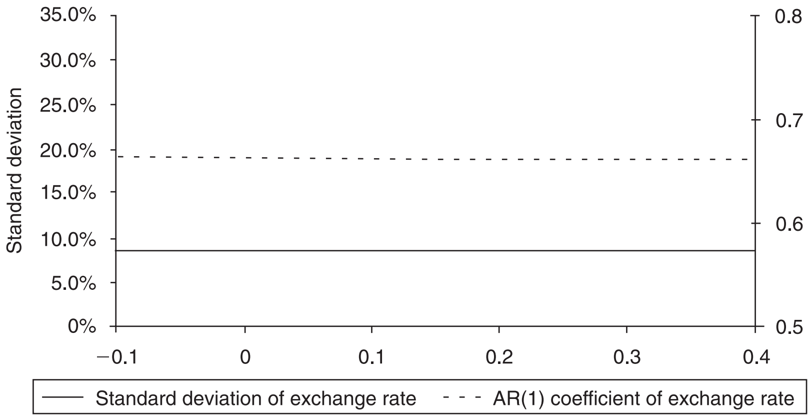

Figure 3.6
Changing the persistence of consumption growth

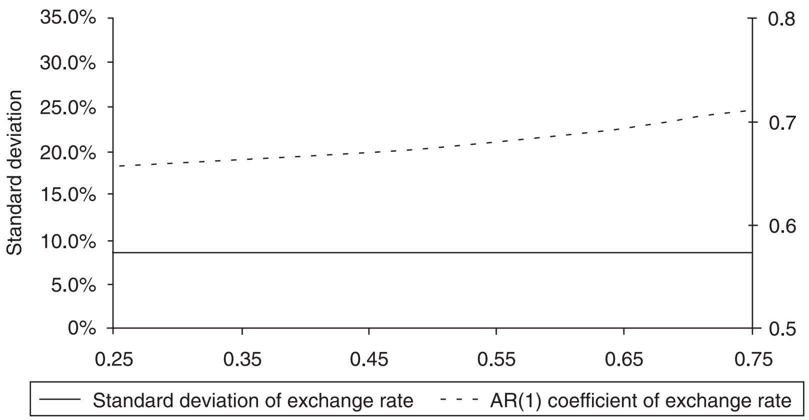

Figure 3.7
Changing the persistence of money growth

exchange rate similar to that found in the data in the low γ and high ϕ region of figure 3.3. In figures 3.4 to 3.7 we change parameters in the consumption and money growth forcing processes. The overall impression is that the results do not change substantially if the parameters of the exogenous forcing process are varied within plausible ranges.

3.4.2 Forecasting Exchange Rates

Since the classic paper by Meese and Rogoff (1983) economists have found it difficult to explain why the driftless random walk exchange rate model outperforms many other models in short-horizon forecasting. The root mean squared forecast error (RMSE) of the random walk model tends to less than those of other models at most forecast horizons for many exchange rates vis-à-vis the US dollar. It is of interest to investigate how well the habit model forecasts the nominal exchange rate out of sample. As an example we will focus on forecasting the dollar–sterling exchange rate.

Mark (1995) and Faust, Rogers, and Wright (2001) estimate the following exchange rate equation at forecast horizon k:

$$s_t - s_{t-k} = \alpha_k + \beta_k z_{t-k} + e_t, \qquad e_t \sim \mathrm{iid}(0, \sigma_e^2), \tag{19}$$

where z_t is the log deviation of the exchange rate from fundamentals. This is an error correction model without the short-run dynamics. They estimate z_t using the "monetary model." Thus

$$z_t = (m_t^1 - m_t^2) - (y_t^1 - y_t^2) - s_t, \tag{20}$$

where y is log of real GDP. We follow Mark (1995) and Faust, Rogers, and Wright (2001) who compute recursive out of sample forecasting exercises. The last 40 observations of a sample of size T are used for evaluation, and we estimate (19) with T-40 observations and produce forecasts for horizons of four, eight, and twelve quarters. Then one observation is added to the end of the estimation sample and repeat the forecasting exercise. This results in 37 four-quarter ahead forecasts, 33 eight-quarter ahead forecasts, and 29 twelve-quarter ahead forecasts. We use US and UK quarterly monetary base data for m_1 and m_2, respectively, for the period 1973:1 to 1998:4.

In the habit model the nominal exchange rate is related to endowment per capita, money per capita, and surplus consumption differentials. We estimate z_t as the negative of the residual from estimating (17):

$$z_t = \alpha_1(m_t^1 - m_t^2) + \alpha_2(c_t^1 - c_t^2) + \alpha_3(x_t^1 - x_t^2) - s_t \tag{21}$$

(where α_1, α_2, and α_3 are unrestricted parameter estimates) using the "fully modified" estimation procedure of Phillips and Hansen (1990). We use US and UK quarterly monetary base per capita data for m_1 and m_2, respectively, and US and UK quarterly real consumption expenditure on nondurables and services per capita data for c_1 and c_2, respectively, for the period 1973:1 to 1998:4.

We calculate log surplus consumption for both the United States and the United Kingdom using (6) through (8). Campbell and Cochrane (1999) choose log surplus consumption serial correlation parameter to match the serial correlation of the log price–dividend ratio. We estimate this parameter to be 0.99 and 0.88 for the United States and United Kingdom, respectively, using quarterly data on the log price–dividend ratio for the period 1973:1 to 1998:4. The standard deviation of the shock to consumption growth σ_c is estimated to be 0.5 and 0.8 percent for the United States and United Kingdom, respectively. A coefficient of relative risk aversion of 2 is commonly used in studies using US data. This has been estimated to be about 4 for the United Kingdom (see Deaton 1992). Thus, given the expression for local relative risk aversion in the habit model in note 11, we can back out γ, assuming the steady state surplus consumption ratios are the same for both goods.

It is assumed that log surplus consumption for both the United States and the United Kingdom starts at the steady state value in 1973:1. We follow Campbell and Cochrane (1999) and set the v_t^j, the shock to consumption growth in good j, in equation (6) to be equal to the log change in consumption minus its mean. The habit model is used to create forecasts as described above.

In the second column of table 3.4, panel A, we report the root mean squared error (RMSE) of forecasts from the monetary model relative to those of the driftless random walk model. As is commonly found with many exchange rates, vis-à-vis the US dollar, these ratios are greater than one and increase with forecast horizon. In contrast, in the third column of panel A of the table we report the root mean squared error of forecasts from the habit model relative to those of the driftless random walk model. The relative RMSEs are less than 1 and decrease with forecast horizon. It would appear that the habit model can produce more accurate forecasts than the driftless random walk model, particularly at longer horizons. Finally, in the fourth column of panel

Table 3.4

Forecast horizon k	Monetary/ random walk	Habit/ random walk	Monetary/ habit
A. Relative RMSE of monetary, habit, and driftless random walk models in out-of-sample forecasting of US dollar–sterling exchange rate[a]			
4 quarters	1.044	0.897	1.164
8 quarters	1.406	0.582	2.415
12 quarters	1.976	0.491	4.022
Forecast horizon k	H_0: Random walk model H_1: Monetary model	H_0: Random walk model H_1: Habit model	H_0: Monetary model H_1: Habit model
B. Probability values for superior predictive ability test of out-of-sample forecasts of US dollar–sterling exchange rate[b]			
4 quarters	0.53	0.00	0.04
8 quarters	0.99	0.00	0.00
12 quarters	1.00	0.01	0.00

a. Data cover the period 1973:1 to 1998:4. The statistics shown are the ratio of the mean square errors of competing models. For example, the column headed "Monetary/random walk" shows the ratio of the mean square of forecasts from the monetary model relative to the driftless random walk model.
b. Data cover the period 1973:1 to 1998:4. The statistics shown are the p-values from the superior predictive ability test that model H_1 has the same forecasting power as model H_0. A small p-value indicates that H_1 has superior predictive ability.

A, we report the root mean squared error of forecasts from the monetary model relative to those of the habit model. These ratios are greater than one and increase with forecast horizon.

In order to test whether the difference in mean squared errors is significant we use a recently developed test of superior predictive ability (SPA). See Hansen (2001) for a thorough discussion.[16] The SPA tests for the best standardized forecasting performance (using mean squared errors) relative to a benchmark model, which we label H_0. The null hypothesis is that none of the competing models is better than the benchmark. The p-values for the SPA tests presented in panel B of table 3.4 are generated using bootstrap methods. At four-, eight-, and twelve-quarter forecast horizons the habit model produces superior forecasts than either the random walk or monetary models at the 1 percent significance level. In summary, by either metric, it would appear that inclusion of log surplus consumption differentials significantly improves forecasts of the dollar exchange rate over either the monetary or random walk models.

Table 3.5
Standard deviations in fixed and floating exchange rate periods

Variable	Fixed period	Floating period
$(x_1 - x_2)$	154%	3174%
$(m_1 - m_2) - (y_1 - y_2)$	8%	45%

Note: Data cover the period 1963:1 to 1998:4. We use US M3 for m_1 and UK M4 for m_2. We use US real GDP for y_1 and UK real GDP for y_2. All data is available from Datastream. In calculating the log surplus consumption ratios local risk aversion was set to 2 and 4 for the United States and United Kingdom respectively.

3.4.3 *Fixed and Floating Exchange Rates*

Flood and Rose (1999) demonstrate that fluctuations in monetary fundamentals are very similar during the fixed and floating exchange rate periods. A natural question to ask is whether the habit model is capable of explaining the large increase in exchange rate volatility since 1973:1. The habit model is capable of explaining this fact provided that the log surplus consumption differential in equation (17) is more volatile after 1973:1. This variable is related to relative recessions. One might expect that this has increased post 1973. In table 3.5 we present the volatility of the log surplus consumption differential and also of the conventional "fundamentals" on the right hand side of equation (17). There is a striking increase in the volatility of the log surplus consumption differential between the fixed and floating rate periods. The corresponding change in the volatility of the "fundamentals" is not nearly so noticeable, as Flood and Rose have already observed.[17] It would appear that the habit model goes some distance to explaining the increase in the fixed and floating period.

3.5 Conclusions

The point of this chapter is simple. Through a flexible price model with a simple twist, it is perfectly possible to explain the volatility of nominal exchange rates so long as preferences are subject to an aggregate consumption externality. The model, proposed here, still has limitations. It only describes an exchange economy. Ljungqvist and Uhlig (2000) have pointed out that there are problems in expanding the Campbell and Cochrane (1999) framework to a production economy. "Consumption bunching" rather than consumption smoothing becomes welfare optimal. However, this only arises if the habit is internalized: our habit is strictly an externality.

The claims being made in this chapter are very traditional. Rather than use a contrivance such as sticky prices, we insist on accounting for exchange rate volatility using a neoclassical framework. Instead of sticky prices, the desired result emanates from a carefully selected preference specification. An Occam's razor argument convinces us that our approach is superior.

Notes

We are grateful for comments from participants at the 2002 CESifo Venice Summer Institute Workshop on Exchange Rate Modelling: Where Do We Stand?

1. The other determinants of the demand for money can be factored out in a general equilibrium framework.

2. Alternatively, the covariances between money and income or home and foreign variables have to assume values not observed in the data.

3. ESTAR stands for exponential smooth transition autoregressive.

4. See also Moore and Roche (2002).

5. The superscript denotes country of origin and the subscript denotes country of use. Uppercase letters denote variables in levels; lowercase letters denote variables in log levels, including growth and interest rates. Greek letters without time subscripts denote parameters. Bars over variables denote steady states.

6. In the standard model γ is the coefficient of relative risk aversion. In the habit model the coefficient of relative risk aversion is time varying (see note 11).

7. Engel (1992) argues that the empirical covariance between real and nominal shocks is low or zero.

8. This is motivated by the observation that habits across similar countries are unlikely to be very different. By contrast, habits across different goods, such as French wine and American computers, are likely to be very different.

9. Recall that the habit is external. This idea is implemented by treating the surplus consumption ratio as exogenous in the optimisation problem.

10. We follow Campbell and Cochrane (1999) and use real consumption expenditure on nondurables and services per capita to proxy for endowments.

11. Defining risk aversion in a multiple-goods model is not trivial (see Engel 1992; Moore 1997). An intertemporal model has as many goods as time periods. In addition our model has two goods in each time period. We evade this problem by only considering its value at the steady state. The local curvature of the utility function with respect to good j is

$$-C_{it}^{j}\frac{\delta^2 U/\delta(C_{it}^{j})^2}{\delta U/\delta C_{it}^{j}} = \frac{\gamma}{\bar{X}_{it}^{j}} = \frac{\gamma}{\bar{X}_{t}^{j}} = \frac{\sqrt{\gamma(1-\phi)}}{\sigma_{c^j}}, \qquad i=1,2, \quad j=1,2.$$

Campbell and Cochrane (1999) show that this expression is positively related to the coefficient of relative risk aversion for the one good case.

12. The Fed does not report weights for Denmark and Greece. Luxembourg is included in the Belgian weights. Thus the EU aggregate is based on twelve countries.

13. See, for example, Backus, Gregory, and Telmer (1993).

14. Campbell and Cochrane choose the serial correlation parameter ϕ to match the serial correlation of log price–dividend ratios but offer no further explanation.

15. A model that implies that spot returns are autocorrelated suggests that they are forecastable. This is hard to believe. Although the autocorrelation coefficients reported for the range of exchange rates in table 3.1 are all positive, many are very imprecise estimates. It is encouraging that the habit model generates the intuition of most economists.

16. The authors would like to thank Peter Hansen for supplying Ox code that calculates the SPA test statistics and associated p-values.

17. Because of lack of UK money base data prior to 1969, we use M4 for the United Kingdom and M3 for the United States for our money variables. The data cover the period 1963:1 to 1998:4.

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4 Real Exchange Rates and Nonlinearities

Mark P. Taylor

If the nominal exchange rate is defined simply as the price of one national currency in terms of another, then the real exchange rate is the nominal exchange rate adjusted for relative national price level differences. When purchasing power parity (PPP) holds, the real exchange rate is a constant, so that movements in the real exchange rate represent deviations from PPP. Hence a discussion of the real exchange rate is tantamount to a discussion of PPP.

In this chapter, I provide a selective survey of the literature on PPP and real exchange rate behavior and discuss the recent research on nonlinearity in real exchange rate adjustment. I begin with the law of one price, which can be viewed as a basic building block of the purchasing power parity condition. Throughout my discussion, I pursue a largely chronological analysis of the issues and puzzles that come up in the literature, as a way of setting out the motivation for the recent interest in nonlinearities in this context.

4.1 The Law of One Price

The law of one price (LOP) in its *absolute* version can be written as

$$P_{i,t} = S_t P^*_{i,t}, \qquad i = 1, 2, \ldots, N, \tag{1}$$

where $P_{i,t}$ denotes the price of good i in terms of the domestic currency at time t, $P^*_{i,t}$ is the price of good i in terms of the foreign currency at time t, and S_t is the nominal exchange rate expressed as the domestic price of the foreign currency at time t. By equation (1), the LOP essentially postulates that the same good should have the same price across countries if prices are expressed in terms of the same currency of denomination. The basic argument as to why the LOP should hold is generally based on the idea of *frictionless* goods arbitrage. As we will see, researchers have generally sought to explain deviations from the LOP in terms of market frictions. This reasoning has in turn introduced nonlinearities.

In its *relative* version, the LOP postulates the relatively weaker condition:

$$\frac{P^*_{i,t+1}S_{t+1}}{P_{i,t+1}} = \frac{P^*_{i,t}S_t}{P_{i,t}}, \qquad i = 1, 2, \dots, N. \tag{2}$$

Obviously the absolute LOP implies the relative LOP, and not vice versa.

Since the LOP can be adequately tested only if goods produced internationally are perfect substitutes, then the condition of no profitable arbitrage should ensure equality of prices in highly integrated goods markets. Nevertheless, the presence of any sort of tariffs, transport costs, and other nontariff barriers and duties will induce a violation of the no-arbitrage condition and, inevitably, of the LOP. Also the assumption of perfect substitutability among goods across different countries is crucial for verifying the LOP. In general, however, product differentiation across countries creates a wedge between domestic and foreign prices of a product which is proportional to the freedom of tradability of the good itself.[1]

Formally, by summing up all the traded goods in each country, the *absolute* version of the PPP hypothesis requires

$$\sum_{i=1}^{N} \alpha_i P_{i,t} = S_t \sum_{i=1}^{N} \alpha_i P^*_{i,t}, \tag{3}$$

where the weights in the summation satisfy $\sum_{i=1}^{N} \alpha_i = 1$. Alternatively, if the price indexes are constructed using a geometric index, then we must form the weighted sum after taking logarithms:

$$\sum_{i=1}^{N} \gamma_i p_{i,t} = s_t + \sum_{i=1}^{N} \gamma_i p^*_{i,t}, \tag{4}$$

where the geometric weights in the summation satisfy $\sum_{i=1}^{N} \gamma_i = 1$ and lowercase letters denote logarithms. The weights α_i or γ_i are based on a national price index and, according to the seminal, Cassellian formulation of PPP, the consumer price index (CPI). If the national price levels are P_t and P^*_t or, in logarithms, p_t and p^*_t, then (according to whether the arithmetic or geometric index is used), we can use (3) or (4) to derive the (absolute) PPP condition:

$$s_t = p_t - p_t^*. \tag{5}$$

From equation (5) it is easily seen that the real exchange rate, defined here in logarithmic form

$$q_t \equiv s_t - p_t + p_t^*, \tag{6}$$

may be viewed as a measure of the deviation from PPP.

Clearly, deriving PPP from the LOP introduces a range of index number problems. For example, equations (3) and (4) implicitly assume that the same weights are relevant in each country, whereas price index weights will typically differ across different countries (even being zero in one country and nonzero in another for some goods and services) and will also tend to shift through time. In practice, researchers often assume that PPP should hold approximately using the price indexes of each country. In the geometric index case, for example, we can rearrange (4) to yield

$$\sum_{i=1}^{N} \gamma_i p_{i,t} = s_t + \sum_{i=1}^{N} \gamma_i^* p_{i,t}^* + \sum_{i=1}^{N} (\gamma_i - \gamma_i^*) p_{i,t}^* \tag{7}$$

or

$$\sum_{i=1}^{N} \gamma_i p_{i,t} = s_t + \sum_{i=1}^{N} \gamma_i^* p_{i,t}^* + u_t, \tag{8}$$

where the γ_i^* denote the weights in the foreign price index. Clearly, the greater the disparity between the relevant national price indexes, the greater is the apparent disparity—represented by u_t—from aggregate PPP, even when the LOP holds for individual goods. Note, however, that because the geometric price indices are homogeneous of degree one (i.e., an equiproportionate increase in all prices will raise the overall price level by the same proportion) then differences in weights across countries will matter less where price impulses affect all goods and services more or less homogeneously. An x percent increase in all prices in the foreign country will lead, for example, to an x percent increase in the foreign price level, where the right-hand side of (8) will be augmented by x and the change in the u_t term will be zero. Thus, assuming domestic prices are constant, we find that an x percent appreciation of the domestic currency is required in order to restore equilibrium.

A similar analysis may be applied when some goods and services are nontraded. Suppose that the LOP applies only among traded goods. An x percent increase in all foreign traded goods prices implies, other things equal, an x percent appreciation of the domestic currency. But if there is also an x percent rise in all *nontraded* foreign goods prices, the PPP condition based on individual national price indexes will also imply an x percent exchange rate movement.

In practice, it is more common for national statistical bureaus to use arithmetic rather than geometric price indexes, although deviations from measured PPP arising from this source are not likely to be large. Considerable differences may arise, however, where price impulses impinge heterogeneously across the various goods and services in an economy and, in particular, where price inflation differs between the traded and nontraded goods sectors.

The choice of the appropriate price index to be used in implementing absolute PPP has been the object of a long debate in the literature, going back at least as far as Keynes (1923). All commonly used price measures include some proportion of nontraded goods, which may induce rejection of PPP or at least of the conditions of homogeneity and proportionality (discussed below) required by PPP. Thus many attempts exist in the literature to construct appropriate price measures for testing PPP. The most influential work in this context has been carried out by Summers and Heston (1991), who developed the International Comparison Programme (ICP) data set, which reports estimates of absolute PPP for a long sample period and a number of countries, using a common basket of goods across countries. The ICP is not, however, of great practical help in much empirical work since it is constructed at infrequent and large time intervals and, for certain time periods, data are only available for several countries. Moreover, since extensive use of extrapolation has been made in order to solve this problem, the data presented in the ICP becomes partially artificial, somehow losing reliability. Overall, price indexes made available by official sources therefore still remain the basis commonly used for implementing absolute PPP, despite the discussed limitations.

In general, however, the difficulty in finding evidence strongly supportive of PPP and the difficulties encountered in moving from the LOP to PPP has provided a strong motivation for researchers to investigate the LOP empirically.

Recent econometric tests of the LOP have often been motivated as a reaction to the rejection of PPP during the recent floating exchange rate regime, which we discuss further below. In general, econometric studies suggest rejection of the LOP for a very broad range of goods and provide strong empirical evidence both that deviations from the LOP are highly volatile and that the volatility of relative prices is considerably lower than the volatility of nominal exchange rates. This is suggested, for example, by two influential studies executed in the 1970s. First, Isard (1977) uses disaggregated data for a number of traded goods (chemical products, paper and glass products, etc.) and for a number of countries, providing strong empirical evidence that the deviations from the LOP are large and persistent and appear to be highly correlated with exchange rate movements. Second, Richardson (1978) finds very similar results to Isard, by using data for four- and seven-digit standard industrial classification (SIC) categories.

Giovannini (1988) uses a partial equilibrium model of the determination of domestic and export prices by a monopolistic competitive firm and argues that the stochastic properties of deviations from the LOP are strongly affected by the currency of denomination of export prices. In particular, Giovannini uses data on domestic and dollar export prices of Japanese goods and provides evidence that deviations from the LOP—found to be large not only for sophisticated manufacturing goods but also for commodities such as screws, nuts and bolts—are mainly due to exchange rate movements, consistent with the earlier relevant literature (see also Benninga and Protopapadakis 1988; Goodwin, Grennes, and Wohlgenant 1990; Bui and Pippinger 1990; Fraser, Taylor, and Webster 1991; Goodwin 1992).

Some of the most influential and convincing work in testing for the LOP is provided by Knetter (1989, 1993). Knetter uses high-quality disaggregated data (7-digit) and provides evidence that large and persistent price differentials exist for traded goods exported to multiple destinations (e.g., for German beer exported to the United Kingdom as compared to the United States).[2] Another interest study in this context is due to Engel (1993), who uncovers a strong empirical regularity: the consumer price of a good relative to a different good within a country tends to be much less variable than the price of that good relative to a similar good in another country. This fact holds for all goods except very simple, homogeneous products. Engel suggests that models of real exchange rates are likely to have predictions regarding this relation, so this fact may provide a useful gauge for discriminating among models.

Parsley and Wei (1996) look for convergence toward the LOP in the absence of trade barriers or nominal exchange rate fluctuations by analyzing a panel of 51 prices from 48 cities in the United States. They find convergence rates substantially higher than typically found in cross-country data, that convergence occurs faster for larger price differences and that rates of convergence are slower for cities further apart. Extending this line of research, Engel and Rogers (1996) use CPI data for both US and Canadian cities and for fourteen categories of consumer prices in order to analyse the stochastic properties of deviations from the LOP. The authors provide evidence that the distance between cities can explain a considerable amount of the price differential of similar goods in different cities of the same country. Nevertheless, the price differentials are considerably larger for two cities across different countries relative to two equidistant cities in the same country. The estimates of Engel and Rogers suggest that crossing the national border—the so-called border effect—increases the volatility of price differentials by the same order of magnitude which would be generated by the addition of 2,500 to 23,000 extra miles between the cities considered. Rogers and Jenkins (1995) find similar results to Engel and Rogers, providing evidence that the "border effect" is effective in increasing not only the volatility of price differentials but also their persistence.

Among the possible explanations of the violation of the LOP suggested by the literature, transport costs, tariffs, and nontariff barriers play a dominant role. An estimate of the wedge driven by the costs of transportation is given, for example, by the International Monetary Fund (IMF 1994): the difference between the value of world exports computed as free on board (FOB) and the value of world imports charged in full, or cost, insurance, and freight (CIF) is estimated at about 10 percent and is found to be highly variable across countries. Moreover the presence of significant nontraded components in the price indexes used by the empirical literature may induce the violation of the LOP. Even if the wholesale price index (WPI) includes a smaller nontraded component relative to the consumer price index, it still includes a significant nontraded component (e.g., the cost of labor employed and insurance). Moreover, even if tariffs have been considerably reduced over time across major industrialized countries, nontariff barriers are still very significant. Governments of many countries often intervene in trade across borders using nontariff barriers in a way that they do not use within their borders (e.g., in the form of strict inspection requirements; see Knetter 1994; Feenstra 1995; Rogoff 1996; Feenstra and Kendall 1997).

4.1.1 Nonlinearities in Deviations from the Law of One Price

Frictions in international arbitrage have important implications and, in particular, imply potential nonlinearities in the deviations from the LOP. The idea that there may be nonlinearities in goods arbitrage dates at least from Heckscher (1916), who suggested that there may be significant deviations from the LOP due to international transaction costs between spatially separated markets. A similar viewpoint can be discerned in the writings of Cassel (e.g., Cassel 1922) and, to a greater or lesser extent, in other earlier writers (Officer 1982). More recently a number of authors have developed theoretical models of nonlinear real exchange rate adjustment arising from transaction costs in international arbitrage (e.g., Benninga and Protopapadakis 1988; Williams and Wright 1991; Dumas 1992; Sercu, Uppal, and Van Hulle 1995; O'Connell 1997; Ohanian and Stockman 1997). In most of these models, proportional or "iceberg" (because a fraction of goods are presumed to "melt" when shipped) transport costs create a band for the real exchange rate within which the marginal cost of arbitrage exceeds the marginal benefit. Assuming instantaneous goods arbitrage at the edges of the band then typically implies that the thresholds become reflecting barriers.

In drawing on recent work on the theory of investment under uncertainty, some of these studies show that the thresholds should be interpreted more broadly than as simply reflecting shipping costs and trade barriers per se, but also as resulting from the sunk costs of international arbitrage and the resulting tendency for traders to wait for sufficiently large arbitrage opportunities to open up before entering the market (see, in particular, Dumas 1992; Dixit 1989; Krugman 1989). O'Connell and Wei (1997) extend the iceberg model to allow for fixed as well as proportional costs of arbitrage. This results in a two-threshold model where the real exchange rate is reset by arbitrage to an upper or lower inner threshold whenever it hits the corresponding outer threshold. Intuitively, arbitrage will be heavy once it is profitable enough to outweigh the initial fixed cost, but will stop short of returning the real rate to the PPP level because of the proportional arbitrage costs. Coleman (1995) suggests that the assumption of instantaneous trade should be replaced with the presumption that it takes time to ship goods. In this model, transport costs again create a band of no arbitrage for the real exchange rate, but the exchange rate can stray beyond the thresholds. Once beyond the upper or lower threshold, the real rate becomes increasingly mean reverting with the distance from the threshold. Within the transaction costs band, when no trade takes place, the process is divergent so that the exchange rate spends most of the time away from parity.

Some empirical evidence of the effect of transaction costs in this context is provided by Davutyan and Pippenger (1990). More recently Obstfeld and Taylor (1997) have investigated the nonlinear nature of the adjustment process in terms of a threshold autoregressive (TAR) model (Tong 1990). The TAR model allows for a transaction costs band within which no adjustment in deviations from the LOP takes place—so that deviations may exhibit unit root behavior—while outside of the band, as goods arbitrage becomes profitable and its effects are felt, the process switches abruptly to become stationary autoregressive. Obstfeld and Taylor provide evidence that TAR models work well when applied to disaggregated data, and yield estimates in which the thresholds correspond to popular rough estimates of the order of magnitude of actual transport costs.

A different story for rationalizing the rejection of the LOP comes from the pricing-to-market (PTM) theory of Krugman (1987) and Dornbusch (1987). Following the developments of theories of imperfect competition and trade, the main feature of this theory is that the same good can be given a different price in different countries when oligopolistic firms are supplying it. This is feasible because there are many industries which can supply separate licences for the sale of their goods at home and abroad.[3] At the empirical level, Knetter (1989, 1993) finds that PTM is very important for German and Japanese firms relative to US companies and that it is a strategy used for a very broad range of goods.[4]

Kasa (1992) argues, however, that the rationale underlying PTM is not price discrimination, as proposed by Krugman and Dornbusch. Kasa argues that PTM is better rationalized by an adjustment cost framework. This is a model in which firms face some sort of menu costs or a model in which consumers face fixed costs when switching between different products (see also Froot and Klemperer 1989).

In an interesting study, Ghosh and Wolf (1994) examine the statistical properties and the determinants of changes in the cover price of *The Economist* newspaper across twelve countries during the recent float. They show that standard tests of PTM may fail to discriminate the alternative hypothesis of menu costs. Their findings suggest a strong violation of the LOP and are consistent with menu-cost-driven pricing behavior.

A final issue which is worth noting is the possibility that the failure of the LOP may be explained by institutional factors typical of this century which have increased the persistence of deviations from the LOP. Nevertheless, Froot, Kim, and Rogoff (1995), using data on prices for grains and other dairy goods in England and the Netherlands for a span of data that goes from the fourteenth to the twentieth century, provide empirical evidence suggesting that the volatility of the LOP is quite stable during the whole period, regardless of the many regime shifts during the sample.

4.2 Purchasing Power Parity

Absolute PPP implies that the nominal exchange rate is equal to the ratio of the two relevant national price levels. Relative PPP posits that changes in the exchange rate are equal to changes in relative national prices. The early empirical literature—until the late 1970s—on testing PPP is based on estimates of equations of the form

$$s_t = \alpha + \beta p_t + \beta^* p_t^* + \omega_t, \tag{9}$$

where ω_t is a disturbance term. A test of the restrictions $\beta = 1, \beta^* = -1$ would be interpreted as a test of absolute PPP, while a test of the same restrictions applied to the equation with the variables in first differences would be interpreted as a test of relative PPP. In particular, a distinction is often made between the test that β and β^* are equal and of opposite sign—the symmetry condition—and the test that they are equal to unity and minus unity respectively—the proportionality condition.

In the earlier literature, researchers did not introduce dynamics in the estimated equation in such a way as to distinguish between short-run and long-run effects, even if it was recognized by researchers that PPP is only expected to hold in the long run. Nevertheless, the empirical literature based on estimation of equations of the form (9) generally suggest rejection of the PPP hypothesis. In an influential study, however, Frenkel (1978), obtains estimates of β and β^* very close to plus and minus unity on data for high-inflation countries, suggesting that PPP represents an important benchmark in long-run exchange rate modeling. Several drawbacks affect, however, this approach. First, Frenkel does not investigate the stochastic properties of the residuals and in particular does not test for stationarity. If the residuals are not stationary, in fact, part of the shocks impinging on the real exchange rate will be permanent, meaning PPP is violated. Second, apart from hyperinflationary economies, PPP tends to be strongly rejected on the basis of estimates of equations such as (9). Frenkel argues, however, that the rejection of PPP may be due only to temporary real shocks and price stickiness in the goods market, but convergence to PPP is expected to occur in the long run.

Another problem in testing PPP on the basis of estimates of equation (9) is the endogeneity of both nominal exchange rates and price levels: indeed, the choice of the variable to be put on the left-hand side of (9) is arbitrary. Krugman (1978) constructs a flexible-price exchange rate model in which the domestic monetary authorities intervene against real shocks using expansionary monetary policies, therefore inducing inflation. The model is estimated by instrumental variables (IV) and ordinary least squares (OLS). The IV estimates of β and β^* are closer to unity in absolute value relative to the OLS estimates, but PPP is still rejected (see also Frenkel 1981).

The crucial problem is, however, that this early literature does not investigate the stationarity of the residuals in the estimated equation. If both nominal exchange rates and relative prices are nonstationary variables (and are not cointegrated), then (9) is a spurious regression and conventional OLS-based statistical inference is invalid (Granger and Newbold 1974). If the error term in (9) is stationary, however, then a strong long-run linear relationship exists between exchange rates and relative prices, but conventional statistical inference is still invalid because of the bias present in the estimated standard errors (Engle and Granger 1987; Banerjee et al. 1986).[5]

The next stage in the development of this literature was explicitly to address the issue of nonstationarity of the variables under consideration, starting with an analysis of whether the real exchange rate itself is stationary—implying evidence of long-run PPP—or whether it tends to follow a unit root process—implying absence of any tendency to converge on a long-run equilibrium level.

4.2.1 Cointegration and Unit Root Tests

The real exchange rate in its logarithmic form may be written as

$$q_t \equiv s_t + p_t^* + p_t. \tag{10}$$

The approach taken by the second stage of tests of PPP undertaken by the empirical literature is based on testing for the nonstationarity of the real exchange rate. Early studies taking this approach include—among others—Roll (1979), Adler and Lehmann (1983), Hakkio (1984), Edison (1985), Frankel (1986), Huizinga (1987), and Meese and Rogoff (1988). From the mid to late 1980s onward, a basic standard approach has been to employ a variant of the augmented Dickey-Fuller (ADF) test for a unit root in the process driving the real rate. This is generally based on an auxiliary regression of the general form

$$\Delta q_t = \gamma_0 + \gamma_1 t + \gamma_2 q_{t-1} + \Xi(L)\Delta q_{t-1} + e_t, \tag{11}$$

where $\Xi(L)$ denotes a pth order polynomial in the lag operator L, and e_t is a white noise process. Testing the null hypothesis that $\gamma_2 = 0$, via an ADF test, is tantamount to testing for a single unit root in the data generating process for q_t and would imply no long-run equilibrium level for q_t. The alternative hypothesis that PPP holds requires that $\gamma_1 < 0$.[6] A variant of this approach is to use a modified version of this test to allow for non-Gaussian disturbances (Phillips 1986; Phillips and Perron 1988).

Empirical studies employing tests of this type for testing PPP during the recent float generally cannot reject the random walk hypothesis for the real exchange rates of the currencies of all the major industrialized countries against one another, therefore suggesting that deviations from PPP are permanent (see also Enders 1988; Taylor 1988; Mark 1990; Edison and Pauls 1993). Two exceptions are Huizinga (1987), who uses variance ratio tests and data for dollar exchange rates against a number of currencies for sample periods shorter than two years, and Chowdhury and Sdogati (1993), who analyze the European Monetary System (EMS) period 1979 to 1990 and find support for PPP for real exchange rates when expressed vis-à-vis the German mark, but not when expressed vis-à-vis the US dollar.[7]

Cointegration, as originally developed by Engle and Granger (1987), seems to be an ideal approach to testing for PPP. While allowing q_t to vary in the short run, a necessary condition for PPP to hold is that the "equilibrium error" (Granger 1986) q_t be stationary over time. If this is not the case, then the nominal exchange rate and the relative price will permanently tend to deviate from each other. Cointegration analysis tells us that any two nonstationary series, which are found to be integrated of the same order, are cointegrated if a linear combination of the two exists that is itself stationary. If this is the case, then the nonstationarity of one series exactly offsets the nonstationarity of the other and a long-run relationship is established between the two variables. In our context, if both the nominal exchange rate s_t and the relative price π_t have a stationary, invertible, nondeterministic ARMA representation after differencing d times. This means that if they are both integrated of order d or $I(d)$, then the linear combination

$$s_t + \kappa\pi_t = z_t \tag{12}$$

will, in general, found to be $I(d)$ as well, provided that the real exchange rate has a random walk component. Nevertheless, the cointegrating parameter α nominal exchange rate and the relative price must be cointegrated of order d, c, or $CI(d, c)$. In the context of PPP testing we want $d = c = 1$, that is, s_t and π_t are both $I(1)$ variables, but z_t is mean reverting. In this case one may feel confident that a strong long-run relationship exists between the two variables considered, since they share a common stochastic trend (Stock and Watson 1988) and "cointegration of a pair of variables is at least a necessary condition for them to have a stable long-run (linear) relationship" (Taylor 1988; Taylor and McMahon 1988).

However, if the no-cointegration hypothesis cannot be rejected, then the estimated regression is just a "spurious" one and has no economic meaning: the analysis is subject to the same drawbacks discussed above. Given that no bounded combination of the levels exists, then the error term in the regression must be nonstationary under the null hypothesis.

The main difference in using cointegration in testing for PPP rather than testing for the nonstationarity of the real exchange rate is that the symmetry and proportionality conditions are not imposed and cannot be tested easily given the bias in the estimated standard errors. Rationales for the rejection of the symmetry and proportionality conditions, based on considerations of measurement errors (in particular, systematic differences between actual measured price indexes and those theoretically relevant for PPP calculations) and barriers to trade, are provided by, inter alios, Taylor (1988), Fisher and Park (1991), and Cheung and Lai (1993a, b).

The Johansen (1988, 1991) maximum likelihood estimator circumvents these problems and enables us to test for the presence of multiple cointegrating vectors. Johansen shows how to test for linear restrictions on the parameters of the cointegrating vectors, and this is of great interest because it makes it possible to test the symmetry and proportionality conditions exactly.[8]

Earlier cointegration studies generally reported the absence of significant mean reversion of the exchange rate toward PPP for the recent floating experience (Taylor 1988; Mark 1990) but were supportive of reversion toward PPP for the interwar float (Taylor and McMahon 1988), for the 1950s US–Canadian float (McNown and Wallace 1989), and for the exchange rates of high-inflation countries (Choudhry, McNown, and Wallace 1991). More recent applied work on long-run PPP among the major industrialized economies has, however, been more favorable toward the long-run PPP hypothesis for the recent float (e.g., Corbae and Ouliaris 1988; Kim 1990; Cheung and Lai 1993a, b).

Overall, cointegration studies highlight some important features of the data. The null hypothesis of no-cointegration is more easily rejected when in the sample period considered the exchange rates are fixed rather than floating. Also, interestingly, stronger evidence supporting PPP is suggested when the WPI is used rather than the CPI and, even more so, when the GDP deflator is used. This is easy to explain since the WPI price level contains a relatively smaller nontradables component and represents therefore a better approximation to the ideal price index required by the PPP hypothesis than the CPI and the GDP deflator.[9]

Another feature of the data suggested by the cointegration literature is that in bivariate systems cointegration is established more frequently than in trivariate systems and in Engle-Granger two-step procedures. The disappointing finding is, however, that the symmetry and proportionality conditions are very often rejected and the parameters estimated in PPP regressions are often far from the theoretical values. While this result may simply be caused by small-sample bias in the case of two-step cointegration procedures, it is difficult to explain rejections occurring in large samples and in estimates obtained using the Johansen procedure. Thus the problem may simply be that longer data sets are needed to detect PPP and mean reversion in the real exchange rate. In general, rejection of PPP may be due to lack of power of conventional econometric tests. Some notable attempts to overcome this problem are discussed in the following sections.

Following an early warning from Frankel (1986, 1990), a number of authors have noted that the tests typically employed during the 1980s to examine the long-run stability of the real exchange rate may have very low power to reject a null hypothesis of real exchange rate instability when applied to data for the recent floating rate period alone (e.g., Froot and Rogoff 1995; Lothian and Taylor 1996, 1997). The argument is that if the real exchange rate is in fact stable in the sense that it tends to revert toward its mean over long periods of time, then examination of just one real exchange rate over a period of twenty-five years or so may not yield enough information to be able to detect slow mean reversion toward purchasing power parity.

Much of the early work on unit roots and cointegration for real exchange rates was published in the late 1980s and was therefore based on data spanning the fifteen years or so since the period of generalized floating beginning in 1973. Using Monte Carlo analysis, Lothian and Taylor (1997) show that for the speeds of mean reversion typically recorded in the literature (Froot and Rogoff 1995; Rogoff 1996), the probability of rejecting the null hypothesis of a random walk real exchange rate when in fact the real rate is mean reverting would only be in region of about 5 percent when using only fifteen years of data. Given that we have, of course, only one data set on real exchange rates available, an alternative way of viewing this is to note that if real exchange rates are mean reverting in this fashion, the probability of *never* being able to reject the null hypothesis of a unit root given the available data is in the region of 95 percent when we have only fifteen years of data available. Even with the benefit of the additional ten years or so of data, which is now available, the power of the test increases only slightly, to a maximum of around 10 percent on the most optimistic view of the speed of mean reversion. Further Lothian and Taylor (1997, pp. 950–951) note that "even with a century of data on the sterling–dollar real exchange rate, we would have less than an even chance of rejecting the unit-root hypothesis."[10]

The Monte Carlo evidence of Shiller and Perron (1985) demonstrates that researchers cannot circumvent this problem by increasing the frequency of observation—say from annual to quarterly or monthly—and thereby increasing the number of data points available. Given that in a spectral analysis sense we are examining the low-frequency components of real exchange rate behavior, this requires a long *span* of data in terms of years in order to improve the power of the test.[11]

This realization led some researchers to do exactly that—examine the behavior of real exchange rates using very long data sets. An alternative means of increasing test power is to keep the same length of data set (e.g., since 1973) but to test for unit roots jointly using a panel of real exchange rates for a number of countries. This literature is discussed below.

4.2.2 *Long-Span Studies of PPP*

The first approach considered in the literature to circumvent the low-power problem of conventional unit-root tests was to employ long-span data sets.[12] For example, using annual data from 1869 to 1984 for the dollar–sterling real exchange rate, Frankel (1986) estimates an AR(1) process for the real rate with an autoregressive parameter of 0.86 and is able to reject the random walk hypothesis. Long-run PPP for the dollar–sterling exchange rate is also examined by Edison (1987) over the period 1890 to 1978, using an error-correction mechanism (ECM) of the form

$$\Delta s_t = \delta_0 + \delta_1 \Delta(p_t - p_t^*) + \delta_2(s_{t-1} - p_{t-1} + p_{t-1}^*) + u_t, \tag{13}$$

which has a long-run constant equilibrium level of real exchange rate. Edison's results provide evidence that PPP holds, but shocks impinging upon the real exchange rate are very persistent and the half-life is about 7.3 years. Glen (1992) also finds mean reversion of the real exchange rates for nine countries and a half-life of 3.3 years over the sample period 1900 to 1987 (see also Cheung and Lai 1994).

Lothian and Taylor (1996) use two centuries of data on dollar–sterling and franc–sterling real exchange rates and provide indirect evidence supporting PPP in the recent floating period. They cannot find any significant evidence of a structural break between the pre– and post–Bretton Woods period using a Chow test and show that the widespread failure to detect mean reversion in real exchange rates during the recent float may simply be due to the shortness of the sample.

Long-span studies have, however, been subject to some criticism in the literature. One criticism relates to the fact that because of the very long data spans involved, various exchange rate regimes are typically spanned. Also real shocks could have generated structural breaks or shifts in the equilibrium real exchange rate (e.g., see Hegwood and Papell 1999). This is, of course, a "necessary evil" with long-span studies of which researchers are generally aware. Researchers using long-span data are generally at pains to test for structural breaks (e.g., see Lothian and Taylor 1996).

Nevertheless, in order to provide a convincing test of real exchange rate stability during the post–Bretton Woods period, it is necessary to devise a test using data for that period alone. This provided the impetus for panel data studies of PPP.

4.2.3 Panel Data Studies of PPP

A different approach undertaken by the literature on testing for PPP in order to circumvent the problem of low-power displayed by conventional unit root tests is to increase the number of exchange rates under consideration.

The first attempt is due to Hakkio (1984), who employs generalized least squares (GLS) and tests the null hypothesis of nonstationarity using data for a system of four exchange rates. Hakkio cannot reject, however, the null hypothesis that all real exchange rates under examination follow a random walk.

Abuaf and Jorion (1990) employ a similar approach in that they examine a system of ten AR(1) regressions for real dollar exchange rates where the first-order autocorrelation coefficient is constrained to be equal across rates, taking account of contemporaneous correlations among the disturbances. The estimation is executed employing Zellner's (1962) "seemingly unrelated" (SUR) estimator, which is basically multivariate GLS using an estimate of the contemporaneous covariance matrix of the disturbances obtained from individual OLS estimation. Thus Abuaf and Jorion test the null hypothesis that the real exchange rates are jointly nonstationary for all ten series over the sample period 1973 to 1987. Their results indicate a marginal rejection of the null hypothesis of joint nonstationarity at conventional nominal levels of significance and are interpreted as evidence in favor of PPP. The study of Abuaf and Jorion (1990) has stimulated a strand of literature that employs multivariate generalizations of unit-root tests in order to increase the test power (e.g., Flood and Taylor 1996; Wu 1996; Frankel and Rose 1996; Coakley and Fuertes 1997; Lothian 1997; O'Connell 1998; Papell 1998). A number of these studies provide evidence supporting long-run PPP, given a sufficiently broad range of countries is considered and even only on post–Bretton Woods data.[13]

Taylor and Sarno (1998) and Sarno and Taylor (1998) argue, however, that the conclusions suggested by some of these studies may be misleading due to an incorrect interpretation of the null hypothesis of the multivariate unit root tests employed by Abuaf and Jorion and the subsequent literature. The null hypothesis in those studies is *joint* nonstationarity of the real exchange rates considered and hence rejection of the null hypothesis may occur even if only one of the series considered is stationary. Therefore, if rejection occurs when a group of real exchange rates is examined, then it may not be very informative and certainly it cannot be concluded that this rejection implies evidence supporting PPP for all them. On the basis of a large number of Monte Carlo experiments calibrated on dollar real exchange rates among the G5 countries, for example, Taylor and Sarno (1998) find that, for a sample size corresponding to the span of the recent float, the presence of a single stationary process together with three unit root processes led to rejection at the 5 percent level of the joint null hypothesis of nonstationarity in about 65 percent of simulations when the root of the stationary process was as large as 0.95, and on more than 95 percent of occasions when the root of the single stationary process was 0.9 or less.[14]

Taylor and Sarno (1998) employ two multivariate tests for unit roots which are shown—using Monte Carlo methods—to be relatively more powerful than traditional univariate tests using data for the G5 over the post–Bretton Woods period. The first test is based on a generalization of the augmented Dickey-Fuller test where, unlike in Abuaf and Jorion (1990), the autocorrelation coefficients are not constrained to be equal across countries and a more general AR(4) regression for each real exchange rate is considered. Although the null hypothesis is rejected, the test does not allow the authors to identify for how many and for which currencies PPP holds. The second test is based on an extension of the Johansen cointegration procedure, employed by the authors as a multivariate unit root test. Given that among a system of N I(1) series, there can be at most $N-1$ cointegrating vectors, if one can reject the hypothesis that there are less than N cointegrating vectors among N series, this is equivalent to rejecting the hypothesis of nonstationarity of *all* of the series. Put another way, the only way there can be N distinct cointegrating vectors among N series is if each of the series is I(0) and so is itself a cointegrating relationship.[15] Thus the null hypothesis under the Johansen procedure as applied by Taylor and Sarno is that there are $(N-1)$ or less cointegrating vectors among the N series concerned in the panel, which implies that at least one of them is nonstationary; rejection of the null in this case implies that *all* of the series in the panel are mean reverting. By rejecting this null hypothesis at the 1 percent nominal level of significance, Taylor and Sarno provide evidence that real exchange rates for the G5 constructed using the CPI price level are mean reverting during the recent floating period.

4.2.4 The Purchasing Power Parity Puzzle

In the previous two sections we have discussed the way in which researchers have sought to address the purchasing power problem in testing for mean reversion in the real exchange rate—either through long-span studies or through panel unit-root studies. As we made clear in our discussion, however, whether or not the long-span or panel data studies do answer the question of PPP holding over the long run remains contentious. As far as the long-span studies are concerned, as noted in particular by Frankel and Rose (1996), the long samples required to generate a reasonable level of statistical power with standard univariate unit-root tests may be unavailable for many currencies (perhaps thereby generating a "survivorship bias" in tests on the available data; Froot and Rogoff 1995). In any case, the tests may potentially be inappropriate because of differences in real exchange rate behavior both across different historical periods and across different nominal exchange rate regimes (e.g., Baxter and Stockman 1989; Hegwood and Papell 1999). As for panel data studies, the potential problem with panel unit-root tests, highlighted by the Monte Carlo evidence of Taylor and Sarno (1998), is that the null hypothesis in such tests is generally that *all* of the series are generated by unit-root processes, so that the probability of rejection of the null hypothesis may be quite high when as few as just one of the series under consideration is a realization of a stationary process.

Even if, however, we were to take the results of the long-span or panel data studies as having solved the first PPP puzzle, a second PPP puzzle then arises as follows. Among the long-span and panel data studies which do report significant mean reversion of the real exchange rate, there appears to be a consensus that the size of the half-life of deviations from PPP is about three to five years (Rogoff 1996). If we take as given that real shocks cannot account for the major part of the short-run volatility of real exchange rates (since it seems incredible that shocks to real factors such as tastes and technology could be so volatile) and that nominal shocks can only have strong effects over a time frame in which nominal wages and prices are sticky, then a second PPP puzzle is the apparently high degree of persistence in the real exchange rate (Rogoff 1996). Rogoff (1996) sums this issue up as follows: "The purchasing power parity puzzle then is this: How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?"

Since Rogoff first noted the PPP puzzle in 1996, researchers have sought to address this as an additional issue in research on real exchange rates. Allowing for underlying shifts in the equilibrium dollar–sterling real exchange rate (Harrod-Balassa-Samuelson, HBS, effects) over the past two hundred years through the use of nonlinear time trends, for example, Lothian and Taylor (2000) suggest that the half-life of deviations from PPP for this exchange rate may in fact be as low as two and a half years.

Recently Taylor (2000b) has shown that empirical estimates of the half-life of shocks to the real exchange rate may be biased upward because of two empirical pitfalls. The first pitfall identified by Taylor relates to temporal aggregation in the data. Using a model in which the real exchange rate follows an AR(1) process at a higher frequency than that at which the data are sampled, Taylor shows analytically that the degree of upward bias in the estimated half-life rises as the degree of temporal aggregation increases—namely as the length of time between observed data points increases. The second pitfall highlighted by Taylor concerns the possibility of nonlinear adjustment of real exchange rates. On the basis of Monte Carlo experiments with a nonlinear artificial data-generating process, Taylor shows that there can also be substantial upward bias in the estimated half-life of adjustment from assuming linear adjustment when in fact the true adjustment process is nonlinear. The time aggregation problem is a difficult issue for researchers to deal with since, as discussed above, long spans of data are required in order to have a reasonable level of power when tests of nonstationarity of the real exchange rate are applied, and long spans of high-frequency data do not exist.[16] On the other hand, Taylor also shows that the problem becomes particularly acute when the degree of temporal aggregation exceeds the length of the actual half-life, so this source of bias may be mitigated somewhat if the researcher believes that the true half-life is substantially greater than the frequency of observation. In any case, the literature to date has only begun to explore the issue of nonlinearities in real exchange rate adjustment.

4.3 Nonlinearities in Real Exchange Rate Movements

The models discussed above in the context of determining the stochastic process of the deviation from the LOP also imply nonlinearity in the real exchange rate. They suggest that the exchange rate will become increasingly mean reverting with the size of the deviation from the equilibrium level. In some models the jump to mean-reverting behavior is sudden, while in others it is smooth, and Dumas (1994) suggests that even in the former case, time aggregation will tend to smooth the transition between regimes. Moreover, if the real exchange rate is measured using price indexes made up of goods prices, each with a different size of international arbitrage costs, one would expect adjustment of the overall real exchange rate to be smooth rather than discontinuous.

Michael, Nobay, and Peel (MNP) (1997) and Taylor, Peel, and Sarno (TPS) (2001) propose an econometric modeling framework for the empirical analysis of PPP that allows for the fact that commodity trade is not frictionless and for aggregation across goods with different thresholds. To state the issues clearly, recall that equilibrium models of exchange rate determination in the presence of transaction costs were proposed by Benninga and Protopapadakis (1988), Dumas (1992), and Sercu, Uppal, and van Hulle (1995). As a result of the costs of trading goods, persistent deviations from PPP are implied as an equilibrium feature of these models (deviations are left uncorrected as long as they are small relative to the costs of trading). A significant insight into the nature of PPP deviations is provided by Dumas (1992), who analyses the dynamic process of the real exchange rate in spatially separated markets under proportional transaction costs. Deviations from PPP are shown to follow a nonlinear process which is mean reverting. The speed of adjustment toward equilibrium varies directly with the extent of the deviation from PPP. Within the transaction band, when no trade takes place, the process is divergent so that the exchange rate spends most of the time away from parity. This implies that deviations from PPP last for a very long time (p. 154), although they certainly do not follow a random walk.[17]

Kilian and Taylor (2001) provide an alternative or complementary analysis of why real exchange rates may exhibit nonlinearities in adjustment, based on a model in which there are heterogeneous agents exerting influence in the foreign exchange market, namely economic fundamentalists, technical analysts, and noise traders. It is assumed that traders take the advice of fundamentalists who themselves may differ in opinion as to what the true equilibrium level of the exchange rate is, and therefore with respect to their forecasts. When there is strong disagreement among the fundamentalists, traders will tend to rely at least partly on the advice of technical analysts who use trend-following forecasts which impart a unit root into the exchange rate.[18]

Thus the nominal—and hence the real—exchange rate will tend to move away from the equilibrium level so long as fundamentalists disagree about the level of that equilibrium. As the exchange rate moves further away from equilibrium, however, there is an increasingly greater degree of agreement among the fundamentalists that the exchange rate is above or below its equilibrium, and hence an increasingly strong tendency of the exchange rate to revert back toward the equilibrium level as traders are swayed by an emerging consensus concerning the likely future direction of real and nominal exchange rate movements.[19]

In the procedures conventionally applied to test for long-run PPP, the null hypothesis is usually that the process generating the real exchange rate series has a unit root, while the alternative hypothesis is that all of the roots of the process lie within the unit circle. Thus the maintained hypothesis in the conventional framework assumes a linear autoregressive process for the real exchange rate, which means that adjustment is both continuous and of constant speed, regardless of the size of the deviation from PPP. As noted above, however, the presence of transaction costs may imply a nonlinear process that has important implications for the conventional unit root tests of long-run PPP. Some empirical evidence of the effect of transaction costs on tests of PPP is provided by Davutyan and Pippenger (1990). More recently Obstfeld and Taylor (1997) have investigated the nonlinear nature of the adjustment process in terms of a threshold autoregressive (TAR) model (Tong 1990) that allows for a transaction costs band within which no adjustment takes place, while outside of the band the process switches abruptly to become stationary autoregressive. Although discrete switching of this kind may be appropriate when considering the effects of arbitrage on disaggregated goods prices (Obstfeld and Taylor 1997), discrete adjustment of the aggregate real exchange rate would clearly be appropriate only when firms and traded goods are identical. Moreover, many of the theoretical studies discussed above suggest that smooth rather than discrete adjustment may be more appropriate in the presence of proportional transaction costs and, as suggested by Teräsvirta (1994), Dumas (1994), and Bertola and Caballero (1990), time aggregation and nonsynchronous adjustment by heterogeneous agents is likely to result in smooth aggregate regime switching.

An alternative characterization of nonlinear adjustment, which allows for smooth rather than discrete adjustment, is in terms of a smooth transition autoregressive (STAR) model (Granger and

Teräsvirta 1993). This is the model employed by Michael, Nobay, and Peel (1997) and Taylor, Peel, and Sarno (2001). In the STAR model, adjustment takes place in every period but the speed of adjustment varies with the extent of the deviation from parity. A STAR model may be written

$$[q_t - \mu] = \sum_{j=1}^{p} \beta_j [q_{t-j} - \mu] + \left[\sum_{j=1}^{p} \beta_j^* [q_{t-j} - \mu] \right] \Phi[\theta; q_{t-d} - \mu] + \varepsilon_t, \tag{14}$$

where $\{q_t\}$ is a stationary and ergodic process, $\varepsilon_t \sim \mathrm{iid}(0, \sigma^2)$ and $(\theta\mu) \in \{\Re^+ \times \Re\}$, where $\Re$ denotes the real line $(-\infty, \infty)$ and $\Re^+$ the positive real line $(0, \infty)$. The transition function $\Phi[\theta; q_{t-d} - \mu]$ determines the degree of mean reversion and is itself governed by the parameter θ, which effectively determines the speed of mean reversion, and the parameter μ which is the equilibrium level of $\{q_t\}$. A simple transition function, suggested by Granger and Teräsvirta (1993), is the exponential function:

$$\Phi[\theta; q_{t-d} - \mu] = 1 - \exp[-\theta^2 [q_{t-d} - \mu]^2], \tag{15}$$

in which case (17) would be termed an exponential STAR or ESTAR model. The exponential transition function is bounded between zero and unity, $\Phi : \Re \to [0, 1]$, has the properties $\Phi[0] = 0$ and $\lim_{x \to \pm\infty} \Phi[x] = 1$, and is symmetrically inverse—bell shaped around zero. These properties of the ESTAR model are attractive in the present modelling context because they allow a smooth transition between regimes and symmetric adjustment of the real exchange rate for deviations above and below the equilibrium level. The transition parameter θ determines the speed of transition between the two extreme regimes, with lower absolute values of θ implying slower transition. The inner regime corresponds to $q_{t-d} = \mu$, when $\Phi = 0$ and (17) becomes a linear AR(p) model:

$$[q_{t-d} - \mu] = \sum_{j=1}^{p} \beta_j [q_{t-j} - \mu] + \varepsilon_t. \tag{16}$$

The outer regime corresponds, for a given θ, to

$$\lim_{[q(t-d)-\mu] \to \pm\infty} \Phi[\theta; q_{t-d} - \mu].$$

In (17) it becomes a different AR(p) model

$$[q_{t-d} - \mu] = \sum_{j=1}^{p} (\beta_j + \beta_j^*)[q_{t-j} - \mu] + \varepsilon_t \tag{17}$$

with a correspondingly different speed of mean reversion so long as $\beta_j^* \neq 0$ for at least one value of j.

It is also instructive to reparameterize the STAR model (17) as

$$\Delta q_t = \alpha + \rho q_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta q_{t-j} + \left\{ \alpha^* + \rho^* q_{t-1} + \sum_{j=1}^{p-1} \phi_j^* \Delta q_{t-j} \right\} \Phi[\theta; q_{t-d}] + \varepsilon_t, \tag{18}$$

where $\Delta q_{t-j} \equiv q_{t-j} - q_{t-j-1}$. In this form the crucial parameters are ρ and ρ^*. Our discussion of the effect of transaction costs above suggests that the larger the deviation from PPP, the stronger will be the tendency to move back to equilibrium. This implies that while $\rho \geq 0$ is admissible, we must have $\rho^* < 0$ and $(\rho + \rho^*) < 0$. That is, for small deviations q_t may be characterized by unit root or even explosive behavior, but for large deviations the process is mean reverting. This analysis has implications for the conventional test for a unit root in the real exchange rate process, which is based on a linear AR(p) model, written below as an augmented Dickey-Fuller regression:

$$\Delta q_t = \alpha' + \rho' q_{t-1} + \sum_{j=1}^{p-1} \phi_j' \Delta q_{t-j} + \varepsilon_t. \tag{19}$$

Assuming that the true process for q_t is given by the nonlinear model (21), estimates of the parameter ρ' in (22) will tend to lie between ρ and $(\rho + \rho^*)$, depending on the distribution of observed deviations from the equilibrium level μ. Hence the null hypothesis $H_0 : \rho' = 0$ (a single unit root) may not be rejected against the stationary linear alternative hypothesis $H_1 : \rho' < 0$, even though the true nonlinear process is globally stable with $(\rho + \rho^*) < 0$. Thus, failure to reject the unit root hypothesis on the basis of a linear model does not necessarily invalidate long-run PPP.[20]

MNP (1997) apply this model to monthly interwar data for the French franc–US dollar, French franc–UK sterling, and UK sterling–US dollar as well as for the Lothian and Taylor (1996) long-span data set. Their results clearly reject the linear framework in favor of an

ESTAR process. The systematic pattern in the estimates of the nonlinear models provides strong evidence of mean-reverting behavior for PPP deviations, and helps explain the mixed results of previous studies. However, the periods examined by MNP are ones over which the relevance of long-run PPP is uncontentious (Taylor and McMahon 1988; Lothian and Taylor 1996).

Using data for the recent float, however, TPS (2001) record empirical results that provide strong confirmation that four major real bilateral dollar exchange rates are well characterized by nonlinearly mean-reverting processes over the floating rate period since 1973. Their estimated models imply an equilibrium level of the real exchange rate in the neighborhood where the behavior of the log level of the real exchange rate is close to a random walk, becoming increasingly mean reverting with the absolute size of the deviation from equilibrium; this is consistent with the recent theoretical literature on the nature of real exchange rate dynamics in the presence of international arbitrage costs. TPS also estimated the impulse response functions corresponding to their estimated nonlinear real exchange rate models by Monte Carlo integration.[21] By taking account of statistically significant nonlinearities, TPS find the speed of real exchange rate adjustment to be typically much faster than the very slow speeds of real exchange rate adjustment hitherto recorded in the literature. These results therefore seem to shed some light on Rogoff's PPP puzzle (Rogoff 1996). In particular, it is only for small shocks occurring when the real exchange rate is near its equilibrium that the nonlinear models consistently yield half-lives in the range of three to five years, which Rogoff (1996) terms "glacial." For dollar–mark and dollar–sterling, in particular, even small shocks of 1 to 5 percent have a half-life under three years. For larger shocks, the speed of mean reversion is even faster.[22]

In a number of Monte Carlo studies calibrated on the estimated nonlinear models, TPS also demonstrate the very low power of standard univariate unit root tests to reject a false null hypothesis of unit root behavior when the true model is nonlinearly mean reverting, thereby suggesting an explanation for the difficulty researchers have encountered in rejecting the linear unit root hypothesis at conventional significance levels for major real exchange rates over the recent floating rate period. Panel unit-root tests, however, displayed much higher power in their rejection of the false null hypothesis against an alternative of nonlinear mean reversion, in keeping with the recent literature. The results of TPS therefore encompass previous empirical work in this area.[23]

4.4 Concluding Remarks

A promising strand of research that goes some way toward resolving both fundamental puzzles in this literature—namely whether PPP holds and whether one can reconcile estimated half-lives of shocks to the real exchange rates with their observed high volatility—has investigated the role of nonlinearities in real exchange rate adjustment toward long-run equilibrium. Taylor, Peel, and Sarno (2001) provide evidence of nonlinear mean reversion in a number of major real exchange rates during the post–Bretton Woods period such that real exchange rates behave more like unit-root processes the closer they are to long-run equilibrium and, conversely, become more mean reverting the further they are from equilibrium. Moreover, while small shocks to the real exchange rate around equilibrium will be highly persistent, larger shocks mean-revert much faster than the "glacial rates" previously reported for linear models (Rogoff 1996). Further TPS reconcile these results with the huge literature on unit roots in real exchange rates through Monte Carlo studies and, in particular, demonstrate that when the true data-generating process implies nonlinear mean reversion of the real exchange rate, standard univariate unit-root tests will have very low power, while multivariate unit-root tests will have much higher power to reject a false null hypothesis of unit-root behavior.

The main conclusion emerging from the recent literature on testing the validity of the PPP hypothesis appears to be that PPP might be viewed as a valid long-run international parity condition when applied to bilateral exchange rates obtaining among major industrialised countries and that mean reversion in real exchange rates displays significant nonlinearities.

Further exploration of the importance of nonlinearities in real exchange rate adjustment therefore seems warranted.

Notes

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1. An example often used in the literature is the product differentiation of MacDonald's hamburgers across countries. An example of a good for which the LOP may be expected to hold is gold and other internationally traded commodities (see Rogoff 1996).

2. Herguera (1994) investigates the implications of product differentiation for the price adjustment mechanism in international trade using an imperfect competition model. In particular, Herguera finds that market structure, product differentiation, and strategic behavior can explain the persistent price differential of perfectly substitutable goods across countries (see also Chen and Knez 1995; Dumas, Jennergren, and Naslund 1995).

3. Froot and Rogoff (1995) note how the PTM theory not only can explain the long-run deviations from the LOP, but has important implications for the transmission mechanism of disturbances from the money market in the presence of nominal rigidities (see also Marston 1990).

4. A potential explanation of this finding is provided by Rangan and Lawrence (1993) who argue that since US firms sell a large part of their exports through subsidiaries, the PTM by US firms may occur at subsidiary level. In this case the comparisons executed by Knetter may lead to an underestimation of the importance of PTM by US firms.

5. Cointegration among the variables may also reduce the problem of endogeneity of the right-hand side variables because of the superconsistency property of OLS in cointegrating regressions; see Engle and Granger (1987).

6. Meese and Rogoff (1988), for example, take this approach; their results provide strong evidence supporting the nonstationarity of the real exchange rate during the floating period.

7. Another result supportive of PPP is due to Whitt (1992). Whitt uses a Bayesian unit root test due to Sims (1988), and is able to reject the null hypothesis that the real exchange rate follows a random walk for a number of countries and for both the pre– and the post–Bretton Woods period.

8. It is also possible to circumvent the problem by simply estimating the regression of the nominal exchange rate on the relative price by fully modified OLS (FM OLS), due to Phillips and Hansen (1990), instead of OLS, since a correction is made for the problem of the bias in the standard errors.

9. The argument that PPP should hold better with the WPI than with the CPI goes back to Keynes (1932) and McKinnon (1971).

10. Engel (2000), using artificial data calibrated to nominal exchange rates and disaggregated data on prices also shows that standard unit-root and cointegration tests applied to real exchange rate data may have significant size biases and also demonstrates that tests of stationarity may have very low power.

11. Similar remarks would apply to variance ratio tests and tests for non-cointegration.

12. As discussed above, alternative unit root tests may also be sufficiently powerful to detect mean reversion in real exchange rates. For example, Diebold, Husted, and Rush (1991) and Cheung and Lai (1993a) apply fractional integration techniques and find evidence supporting long-run PPP. See also Taylor (2000a).

13. Flood and Taylor (1996) find strong support for mean reversion toward long-run PPP using data on 21 industrialized countries over the floating rate period and regressing five-, ten-, and twenty-year average exchange rate movements on average inflation differentials against the United States.

14. Note that the artificial data generating process is calibrated on quarterly data, so that roots of this magnitude are plausible; see Taylor and Sarno (1998) for further details. O'Connell (1998) points out an additional problem with panel unit-root tests, namely that they typically fail to control for cross-sectional dependence in the data, and shows that this may lead to considerable size distortion, raising the significance level of tests with a nominal size of 5 percent to as much as 50 percent.

15. This assumes that the underlying process must be either $I(0)$ or $I(1)$.

16. A possible solution to this would be to use panels of high-frequency data in order to increase test power, although care must in this case be taken to avoid the panel unit-root problem highlighted by Taylor and Sarno (1998).

17. Dumas (1992) conjectures that the Roll (1979) ex ante PPP hypothesis holds as a limiting case of his model as the degree of risk aversion tends to zero.

18. See Allen and Taylor (1990), Taylor and Allen (1992), and Sarno and Taylor (2001a), for a discussion of the importance of the influence of technical analysis in the foreign exchange market.

19. Taylor (2002) also provides some evidence that official foreign exchange intervention may impart nonlinearity into real exchange rate movements.

20. In empirical applications, Granger and Teräsvirta (1993) and Teräsvirta (1994) suggest choosing the order of the autoregression, p, through inspection of the partial autocorrelation function, PACF; the PACF is to be preferred to the use of an information criterion since it is well known that the latter may bias the chosen order of the autoregression toward low values, and any remaining serial correlation may affect the power of subsequent linearity tests. Granger and Teräsvirta (1993) and Teräsvirta (1994) then suggest applying a sequence of linearity tests to specifically designed artificial regressions for various values of d (see also Luukkonen, Saikkonen and Teräsvirta 1988). These can be interpreted as second or third-order Taylor series expansions of the STAR model. This allows detection of general nonlinearity through the significance of the higher order terms, with the value of d selected as that giving the largest value of the test statistic.

21. Note that, because of the nonlinearity, the half-lives of shocks to the real exchange rates vary both with the size of the shock and with the initial conditions.

22. Half-lives estimated using ESTAR models fitted to mark-based European real exchange rate series (Taylor and Sarno 1999) were generally slightly lower than those for dollar-based real exchange rates. This is unsurprising, given the proximity of the European markets involved and the fact that they are operating within a customs union, and accords with previous evidence on the mean-reverting properties of European real exchange rates (e.g., Canzoneri, Cumby, and Diba 1999; Cheung and Lai 1998). In a complementary study Taylor and Peel (2000) fit ESTAR models to deviations of the nominal exchange rate from the level suggested by "monetary fundamentals," and find that the model performs well for dollar–mark and dollar–sterling over the recent float.

23. In their fitted ESTAR models, the real exchange rate will be closer to a unit root process the closer it is to its long-run equilibrium. Somewhat paradoxically, failure to reject a

unit root may therefore indicate that the real exchange rate has on average been relatively close to equilibrium, rather than implying that no such long-run equilibrium exists.

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5 Heterogeneity of Agents and the Exchange Rate: A Nonlinear Approach

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The rational expectations efficient market model developed during the 1970s has dominated our thinking about exchange rates. This model led to the propositions, first, that exchange rate changes can only occur because of unexpected movements (news) in the underlying fundamental economic variables (inflation, growth of output, interest rates, etc.) and, second, that the link between exchange rates and fundamentals is a stable one. Well-known examples of the rational expectation efficient market model are the monetary model, the Dornbusch model (Dornbusch 1976), and the portfolio balance model. Although these models continue to be popular and maintain a prominent place in textbooks, they have failed empirically. The most notorious empirical rejection was made by Meese and Rogoff at the beginning of the 1980s (Meese and Rogoff 1983). This led to a large empirical literature that uncovered a number of empirical puzzles concerning the behavior of the exchange rate, which cannot be explained by the "news" models. It has become increasingly clear that this model performs poorly in explaining the many empirical anomalies observed in the foreign exchange markets.

The empirical failure of the exchange rate models of the 1970s has led to new attempts to model the exchange rate. These attempts have led to three different modeling approaches. The first one uses the Obstfeld–Rogoff framework of dynamic utility optimization of a representative agent. The models that came out from this approach have a high content of intellectual excitement. However, up to now they have led to few testable propositions.

A second approach starts from the analysis of the microstructure of the foreign exchange market. This approach has led to new insights into the way information is aggregated and is important for the understanding of the short-term behavior of the exchange rate.

Finally, a third approach recognizes that heterogeneous agents have different beliefs about the behavior of the exchange rate. These different beliefs introduce nonlinear features in the dynamics of the exchange rate. In this chapter we present a simple model of the exchange rate that incorporates these nonlinear features, and we analyze their implications for the dynamics of the exchange rate. In addition we make use of recent empirical evidence that strongly suggests that the adjustment toward PPP is nonlinear in nature. It will be shown that our simple nonlinear model is capable of solving the empirical puzzles observed in the foreign exchange market.

The chapter is organized as follows: In section 5.1 we present the theoretical model. In sections 5.2 to 5.6 we analyze its features. In section 5.7 we show the empirical relevance of this model.

5.1 A Simple Nonlinear Exchange Rate Model

In this section we develop a simple nonlinear exchange rate model. We start from a well-known model of the exchange rate, which is often used in the literature. We then introduce heterogeneous agents who use this model as a benchmark to define their beliefs about the future exchange rate.

The exchange rate can be written as follows:

$$s_t = f_t + a[E_t s_{t+1} - s_t], \tag{1}$$

where f_t represents the fundamental economic variables driving the exchange rate (the fundamental for short) in period t, s_t is the exchange rate in period t, s_{t+1} is the exchange rate in period $t+1$, and E is the expectations operator. Underlying the fundamental variables one can specify a whole model of the economy, such as a monetary model, or a more elaborate one like the Obstfeld-Rogoff new open economy macro model (Obstfeld and Rogoff 1996). We leave this for further research. Here we concentrate on the simplest possible exchange rate modeling. For the sake of simplicity we assume that the fundamentals are determined exogenously.[1]

Equation (1) can be rewritten as follows:

$$s_t = \frac{1}{1+a} f_t + \frac{a}{1+a}[E_t s_{t+1}]. \tag{2}$$

We use this model to define the fundamental equilibrium exchange rate. This is the rational expectations solution of equation (2). It will be used as a benchmark against which the beliefs of different agents are measured.

In the absence of bubbles the fundamental solution to (2) is given by

$$s_t^* = \frac{1}{1+a}\sum_{i=0}^{\infty}\left(\frac{a}{1+a}\right)^i E_t f_{t+i}. \tag{3}$$

For the sake of simplicity we will assume that f_t follows a random walk process without drift. We then find the following fundamental solution of the exchange rate:

$$s_t^* = f_t. \tag{4}$$

In some applications we will assume that f_t is a constant.

We now introduce the assumption that the agents have heterogeneous beliefs, and we classify them according to their beliefs. Let us assume that there are N_h individuals of type h belief (where $\sum N_h = N$). We can then characterize the beliefs of type h agents as follows:[2]

$$E_{h,t}s_{t+1} = s_t^* + g_h(s_{t-1}, s_{t-2}, \ldots), \tag{5}$$

where $E_{h,t}$ represents the expectations operator of type h agent at time t. Thus agents' beliefs can be classified depending on how they view the process by which the market price will grope toward the fundamental exchange rate s_t^*. They all use information on past exchange rates to forecast these future developments.

The market expectation can then be written as follows:

$$E_t s_{t+1} = \sum_{h=1}^{H} n_h E_{h,t}[s_{t+1}] = s_t^* + \sum_{h=1}^{H} n_h g_h(s_{t-1}, s_{t-2}, \ldots). \tag{6}$$

Note that $n_h = N_h/N$, so n_h can be interpreted as the weight of agents of type h in the market. An important characteristic of our model is that these weights will be made endogenous.

The realized market rate in period $t+1$ equals the market forecast made at time t plus some white noise error (i.e., the news that could not be predicted at time t):

$$s_{t+1} = s_t^* + \sum_{h=1}^{H} n_h g_h(s_{t-1}, s_{t-2}, \ldots) + \varepsilon_{t+1}. \tag{7}$$

In the previous discussion the nature of the beliefs of agents was specified in very general terms. We further simplify the model by assuming that there are only two types of agents in the foreign exchange market, which we will call *fundamentalists* and *chartists*.[3]

The *fundamentalists* base their forecasts on a rule like in equation (5). That is to say, they compare the past market exchange rates with the fundamental rate, and they forecast the future market rate to move toward the fundamental rate. In this sense they follow a *negative feedback* rule.[4] We will make the additional assumption that they expect the speed with which the market rate returns to the fundamental rate to be determined by the speed of adjustment in the goods market.

There is an increasing amount of empirical evidence indicating that the speed of adjustment in the goods market follows a nonlinear dynamics; namely the speed with which prices adjust toward equilibrium depends positively on the size of the deviation from equilibrium (see Kilian and Taylor 2001; Taylor, Peel, and Sarno 2001; Nobay and Peel 1997). We will assume that this adjustment process is quadratic in nature.[5] Fundamentalists take this nonlinear dynamic adjustment into account in making their forecast. This leads us to specify the following rule for the fundamentalists:

$$E_{f,t}(\Delta s_{t+1}) = -\psi(s_t - s_t^*), \tag{8}$$

where $E_{f,t}$ is the forecast made in period t by fundamentalists and ψ is a function of the size of the deviation from the fundamental variable.

We assume the following simple specification:

$$\psi = \theta|s_t - s_t^*|,$$

where $\theta > 0$. Thus, when the size of the deviation from equilibrium is large, the fundamentalists expect a faster speed of adjustment toward the fundamental rate than when the size of the deviation is small. The economics behind this nonlinear specification is that in order to profit from arbitrage opportunities in the goods market, some fixed investment must be made; for example, trucks must be bought and planes be chartered. These investments become profitable with sufficiently large deviations from the fundamental exchange rate. Note that we do not model the goods market explicitly; rather we assume that in order to form their expectations about the exchange rate, the fundamentalists take into account the dynamics of the goods market and the speed of adjustment of goods prices.

The *chartists* are assumed to follow a *positive feedback* rule: they extrapolate past movements of the exchange rate into the future. Their forecast is written as

$$E_{c,t}(\Delta s_{t+1}) = \beta \sum_{i=0}^{T} \alpha_i \Delta s_{t-i}, \tag{9}$$

where $E_{c,t}$ is the forecast made by chartists using information up to time t; Δs_t is the change in exchange rate.

As can be seen, the chartists compute a moving average of the past exchange rate changes, and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β. Note that in contrast to the general rule as given by equation (5) (and also in contrast to fundamentalists), they do not take into account information concerning the fundamental exchange rate. In this sense they can be considered to be pure *noise traders*.[6]

Our choice to introduce chartists' rules of forecasting is based on empirical evidence. The evidence that chartism is used widely to make forecasts is overwhelming (see Cheung and Chinn 1989; Taylor and Allen 1992). Therefore we give a prominent role to chartists in our model. It remains important, however, to check if the model is internally consistent. In particular, the chartists' forecasting rule must be shown to be profitable within the confines of the model. If these rules turn out to be unprofitable, they will not continue to be used. We return to this issue when we let the number of chartists be determined by the profitability of the chartists' forecasting rule.

In a similar logic as in equation (7) the market exchange rate can now be written as

$$\Delta s_{t+1} = -n_{ft}\theta |s_t - s_t^*|(s_t - s_t^*) + n_{ct}\beta \sum_{i=0}^{T} a_i \Delta s_{t-i} + \varepsilon_{t+1}, \tag{10}$$

where n_{ft} and n_{ct} are the weight of fundamentalists and the weight of chartists.

We now specify the dynamics that governs the weights of chartists and fundamentalists, namely n_{ct} and n_{ft}. In order to do so, we describe how the number of chartists and fundamentalists change from period $t-1$ to period t:

$$N_t^c = N_{t-1}^c + N_{t-1}^f p_t^{fc} - N_{t-1}^c p_t^{cf}, \tag{11}$$

$$N_t^f = N_{t-1}^f + N_{t-1}^c p_t^{cf} - N_{t-1}^f p_t^{fc}, \tag{12}$$

where N_t^c and N_t^f are the number of chartists and fundamentalists in period t. p_t^{cf} represents the probability of a chartist to become a fundamentalist in period t; p_t^{fc} is the probability of a fundamentalist to become a chartist in period t.

These probabilities are assumed to be a function of the relative profitability of the forecasting rules and the *risk* associated with their use. The probabilities are specified as follows:

$$p_t^{fc} = \imath_1 \frac{\exp(\pi_{c,t-1})}{\exp(\pi_{c,t-1}) + \exp(\pi_{f,t-1})} \exp(-z(s_{t-1} - f_{t-1})^2), \tag{13}$$

$$p_t^{cf} = \imath_2 \frac{\exp(\pi_{f,t-1})}{\exp(\pi_{c,t-1}) + \exp(\pi_{f,t-1})} (1 - \exp(-z(s_{t-1} - f_{t-1})^2)), \tag{14}$$

where $\pi_{c,t-1}$ is the profit of the chartists' forecasting rule in period $t-1$ and $\pi_{f,t-1}$ is the profit of the fundamentalists' forecasting rule. The chartists make a profit when they correctly forecast the direction of the exchange rate movements. They make a loss if they wrongly predict the direction of the exchange rate movements. The profit (the loss) they make equals the one-period return of the exchange rate. Thus the ratio $\exp(\pi_{c,t-1})/[\exp(\pi_{c,t-1}) + \exp(\pi_{f,t-1})]$ represents the profit of chartists relative to the total market profits. The functional form assures that this ratio is between 0 and 1. We will call this ratio the *relative profitability of chartism*. The parameter $\imath_1$ measures the sensitivity of the probability of switching from fundamentalism to chartism with respect to the relative profitability of chartism. Note also that $0 < \imath_1 < 1$.

The term $\exp(-z(s_{t-1} - f_{t-1})^2)$ captures the *risk* associated with the use of chartists' forecasting. We postulate that when the size of the deviation of the exchange rate from its fundamental value (the misalignment) increases the risk of using a chartist extrapolative forecasting rule increases. This reduces the probability that a fundamentalist switches to a chartist rule. The parameter z measures how sensitive the risk term is with respect to the size of the misalignment. The logic behind this formulation is based on the mean-variance utility framework. In particular, it implies that when the risk increases because of the increasing misalignment, the profit of chartism must increase in order to induce a given fraction of agents to switch to chartism.

In figure 5.1 we show this relationship between "risk" and profitability of the forecasting rule that is implicit in equation (13). On the vertical axis we set out the relative profitability of chartism. On the horizontal axis we set out the risk factor as measured by misalignment.

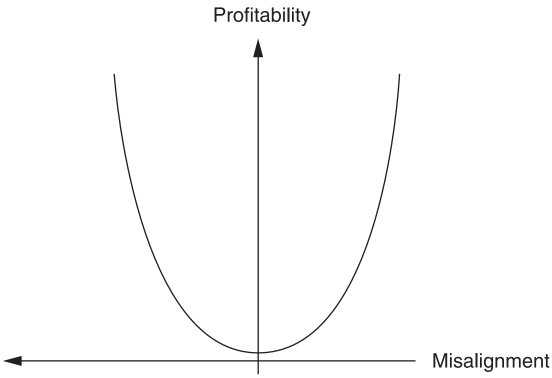

Figure 5.1
Risk and profitability of chartism

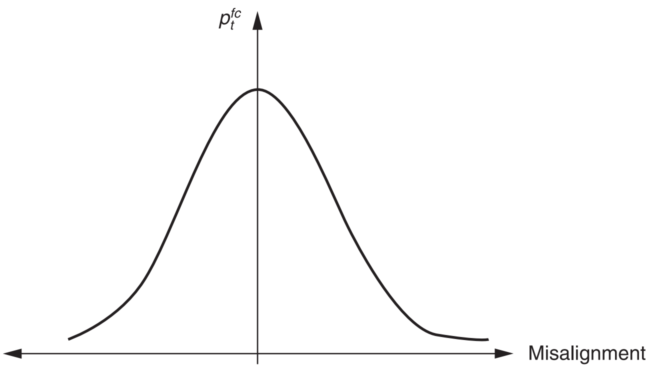

Figure 5.2
Profitability of switching to chartism

The curve presented in figure 5.1 is obtained by fixing the probability of switching from fundamentalism to chartism in equation (13) and plotting the relative profitability against the misalignment that keeps this probability unchanged. We see that when the risk increases, an increasingly larger relative profitability is necessary to induce the same fraction of fundamentalists to switch to Chartism.

In figure 5.2 we give a second graphical interpretation of equation (13). It shows the probability of switching from fundamentalism to chartism on the vertical axis as a function of the degree of misalignment (risk) on the horizontal axis. The bell-shaped curve is obtained by assuming a given level of relative profitability of chartism. We observe that as the degree of misalignment increases the probability of switching to chartism declines and goes to zero asymptotically.

Equation (14) which defines the probability of a chartist to switch to fundamentalism, can be interpreted in a similar way. Note that in this case an increasing misalignment makes the use of fundamentalist forecasting less risky.

Finally, we compute the chartists weight $n_{c,t}$ and the fundamentalists weight $n_{f,t}$:

$$n_{c,t} = \frac{N_{c,t}}{N_{c,t} + N_{f,t}}, \tag{15}$$

$$n_{f,t} = \frac{N_{f,t}}{N_{c,t} + N_{f,t}}. \tag{16}$$

We assume that $N_{c,t} + N_{f,t} = N$, which is a constant.

5.2 The Model with Transactions Costs

There is an increasing body of theoretical literature stressing the importance of transactions costs as a source of nonlinearity in the determination of the exchange rate (Dumas 1992; Sercu, Uppal, and Van Hulle 1995; Obstfeld and Rogoff 2000). The importance of transaction costs has also been confirmed empirically (Taylor, Peel, and Sarno 2001; Kilian and Taylor 2001). Therefore we will develop a version of the previous model in which the transaction costs play a role.

We take the view that if transaction costs exist, the fundamentalists take this information into account. Therefore, if the exchange rate is within the transaction costs band, the fundamentalists behave differently than if the exchange rate moves outside the transaction costs band.

Consider the first case, when the exchange rate deviations from the fundamental value are smaller than the transaction costs. In this case the fundamentalists know that arbitrage in the goods market does not apply. As a result they expect the changes in the exchange rate to follow a white noise process ε_t. The best they can do is to forecast no change. More formally, when $|s_t - s_t^*| < C$, then $E_{f,t}(\Delta s_{t+1}) = 0$.

In the second case, when the exchange rate deviation from its fundamental value is larger than the transaction costs C (assumed to be of the “iceberg” type). Then the fundamentalists follow the same forecasting rule as in equation (8). More formally, when $|s_t - s_t^*| > C$ holds, then equation (8) applies.

Formulation (8) implies that when the exchange rate moves outside the transaction costs band, market inefficiencies other than transaction costs continue to play a role. As a result these inefficiencies prevent the exchange rate from adjusting instantaneously. In our model these inefficiencies are captured by the fact that the speed of adjustment in the goods market is not infinite (equation 8).

5.3 Solution of the Model

In this section we investigate the properties of the solution of the model. We first study the deterministic solution of the model. We do this because we want to analyze the intrinsic characteristics of the solution that are not clouded by exogenous noise. We use simulation techniques since the nonlinearities do not allow for a simple analytical solution. We select "reasonable" values of the parameters, namely those that come close to empirically observed values. We will, however, analyze how sensitive the solution is to different sets of parameter values.

We first concentrate on the fixed point solutions of the model. We find that for a relatively wide range of parameters the solution converges to a fixed point (a *fixed-point attractor*). However, there are many such fixed points (attractors) to which the solution converges, depending on the initial conditions. We illustrate this feature in figure 5.3, where we show the exchange rate in time domain for a particular set of parameters and different initial conditions. We find that the exchange rate converges to a different fixed point, depending on the initial conditions. For each different initial condition we obtain a different fixed-point solution. (In the next section we perform a sensitivity analysis to check the general nature of this result.) We show this feature in figure 5.4 by plotting the fixed point solutions (attractors) as a function of the different initial conditions. On the horizontal axis we set out the different initial conditions. These are initial shocks to the deterministic system. The vertical axis shows the solutions corresponding to these different initial conditions. Note that this characteristic of many fixed-point attractors is a natural result of the nonlinear nature of our model, in particular, of the existence of transactions costs. We return to this to give an interpretation to this phenomenon.

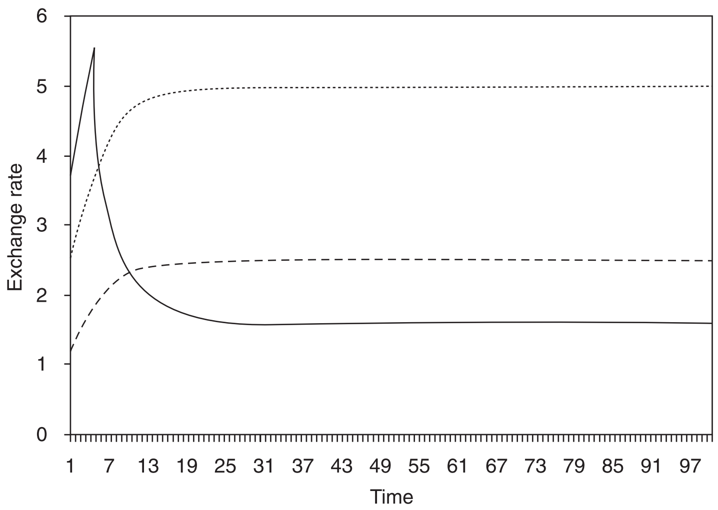

Figure 5.3
Exchange rates in time domains ($c = 5$, beta $= 1$, theta $= 0.2$) for different initial conditions

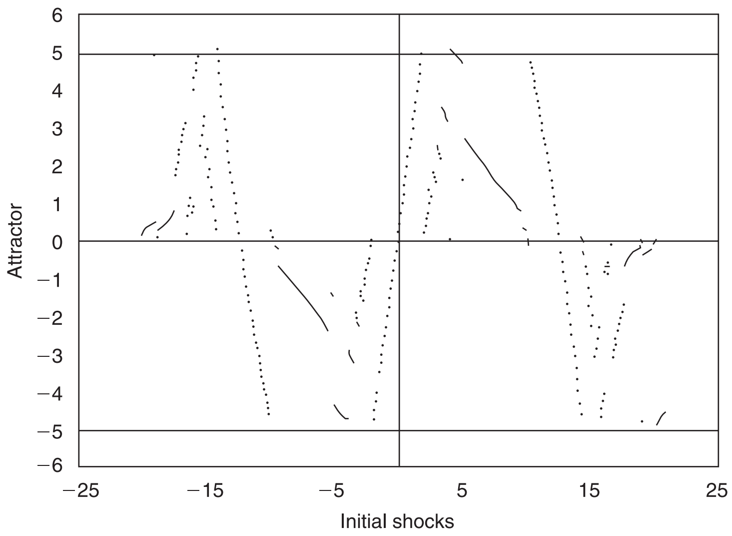

Figure 5.4
Fixed attractors ($c = 5$, beta $= 1$, theta $= 0.2$)

5.4 Sensitivity Analysis

We obtain a multiplicity of fixed-point solutions for a relatively broad range of parameters. In the appendix at the end of this chapter we present a table describing the nature of the solutions for different combinations of parameter values. We find that the extrapolation parameter of the chartists, β is of crucial importance. In particular, the extrapolation parameter must lie below a certain critical value, that depends itself on the other parameter values, to obtain multiplicity of fixed point solutions. In figure 5.5 we show the fixed-point attractors for two different combinations of parameter values.

It can be seen that we obtain a multiplicity of fixed-point attractors, each one, depending on the initial shock. In addition there are many discontinuities in the location of these fixed points. It should also be noted that the fixed-point attractors lie within the transaction costs band. The intuition is that any fixed-point solution outside the transaction costs band would create an inconsistency, which can be described as follows. Outside the transaction costs band the fundamentalists' behavior leads to a mean reverting process of the exchange rate, moving the latter toward the transaction costs band. Thus, if a fixed-point solution were observed outside the transactions cost band, this would mean that the fundamentalists would fail to move the exchange rate toward the band. Once inside the band, the fundamentalists' dynamics disappears. The only dynamics then comes from the chartists who drive the exchange rate to some attractor within the band. The exact position of this attractor depends on the entry point of the exchange rate in the transactions cost band, and this depends on the initial shock.

The existence of fixed-point solutions depends on the parameters of the model. As mentioned earlier, the chartists' extrapolation parameter β is of crucial importance. For values of β that are above some critical value the trajectories of the exchange rate become chaotic. This critical value depends on the other parameter values of the model. We show this in more detail in the appendix. We find that for a wide range of the other parameters this critical value is located around 2. In figure 5.6 we show the exchange rate returns trajectory in phase space for some sets of parameters.

We can observe that increasing betas lead to a greater complexity of the exchange rate. The intuition is that high extrapolation parameters lead the exchange rate far outside the transaction costs band, forcing the fundamentalists to become active in the market. The mean-reverting

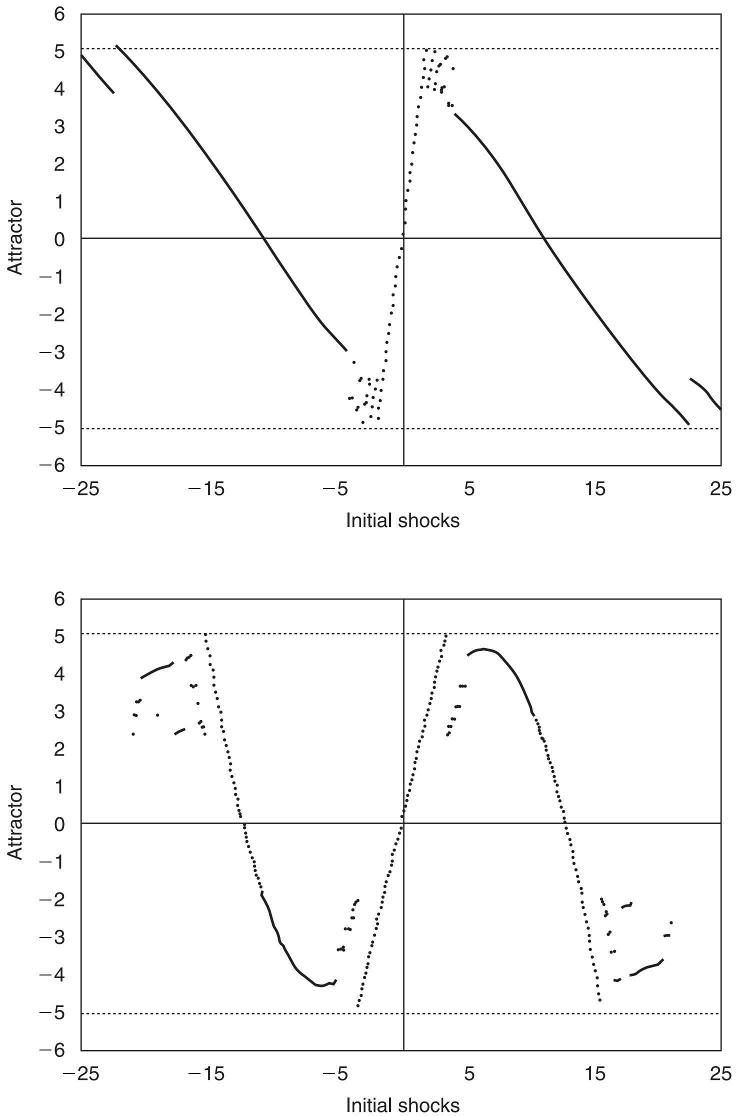

Figure 5.5
Fixed attractors ($c = 5$, beta = 1, theta = 0.05); fixed attractors ($c = 5$, beta = 0.5, theta = 0.2)

process then brings back the exchange rate within the band from which it is pushed away again because of the strong extrapolative behavior of chartists. Thus the exchange rate will be pushed in and out of the transaction costs band, and it never settles within it.

It is also important to analyze the dynamics of the weights of chartists and fundamentalists. We find that when the exchange rate converges toward a fixed-point attractor the weights of chartists and fundamentalists converge to 0.5. This can be explained as follows: When the exchange rates reach a fixed-point solution, chartists and fundamentalists expect no change anymore. They do not buy or sell, and thus make neither profits nor losses. As a result the profit related selection rule of the model (see equations 13 and 14) assures that they will be equally represented in the market.

Things are very different when the exchange rate follows a chaotic pattern. In this case the chartists and fundamentalists weights will show cyclical movements. In general, the chartists' weight will tend to fluctuate around a value larger than 0.5 provided the chartist extrapolation is not too high (i.e., not exceeding values around 3.5). Our interpretation is that the noise produced by chartists' strategies increases the profitability of trading on this noise. However, when the extrapolation parameter becomes too high, it leads to large misalignments. The latter make the fundamentalists forecasting rule profitable and increase the weight of fundamentalists in the market.

Chaotic patterns of the exchange rate are obtained only for relatively high values of beta, which exceed 1. Empirical evidence of the degree of extrapolation by chartists is hard to find. However, common sense suggests that high extrapolative parameters, namely those that are very much larger than one, are unlikely to occur in reality. Thus deterministic chaos is unlikely to be observed. This might explain why it has been difficult to detect deterministic chaos from real life data (Granger 1994; Guillaume 1996; Schittenkopf, Dorffner, and Dockner 2001).

In the next section we restrict the analysis of the model for parameter values that do not lead to deterministic chaos. We will show that in combination with stochastic shocks this model is capable of producing a dynamics that exhibits many of the features of chaotic dynamics despite the fact that the deterministic solutions of the model are fixed points. Thus our model is also in the spirit of Malliaris and Stein (1999) who find that models of deterministic chaos perform poorly in describing the dynamics of price changes in the financial markets. As we will show in section 5.8, our model performs best in mimicking the

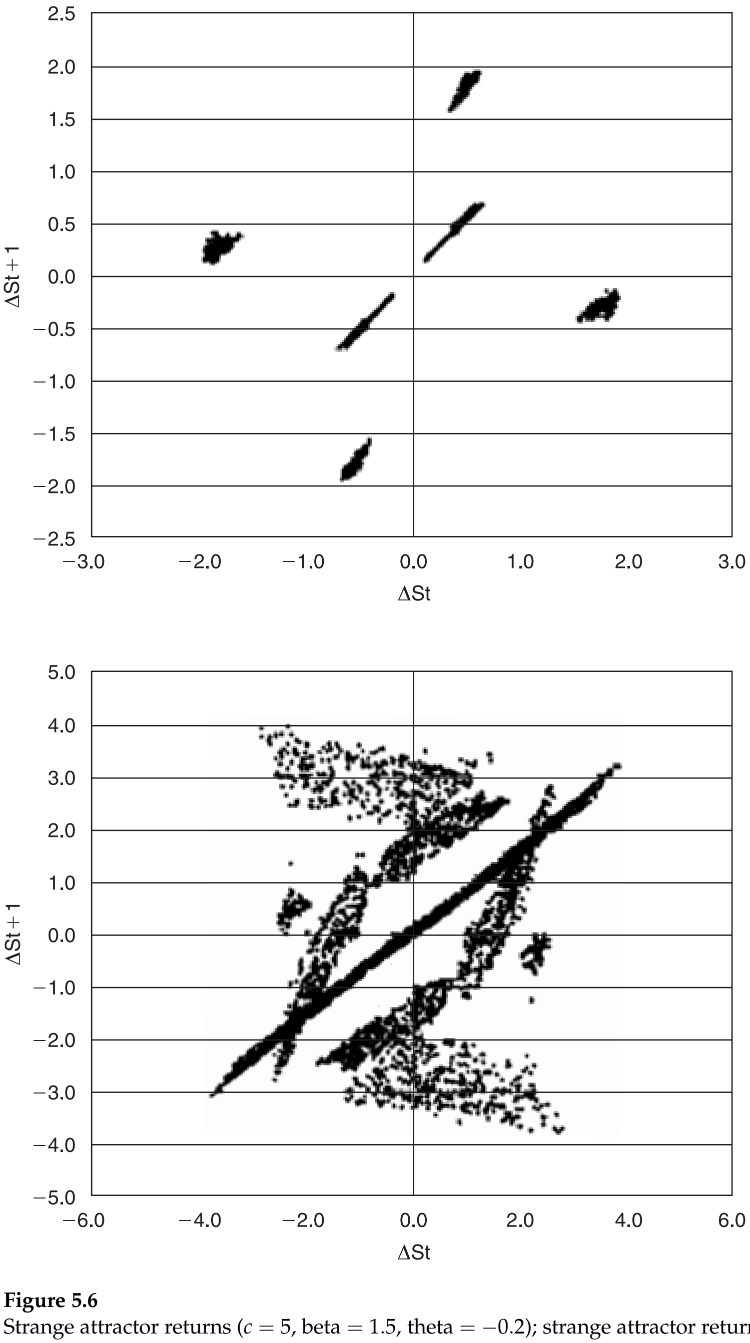

Figure 5.6
Strange attractor returns ($c = 5$, beta = 1.5, theta = −0.2); strange attractor returns ($c = 5$, beta = 2, theta = −0.2); strange attractor returns ($c = 5$, beta = 3, theta = −0.2); strange attractor returns ($c = 5$, beta = 4, theta = −0.2)

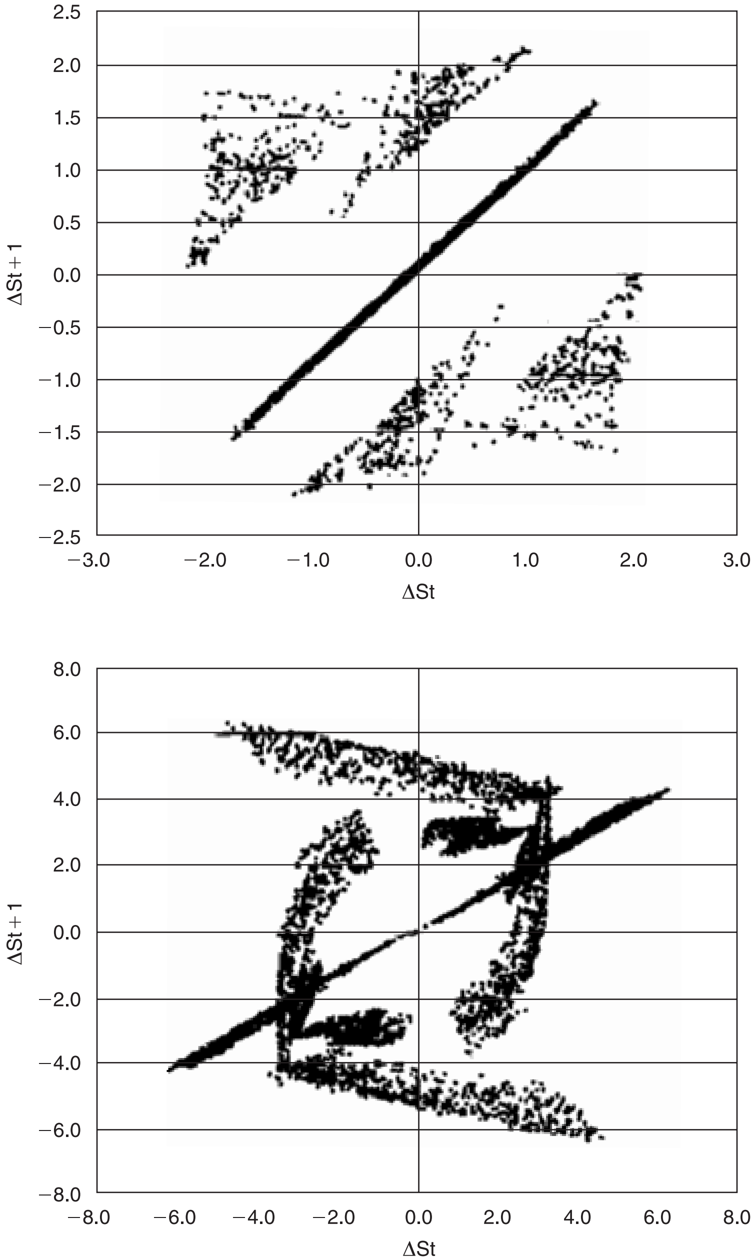

Figure 5.6
(continued)

dynamics of the exchange rates when we calibrate it in such a way that deterministic chaos is excluded.

5.5 The Stochastic Version of the Model

We now introduce stochastic disturbances to the model. In our model these disturbances affect the fundamental, which is assumed to be a random walk. In addition, as can be seen from equation (10), there is exogenous noise leading to forecast errors of chartists and fundamentalists. We simulate the model with a certain combination of parameter values that we refer to as the "standard case." This includes setting $c = 5$, beta $= 1$, theta $= 0.2$, and iota $= 0.001$.

A first feature of the solution of the stochastic version of the model is the sensitivity to initial conditions. To show this, we first simulate the model with the "standard" parameter values, and then we simulate the model with the same parameters setting but with a slightly different initial condition. In both cases we use identical stochastic disturbances. We show the time paths of the (market) exchange rate in figure 5.7.

We observe that after a certain number of periods the two exchange rates start following a different path. This result is related to the presence of many fixed-point attractors in the deterministic part of the

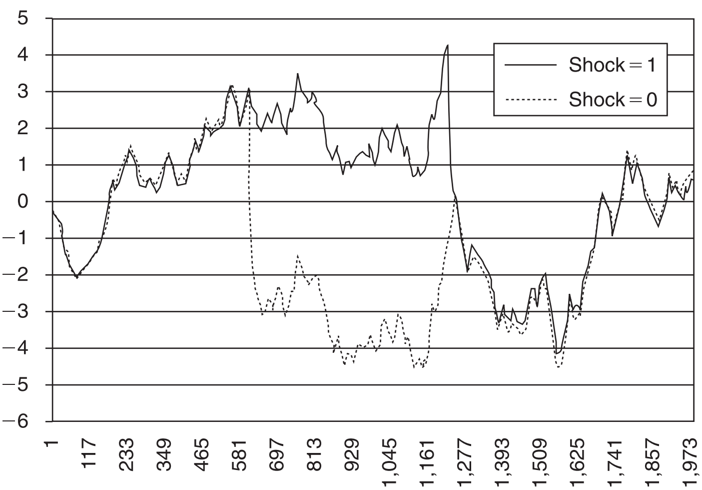

Figure 5.7
Sensitivity to initial conditions ($c = 5$, beta $= 1$, theta $= -0.2$) of different initial shocks

model; these attractors are themselves dependent on the initial conditions (see figure 5.4, which shows how slight differences in initial conditions can lead to fixed-point attractors that are very far apart). As a result the two exchange rates can substantially diverge because attracted by fixed points that are far away from each other. The nice aspect of this result is that we obtain a result that is typical for chaotic systems, however, without chaos being present in the deterministic part of the model. This feature is due to the existence of many fixed points located in different basins of attraction.[7]

5.6 The Effect of Permanent Shocks

In this section we analyze how a permanent shock in the fundamental exchange rate affects the market exchange rate. In linear models a permanent shock in the fundamental has a predictable effect on the exchange rate, namely the coefficient that measures the effect of the shock in the fundamental on the exchange rate converges after some time to a fixed number. Things are very different in our nonlinear model. We illustrate this by showing how a permanent increase in the fundamental is transmitted to the exchange rate. We assumed that the fundamental rate increases by 10, and we computed the effect on the exchange rate by taking the difference between the exchange rate with the shock and the exchange rate without the shock. The simulations of these two exchange rates are done using the same exogenous noise. In a linear model we would find that in the long run the exchange rate increases by exactly 10. This is not the case in our model. We present the evidence in figure 5.8 where we show the effect of a permanent shock of 10 in the fundamental rate on the exchange rate for our standard set of parameter values.

The most striking feature of these results is that the effect of the permanent shock does not converge to a fixed number. In fact it follows a complex pattern. The complexity of this effect is shown in the strange attractor of the effects of the shock (lower panel). Thus in a nonlinear world it is very difficult to predict what the effect will be of a given shock in the fundamental, even in the long run. Such predictions can only be made in a statistical sense; that is, our model tells is that *on average* the effect of a shock of 10 in the fundamental will be to increase the exchange rate by 10. In any given period, however, the effect could deviate substantially from this average prediction.

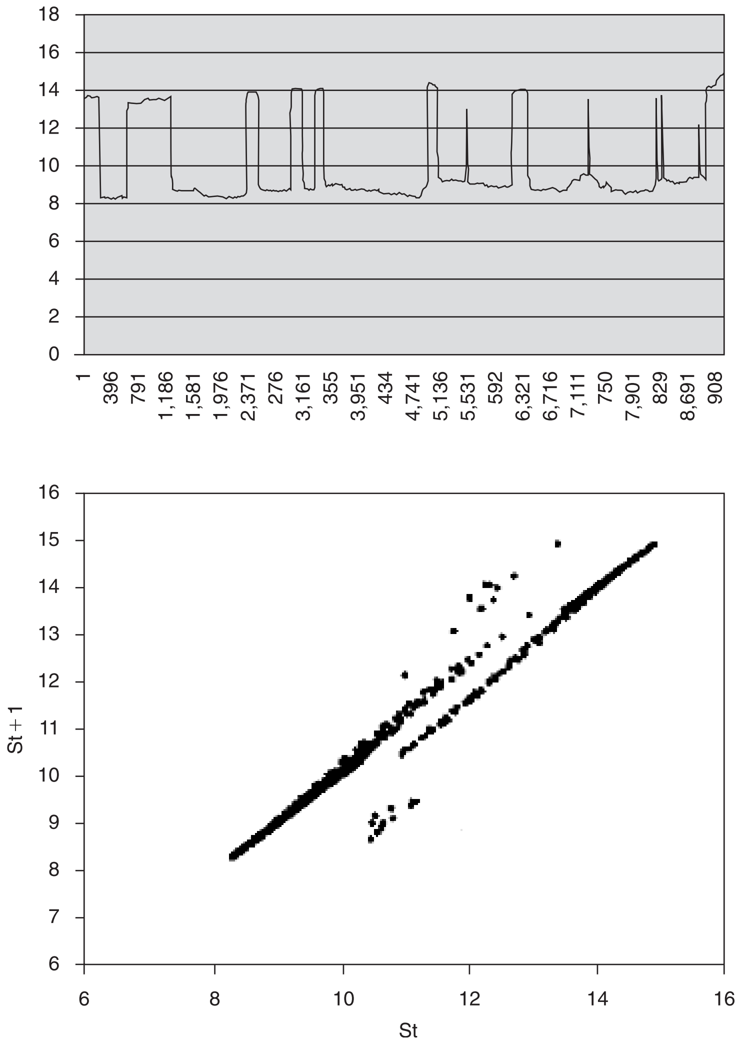

Figure 5.8
Effect of shock in fundamental exchange rate of 10 ($c = 5$, beta = 1, theta = −0.2); strange attractor effect of permanent shock ($c = 5$, beta = 1, theta = −0.2)

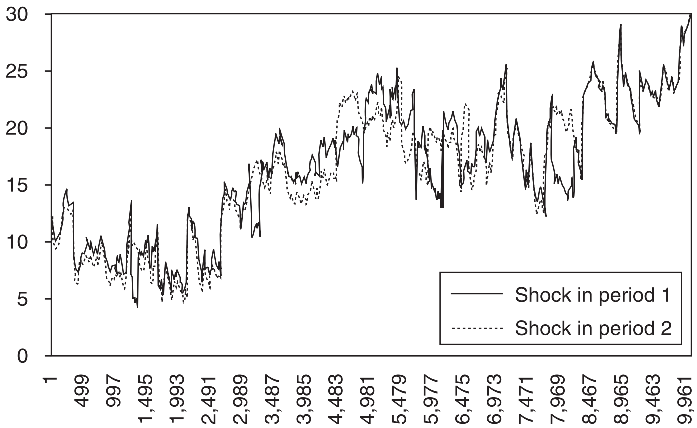

Figure 5.9
Exchange rate after shock in fundamental in two periods ($c = 5$, beta $= 1.1$, theta $= -0.2$)

The importance of the initial conditions for the effect of a permanent shock in the fundamental can also be seen by the following experiment. We simulated the same permanent shock in the fundamental but applied it in two different time periods. In the first simulation we applied the shock in the first period; in the second simulation we applied it in the next period. The exogenous noise was identical in both simulations. Thus the only difference is in the timing of the shock. We show the results in figure 5.9.

We observe that the small difference in timing changes the future history of the exchange rate. As a result the effect of the shock measured at a particular point in time can be very different in both simulations. Thus history matters. The time at which the permanent shock occurs influences the effects of the shock.

Note, however, that in a statistical sense timing does not matter. When we compute the *average* effect of the same shock in the two simulations over a sufficiently long period of time we obtain the same result, meaning the exchange rate increases by 10 on average. The time period needed to make valid statistical inferences clearly is large. We illustrate this by the frequency distributions of the effects of the same shock in the two simulations obtained over two different simulation runs, the first one containing 1,000 periods and the second one 10,000 periods.

The remarkable aspect of figure 5.10 is that when computed over a sample of 1,000 periods, the distribution of the effects is irregular and

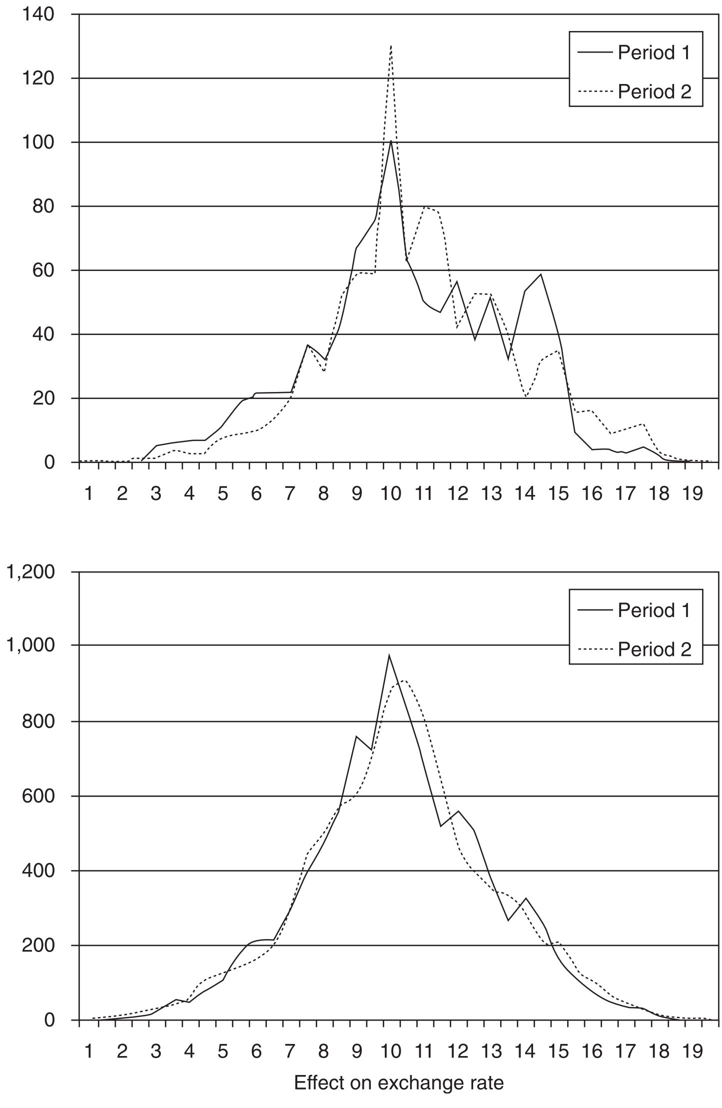

Figure 5.10
Frequency distribution for 1,000 periods of effect of shock in fundamental in two different periods; frequency distribution for 10,000 periods of effect of shock in fundamental in two different periods

quite different for the two simulations. Only when the sample becomes very large (10,000) do we obtain well-behaved distributions permitting statistical inferences about the effect of the same shock.

Our results help to explain why in the real world it appears so difficult to predict the effects of changes in the fundamental exchange rate on the market rate, and why these effects seem to be very different when applied in different periods. In fact this is probably one of the most intriguing empirical problems. Economists usually explain the difficulty of forecasting the effects of a particular change in one exogenous variable (e.g., an expansion of the money stock) by invoking the ceteris paribus hypothesis. By this hypothesis, there are usually other exogenous variables that change unexpectedly and prevent us from isolating the effect of the first exogenous variable. In our model the uncertainty surrounding the effect of a disturbance in an exogenous variable is not due to the failure of the ceteris paribus hypothesis. No other exogenous variable is allowed to change. The fact is that the change in the exogenous variable occurs at a particular time that is different from all other times. Initial conditions (history) matters to forecast the effect of shocks. Since each initial condition is unique, it becomes difficult to forecast the effect of a shock at any given point in time with sufficient precision.

Finally, it should be stressed that the uncertainty about the effect of a permanent shock in the fundamental only holds in a particular environment that is related to a low variance of the noise. In a later section we will analyse how different environments concerning the variance of shocks affect the results.

5.7 Empirical Relevance of the Model

In this section we analyze how well our model mimics the empirical anomalies and puzzles that have been uncovered by the flourishing empirical literature. We start with the "disconnect puzzle." We calibrate the model in such a way that it best replicates the observed statistical properties of the exchange rate movements. This calibration is such that it excludes deterministic chaos. Put differently, we have to choose parameter values that do not lead to deterministic chaos to make the model empirically attractive. A similar result was also obtained by Malliaris and Stein (1999).[8]

5.7.1 The Disconnect Puzzle

The first and foremost empirical puzzle has been called the "disconnect" puzzle (see Obstfeld and Rogoff 2000), by which the exchange rate is described to be disconnected from its underlying fundamentals most of the time. This puzzle was first analyzed by John Williamson (1985), who called it the misalignment problem. The puzzle was also implicit in the celebrated Meese and Rogoff studies of the early 1980s, documenting that there is no stable relationship between exchange rate movements and the *news* in the fundamental variables. Goodhart (1989) and Goodhart and Figlioli (1991) found that most of the changes in the exchange rates occur when there is no observable news in the fundamental economic variables. This finding contradicted the theoretical models (based on the efficient market hypothesis), which imply that the exchange rate can only move when there is news in the fundamentals.

Our model is capable of mimicking this empirical regularity. In figure 5.11 we show the market exchange rate and the fundamental rate for a combination of parameters that does not produce deterministic chaos.

We observe that the market rate can deviate from the fundamental value substantially and in a persistent way. Moreover it appears that

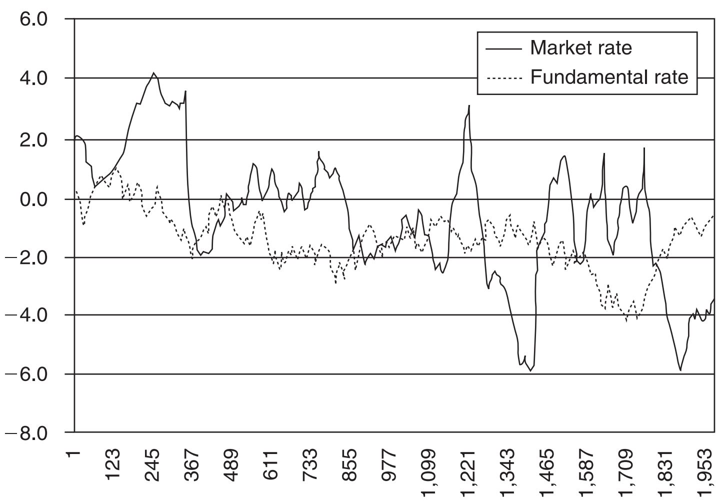

Figure 5.11
Market and fundamental exchange rates ($c = 5$, theta $= -0.2$, beta $= 1.0$)

the exchange rate movements are often disconnected from the movements of the underlying fundamental. In fact they often move in opposite directions.

We show the nature of the disconnect phenomenon in a more precise way by applying a cointegration analysis to the simulated exchange rate and its fundamental using the same parameter values as in figure 5.11 for a sample of 8,000 periods. We found that there is a cointegration relationship between the exchange rate and its fundamental.[9] Note that in our setting there is only one fundamental variable. This implies that no bias from omitted variables can occur.

In the next step we specify a EC model in the following way:

$$\Delta s_t = \mu(s_{t-1} - \gamma s^*_{t-1}) + \sum_{i=1}^{n} \lambda_i \Delta s_{t-i} + \sum_{i=1}^{n} \varphi_i \Delta s^*_{t-i}. \tag{17}$$

The first term on the right-hand side is the error correction term. The result of estimating this equation is presented in table 5.1 where we have set $n = 4$.[10]

We find that the error correction coefficient (μ) is very low. This suggests that the mean reversion toward the equilibrium exchange rate takes a very long time. In particular, only 0.2 percent of the adjustment takes place each period. It should be noted that in the simulations we have assumed a speed of adjustment in the goods market equal to 0.2. This implies that each period the adjustment in the goods market is 20 percent. Thus the nominal exchange rate is considerably slower to adjust toward its equilibrium than what is implied by the speed of adjustment in the goods market. This slow adjustment of the nominal exchange rate is due the chartist extrapolation behavior. We will come back to this issue in more details further below. From table 5.1 we also

Table 5.1
Parameter estimates of EC model (equation 17)

Error correction term		Δs_{t-i}				Δs^*_{t-i}			
μ	γ	λ_1	λ_2	λ_3	λ_4	φ_1	φ_2	φ_3	φ_4
−0.002	0.99	0.24	0.16	0.11	0.07	0.01	0.01	−0.01	0.01
−5.08	*11.83*	*21.93*	*14.11*	*9.09*	*5.71*	*1.53*	*1.17*	*−1.40*	*1.44*

Note: The sample consists of 8,000 periods. The numbers in italics are t-statistics. $R^2 = 0.19$.

note that the changes in fundamentals have a small and insignificant impact on the change in exchange rate. In contrast, the past changes in the exchange rate play a significant role in explaining the change in exchange rate.[11] These results are consistent with the empirical findings using VAR approach, which suggests that the exchange rate is driven by its own past (see De Boeck 2000).

We also performed a cointegration analysis for shorter sample periods (1,000 periods). We find that in some sample periods the exchange rate and its fundamental are cointegrated; in other sample periods we do not find cointegration. This is in line with the empirical evidence indicating that in some periods the exchange rate seems to be disconnected from its fundamental while in other periods it tightly follows the fundamentals.[12]

Thus our model generates an empirical regularity (the "disconnect" puzzle) that has also been observed in reality. We can summarize the features of this puzzle as follows. First, over the very long run the exchange rate and its fundamentals are cointegrated. However, the speed with which the exchange rate reverts to its equilibrium value is very slow. Second, in the short run the exchange rate and its fundamentals are "disconnected," meaning they do not appear to be cointegrated. Our model closely mimics these empirical regularities.

5.7.2 The "Excess Volatility" Puzzle

In this section we discuss another important empirical regularity that has been called the "excess volatility" puzzle. By this it is meant that the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables. Baxter and Stockman (1989) and Flood and Rose (1995) found that while the movements from fixed to flexible exchange rates led to a dramatic increase in the volatility of the exchange rate, no such increase could be detected in the volatility of the underlying economic variables. This contradicted the "news" models that predicted that the volatility of the exchange rate can only increase when the variability of the underlying fundamental variables increases (see Obstfeld and Rogoff 1996 for a recent formulation of this model).

In order to deal with this puzzle, we compute the noise-to-signal ratio in the simulated exchange rate. We derive this noise-to-signal ratio as follows:

$$\mathrm{var}(s) = \mathrm{var}(f) + \mathrm{var}(n), \tag{18}$$

where var(s) is the variance of the simulated exchange rate, var(f) is the variance of the fundamental, and var(n) is the residual variance (noise) produced by the nonlinear speculative dynamics which is uncorrelated with var(f). Rewriting (19), we obtain

$$\frac{\mathrm{var}(n)}{\mathrm{var}(f)} = \frac{\mathrm{var}(s)}{\mathrm{var}(f)} - 1. \tag{19}$$

The ratio var(n)/var(f) can be interpreted as the noise-to-signal ratio. It gives a measure of how large the noise produced by the nonlinear dynamics is with respect to the exogenous volatility of the fundamental exchange rate. We simulate this noise-to-signal ratio for different values of the extrapolation parameter beta. In addition, since this ratio is sensitive to the time interval over which it is computed, we checked how it changes depending on the length of the time interval. In particular, we expect that the noise-to-signal ratio is larger when it is computed on a short- than on a long-time horizon. We show the results in table 5.2.

First, we find that with increasing beta the noise-to-signal ratio increases. This implies that when the chartists increase the degree with which they extrapolate the past exchange rate movements the noise, namely the volatility in the exchange rate, which is unrelated to fundamentals, increases. Thus the signal about the fundamentals that we can extract from the exchange rate becomes more clouded when the chartists extrapolate more. Second, we find that when the time horizon increases, the noise-to-signal ratio declines. This is so because over

Table 5.2
Noise-to-signal ratio for different values of beta

Sample periods beta	Noise-to-signal ratio 100	500	1,000	10,000
0.5	0.69	0.42	−0.13	−0.36
0.6	1.47	0.94	0.17	−0.29
0.7	2.76	2.57	0.77	−0.18
0.8	4.66	4.43	2.40	−0.06
0.9	10.1	6.97	4.42	0.07
1	25.05	9.05	5.47	0.28
1.1	66.99	16.03	7.06	0.30
1.2	111.8	21.30	9.83	0.43

long time horizons most of the volatility of the exchange rate is due to the fundamentals' volatility and very little to the endogenous noise. In contrast, over short-time horizons the endogenous volatility is predominant and the signal that comes from the fundamentals is weak. This is consistent with the empirical finding concerning misalignments we discussed before.

5.7.3 Fat Tails and Excess Kurtosis

It is well known that the exchange rate changes do not follow a normal distribution. Instead, it has been observed that the distribution of exchange rate changes has more density around the mean than the normal and exhibits fatter tails than the normal (see de Vries 2001). This phenomenon was first discovered by Mandelbrot (1963), in commodity markets. Since then, fat tails and excess kurtosis have been discovered in many other asset markets including the exchange market. In particular, in the latter the returns have a kurtosis typically exceeding 3 and a measure of fat tails (Hill index) ranging between 2 and 5 (see Koedijk, Stork, and de Vries 1992). It implies that most of the time the exchange rate movements are relatively small but that occasionally periods of turbulence occur with relatively large exchange rate changes. However, it has also been detected that the kurtosis is reduced under time aggregation. This phenomenon has been observed for most exchange rates. We checked whether this is also the case with the simulated exchange rate changes in our model.

The model was simulated using normally distributed random disturbances (with mean = 0 and standard deviation = 1). We computed the kurtosis and the Hill index of the simulated exchange rate returns. We computed the Hill index for five different samples of 2,000 observations. In addition we considered three different cutoff points of the tails (2.5, 5, and 10 percent). We show the results of the kurtosis in table 5.3 and of the Hill index in table 5.4. We find that for a broad range of parameter values the kurtosis exceeds 3 and the Hill index indicates the presence of fat tails. Finally we check if the kurtosis of our simulated exchange rate returns declines under time aggregation. To do so, we chose different time aggregation periods, and we computed the kurtosis of the time-aggregated exchange rate returns. We found that the kurtosis declines under time aggregation. In table 5.5 we show the results for some sets of parameter values.

Table 5.3
Kurtosis index

	Kurtosis of simulated exchange rate returns			
Beta, theta	0.05	0.1	0.2	0.3
0.5	25.6	119.3	454.7	720.2
1	15.1	53.8	141.6	410.0
1.5	5.3	9.8	14.3	4.5
2	4.6	2.0	2.9	3.3

Table 5.4
Measure of fat tails: Hill index

		Median Hill index (5 samples 2,000 observations)		
Parameter values	Kurtosis	2.5% tail	5% tail	10% tail
$c = 5$, beta = 1, theta = 0.2	141.5	2.63 (2.29–3.49)	3.1 (2.85–4.04)	3.50 (3.06–3.61)
$c = 5$, beta = 0.5, theta = 0.2	454.7	4.14 (3.41–5.14)	4.45 (3.89–4.86)	4.11 (3.77–4.38)
$c = 5$, beta = 1.3, theta = 0.2	95.9	1.61 (1.19–2.61)	1.36 (1.34–2.10)	1.72 (1.28–1.84)
$c = 5$, beta = 1, theta = 0.05	15.13	4.74 (3.36–4.84)	4.91 (3.8–5.11)	4.22 (3.9–4.5)

Table 5.5
Kurtosis under time aggregation

Parameter values		1 period returns	10 period returns	25 period returns	50 period returns
C = 5, theta = −0.2, beta = 1	Skewness	0.98	1.18	0.88	0.64
	Kurtosis	141.6	22.2	11.6	6.4
C = 5, theta = 0.2, beta = 1.2	Skewness	0.71	0.27	0.22	0.1
	Kurtosis	112.5	18.8	10.3	5.9
C = 5, theta = 0.2, beta = 1.3	Skewness	0.63	0.07	0.05	0.21
	Kurtosis	95.9	17.7	10.2	6.00
C = 5, theta = 0.1, beta = 1	Skewness	5.14	1.64	0.92	0.57
	Kurtosis	410.0	54	24.0	11.9
C = 5, theta = 0.3, beta = 0.5	Skewness	10.63	3.57	2.14	1.28
	Kurtosis	720.2	71.6	28.4	14.0

Another empirical regularity of the distribution of the exchange rate returns pertains to the measure of skewness, namely the degree of asymmetry of a distribution around its mean.[13] It has been observed that most exchange rates present a (very) small degree of (positive or negative) skewness (see Lux 1998). Our simulated exchange rate returns also mirror this feature. The results are shown in table 5.5.

5.7.4 *Volatility Clustering*

The last empirical regularity we investigate concerns the clustering of volatility. It has been widely observed that the exchange rate returns show a GARCH structure, that there is time dependency in the volatility of the exchange rate returns (see Kirman 2002; Lux and Marchesi 2000). To check if our model is capable of reproducing this statistical property, we tested if our simulated exchange rate returns have a GARCH structure. We first computed the autocorrelation function of the absolute returns of our simulated exchange rate for different parameter values. In figure 5.12 we show the autocorrelation function for a particular set of parameters. In the appendix we show more autocorrelation functions for different values of the chartists extrapolation parameter. We find that the autocorrelation parameters die out slowly. This implies that volatility in the exchange rate returns has a long memory.

Moreover we performed an ARCH test on the residuals of the simulated exchange rate returns and we rejected the null hypothesis of homoskedasticity. Then we tested for GARCH effects in the exchange rate returns. To do so, we chose the simplest possible GARCH specification, namely GARCH $(1,1)$:

$$\Delta s_t = a + \varepsilon_t,$$

$$\sigma_t^2 = b + \alpha\varepsilon_{t-1}^2 + \delta\sigma_{t-1}^2,$$

where ε_t is the error term, a is a constant, and σ_t^2 is the conditional variance of the returns. We estimated this model using the simulated exchange rate returns. We present the results in table 5.6 for different values of the extrapolation parameter beta. Although the appropriate orders of lags in the GARCH specification could be identified by the Box-Jenkins methodology applied at the squared residuals ε_t^2, we will see that our particularly simple specification fits the empirical evidence rather well.

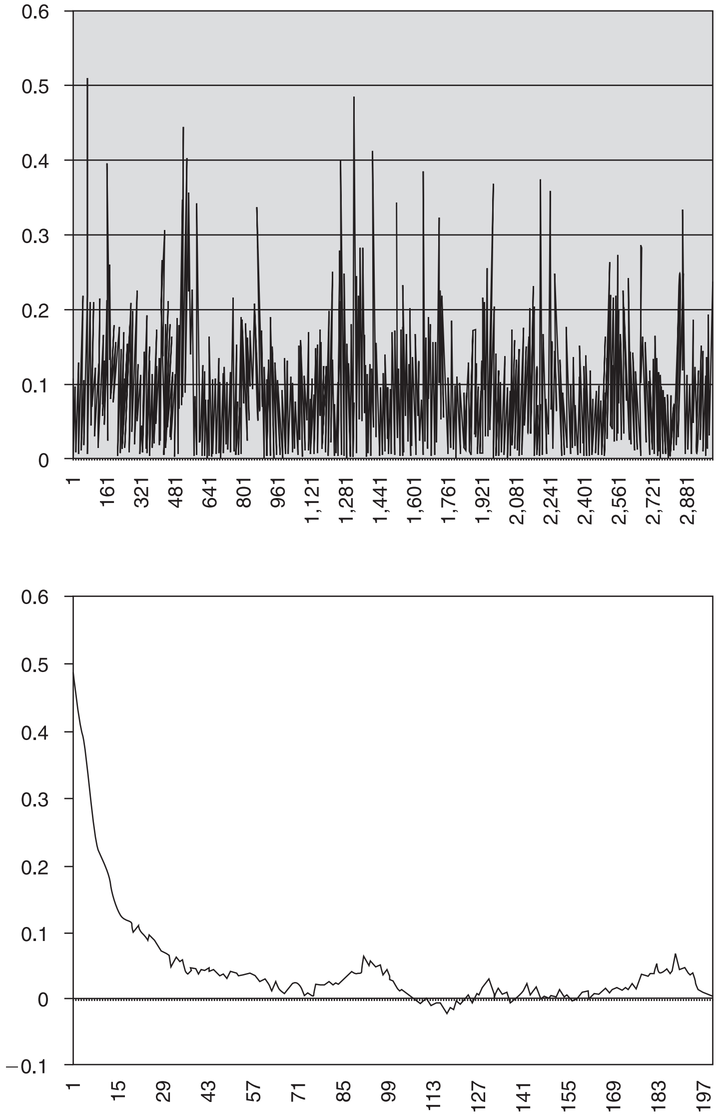

Figure 5.12
Absolute returns ($c = 5$, theta = 0.2, beta = 0.9, iota = 0.001, $n = 0.01$); autocorrelation function ($c = 5$, theta = 0.2, beta = 1, $i = 0.01$, $n_c = 0.001$)

Table 5.6
GARCH model

	Coefficient	z-Statistic
Parameter set: $c = 5$, beta $= 1$, theta $= 0.2$		
a	−0.005	−15.5
	(0.0003)	
b	0.001	24.4
	(0.00004)	
α	0.47	55.9
	(0.008)	
δ	0.50	59.8
	(0.008)	
R^2	−0.004	
Parameter set: $c = 5$, beta $= 1.1$, theta $= 0.2$		
a	−0.001	−3.1
	(0.0003)	
b	0.001	24.5
	(0.00004)	
α	0.56	77.6
	(0.007)	
δ	0.48	85.0
	(0.006)	
R^2	−0.0001	
Parameter set: $c = 5$, beta $= 0.9$, theta $= 0.2$		
a	−0.0004	−0.68
	(0.0006)	
b	0.002	21.0
	(0.00001)	
α	0.28	28.6
	(0.01)	
δ	0.38	13.8
	(0.03)	
R^2	−0.0003	

Note: Numbers in brackets are standard errors.

We observe that the GARCH coefficients, α and δ, are significantly different from zero, implying that there is volatility clustering in the exchange rate returns. In addition we find that for values of beta larger or equal to one the sum of the α and δ, which is a measure of the degree of the inertia of the volatility, is close to one, suggesting that the effect of volatility shocks dies out slowly. Thus our model is capable of reproducing a widely observed phenomenon of clustering and persistence in volatility.

5.8 Low and High Variances of Shocks

In linear models the size of the shocks does not affect the nature of the dynamics. In nonlinear models things are different. The size of the shocks matters. This is also the case in our exchange rate model. To illustrate this, we simulated the model under two different assumptions about the variance of the shocks in the fundamental exchange rate. In the first case we assume low variance of these shocks, and in the second case we assume a high variance (ten times higher). The results of our simulations are presented in figures 5.13 and 5.14. (The simulations shown here are representative for a wide range of parameter values.)

Two conclusions follow from a comparison of the low- and high-variance cases. First, in the low-variance case we observe sustained deviations from the equilibrium exchange rate; this is not the case when the equilibrium exchange rate is subject to large shocks (compare upper panels of figure 5.13 and 5.14). Second, the sensitivity to small changes in initial conditions is clearly visible when the variance of the exchange rate is low (see lower panel of figure 5.13). When this variance is high, no such sensitivity can be observed (lower panel of figure 5.14). It is important to stress that the transactions cost band is the same in both cases. Thus, when the shocks are small relative to the given band of transactions costs, the movements of the exchange rate show more complexity than when the shocks are large.

The previous results are confirmed by a cointegration analysis like the one we performed in section 5.7.1 (see table 5.1) where this analysis refers to a low-variance environment. We show the results for the high-variance regime in table 5.7. These results contrast with those obtained in table 5.1. First, the error correction coefficient is much larger in the

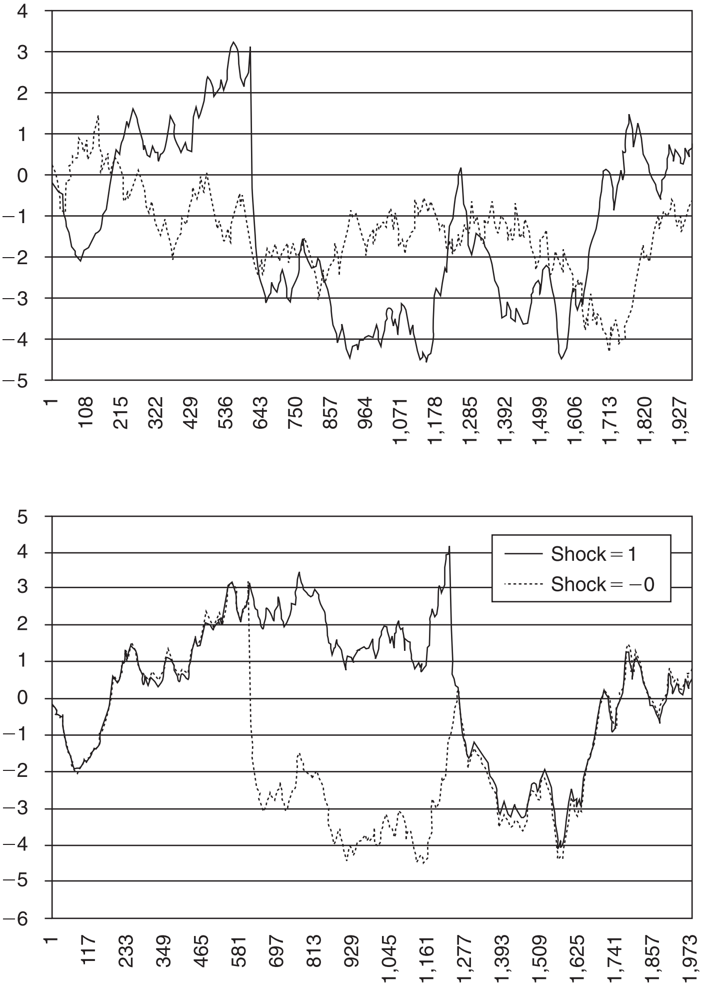

Figure 5.13
Low variance of equilibrium exchange rate; market rate and fundamental rate ($c = 5$, theta = 0.2, beta = 1); sensitivity to initial conditions ($c = 5$, beta = 1, theta = −0.2) of different initial shocks

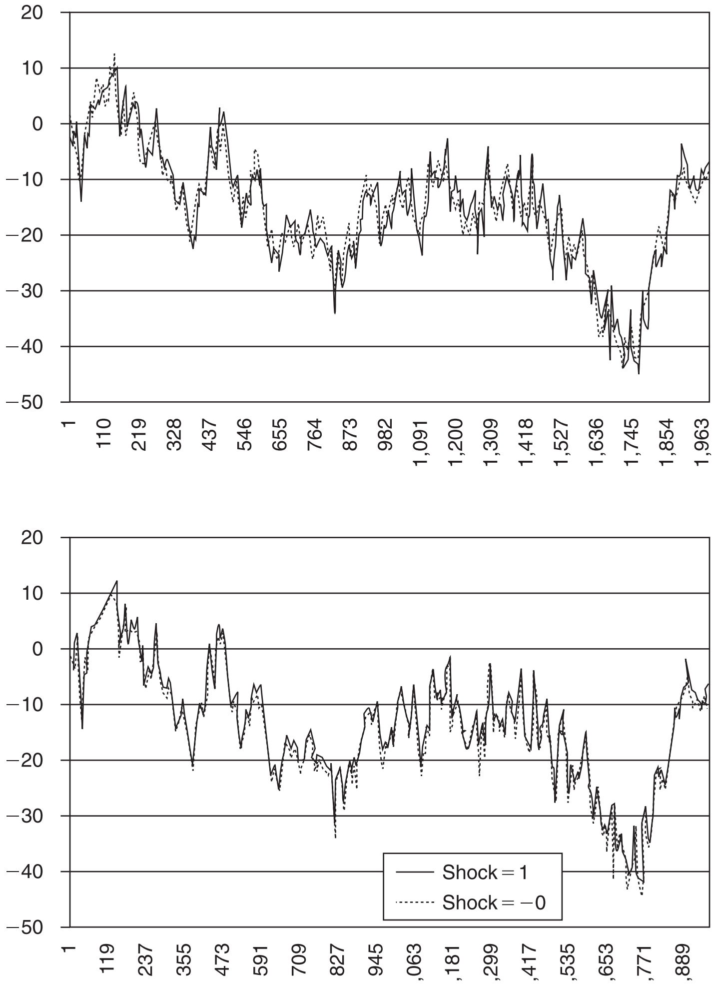

Figure 5.14
High variance of equilibrium exchange rate; market rate and fundamental rate ($c = 5$, theta = 0.2, beta = 1); sensitivity to initial conditions ($c = 5$, beta = 1, theta = −0.2) of different initial shocks

Table 5.7
Parameter estimates EC model with high variance of shocks (equation 17)

Error correction term		Δs_{t-I}				Δs^*_{t-i}			
μ	γ	λ_1	λ_2	λ_3	λ_4	φ_1	φ_2	φ_3	φ_4
−0.098	0.998	0.18	0.15	0.09	0.07	0.01	0.02	−0.004	0.00
−27.13	*566.7*	*16.6*	*13.6*	*8.15*	*6.29*	*9.94*	*1.92*	*−0.42*	*0.05*

Note: The sample consists of 8,000 periods. The numbers in italics are t-statistics. $R^2 = 0.24$.

high-variance regime of table 5.7 than in the low-variance regime of table 5.1. In particular, 10 percent of the adjustment is realized each period in the high-variance regime in contrast with only 0.2 percent in the low-variance case. Second, the impact of past changes in fundamentals is significantly higher in the high-variance than in the low-variance case.

As in the low-variance case we also performed a cointegration analysis over shorter sample periods. The results contrast with the low-variance case. For sample periods of 1,000 we find that exchange rate and its fundamentals are cointegrated, while we do not find cointegration in the low-variance case.

These results confirm what we observed from figures 5.13 and 5.14. That is, in a regime of high variance of shocks the exchange rate is linked much tighter to the fundamentals, and the speed of adjustment toward the equilibrium is significantly higher than in low-variance regimes. The intuition of this result is that when the fundamental shocks are small, the exchange rate regularly switches from the dynamics inherent in the band to the one prevalent outside the band. This nonlinearity produces a lot of noise and complexity in the dynamics of the exchange rate.

When the shocks are large relative to transactions cost band the dynamics outside the band mostly prevails, leading to a tighter link between the exchange rate and the fundamental. This feature has also been found to hold empirically. In particular, it has been found that the PPP-relationship holds much tighter in high-inflation countries than in low-inflation countries (see De Grauwe and Grimaldi 2001). Put differently, in high-inflation countries the link between the exchange rate and one of its most important fundamentals is tighter than in low-inflation countries.

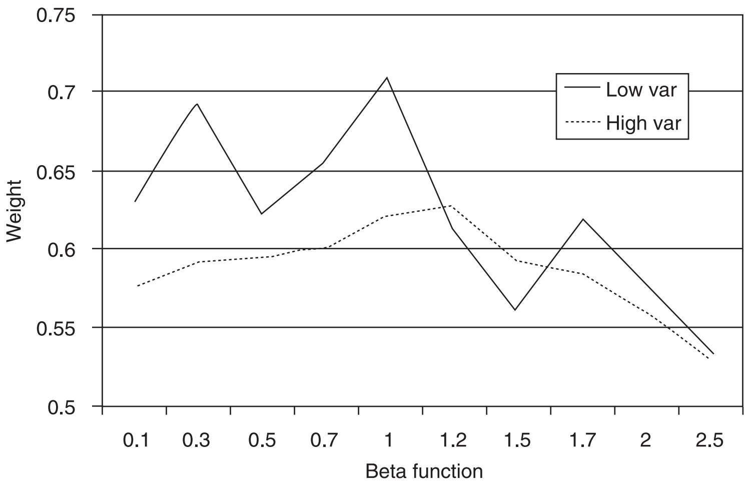

Figure 5.15
Weight of chartism in low- and high-variance environment

5.9 Is Chartism Evolutionary Stable?

An important issue is whether chartism survives in our model. Put differently, we ask the question under which conditions chartism is profitable such that it does not disappear. It should be noted that there is a broad literature that shows that technical analysis is used widely, also by large players (see Wei and Kim 1997).

We investigate this issue by analyzing how chartism evolves under different conditions. In figure 5.15 we show the average chartists weight for increasing values of the extrapolation parameter beta in two different environments concerning the variance of the shocks in fundamentals. We obtained the chartists weights by simulating the model over 10,000 periods and computing the average weight over the last 5,000 periods. Our first finding is that chartism does not disappear. That is, in all simulations for many different parameters configurations we find that the weight attached to chartists never goes to zero. Second, for a wide range of parameter values we find that the chartists weight fluctuates around a market share, which exceeds 50 percent. For high values of beta the chartist weight settles around 50 percent. This result is consistent with the empirical evidence of the importance of chartism in foreign exchange market (Taylor and Allen 1992). And it also suggests that chartism is evolutionary stable.

Finally one could ask the question if fundamentalism disappears in the long run. It is clear that it does not. The fundamentalists' weight complements the chartists' one. Thus, for example, for beta $= 0.6$, the chartists' weight is (approximately) 0.65 (see figure 5.15). This implies a fundamentalists' weight equal to (approximately) 0.35.

It should also be noted that in our model chartists and fundamentalists can both make profits (losses). A third agent, for example, the hedgers, which we do not model, will take the opposite position.

5.10 Conclusion

In this chapter we developed an exchange rate model that has the following features. First, it introduces nonlinearities in the arbitrage dynamics of the goods market. The most important nonlinearity comes from the existence of transaction costs. Second, it allows for heterogeneity of the agents' beliefs. In particular, it is assumed that agents use two different forecasting rules. The first one (*chartists' rule*) is a positive feedback rule that tends to destabilize the market. The second one (*fundamentalists'* rule) is a negative feedback rule, which is stabilizing. This simple model is capable of creating a remarkably complex dynamics.

The model generates a multitude of fixed-point attractors depending on the initial conditions. Put differently, for each initial condition there is a unique solution. By adding exogenous noise the model produces a complex dynamics that resembles a chaotic dynamics, although the deterministic part of the model is not chaotic. This feature has interesting implications. First, there is sensitivity to initial conditions, which implies that a small disturbance can drive the exchange rate on a different path. Second, the effect of a permanent shock in the fundamental exchange rate has a complex structure that might even be chaotic. This implies that the effect of a permanent shock is largely unpredictable, meaning one cannot forecast how the shock will affect the exchange rate in any particular point of time but can predict the *average* effect.

The model can generate a chaotic dynamics of the exchange rate in its deterministic part, but we need a very large degree of extrapolation by chartists to obtain chaotic dynamics. This can explain why the empirical evidence in favour of deterministic chaos in the foreign exchange market is weak. Put differently, in order to have deterministic chaos the extrapolation made by chartists should be quite high. Since there is little evidence for deterministic chaos this suggests that the intensity with which chartists extrapolate past movements of the exchange rate is relatively weak.

The empirical relevance of the model is a measure of its quality. Therefore we analyzed to what extent our model is capable of reproducing the exchange rate puzzles that we observe in reality. The first puzzle we analyzed is the "disconnect puzzle." This puzzle relates to the fact that the exchange rate movements are disconnected, most of the time, from the movements of the underlying fundamental variables. In our model "disconnection" is a natural outcome of the complex dynamics.

The second puzzle relates to the presence of excess volatility of the exchange rate compared to the volatility of its fundamentals. We obtain this feature in our model as a result of the chartists' behavior.

Third, fat tails and excess kurtosis that have been detected in the exchange rate returns are generated by our model. In other words, our model generates a complex dynamics of the exchange rate with *intermittency* of high- and low-turbulence periods.

A fourth empirical regularity concerns the volatility clustering and persistence of exchange rate returns. We found GARCH effects in the simulated exchange rate returns that come close to the observed GARCH effects in the real life exchange rate returns.

Fifth, the empirical evidence suggests that in environments with high variance of the fundamental exchange rate, the link between exchange rate changes and its fundamentals is tighter than in low-variance environments. We also obtain such a result in our model.

Finally, the results of our chapter also shed some light on the relevance of chaotic dynamics in the foreign exchange markets. Our main finding is that empirical puzzles observed in the foreign exchange markets cannot be explained by the existence of deterministic chaos. Instead, models that produce different fixed-point solutions that are located in different basins of attraction come much closer in explaining the empirical evidence when they are combined with noise in the exogenous variables. Such models are also capable of producing complexity in the movements of the exchange rates, such as sensitivity to initial conditions.

Appendix 5.1: Sensitivity Analysis

$c = 5$, $\theta = 0.2$

	ι			
β	0.001	0.01	0.1	1
0.5	F	F	F	F
1	F	F	F	F
1.3	F	F	F	F
1.4	F*	F*	F	F
1.5	F*	F*	F	F
1.7	F*	F*	C	F
2	C	C	F*	F
4	C	C	F*	U
5	C	C	C	F

$c = 5$, $\theta = 0.05$

	ι			
β	0.001	0.01	0.1	1
0.5	F	F	F	F
1	F	F	F	F
1.3	F	F	F	F
1.4	F	F	F	F
1.5	F	F	F	F
1.7	F*	F	F	F
2	F*	F*	F	F
4	C	C	C	U
5	C	C	C	U

Note: β = the extrapolation parameter of chartists, θ = the speed of adjustment in the goods market, and ι = the intensity of the switching rule. F = fixed point; F* = fixed point after period of chaotic dynamics; C = chaotic dynamics; U = unstable.

Appendix 5.2: Autocorrelation Functions of Simulated Absolute Returns for Different Values of the Extrapolation Parameter Beta

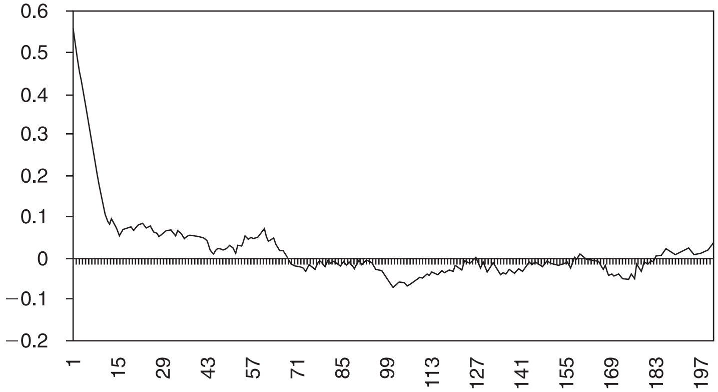

Figure 5.16
Autocorrelation function ($c = 5$, theta $= 0.2$, beta $= 0.9$, $i = 0.001$, $n_c = 0.01$)

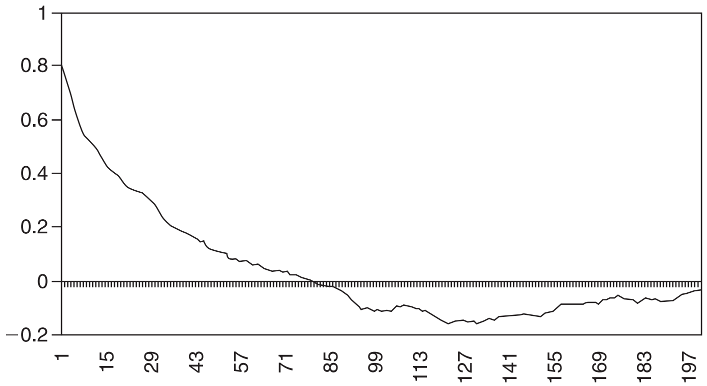

Figure 5.17
Autocorrelation function ($c = 5$, theta $= 0.05$, beta $= 1.1$, $i = 0.001$, $n_c = 0.001$)

Notes

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1. This means that we assume implicitly that there is no feedback from the exchange rate to the goods market. We leave it for further research to explore this possibility.

2. See Brock and Hommes (1998) for such a formulation.

3. This way of modeling the foreign exchange market was first proposed by Frankel and Froot (1988). It was further extended by De Long et al. (1990) and De Grauwe et al. (1993), and more recently by Kilian and Taylor (2001). For evidence about the use of chartism, see Taylor and Allen (1992).

4. Note that this is also the approach taken in the Dornbusch model.

5. See Kilian and Taylor (2001). See also De Grauwe and Grimaldi (2001) in which we showed that a quadratic specification fits the data rather well.

6. See De Long et al. (1990).

7. This phenomenon has been called *global stability of an adjustment process*. An adjustment process is called globally stable if, for any set of initial conditions, there is a rest point to which the system converges. Different initial conditions lead to different rest points. In other words, there are as many stable point to which the system converges as initial conditions.

8. See also Granger (1994) on this issue.

9. We first performed a unit root test on the simulated exchange rate. We could not reject the existence of a unit root. Next, we tested for cointegration using the Johansen cointegration procedure (see Johansen 1991). We assumed that there is no deterministic trend in the data. However, we do allow the intercept to be different from zero.

10. The number of lags has been chosen according to the information criteria, such that the error term is white noise.

11. It should be noted that our results are akin to what was found in stock markets, which is that in the short-run the exchange rate underreacts to news while it overreacts in the long run. See Schleiffer (2000).

12. See Obsteld and Rogoff (2000). See also De Grauwe (2000) for a survey of the empirical evidence. In De Grauwe and Vansteenkiste (2001) we present additional empirical evidence.

13. Positive skewness indicates a distribution with an asymmetric tail extending toward more positive values. Negative skewness indicates a distribution with an asymmetric tail extending toward more negative values.

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6 Dynamics of Endogenous Business Cycles and Exchange Rate Volatility

Volker Böhm and Tomoo Kikuchi

International data show that exchange rates exhibit higher volatility than price ratios for countries with floating currencies and open capital markets. This seems to indicate that short-run exchange rate volatility would have to be explained under some price rigidities. Moreover real exchange rates exhibit much less volatility under a fixed exchange rate regime than under a floating regime. This implies that the variability in real exchange rates is not entirely attributable to real shocks as postulated in real business cycle models but is related to nominal rigidities or disturbances.

Given these empirical observations it seems apparent that theoretical models should attempt to identify those forces (e.g., monetary shocks or others) that interact with sticky prices *and* that generate the observed volatility in real exchange rates. Some models assuming nominal rigidities within Keynesian open economy models were developed during the 1960s and 1970s (e.g., Mundell 1968; Dornbusch 1976). Dornbusch focused on the effects of slow price level adjustments in response to movements in money supply. He showed that a permanent monetary expansion causes the nominal exchange rate to depreciate so much that it overshoots its long-run value. During the adjustment process, commodity prices increase and the exchange rate is appreciated until the purchasing power parity (PPP) holds in the long-run equilibrium. A particular feature of this model is the saddle-path solution. The overshooting result hinges on the assumption that the nominal exchange rate adjusts directly to the stable manifold of a new system after an unexpected monetary expansion. Furthermore the model's lack of micro foundations deprives it from any welfare consideration by which to evaluate alternative macroeconomic policies. Since the late 1970s most of the theoretical models incorporated explicit micro foundations for the private sector's consumption, investment, and production decisions. These models developed implications of dynamic optimization by the private sector while assuming away the observed stickiness of prices.

A new wave of research as documented in the survey by Lane (2001) on new open economy macroeconomics offers a variety of approaches that incorporate price rigidities and market imperfections into a dynamic general equilibrium model with well-specified micro foundations. In contrast to earlier ones with perfect competition, these models with monopolists derive rather than assume incomplete price adjustment. In the literature, such as Svensson and Wijnbergen (1989) and Obstfeld and Rogoff (1995), firms are assumed to set their prices one period in advance. However, the economy is predicted to adjust to its new long-run equilibrium in a single period in which the law of one price holds and the real exchange rate is constant. This has the counterfactual implication that the price level jumps discretely. Contributions, such as Chari, Kehoe, and McGrattan (2002), Betts and Devereux (2000), and Kollmann (2001), permit smooth price-level adjustment by allowing firms to adjust their price stochastically. As a consequence only a fixed fraction of firms adjusts its price each period so that the price level changes only gradually over time. The firms can also price discriminate across countries, so there are deviations from the law of one price. Under their circumstances the pricing to market can generate a quantitatively significant increase in exchange rate volatility. Kollmann (2001) and Chari, Kehoe, and McGrattan (2002) examine quantitative dynamic general equilibrium models which produce exchange rate volatility higher than in standard real business cycle models with flexible prices and wages. These recent contributions attempt to address two of the so called "pricing puzzles" (see Obstfeld and Rogoff 2001). The first is the PPP puzzle Rogoff (1996), which highlights the weak connection between exchange rates and national price levels. The second one is the so-called exchange rate disconnect puzzle (Rogoff 1998), which alludes to the high volatility of exchange rates apparently disconnected from fundamental macroeconomic variables. However, these models cannot explain persistent fluctuations of any variables without exogenous monetary shocks nor can they identify sources for the difference of the volatility of domestic macroeconomic variables versus exchange rates.

This chapter differs from these recent developments in the new open economy macroeconomics in choosing a competitive but non-Walrasian approach rather than introducing imperfect competition as a source for price stickiness. The presence of non-Walrasian prices in a dynamic context essentially means that prices respond imperfectly to market situations implying sluggish price adjustments. At the same time trading mechanisms on all markets yield economic realizations of all relevant variables at all times, meaning feasible trades at non-Walrasian prices need to be defined. Within the context of perfect competition such trading rules correspond precisely to those developed by the so called fixed price literature, such as Barro and Grossman (1971), Benassy (1975), and Malinvaud (1977). Here these *temporary fixed-price* situations are integrated into a *dynamic flex price* setting by amending the allocation rules with appropriate price adjustment rules.

The chapter presents an extension of the closed economy model of Böhm (1993) and Böhm, Lohmann, and Lorenz (1997) to a small open economy framework in which the expectation formation mechanism in the foreign exchange market is derived explicitly under the uncovered interest rate parity (UIP). This expectation formation works as a channel transmitting domestic business cycles into exchange rate fluctuations, and vice versa. The chapter offers a general model with explicit dynamics in which endogenous business cycles of a small domestic economy and permanent exchange rate fluctuations interact without exogenous shocks. In section 6.1, microeconomic assumptions of economic agents are introduced and their behavioral consequences derived. In section 6.2, we describe temporary feasible allocations. In section 6.3, we analyze the dynamics of the model, and in section 6.4 present the simulation results.

6.1 Behavioral Assumptions

The model describes a simple dynamic prototype economy with markets for labor, output, and bonds with sluggish price and wage adjustment (e.g., in Böhm, Lohmann, and Lorez 1997)[1] extended to the case of a small open economy (see also Neary 1990). There exists a stationary set of economic agents of the domestic economy consisting of young and old consumers, a single producer, a government, and a central bank.

6.1.1 Domestic Consumers

The private domestic consumption sector consists of overlapping generations of consumers with two-period lives. For simplicity, it is assumed that each generation consists of a single consumer only. Each young consumer supplies labor $L_{max} > 0$ inelastically in the first period of his life and receives wage income $w_t L_t$ for his current employment L_t and all profits Π_t from the producer. A proportional income tax is paid at the rate $0 \leq tax \leq 1$, after which his net nominal income is equal to

$$Y_t^{net} = (1 - tax)(w_t L_t + \Pi_t) = (1 - tax) y_t p_t.$$

The intertemporal preferences of each young consumer are described by a two-period utility function that satisfies the following property:

Assumption 1 The utility function $u : \mathbb{R}_+^2 \to \mathbb{R}$ is C^2, strictly quasi-concave, strictly monotonically increasing, and homothetic.

For the subsequent numerical analysis the intertemporal CES utility function of the form

$$u(x_t, x_{t+1}) = \begin{cases} \dfrac{1}{\rho}(x_t^\rho + \delta x_{t+1}^\rho) & \text{if } \rho \neq 0, \\ \ln x_t + \delta \ln x_{t+1} & \text{if } \rho = 0, \end{cases} \tag{1}$$

will be used, with $\delta > 0$ as the time discount factor and $-\infty < \rho < 1$ as the parameter of substitution.

Consumption in the second period of each consumer's life is financed through savings in the form of holding retradable government bonds. Let $r_{t,t+1}^e$ denote the consumer's expected nominal rate of return for $t + 1$ on bonds at time t and $p_{t,t+1}^e$ the expected commodity price for $t + 1$ at time t. Then the consumption/savings decision of a young consumer is given by the *notional* commodity demand

$$x_t^{dy} = \arg\max_x \left\{ u\left(x, \frac{(Y_t^{net} - p_t x)(1 + r_{t,t+1}^e)}{p_{t,t+1}^e} \right) \right\}. \tag{2}$$

Given the homotheticity of the utility function, *notional* commodity demand can be written as

$$x_t^{dy} = c(R_{t,t+1}^e) \frac{Y_t^{net}}{p_t}, \tag{3}$$

and notional bond demand is

$$B_t^d = (1 - c(R_{t,t+1}^e)) \frac{Y_t^{net}}{s_t} \tag{4}$$

with $\theta^e_{t,t+1} := (p^e_{t,t+1}/p_t) - 1$ as the expected rate of inflation, $R^e_{t,t+1} := (1 + r^e_{t,t+1})/(1 + \theta^e_{t,t+1})$ as the expected real return on real savings, and s_t as the nominal bond price.

For the CES utility function the propensity to consume out of real net income $0 < c(R^e_{t,t+1}) < 1$ is given by

$$c(R^e_{t,t+1}) = \begin{cases} \dfrac{1}{1 + \delta^{1/(1-\rho)}(R^e_{t,t+1})^{\rho/(1-\rho)}} & \text{if } \rho \neq 0, \\ \dfrac{1}{1+\delta} & \text{if } \rho = 0, \end{cases} \tag{5}$$

confirming the fact that for $\rho = 0$ (i.e., for the standard Cobb-Douglas utility function) the propensity to consume is independent of the expected real rate of return.

A typical old consumer in period t with bond holdings B_t receives a nominal interest payment $d \geq 0$ per bond and sells his holdings at the nominal price s_t. He spends his total nominal revenue for consumption, implying a *notional* consumption demand of

$$x^{do}_t = \frac{B_t}{p_t}(s_t + d). \tag{6}$$

6.1.2 *Domestic Production*

The production sector consists of a single infinitely lived firm that produces a homogeneous commodity y using domestic labor L as the only instantaneous input without possibility for inventory.

Assumption 2 The production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is C^2, strictly monotonically increasing, strictly concave, and satisfies the Inada conditions.

For the numerical analysis the isoelastic production function

$$y = f(L) = \frac{A}{B} L^B \tag{7}$$

will be used, with $A > 0$ as a scaling parameter and $0 < B < 1$ as the elasticity of production.[2] Maximizing profits in any period t at given price p_t and wage w_t yields as *notional* demand for labor and as *notional* commodity supply

$$
\begin{aligned}
L_t^* &= L^*\left(\frac{w_t}{p_t}\right) := \arg\max_L (p_t f(L) - w_t L), \\
y_t^* &= y^*\left(\frac{w_t}{p_t}\right) := f\left(L^*\left(\frac{w_t}{p_t}\right)\right).
\end{aligned}
\tag{8}
$$

For the isoelastic production function one obtains

$$
\begin{aligned}
L_t^* &= L^*\left(\frac{w_t}{p_t}\right) = \left(\frac{p_t A}{w_t}\right)^{1/(1-B)}, \\
y_t^* &= y^*\left(\frac{w_t}{p_t}\right) = \frac{A}{B}\left(\frac{p_t A}{w_t}\right)^{B/(1-B)}.
\end{aligned}
\tag{9}
$$

6.1.3 The Government and Foreign Demand

The government has two ways of financing its demand $g > 0$. First, it levies a proportional tax at the rate $0 \leq tax \leq 1$ on the young household's income (wages and profits). Second, the government conducts open market operations to issue new bonds or collect old ones at the rate Δ in every period. Since existing bonds are held and sold by old consumers, total bond supply in period t is equal to

$$B_t^s = B_t(1 + \Delta), \qquad -1 < \Delta < \infty. \tag{10}$$

The government services its bonds by paying a nominal amount $d > 0$ per bond each period to the asset holder. For the discussion of volatility issues of the exchange rate in this chapter, Δ and d, as well as g and *tax* will be treated as exogenous parameters. Issues of budget balance for the government are neglected.

To complete the description of the real part of the economy, it is assumed that the international real link of the economy is given by an exogenously given nominal demand E in foreign currency units in every period t inducing a nominal demand equal to $E \cdot X_t$ in domestic units if X_t is the exchange rate.

6.2 Temporary Feasible States

One of the distinctive features of the model is the assumption of an instantaneous adjustment of the bond price to clear the bond market in contrast to sluggish price adjustment in the real markets. Thus the domestic nominal wage w and the commodity price p are assumed to be given at the beginning of the period, and they remain unchanged during the period. As a consequence transactions take place through rationing when prices and wages are not at their Walrasian values. Then a possible imbalance of demand and supply gives rise to domestic price adjustments in the real markets at the end of the period. In contrast, domestic as well as foreign asset markets are assumed to clear instantaneously. This assumption induces a particular sequential structure of the determination of the foreign exchange rate as well as of the domestic bond price. If one uses, as will be done here, the concept of the UIP as an equilibrium device for the foreign exchange market combined with perfect foresight (see section 6.3.2 below), the sequential structure of the determination of expectations, of nominal prices, and of nominal exchange rates implies that the nominal exchange rate is also given at the beginning of the period. Thus, all prices except the one for domestic bonds is given and unchanged during one period.

6.2.1 Effective Aggregate Demand and the Domestic Bond Price

Aggregate demand y for domestic production in any period t must satisfy

$$y = c(R^e_{t,t+1})(1 - tax)y + \frac{B_t s_t}{p_t} + \frac{B_t d}{p_t} + \frac{EX_t}{p_t} + g. \tag{11}$$

Solving (11) for y one obtains the *effective* aggregate demand function

$$y^{eff}(s) := \frac{g + (s + d)B_t/p_t + EX_t/p_t}{1 - c(R^e_{t,t+1})(1 - tax)}, \tag{12}$$

depending among other things on the bond price, where (B_t, X_t, p_t) and $R^e_{t,t+1}$ are given at the beginning of the period. If no disequilibrium is allowed in the bond market, the actual equilibrium bond price in any period must be determined simultaneously with actual real income. Therefore equality of *notional* demand for bonds (4) by young consumers with bond supply (10) implies the bond market-clearing condition

$$p_t y(1 - tax) - p_t y(1 - tax)c(R^e_{t,t+1}) = sB_t(1 + \Delta). \tag{13}$$

As a consequence the solution (s^d, y^d) of (12) and (13) simultaneously determines *effective* aggregate demand y^d and a corresponding demand consistent bond price s^d as

$$y^d = \frac{1+\Delta}{tax + \Delta[1 - c(R^e_{t,t+1})(1-tax)]}\left(\frac{B_t d}{p_t} + \frac{EX_t}{p_t} + g\right), \tag{14}$$

$$s^d = \frac{(1 - c(R^e_{t,t+1}))(1-tax)}{B_t(tax + \Delta[1 - c(R^e_{t,t+1})(1-tax)])/p_t}\left(\frac{B_t d}{p_t} + \frac{EX_t}{p_t} + g\right). \tag{15}$$

Notice that the two solutions are homogeneous of degree zero in (B_t, X_t, p_t). Therefore let $b_t := B_t/p_t$ denote real bonds and $x_t := X_t/p_t$ denote the real exchange rate. Then, for given parameters (g, tax, d, Δ) equations (14) and (15) define *effective* aggregate demand and the associated demand consistent bond price

$$y^d = \mathscr{D}(b, x, R^e) := \frac{(1+\Delta)(bd + xE + g)}{tax + \Delta(1 - c(R^e)(1-tax))}, \tag{16}$$

$$s^d = \mathscr{S}^d(b, x, R^e) := \frac{(1 - c(R^e))(1-tax)(bd + xE + g)}{b(tax + \Delta(1 - c(R^e)(1-tax))}, \tag{17}$$

as functions of real bonds, the real exchange rate, and the expected rate of return for any period t. The time subscript has been eliminated since all arguments refer to the same (current) time period.

6.2.2 *Feasible Allocations and Bond Market Equilibrium*

Let $v := (b, \alpha, x, R^e)$ denote the temporary state vector given at the beginning of an arbitrary period where $\alpha := w/p$ denotes the real wage. To obtain feasible transactions in the commodity market and in the labor market the minimum rule is employed for the rationing mechanism. Actual output y is the minimum of aggregate commodity demand y^d, notional supply of goods y^*, and of capacity output $y_{max} = f(L_{max})$. Notional supply is a function of the real wage alone. Therefore feasible output y and employment L are functions of v given by

$$y = \mathscr{Y}(v) := \min\{\mathscr{D}(v), y^*(\alpha), y_{max}\}, \tag{18}$$

$$L = \mathscr{L}(v) := f^{-1}(\mathscr{Y}(v)). \tag{19}$$

The trading rules (18) and (19) induce three typical disequilibrium states for the domestic economy known as *classical unemployment*, *Keynesian unemployment*, and *repressed inflation*. For each situation the equilibrium bond price has to be determined, since spillovers from the rationing scheme have an impact on the demand for bonds.

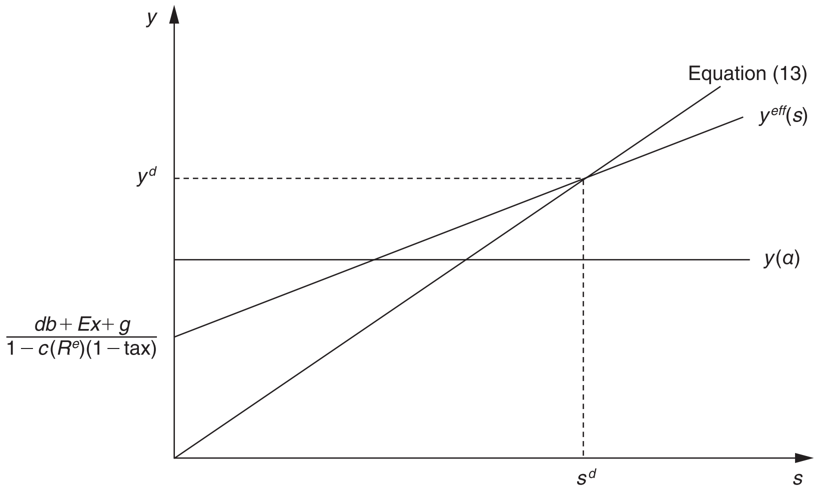


Figure 6.1
Excess demand in the commodity market

Let $y(\alpha) := \min\{f(L_{max}), y^*(\alpha)\}$ denote aggregate effective supply. If $y = y^d < y(\alpha)$, meaning if there is no excess demand in the commodity market, the effective demand and the associated demand consistent bond price $s_t = s_t^d$ obtains. This state in which the supply side of each market is rationed is referred to as the *Keynesian unemployment regime* **K**. The other two cases imply demand rationing on the commodity market either with unemployment (called the *classical unemployment regime* **C**) or with demand rationing on the labor market (called the *repressed inflation regime* **I**).

Let $y = y(\alpha) < y^d$, a situation depicted in figure 6.1. Since there is excess demand in the commodity market, several agents will be affected by potential rationing. As in Böhm, Lohmann, and Lorenz (1997) we assume that young consumers are rationed first before old consumers, the government, and before the foreign demand. Such a rule implies that consumption by young consumers will be given by

$$c^{young} := \max\{0, y(\alpha) - b(s+d) - g - Ex\}. \tag{20}$$

This yields *effective real savings* for bonds by young consumers as a function of the bond price s:

$$\begin{aligned} S^{eff}(b, x, y(\alpha), s) &:= y(\alpha)(1 - tax) - \max\{0, y(\alpha) - b(s+d) - g - Ex\} \\ &= \min\{y(\alpha)(1 - tax), b(s+d) + g + Ex - tax\, y(\alpha)\}, \end{aligned}$$

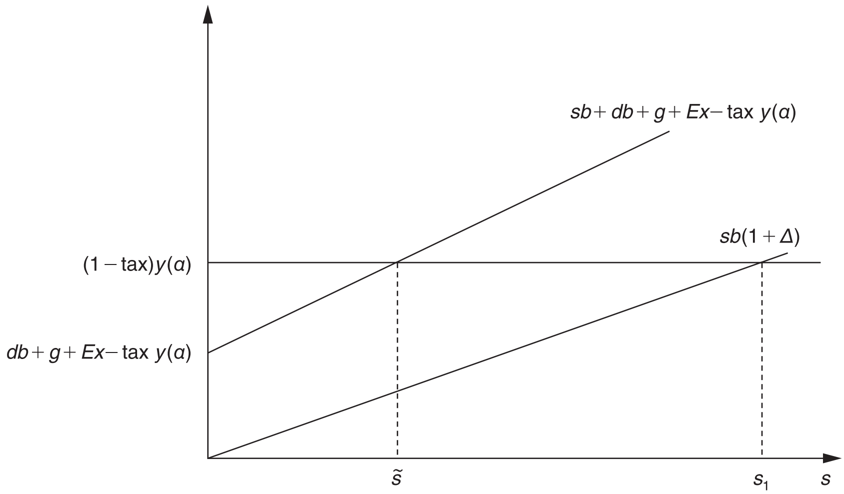


Figure 6.2
Bond market equilibrium for $\Delta \leq 0$

implying the modified bond market-clearing condition

$$S^{\mathit{eff}}(b, x, y(\alpha), s) = sb(1 + \Delta). \tag{21}$$

There are two possible situations, young consumers being rationed completely or only partially. The equilibrium bond price under demand rationing is

$$S^{\mathit{eff}} := \begin{cases} \min\left\{ \dfrac{y(\alpha)(1 - tax)}{b(1+\Delta)}, \dfrac{g + bd + Ex - tax\ y(\alpha)}{b\Delta} \right\}, & y^d > y(\alpha), \\ & \Delta > 0, \\ \dfrac{y(\alpha)(1 - tax)}{b(1+\Delta)}, & y^d > y(\alpha), \\ & \Delta \leq 0. \end{cases} \tag{22}$$

Figures 6.2 and 6.3 provide a geometric description of the determination of the bond price under demand rationing. At $\tilde{s}$, young consumers are completely rationed. For $\Delta > 0$, two situations are possible, one with and without complete rationing of young consumers. At s_1, young consumers are completely rationed and invest their entire net income in the bond market. At s_2, they are only partially rationed and invest their remaining income after consumption. The equilibrium bond price in all possible cases is determined by the function

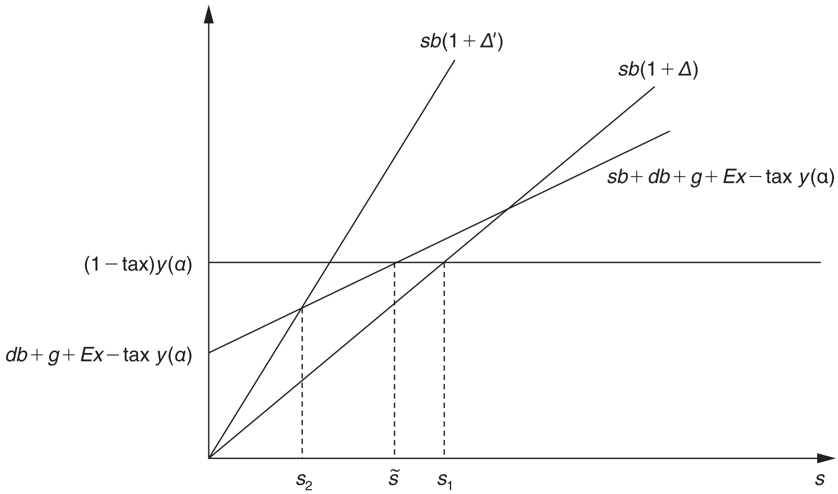


Figure 6.3
Bond market equilibrium for $\Delta > 0$

$$s = \mathscr{S}(v) := \begin{cases} \dfrac{(1-c(R^e))(1-tax)(bd+xE+g)}{b(tax+\Delta(1-c(R^e)(1-tax)))}, & y^d \leq y(\alpha), \\ \min\left\{\dfrac{y(\alpha)(1-tax)}{b(1+\Delta)}, \dfrac{g+bd+Ex-tax\,y(\alpha)}{b\Delta}\right\}, & y^d > y(\alpha), \\ & \Delta > 0, \\ \dfrac{y(\alpha)(1-tax)}{b(1+\Delta)}, & y^d > y(\alpha), \\ & \Delta \leq 0. \end{cases} \tag{23}$$

Together with the results from the determination of output and employment one obtains the following lemma:

Lemma 1 Given the parameters $(g, tax, d, \Delta, L_{max}, E)$, any temporary state vector $v := (\alpha, b, x, R^e) \gg 0$ induces a unique positive temporary feasible allocation (y, L) given by equations (18) and (19) and a positive market clearing bond price by equation (23), if $\Delta > -1$. The functions $\mathscr{Y}, \mathscr{L}, \mathscr{S}$ are continuous and piecewise differentiable functions of the state vector v.

Figure 6.4 depicts the partition of the state space into the regions of the three regimes. Its characteristics can be derived from equations (18), (19), and (23) directly. The area of the possible Keynesian unemployment regime (marked **K**) is defined by all values $(b, \alpha, x) \in \mathbb{R}^3_+$

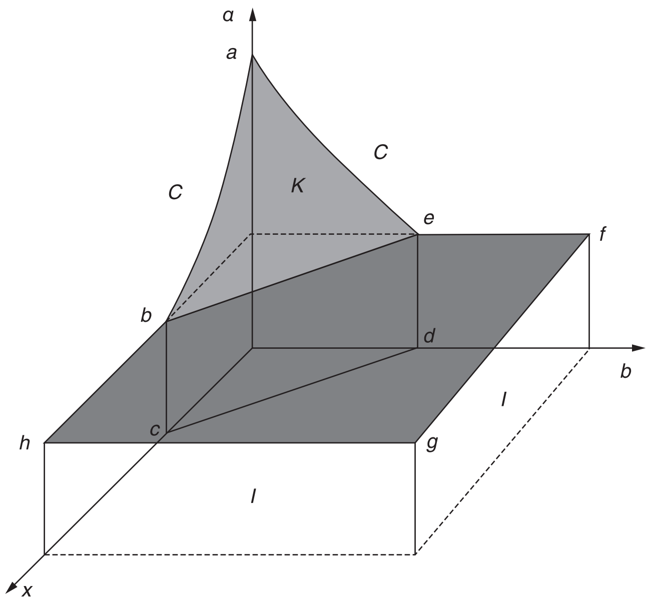


Figure 6.4
Partition of state space into three regimes

behind the plane *bcde* and below the surface *abe*. Above the surface *abe* and above the plane *befgh* lie all states of the classical unemployment regime **C**, while below the plane *befgh* are all repressed inflation states of the regime **I**.

6.3 Dynamics and Expectations Formation

Existence and uniqueness of temporary feasible states provide the basis for a well defined forward recursive structure defining the dynamic development of the economy over time. This requires the description of the dynamical evolution of all state variables of a dynamical system in the mathematical sense. The dynamic equations for the real wage and real bonds are derived first. The dynamics of the exchange rate and the expectations processes are discussed afterward.

6.3.1 *Adjustment of Prices and Wages*

Any temporary state vector $v = (b, \alpha, x, R^e)$ uniquely determines the state of the economy for given parameters which in most cases is not the Walrasian equilibrium. This means that quantity constraints occur on the labor and/or on the commodity market which lead to price and/or wage adjustment at the end of the period according to the size of rationing. The adjustments are assumed to follow the so-called law of supply and demand. This means that if supply exceeds demand in a market, its price goes down, and vice versa. One possible formulation of this principle uses the definition of disequilibrium signals for the labor market $s^l \in [-1, 1]$ and for the goods market $s^c \in [-1, 1]$, measuring the sign and the size of rationing. Their dependence on the temporary state vector $v = (b, \alpha, x, R^e)$ is described by two functions σ^l and σ^c:

$$\sigma^c : \mathbb{R}^4_{++} \to [-1, +1] : s^c = \sigma^c(v),$$

$$\sigma^l : \mathbb{R}^4_{++} \to [-1, +1] : s^l = \sigma^l(v).$$

The signs of the signals correspond to the signs of the respective excess demand functions. On the basis of any pair of disequilibrium signals (s^c_t, s^l_t) in period t, a price adjustment function P and a wage adjustment function W are defined to obtain

$$P : [-1, 1] \to (-1, +\infty), \quad \frac{p_{t+1}}{p_t} = 1 + P(s^c_t), \tag{24}$$

$$W : [-1, 1] \to (-1, +\infty), \quad \frac{w_{t+1}}{w_t} = 1 + W(s^l_t). \tag{25}$$

P and W are continuous, strictly monotonically increasing, and satisfy $W(0) = P(0) = 0$. Together with the signaling function they induce two mappings for price $\mathscr{P} := P \circ \sigma^c$ and wage $\mathscr{W} := W \circ \sigma^l$ adjustment:

$$p_{t+1} = p_t[1 + P(\sigma^c(v_t))] = p_t(1 + \mathscr{P}(v_t)), \tag{26}$$

$$w_{t+1} = w_t[1 + W(\sigma^l(v_t))] = w_t(1 + \mathscr{W}(v_t)). \tag{27}$$

If one uses a linear adjustment rule, too much instability of interior steady states with frequent divergence to the boundary is created. The following nonlinear functional form of price and wage adjustment will be used in the numerical simulations below.

$$\mathscr{P}(v) = \begin{cases} \gamma \tanh\left(\dfrac{y(\alpha)^{\mathit{eff}} - y}{y(\alpha)^{\mathit{eff}}}\right) & \text{if } y^d > y \\ \kappa \tanh\left(\dfrac{y - y^*}{y^*}\right) & \text{otherwise} \end{cases} \tag{28}$$

with $0 < \gamma < 1$ and $0 < \kappa < 1$ as adjustment speeds and $y(\alpha)^{eff}$ as the modified effective aggregate demand. This is defined as

$$y(\alpha)^{eff} := y(\alpha)(1 - tax)c(R^e) + b(s + d) + g + Ex, \tag{29}$$

satisfying $y^{eff}(\alpha) > y(\alpha)$. Given the disequilibrium signals the tangent hyperbolic function delivers a symmetric price and wage adjustment downward and upward with a maximum derivative equal to one. If **K** or **K** $\cap$ **I** holds, excess supply in the commodity market gives a downward pressure for the price. If $y^d = y^* = y$, meaning if **C** $\cap$ **K** holds, the commodity market is in equilibrium and there is no price adjustment. In all other cases, meaning if **I**, **C** or **C** $\cap$ **I** holds, there is excess demand in the commodity market and an upward pressure for the price. Apart from the situation when $y^d = y^{max} < y^*$ the signal functions $\sigma^l(v)$ and $\sigma^c(v)$ are continuous.[3]

Applying the same principle to the labor market, one obtains

$$\mathscr{W}(v) = \begin{cases} \lambda \tanh\left(\dfrac{L - L_{max}}{L_{max}}\right) & \text{if } L_{max} > L, \\ \mu \tanh\left(\dfrac{L^* - L}{L^*}\right) & \text{otherwise,} \end{cases} \tag{30}$$

with $0 < \lambda < 1$ and $0 < \mu < 1$ as adjustment speeds. If **I** or **K** $\cap$ **I** holds, excess demand in the labor market gives an upward pressure for the wage rate. If $L_{max} = L^* = L$; that is, if **C** $\cap$ **I** holds, the labor market is in equilibrium and there is no wage adjustment. In all other cases (**C**, **K** or **C** $\cap$ **K**) there is excess supply in the labor market and a downward pressure for the price.

Together the two adjustment functions imply the dynamic equation for the real wage

$$\alpha_{t+1} = \alpha_t \frac{1 + \mathscr{W}(v_t)}{1 + \mathscr{P}(v_t)}. \tag{31}$$

The assumptions concerning bond market equilibrium imply that total final bond holdings by young consumers in period t are equal to $B_{t+1} = B_t(1 + \Delta)$. Therefore the dynamics of real bonds are given by

$$b_{t+1} = b_t \frac{1 + \Delta}{1 + \mathscr{P}(v_t)}. \tag{32}$$

6.3.2 *Uncovered Interest Parity and Expectations Formation*

One of the interpretations of the uncovered interest parity (UIP) is that of a condition of expected no arbitrage to hold in perfect international capital markets. When applying the UIP to a dynamic model, it is important to take proper account of the sequential structure of the available information and of expectations formation. Let r^f denote the nominal rate of return for holding foreign assets. Under the UIP expected returns on domestic and foreign capital markets are assumed to be the same. When purchasing foreign bonds in t, the amount of domestic investment has to be converted at the spot exchange rate X_t into the foreign currency. One period later the principle and the interest have to be reconverted into domestic currency at the future spot exchange rate X_{t+1}. Thus, under expected no arbitrage, the expected returns denominated in either currency have to be the same, implying the following form of the UIP,

$$1 + r^e_{t,t+1} = (1 + r^f)\frac{X^e_{t,t+1}}{X_t}. \tag{33}$$

Rearranging terms one obtains an equation determining the nominal exchange rate

$$X_t = X^e_{t,t+1}\frac{1 + r^f}{1 + r^e_{t,t+1}} \tag{34}$$

as a function of expectations formed prior to the realization of the current exchange rate. Thus the sequential structure of the expectations formation implicit in the condition of the UIP reveals that the dynamic equation determining the actual exchange rate is a function of expectations alone independent of the previous actual exchange rate.

Moreover, when forming expectations for $t+1$ agents can use observable information only up to $t-1$, implying a so called "expectational lead" for the functional relationship.[4] Figure 6.5 shows the timing of expectations and of exchange rate determination under UIP. In most models imposing the UIP, it is assumed that agents have perfect foresight with respect to the exchange rate, a property that can be guaranteed here by deriving an explicit perfect forecasting rule. Considering the timing of expectations formation, the perfect foresight property implies that the difference between the forecast $X^e_{t-1,t}$ made in

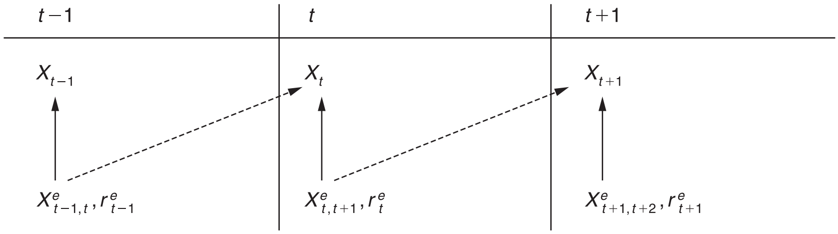


Figure 6.5
Timing of expectations and exchange rates under UIP

$t-1$ and the actual value X_t must be equal to zero. Put differently, one must have

$$X^e_{t,t+1}\frac{1+r^f}{1+r^e_{t,t+1}} - X^e_{t-1,t} = 0.$$

Solving for $X^e_{t,t+1}$ yields the unique explicit functional form of the perfect predictor ψ_* for the exchange rate:

$$X^e_{t,t+1} = \psi_*(X^e_{t-1,t}, r^e_{t,t+1}) := \frac{1+r^e_{t,t+1}}{1+r^f} X^e_{t-1,t}. \tag{35}$$

Therefore the assumption of perfect foresight together with UIP requires that the prediction of the exchange rate is a function of previous predictions and not of previous exchange rates. In other words, the prediction of the exchange rate today guarantees that the prediction of yesterday will be correct.[5]

As for the exchange rate, one could ask whether it is possible to derive a perfect predictor as well for the expectations formation for the domestic rate of return r^e as well as for the inflation rate θ^e. In principle, this might be possible using the techniques from Böhm and Wenzelburger (2004). However, at this stage an explicit solution for a perfect predictor cannot be calculated due to the nonlinearities of the equations and to the dependence on the state variables involved. Therefore an adaptive (but not perfect) forecasting rule will be used for the domestic rate of return and for the inflation rate. Observe, however, that the resulting dynamics will, in general, depend on the choice of the adaptive scheme.

Let s_{t-1} denote the purchase price for bonds, s_t the selling price, and d the dividend payment. Then the rate of return on domestic bonds effective in period t is given by

$$r_t = \frac{d + s_t}{s_{t-1}} - 1. \tag{36}$$

The forecasting rule for the rate of return is assumed to follow the simple adaptive principle $r^e_{t,t+1} = r_{t-1}$, inducing a predictor ψ_r of the form

$$r^e_{t,t+1} = \psi_r(s_{t-1}, s_{t-2}) := \frac{d + s_{t-1}}{s_{t-2}} - 1. \tag{37}$$

Notice that two special features are present in any adaptive scheme using past data. First, resulting from the sequential structure, the prediction $r^e_{t,t+1}$ has to be made prior to the realization of the bond price s_t, implying that the value for r_t is not available as information. Second, the definition of r_{t-1} implies an additional delay of order two with respect to bond prices, thus increasing the dimensionality of the dynamical system.

Similarly assume that the adaptive scheme for prices defines the expected inflation rate $\theta^e_{t,t+1} := p^e_{t,t+1}/p_t - 1$ for period $t+1$ as a function of the last $\tau \geq 1$ inflation rates:

$$\theta^e_{t,t+1} = \Psi(\theta_t, \ldots, \theta_{t-\tau-1}) \qquad \text{with } \theta_{t-k+1} := \frac{p_{t-k+1}}{p_{t-k}} - 1, \quad k = 1, \ldots, \tau. \tag{38}$$

The function $\Psi : (-1, \infty)^\tau \to (-1, \infty)$ is assumed to be continuous satisfying the following property:

$$\Psi(\theta, \theta, \ldots, \theta) = \theta \qquad \forall \theta > -1. \tag{39}$$

The class of such functions includes most of the commonly used adaptive prediction mechanisms with finite memory.

6.3.3 Dynamical System

It is apparent that the evolution of the economic model will be governed by an interaction of the adjustment equations with the expectations formation rules inducing two strong expectations feedbacks. These are decisive in the stability and in the long-run behavior of the economy. Combining the dynamic equations for the domestic economy with the appropriate mappings for the expectations processes ψ_*, ψ_r, and Ψ for the expected inflation rate, the expected interest rate, and for the exchange rate

$$
\begin{aligned}
X^e_{t,t+1} &= \psi_*(X^e_{t-1,t}, r^e_{t,t+1}), \\
r^e_{t,t+1} &= \psi_r(s_{t-1}, s_{t-2}), \\
\theta^e_{t,t+1} &= \Psi(\theta_t, \ldots, \theta_{t-\tau-1}), \\
R^e_{t,t+1} &:= \frac{1 + r^e_{t,t+1}}{1 + \theta^e_{t,t+1}},
\end{aligned}
\tag{40}
$$

one obtains as the vector of state variables

$$\underline{v_t} = (b_t, \alpha_t, x^e_{t-1,t}, s_{t-1}, s_{t-2}, \theta_t, \ldots, \theta_{t-\tau-1})$$

where $x^e_{t-1,t} = X^e_{t-1,t}/p_t$. Then the dynamical system is defined by the following mapping:

$$
\begin{aligned}
b_{t+1} &= \mathscr{B}(\underline{v}_t) := \frac{b_t(1+\Delta)}{1+\mathscr{P}(\underline{v}_t)}, \\
\alpha_{t+1} &= \mathscr{A}(\underline{v}_t) := \alpha_t \cdot \frac{1+\mathscr{W}(\underline{v}_t)}{1+\mathscr{P}(\underline{v}_t)}, \\
x^e_{t,t+1} &= \Phi(\underline{v}_t) := \frac{x^e_{t-1,t}}{1+\mathscr{P}(\underline{v}_t)} \frac{1+r^e_{t,t+1}}{1+r^f}, \\
s_t &= \mathscr{S}(\underline{v}_t), \\
\theta_{t+1} &= \Upsilon(\underline{v}_t) := \mathscr{P}(\underline{v}_t).
\end{aligned}
\tag{41}
$$

The vector of past bond prices and past inflation rates $(s_{t-1}, s_{t-2}, \theta_t, \ldots, \theta_{t-\tau-1})$ is just shifted by one time step. Therefore the dynamic behavior of the economy is described by a sequence $\{b_t, \alpha_t, x^e_{t-1,t}, s_{t-1}, s_{t-2}, \theta_t, \ldots, \theta_{t-\tau-1}\}_{t_0}^T$ implying that the state space of the dynamical system equal is a subset of $\mathbb{R}^{5+\tau}$. The system exhibits a highly nonlinear structure. This arises not only from the many nonlinear functional relationships but also from the regime switching that occurs induced by the temporary state variables. In the case of a Cobb-Douglas utility function, which implies a constant marginal propensity to consume with no domestic expectations feedback, the dynamical system has dimension five with a one-period delay in the bond price. Figure 6.6 illustrates the sequential structure when there is no expectations feedback on domestic consumption, where the solid arrows identify the individual mappings of (41). Figure 6.7 illustrates the time one map of the dynamical system (41). The vertical arrows indicate that the expectation of inflation rates, of interest rates, and of exchange rates at time t

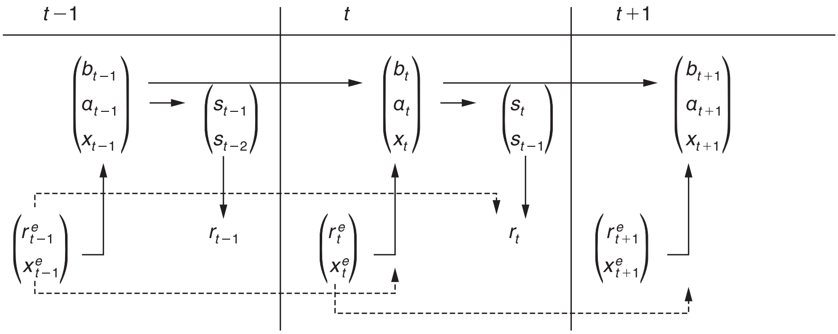


Figure 6.6
Sequential structure of prices, exchange rates, and expectations under UIP

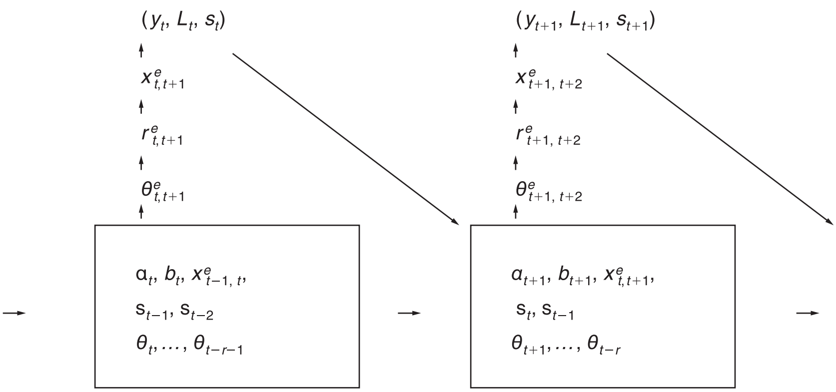


Figure 6.7
Structure of time one map

for period $t + 1$ are formed on the basis of past realizations of the economic variables. On the other hand, the allocation and the corresponding rationing situation at t depend on these forecasts. It is obvious that the system possesses a lag structure: the bond price, the inflation rate, and the expectation of the exchange rate influence the actual market process at least over two periods.

6.3.4 Stationary States

Due to the high dimensionality of the dynamical system (41), it is apparent that a full analytic characterization of its stationary states and their stability properties may be beyond reach. Fortunately, many features for the associated model of a closed economy are well understood (see Böhm, Lohmann, and Lorenz 1997 and Kaas 1995) and some of them carry over directly to the small open economy case, as those stated in the next lemma. Note that all forecasting rules of the model imply perfect foresight in steady states.

Lemma 2 Given a feasible list of the parameters $(g, tax, \Delta, r^f, L_{max}, E)$ of the system (41), let $(b, \alpha, \theta, s, x) \gg 0$ denote a stationary state with perfect foresight. Then:

1. $-1 < \theta < 0$ if and only if $\Delta < 0$, and the state is of the Keynesian unemployment type **K**,
2. $\theta = 0$ if and only if $\Delta = 0$, and the state is of the Walrasian type **W**,
3. $\theta > 0$ if and only if $\Delta > 0$ and the state is of the repressed inflation type **I**.

In other words, the sign of the policy parameter Δ determines uniquely the type of an interior long-run disequilibrium state independent of the specific functional forms and the mechanisms. This is one of the fundamental insights into the structural features of this class of models. As a consequence this implies among other things that the local stability properties of any stationary state are regime specific.[6]

6.4 Numerical Analysis

For the numerical analysis it is necessary to use specific functional forms for the intertemporal preferences and for the technology as the ones introduced above. Those have proved to generate tractable results for the closed economy model. Therefore, for the remainder of this chapter, consider the economy with intertemporal preferences of the CES type (1) with $\rho = 0$, isoelastic production (7), and with hyperbolic price and wage adjustments of the form (28) and (30). The assumption on preferences eliminates the expectations feedback on domestic consumption implying a constant propensity to consume and no role for the inflation predictor Ψ. Therefore the dimension of the dynamical system is five with the state variables $(b, \alpha, s_{-2}, s_{-1}, x^e)$. Even for this special case a full derivation of the eigenvalues when $\Delta \neq 0$ has not yet been obtained. The following partial results and numerical simulations are designed to demonstrate that

1. there is wide (robust) confirmation of excess volatility of exchange rates compared to domestic real variables in the nonperiodic as well as in the periodic case,

2. the closed economy exhibits period doubling bifurcations for some large sets of parameters while for other ranges of parameters fold bifurcations (saddle-point properties) seem to be the cause of the non-periodic fluctuations,

3. there is a general overall loss of stability of the domestic economy after introducing foreign demand.

6.4.1 Bifurcation Analysis

Table 6.1 provides a complete list of all parameters used. For this analysis the numerical investigations were restricted to the relationship

Table 6.1
Standard parameter set

Parameter	Description/origin	Value
γ	Adjustment speed p_t	0.2
κ	Adjustment speed p_t	0.2
λ	Adjustment speed w_t	0.2
μ	Adjustment speed w_t	0.2
g	Government demand	0.3
tax	Income tax rate	0.3
Δ	New issues of bonds	0.05
d	Nominal interest	0.01
A	Scaling parameter	1
b	**Elasticity of production**	**Different values**
L_{max}	Constant labor supply	1
δ	Time discount factor	1
ρ	Parameter of substitution	0
τ	Expectational lag	10
r^f	Foreign rate of return	0.01
E	Foreign demand for goods	0.1
b_0	Initial real bond	0.6
α_0	Initial real wage	0.6
s_0	Initial bond price	0.75
x_0^e	Initial real expected exchange rate	0.5

between the production elasticity and the adjustment speeds, since this unveils already some new and important qualitative features. For all time series results the same values were used for the adjustment speeds in analyzing the influence of four different values of the labor elasticity.

Openness and Loss of Stability

It is known from the closed economy analogue with money only that the occurrence of endogenous business cycles and bifurcations in such non-Walrasian models is caused by a combination of the adjustment speeds of prices and wages and the labor demand elasticity given by $1/(B-1)$ (see Kaas 1995). This elasticity becomes large for values of the production elasticity B close to one. The same phenomenon occurs here as well, but with some substantial differences to the closed economy. When $E > 0$, unstable steady states and non periodic behavior predominate for the open economy where the closed economy would exhibit stable steady states for the same set of parameter values. This is caused primarily by the perfect predictor in conjunction with the UIP assumption. Taken by itself first-order effects of the exchange rate tend to induce a derivative equal to plus one near the steady state equivalent to a saddle type property. If second-order effects are positive there will be at least one root larger than one. This is precisely the reason why in the original model by Dornbusch (1976) the steady state is a saddle under the UIP hypothesis since there secondary effects are ignored.

Proposition 1 There exists a large critical set of parameter values $(g, tax, \Delta, r^f, L_{max})$ for which the stationary states of the system (41) undergo a period doubling bifurcation when $E = 0$. When $E > 0$, there exists a large open set of parameter values such that the stationary states are locally unstable and the system displays finite as well as complex cycles.

Figure 6.8 shows a distinctive destabilizing effect of international trade under UIP. While for the closed economy figure 6.8a the typical period doubling bifurcation and endogenous cycles occur only for very high values of B, figure 6.8b shows a large range of low B for which the open economy exhibits endogenous cycles. For high values the bifurcation scenarios appear to be very similar.

Figure 6.9 provides evidence of the robustness of the bifurcation scenario of figure 6.8 in the form of a so-called cyclogram over B and

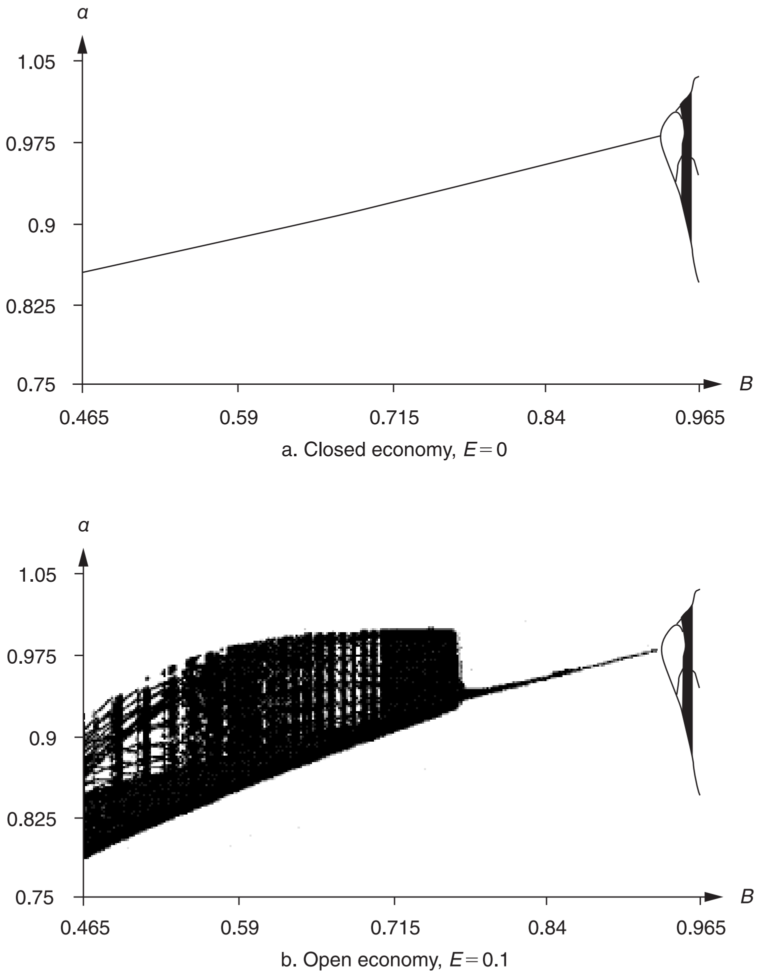


Figure 6.8
Bifurcation diagram for α

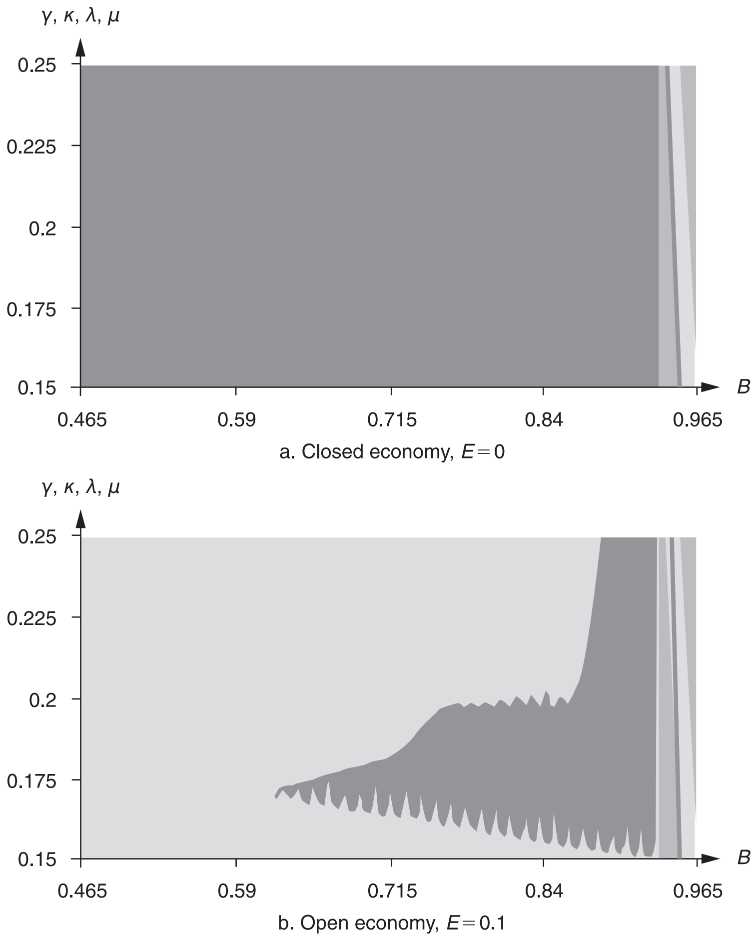


Figure 6.9
Cyclogram for α

adjustment speeds $\gamma = \kappa = \lambda = \mu$ simultaneously. A cyclogram is a qualitative multidimensional bifurcation diagram. For each pair of parameters $(B, \gamma = \kappa = \lambda = \mu)$ the respective color assignment indicates the order of the cycle of the limiting behavior of the system. According to the codes given in figure 6.10 the color "yellow" indicates nonperiodic limiting behavior. One easily verifies the features of the bifurcation diagram figure 6.8 by traversing horizontally at the value $\gamma = \kappa = \lambda = \mu = 0.2$ in figure 6.9. Figure 6.8b and figure 6.9b show that nonperiodic behavior predominates when $E > 0$. Most important,

Order 1	Order 2	Order 3	Order 4	Order 5	Order 6	Order 7	Order 8
Order 9	Order 10	Order 11	Order 12	Order 13	Order 14	Order 15	Order 16
Order 17	Order 18	Order 19	Order 20	Order 21	Order 22	Order 23	Order 24
Order 25	Order 26	Order 27	Order 28	Order 29	Order 30	oor	Chaotic

Figure 6.10
Color code for cyclogram

however, the two subfigures reveal that for the closed economy the production elasticity alone determines the bifurcation points, whereas for the open economy there is a stability trade-off (nonvertical boundary of the yellow/red region) between the elasticity of production and the adjustment speeds.

Exchange Rate Volatility

Let $\hat{w}_t := w_{t+1}/w_t$, $\hat{p}_t := p_{t+1}/p_t$, and $\hat{X}_t := X_{t+1}/X_t$ denote the growth factors of wages, prices, and of the nominal exchange rates respectively. The time series in figure 6.11 show the typical comovements of the growth factors of the domestic variables, which move procyclically with the bond price as well. $B = 0.5$ induces a very long finite cycle, while $B = 0.7$ shows a quasi-periodic or complex time series. Notice that the exchange rate fluctuates substantially more than the other variables. In addition figures 6.12 and 6.13 and table 6.2 provide statistical information of the long-run behavior, confirming the higher volatility of the exchange rate by a distinctively higher standard deviation.

For an investigation of the bifurcation effects induced by the elasticity of production we consider two further cases. For $B = 0.958$ the limiting behavior is described by a complex (nonperiodic) orbit with time series given in figure 6.14a, while for $B = 0.96$ the limiting behavior of the economy is a finite cycle of order three as one observes in figure 6.14b. These figures portray typical time series of the rates of change showing very clearly the higher volatility of the exchange rate as compared to the domestic variables, especially the price level. Figures 6.15, 6.16, and table 6.3 supply additional evidence of the distinct volatility features by showing a projection of the attractor into the $\hat{X}$-$\hat{p}$-space, the marginal densities (histograms) of the exchange rate, of the rate of inflation, as well as a table of descriptive statistics. These indicate that:

- price and exchange rate changes do not reveal a clear cut long-run correlation;

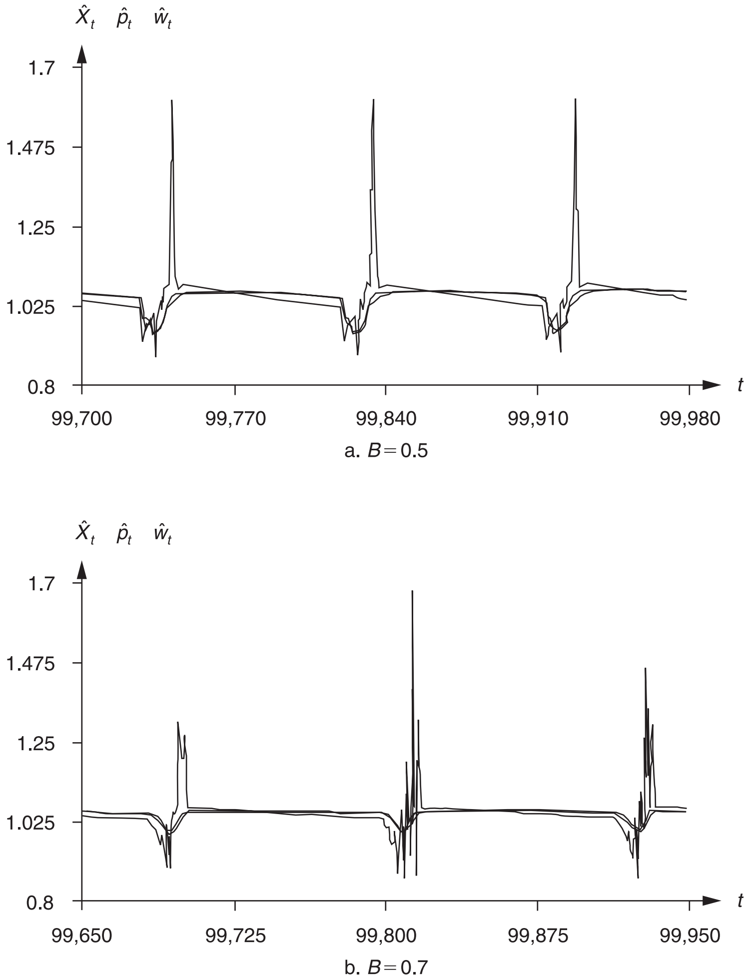


Figure 6.11
Time series of $\hat{X}$, $\hat{p}$, and $\hat{w}$ for relatively low values of B

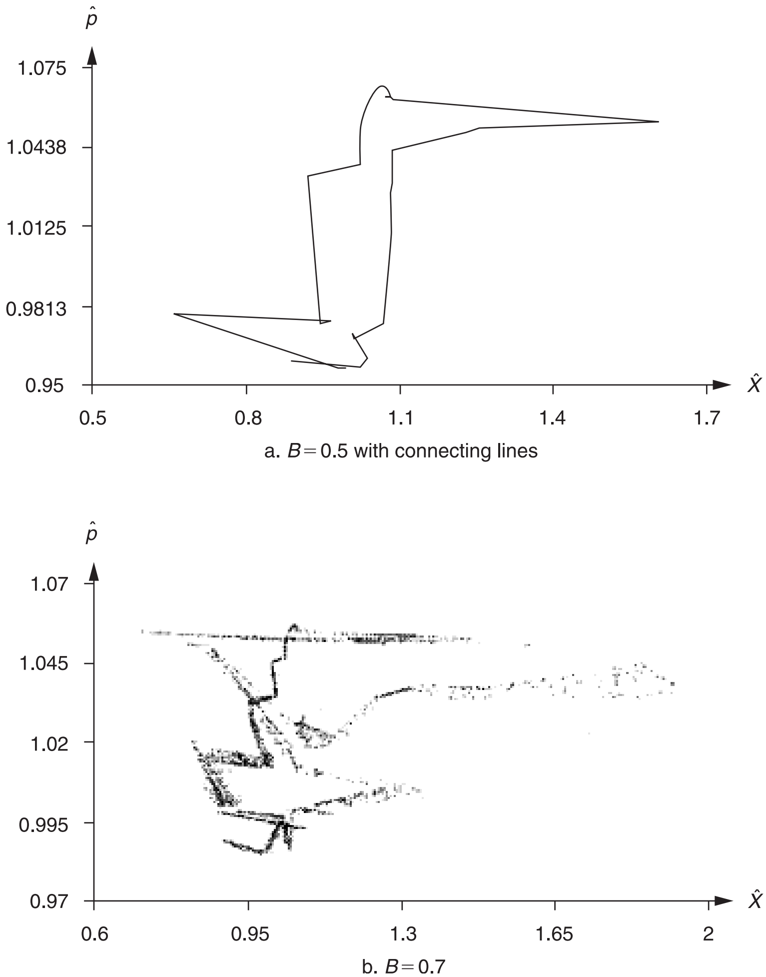


Figure 6.12
Attractor plots in $\hat{X}$-$\hat{p}$ space for relatively low values of B

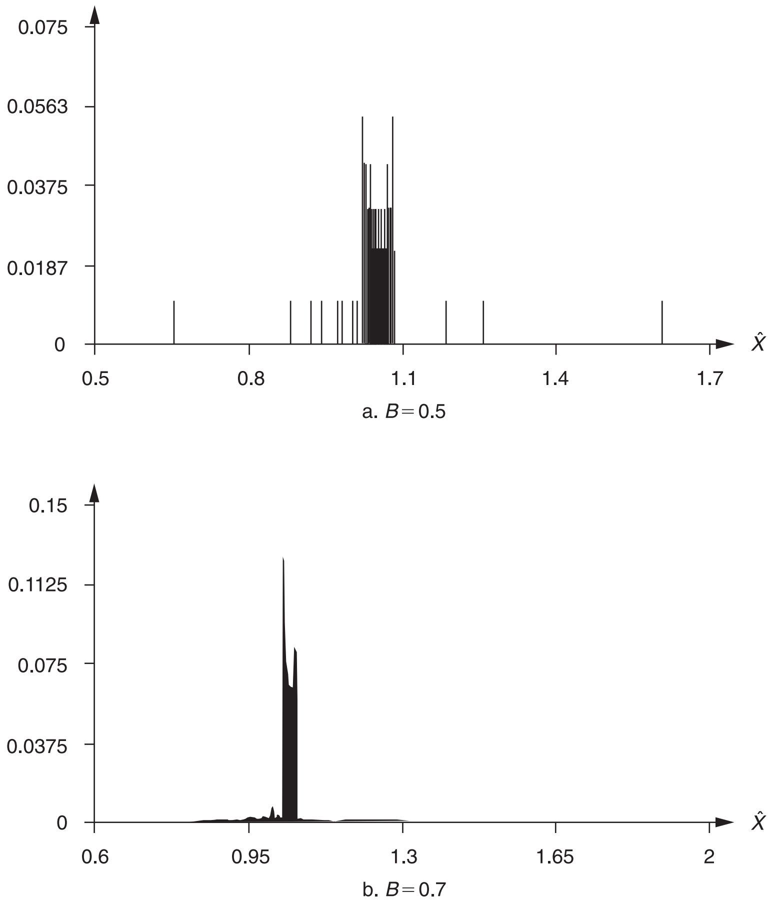


Figure 6.13
Density plots of $\hat{X}$ for relatively low values of B

Table 6.2
Descriptive statistics $\hat{X}$, $\hat{p}$, and $\hat{w}$ for relatively low values of B

	$B = 0.5$		$B = 0.7$	
Variable	Mean	Standard deviation	Mean	Standard deviation
$\hat{X}$	1.0532	0.0831	1.0514	0.0551
$\hat{p}$	1.0504	0.0306	1.0513	0.0177
$\hat{w}$	1.0504	0.0299	1.0501	0.0171

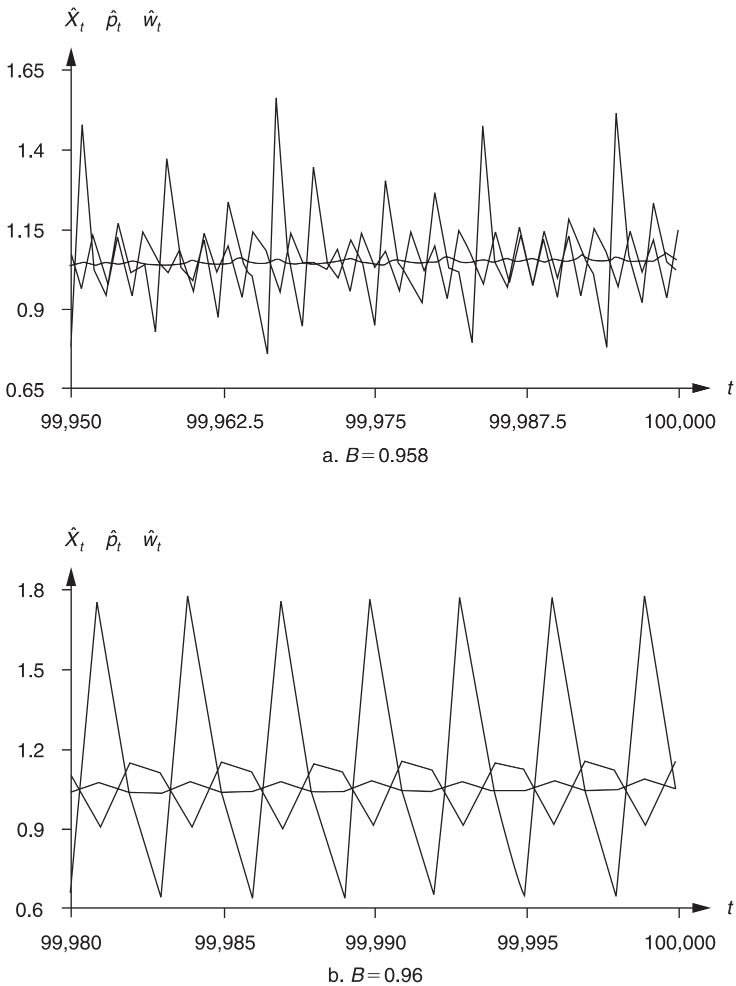


Figure 6.14
Time series of $\hat{X}$, $\hat{p}$, and $\hat{w}$ for high values of B

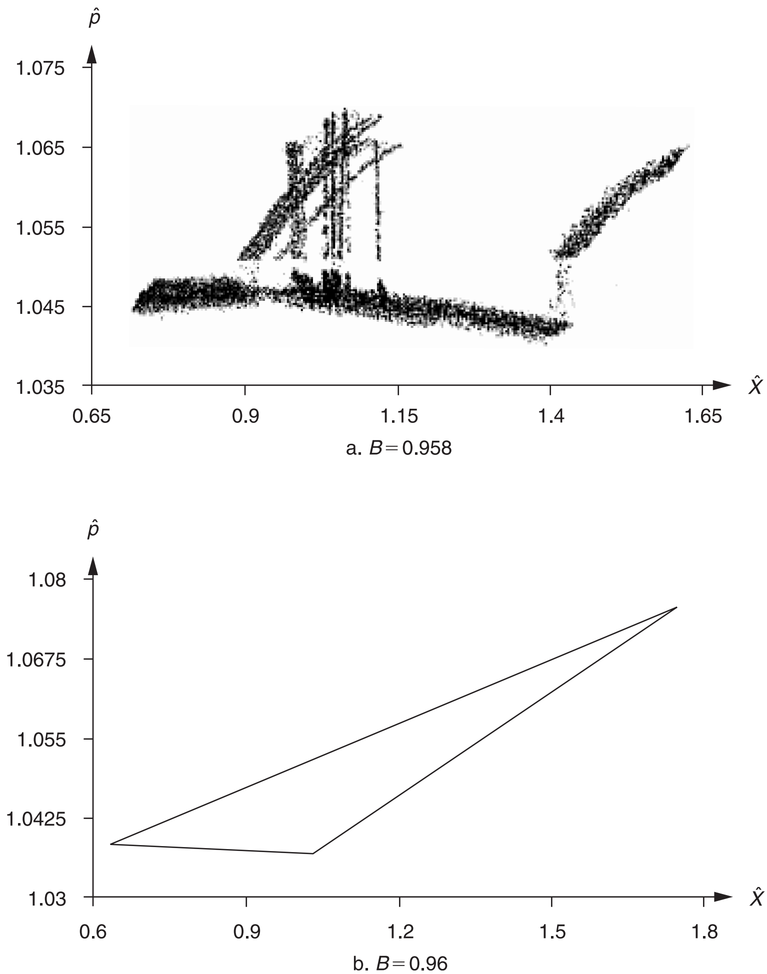


Figure 6.15
Attractor plots in $\hat{X}$-$\hat{p}$ space for high values of B

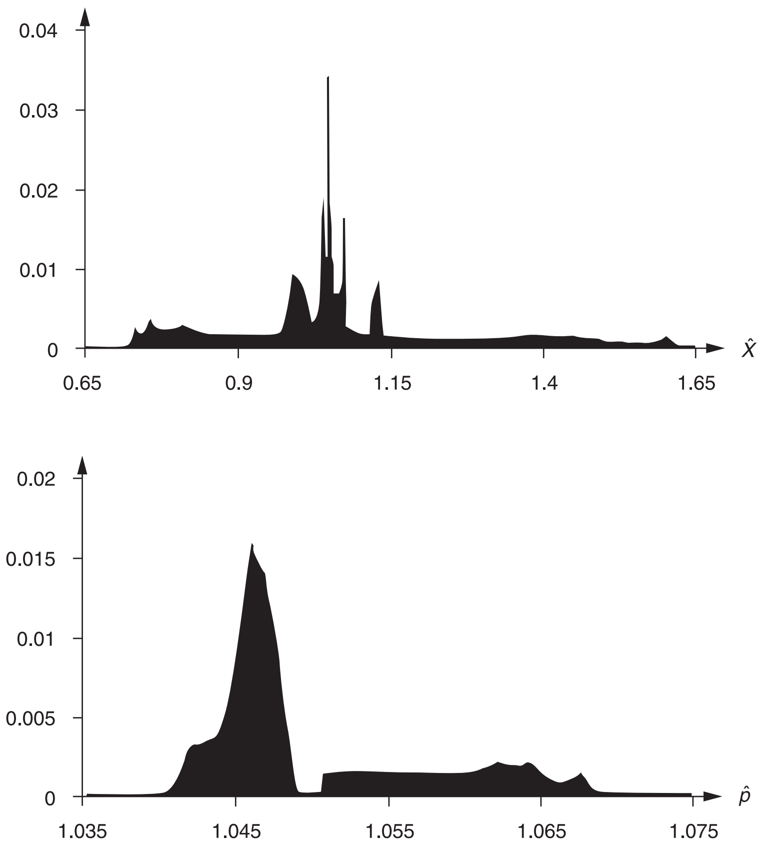


Figure 6.16
Density plots of $\hat{X}$ and $\hat{p}$ for $B = 0.958$

Table 6.3
Descriptive statistics $\hat{X}$, $\hat{p}$, and $\hat{w}$ for high values of B

	$B = 0.958$			$B = 0.96$		
Variable	Mean	Standard deviation	Skewness	Mean	Standard deviation	Skewness
$\hat{X}$	1.0670	0.1956	0.7669	1.1451	0.4689	0.3281
$\hat{p}$	1.0500	0.0072	0.1203	1.0501	0.0183	0.6365
$\hat{w}$	1.0527	0.0752	−0.2174	1.0558	0.1076	−0.6422

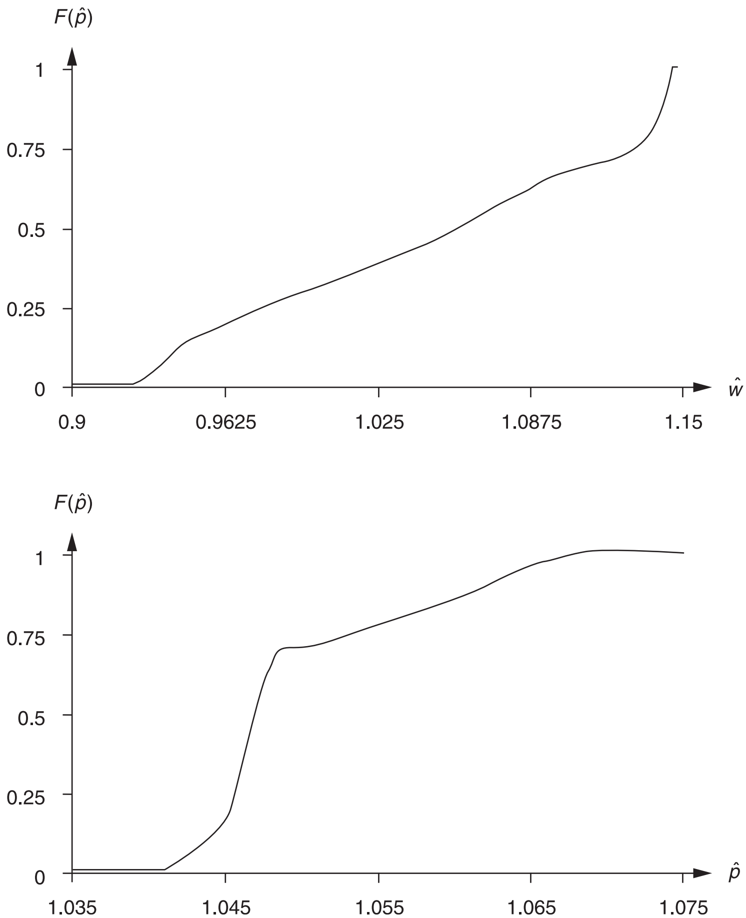


Figure 6.17
Cumulative marginal distribution function of $\hat{w}$ and $\hat{p}$ for $B = 0.958$

- the standard deviation of the growth factor of the exchange rate is roughly twenty-five times as large as that of domestic prices;
- the skewness of the exchange rate and of prices are positive while the skewness of wages is negative.

The calculation of the associated cumulative marginal distribution functions $F(\hat{w})$ and $F(\hat{p})$ yields the two curves depicted in figure 6.17. These curves indicate that there is always a positive inflation and that the probability of increasing wage rate is almost 0.6. Therefore about

60 percent of the simulated trajectory is located in the repressed inflation regime, **I** (i.e., with full employment and demand rationing), while the others will be of the Keynesian and of the classical type. Finally, there seems to be a positive correlation between domestic prices and wages for relatively low values of B and a negative correlation for higher values (see figure 6.18 for $B = 0.5, 0.7, 0.958, 0.96$). The higher variance for wages relative to that for prices is linked to the high value of B.

6.5 Conclusions

The results of this chapter provide a first explicit account of possible dynamics of a small open economy in its relationship to perfect international capital markets under the UIP hypothesis. They show that the structural nonlinear relationships between asset markets and real markets can generate permanent endogenous fluctuations. These are the result of the interaction of a strong expectations feedback with sluggish domestic price and wage adjustments under fully competitive/price-taking behavior in all markets. No elements of market imperfections are present. Moreover the cyclical recurrence arises within a deterministic model when no random perturbations are present. The numerical analysis shows examples which confirm some typical empirically observed high volatility of the nominal exchange rate relative to that of domestic variables. This result directs us closer toward a possible answer to one of the pricing puzzles.

The model demonstrates that the channels between domestic real markets and competitive international financial markets induce clear dynamic correlations between real and monetary phenomena whose qualitative properties depend heavily on particular structurally given values of the domestic economy, especially the elasticity of production. It has to be taken as one of the surprising findings that the introduction of competitive international capital markets within this class of models under the UIP hypothesis induces strong destabilizing forces often making stable regular periodic behavior impossible. With these results further research should investigate the structural relationships between domestic macroeconomic variables and international capital markets as well as its policy implications. Moreover a more detailed analytical investigation of the stability properties of this class of dynamic models will identify better the sources of the volatility and of the fluctuations.

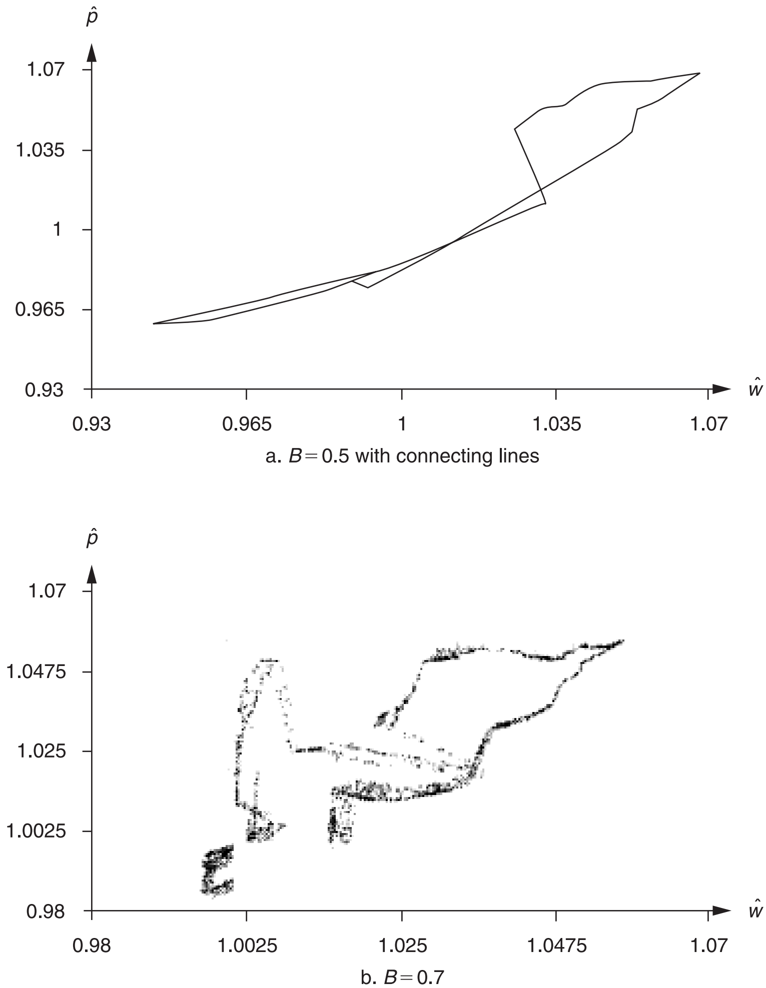
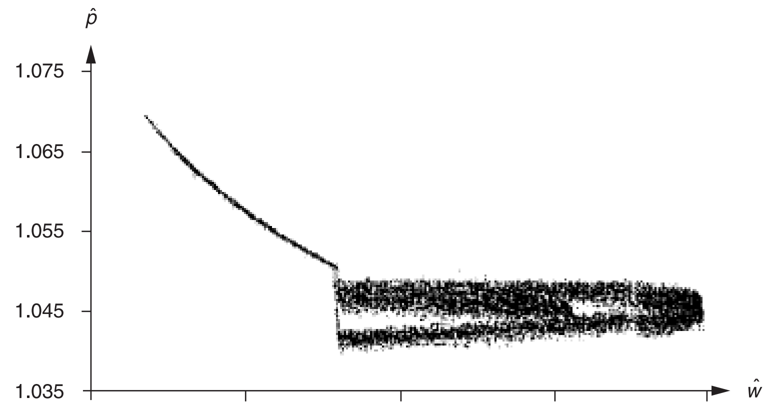
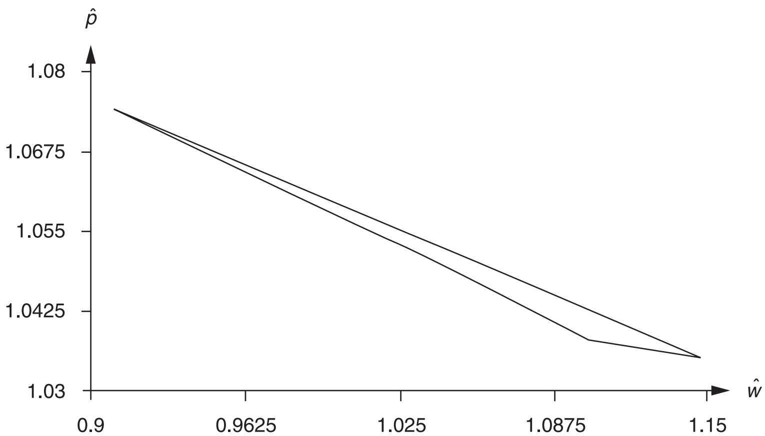


Figure 6.18
Attractor plots in $\hat{w}$-$\hat{p}$ space



c. $B = 0.958$



d. $B = 0.96$ with connecting lines

Figure 6.18
(continued)

Notes

The research for this chapter is part of the project Endogene stochastische Konjunkturtheorie von Realgüter—und Finanzmärkten supported by the Deutsche Forschungsgemeinschaft under contract number Bo. 635/9–1,3. We are indebted to J. Wenzelburger and T. Pampel for useful discussions and M. Meyer for computational assistance. We acknowledge discussions with A. Förster at the initial stage of the project.

1. The model of a closed economy with instantaneous bond market clearing possesses essentially the same temporal and dynamic structure as the one with money alone.

2. The parameter B chosen for the specific form should not be confused with the nominal stock of bonds denoted B_t.

3. Note that if we assume the price adjustment function to be continuous at $y^d = y^{max} < y^*$, there is no price adjustment in commodity market even though the commodity market is not in equilibrium. To avoid this, we assume that $\sigma^l(v) > \sigma^c(v)$ in this particular case. See Böhm, Lohmann, and Lorenz (1997) for details.

4. Such expectational leads with independence occur in a natural way in many intertemporal equilibrium models when the sequential structure of the expectations formation process is made explicit.

5. This special property of the perfect predictor follows directly from the two structural properties of the exchange rate mapping (34), the presence of an expectational lead and of the independence of the previous actual exchange rate (for a general treatment, see Böhm and Wenzelburger 2004).

6. Note that there is no stationary state of the classical type **C** since the real wage α always decreases in that regime.

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7 The Euro, Eastern Europe, and Black Markets: The Currency Hypothesis

Hans-Werner Sinn and Frank Westermann

Speculating with the euro has been disappointing for many professional investors because the movements of the exchange rate did not seem to follow conventional wisdom. The euro declined when the US economy went into recession, and it began to rise when the European stock marked slumped in early 2002.

In this chapter we elaborate on an explanation that one of us had suggested in two newspaper articles.[1] According to this explanation the euro weakened before the physical currency conversion because holders of black money and eastern Europeans fled from the old European currencies, and it strengthened thereafter because these groups of money holders developed a new interest in the euro.[2]

Although we regard an episode in economic history, we also attempt to contribute to the theory of the exchange rate by explicitly introducing currency stocks in addition to interest-bearing assets in the international portfolio of wealth owners. The inclusion of currency stocks is a simple, though uncommon, extension of the portfolio balance approach. It leads to an explanation for the negative correlation of the stock of deutschmarks in circulation and the value of the deutschmark, which Frankel (1982, 1993) once called the "mystery of the multiplying marks." Also, by this means, we can modify traditional interpretations of the portfolio balance approach, leading to new kinds of predictions for the exchange rate.

By the portfolio balance approach, it is often argued that the exchange rate is the relative price of interest-bearing assets and thus reflects the profitability of the economies involved. Given the stocks of these assets, an increase in the profit expectations for US firms, for example, implies a change in the desired composition of the portfolio in the direction of US assets. Since the composition of the portfolio cannot change in the short run, the dollar appreciates until any preference for portfolio restructuring in the aggregate disappears.

The problem with this interpretation is not only that it no longer fitted when the US slump began in 2000 or when European share prices fell, but also that it abstracts from the role of currency in the portfolios of international investors. After all, the exchange rate is the relative price of two currencies rather than shares, and shares have their own prices, which are quoted instantaneously at the stock exchange. When share prices are flexible, a profit or demand based portfolio interpretation cannot easily explain the exchange rate because there are two prices for shares, one of which seems to be redundant. If, for example, the profit expectations of the new economy are captured by the Nasdaq, there is no need for the price of the dollar to capture them too.

To determine the exchange rate in the presence of flexible share prices, other assets whose prices are not flexible are required. In the formal model derived below, interest-bearing assets whose rates of return are controlled by a central bank via passive interventions and money balances whose rates of return are fixed at a level of zero are considered in addition to stocks. We use this model to develop a new theory of the exchange rate that we call the "currency hypothesis." This is because we see the exchange rate basically as the ratio of marginal utilities of money holding. By the currency hypothesis we are able to explain the startling empirical development of the euro exchange rate with a changed demand for money balances. It is well known that the traditional portfolio balance model, which does not contain national money balances, has been relatively unsuccessful in explaining the exchange rate (Taylor 1995). Our version of the portfolio balances model reconciles the theory with the development of different exchange rates. In particular, we use it to explain the development of the deutschmark–dollar exchange rate in the period from the fall of the Iron Curtain to the physical introduction of the euro. It is this period that is identified by a unique historical experiment that creates huge shifts in the demand for deutschmarks.

7.1 Eurosclerosis, New Economy, and the Euro

To detect the flaw of traditional exchange rate explanations it is useful to start with the development of the euro. Figure 7.1 depicts the time path of the euro in terms of dollars from 1990 to July 2002. A synthetic

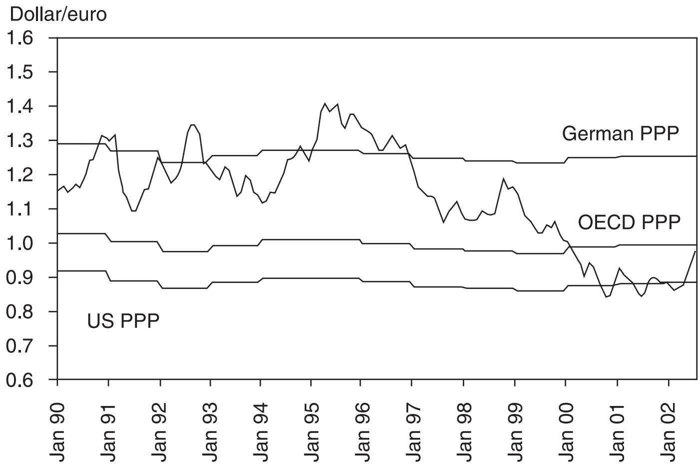

Figure 7.1
The development of the euro. Exchange rates are monthly data, while PPPs are given at an annual frequency. Different PPPs are computed with respect to the different consumption baskets in the United States, the OECD, and Germany. The latest data point is from July 1, 2002, with a value of 0.989 for the euro. (Sources: Federal Reserve Bank of St. Louis, Economic and Financial database, www.stls.frb.org/fred/; March 2002, and CESifo homepage, www.cesifo.de.)

euro was constructed for the years before 1999 by way of an official final exchange rate with the deutschmark. The diagram also shows the purchasing power parity (PPP) in accord with OECD, US, and German commodity baskets.

As the figure shows, the euro was strong, hovering around the upper PPP bound, until 1996. From 1997 onward it began a decline only to recover in February 2002, which was the month when the conversion of the old euro currencies into the physical euro was completed.

Many reasons for the long period of decline in the value of the euro are given in the literature, including labor market rigidities,[3] the European welfare net,[4] the Kosovo war,[5] Italy's ability to violate the Maastricht rules,[6] the excellent growth performance of the US economy,[7] and the initially high US interest rates.[8] However, the most frequent argument, which also underlies some of the media assessments, is the high volume of capital flows into the United States in recent years, in particular, the high volume of direct investment flowing into the new American economy.[9] We call this the economic prosperity view.

As figure 7.2 shows, capital flows into the United States were huge in the 1990s, and they have continued to increase until 2002, reaching

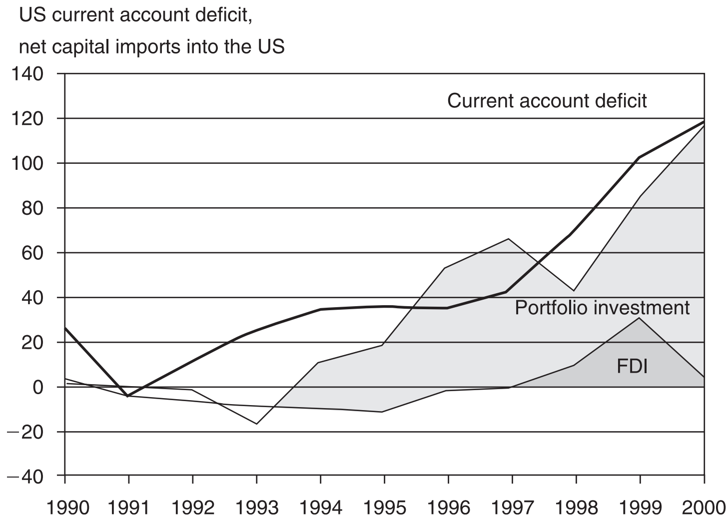

Figure 7.2
Capital imports into the United States and current account deficit. FDI = Foreign direct investment. The current account is defined as the sum of the capital account and the balance of payments (which is near 0 in the United States). The capital account is the sum of net direct investment, net portfolio investment and other investment. Other investment includes international credit and repayments of credits, participation of governments in international organizations and international real estate purchases. (Source: IMF, International Financial Statistics, CD-ROM, March 2001.)

a level of more than 4 percent of US GDP. In most years the capital flow was predominantly portfolio rather than direct investment, but in 1998 and 1999 the direct investment was also substantial, peaking at about a third of total US capital imports. In view of the size of the US capital imports it is understandable that many observers have attributed the strength of the dollar to the prosperous investment opportunities in the new American economy, and in contrast to the meagre outlook for an apparently desolate Europe suffering from a so-called Eurosclerosis.

However, there are two problems with this interpretation: a possible confusion between supply and demand and a theoretical mistake in the reasoning underlying the economic prosperity view. Let us consider these problems in more detail.

The economic prosperity view implicitly uses the traditional portfolio balance model that threatens the exchange rate in terms of the relative prices of European and American assets.[10] Capital flow into the

United States is assumed to result from an increase in demand for American assets by European investors. The increase in demand, it argues, drives up the value of the dollar because the price of the dollar is the price of American assets.

However, if an observable capital flow results in Europeans buying American assets, the reason could also be an increase in the supply of such assets. The supply of American assets is equivalent to an excess of planned investments over planned savings, and this is the same thing as a planned current account deficit or an excess of planned commodity imports over exports. A planned current account deficit is a net supply of American assets in the international capital markets. If the planned current account deficit goes up and if the price of the dollar is the price of American assets, the value of the dollar will fall rather than rise as capital flows into the US increase.

As usual, an increase in trading volume in a market says little about whether this increase is demand or supply driven. The signal for it being demand driven is the strength of the dollar. However, this is not a compelling argument for the economic prosperity view. As we will see, there are other reasons for the dollar's strength, and there are two empirical observations that support the supply-side rather than the demand-side explanation of the capital flows.

The startling decline in savings by US households is one of these observations. At the start of the 1990s the savings rate was about 5 percent; then it fell continually until in 1999 and 2000 it became negative.[11] By contrast, the euroland savings rate was nearly 11 percent in 2000. The negative savings rate meant that American households were no longer buying assets but were selling them to finance their excess absorption in resources. Given the high American investment volume, the increase in the current account deficit and the increase in the supply of assets in international capital markets were the only way to replace the American lack of savings. This development is illustrated in figure 7.3.

A further piece of information that contradicts the economic prosperity view is the poor performance of the US stock market in 1999 and early 2000. If the economic prosperity view is correct, not only the dollar but also American share prices should have increased relative to their European counterparts. But this was not the case as was already pointed out by De Grauwe (2000). Although the European stock market index performed better than the American one, the dollar was rising. A similar phenomenon occurred in the first half of 2002.

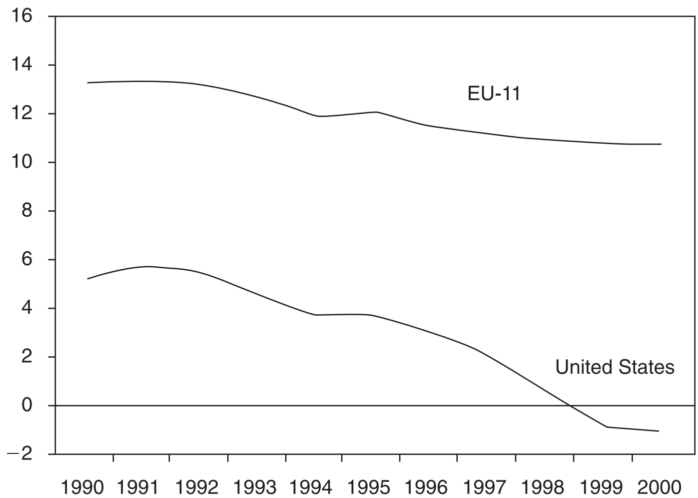

Figure 7.3
Savings rates compared. The savings rate is defined as private household savings divided by disposable household income. (Source: OECD Economic Outlook, OECD Statistical Compendium, CD-ROM.)

Newspapers attributed the new strength of the euro to a growing disinterest in American shares, but in fact the European share prices fell sharply relative to American share prices in the same period.

7.2 The Flaw in the Theoretical Argument

A larger problem with the economic prosperity view and the traditional portfolio balance model is that it does not seem to have a theoretical basis. The exchange rate is the price of a currency, and not the price of shares or other interest-bearing assets. It is true that the price of the dollar is a component of the price of American shares, if seen from the viewpoint of European investors, but the US share price itself is another component. This is a trivial but important point that may ultimately contribute to unraveling the puzzle.

Suppose that the return on US investment rises because of the new economy effect or for whatever other reason. This increase will raise demand for US shares among European investors and raise the price of American shares compared to the prices of European shares. But does this call for a revaluation of the dollar? Why is it not enough if the dollar price of American shares goes up relative to the euro price of European shares? Obviously there are two relative prices for the same thing, and one is redundant.

The traditional portfolio balance approach downplays the redundancy problem by assuming that the rates of return for the trading countries' assets are fixed or determined by monetary policy.[12] The only way to reach a portfolio equilibrium, namely a situation where the aggregate of all investors is content with the assets they possess, is an exchange rate adjustment. However, if share prices are flexible, the exclusive focus on the exchange rate adjustment in the establishment of a portfolio equilibrium no longer makes sense.

The necessary amendments of the traditional portfolio balance model can best be understood by following the layman's argument for why a higher demand for US shares by European investors will drive up the share prices. It goes as follows: The investors sell their European shares in Europe against euros, and then they sell the euros obtained against dollars in the currency exchange market in order to use these dollars for the purchase of American shares. As this involves a demand for dollars and a supply of euros, so it is maintained, the value of the euro in terms of dollars must fall.

The fallacy of this view is that it overlooks the implications of the additional demand for US shares on share prices and the repercussions on foreign exchange markets. In the short run the volume of outstanding US shares is given. Thus the portfolio reshuffling planned by European investors will be possible only to the extent that American investors are crowded out and give their shares to the Europeans. The American investors, on the other hand, may not wish to keep the dollars they receive but to buy other things instead. If it is shares, they will go abroad because only there do they find the supply they need to satisfy their demand, and in particular, they will go to Europe where shares are cheap because they are sold by the European investors. Thus they will supply the dollars they received from the European investors in the currency exchange market and feed the demand for euros instead. If the original purchase of dollars drove up the dollar, this will instead drive up the euro and eliminate the effect on the exchange rate.

With the passage of time the crowding out of American share holders will become weaker because the share price increase induces an additional flow of new issues of shares to finance more investment. However, because an increase in planned net investment is equivalent to an increase in the planned current account deficit, this will not generate a positive revaluation effect on the dollar. It will, however, imply a smaller share price increase.

The real possibility to generate a revaluation effect is if the crowded-out American shareholders do not go into foreign shares because they have a home bias in their preferences. There are two alternatives.

One is that the crowded-out American shareholders prefer to go into US money instead of European shares. This is the clearest case where a revaluation of the dollar occurs. However, it hardly supports the naive view that an increased demand for American assets drives up the dollar simply because there is a transitional demand for dollars in the process of portfolio conversion.

The alternative is that the crowded-out American shareholders prefer to go into American bonds instead of European shares. If the central bank does not stabilize the interest rate by open market operations, this will drive down the interest rate and crowd out previous bondholders. If these then choose European bonds or shares instead of the American bonds they sold, there is again a countervailing supply of dollars in the exchange market. However, if the central bank stabilizes the interest rate by selling bonds and buying the dollars that the crowded-out shareholders do not want, the countervailing effect will be mitigated, and on balance, an appreciation of the dollar will remain.

The lesson from these considerations is that the dollar appreciates when more dollars are demanded or fewer dollars are supplied, not when more American interest-bearing assets are demanded. It is surprising how frequently this simple fact has been overlooked in the literature on the determinants of the exchange rate.

One of the reasons why the layman's argument overlooks the possible repercussions resulting from the actions of crowded-out shareholders is that it focuses on transitional demand and supply flows in the currency exchange markets rather than on ultimate preferences for stocks of assets such as shares, bonds, and currencies. To analyze what is happening to the exchange rate, we need a portfolio balance model enriched with stock demands for domestic and foreign currency. According to such a model, the interest rate, the price of shares, and the exchange rate are determined by the need to equate desired with actual wealth portfolios. At any point in time the actual portfolio of assets is given in the aggregate, and thus a desire to restructure this portfolio cannot be fulfilled. Instead, asset prices, rates of return, and exchange rates have to adjust until people's preferences fit the given actual stocks of assets available, notwithstanding the fact that from a microeconomic point of view, it is always possible to adjust the portfolio to the preferences.

A Friedmanian thought experiment exemplifies the merit of a currency-augmented portfolio balance approach in the present case. Suppose that the European investors who wish to replace their European shares with American ones pack these shares into coffers, fly to the United States, and negotiate directly with the American shareholders. They then find an exchange rate between European and American shares, and hence relative rates of return, at which the American shareholders are willing to participate in the deal. In general equilibrium, this direct deal cannot result in any exchange rate other than the one brought about by a transitional conversion of European shares into euros, of euros into dollars, and of dollars into US shares. Thus the thought experiment confirms that the dollar–euro exchange rate cannot be effected if the American shareholders who sold their shares are happy to hold European shares instead.

If the dollar appreciates, it must be because American shareholders are not happy with all the European shares they purchased and convert them into other assets in a way that increases the demand for of US money balances or reduces the supply of such money balances. As explained above, the first of these cases is the straightforward move from European shares into American money. The second case results from the wish to convert European shares into American bonds (or bills). If this induces the Fed to supply more bonds and reduce the stock of currency in circulation so as to defend the short-term interest rate, US currency will become more scarce and the dollar will appreciate.

7.3 Why Money Matters

To clarify the role of currency in the determination of the exchange rate more formally, we now specify a simple two-country portfolio balance model with a representative international investor who chooses among three types of assets in each of the two countries: shares S, bonds (or bills) B, and money M.[13] The two countries are the United States and Europe. In a market equilibrium the share prices, the exchange rate, and the interest rates are determined so as to equate the desired portfolio structure resulting from the investor's optimization to the actual one, which is taken as given.[14]

The units of account for measuring the volumes of shares, bonds, and money are the respective national currencies. The volume of shares S is expressed in terms of the nominal share value. The market value of a share is a multiple P of the nominal value. We call this multiple the share price. When r denotes the rate of return on nominal share values, $r \cdot S$ is the dividends distributed and r/P is the effective rate of return on shares (without a potential return from share appreciation). Let i denote the rate of interest on bonds. Variables that refer to the United States are labeled with an asterisk; variables without an asterisk refer to Europe and are expressed in terms of euros. The exchange rate e is the price of euros in terms of dollars.

The representative international investor is meant to reflect the aggregate of all wealthly Americans and Europeans. He optimizes his portfolio for a given investment period, which may or may not be part of a multiple-period setting. At the beginning of the period he has a given endowment of assets that constitutes his total wealth W in terms of euros, but he chooses to re-optimizes his portfolio structure, taking the two share prices, the exchange rate, and the two interest rates as given.[15] The investor's budget constraint in terms of euro expenses for the six types of assets available is

$$W = S^* \frac{P^*}{e} + B^* \frac{1}{e} + M^* \frac{1}{e} + SP + B + M. \tag{1}$$

Note that the choice of numéraire is arbitrary but meaningless. Nothing would change by choosing the dollar as the numéraire.

Among other things, the investor's decisions depend on expectations of end-of-period share prices and of the end-of-period exchange rate, which we denote $\tilde{P}$ and $\tilde{e}$. The model predicts that changed expectations about these variables will immediately translate into their current counterparts, but we fix the expectations throughout this chapter in order to concentrate on the fundamentals affecting the exchange rate. Our discussion focuses on changed stocks of assets due to government policies, changed real returns, and changed preferences for certain types of assets, given the expectations. The investor's utility is assumed to be given by the sum of end-of-period wealth plus a liquidity service

$$U\left(\frac{\sigma^* S^* \tilde{P}^*}{\tilde{e}}, \frac{\beta^* B^*}{\tilde{e}}, \frac{\mu^* M^*}{\tilde{e}}, \sigma S\tilde{P}, \beta B, \mu M\right),$$

which depends on the respective expected stock values $S^*\tilde{P}^*/\tilde{e}$, $B^*/\tilde{e}, M^*/\tilde{e}, S\tilde{P}, B$, and M.[16] The liquidity service is meant to capture all considerations important for the choice of assets other than their contribution to the pecuniary return, including risk characteristics, Baumol-Tobin type transactions costs, the timing of planned commodity purchases, and the like. The Greek symbols $\sigma^*, \beta^*, \mu^*, \sigma, \beta$, and μ denote parameters of the utility function, which allow us in a simple fashion to represent arbitrary preference changes including those that generate cross-price effects among different assets. We assume that U is an increasing, separable, and strictly concave function and that the parameters are unity before a preference change takes place.

Formally, the investor's decision can be depicted by maximizing the Lagrangean

$$\begin{aligned} L = {} & S^*\frac{1}{\tilde{e}}(\tilde{P}^* + r^*) + B^*\frac{1}{\tilde{e}}(1+i^*) + M^*\frac{1}{\tilde{e}} + S(\tilde{P}+r) + B(1+i) + M \\ & + U\left(\frac{\sigma^* S^* \tilde{P}^*}{\tilde{e}}, \frac{\beta^* B^*}{\tilde{e}}, \frac{\mu^* M^*}{\tilde{e}}, \sigma S\tilde{P}, \beta B, \mu M\right) \\ & + \lambda\left(W - S^*\frac{P^*}{e} - B^*\frac{1}{e} - M^*\frac{1}{e} - SP - B - M\right) \end{aligned}$$

with respect to the six different asset volumes considered in the model. Here the first line is end-of-period wealth in terms of euros, the second gives the liquidity services, and the third contains the investor's budget constraint where λ is the Lagrangean multiplier. The marginal conditions resulting from this optimization approach are

$$\frac{e}{\tilde{e}} \cdot \frac{\tilde{P}^*(1+\sigma^* U_{S^*}) + r^*}{P^*} = \lambda, \tag{2}$$

$$\frac{e}{\tilde{e}}(1 + i^* + \beta^* U_{B^*}) = \lambda, \tag{3}$$

$$\frac{e}{\tilde{e}}(1 + \mu^* U_{M^*}) = \lambda, \tag{4}$$

$$\frac{\tilde{P}(1+\sigma U_S) + r}{P} = \lambda, \tag{5}$$

$$1 + i + \beta U_B = \lambda, \tag{6}$$

and

$$1 + \mu U_M = \lambda. \tag{7}$$

These equations are similar insofar as they all show that in the optimum the sum of each asset's own rate of return factor plus the marginal liquidity service, possibly corrected by a growth factor reflecting the expected exchange rate adjustment, equals a common yardstick, the Lagrangean multiplier λ. In the case of US shares (2), the rate of return factor is a combination of the growth factor of the dollar in terms of euros, $e/\tilde{e}$, of the growth factor of the US share price, $\tilde{P}^*/P^*$, and the effective rate of return on US shares, r^*/P^*. In the case of dollar currency (4), the rate of return factor is just the growth factor of the dollar in terms of euros, and in the case of euro currency, it is simply one. The other cases should be self-explanatory. In general, an asset's pecuniary rate of return factor is smaller, the larger this asset's marginal liquidity service. As the rate of return on shares tends to be higher than that on bonds and the latter higher than that on cash, the marginal liquidity services will presumably follow the adverse ordering.

Let a bar above a variable indicate the given asset stocks in the economy. The investor's wealth in terms of euros with which he enters the period is then determined by

$$\bar{S}^* \frac{P^*}{e} + \bar{B}^* \frac{1}{e} + \overline{M}^* \frac{1}{e} + \bar{S}P + \bar{B} + \overline{M} \equiv W. \tag{8}$$

Equations (1) through (8) define the demand functions for all six assets. The asset prices, the exchange rate, and the interest rate follow if we assume that, for each asset, demand equals supply:

$$S^* = \bar{S}^*,\ B^* = \bar{B}^*,\ M^* = \overline{M}^*,\ S = \bar{S},\ B = \bar{B},\ M = \overline{M}. \tag{9}$$

In total, there are now 14 equations, one of which is redundant. They explain six asset stocks, two interest rates, two share prices, one exchange rate, the Lagrangean multiplier, and the wealth level, in a total of 13 variables.

There is no need to explicitly solve for all of these variables because a number of useful observations can easily be derived by inspecting the equations. One concerns the economic prosperity view. Suppose that σ^* in equation (2) increases and/or σ in equation (5) declines while the marginal utilities of money holding remain constant. Equations (4) and (7) then fix the exchange rate e and the Lagrangean multiplier λ. As U_{S^*} and U_S are fixed by the given levels of S^* and S, it follows from (2) and (5) that the changed preferences for share holdings will be accommodated only by an increase in the price of US shares P^* and/or a decline in the price of European shares P. No exchange rate movements are necessary to maintain a portfolio equilibrium.

Changes in the nominal rates of return r and r^* in favor of American assets would, as the reader can easily verify for himself, have very similar effects. If the money demands do not change, they would not, as the economic prosperity view predicts, result in an appreciation of the dollar but, once again, only in an increase in the US share price relative to the European one.

A similar remark applies to the rates of interest on bonds. Again, the exchange rate e and the Lagrangean multiplier λ are fixed by (4) and (7) independently of these interest rates. An increase in the preference for US bonds as reflected by an increase in β^* will, according to (3), only result in a fall in the US interest rate, and similarly an increase in the preference for European bonds will reduce the European interest rate according to (6) without affecting the exchange rate.

The crucial equations for the determination of the exchange rate are (4) and (7). Together they imply that the value of the euro is explained by the marginal liquidity services of euros and dollars in the international wealth portfolio:

$$e = \tilde{e} \cdot \frac{1 + \mu U_M}{1 + \mu^* U_{M^*}}. \tag{10}$$

No pecuniary rates of return of the assets on which the portfolio balance approach focuses enter this formula, since these rates are endogenous to the market equilibrium. This reiterates the point made above, which is less trivial than it sounds: the currency exchange rate is the exchange rate between two types of money, and not the exchange rate between interest-bearing assets.

The remarkable aspect of these neutrality results is that preference changes concerning interest-bearing assets will result in price and rate of return changes that are large enough to compensate for these changes but do not affect the exchange rate. For exchange rate movements to come along with such preference changes, it would be necessary that preference changes for money balances be involved too. Consider, for example, the home bias discussed in the previous section implying that crowded-out American shareholders like to go into American money. In the aggregate model considered here, this can be captured by the assumption that the increased preference for American shares comes along with an increased preference for US money, meaning an increase of μ^*. According to equation (10) this would indeed imply a weakening of the euro.

Thus far we assumed that the stocks of assets are given in the portfolios and that the pecuniary rates of return are flexible. Rate of return adjustments will then be able to accommodate the preference changes with regard to bonds and shares but not with regard to money holdings, because the pecuniary return of money is fixed at zero. Only a changed preference for money holding needs an exchange rate adjustment to keep the desired portfolio structure in line with the given actual one.

Things are different, though, when other rates of return are fixed too. The relevant case here is that the two central banks fix the national interest rates and accommodate any changes in preferences for money and bonds with appropriate open market policies that change the composition of the outstanding stocks of bonds and money balances. This will affect the marginal liquidity services of money balances and will have repercussions on the exchange rate according to equation (10).

From equations (3), (4), (6), and (7) it follows that the national interest rates are given by

$$i^* = \mu^* U_{M^*} - \beta^* U_{B^*} \quad \text{and} \quad i = \mu U_M - \beta U_B. \tag{11}$$

Given the stocks of money and bonds and hence given U_{M^*}, U_{B^*}, U_M, and U_B, a national interest rate obviously decreases with a decrease in the preference for the respective national money (decrease of μ^* or μ) and/or an increase in the preference for national bonds (increase of β^* and β), as was explained. To prevent this from happening and to fix the interest rates, the central banks have to accept any exchange between the national stocks of money and bonds that the public wants to carry out at the given interest rates; that is, they have to intervene passively by supplying more of the respective stock in demand and withdrawing the other one from the market.

Passive intervention of this type will make the exchange rate reactive to changed preferences for bond holdings and protect it partly from changes in the preference for money holdings. Consider, for example, the case of an increased preference for US bonds, as is reflected by an increase in β^*. To avoid a decrease in the US interest rate, the Federal Reserve Bank will react by selling bonds against US currency, which increases U_{M^*} and lowers e according to (10). The dollar appreciates after an increase in the demand for US bonds. Similarly a depreciation of the euro, e, could be brought about by a reduced preference for European bonds if the European Central Bank fixed the interest rate by buying bonds and selling euros—or, as discussed in the previous section, by an increased preference for American bonds which the Fed accommodates with a contractionary open market policy.

Things would be similar if the central banks intervened also to keep the effective rate of return on shares constant, but of course they don't. This is the crucial point overlooked in the existing portfolio balance literature. If the central bank intervenes only to keep the interest rate constant and if no more than the preference for shares changes as is reflected by σ^* and σ, equations (2) through (7) continue to ensure an isolation of the exchange rate. This confirms the above criticism of the economic prosperity explanation of the euro's weakness and of the traditional portfolio-balance approach as such. Even when the central bank intervenes passively to keep the interest rate constant, changes in profit expectations, in preferences for share holdings, or in preferences for direct investment cannot influence the exchange rate unless they also imply changes in preferences for bonds or money balances.

Let us now discuss the reason why a passive intervention might partially protect the exchange rate against changes in liquidity preferences. Suppose that the preference for euro currency declines, as is represented by a reduction of μ. According to (10), this will depreciate the euro, and according to (11), it will reduce the European interest rate. To prevent the interest reduction, the European Central Bank will buy back money balances against private bonds. In itself, this will increase U_M and increase e, meaning it will stabilize the exchange rate. The stabilization will not be perfect, though, because the increase in the stock of bonds results in a reduction in the marginal utility from bond holding, U_B. According to (11), a constancy of the interest rate therefore implies that the marginal utility from money holding, μU_M, will not be pushed back to where it was before the preference change and that there is a negative net effect on the euro.

This can also be seen by deriving a modified interest parity condition from equations (3) and (6), which relates the exchange rate to the national interest rates and the marginal liquidity premia for bonds:[17]

$$e = \tilde{e} \cdot \frac{1 + i + \beta U_B}{1 + i^* + \beta^* U_{B^*}}. \tag{12}$$

As the passive intervention triggered off by the decline in μ increases the stock of bonds held by the public, B, and thus reduces the bonds' marginal liquidity service U_B, equation (12) ensures that the net effect on the exchange rate is negative. A similar result holds for an increase in μ^*. As the reader may verify for himself, a negative net effect on e and a decrease of M^* can also result from an increase in the preference for dollar currency if the dollar interest rate is given.

The effect has a certain similarity with an active intervention in the exchange market. If such an intervention is sterilized in the sense that it leaves the interest rates fixed in the two countries, it will involve a sale of dollar currency and dollar bonds against euro currency and euro bonds so as to keep the respective national differences in the marginal liquidity services of money and bonds constant, as is indicated by (11). The decline in the marginal utility of US bonds, and the respective increase in the marginal utility of European bonds that results from this change in the structure of the market portfolio, raises the fraction on the right-hand side of (12) and hence the value of the euro.[18]

It is a common feature of the active and passive interventions that a decline in the stock of euro currency exhibits a positive effect on the value of the euro. However, the distinguishing feature is that this effect comes independently when the central bank intervenes actively in the foreign exchange market while it is only an induced compensating effect, which cannot offset the primary effect when the central bank intervenes passively by fixing the interest rate. Thus the correlation between the stock of euro currency and the value of the euro should be negative in the case of active intervention with a given interest rate, and positive in the case of passive intervention after a change in the currency preference. As we showed above that a negative correlation would also characterize the case of passive intervention after a change in bond preferences, it seems that the sign of the correlation between the currency stocks and the exchange rate might be a clue for finding the causes of the weak euro.[19]

It is essential for our theory that American and European bonds be imperfect substitutes in the international portfolio. If they were perfect substitutes, a preference shift would be made from European to American currency. The shift would be accommodated by a contractionary open market policy in Europe and an expansionary one in the United States, so as to keep the interest rates constant and not affect the exchange rate. The simplest way to depict this possibility in our model would be to assume that bonds do not deliver marginal liquidity services in addition to their pecuniary return, such that $\beta^* U_{B^*} = \beta U_B = 0$.

Equations (10) through (12) would then imply that fixing the interest rates eliminates any effect of a changed preference for money holding on the exchange rate. Similarly equation (12) would imply that the ECB tried the impossible when it intervened in the foreign exchange market to stabilize the euro without changing the European interest rate. However, we find it hard to believe that bonds denominated in different currencies and separated by a flexible and risky currency exchange rate will even come close to being perfect substitutes. This is the old dichotomy between the portfolio balance and the monetary approaches, which can only be solved empirically.

Feldstein and Horioka (1980) and Dooley, Frankel, and Mathieson (1987) have argued that a high correlation between savings and investment points to a rather limited international substitutability of assets, and within our model we will also be able to provide supporting evidence for a limited substitutability.[20] If American and European bonds are perfect substitutes, the value of the euro and the stock of euro currency should be uncorrelated both in the presence of demand and supply shocks if one controls for the interest rates. On the other hand, if they are imperfect substitutes, then controlling for the interest rates, there should be a negative correlation when supply shocks dominate and a positive correlation if demand shocks dominate. These are clear-cut predictions, and we will show that during the historical period considered there was indeed a very significant positive correlation.

7.4 Black Money and Deutschmarks Circulating Abroad

The deutschmark provides a particularly striking example of the positive correlation between the stock of currency in circulation and the foreign exchange value of this currency: in the late 1980s and early 1990s the Bundesbank and the public had regularly been surprised, if not alarmed, by the fact that the German monetary base grew much more rapidly than was anticipated, typically exceeding the projection corridor the Bundesbank had published. During this period there was a persistent revaluation pressure for the deutschmark. The pressure even led to the collapse of EMS in 1992, which implied a sudden revaluation of the deutschmark relative to most of the European currencies and the dollar.[21] Since 1997, however, this trend has been reversed (see figure 7.1), and so has the trend in the growth rate of money balances. When the external value of the deutschmark began to decline,

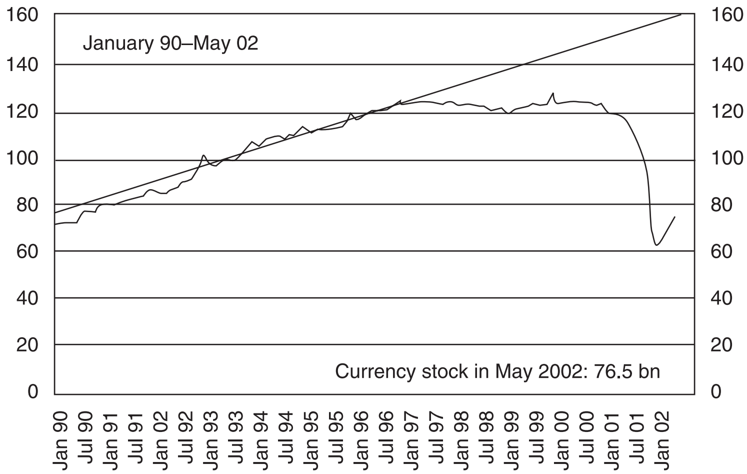

Figure 7.4
German currency in circulation (monthly data, billion). (Source: Deutsche Bundesbank homepage, 2002.)

the growth rate of the German monetary base began to decline relative to its trend, and during the year 2000 even the base itself began to fall with a gradually accelerating speed. Figure 7.4 illustrates this development.

The development of the stock of all euro currencies, as depicted in figure 7.5, paralleled that of the stock of deutschmark currency. No econometric approach is need to uncover the movements. Obviously the stock of euro currencies in circulation was falling against the trend from about 370 billion € to about 250 billion €, which is a decline of 120 billion € or one-third. This is ten times more than the numbers monetary theorists usually try to interpret.

The numbers are also huge if compared with previous intervention and speculation volumes. George Soros is said to have succeeded to tilt the EMS with only a few billion pounds, and the ECB's frequent interventions to stabilize the euro had probably not exceeded 4 billion euros in total.

It can only be guessed what the reasons for the euro currencies returning to the ECB were. We believe that it has do to with the announcement and anticipation of the physical currency conversion, which induced a flight from euro currencies into other assets including other currencies. There are two categories of flight money: deutsch-

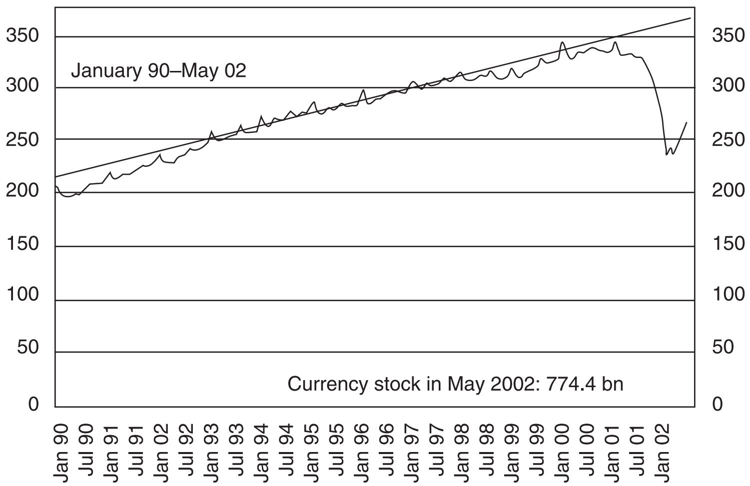

Figure 7.5
Euro zone currency in circulation (monthly data, billion). (Sources: September 1997–May 2002 Deutsche Bundesbank (2002), January 1990–August 1997 Ifo estimate based on monthly changes.)

marks that were legally and illegally held for transactions purposes outside Germany, and stocks of black money denominated in all euro currencies that were held by west Europeans. Other reasons that relate to the more technical aspects of the currency conversion could have been important in the very last moment before the conversion, but the deviation from the trend began too early for these reasons to have a considerable explanatory weight.

The first category must have been substantial because the German currency was the only one among the euro currencies that served as a means of transactions in other countries, in particular, in eastern Europe and Turkey but also in other parts of the world. In a Bundesbank discussion paper published by Seitz (1995), the accumulated stock of deutschmark currency outside Germany was estimated to be between 60 and 90 billion in 1995, which is equivalent to 30 to 45 billion €. At the time this number was between 25 and 35 percent of the German monetary base and between 10 and 15 percent of the monetary base of what later would be the euro countries.[22]

The deutschmarks circulating abroad began to return after the firm announcement of the currency union at the Dublin meeting in 1996. Foreign money holders had heard about the abolishment of the deutschmark and were afraid of sustaining a conversion loss. Even in Germany, many people were afraid of losing part of their wealth, despite the frequent advertising campaigns for the euro. The uncertainty of ordinary people elsewhere in the world must have been much bigger, since they were not informed about the conditions of the conversion and probably wondered what all this euro business was about. No doubt they heard that the deutschmark was to be abolished in 2002 and had wind of the talk about a new currency replacing it. But they did not know who would carry out the conversion, what the exchange rate would be, and what commission fees would be charged. Those people afraid of sustaining a loss continued to hoard deutschmarks and hurried into the dollar or other currencies, including their own, which were free of this kind of uncertainty. The recipients of the deutschmarks, typically banks and other financial institutions, then returned the deutschmarks to the Bundesbank in exchange for interest-bearing assets, typically short-term securities that were counted as part of M3.

It is interesting in this regard that that the ECB announced in its *Bulletin* of November 2001 a redefinition of its stock of M3 because a growing proportion of such securities had been accumulated by foreigners and was nevertheless counted as part of M3. Short-term securities with a maturity of up to two years that were being held by foreigners were decided no longer to be included in the definition of M3. According to the ECB's own information this amounted to an adjustment of the published increments of M3 on the order of 40 billion € in one year. An analogous comparison between the old and new M3 figures for the period back to January 1999 shows that the effect could even have been on the order of 100 billion €. It is unclear how much of this can be attributed to the returning deutschmarks, but the figures must be seen as a clue to the forces at work.

Further evidence comes from two surveys. One was conducted by us, using the Ifo Institute's Economic Survey International, a quarterly transnational poll among country experts. We asked 150 experts in eastern Europe, typically economists working for international companies, about a potential shift in the interest of ordinary people from the deutschmark to the dollar. Of the 71 people from 15 countries who responded to the poll, a majority of 54 percent reported that the public showed a growing interest in the dollar, 78 percent thought that the public had not been sufficiently informed about the introduction of the euro, and another 54 percent said that the public was at least partially worried about losses if they did not soon exchange their German marks into a permanent currency such as the dollar.

Another, much more extensive survey with thousands of east Europeans was conducted by the Austrian Central Bank (Stix 2001). The survey was taken at various times over two years in Croatia, Hungary, Slovenia, the Czech Republic, and Slovakia. It affirmed that the decline of the share of D-mark in circulation in the total euro money supply was due to the deutschmarks returning from abroad and that as late as May 2001 no less than 41 percent of the holders of deutschmarks who had made up their minds planned to exchange their stocks not into euros but into other currencies.

Let us now turn to the second reason for the flight of cash, namely the flight of black money in the run-up to the physical conversion of euro currencies. According to the European laws against money laundering the official conversion of larger sums of old cash into euros was not possible without registration. People who held stocks of black European monies therefore had to find ways to gradually convert them outside the banking system before the official conversion date, but they could not convert them into the euro because this currency existed only in a virtual form. Thus they had to go into the dollar, the pound, or other currencies that were not part of the euro group, and the sellers of these international currencies then exchanged the surplus stocks of euro currencies against interest-bearing assets that, after a substitution chain, ultimately came from ECB, which tried to stabilize the interest rate as explained above.

Unfortunately, no official statistics are available that allow a precise distinction between the two sources of the decline in currencies as depicted by figures 7.4 and 7.5. Neither black stocks of money balances nor currency stocks held in eastern Europe are easily observable. Nevertheless, there is indirect evidence that provides rough estimates of the relative magnitudes involved.

Consider first the results of Schneider and Ernste (2000) on the size of the black economy in Europe. According to these authors, the share of the black economy in the euro countries is about 14 percent of the actual GDP including the black activities. Based on this figure and the trend value of 370 billion euro, as shown in figure 7.5, the potential stock of black currency at the time of currency conversion can be expected to have been 52 billion € or more.

Figures 7.4 and 7.5 make it clear that roughly this sum could have contributed to the net decline of the currency in circulation until the time of physical currency conversion. As the results of Schneider and Ernste reveal that Germany's black market share in GDP is close to the European average and as German GDP is about 31 percent of the total of all euro countries, the reduction in the stock of deutschmarks in circulation would have had to be 31 percent of 120 billion €, in other words, 36 billion € if it was exclusively explained by the black market effect. However, figure 7.5 reveals that the decline against the trend of the stock of deutschmarks in circulation was much higher, about 90 billion €. This clearly points to the importance of the eastern European effect. Assuming that the 30 billion € decline of non-German currency in circulation, revealed by figures 7.5 and 7.6, can be explained fully by the black market effect[23] in the non-German euro countries, which produce 69 percent of the GDP and should therefore hold 69 percent of the stock of black money, the total black market effect for all euro countries can be taken to be about 45 billion €. Thus the remainder of the total decline of 120 billion €, which is 75 billion €, can be seen to reflect the stock of deutschmark currency that returned from eastern Europe and other parts of the world, or did not flow there in the first place because of the expected euro introduction.

These are only rough estimates. Whatever the true relative importance of the two effects may be, the fact that ordinary people outside Germany and west European holders of black money had lost their interest in euro currencies in the run-up to the currency conversion is beyond doubt. There was exactly the kind of reduced preference for euros that was modeled by a decline of the utility parameter μ in the previous section.

Our theory indicates that this reduced preference would have lowered the value of the euro and the European interest rate if the ECB had not intervened. The euro and the interest rate would have adjusted such that the existing stocks of money balances continued to be held in the international wealth portfolio. However, the ECB intervened passively so as to stabilize the interest rate. As explained in the theory section, this mitigated the decline of the euro without eliminating it, while the stock of circulating currency fell.

The mechanism through which this actually happened is that the euro currency held by foreigners and black market agents went to international financial agencies (banks and investors) that held both euro and dollar currencies. Some of the dollars delivered by these agencies may have come from the Fed in exchange for US securities and some of the euros received by them went to ECB in exchange for

European securities. In the end, the euro declined, and there was less US currency and more European currency in the international portfolio of these financial agencies, and more US currency and less European currency in the aggregate international portfolio of all private agents taken together, including eastern Europeans and black market agents. This interpretation fits the observed decline of the stock of outstanding deutschmarks as shown in figures 7.4 and 7.5 and the simultaneous decline of the euro as shown in figure 7.1.

It even fits the rise of the euro after February 2002 when the currency conversion was completed (see figure 7.1). As was predicted by us in the journal articles and other contributions,[24] currency demand by eastern Europeans and holders of black money went up immediately after the physical conversion, forcing the ECB to pump more money into the economy so as to maintain its interest target, and the euro began to appreciate rapidly, taking by surprise the analysts who believed in a correlation between the strong US recovery and the value of the dollar. The development after the physical currency conversion mirrors that of the virtual conversion before it: the euro has been gradually taking the places emptied by the old euro currencies, in particular, the place of the deutschmark in eastern Europe. In a recent paper the ECB (Padua-Schioppa 2002) estimated that until May 2002 no less than 18 billion € were transferred to countries in eastern Europe. The fall of the Iron Curtain bolstered the deutschmark in the early 1990s. Fear of its conversion into the euro weakened it after 1997 and with it the euro itself. By the same logic, the euro has started to gain strength in the period since the conversion.

7.5 A Quantitative Assessment of the Effect

An important question is whether a decline of the monetary base by about 120 billion € against the trend can cause effects large enough to explain the actual exchange rate movements. The search for its answer requires an empirical determination of the corresponding reaction coefficients. Here we take two different approaches. First, we review the evidence from recent studies of micro data on the effect of money demand on the exchange rate. Second, we estimate a modified portfolio balance model, using macro data.

Recent contributions by Evans and Lyons (1999, 2001) on the "micro structure of the exchange rate" conclude that each billion of additional sterilized dollar currency demand raises the dollar exchange rate by up to half a cent in the short run and about 30 cents in the long run. If these figures apply equally to the euro, then our theory explains the depreciation of the euro by about 36 cents in the period 1997 to 2000. This is extremely close to the actual depreciation, which was 34 cents during this period.

In order to assess the co-movement of the exchange rate and relative money supplies from macro data, we now analyze empirically the determinants of the exchange rate. The question in the context of our model is whether the currency in circulation has a significant positive partial effect on the exchange rate of the euro in the presence of the other variables. The co-integration technique is used to study the empirical long-run relationship among the five variables relevant to our model: the exchange rate, relative money supplies, relative interest rates, relative bonds, and relative share prices. We analyze the co-movements for the period from 1984 to the end of 2001 for German, Japanese, UK, and Swiss exchange rates with respect to the United States.

The Johansen (1991) procedure is used to test for the presence of co-integration.[25] The Johansen test results are reported in panel A of table 7.1, along with the robustness of this model and some econometric issues. The long-run coefficients in the table were the exchange rates normalized to one. All variables are defined as in the theoretical model above.

The empirical results are consistent with our impression from the data analysis and the discussion in the previous sections. We first focus on the long-run coefficients. In all countries, except Switzerland, which used to control money supply rather than interest rates, the currency in circulation has a positive effect on the exchange value of the domestic currency. Because American and European bonds are perfect substitutes, this contradicts the view that a policy of fixing the interest rates eliminates the effect of currency demand changes on the exchange rate.

The positive correlation between the monetary base and the foreign exchange value of the currency had also been observed in earlier work by Frankel (1982, 1993), who called it the "mystery of the multiplying marks" and attributed it to model misspecifications or wealth effects in the monetary model of the exchange rate. Indeed, the positive correlation seems puzzling if the monetary base is seen as resulting from a supply policy of the central bank and active interventions. However, according to our model, the positive correlation has a straightforward explanation in the historical episode considered here if variations in the

Table 7.1
Currency augmented portfolio balance model Johansen co-integration results, 1984:1 to 2001:4

Variable	GER	UK	JAP	SWI
A. Long-run coefficients				
tr	80.57	62.18	82.55	115.25
cv	68.52	47.21	68.52	68.52
$\ln M - \ln M^*$	0.804	1.622	0.145	−7.825
	(0.414)	(0.219)	(0.993)	(9.145)
	(1.943)	(7.386)	(0.146)	(−0.856)
$\ln i - \ln i^*$	0.009	0.013	0.014	0.109
	(0.014)	(0.006)	(0.085)	(0.166)
	(0.680)	(2.090)	(0.166)	(0.659)
$\ln B - \ln B^*$	−0.129		0.024	2.443
	(0.197)		(0.164)	(2.636)
	(−0.654)		(0.151)	(0.926)
$\ln P - \ln P^*$	−1.179	−0.079	−0.025	3.970
	(0.257)	(0.091)	(0.153)	(5.247)
	(−4.580)	(−0.874)	(−0.164)	(0.756)
B. Reversion coefficients				
$\Delta(\ln e)$	0.134	−0.239	−0.003	−0.009
	(0.044)	(0.106)	(0.001)	(0.020)
	(3.009)	(−2.247)	(−1.838)	(−0.448)
$\Delta(\ln M - \ln M^*)$	0.155	0.140	0.988	−0.003
	(0.039)	(0.081)	(0.191)	(0.026)
	(3.922)	(1.725)	(5.168)	(−0.119)
$\Delta(\ln i - \ln i^*)$	0.240	−1.166	3.855	0.150
	(0.457)	(1.569)	(2.718)	(0.378)
	(0.524)	(−0.743)	(1.418)	(0.396)
$\Delta(\ln B - \ln B^*)$	0.019		0.022	0.036
	(0.035)		(0.189)	(0.012)
	(0.550)		(0.120)	(3.000)
$\Delta(\ln P - \ln P^*)$	−0.060	−0.176	−0.158	0.092
	(0.062)	(0.059)	(0.397)	(0.026)
	(−0.972)	(−2.961)	(−0.399)	(3.496)

Note: Bond data were not available for the United Kingdom. The Swiss data start in 1989, as stock market data were not available before. *tr* denotes the likelihood ratio test statistic for the null hypothesis of zero cointegrating vectors against the alternative of one cointegrating vector. The asymptotic critical values are denoted by "cv." In all cases, except for Switzerland, there exists only one cointegrating vector. Standard errors and *t*-statistics are in parentheses.

foreign and black market demand for a country's currency are taken into account.

The other estimates are also broadly in line with our theoretical model. The positive effect of the interest rate (for Germany, Japan, and Switzerland) on the value of the domestic currency can have two explanations. One is that it results from an increased preference for the domestic currency which, as indicated by (10) and (11), will imply a revaluation and an increase of the interest rate if the central bank does not intervene. The other is that the central bank actively intervenes by tightening the money supply. According to (11), this increases the difference of the marginal liquidity premia of money and bonds and hence the interest rate, and according to (10), it implies a revaluation.

Bonds have a smaller negative effect in Germany, although it is not statistically significant and may be the counterpart of the positive effect of money holdings, since interventions imply that bonds and money balances vary inversely.

The significant negative coefficient of share prices supports the puzzle established by De Grauwe (2000), that the value of an economy's currency varies inversely with its prosperity, which is the opposite of what the economic prosperity view predicts. By our model, the explanation for the negative correlation is that domestic shareholders whose preferences imply a home bias switch between domestic shares and domestic money, depending on the information they receive. This changes the marginal liquidity premium on domestic money balances conversely to share prices. According to equation (10) the domestic currency appreciates when share prices are low, and vice versa.

Given the co-integration result, we use a vector error correction model to explore the reaction to a deviation from the long-run equilibrium.[26] The responses of each of the variables to deviations from the long-run equilibrium are captured by the revision coefficients reported in table 7.1. In the cases of Germany, the United Kingdom, and Japan, the exchange rate and the relative money bases react to the deviations from the equilibrium, while most others do not.

It is known from the work of Meese and Rogoff (1983) and Taylor (1995) that the empirical research on exchange rate determination suffers from instability of the parameters over time, and poor out-of-sample performance. This problem also applies to our empirical exercise. In order to check the robustness of our estimation procedures, a set of appropriate tests was performed, using several estimation procedures that addressed econometric problems associated with this type of regression exercise. For example, we estimated an ARCH model, correcting for conditional heteroscedasticity, examined alternative lag structures in the co-integration exercise, and implemented an instrumental variables approach, aiming to reduce the endogeneity problem by way of lagged values as instruments. While most of our variables were affected by these alternative specifications, our main variable of interest, the relative money stocks, remained remarkably robust, exhibiting in most cases the significant positive correlation with the exchange rate predicted by our theory.

7.6 Conclusions

In this chapter we provide a criticism of the portfolio balance approach, and we attempt to develop a new theory of the exchange rate that we call the currency hypothesis. We take an explicit two-country portfolio model with money, bonds, and shares and show that there is little reason to expect the demand for shares to translate into the exchange rate because this demand is already reflected in the share price. We argue that what counts most is the stock demand for money in the narrow sense of the word. The exchange rate is the price of one type of money in terms of another and not the price of interest-bearing assets, as both portfolio managers and economists who developed the portfolio balances approach have claimed.

This theoretical result is confirmed by a number of empirical tests of exchange rates among various currencies. The tests demonstrated a strong and robust positive correlation between a country's stock of currency in circulation and the respective exchange value of this currency.

Our currency hypothesis is motivated historically by our observing the movements of the exchange value of the deutschmark and the euro from the time of the fall of the Iron Curtain to the physical conversion of the euro. We explain these co-movements in quantitative terms, using the "microstructure of the exchange rate" approach. With the fall of the Iron Curtain, the deutschmark became popular in eastern Europe in the early 1990s, leading to an unprecedented monetary expansion and the appreciation crisis of 1992. Fear of loss in its conversion into the euro reduced the demand for deutschmarks and weakened both the deutschmark and the euro after 1997. By the same logic, and predicted accurately by us in earlier contributions on this topic, the euro has gained in strength since the time of the physical conversion. A good reason for the appreciation of the euro is that it is ideally suited for black market operations and is finding friends in eastern Europe and elsewhere.

Notes

Earlier work along these lines was presented at a workshop on Exchange Rate and Monetary Policy Issues in Vienna, April 2001, and at the CESifo Macro and International Finance Area Conference in Munich, May 2001. We gratefully acknowledge useful comments by Paul De Grauwe, Walter Fisher, Huntley Schaller, and Haakon Solheim.

1. Hans-Werner Sinn, *Handelsblatt*, November 6, 2000, *Financial Times*, April 4, 2001, and *Süddeutsche Zeitung*, April 6, 2000. See also Paul Krugman's comment on Sinn in *New York Times*, April 1, 2001, and *Bundesbank Geschäftsbericht* of April 4, 2001.

2. Alternative explanations can be found in Alquist and Chinn (2002) and Corsetti (2000).

3. *Economist*, June 5, 1999, p. 13; April 20, 2000, pp. 25–26. *Der Spiegel*, October 2000, "Interner Bericht des Finanzministeriums fordert tiefgreifende Reformen zur Stabilisierung des Euro."

4. *Economist*, June 5, 1999, p. 14.

5. ECB, *Monthly Bulletin*, June 1999, p. 39.

6. Ibid.

7. Ibid.

8. *Der Spiegel*, online, Interview with Karl Otto Pöhl, June 19, 2000.

9. "Interner Bericht des Finanzministeriums . . . ," ibid. See also "Prospects for sustained growth in the Euro area," ch. 2, *European Economy*, vol. 71, 2000. Office for Official Publications of the EC, Luxembourg, pp. 62–67.

10. The literature ranges from Branson (1977), Branson, Halttunen, and Masson (1977), Branson and Henderson (1985), Girton and Henderson (1976), and Henderson (1980) to Dooley and Isaard (1982), Sinn (1983a), MacDonald and Taylor (1992), and Mann and Meade (2002), to mention only a few of the relevant papers. For a description of current research and further references, see Isaard (1995).

11. The officially measured savings rate does not include capital gains. This is not a problem in the present context where the savers' willingness to absorb assets offered in the capital market is concerned.

12. See note 8.

13. We also formulated a more elaborate model distinguishing, among other things, between American and European investors, but the more parsimonious model presented here is sufficient for the points we wish to make.

14. An increase in the portfolio volume will not affect share prices, interest rates, and the exchange rate if preferences are homothetic and growth does not change the actual portfolio structure. For simplicity we assume that this is the case.

15. This is the general structure of a multi-period stochastic portfolio decision problem. See Sinn (1983b) for a more extensive elaboration on this problem. Here we cut things short by considering one period only and simplifying the utility function.

16. See Fried and Howitt (1983) for a discussion of the potential liquidity services and a formulation along these lines.

17. Equation (12) specifies the interest rates rather than the exchange rates when the respective asset stocks are given and the ECB does not intervene. According to (3) and (6), in equilibrium the interest rates on American and European bonds have to adjust such that they complement the marginal liquidity services of bonds to generate the required overall return factor λ. This then automatically satisfies the interest parity condition without giving equation (12) much explanatory power for the determination of the exchange rate. When central banks intervene passively to fix the interest rates, the explanatory power increases.

Although (12) refers to the spot rate e, it can also be used to determine the forward rate $\tilde{e}_f$ by way of the covered interest parity condition $\tilde{e}_f = e \cdot (1+i^*)/(1+i)$. The forward rate is not the same as the expected future spot rate. The relationship between these rates is found by substituting (12) into the preceding equation:

$$\tilde{e}_f = \tilde{e}\frac{1+\beta U_B/(1+i)}{1+\beta^* U_{B^*}/(1+i^*)}.$$

This expression shows that a reduced preference for euro currency combined with the adjustment to the interest rate reduces the euro's forward rate relative to its expected future spot rate without affecting the forward premium or the swap rate.

18. In practice, the interventions by the ECB involved the sale of US treasury bonds, which required the Fed to react with an expansionary open market policy increasing the money supply so as to avoid an increase in the US interest rate.

19. It should be noted that the positive correlation between the stock of money balances and the foreign exchange value of this money that the currency hypothesis predicts refers to high-powered base money (M0) rather than broader money aggregates. There are two reasons why an extension of the argument to M1, M2, or M3 is not possible. First, demand, savings, and time deposits may be implicitly or explicitly interest bearing and may therefore classify as part of B rather than M in our model. Second, even if demand deposits and cash are considered as close substitutes by the public, M1 may not be positively correlated with M0. Suppose that the demand for euro cash declines. In that case, the cash will return to the banks in exchange for demand deposits. The money multiplier will increase and induce the banks to expand M1 by giving out more loans to their clients. This will contribute to the decline in the marginal utility of money and the downward effects on the exchange rate and the interest rate. Thus, before and without passive intervention by the ECB, there is a negative correlation between M1 and the exchange rate and none between M0 and the exchange rate. If the ECB intervenes to reestablish the targeted interest level, it can only partly offset the exchange rate effect, and it reduces M0 as was shown above. However, the net effect on M1 will be unclear. Indeed, M1 remained remarkably stable during the collapse of euro base money in the years before currency conversion.

20. See also chapter 1 by Evans and Lyons in this volume.

21. For analyses of this episode see Eichengreen and Wyplosz (1993), De Grauwe (1994), and Sinn (1999).

22. No less than 60 percent of the US monetary base is said to circulate outside the United States (see Porter and Judson 1996). The outstanding deutschmarks were a source of a significant seignorage profit made by the Bundesbank, as was calculated by Sinn and Feist (1997, 2000). When the euro was introduced, the deutschmark constituted a much larger fraction of the euro-11 monetary base than the share in the ECB profit remittances, which was only 31 percent, according to the average of Germany's GDP and population shares. Sinn and Feist calculated that this implied a seignorage loss which was equivalent to a one-off capital levy of nearly 60 billion DM or 30 billion € on the German Bundesbank.

23. It is also possible that some of the decline was due to other, more technical, reasons such as the ordinary citizen's attempt to minimize the stock of money balances at the time of currency conversion. However, all countries would have been affected in proportion to their GDP size. In this case the idiosynchratic component of the reduction in money demand applied to Germany was on the order of 75 billion €. Note that our estimates of the composition of the decline in money balances have only an informative character. None of our arguments for why a decline in money balances reduces the exchange value depends on the causes of this decline.

24. See, in particular, the articles in *Handelsblatt* and *Financial Times* published in 2000, as cited in note 1, as well as Sinn and Westermann (2001).

25. All series are nonstationary in levels and stationary in first differences. We let $\boldsymbol{x}_t$ be a 5×1 vector containing the variables $\{e, \ln M\text{–}\ln M^*, \ln i\text{–}\ln i^*, \ln B\text{–}\ln B^*, \ln P\text{–}\ln P^*\}$. The Johansen test statistics are devised from the sample canonical correlations (Anderson 1958; Marinell 1995) between $\Delta\boldsymbol{x}_t$ and $\boldsymbol{x}_{t-p}$, where t is time and p denotes the lag length, adjusting for all intervening lags. To implement the procedure, we first obtain the least squares residuals from

$$\Delta\boldsymbol{x}_t = \boldsymbol{\mu}_1 + \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j \Delta\boldsymbol{x}_{t-j} + \boldsymbol{\varepsilon}_{1t},$$

$$\boldsymbol{x}_{t-p} = \boldsymbol{\mu}_2 + \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j \Delta\boldsymbol{x}_{t-j} + \boldsymbol{\varepsilon}_{2t},$$

where $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are constant vectors, $\boldsymbol{\Gamma}$ is a matrix of parameters, and ε_1 amd ε_2 are vectors of the error terms. The lag parameter p is identified by the Akaike information criterion. Next, we compute the eigenvalues, $\lambda_1 \geq \cdots \geq \lambda_n$, of $\boldsymbol{\Omega}_{21}\boldsymbol{\Omega}_{11}^{-1}\boldsymbol{\Omega}_{12}$ with respect to $\boldsymbol{\Omega}_{22}$ and the associated eigenvectors, $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$, where the moment matrices

$$\boldsymbol{\Omega}_{lm} = T^{-1} \sum_t \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t'$$

for $l, m = 1, 2$, and n is the dimension of $\boldsymbol{x}_t$ (i.e., $n = 5$ in this exercise). $\lambda_1 \ldots \lambda_n$ are the squared canonical correlations between $\Delta\boldsymbol{x}_t$ and $\boldsymbol{x}_{t-p}$, adjusting for all intervening lags. The trace statistic,

$$tr = -T \sum_{j=r+1}^{n} \ln(1 - \lambda_j),$$

where $0 \leq r \leq n$, tests the hypothesis that there are at most r cointegration vectors. The eigenvectors, $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_r$ are sample estimates of the co-integration vectors.

26. Specifically, the changes in each of the five variables are modeled using $\Delta\boldsymbol{x}_t = \boldsymbol{\mu} + \sum_{j=1}^{p} \boldsymbol{\Gamma}_j \Delta\boldsymbol{x}_{tj} + \boldsymbol{a}\boldsymbol{ec}_{t-1} + \boldsymbol{\varepsilon}_t$, where $\boldsymbol{ec}_t$ is the error correction term.

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8 What Do We Know about Recent Exchange Rate Models? In-Sample Fit and Out-of-Sample Performance Evaluated

Yin-Wong Cheung, Menzie D. Chinn, and Antonio Garcia Pascual

In contrast to the intellectual ferment that followed the collapse of the Bretton Woods era, the 1990s were marked by a relative paucity of new *empirical* models of exchange rates. The sticky-price monetary model of Dornbusch and Frankel remained the workhorse of policy-oriented analyses of exchange rate fluctuations among the developed economies. However, while no completely new models were developed, several approaches gained increased prominence. Some of these approaches were inspired by new empirical findings, such as the correlation between net foreign asset positions and real exchange rates. Others, such as those based on productivity differences, were grounded in an older theoretical literature but given new respectability by the new international macroeconomics (Obstfeld and Rogoff 1996) literature. None of the empirical models, however, were subjected to rigorous examination of the sort that Frankel (1979) and Meese and Rogoff (1983a, b) conducted in their seminal works.

Consequently, instead of re-examining the usual suspects—the flexible price monetary model, purchasing power parity, and the interest differential[1]—we vary the set of performance criteria and expand the set to include the mean squared error, and the direction-of-change statistic. The later dimension is potentially more important from a market timing perspective, besides serving as another indicator of forecast attributes.

To summarize, in this study, we compare exchange rate models along several dimensions:

- Four models are compared against the random walk. Only one of the structural models—the benchmark sticky-price monetary model of Dornbusch and Frankel—has been the subject of previous systematic analyses. The other models include one incorporating productivity differentials in a fashion consistent with a Balassa-Samuelson formulation, an interest rate parity specification, and a representative behavioral equilibrium exchange rate model.

- The behavior of US dollar-based exchange rates of the Canadian dollar, British pound, German mark, Swiss franc, and Japanese yen are examined. We also examine the corresponding yen-based rates to ensure that our conclusions are not driven by dollar specific results.
- The models are estimated in two ways: in first-difference and error correction specifications.
- In sample fit is assessed in terms of how well the coefficient estimates conform to theoretical priors.
- Forecasting performance is evaluated at several horizons (1-, 4- and 20-quarter horizons), for a recent period not previously examined (post-1992).
- We augment the conventional metrics with a direction-of-change statistic and the "consistency" criterion of Cheung and Chinn (1998).

In accordance with previous studies, we find that no model consistently outperforms a random walk according to the mean squared error criterion at short horizons. However, at the longest horizon we find that the proportion of times the structural models incorporating long-run relationships outperform a random walk is more than would be expected if the outcomes were merely random. Using a 10 percent significance level, a random walk is outperformed 17 percent of the time along a MSE dimension and 27 percent along a direction of change dimension.

In terms of the "consistency" test of Cheung and Chinn (1998), we obtain slightly less positive results. The actual and forecasted rates are cointegrated more often than would occur by chance for all the models. While in many of these cases of cointegration, the condition of unitary elasticity of expectations is rejected; only about 5 percent fulfill all the conditions of the consistency criteria.

We conclude that the question of exchange rate predictability remains unresolved. In particular, while the oft-used mean squared error criterion provides a dismal perspective, criteria other than the conventional ones suggest that structural exchange rate models have some usefulness. Furthermore, structural models incorporating restrictions at long horizons tend to outperform random walk specifications.

8.1 Theoretical Models

The universe of empirical models that have been examined over the floating rate period is enormous. Consequently any evaluation of these models must necessarily be selective. The models we have selected are prominent in the economic and policy literature, and readily implementable and replicable. To our knowledge, with the exception of the sticky-price model, they have also not previously been evaluated in a systematic fashion. We use the random walk model as our benchmark naive model, in line with previous work, but we also select one model—the Dornbusch (1976) and Frankel (1979) model—as a representative of the 1970s vintage models. The sticky-price monetary model can be expressed as follows:

$$s_t = \beta_0 + \beta_1 \hat{m}_t + \beta_2 \hat{y}_t + \beta_3 \hat{i}_t + \beta_4 \hat{\pi}_t + u_t, \tag{1}$$

where s is exchange rate in log, m is log money, y is log real GDP, i and π are the interest and inflation rate, respectively, the caret (ˆ) denotes the intercountry difference, and u_t is an error term.

The characteristics of this model are well known, so we will not devote time to discuss the theory behind the equation. We will observe, however, that the list of variables included in (1) encompasses those employed in the flexible price version of the monetary model, as well as the micro-based general equilibrium models of Stockman (1980) and Lucas (1982).

Second, we assess models that are in the Balassa-Samuelson vein, in that they accord a central role to productivity differentials in explaining movements in real, and hence also nominal, exchange rates (see Chinn 1997). Such models drop the purchasing power parity assumption for broad price indexes and allow the real exchange rate to depend on the relative price of nontradables, itself a function of productivity (z) differentials. A generic productivity differential exchange rate equation is

$$s_t = \beta_0 + \beta_1 \hat{m} + \beta_2 \hat{y} + \beta_3 \hat{i} + \beta_5 \hat{z}_t + u_t. \tag{2}$$

The third set of models we examine we term the "behavioral equilibrium exchange rate" (BEER) approach. We investigate this model as a proxy for a diverse set of models that incorporate a number of familiar relationships. A typical specification is

$$s_t = \beta_0 + \hat{p}_t + \beta_6 \hat{\omega}_t + \beta_7 \hat{r}_t + \beta_8 \hat{g}debt_t + \beta_9 tot_t + \beta_{10} nfa_t + u_t, \tag{3}$$

where p is the log price level (CPI), ω is the relative price of nontradables, r is the real interest rate, *gdebt* is the government debt to GDP ratio, *tot* is the log terms of trade, and *nfa* is the net foreign asset ratio. A unitary coefficient is imposed on $\hat{p}_t$. This specification can be thought of as incorporating the Balassa-Samuelson effect, the real interest differential model, an exchange risk premium associated with government debt stocks, and additional portfolio balance effects arising from the net foreign asset position of the economy.[2] Evaluation of this model can shed light on a number of very closely related approaches, including the macroeconomic framework of the IMF (Isard et al. 2001) and Stein's NATREX (Stein 1999). The empirical determinants in both approaches overlap with those of the specification in equation (3).

Models based on this framework have been the predominant approach to determining the level at which currencies will gravitate to over some intermediate horizon, especially in the context of policy issues. For instance, the behavioral equilibrium exchange rate approach is the model that is most used to determine the long-term value of the euro.

The final specification assessed is not a model per se; rather it is an arbitrage relationship—uncovered interest rate parity:

$$s_{t+k} - s_t = \hat{i}_{t,k}, \tag{4}$$

where $\hat{i}_{t,k}$ is the interest rate of maturity k. Unlike the other specifications, this relation does not need to be estimated in order to generate predictions.

Interest rate parity at long horizons has recently gathered empirical support (Alexius 2001; Chinn and Meredith 2002), in contrast to the disappointing results at the shorter horizons. MacDonald and Nagayasu (2000) have also demonstrated that long-run interest rates can predict exchange rate levels. On the basis of these findings, we anticipate that this specification will perform better at the longer horizons than at the shorter.[3]

8.2 Data and Full-Sample Estimation

8.2.1 Data

The analysis uses quarterly data for the United States, Canada, the United Kingdom, Japan, Germany, and Switzerland over the 1973:2 to

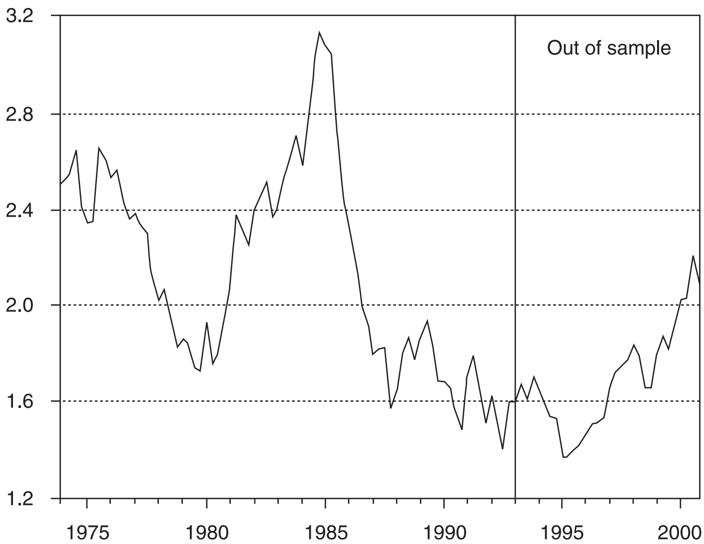

Figure 8.1
German mark–US dollar exchange rate

2000:4 period. The exchange rate, money, price and income variables are drawn primarily from the IMF's *International Financial Statistics*. The productivity data were obtained from the Bank for International Settlements, while the interest rates used to conduct the interest rate parity forecasts are essentially the same as those used in Chinn and Meredith (2002). See appendix A for a more detailed description.

The out-of-sample period used to assess model performance is 1993:1 to 2000:4. Figures 8.1 and 8.2 depict, respectively, the dollar based German mark and yen exchange rates, with the vertical line indicating the beginning of the out-of-sample period. The out-of-sample period spans a period of dollar depreciation and then sustained appreciation.[4]

8.2.2 Full-Sample Estimation

Two specifications of the theoretical models were estimated: (1) an error correction specification, and (2) a first-differences specification. Since implementation of the error correction specification is relatively involved, we will address the first-difference specification to begin

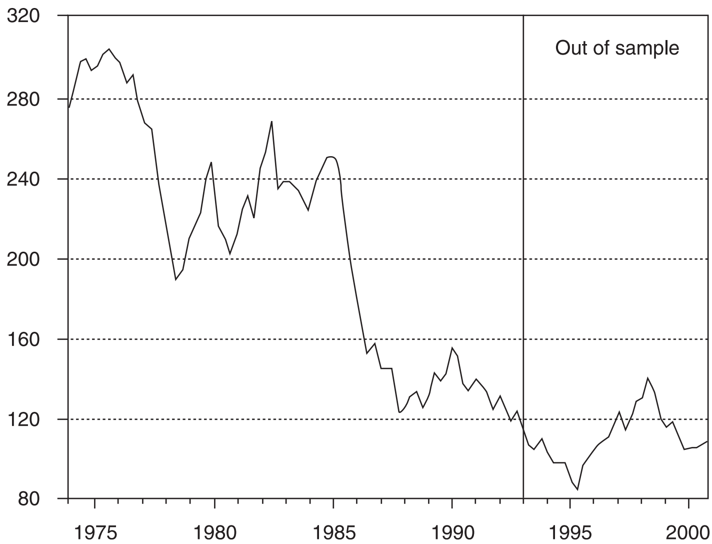

Figure 8.2
Japanese yen–US dollar exchange rate

with. Consider the general expression for the relationship between the exchange rate and fundamentals:

$$s_t = X_t\Gamma + u_t, \tag{5}$$

where X_t is a vector of fundamental variables under consideration. The first-difference specification involves the following regression:

$$\Delta s_t = \Delta X_t\Gamma + u_t. \tag{6}$$

These estimates are then used to generate forecasts one and many quarters ahead. Since these exchange rate models imply joint determination of all variables in the equations, it makes sense to apply instrumental variables. However, previous experience indicates that the gains in consistency are far outweighed by the loss in efficiency, in terms of prediction (Chinn and Meese 1995). Hence we rely solely on OLS.

One exception to this general rule is the UIP model. In this case the arbitrage condition implies a relationship between the change in the exchange rate and the level of the interest rate differential. Since no long-run condition is implied, we simply estimate the UIP relationship as stated in equation (4).

8.2.3 *Empirical Results*

The results of estimating the sticky-price monetary model in levels are presented in panel A of table 8.1. Using the 5 percent asymptotic critical value, we find that there is evidence of cointegration for the dollar-based exchange rates for all currencies save one. The German mark stands out as a case where it is difficult to obtain evidence of cointegration; we suspect that this is largely because of the breaks in the series for both money and income associated with the German reunification. The evidence for cointegration is more attenuated when the finite sample critical values (Cheung and Lai 1993) are used. Then only the Canadian dollar and yen have some mixed evidence in favor of cointegration.

This ambiguity is useful to recall when evaluating the estimates for the British sterling; the coefficient estimates do not conform to those theoretically implied by the model, as the coefficients of money, inflation and income are all incorrectly signed (although the latter two are insignificantly so). Only the interest rate coefficient is significant and correctly signed. In contrast, both the yen and franc broadly conform to the monetary model. Money and inflation are correctly signed, while interest rates enter in correctly only for the yen. Finally, the Canadian dollar presents some interesting results. The coefficients are largely in line with the monetary model, although the income coefficient is wrongly signed, with economic and statistical significance.

The use of the first-difference specification is justified when there is a failure to find evidence of cointegration (the German mark), or alternatively one suspects that estimates of the long-run coefficients are insufficiently precisely estimated to yield useful estimates. In panel B of table 8.1, the results from the first-difference specification are reported. A general finding is that the coefficients do not typically enter with both statistical significance and correct sign. One partial exception is the interest differential coefficient. Higher interest rates, if all else constant is held constant, appear to appreciate the currency in four of five cases, although the yen–dollar rate estimate is not statistically significant. The British sterling–dollar rate estimate is positive (while the inflation rate coefficient is not statistically significant), a finding that is more consistent with a flexible price monetary model than a sticky-price one. Otherwise, the fit does not appear particularly good.

These mixed results are suggestive of alternative approaches; the first we examine is the productivity-based model. Our interpretation

Table 8.1
Full-sample estimates of sticky-price model

	Sign	BP/$	Can$/$	DM/$	SF/$	Yen/$
A. In levels[a]						
Cointegration (asy)		1, 1	3, 1	0, 0	1, 1	1, 1
Cointegration (fs)		0, 0	1, 0	0, 0	0, 0	0, 1
Money	[+]	−2.89*	1.10*	2.14*	3.61*	1.29
		(1.01)	(0.25)	(0.74)	(0.74)	(0.96)
Income	[−]	1.64	9.70*	0.93	−1.10	0.77
		(3.94)	(1.87)	(1.87)	(1.72)	(1.97)
Interest rate	[−]	−19.49*	−6.44*	−5.86	2.09	−17.11*
		(4.01)	(3.27)	(4.14)	(5.73)	(4.72)
Inflation rate	[+]	−7.11	10.74*	24.29*	40.96*	26.56*
		(4.60)	(3.11)	(4.27)	(6.79)	(4.03)
B. In first differences[b]						
Money	[+]	−0.21	−0.00	0.16	−0.02	0.44
		(0.12)	(0.06)	(0.22)	(0.14)	(0.24)
Income	[−]	−2.02*	−0.48	−0.51	0.59	−0.00
		(0.42)	(0.29)	(0.43)	(0.52)	(0.39)
Interest rate	[−]	0.83*	−0.42*	−0.91*	−0.82*	−0.28
		(0.41)	(0.10)	(0.45)	(0.37)	(0.33)
Inflation rate	[+]	−0.15	−0.07	1.26	1.29	0.32
		(0.48)	(0.20)	(1.09)	(0.81)	(0.44)

Note: "Sign" indicates coefficient sign implied by theoretical model. * indicates significantly different from zero at the 5% marginal significance level. Estimates for DM include shift and impulse dummies for German monetary and economic unification.

a. Long-run cointegrating estimates from Johansen procedure (standard errors in parentheses), where the VECM includes two lags of first differences. The number of cointegrating vectors is implied by the trace and maximal eigenvalue statistics, using the 5% marginal significance level; "asy" denotes asymptotic critical values and "fs" denotes finite sample critical values of Cheung and Lai (1993) that are used.

b. OLS estimates (Newey-West standard errors in parentheses, truncation lag = 4).

of the model simply augments the monetary model with a productivity variable. The results for this model are presented in table 8.2. From the asymptotic critical values, the evidence of cointegration in panel A of table 8.2 is comparable to that reported in panel A of table 8.1. For both the British sterling and Canadian dollar, there is evidence of multiple cointegrating vectors. However, in using the finite sample critical values, we find that the number of implied vectors drops to one (or zero) in this case.

In all cases the interest coefficient is correctly signed, and significant in most cases. Furthermore the money and inflation variables are correctly signed in most cases. The productivity coefficients are significant and consistent with the productivity in three cases—the Swiss franc, German mark, and yen. The latter two currencies have previously been found to be influenced by productivity trends.[5]

Estimates of the first-difference specifications do not yield appreciably better results than their sticky-price counterparts. Interest differentials tend to be important, once again, while productivity fails to evidence any significant impact for three of five rates. To the extent that one thinks that productivity is a slowly trending variable that influences the real exchange rate over long periods, this result is unsurprising. While this variable has the correct sign for the German mark–dollar rate, it has the opposite for the sterling–dollar rate.

The Canadian dollar appears to be as resilient to being modeled using this productivity specification as the others. Chen and Rogoff (2002) have asserted that the Canadian dollar is mostly determined by commodity prices; hence it is not surprising that both models fail to have any predictive content.

The BEER model results are presented in table 8.3. There are no estimates for the Swiss franc and the yen because we lack quarterly data on government debt and net foreign assets. Overall, the results are not uniformly supportive of the BEER approach.[6] Although there are some instances of correctly signed coefficients, none show up correctly signed across all three currencies. Moving to a first-difference specification does not improve the results. Besides those on the relative price and real interest rate differentials, very few coefficient estimates are in line with model predictions. For the DM/$ rate, the real interest rate and debt variables possess the correctly signed coefficients, as do the relative price and net foreign assets for the Canadian dollar, but these appear to be isolated instances.[7]

Table 8.2
Full-sample estimates of productivity model

	Sign	BP/$	Can$/$	DM/$	SF/$	Yen/$
A. In levels[a]						
Cointegration (asy)		1, 2	2, 2	0, 0	1, 1	1, 1
Cointegration (fs)		0, 0	1, 0	0, 0	0, 0	0, 1
Money	[+]	0.97*	6.81*	0.62*	2.00*	0.18
		(0.47)	(1.45)	(0.33)	(0.30)	(0.54)
Income	[−]	−4.11*	25.76*	−0.68	−1.04	2.77*
		(1.23)	(6.62)	(0.81)	(0.76)	(1.29)
Interest rate	[−]	−10.63*	−34.53*	−9.35*	3.67	−12.07*
		(1.65)	(11.16)	(2.57)	(2.54)	(2.67)
Inflation rate	[+]	9.86*	70.63*	9.18*	15.36*	12.09*
		(1.63)	(12.00)	(1.85)	2.79	(2.49)
Productivity	[−]	3.56*	16.78*	−5.66*	−4.43*	−2.65*
		(0.68)	(5.60)	(1.11)	(1.46)	(0.76)
B. In first differences[b]						
Money	[+]	0.40*	−0.00	0.16	−0.01	0.43
		(0.16)	(0.06)	(0.22)	(0.14)	(0.24)
Income	[−]	−1.59*	−0.47	−0.51	0.70	0.00
		(0.39)	(0.29)	(0.43)	(0.51)	(0.40)
Interest rate	[−]	−0.57	−0.42*	−0.91*	−0.82*	−0.28
		(0.46)	(0.10)	(0.45)	(0.41)	(0.32)
Inflation rate	[+]	1.10*	−0.08	1.26	1.19	0.37
		(0.50)	(0.20)	(1.09)	(0.81)	(0.45)
Productivity	[−]	1.11*	−0.03	−5.66*	−0.25	−0.32
		(0.21)	(0.15)	(1.11)	(0.21)	(0.31)

Note: "Sign" indicates coefficient sign implied by theoretical model. * indicates significantly different from zero at the 5% marginal significance level. Estimates for DM include shift and impulse dummies for German monetary and economic unification.
a. Long-run cointegrating estimates from Johansen procedure (standard errors in parentheses), where the VECM includes two lags of first differences. The number of cointegrating vectors is implied by the trace and maximal eigenvalue statistics, using the 5% marginal significance level; "asy" denotes asymptotic critical values and "fs" denotes finite sample critical values of Cheung and Lai (1993) that are used.
b. OLS estimates (Newey-West standard errors in parentheses, truncation lag = 4).

Table 8.3
Full-sample estimates of BEER model

	Sign	BP/$	Can$/$	DM/$
A. In levels[a]				
Cointegration (asy)		2, 2	4, 2	1, 1
Cointegration (fs)		1, 2	2, 1	0, 0
Relative price	[−]	1.27*	−1.05*	−9.38*
		(0.38)	(0.34)	(1.36)
Real interest rate	[−]	−3.13*	2.03*	−2.37
		(1.07)	(0.91)	(2.09)
Debt	[+]	−1.06*	−2.62*	0.04
		(0.30)	(0.51)	(0.72)
Terms of trade	[−]	−0.92	0.75*	−0.13
		(0.82)	(0.24)	(1.04)
Net foreign assets	[−]	5.65*	−1.39*	−4.88*
		(0.56)	(0.40)	(0.76)
B. In first differences[b]				
Relative price	[−]	−0.55	−0.44*	−0.38
		(0.56)	(0.17)	(0.59)
Real interest rate	[−]	−0.17	−0.15	−1.04*
		(0.16)	(0.11)	(0.34)
Debt	[+]	−0.38	0.18	1.52*
		(0.27)	(0.22)	(0.64)
Terms of trade	[−]	0.09	0.02	0.59*
		(0.31)	(0.06)	(0.27)
Net foreign assets	[−]	2.61*	−1.19*	3.14*
		(0.49)	(0.25)	(0.72)

Note: "Sign" indicates coefficient sign implied by theoretical model. * indicates significantly different from zero at the 5% marginal significance level. Estimates for DM include shift and impulse dummies for German monetary and economic unification.
a. Long-run cointegrating estimates from Johansen procedure (standard errors in parentheses), where the VECM includes 2 lags of first differences (4 lags for DM). The number of cointegrating vectors is implied by the trace and maximal eigenvalue statistics, using the 5% marginal significance level; "asy" denotes asymptotic critical values and "fs" denotes finite sample critical values of Cheung and Lai (1993) that are used.
b. OLS estimates (Newey-West standard errors in parentheses, truncation lag = 4).

Table 8.4
Uncovered interest parity estimates

		BP/$	Can$/$	DM/$	SF/$	Yen/$
Horizon						
3 month		−2.19*	−0.48*	−0.70	−1.28*	−2.99*
		(1.08)	(0.51)	(1.09)	(1.04)	(0.96)
	Adj R^2	0.04	−0.00	−0.01	0.01	0.06
	SER	0.21	0.08	0.26	0.29	0.28
1 year		−1.42*	−0.61*	−0.58*	−1.05*	−2.60*
		(0.99)	(0.49)	(0.66)	(0.52)	(0.69)
	Adj R^2	0.06	0.03	0.00	0.04	0.17
	SER	0.11	0.04	0.14	0.14	0.13
5 year		0.44	0.24	0.52	−1.18*	1.19
		(0.36)	(0.47)	(0.75)	(0.97)	(0.38)
	Adj R^2	0.02	−0.00	0.02	0.04	0.13
	SER	0.04	0.02	0.06	0.04	0.05

Note: OLS estimates (Newey-West standard errors in parentheses, truncation lag = $k-1$). SER is standard error of regression. * indicates significantly different from *unity* at the 5 percent marginal significance level.

Although we do not use estimated equations to conduct the forecasting of the UIP model, it is informative to consider how well the data conform to the UIP relationship. As is well known, at short horizons, the evidence in favor of UIP is lacking.[8] The results of estimating equation (4) are reported in table 8.4. Consistent with Chinn and Meredith (2002), the short-horizon data (1 quarter and 4 quarter maturities) provide almost uniformly negative coefficient estimates, in contradiction to the implication of the UIP hypothesis. At the five-year horizon, the results are substantially different for all cases, save the Swiss franc. Now all the coefficients are positive; moreover in no case except the franc is the coefficient estimate significantly different from the theoretically implied value of unity.

8.3 Forecast Comparison

8.3.1 Estimation and Forecasting

We adopt the convention in the empirical exchange rate modeling literature of implementing "rolling regressions." That is, estimates are applied over a given data sample, out-of-sample forecasts produced, then the sample is moved up, or "rolled" forward one observation before the procedure is repeated. This process continues until all the out-of-sample observations are exhausted. This procedure is selected over recursive estimation because it is more in line with previous work, including the original Meese and Rogoff paper. Moreover the power of the test is kept constant as the sample size over which the estimation occurs is fixed, rather than increasing as it does in the recursive framework.

The error correction estimation involves a two-step procedure. In the first step, the long-run cointegrating relation implied by (5) is identified using the Johansen procedure, as described in section 8.2. The estimated cointegrating vector ($\tilde{\Gamma}$) is incorporated into the error correction term, and the resulting equation

$$s_t - s_{t-k} = \delta_0 + \delta_1(s_{t-k} - X_{t-k}\tilde{\Gamma}) + u_t \tag{7}$$

is estimated via OLS. Equation (7) can be thought of as an error correction model stripped of the short-run dynamics. A similar approach was used in Mark (1995) and Chinn and Meese (1995), except for the fact that, in those two cases, the cointegrating vector was imposed a priori.

One key difference between our implementation of the error correction specification and that undertaken in some other studies involves the treatment of the cointegrating vector. In some other prominent studies (MacDonald and Taylor 1994) the cointegrating relationship is estimated over the entire sample, and then out-of-sample forecasting undertaken, where the short-run dynamics are treated as time varying *but the long-run relationship is not*. While there are good reasons for adopting this approach—in particular, one wants to use as much information as possible to obtain estimates of the cointegrating relationships—the asymmetry in the estimation approach is troublesome, and makes it difficult to distinguish quasi–ex ante forecasts from true ex ante forecasts. Consequently our estimates of the long-run cointegrating relationship vary as the data window moves.

It is also useful to stress the difference between the error correction specification forecasts and the first-difference specification forecasts. In the latter, ex post values of the right-hand side variables are used to generate the predicted exchange rate change. In the former, contemporaneous values of the right-hand side variables are not necessary, and the error correction predictions are true ex ante forecasts. Hence we are affording the first-difference specifications a tremendous informational advantage in forecasting.[9]

8.3.2 *Forecast Comparison*

To evaluate the forecasting accuracy of the different structural models, the ratio between the mean squared error (MSE) of the structural models and a driftless random walk is used. A value smaller (larger) than one indicates a better performance of the structural model (random walk). We also explicitly test the null hypothesis of no difference in the accuracy of the two competing forecasts (structural model vs. driftless random walk). In particular, we use the Diebold-Mariano statistic (Diebold and Mariano 1995), which is defined as the ratio between the sample mean loss differential and an estimate of its standard error; this ratio is asymptotically distributed as a standard normal.[10] The loss differential is defined as the difference between the squared forecast error of the structural models and that of the random walk. A consistent estimate of the standard deviation can be constructed from a weighted sum of the available sample autocovariances of the loss differential vector. Following Andrews (1991), a quadratic spectral kernel is employed, together with a data-dependent bandwidth selection procedure.[11]

We also examine the predictive power of the various models along different dimensions. One might be tempted to conclude that we are merely changing the well-established “rules of the game” by doing so. However, there are very good reasons to use other evaluation criteria. First, there is the intuitively appealing rationale that minimizing the mean squared error (or relatedly mean absolute error) may not be important from an economic standpoint. A less pedestrian motivation is that the typical mean squared error criterion may miss out on important aspects of predictions, especially at long horizons. Christoffersen and Diebold (1998) point out that the standard mean squared error criterion indicates no improvement of predictions that take into account cointegrating relationships vis à vis univariate predictions. But surely any reasonable criteria would put some weight on the tendency for predictions from cointegrated systems to “hang together.”

Hence, our first alternative evaluation metric for the relative forecast performance of the structural models is the direction-of-change statistic, which is computed as the number of correct predictions of the direction of change over the total number of predictions. A value above (below) 50 percent indicates a better (worse) forecasting performance than a naive model that predicts the exchange rate has an equal chance to go up or down. Again, Diebold and Mariano (1995) provide a test statistic for the null of no forecasting performance of the structural model. The statistic follows a binomial distribution, and its studentized version is asymptotically distributed as a standard normal. Not only does the direction-of-change statistic constitute an alternative metric, it is also an approximate measure of profitability. We have in mind here tests for market-timing ability (Cumby and Modest 1987).[12]

The third metric we used to evaluate forecast performance is the consistency criterion proposed in Cheung and Chinn (1998). This metric focuses on the time series properties of the forecast. The forecast of a given spot exchange rate is labeled as consistent if (1) the two series have the same order of integration, (2) they are cointegrated, and (3) the cointegration vector satisfies the unitary elasticity of expectations condition. Loosely speaking, a forecast is consistent if it moves in tandem with the spot exchange rate in the long run. Cheung and Chinn (1998) provide a more detailed discussion on the consistency criterion and its implementation.

8.4 Comparing the Forecast Performance

8.4.1 The MSE Criterion

The comparison of forecasting performance based on MSE ratios is summarized in table 8.5. The table contains MSE ratios and the p-values from five dollar-based currency pairs, four structural models, the error correction and first-difference specifications, and three forecasting horizons. Every cell in the table has two entries. The first one is the MSE ratio (the MSEs of a structural model to the random walk specification). The entry underneath the MSE ratio is the p-value of the hypothesis that the MSEs of the structural and random walk models are the same. Because of the lack of data, the behavioral equilibrium exchange rate model is not estimated for the dollar–Swiss franc, dollar–yen exchange rates, and all yen-based exchange rates. Altogether there are 153 MSE ratios. Of these 153 ratios, 90 are computed from the error correction specification and 63 from the first-difference one.

Note that in the tables only "error correction specification" entries are reported for the interest rate parity model. This model is not

Table 8.5
MSE ratios from the dollar-based and yen-based exchange rates

Specification	Horizon	S–P	IRP	PROD	BEER	S–P	IRP	PROD
Panel A		*BP/$*				*BP/yen*		
ECM	1	1.0469	1.0096	1.0795	1.1597	0.9709	1.0421	1.0266
		0.3343	0.6613	0.1827	0.0909	0.5831	0.6269	0.7905
	4	1.0870	0.7696	1.1974	1.5255	1.1466	1.0008	1.4142
		0.5163	0.3379	0.2571	0.0001	0.3889	0.9975	0.3171
	20	0.4949	0.9810	0.7285	1.2841	1.2020	0.7611	1.7493
		0.1329	0.9581	0.5225	0.4016	0.1302	0.5795	0.0295
FD	1	1.0357		1.1678	1.8876	0.9655		1.0000
		0.7095		0.4255	0.0092	0.7175		1.0000
	4	1.2691		1.3830	3.7789	1.1191		1.1114
		0.3260		0.1038	0.0004	0.6543		0.6886
	20	6.0121		2.2029	18.370	4.5445		4.7881
		0.0000		0.0021	0.0000	0.0000		0.0000
Panel B		*CAN$/$*				*CAN$/yen*		
ECM	1	1.0365	1.0849	1.0537	1.2644	0.9617	1.0096	0.9948
		0.3991	0.0316	0.3994	0.0018	0.2537	0.8710	0.9269
	4	1.0681	1.0123	1.1194	1.5570	0.9716	1.0045	1.1185
		0.2531	0.9592	0.2015	0.0002	0.7037	0.9814	0.4038
	20	0.6339	0.1881	1.0204	1.7609	1.1694	0.6462	4.8827
		0.0248	0.0001	0.9276	0.0302	0.2747	0.4125	0.1130
FD	1	1.0474		1.0842	0.5424	1.0106		0.9827
		0.6214		0.3971	0.1544	0.9144		0.8456
	4	0.9866		1.0519	1.2907	1.1578		1.1663
		0.9531		0.8232	0.5046	0.5751		0.5827

	20	0.2051		0.2937	4.7274	12.181		12.12
		0.0318		0.1018	0.0000	0.0000		0.0000
Panel C		*DM/$*				*DM/yen*		
ECM	1	0.9990	1.0705	0.9867	1.0810	1.0447	0.9662	0.9983
		0.5440	0.0383	0.5858	0.1951	0.3200	0.4790	0.0528
	4	0.9967	1.2090	0.9298	1.0484	1.0006	0.8571	1.0003
		0.5861	0.0694	0.2956	0.3109	0.5779	0.3238	0.7265
	20	1.0242	1.0073	1.0410	0.6299	1.0034	0.5485	0.9921
		0.0004	0.9354	0.0030	0.0891	0.6003	0.0480	0.1126
FD	1	1.0354		1.1208	0.4649	1.0227		1.0060
		0.3020		0.1959	0.0009	0.7181		0.9219
	4	1.1184		1.1782	0.3331	1.0859		1.0045
		0.2019		0.0029	0.0059	0.1849		0.9625
	20	2.0817		1.9828	1.2906	0.9521		0.8569
		1.1915		0.0000	0.2550	0.7217		0.3572
Panel D		*SF/$*				*SF/yen*		
ECM	1	0.9784	1.1101	1.1200		0.9961	0.9985	1.0515
		0.7773	0.0692	0.1614		0.9333	0.9522	0.2892
	4	0.8864	1.2871	1.0409		1.0627	0.9276	1.0140
		0.4152	0.0689	0.7438		0.2595	0.3983	0.7786
	20	1.2873	1.4894	0.9651		0.8331	0.9031	0.9216
		0.1209	0.0000	0.8684		0.2925	0.4856	0.1019
FD	1	1.3115		1.3891		0.9350		0.9338
		0.1641		0.1734		0.1643		0.1765
	4	1.6856		1.8437		1.0114		0.9666
		0.0774		0.0713		0.8595		0.7366
	20	5.6773		5.9918		0.9208		0.8852
		0.0000		0.0000		0.0000		0.0001

Table 8.5
(continued)

Specification	Horizon	S–P	IRP	PROD	BEER	S–P	IRP	PROD
Panel E		*Yen/$*						
ECM	1	0.9821	1.0681	0.9973				
		0.8799	0.2979	0.9647				
	4	0.8870	1.2047	0.9460				
		0.6214	0.2862	0.7343				
	20	0.8643	0.9824	0.8500				
		0.4299	0.9661	0.3856				
FD	1	1.0022		0.9456				
		0.9840		0.4427				
	4	1.0240		1.0624				
		0.8207		0.5342				
	20	2.7132		2.2586				
		0.0000		0.0001				

Note: The results are based on dollar-based and yen-based exchange rates and their forecasts. Each cell has two entries. The first is the MSE ratio (the MSEs of a structural model to the random walk specification). The entry underneath the MSE ratio is the *p*-value of the hypothesis that the MSEs of the structural and random walk models are the same (Diebold and Mariano 1995). The notation used in the table is ECM: error correction specification; FD: first-difference specification; S–P: sticky-price model; IRP: interest rate parity model; PROD: productivity differential model; and BEER: behavioral equilibrium exchange rate model. The forecasting horizons (in quarters) are listed under the heading “horizon.” The forecasting period is 1993:1 to 2000:4. Due to data unavailability, the BEER model was not estimated for the Japanese yen and Swiss franc.

estimated; rather the predicted spot rate is calculated using the uncovered interest parity condition. To the extent that long-term interest rates can be considered the error correction term, we believe this categorization is most appropriate.

Overall, the MSE results are not favorable to the structural models. Of the 153 MSE ratios, 109 are not significant (at the 10 percent significance level), and 44 are significant. That is, for the majority of the cases one cannot differentiate the forecasting performance between a structural model and a random walk model. For the 44 significant cases, there are 32 cases in which the random walk model is significantly better than the competing structural models and only 11 cases in which the opposite is true. As 10 percent is the size of the test and 12 cases constitute less than 10 percent of the total of 153 cases, the empirical evidence can hardly be interpreted as supportive of the superior forecasting performance of the structural models. One caveat is necessary, however. When one restricts attention to the long-horizon forecasts, it turns out that those incorporating long-run restrictions outperform a random walk more often than would be expected to occur randomly: five out of 30 cases, or 17 percent, using a 10 percent significance level.

Inspecting the MSE ratios, one does not observe many consistent patterns, in terms of outperformance. It appears that the BEER model does not do particularly well except for the DM/$ rate. The interest rate parity model tends to do better at the 20-quarter horizon than at the 1- and 4-quarter horizons—a result consistent with the well-known bias in forward rates at short horizons.

In accordance with the existing literature, our results are supportive of the assertion that it is very difficult to find forecasts from a structural model that can consistently beat the random walk model using the MSE criterion. The current exercise further strengthens the assertion as it covers both dollar- and yen-based exchange rates and some structural models that have not been extensively studied before.

8.4.2 The Direction-of-Change Criterion

Table 8.6 reports the proportion of forecasts that correctly predicts the direction of the exchange rate movement and, underneath these sample proportions, the p-values for the hypothesis that the reported proportion is significantly different from 0.5. When the proportion statistic is significantly larger than 0.5, the forecast is said to have the ability to predict the direct of change. On the other hand, if the statistic is

Table 8.6
Direction-of-change statistics from the dollar-based and yen-based exchange rates

Specification	Horizon	S–P	IRP	PROD	BEER	S–P	IRP	PROD
Panel A		*BP/$*				*BP/yen*		
ECM	1	0.5312	0.4849	0.5313	0.4062	0.5625	0.4546	0.6563
		0.7236	0.8618	0.7237	0.2888	0.4795	0.6015	0.0771
	4	0.5862	0.5455	0.4483	0.3448	0.5517	0.6364	0.5517
		0.3531	0.6015	0.5775	0.0946	0.5774	0.1172	0.5775
	20	0.8461	0.7273	0.7692	0.3846	0.5384	0.5758	0.2308
		0.0125	0.0090	0.0522	0.4053	0.7815	0.3841	0.0522
FD	1	0.5937		0.4688	0.4062	0.5937		0.4375
		0.2888		0.7237	0.2888	0.2888		0.4795
	4	0.5517		0.5172	0.3448	0.6551		0.5862
		0.5774		0.8527	0.0946	0.0946		0.3532
	20	0.3076		0.1539	0.3076	0.0000		0.0000
		0.1655		0.0126	0.1655	0.0000		0.0000
Panel B		*CAN$/$*				*CAN$/yen*		
ECM	1	0.4062	0.3939	0.3438	0.3125	0.5937	0.4849	0.6250
		0.2888	0.2230	0.0771	0.0338	0.2888	0.8618	0.1573
	4	0.4827	0.4242	0.4828	0.1724	0.6206	0.5758	0.5172
		0.8526	0.3841	0.8527	0.0004	0.1936	0.3841	0.8527
	20	0.7692	1.0000	0.4615	0.0769	0.5384	0.7273	0.2308
		0.0522	0.0000	0.7815	0.0022	0.7815	0.0090	0.0522
FD	1	0.5312		0.5625	0.6250	0.5000		0.4375
		0.7236		0.4795	0.1573	1.0000		0.4795
	4	0.7586		0.7241	0.5862	0.5172		0.4828
		0.0053		0.0158	0.3531	0.8526		0.8527

	20	1.0000		1.0000	0.0000	0.3076		0.3077
		0.0000		0.0000	0.0000	0.1655		0.1655
Panel C		*DM/$*				*DM/yen*		
ECM	1	0.5000	0.3030	0.3750	0.5625	0.6250	0.5152	0.5000
		1.0000	0.0236	0.1573	0.4795	0.1573	0.8618	1.0000
	4	0.5517	0.3030	0.3103	0.4827	0.4137	0.6667	0.3793
		0.5774	0.0236	0.0411	0.8526	0.3531	0.0555	0.1937
	20	0.0769	0.5152	0.2308	0.2307	0.6923	0.8485	0.6154
		0.0022	0.8618	0.0522	0.0522	0.1655	0.0001	0.4054
FD	1	0.5000		0.4063	0.8125	0.4687		0.5000
		1.0000		0.2888	0.0004	0.7236		1.0000
	4	0.3448		0.2759	0.7931	0.4827		0.4483
		0.0946		0.0158	0.0015	0.8526		0.5775
	20	0.0769		0.0769	0.3076	0.3076		0.4615
		0.0022		0.0023	0.1655	0.1655		0.7815
Panel D		*SF/$*				*SF/yen*		
ECM	1	0.5625	0.3030	0.5625		0.6562	0.6061	0.4688
		0.4795	0.0236	0.4795		0.0771	0.2230	0.7237
	4	0.5517	0.3636	0.5517		0.4827	0.5758	0.4138
		0.5774	0.1172	0.5775		0.8526	0.3841	0.3532
	20	0.5384	0.4546	0.6923		0.5384	0.5000	0.6154
		0.7815	0.6698	0.1655		0.7815	1.0000	0.4054
FD	1	0.4062		0.4375		0.5937		0.6875
		0.2888		0.4795		0.2888		0.0339
	4	0.4137		0.5172		0.5517		0.5862
		0.3531		0.8527		0.5774		0.3532
	20	0.2307		0.2308		0.5384		0.6154
		0.0522		0.0522		0.7815		0.4054

Table 8.6
(continued)

Specification	Horizon	S–P	IRP	PROD	BEER	S–P	IRP	PROD
Panel E		*Yen/$*						
ECM	1	0.6562	0.3636	0.5625				
		0.0771	0.1172	0.4795				
	4	0.5517	0.5152	0.4828				
		0.5774	0.8618	0.8527				
	20	0.7692	0.5152	0.6923				
		0.0522	0.8618	0.1655				
FD	1	0.6875		0.6563				
		0.0338		0.0771				
	4	0.6551		0.6207				
		0.0946		0.1937				
	20	0.0000		0.0000				
		0.0000		0.0000				

Note: The table reports the proportion of forecasts that correctly predict the direction of the dollar-based and yen-based exchange rate movements. Under each direction-of-change statistic, the *p*-values for the hypothesis that the reported proportion is significantly different from 0.5 is listed. When the statistic is significantly larger than 0.5, the forecast is said to have the ability to predict the direct of change. If the statistic is significantly less than 0.5, the forecast tends to give the wrong direction of change. The notation used in the table is ECM: error correction specification; FD: first-difference specification; S–P: sticky-price model; IRP: interest rate parity model; PROD: productivity differential model; and BEER: behavioral equilibrium exchange rate model. The forecasting horizons (in quarters) are listed under the heading "horizon." The forecasting period is 1993:1 to 2000:4. Due to data unavailability, the BEER model was not estimated for the Japanese yen and Swiss franc.

significantly less than 0.5, the forecast tends to give the wrong direction of change. If a model consistently forecasts the direction of change incorrectly, traders can derive a potentially profitable trading rule by going against these forecasts. Thus, for trading purposes, information regarding the significance of "incorrect" prediction is as useful as the one of "correct" forecasts. However, in evaluating the ability of the model to describe exchange rate behavior, we separate the two cases.

There is mixed evidence on the ability of the structural models to correctly predict the direction of change. Among the 153 direction-of-change statistics, 23 (27) are significantly larger (less) than 0.5 at the 10 percent level. The occurrence of the significant outperformance cases is slightly higher (15 percent) than the one implied by the 10 percent level of the test. The results indicate that the structural model forecasts *can* correctly predict the direction of the change, although the proportion of cases where a random walk outperforms the competing models is higher than what one would expect if they occurred randomly.

Let us take a closer look at the incidences in which the forecasts are in the right direction. About half of the 23 cases are in the error correction category (12). Thus it is not clear if the error correction specification—which incorporates the empirical long-run relationship—is a better specification for the models under consideration.

Among the four models under consideration, the sticky-price model has the highest number (10) of forecasts that give the correct direction-of-change prediction (18 percent of these forecasts), while the interest rate parity model has the highest proportion of correct predictions (19 percent). Thus, at least on this count, the newer exchange rate models do not significantly edge out the "old fashioned" sticky-price model save perhaps the interest rate parity condition.

The cases of correct direction prediction appear to cluster at the long forecast horizon. The 20-quarter horizon accounts for 10 of the 23 cases while the 4-quarter and 1-quarter horizons have, respectively, 6 and 7 direction-of-change statistics that are significantly larger than 0.5. Since there have been few studies utilizing the direction-of-change statistic in similar contexts, it is difficult to make comparisons. Chinn and Meese (1995) apply the direction-of-change statistic to three-year horizons for three conventional models, and find that performance is largely currency-specific: the no-change prediction is outperformed in the case of the dollar–yen exchange rate, while all models are outperformed in the case of the dollar–sterling rate. In contrast, in our study at the 20-quarter horizon, the positive results appear to be concentrated in the

Table 8.7
Cointegration between exchange rates and their forecasts

Specification	Horizon	S–P	IRP	PROD	BEER	S–P	IRP	PROD
Panel A		*BP/$*				*BP/yen*		
ECM	1	2.12	14.25*	2.41	19.26*	8.70	5.35	5.06
	4	4.88	5.72	6.98	18.13*	26.54*	3.99	7.26
	20	9.69*	8.71	16.45*	6.54	6.27	5.25	4.02
FD	1	8.51		19.05*	7.66	15.85*		5.50
	4	8.30		7.32	4.53	5.34		5.38
	20	2.78		7.73	1.87	8.77		8.80
Panel B		*CAN$/$*				*CAN$/yen*		
ECM	1	6.74	6.03	3.41	6.32	6.94	6.59	7.77
	4	6.31	5.87	1.97	5.80	2.85	4.18	1.13
	20	6.58	7.03	8.96	4.53	7.22	9.51	4.29
FD	1	14.42*		15.60*	12.53*	15.07*		13.87*
	4	10.97*		7.22	6.22	5.64		4.20
	20	3.87		4.08	1.93	6.31		6.50
Panel C		*DM/$*				*DM/yen*		
ECM	1	2.78	11.18*	3.11	8.38	2.43	5.71	5.57
	4	4.74	11.72*	2.83	6.42	14.77*	4.39	9.50
	20	1.17	1.01	11.09*	3.30	7.12	13.97*	6.45
FD	1	14.99*		7.21	7.63	14.28*		16.37*
	4	8.37		7.36	3.02	42.41*		3.58
	20	1.37		1.20	5.17	5.55		5.84

Panel D		*SF/$*				*SF/yen*		
ECM	1	1.08	6.88	3.24	—	5.12	2.76	10.31*
	4	22.52*	6.84	34.23*	—	1.57	108.57*	3.25
	20	0.69	6.93	0.49	—	4.05	4.72	6.39
FD	1	2.73		1.02	—	4.40		47.89*
	4	5.21		1.65	—	1.81		3.10
	20	2.90		2.78	—	7.83		7.01
Panel E		*Yen/$*						
ECM	1	14.82*	12.20*	4.84	—			
	4	5.73	10.93*	5.33	—			
	20	14.99*	1.05	13.16*	—			
FD	1	20.48*		25.39*	—			
	4	5.61		42.86*	—			
	20	15.06*		13.17*	—			

Note: The table reports the Johansen maximum eigenvalue statistic for the null hypothesis that a dollar-based (or a yen-based) exchange rate and its forecast are not cointegrated. * indicates the 10% marginal significance level. Tests for the null of one cointegrating vector were also conducted, but in all cases the null was not rejected. The notation used in the table is ECM: error correction specification; FD: first-difference specification; S–P: sticky-price model; IRP: interest rate parity model; PROD: productivity differential model; and BEER: behavioral equilibrium exchange rate model. The forecasting horizons (in quarters) are listed under the heading "horizon." The forecasting period is 1993:1 to 2000:4. The dash indicates that the statistics were not generated due to unavailability of data.

yen–dollar and Canadian dollar–dollar rates.[13] It is interesting to note that the direction-of-change statistic works for the interest rate parity model almost only at the 20-quarter horizon, thus mirroring the MSE results. This pattern is entirely consistent with the finding that uncovered interest parity holds better at long horizons.

8.4.3 The Consistency Criterion

The consistency criterion only requires the forecast and actual realization comove one-to-one in the long run. One could argue that the criterion is less demanding than the MSE and direction-of-change metrics. Indeed, a forecast that satisfies the consistency criterion can (1) have a MSE larger than that of the random walk model, (2) have a direction-of-change statistic less than 0.5, or (3) generate forecast errors that are serially correlated. However, given the problems related to modeling, estimation, and data quality, the consistency criterion can be a more flexible way to evaluate a forecast. In assessing the consistency, we first test if the forecast and the realization are cointegrated.[14] If they are cointegrated, then we test if the cointegrating vector satisfies the $(1, -1)$ requirement. The cointegration results are reported in table 8.7. The test results for the $(1, -1)$ restriction are reported in table 8.8.

Thirty-eight of 153 cases reject the null hypothesis of no cointegration at the 10 percent significance level. Thus 25 percent of forecast series are cointegrated with the corresponding spot exchange rates. The error correction specification accounts for 20 of the 38 cointegrated cases and the first-difference specification accounts for the remaining 18 cases. There is no evidence that the error correction specification gives better forecasting performance than the first-difference specification.

Interestingly the sticky-price model garners the largest number of cointegrated cases. There are 54 forecast series generated under the sticky-price model. Fifteen of these 54 series (i.e., 28 percent) are cointegrated with the corresponding spot rates. Twenty-six percent of the interest rate parity and 24 percent of the productivity model are cointegrated with the spot rates. Again, we do not find evidence that the recently developed exchange rate models outperform the "old" vintage sticky-price model.

The yen–dollar has 10 out of the 15 forecast series that are cointegrated with their respective spot rates. The Canadian dollar–dollar pair, which yields relatively good forecasts according to the direction-of-change metric, has only 4 cointegrated forecast series. Evidently the forecasting performance is not just currency specific; it also depends on the evaluation criterion. The distribution of the cointegrated cases across forecasting horizons is puzzling. The frequency of occurrence is inversely proportional to the forecasting horizons. There are 19 of 51 one-quarter ahead forecast series that are cointegrated with the spot rates. However, there are only 11 of the 4-quarter ahead and 8 of the 20-quarter ahead forecast series that are cointegrated with the spot rates. One possible explanation for this result is that there are fewer observations in the 20-quarter ahead forecast series, and this effects the power of the cointegration test.

The results of testing for the long-run unitary elasticity of expectations at the 10 percent significance level are reported in table 8.8. The condition of long-run unitary elasticity of expectations, that is, the $(1, -1)$ restriction on the cointegrating vector, is rejected by the data quite frequently. The $(1, -1)$ restriction is rejected in 33 of the 38 cointegration cases. That is 13 percent of the cointegrated cases display long-run unitary elasticity of expectations. Taking both the cointegration and restriction test results together, 3 percent of the 153 cases meet the consistency criterion.

8.4.4 Discussion

Several aspects of the foregoing analysis merit discussion. To begin with, even at long horizons, the performance of the structural models is less than impressive along the MSE dimension. This result is consistent with those in other recent studies, although we have documented this finding for a wider set of models and specifications. Groen (2000) restricted his attention to a flexible price monetary model, while Faust et al. (2001) examined a portfolio balance model as well; both remained within the MSE evaluation framework.

Expanding the set of criteria does yield some interesting surprises. In particular, the direction-of-change statistics indicate more evidence that structural models can outperform a random walk. However, the basic conclusion that no economic model is consistently more successful than the others remains intact. This, we believe, is a new finding.

Even if we cannot glean from this analysis a consistent "winner," it may still be of interest to note the best and worst performing combinations of model/specification/currency. The best performance on the MSE criterion is turned in by the interest rate parity model at the

Table 8.8
Results of (1, −1) restriction test

Specification	Horizon	S–P	IRP	PROD	BEER	S–P	IRP	PROD
Panel A		*BP/$*				*BP/yen*		
ECM	1	—	39.66	—	0.32	—	—	—
		—	0.00	—	0.57	—	—	—
	4	—	—	—	19.99	49.55	—	—
		—	—	—	0.00	0.00	—	—
	20	445.3	—	458.91	—	—	—	—
		0.00	—	0.00	—	—	—	—
FD	1	—		1.56	—	24.73		—
		—		0.21	—	0.00		—
	4	—		—	—	—		—
		—		—	—	—		—
	20	—		—	—	—		—
		—		—	—	—		—
Panel B		*CAN$/$*				*CAN$/yen*		
ECM	1	—	—	—	—	—	—	—
		—	—	—	—	—	—	—
	4	—	—	—	—	—	—	—
		—	—	—	—	—	—	—
	20	—	—	—	—	—	—	—
		—	—	—	—	—	—	—
FD	1	16.58		15.73	1263	17.17		28.50
		0.00		0.00	0.00	0.00		0.00
	4	132.5		—	—	—		
		0.00		—	—	—		

	20	—		—	—	—		
		—		—	—	—		
Panel C		*DM/$*				*DM/yen*		
ECM	1	—	164.5	—	—	—	—	—
		—	0.00	—	—	—	—	—
	4	—	392.97	—	—	11.20	—	—
		—	0.00	—	—	0.00	—	—
	20	—	—	535.13	—	—	5.06	—
		—	—	0.00	—	—	0.02	—
FD	1	6.73		—	—	3.40		3.40
		0.00		—	—	0.06		0.07
	4	—		—	—	3.88		—
		—		—	—	0.04		—
	20	—		—	—	—		—
		—		—	—	—		—
Panel D		*SF/$*				*SF/yen*		
ECM	1	—	—	—		—	—	4.56
		—	—	—		—	—	0.03
	4	3.34	—	9.77		—	313.12	—
		0.06	—	0.00		—	0.00	—
	20	—	—	—		—	—	—
		—	—	—		—	—	—
FD	1	—		—		—		31.07
		—		—		—		0.00
	4	—		—		—		—
		—		—		—		—
	20	—		—		—		—
		—		—		—		—

Table 8.8
(continued)

Specification	Horizon	S–P	IRP	PROD	BEER	S–P	IRP	PROD
Panel E		*Yen/$*						
ECM	1	62.10	209.36	—				
		0.00	0.00	—				
	4	—	33.58	—				
		—	0.00	—				
	20	876.4	—	1916				
		0.00	—	0.00				
FD	1	0.582		1.03				
		0.445		0.31				
	4	—		1.14				
		—		0.29				
	20	436.4		289.22				
		0.00		0.00				

Note: The likelihood ratio test statistic for the restriction of $(1, -1)$ on the cointegrating vector and its p-value are reported. The test is only applied to the cointegration cases present in table 8.3. The notation used in the table is ECM: error correction specification; FD: first-difference specification; S–P: sticky-price model; IRP: interest rate parity model; PROD: productivity differential model; and BEER: behavioral equilibrium exchange rate model. The forecasting horizons (in quarters) are listed under the heading “horizon.” The forecasting period is 1993:1 to 2000:4.

20-quarter horizon for the Canadian dollar–yen exchange rate, with a MSE ratio of 0.19 (*p*-value of 0.0001). The worst performances are associated with first-difference specifications; in this case the highest MSE ratio is for the first differences specification of the sticky-price exchange rate model at the 20-quarter horizon for the Canadian dollar–US dollar exchange rate. However, the other catastrophic failures in prediction performance are distributed across first-difference specifications of the various models, so the key determinant in this pattern of results appears to be the difficulty in estimating *stable* short-run dynamics. (We take here into account the fact that these predictions utilize ex post realizations of the right-hand side variables.)

Overall, the inconstant nature of the parameter estimates appears to be closely linked with the erratic nature of the forecasting performance. This applies to the variation in long-run estimates and reversion coefficients, but perhaps most strongly to the short-run dynamics obtained in the first-differences specifications.

8.5 Concluding Remarks

In this chapter we systematically assess the in-sample fit and out-of-sample predictive capacities of models developed during the 1990s. These models are compared along a number of dimensions, including econometric specification, currencies, and differing metrics.

Our investigation does not reveal that any particular model or any particular specification fit the data well, in terms of providing estimates in accord with theoretical priors. Of course, this finding is dependent on a very simple specification search, and we used theory to discipline variable selection and information criteria to select lag lengths.

On the other hand, some models seem to do well at certain horizons, for certain criteria. Indeed, it may be that one model will do well for one exchange rate and not for another. For instance, the productivity model does well for the mark–yen rate along the direction-of-change and consistency dimensions (although not by the MSE criterion), but that same conclusion cannot be applied to any other exchange rate.

Similarly we fail to find any particular model or specification that out-performed a random walk on a consistent basis. Again, we imposed the disciplining device of using a given specification, and a given out-of-sample forecasting period. Perhaps most interestingly, there is little apparent correlation between how well the in-sample estimates accord with theory and out-of-sample prediction performance.

The only link between in-sample and out-of-sample performance is an indirect one, for the interest parity condition. It is well known that interest rate differentials are biased predictors of future spot rate movements at short horizons. However, the improved predictive performance at longer horizons does accord with the fact that uncovered interest parity is more likely to hold at longer horizons than at short horizons.

In sum, while the results of our study have been fairly negative regarding the predictive capabilities of newer empirical models of exchange rates, in some sense we believe the findings pertain more to difficulties in estimation, rather than the models themselves. And this may point the direction for future research avenues.[15]

Appendix A: Data

Unless otherwise stated, we use seasonally adjusted quarterly data from the *IMF International Financial Statistics* ranging from the second quarter of 1973 to the last quarter of 2000. The exchange rate data are end of period exchange rates. Money is measured as narrow money (essentially M1), with the exception of the United Kingdom, where M0 is used. The output data are measured in constant 1990 prices. The consumer and producer price indexes also use 1990 as base year.

The three-month, annual, and five-year interest rates are end-of-period constant maturity interest rates and are obtained from the IMF country desks. See Meredith and Chinn (1998) for details. Five-year interest rate data were unavailable for Japan and Switzerland; hence data from Global Financial Data http://www.globalfindata.com/ were used, specifically, five-year government note yields for Switzerland and five-year discounted bonds for Japan.

The productivity series are labor productivity indexes, measured as real GDP per employee, converted to indexes (1995 = 100). These data are drawn from the Bank for International Settlements database.

The net foreign asset (NFA) series is computed as follows. Using stock data for year 1995 on NFA (Lane and Milesi-Ferretti 2001) at http://econserv2.bess.tcd.ie/plane/data.html, and flow quarterly data from the IFS statistics on the current account, we generated quarterly stocks for the NFA series (with the exception of Japan, for which there is no quarterly data available on the current account).

To generate quarterly government debt data, we follow a similar strategy. We use annual debt data from the IFS statistics, combined with quarterly government deficit (surplus) data. The data source for Canadian government debt is the Bank of Canada. For the United Kingdom, the IFS data are updated with government debt data from the public sector accounts of the UK Statistical Office (for Japan and Switzerland, we have very incomplete data sets, and hence no behavioral equilibrium exchange rate models are estimated for these two countries).

Appendix B: Evaluating Forecast Accuracy

The Diebold-Mariano statistics (Diebold and Mariano 1995) are used to evaluate the forecast performance of the different model specifications relative to that of the *naive* random walk.

Given the exchange rate series x_t and the forecast series y_t, the loss function L for the mean square error is defined as

$$L(y_t) = (y_t - x_t)^2. \tag{A1}$$

Testing whether the performance of the forecast series is different from that of the naive random walk forecast z_t is equivalent to testing whether the population mean of the loss differential series d_t is zero. The loss differential is defined as

$$d_t = L(y_t) - L(z_t). \tag{A2}$$

Under the assumptions of covariance stationarity and short-memory for d_t, the large-sample statistic for the null of equal forecast performance is distributed as a standard normal, and can be expressed as

$$\frac{\bar{d}}{\sqrt{2\pi \sum_{\tau=-(T-1)}^{(T-1)} l(\tau/S(T)) \sum_{t=|\tau|+1}^{T} (d_t - \bar{d})(d_{t-|\tau|} - \bar{d})}}, \tag{A3}$$

where $l(\tau/S(T))$ is the lag window, $S(T)$ is the truncation lag, and T is the number of observations. Different lag-window specifications can be applied, such as the Barlett or the quadratic spectral kernels, in combination with a data-dependent lag-selection procedure (Andrews 1991).

For the direction-of-change statistic, the loss differential series is defined as follows: d_t takes a value of one if the forecast series correctly predicts the direction of change, otherwise it will take a value of zero.

Hence a value of $\bar{d}$ significantly larger than 0.5 indicates that the forecast has the ability to predict the direction of change; on the other hand, if the statistic is significantly less than 0.5, the forecast tends to give the wrong direction of change. In large samples, the studentized version of the test statistic,

$$\frac{\bar{d} - 0.5}{\sqrt{0.25/T}}, \tag{A4}$$

is distributed as a standard normal.

Notes

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1. A recent review of the empirical literature on the monetary approach is provided by Neely and Sarno (2002).

2. See Clark and MacDonald (1999), Clostermann and Schnatz (2000), Yilmaz and Jen (2001), and Maeso-Fernandez et al. (2001) for recent applications of this specification. On the portfolio balance channel, Cavallo and Ghironi (2002) provide a role for net foreign assets in the determination of exchange rates in the sticky-price optimizing framework of Obstfeld and Rogoff (1995).

3. Despite this finding, there is little evidence that long-term interest rate differentials—or equivalently long-dated forward rates—have been used for forecasting at the horizons we are investigating. One exception from the professional literature is Rosenberg (2001).

4. The findings reported below are not very sensitive to the forecasting periods (Cheung, Chinn, and Garcia Pascual 2002).

5. For the pound, the productivity coefficient is incorrectly signed, although this finding is combined with a very large (and correctly signed) income coefficient, which suggests some difficulty in disentangling the income from productivity effects.

6. Overall, the interpretation of the results is complicated by the fact that, for the level specifications, multiple cointegrating vectors are indicated using the asymptotic critical values. The use of finite sample critical values reduces the implied number of cointegrating vectors, as indicated in the second row, to one or two vectors. Hence we do not believe the assumption of one cointegrating vector does much violence to the data.

7. One substantial caveat is necessary at this point. BEER models have almost uniformly been couched in terms of multilateral exchange rates; hence the interpretation of the BEERs in a bilateral context does not exactly replicate the experiments conducted by BEER exponents. On the other hand, the fact that it is difficult to obtain the theoretically

implied coefficient signs suggests that some searching is necessary in order to obtain a "good" fit.

8. Two recent exceptions to this characterization are Flood and Rose (2002) and Bansal and Dahlquist (2000). Flood and Rose conclude that UIP holds much better for countries experiencing currency crises, while Bansal and Dahlquist find that UIP holds much better for a set of non-OECD countries. Neither of these descriptions applies to the currencies examined in this study.

9. We opted to exclude short-run dynamics in equation (7) because, on the one hand, the use of equation (7) yields true ex ante forecasts and makes our exercise directly comparable with, for example, Mark (1995), Chinn and Meese (1995), and Groen (2000), and on the other, the inclusion of short-run dynamics creates additional demands on the generation of the right-hand-side variables and the stability of the short-run dynamics that complicate the forecast comparison exercise beyond a manageable level.

10. In using the DM test, we are relying on asymptotic results, which may or may not be appropriate for our sample. However, generating finite sample critical values for the large number of cases we deal with would be computationally infeasible. More important, the most likely outcome of such an exercise would be to make detection of statistically significant out-performance even more rare, and leaving our basic conclusion intact.

11. We also experimented with the Bartlett kernel and the deterministic bandwidth selection method. The results from these methods are qualitatively very similar. In appendix B we provide a more detailed discussion of the forecast comparison tests.

12. See also Leitch and Tanner (1991), who argue that a direction of change criterion may be more relevant for profitability and economic concerns, and hence a more appropriate metric than others based on purely statistical motivations.

13. Using Markov switching models, Engel (1994) obtains some success along the direction of change dimension at horizons of up to one year. However, his results are not statistically significant.

14. The Johansen method is used to test the null hypothesis of no cointegration. The maximum eigenvalue statistics are reported in the manuscript. Results based on the trace statistics are essentially the same. Before implementing the cointegration test, both the forecast and exchange rate series were checked for the $I(1)$ property. For brevity, the $I(1)$ test results and the trace statistics are not reported.

15. Our survey is necessarily limited, and we leave open the question of whether alternative statistical techniques can yield better results, for example, nonlinearities (Meese and Rose 1991; Kilian and Taylor 2001), fractional integration (Cheung 1993), and regime switching (Engel and Hamilton 1990), cointegrated panel techniques (Mark and Sul 2001), and systems-based estimates (MacDonald and Marsh 1997).

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9 The Euro–Dollar Exchange Rate: Is It Fundamental?

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The evolution of the euro exchange rate vis-à-vis the main international currencies, and particularly, the US dollar, has given rise to a growing literature. Contrary to the more or less general expectations of appreciation, the euro has been in its first three years of existence depreciating against the dollar. Many arguments have been given in search of fundamentals, but the results are up to now puzzling (e.g., see De Grauwe 2000 or Meredith 2001). Two arguments can be put forth to support this fact. First, an analysis based on fundamentals cannot be performed on a short-term basis. Although the operators in the money markets seem to be working in a chartist world, from a policy-oriented view the data span has to be long enough to capture the long-run equilibria relationships, and the econometric framework based on cointegration is the most appropriate methodology for this purpose. Second, and related to the preceding argument, the absence of historical data for the euro makes it necessary to use aggregate variables in order to expand the series backward (ECB 2000). This "synthetic" euro and the aggregate euro area variables have an important qualification: they summarize the evolution of the legacy currencies that developed in the framework of rather heterogeneous economic environments.[1] This heterogenous character and its contribution to the "strength" of the euro were pointed out by De Grauwe (1997). In this chapter we propose a complementary approach that shows how to overcome these problems. Our main attempt will be to compare the behavior of the bilateral real exchange rates for the individual euro-area countries in a panel with the performance of a model estimated using aggregate euro-area variables for the period 1970 to 1998 in terms of quarterly data. To make the results fully comparable, we restrict the countries analyzed to those with information available for the whole period. Concerning the econometric techniques applied, we first use the pooled mean group (PMG) estimator proposed by Pesaran, Shin, and Smith (1999) for nonstationary regressors and estimate a panel for a group of euro-area currencies. This method constrains the long-run coefficients to be identical but allows error variances and short-run parameters to differ. By this method we also will be able to capture the long-run relationships consistently with the medium- and long-run orientation of the fundamental exchange rate models and the objectives of European monetary policy. Also it should enable us to understand the different responses of the euro-area countries. Second, we estimate an aggregate bilateral model for the euro–dollar real exchange rate. We use the standard Johansen cointegration analysis method to arrive at the long-run determinants of the real exchange rate based on the current values of the variables. With this framework we are further able to test for regime shifts or structural breaks. However, we must bear in mind that these changes can only be detected with a significant delay. Thus, even if the creation of the European Monetary Union has provoked a change in regime, it is still too early to be able to detect it using the available techniques.

The remainder of the chapter is organized as follows. In section 9.1 we provide an overview of the recent empirical literature on the issue of exchange rate determination in the euro case. In section 9.2, we describe the theoretical models and in section 9.3 present the econometric results. Finally, in section 9.4 we report the main results and conclusions.

9.1 Recent Empirical Literature[2]

A traditional starting point for estimating equilibrium exchange rate has been the PPP theory, either in its absolute or relative version. However, due to a different bulk of factors well documented in the literature, the speed of adjustment of the current value of exchange rate to the long-run equilibrium is very slow. Therefore other approaches have been implemented over time. Basically these approaches can be classified in accord with two strands of literature: fundamental equilibrium exchange rate (FEER) or behavioral equilibrium exchange rates (BEER).[3] The caveat to the first approach is its normative nature. This is due to the fact that under the FEER approach the exchange rate has to be consistent with internal and external balance. Thus we think, as Clark and MacDonald (1999) point out, that the behavioral approach may be a better empirical approach to exchange rate modeling because the computation is based on current levels of the fundamental factors. The problem is to determine the correct combination of fundamental variables, and the answer is largely empirical. Over the past two years different econometric techniques were implemented in several studies in line with the behavioral approach. Alberola et al. (1999) using cointegration techniques for individual currencies as well as for a panel of currencies found only a long-run relationship with net foreign assets and relative sectoral prices (the Balassa-Samuelson effect), and Ledo and Taguas (1999) found that the deviations from PPP can be explained largely by productivity differentials and interest rate differentials in an error correction model. Additionally Closterman and Schnatz (2000) found an equilibrium relationship for the bilateral euro–dollar exchange rate that includes the productivity differential, the interest rate differential, the real oil price, and the relative fiscal position. Makrydakis et al. (2000) found a relation with the productivity differential and the real interest rate differential as in Alquist and Chinn (2001). Finally Maeso-Fernández et al. (2001) found the euro to be mainly affected by productivity developments, real interest rate differentials, and external shocks due to oil dependence of the euro area. All the models taken together appear to encompass useful information, so any assessment about the evolution of the real exchange rate should start to build in some way on this broad-based multifaceted range of analysis (ECB 2002).

9.2 Theoretical Models: An Eclectic Nested Approach

As in the euro–dollar case discussed above, the most recent empirical evidence on real exchange rates has not been able to secure a position among traditional theoretical models. In his search of an answer to the problems associated with modeling exchange rates and, in particular, real exchange rates, MacDonald (1998) has proposed an eclectic approach to model real exchange rates. Meese and Rogoff (1988), in their study of the link between real exchange rates and real interest rate differentials, have tried to solve some of the problems related to the monetary models. They define the real exchange rate, q_t, as $q_t \equiv e_t - p_t + p_t^*$, where e_t is the price of a unit of foreign currency in terms of domestic currency and p_t and p_t^* are the logarithms of domestic and foreign prices. Three assumptions are made: first, that when a shock occurs, the real exchange rate returns to its equilibrium value at a constant rate; second, that the long-run real exchange rate, $\hat{q}_t$, is a nonstationary variable; finally, that uncovered real interest rate parity is fulfilled.

Combining the three assumptions above, the real exchange rate can be expressed in the following form:

$$q_t = -\varphi(R_t - R_t^*) + \hat{q}_t, \tag{1}$$

where R_t^* and R_t are, respectively, the real foreign and domestic interest rates for an asset of maturity k. This leaves relatively open the question of which are the determinants of $\hat{q}_t$, which is a nonstationary variable.

Meese and Rogoff real exchange rate model has been very influential in the empirical literature. As Edison and Melick (1995) show in their paper, the implementation of the empirical tests depends on the treatment of the expected real exchange rate derived from equation (1). The simplest model will assume that the expected real exchange rate is constant, while the models including other variables will specify it using other determinants.

The model was first tested, in its simplest version, by Campbell and Clarida (1987) and Meese and Rogoff (1988). The former found little of the movement in real exchange rates to be explained by movements in real interest differentials. Meese and Rogoff (1988), using cointegration techniques (Engle and Granger single-equation tests), could not find a long-run relationship between the two variables. However, Baxter (1994) found more encouraging results, and in a recent paper, MacDonald and Nagayasu (2000) tested this relationship for 14 industrialized countries using both long- and short-term real interest rate differentials and time series as well as panel cointegration methods. After obtaining evidence of statistically significant long-run relationships and plausible point estimates using panel tests, they concluded that the failure of previous researches was probably due to the estimation method used rather than to any theoretical deficiency.

In a second group of papers, the assumption that the expected real exchange rate is constant is relaxed, and additional variables are introduced in an attempt to explain it. This approach was first introduced by Hooper and Morton (1982), who modeled the expected real exchange rate as a function of cumulated current account. Edison and Pauls (1993) and Edison and Melick (1995) estimate the same model using cointegration techniques. In the second paper they find evidence of a cointegrating relationship, after Edison and Pauls (1993) failed to find a statistical link between real exchange rates and real interest rates using the Engle-Granger methodology. However, the estimated error correction models are more supportive of such a relation. Wu (1999) has recently also obtained good results (even in forecasting ability) for this type of specification applied to Germany and Japan in relation to the dollar exchange rate and using the Johansen technique.

MacDonald (1998) used this approach, dividing the real exchange rate determinants into two components: the real interest rate differential and a set of fundamentals that explains the behavior of the long-run (equilibrium) real exchange rate, which include productivity differentials, the effect of relative fiscal balances on the equilibrium real exchange rate, the private sector savings, and the real price of oil. We will describe this eclectic approach in more detail because it forms the basis of our analysis.

MacDonald assumes that PPP holds for nontraded goods, so he arrives at the following expression for the long-run equilibrium real exchange rate:

$$\hat{q}_t \equiv q_t^T + \alpha_t(p_t^T - p_t^{NT}) - \alpha_t^*(p_t^{T^*} - p_t^{NT^*}), \tag{2}$$

where q_t^T is the real exchange rate for traded goods; $(p_t^T - p_t^{NT}) - (p_t^{T^*} - p_t^{NT^*})$ is the relative price of traded to nontraded goods between the home and the foreign country and α and α^* are the weights.

By way of (2), MacDonald identifies two potential sources of variation in the equilibrium real exchange rate:

1. Movements in the relative prices of traded to nontraded goods between the home and foreign country (second and third terms in equation 2). These differences are mostly concentrated in nontraded goods. In particular, according to the traditional Balassa-Samuelson effect, productivity differences in the production of traded goods across countries can introduce a bias in the overall real exchange rate. This is because productivity advances tend to concentrate in the traded goods sectors. Because of the linkages between prices of goods and wages (and wages across sectors), provided that there is internal factor mobility (from the nontraded to the traded goods sectors and conversely), the real exchange rate tends to appreciate in fast growing economies.

2. Nonconstancy of the real exchange rate for traded goods, q_t^T, (the first term in equation 2). Two additional factors may introduce variability in q_t^T: international differences in savings and investment and changes in the real price of oil.

a. The real exchange rate for traded goods is also, following MacDonald (1998), a major determinant of the current account and is in turn driven by the determinants of savings and investment. We can separate two variables that may capture this effect:

- Fiscal policy, whose relation with the real exchange rate depends on the approach. According to the Mundell-Fleming model, an expansionary fiscal policy reduces national savings, increases the domestic real interest rate, and generates a permanent appreciation. In contrast, the portfolio balance models consider permanent fiscal expansion to cause a decrease in net foreign assets and a depreciation of the currency.
- Private sector net savings, whose effect on the real exchange rate is influenced by demographic factors. This way the cross-country variations of saving rates are seen to affect the relative net foreign asset position.

b. Increases in the real price of oil tends to appreciate the currencies of the net oil exporters or, in general, the currencies of the less energy dependent countries.

MacDonald's proposal does not rely exclusively on the monetary approach to exchange rate determination, although it captures the majority of the fundamental variables mentioned in the literature and makes them compatible with it. Accordingly, the above-mentioned factors can be summarized in the following empirical specification:

$$\begin{aligned} q_t &= -\varphi(R_t - R_t^*) + \hat{q}_t \\ &= f(\underset{(-)}{(R_t - R_t^*)}, \underset{(-)}{(a_t - a_t^*)}, \underset{(-/+)}{(g_t - g_t^*)}, \underset{(-)}{oil_t}, \underset{(-)}{dnfa_t}), \end{aligned} \tag{3}$$

where $(a_t - a_t^*)$ is the difference between the domestic and foreign economies productivity,[4] $(g_t - g_t^*)$ is the public expenditure differential, oil_t[5] is the real oil price and $dnfa_t$ is the relative net foreign asset position of the economy.

9.3 Empirical Results

Two different econometric techniques have been applied to the same data set. First, using dynamic panel techniques, we estimate the real exchange rate of the dollar versus a group of seven individual countries. In addition we study separately the euro countries in the sample from the rest. Second, using time series techniques, we explain the dollar–euro real exchange rate in terms of euro-area aggregated variables.

9.3.1 *Panel Analysis: the Dollar in the World*

As we noted earlier in our theoretical discussion, we examined a wide set of explanatory (fundamental) variables in order to assess the main factors behind the behavior of the dollar's real exchange rate. This first part of the analysis involves eight countries: the United States as the domestic country, Japan, Canada, the United Kingdom, and four euro-area countries (those with information available for the sample period and variables of interest). As a result in this first part of the analysis we do not strictly estimate a model for the dollar versus the euro area. We have chosen to include countries, such as the United Kingdom, Canada, and Japan, that do not participate in EMU in order to capture the behavior of the most important world currencies. Our method in this part of the analysis allows for both group and individual approaches.

We consider first the entire group of countries (where $N = 7$) and then divide the panel into the euro area countries ($N = 4$: Germany, Spain, France, and Italy) and non-euro area countries ($N = 3$: Canada, Japan, and the United Kingdom). The data are quarterly and the sample goes from 1970:1 to 1998:4.[6]

In choosing our model specification, we tried to follow as close as possible the general to specific methodology. Our starting point was the models described in the previous section, and to make the estimated models comparable, we used a general specification:

$$rerdol_{it} = f(\underset{(-)}{dpro_{it}}, \underset{(-)}{drr_{it}}, \underset{(-)}{oildep_{it}}, \underset{(-)}{dnfa_{it}}, \underset{(+/-)}{dpex_{it}}),$$

where $rerdol_{it}$ is the real exchange rate of the dollar versus all the currencies defined as the units of domestic currency necessary to buy a unit of foreign currency in real terms; $dpro_{it}$ is the relative productivity of the United States versus that of the other countries: an increase in the value of this variable tends to appreciate the currency; drr_{it} is the real interest rate differential between the United States and the other countries analyzed: an increase in this differential appreciates the currency; $oildep_{it}$ is the real price of oil adjusted by the relative dependency on oil imports by each country compared to that of the United

States: in this case, the dollar will appreciate when the oil dependency of the foreign countries is increasing; $dnfa_{it}$ is the difference in the net foreign asset position over GDP of the United States versus the other countries, and the sign should be negative (the currencies of countries increasing its net foreign asset position tend to appreciate); $dpex_{it}$ is the difference in public expenditure over GDP between the United States and each of the other countries. In the last instance, there are two competing theories explaining the relation of public expenditure to the GDP with respect to the real exchange rate. The relation is positive (depreciation) if the portfolio balance model prevails, but it is negative according to the Mundell-Fleming approach.

The models we used are the following:[7]

Model 1 Eclectic model:

$$rerdol_{it} = \alpha_i + \beta_{1i}drr_{it} + \beta_{2i}dpex_{it} + \beta_{3i}dpro_{it} + \beta_{4i}dnfa_{it} + \beta_{5i}oildep_{it}.$$

Model 2 Restricted eclectic model:

$$rerdol_{it} = \alpha_i + \beta_{1i}drr_{it} + \beta_{2i}dpex_{it} + \beta_{3i}dpro_{it} + \beta_{4i}dnfa_{it}.$$

Model 1 follows the general specification described above. Model 2 is a version of model 1 with the oil dependence variable excluded. In what follows, we show how these empirical models were tested.

Order of Integration of the Variables

Bearing all these considerations in mind, we should start the analysis with the study of the order of integration of the variables. Several panel unit root tests are already available in the literature, from the early works of Levin and Lin (1992)[8] to the Im, Pesaran, and Shin (1995) tests. However, because of its higher power we applied the *LM* test for the null of stationarity proposed by Hadri (2000) with heterogeneous and serially correlated errors. These tests can be considered a panel version of the *KPSS* tests applied in the univariate context. Hadri (2000) provides two models (with and without a deterministic trend) that can be decomposed into the sum of a random walk and a stationary disturbance term. He tests the null hypothesis that all the variables (y_{it}) are stationary (around deterministic levels or around deterministic trends), so that for the N elements of the panel the variance of the errors is such that

$$H_0 : \sigma_{u1}^2 = \cdots = \sigma_{uN}^2 = 0 \tag{4}$$

Table 9.1
Hadri (2000) stationarity tests ($l = 2$)

Variables	η_μ	η_τ
$rerdol_{it}$	23.72**	175.45**
$dpex_{it}$	14.30**	262.49**
$dnfa_{it}$	47.05**	1655.32**
$dpro_{it}$	29.79**	801.21**
drr_{it}	18.23**	167.71**
$oildep_{it}$	18.38**	149.01**

Note: The statistic η_μ does not include a time trend, whereas η_τ does, and both are normally distributed. The two asterisks denote rejection of the null hypothesis of stationarity at 5 percent. The number of lags selected is $l = 2$.

against the alternative H_1: that some $\sigma^2_{ui} > 0$. This alternative allows for heterogeneous σ^2_{ui} across the cross sections and includes the homogeneous alternative ($\sigma^2_{ui} = \sigma^2_u$ for all i) as a special case. It also allows for a subset of cross sections to be stationary under the alternative. The two statistics are called η_μ for the null of stationarity around an intercept and η_τ when the null is stationarity around a deterministic trend.

The results of the tests applied to the four variables are presented in table 9.1. The null hypothesis of stationarity can be easily rejected in the two cases (with and without time trend), so that all the panel variables can be considered nonstationary.

Long-Run Relationships: "Pooled Mean Group" Estimation Results

Once we have determined the order of integration of the variables for the analysis of the real exchange rate of the dollar, we can follow the methodology proposed by Pesaran, Shin, and Smith (1999) and compute the pooled mean group estimators.[9] This estimation technique is well suited in our case because we are interested in considering different groups of countries and comparing the estimation results (i.e., the whole group, the euro area countries, and the non–euro area countries).

The pooled mean group (PMG) estimator involves both pooling and averaging. This estimator allows the intercepts, short-run coefficients, and error variances to differ across groups, but the long-run coefficients are constrained to be the same. Due to the high level of economic integration achieved among the euro-area countries, we chose to impose equality in the long-run parameters (or rather in most of them) but allow the short-run slope coefficients and the dynamic specification (i.e., the number of lags included) to differ across groups.

Then we estimated the panel, using a maximum likelihood approach. The *ML* estimators that result are the pooled mean group (*PMG*) estimators. This is because they are both pooled, as implied by the homogeneity restrictions on the long-run coefficients, and averaged across groups to obtain means of the estimated error-correction coefficients and the other short-run parameters of the model.

The empirical model takes on the eclectic form presented above, starting with model 1, which includes the main explanatory variables proposed by the literature on real exchange rates. Other theoretical models are restricted versions of model 1.

Many empirical specifications have been estimated and compared through likelihood-based information criteria, such as the *AIC* and the *SBC*. In addition in each specification we have tested two important questions: the homogeneity restriction using a likelihood ratio test; the existence of discrepancies between the pooled mean group estimates and the mean group estimates, which differ also in the degree of heterogeneity allowed. The Hausman test permits us to decide whether these discrepancies recommend the exclusion of the homogeneity restriction in some of the long-run parameters. Thus the second test complements the first one because, if homogeneity is rejected using the *LR* test, the Hausman test for the individual variables helps identify the variable source of the heterogeneity. Concerning the dynamics of the model, the short-run has been modeled using up to two lags, as derived in the application of the Schwarz Bayesian criterion for lag selection.

In the second and third columns of table 9.2 we present the information criteria used in the selection of the two models, and show the corresponding *LR* homogeneity test results along with the concrete hypotheses tested for the three groups of countries analyzed.

In model 1, all the variables were considered, and it has higher AIC and SBC than model 2. No null hypothesis of homogeneity in the long-run parameters could be accepted for any of the groups of countries analyzed (e.g., see, for $N = 7$, $\chi^2(18) = 67.81$ with a probability of [0.00]). Also the long-run parameter of the variable $oildep_t$ is nonsignificant. Where some heterogeneity was allowed, specifically in the oil dependency variable, the results did not improve.[10]

Model 2 is a restricted version of model 1, where $oildep_t$ has been excluded. The information criteria are smaller, and after we imposed the condition that not all the long-run parameters must be equal for all the countries, the restrictions for the rest of the variables in the three

Table 9.2
Comparison of the specified models

	AIC	SBC	LR test	Variables				
				drr_t	$dpex_t$	$oild_t$	$dnfa_t$	$dpro_t$
$N = 7$								
Model 1	1714	1686	$\chi^2(18) = 67.81[0.00]$	$\neq$	$= \forall$	$\neq$	$= \forall$	$= \forall$
Model 2	1691	1665	$\chi^2(12) = 20.68[0.05]$**	$\neq$	$= \forall$	—	$\neq$	$= \forall$
$N = 4$: Euro-area								
Model 1	1036	1018	$\chi^2(6) = 17.95[0.00]$	$\neq$	$\neq$	$\neq$	$= \forall$	$= \forall$
Model 2	998	982	$\chi^2(9) = 15.87[0.07]$**	$\neq$	$= \forall$	—	$= \forall$	$= \forall$
			$\chi^2(6) = 11.37[0.07]$**	$\neq$	$= \forall$	—	$= \forall$	$\neq$
$N = 3$: Non-euro								
Model 1	763.74	748.40	$\chi^2(4) = 28.51[0.00]$	$\neq$	$\neq$	$\neq$	$= \forall$	$= \forall$
Model 2	763	750	$\chi^2(4) = 8.55[0.07]$**	$\neq$	$= \forall$	—	$\neq$	$= \forall$

Note: *AIC* stands for Akaike Information Criterium, *SBC* for Swartz Bayesian criterium and *LR test* is the likelihood ratio test for equality of either some or all the long-run parameters (probability values appear in parentheses). Two asterisks denote acceptance of the restriction on the long-run parameters at 5 percent significance level. $\neq$ stands for the assumption of different parameter values for all the N members of the panel. The homogeneity hypothesis is represented by the symbols $= \forall$.

configurations could be accepted. For example, for $N = 7$, the homogeneity restriction is accepted for $dpro_t$ and $dpex_t$ ($\chi^2(12) = 20.68$ with a probability of [0.05]), although it is necessary to allow for some heterogeneity in the real interest rate and in the net foreign asset differential. The estimates and the associated t-statistics are presented in the first column of table 9.3, where all variables but drr_t are significant. It should be noted that the error correction coefficient is highly significant and of a reasonable magnitude (-0.120). Thus the adjustment toward equilibrium will take approximately two years. In tables 9.4 and 9.5 the information concerns the long-run relations among the countries as well as the misspecification tests. As is evident, apart from some normality departures in some of the countries, the individual equations pass the misspecification tests. Moreover the $\bar{R}^2$ in almost every case (Canada excepted) is over 0.80.

The estimated parameters conform to the theory and are of correct sign. Thus the increase in the real interest differential causes the currency to appreciate ($\beta_1 < 0$). The expansionary fiscal policy in the United States relative to the other countries causes the currency ($\beta_2 > 0$) to depreciate, whereas an increase in relative productivity

Table 9.3
Pooled mean group estimates

Variables	All countries ($N = 7$)	Euro-area ($N = 4$)		Non-euro ($N = 3$)
Model 2: $rerdol_{it} = \alpha_i + \beta_{1i} dpro_{it} + \beta_{2i} drr_{it} + \beta_{3i} dnfa_{it} + \beta_{4i} dpex_{it}$				
drr_t	-0.005^a	-0.007^a	-0.006^a	-0.008^a
	(−1.58)	(−1.92)	(−2.38)	(−2.23)
$dpex_t$	0.003	0.003	0.002	0.008
	(2.95)	(2.48)	(2.09)	(2.72)
$dpro_t$	−0.851	−0.870	-0.749^a	−0.836
	(−27.02)	(−22.34)	(−7.12)	(−15.47)
$dnfa_t$	-0.327^a	−0.314	−0.288	-0.266^a
	(−5.57)	(−6.94)	(−6.58)	(−1.59)
ecm_{t-1}	−0.120	−0.126	−0.134	−0.149
	(−3.83)	(−2.99)	(−3.15)	(−4.77)

Note: Student's t is in parentheses.
Superscript "a" indicates that the corresponding variable was not subject to the restriction of equal long-run parameters for all the members of the group. Thus its estimate is the mean group estimate, instead of the PMGE.

causes the currency ($\beta_3 < 0$) to appreciate, due to the Balassa-Samuelson effect. Finally, an increase in the relative net foreign assets position also induces appreciation ($\beta_4 < 0$). Notice that in the long-run parameter estimates of drr_t and $dnfa_t$, we do not impose equality of all the cross-sectional elements. The individual country estimates are presented in detail in table 9.4.

Although with larger N this technique has more advantages, due to our focus on the euro area, we have also estimated the dynamic panel data for the four EMU countries with the information available, as well as for the other three countries considered. The long-run parameters estimates, also presented in table 9.3, are very similar to those obtained for the larger group.

Recall from table 9.2 the information criteria (also smaller than in model 1), as well as the LR tests for homogeneity in the long-run parameters, for the euro-area countries. In this case, after imposing that drr_t is heterogeneous for the members of the group, we accept the homogeneity of the other three explanatory variables. As an additional test for homogeneity, we used the Hausman test for the variable $dpro_t$, which did not accept the similarity between the coefficient estimated using the PMG estimator and the MG estimator, where heterogeneity

Table 9.4
Individual countries estimates

Countries	$N = 7$					$N = 4$					$N = 3$				
	drr_t	$dpro_t$	$dnfa_t$	$dpex_t$	ecm_{t-1}	drr_t	$dpro_t$	$dnfa_t$	$dpex_t$	ecm_{t-1}	drr_t	$dpro_t$	$dnfa_t$	$dpex_t$	ecm_{t-1}
Model 2															
Germany	−0.005	−0.851	−0.328	0.003	−0.120	−0.006	−0.74	−0.288	0.002	−0.128	—	—	—	—	—
	(−1.58)	(−27.02)	(−5.57)	(2.95)	(−3.83)	(−1.91)	(−8.52)	(−6.57)	(2.09)	(−3.92)					
Spain	0.0001	−0.851	−0.372	0.003	−0.117	0.0001	−0.891	−0.288	0.002	−0.123	—	—	—	—	—
	(0.08)	(−27.02)	(−2.81)	(2.95)	(−2.99)	(0.09)	(−8.39)	(−6.57)	(2.09)	(−3.10)					
France	−0.005	−0.851	−0.330	0.003	−0.215	−0.005	−0.907	−0.288	0.002	−0.246	—	—	—	—	—
	(−3.46)	(−27.02)	(−5.27)	(2.95)	(−4.52)	(−4.24)	(−20.85)	(−6.57)	(2.09)	(−4.90)					
Italy	−0.004	−0.851	0.127	0.003	−0.096	−0.011	−0.454	−0.288	0.002	−0.039	—	—	—	—	—
	(−1.46)	(−27.02)	(1.16)	(2.95)	(−2.72)	(−1.08)	(−1.30)	(−6.57)	(2.09)	(−1.60)					
Canada	−0.007	−0.851	−0.350	0.003	−0.144	—	—	—	—	—	−0.008	−0.836	−0.383	0.007	−0.139
	(−3.73)	(−27.02)	(−7.07)	(2.95)	(−4.15)						(−3.79)	(−15.47)	(−15.47)	(2.72)	(−4.18)
Japan	−0.009	−0.851	−0.430	0.003	−0.126	—	—	—	—	—	−0.011	−0.836	−0.478	0.007	−0.100
	(−2.51)	(−27.02)	(−6.95)	(2.95)	(−3.33)						(−2.26)	(−15.47)	(−5.73)	(2.72)	(−2.95)
United Kingdom	−0.003	−0.851	0.043	0.003	−0.217	—	—	—	—	—	−0.003	−0.836	0.063	0.007	−0.207
	(−2.12)	(−27.02)	(2.07)	(2.95)	(−3.91)						(−2.28)	(−15.47)	(2.53)	(2.72)	(−3.84)

Table 9.5
Individual countries specification tests

	$\bar{R}^2$	Correlation	FF	NO	HE
Model 2 ($N = 7$)					
Germany	0.882	0.71	17.43*	34.03*	36.82*
Spain	0.829	0.10	1.67	36.63*	0.03
France	0.890	0.18	1.87	4.39	1.07
Italy	0.850	3.71	1.21	35.98*	0.08
Canada	0.578	1.28	0.52	2.36	0.13
Japan	0.869	0.01	0.54	5.68	0.00
United Kingdom	0.844	1.09	0.33	25.79*	0.52

is allowed.[11] Once the two variables are not constrained to be homogeneous, the model passes the Hausman test. Note that in table 9.3 the estimation results for the two cases are very similar. All the variables are significant and the error correction term is slightly larger in the second case.

For the other three countries (Canada, Japan, and United Kingdom), the homogeneity of all the variables is rejected. Only after allowing heterogeneity in drr_t and $dnfa_t$ the homogeneity of the other long-run parameters can be accepted. Model 2 and model 1's AIC and SBC are similar, but only in model 2 the partial homogeneity is accepted after the restrictions are imposed, this being the test $\chi^2(4) = 8.55$ with a probability of [0.07]. Thus model 2 seems adequate also for $N = 3$. The long-run estimates of the parameters have similar magnitude if compared with the larger model. The only exception is $dpex_t$, whose value is 0.008 in contrast with 0.003. The error correction coefficient takes the value of -0.149 and an associated student t of -4.77.

9.3.2 Aggregate European Results: The Euro and the Dollar

The preceding panel analysis gives some clues about the behavior of the dollar in terms of major world currencies. As we expected, the results do not fit a simple model (e.g., the Meese and Rogoff 1988 real interest differential), but a rather eclectic specification as it includes variables both from the demand and the supply sides of the economy. In our results the role of productivity differentials supports the fulfillment of the Balassa-Samuelson effect. The real interest rate differential

Table 9.6
Cointegration test statistics

r	Eigenvalues	Trace	Trace (R)	Trace 95%
0	0.3748	122.7**	97.28*	94.2
1	0.3420	81.78**	64.86	68.5
2	0.2791	45.36	35.97	47.2
3	0.1085	16.88	13.39	29.7
4	0.0699	6.883	5.429	15.4
5	0.0065	0.571	0.452	3.8

Note: The critical values are given with 95 percent critical values based on a response surface fitted to the results of Osterward-Lenum (1992). (R) stands for the small-sample correction of the *trace* tests statistics proposed by Reimers (1992). * and ** denotes rejection of the null hypothesis at 5 and 1 percent significance level respectively.

is also present, although this is not the exclusive determinant of real exchange rate behavior: the fiscal policies and the net foreign assets of the countries are among the explanatory variables. The only variable that did not show a significant contribution was the real oil price. The additional conclusion that can be drawn from the dynamic panel analysis is that overall, the model estimated for the dollar real exchange rate does not change much with the different configurations of the countries (besides the minor exceptions already mentioned).

Once the panel analysis has been completed for the European countries separately, we focus on the "synthetic" euro-area variables. The two approaches are complementary as the use of panels allows for heterogeneity. In fact the lack of heterogeneity is one of the main criticisms of aggregate analyses. If the results from these two complementary methodologies do not show important discrepancies, we can be more confident in using the aggregate series for inference and policy analysis.

For this part of the analysis we use the Johansen (1995) method for the estimation and identification of cointegrated systems where differentials are no longer calculated for the United States relative to every other country but relative to a representative euro-area variable.

First in the analysis we studied the order of integration of the variables, using a stationarity testing strategy in the context of the VAR system. All the variables turned out to be $I(1)$.[12] Table 9.6 shows the trace test statistics for the determination of the number of cointegration relationships.[13] The Reimers adjusted *trace* test statistics are also shown. Clearly, the trace test statistic fails to reject the existence of two cointegration vectors, whereas the Reimers adjusted test fails to reject one cointegration vector. To gain insight on the appropriate number of cointegration vectors, we need to add to this analysis information about the roots of the companion matrix: three are almost unity and other two are pretty close to unity, implying that five is the number of common stochastic trends. Moreover, for $r = 1$, the largest roots are removed, leaving no near unit root in the model, so this must be the appropriate choice for r. In addition, from the time path plot for each of the feasible cointegration vectors, only the first one seems to be stationary. The recursive analysis of the system provides other useful information regarding the existence of cointegration: the recursive time path of the nonadjusted trace statistic suggests that at most there exist two cointegration vectors though one is the most sensible outcome. From all this evidence, the most feasible choice is the existence of one cointegration vector, that is, $p - r = 5$, where p is the number of common stochastic trends.

We can proceed to identify the cointegration vector by imposing the overidentifying restriction that the variable for energy dependence ($oildep_t$) is excluded from the long-run: the LR statistic is $\chi^2(1) = 3.43$ with a probability value of 0.06. The resulting cointegration vector takes the form (standard errors in parentheses):

$$q_t = \underset{(0.001)}{0.011} dpex_t - \underset{(0.001)}{0.007} drr_t - \underset{(0.033)}{0.77} dpro_t - \underset{(0.032)}{0.36} dnfa_t. \tag{5}$$

At this stage of the analysis we can already compare the results obtained using the PMG in the dynamic panel with the time series model using aggregate variables. Taking into account the results presented in table 9.3 for model 2, we can observe that the results are very similar. First, the variable relative oil dependency ($oildep_t$) that turned out not to be significant in the panel analysis can be also excluded from the time series cointegration vector. Second, the four variables have the same signs even if we are using quite different estimation techniques. Moreover the parameters' estimates are not very different in magnitude, the only exception being the case of $dpex_t$, where the time series value is 0.011 and 0.002 for the panel. In other cases the parameters are almost equal, as for the real interest differential (−0.007 for the aggregate model and −0.006 for the panel) or the productivity differential (−0.77 in the time series model and −0.749 in the panel).[14] Finally, the net foreign asset position is also in a similar range: −0.36 in the aggregate model and −0.288 in the panel.

Once we have identified the cointegration vector, we formally test for weak exogeneity of the variables in the system. According to our results all the variables appear to be weakly exogenous with the only exception of the real exchange rate. The joint hypothesis of weak exogeneity and the identifying restrictions on the cointegration space β are accepted: the LR statistic value is $\chi^2(6) = 11.16$ with a probability of 0.08. We present next the error correction model (ECM hereafter) for the univariate partial model (t-values in brackets):

$$\Delta q_t = \underset{[5.010]}{0.291} - \underset{[-7.675]}{0.375}\Delta dpro_t - \underset{[-2.999]}{0.185}\Delta dpro_{t-1} - \underset{[-2.068]}{0.105}\Delta dpro_{t-2}$$

$$-\underset{[-2.002]}{0.002}\Delta drr_{t-3} - \underset{[-5.007]}{0.184}ecm_{t-1} + \varepsilon_t. \tag{6}$$

Misspecification tests

Residual correlation: $F(5, 76) = 1.0856[0.3752]$

ARCH: $F(4, 73) = 0.8310[0.5098]$

Normality: $\chi^2(2) = 1.1128[0.5733]$

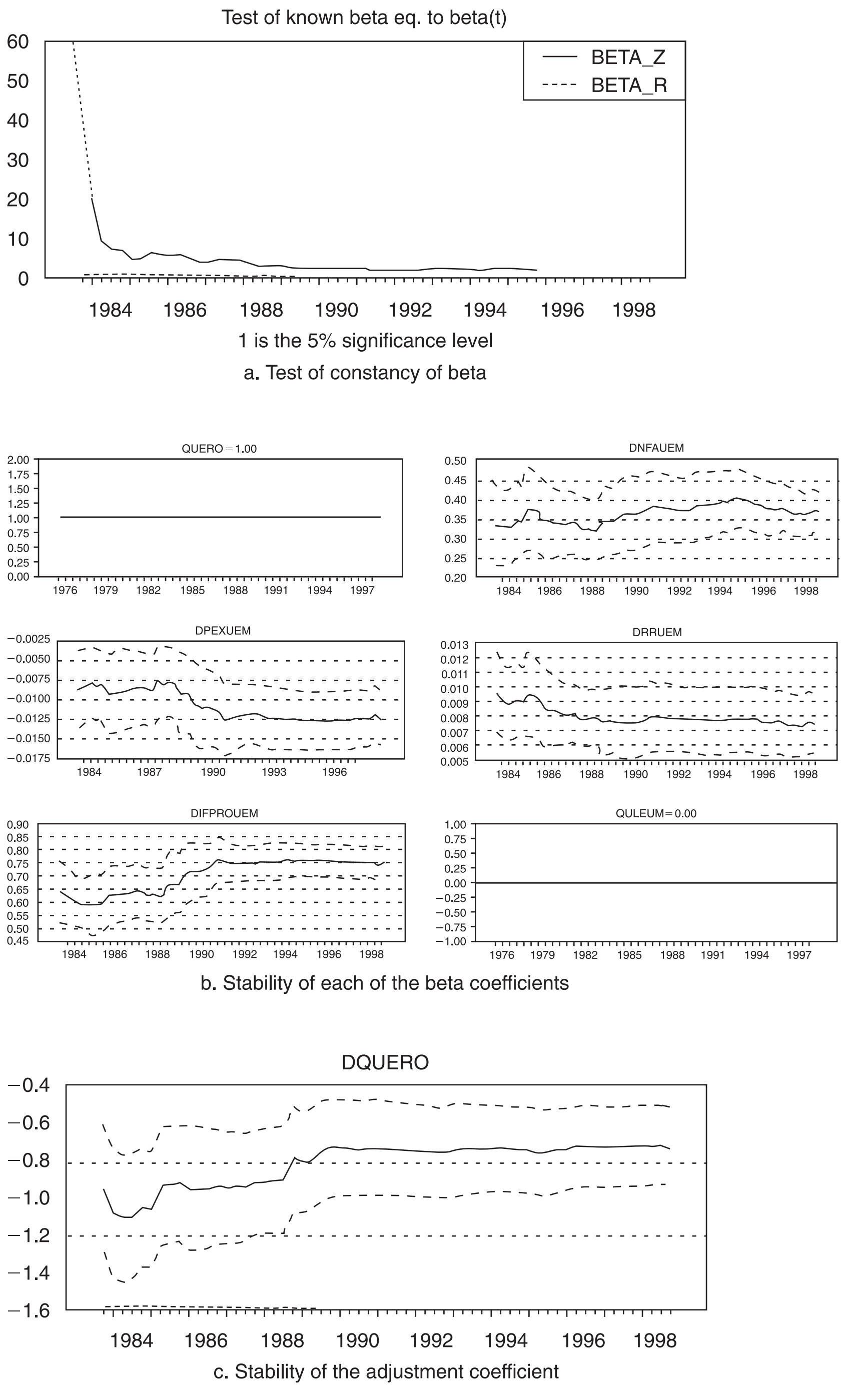
Heteroscedaticity (squares): $F(10, 70) = 1.0960[0.3774]$

Heteroscedaticity (squares and cross products): $F(20, 60) = 1.1588[0.3203]$

In the equation above ε_t is a vector of disturbances and ecm_{t-1} is the cointegration vector (5). None of the misspecification tests reported here rejects the null hypothesis that the model is correctly specified.

In addition we apply the Hansen and Johansen (1993) approach to test for parameter instability in the cointegration vector. Specifically we test both whether the cointegration space and each of the parameters in the cointegration vector are stable. We also test for the stability of the loading parameters. If both α and β appear to be stable, we can conclude that our error correction model is well specified for the period analyzed.

Panel a of figure 9.1 shows the plot of the test for constancy of the cointegration space. The test statistic has been scaled by the 95 percent quantile in the χ^2-distribution so that unity corresponds to the 5 percent significance level. The test statistic for stability is obtained using both the Z-representation and the R-representation of our model. In the former, stability is analyzed by the recursive estimation of the whole model, and in the latter the short-run dynamics are fixed and only the long-run parameters are re-estimated. Thus the



a. Test of constancy of beta

b. Stability of each of the beta coefficients

c. Stability of the adjustment coefficient

Figure 9.1
Stability of the cointegration space

R-representation is the relevant one to assess the stability of the cointegration space, which is clearly accepted.

Panels b and c of figure 9.1 show, respectively, the stability tests for each of the beta coefficients and for the loadings to the cointegration vector. In all cases, the recursively estimated coefficients lay within the 95 percent confidence bounds showing a remarkable stability.

To summarize, we can conclude that the cointegration space is stable, that is, the long-run parameters as well as the loadings do not show signs of instability.

Finally, panel b of figure 9.2 presents several recursive tests of parameter stability for the parsimonious conditional model. Accordingly, our model is stable not only concerning the cointegration space but also the model as a whole.

As for the real exchange rate ECM presented in equation (6), we should note that the error correction parameter presents the correct sign and magnitude (taking into account that the data are quarterly), and passes the Banerjee, Dolado, and Mestre (1992) cointegration test. In addition two of the variables appear in the dynamics of the real exchange rate. The first is, with three lags, the real interest rate differential (drr_t), although it is borderline significative. The negative parameter for this variable, as in the panel analysis, is the one expected from the theory. Second is the productivity differential measure, contemporaneous and lagged from one to two periods, with the same negative sign found in the long-run time series analysis and in the panel analysis reported in section 9.3.1 above. The important role that the productivity differential has in driving the system toward the equilibrium should be emphasized and also the fact that the adjustment starts in the same quarter where the shocks have occurred.

We can again compare the error correction model of the aggregate European variables with the results for the panel. As in the time series case, the contemporaneous effects coming from the productivity differential are very important and of the same sign (with a *t*-statistic of -17.56), but the rest of the variables are not significant. Concerning the error correction coefficient, its magnitude is smaller in the panel (-0.134).

Although there is no consensus in the profession on a particular model specification of exchange rate equations inspired by the New Open Macroeconomics literature (Sarno 2002), the results obtained in this chapter are compatible with these models. In particular, according to Lane (2002), net foreign assets positions are an important form of

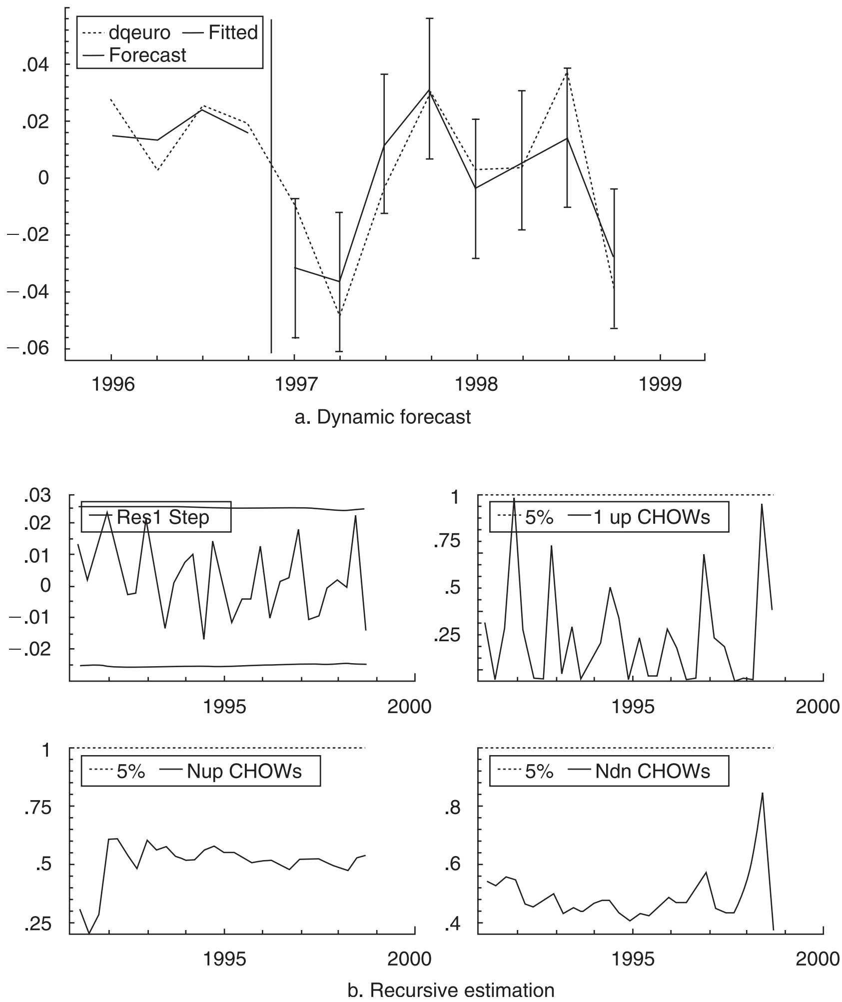


Figure 9.2
Dynamic forecast and recursive estimation

international macroeconomic interdependence. The influence of net foreign asset positions on the values of the real exchange rate has also been studied recently in Cavallo and Ghironi (2002) and Lane and Milesi-Ferretti (2001). In this chapter, we have used the net foreign asset dataset constructed in Lane and Milesi-Ferretti (2001), that is, the "adjusted cumulative current account," and our results are compatible with the most recent empirical literature besides the previous empirical work.[15]

To complete our analysis, we check the predictive ability of the euro-area model. Table 9.7 presents ex post and ex ante forecasting results. To compute the ex post forecasts, we left out eight observations (two years) and re-estimated the model. From the one-step static forecast analysis, our model appears to deliver sensible and stable forecasts. The estimates for the dynamic forecast are carried out recursively: the estimation period is successively extended quarter by quarter so that the real exchange rate is forecasted for up to eight quarters into the future. Panel a of figure 9.2 shows graphically the predictive performance of our model. This graph plots the dynamic forecasts for the period 1997:1 to 1998:4 estimated by full-information maximum likelihood. The forecasts lie within the 95 percent confidence interval, shown by the vertical error bars of plus or minus twice the forecast's standard error. Moreover the fit of the model is good, and there are no large departures from the actual values.

Finally, the forecast quality of our model is also assessed by comparing its forecast accuracy with a random walk model for the real exchange rate. For this purpose we obtain the ratio between the root mean squared error (RMSE) corresponding to our VECM relative to the random walk. If the VECM presents a better predictive performance, that is, lower RMSE, this ratio will be below 1. In addition, following Diebold (1998), we carried out a formal test to gain insight into whether the random walk model can generate significantly better forecasts from a statistical point of view. Thus, rejection of the null for this test implies that the random walk model does not provide significantly better forecasts than our VECM. Table 9.7 presents the ratio of the two RMSE for a forecast horizon up to eight quarters as well as the significance level for the Diebold and Mariano test statistic, which is indicated by asterisks in the third column. By these results the VECM outperforms the random walk model even in the shorter horizons, as can be seen from RMSE ratios, which are well below 1. Moreover the predictive performance of our model is statistically shown, rejecting

Table 9.7
Static and dynamic forecasting

A. One-step (ex post) forecast analysis: 1997:1 to 1998:4		
Parameter constancy		
ξ_1	$\chi^2(8) = 10.679$ [0.2205]	$F(8,73) = 1.3349$ [0.2402]
ξ_2	$\chi^2(8) = 8.9096$ [0.3500]	$F(8,73) = 1.1137$ [0.3643]
ξ_3	$\chi^2(8) = 9.5206$ [0.3003]	$F(8,73) = 1.1901$ [0.3169]
Forecast tests: $\chi^2(1)$		
	Using ξ_1	Using ξ_2
1997:1	3.4134 [0.0647]	2.6766 [0.1018]
1997:2	1.0618 [0.3028]	0.8527 [0.3558]
1997:3	1.3785 [0.2404]	1.0663 [0.3018]
1997:4	0.0069 [0.9337]	0.0062 [0.9369]
1998:1	0.2791 [0.5973]	0.2503 [0.6168]
1998:2	0.0428 [0.8361]	0.0380 [0.8454]
1998:3	3.6488 [0.0561]	3.2989 [0.0693]
1998:4	0.8479 [0.3571]	0.7203 [0.3960]
Forecast horizon	RMSE (ratio)	Significance
B. Forecast quality: 1997:1 to 1998:4		
1997:1	0.2509	
1997:2	0.2176	
1997:3	0.1887	
1997:4	0.1821	***
1998:1	0.1716	***
1998:2	0.1676	***
1998:3	0.1665	***
1998:4	0.1728	***

Note: ξ_1, ξ_2, and ξ_3 are indexes of numerical parameter constancy. The former ignores both parameter uncertainty and intercorrelation between forecasts errors at different time periods. ξ_2 is similar to ξ_1 but takes parameter uncertainty into account. ξ_3 takes both parameter uncertainty and intercorrelations between forecasts errors into account. *Forecast test* are the individual test statistics underlying ξ_1 and ξ_2. *** stands for 1 percent error probability.

for all the forecast horizons the superiority of the random walk model with a probability as low as 1 percent.

9.4 Conclusions

In this chapter we apply two different but complementary techniques and approaches to the study of the evolution of the dollar real exchange rate in relation with the euro-area currencies. First, using panel techniques, we study the long-run relationship between the bilateral real exchange rate of the dollar versus the currencies of five European countries, Canada, and Japan. Second, in a time series framework, we use euro-area aggregate or "synthetic" variables to study the behavior of the dollar–euro real exchange rate. Our aim was to compare the results obtained from the two approaches and for the same time span. Given that the lack of heterogeneity is one of the main criticisms commonly associated with aggregate analyses, in using a panel analysis, we allow for individual country differences. The similarity of the results obtained by the two methods adds robustness to the euro-area measures. Heterogeneity is a feature not evident in other papers dealing with the real exchange rate of the euro.

We maintain this distinction in summarizing the most important empirical results. First, concerning the dynamic panel analysis, we use the methodology of Pesaran et al. (1999), which allows for short-run heterogeneity for the individual components of each panel and a formal test of homogeneity in the long-run parameters. We find that both the supply- and demand-side factors can be accounted for to explain the bilateral real exchange rate of the US dollar. In particular, the estimated error correction models support a specification that includes relative productivity, the real interest rate differential, the difference in public expenditure, and the relative net foreign asset position. This type of relation holds not only for the euro countries but also for the whole group and for the rest-of-the-world countries.

We arrived at the same long-run specification using the Johansen technique in a time series context. Therefore, we showed that even if we allow a larger degree of heterogeneity in the panel and even if we use different estimation techniques, the results appear to be almost identical. In addition, in the aggregate time series empirical model, the cointegration vector passed all the applied stability tests. Last, the estimated VECM was remarkably predictive in performance and provided better forecasts than the random walk for both the short and the medium terms.

The long-run results showed the dollar–euro exchange rate to depreciate if American fiscal policy becomes more expansionary than European fiscal policy. However, productivity growth and real interest rate differentials, together with the accumulated net foreign assets, will appreciate the currency.

Appendix: Data Sources

We used quarterly data for the period 1970:1 to 1998:4 from France, Germany, Italy, Spain, and the United Kingdom. We included data from the United States (the home country) and Canada and Japan. The data were obtained from the magnetic tapes of the International Monetary Fund International Financial Statistics (IFS) with the exception of employment and oil balances data, which came from the International Sectoral Database (OECD). The net foreign assets data were taken from Lane and Milesi-Ferretti (2001), L-M hereafter. The nominal exchange rate for the euro relative to the US dollar was from the database for European variables of the Banco Bilbao Vizcaya Argentaria (BBVA).

The panel data were constructed as follows:

***rerdol**$_{it}$*: Bilateral real exchange rate of the US dollar relative to the other currencies considered. The nominal exchange rate, s_t, has been defined as currency units of US dollar to purchase a unit of currency j:

$$rerdol_t = \log\left(\frac{p_t^{USA}}{s_t \times p_t^j}\right),$$

where p_t^{USA} and p_t^j are respectively the CPI for the Unites States and the foreign country. (Source: IFS)

***drr**$_{it}$*: Real interest rate differential. The nominal interest rates are *call money rates* as defined by the IMF. In order to obtain the real variables, the expected inflation rate is the smoothed variable based on CPI indexes using the Hodrick and Prescott filter:

$$\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}} \times 100,$$

$$\pi_t^e = \pi_t - \pi_t^t,$$

$$rr_t = r_t - \pi_t^e,$$

$$drre_t = rr_t^{USA} - rr_t^j,$$

where π_t^e is expected inflation filtered using the HP filter, π_t^t is the transitory component of inflation, rr_t^{USA} is the American real interest rate, and rr_t^j the foreign rate. (Source: IFS)

dpro$_{it}$: Apparent productivity differential in labor,

$dpro_t = pro_t^{USA} - pro_t^j$,

where pro_t^{USA} and pro_t^j are respectively the American and the foreign apparent labor productivity. This is calculated as

$$pro_t^j = \log\left(\frac{gdp_t^j}{employment_t^j}\right) \times \frac{1}{s_t},$$

with

$$pro_t^{USA} = \log\left(\frac{gdp_t^{USA}}{employment_t^{USA}}\right).$$

(Source: IFS and OCDE)

dpex$_{it}$: Public expenditure differential, calculated as

$dpex_t = pex_t^{USA} - pex_t^j$,

where pex_t^{USA} and pex_t^j are respectively the American and the foreign government spending. The government spending is calculated relative to GDP:

$$pex_t = \frac{pexn_t}{gdpn_t} \times 100,$$

where $pexpn_t$ is nominal public expenditure. (Source: IFS)

dnfa$_{it}$: Net foreign assets differential,

$dnfa_t = rnfa_t^{USA} - rnfa_t^j$,

where $rnfa_t^{USA}$ and $rnfa_t^j$ stands respectively for the American and the foreign's net foreign asset position relative to the GDP in US dollar:

$$rnfa_t^j = \frac{nfa_t^j}{gdp_t^j \times (1/s_t)}$$

and

$$rnfa_t^{USA} = \frac{nfa_t^{USA}}{gdp_t^{USA}}$$

(Source: L-M)

oildep$_{it}$: Relative oil dependence,

$$oildep_t = \frac{bal_t^j}{bal_t^{USA}} \times \frac{brent\ price}{cpi_t^{USA}} \times 100,$$

where bal_t^{USA} and bal_t^j are measures of energetic dependence for the United States and the foreign country respectively. This is obtained as

$$bal_t = \frac{Net\ oil\ imports}{gdpn_t}.$$

(Source: IFS and OCDE)

For the time series analysis, differentials are no longer calculated for the United States relative to every other country but relative to a representative European variable. The latter is obtained as the weighted average of the corresponding national values already used in the panel analysis. The weights are the share of national GDP relative to the GDP for our idyosincratic euro area. The GDP are in constant terms and PPP, as reported by the OECD, the base year being 1993. The bilateral real exchange rate (q_t) of the US dollar relative to the euro is obtained as in the panel, where s_t is defined as units of dollars required to purchase a euro. The sources for s_t are BBVA (from 1970:1 to 1997:4) and IFS for the rest of the sample.

Notes

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1. See ECB (2002).

2. For a complete overview of different empirical approaches, see Williamson (1994), and more recently, MacDonald (2000).

3. For simplicity, we are omitting the NATREX and the PEER approaches. We consider the first to be clearly connected to the FEER approach and the second to the BEER approach.

4. The breakdown between traded and nontraded goods has not been possible for the sample period, the OECD data available only reaching 1992.

5. Hamilton (1983) found that the energy price can account for innovations in many US macroeconomic variables. Amano and van Norden (1998) find a stable link between the

effective real exchange rate of the dollar and the oil price shocks. They also think that these shocks account for most of the major movements in the terms of trade. According to them, the correlations between the terms of trade and the one-period lagged price of oil are −0.57, −0.78, and −0.92 for the United States, Japan, and Germany, respectively.

6. A detailed description of the variables can be found in appendix A.

7. In addition, other specifications have been estimated in the empirical part of the model. In particular there are the simplest version of the Meese and Rogoff (1988) model ($rerdol_{it} = \alpha_i + \beta_{1i} drr_{it}$) and the Rogoff (1992) intertemporal model ($rerdol_{it} = \alpha_i + \beta_{1i} dpex + \beta_{2i} dpro_{it} + \beta_{3i} oildep_{it}$). In the first case, although the information criteria were encouraging, the model was not very explanatory (with $\bar{R}^2$ under 0.10 for the individual countries). As for the Rogoff (1992) model, none of the hypotheses concerning the long-run parameters were accepted, and the information criteria did not recommend its choice. The results, although not reported in this chapter, are available upon request.

8. Finally published as Levin, Lin, and Chu (2002).

9. Groen (2000) and Mark and Sul (2001) have also recently applied panel techniques to estimate models for the dollar exchange rate determination. In particular, Groen (2000) applies a panel version of the Engle and Granger two-step procedure under the homogeneity restriction on the long-run parameters. Mark and Sul (2001) apply dynamic OLS estimators, and also impose homogeneity in the cross sections.

10. All the results concerning this specification are available upon request.

11. The p-values associated with the test for each of the variables are the following: $dpex_t$ [0.40], $dnfa_t$ [0.51], and $dpro_t$ [0.00].

12. The results are available upon request.

13. The model has been specified with the constant unrestricted. Previous to this choice, the different possible specifications for the deterministic components were compared using the procedure suggested by Johansen (1996).

14. The magnitude of this parameter also lies in the range commonly found in the empirical literature, as reported by Gregorio and Wolf (1994). According to them, this range is $(-0.1, -1.0)$.

15. We should note that the real exchange rate is defined in our chapter in the opposite way. More precisely, an increase in the real exchange rate corresponds to a real depreciation.

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10 Dusting off the Perception of Risk and Returns in FOREX Markets

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After the demise of the Bretton Woods system in early 1973, many industrialized countries turned to a (semi-) floating exchange rate regime.[1] Academics try to explain causes of exchange rate fluctuations and search for policy recommendations. There are numerous papers attempting at explaining the movement of exchange rates.[2] Many theoretical versions, however, fail to determine exchange rates in practice. Empirical investigations have been carried out to test the exchange rate theories and the predictability of exchange rates.[3] The empirical support for the theories has been rather weak.

In this chapter a nonlinear model for exchange rates is proposed, based on the monetary exchange rate theory and the theory of financial asset pricing, so as to provide alternative insights about the anomalous behavior of exchange rates. This model is inspired by the pioneering work of Hodrick (1989), who introduced the volatilities of macroeconomic fundamentals in the exchange rate model as additional risk factors. Unlike Hodrick (1989), I incorporate macroeconomic risk into the flexible-price and the sluggish-price monetary models. This allows the long-run and short-run effects of the fundamental uncertainty to be examined. The empirical results are rather striking and supportive compared to those of Hodrick (1989).

As I show in this chapter, in the long run the nonlinear model explains how an increase in domestic money supply or a decrease in domestic real income leads to depreciation of the domestic currency, and vice versa for the foreign variables. Time-varying conditional variances of the macroeconomic variables, representing macroeconomic risk, can be related to the deviation of the exchange rate from its fundamental-based value. Macroeconomic uncertainty influences the perception of FOREX risk and consequently influences market expectations about compensation for risk bearing. Due to risk aversion, high risk is accompanied by high expected future returns, or equivalently a current depreciation of the currency. In the short run, the nonlinear model is shown to provide evidence for correction of equilibrium errors toward the long-run equilibrium.

These results indicate that macroeconomic sources of FOREX risk is a missing factor in exchange rate studies and that the monetary-approach models is potentially still useful. In section 10.1, I give the motivation of my work. In section 10.2, I discuss the nonlinear dynamic model. I report its econometric results in section 10.3, and make some concluding remarks in section 10.4.

10.1 Motivation for the Model

The monetary approach to model exchange rates has been viewed as one of the most dismal failures in modern economics (see Flood and Rose 1999). Nevertheless, it can hardly be denied that for our anticipation of exchange rates we rely on economic fundamentals, and often in the manner predicted by the monetary-based exchange rate models. My work was inspired by Dornbusch (1976) and Hodrick (1989). I use their model for exchange rates to reconsider the expectation assumptions used in the traditional exchange rate models by exploiting the statistical regularity of time-varying conditional variances of fundamental growth rates. As suggested by Dornbusch (1976), a fundamental change from its equilibrium level may cause a short-run overshooting in the exchange rate.

Volatility in the macroeconomic variables may consequently induce volatility in the exchange rate. In turn the uncertainty in macroeconomic fundamentals may influence the perception of risk in the markets, and subsequently through the risk premium it may price returns on the exchange rate, as stated in Hodrick (1989). This seems like a natural way to explain the exchange rate risk premium, as arising from variation in conditional variances of exchange rate returns, but Hodrick (1989) finds little support for the idea. After more than a decade since his research and almost three decades of floating exchange rate regime, it is time to reinvestigate the hypothesis in Hodrick (1989).

In the literature, exchange rates rely on two factors: the current fundamental levels, $\tilde{f}_t$, and the expectation of future exchange rates, $E_t[e_{t+1}]$.[4] A general framework of the models in the exchange rate literature can be summarized as shown in Cuthbertson (1999):[5]

$$e_t = E_t[e_{t+1}] - \alpha\tilde{f}_t, \tag{1}$$

where e_t is the logarithm of the nominal exchange rate, $\tilde{f}_t$ represents the fundamentals that may differ in each model, and $E_t[\cdot]$ is the conditional expectation operator. Apart from many possible estimation problems,[6] as expectations about the future exchange rate are likely to be a self-fulfilling prophecy, the expectation formation deserves considerable attention.

In the context of the present value relation, it is known that persistent movement in an asset's expected return tends to have dramatic effects on the asset price, as it makes the price more volatile than in the case of a constant expected return.[7] This also holds for the currency price, for which the expected return is represented by the expected price change. However, the source of the expectation variation is an unresolved issue. In this chapter, I provide an alternative explanation for the expectation formation in the exchange rate models. According to the exchange rate literature, the fundamental solution of the exchange rate is determined by the expected present value of macroeconomic fundamentals, discounted at a constant rate (following from Cuthbertson 1999 in this case is equal to one):[8]

$$e_t = -\sum_{i=0}^{\infty} \alpha E_t[\tilde{f}_{t+i}]. \tag{2}$$

By comparing this equation (i.e., $e_t = -\alpha\tilde{f}_t - \sum_{i=1}^{\infty} \alpha E_t[\tilde{f}_{t+i}]$) to equation (1), we find that the expected future fundamentals are used to determine the expected future exchange rate. However, in practice, the structure of expectation formation is not known, and the infinite horizon is not easily specified. It is often assumed that the fundamental processes are a random walk process, $E_t[x_{t+1}] - x_t = 0$. As a consequence the models are left with the current values of the fundamentals as representatives of the expected future fundamentals (e.g., see Meese and Rogoff 1983). As there is no expected change in the fundamentals, these rational expectation models imply zero expected exchange rate returns. Yet empirically positive correlations of exchange rate returns are found at short horizons, whereas negative serial correlations are reported at longer horizons (e.g., see Cuthbertson 1999).

Moreover there is some evidence for predictability of the exchange rate at long horizons once the fundamentals are brought into the analysis.[9] It is unlikely that the expected returns are zero. In particular, patterns of time variation in the mean and the variance of the fundamental changes have actually been observed. Like exchange rate returns, there is strong evidence of time-varying conditional variances of the fundamentals, although this is not well documented.[10] As there exists systematic fundamental volatility, I investigate in this chapter whether the fundamental uncertainty (e.g., through the risk premium) can determine expected exchange rate returns and thus the exchange rate movement.

This doctrine is similar to the well-known theme of asset pricing models, such as the capital asset pricing model (CAPM) developed by Markowitz (1959), Sharpe (1964), and Lintner (1965) and the arbitrage pricing theory of Ross (1976). The theory's goal is mainly to quantify the assets' equilibrium expected returns from the risk of bearing the assets. To relate exchange rate risk and return, Fama (1984) finds that the variation in the risk premium in the forward exchange market is more pronounced than the expected depreciation rate (i.e., expected exchange rate return). Frankel and Meese (1987) indicate that changes in conditional variance of the exchange rate have substantial impacts on the level of the exchange rate. Hodrick's (1989) model theoretically predicts that changes in the macroeconomic variances affect risk premia and therefore, exchange rates. Yet the empirical results are not supportive.

10.2 The Model

The present value of the exchange rate for the flexible-price model can be written as

$$e_t = \varsigma_0 \sum_{i=0}^{\infty} \varsigma_2^i + \varsigma_1 \sum_{i=0}^{\infty} \varsigma_2^i E_t[\tilde{f}_{t+i}], \tag{3}$$

where $\tilde{f}_t = \tilde{m}_t - (1+\gamma)\tilde{y}_t$, and $\tilde{m}_t$ and $\tilde{y}_t$ are the logarithms of the domestic money supply and real income with respect to the foreign levels. For the sluggish-price model, inertia is introduced into the price mechanism and thus the exchange rate equation. Cuthbertson (1999) shows that with the UIP condition the Dornbusch model gives rise to a form similar to equation (2):

$$e_t = \vartheta_1 e_{t-1} + \lambda \sum_{i=0}^{\infty} \vartheta_2 E_{t-1}[\tilde{k}_{t+i}], \qquad (\vartheta_1, \vartheta_2) < 1, \tag{4}$$

where $\tilde{k}_t \equiv (1/\varphi)\tilde{f}_t + [(1-\theta)/\theta\varphi]\tilde{f}_{t-1}$. The exchange rate now depends on $\tilde{k}_t$, namely current and lagged values of money supply and real income, and on its expected future values.[11]

Since the exchange rate is a discounted sum of expected future fundamentals (i.e., equations 2, 3, and 4), if the expectation of $\tilde{f}$ (or $\tilde{k}$ in a case of the sticky-price model) can be specified, an explicit process of the exchange rate can be found. A number of methods to incorporate the fundamentals' variances into their expectations are discussed in appendix C. Here we assume that the fundamental series can be explained by their historical values and their time-varying second moment.[12] Therefore the expected future fundamentals do not only depend on the current fundamental levels but also the expected variances of the fundamentals, representing the volatility of the fundamentals. An explicit solution of the flexible-price model can then be written as

$$e_t = a_0 + a_1\tilde{m}_t + a_2\tilde{y}_t + a_3h_{\tilde{m},t} + a_4h_{\tilde{y},t}. \tag{5}$$

In addition to the current fundamental values, the exchange rate is determined by time-varying conditional variances of the fundamentals, h_t.

For the sticky-price model the closed-form solution is

$$\begin{aligned} e_t = {} & b_0 + b_1e_{t-1} + b_2\tilde{m}_t + b_3\tilde{y}_t + b_4\tilde{m}_{t-1} + b_5\tilde{y}_{t-1} + b_6h_{\tilde{m},t} \\ & + b_7h_{\tilde{y},t} + b_8h_{\tilde{m},t-1} + b_9h_{\tilde{y},t-1}, \end{aligned} \tag{6}$$

in which the present and lagged values of the fundamentals and their time-varying conditional variances are included in the exchange rate determination. The levels of macroeconomic fundamentals are well known to be insufficient for explaining exchange rate movements. In addition to the traditional monetary models, we introduce macroeconomic risk to describe the deviation of the excessive volatile exchange rate relative to the conventional prediction based on economic fundamentals.

In this chapter the expectations of future fundamentals are reformulated by exploiting the systematic pattern of fundamental volatility, instead of assuming a random walk process. Equations (5) and (6) similarly predict that, ceteris paribus, an increase in money supply and a decrease in industrial production, relative to the foreign levels, tend to depreciate the domestic currency. Besides, we explain anomalous movements of the exchange rate, relative to the traditional paradigm, by the presence of volatility clusters in the fundamentals.[13] To capture the currency price volatility, time variation in conditional variances of the fundamentals, captured by a $\mathrm{GARCH}(1,1)$ model,[14] are incorporated to describe expected exchange rate returns.

The modified flexible-price model in equation (5) is used to characterize the long-run equilibrium of exchange rates, while the modified sticky-price model in equation (6) corrects for fundamental disequilibrium. The idea to examine the long-run impacts of macroeconomic risk on the exchange rate may seem controversial at first, as one would think that the exchange rate volatility is considered as a short-term phenomenon and has nothing to do with the long run. In fact, asset pricing models, such as CAPM, are used for the long-run equilibrium price determination. Intuitively the models say that one who holds risky assets expects to be compensated at least in the long run.

10.3 Specification and Estimation

With regard to the exchange rate level, although many developments can cause permanent changes in the exchange rate, the cointegration relationship between the spot rates and macroeconomic fundamentals implies that there is some long-run equilibrium relation tying the exchange rate to its macroeconomic fundamentals (see Hamilton 1994).[15] Moreover persistent movements in the fundamental volatility are likely to have larger impacts on exchange rate risk and returns than temporary movements. To model the exchange rate, we are therefore concerned with the cointegration among the variables in equation (5), whereas equation (6) is applied as an error-correction model to explain the adjustment toward the long-run equilibrium.

Like other macroeconomic studies this empirical study involves nonstationary and trending variables, such as exchange rates, money supply, and industrial production. Furthermore some GARCH series, as a proxy of time variation in conditional variances h_t, may appear to be $I(1)$ as the variance process is close to an integrated GARCH model, namely IGARCH. There are several ways to manipulate such series, to use transformations to reduce them to stationarity, such as to use a vector autoregressive (VAR) model or to analyze the relationship between these trending variables. Hodrick (1989) takes first differences to make the series stationary. However, in the existence of a cointegration relationship differencing the data might not be appropriate since counterproductively, it would obscure the long-run relationships between the variables.

As mentioned, the latter option allows us to distinguish between a long-run relationship, in which the variables drift together at roughly the same rate, and the short-run dynamics that capture the relationship between deviations of the variables from the long-run trend (see Stock and Watson 1988; Greene 2000). It should also be noted that the analysis involves generated regressors, in the form of the estimated conditional variances. According to Pagan (1984), the two-step procedure to estimate the conditional variances from the ARCH models and exogenously use the estimated variances in the OLS regression can produce consistency in estimated coefficients if the ARCH processes provide consistent estimates of true conditional variances (see also Hodrick 1989). Unlike Hodrick (1989), we will use a GARCH$(1, 1)$ model with a student t error distribution to estimate conditional variances h_t.[16]

For the empirical study we take our macroeconomic series from six OECD countries—Canada, France, Italy, Japan, the United Kingdom, and the United States. Our theoretical constructs will focus on exchange rates, money supply, and industrial production.[17] To see the role of economic fundamental uncertainty in determining the exchange rate risk and expected returns, we consider the price of a US dollar in terms of the domestic currency, as the US dollar has been recognized as a vehicle currency.[18] The US variables are thus treated as foreign variables in the exchange rate models. Hodrick (1989), however, finds no evidence for fundamental volatility to price exchange rates because of the weak evidence of ARCH in monthly exchange rates. By expanding the period employed in Hodrick (1989), we have stronger evidence of ARCH in monthly observations.[19] So we can reexamine the question posed in Hodrick (1989).

To investigate the exchange rate determination based on equations (5) and (6), we need to look at the domestic and the foreign variables separately, not in relative terms.[20] Thus the regression equations become

$$\begin{aligned} e_t = {} & a_0 + a_{1,d} m_t + a_{1,f} m_t^* + a_{2,d} y_t + a_{2,f} y_t^* + a_{3,d} \hat{h}_{m,t} \\ & + a_{3,f} \hat{h}_{m^*,t} + a_{4,d} \hat{h}_{y,t} + a_{4,f} \hat{h}_{y^*,t} \end{aligned} \tag{7}$$

and

$$\begin{aligned} e_t = {} & b_0 + b_1 e_{t-1} + b_{2,d} m_t + b_{2,f} m_t^* + b_{3,d} y_t + b_{3,f} y_t^* \\ & + b_{4,d} m_{t-1} + b_{4,f} m_{t-1}^* + b_{5,d} y_{t-1} + b_{5,f} y_{t-1}^* + b_{6,d} \hat{h}_{m,t} \\ & + b_{6,f} \hat{h}_{m^*,t} + b_{7,d} \hat{h}_{y,t} + b_{7,f} \hat{h}_{y^*,t} + b_{8,d} \hat{h}_{m,t-1} \\ & + b_{8,f} \hat{h}_{m^*,t-1} + b_{9,d} \hat{h}_{y,t-1} + b_{9,f} \hat{h}_{y^*,t-1}, \end{aligned} \tag{8}$$

where e is the logarithm of the nominal exchange rate (i.e., the price of a unit of foreign currency in terms of domestic currency), x represents a domestic variable, and x^* represents a foreign (US) variable.[21]

The method of investigation is as follows: An augmented Dickey-Fuller test is firstly applied to test the null hypothesis that the variables in equation (7) contain a unit root, namely using an $I(1)$ series, and whether the series are integrated to the same order. If the variables are integrated to different orders, a cointegration model would not be appropriate. Second, the Johansen (1988) test is used to identify the number of cointegration vectors from groups of the variables. Then, by an augmented Engle and Granger (1987) test, we check if the error term of the cointegration equation is an $I(0)$ series. Later, we advance to a dynamic OLS estimation of equation (7) and the short-run dynamic equation (8).

The first step is to identify the appropriate degree of differencing for each series. Suppose that the series of interest is z_t. Then the augmented Dickey-Fuller test is based on the regression of the following equation, with or without the presence of a trend t:

$$\chi(L)\Delta z_t = \mu + \tau t + \beta z_{t-1} + \upsilon_t,$$

where

$$\chi(L) = \mathbf{I}_n - \chi_1 L - \chi_2 L^2 - \cdots - \chi_p L^p$$

and υ_t is an error term. This augmented specification is used to test the null hypothesis of a unit root in the series, which is $H_0 : \beta = 0$ against $H_1 : \beta < 0$. Table 10.1 shows the results from the augmented Dickey-Fuller tests of the null hypotheses (1) that the logarithmic level of the series is $I(1)$ and (2) that the logarithmic first difference of the series contains a unit root. The table displays $\hat{\beta}$, and throughout this chapter an asterisk, two asterisks, and three asterisks indicate significance at the 10, 5, and 1 percent levels of significance, respectively.

According to table 10.1, the economic series are likely to be $I(1)$ series. At the 1 percent level of significance, first-differencing is appropriate to induce stationary in the natural logarithms of the exchange rate, money supply, and industrial productivity. The estimated GARCH processes of the macroeconomic variables are shown to be $I(0)$, except for the estimated series of the French money supply. The estimated GARCH processes of the Canadian money supply and real income exhibit trend stationarity at the 5 percent significance level. Therefore the model represented by equation (7) involves the variables that can

Table 10.1
Results of the augmented Dickey-Fuller unit root test, $\chi(L)\Delta z_t = \mu + \tau t + \beta z_{t-1} + \upsilon_t$

		Canada	France	Italy	Japan	United Kingdom	United States
Exchange rate	e	−0.709	−1.933	−1.920	−2.338	−2.560	
	Δe	−7.729***	−6.873***	−6.834***	−7.212***	−7.458***	
Money supply	m	−1.591	−2.200	−0.965	−3.275*	−3.366*	1.313
	Δm	−10.710***	−10.997***	−12.714***	−12.560***	−9.798***	−7.873***
	$\hat{h}_m$	−3.185**	−2.400	−8.218***	−3.492***	−4.903***	−4.043***
	$\Delta\hat{h}_m$	−7.824***	−8.639***				
Industrial production	y	−2.403	−2.783	−3.143	−1.108	−2.845	−2.915
	Δy	−6.350***	−8.125***	−8.754***	−5.471***	−7.944***	−6.461***
	$\hat{h}_y$	−3.179**	−5.676***	−6.538***	−3.919***	−4.413***	−5.563***
	$\Delta\hat{h}_y$	−11.059***					

Note: The results are of the augmented Dickey-Fuller unit root test. The test is based on the augmented equation displayed on top of the table. The specification, with or without a trend t depending on its significance, is used to test the null hypothesis of a unit root in the series (i.e., $H_0 : \beta = 0$) against the alternative hypothesis of no unit root ($H_1 : \beta < 0$). The test is applied to the natural logarithmic levels of exchange rate (e), money supply (m), and real income (y), and also to the estimated GARCH series (h) of money growth and income growth. For the series that cannot reject the unit root at the 1 percent level, the test is also applied to first differences of these series. *, **, and *** indicate significance at the 10, 5, and 1 percent levels respectively.

Table 10.2
Results of the Johansen cointegration test

$e_t = a_0 + a_{1,d}m_t + a_{1,f}m_t^* + a_{2,d}y_t + a_{2,f}y_t^* + a_{3,d}\hat{h}_{m,t} + a_{3,f}\hat{h}_{m^*,t} + a_{4,d}\hat{h}_{y,t} + a_{4,f}\hat{h}_{y^*,t}$

	Canada	France	Italy	Japan	United Kingdom
Hypothesized Number of ranks	2**	1**	1**	1**	2**

Note: The results are of the Johansen cointegration test for the group of the $I(1)$ variables in the modified flexible-price model (as shown on top of the table). The test is conducted under the null hypothesis that the cointegrating rank is r or lower. The table shows the number of cointegrating vectors, that cannot be rejected. *, **, and *** indicate significance at the 10, 5, and 1 percent levels respectively.

individually be either $I(0)$ or $I(1)$. The modified exchange rate equation is then tested for a cointegration relationship, that is, if there exists a stationary linear combination of these variables. The $I(0)$ variables are introduced as exogenous regressors in the cointegration function.

The second step is to examine if there is any cointegration relationship among these $I(1)$ series. The Johansen (1988) test is used to serve this purpose.[22] Table 10.2 reports the number of significant cointegration vectors. The likelihood ratio (LR) test can reject the null hypothesis of no cointegration in every country. At the 5 percent significance level, the LR test indicates 1 cointegration relationship for France, Italy, and Japan, and 2 cointegration relationships in the case of Canada and the United Kingdom.

As the Johansen test predicts cointegration relationship(s) for every country, an alternative method by Engle and Granger (1987) is used to assess whether linear combinations, based on the flexible-price model in equation (7), are stationary. From equation (7), the model for the exchange rate that is suitable for regression analysis can be rewritten as

$$e_t = \hat{a}_0 + \hat{a}_{1,d}m_t + \hat{a}_{1,f}m_t^* + \hat{a}_{2,d}y_t + \hat{a}_{2,f}y_t^* + \hat{a}_{3,d}\hat{h}_{m,t} + \hat{a}_{3,f}\hat{h}_{m^*,t} + \hat{a}_{4,d}\hat{h}_{y,t} + \hat{a}_{4,f}\hat{h}_{y^*,t} + \varepsilon_t, \tag{9}$$

where ε_t is an error term. In equation (9) the cointegration function represents the long-run movement of exchange rates. OLS estimation is applied because it has been proved to yield asymptotically super-consistent estimators when estimating cointegration relationships (see Greene 2000). The Engle and Granger (1987) two-step procedure test is applied to examine the stationarity of the residual term ε_t.

To correct for autocorrelation in the equilibrium error series, an augmented Engle and Granger test is based on estimating

Table 10.3
Results of the augmented Engle and Granger cointegration test
$\Delta\varepsilon_t = \phi_0\varepsilon_{t-1} + \phi_1\Delta\varepsilon_{t-1} + \cdots + \varepsilon_t$

	Canada	France	Italy	Japan	United Kingdom
$\hat{\phi}_0$	−3.415**	−4.904***	−3.722***	−3.129**	−4.522***

Note: The results are of the augmented Engle and Granger cointegration test on the equilibrium error ε_t. It is to test the significance of the null hypothesis that the error series contains a unit root (i.e., $H_0 : \phi_0 = 0$, $H_1 : \phi_0 < 0$). If the null hypothesis cannot be rejected, there is no cointegration relationship. *, **, and *** indicate significance at the 10, 5, and 1 percent levels respectively.

$$\Delta\varepsilon_t = \phi_0\varepsilon_{t-1} + \phi_1\Delta\varepsilon_{t-1} + \cdots + \varepsilon_t$$

by the Newey-West approach. If the null hypothesis of a unit root in the residual series ($H_0 : \phi_0 = 0$, $H_1 : \phi_0 < 0$) cannot be rejected, there is no cointegration relationship among the variables in the model. Table 10.3 shows $\hat{\phi}_0$. Asterisks indicate that the null hypothesis of unit root can be rejected at the 5 percent significance level for Canada and Japan, and at the 1 percent level for France, Italy and the United Kingdom. The evidence in tables 10.2 and 10.3 demonstrates the cointegration in these countries.

To test our assumption regarding the expectation formation that incorporates macroeconomic uncertainty, we apply the two-step cointegration approach as proposed by Engle and Granger (1987).[23] We first deal with the modified flexible-price model and then the modified sluggish-price model. The Stock and Watson (1993) dynamic OLS estimation method is employed to regress the logarithm of the exchange rate against the logarithms of money supply and industrial production, and the estimated conditional variances—from a GARCH$(1, 1)$ model—of the growth rates of money supply and industrial production. The US variables are used as the foreign variables. Although the OLS estimation has proved to asymptotically yield superconsistent estimates, because of the possibility that the explanatory variables are contemporaneously correlated with the disturbance term, the OLS regression coefficients are likely to be inconsistent.[24] The dynamic OLS procedure, on the other hand, is robust to small sample size and simultaneity bias.

To eliminate the effects of these correlations, we apply the Stock and Watson (1993) dynamic OLS approach by adding the one-period leads and lags of the first differences of the regressors mentioned above.[25] The method is also known for being a robust single equation that corrects for stochastic-regressor endogeneity. According to equation (7) the dynamic OLS equation is

$$\begin{aligned} e_t = {} & \hat{a}_0 + \hat{a}_{1,d} m_t + \hat{a}_{1,f} m_t^* + \hat{a}_{2,d} y_t + \hat{a}_{2,f} y_t^* + \hat{a}_{3,d} \hat{h}_{m,t} + \hat{a}_{3,f} \hat{h}_{m^*,t} \\ & + \hat{a}_{4,d} \hat{h}_{y,t} + \hat{a}_{4,f} \hat{h}_{y^*,t} + \hat{a}_{5,d} \Delta m_{t+1} + \hat{a}_{5,f} \Delta m_{t+1}^* + \hat{a}_{6,d} \Delta y_{t+1} \\ & + \hat{a}_{6,f} \Delta y_{t+1}^* + \hat{a}_{7,d} \Delta \hat{h}_{m,t+1} + \hat{a}_{7,f} \Delta \hat{h}_{m^*,t+1} + \hat{a}_{8,d} \Delta \hat{h}_{y,t+1} \\ & + \hat{a}_{8,f} \Delta \hat{h}_{y^*,t+1} + \hat{a}_{9,d} \Delta m_{t-1} + \hat{a}_{9,f} \Delta m_{t-1}^* + \hat{a}_{10,d} \Delta y_{t-1} \\ & + \hat{a}_{10,f} \Delta y_{t-1}^* + \hat{a}_{11,d} \Delta \hat{h}_{m,t-1} + \hat{a}_{11,f} \Delta \hat{h}_{m^*,t-1} + \hat{a}_{12,d} \Delta \hat{h}_{y,t-1} \\ & + \hat{a}_{12,f} \Delta \hat{h}_{y^*,t-1} + \xi_t, \end{aligned} \tag{10}$$

where ξ_t denotes the error term. Table 10.4 contains the estimated parameters from equation (10), $\hat{a}_{i,d}$ and $\hat{a}_{i,f}$ when $i = 1, \ldots, 4$. An asterisk, two asterisks, and three asterisks indicate significance at the 10, 5, and 1 percent level of significance, respectively.

Apart from allowing us to examine the long-run impacts of macroeconomic risk on exchange rates, adding the estimated macroeconomic risk into a cointegration equation may help reduce the problem of omitted variables.[26] From table 10.4 the estimated coefficients of money supply and real income have signs as expected in the literature. In the long run an increase in the domestic money supply or a decrease in the foreign money supply tends to depreciate the domestic currency,

Table 10.4
Parameters of the modified flexible-price model

$e_t = \hat{a}_0 + \hat{a}_{1,d} m_t + \hat{a}_{1,f} m_t^* + \hat{a}_{2,d} y_t + \hat{a}_{2,f} y_t^* + \hat{a}_{3,d} \hat{h}_{m,t} + \hat{a}_{3,f} \hat{h}_{m^*,t} + \hat{a}_{4,d} \hat{h}_{y,t} + \hat{a}_{4,f} \hat{h}_{y^*,t} + \cdots$

	Canada	France	Italy	Japan	United Kingdom
$\hat{a}_0$	−3.128***	6.740***	6.210***	7.472***	3.868***
$\hat{a}_{1,d}$	−0.116	0.753***	0.929***	0.774***	0.321**
$\hat{a}_{1,f}$	0.155**	−0.580***	−0.666***	−1.101***	0.097
$\hat{a}_{2,d}$	0.535**	−1.886***	−2.056***	0.538**	−1.266**
$\hat{a}_{2,f}$	0.076	0.479*	0.693***	−1.444***	0.029
$\hat{a}_{3,d}$	116.328***	−530.64***	−497.51*	−224.703*	−877.01**
$\hat{a}_{3,f}$	1356.045**	−3185.05**	−4555.66**	−1840.97	−6340.388***
$\hat{a}_{4,d}$	178.947***	376.15**	46.59**	270.166*	−172.19*
$\hat{a}_{4,f}$	−25.750	6.369	−433.14	−555.754**	−777.12***

Note: The estimation results are of the modified flexible-price model, based on the Stock and Watson (1993) dynamic OLS approach. The estimated parameters are $\hat{a}_{i,d}$ and $\hat{a}_{i,f}$ when $i = 1, \ldots, 4$. An asterisk, two asterisks, and three asterisks indicate significance at the 10, 5, and 1 percent levels of significance respectively.

except for Canada. Higher domestic output or lower foreign output is likely to appreciate the domestic currency (although there are exceptions for Canada and Japan).

For Canada and the United Kingdom, at the 5 percent significance level the Wald test cannot reject the null hypothesis that the coefficients of domestic and foreign macroeconomic variables, like money supply and real income, are significantly equal. When we restrict the domestic and foreign coefficients of money supply and real income to be equal in these countries, higher money supply or lower real income relative to the US tends to depreciate the domestic currencies while the coefficients of macroeconomic risk are similar to those in table 10.4.[27]

Significantly, an increase in the money supply volatility, both domestic and foreign, depreciates the Canadian dollar but appreciates other currencies. For uncertainty in real income, the results significantly show that with an increase in the domestic volatility, the domestic currency depreciates (except in the United Kingdom). In contrast, with an increase in the foreign volatility, the domestic currency appreciates. Higher uncertainty in the US real income or the US money supply raises the expected future returns on US dollars by pushing down the current US dollar price. It consequently causes the domestic currency to appreciate (except in Canada). By the same argument, uncertainty in the domestic real income is positively related to the US dollar exchange rates. It leads to an upward bias in the variation of actual exchange rates from the prediction of the traditional model.

From table 10.4, macroeconomic uncertainty, represented by the conditional variances of money supply and real income, relates significantly to the deviation of the exchange rate from its fundamentally based value. Uncertainty about the economy appears to lower the demand for the currency and subsequently leads to depreciation, relative to the fundamental benchmark value. From an asset pricing perspective, higher risk should be accompanied by higher expected future returns, leading to a current depreciation of the currency. Theory coherently predicts that higher variability of domestic fundamentals should result in higher current depreciation of the domestic currency. However, the opposite impact can also be observed in some cases of uncertainty in money growth.

For every country except Canada, higher volatility in the domestic money supply tends to increase the domestic currency prices. This might be because the volatile money supply (e.g., due to volatile capital flows or active domestic monetary policy) does not necessarily imply a negative outlook on the domestic currency.[28] Because of this positive effect of macroeconomic risk, economic agents prefer to hold their local currencies and will pay a higher price. These cases also reveal a strong preference for domestic currency that is parallel to the equity home bias, meaning the tendency to underinvest in (more attractive) foreign assets, that has been long studied in finance.[29] On the other hand for Canada there exists a negative risk premium toward the US dollar, which shows a positive reaction toward an active US monetary policy.[30]

The modified sticky-price model extends the cointegration relationship between the exchange rate and its fundamentals by adding the long-run equilibrium error adjustment. By rearranging equation (8), we obtain a form of the error-correction model:

$$\begin{aligned}\Delta e_t &= \hat{b}_0 + \hat{b}_{2,d}\Delta m_t + \hat{b}_{2,f}\Delta m_t^* + \hat{b}_{3,d}\Delta y_t + \hat{b}_{3,f}\Delta y_t^* + \hat{b}_{6,d}\Delta \hat{h}_{m,t} \\ &\quad + \hat{b}_{6,f}\Delta \hat{h}_{m^*,t} + \hat{b}_{7,d}\Delta \hat{h}_{y,t} + \hat{b}_{7,f}\Delta \hat{h}_{y^*,t} + (\hat{b}_1 - 1) \\ &\quad \times \left\{ \begin{matrix} e_{t-1} - \hat{c}_{1,d} m_{t-1} - \hat{c}_{1,f} m_{t-1}^* - \hat{c}_{2,d} y_{t-1} - \hat{c}_{2,f} y_{t-1}^* \\ -\hat{c}_{3,d}\hat{h}_{m,t-1} - \hat{c}_{3,f}\hat{h}_{m^*,t-1} - \hat{c}_{4,d}\hat{h}_{y,t-1} + \hat{c}_{4,f}\hat{h}_{y^*,t-1} \end{matrix} \right\} + v_t,\end{aligned}$$

where

$\hat{c}_{1,d} = -(\hat{b}_{4,d} + \hat{b}_{2,d})/(\hat{b}_1 - 1)$, $\hat{c}_{1,f} = -(\hat{b}_{4,f} + \hat{b}_{2,f})/(\hat{b}_1 - 1)$,
$\hat{c}_{2,d} = -(\hat{b}_{5,d} + \hat{b}_{3,d})/(\hat{b}_1 - 1)$, $\hat{c}_{2,f} = -(\hat{b}_{5,f} + \hat{b}_{3,f})/(\hat{b}_1 - 1)$,
$\hat{c}_{3,d} = -(\hat{b}_{8,d} + \hat{b}_{6,d})/(\hat{b}_1 - 1)$, $\hat{c}_{3,f} = -(\hat{b}_{8,f} + \hat{b}_{6,f})/(\hat{b}_1 - 1)$,
$\hat{c}_{4,d} = -(\hat{b}_{9,d} + \hat{b}_{7,d})/(\hat{b}_1 - 1)$ and $\hat{c}_{4,f} = -(\hat{b}_{9,f} + \hat{b}_{7,f})/(\hat{b}_1 - 1)$.

Provided that the relationship between the exchange rate and the fundamentals is stable, the set of coefficients c in this equation is equivalent to the set of coefficients a in the modified flexible-price model. Thus we can test the short-run dynamic equation[31]

$$\begin{aligned}\Delta e_t &= \hat{b}_0 + \hat{b}_{2,d}\Delta m_t + \hat{b}_{2,f}\Delta m_t^* + \hat{b}_{3,d}\Delta y_t + \hat{b}_{3,f}\Delta y_t^* + \hat{b}_{6,d}\Delta \hat{h}_{m,t} \\ &\quad + \hat{b}_{6,f}\Delta \hat{h}_{m^*,t} + \hat{b}_{7,d}\Delta \hat{h}_{y,t} + \hat{b}_{7,f}\Delta \hat{h}_{y^*,t} + (\hat{b}_1 - 1)\varepsilon_{t-1} + v_t.\end{aligned}$$

Note that since first differencing is sufficient to produce stationary series and since there exists the cointegration relationship shown in tables 10.2 and 10.3, the residual term v_t is an $I(0)$ series.

As stated by Greene (2000), the movement of the exchange rate from the previous period associates with the changes in the fundamentals along the long-run equilibrium corrected for the previous deviation

Table 10.5
Parameters of the modified sticky-price model

$$\Delta e_t = \hat{b}_0 + (\hat{b}_1 - 1)\varepsilon_{t-1} + \hat{b}_{2,d}\Delta m_t + \hat{b}_{2,f}\Delta m_t^* + \hat{b}_{3,d}\Delta y_t + \hat{b}_{3,f}\Delta y_t^* + \hat{b}_{6,d}\Delta \hat{h}_{m,t} + \hat{b}_{6,f}\Delta \hat{h}_{m^*,t} + \hat{b}_{7,d}\Delta \hat{h}_{y,t} + \hat{b}_{7,f}\Delta \hat{h}_{y^*,t} + v_t$$

	Canada	France	Italy	Japan	United Kingdom
$\hat{b}_0$	0.001	2.37E-4	0.003	−0.002	0.001
$(\hat{b}_1 - 1)$	−0.023**	−0.056***	−0.027	−0.063***	−0.087***
$\hat{b}_{2,d}$	0.042	−0.024	−0.083	0.021	−0.067
$\hat{b}_{2,f}$	−0.055	−0.071	0.040	−0.189*	0.003
$\hat{b}_{3,d}$	0.070	−0.109	0.009	−0.050	−0.078
$\hat{b}_{3,f}$	0.207**	0.502**	0.481**	0.084	0.675***
$\hat{b}_{6,d}$	9.119	−10.505	−56.871	75.438	−159.643*
$\hat{b}_{6,f}$	138.630	33.414	59.534	−210.013	−229.840
$\hat{b}_{7,d}$	17.408	22.570	−3.041	12.068	−10.498
$\hat{b}_{7,f}$	−28.122	−75.073	−4.561	−21.820	−42.507

Note: The estimation results are of the modified sticky-price model, based on the linear OLS regression. An asterisk, two asterisks, and three asterisks indicate significance at the 10, 5, and 1 percent levels of significance respectively.

from the long-run equilibrium. This equation contains an equilibrium relationship in the first two lines and an adjustment for the deviation from the previous equilibrium in the last line. Table 10.5 shows that there exists a correction mechanism of equilibrium errors toward the long-run equilibrium, as $(\hat{b}_1 - 1)$ is significantly negative, except in the case of Italy. The error correction term, ε_{t-1}, is significantly negative at the 5 percent significance level in the case of Canada and at the 1 percent significance level for France, Japan, and the United Kingdom. For Italy, at the monthly horizon, the adjustment toward long-run equilibrium is not significant but the sign of $(\hat{b}_1 - 1)$ is still negative. Furthermore, in the short run, the exchange rate can be significantly explained by changes in the US real income. Yet other macroeconomic fundamentals as well as their uncertainty fail to explain the exchange rate in the short run.

10.4 Conclusion

The expectations regarding macroeconomic circumstances can influence the exchange rate in the manner predicted by the monetary models, but the random walk assumption is too naive for market expectations. In this chapter, I propose an alternative expectation formation process for the macroeconomic variables by introducing additional risk factors, based on the volatility of the macroeconomic fundamentals. As the fundamentals empirically exhibit a mean-reverting process with persistent memory in the standard deviation (representing the adjustment and speed toward the mean), a nonlinearity in the expectation formation process is present. To capture the exchange rate volatility, in addition to the traditional fundamentals, such as money supply and real income, time variation in the second moments of these fundamentals is incorporated to describe the expected exchange rate returns.

The result shows significant cointegration between the variables in the modified flexible-price monetary model, as well as a correction of equilibrium errors toward the long-run equilibrium in the modified sticky-price model. In the long run, an increase in the domestic money supply or a decrease in the foreign money supply tends to depreciate the domestic currency. Higher domestic output or lower foreign output is likely to appreciate the domestic currency. The impacts of macroeconomic sources of risk are also significant. In general, uncertainty about the economy lowers the demand for the currency and subsequently depreciates the currency, relative to the fundamental-based value. From an asset pricing perspective, increased risk is accompanied by increased expected future returns, leading to a current depreciation of the currency. The findings in this chapter indicate that macroeconomic sources of FOREX risk are a missing factor in exchange rate studies and that the monetary exchange rate models are still potentially useful.

Appendix A: Data Sources

The data applied in this chapter are monthly observations of exchange rates, money supply and industrial production, starting from June 1973 (with the breakdown of the Bretton Woods system) to December 1998. There are six OECD countries studied: Canada, France, Italy, Japan, the United Kingdom, and the United States. Both European and non-European countries, with possible different economic mechanisms, are selected based on the availability of the required data. The US dollar is used as a vehicle currency and the US variables are used as the foreign variables.

The main data source is the IMF International Financial Statistics (IFS), except for M1 of the United States. This time series is from the US Federal Reserve Bank at St. Louis. It is compared with available quarterly series from the IFS and they are very similar. The US dollar exchange rates (domestic currency prices per one US dollar) from the IFS are coded AE. Monetary aggregation is represented by seasonally unadjusted M1 data from IFS coded 34, except for the United Kingdom. For the purpose of this chapter, liquidity under the central bank's controllability is preferable. For the United Kingdom, M_0, is used and coded 59, instead of another available choice M_4. Seasonally adjusted industrial production, coded 66, is used as a proxy for real income. If necessary, a seasonal adjustment can be made by way of an additive seasonal moving average approach.

Appendix B: The Reduced-Form Solutions of the Exchange Rate Models

The flexible-price model is derived from the simple quantity equation $M_tV_t = P_tY_t$. In logarithms, the quantity equation reveals that

$$m_t + v_t = p_t + y_t, \tag{A1}$$

where m_t, v_t, p_t, and y_t are the logarithms of the money supply, the money velocity, the price level, and the real income at period t respectively. We can assume that purchasing power parity (PPP) and uncovered interest parity (UIP) hold.

The stochastic PPP assumption, which is a more specific version of the no-arbitrage assumption, is defined as

$$p_t = \tau + p_t^* + e_t + \omega_t. \tag{A2}$$

In equation (A2), e_t, p_t, and p_t^* are the logarithms of the nominal exchange rate, namely the price of a unit of foreign currency, the domestic price level and the foreign price level respectively. An asterisk denotes a foreign variable, which in this case is a US variable. While τ is a constant, ω_t represents a stationary, zero-mean disturbance term, sometimes referred to as the real exchange rate.

According to the UIP condition, the interest rate differential between domestic and foreign assets is supposed to be equal to the expected rate of depreciation of the domestic currency. The expected change in currency price that satisfies equilibrium in the capital markets can thus be written as

$$E_t[e_{t+1}] - e_t = i_t - i_t^*, \tag{A3}$$

where i_t and i_t^* are the domestic interest rate and the foreign interest rate respectively. $E_t[\cdot]$ is the conditional expectation operator.

The velocity of money circulation is presumed to be a stable function of real income and the interest rate. The logarithm of money velocity is linearly specified as a decreasing function of the logarithm of real income and an increasing function of the interest rate:

$$v_t = \theta - \gamma y_t + \varphi i_t + \varpi_t, \tag{A4}$$

where θ is a constant and ϖ_t is a stationary, zero-mean disturbance.

Suppose that (A1) holds at home and in foreign countries with an identical income elasticity, γ, and interest semi-elasticity, φ. Combine (A1) with (A2), (A3), and (A4) and rework for the foreign country:

$$e_t = -\tau + \frac{1}{1+\varphi}\tilde{m}_t - \frac{(1+\gamma)}{1+\varphi}\tilde{y}_t + \frac{\varphi}{1+\varphi}E_t[e_{t+1}] + \varepsilon_t, \tag{A5}$$

where $\tilde{x}_t = x_t - x_t^*$ and $\varepsilon_t = \varpi_t - \varpi_t^* - \omega_t$.

To solve this linear equation with rational expectation, we apply the law of iterated expectations (see Samuelson 1965; Blanchard and Fischer 1993). For simplicity, we rewrite equation (A5) as

$$e_t = \varsigma_0 + \varsigma_1 \tilde{f}_t + \varsigma_2 E_t[e_{t+1}] + \varepsilon_t, \tag{A6}$$

where $\varsigma_0 = -\tau$, $\varsigma_1 = 1/(1+\varphi)$, $\varsigma_2 = \varphi/(1+\varphi)$, and $\tilde{f}_t = \tilde{m}_t - (1+\gamma)\tilde{y}_t$. Note that $\varsigma_2 = 1 - \varsigma_1$ and that ς_1 and $\varsigma_2 \in (0,1)$ as $0 < \varphi < 1$ (see Flood, Rose, and Mathieson 1991; Flood and Rose 1995). Equation (A6) implies that the exchange rate depends on its expected rate for the next period, $E_t[e_{t+1}]$, and on the current fundamentals, $\tilde{f}_t$, with the weights summing up one. According to the law of iterated expectations, we have

$$e_t = \varsigma_0 \sum_{i=0}^{T} \varsigma_2^i + \varsigma_1 \sum_{i=0}^{T} \varsigma_2^i E_t[\tilde{f}_{t+i}] + \varsigma_2^{T+1} E_t[e_{t+T+1}] + \sum_{i=0}^{T} \varsigma_2^i E_t[\varepsilon_{t+i}]. \tag{A7}$$

We then assume that as the horizon T increases, the exchange rate at $T+1$ periods becomes negligeable, or equivalently the rational bubble shrinks to zero and that $E_t[\varepsilon_{t+i}] = 0$.

As T tends to infinity,

$$\lim_{T\to\infty} \varsigma_2^{T+1} E_t[e_{t+T+1}] = 0, \tag{A8}$$

and the solution becomes

$$e_t = \varsigma_0 \sum_{i=0}^{\infty} \varsigma_2^i + \varsigma_1 \sum_{i=0}^{\infty} \varsigma_2^i E_t[\tilde{f}_{t+i}]. \tag{A9}$$

This equation is comparable to equation (2), and implies that the elasticity of the exchange rate, with respect to its expected fundamentals, declines as we look farther into the future as

$$\lim_{t\to\infty} \varsigma_2^t = 0. \tag{A10}$$

Moreover, for equation (A8) to converge, it requires that the logarithm of fundamentals, $\tilde{f}$, grow at rate lower than $\varsigma_1/(1-\varsigma_1)$ (i.e., $1/\varphi$); otherwise, the solution (A9) would be explosive.

The sluggish-price model is an extension of the flexible-price model with inertia introduced into the price mechanism, instead of relying on perfectly flexible prices. Empirically there are deviations from purchasing power parity in equation (A2) where ω_t are large and persistent. There is also strong correlation between nominal and real exchange rates. In Dornbusch's (1976) sluggish-price model the expected exchange rate return is formed as the discrepancy between the long-run rate $\bar{e}$, to which the economy will eventually converge, and the current spot rate e. Mathematically,

$$E[e] - e = \delta(\bar{e} - e), \qquad 0 < \delta < 1.$$

To allow for sticky prices, the Phillips curve equation is substituted in equation (A2) in the place of purchasing power parity (e.g., see Obstfeld and Rogoff 1984; Flood and Rose 1995). It is conventional to assume that in addition to the PPP condition, prices respond to the lagged excess demand in the good markets, $y_t - \bar{y}_t$, and shocks to the good markets, g_t:

$$p_{t+1} - p_t = \mu(y_t - \bar{y}_t) + g_t + E_t[\hat{p}_{t+1} - \hat{p}_t], \qquad 0 < \mu < 1, \tag{A11}$$

where $\bar{y}$ is the long-run output level, g_t has zero mean and constant variance, and $\hat{p}_t$ is the price level at time t if prices were flexible and the good markets cleared.

$$y_t - \bar{y}_t = \Theta(e_t + p_t^* - p_t) + \Phi r_t. \tag{A12}$$

The excess demand is defined as an increasing function of real exchange rate, $\Theta > 0$, and a decreasing function of the ex ante expected real interest rate, namely $r_t \equiv i_t - E_t[p_{t+1} - p_t]$, $\Phi < 0$. Thus, by substituting equation (A12) into equation (A11), we get

$$p_{t+1} - p_t = \mu[\Theta(e_t + p_t^* - p_t) + \Phi r_t] + g_t + E_t[\hat{p}_{t+1} - \hat{p}_t]. \tag{A13}$$

Equation (A13) displays the long-run equilibrium (when the purchasing power parity holds and thus, the left-hand side, LHS, is equal to the last term on the right-hand side, RHS) and its short-run dynamics (represented by deviations from the purchasing power parity by the first and the second terms on the RHS).

As in the long run $\hat{p} = p$, $\hat{p}$ can be defined by

$$\mu[\Theta(e_t + p_t^* - \hat{p}_t) + \Phi r_t] + g_t = 0,$$

and thus

$$p_{t+1} - p_t = \mu[\Theta(e_t + p_t^* - p_t) + \Phi r_t] + g_t + E_t[p_{t+1}^* - p_t^*] \\ + E_t[e_{t+1} - e_t] + \frac{\Phi}{\Theta} E_t[r_{t+1} - r_t] + \frac{1}{\mu\Theta} E_t[g_{t+1} - g_t].$$

Therefore, instead of using the purchasing power parity condition in equation (A2), we substitute the price equation,

$$\tilde{p}_t = p_t - p_t^* = e_t + \frac{1}{\Theta\mu} E_t[e_{t+1} - e_t] + \frac{1}{\Theta\mu} E_t[p_{t+1}^* - p_t^*] - \frac{1}{\Theta\mu}(p_{t+1} - p_t) \\ + \frac{1}{\mu}\frac{\Phi}{\Theta^2} E_t[r_{t+1} - r_t] + \frac{\Phi}{\Theta} r_t + \frac{1}{\mu^2\Theta^2} E_t[g_{t+1} - g_t] + \frac{1}{\Theta\mu} g_t, \tag{A14}$$

into the money demand equation, derived from the quantity equation (A1) and the assumption of money circulation (A4):

$$\tilde{p}_t = \tilde{m}_t - (1+\gamma)\tilde{y}_t + \varphi\tilde{i}_t + \tilde{\varpi}_t.$$

Hence

$$e_t = \tilde{m}_t - (1+\gamma)\tilde{y}_t + \varphi\tilde{i}_t + \tilde{\varpi}_t - \frac{1}{\Theta\mu} E_t[e_{t+1} - e_t] \\ - \frac{1}{\Theta\mu} E_t[p_{t+1}^* - p_t^*] + \frac{1}{\Theta\mu}(p_{t+1} - p_t) - \frac{1}{\mu}\frac{\Phi}{\Theta^2} E_t[r_{t+1} - r_t] \\ - \frac{\Phi}{\Theta} r_t - \frac{1}{\mu^2\Theta^2} E_t[g_{t+1} - g_t] - \frac{1}{\Theta\mu} g_t. \tag{A15}$$

To present the model in a common form as in equation (2), we assume the UIP condition (A3) and the price process in equation (A14). As a consequence the exchange rate equation becomes

$$e_t = \tilde{k}_t + E_t[e_{t+1}] + \frac{(1-\theta)\varphi - \theta}{\theta\varphi} E_{t-1}[e_t] - \frac{(1-\theta)\varphi - \theta + 1}{\theta\varphi} e_{t-1} + \psi_t. \tag{A16}$$

where

$\tilde{k}_t = \frac{1}{\varphi}\tilde{f}_t + \frac{1-\theta}{\theta\varphi}\tilde{f}_{t-1}$,
$\psi_t = \frac{1}{\varphi}p_t^* - \frac{1}{\varphi}E_{t-1}[p_t^*] - \frac{\Omega}{\varphi}E_{t-1}[r_t] - \frac{(1-\theta\Omega)}{\theta\varphi}r_{t-1} - \frac{1}{\varphi}g_{t-1} - \frac{\theta}{\varphi}E_{t-1}[g_t - g_{t-1}]$,

and the fundamental $\tilde{f}_t$ is defined as $\tilde{f}_t = \tilde{m}_t - (1+\gamma)\tilde{y}_t$. The coefficients are assigned by $\theta = 1/\Theta\mu$ and $\Omega = \Phi/\Theta$. To apply the law of iterated expectations to this second-order difference equation, we define $A_t = e_t + \{[(1-\theta)\varphi - \theta + 1]/\theta\varphi\}e_{t-1}$. Equation (A16) can then be rewritten as

$$A_t = \tilde{k}_t + E_{t-1}[A_{t+1}] - \frac{1}{\theta\varphi}E_{t-1}[e_t] + \kappa_t + \psi_t, \tag{A17}$$

where $\kappa_t = E_t[e_{t+1}] - E_{t-1}[e_{t+1}]$.

By the law of iterated expectations, we get

$$A_t = \tilde{k}_t + \sum_{i=1}^{T} E_{t-1}[\tilde{k}_{t+i}] - \frac{1}{\theta\varphi}\sum_{i=0}^{T} E_{t-1}[e_{t+i}] + E_{t-1}[A_{t+T+1}]$$
$$+ \sum_{i=0}^{T} E_{t-1}[\kappa_{t+i}] + \sum_{i=0}^{T} E_{t-1}[\psi_{t+i}].$$

For simplicity, we presume that the expected exchange rate in any one period, namely $E_{t-1}[A_{t+T+1}]$, is only a small component in determining the current spot rate, and it becomes negligeable as the horizon T rises. Furthermore, when $i \geq 0$, $E_{t-1}[\kappa_{t+i}] = E_{t-1}[\psi_{t+i}] = 0$. As a consequence, as T tends to infinity, the solution becomes

$$e_t = \frac{(1+\varphi)(\theta-1)}{\theta\varphi} e_{t-1} + \tilde{k}_t + \sum_{i=1}^{\infty} E_{t-1}[\tilde{k}_{t+i}] - \frac{1}{\theta\varphi}\sum_{i=0}^{\infty} E_{t-1}[e_{t+i}]. \tag{A18}$$

Equation (A18) is like equation (A9) in the flexible-price model, except there is inertia in the exchange rate equation. The exchange rate now depends on $\tilde{k}_t$, namely current and lagged values of money supply and income, and its expected future fundamentals. Additionally equation (A18) is rather similar to the sticky-price concept stated earlier and also a solution (4) from Cuthbertson (1999).

Appendix C: Adding Stochastic Volatility to the Fundamental Expectations

There are many ways to incorporate the second moments of the fundamentals into their expectations. In this appendix I show a few possible ways. In developing an explicit solution for the exchange rate, Hodrick (1989) assumes a conditionally lognormal data-generating process for the fundamentals, and applies the fact that if x has a lognormal distribution with $\log(x) \sim N(\mu, \sigma^2)$, its expectation reads $E[x] = e^{\mu+\sigma^2/2}$. Hence, by loglinearization of his general equilibrium model, we get $\log(E[x]) = \mu + \frac{1}{2}\sigma^2$, which is explored in Hodrick (1989).

We could equivalently adopt the modified form of uncovered interest parity (UIP) that adjusts for a risk premium, and we could specify a risk premium as a function of time-varying fundamental variances. This is similar to the portfolio balance model, in which the UIP condition incorporates the risk premium as a function of relative asset holding in domestic and foreign bonds. By combining equation (A5) with a modified version of UIP that has a time-varying risk premium ρ_t, and applying the law of iterated expectations, we can express the exchange rate as

$$e_t = E_t[\tilde{m}_{t+i}] - \beta E_t[\tilde{y}_{t+i}] + \alpha E_t[\rho_{t+i}].$$

From the equation above, the exchange rates are determined by two components: the expectation regarding the future fundamental values and the expectation regarding risk from holding the currency. Intuitively, a deviation from its expected fundamental value needs an extra compensation. So, using a risk premium, we can characterize risk in the FOREX markets by macroeconomic uncertainty.

Another technical approach is to apply Taylor's theorem. To make our point, we consider money supply process based on Lucas (1982) and Obstfeld (1987). Suppose $m_t = w_t + m_{t-1}$, where m_t is the logarithmic level of money supply and w_t is the stochastic growth rate of money supply. Obstfeld (1987) assumes that w_t exhibits a jump process, meaning $w_t = d_t \mu_t$, where d_t represents a dummy variable for the occurrence of a Poisson event and μ_t denotes the volume of change. To describe money growth w_t, there are a number of possible Poisson processes, ranging from the simplest one with a constant probability to the one with unstable probability behavior where d_t is a Markov chain with an unabsorbing state.

In practice, we know that the logarithmic first difference of the fundamentals, $m_t - m_{t-1} = \Delta m_t = d_t \mu_t$, is likely to be mean reverting. Hence, to proxy the movement of the variable Δm_t around its mean, we can apply Taylor's theorem to an arbitrary function (see Chiang 1984). If the mean is close to zero, we can use Maclaurin's series by expanding the function around the point $\Delta x = 0$. To include the variance term in the fundamental expectation, we can expand the series to the second degree, which is rather conventional for Taylor's expansion. As a result we can proxy the expected movement of the macroeconomic series by a nonlinear function.

Appendix D: The Closed-Form Solutions

To introduce time-varying conditional variances of the macroeconomic variables into the exchange rate model, we assume that there is a relationship between the first and the second moments of the fundamentals. The fundamentals are assumed to have somewhat similar to ARCH-in-Mean (ARCH-M) processes. The ARCH-M model, initiated by Engle, Lilien, and Robins (1987), is originally used to describe the risk and return relationship of assets, as suggested in finance theory. For macroeconomic variables, there is rather weak evidence of ARCH-M process.[32] An approximate linear relationship between the fundamental expectation and its variance is, however, intuitive.

Similar to the ARCH-M model, the whole sequence of future fundamentals can be represented by its current value and its variance. If x_t is the time series of interest, the model may be written as

$$x_{t+1} = \gamma_0 + \gamma_1 x_t + \gamma_2 h_{t+1} + u_{t+1}, \tag{A19}$$

where x represents a macroeconomic variable, h is the conditional variance of the variable x, presumably time varying, and u is a residual term. As the fundamentals empirically exhibit mean-reverting processes with persistent memory in standard deviations, time variation in the conditional variance may represent the adjustment and speed toward the mean.

In equation (29) the first component is like a random walk or an AR(1) process, which is often assumed for macroeconomic variables. The second component shows that macroeconomic uncertainty plays a role in the fundamental expectation formations. For example, the fundamental variances may represent economic circumstances, namely whether the economy is in volatile or tranquil periods, in which the expectations may be different. In turmoil (disequilibria), the monetary variables, such as money supply and interest rates, may be altered more often, and the state variables, such as income, unemployment rate, and inflation rate, may be more volatile than in regular periods.

To capture time-varying conditional variances, for simplicity, we use a GARCH(1, 1) model:

$$h_{t+1} = \lambda_0 + \lambda_1 u_t^2 + \lambda_2 h_t.$$

A GARCH(1, 1) model is often used to capture time-varying conditional variances of economic variables (see Bollerslev 1987). By way of the law of iterated expectations, the expected future fundamentals can be described as

$$\begin{aligned} E_t[x_{t+i}] &= \gamma_0 \sum_{s=0}^{i-1} \gamma_1^s + \gamma_1^i x_t + \gamma_2 \sum_{s=0}^{i-1} \gamma_1^s E_t[h_{t+i-s}], \\ E_t[h_{t+i-s}] &= \lambda_0 \sum_{k=0}^{i-s-1} (\lambda_1 + \lambda_2)^k + (\lambda_1 + \lambda_2)^{i-s} h_t. \end{aligned} \tag{A20}$$

Reorganizing gives a process of x as a function of its current value and its conditional variance as

$$E_t[x_{t+i}] = \alpha_0 + \alpha_1 x_t + \alpha_2 h_t, \tag{A21}$$

where

$$\alpha_0 = \textstyle\sum_{s=0}^{i-1} \gamma_1^s [\gamma_0 + \gamma_2 \lambda_0 \sum_{k=0}^{i-s-1} (\lambda_1 + \lambda_2)^k],$$
$$\alpha_1 = \gamma_1^i,$$
$$\alpha_2 = \gamma_2 \textstyle\sum_{s=0}^{i-1} \gamma_1^s [(\lambda_1 + \lambda_2)^{i-s}].$$

Substitute the expectations for money supply and real income into equation (A9), and rework with inertia in equation (A18). The results are equations (5) and (6), respectively.

Notes

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1. This discussion is based on my doctoral thesis (2002).

2. For example, for the monetary-approach partial equilibrium models Frenkel (1976), Mussa (1976), and Bilson (1978) discuss the flexible-price model, while Dornbusch (1976), Frankel (1979), Mussa (1979), and Buiter and Miller (1982) consider the sticky-price model. The general equilibrium asset-pricing models are studied by Stockman (1980), Lucas (1982), Svensson (1985a, b), and Hodrick (1989), and extended into the continuous-time stochastic framework by Bakshi and Chen (1997) and Basak and Gallmeyer (1998).

3. Among the empirical studies are those by Frenkel (1976), Bilson (1978), Hodrick (1978, 1989), Meese and Rogoff (1983, 1988), Backus (1984), Meese (1990), MacDonald and Taylor (1994), Chinn and Meese (1995), Mark (1995), and Flood and Rose (1995, 1999).

4. Mathematical applications are partially adopted from Cuthbertson (1999).

5. This equation is derived from the uncovered interest parity (UIP) and from an assumption corresponding to the monetary models that the interest rate differential depends on the fundamentals $\tilde{f}_t$:

$i_t - i_t^* = \alpha \tilde{f}_t$.

6. For a summary, see Meese (1990).

7. See Campbell, Lo, and MacKinlay (1997, ch. 7).

8. This solution is derived by applying to equation (1) the law of iterated expectations, that is $E_t[E_{t+1}[X]] = E_t[X]$. Suppose that the discount rate is lower than one, that it is governed by an interest semi-elasticity to money demand smaller than one. The expectation would be assigned a lower exponential weight (to the power i) as looking forward (to time $t+i$). From the limit theorem, at infinity $T \to \infty$ the bubble term (with a weight to the power $T+1$) vanishes (See also Blanchard and Fischer 1993, ch. 5).

9. For instance, MacDonald and Taylor (1994) find cointegration between exchange rates and monetary variables in the fundamental exchange rate models. Chinn and Meese (1995), as well as Mark (1995), find evidence that for long horizons the monetary-based exchange rate model overcomes the random walk model in predicting exchange rates. Groen (1999) shows that at a pooled time series level, there is cointegration between exchange rates and macroeconomic variables in the monetary model.

10. The exceptions include the studies by Cragg (1982), Engle (1982, 1983), Obstfeld (1987), Hodrick (1989), Arnold (1996), and Bekaert (1996).

11. In appendix B, I provide the derivation (in detail) of the reduced-form solutions of the flexible-price and sluggish-price models. It should be noted that for the sluggish-price model one actually works with a more complex assumption of price inertia. As a result the solution can be tedious (but similar), compared to equation (4). Importantly, it facilitates our closed-form derivation shown next.

12. The argument and derivation are in appendix D.

13. The nonlinearity in the model seems to coincide with the idea of nonlinear bubbles. For example, in Froot and Obstfeld (1991) the bubble is a nonlinear function of stock's dividend.

14. A GARCH$(1,1)$ model (with a Student's t distribution, if necessary) is used to capture fundamental uncertainty. The model, originated by Bollerslev (1986), suggests a form of heteroskedasticity in which the conditional variance changes over time as a function of past errors and past conditional variances. Therefore a turbulent (tranquil) period

is likely to be followed by turbulent (tranquil) periods. Alternatively, regative news has persistent effect in some periods.

15. For empirical results, see, for example, MacDonald and Taylor (1994) and Groen (1999).

16. Hodrick (1989) applies the ARCH-LR test and models fundamental volatility by using an ARCH(1) model with a normal distribution. In contrast, we specify the conditional variance model by using a GARCH$(1,1)$-t model, suggested in Bollerslev (1987). First, this is because the GARCH$(1,1)$ model is considered to be a parsimonious model of conditional variance that adequately fits many economic time series. See, for example, Bollerslev (1986) for the merit of the GARCH$(1,1)$ model in allowing long memory. Second, heteroskedasticity may be a reason for a heavy-tailed distribution, see, for example, de Haan, Resnick, Rootzen, and de Vries (1989) and Embrechts, Kluppelberg, and Mikosch (1999); Bollerslev (1987) shows the adequacy of the GARCH$(1,1)$-t model for fat-tail distributed economic series. Additionally my empirical results show highly significant GARCH coefficients and significantly reject the null hypothesis of normally distributed error terms.

17. For more detail, the reader is referred to appendix A. I also studied Austria, Germany, and the Netherlands. There is no evidence of a cointegration relationship in the Netherlands. However, there are ambiguous cointegration test results between the Johansen (1988) test and the augmented Engle and Granger (1987) test in the case of Austria and Germany.

18. This definition is given in Krugman and Obstfeld (1997). The US dollar is broadly accepted and held as a financial asset.

19. There are many studies investigating ARCH properties in the logarithmic changes in exchange rates. At short horizons, strong findings in weekly and daily intervals respectively have been reported by Engle and Bollerslev (1986) and Baillie and Bollerslev (1987), but due to temporal aggregation (see Drost and Nijman 1993) rather weak evidence for monthly data has been reported by Baillie and Bollerslev (1989) and Hodrick (1989).

Within our sample, we find rather strong evidence of ARCH in monthly exchange rate returns and fundamental growth rates. The results of the ARCH(1)-LM test, the ARMA-GARCH modeling method and the estimated coefficients of GARCH models are available upon request.

20. These reduced-form equations are the unrestricted monetary models of equations (5) and (6). This follows from the discussion in Meese (1990) and MacDonald and Taylor (1994) regarding the failure of the monetary models due to imposing inappropriate coefficient restrictions. Meese (1990) states that although most models are formulated in relative terms to simplify exposition, in estimation there is no need to impose the constraints on structural parameters. Furthermore MacDonald and Taylor (1994) show that their unrestricted flexible-price monetary model is valid in explaining the long-run exchange rate.

21. If the theoretical specifications are correct, one would expect the coefficients of domestic and foreign variables to be equal (in absolute term but with opposite signs). In practice, the coefficient restrictions are rejected by the data in three out of five countries. Only in the case of Canada and the United Kingdom, at the 5 percent significance level, the Wald test cannot reject the restriction that the coefficients of the domestic and foreign (US) variables are equal in money supply and real income.

22. For more detail, the reader is referred to Hamilton (1994) and Greene (2000).

23. Based on the method of Engle and Granger (1987), the long-run equilibrium relationship is first estimated. The estimated parameters of the cointegration vector are, subsequently, used in the error correction equation. See, for example, Engle and Granger (1987), Phillips and Loretan (1991), and MacDonald and Taylor (1994). The estimated coefficients of the long-run and short-run relationships are presented in tables 10.4 and 10.5, respectively.

24. However, if the explanatory variables and the disturbance term are not independent but they are contemporaneously uncorrelated, the OLS retains its desirable properties; see Dougherty (1992).

25. According to Hamilton (1994), the similar method has been suggested by Saikkonen (1991) and Phillips and Loretan (1991).

26. It should also be stressed that this approach is not exposed to the simultaneity bias. To avoid the simultaneity bias (or other violation of the fourth Gauss-Markov condition, e.g., from stochastic regressors or measurement errors), we use instrumental variables that are highly correlated with the regressors but not correlated to the error terms. In this chapter, rather than using the true conditional variances, whose random components may be correlated with error terms in the exchange rate equation, I use the predicted values of the endogenous explanatory variables, namely the GARCH forecast of volatility. By using the forecasts that are functions of the squared lagged residual and the estimated variances from the previous period, one can eliminate the random components in the fundamentals' conditional variances.

27. The regression result for Canada is

$$e_t = 0.889^{***} + 0.343^{***}\tilde{m}_t - 0.304\tilde{y}_t + 149.477^{***}\hat{h}_{m,t} + 1739.14^{**}\hat{h}_{m^*,t} + 141.418^{***}\hat{h}_{y,t} - 440.945^{***}h_{y^*,t} + \xi_t.$$

The regression result for the United Kingdom is

$$e_t = 0.320 + 0.119\tilde{m}_t - 0.566^{*}\tilde{y}_t - 1500.33^{***}\hat{h}_{m,t} - 5263.13^{***}\hat{h}_{m^*,t} - 315.254^{***}\hat{h}_{y,t} - 341.454h_{y^*,t} + \xi_t.$$

28. An increase in risk is not always a bad thing if society at large receives (nonmarketable) gains from the higher risk, as noted in Cumperayot et al. (2000).

29. For example, see Levy and Sarnat (1970) and Solnik (1974).

30. The ambiguous result for the impact of volatile money growth on the exchange rate is an interesting topic for further research.

31. The estimation is based on the two-step method of Engle and Granger (1987). Thus the estimated parameters of the cointegration vector in table 10.4 are used in this error-correction model. See, for instance, Engle and Granger (1987) and MacDonald and Taylor (1994).

32. For example, in our data set only Canada and the United Kingdom show weak evidence of this feature.

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